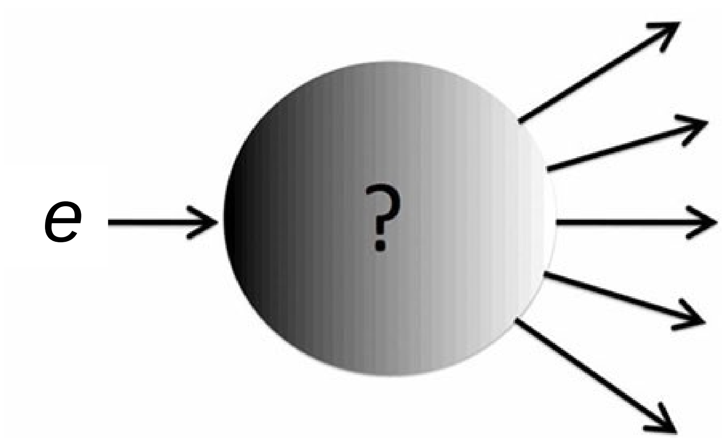
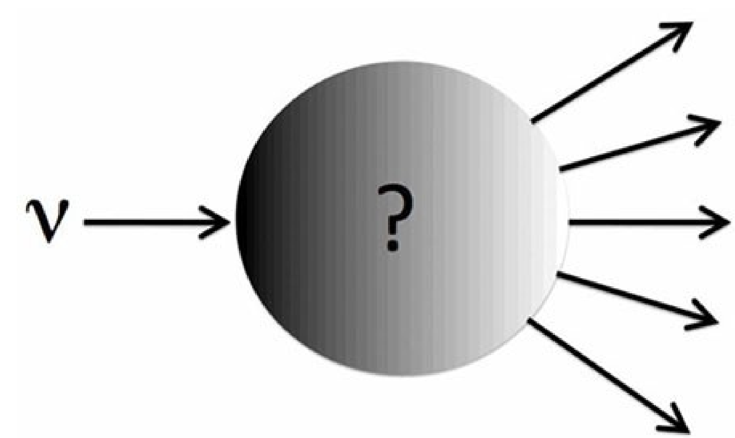
INT Workshop 13-54W
"Neutrino-Nucleus Interactions for Current and Next Generation Neutrino Oscillation Experiments"
Seattle, December 3-13, 2013

Scaling of Inclusive Electroweak Interactions with Nuclei
Part I

Maria Barbaro, University of Turin and INFN, ITALY





Outline

- **Review of Scaling in inclusive electron scattering (e,e')**
- Quasi Elastic Peak
- Definitions: scaling of 1st kind (y-scaling)

  scaling of 2nd kind and "Super-Scaling"

  scaling of 0th kind
- Non-QE scaling
- Inelastic scattering
- Scaling violations: Meson Exchange Currents (Quique Amaro)
- **Predicting CCν cross sections using scaling ("SuSA")**
- Definition: the "SuSA" approach and scaling of 3rd kind
- Comparison with electron scattering data
- Comparison with neutrino scattering data (Juan Caballero)

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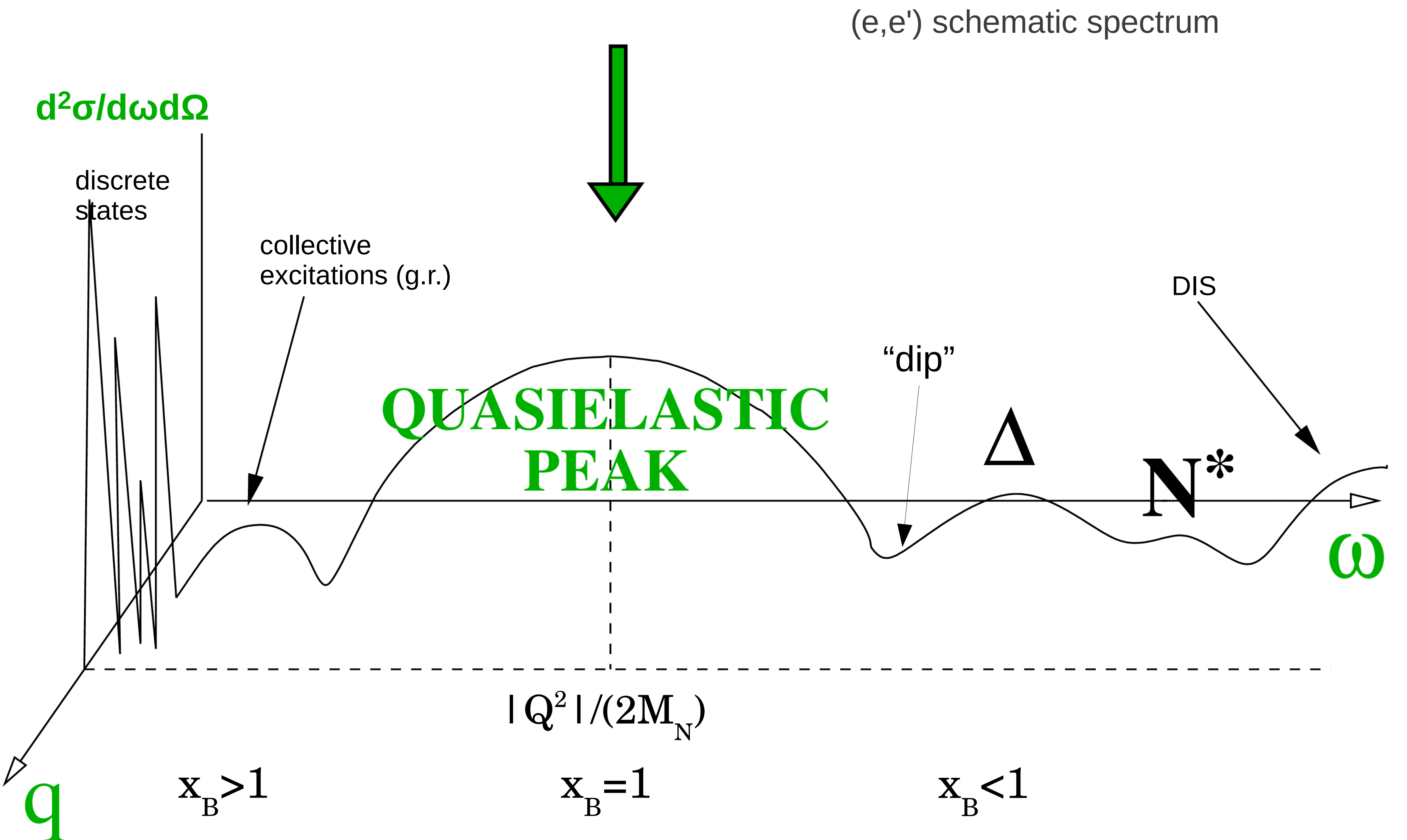

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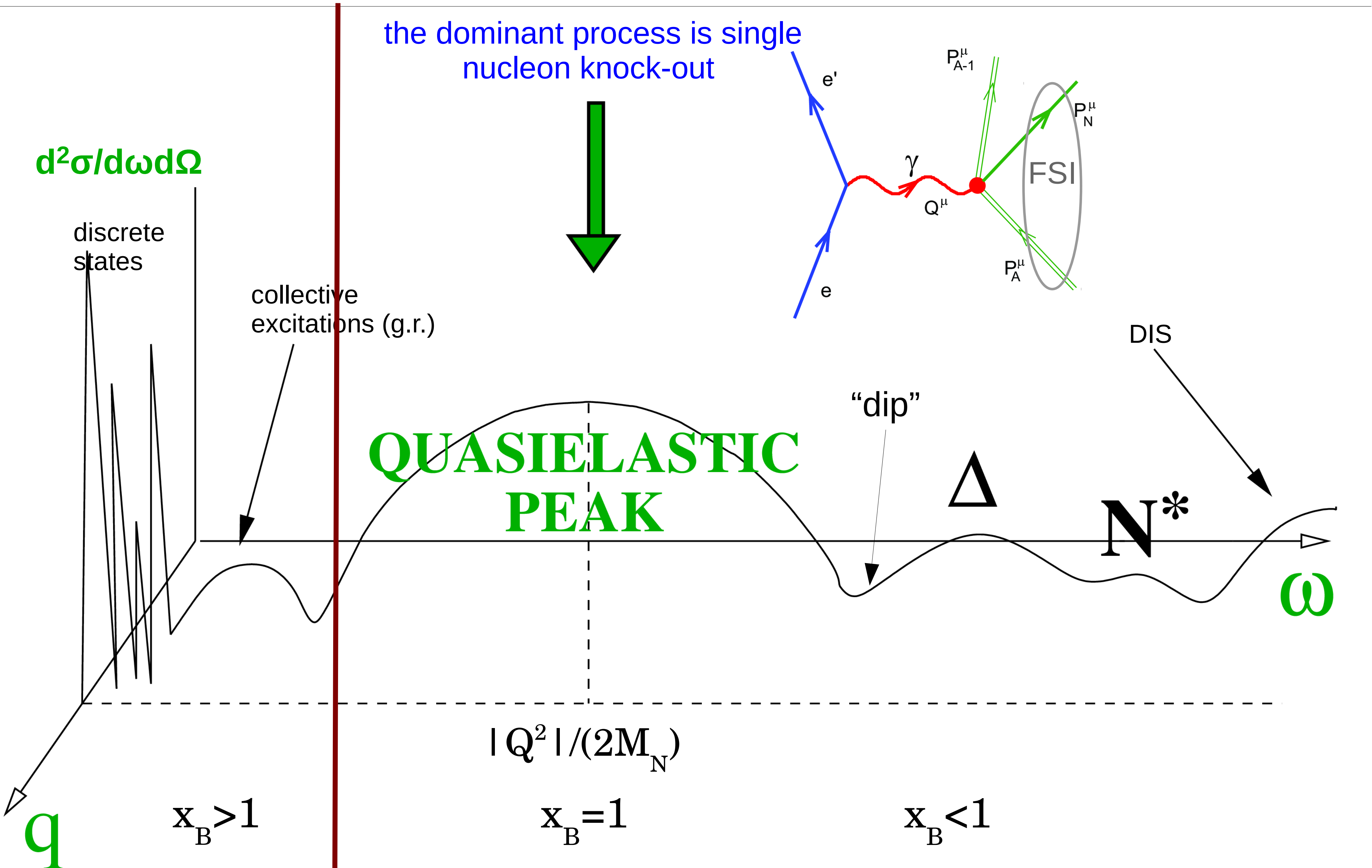

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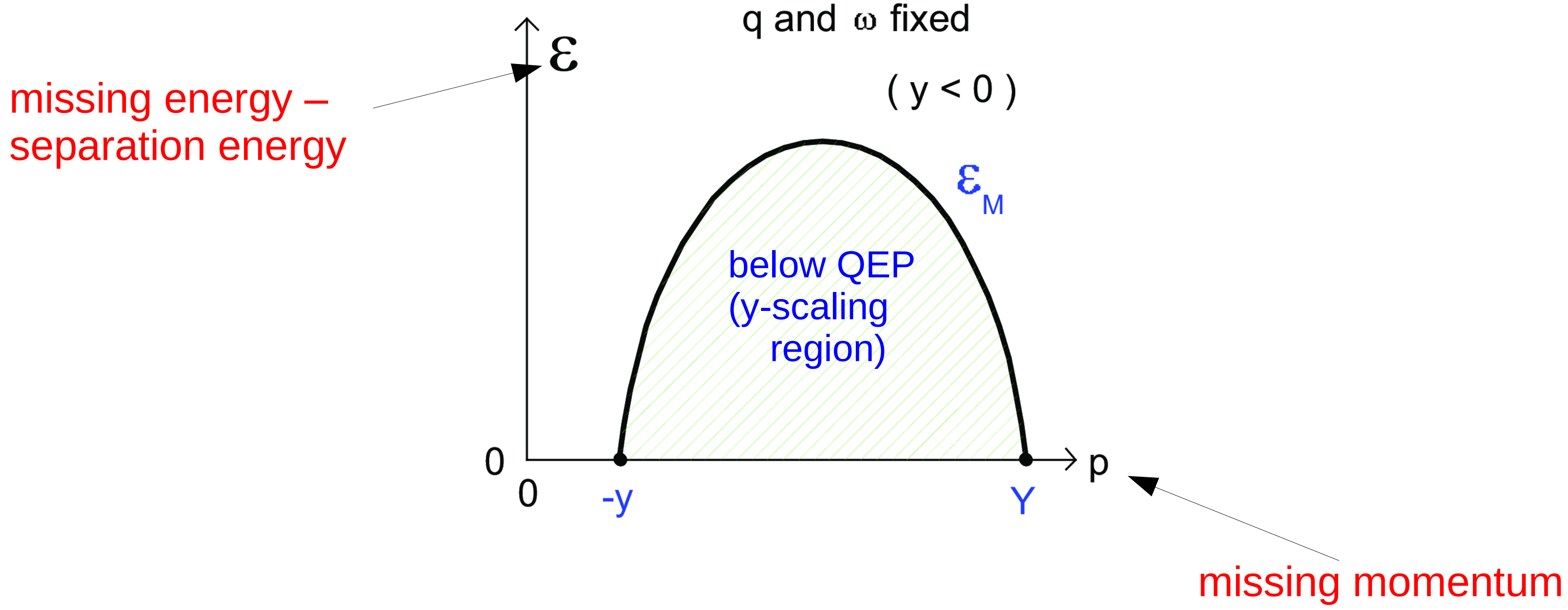
Quasielastic kinematics and y-scaling

For each value of q and ω, evaluating the *(e,e')* cross section implies an integral over the kinematically allowed region for the semi-inclusive reaction *(e,e'N):*



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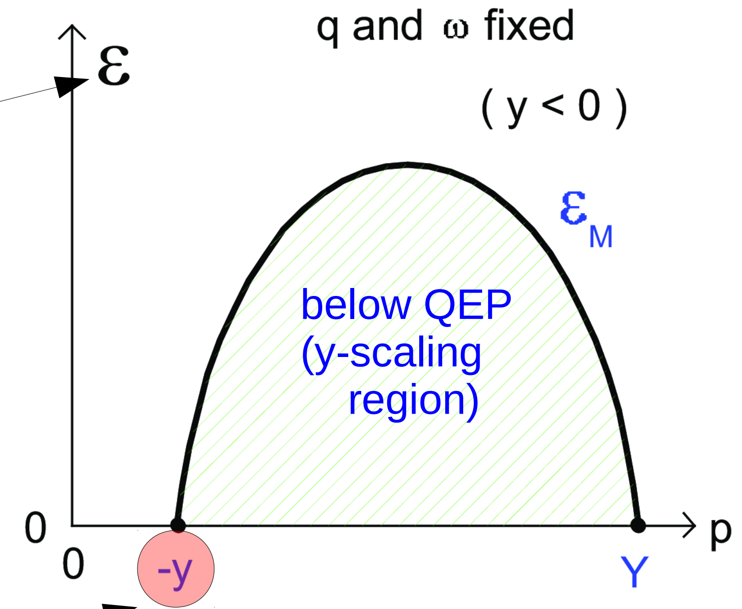
missing energy – separation energy



missing momentum

y scaling variable: -y(q,ω) is the lowest value of the missing momentum at the lowest missing energy kinematically allowed for semi-inclusive knockout of nucleons from the nucleus.

$$y \cong y_\infty = \sqrt{\widetilde{\omega}(2m_N + \widetilde{\omega})} - q.$$

$$\widetilde{\omega} \equiv \omega - E_S.$$

at the QEP: *y=0*

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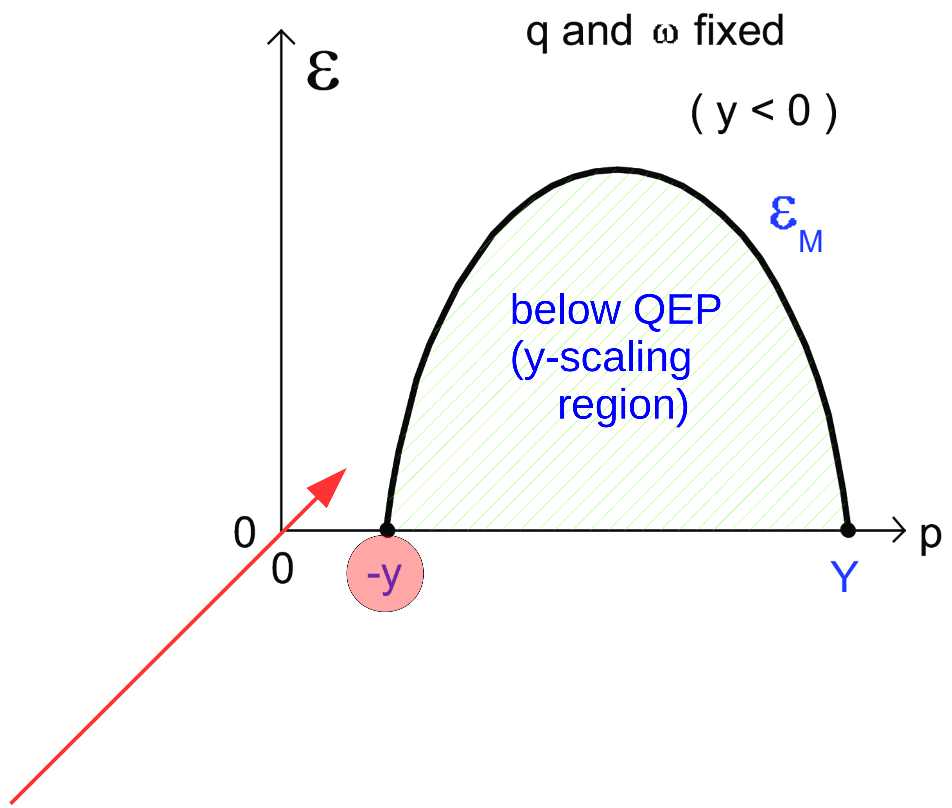

The semi-inclusive cross section is typically largest at small *p* and *ε*

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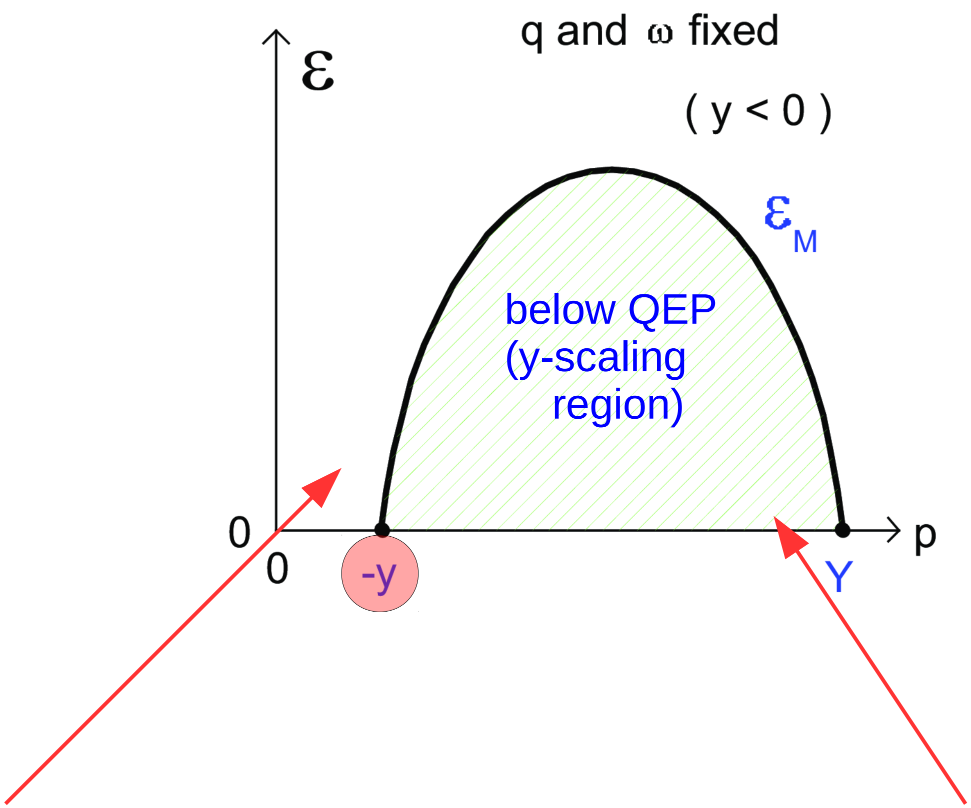

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and very small at large p and small ε

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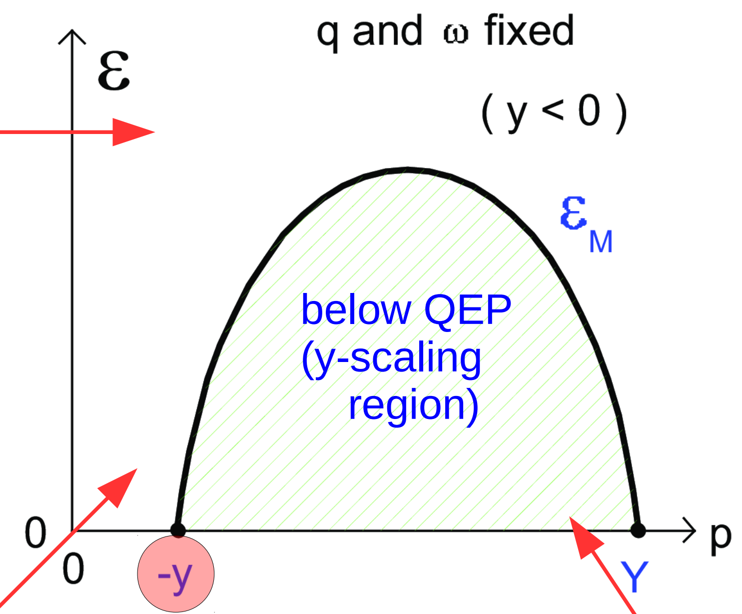
for given *y<0*, the region of low *p* and high *ε* is unaccessible



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and very small at large *p* and small *ε*

The y-scaling function

- Instead of *(q,ω)* use the variables *(q,y)*
- Typical parametrizations for the off-shell single-nucleon cross sections

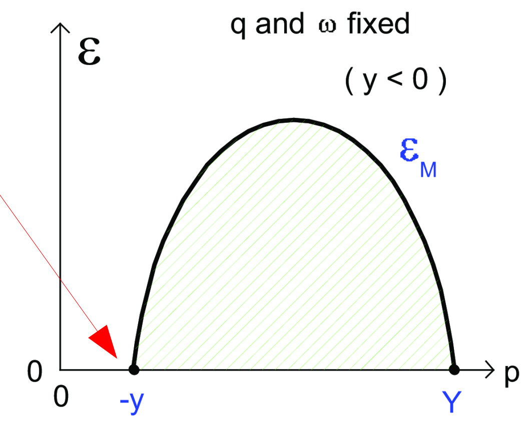
$$\sigma^{off}_{eN}(q,\ \omega,\ p,\ \mathcal{E},\ \Phi_N)$$

vary slowly as functions of *(p, Ɛ)* for fixed *(q, ω, Φ_N)*. This suggests integrating over Φ_N (leaving only R_L and R_T) and then removing the result evaluated at an "optimal" choice of *p* and *Ɛ*.

- From the above analysis the "optimal" choice is *p=|y|* and *Ɛ=0*

$$\Sigma_{eN}^{eff} = \frac{1}{A}\left(Z\,\overline{\sigma}_{ep}^{elastic} + N\,\overline{\sigma}_{en}^{elastic} \right)_{p=-y,\,\varepsilon=0}$$

Effective single nucleon cross section

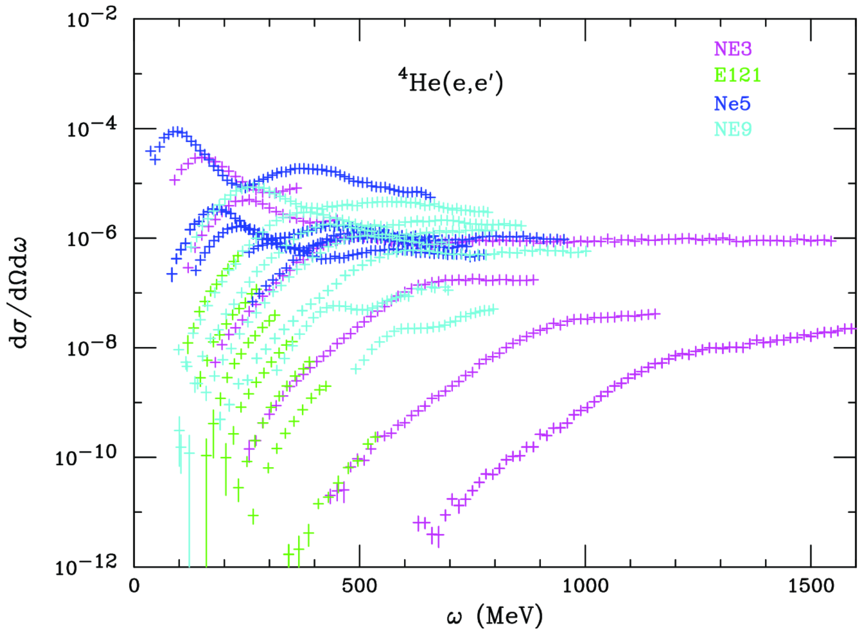


- **Scaling Function**

$$F(q,y) \equiv \frac{1}{A\Sigma_{eN}^{eff}} \frac{d^2\sigma}{d\Omega\, d\omega}$$

y-scaling of (e,e') data

Example: ^{4}He SLAC data



Inclusive cross section for various beam energies and scattering angles

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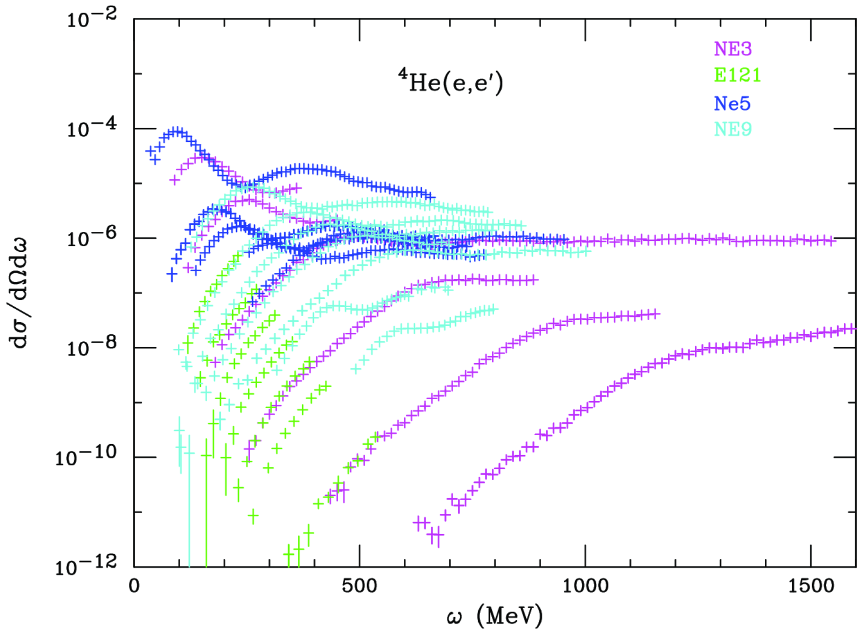

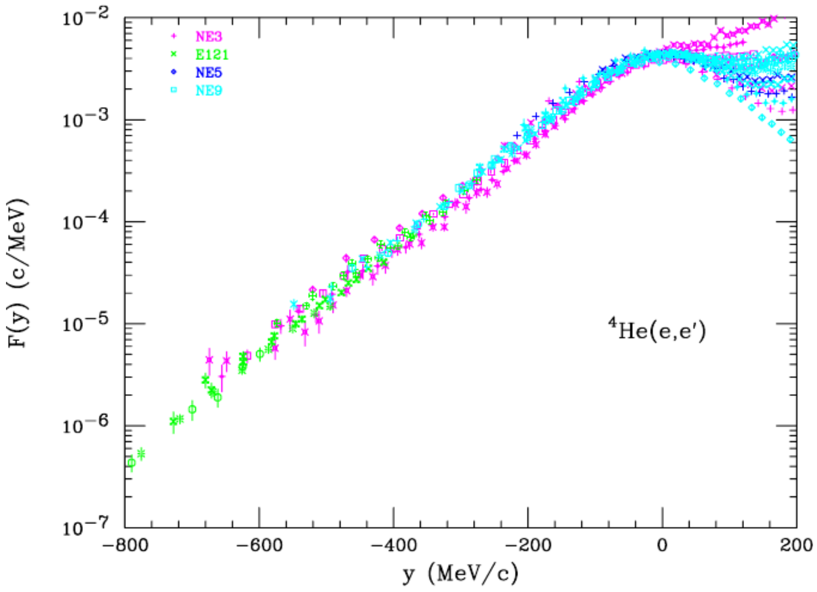

Inclusive <u>cross section</u> for various beam energies and scattering angles

<u>Scaling function</u> plotted as a function of *y* for various values of *q*

[Day,McCarthy,Donnelly,Sick,Ann.Rev.Nucl.Part.Sci.40(1990); Donnelly & Sick, PRC60(1999),PRL82(1999)]

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Example: ^{4}He SLAC data

ω<ω$_{QEP}$
(x>1, y<0)

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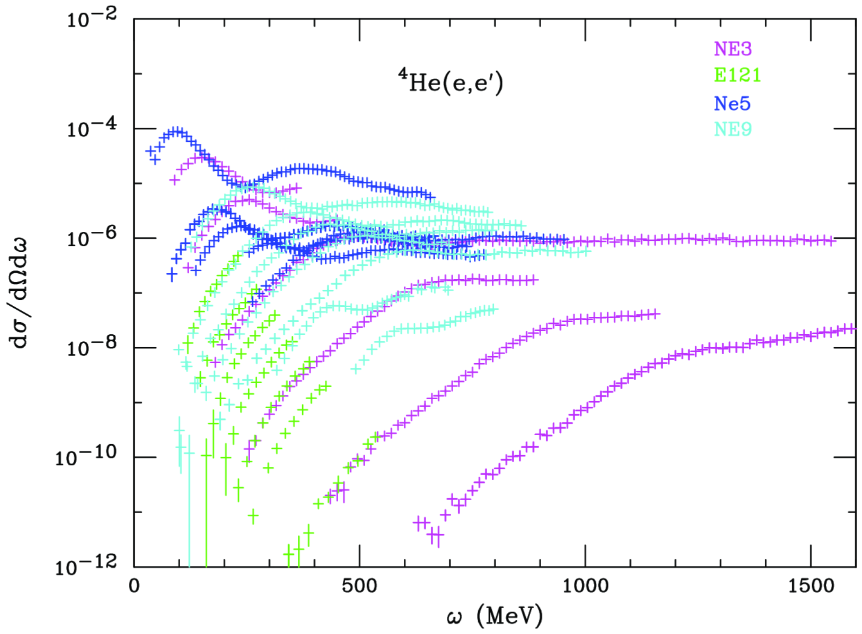

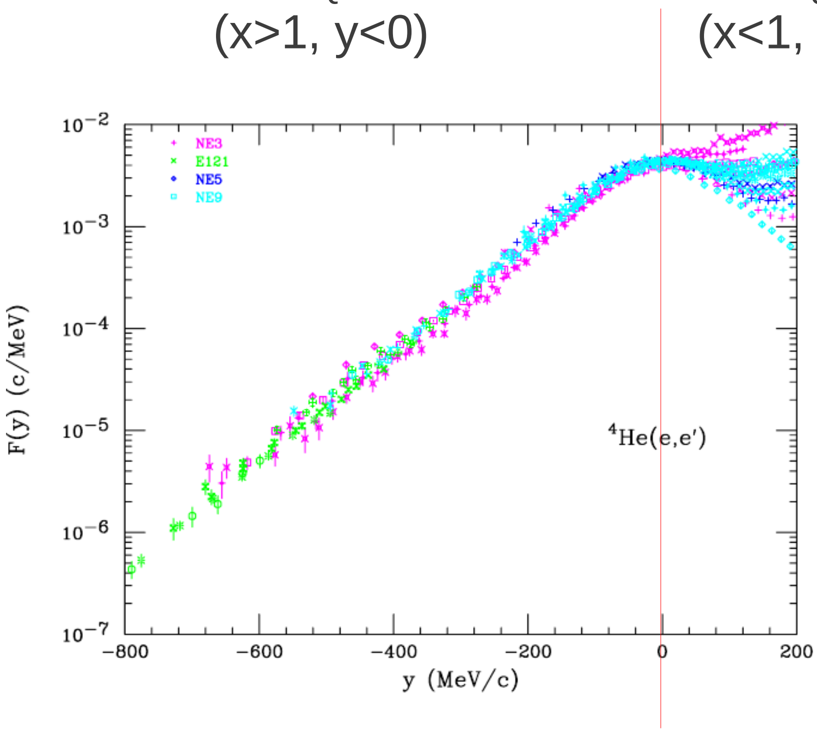

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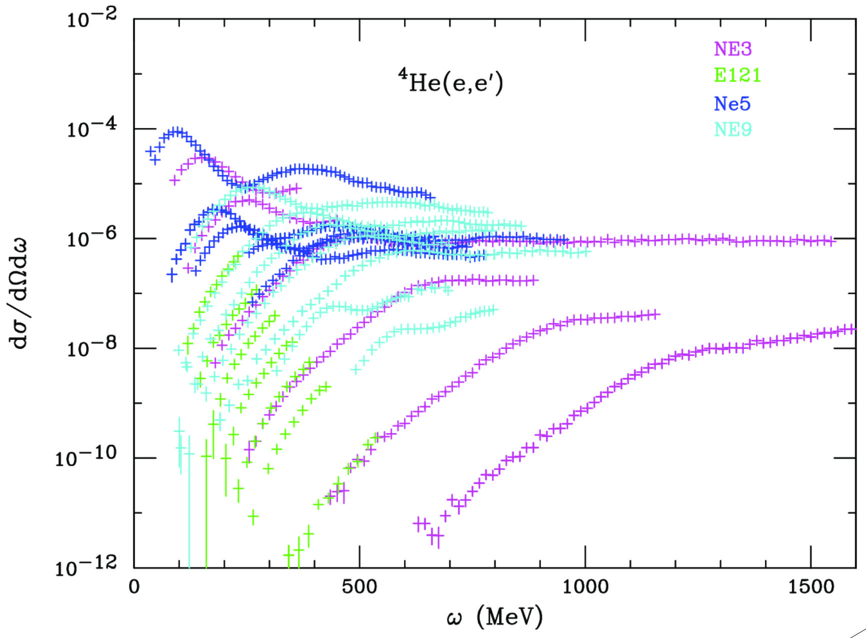
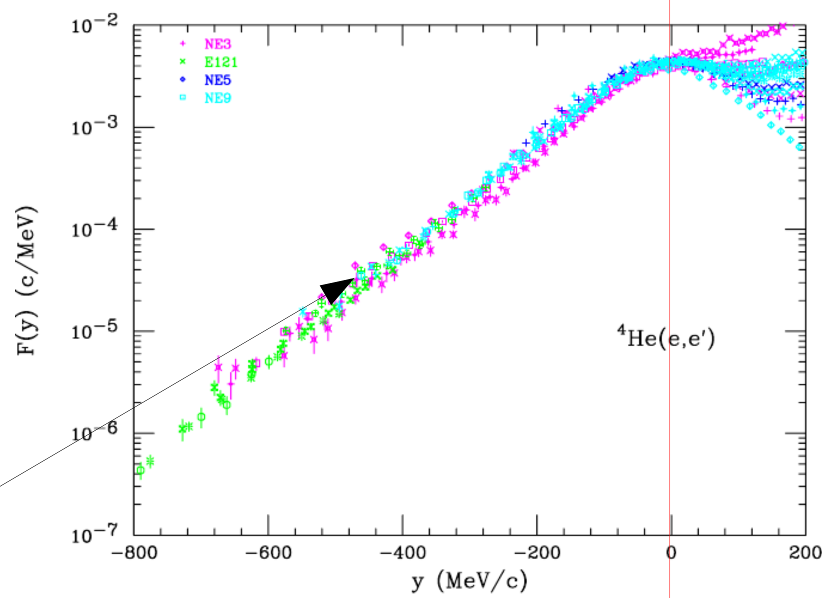




Independence of *q* of the scaling function *F(q,y)*:

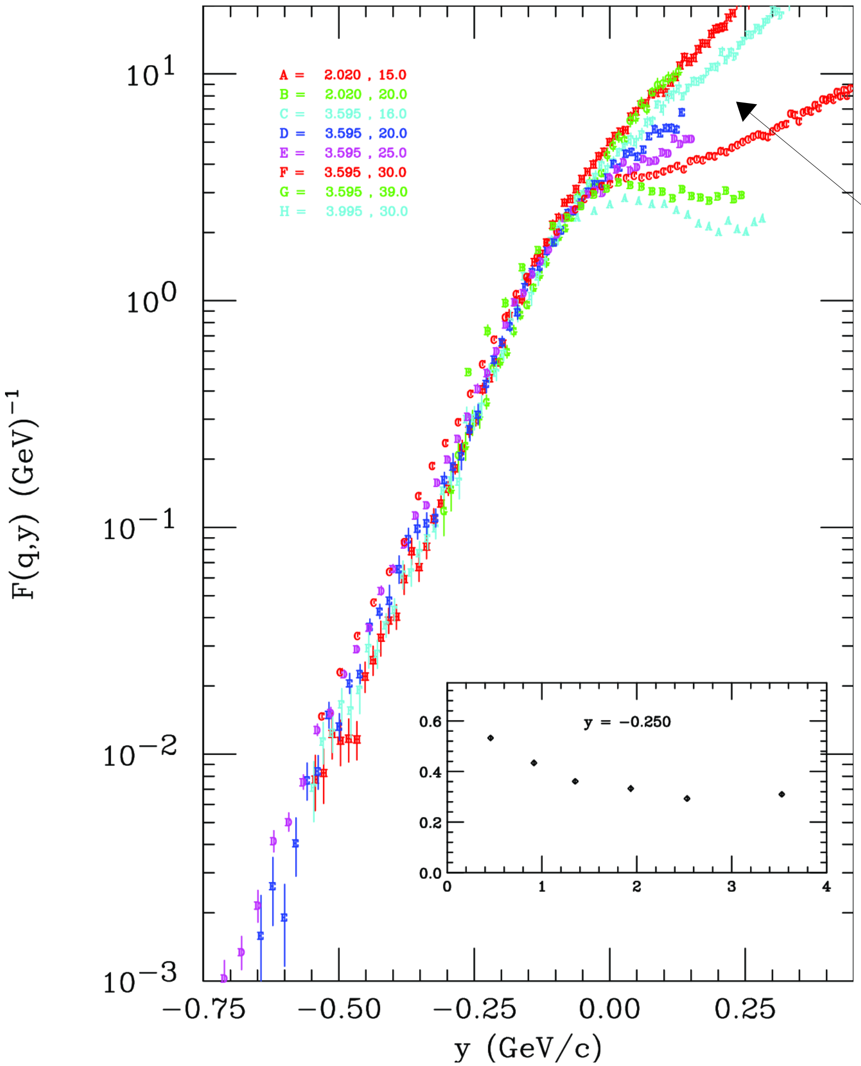
$$F(y,q) \longrightarrow F(y) \equiv F(y,\infty) \quad \text{for } q \to \infty$$

Scaling of the first kind
(or y-scaling)

[Day,McCarthy,Donnelly,Sick,Ann.Rev.Nucl.Part.Sci.40(1990); Donnelly & Sick, PRC60(1999),PRL82(1999)]

y-scaling of (e,e') data

Another example: ^{57}Fe data



Scaling is violated at *y>0* due to resonances, meson production, deep inelastic scattering....

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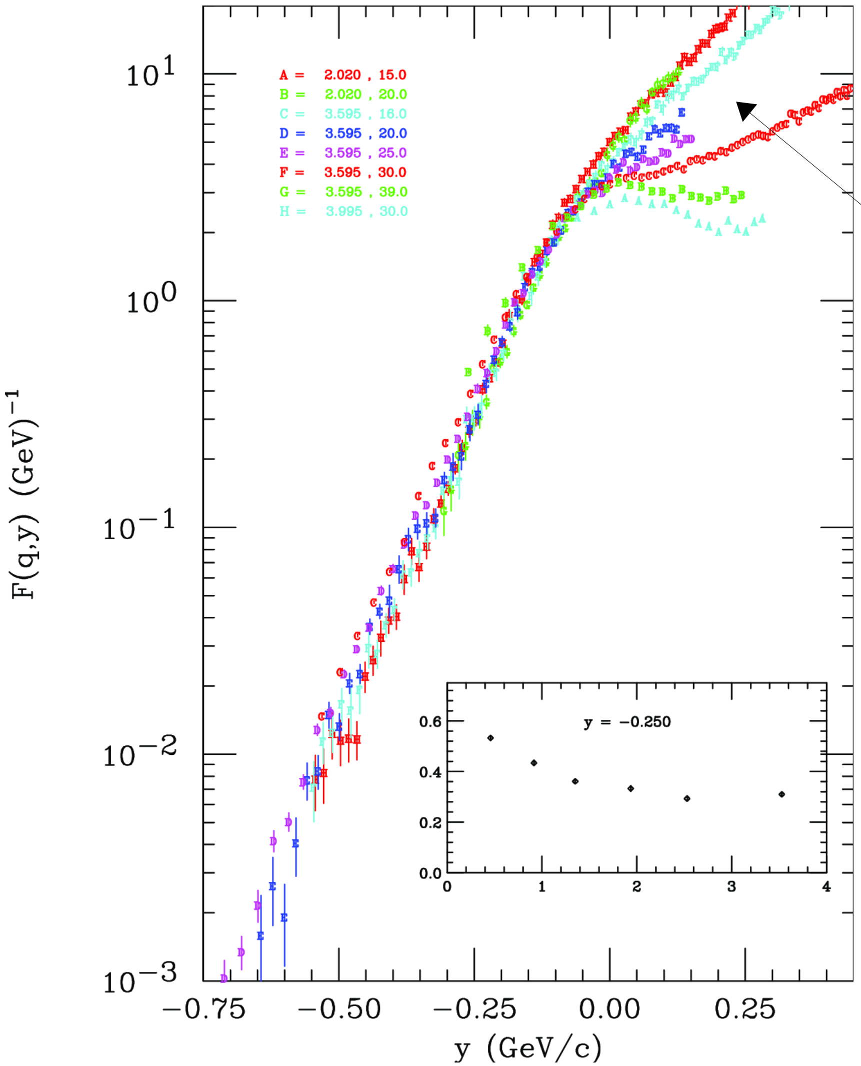

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Scaling function at fixed *y* plotted versus Q^2 : scaling is approached “from above”

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2^{nd} kind scaling

- Let us now introduce a characteristic momentum k_A for a given nucleus with mass number A and define the dimensionless function

$$f(q,y) = k_A * F(q,y)$$

- Correspondingly we introduce a dimensionless scaling variable ψ and plot $f(q,\psi)$ vs ψ for different values of q

- The **Relativistic Fermi Gas** model is used to motivate the choice of the scaling variable ψ:

$$\psi(\lambda,\tau)=\frac{1}{\sqrt{\xi_F}}\frac{\lambda-\tau}{\sqrt{\tau(1+\lambda)+\kappa\sqrt{\tau(1+\tau)}}}$$

$$\lambda=\frac{\omega}{2m_N},\ \kappa=\frac{q}{2m_N},\ \tau=\kappa^2-\lambda^2$$

dimensionless variables

$k_A=k_F$ Fermi momentum, ξ_F Fermi kinetic energy

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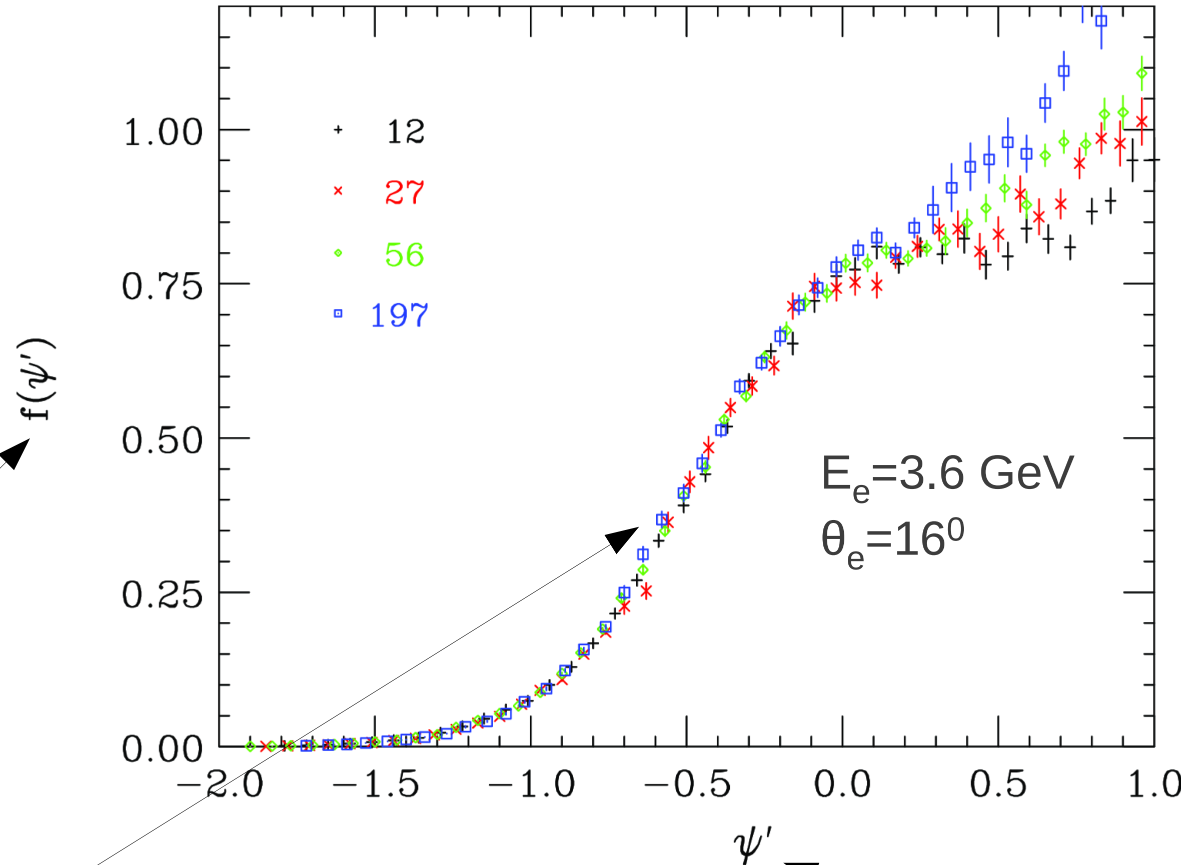
- A phenomenological energy shift E_{shift} (typically ~20 MeV) is introduced in order to give the right position of the QEP: $\omega \rightarrow \omega' = \omega - E_{shift}$ which implies $\psi \rightarrow \psi'$

2^{nd} kind scaling

Plotting f(q,ψ') at fixed kinematics (q) for different nuclei (A) one gets



k_A*F

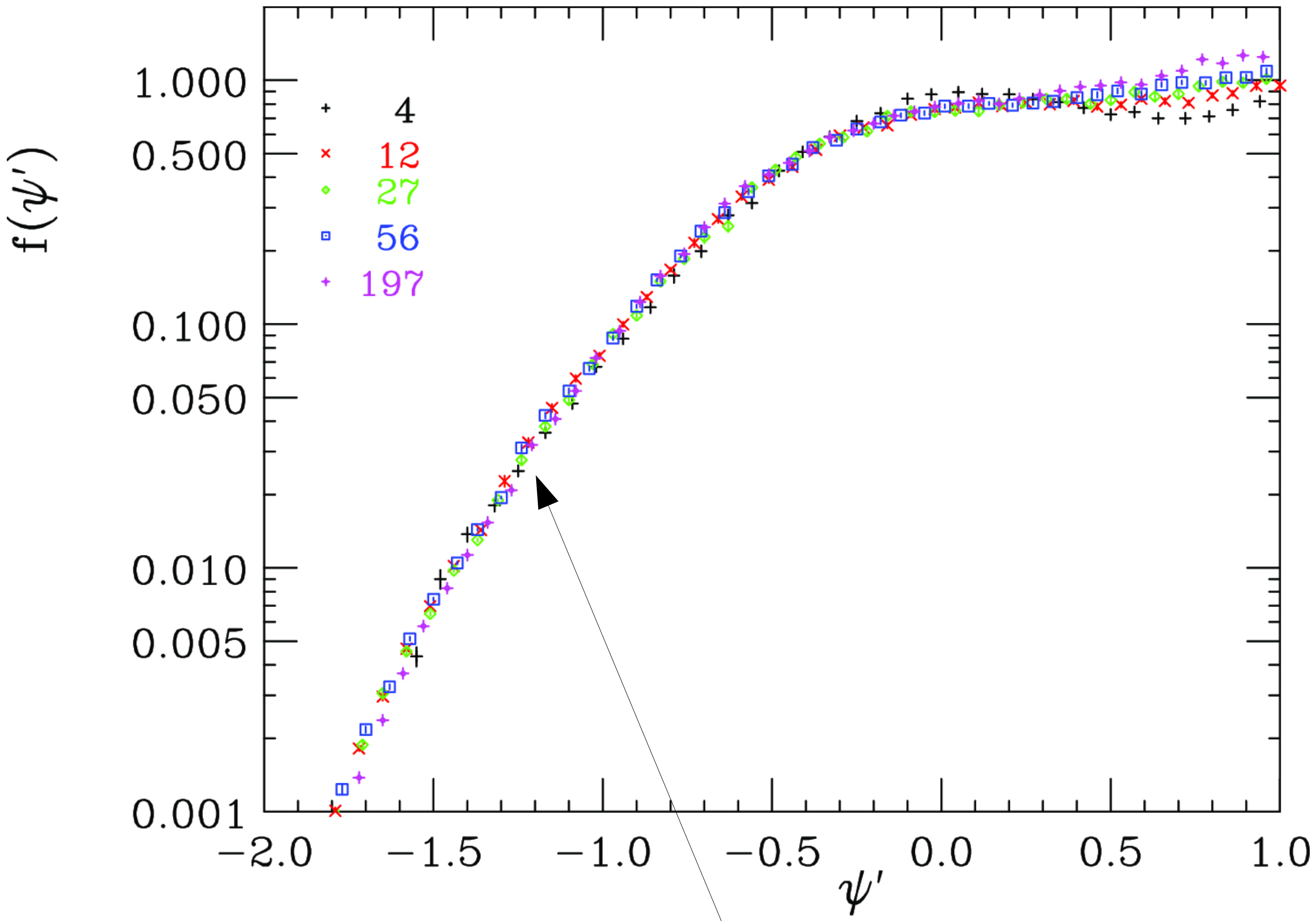
k_A = characteristic momentum scale for each nucleus

~y/k_F

Second kind scaling = A-independence for ψ' < 0

2nd kind scaling

In semi-logarithmic scale:



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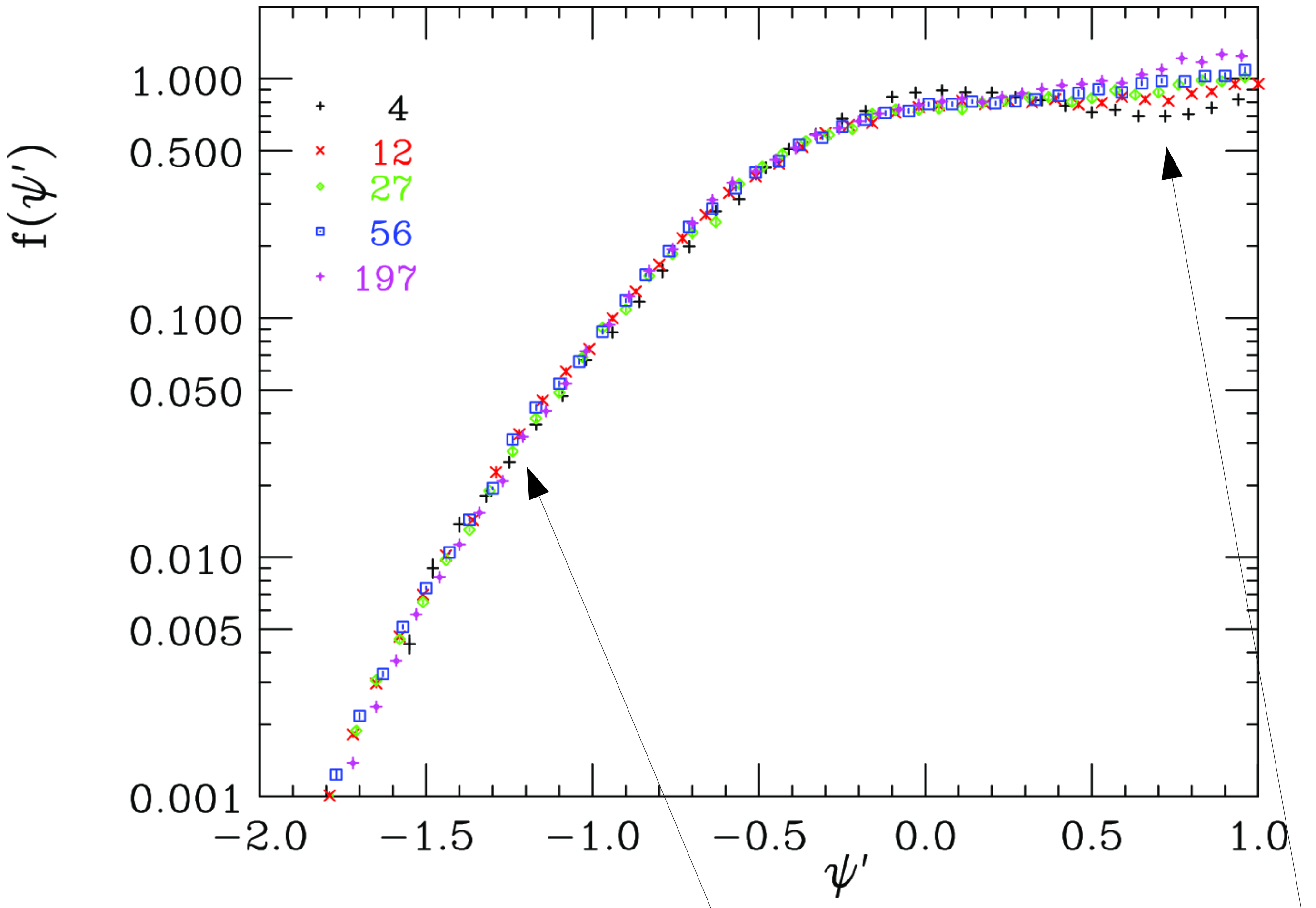
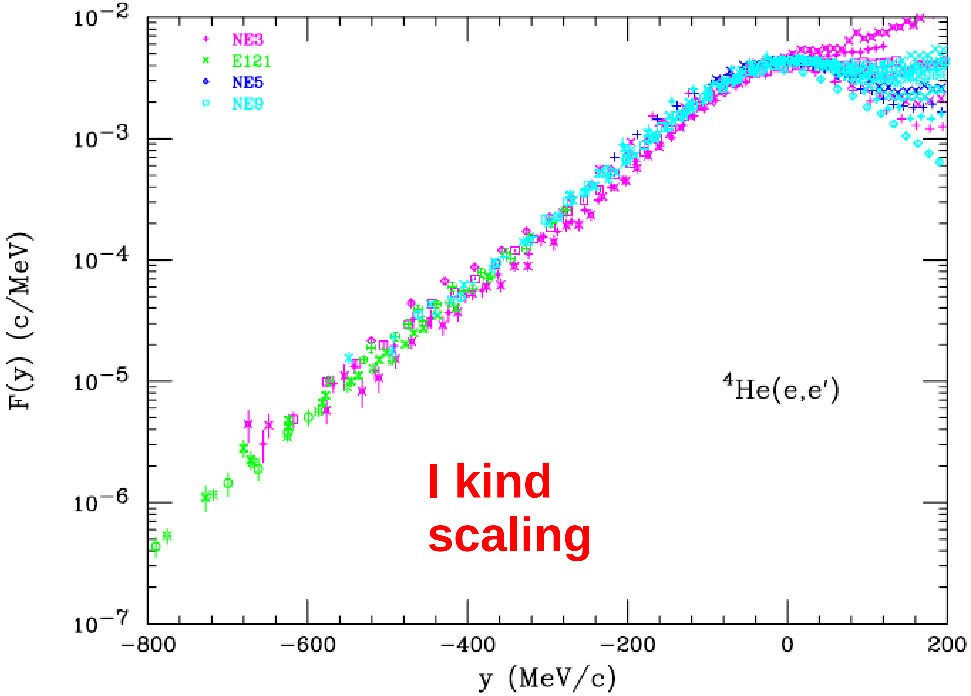
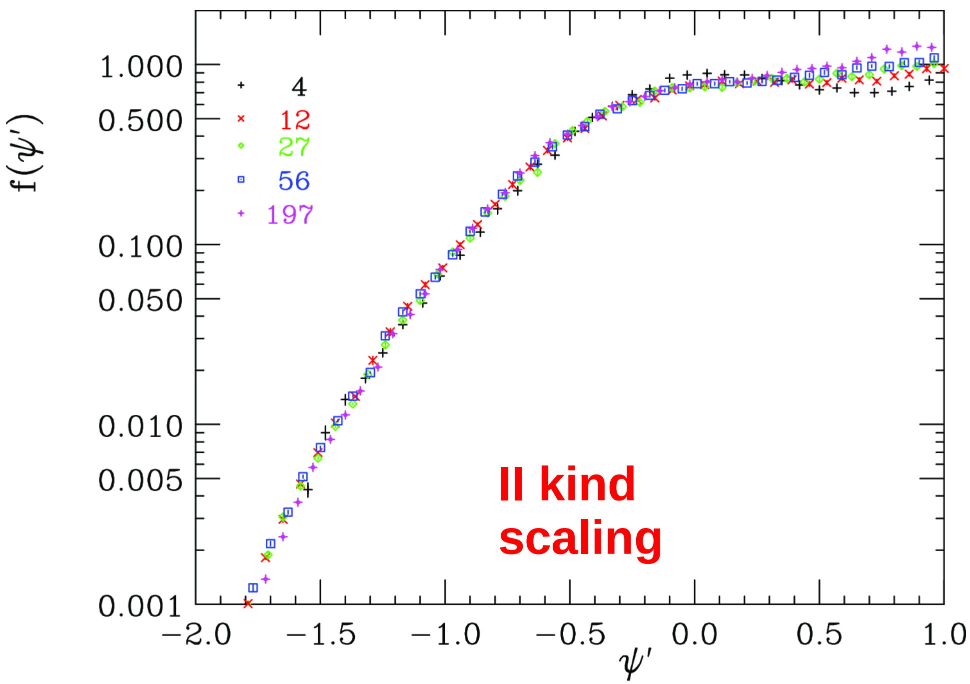


Violations for $\psi' > 0$ due to non QE contributions

Second kind scaling = A-independence for $\psi' < 0$

Super-Scaling





We define **"Super-Scaling"** the simultaneous occurrence of

I kind scaling (independence of q)

and

II kind scaling (independence of A)

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0^{th} kind scaling and L/T separation

The scaling analysis can be performed il the longitudinal and transverse channels separately, using the (few) existing L/T separated (e,e') data

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{Mott}(v_L R_L + v_T R_T)$$

$$v_L = |Q^2/q^2|^2$$

$$v_T = \frac{1}{2}|Q^2/q^2| + \tan^2\frac{\theta_e}{2}$$

kinematical factors

R_L, R_T **Response Functions**

Longitudinal and Transverse scaling functions:

$$F_L(q,y) \equiv \frac{R_L(q,\omega)}{A\Sigma^{eff}_{eN,L}/\sigma_{Mott} v_L} \equiv f_L(q,y)/k_A$$

$$F_T(q,y) \equiv \frac{R_T(q,\omega)}{A\Sigma^{eff}_{eN,T}/\sigma_{Mott} v_T} \equiv f_T(q,y)/k_A$$

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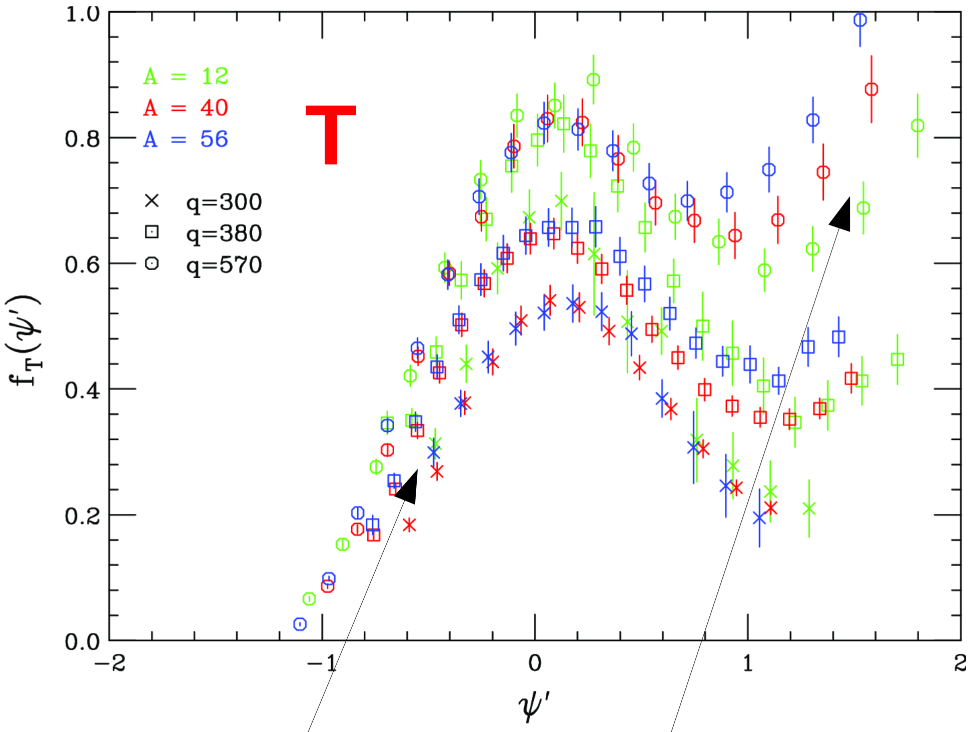
How does it work versus data?

L and T scaling functions



Inelastic contribution (mainly T)
+ MEC (dominantly T)

some violations even
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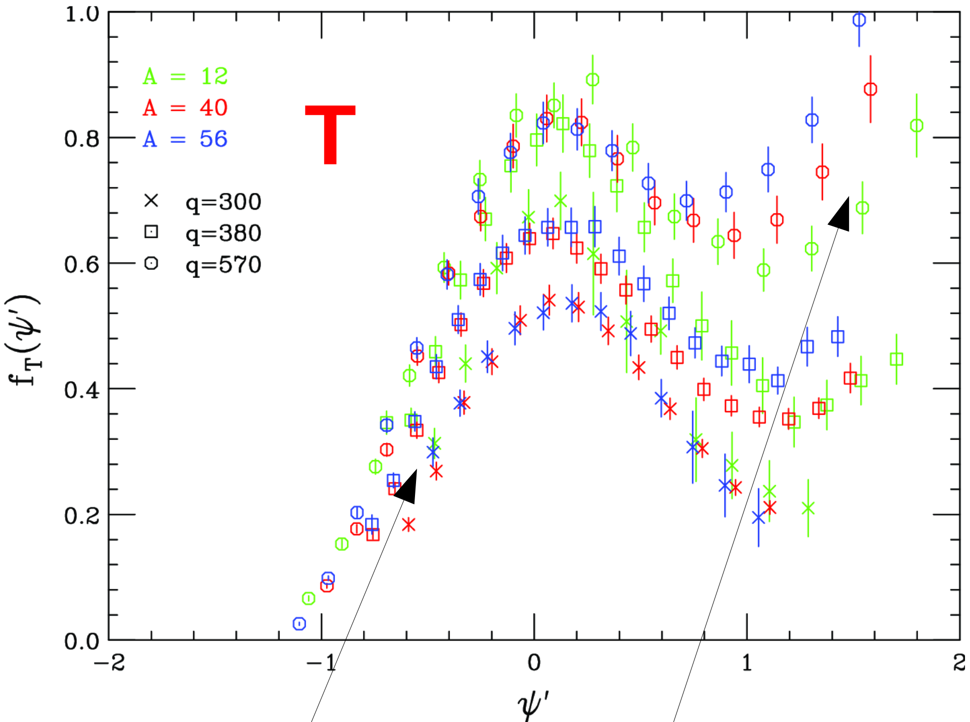

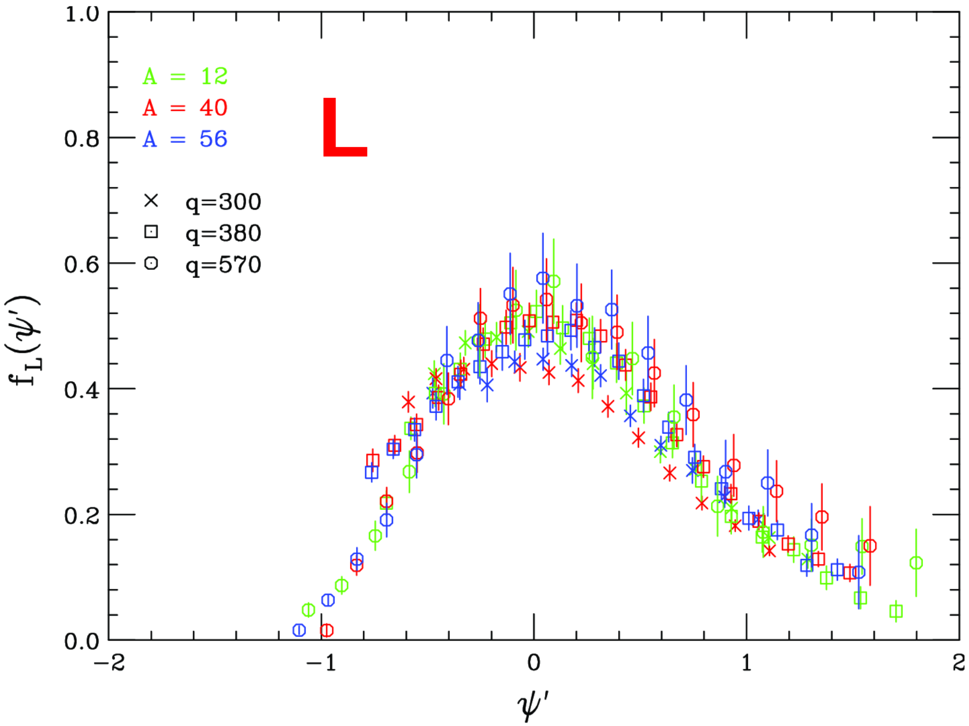

Inelastic contribution (mainly T)
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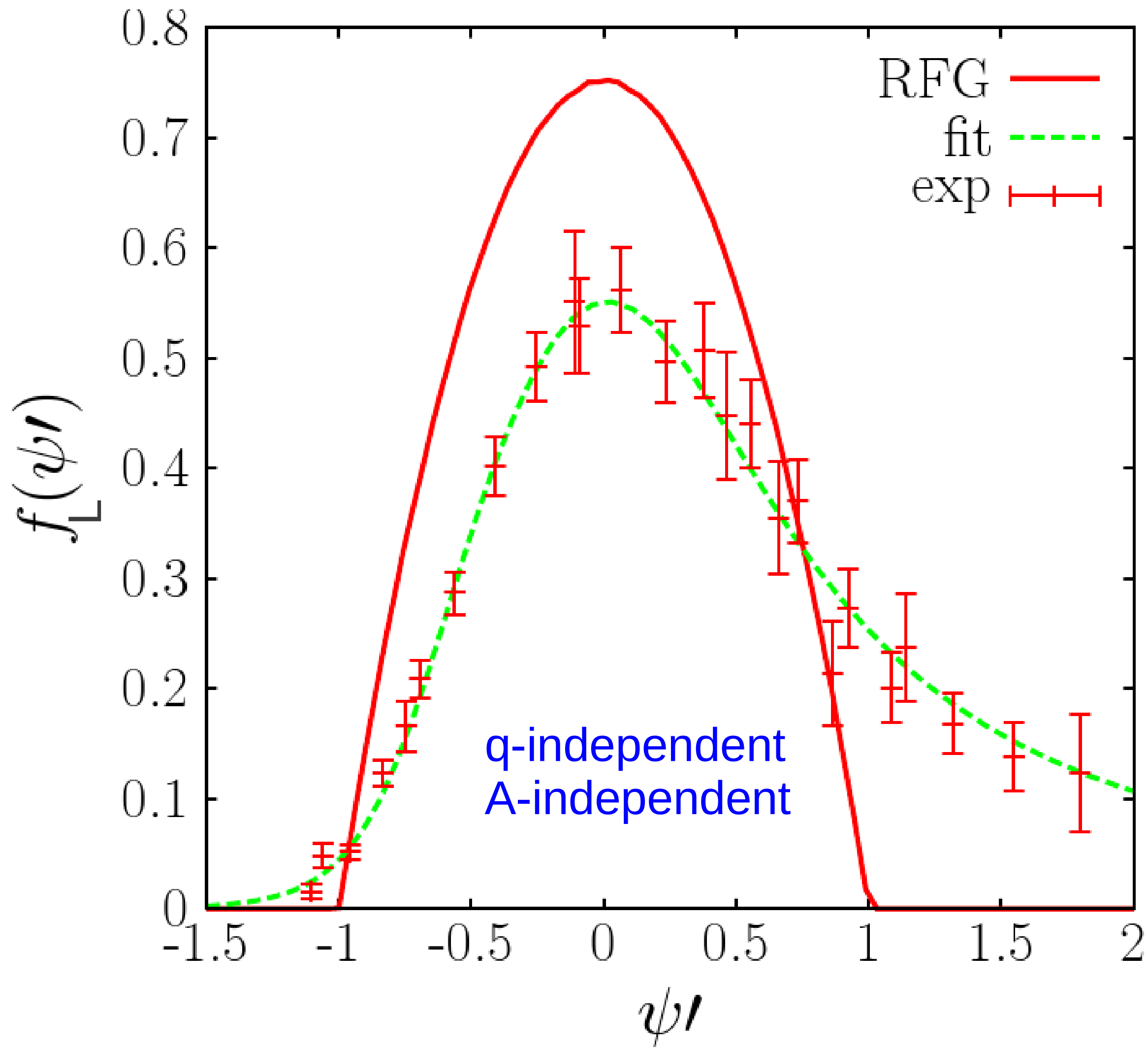
in contrast, the L results show a **universal behavior**

some violations even below the QEP

Phenomenological super-scaling function

A **phenomenological super-scaling function** has been extracted from the *longitudinal* (e,e') word data [Jourdan,NPA603, 117 ('96)]



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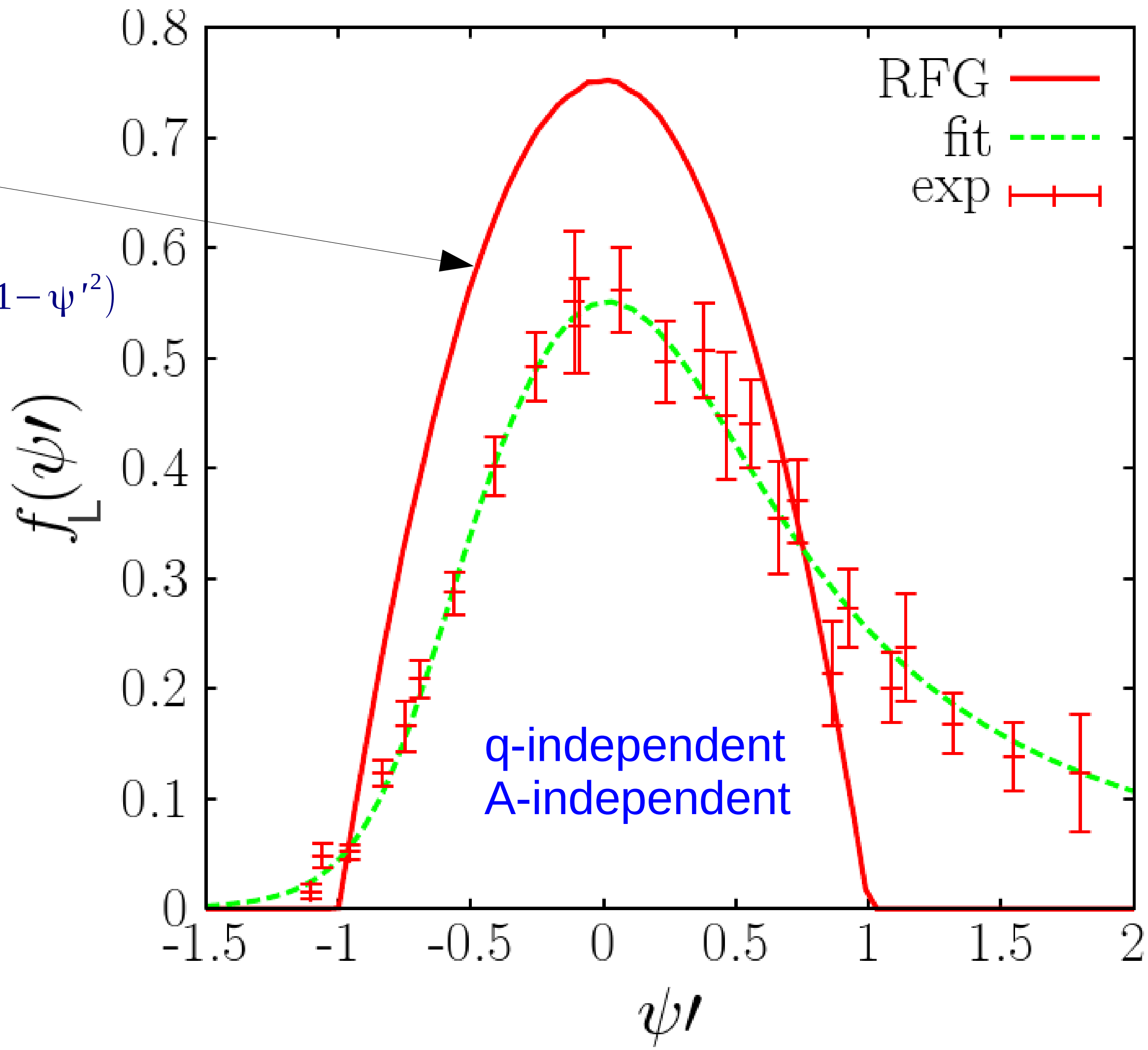
$$f_{RFG}(\psi')=\frac{3}{4}(1-\psi'^2)\theta(1-\psi'^2)$$



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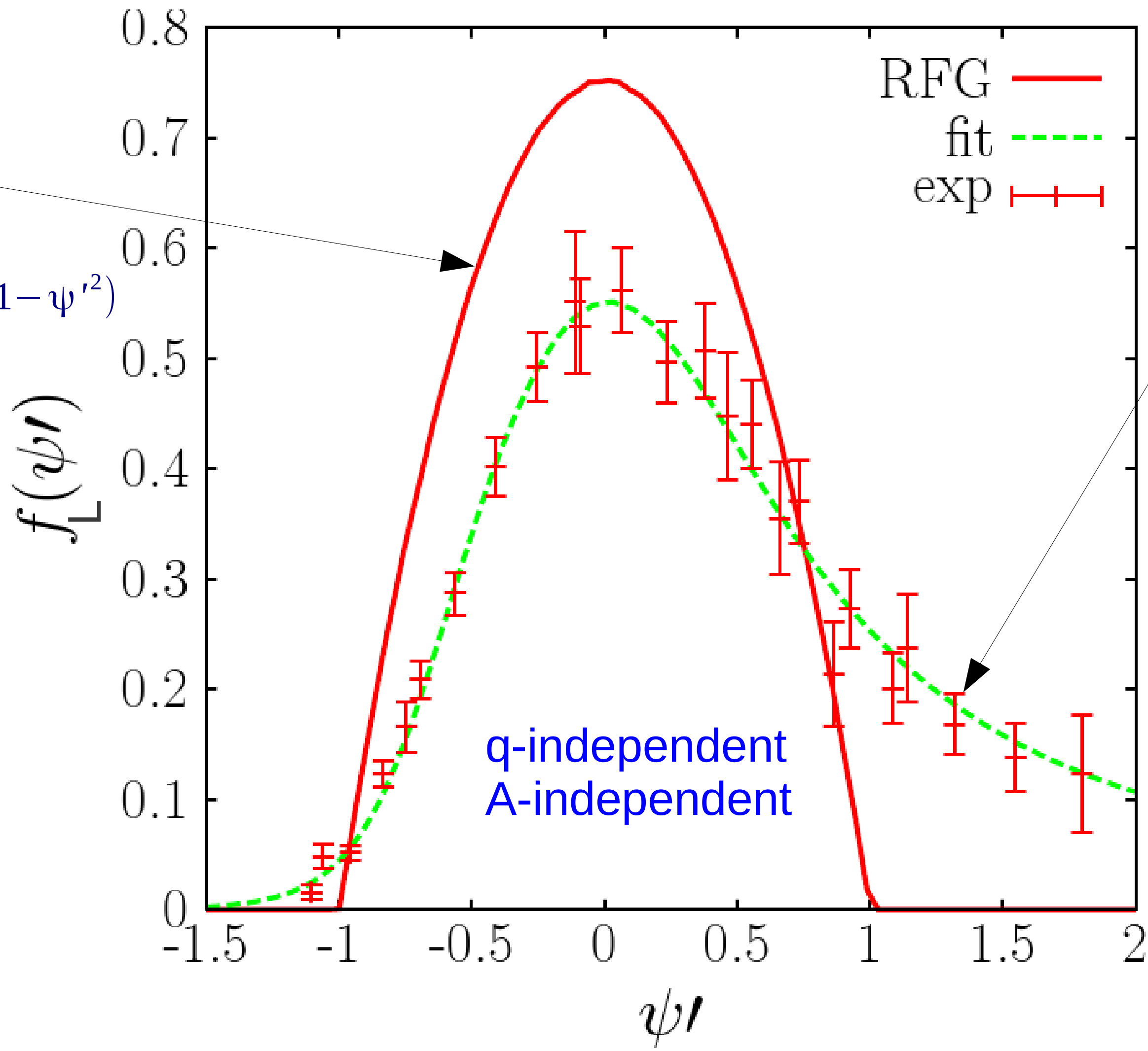

Asymmetric shape with long high energy tail

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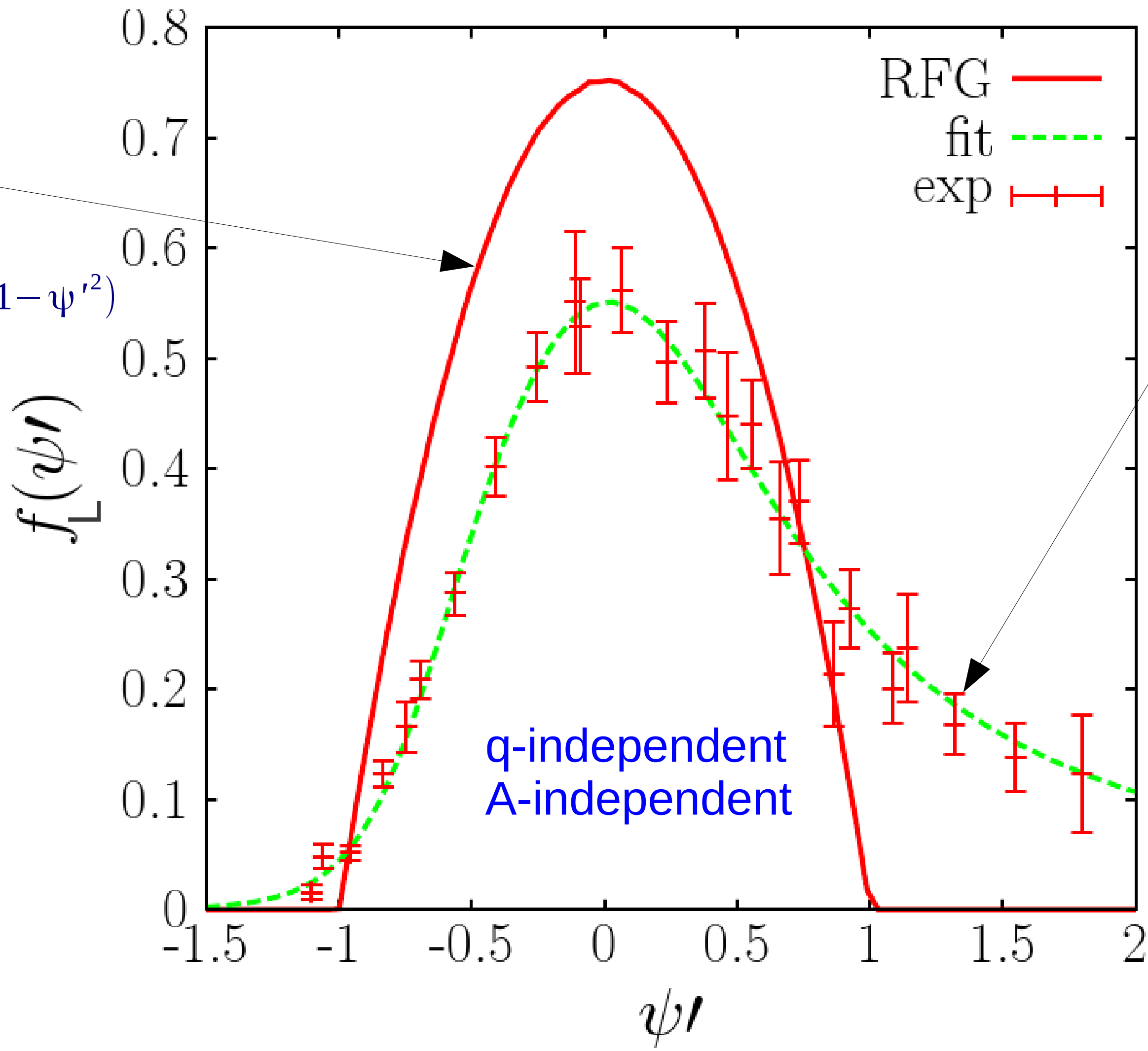


Asymmetric shape with long high energy tail

Strong constraint on nuclear models, which can be tested against this function (see Juan's talk)

0^{th} kind scaling (L/T)

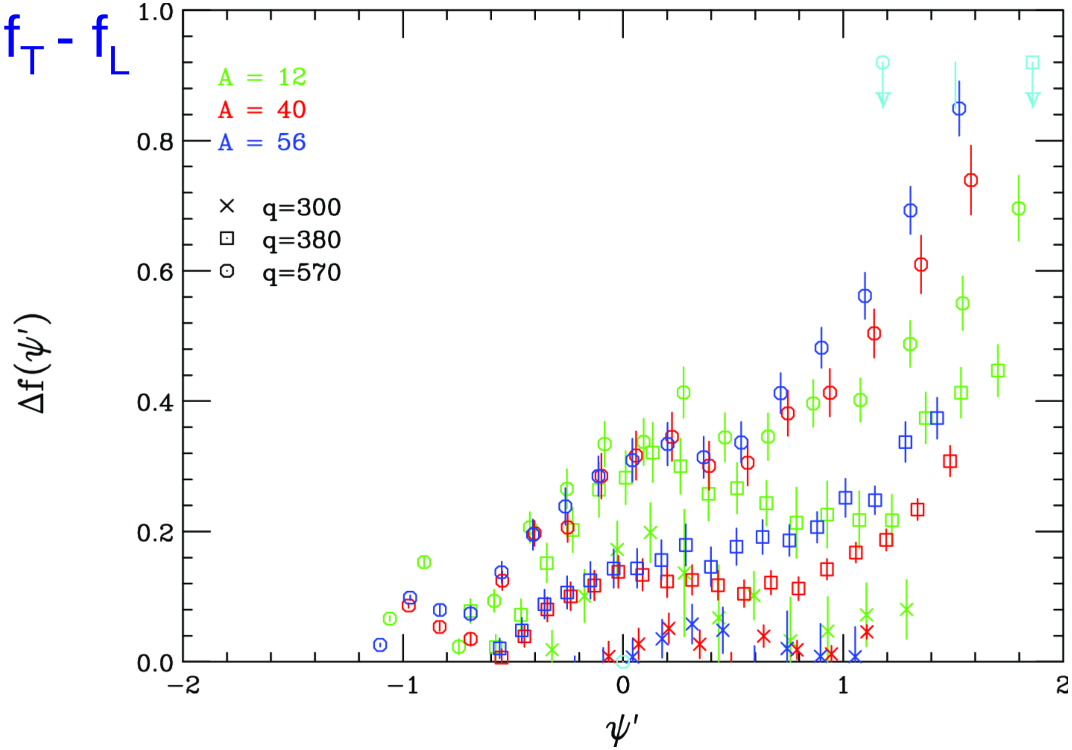
- In the RFG model

$$(f_L)_{RFG} = (f_T)_{RFG} = f_{RFG}$$

also called "scaling of 0^{th} kind".

- From the L/T separated data:

$f_T - f_L$



Violations from:

- resonances

- meson production

- tail of DIS

- Meson Exchange Currents

Ignoring these processes one can assume 0^{th} kind scaling: $f_L=f_T$

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Non-QE scaling

[MBB, J.A.Caballero, T.W.Donnelly, C.Maieron, Phys.Rev.C69, 035502 (2004);
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and define a new scaling variable to be used in the inelastic domain:

$$\psi_{QE}(\lambda,\tau) = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{\tau(1+\lambda) + \kappa\sqrt{\tau(1+\tau)}}} \Rightarrow \psi_X(\lambda,\tau) = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau\rho_X}{\sqrt{\tau(1+\lambda\rho_X) + \kappa\sqrt{\tau(1+\tau\rho_X^2)}}}$$

Recall: $\lambda = \omega/2m_N$
$\kappa = q/2m_N$
$\tau = |Q^2|/4m_N^2$

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$$R_{QE}^{L,T} = \frac{N}{\eta_F^3 \kappa m_N} \xi_F f_{model}(\psi_{QE}) U_{QE}^{L,T} \;\Rightarrow\; R_{inel}^{L,T} = \frac{N}{\eta_F^3 \kappa m_N} \xi_F \int_{\mu_{thresh}}^{1+2\lambda-\epsilon_S} d\mu_X \mu_X f_{model}(\psi_X) U^{L,T}$$

from Bodek-Ritchie parametrization *[PRD23 (1981); PRD24 (1981)]*

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from Bodek-Ritchie parametrization (for $U^{L,T}$)
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- **2p-2h meson-exchange current** effects must be added to have a complete descritpion of the (e,e') spectrum (Quique Amaro's talk)

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- CCν reactions are purely isovector, while (e,e') is both isoscalar and isovector

$$f_L \sim \tfrac{1}{2} f_L^{(T=0)} + \tfrac{1}{2} f_L^{(T=1)}$$

Thus in going from electron- to CCν-scattering we have to invoke a **3rd kind of scaling**:

$$f_L^{(T=0)} = f_L^{(T=1)}$$ Isospin-independence

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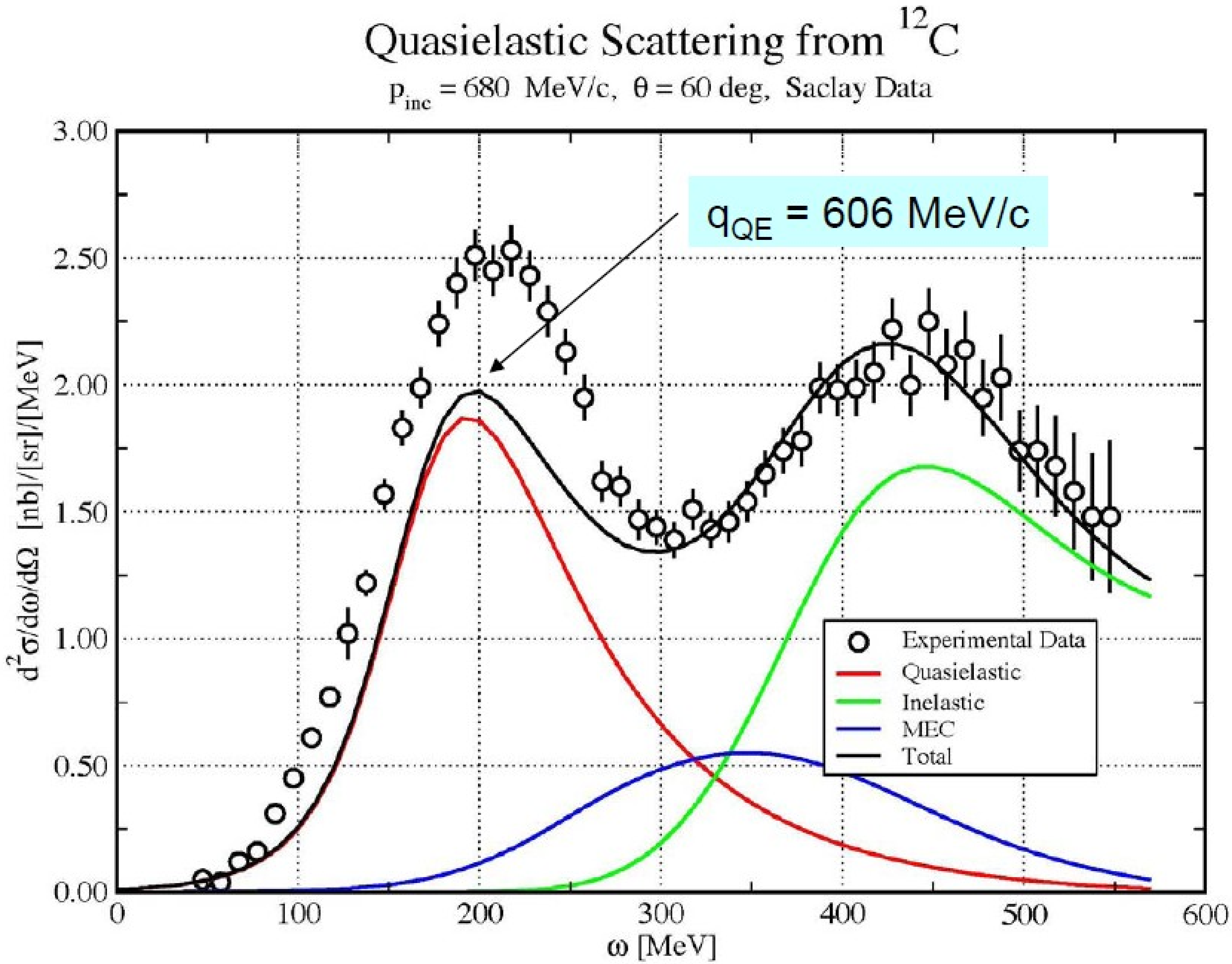
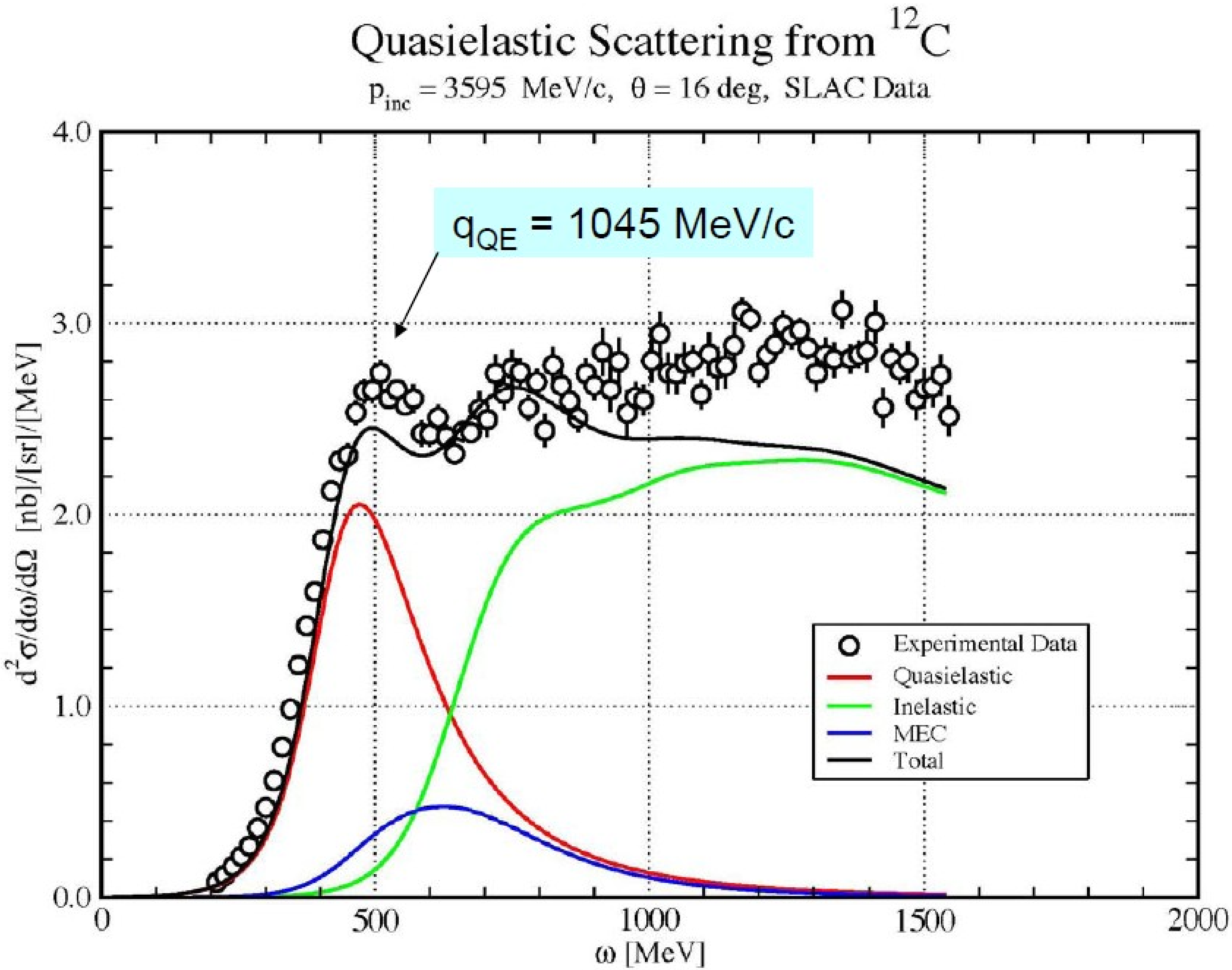
Test of SuSA vs (e,e') data: intermediate energy

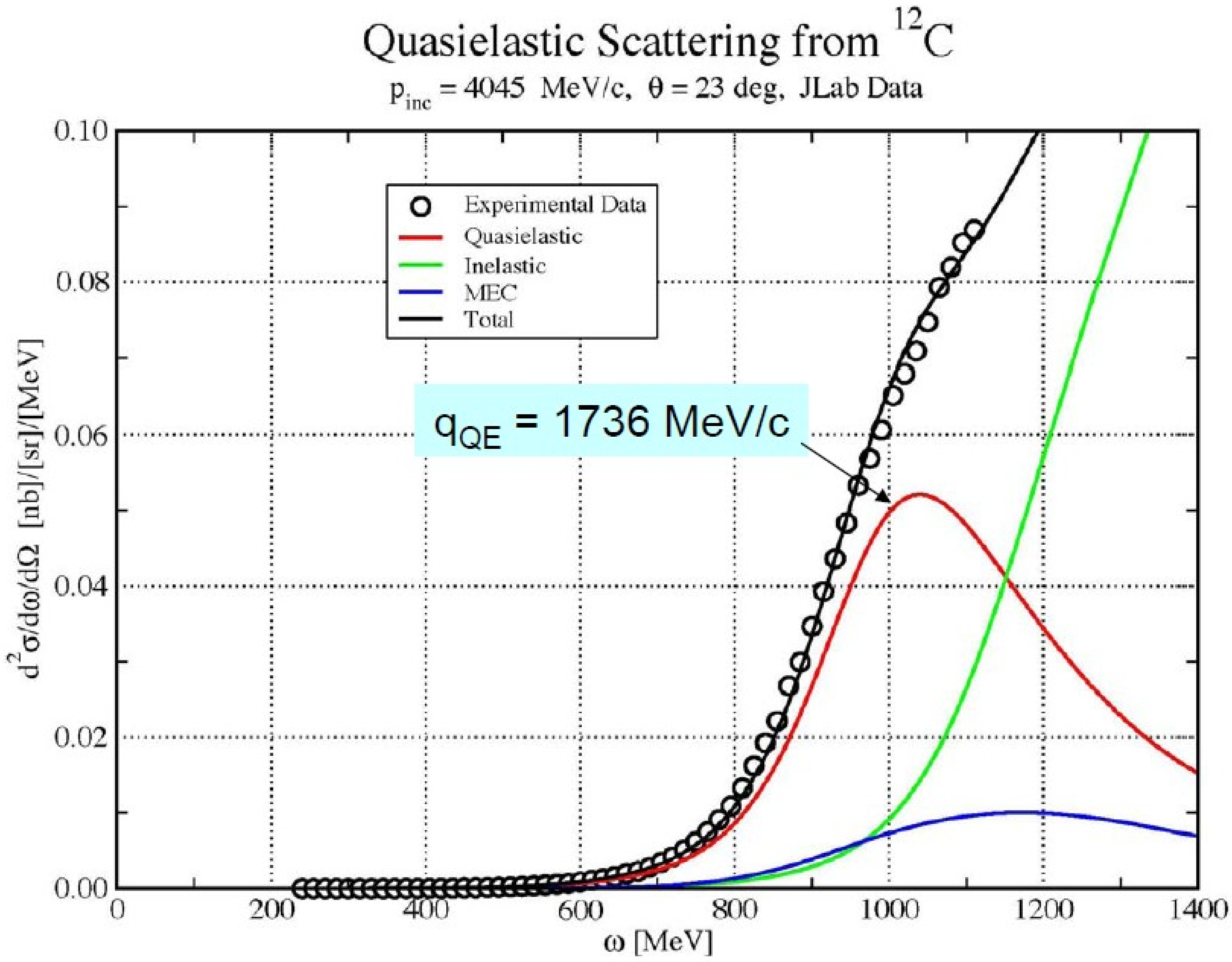


- Some strength is missing at the QEP
- 2p2h MEC are large in the “dip” region

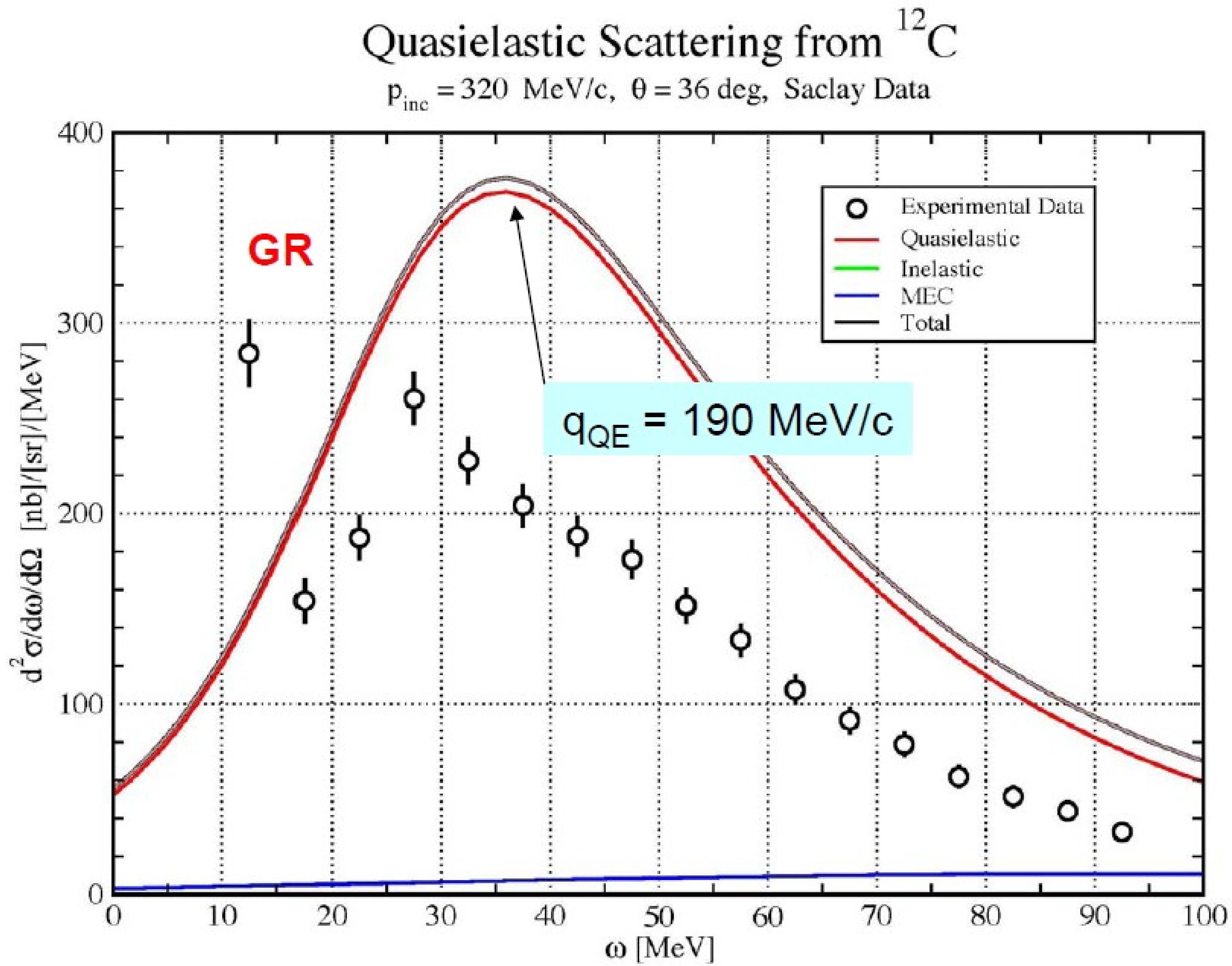
Test of SuSA vs (e,e') data: higher energy



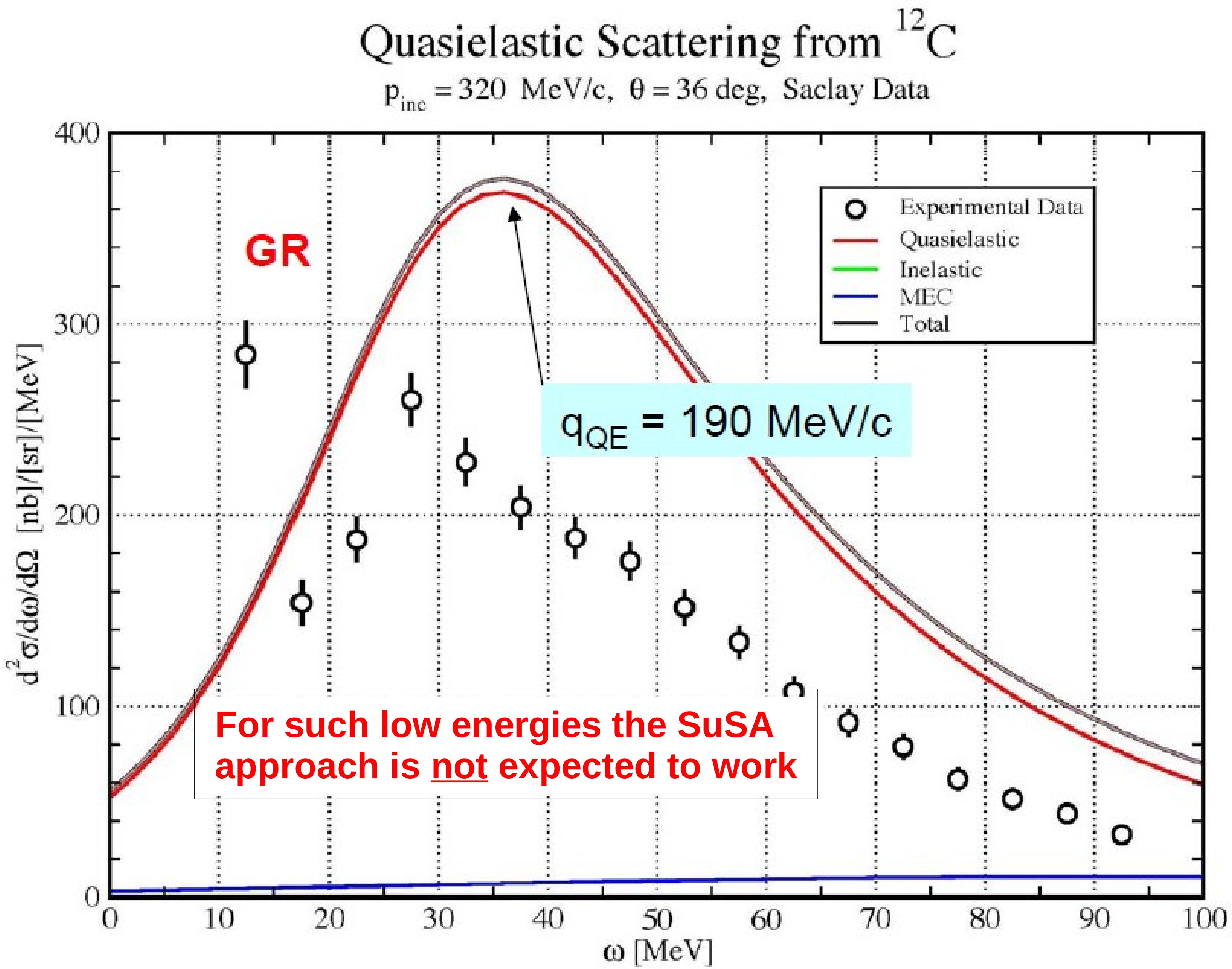
Test of SuSA vs (e,e') data: very high energy



Test of SuSA vs (e,e') data: low energy

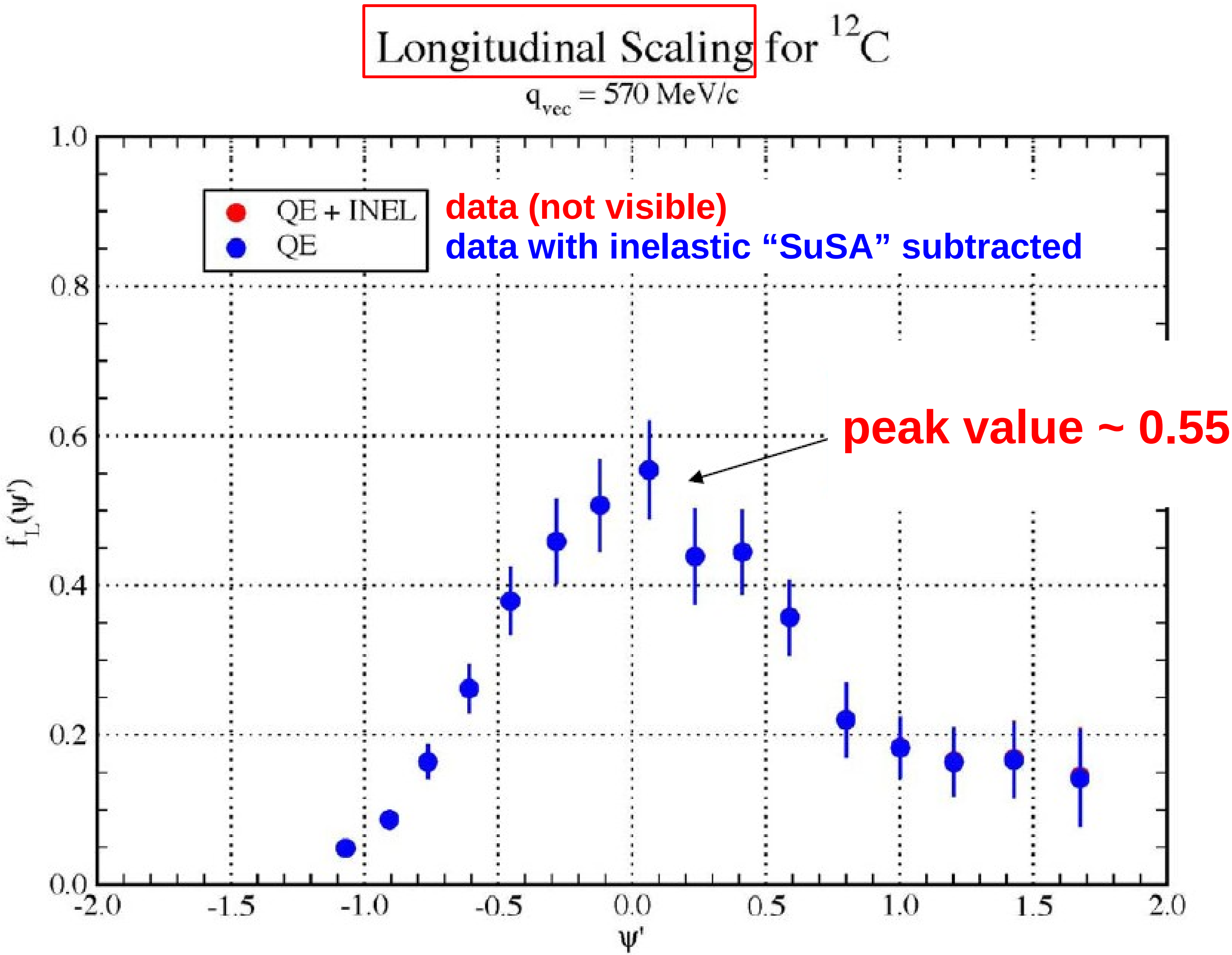


Test of SuSA vs (e,e') data: low energy



For such low energies the SuSA approach is not expected to work

Test of SuSA vs (e,e') data: L/T separation



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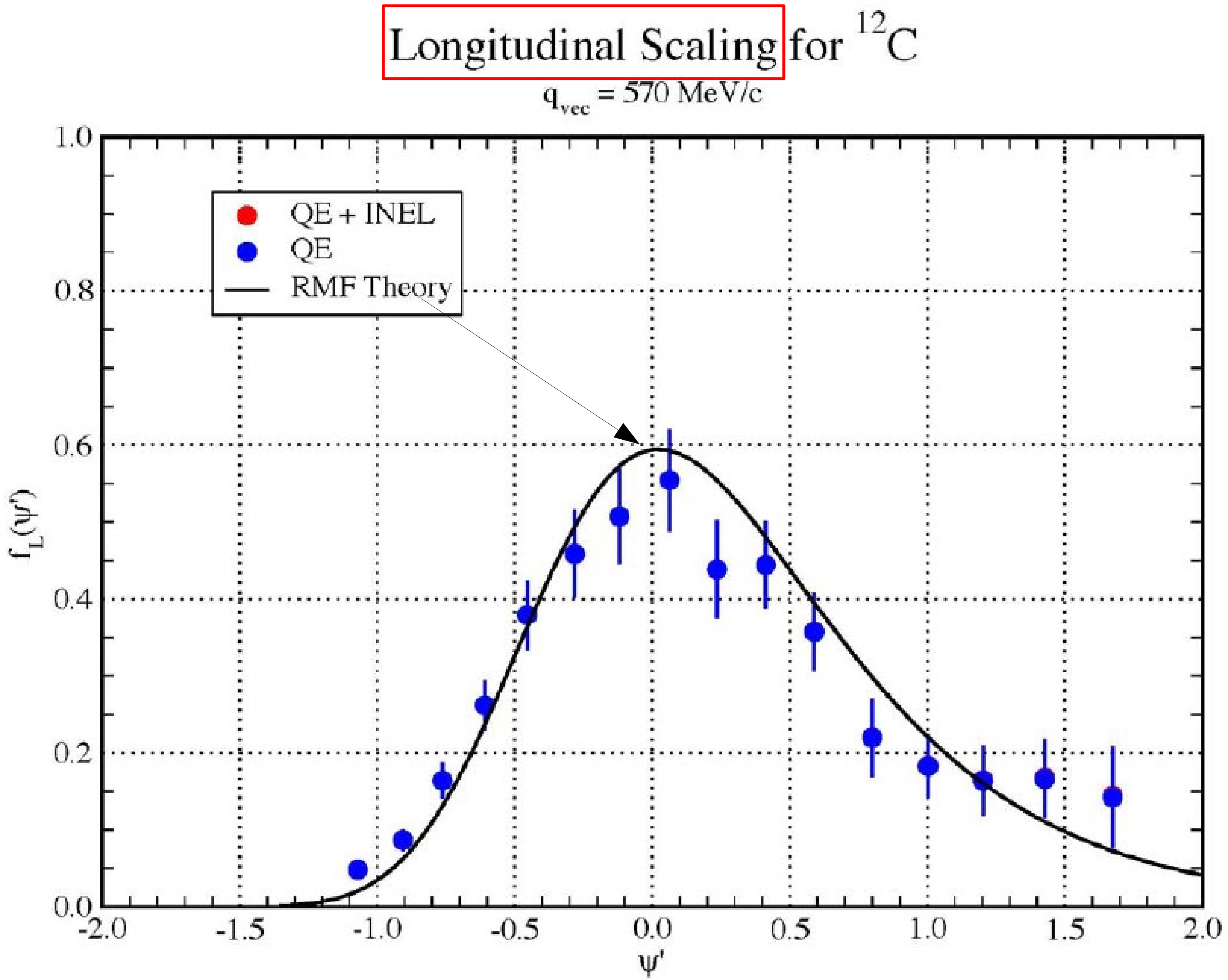

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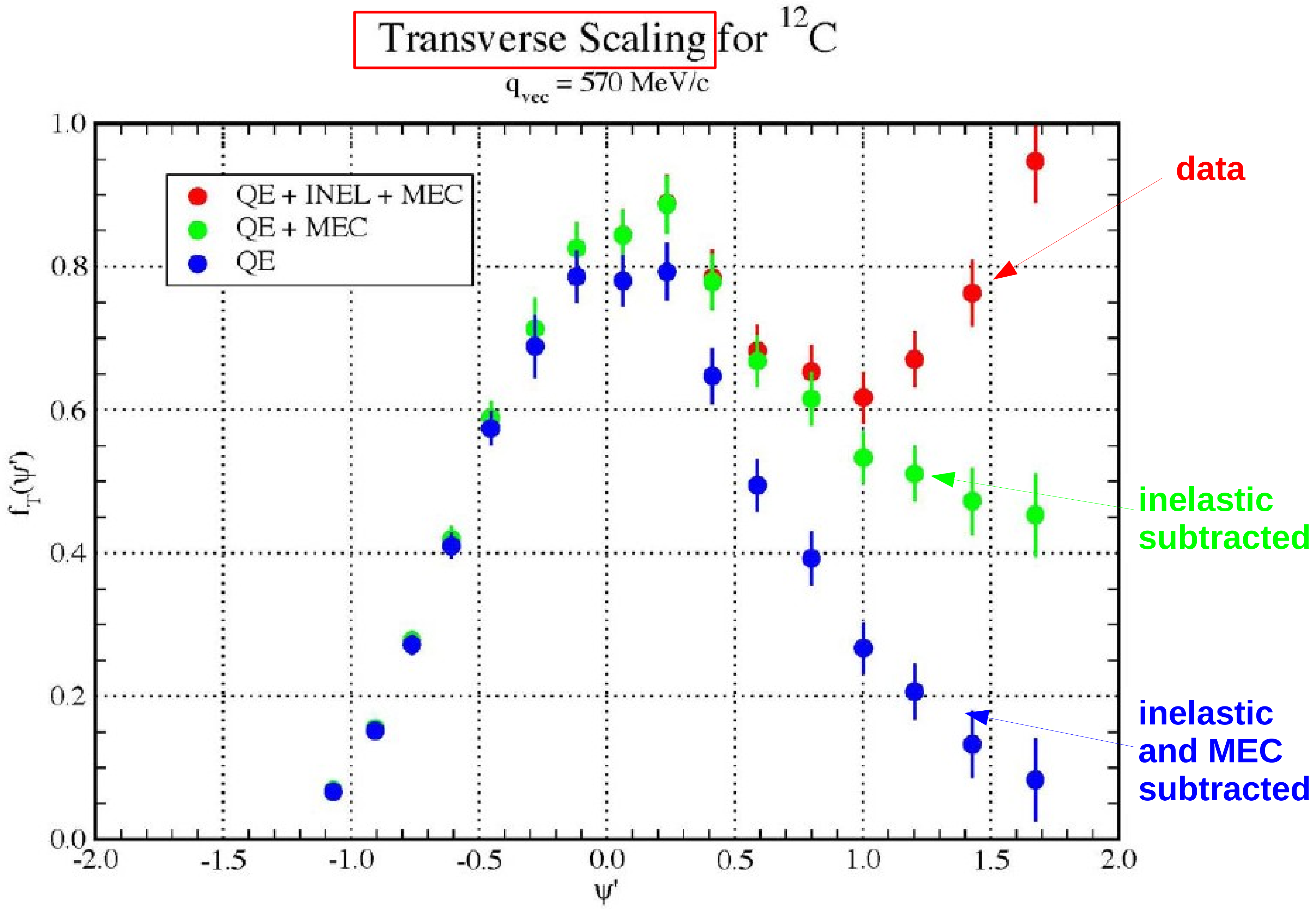

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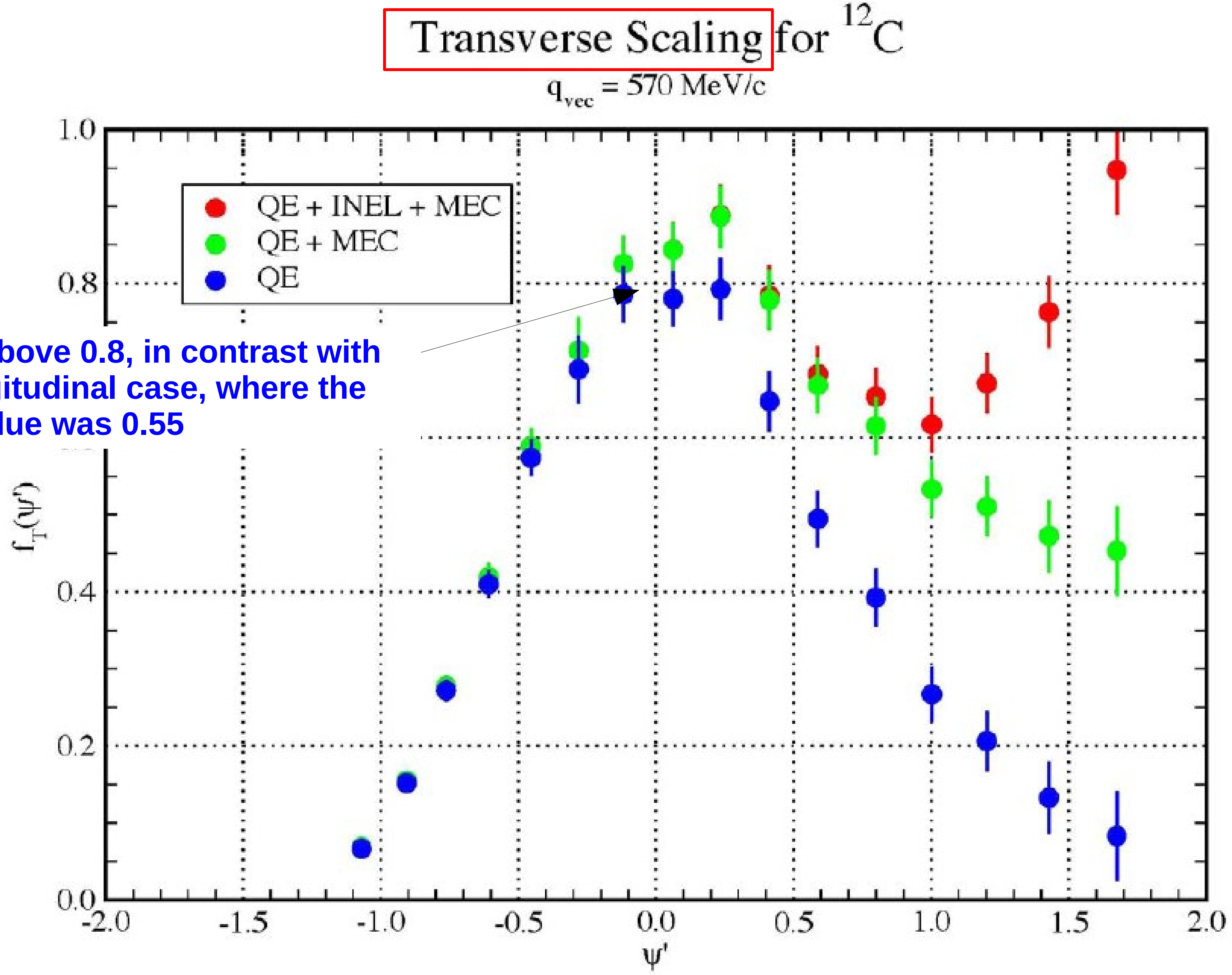

peaks above 0.8, in contrast with the longitudinal case, where the peak value was 0.55

first clear evidence of violation of scaling of 0th kind

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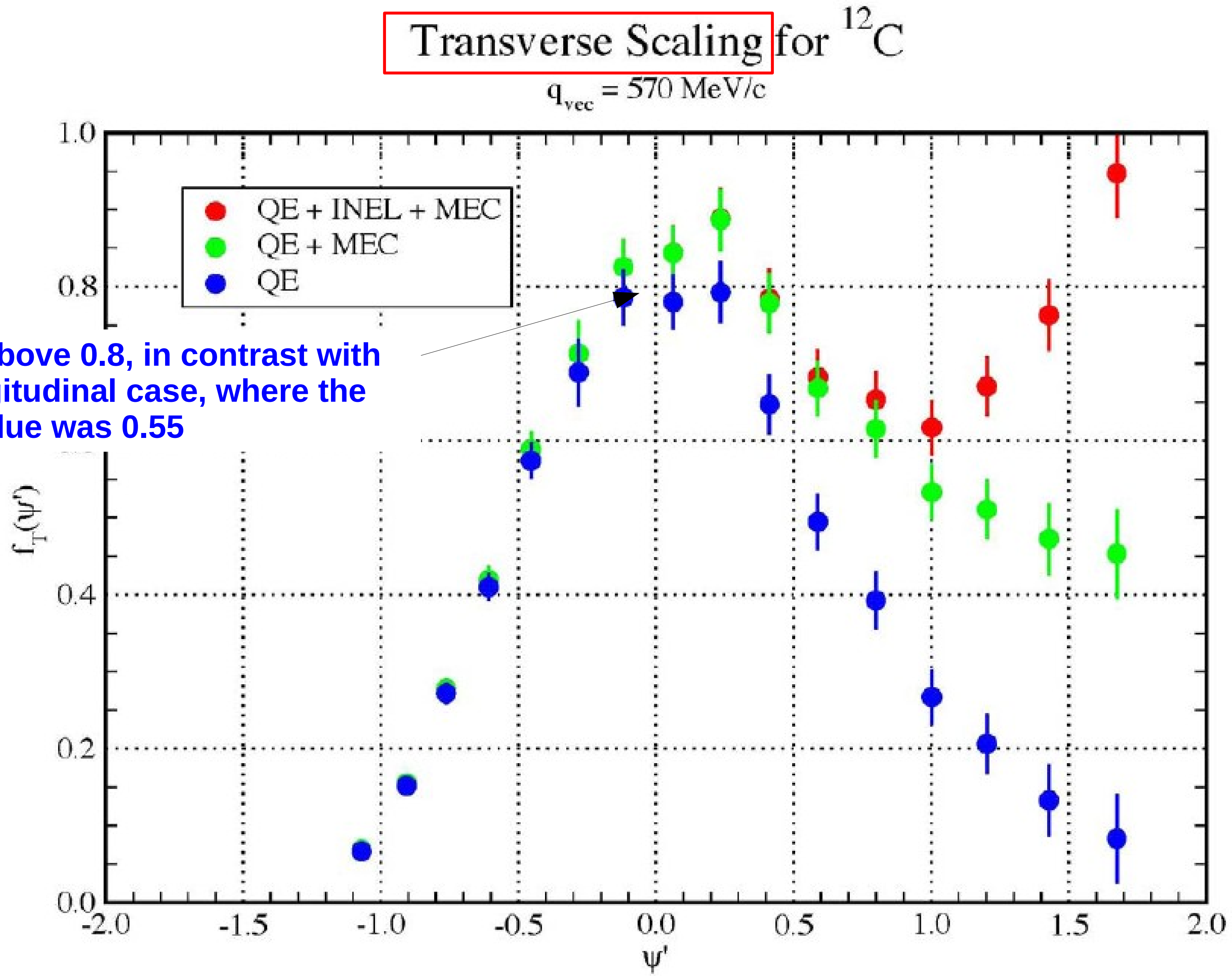

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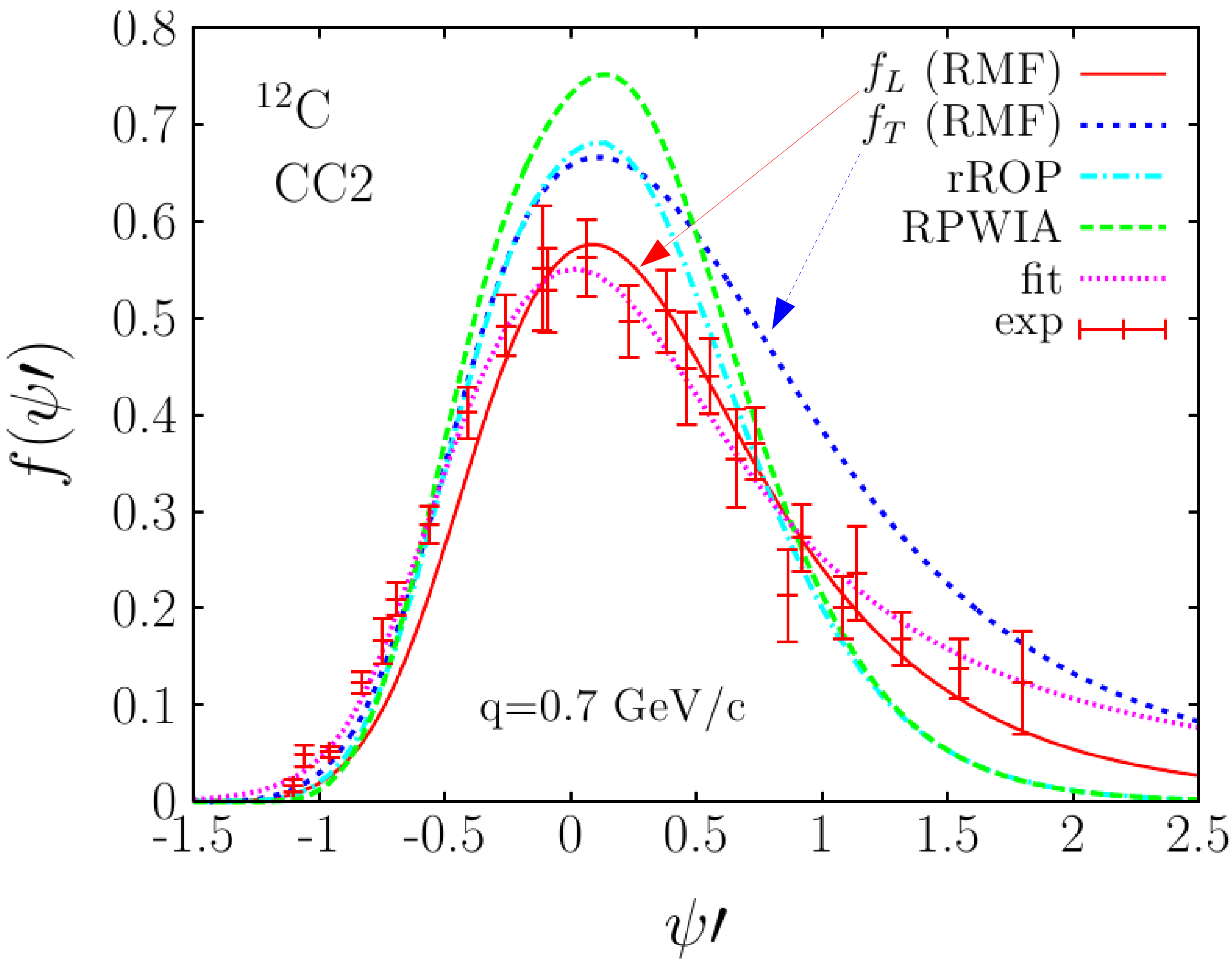
The RMF model predicts violation of scaling of 0th kind and gives better agreement with data than any other model so far. Transverse enhancement due to relativistic effects absent in other models.

Models vs the longitudinal scaling function



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[*Off-shell effects in the relativistic mean field model and their role in CC (anti)neutrino scattering at MiniBooNE kinematics M.V. Ivanov et al., Phys.Lett. B727 (2013) 265-271*]

- Work is in progress to implement this violation in the SuSA approach ("SuSA version 2")

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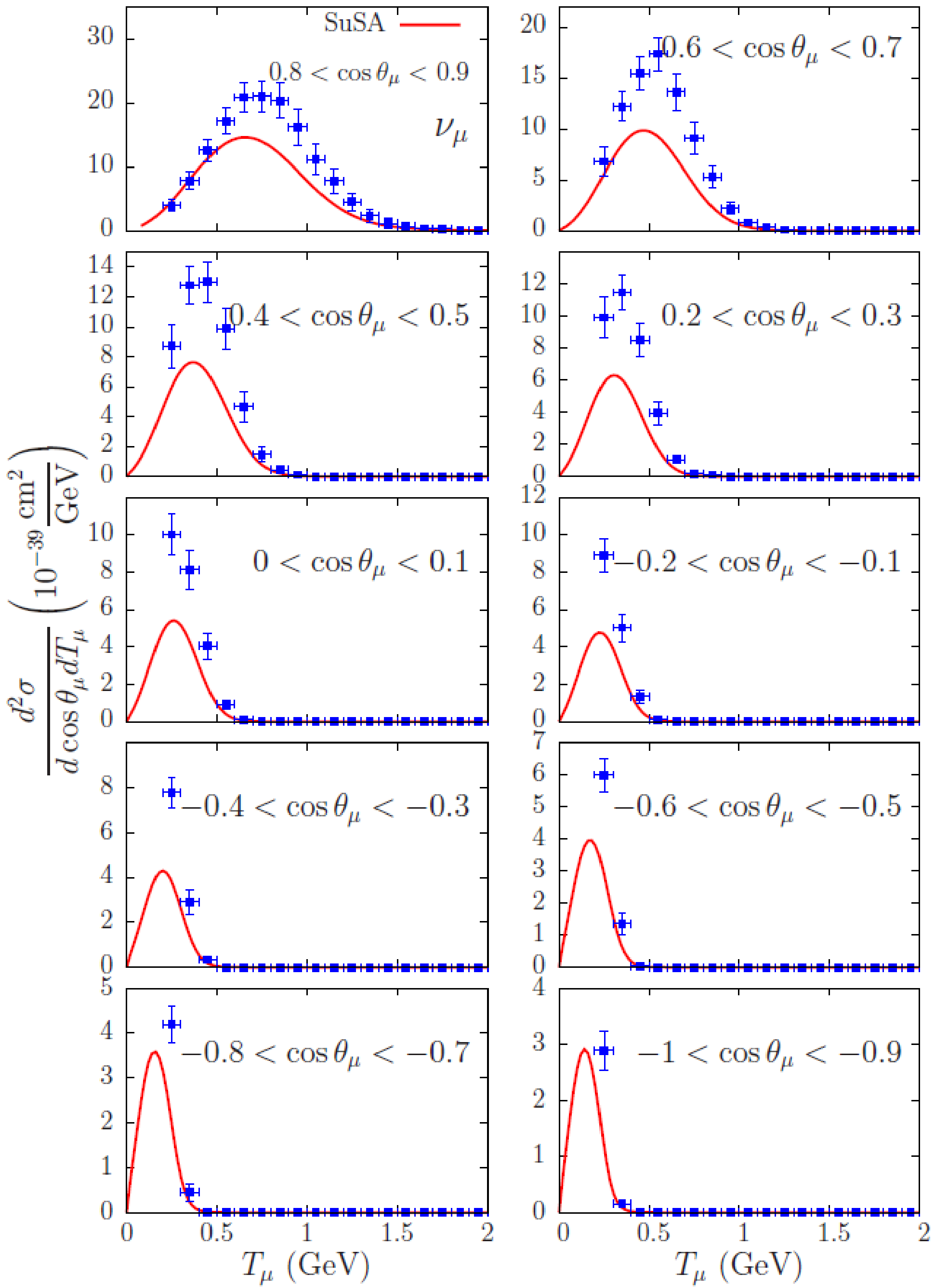
SuSA versus MiniBooNE ν data



The SuSA model underestimates the data

J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, C.F.Williamson, Physics Letters B 696 (2011) 151–155

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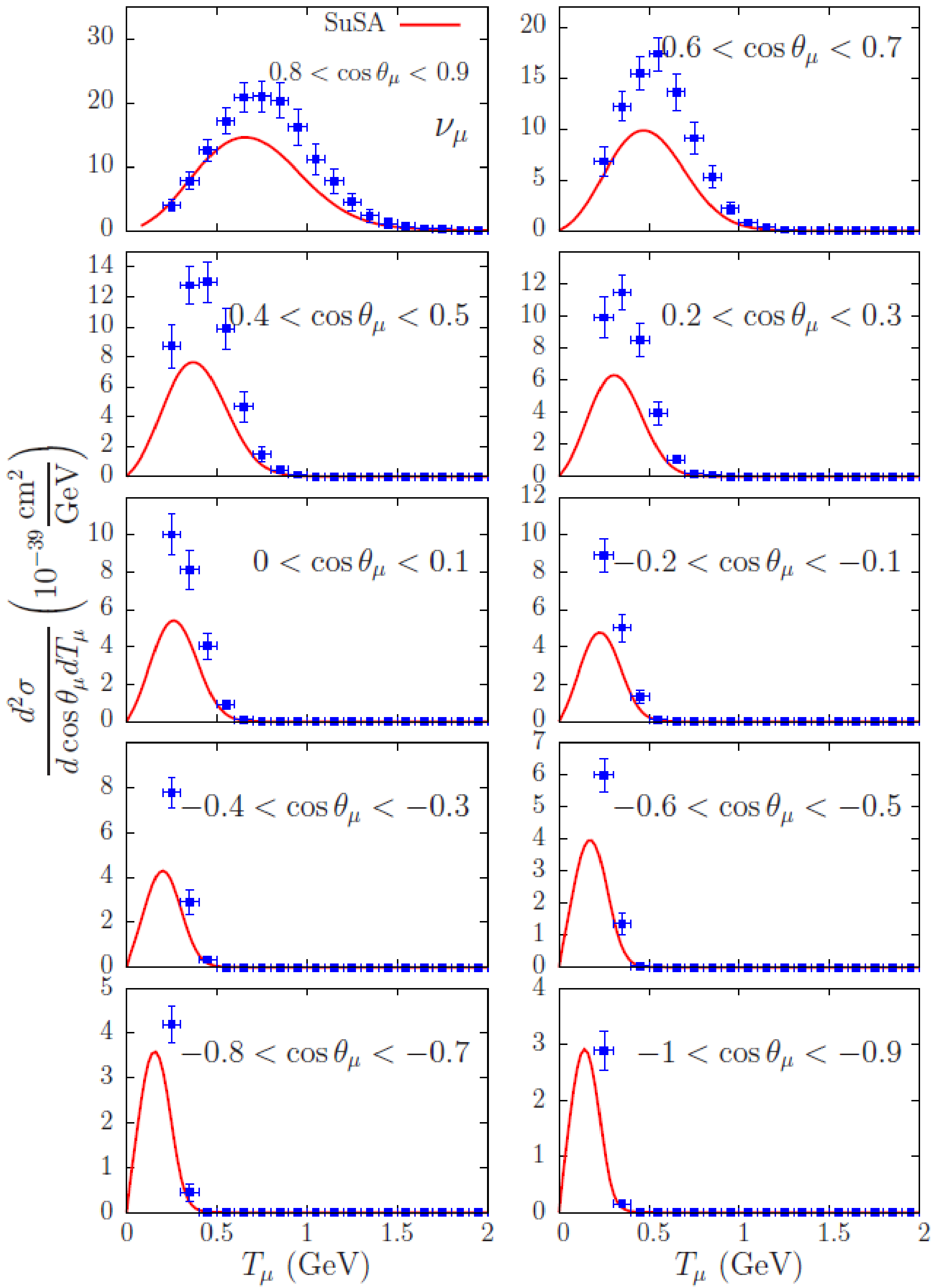
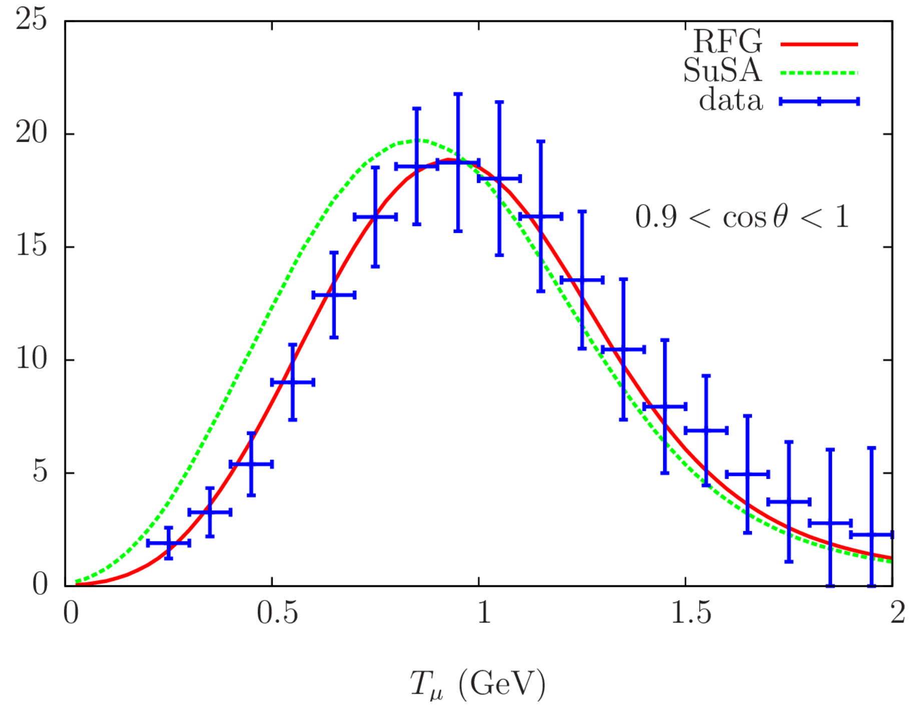


The SuSA model underestimates the data except for the first angular bin:



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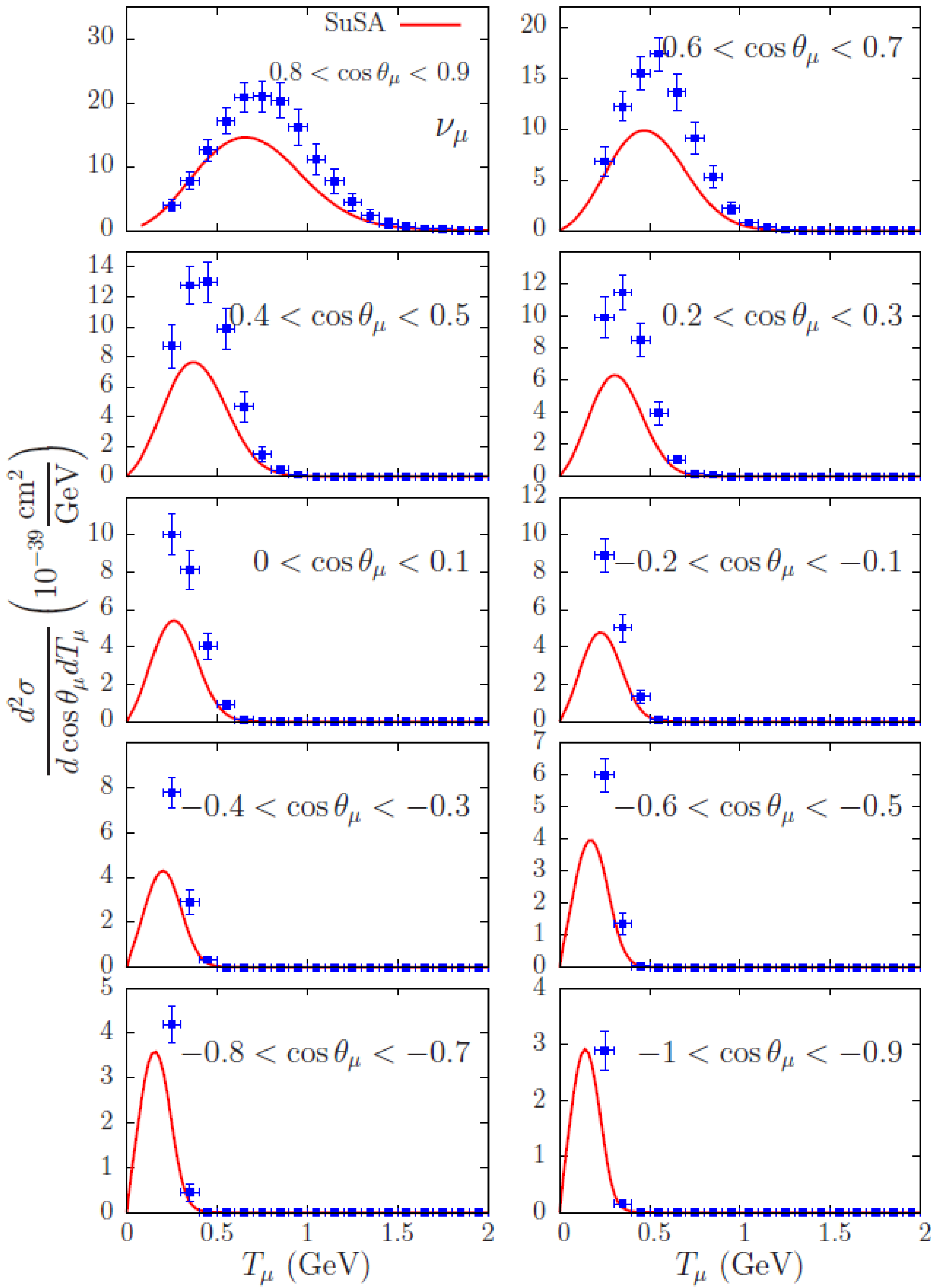

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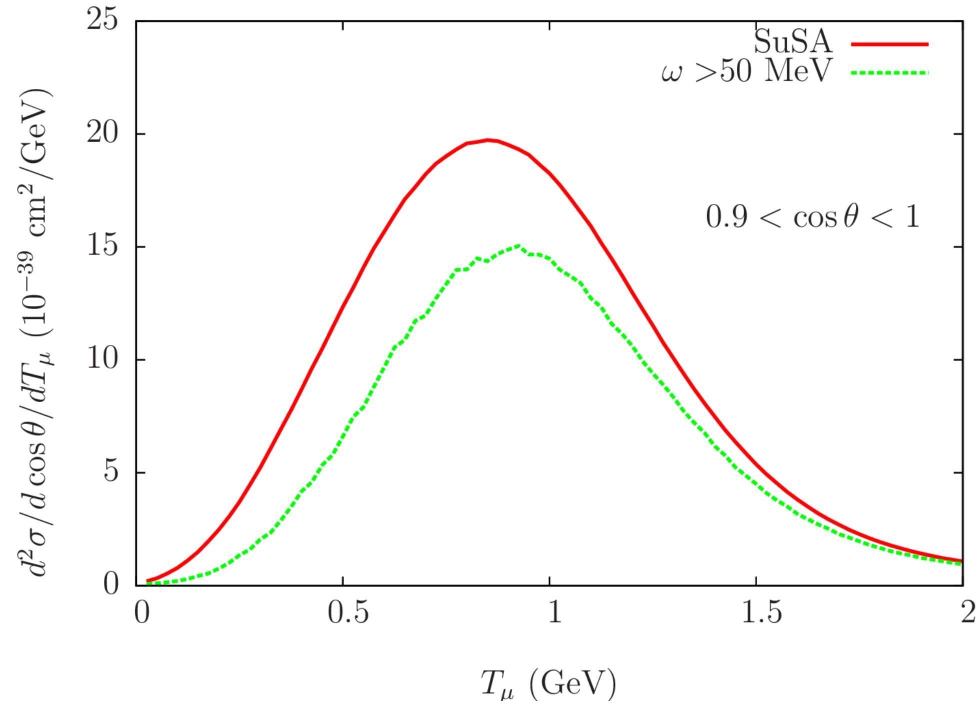

...which is however very sensitive to low excitation energies (ω<50 MeV) and requires a totally different nuclear modeling (discrete sates, giant resonance, etc.)...

J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, C.F.Williamson, Physics Letters B 696 (2011) 151–155

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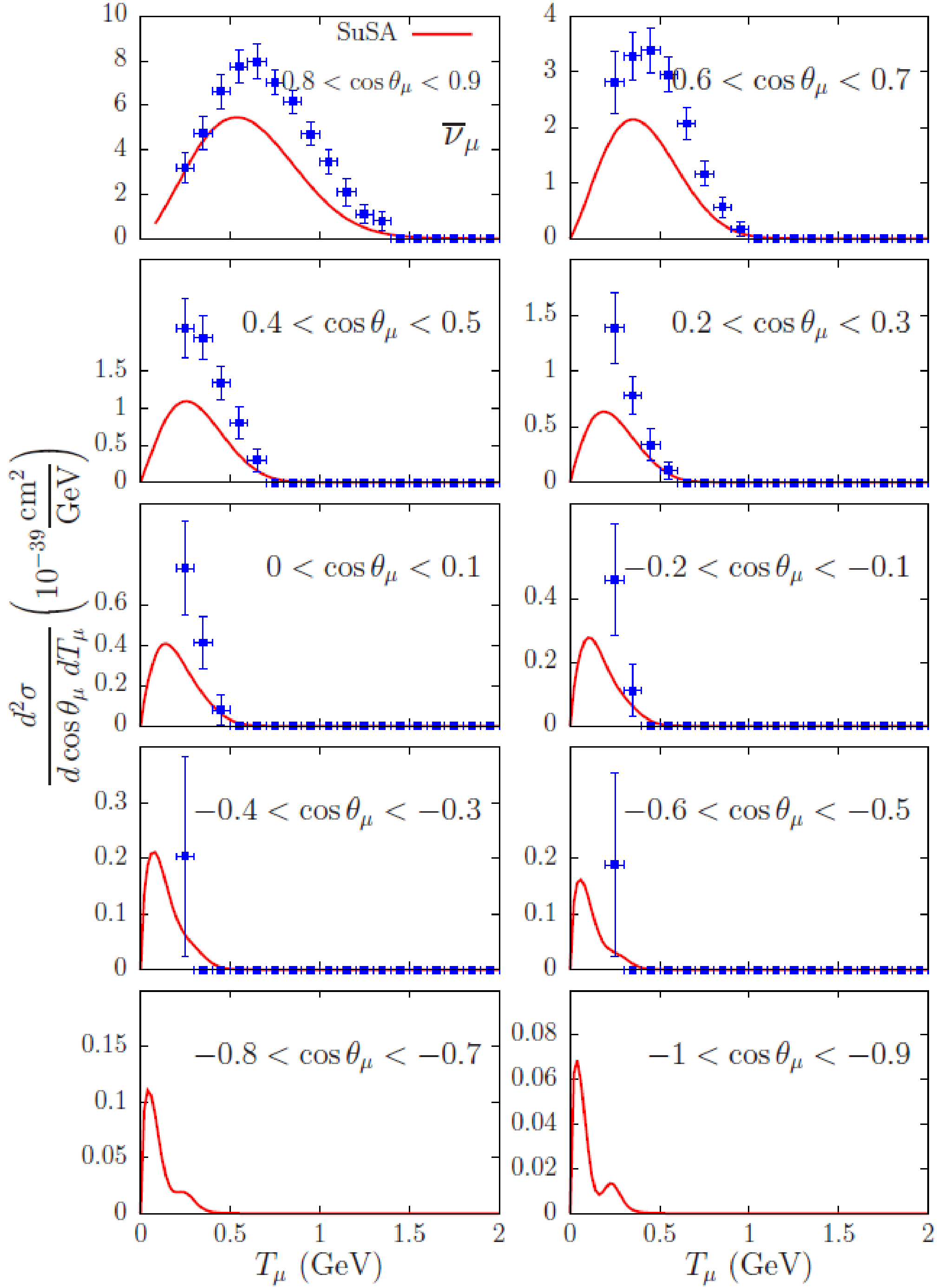
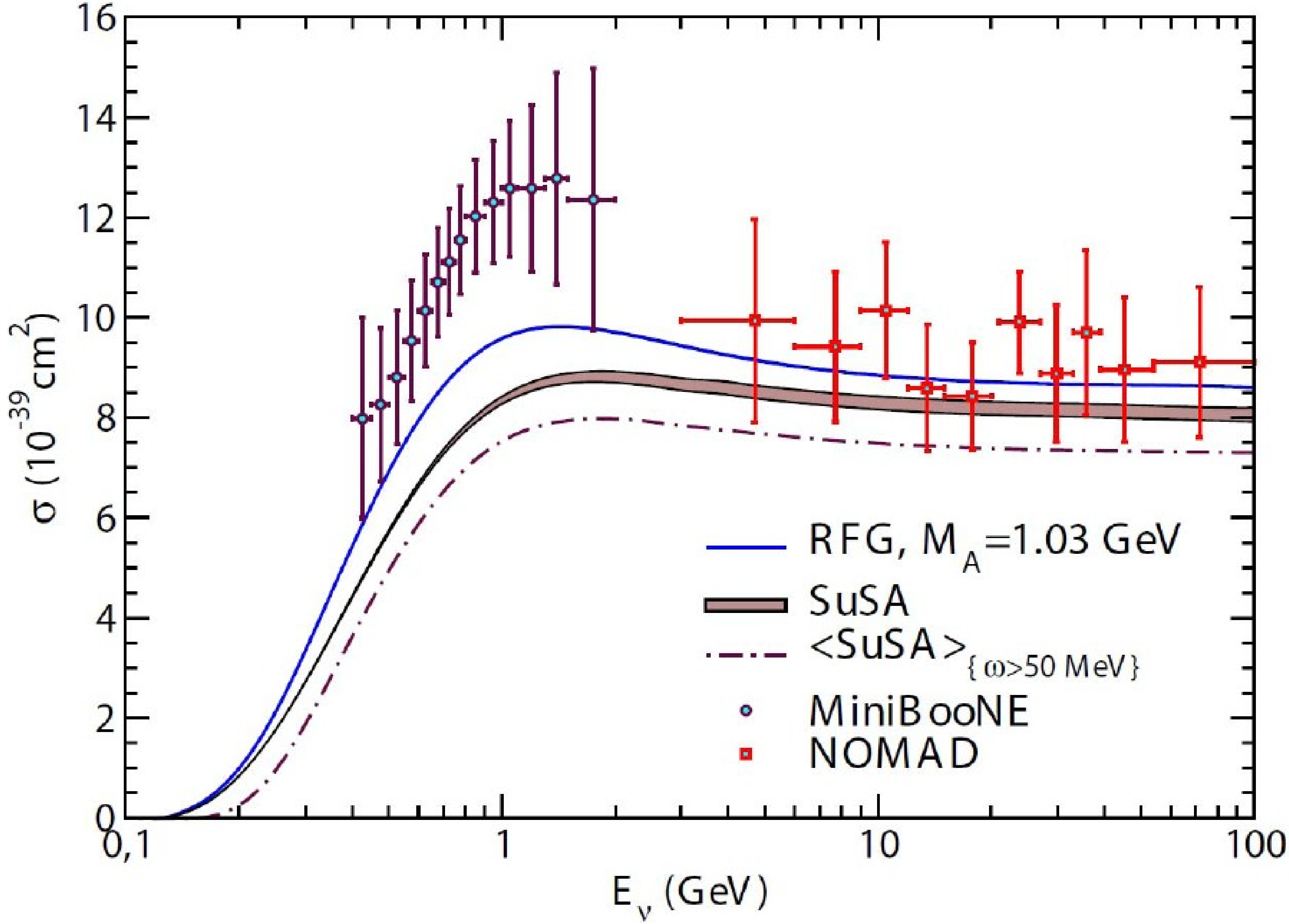


J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, Physical Review Letters 108, 152501 (2012)

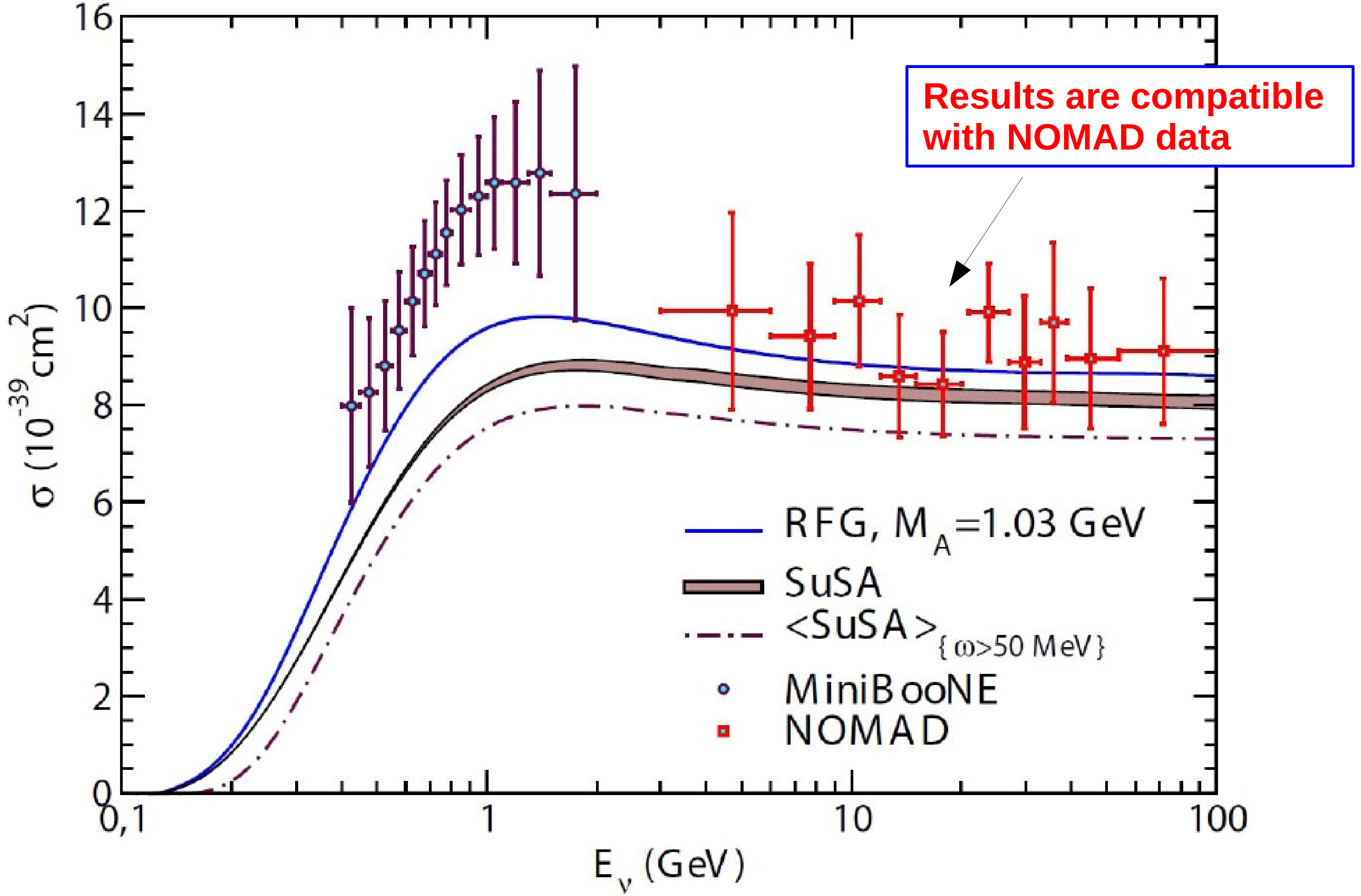
From low to high neutrino energies

The SuSA model can be applied to high energy (NOMAD kinematics) for CCQEν:



G.D.Megias, J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, Phys.Lett. B725 (2013) 170-174

From low to high neutrino energies



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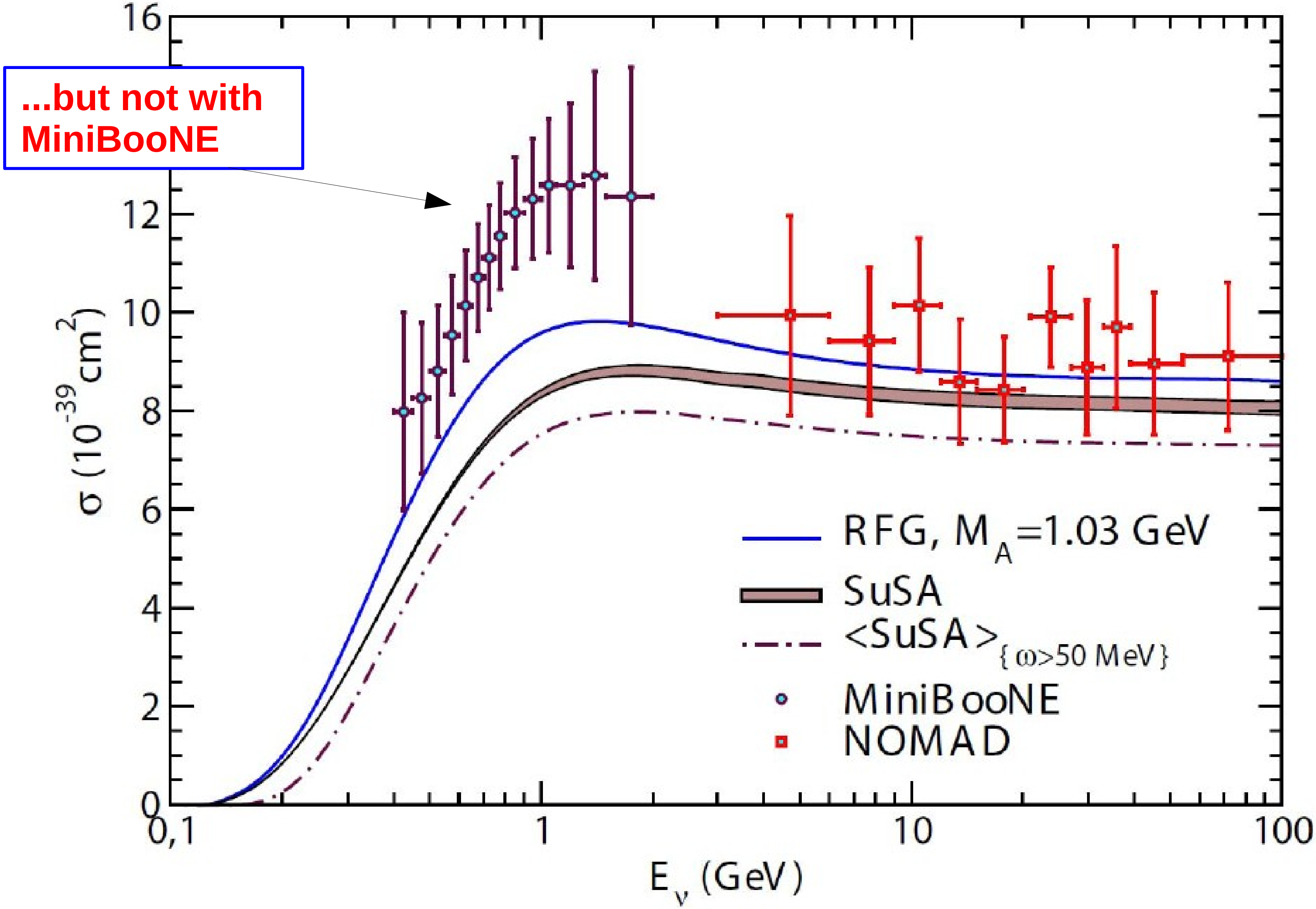

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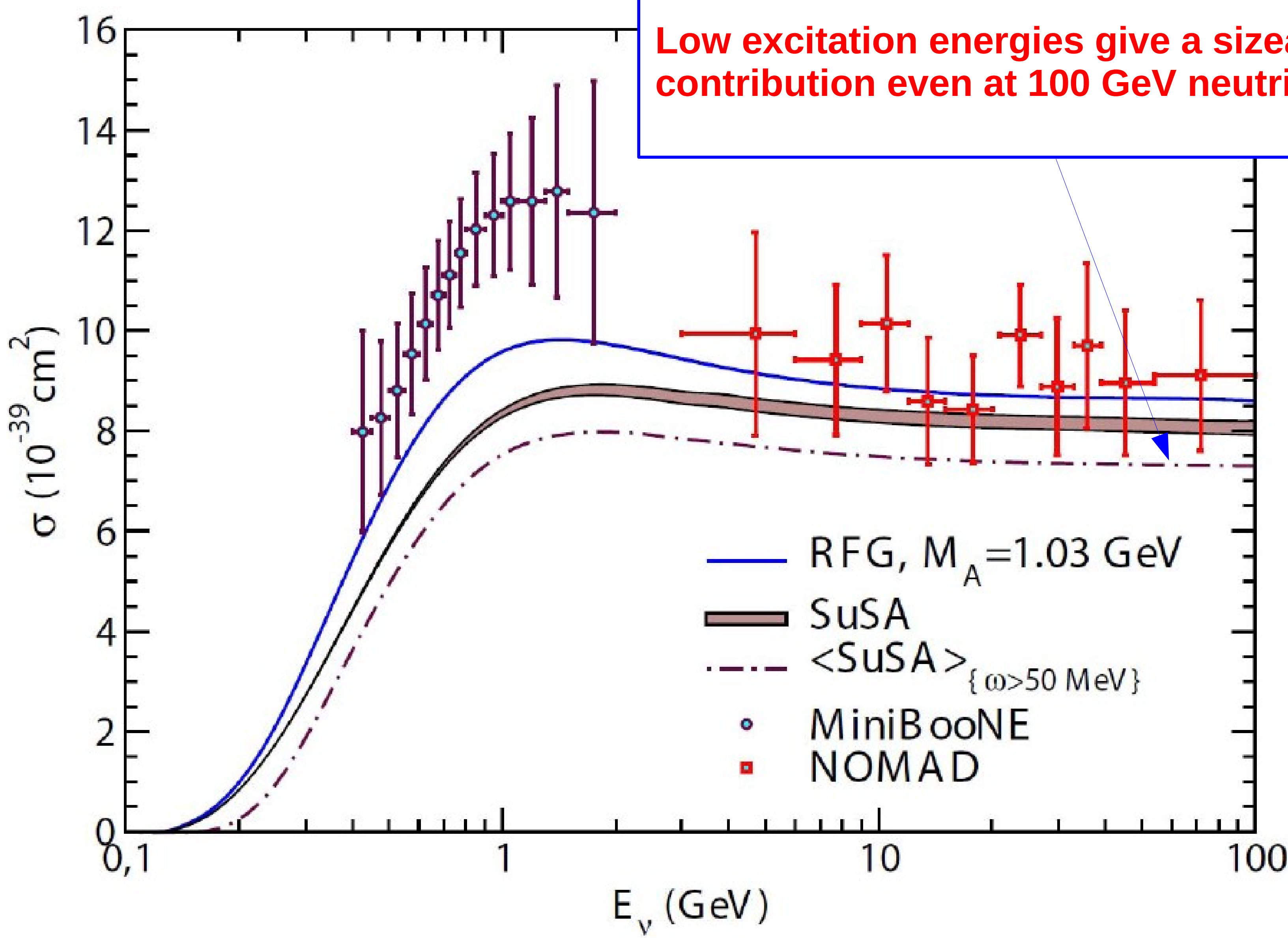
Low excitation energies give a sizeable contribution even at 100 GeV neutrino energy!



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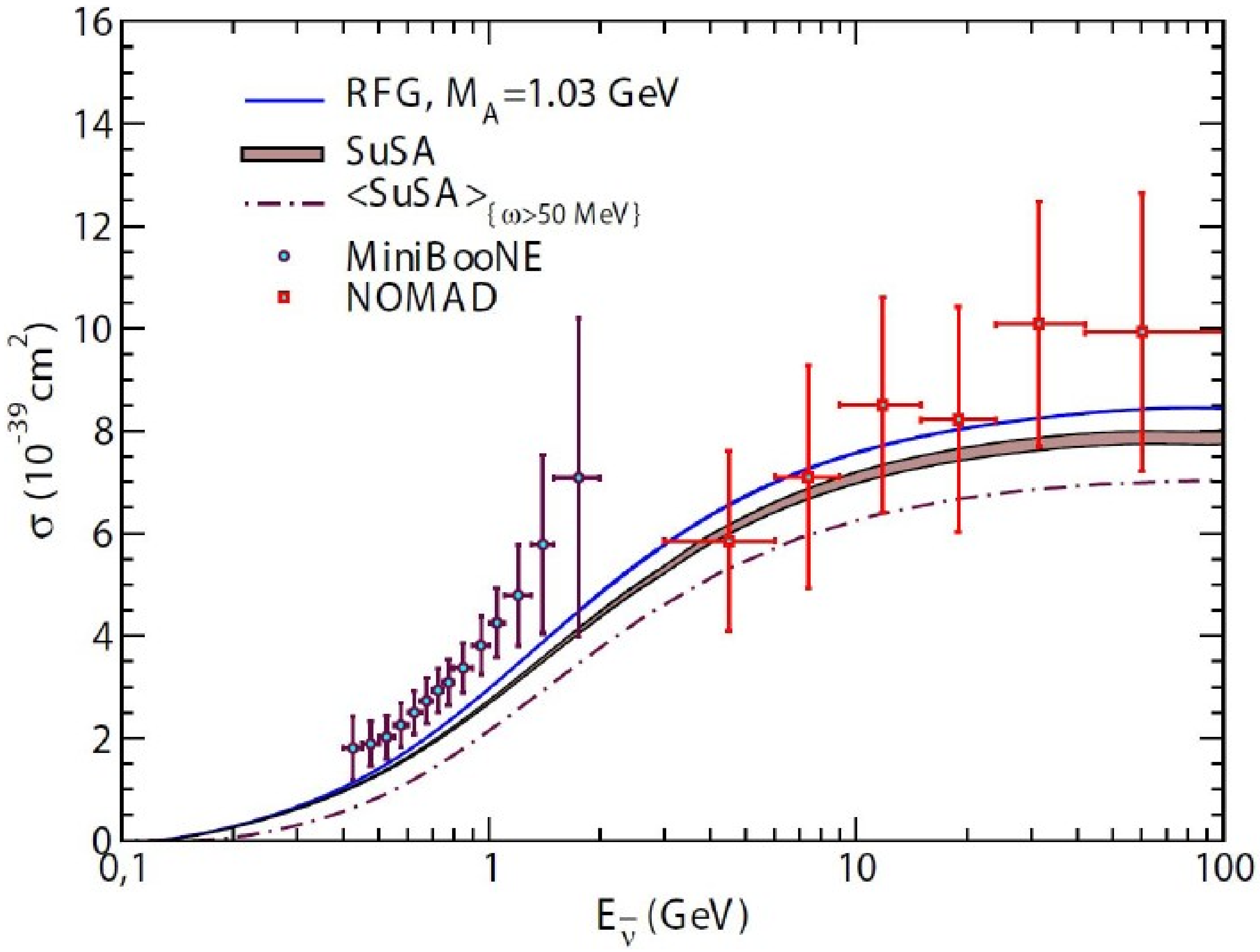
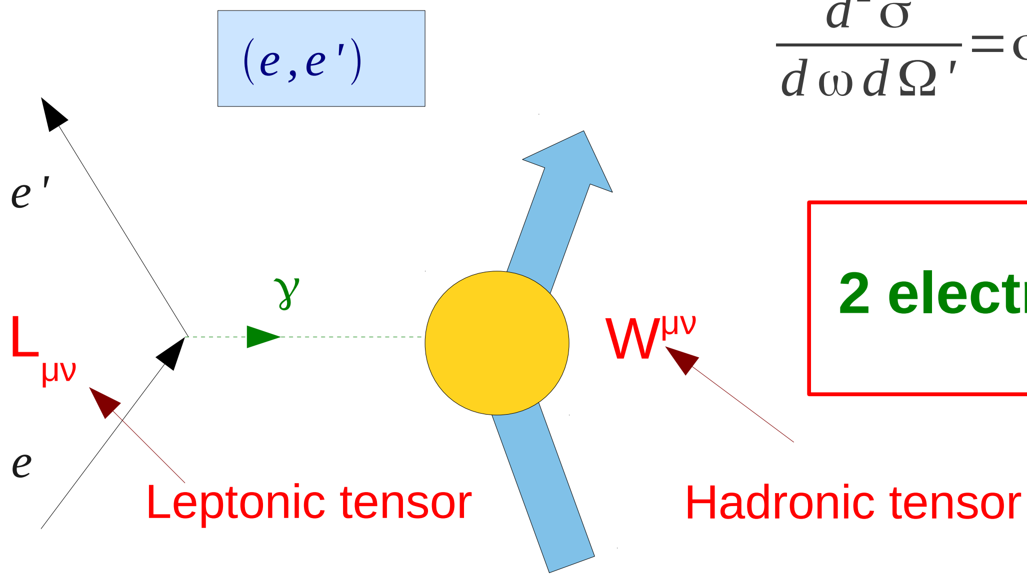
From low to high neutrino energies

Similarly for CCQE antineutrino:



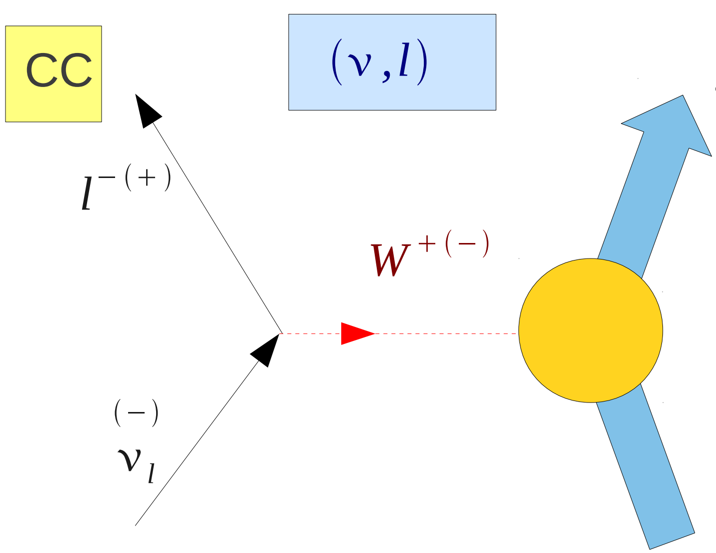
G.D.Megias, J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, Phys.Lett. B725 (2013) 170-174

L, T and T' separate contributions



$$\frac{d^2\sigma}{d\omega\, d\Omega'} = \sigma_{Mott}\left(v_L R_L + v_T R_T\right)$$

2 electromagnetic response functions



$$\frac{d^2\sigma}{d\omega\, d\Omega'} = \sigma_0\left(V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} + V_T R_T \pm 2V_{T'}R_{T'}\right)$$

$V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL} \rightarrow V_L R_L$

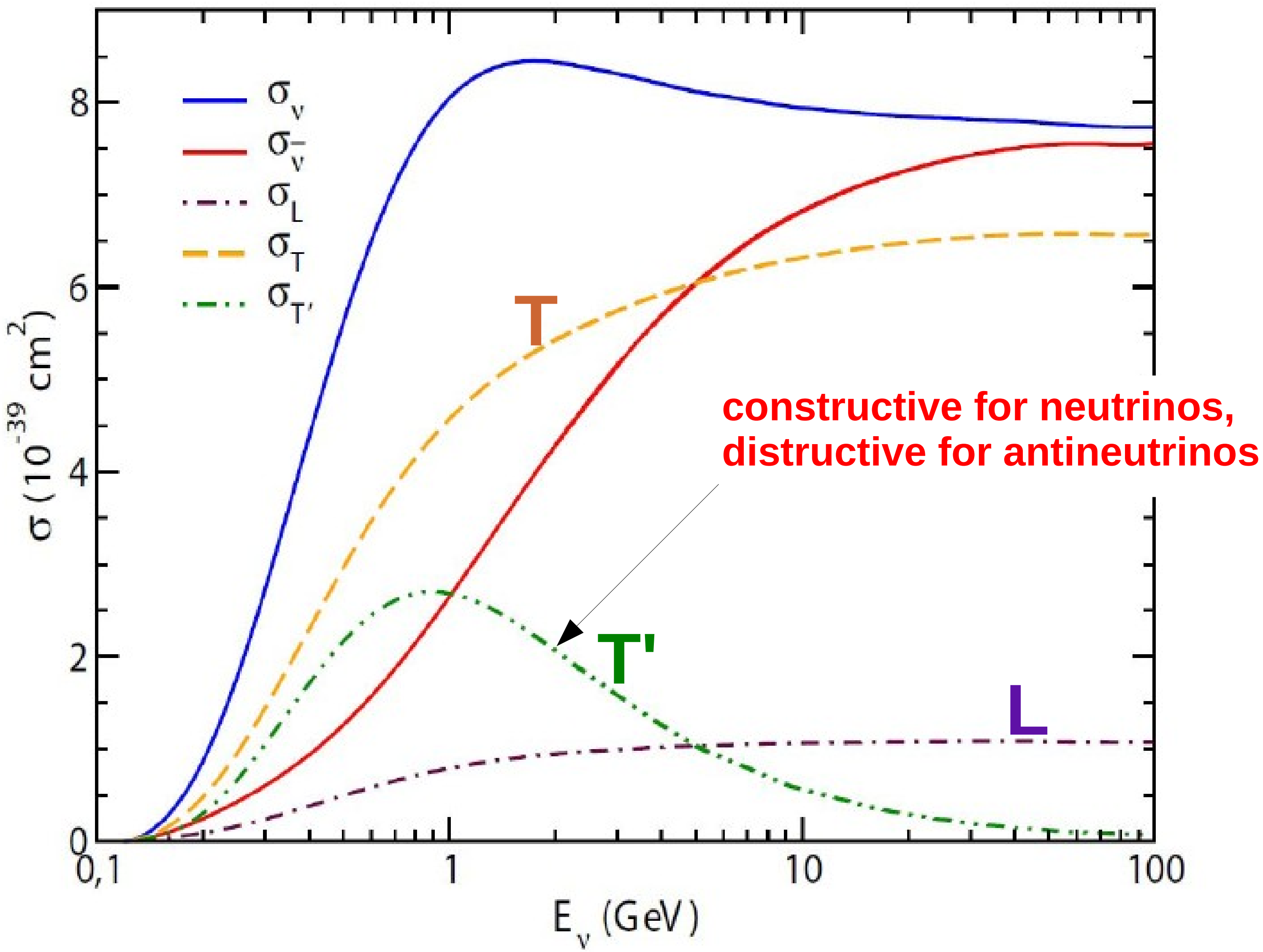
$+\nu$
$-\nu$

$l = \mu, e, \tau$

5 (3) weak response functions

Purely isovector
Typically transverse (CC,CL,LL small)
Have VV, AA and VA components generated by $J_\mu = J_\mu^V + J_\mu^A$

L, T and T' separate contributions



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Collaborators

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Madrid: Elvira Moya, Jose Manuel Udias;
Valencia: Luis Alvarez-Ruso)

Italy (Torino: M.B.B., Arturo De Pace, Alfredo Molinari;
Pavia: Carlotta Giusti, Andrea Meucci;
Chiara Maieron)

Switzerland (Basel: Ingo Sick)

Bulgaria (Sofia: Anton Antonov, Martin Ivanov)

References

Super-Scaling in Quasielastic Electron Scattering

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  T.W. Donnelly, Ingo Sick
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- **Superscaling in inclusive electron - nucleus scattering**
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- **Scaling in electron scattering from a relativistic Fermi gas**
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  M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C. Maieron
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  C. Maieron, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C.F. Williamson
  Phys.Rev. C80 (2009) 035504

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Application to Neutrino Reactions (I)

- **Using electron scattering superscaling to predict charge-changing neutrino cross sections in nuclei**
  J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, A. Molinari, I. Sick
  Phys.Rev. C71 (2005) 015501

- **Superscaling in charged current neutrino quasielastic scattering in the relativistic impulse approximation**
  J.A. Caballero, Jose Enrique Amaro, M.B. Barbaro, T.W. Donnelly, C. Maieron, J.M. Udias
  Phys.Rev.Lett. 95 (2005) 252502

- **Superscaling and neutral current quasielastic neutrino-nucleus scattering**
  Jose Enrique Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly
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- **Quasielastic Charged Current Neutrino-nucleus Scattering**
  J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly
  Phys.Rev.Lett. 98 (2007) 242501

- **Final-state interactions and superscaling in the semi-relativistic approach to quasielastic electron and neutrino scattering**
  J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, J.M. Udias
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Application to Neutrino Reactions (II)

Scaling and isospin effects in quasielastic lepton-nucleus scattering in the Relativistic Mean Field Approach
J.A. Caballero, J.E. Amaro, M.B. Barbaro, T.W. Donnelly, J.M. Udias
Phys.Lett. B653 (2007) 366-372

Meson-exchange currents and quasielastic neutrino cross sections in the SuperScaling Approximation model
J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C.F. Williamson
Phys.Lett. B696 (2011) 151-155

Relativistic analyses of quasielastic neutrino cross sections at MiniBooNE kinematics
J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, J.M. Udias
Phys.Rev. D84 (2011) 033004

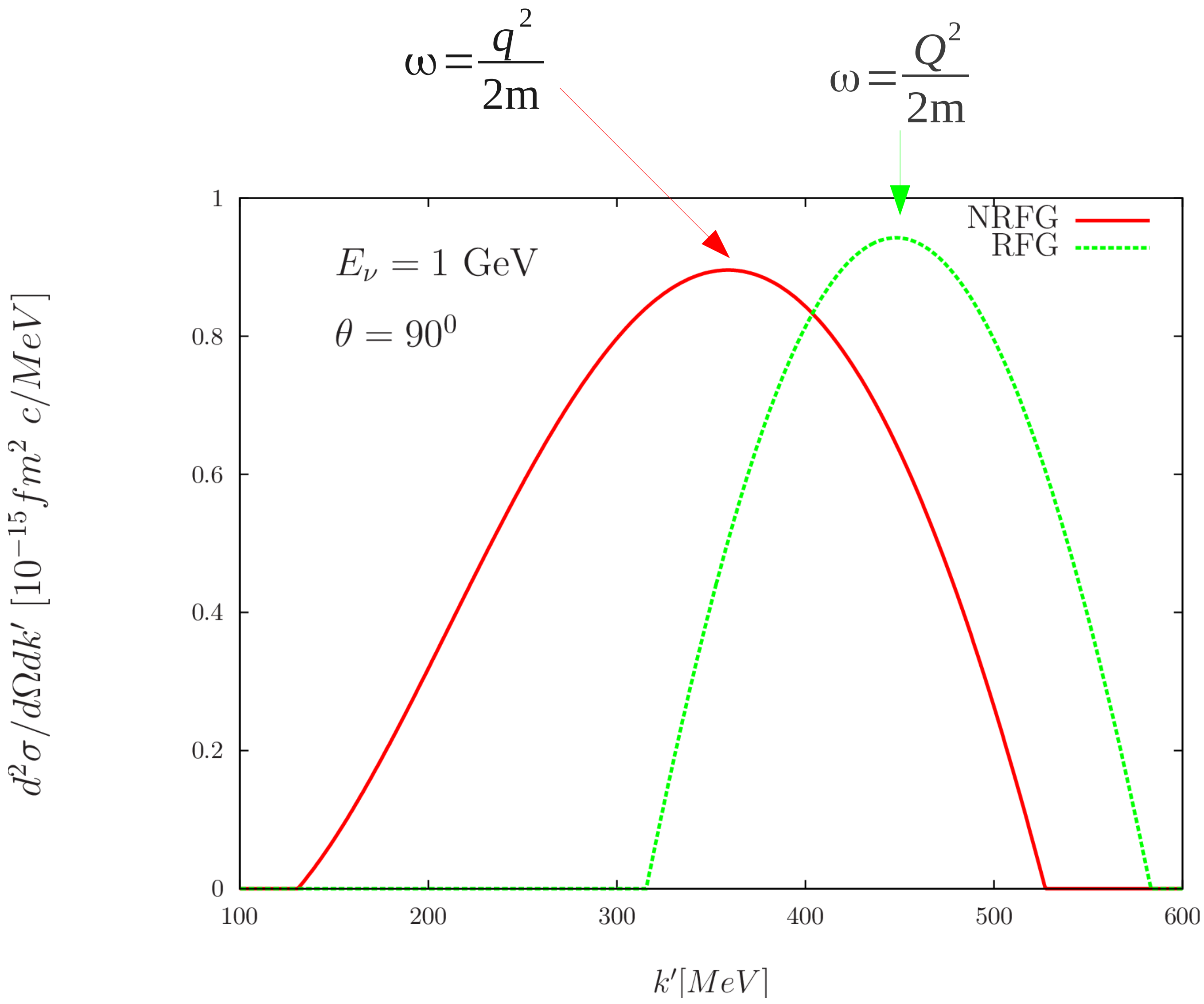
Meson-exchange currents and quasielastic antineutrino cross sections in the SuperScaling Approximation
J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly
Phys.Rev.Lett. 108 (2012) 152501

Neutrino and antineutrino CCQE scattering in the SuperScaling Approximation from MiniBooNE to NOMAD energies
G.D. Megias, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly
Phys.Lett. B725 (2013) 170-174


Thank You
Susa Valley, Italy
Sacra di San Michele

Backup Slides

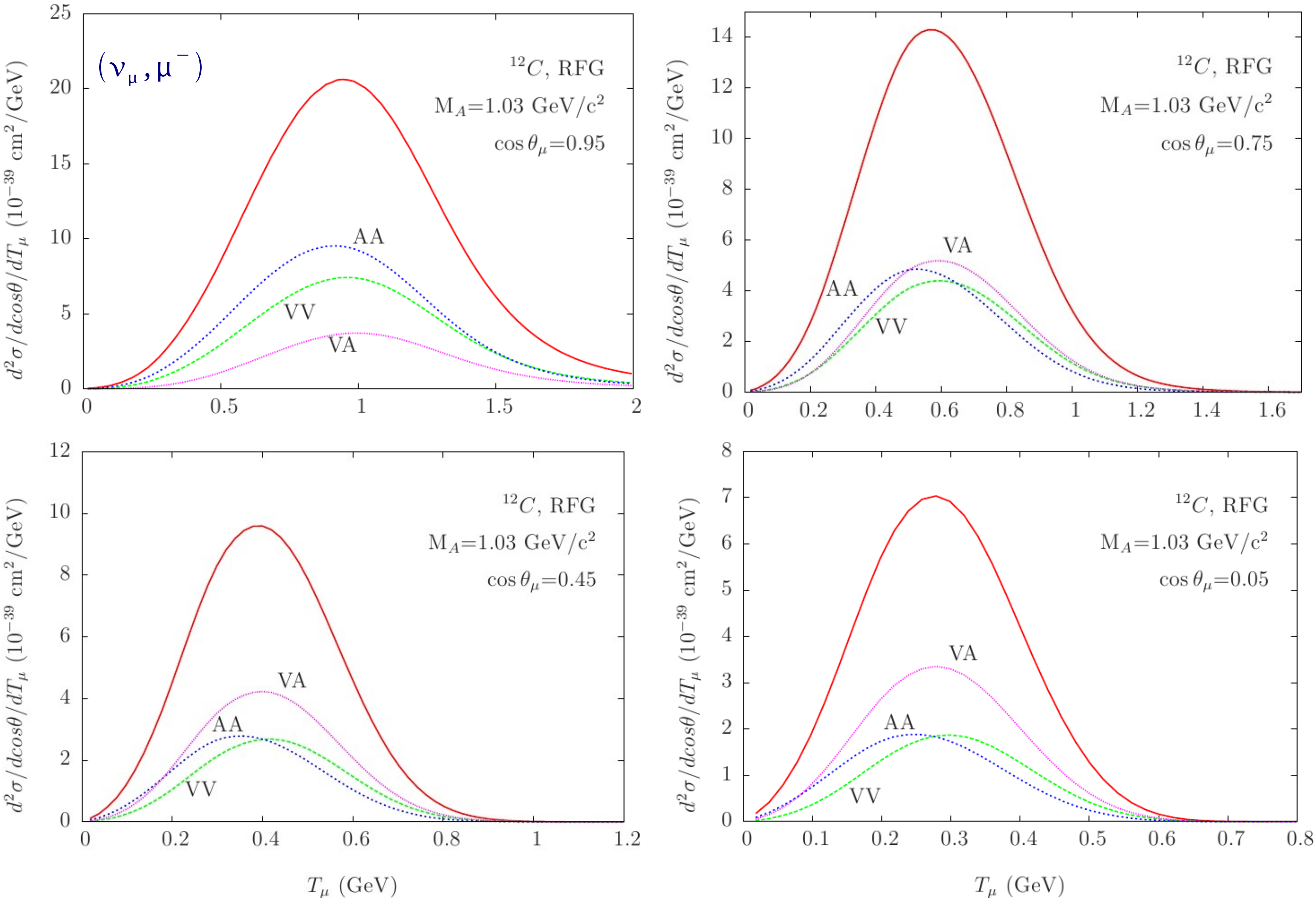
Relativistic effects



- Kinematics modifies the response region
- Boost factors modify the height of the QEP (with opposite sign in the L and T channels)

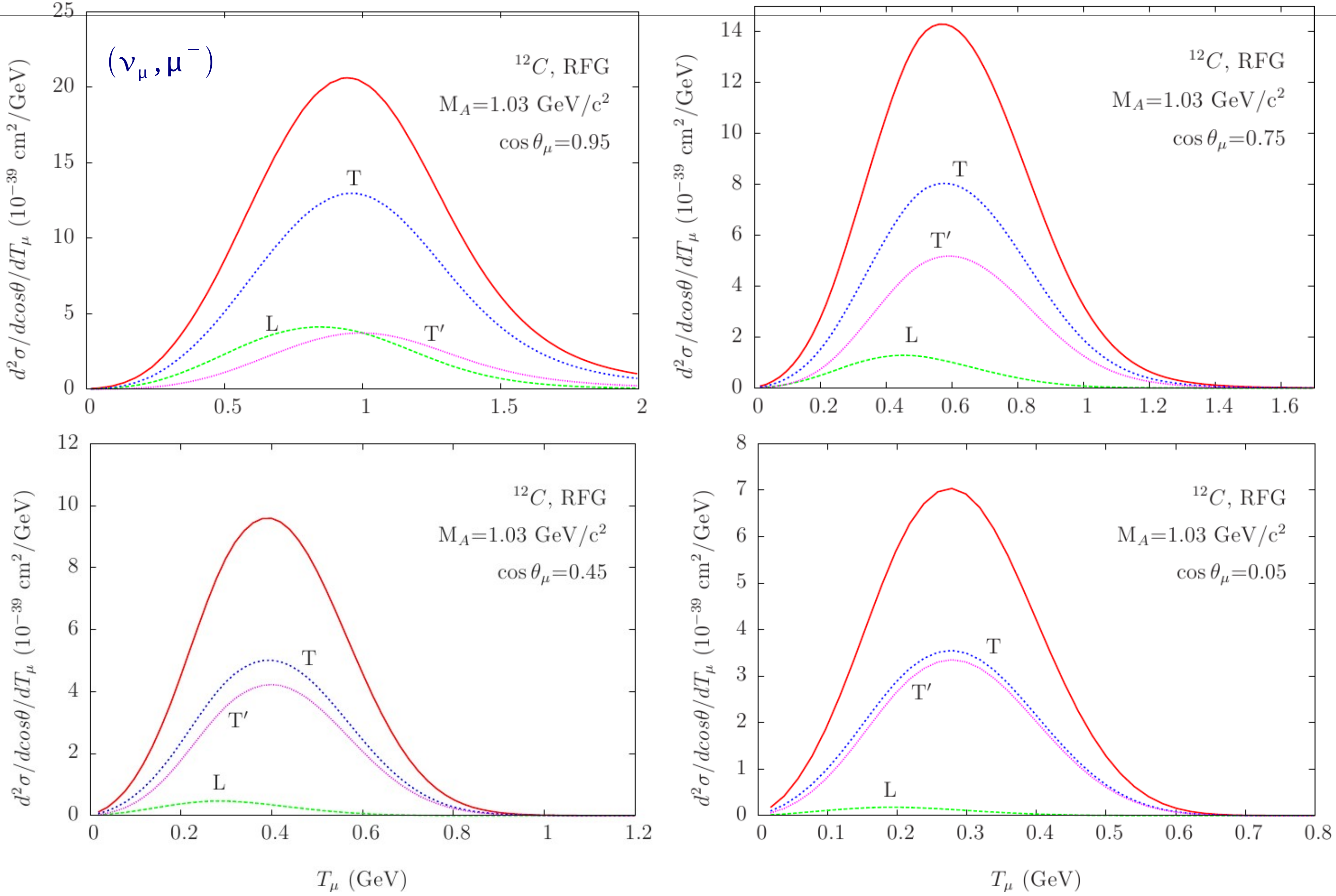
VV-AA-VA separation

MiniBooNE kinematics

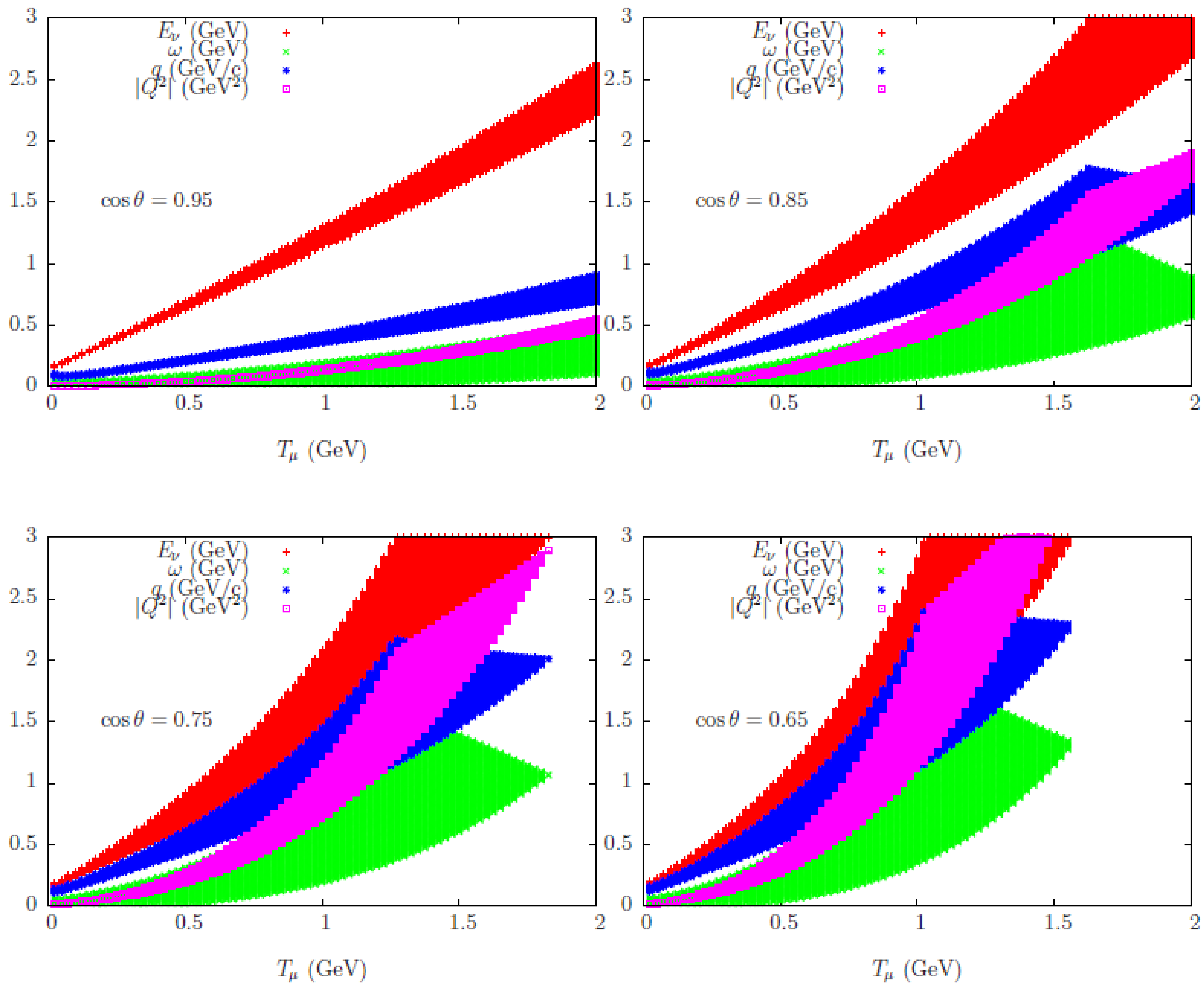


L-T-T' separation

MiniBooNE kinematics

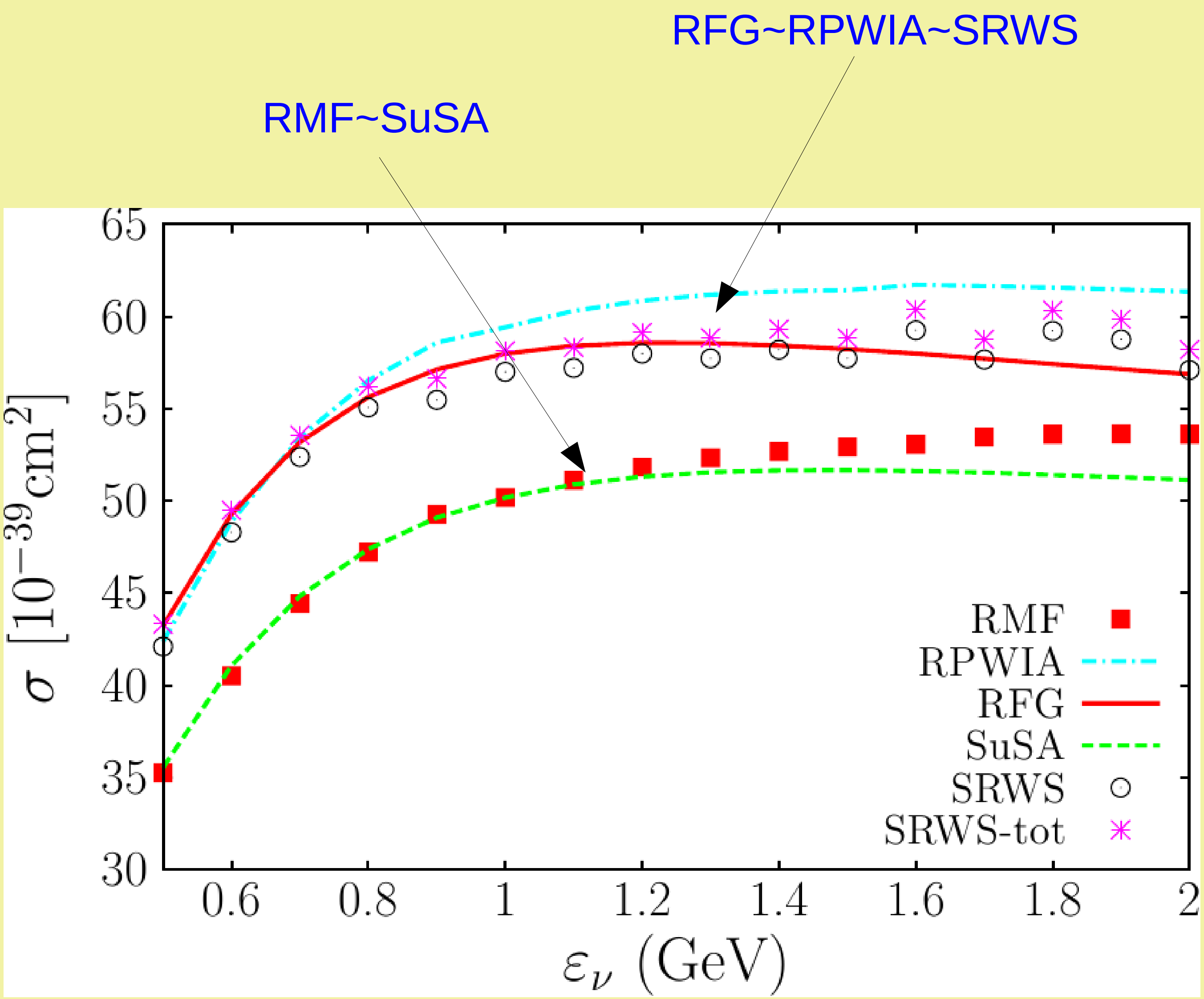


Kinematics



E_ν (GeV)
ω (GeV)
q (GeV/c)
$|Q^2|$ (GeV^2)
$\cos\theta = 0.95$
$\cos\theta = 0.85$
$\cos\theta = 0.75$
$\cos\theta = 0.65$
T_μ (GeV)

Total CCQE cross sections in RIA

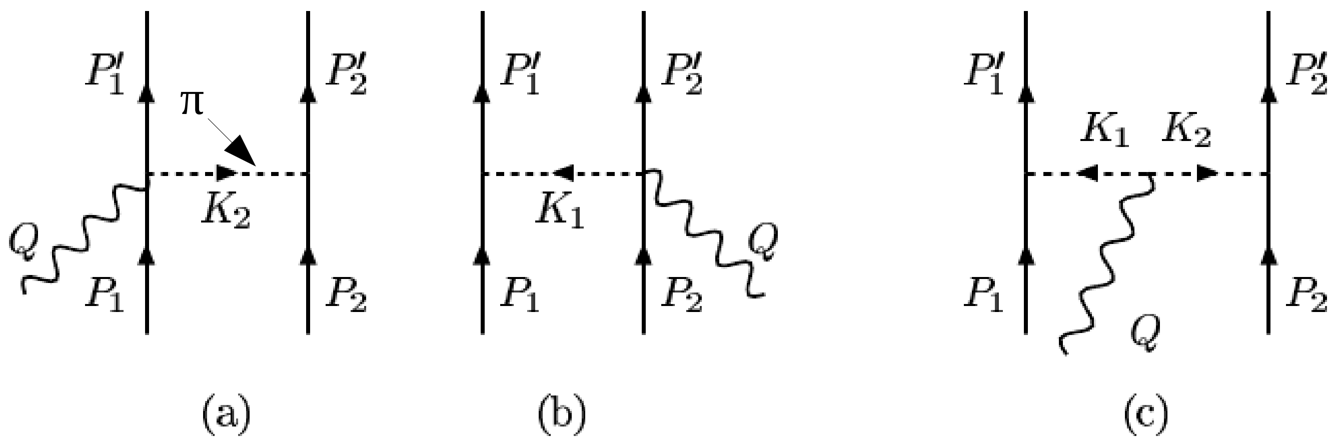


[Amaro et al., PRL 98, 242501 (2007)]

Meson Exchange Currents

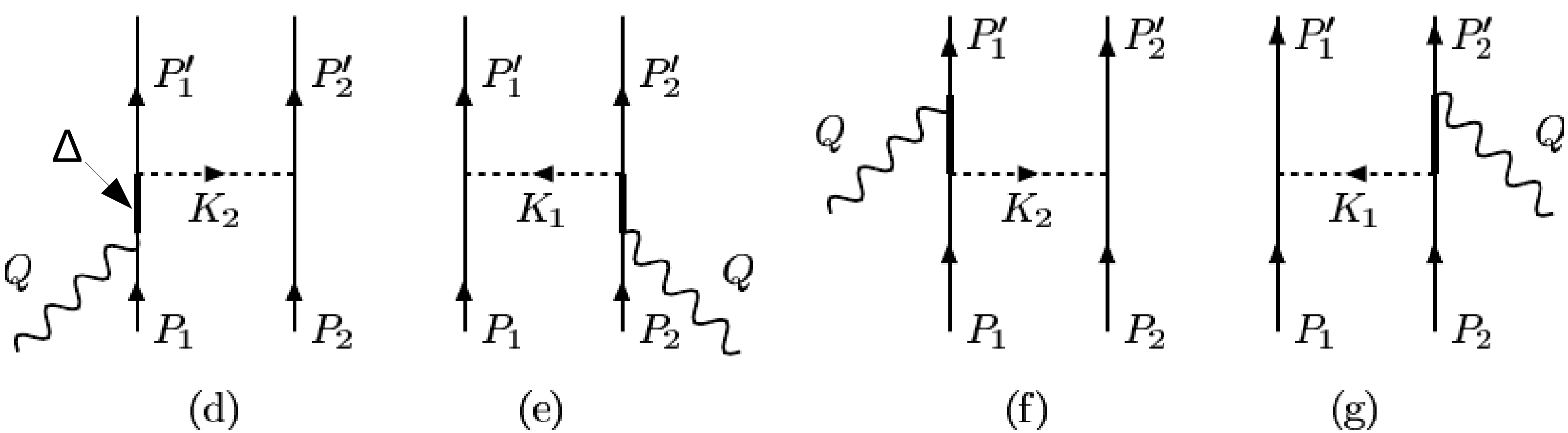
MEC are two-body currents involving 2 nucleons exchanging a meson.
Currents induced by the pion mainly (up to higher order relativistic corrections) occur in transverse channel and violate superscaling.

“contact” or “seagull”



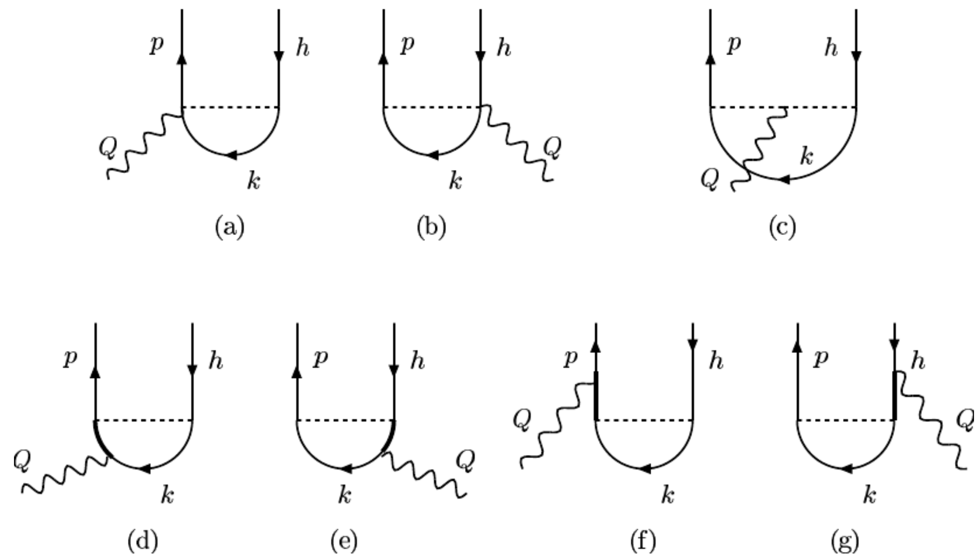
“pion-in-flight”

“Δ-MEC”



Meson Exchange Currents: 1p1h and 2p2h many-body diagrams

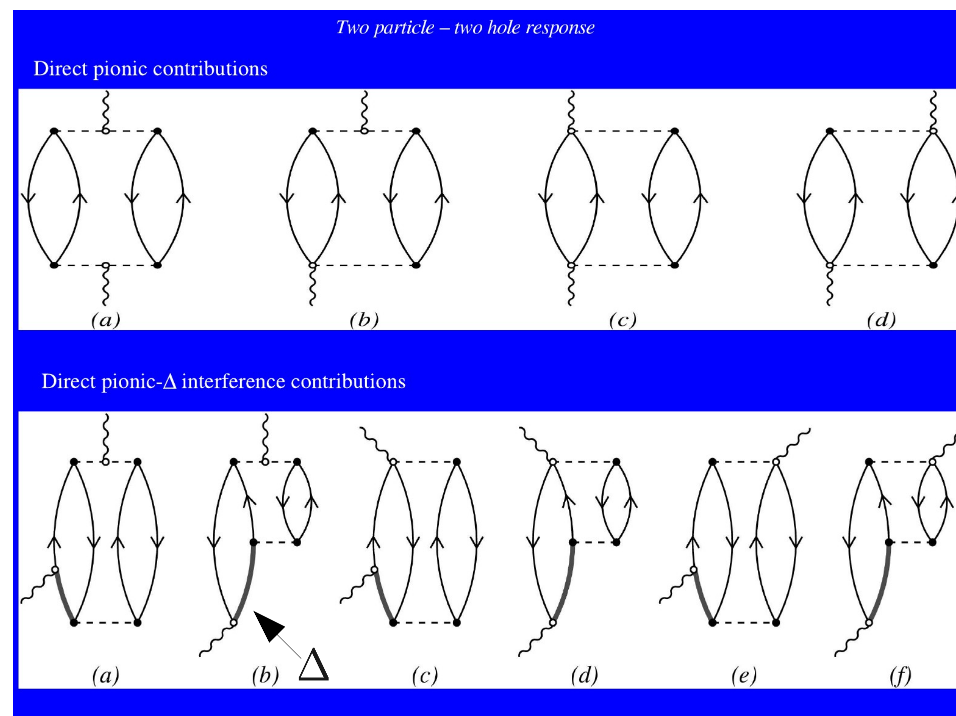
1p-1h sector:



Only contribute inside the RFG response region $-1<\psi<1$.
The net contribution to (e,e') QEP is small due to cancellations between MEC and correlations
[Amaro et al., Phys.Rept.368(2002),NPA723 (2003)]

2p-2h sector

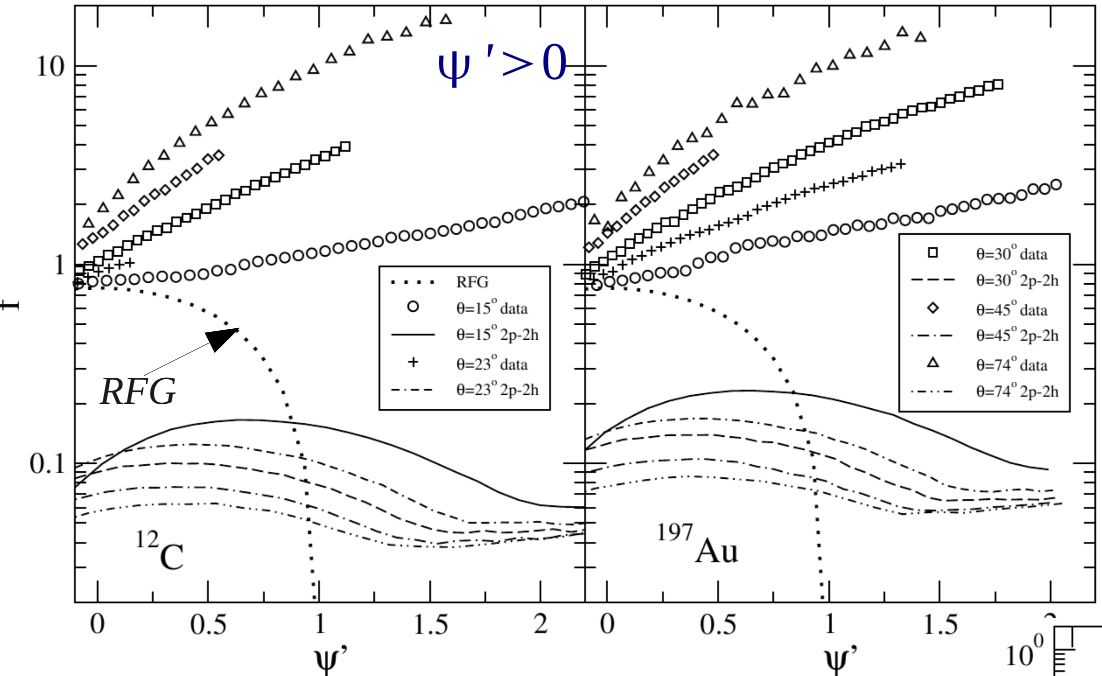
(just a subset of all possible many-body diagrams involving two pionic lines) ⇒



Contribute also outside the RFG response region: $\psi<-1$ and $\psi>1$

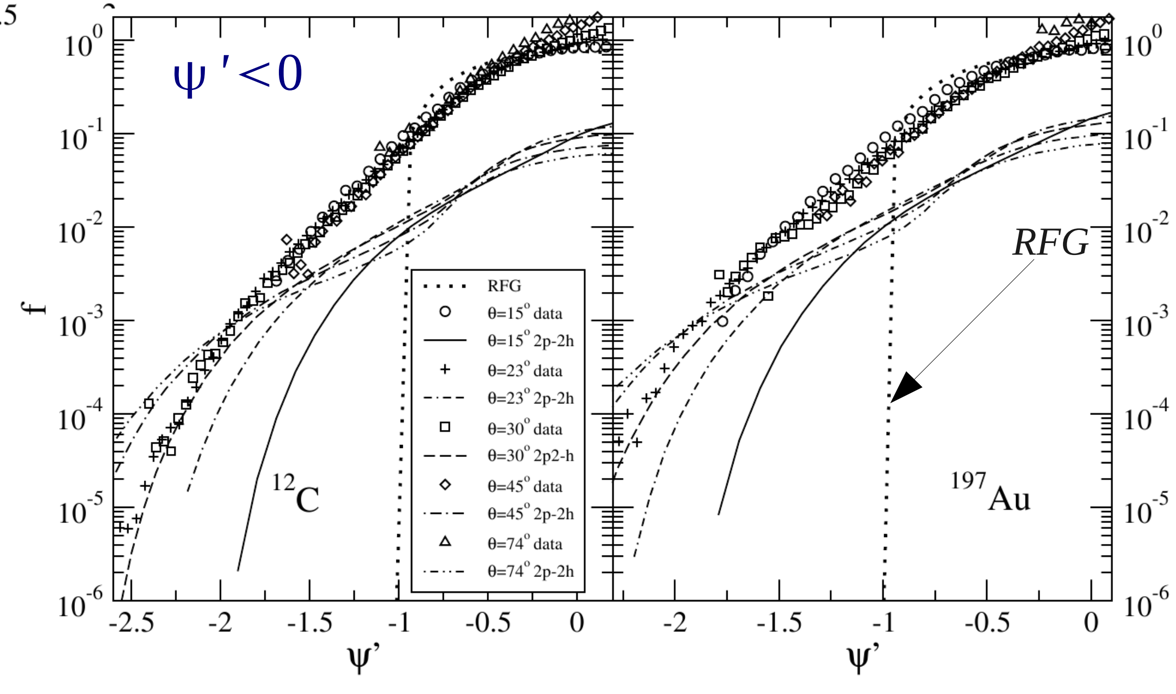
2p-2h MEC in electron scattering

De Pace et al., NPA741, 249 (2004), RFG-based calculation



(a) (b)

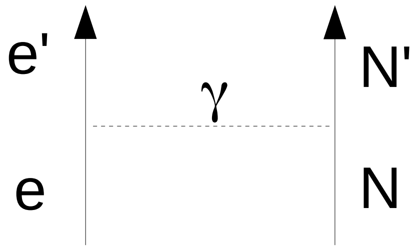
Scaling is broken both above and below the QEP

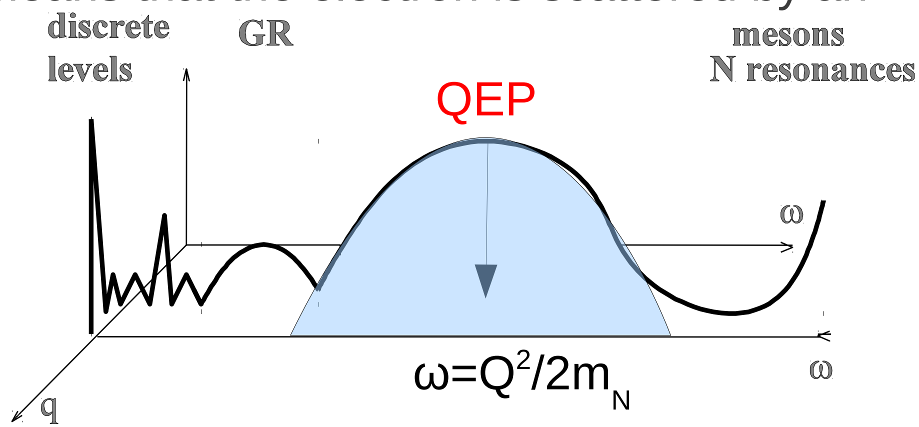


2p-2h MEC give a positive contribution of ~10-20% outside the QEP, filling the “dip” between the QE and Δ peak.

Why can MEC be relevant in quasielastic neutrino scattering?

In (e,e') experiments E_e is well-known and "QE" means that the electron is scattered by an individual nucleon moving inside the nucleus

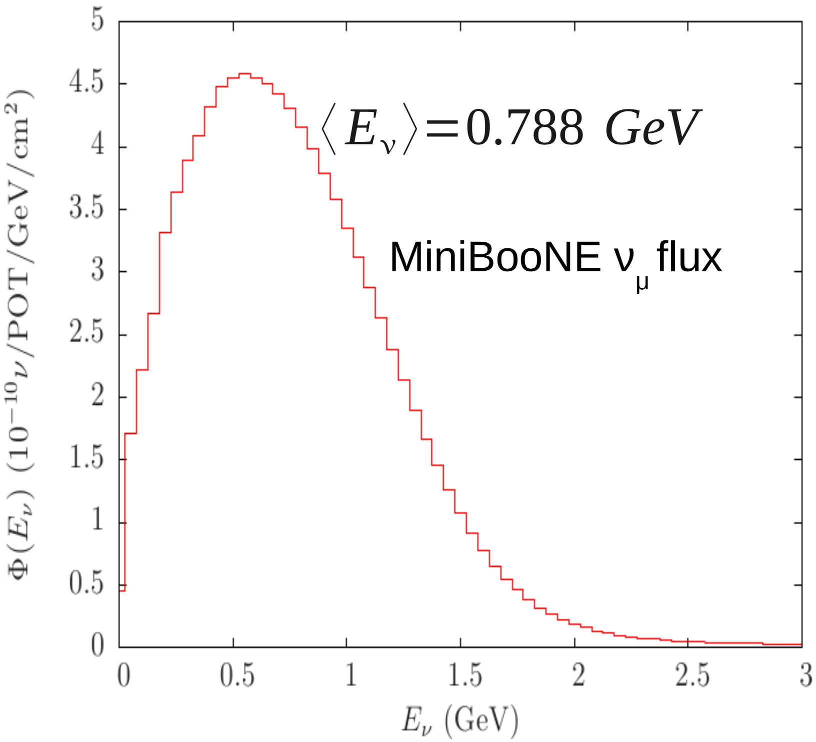




In (ν_μ,μ) the neutrino beam is not monochromatic, but it spans a wide range of energies

"Flux-averaged" cross section:

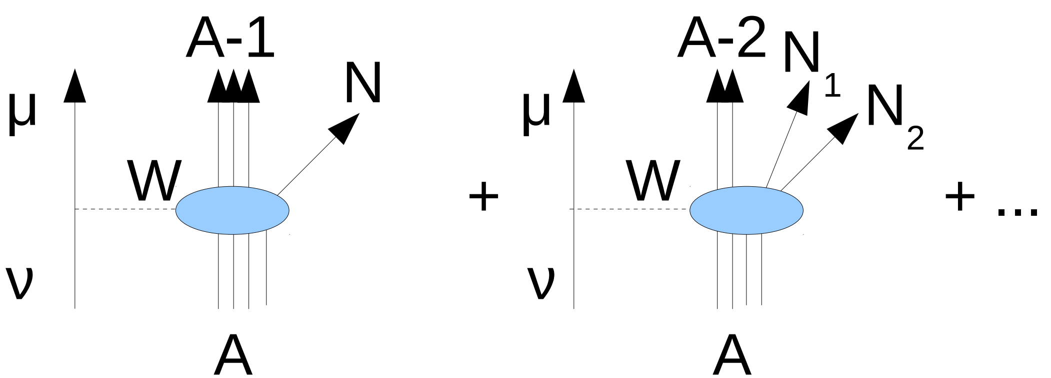
$$< \frac{d^2\sigma}{d\cos\theta\, dT_\mu} > = \frac{1}{\Phi_{tot}} \int \frac{d^2\sigma(E_\nu)}{d\cos\theta\, dT_\mu} \phi(E_\nu)\, dE_\nu$$



Different regions in the (q,ω) plane, corresponding to different reaction mechanisms, contribute to each experimental point (θ,T_μ).

"QE"=no pions in the final state

Processes involving scattering off two or more nucleons must also be considered [Martini et al, Nieves et al]



2p-2h MEC in CCQE neutrino scattering

We apply the calculation of NPA741, 249 (2004), which is fully relativistic and RFG-based, to modify the polar-vector transverse response function using CVC

$$R_T = R_T^{VV} + R_T^{AA}$$

within the SuSA approach.

We neglect the MEC contribution to the axial response because the 2p2h sector is not directly reachable in lowest order for the axial-vector matrix elements:

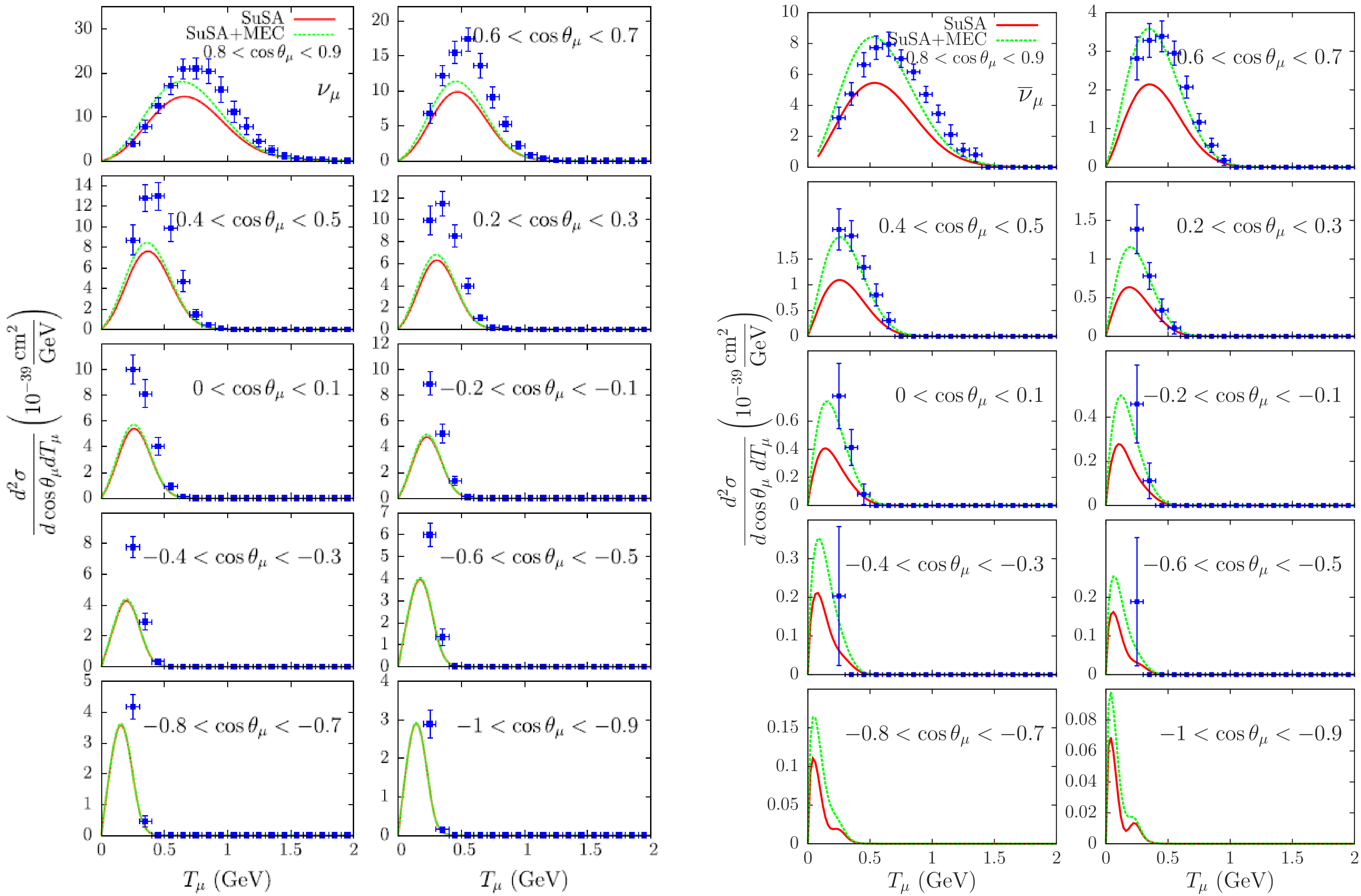
$$\kappa = q/2m_N$$

$$J_0^{V(MEC)} \sim O(\kappa^2) \qquad J_0^{A(MEC)} \sim O(\kappa)$$

$$\vec{J}^{V(MEC)} \sim O(\kappa) \qquad \vec{J}^{A(MEC)} \sim O(\kappa^2)$$

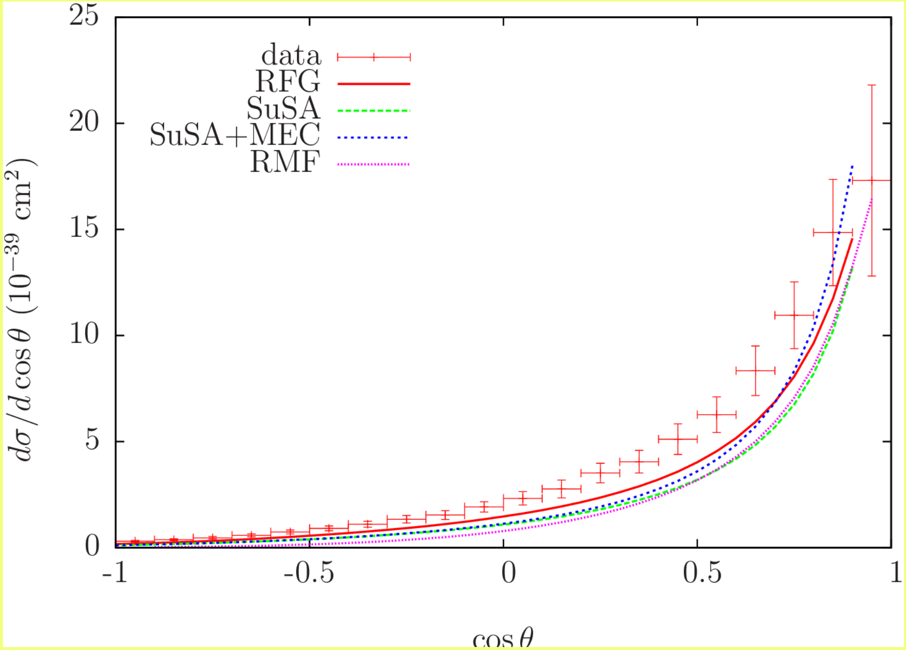
N.B. A fully consistent gauge invariant calculation would require also the inclusion of the associated correlation diagrams, not explicitly included in present calculation.
However these are
1) hard to compute because in RFG because of divergences that need to be renormalized [Amaro et al., Phys.Rev.C82:044601 (2010)]
2) possibly already included in the phenomenological susperscaling function, since they also contribute to longitudinal channel

MiniBooNE data



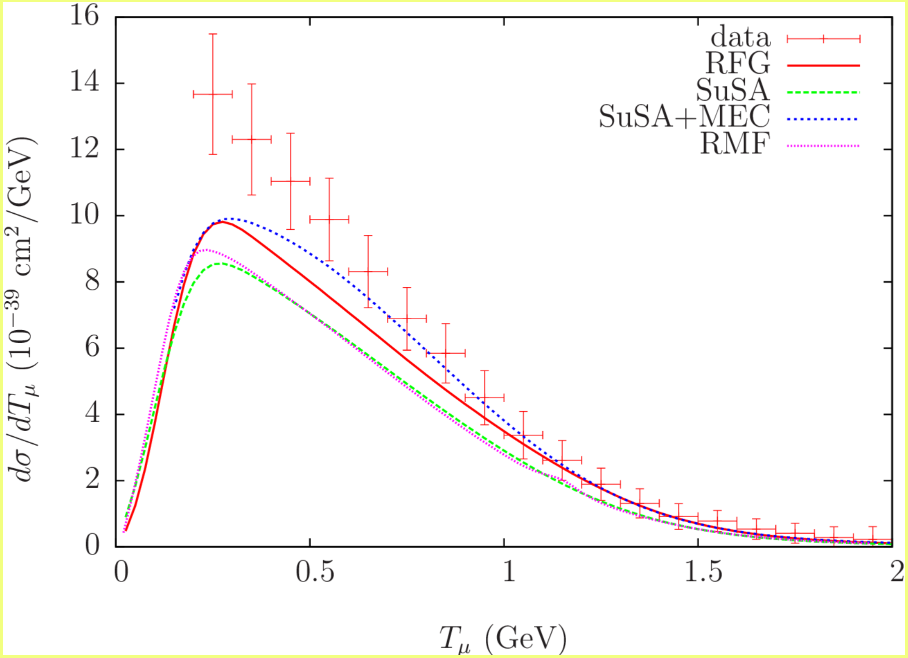
J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, C.F.Williamson, Physics Letters B 696 (2011) 151–155

Comparison with MiniBooNE single differential CC cross sections

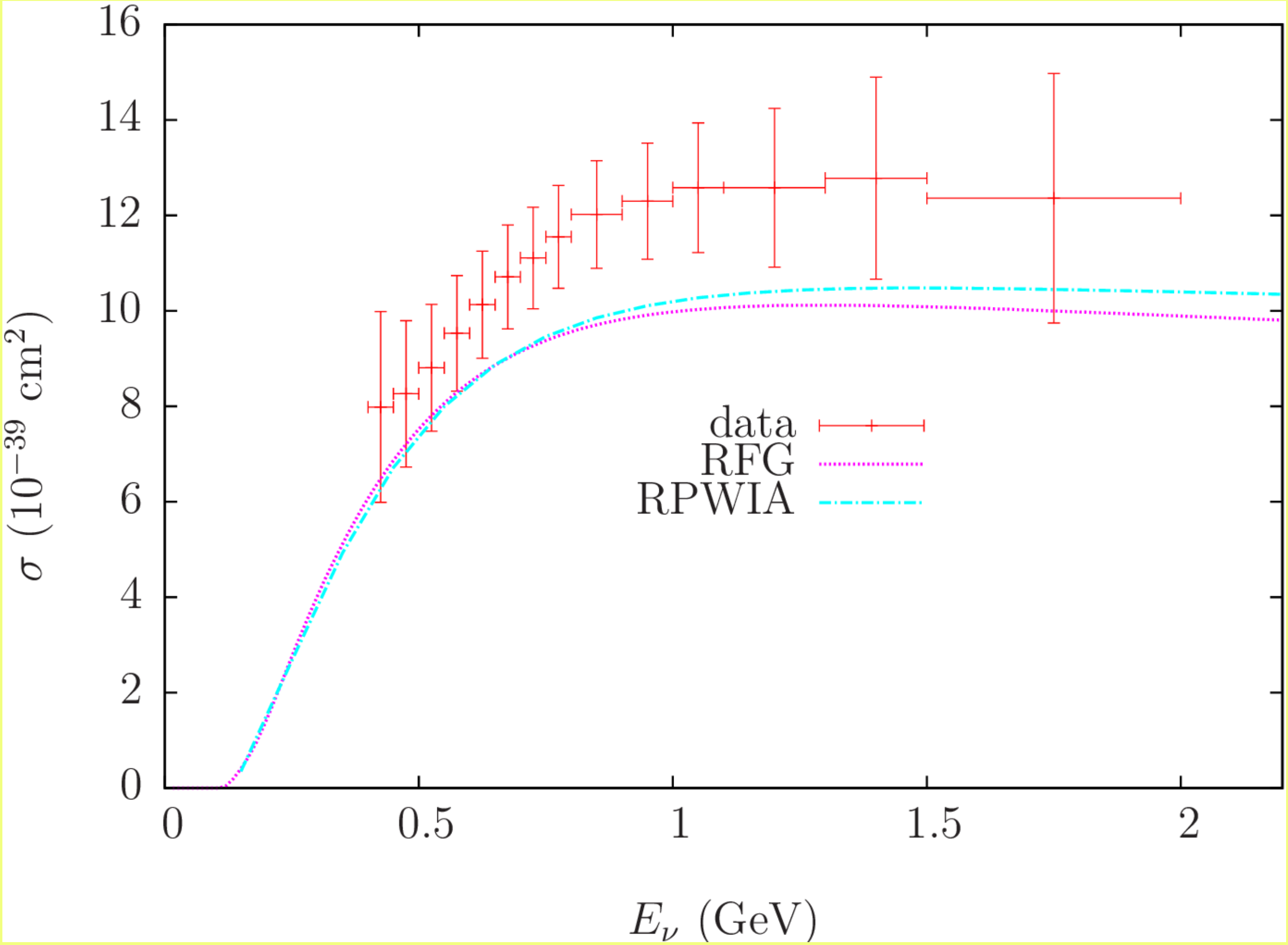


Strength is missing at larger scattering angles ($\theta > 50^0$)

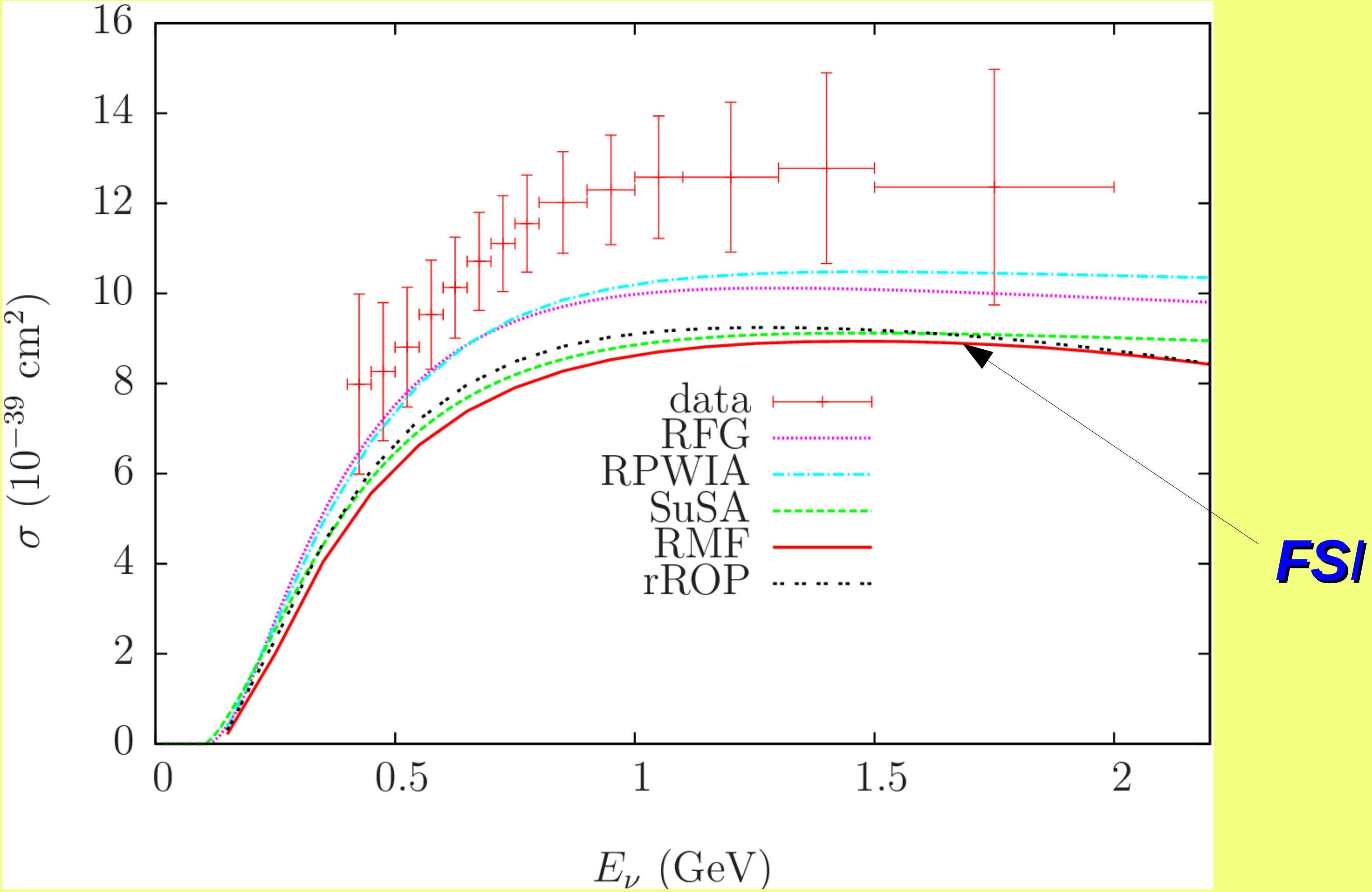
and lower muon energies (T_μ<0.4 GeV)



Total CC cross section

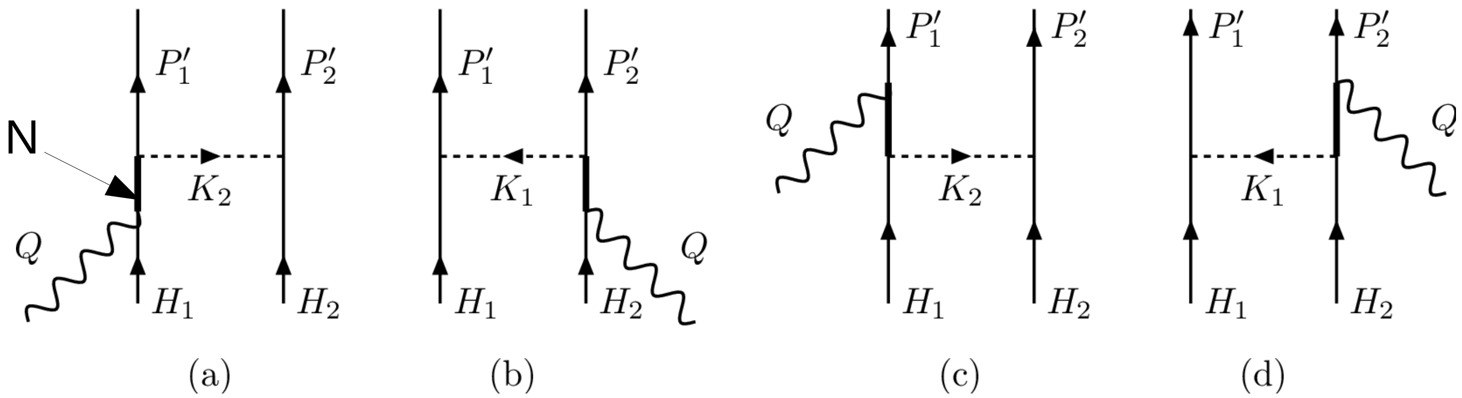


Total CC cross section



Correlation Currents

In order to preserve gauge invariance correlation diagrams, where the virtual boson attaches to one of two interacting nucleons, must be also considered:



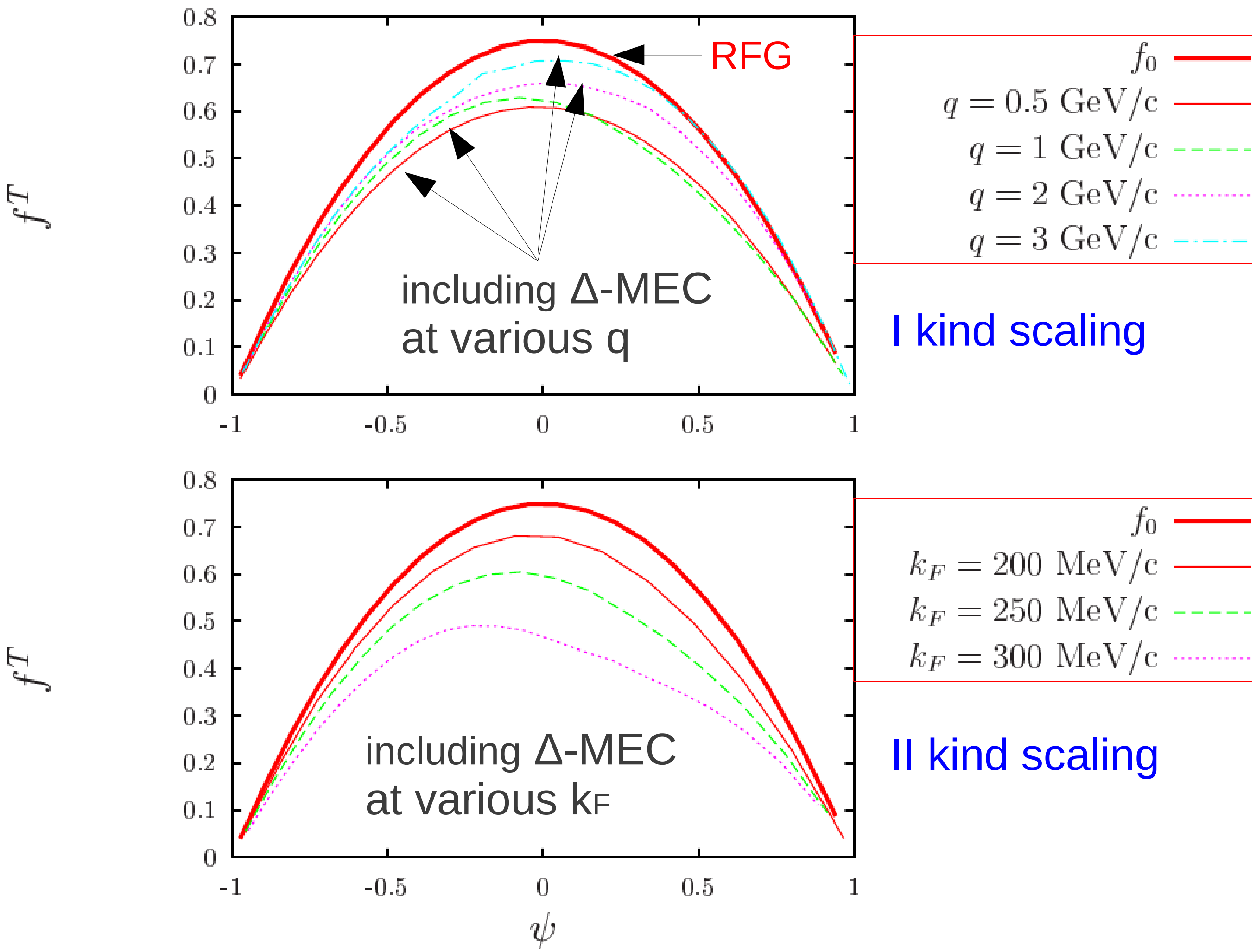
The total two-body current is conserved:

$$\partial^{\mu} J^{(2)}_{\mu} = 0$$

Correlation currents contribute to both longitudinal and transverse channels.

1p-1h MEC in electron scattering

Amaro et al., NPA723, 181 (2003)



I kind scaling

II kind scaling

The response is calculated on the RFG basis and is mainly transverse (although relativistically there is a small L contribution)

The Δ-MEC give the dominant contribution

Both kinds of scaling are violated

The net contribution to the cross section is negative

However correlation diagrams needed to preserve gauge invariance give a positive contribution which roughly compensate MEC

The total contribution of MEC+correlations in the 1p-1h sector is small

Expressions for the π-exchange currents

$$J^{\mu}_{(2)} = J^{\mu}_{(s)} + J^{\mu}_{(\pi)} + J^{\mu}_{(\Delta)}$$

"Seagull":

$$J^{\mu}_{(s)}(p'_1, p'_2; p_1, p_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab} \bar{u}(p'_1)\tau_a\gamma_5\gamma^\nu K_{1\nu} u(p_1) \frac{F_1^V}{K_1^2 - m_\pi^2} \bar{u}(p'_2)\tau_b\gamma_5\gamma^\mu u(p_2) + (1 \Leftrightarrow 2)$$

"Pion-in-flight":

$$J^{\mu}_{(\pi)}(p'_1, p'_2; p_1, p_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab} \bar{u}(p'_1)\tau_a\gamma_5\gamma^\nu K_{1\nu} u(p_1) \frac{F_\pi (K_1 - K_2)^\mu}{(K_1^2 - m_\pi^2)(K_2^2 - m_\pi^2)} \bar{u}(p'_2)\tau_b\gamma_5\gamma^\rho K_{2\rho} u(p_2)$$

"Δ-MEC":

$$J^{\mu}_{(\Delta)}(p'_1, p'_2; p_1, p_2) = \frac{f_{\pi N\Delta} f}{m_\pi^2} \bar{u}(p'_1) T^\mu_a(1)\gamma_5 u(p_1) \frac{1}{K_2^2 - m_\pi^2} \bar{u}(p'_2)\tau_a\gamma_5\gamma^\nu K_{2\nu} u(p_2) + (1 \Leftrightarrow 2)$$

$$T^\mu_a(1) = K_{2\alpha}\Theta^{\alpha\beta} G^{\Delta}_{\beta\rho}(H_1 + Q) S^{\rho\mu}_f(H_1) T_a T_3 + T_3 T_a S^{\rho\mu}_b(P'_1) G^{\Delta}_{\rho\beta}(P'_1 - Q)\Theta^{\beta\alpha} K_{2\alpha}$$

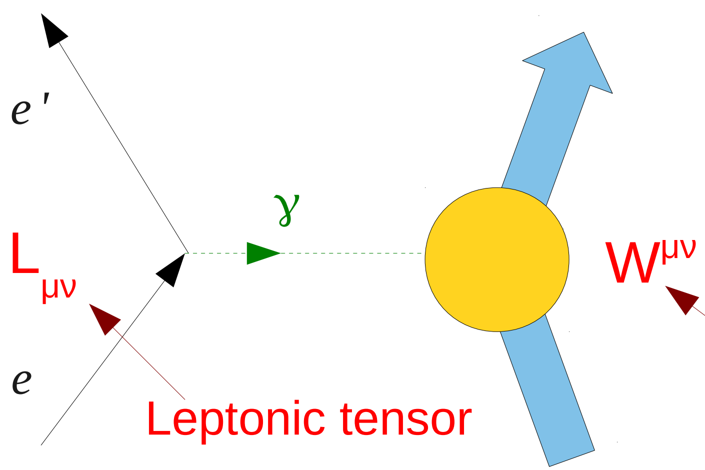
$$\Theta^{\alpha\beta} = g^{\alpha\beta} - \frac{1}{4}\gamma^\alpha\gamma^\beta$$

Rarita-Schwinger propagator

Forward and backward Δ-electroexcitation tensors

Formalism: (l,l') inclusive scattering

(e,e')

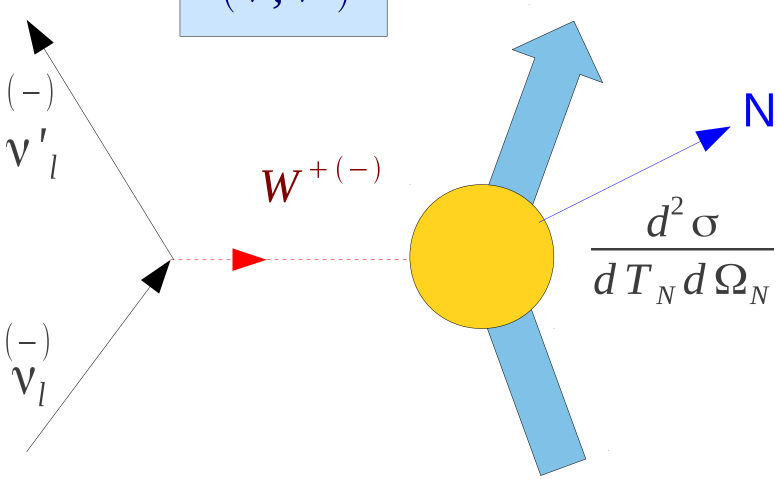


$$\frac{d^2\sigma}{d\epsilon' d\Omega'} = \sigma_{Mott}(v_L R_L + v_T R_T)$$

2 electromagnetic response functions

NC

(ν,ν')

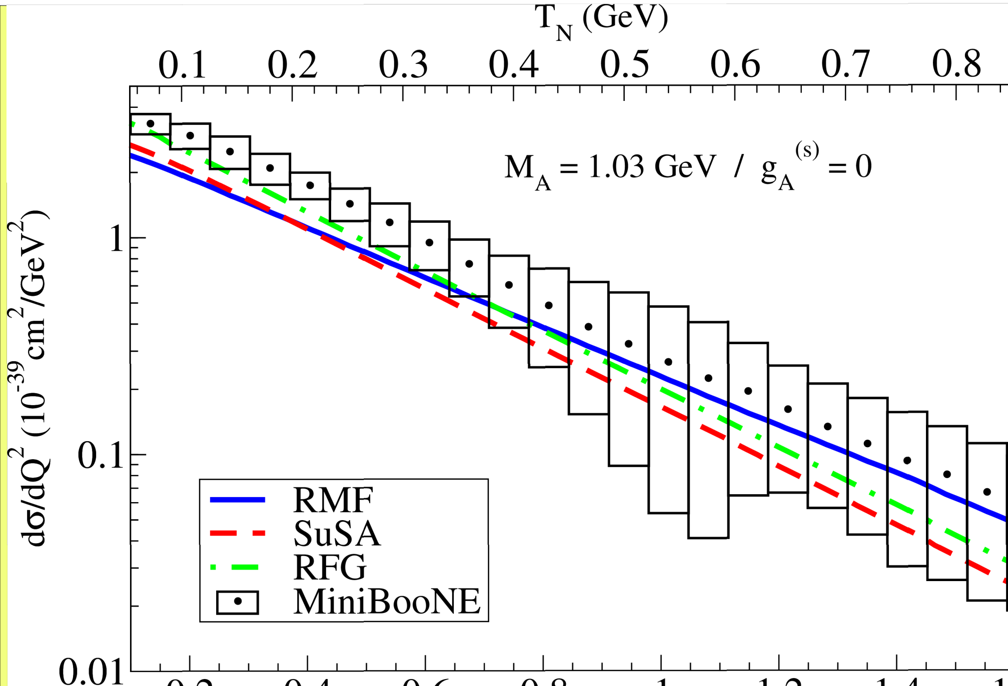


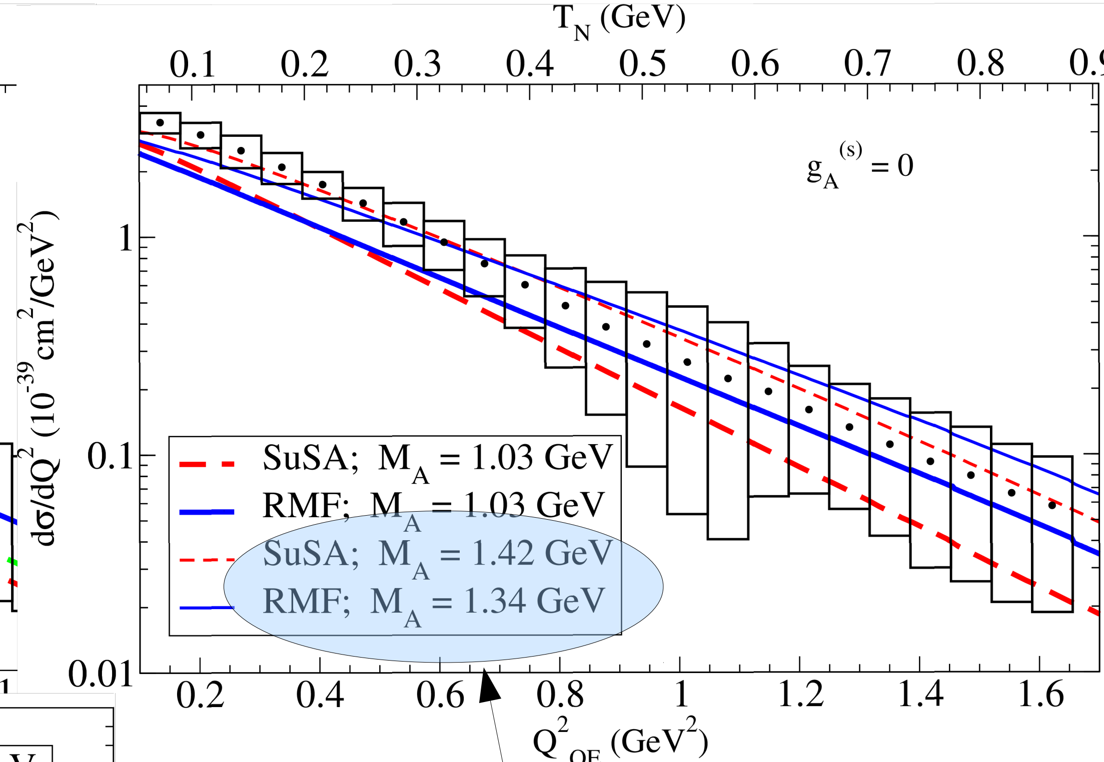
$$\frac{d^2\sigma}{dT_N d\Omega_N} = \sigma_0 [V_L R_L + V_T R_T + V_{TT} R_{TT} + V_{TL} R_{TL} \pm (2 V_{T'} R_{T'} + 2 V_{TL'} R_{TL'})]$$

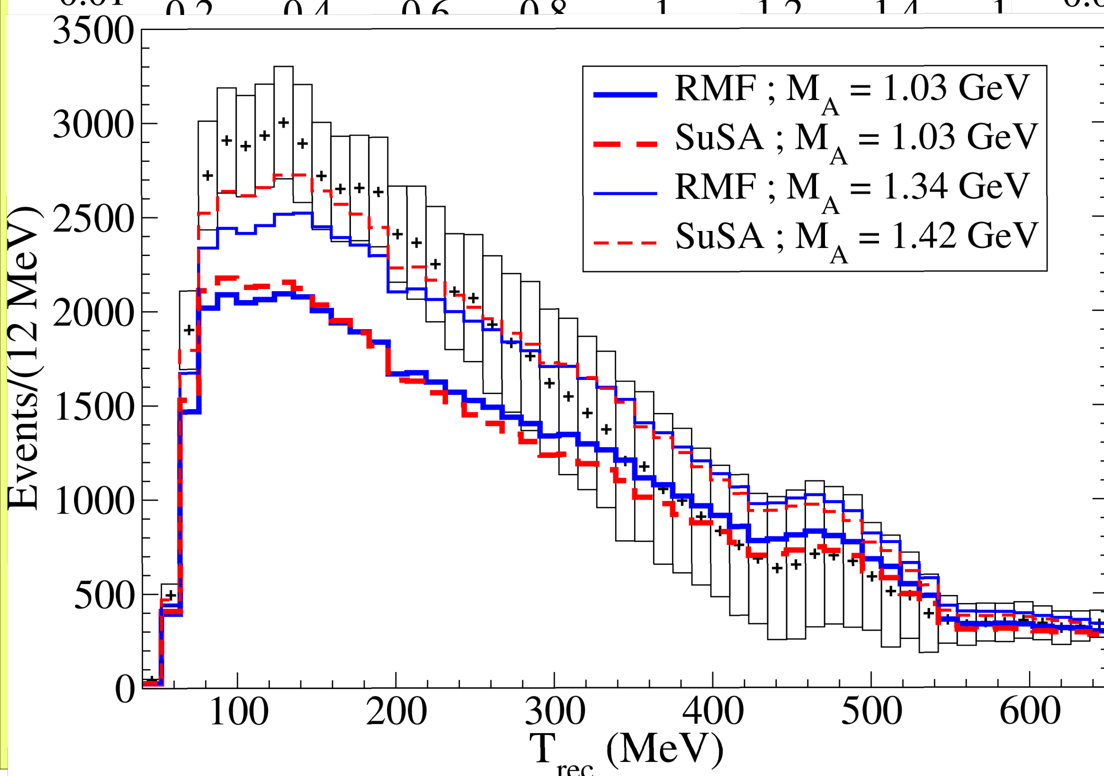
6 weak response functions

$l = \mu, e, \tau$

NC neutrino cross section

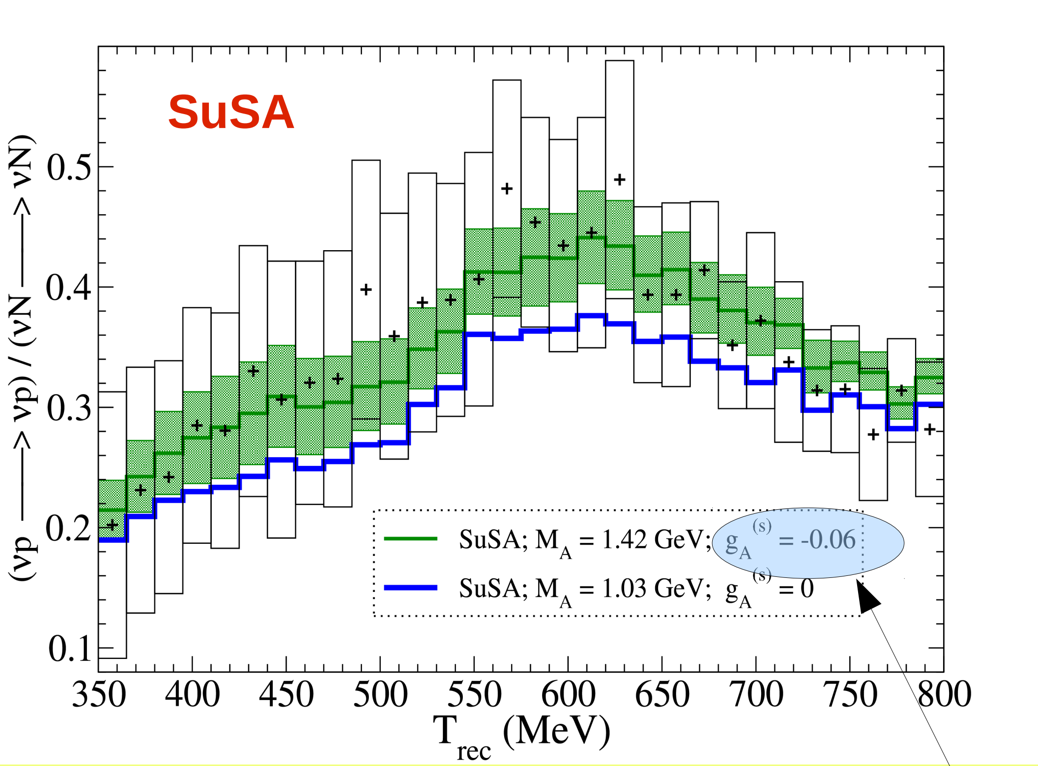


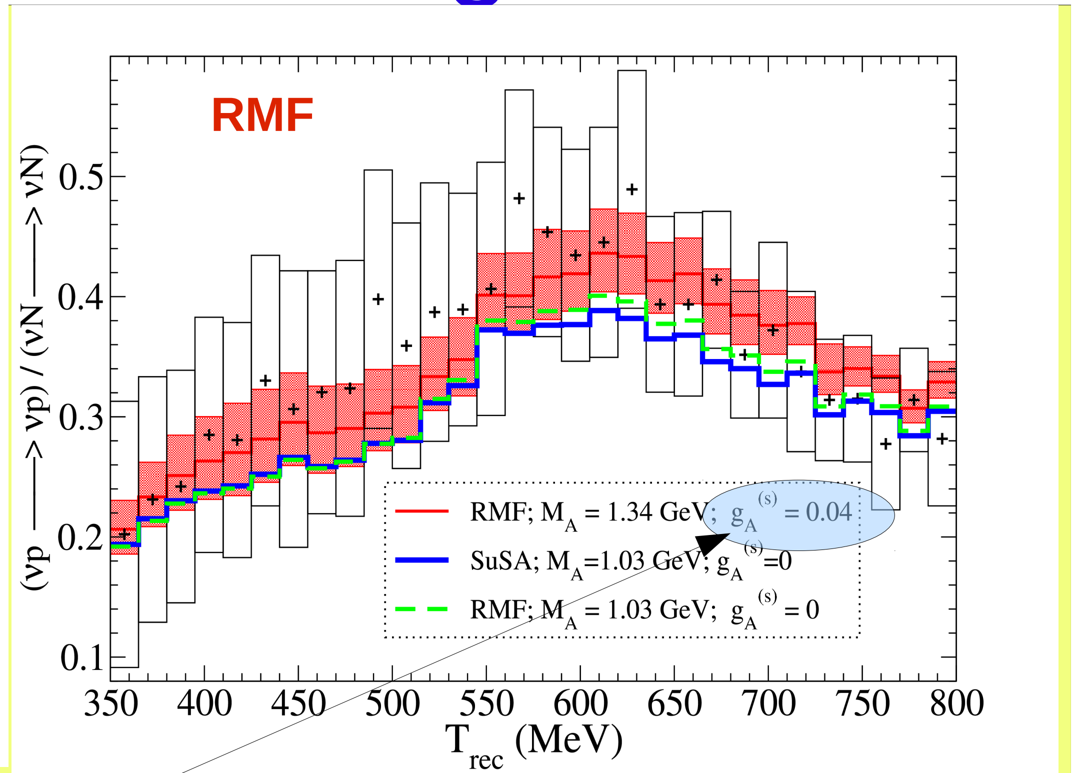




Best fit of axial mass at $g_A^{(s)}$=0 in SuSA and RMF

NC p/N ratio: axial strangeness





Best fits of $g_A^{(s)}$ at fixed M_A

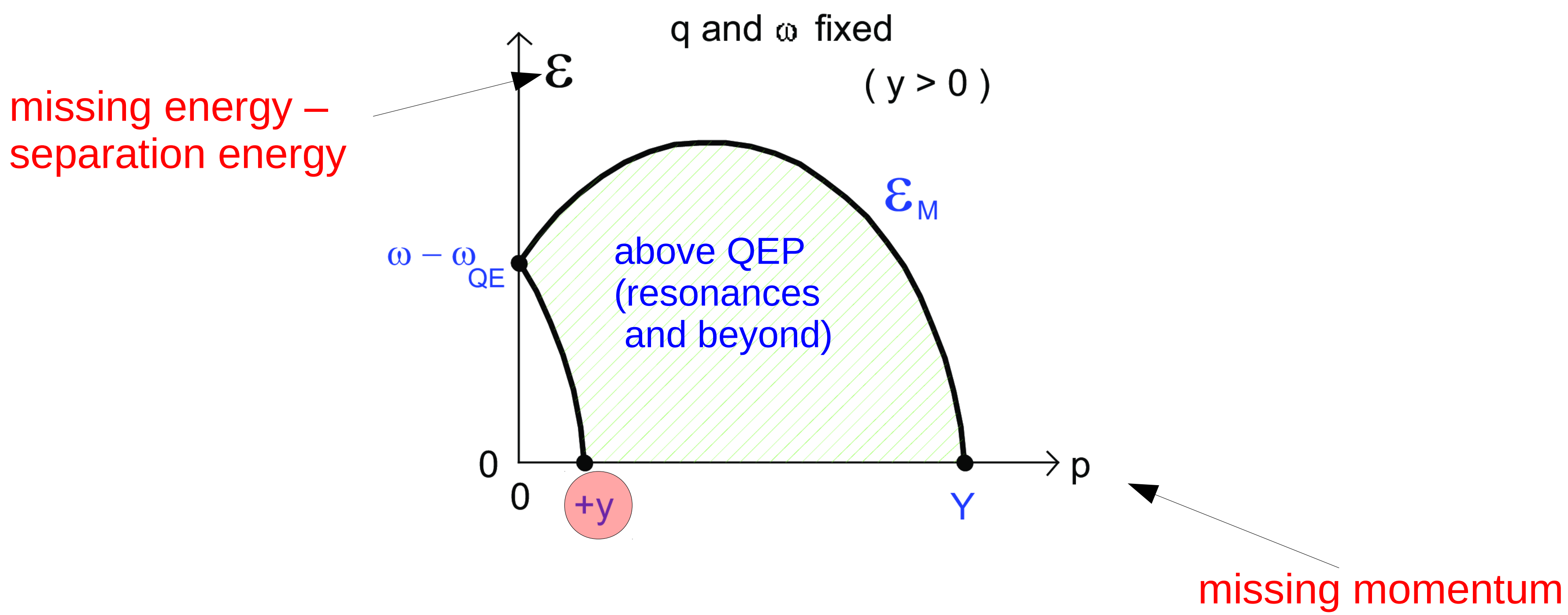
$g_A^{(s)}$ = -0.06±0.31 SuSA (χ^2/DOF=31.3/29)

$g_A^{(s)}$ = +0.04±0.28 RMF (χ^2/DOF=33.6/29)

The dependence upon the nuclear model is essentially canceled in the ratio

Quasielastic kinematics and y-scaling

For each value of q and ω, evaluating the *(e,e')* cross section implies an integral over the kinematically allowed region for the semi-inclusive reaction *(e,e'N)*:



$$y \cong y_\infty = \sqrt{\widetilde{\omega}(2m_N + \widetilde{\omega})} - q,$$

$$\widetilde{\omega} \equiv \omega - E_S,$$

Formalism: Quasi-elastic peak

- Dominant reaction mechanism is CCQE

$$\nu_\mu + n \rightarrow \mu^- + p$$

- Single nucleon current: $j^\mu = j_V^\mu - j_A^\mu$

$$j_V^\mu = \bar{u}(P')(\tilde{F}_1 \gamma^\mu + \frac{i}{2m_N}\tilde{F}_2 \sigma^{\mu\nu} Q_\nu) u(P) \rightarrow \quad R_L^{VV} = \frac{\kappa^2}{\tau}[\tilde{G}_E^{(1)}]^2, \quad R_T^{VV} = 2\tau[\tilde{G}_M^{(1)}]^2$$

$$j_A^\mu = \bar{u}(P')(\tilde{G}_A \gamma^\mu + \frac{1}{2m_N}\tilde{G}_P Q^\mu)\gamma_5 u(P) \rightarrow \quad R_{LL}^{AA} = \frac{\kappa^2}{\lambda^2} R_{CC}^{AA} = \frac{-\kappa}{\lambda} R_{CL}^{AA} = \frac{\kappa^2}{\tau}[\tilde{G}_A^{(1)} - \tau \tilde{G}_P^{(1)}]^2$$

$$R_T^{AA} = 2(1+\tau)[\tilde{G}_A^{(1)}]^2, \quad R_{T'}^{VA} = 2\sqrt{\tau(1+\tau)}\,\tilde{G}_M^{(1)}\tilde{G}_A^{(1)}$$

$\kappa = q/(2\mathrm{m}_N), \ \lambda = \omega/(2\mathrm{m}_N), \ \tau = \kappa^2 - \lambda^2$ *dimensionless variables*

- The nuclear weak responses Ri are

  - purely isovector

  - typically transverse and

  - have vector-vector (VV), axial-axial (AA) and vector-axial (VA) contributions

Scaling in the Delta region

1) subtract the QE contribution obtained from Superscaling hypothesis

$$\left[\frac{d^2\sigma}{d\omega d\Omega}\right]_{'\Delta'} = \left[\frac{d^2\sigma}{d\omega d\Omega}\right]_{exp} - \left[\frac{d^2\sigma}{d\omega d\Omega}\right]_{QE}$$

2) divide by the elementary N $\rightarrow \Delta$ cross section

$$F_{'\Delta'} = \frac{\left[\frac{d^2\sigma}{d\omega d\Omega}\right]_{'\Delta'}}{\sigma_M (v_L G_L^\Delta + v_T G_T^\Delta)}$$

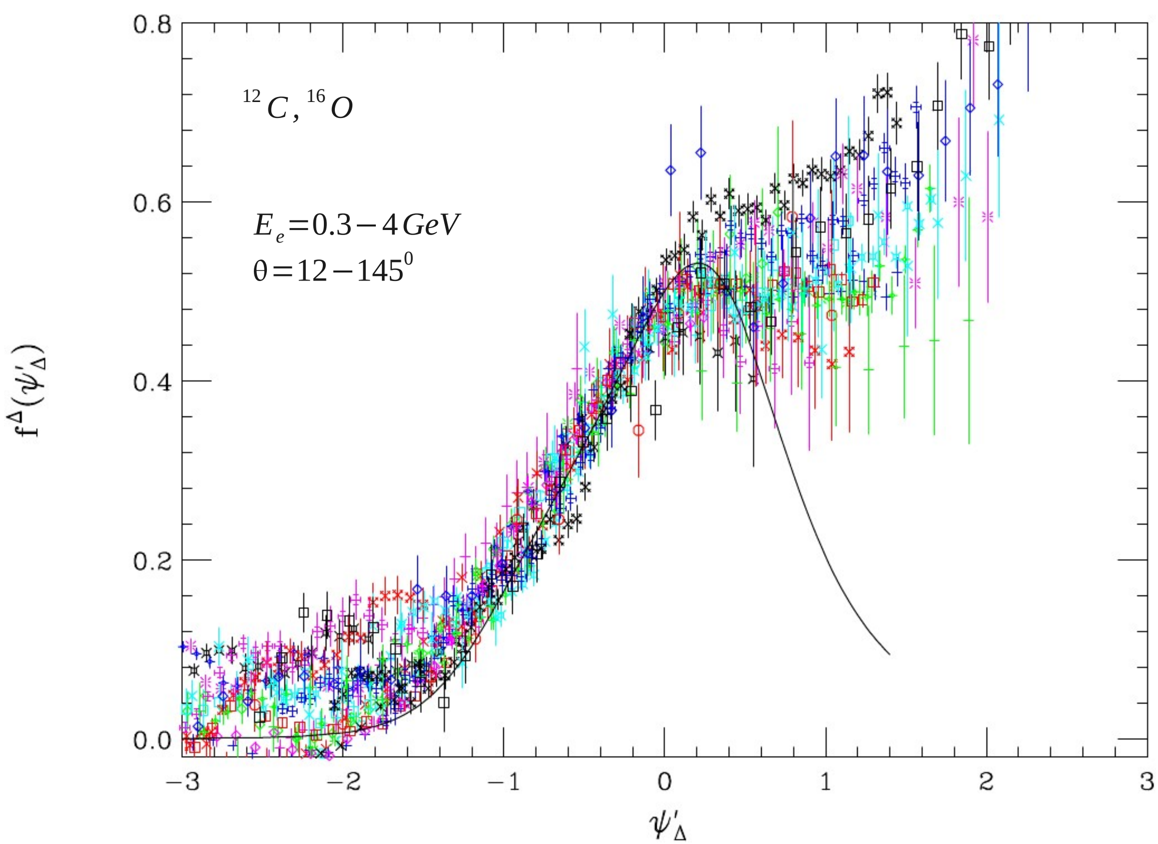
3) multiply by the Fermi momentum

$$f_{'\Delta'} = k_F F_{'\Delta'}$$

4) plot versus the appropriate scaling variable

$$\psi_\Delta = \psi(q\rho, \omega\rho)$$

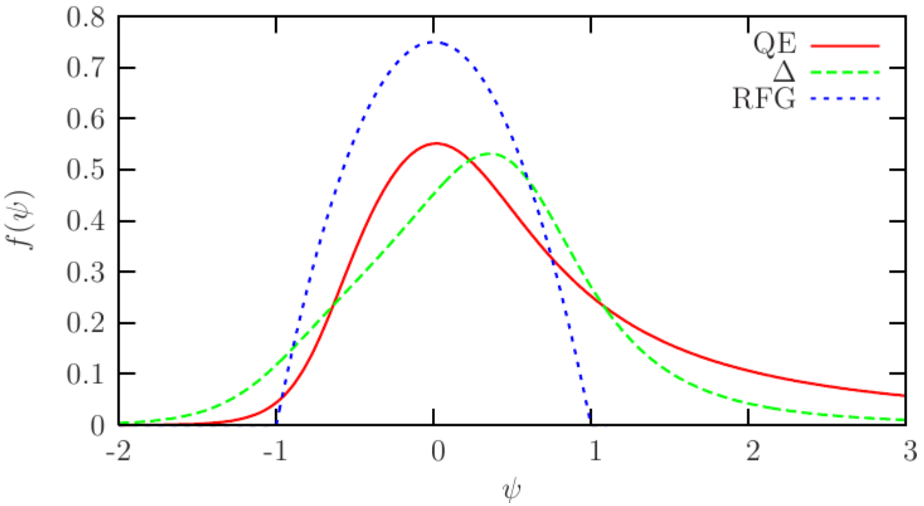
$$\rho = 1 + \frac{1}{4\tau}(m_\Delta^2 / m_N^2 - 1)$$ inelasticity



Amaro, Barbaro, Caballero, Donnelly, Molinari, Sick, PRC71 (2005)

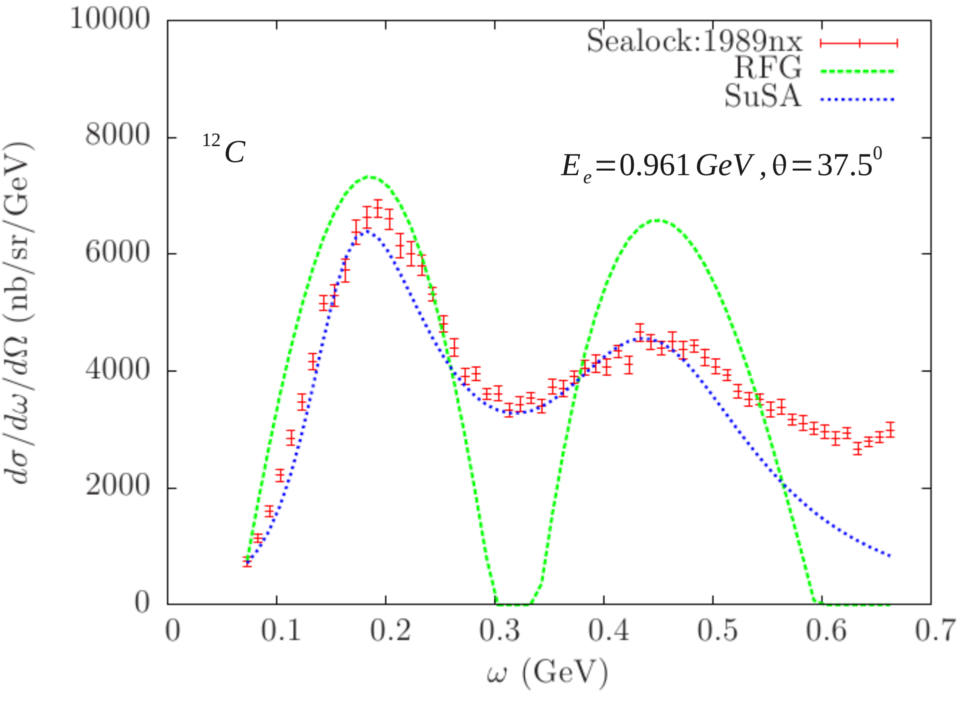
This approach can work only at $\Psi_\Delta<0$, since at $\Psi_\Delta>0$ other resonances and the tail of DIS contribute

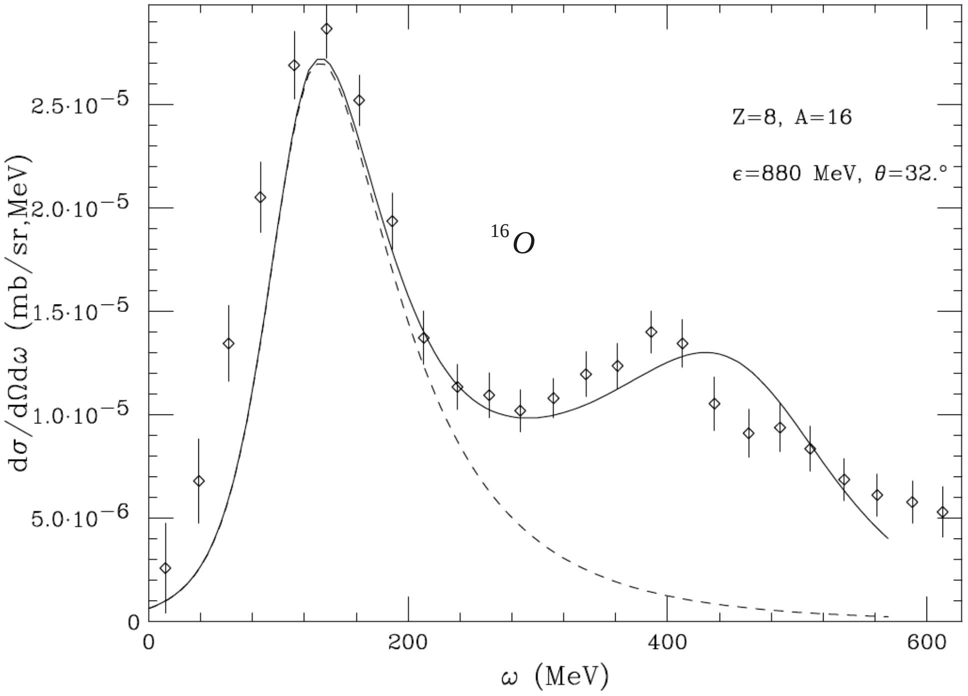
Test of the super-scaling function



$$R_L(q,\omega)=G_L(q,\omega)f_{QE}(\psi)+G_L^{\Delta}(q,\omega)f_{\Delta}(\psi_{\Delta})$$
$$R_T(q,\omega)=G_T(q,\omega)f_{QE}(\psi)+G_T^{\Delta}(q,\omega)f_{\Delta}(\psi_{\Delta})$$

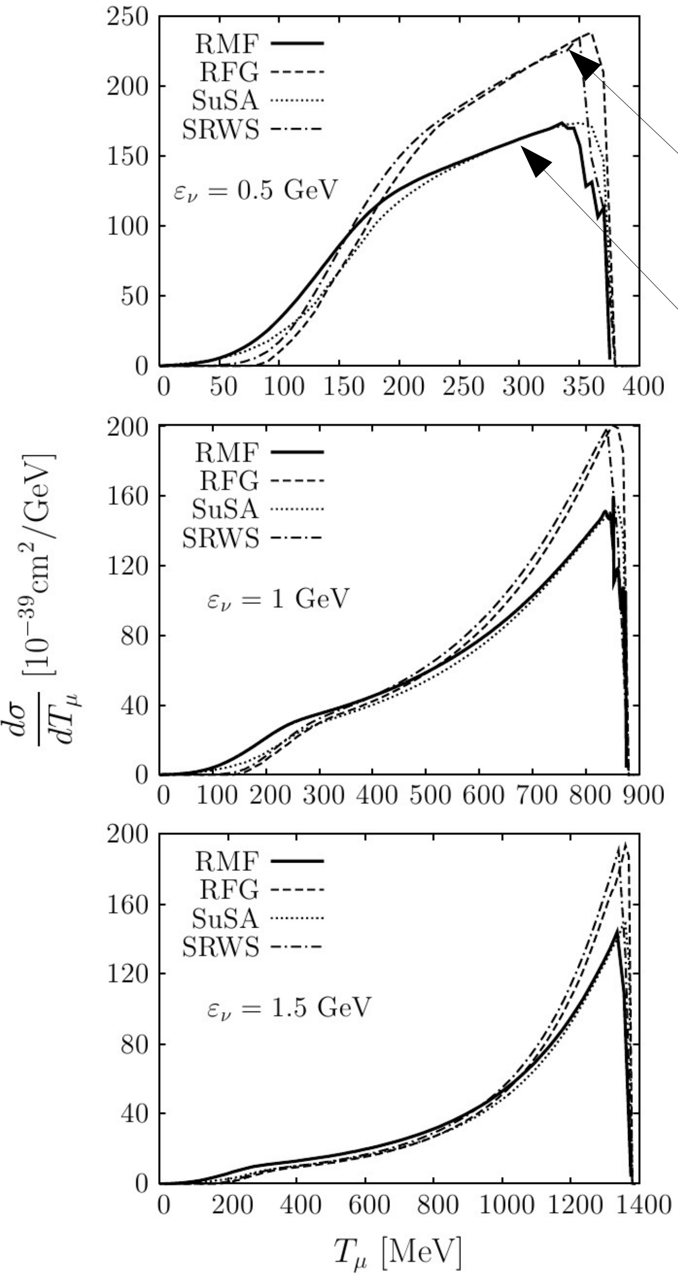
$$\frac{d^2\sigma}{d\omega\, d\Omega}=\sigma_{Mott}(v_L R_L+v_T R_T)$$





Amaro et al., PRC71, 015501 (2005)

Integrated cross sections

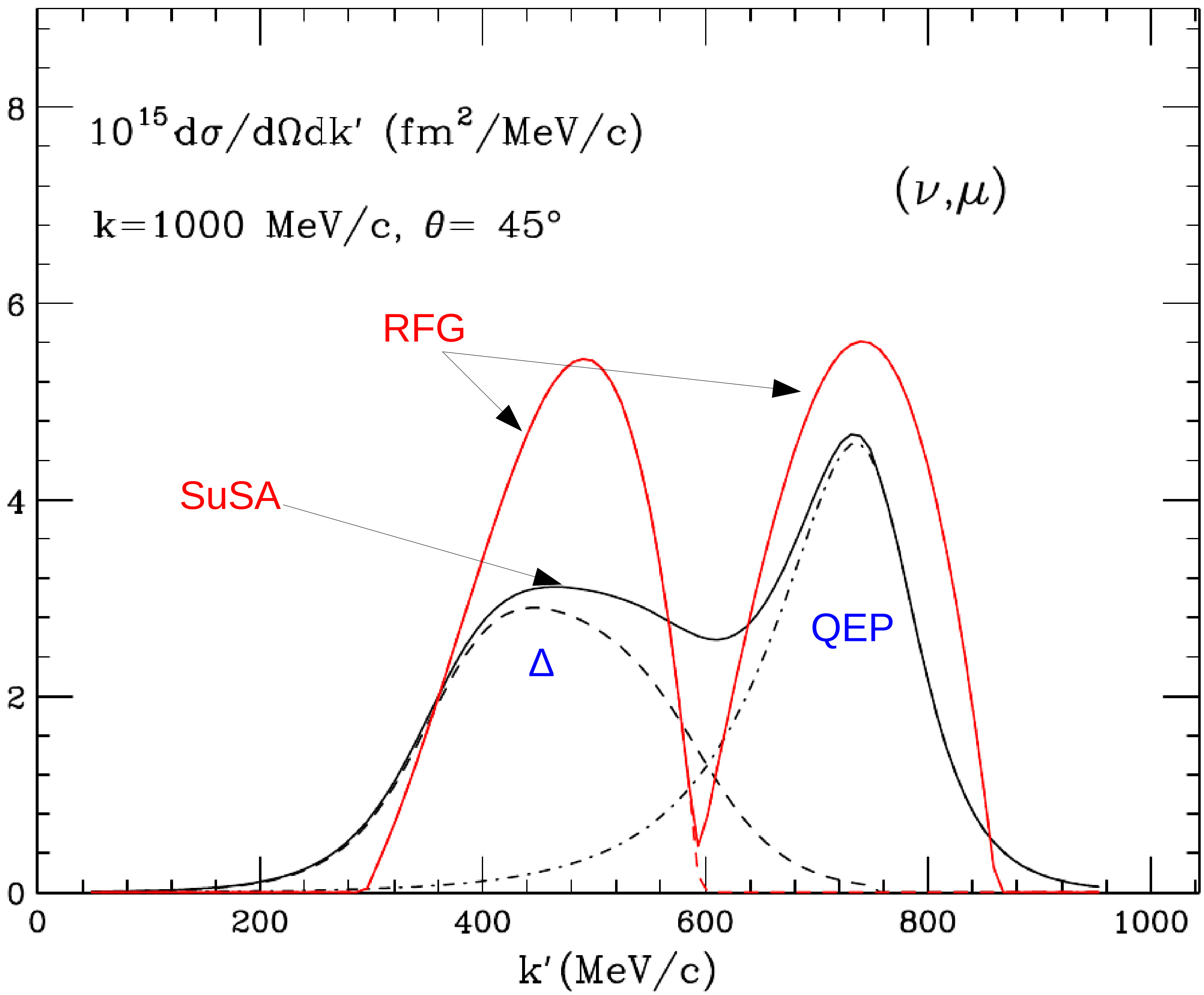


[Amaro et al., Phys. Rev. Lett. 98, 242501 (2007)]

Relativistic Fermi Gas ≈ Semi-relativistic Shell Model

Relativistic Mean Field ≈ SuperScaling Approach

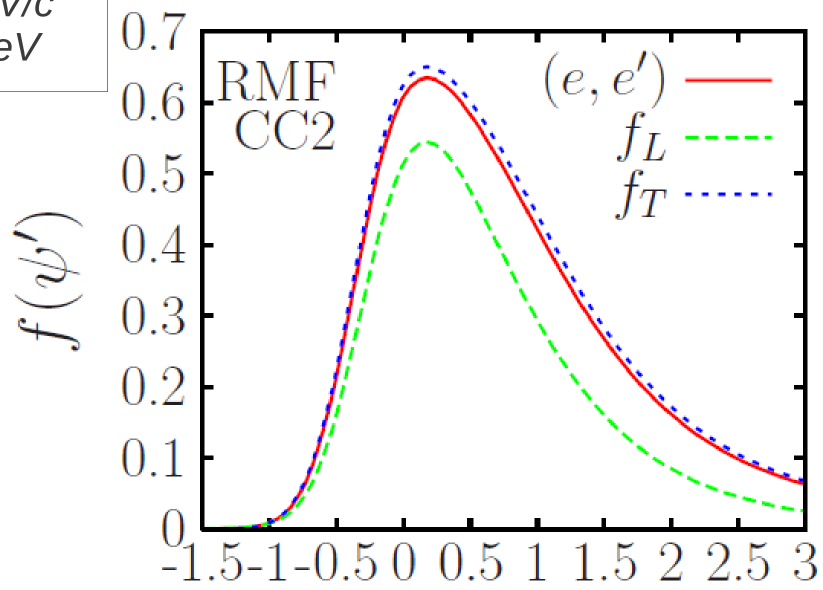
CC neutrino cross section in SuSA model



Amaro et al., PRC71, 015501 (2005)

Transverse enhancement in the RMF model

^{12}C
q=1 GeV/c
E_e=1 GeV



Fully Relativistic Mean Field (RMF) calculation: the L/T difference originates from the dynamical enhancement of the lower components due to the presence of strong potentials.

J.A.Caballero, J.E.Amaro, MBB, T.W.Donnelly, J.M.Udias, Phys.Lett.B653:366-372,2007

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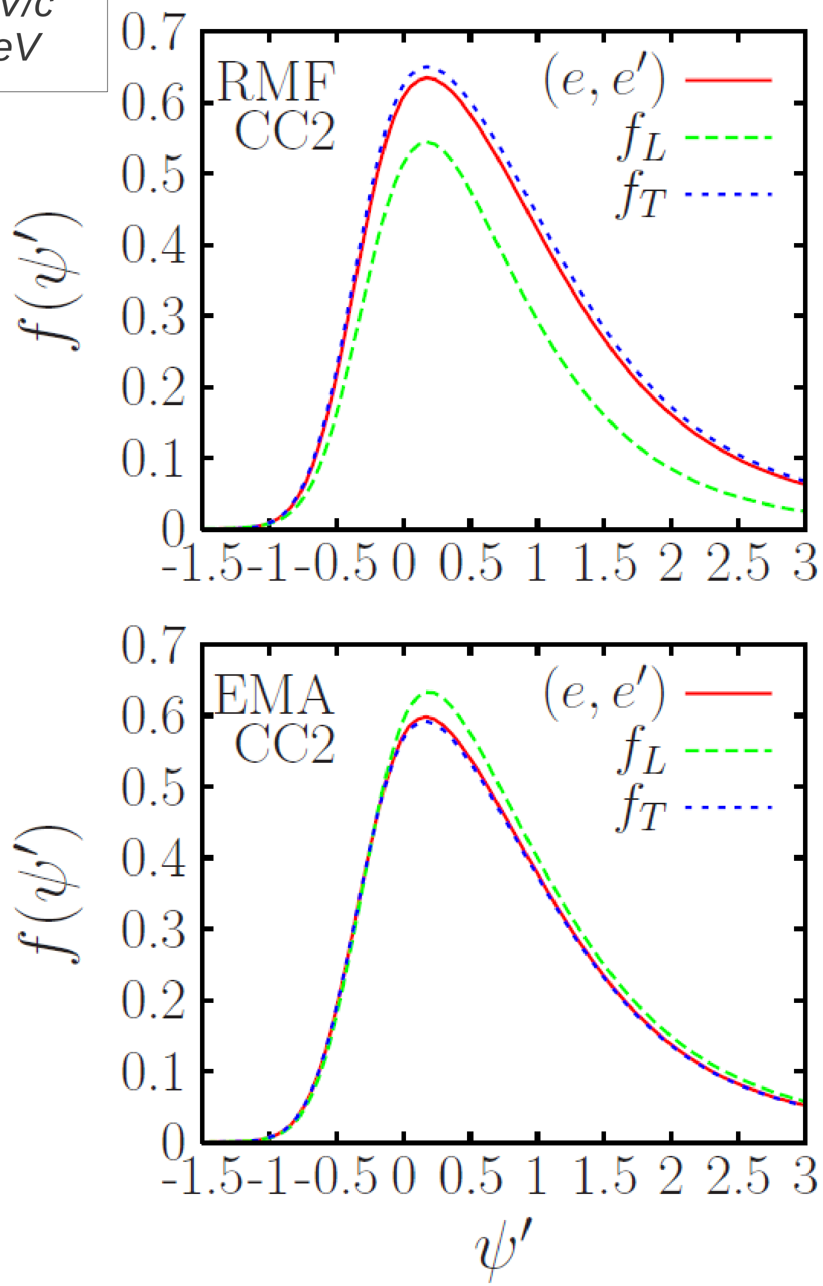

Fully Relativistic Mean Field (RMF) calculation: the L/T difference originates from the dynamical enhancement of the lower components due to the presence of strong potentials.

"Effective Momentum Approach" (EMA): the relationship between upper and lower components is forced to be the same as for free spinors: the L/T difference disappears.

J.A.Caballero, J.E.Amaro, MBB, T.W.Donnelly, J.M.Udias, Phys.Lett.B653:366-372,2007