

# **Scaling of Inclusive Electroweak Interactions with Nuclei**

## **Part II: Relativistic Impulse Approximation**

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*Neutrino Interactions-Nucleus Interaction for Current and Next Generation Neutrino Oscillation Experiments:*

**INT Workshop INT-13-54W, Seattle, December 3-13, 2013**

# OUTLINE

- **Quasielastic  $(e, e')$  data & Scaling/Superscaling**
- **The Relativistic Impulse Approximation**
  - *The Relativistic Mean Field (RMF)*
  - *Analysis of electron scattering & Scaling behavior*
  - *Application to neutrino (CC and NC) processes*
  - *Comparison with the SuperScaling Approach (SuSA)*
  - *MiniBooNE, MINER $\nu$ A & NOMAD experiments*
- **SUMMARY AND CONCLUSIONS**

# **QUASIELASTIC $(e, e')$ DATA & SCALING**

# The SuperScaling Approach (SuSA)

- *Scaling of the first kind below the QE peak ( $\psi \leq 0$ )*
- *Excellent scaling of the second kind in the same region*
- *Breaking of scaling above the QE peak ( $\psi > 0$ )  $\implies$  Effects beyond the IA  
(mainly located in the T channel)*
- **LONGITUDINAL RESPONSE SUPERSCALES**

# The SuperScaling Approach (SuSA)

- Scaling of the first kind before

PRC60 (1999) 065502

PRL82 (1999) 3212

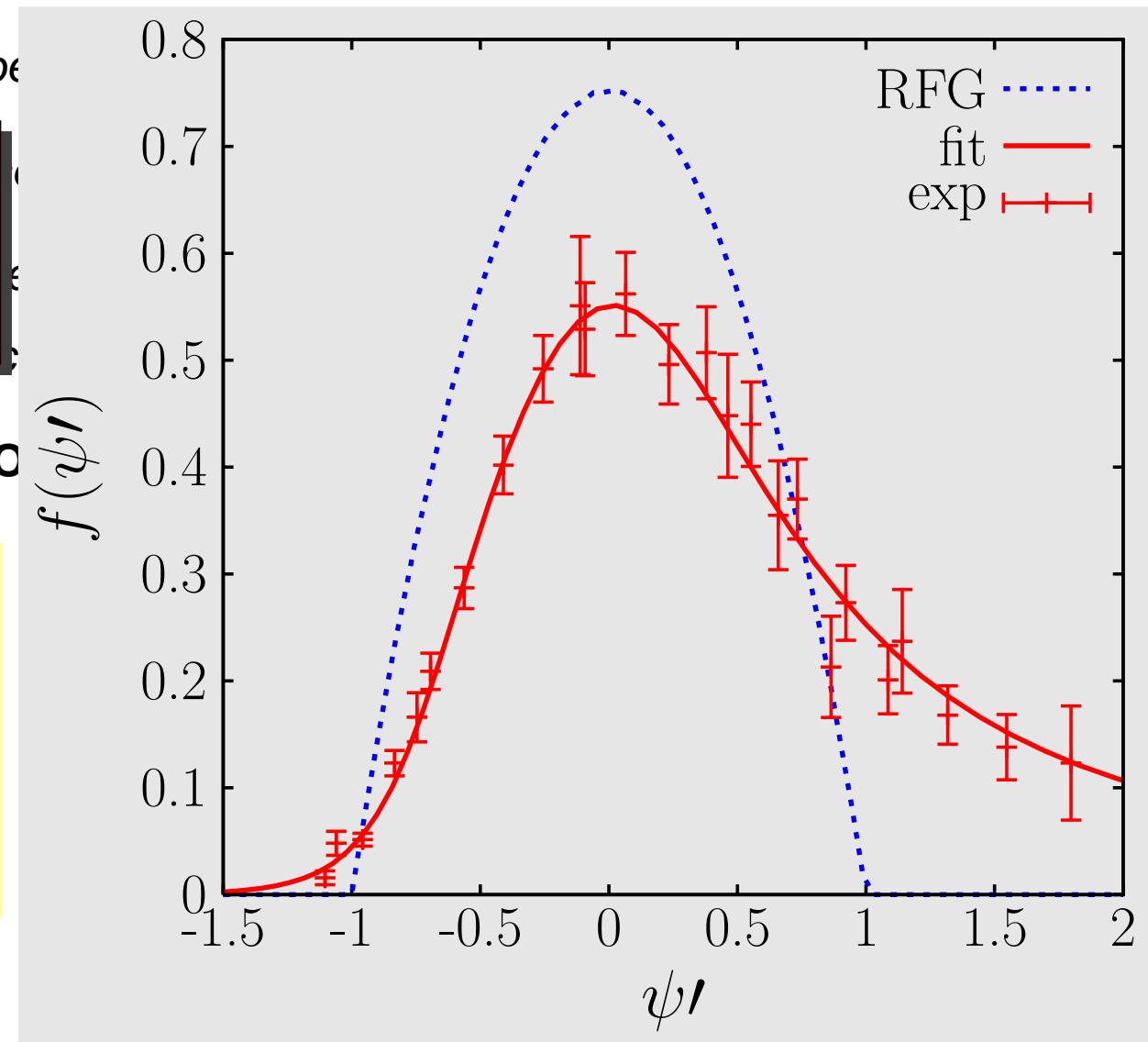
PRC65 (2002) 025502

- LONGITUDINAL RESPONSE

Experimental superscaling function: asymmetric shape with a long tail extended to positive  $\psi$ -values



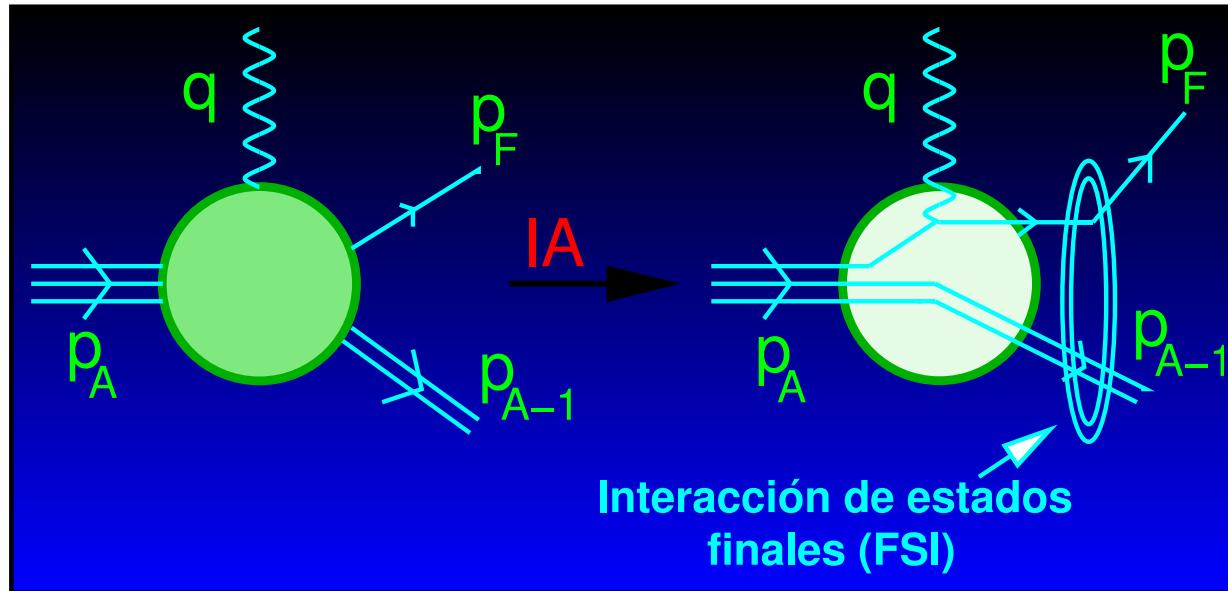
strong constraints to models



## **THE MODEL: RELATIVISTIC IMPULSE APPROXIMATION**

**APPLICATION TO  $(e, e')$  PROCESSES**

# Relativistic Impulse Approximation (RIA)



Nuclear Current  $\Rightarrow$  One-body operator

$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p} \overline{\Psi}_F(\vec{p} + \vec{q}) \hat{J}_N^\mu \Psi_B(\vec{p})$$

Scattering off a nucleus  $\Rightarrow$  incoherent sum of single-nucleon scattering processes

# Ingredients in RIA: nucleon w.f. & current operator

*Solutions of Dirac equation with phenomenological relativistic potentials*

- $\Psi_B$ : Bound nucleon w.f.  $\implies$  **Relativistic Mean Field (RMF)**
- $\Psi_F$ : Ejected nucleon w.f.  $\implies$  **Final State Interactions (FSI)**

**RMF  $\leftrightarrow$  rROP  $\leftrightarrow$  RPWIA  $\leftrightarrow$  RGFA**

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**RMF  $\Leftrightarrow$  rROP  $\Leftrightarrow$  RPWIA  $\Leftrightarrow$  RGFA**

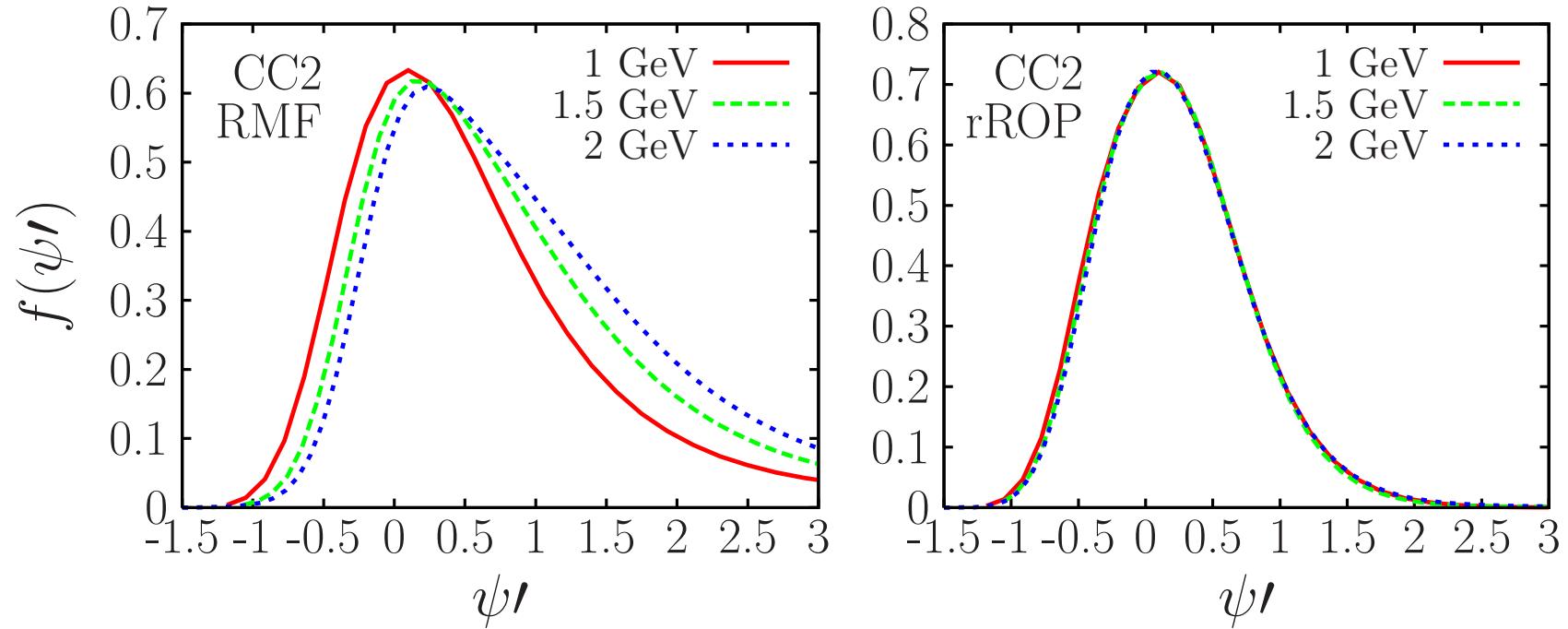
- **Electromagnetic current:**  $(e, e')$

$$\hat{J}_{cc1}^\mu = (F_1 + F_2)\gamma^\mu - \frac{F_2}{2m_N}(\bar{P} + P_N)^\mu$$

$$\hat{J}_{cc2}^\mu = F_1\gamma^\mu + \frac{iF_2}{2m_N}\sigma^{\mu\nu}Q_\nu$$

**Off-shell & Gauge ambiguities ( $Q_\mu J^\mu \neq 0$ )**

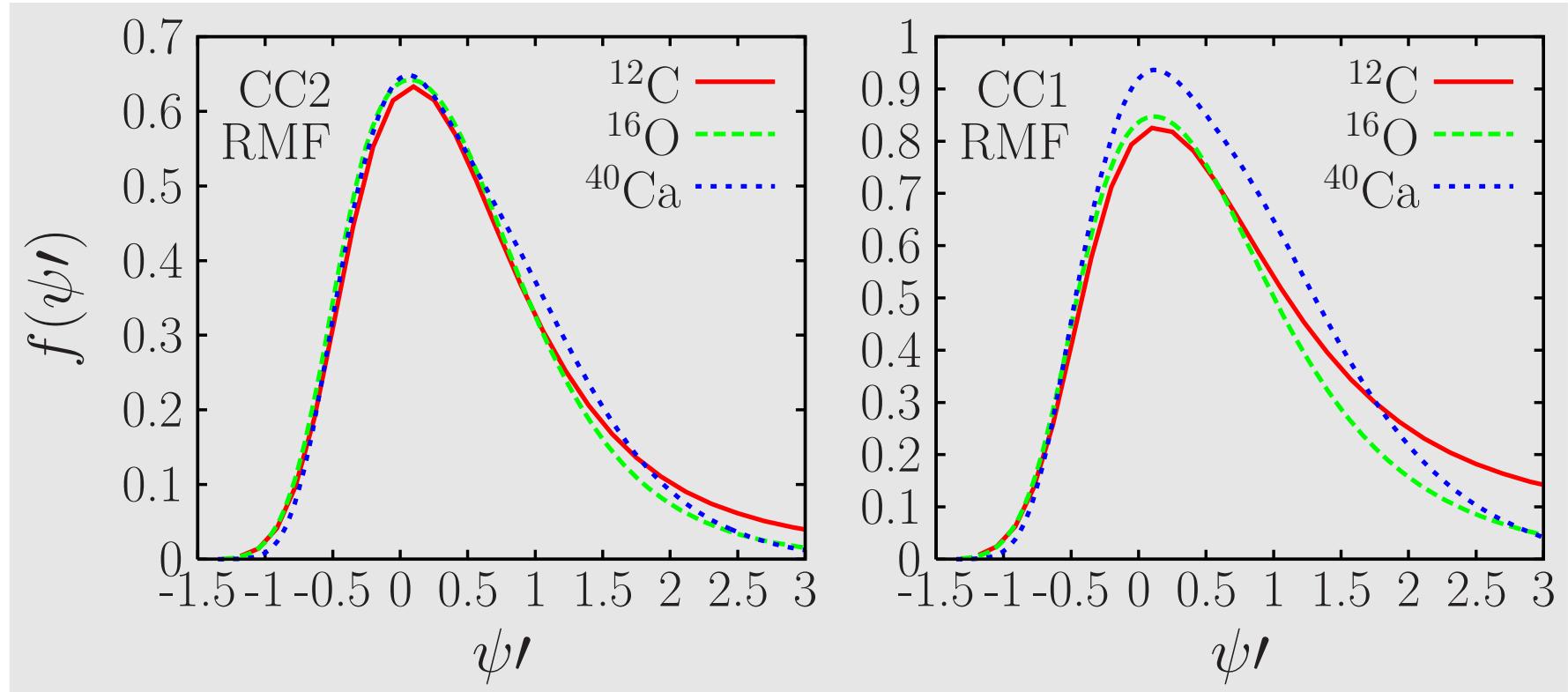
# Scaling of the first kind in $^{12}C(e, e')$



**RMF:** shift in  $\psi' < 0$  and breakdown of scaling at roughly  $\sim 25 - 30\%$  for  $\psi' > 0$  (compatible with data).

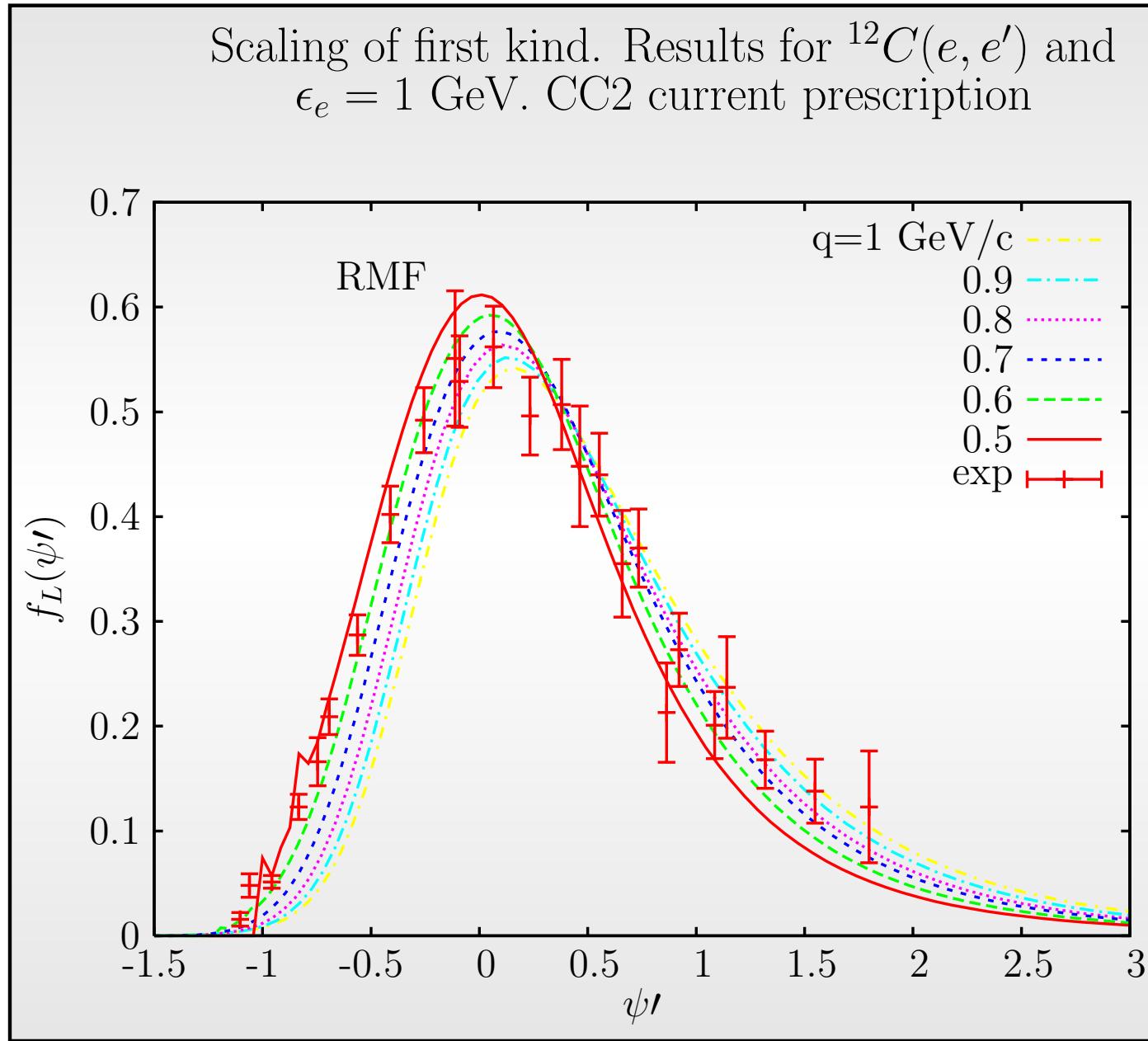
Scaling of the first kind: excellent in **rROP** approach (and **RPWIA**)

# Scaling of the second kind in RIA

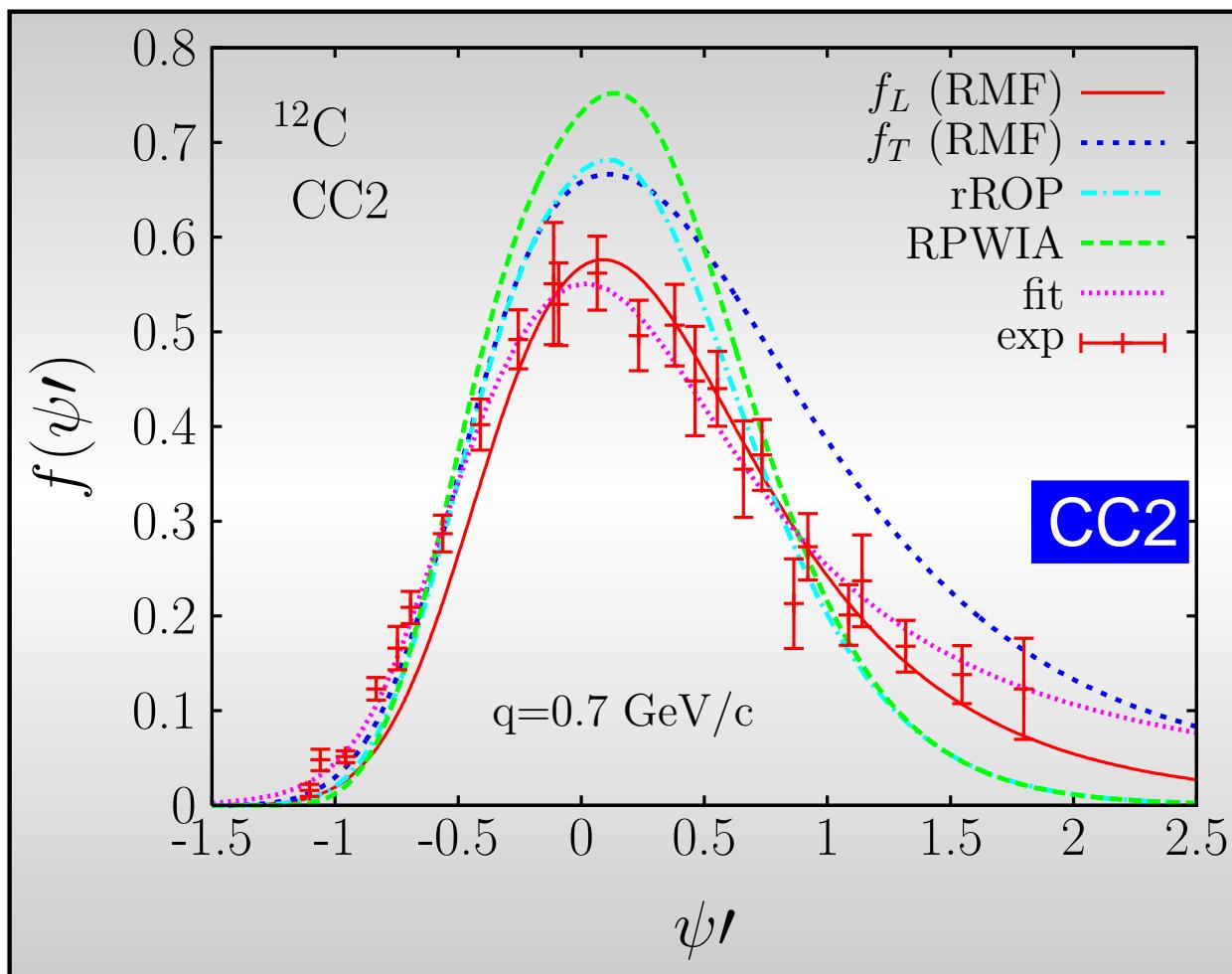


**Scaling of 2<sup>a</sup> kind: excellent with the CC2 current operator**

# How Scaling of the 1<sup>er</sup> kind behaves (RMF)



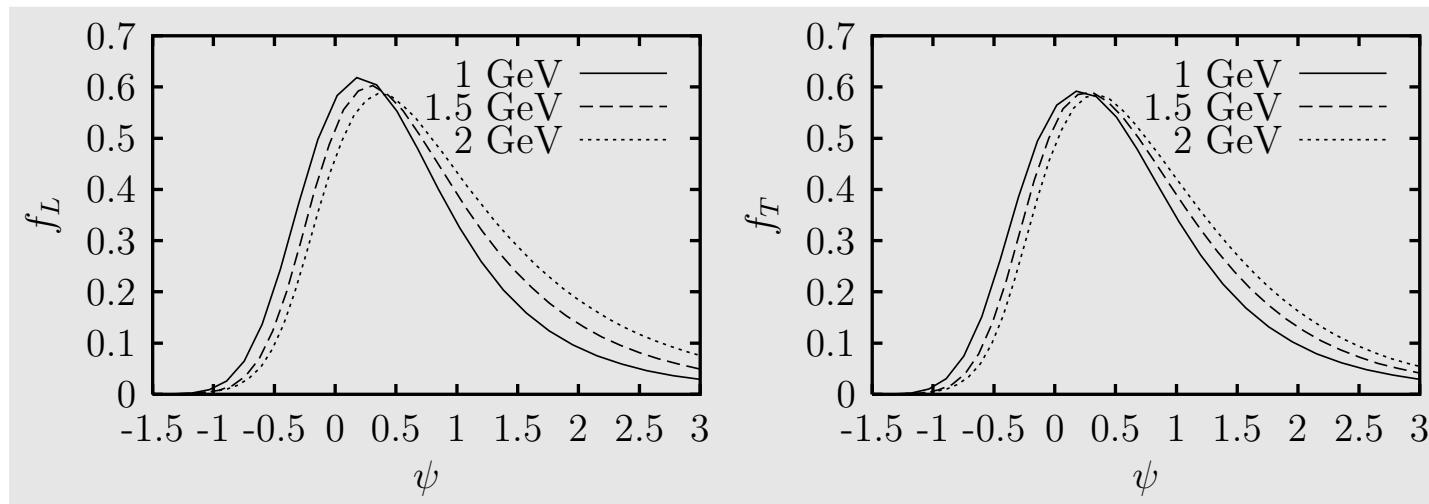
# RMF: Comparison with $(e, e')$ data



Only the description of FSI provided by RMF leads to an asymmetric function  $f(\psi')$  in accordance with the behavior shown by data. Moreover,  $f_T > f_L$

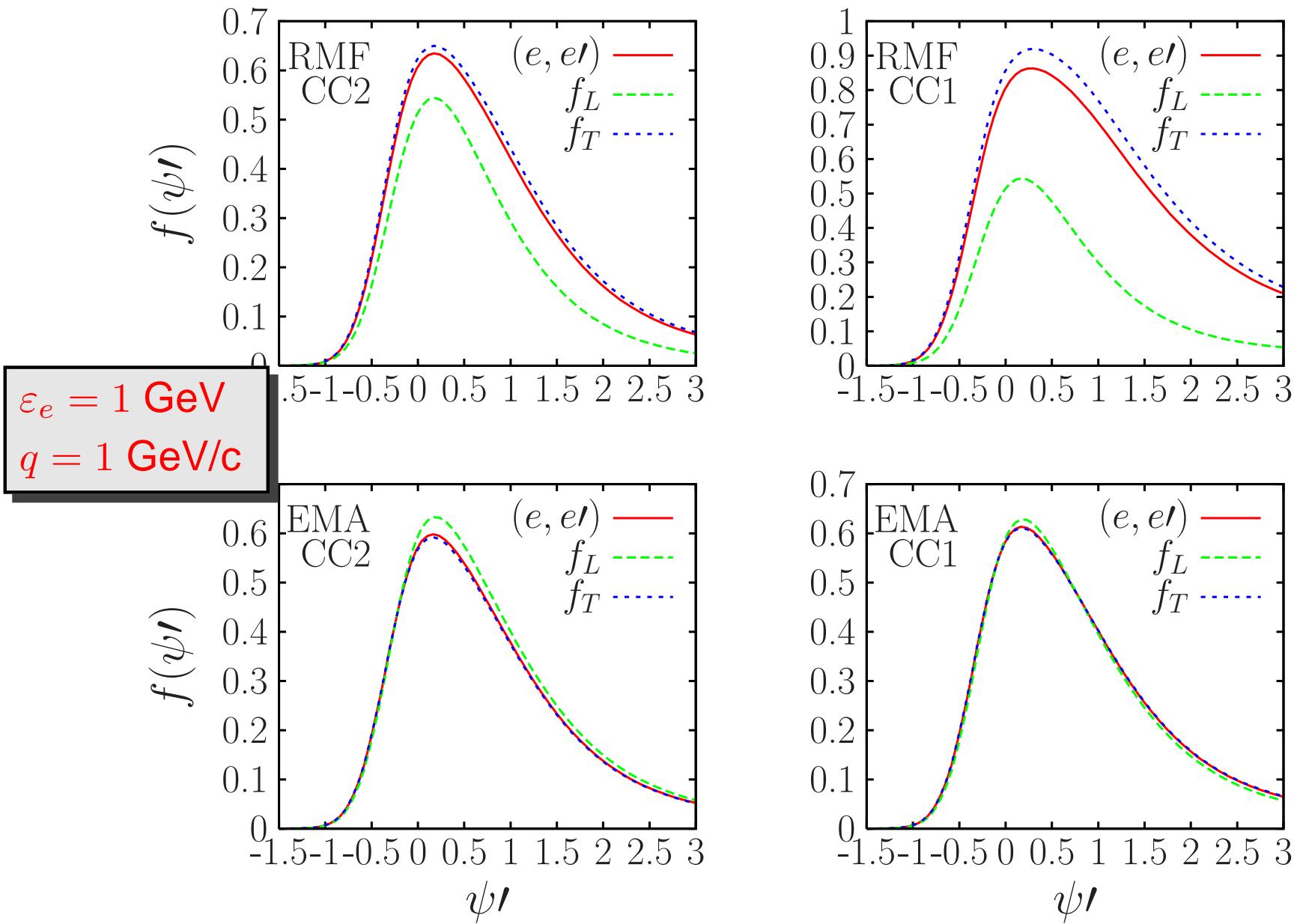
# Scaling of the $0^{th}$ kind

- RFG: by construction  $f_L(\psi) = f_T(\psi) = f(\psi)$
- RPWIA:  $f_L(\psi) = f_T(\psi) = f(\psi)$ —symmetric
- Semi-relativistic (SR) and/or NR approaches with FSI:
  - Woods-Saxon potential: symmetric scaling functions.
  - Dirac Equation-Based (DEB) potential: leads to an asymmetric function  $f(\psi)$ .

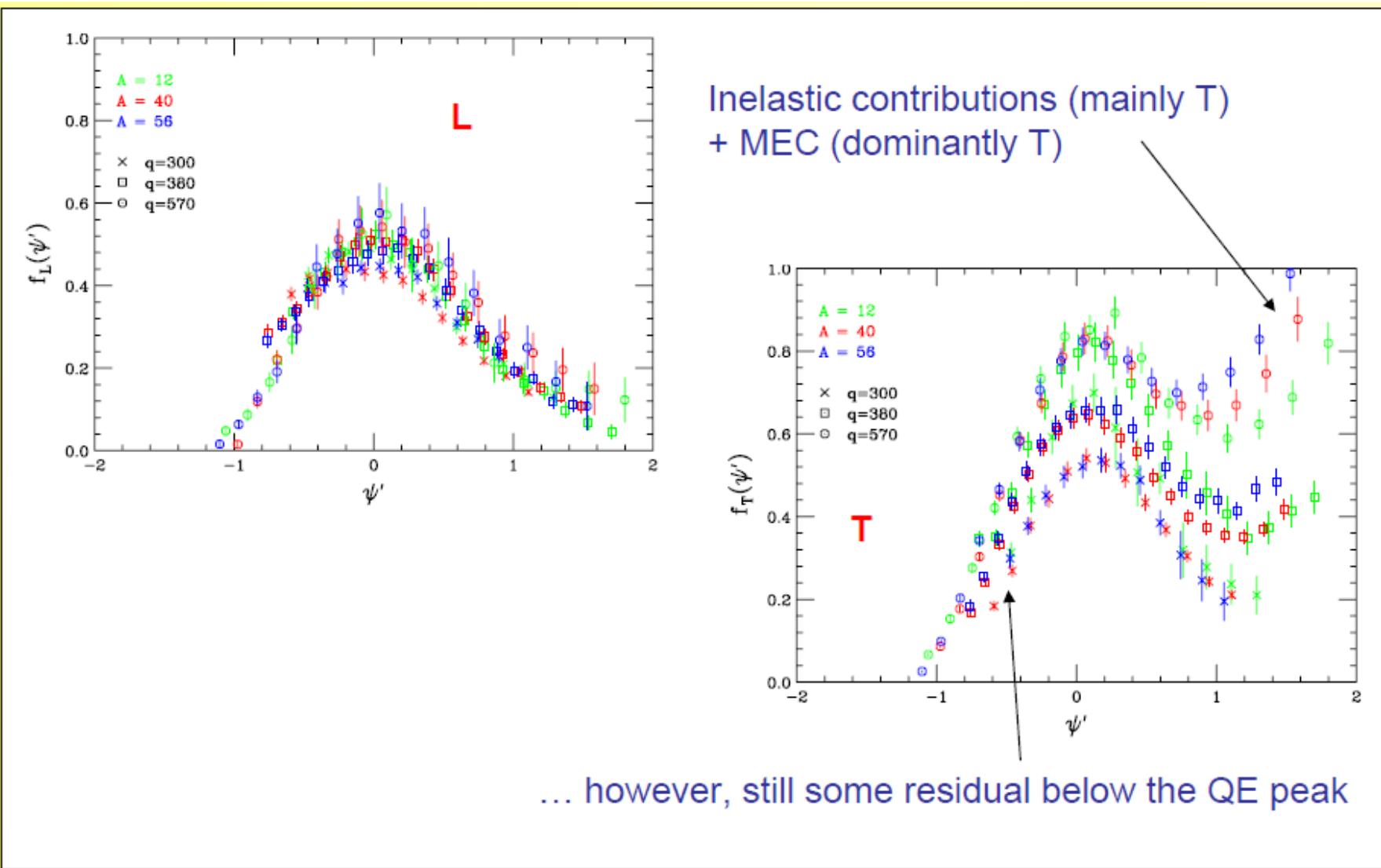


In all cases:  $f_L(\psi) = f_T(\psi) = f(\psi)$

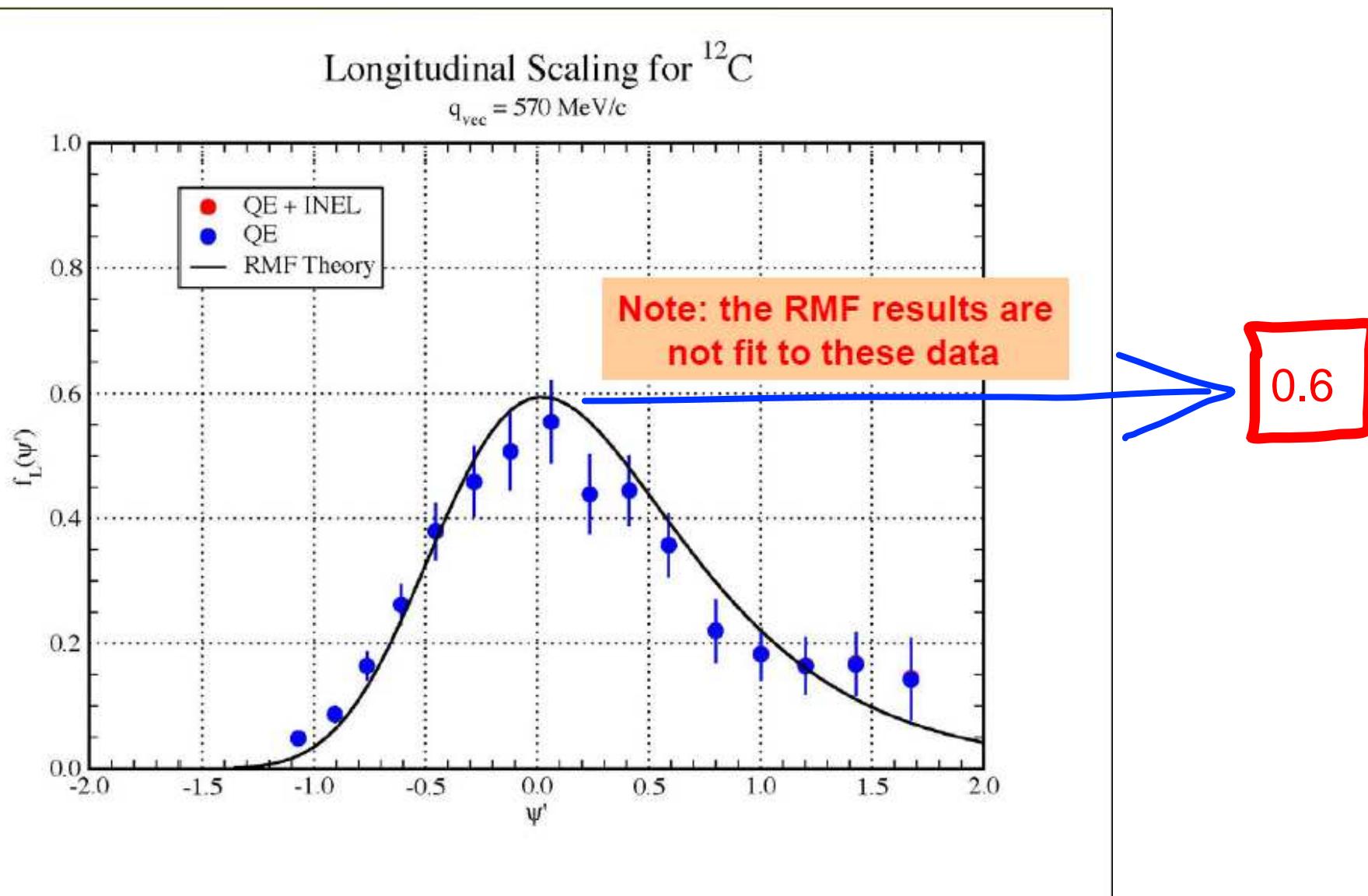
# Scaling of the 0<sup>th</sup> kind in RMF



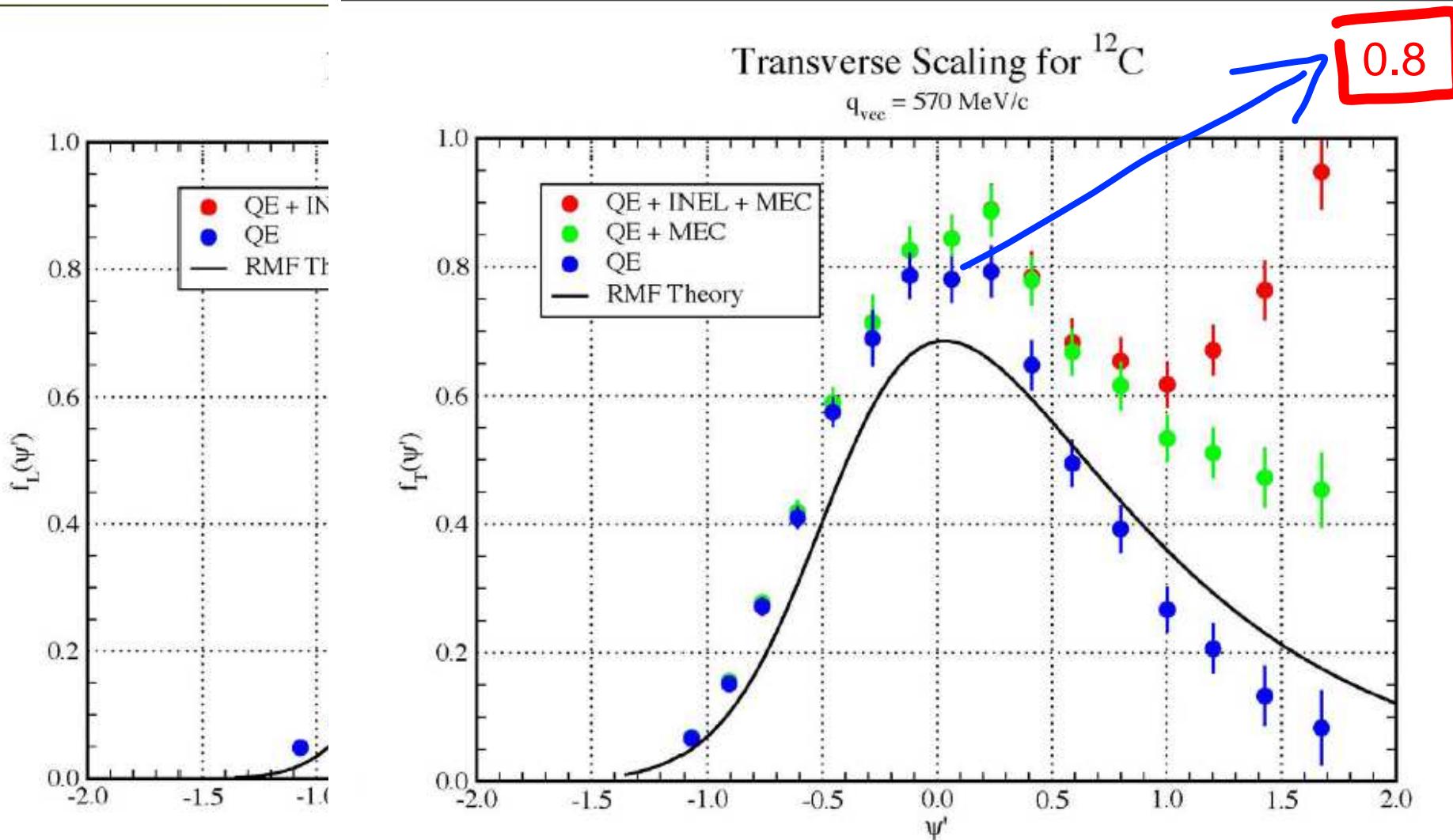
# Analysis of data: $L/T$ separation



# Scaling in QE $L/T$ -channels



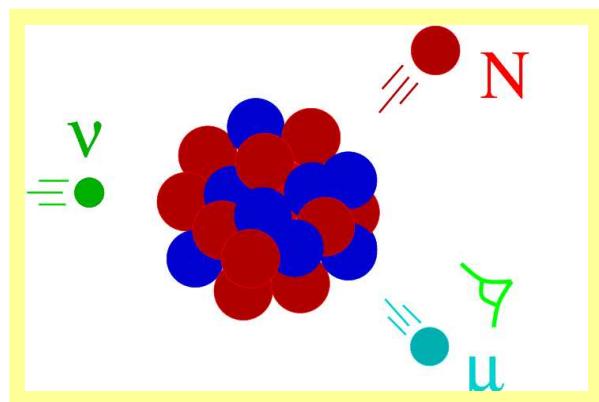
# Scaling in QE $L/T$ -channels



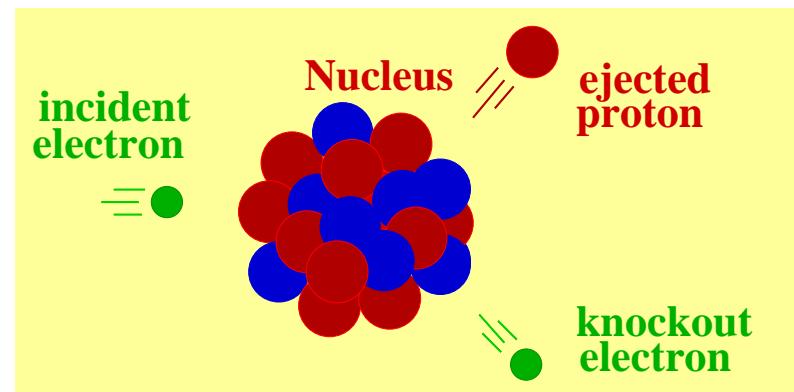
# **INTERACTION OF NEUTRINOS with NUCLEI**

# Quasielastic $(e, e')$ versus $(\nu, \mu)$ reactions

## Neutrinos



## Electrons



*Kinematics in electron and neutrino INCLUSIVE scattering are very similar. One should check models of neutrino scattering against inclusive electron data.*

**However, caution with models that do not succeed in reproducing the experimental  $(e, e')$  scaling function.**

# General formalism for $(\nu, \mu)$ processes

## Double differential cross section

$$\left[ \frac{d\sigma}{dk_\mu d\Omega} \right]_\chi = \sigma_0 \mathcal{F}_\chi^2 \quad ; \quad \sigma_0 = \frac{(G_F^2 \cos \theta_c)}{2\pi^2} \frac{v_0}{2} \left( k_\mu \cos \frac{\tilde{\theta}}{2} \right) \quad ; \quad \chi = +(-) \equiv \nu_\mu (\bar{\nu}_\mu)$$

## Nuclear structure information

$$\mathcal{F}_\chi^2 = \hat{V}_L R_L + \hat{V}_T R_T + \chi [2 \hat{V}_{T'} R_{T'}]$$

## Rosenbluth-like decomposition

$$R_L = R_L^{VV} + R_L^{AA}$$
$$R_T = R_T^{VV} + R_T^{AA}$$
$$R_{T'} = R_{T'}^{VA}$$

## Leptonic ( $j^\mu$ ) & hadronic currents ( $J^\mu$ )

$$j^\mu = j_V^\mu + j_A^\mu \quad ; \quad J^\mu = J_V^\mu + J_A^\mu$$

## Weak nuclear current

$$J_V^\mu = \bar{u}(P') \left[ F_1^V \gamma^\mu + \frac{i}{2m_N} F_2^V \sigma^{\mu\nu} Q_\nu \right] u(P)$$
$$J_A^\mu = \bar{u}(P') \left[ G_A \gamma^\mu + \frac{1}{2m_N} G_P Q^\mu \right] u(P)$$

## Nuclear responses

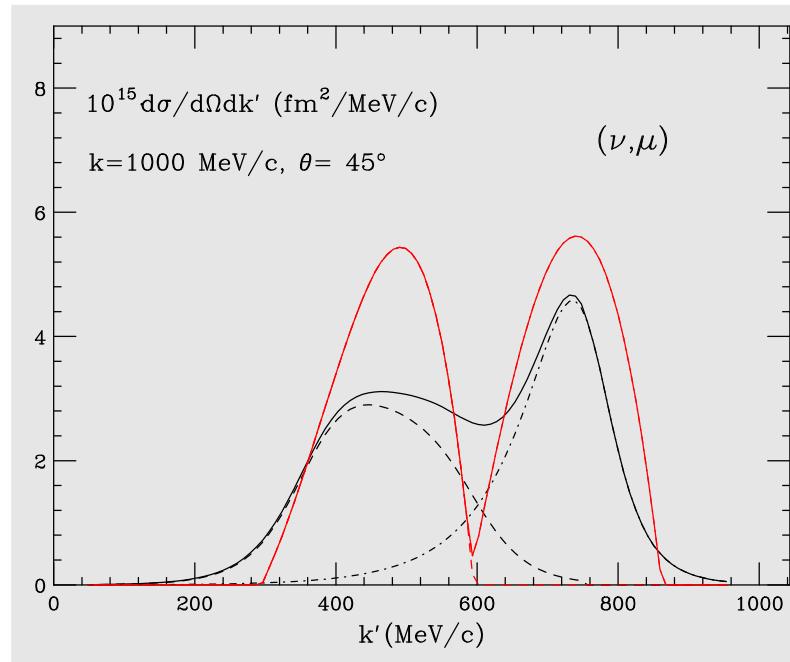
Composed of VV (vector-vector), AA (axial-axial) and VA (vector-axial) components arising from the V and A weak nuclear currents.

# Scaling applied to $(\nu, \mu)$ processes

## SuSA (“SuperScaling Analysis”)

- Hypothesis: the universal character of the function  $f_{exp}(\psi')$  extracted from the analysis of  $L (e, e')$  data  $\implies$  it can be applied to CC  $(\nu_\mu, \mu)$  processes.

Prediction of “realistic”  $(\nu, \mu)$  cross sections  
[MiniBoone, MINER $\nu$ A, NOMAD]

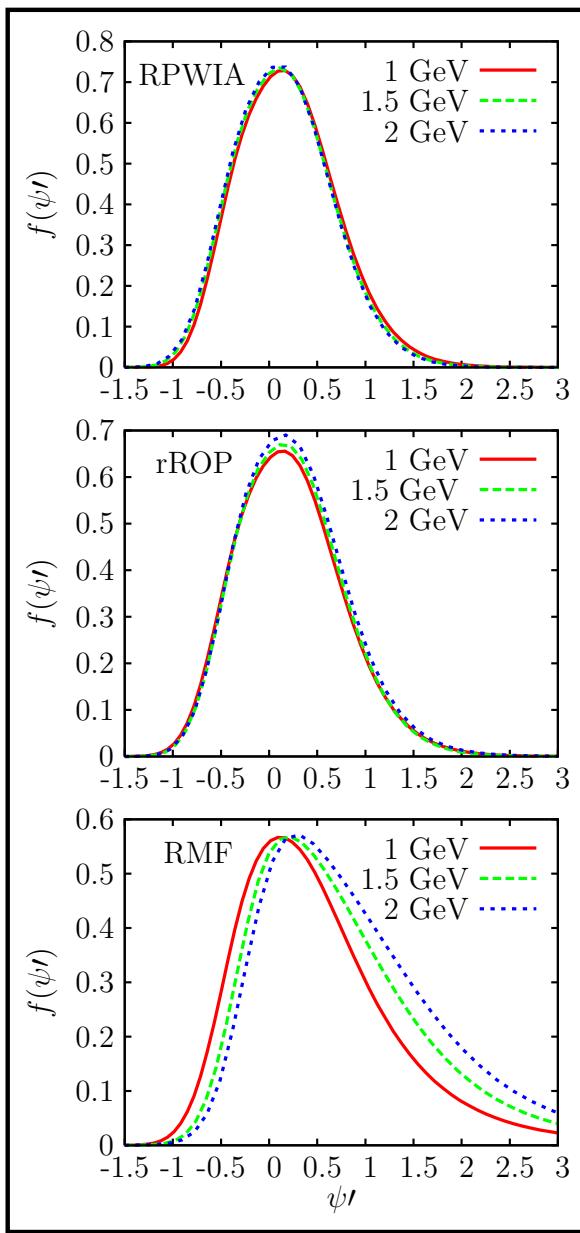


# RMF applied to $(\nu, \mu)$ & Scaling

## PROCEDURE

- Evaluate the inclusive  $(\nu, \mu)$  cross section with a specific RIA model and divide it by the corresponding single-nucleon cross section [weighted by the appropriate proton ( $Z$ ) and neutron ( $N$ ) numbers]  $\implies$  **THEORETICAL SCALING FUNCTION**
- Does the theoretical RIA scaling function satisfy scaling properties?
  - Scaling of the first kind:  $f(q, \psi) \xrightarrow{q \rightarrow \infty} f(\psi)$
  - Scaling of the second kind:  $f(\psi)$  – independent on the nucleus

# RMF applied to $(\nu, \mu)$ & Scaling



## PROCEDURE

$\nu, \mu)$  cross section with a specific RIA model and divide it by the nucleon cross section [weighted by the appropriate proton ( $Z$ ) and

THEORETICAL SCALING FUNCTION

A scaling function satisfy scaling properties?

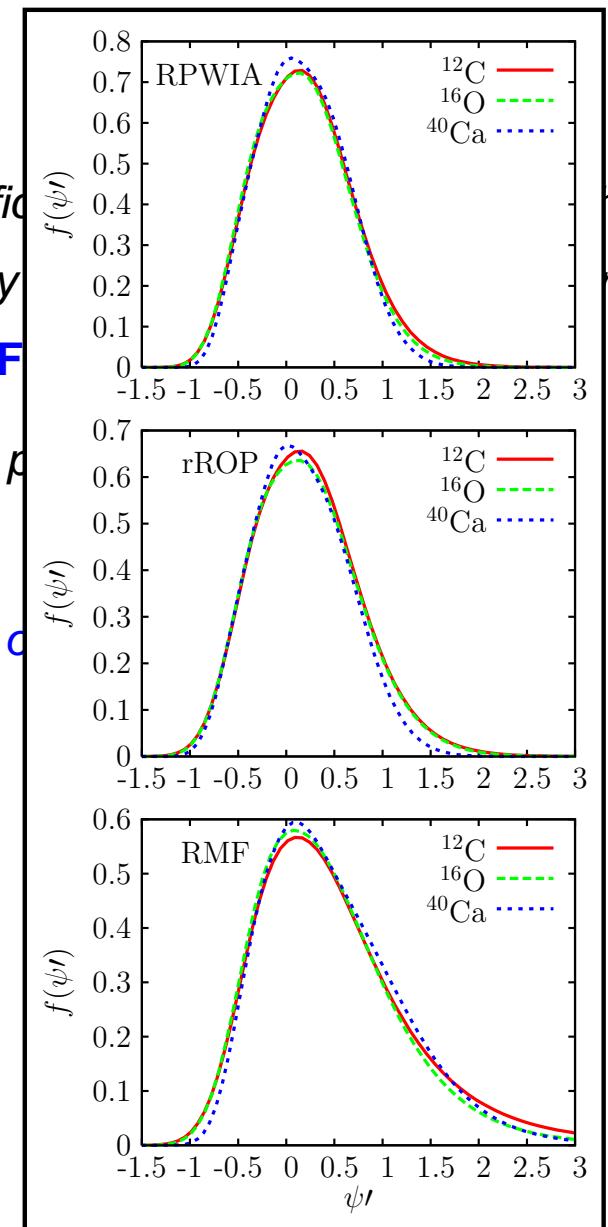
kind:  $f(q, \psi) \xrightarrow{q \rightarrow \infty} f(\psi)$

second kind:  $f(\psi)$  – independent on the nucleus

# RMF applied to $(\nu, \mu)$ & Scaling

## PROCEDURE

- Evaluate the inclusive  $(\nu, \mu)$  cross section with a specific corresponding single-nucleon cross section [weighted by neutron ( $N$ ) numbers]  $\Rightarrow$  THEORETICAL SCALING FUNCTION
- Does the theoretical RIA scaling function satisfy scaling properties?
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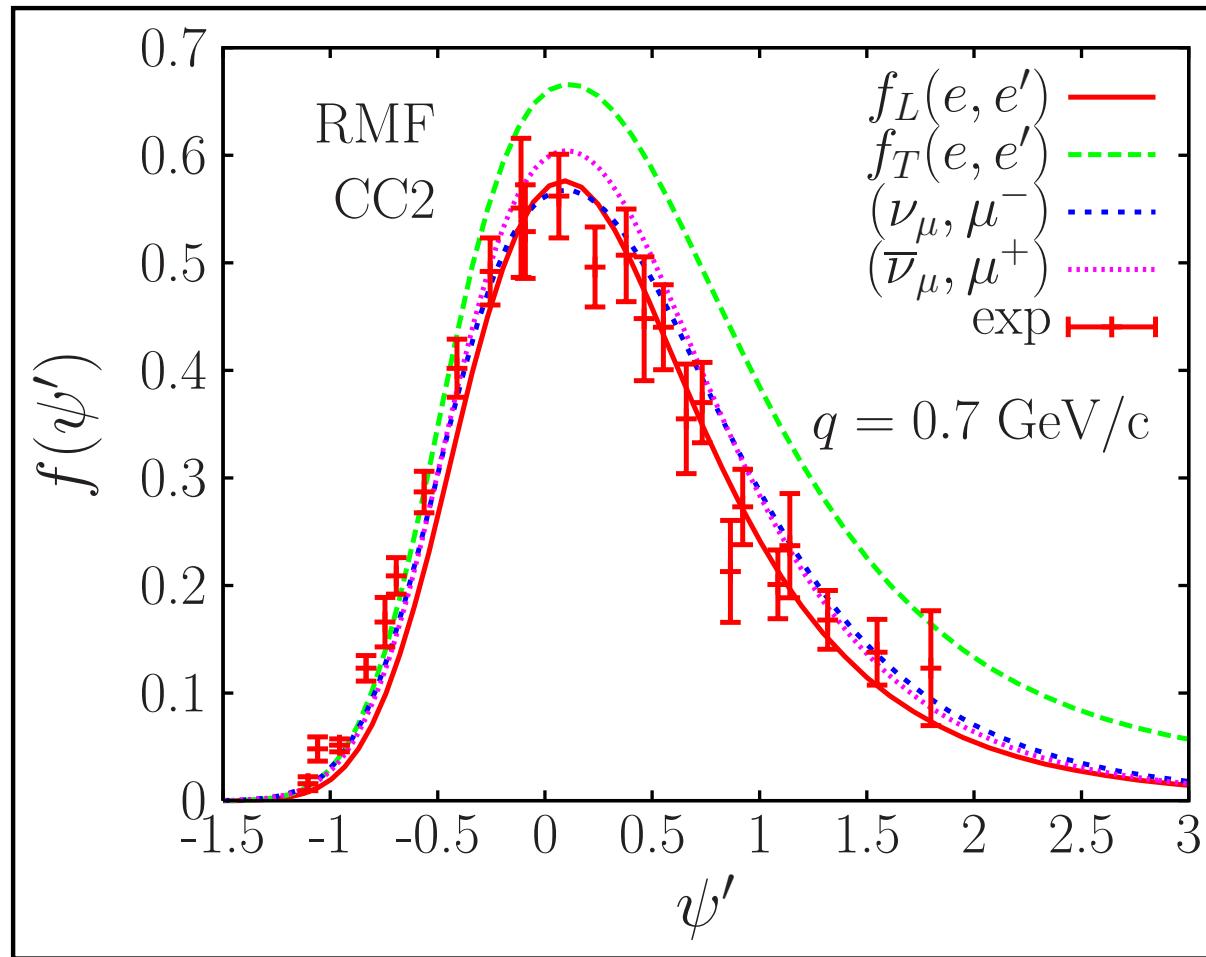
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  - Scaling of the first kind:  $f(q, \psi) \xrightarrow{q \rightarrow \infty} f(\psi)$
  - Scaling of the second kind:  $f(\psi)$  – independent on the nucleus
- Is the function  $f(\psi)$  obtained from  $(\nu, \mu)$  cross sections evaluated within RIA consistent with the function  $f(\psi)$  obtained from  $(e, e')$  calculations (with the same model)?, and with  $f_{exp}(\psi)$ ?

Similar scaling function  $f(\psi)$  for  $(e, e')$  and  $(\nu, \mu)$  processes?

# $(e, e')$ vs $(\nu, \mu)$ . SuSA vs RMF



**Basic result:** the function  $f(\psi)$  evaluated for  $(\nu, \mu)$  processes agrees better with the contribution  $f_L(\psi)$  [corresponding to  $(e, e')$ ] than with  $f_T(\psi)$ .

# ISOSPIN: isoscalar vs isovector (3<sup>er</sup> kind scaling)

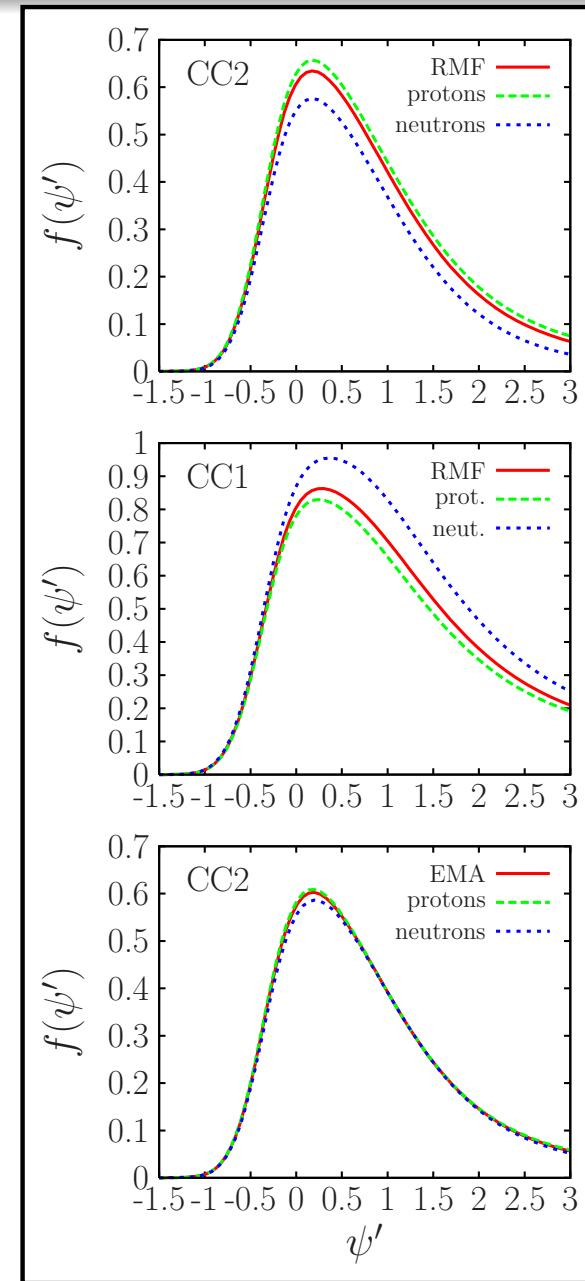
Note that  $(\nu, \mu)$  are pure isovector, whereas  $(e, e')$  contains both isoscalar and isovector. Thus, SuSA applied to CC neutrino implies Scaling of the 3rd kind, i.e., isospin nature in the scaling functions is assumed to be universal.

Analysis of the separate  $VV$ ,  $AA$  and  $VA$  contributions in  $(\nu, \mu)$  reactions.

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Analysis of the separate  $VV$ ,  $AA$  and  $VA$  contributions in  $(\nu, \mu)$  reactions.



# SuperScaling Approach based on RMF: SuSAv2

## Present SuSA

Based on the superscaling function extracted from QE electron-nucleus scattering data.

## Longitudinal

Description of nuclear responses built only on the longitudinal scaling function. Assumption of  $f_L(\psi) \approx f_T(\psi)$ , scaling of 0<sup>th</sup> kind.

## Isoscalar + Isovector Structure

The scaling function based on QE electron scattering data takes into account isovector and isoscalar currents to describe the interaction between the electron and the nucleus.

...

## SuSAv2

The Relativistic Mean Field model (RMF) is employed to improve the data analysis, where RMF accounts for FSI.

...

## Longitudinal + Transversal

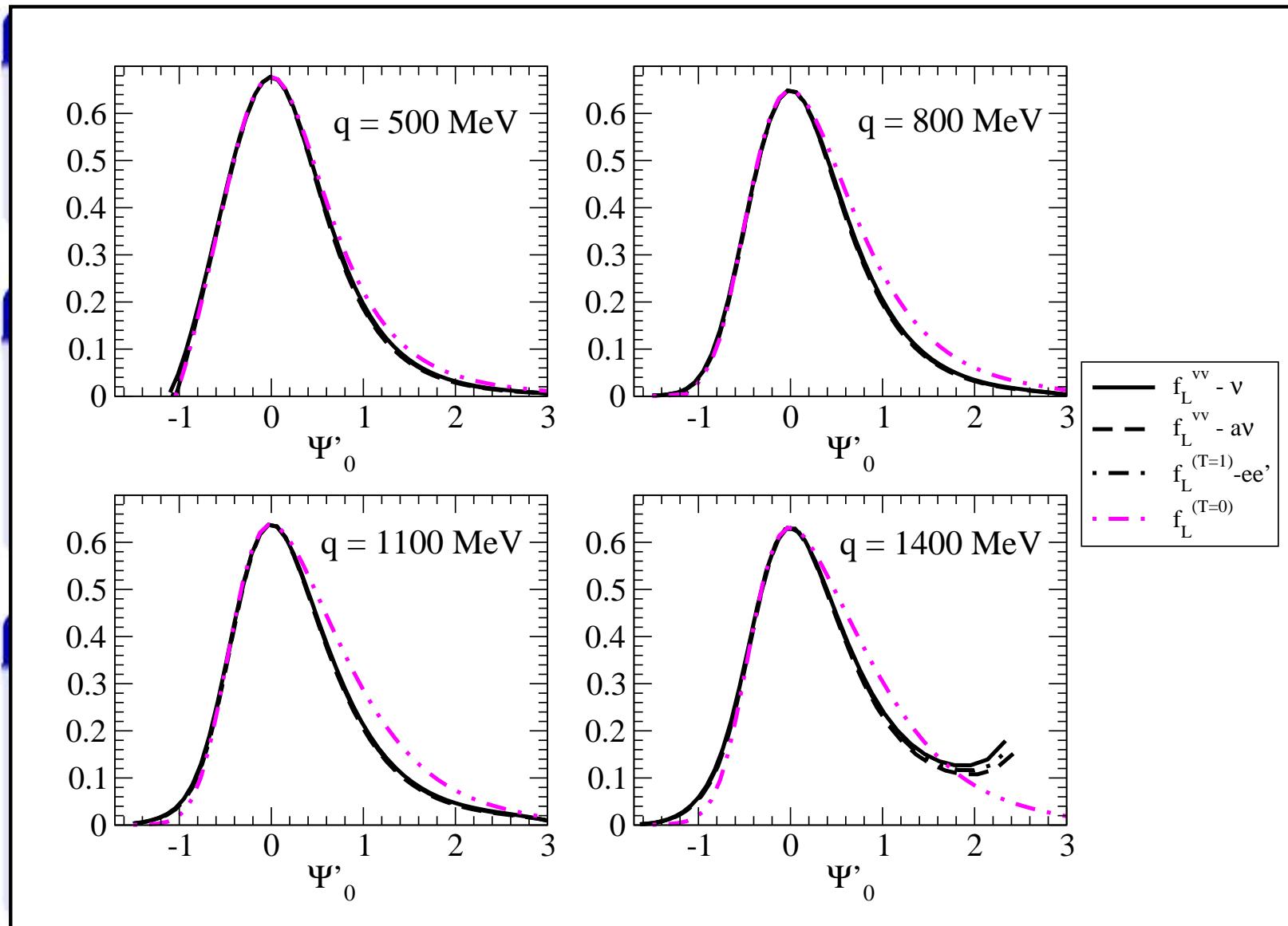
Differences between transverse and longitudinal scaling functions are introduced in order to describe properly the nuclear responses.

...

## Isovector structure

We separate the scaling function into isovector and isoscalar structure so as to employ a purely isovector scaling function for CCQE neutrino-nucleus processes where isospin changes.

# SuperScaling Approach based on RMF: SuSAv2

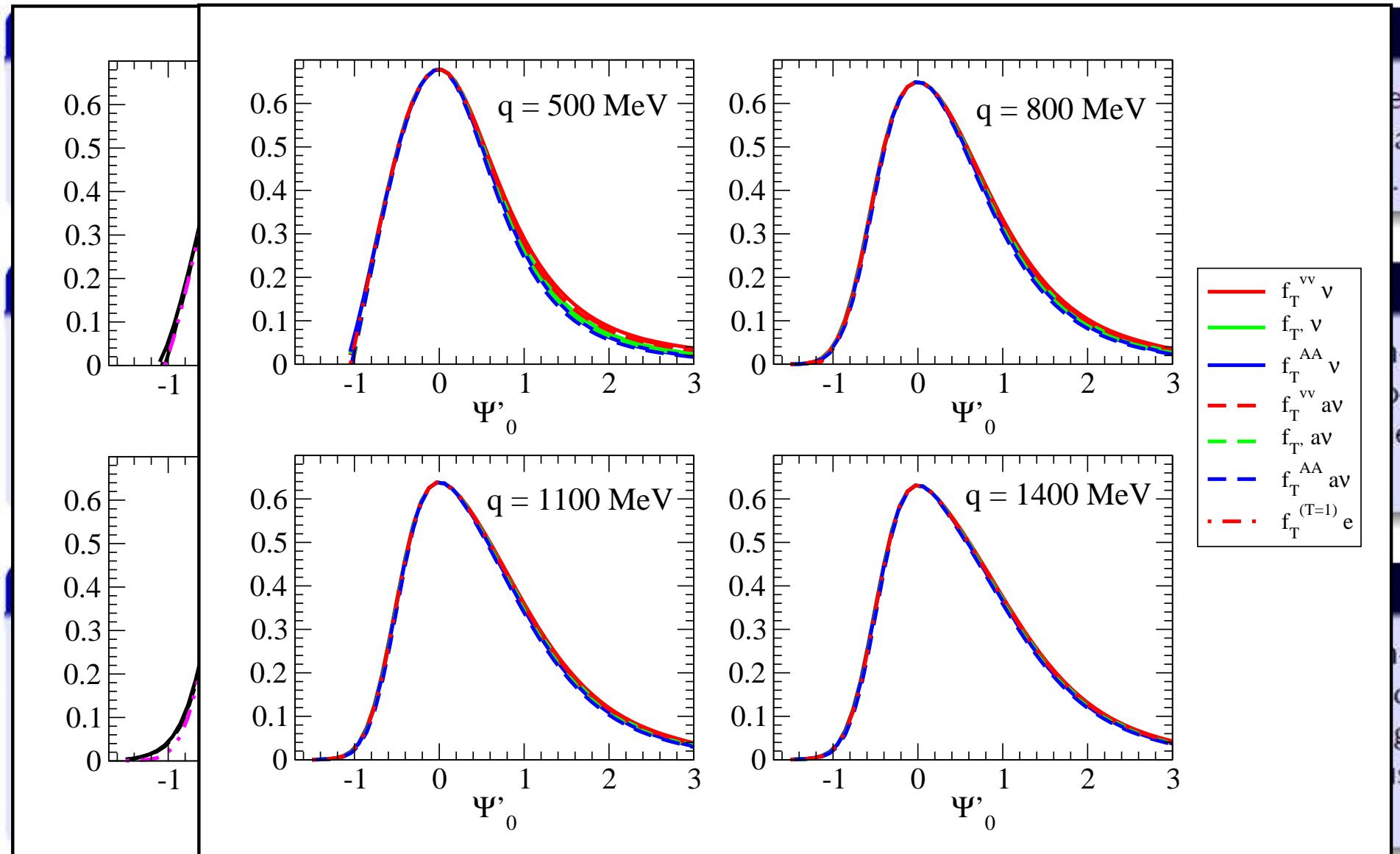


Field model  
prove the data  
counts for FSI.

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structure so  
vector scaling  
neutrino-nucleus  
changes.

# SuperScaling Approach based on RMF: SuSAv2

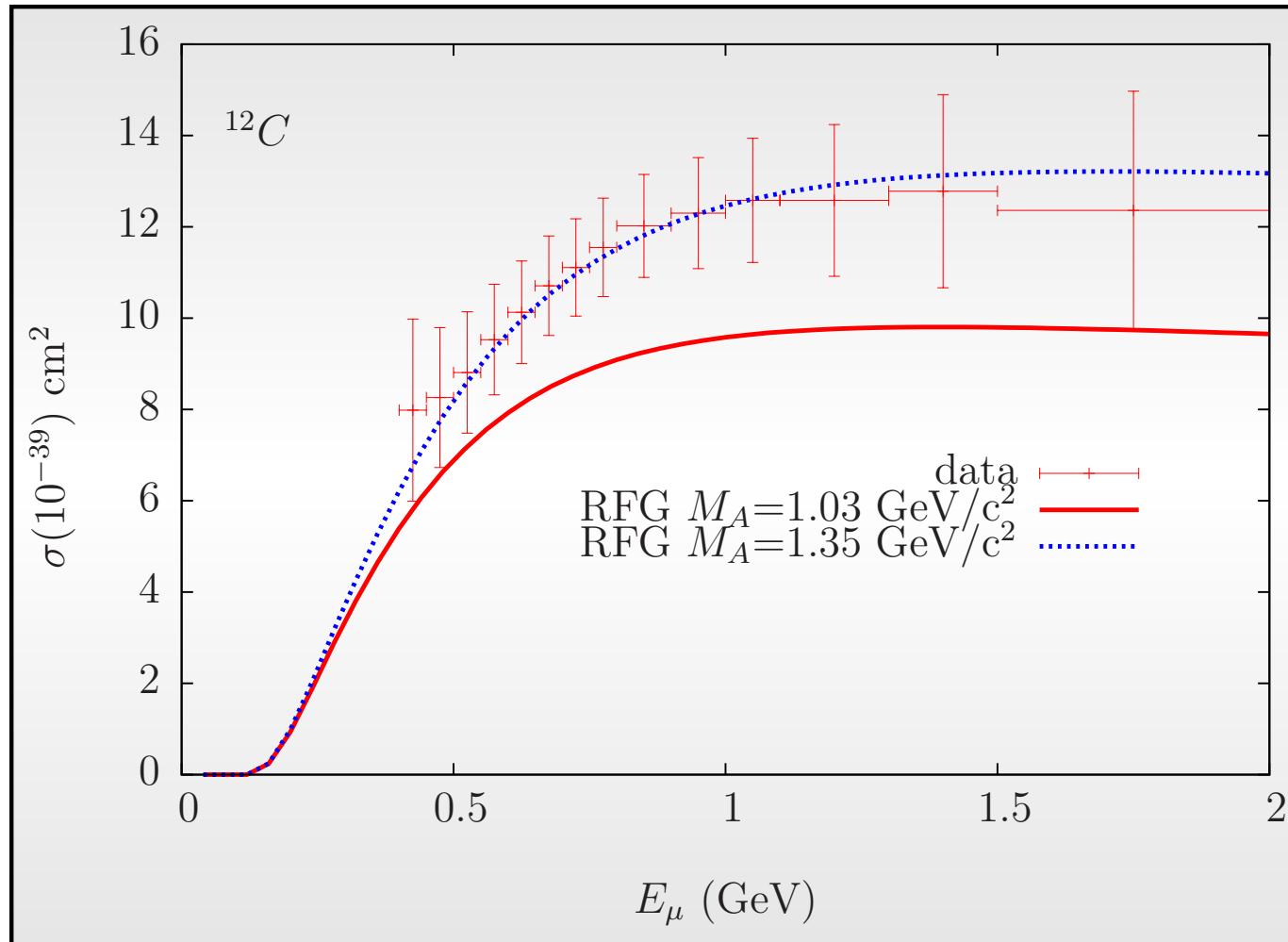


# **COMPARISON WITH DATA:**

## **MiniBooNE, Miner $\nu$ A & NOMAD**

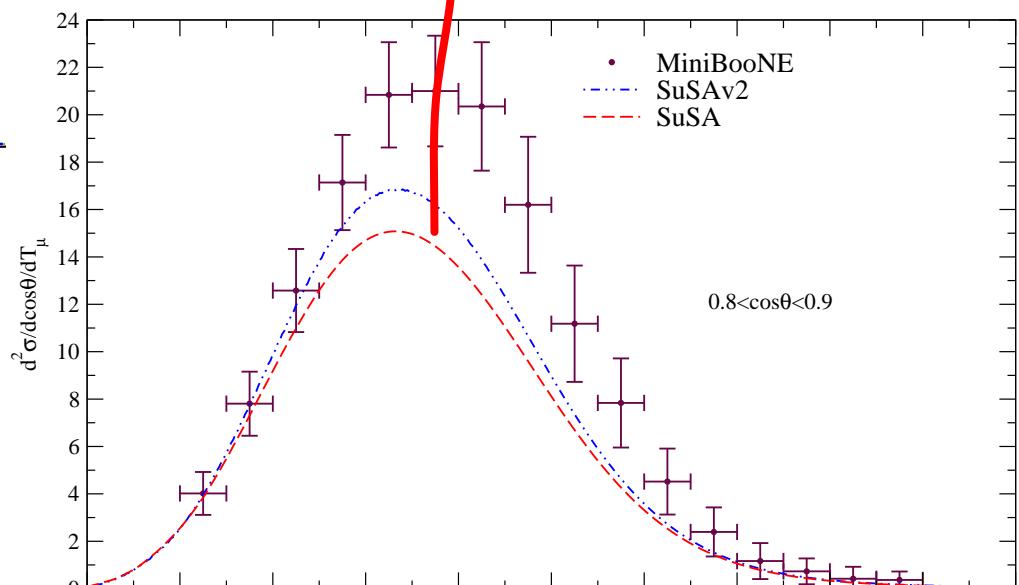
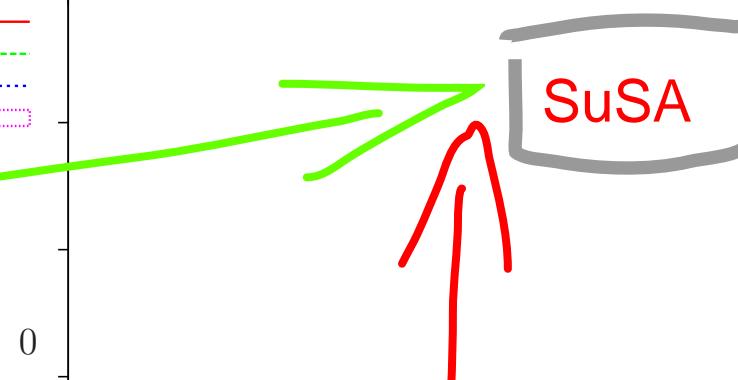
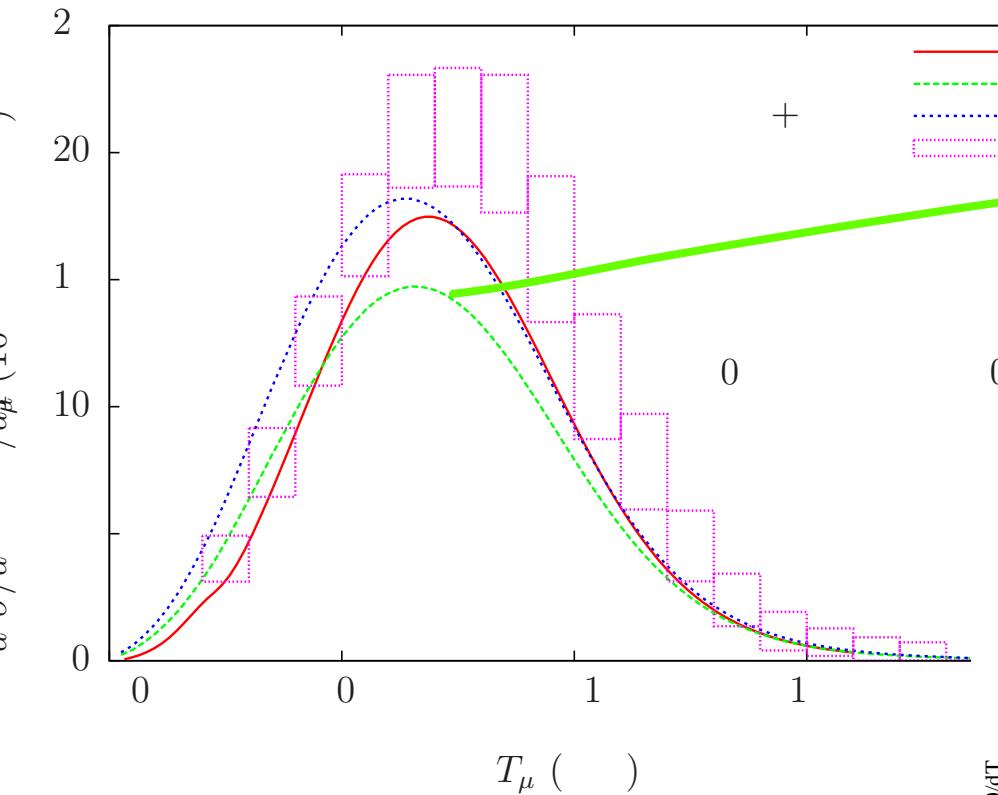
# MiniBooNE experiment (PRD 81, 092005 (2010))

## Flux-unfolded MiniBooNE $\nu_\mu$ CCQE cross sections/neutron



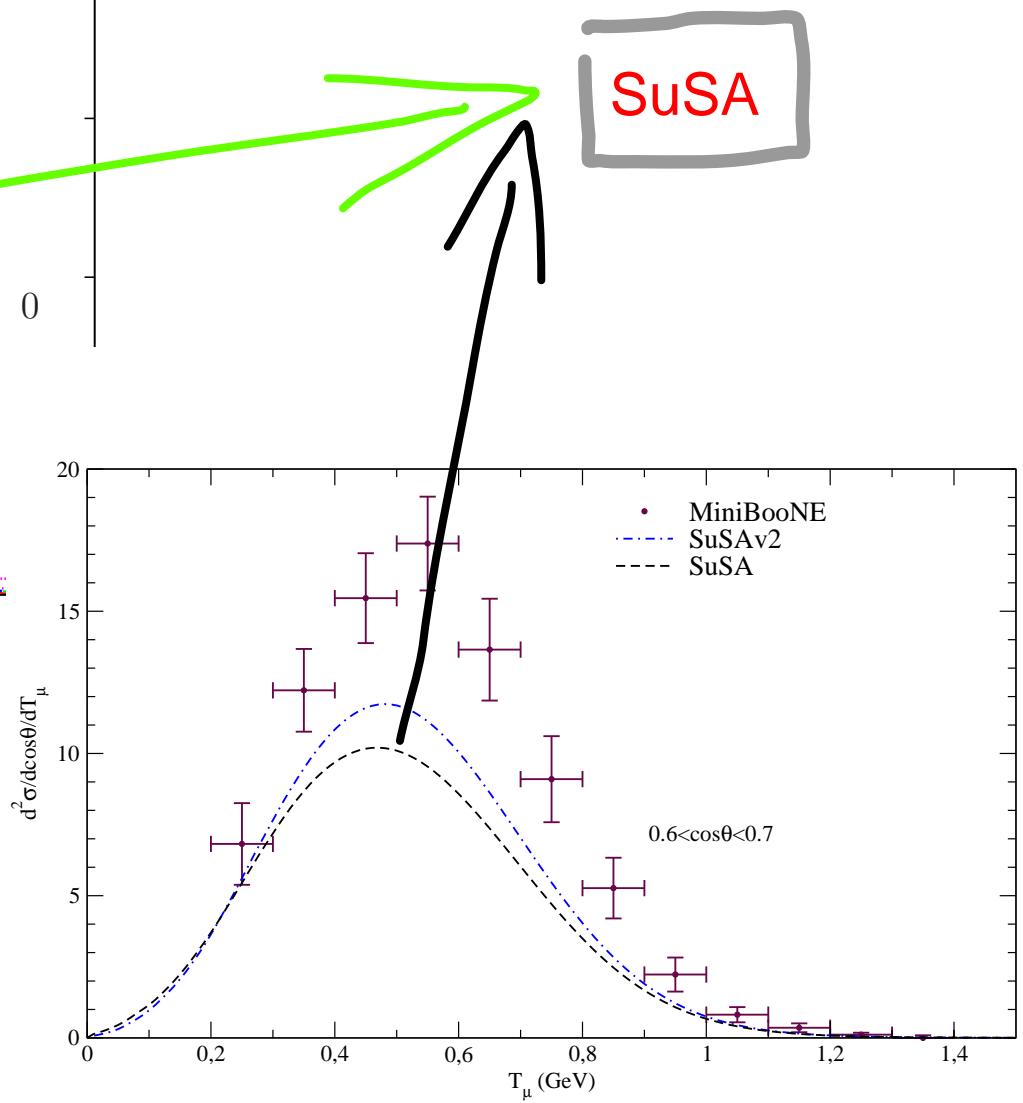
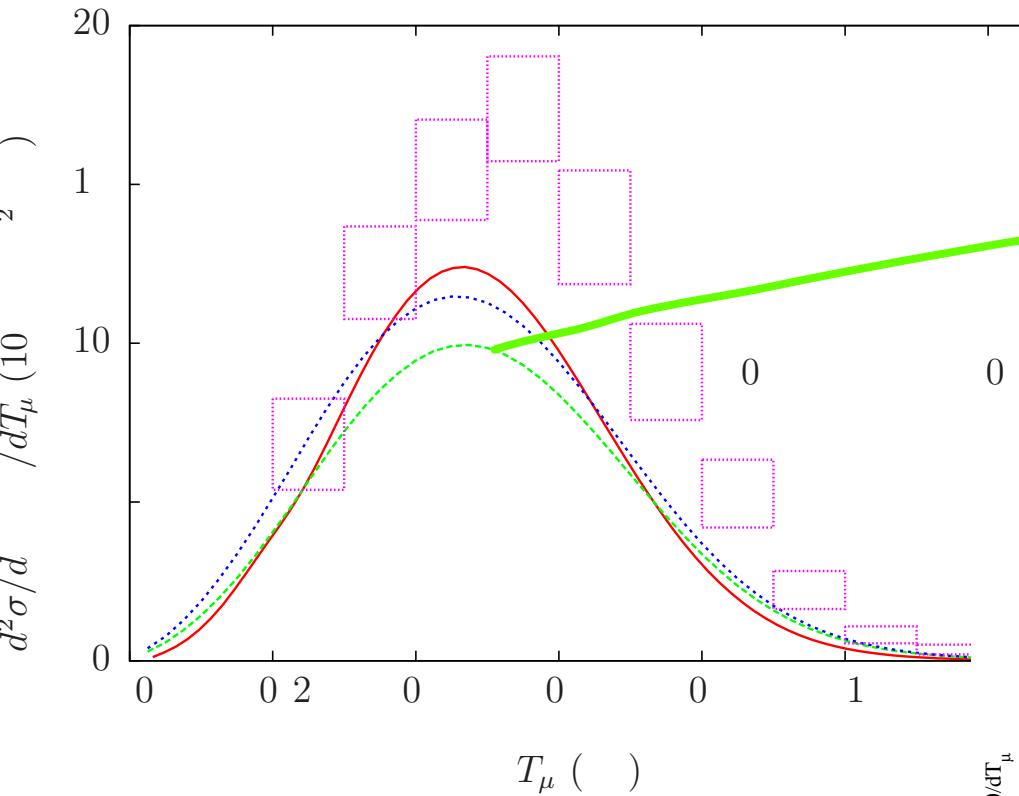
“Unexpected large value extracted for the axial mass  $M_A$ ?”

# Flux-averaged double-differential CCQE: SuSA, SuSAv2 & RMF

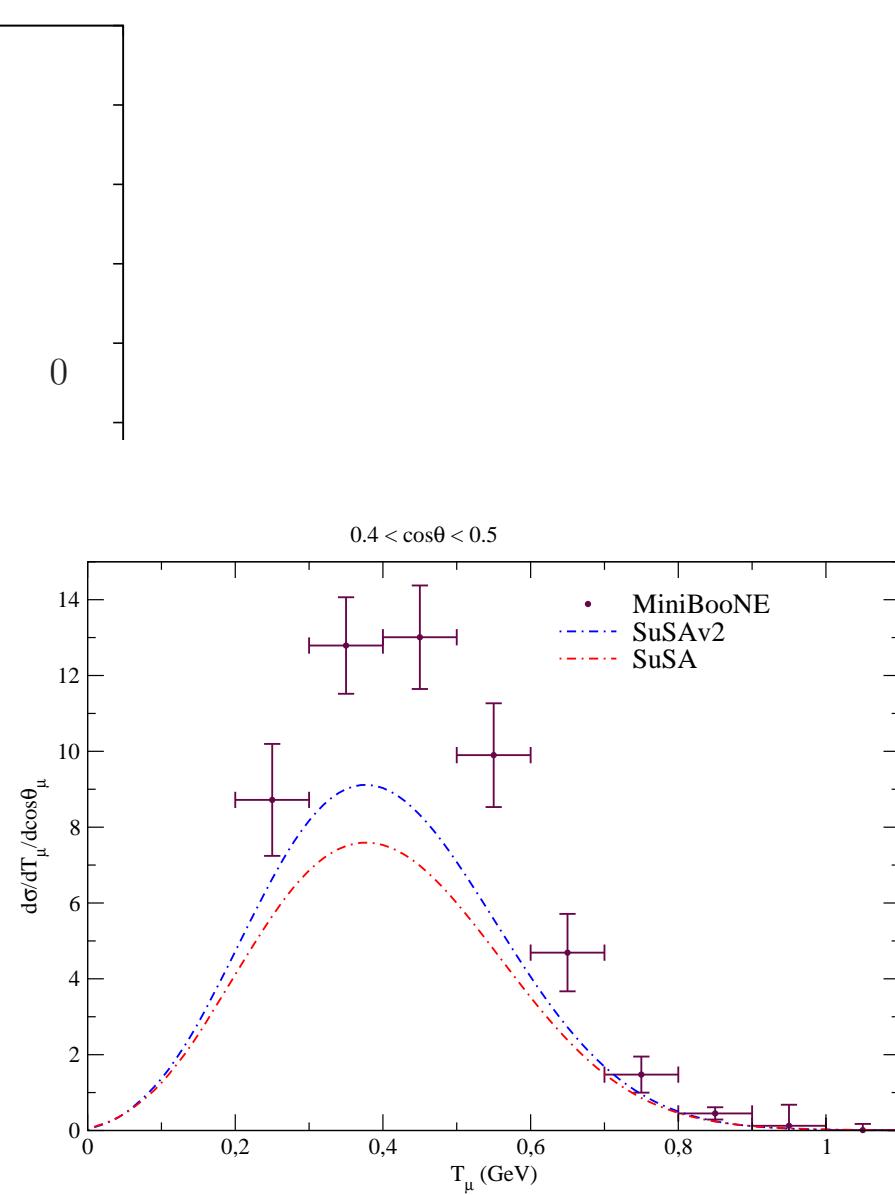
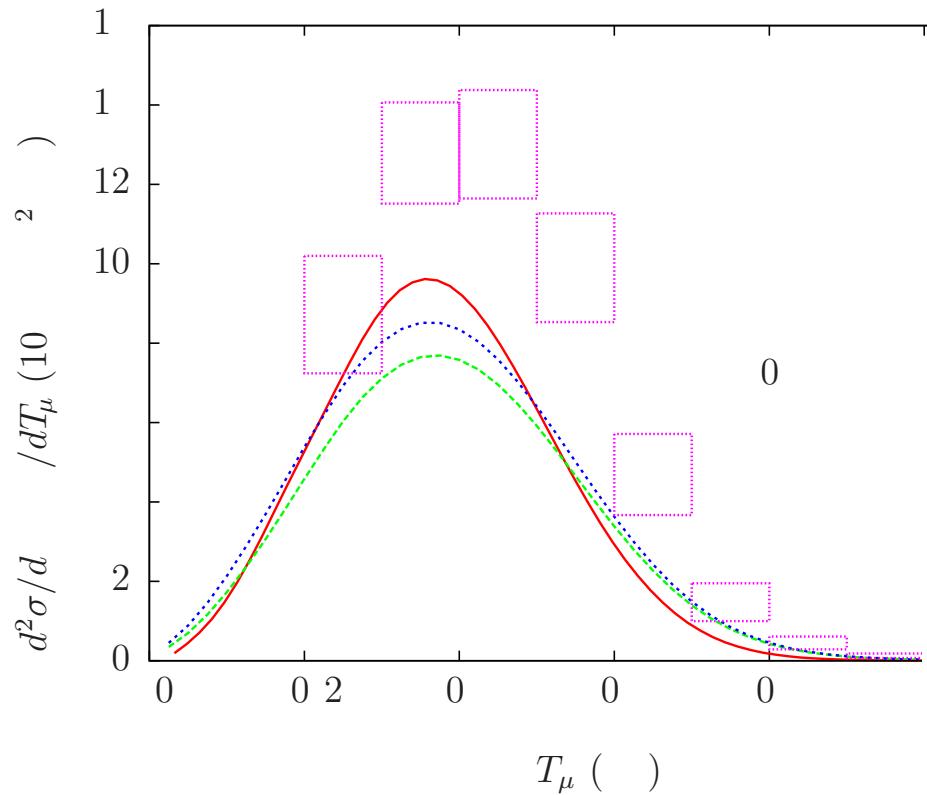


**SuSA (particularly, SuSAv2) and RMF agree, but underestimate data. 2p2h increase results but are still below data [Phys. Rev. D84 (2011) 033004]**

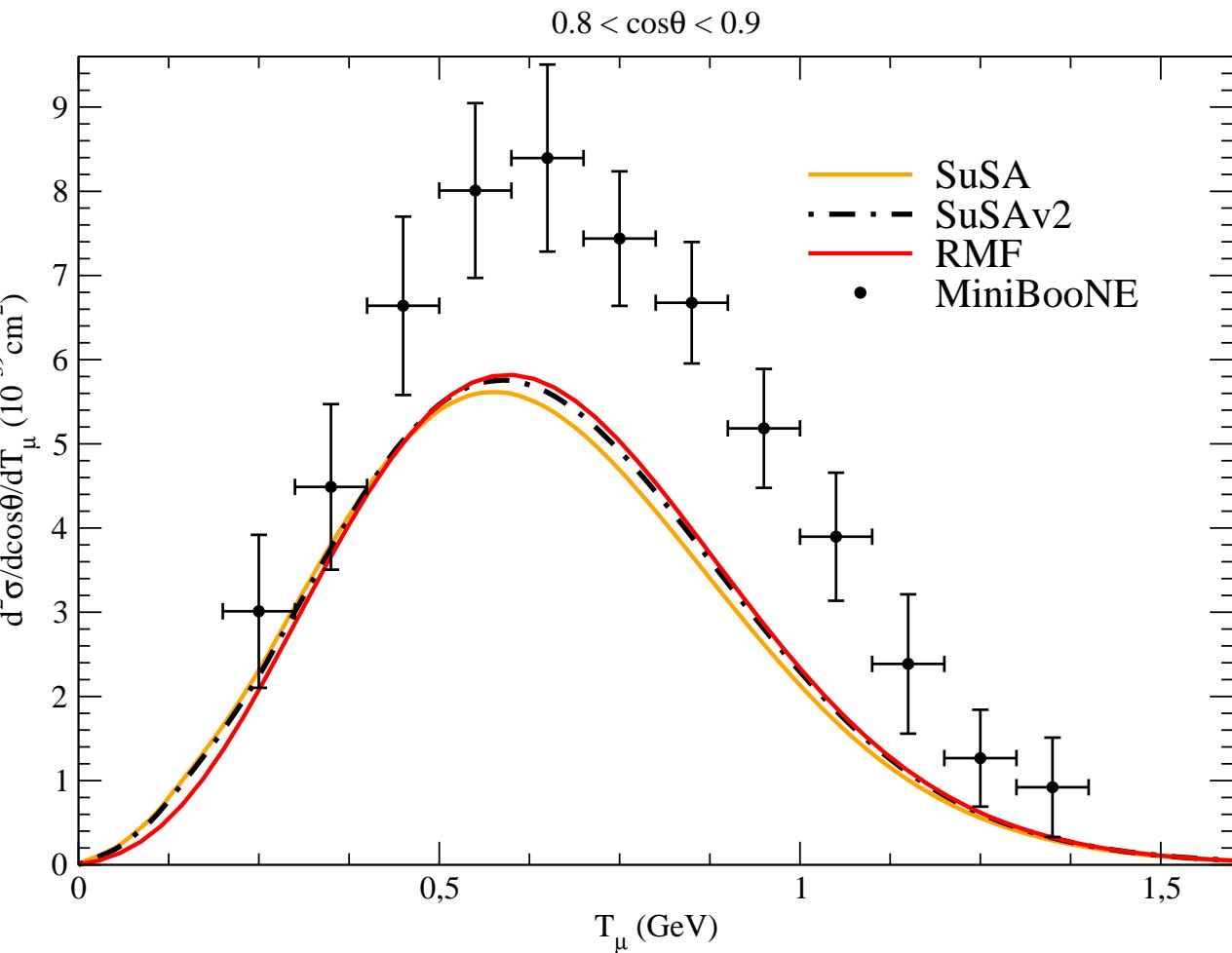
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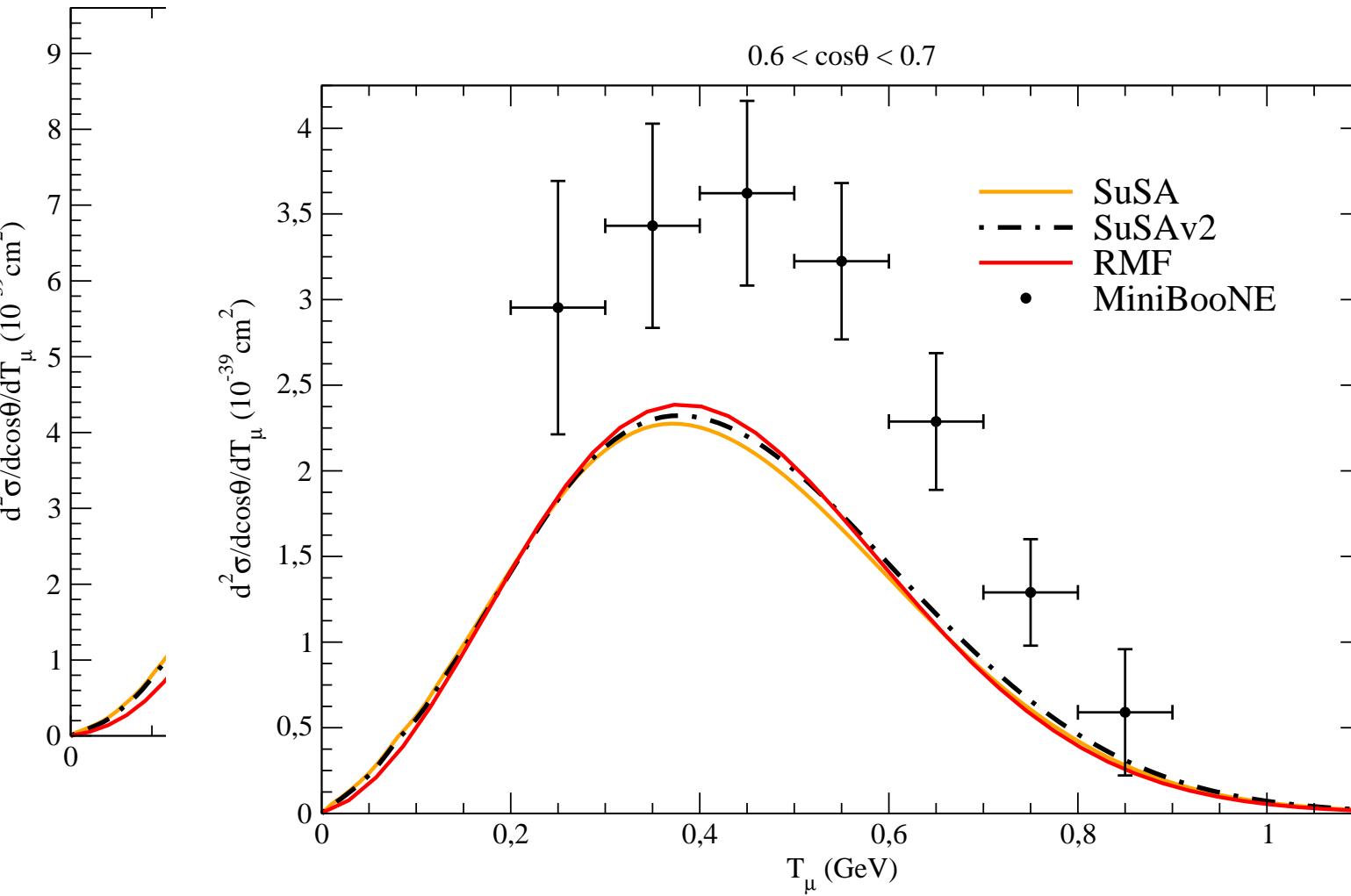
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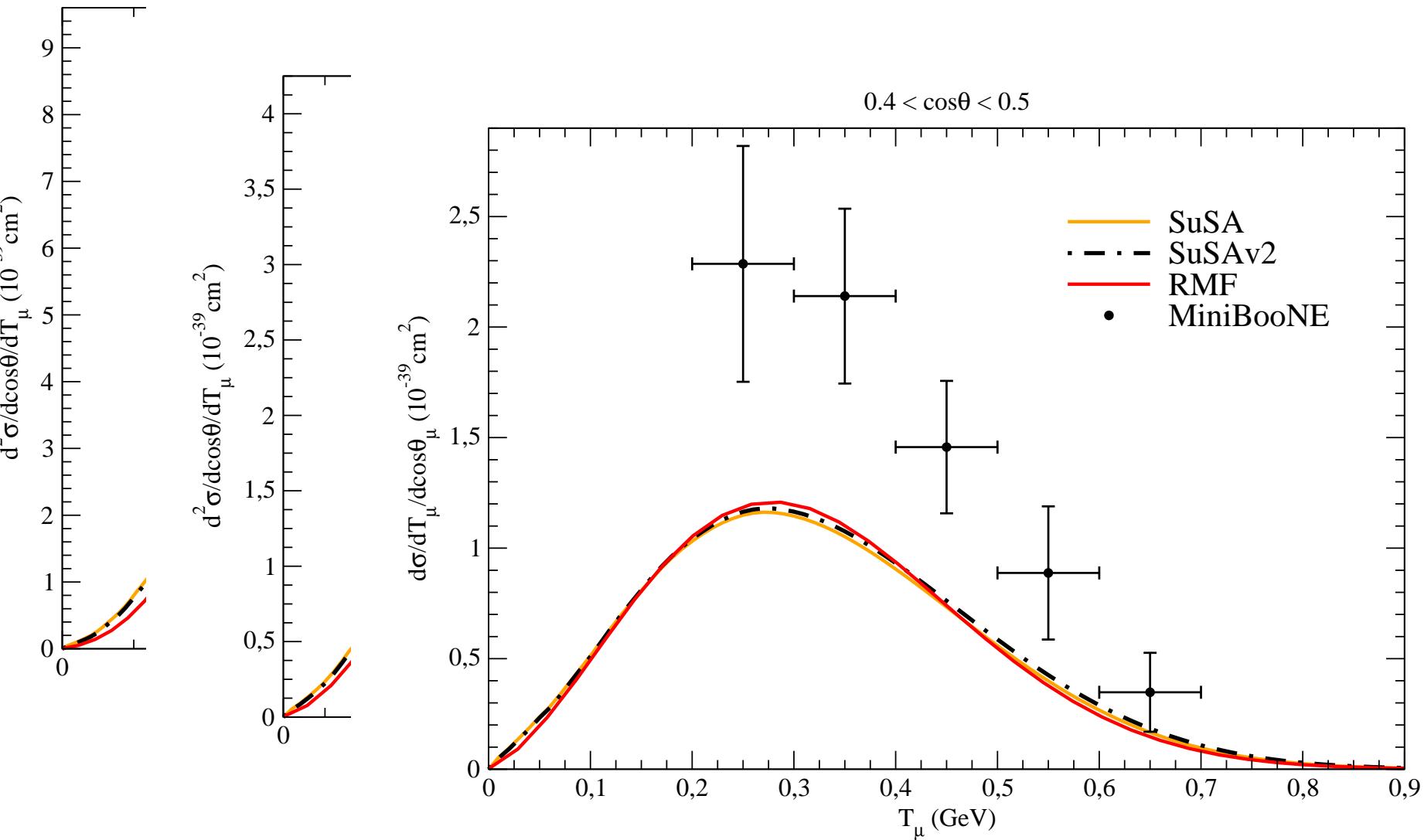
# Antineutrinos: strong enhancement of 2p2h effects?



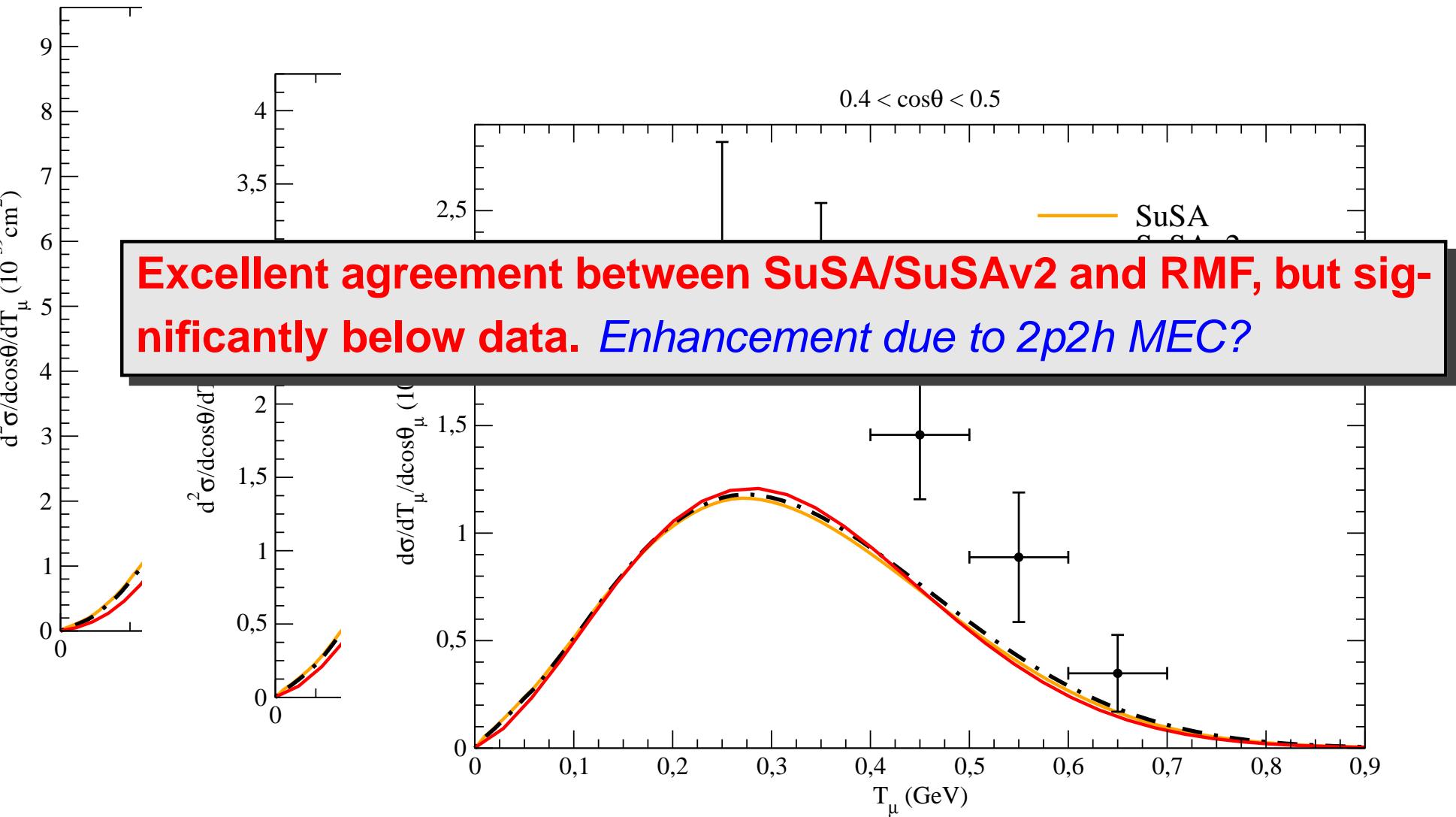
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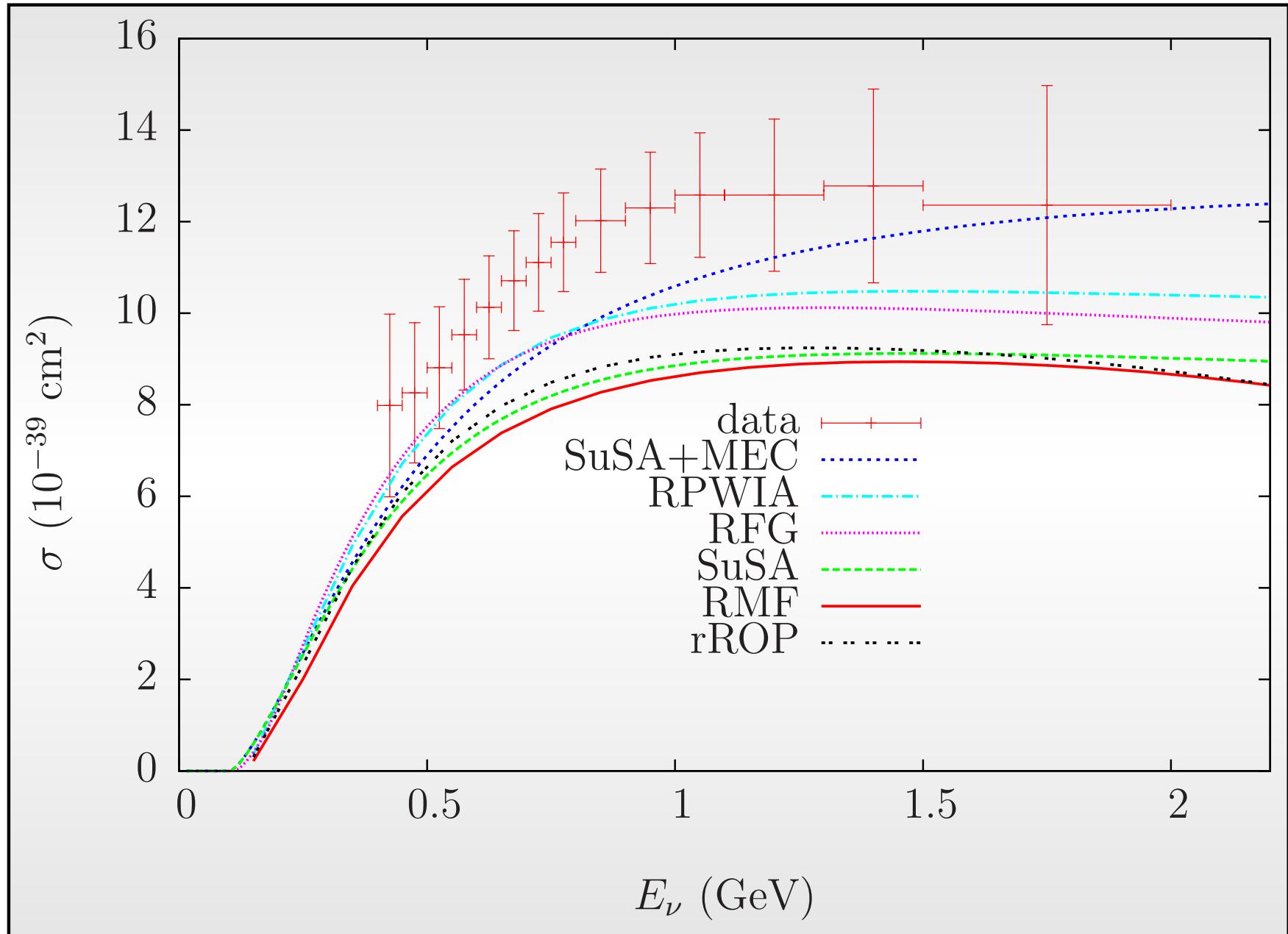
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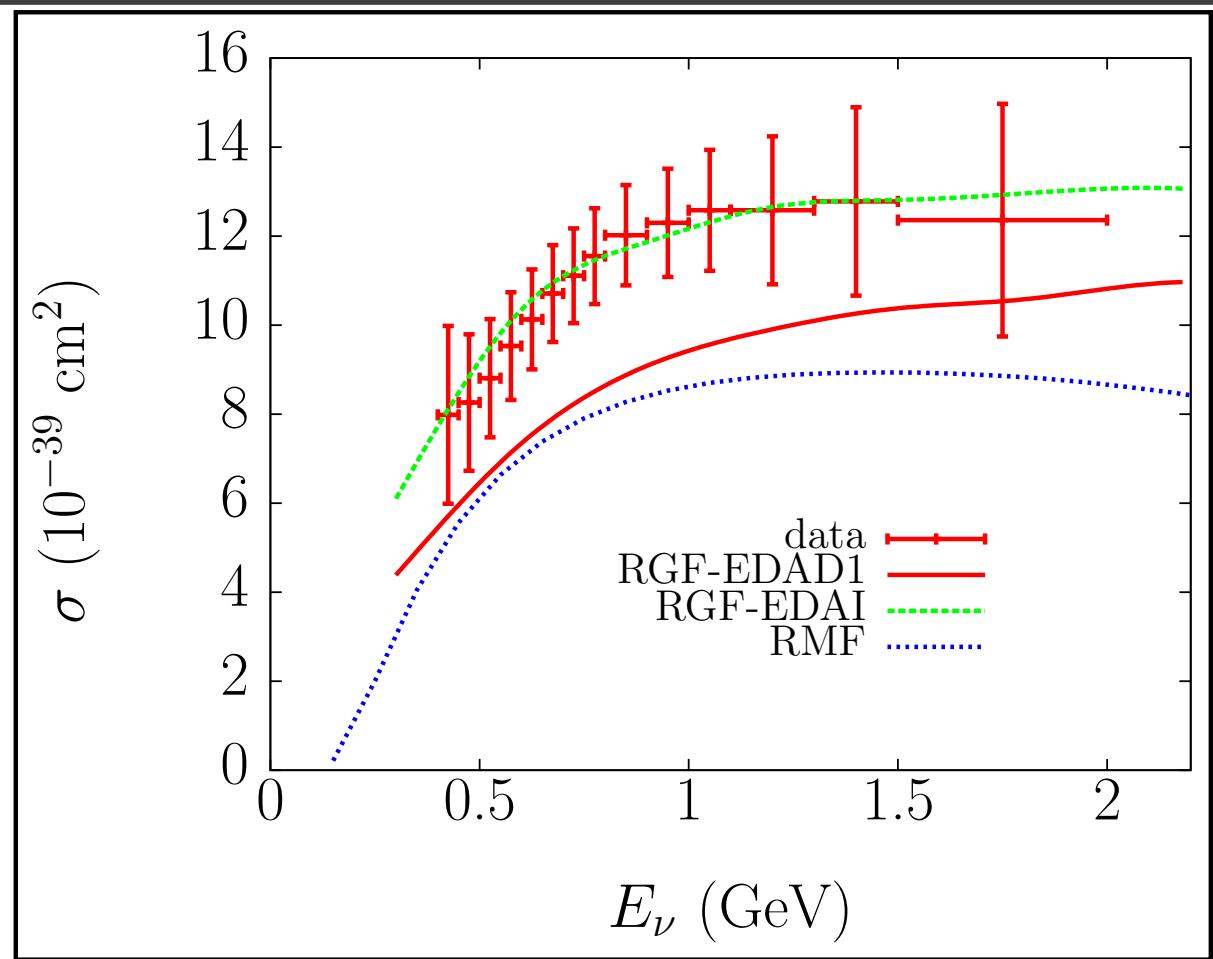
# Total integrated $\nu$ -CCQE cross section



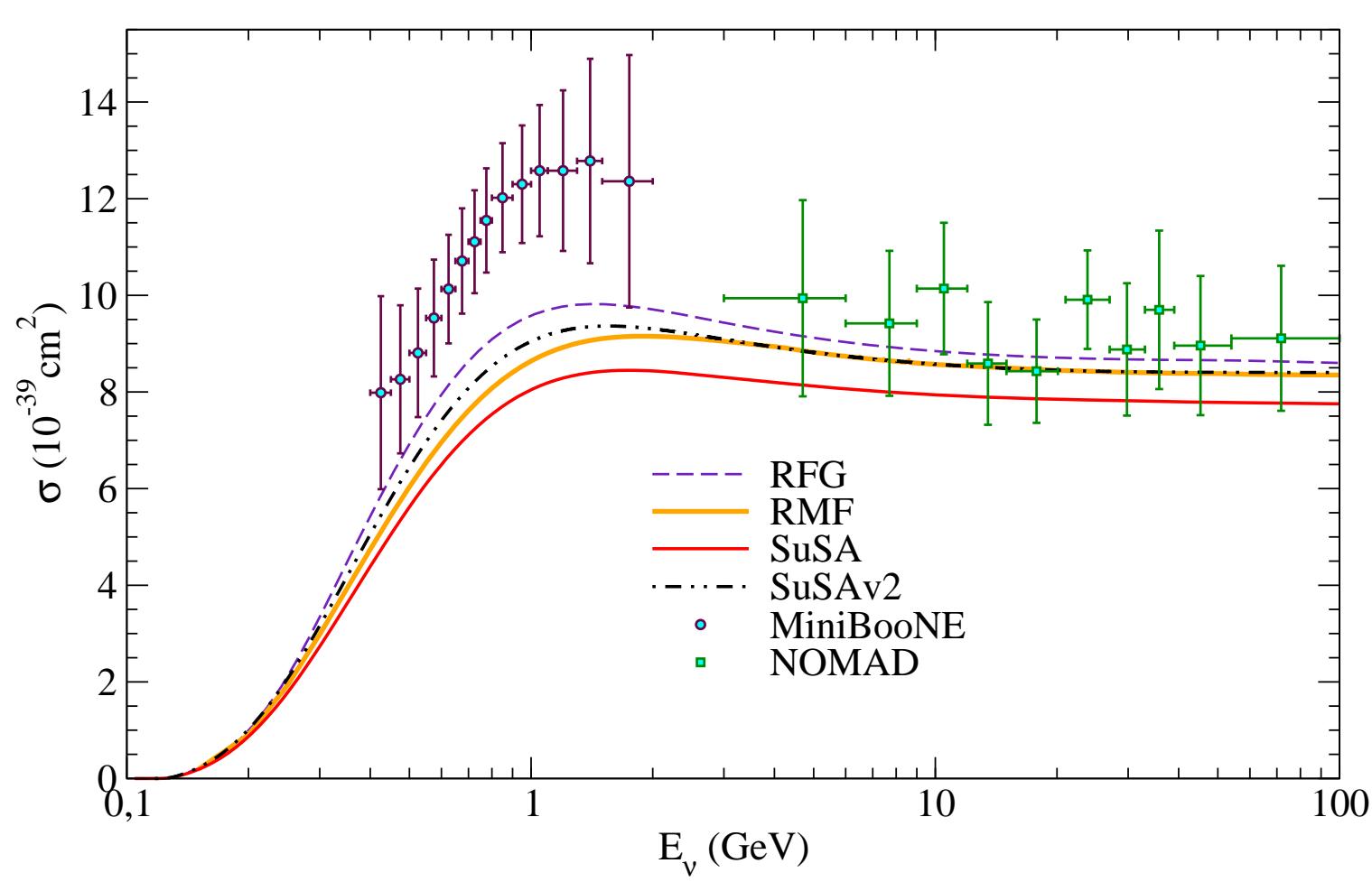
# Other Relativistic Approaches (Pavia Group)

Relativistic Green Function Approach (RGF): *FSI treated consistently in the exclusive & inclusive channels (same relativistic complex optical potential used in both cases, but flux conserved in the latter)*

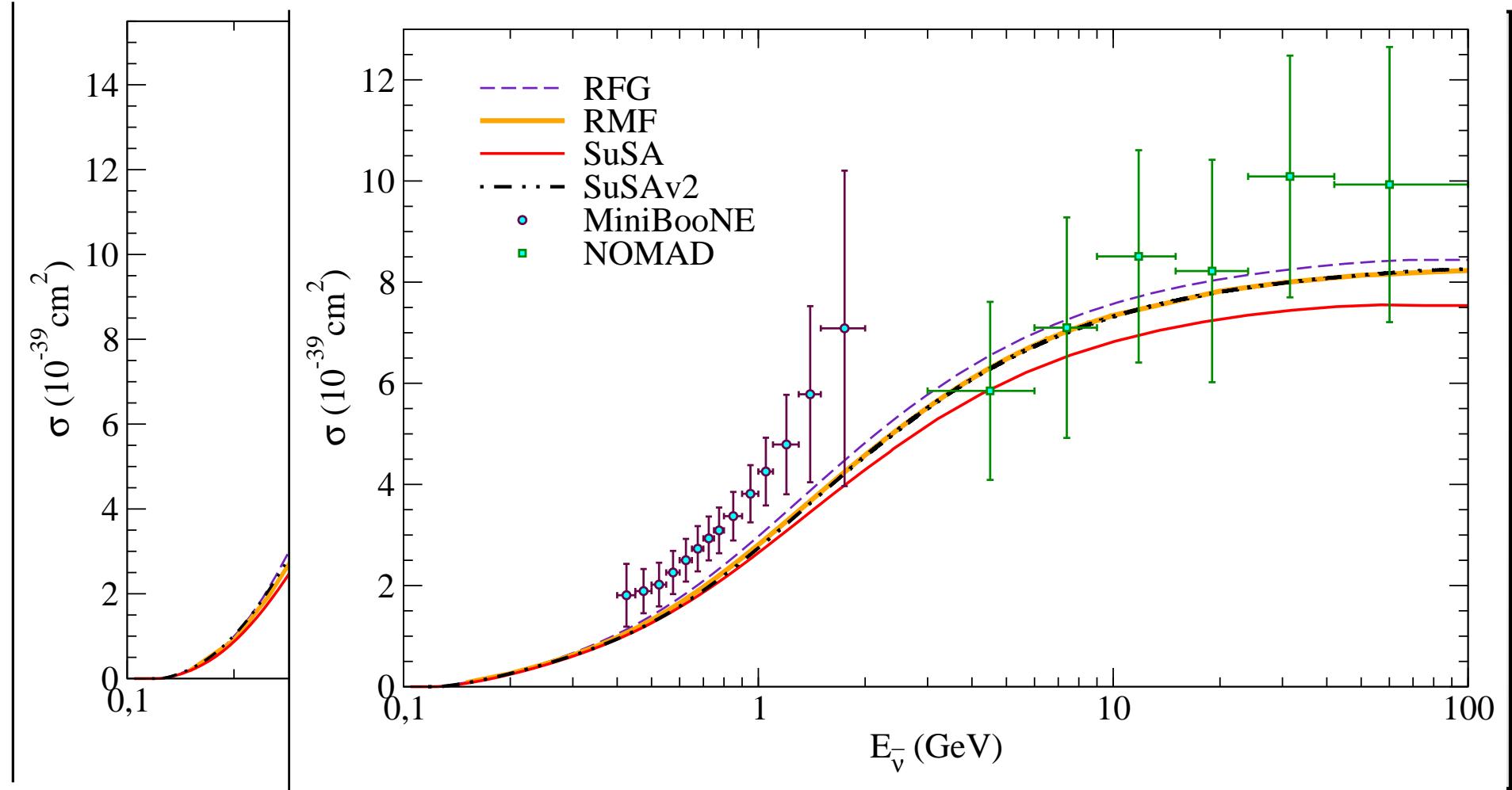
Phys. Rev. Lett. 107,  
172501 (2011)



# NOMAD: high-energy. SuSA vs RMF

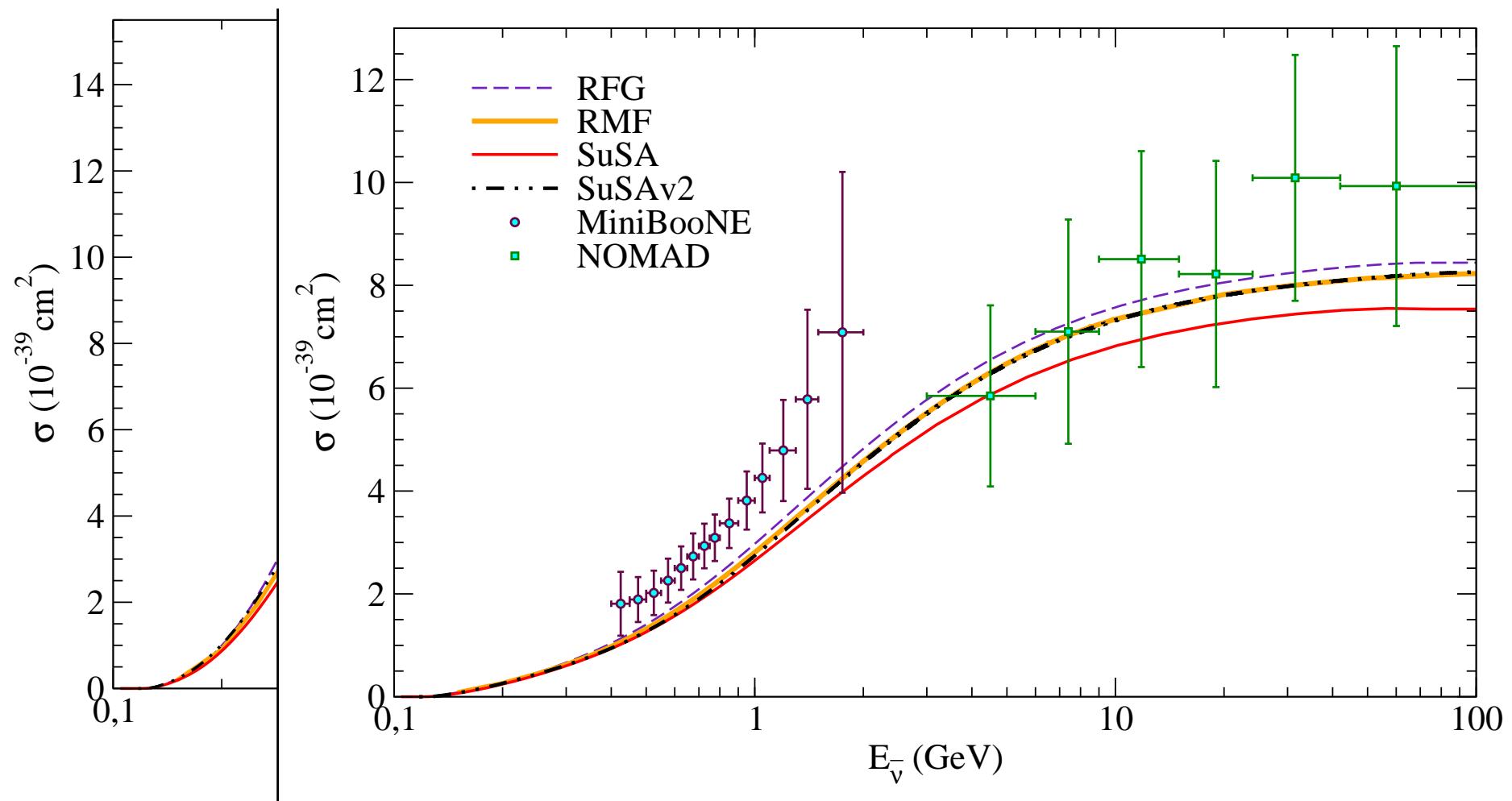


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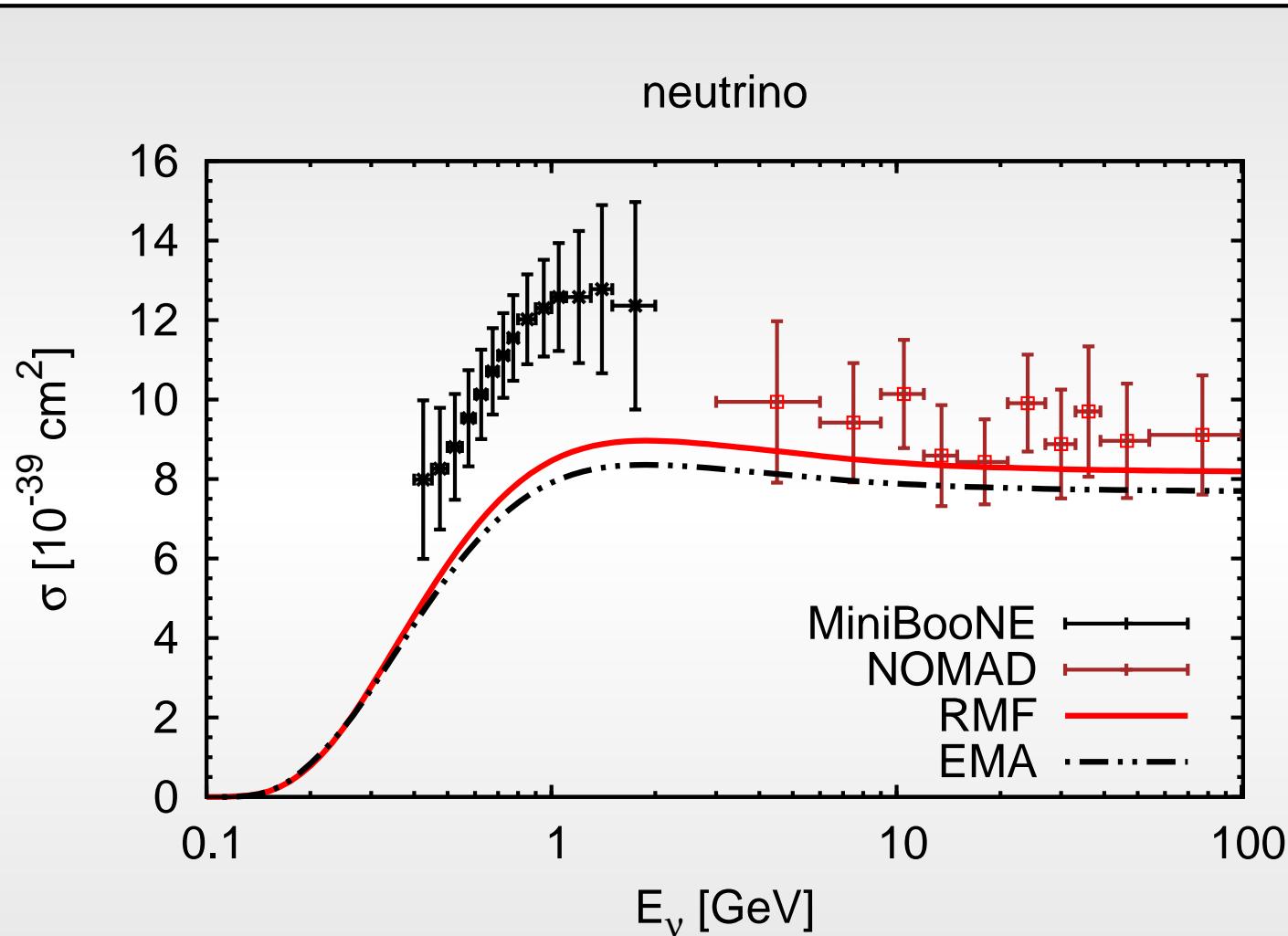


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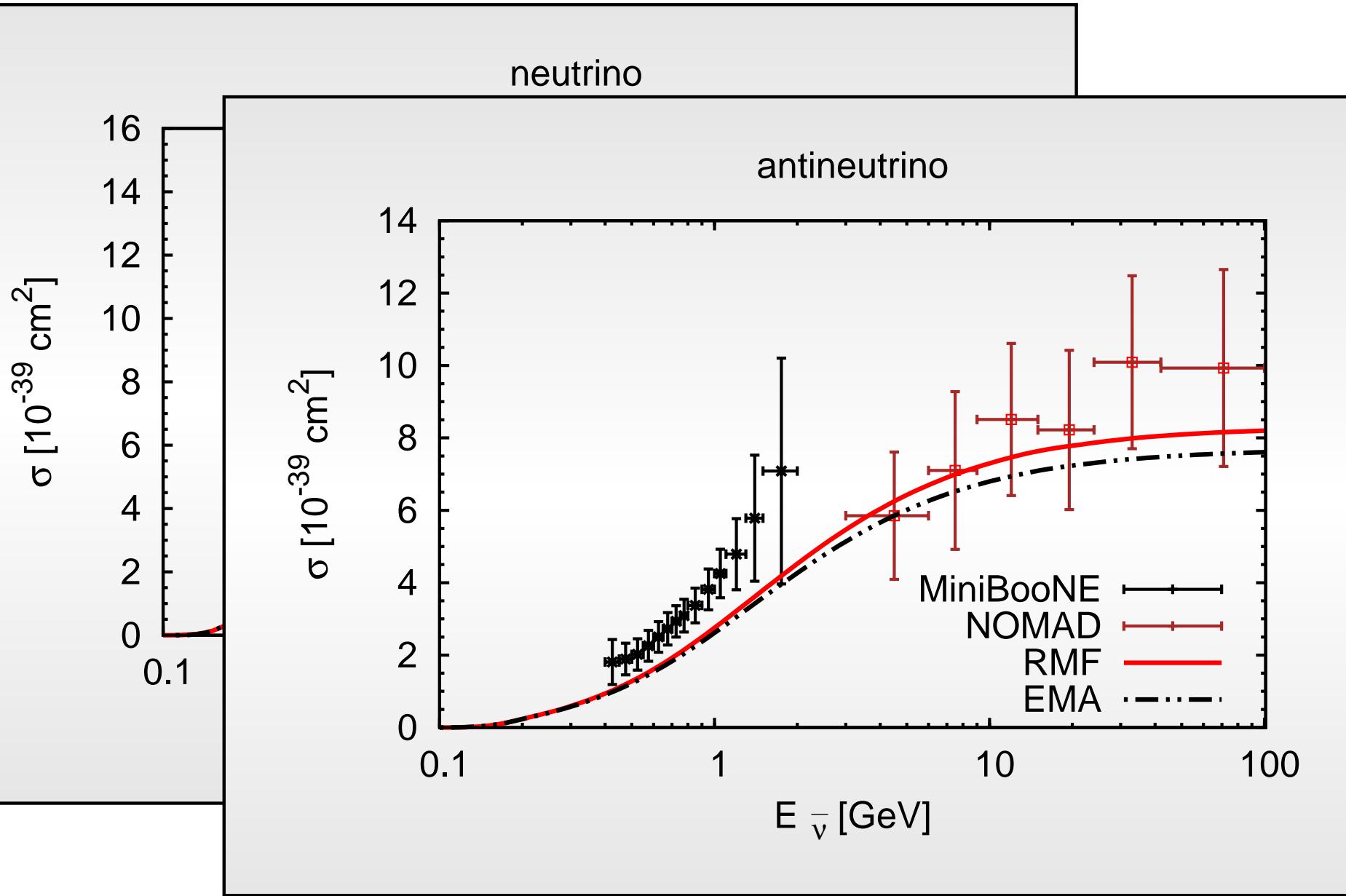
Excellent agreement between SuSA/SuSAv2 and RMF for all energies. Both models reproduce NOMAD but underpredict MiniBooNE (standard value of the axial mass). [PLB 725 (2013) 170]



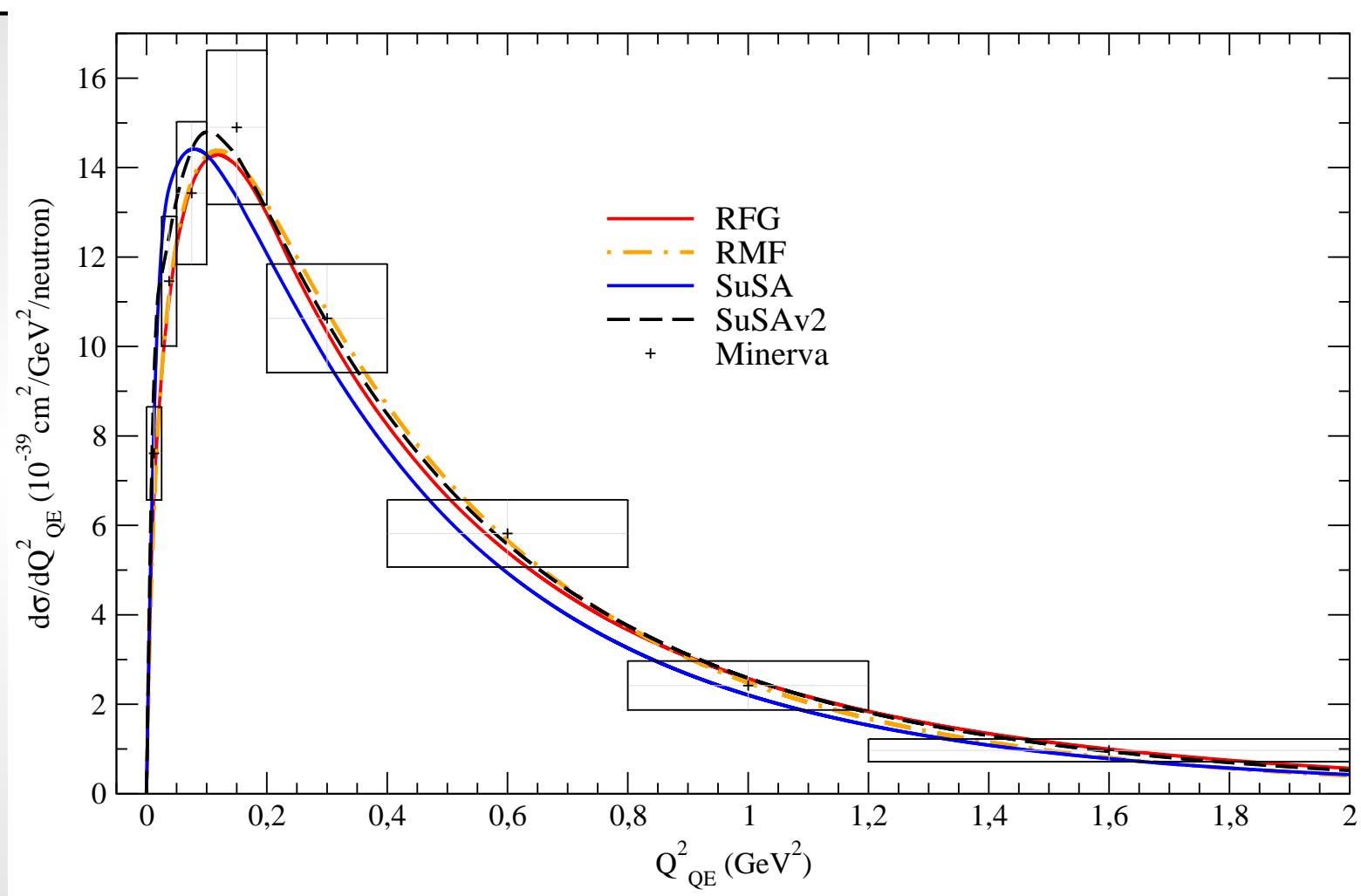
# NOMAD: spinor distortion in RMF



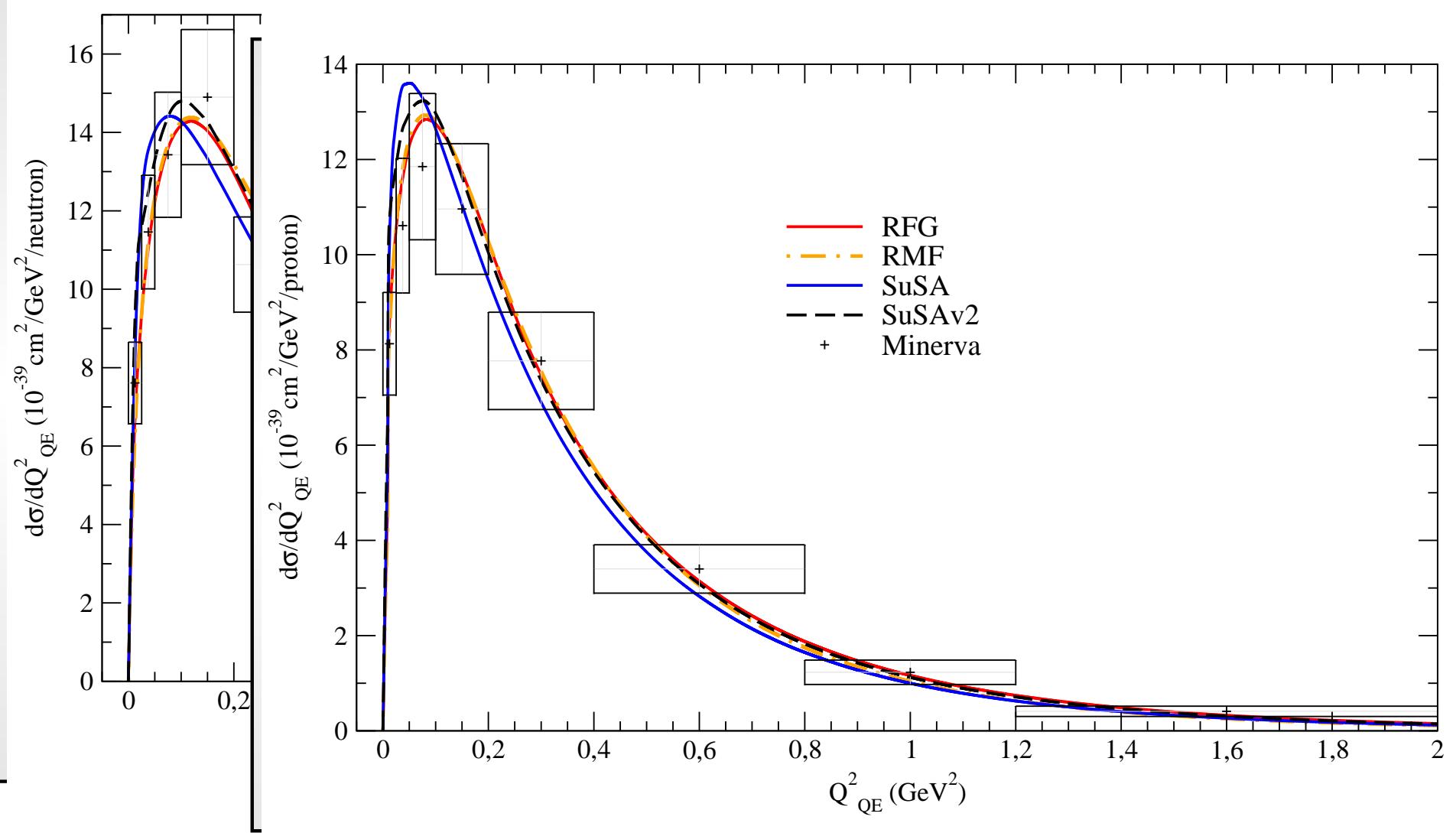
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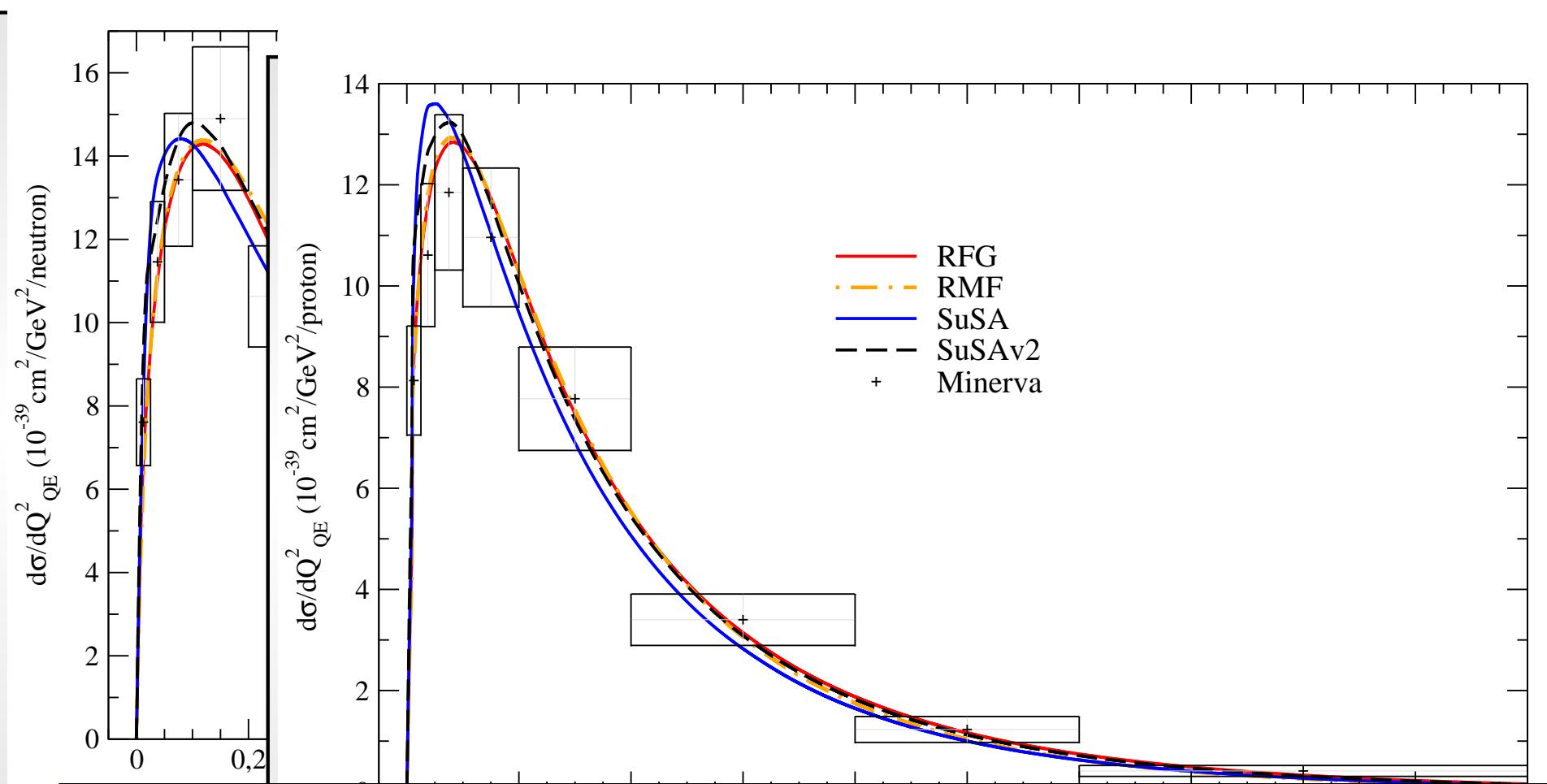
# MINER $\nu$ A: SuSA vs RMF



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# MINER $\nu$ A: SuSA vs RMF



*SuSA/SuSAv2 and RMF are able to reproduce in an excellent way the data without need to increase the value of the axial mass. Consistency with NOMAD but not with MiniBooNE.  
Are effects due to MEC, multinucleon... particularly significant at MiniBooNE kinematics?*

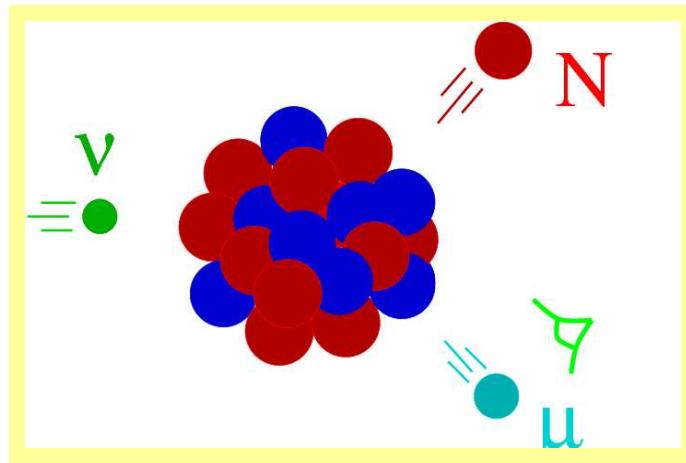
# APPLICATION TO NEUTRAL CURRENT NEUTRINO PROCESSES: $(\nu, N)\nu'$

# NC vs CC neutrino-nucleus QE scattering

The dominant processes in the QE region are assumed to be:

## Charged-Current

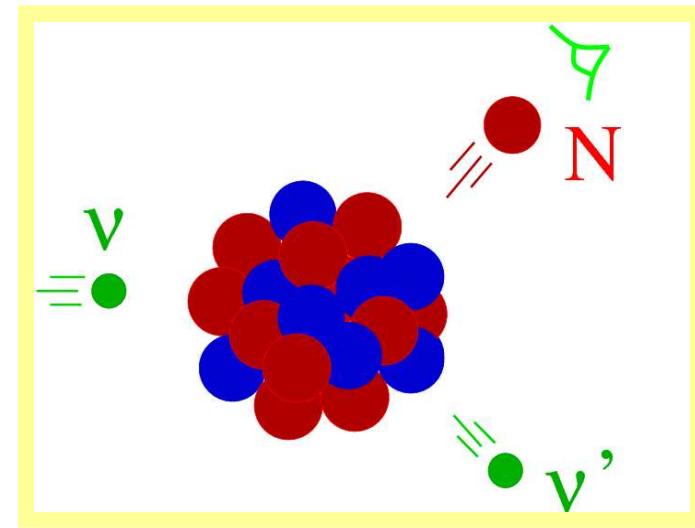
$$\nu + A \Rightarrow \mu + N + B$$



Outgoing lepton detected, fixed  $Q^2$   
as in  $(e, e')$ : t-channel kinematics

## Neutral-Current

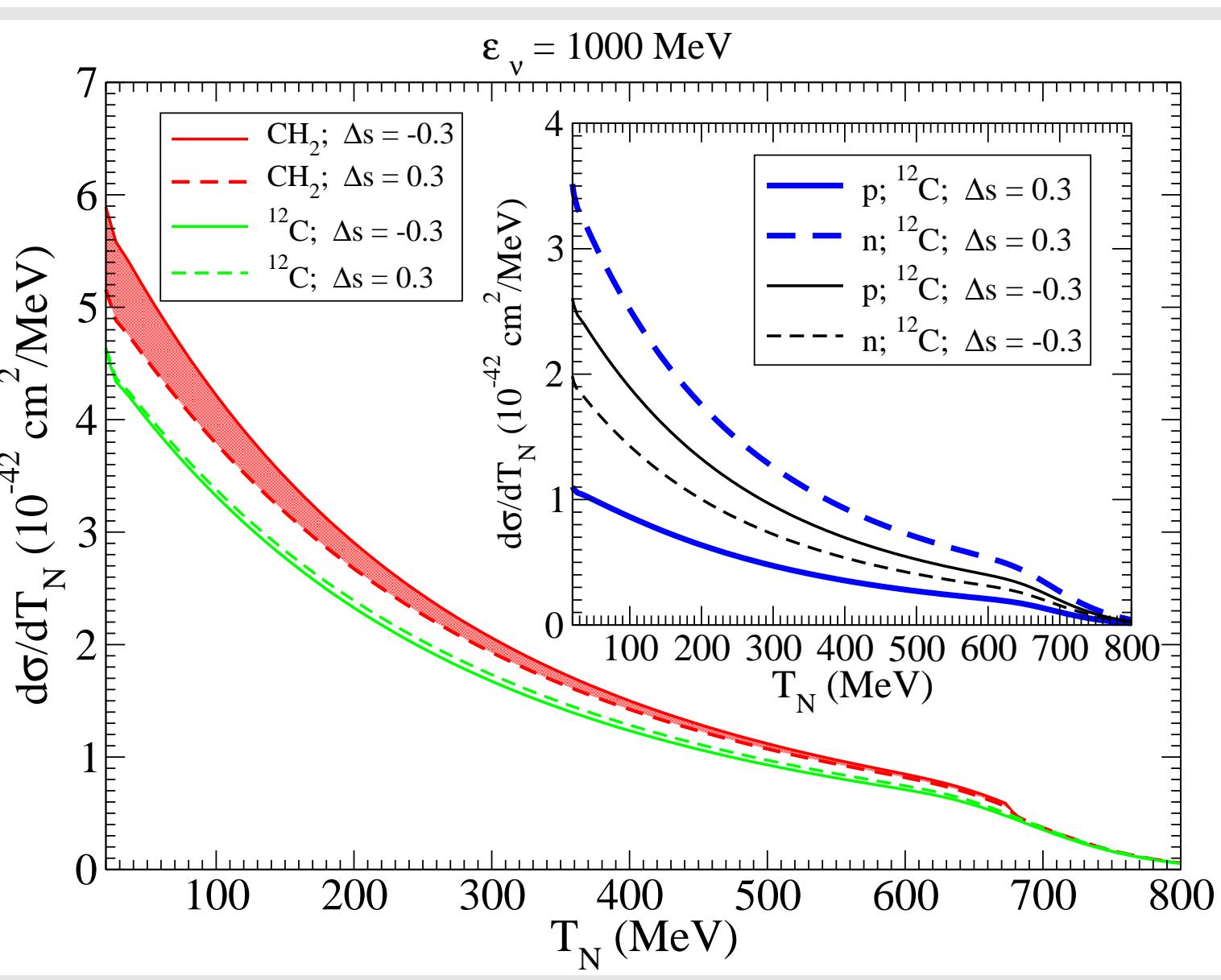
$$\nu + A \Rightarrow \nu' + N + B$$



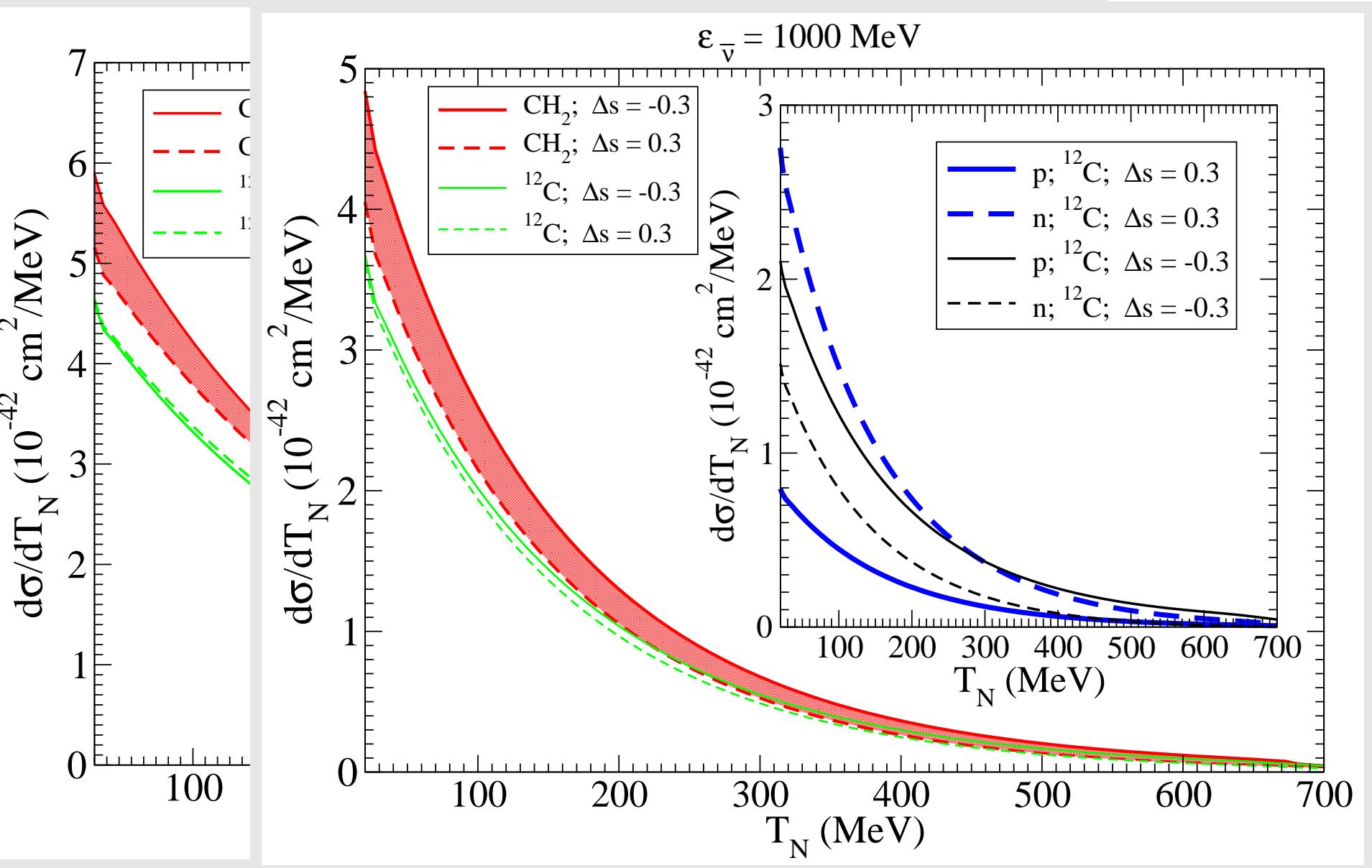
Only the nucleon is detected,  $Q^2$  is not  
fixed: u-channel kinematics

Very different kinematics in both processes. Do they reveal different sensitivity to the nuclear dynamics underlying scaling?

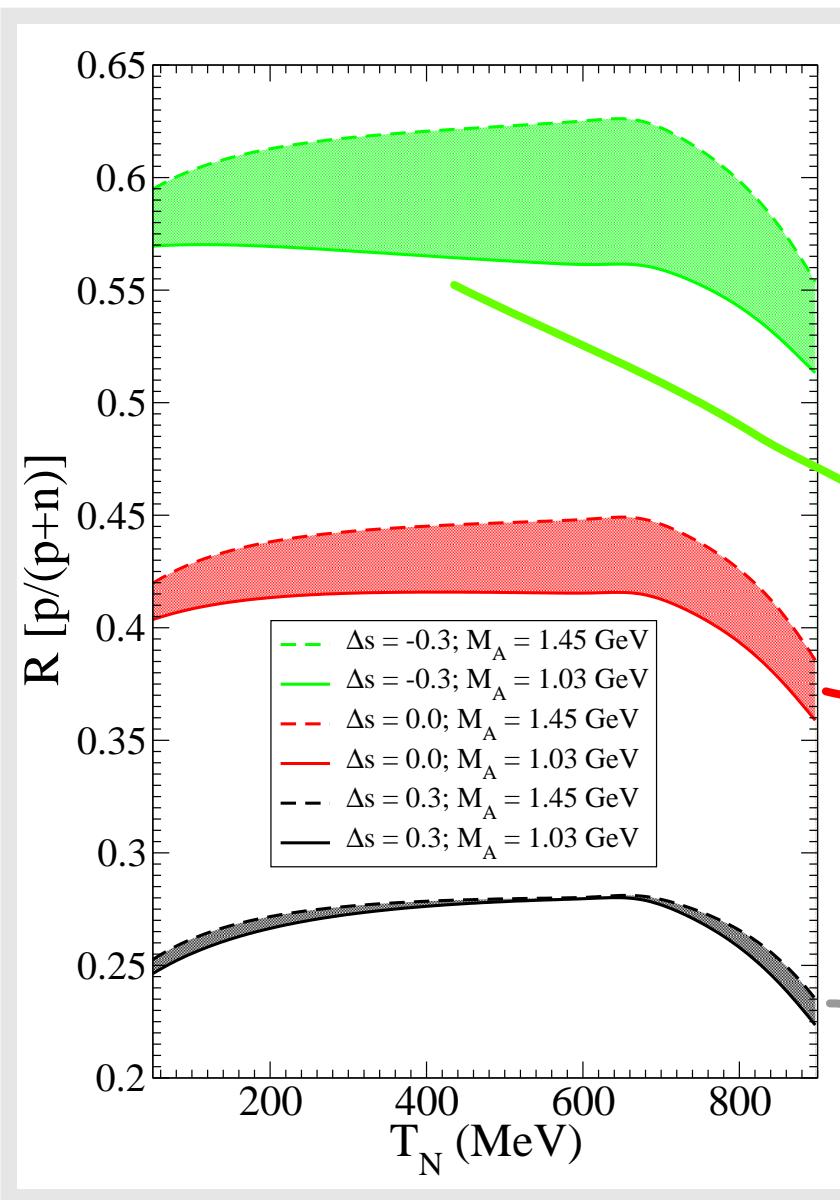
# NCQE & axial strangeness



# NCQE & axial strangeness



# P/N RATIO: axial strangeness vs axial mass



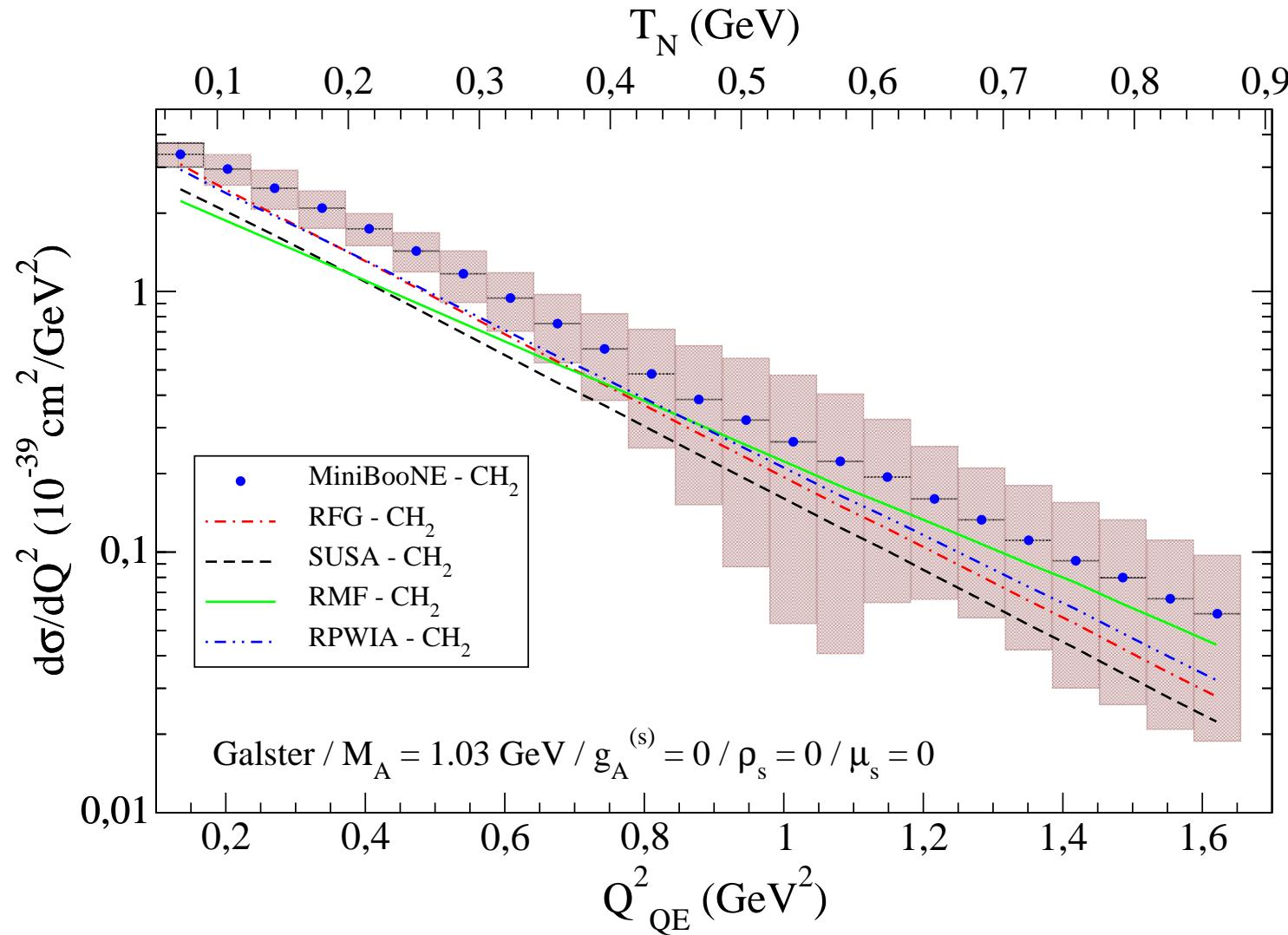
Correlation between axial strangeness & axial mass: for  $\Delta s$ -positive (grey band) NCQE is almost insensitive to  $M_A$ , whereas for  $\Delta s$ -negative (green) uncertainty due to  $M_A$  is  $\sim 10 - 12\%$ .

Negat. Strang.

Zero Strang.

Posit. Strang.

# Comparison with MiniBooNE data



# SUMMARY

- *The RIA/RMF describes in a reasonable way QE ( $e, e'$ ) data, satisfying scaling behavior and providing an asymmetric superscaling  $L$  function in accordance with data.*
- *Contrary to most NR/SR models (likewise RFG), RMF violates scaling of zeroth order, i.e.,  $f_T > f_L$ . This seems to be consistent with ( $e, e'$ ) data analysis.*
- *RMF applied to neutrino scattering also satisfies scaling/superscaling properties.*
- *RMF provides results in excellent agreement with SuSA/SuSAv2 approaches.*
- *Significant discrepancy with MiniBooNE data: **Important enhancement of 2p-2h effects?***
- *RMF results (likewise SuSA/SuSAv2) in accordance with NOMAD and MinerVA data (without need to increase the axial mass).*
- *Correlation between axial strangeness and axial mass in NC processes. Some discrepancy with MiniBooNE data: **role of 2p-2h?, strangeness? ...***

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