Inclusive Electron- and Neutrino-Nucleus Scattering: Correlations and Currents

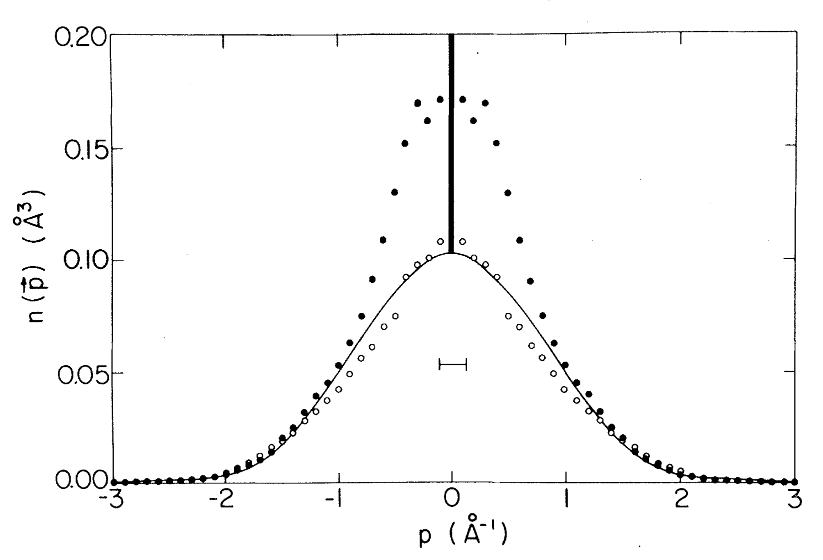
- Motivation
- Interactions and currents
- Review of electron scattering
  - Sum rules
  - Euclidean Response
- Electron/Neutrino Scattering from the Deuteron
- Sum Rules for A=12
- Near Future

A. Lovato (ANL)
S. Gandolfi (LANL)
S. Pieper (ANL)
R. Schiavilla (Jlab/ODU
G. Shen (LANL - UW)
J. Carlson

NUCLEI
Nuclear Computational Low-Energy Initiative

Response Functions
probes of structure and dynamics

Extraction of condensate fraction in liquid He



Sears, et al, PRL, 1982

Feynman PRL, 1952

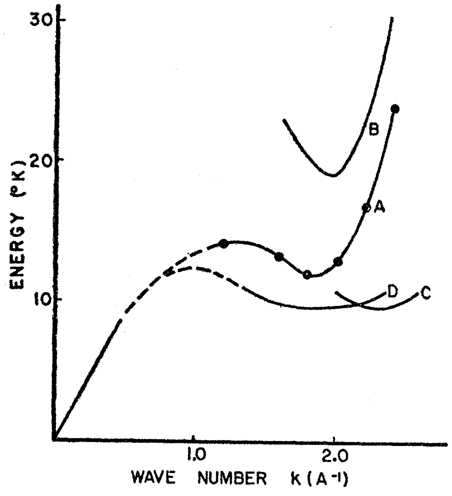


FIG. 6. The energy spectrum of excitations. Curve A is the spectrum $E_2(k)$ computed from Eq. (61). Curve B is the spectrum $E_1(k)$ computed with the simpler wave function (5). Curve C is the Landau-type spectrum used by de-Klerk *et al.*[4] to fit the second sound and specific heat data. Curve D is a Landau-type spectrum with p_0 taken the same as in A, and μ and Δ chosen to fit the specific heat data. For small k, all curves are asymptotic to the line $E=\hbar ck$.

phonon-roton spectrum in liquid He

S(Q) and g(r) for Simple Liquids

The 1994 Nobel Prize – Shull & Brockhouse

more examples:

High Tc

Cold Atomic Gases, ...

Fig. 5.1 The structure factor $S(\kappa)$ for ^{36}Ar at 85 K. The curve through the experimental points is obtained from a molecular dynamics calculation of Verlet based on a Lennard-Jones potential. (After Yarnell *et al.*, 1973.)

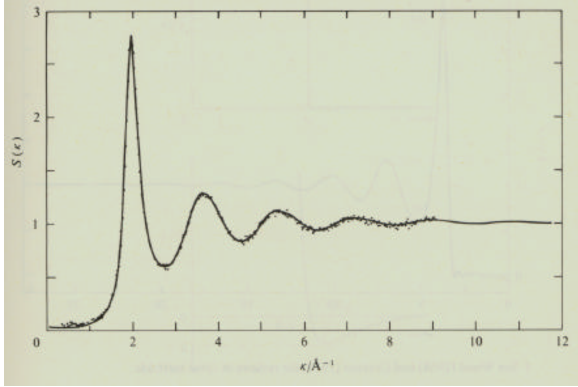
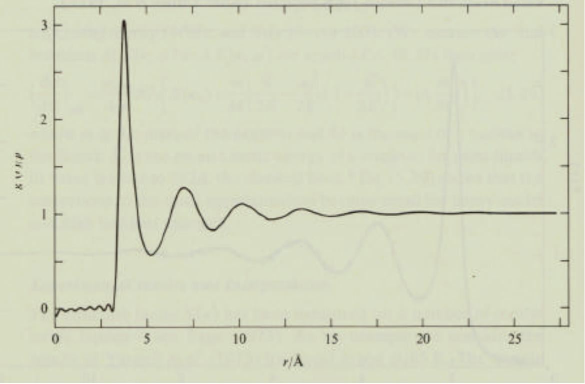
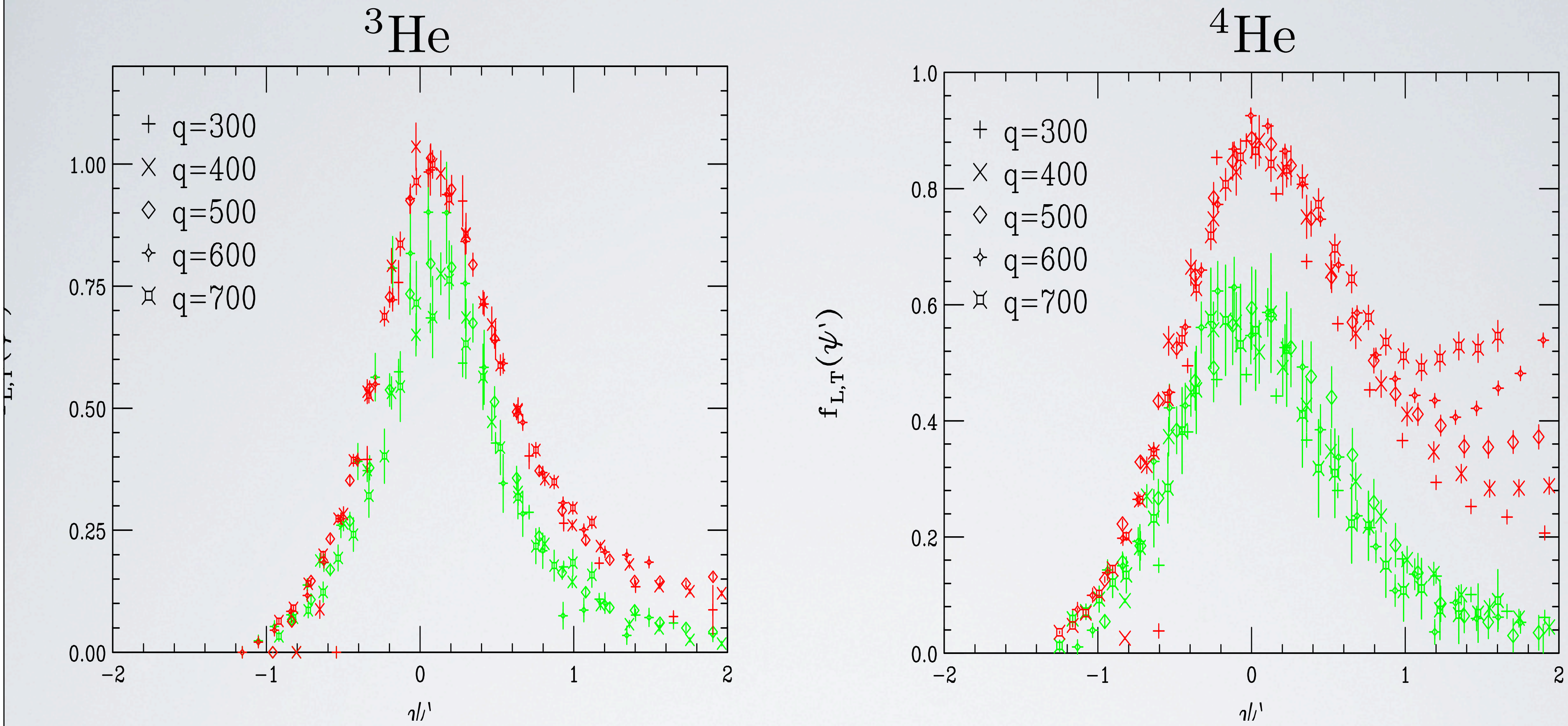


Fig. 5.2 The pair-distribution function $g(r)$ obtained from the experimental results in Fig. 5.1. The mean number density is $\rho = 2.13\times10^{28}$ atoms m^{-3}. (After Yarnell *et al.*, 1973.)



(e, e') Inclusive Response: Scaling Analysis

Donnelly and Sick (1999)



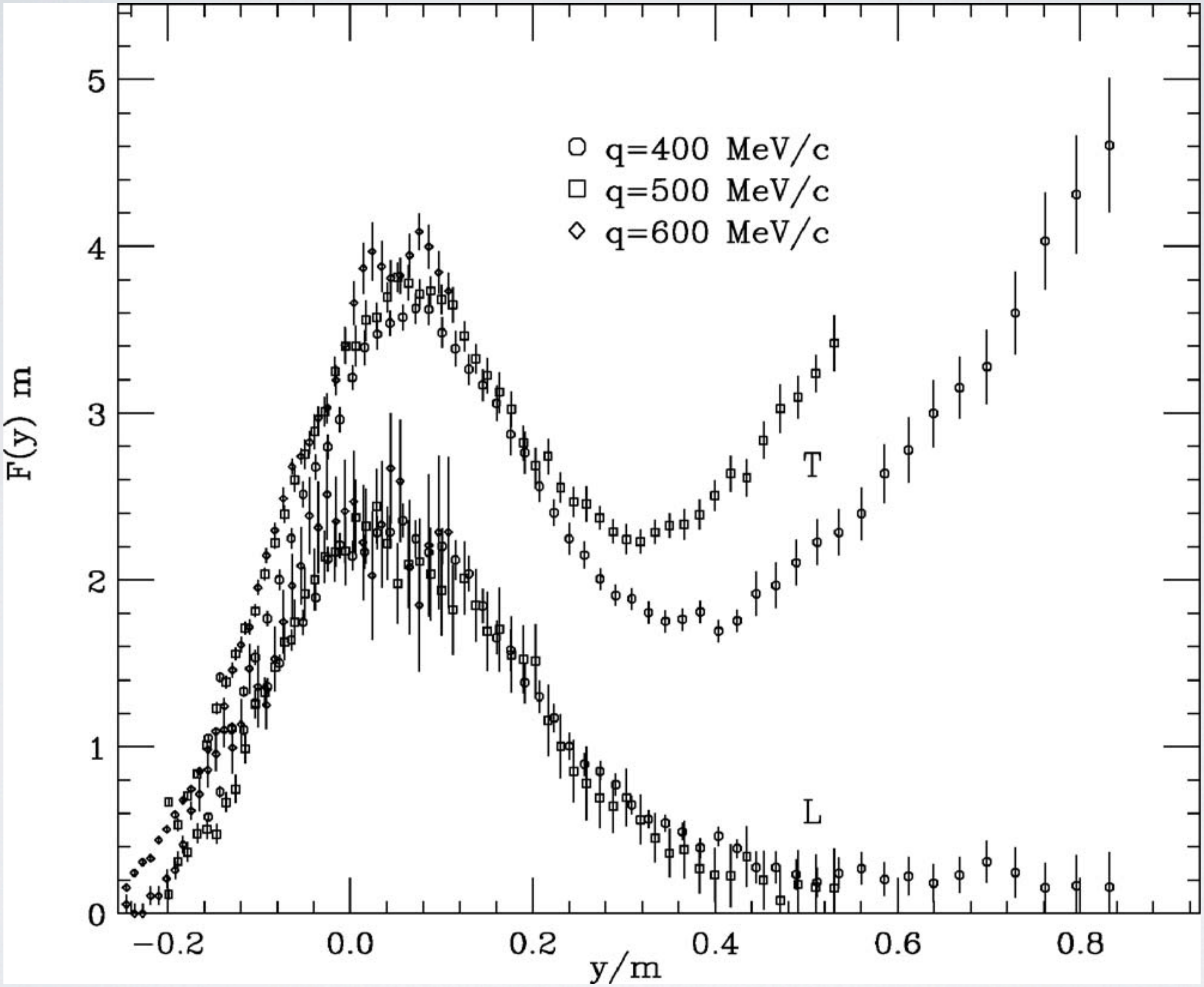
Single nucleon couplings factored out

Momenta of order inverse internucleon spacing:

Large enhancement of transverse over longitudinal response

Requires beyond single nucleon physics

Longitudinal/Transverse separation in ^{12}C

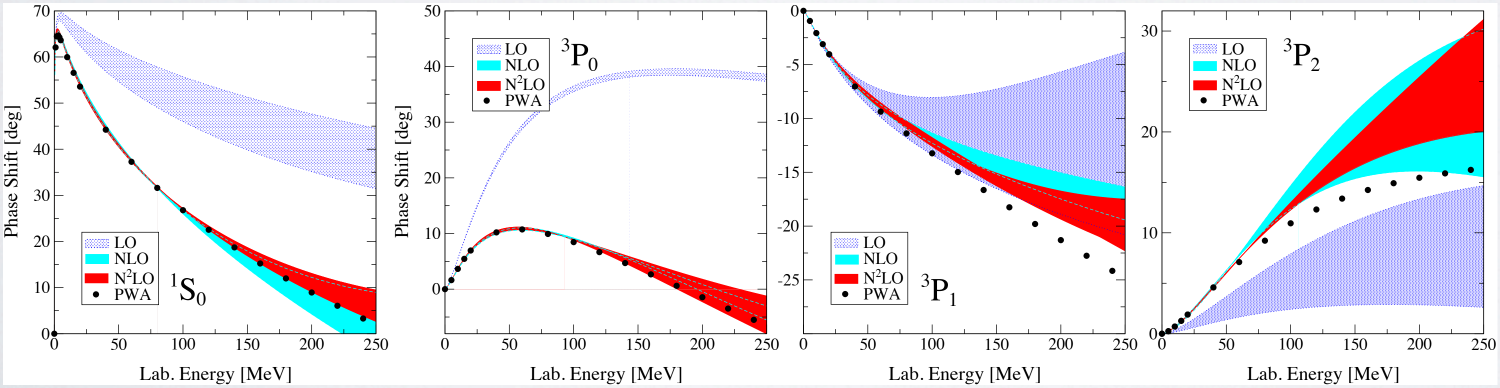


from Benhar, Day, Sick, RMP 2008
data Finn, et al 1984

Nuclear Interactions:

AV18 : excellent fit of NN data
pion exchange plus phenomenology
TNI: Two-pion exchange plus three-pion-exchange
plus phenomenological short-range repulsion

Chiral Interactions: LO, NLO, N2LO, N3LO
increasing order results in better fits to data
uncertainty estimates
Consistency of two plus three nucleon interactions
New local interactions at LO..N2LO



Gezerlis, et al.,
PRL 2013

QMC methods

Basic Idea: project specific low-lying states from
initial guess (or source)

$$\Psi_0 \ = \exp\left[-H\tau\right]\,\Psi_T$$

Use Feynman path integrals to compute propagator

$$\exp\left[-H\tau\right] = \prod \exp[-H\delta\tau]$$

$$\exp[-H\delta\tau] \approx \exp[-T\delta\tau]\,\exp[-V\delta\tau]$$

diffusion branching

Applications: condensed matter (Helium, electronic systems, ...
nuclear physics (light nuclei, neutron matter, SMMC...)
atomic physics (cold atoms,...)

Various formulations: DMC/GFMC, AFMC, AFDMC, Lattice

GFMC Algorithm:

Branching random walk in 3A (36 for ^{12}C) dimensions

Asynchronous Dynamic Load Balancing (ADLB) Library

Each step moves A particles and updates

$2^A \times \binom{A}{Z}$ complex amplitudes (2 GB for ^{12}C gs)

significant linear algebra for each step

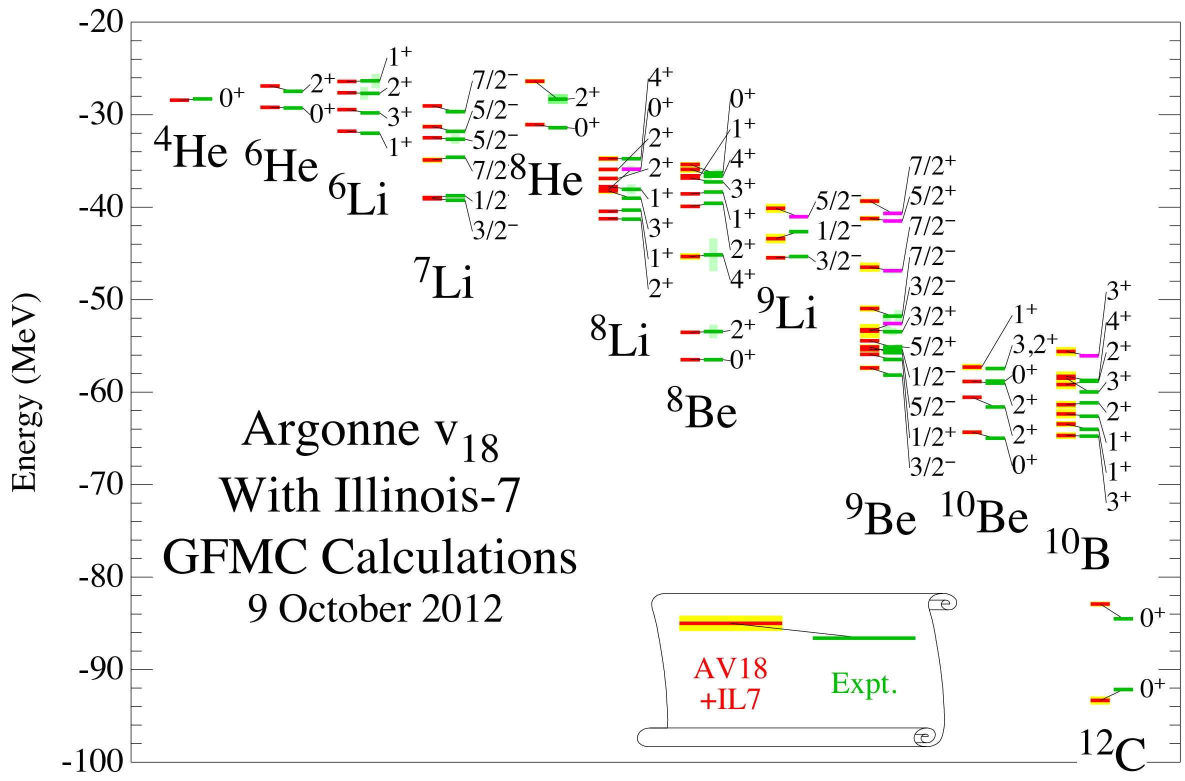
tuned by physicists and math/CS staff at ANL

Similar branching random walks with linear algebra used in condensed matter physics (lattice calculations)

up to
~2M threads

Other methods: NCSM, Coupled Cluster, ...

Spectra of Light Nuclei

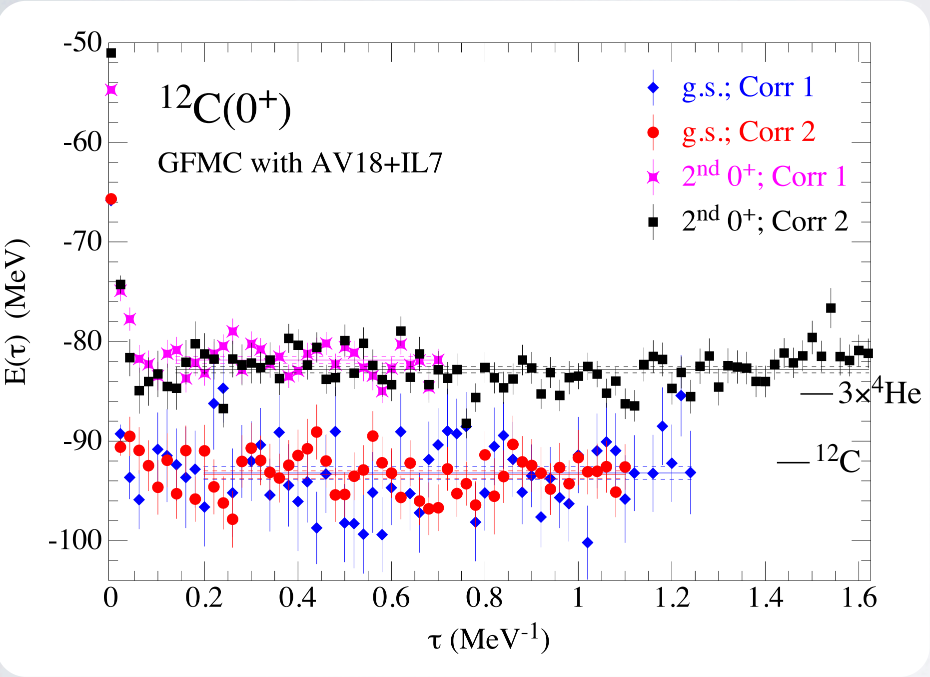


Spectra must be correct to describe low-energy transitions, reactions, etc.

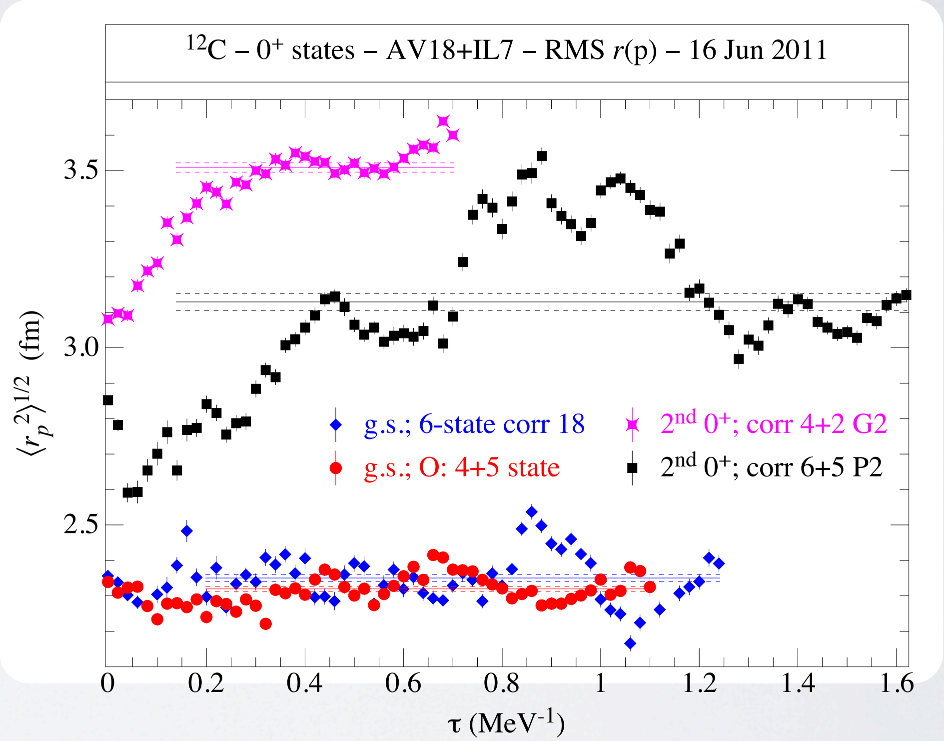
Carbon-12

Ground and Hoyle State
AV18 + IL7 interaction

Energy



RMS radius

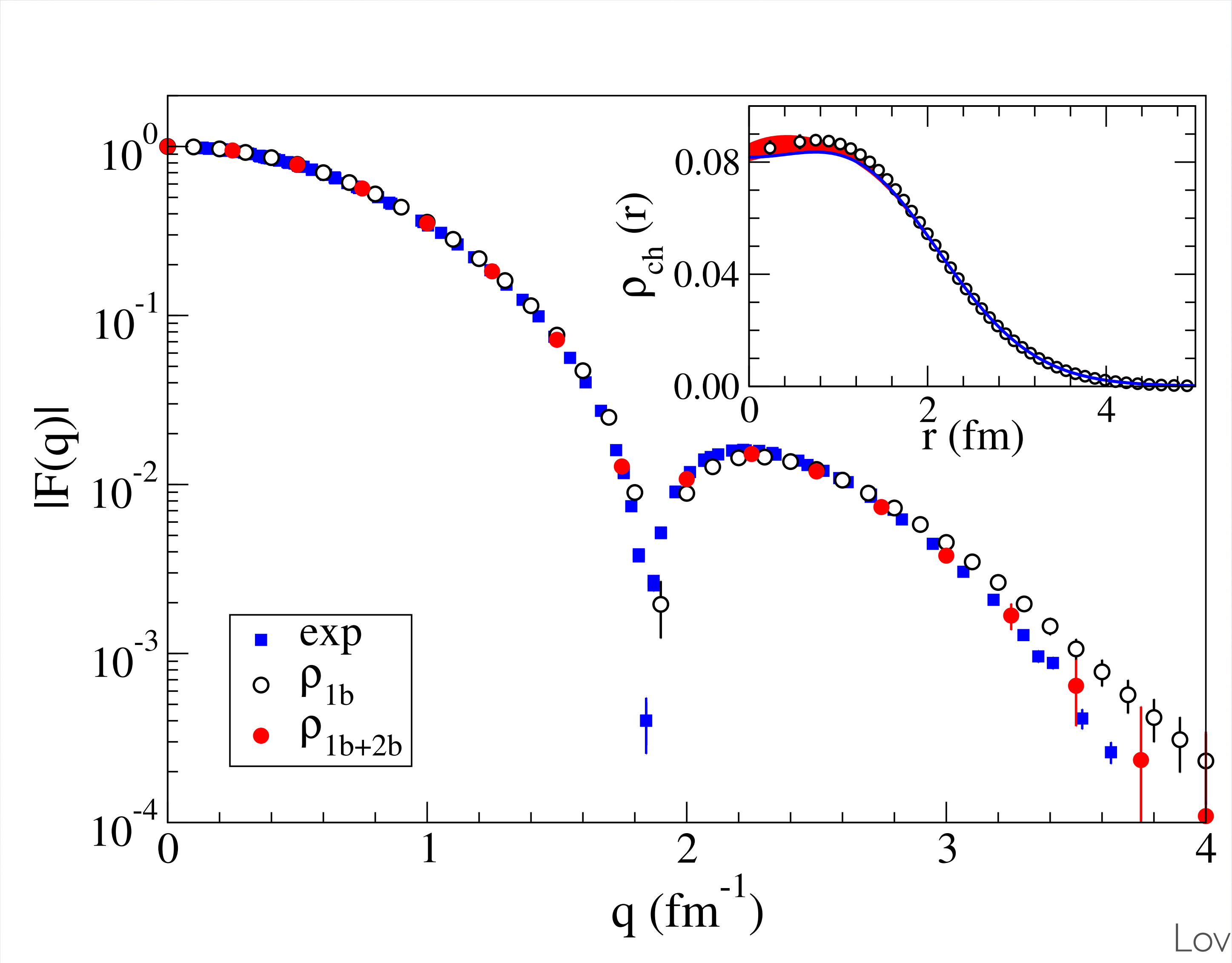


0+ excited state near triple-alpha threshhold postulated by Fred Hoyle to explain nuclear abundances

Pieper, Carlson, Lusk, ...

see also papers by D. Lee, Meissner, et al

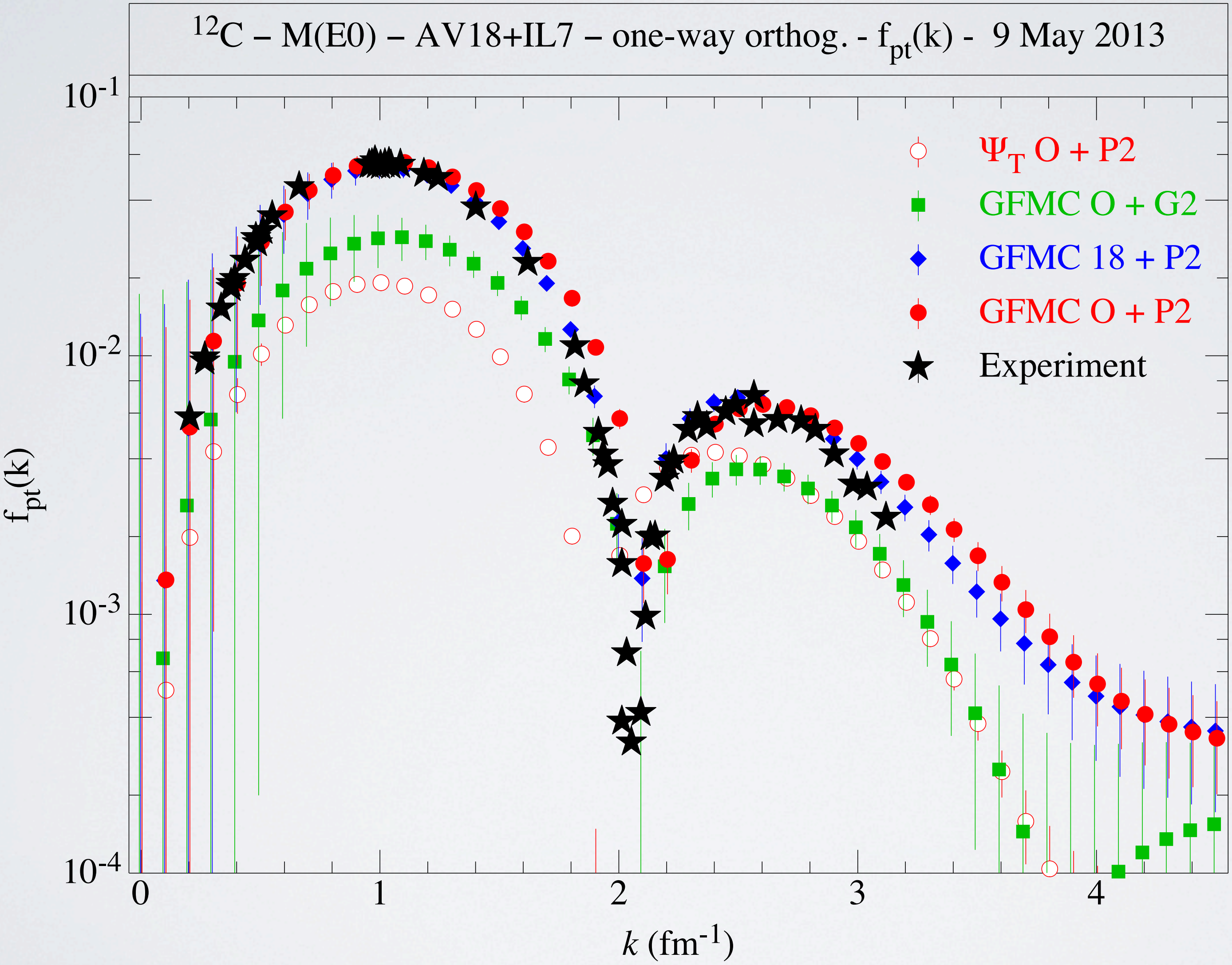
^{12}C Electromagnetic Charge Form Factor



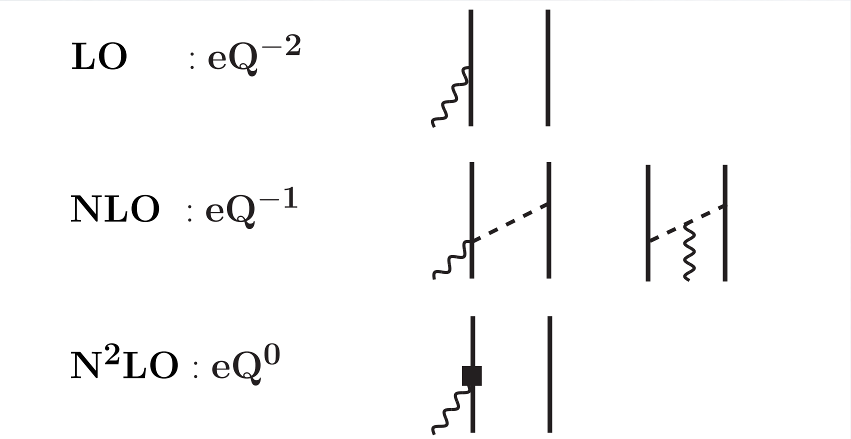
Small role for two-nucleon currents
Excellent agreement with data

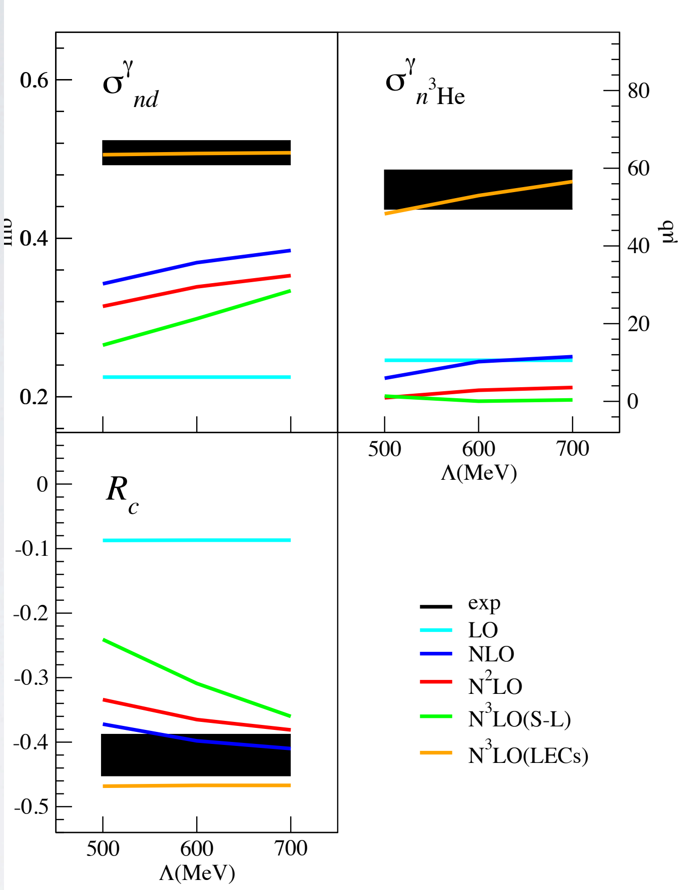
Lovato, Gandolfi, Butler,
Carlson, Lusk, Pieper, Schiavilla
PRL 2013

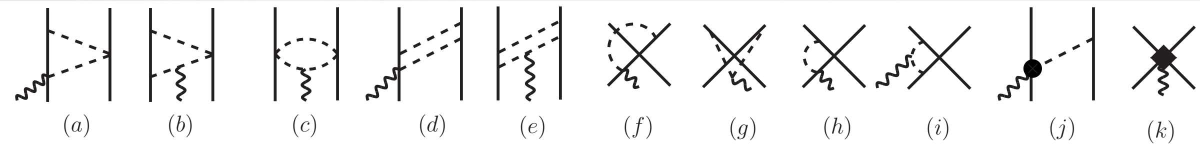
Ground State - Hoyle State Transition form factor



Nuclear Electromagnetic Currents







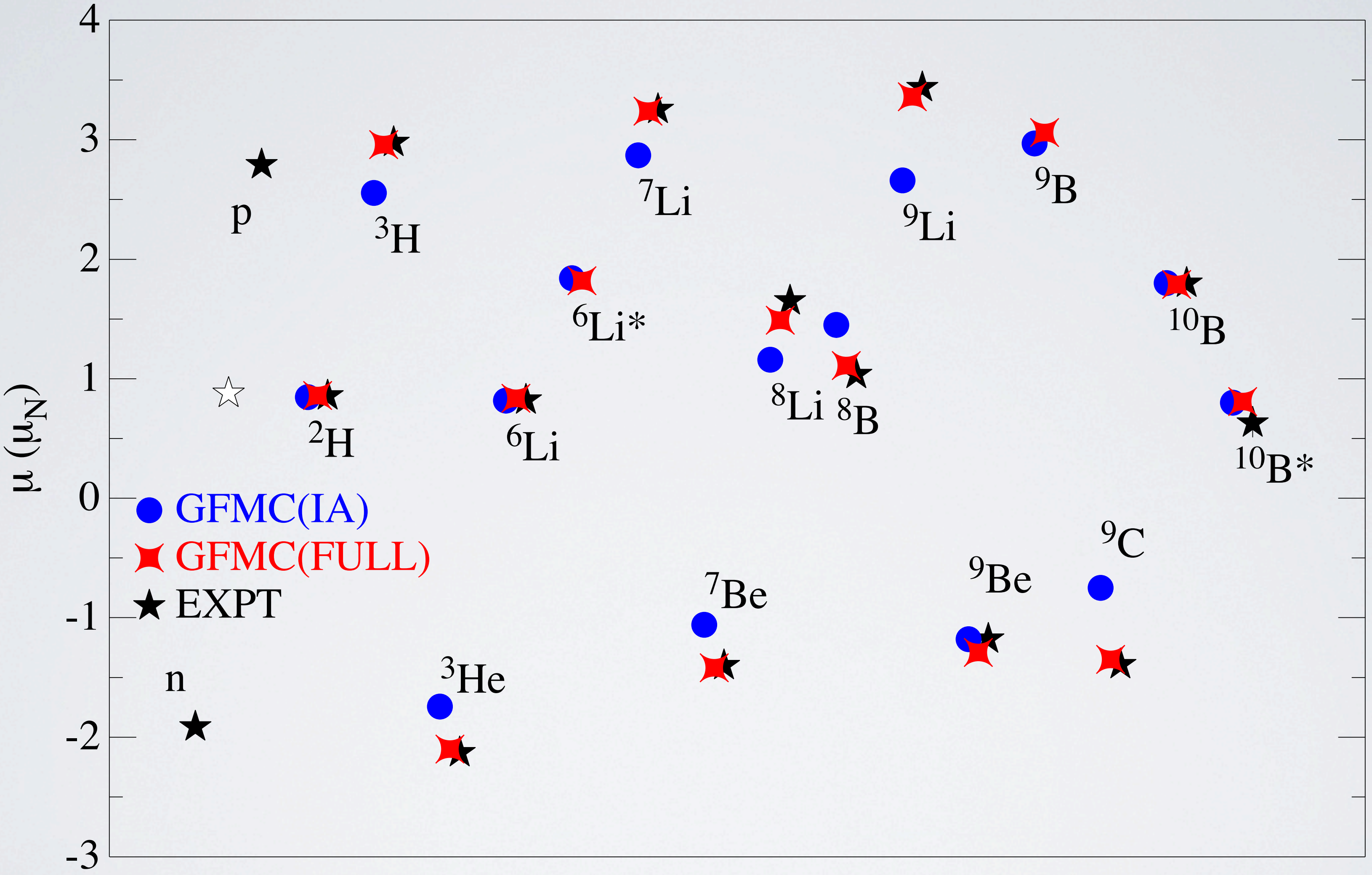
N3LO (eQ)

Coupling constants adjusted to
μ(D), μ_S(A=3) : isoscalar
np capture, μ_V(A=3): isovector

Pastore S, Schiavilla R, Goity J L. Phys. Rev. C, 2008, **78**: 064002
Pastore S, Girlanda L, Schiavilla R, Viviani M, Wiringa R B. Phys. Rev. C, 2009, **80**: 034004

M. Piarulli, L. Girlanda, L. E. Marcucci, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C 87, 014006 (2013).

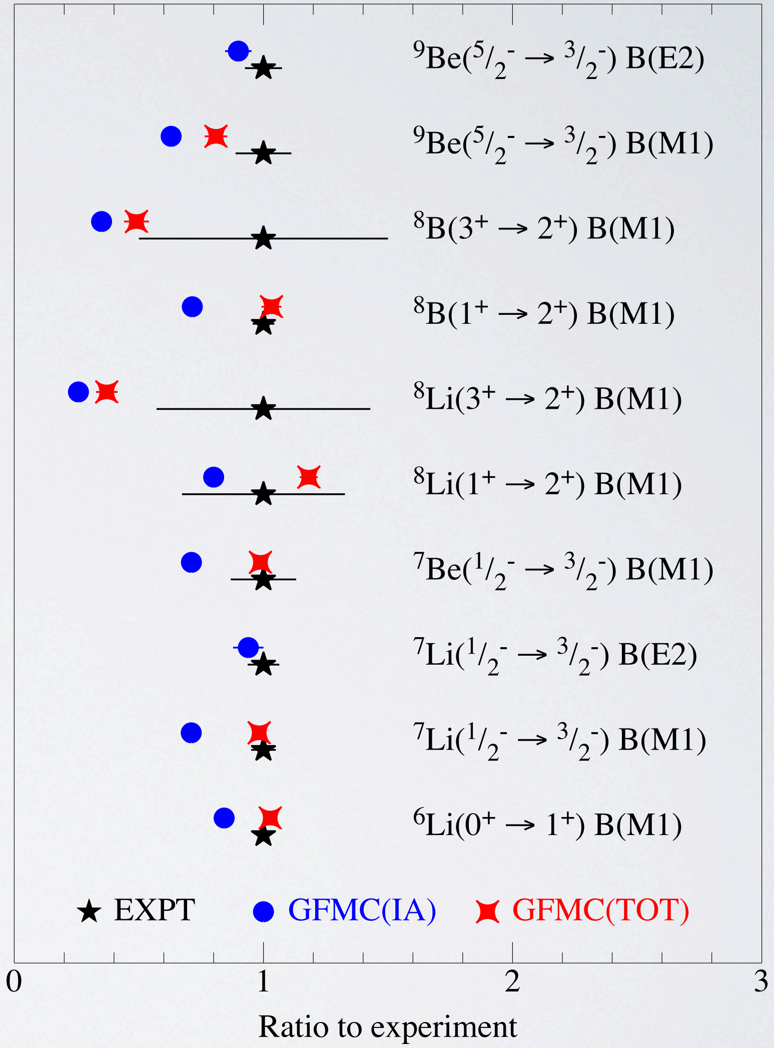
A ≤ 10 Magnetic Moments with Chiral EFT currents



Pastore, et al, Phys. Rev. C 87, 035503 (2013) ; arXiv:1302.5091

$A \leq 9$ $M1$ TRANSITIONS W/ χEFT EXCHANGE CURRENTS

- dominant contribution is from OPE
- five LECs at N3LO
- d_2^V and d_1^V are fixed assuming Δ resonance saturation
- d^S and c^S are fit to experimental μ_c and $\mu_S(^3\text{H}/^3\text{He})$
- c^V is fit to experimental $\mu_V(^3\text{H}/^3\text{He})$
- $\Lambda = 600$ MeV

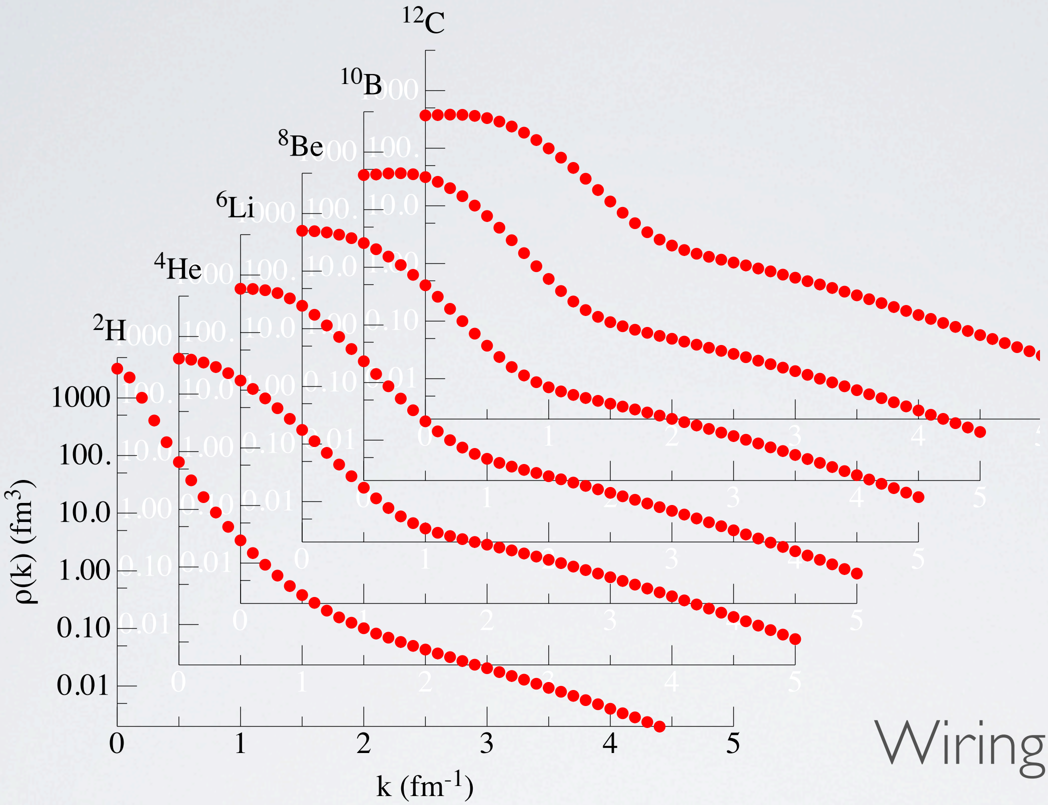


Pastore, Pieper, Schiavilla & Wiringa

PRC **87**, 035503 (2013)

Two-nucleon currents critical to understand low-energy transitions

Higher resolution: Momentum Distributions



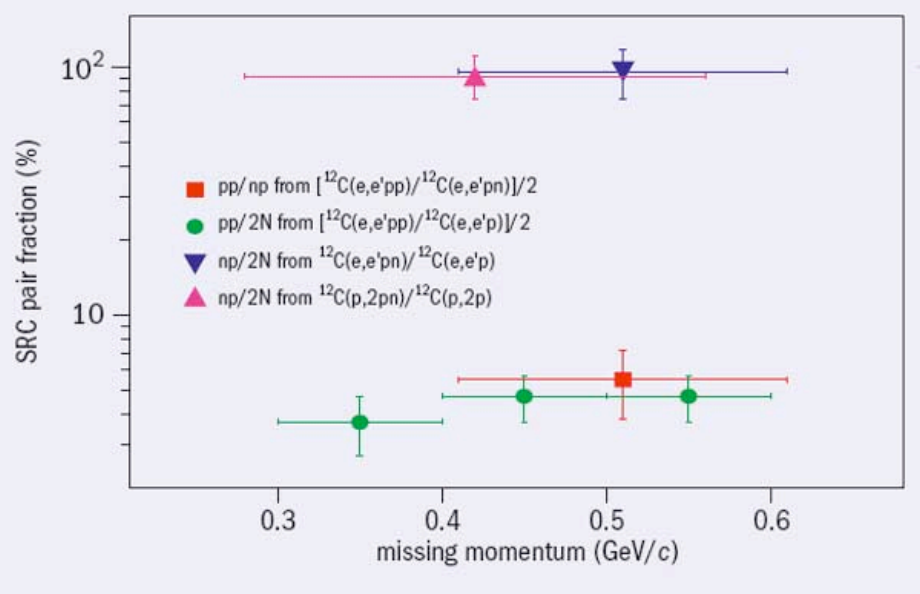
Wiringa, et al, 2013

proton momentum distributions A=2-12

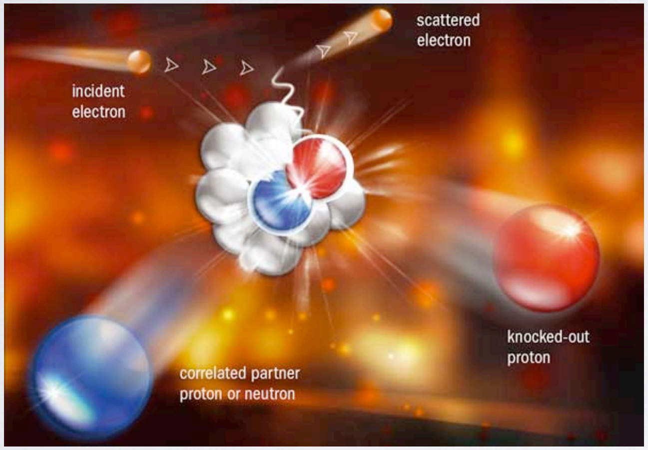
High momentum components dominated by two-nucleon physics

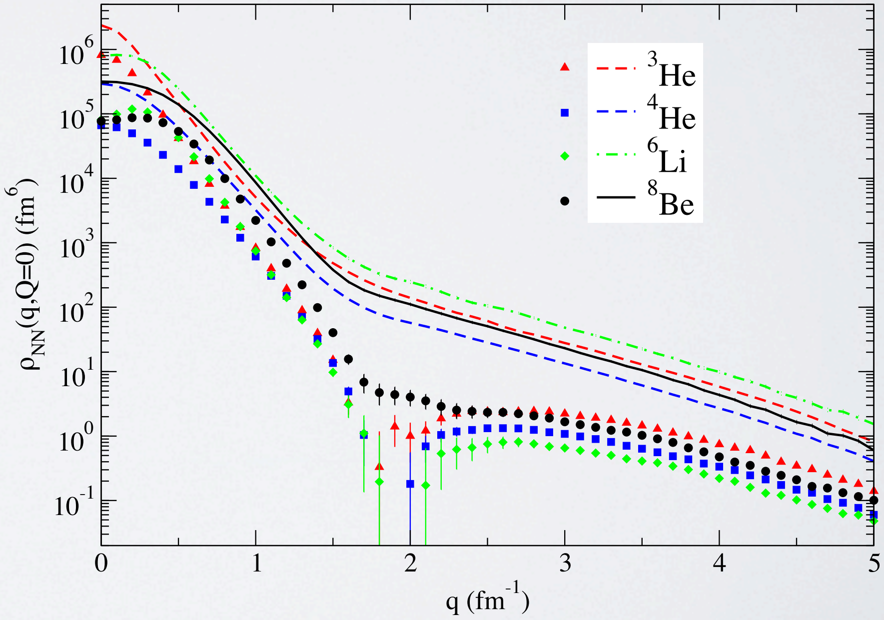
strength at ~ 2 fm^{-1} due to tensor correlations

Back-to-back pairs: pn vs pp,nn in ^{12}C



JLAB, BNL
back-to-back pairs in ^{12}C



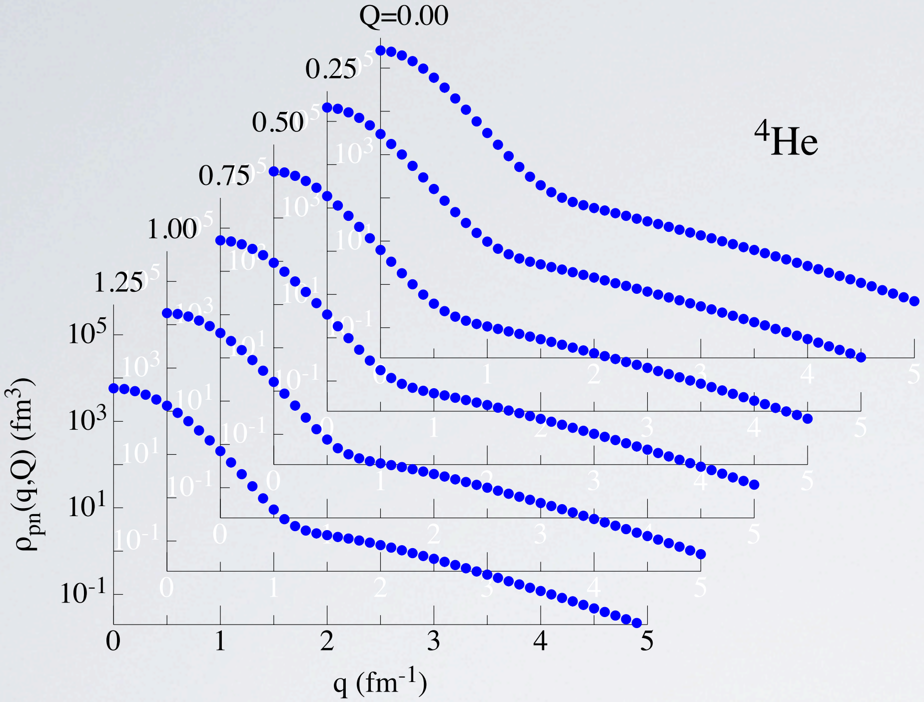


np pairs dominate
over nn and pp

E Piasetzky *et al.* 2006 **Phys. Rev. Lett. 97** 162504.
M Sargsian *et al.* 2005 **Phys. Rev. C 71** 044615.
R Schiavilla *et al.* 2007 **Phys. Rev. Lett. 98** 132501.
R Subedi *et al.* 2008 **Science 320** 1475.

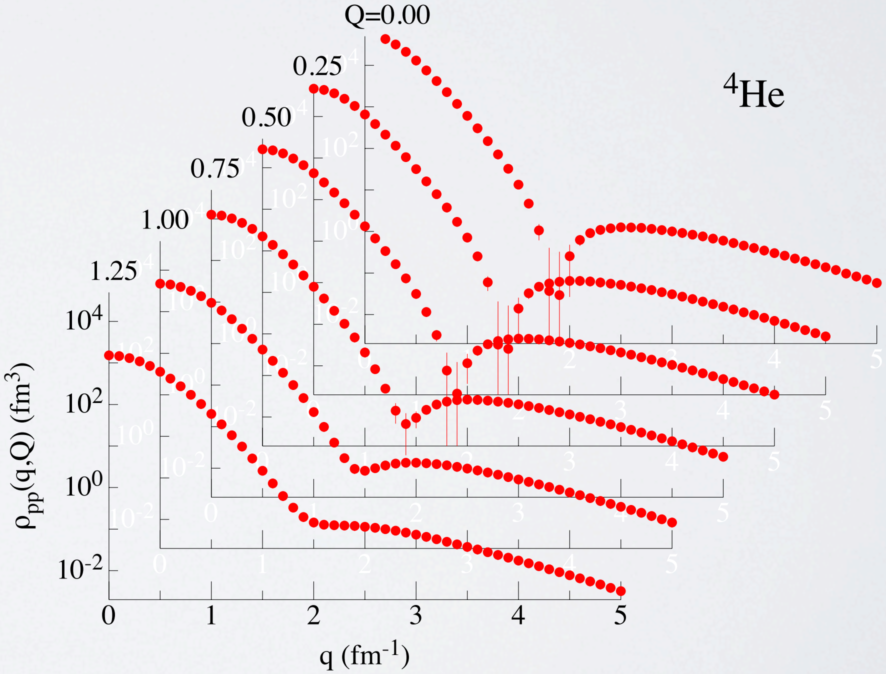
http://www.phy.anl.gov/theory/research/momenta2/

Neutron-Proton pairs



Most pairs at high q have low CM Q

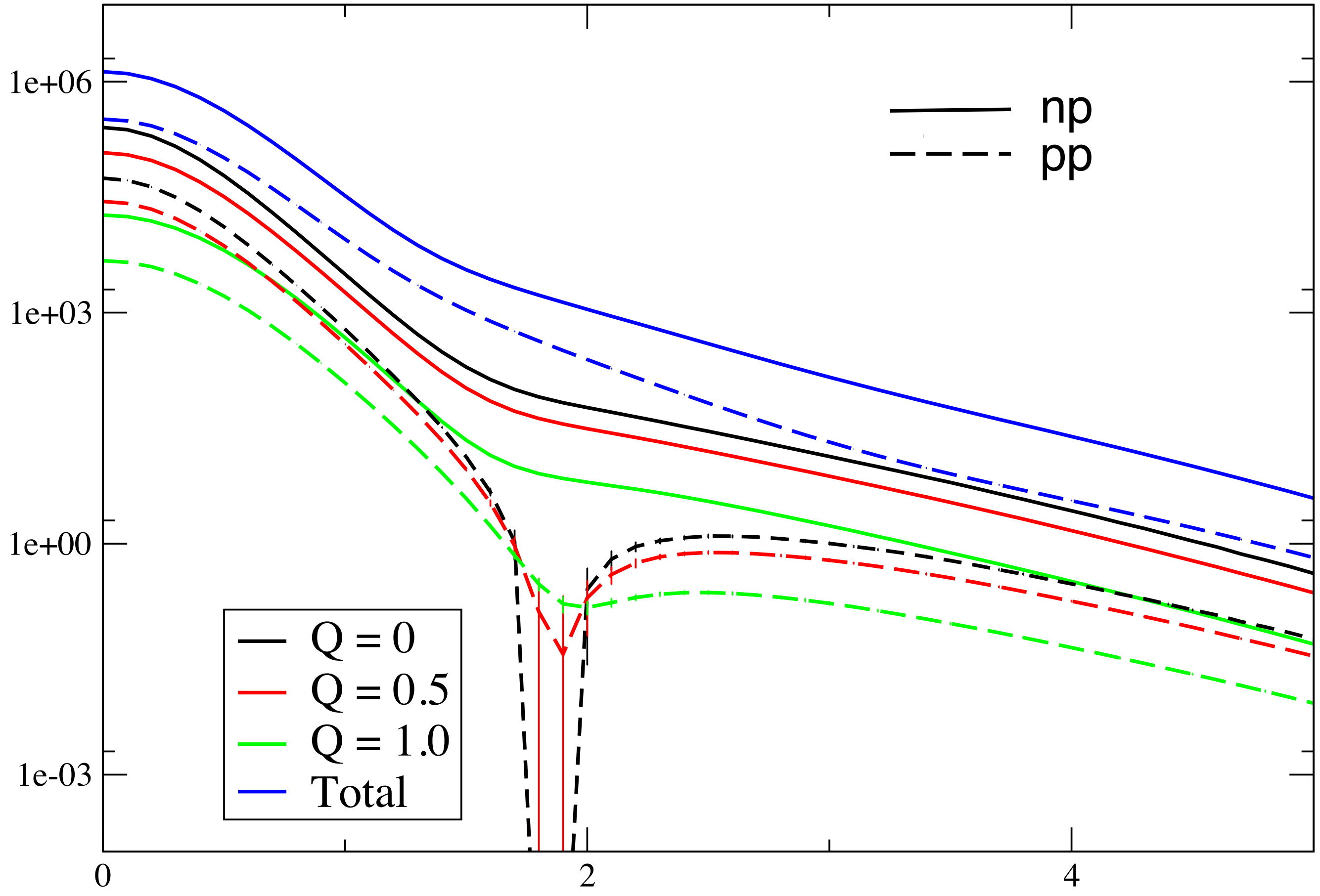
Proton-Proton pairs



np pairs dominate for ^{4}He, ^{12}C

$$\rho(p,q) = \langle 0|\ \exp[iq\cdot(r-r')]\ \exp[iQ\cdot(R+R')]\ |0\rangle$$

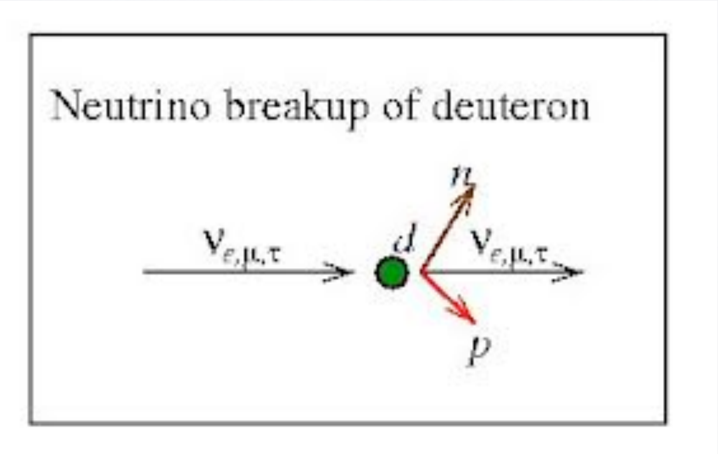
pair momenta vs Q: pn vs pp,nn in ^{4}He



Inclusive Scattering and Response Functions

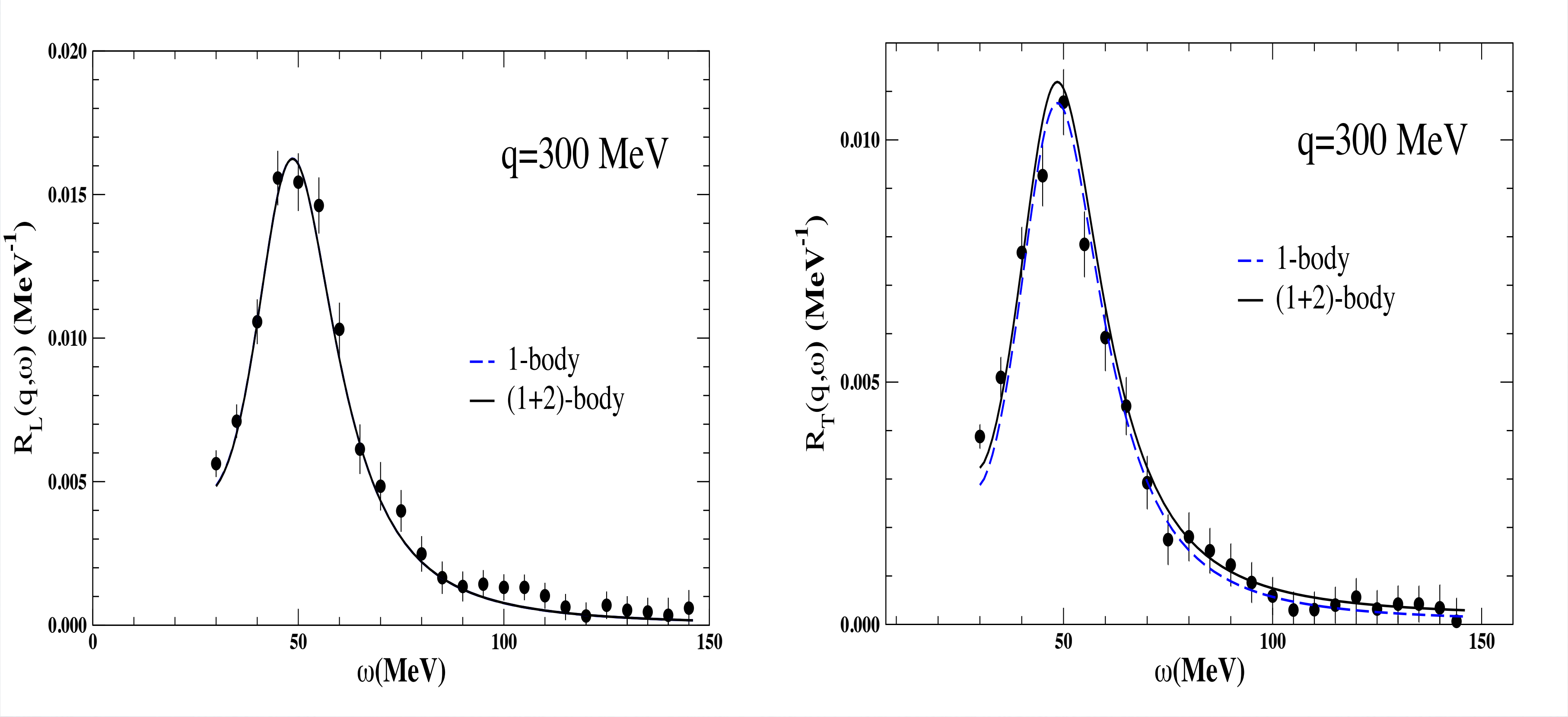
$$R_{L,T}\ (q,\omega)\ = \sum_f\ \delta(\omega\ +\ E_0\ + E_f)\ |\ \langle\ f\ |\mathcal{O}_{\mathcal{L},\mathcal{T}}\ |\ 0\ \rangle\ |^2$$

knowledge of response ⇨ inclusive cross-sections
requires knowledge of all final states



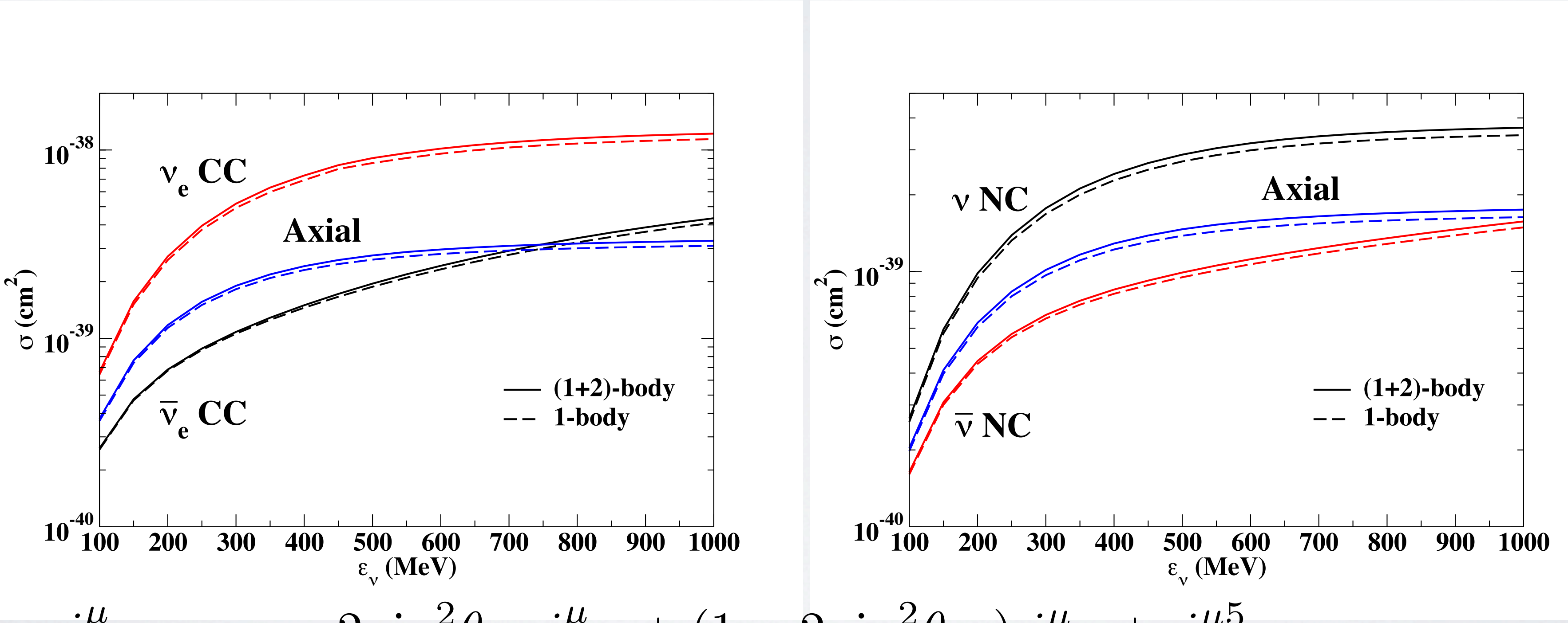
Start with the deuteron, can enumerate all final states.
Use for test of Monte Carlo codes
Accurate predictions: could use to make absolute flux measurements

Electron Scattering on Deuterium



ν-Deuteron Scattering up to GeV Energy

Shen *et al.* (2012)



$$j^\mu_{NC} \;=\; -2\sin^2\theta_W\, j^\mu_{\gamma,S} + (1-2\sin^2\theta_W)\, j^\mu_{\gamma,z} + j^{\mu 5}_z$$

$$j^\mu_{CC} \;=\; j^\mu_\pm + j^{\mu 5}_\pm \qquad j_\pm = j_x \pm i\, j_y \qquad \left[\, T_a \,,\, j^\mu_{\gamma,z} \,\right] = i\, \epsilon_{azb}\, j^\mu_b(1)$$

j^μ_{CC} reproduces well known weak transitions in $A \leq 7$ nuclei and μ-capture rates in d and ^{3}He [Schiavilla and Wiringa (2002); Marcucci *et al.* (2012)]

Deuterons: Neutral Current Comparison of 1-body PW to isolated p + n and ratio

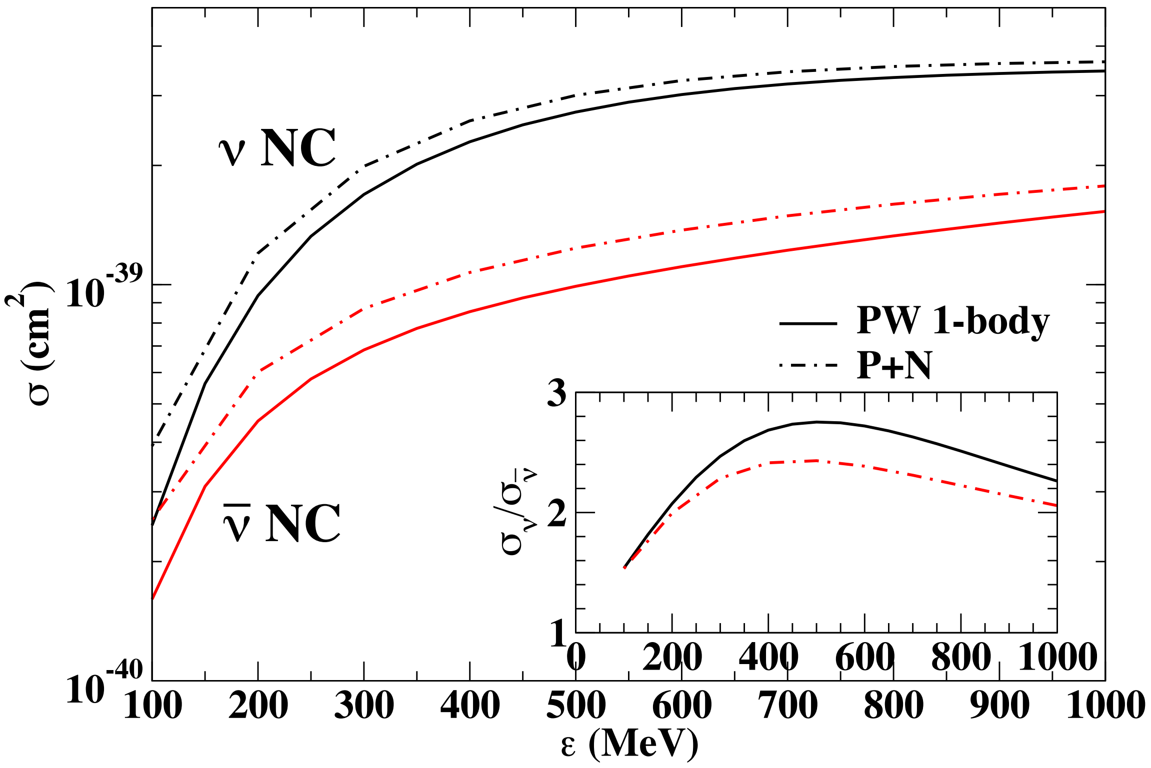
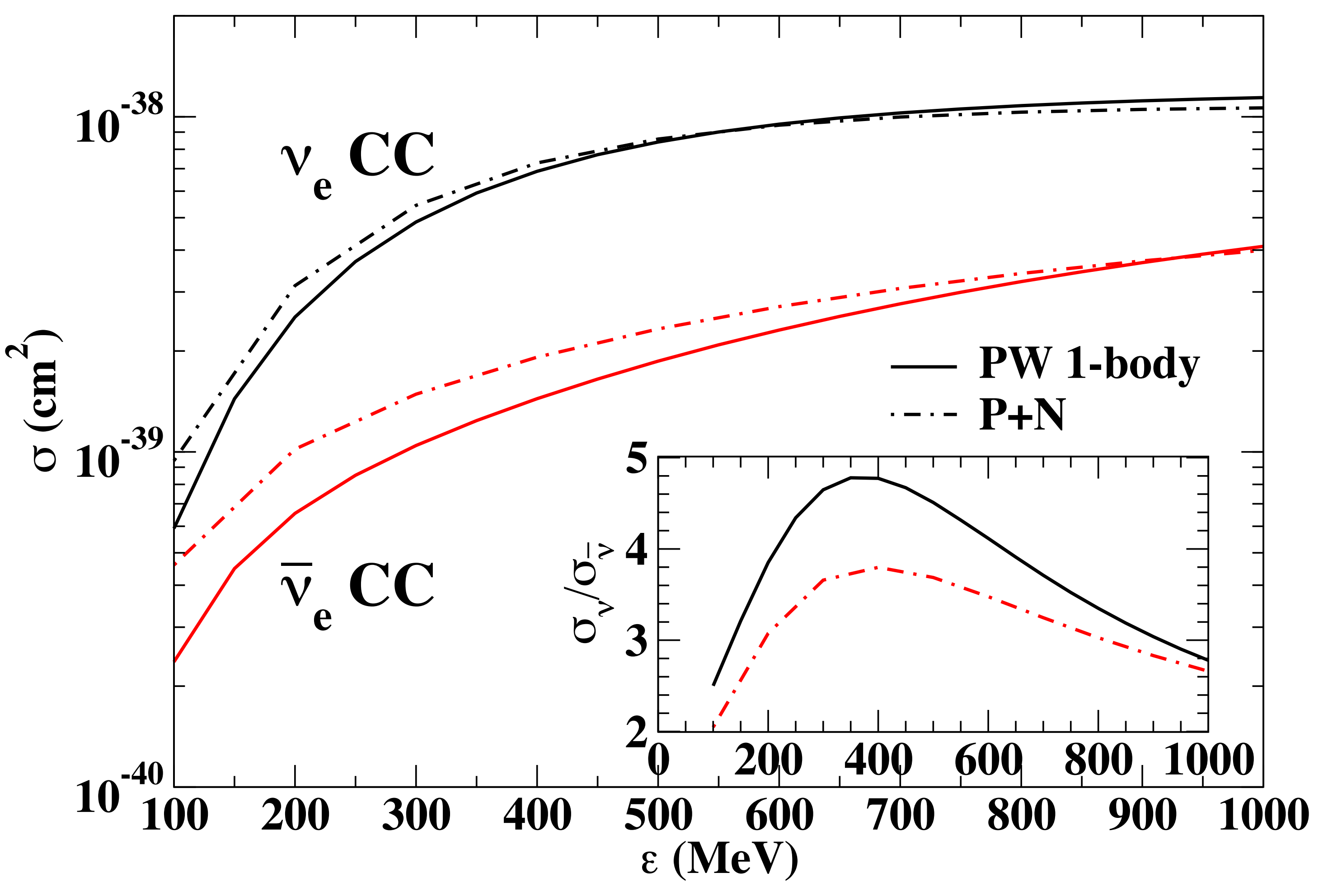


FIG. 16: (color online) The “model” (P+N) NC cross sections for neutrino and antineutrino are compared with plane-wave one-body (PW 1-body) results, see text for explanation. Inset: ratio of neutrino NC versus antineutrino NC cross section.

Charged Current on Deuteron



Heavier Nuclei (A>2)

Easy to calculate Sum Rules: ground-state observable

$$S(q) \ = \ \int d\omega \ R(q,\omega) = \langle 0|O^\dagger(q) \ O(q)|0\rangle$$

Sum Rules are independent of final states (and FSI)

$$E(q) \ = \ \int d\omega \ \omega \ R(q,\omega) \ = \langle 0|O^\dagger(q) H O(q)|0\rangle$$

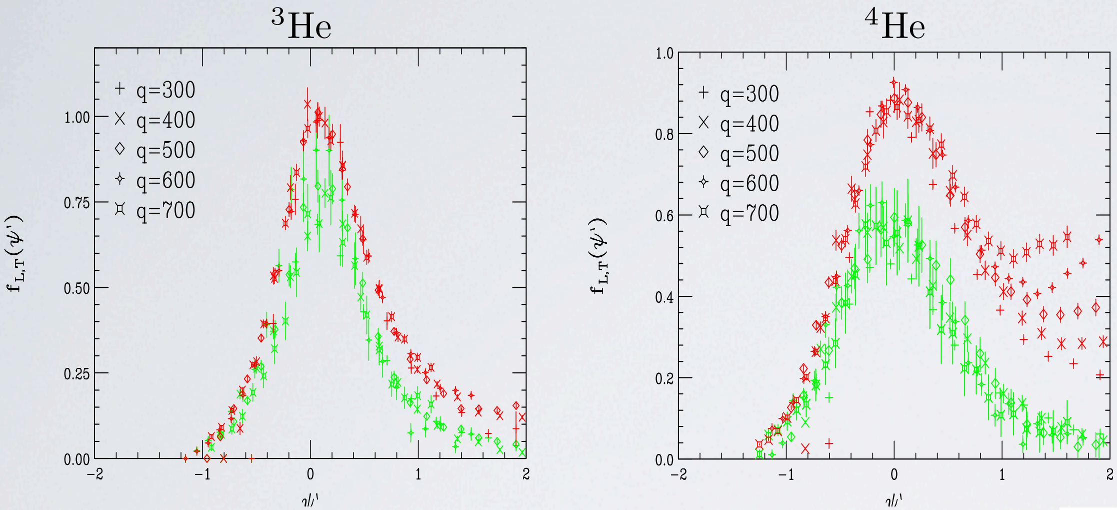
For spin-isospin independent interactions E(q) = $q^2/2m$

For nuclear physics E(q) > $q^2/2m$,
not reproduced by spectral function alone

Longitudinal and Transverse Electromagnetic Response in A=3,4, 12

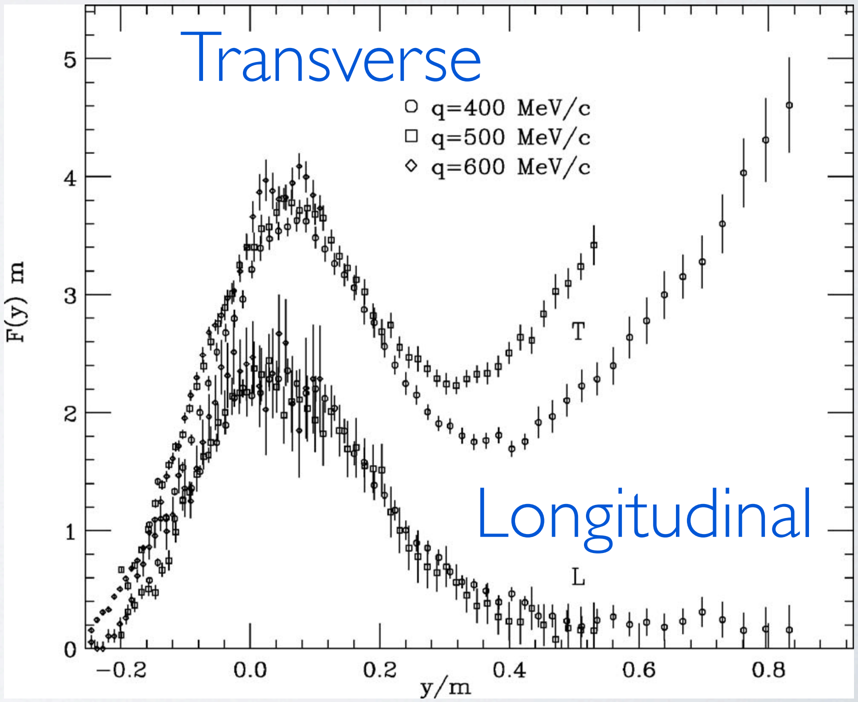
(e, e') Inclusive Response: Scaling Analysis

Donnelly and Sick (1999)



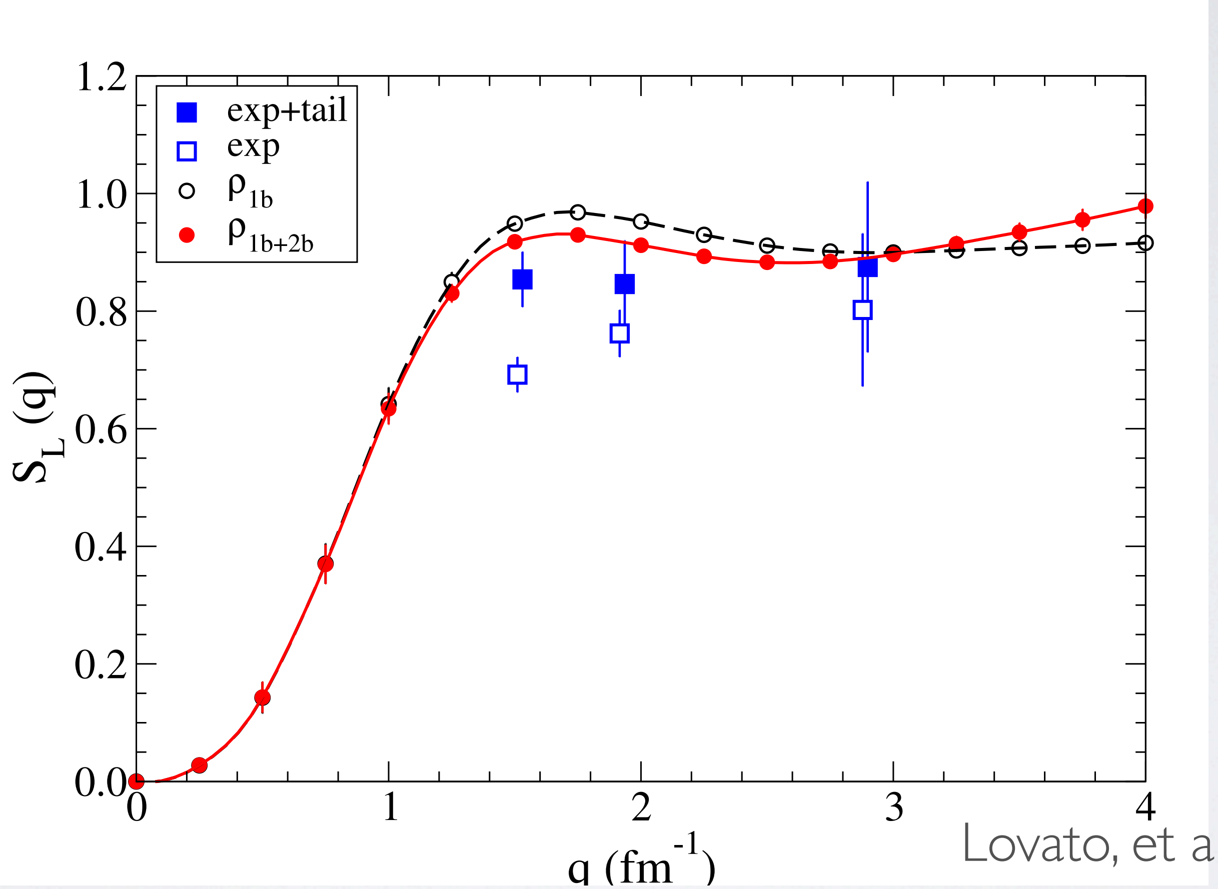
Transverse / Longitudinal enhancement requires more than single-nucleon physics

^{12}C



Carbon-12 : Electron Scattering
Longitudinal Sum Rule

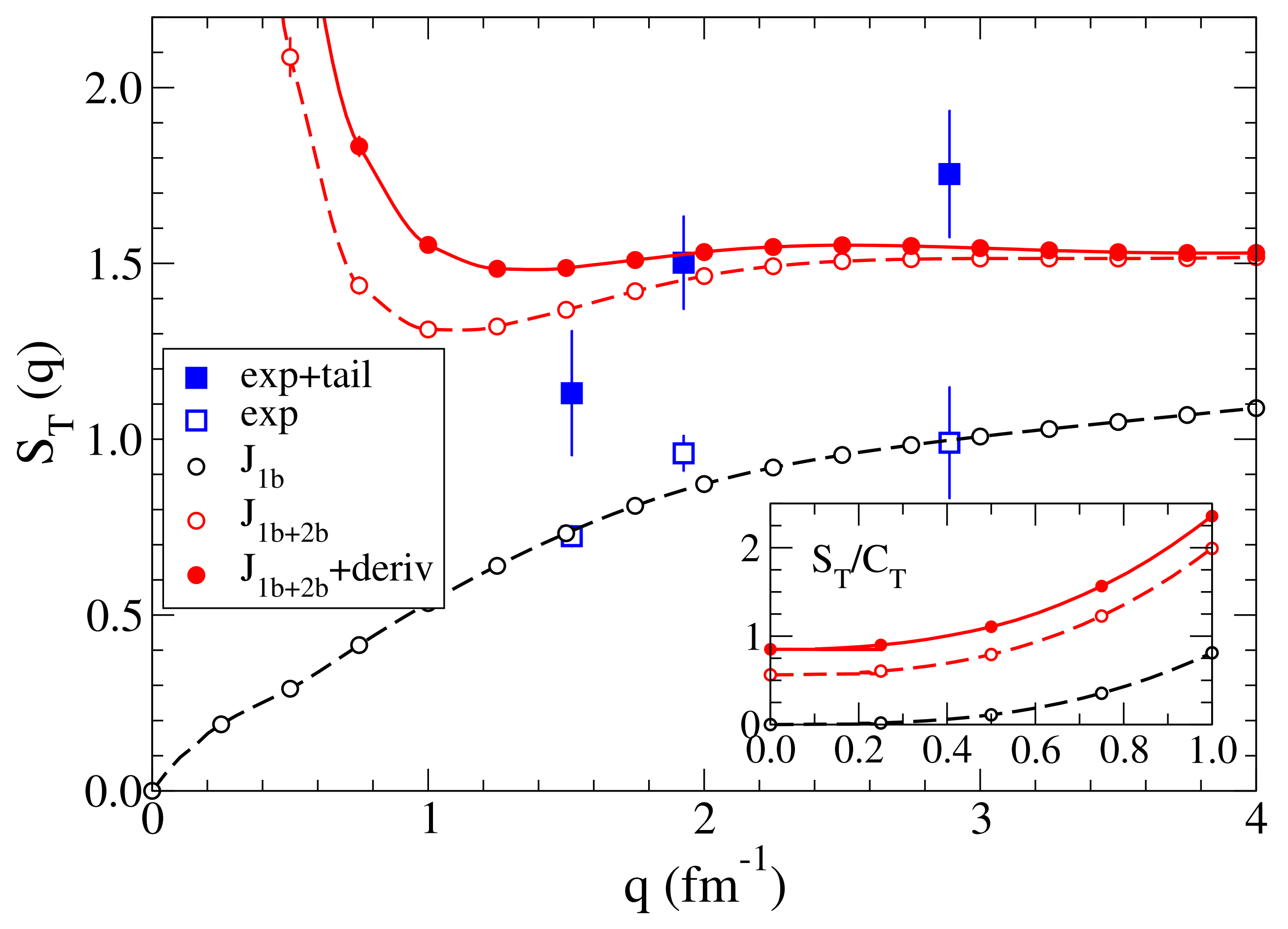
$$S_L(q) \;=\; \langle 0|\; \rho^\dagger(q)\; \rho(q)\; |0\rangle$$



Lovato, et al, PRL 2013

new Jlab experiment soon, also neutrino experiments
again small role for two-nucleon currents

Transverse Sum Rule



Lovato, Gandolfi, Butler, Carlson, Lusk, Pieper, Schiavilla PRL 2013

Two-nucleon currents contribute ~ 50% enhancement
Jlab experiments, neutrino experiments

Sum Rules and Euclidean Response

Real-time response

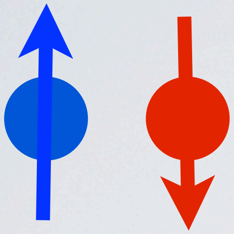
$$R(q,\omega) \;=\; \langle 0|\; \mathbf{j}^\dagger(q)\; |f\rangle\langle f|\; \mathbf{j}(q)\; |0\rangle\; \delta(w-(E_f-E_0))$$

$$R(q,\omega) \;=\; \int dt\; \langle 0|\; \mathbf{j}^\dagger(q)\;\; \exp[iHt]\; \mathbf{j}(q)\; |0\rangle\;\; \exp[i\omega t]$$

Short time 't' : sum rules
Long time: higher energy resolution
No general method for strongly-correlated quantum
systems, typically use model final states

Short-time theories well known -
operator product expansion, ...

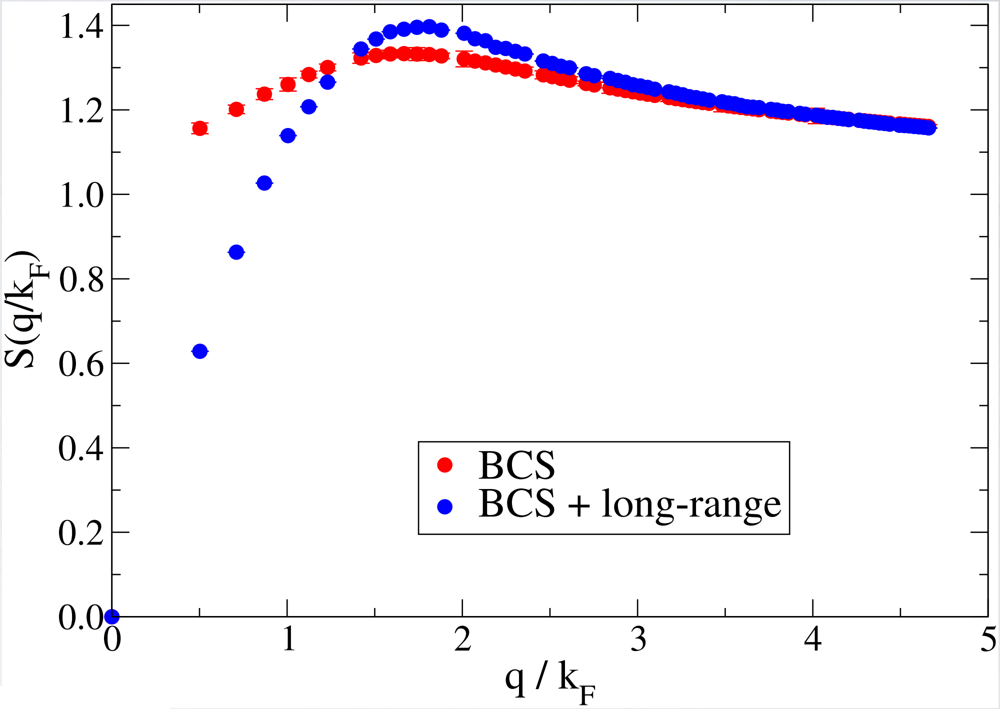
Example: Unitary Fermi Gas



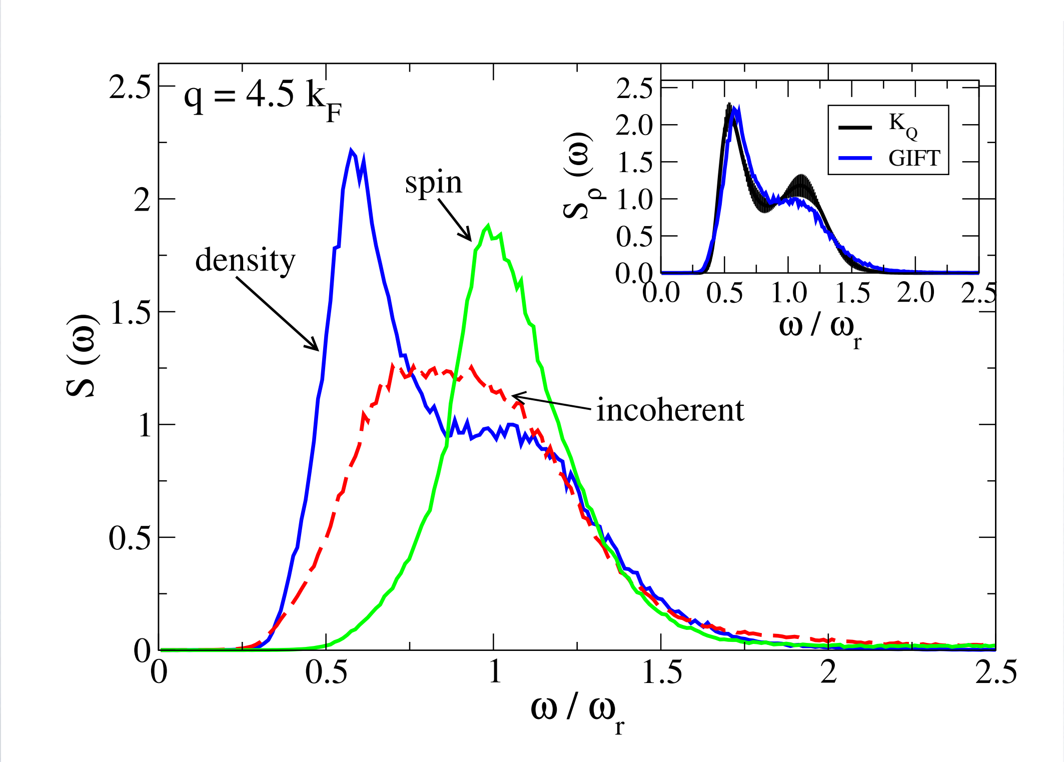
Cold Atoms

zero-range interaction

infinite scattering length



strength > 1



peaks at $q^2 / 2m$; $q^2 / 4m$

latter not reproducible with PWIA or spectral fn

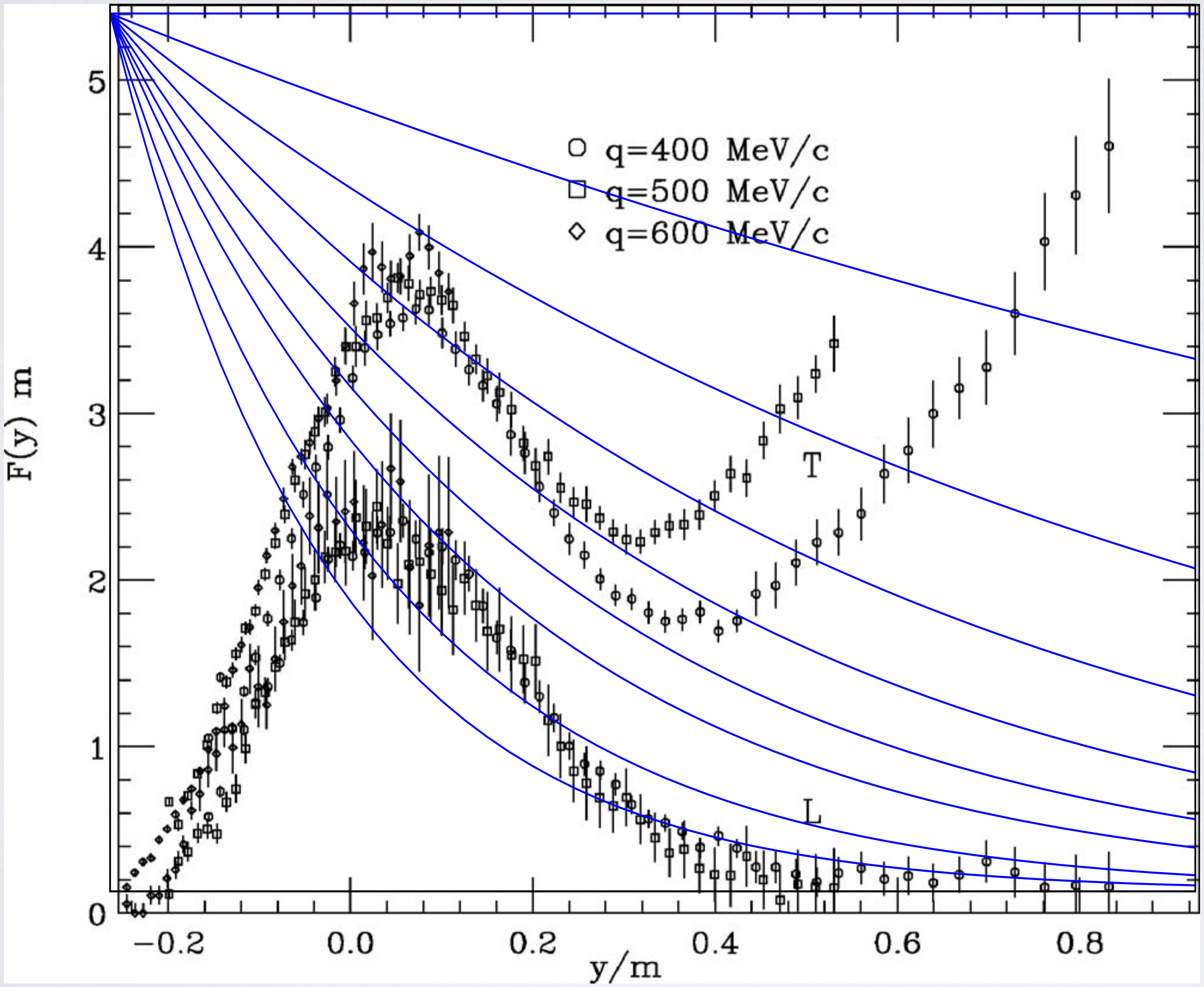
Imaginary-time correlator (Euclidean Response)

$$R(q,\tau) \;=\; \langle 0|\; \mathbf{j}^\dagger(q) \;\; \exp[-H\tau]\; \mathbf{j}(q)\; |0\rangle$$

Converts quantum dynamics to statistical mechanics

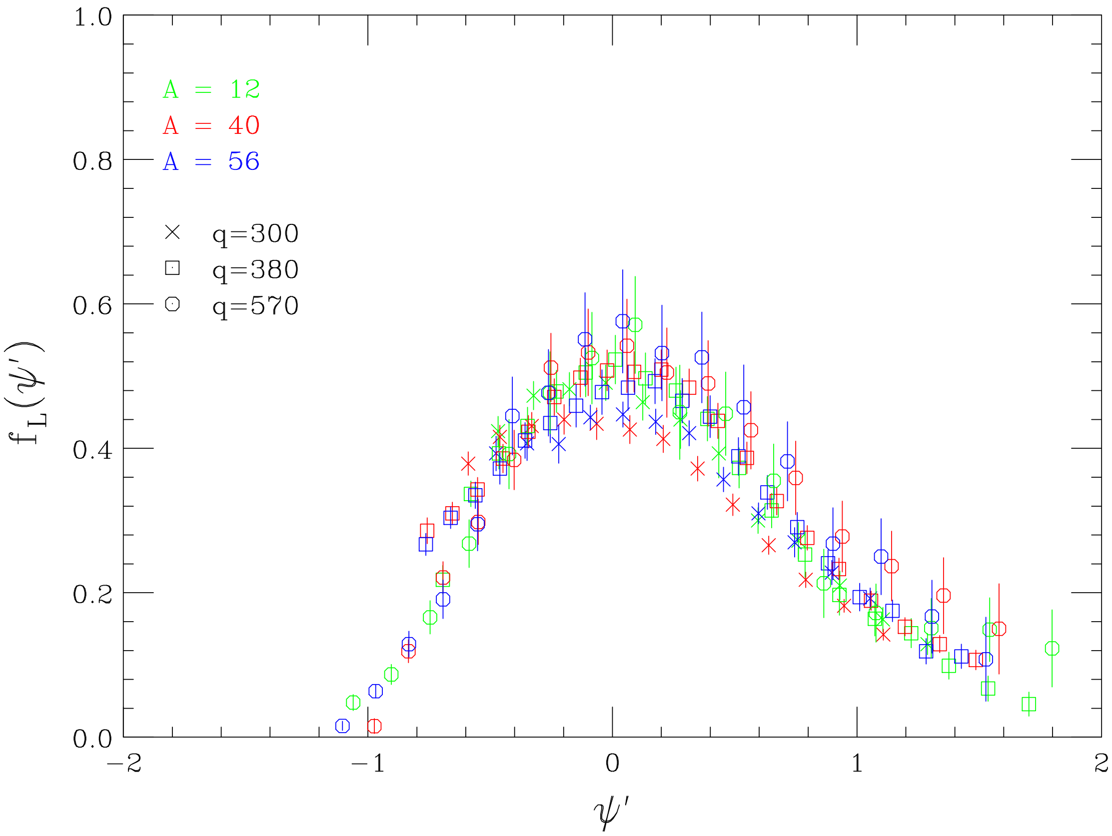
short time : sum rules (high energy)

long 'time' : low energy response (collective modes,...)



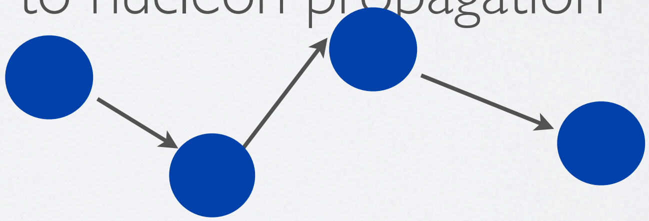
τ = inverse T

Why do FSI add to high-energy response? Longitudinal electron scattering



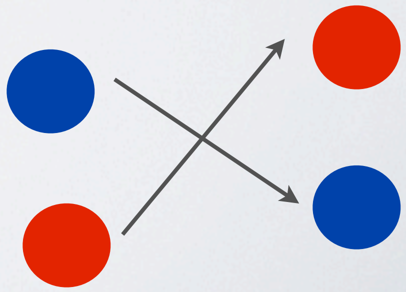
PWIA (or spectral function):

- response tied to charge propagation
- charge propagation charged to nucleon propagation
  - (momentum distribution)



Full Interacting system:

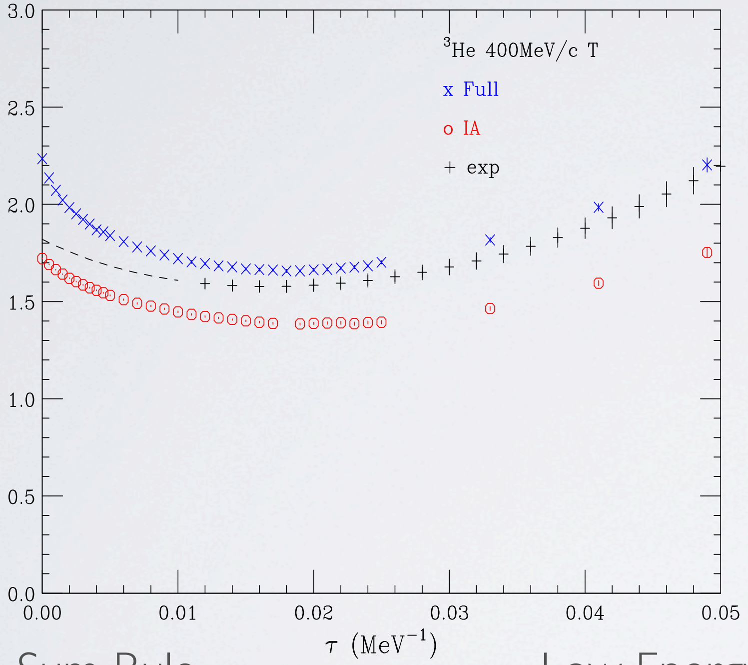
- charge can propagate through pion exchange:
- faster response (low 'mass') adds to high-energy tail



Towards (short-time) Dynamics: Euclidean Response

^{3}He and ^{4}He Transverse Euclidean Response Functions

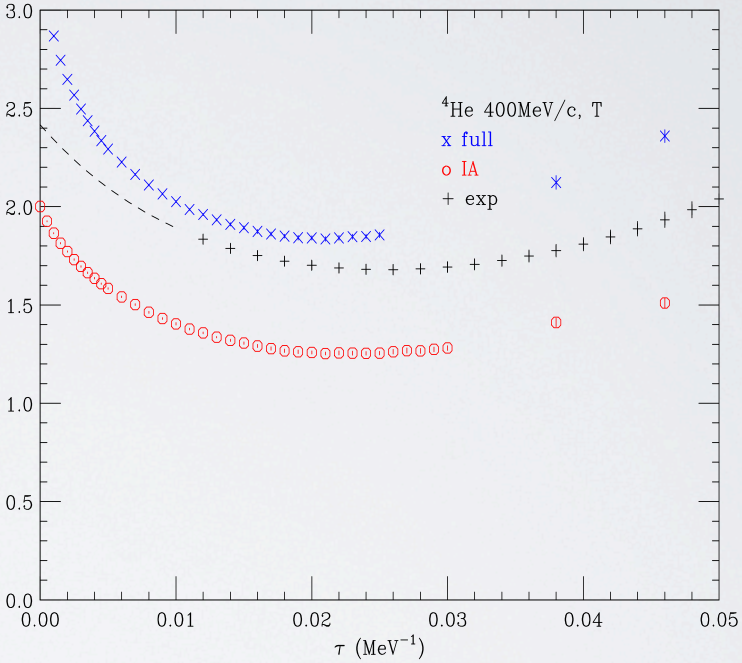
^{3}He Transverse



Sum Rule

Low Energy

^{4}He Transverse



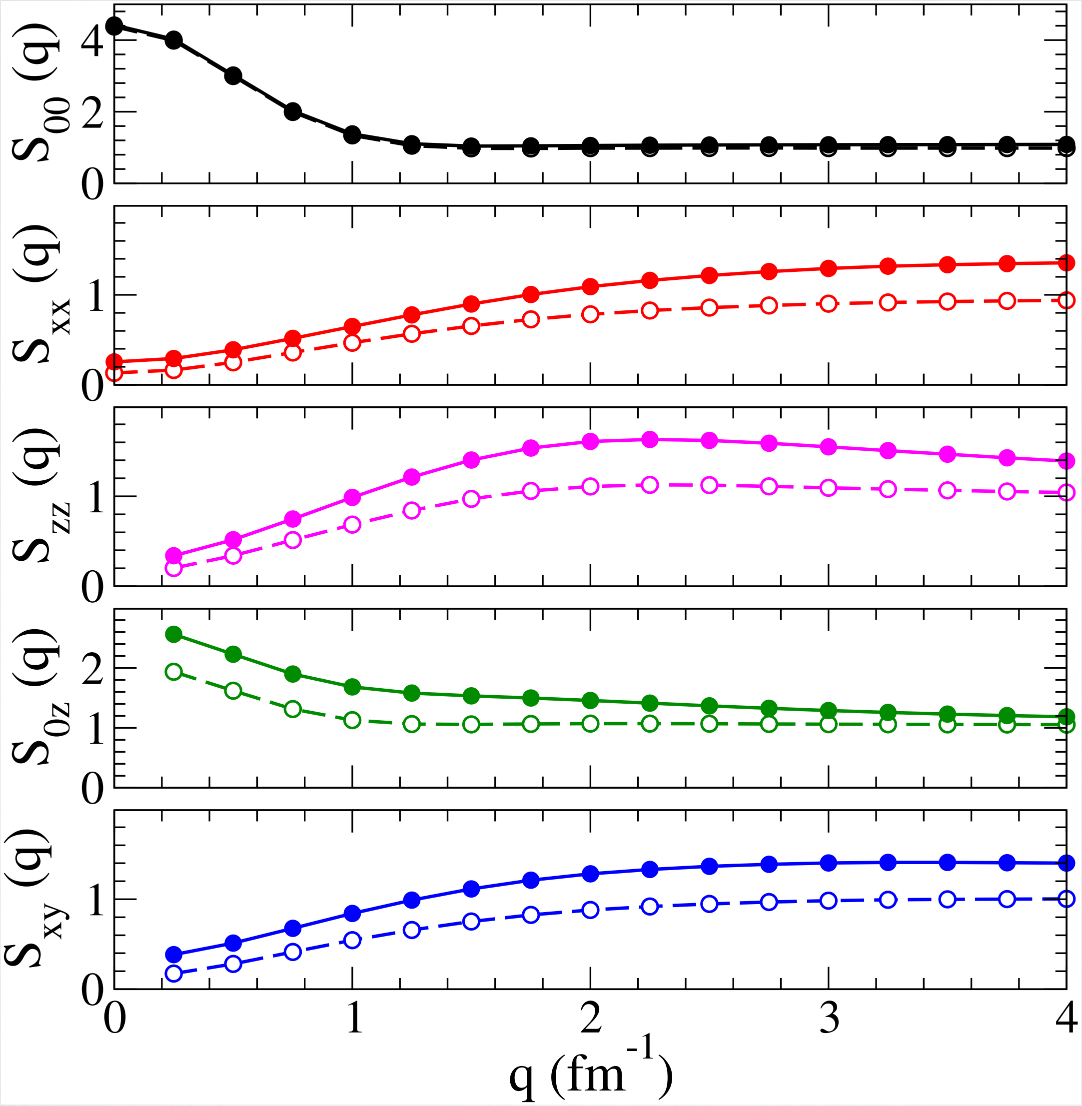
- Excess strength in quasielastic region ($\tau > 0.01$ MeV^{-1})
- Larger in $A = 4$ than in $A = 3$, as already inferred from S_T

Neutrino Scattering:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon'\mathrm{d}\Omega}\right)_{\nu/\overline{\nu}} = \frac{G_F^2}{2\pi^2}\,k'\epsilon'\cos^2\frac{\theta}{2}\left[R_{00}+\frac{\omega^2}{q^2}\,R_{zz}-\frac{\omega}{q}R_{0z}+\left(\tan^2\frac{\theta}{2}+\frac{Q^2}{2\,q^2}\right)R_{xx}\mp\tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2}+\frac{Q^2}{q^2}}\,R_{xy}\right]$$

$$\begin{aligned}
R_{00}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega+m_A-E_f)\,|\langle f|\,j^0(\mathbf{q},\omega)\,|i\rangle|^2\ , \\
R_{zz}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega+m_A-E_f)\,|\langle f|\,j^z(\mathbf{q},\omega)\,|i\rangle|^2\ , \\
R_{0z}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega+m_A-E_f)\Big[\langle f|\,j^0(\mathbf{q},\omega)\,|i\rangle \\
&\quad \times\langle f|\,j^z(\mathbf{q},\omega)\,|i\rangle^* + \text{c.c.}\Big]\ , \\
R_{xx}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega+m_A-E_f)\Big[\,|\langle f|\,j^x(\mathbf{q},\omega)\,|i\rangle|^2 \\
&\quad +|\langle f|\,j^y(\mathbf{q},\omega)\,|i\rangle|^2\,\Big]\ , \\
R_{xy}(q,\omega) &= \overline{\sum_i}\sum_f \delta(\omega+m_A-E_f)\Big[\langle f|\,j^x(\mathbf{q},\omega)\,|i\rangle \\
&\quad \times\langle f|\,j^y(\mathbf{q},\omega)\,|i\rangle^* - \text{c.c.}\Big]\ ,
\end{aligned}$$

Neutrino/Anti-neutrino Scattering
5 response functions
Neutral current sum rules for 12C



Present and Near Future

- Calculations of neutral and charged current scattering on the deuteron (neutrinos and anti-neutrinos) completed
- Codes for neutral current and nearly charged current completed for use in Quantum Monte Carlo calculations
- Calculations of Sum Rules (NC) completed
- Calculations for Euclidean response expected in ~ 1 year
- Studying quasi-analytic approaches to dynamic response in high q, omega region

Thanks to:
ANL devoting ~50-100M core-hours to this project plus staff/postdoc time
INCITE award to NUCLEI project amount largest in country
 - neutrino scattering is an important goal
LANL support through LDRD-DR project (PI: Mauger)