Consistent treatment of one- and two-nucleon currents within the spectral function formalism

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Preamble: the lepton-nucleus x-section

★ Double differential cross section

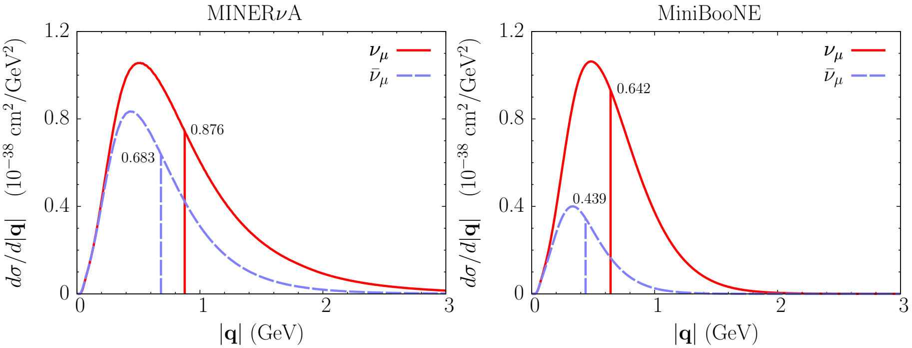
$$\frac{d\sigma_A}{d\Omega_{k'} dk'_0} \propto L_{\mu\nu} W_A^{\mu\nu}$$

- $L_{\mu\nu}$ is fully specified by the lepton kinematical variables (warning: not all of them are known in the case of ν scattering)
- The determination of the target response tensor

$$W_A^{\mu\nu} = \sum_N \langle 0 | J_A^{\mu\,\dagger} | N \rangle \langle N | J_A^\nu | 0 \rangle \delta^{(4)}(P_0 + k - P_N - k')$$

requires a *consistent* description of both the target initial and final states and the nuclear current. Approximations are needed in the kinematical regime in which reletivistic effects are important.

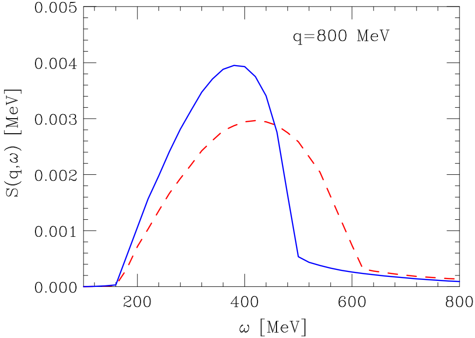
Why worry about relativity



(Figure courtesy of A. Ankowsky)

- ★ Unlike the initial state, the nuclear current and the final hadronic state *can not* be described using non relativistic many-body theory
- ★ In neutrino experiments low- and large-$|\mathbf{q}|$ contributions to the observables are inextricably tangled

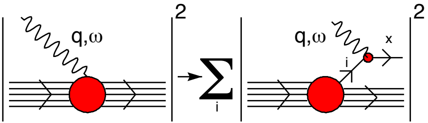
Relativistic vs non relativistic kinematics



- ⋆ Response of uniform isospin symmetric nuclear matter to a scalar probe delivering momentum $|\mathbf{q}| = 800$ MeV.
- ⋆ Calculation carried out using a realistic spectral function. Solid line: relativistic kinematics. Dashed line: non relativistic kinematics

Enter the factorization *ansatz*

★ At $|\mathbf{q}|^{-1} \ll d \sim 1.2$ fm use the impulse approximation (IA)



▹ neglect the contribution of the two-nuleon current

$$J_A^{\mu}(q) \approx \sum_{i=1}^{A} j_i^{\mu}(q)$$

▹ write the final state in the factorized form

$$|N\rangle \to |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle \ .$$

▹ at zero-th order, neglect final state interactions (FSI) between the outgoing nucleon and the spectator particles

Spectral function

- ⋆ within the IA scheme the nuclear matrix element of the one-nucleon current reduces to

$$\langle N | j_i^\mu | 0 \rangle = \int d^3k \, M_m(\mathbf{k}) \, \langle \mathbf{p} | j_i^\mu | \mathbf{k} \rangle \ ,$$

with

$$M_n(\mathbf{k}) = \{ \langle n_{(A-1)}, \mathbf{p}_n | \otimes \langle \mathbf{k} | \} \, | 0 \rangle \ .$$

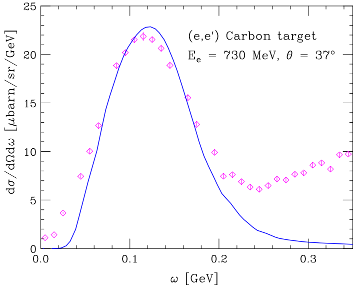
- ⋆ The nuclear spectral function, yielding the probability of removing a nucleon of momentum $\mathbf{k}$leaving the residual system with excitation energy E, is defined as

$$P(\mathbf{k}, E) \sum_n |M_n(\mathbf{k})|^2 \delta(E_0 + E - E_n)$$

IA results compared to electron scattering data

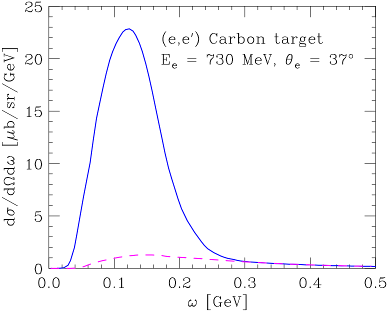
- ⋆ Nuclear x-section $$d\sigma_A = \int d^3k dE \, d\sigma_N \, P(\mathbf{k}, E)$$

- ⋆ QE (nucleon-only final states) only



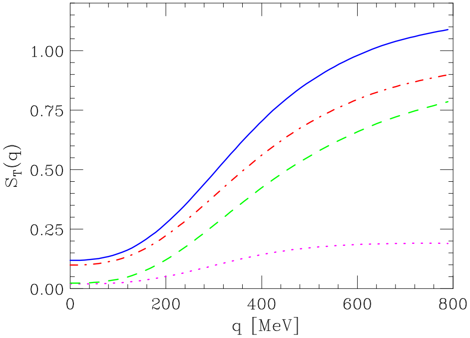
- ⋆ Position and width of the peak are reproduced

- ⋆ Correlation tail, arising from 2p2h final states, cleary visible



Enter the two-nucleon current

- ★ Two-nucleon current contributions can be accurately computed within the non relativistic approximation, using the GFMC approach
- ★ Energy loss integral of the transvers electromagnetic response of carbon (arXiv:1312.1210)



- ★ Interference between one- and two-nucleon currents important

Including MEC contribution

- ⋆ Highly accurate and consistent calculations can only be carried out in the non relativistic regime
- ⋆ Using fully relativistic MEC and a realistic description of the nuclear ground state, including correlations, requires the factorization *ansatz* underlying the IA
  - ▹ Rewrite the final state $|N\rangle$ in the factorized form

$$|N\rangle \to |\mathbf{p}, \mathbf{p}'\rangle \otimes |n_{(A-2)}, \mathbf{p}_n\rangle$$

$$\langle N|{j_{ij}}^{\mu}|0\rangle \to \int d^3k d^3k' M_n(\mathbf{k}, \mathbf{k}') \, \langle \mathbf{p}\mathbf{p}'|{j_{ij}}^{\mu}|\mathbf{k}\mathbf{k}'\rangle$$

The amplitude

$$M_n(\mathbf{k}, \mathbf{k}') = \{\langle n_{(A-2)}|\langle \mathbf{k}, \mathbf{k}'|\} \otimes |0\rangle$$

is independent of q and can be obtained from non relativistic many-body theory

Two-nucleon spectral function

- ⋆ Calculations have been carried out for uniform isospin-symmetric nuclear matter using CBF perturbation theory [PRC 62, 034304 (2000)]

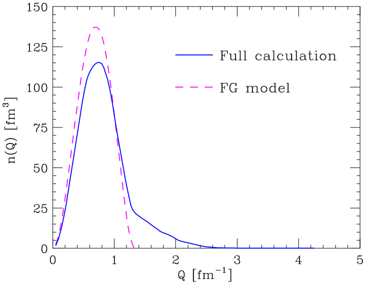
$$P(\mathbf{k},\mathbf{k}',E) = \sum_n |M_n(k,k')|^2 \delta(E + E_0 - E_n)$$

$$n(\mathbf{k},\mathbf{k}') = \int dE \, P(\mathbf{k},\mathbf{k}',E)$$

- ⋆ Relative momentum distribution

$$n(\mathbf{Q}) = 4\pi|\mathbf{Q}|^2 \int d^3q \, n\left(\frac{\mathbf{Q}}{2} + \mathbf{q}, \frac{\mathbf{Q}}{2} - \mathbf{q}\right)$$

$$\mathbf{q} = \mathbf{k} + \mathbf{k}' \quad , \quad \mathbf{Q} = \frac{\mathbf{k} - \mathbf{k}'}{2}$$



Summary & Outlook

- ⋆ Results of exact non relativistic calculations suggest that interference between one and two-nucleon current contributions to the excitation of 2p2h final states is important
- ⋆ The spectral function formalism can be generalized to allow for a consistent treatment of processes involving one- and two- nucleon currents
- ⋆ The implementation of this scheme in Monte Carlo generators does not involve conceptual difficulties