SuSA-based calculations of multi-nucleon effects in scattering

Meson-exchange currents

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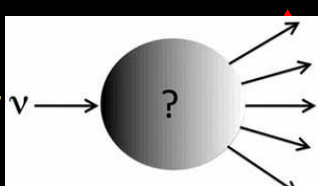


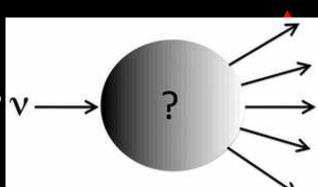
MEC in QE neutrino scattering

Results from the papers:

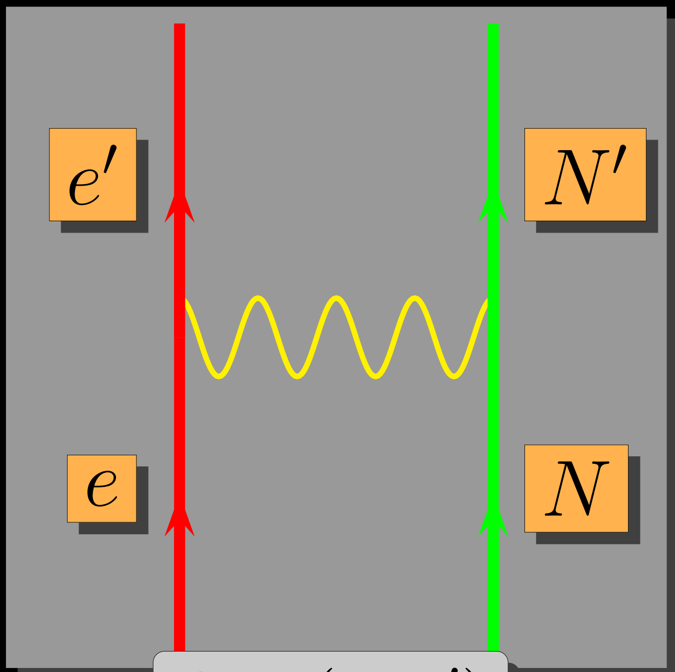
1. Meson-exchange currents and quasielastic Neutrino cross sections in the superscaling approximation model.
   J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C.F. Williamson.
   Physics Letters B 696 (2011) 151.
2. Meson-Exchange Currents and Quasielastic Antineutrino Cross Sections in the Superscaling Approximation
   J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly.
   Physical Review Letters 108, 152501 (2012)

Outline

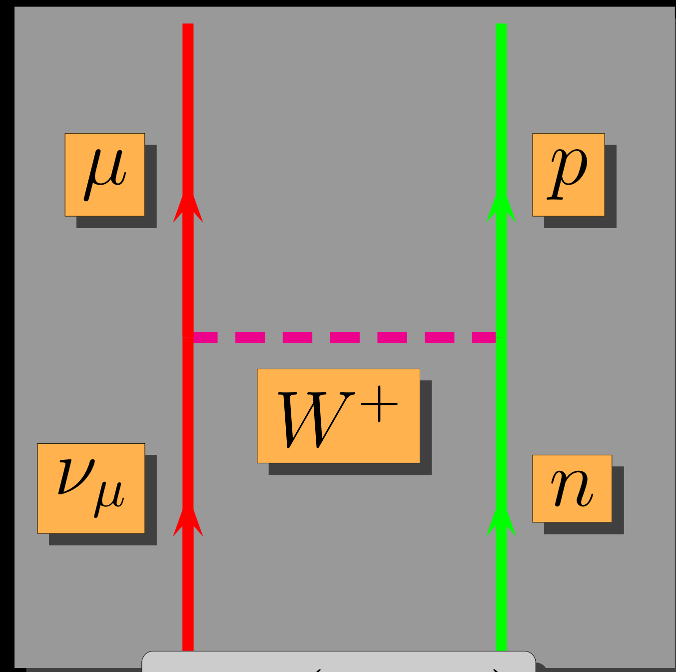
1. General formalism
   - (e, e')
   - (ν_l, l^-)
2. MEC in the 2p-2h channel
3. MEC results for MiniBooNE kinematics
4. Open problems
   - Correlation currents
   - MEC in the 1p-1h channel for high q



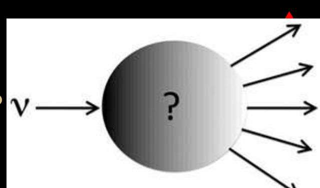
1 General formalism



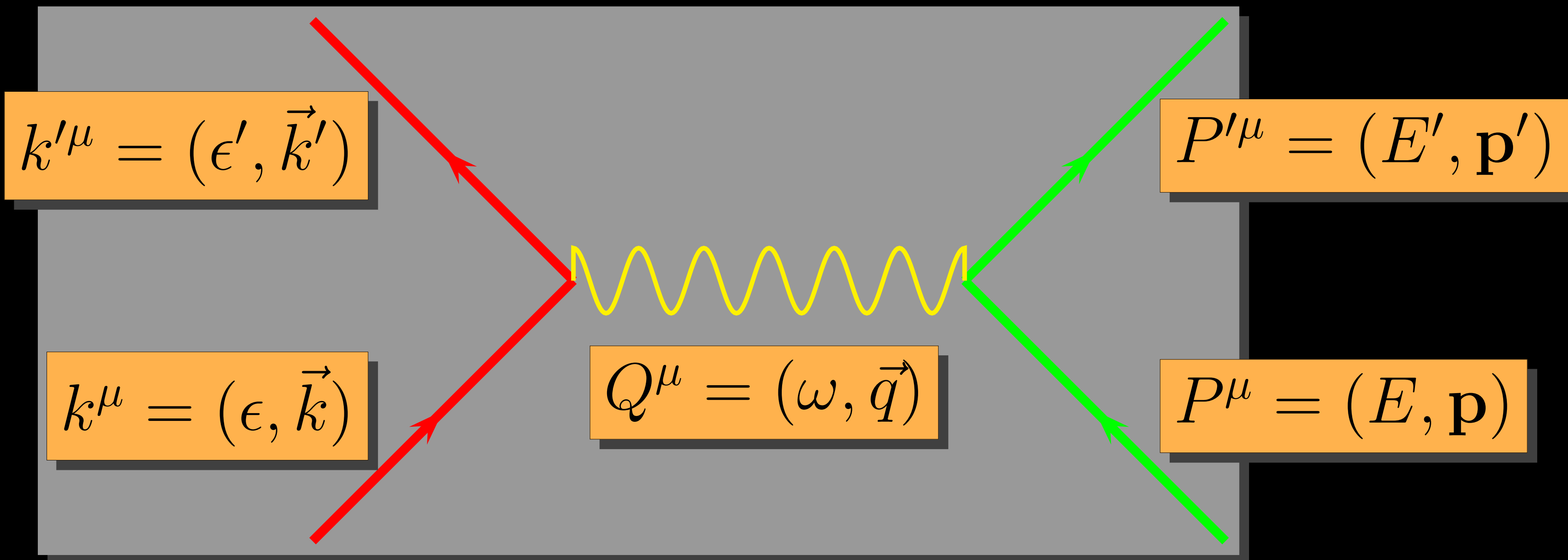
QE (e, e')



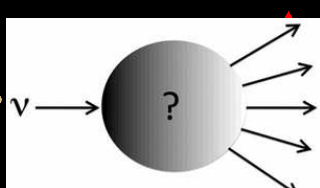
QE (ν_l, l^-)



Kinematics



$$Q^2 = \omega^2 - q^2 < 0$$



(e, e') *formalism*

$$\frac{d\sigma}{d\epsilon' d\Omega'} = \sigma_{Mott} \left(v_L R_L + v_T R_T \right)$$

Electron kinematical factors

$$v_L = \rho^2, \qquad v_T = \frac{1}{2}\rho + \tan^2\frac{\theta}{2}, \qquad \rho \equiv \frac{|Q^2|}{q^2}$$

Response functions: Components of the hadronic tensor $\Longrightarrow$

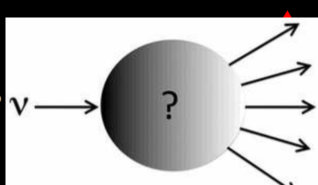
$$\begin{aligned} R_L &= W^{00} \\ R_T &= W^{11} + W^{22} \end{aligned}$$

(e, e') *formalism (II)*

Hadronic tensor for (e, e')

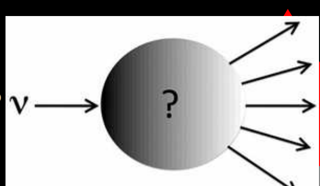
$$W^{\mu\nu}(q,\omega) = \overline{\sum_{fi}} \delta(E_f - E_i - \omega)\langle f|J^{\mu}(Q)|i\rangle^* \langle f|J^{\nu}(Q)|i\rangle$$

$J^{\mu}(Q)$: electromagnetic nuclear current



(ν_l, l^-) *formalism*

Cross section:	$\frac{d\sigma}{d\Omega' d\epsilon'} = \sigma_0 \mathcal{F}_+^2$
Similar to σ_{Mott}:	$\sigma_0 = \frac{G^2 \cos^2\theta_c}{2\pi^2} k'\epsilon' \cos^2\frac{\tilde{\theta}}{2}$
Fermi constant:	$G = 1.166 \times 10^{-11} \quad \mathrm{MeV}^{-2}$
Cabibbo angle:	$\cos\theta_c = 0.975$
Generalized scattering angle:	$\tan^2\frac{\tilde{\theta}}{2} = \frac{\lvert Q^2 \rvert}{(\epsilon+\epsilon')^2 - q^2}$



(ν_l, l^-) formalism (II)

1

Nuclear structure information:

$$\mathcal{F}_+^2 = \widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T + 2\widehat{V}_{T'} R_{T'}$$

kinematical factors $\widehat{V}_K$ from the leptonic tensor

$$\begin{aligned}
\widehat{V}_{CC} &= 1 - \delta^2 \tan^2 \frac{\tilde\theta}{2} \\
\widehat{V}_{CL} &= \frac{\omega}{q} + \frac{\delta^2}{\rho'} \tan^2 \frac{\tilde\theta}{2} \\
\widehat{V}_{LL} &= \frac{\omega^2}{q^2} + \left(1 + \frac{2\omega}{q\rho'} + \rho\delta^2\right) \delta^2 \tan^2 \frac{\tilde\theta}{2} \\
\widehat{V}_{T} &= \tan^2 \frac{\tilde\theta}{2} + \frac{\rho}{2} - \frac{\delta^2}{\rho'} \left(\frac{\omega}{q} + \frac{1}{2}\rho\rho'\delta^2\right) \tan^2 \frac{\tilde\theta}{2} \\
\widehat{V}_{T'} &= \frac{1}{\rho'} \left(1 - \frac{\omega\rho'}{q}\delta^2\right) \tan^2 \frac{\tilde\theta}{2}
\end{aligned}$$

Adimensional variables:

$$\begin{aligned}
\delta &= \frac{m'}{\sqrt{|Q^2|}} \\
\rho &= \frac{|Q^2|}{q^2} \\
\rho' &= \frac{q}{\epsilon + \epsilon'} .
\end{aligned}$$

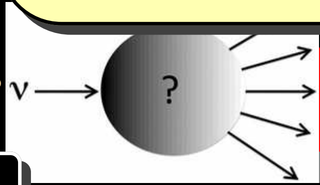
(ν_l, l^-) formalism (III)

Weak response functions

$$
\begin{aligned}
R_{CC} &= W^{00} \\
R_{CL} &= -\frac{1}{2}\left(W^{03}+W^{30}\right) \\
R_{LL} &= W^{33} \\
R_{T} &= W^{11}+W^{22} \\
R_{T'} &= -\frac{i}{2}\left(W^{12}-W^{21}\right)
\end{aligned}
$$

Weak CC hadronic tensor:

$$
W^{\mu\nu}(q,\omega) = \overline{\sum_{fi}} \delta(E_f - E_i - \omega)\langle f|J^{\mu}(Q)|i\rangle^{*}\langle f|J^{\nu}(Q)|i\rangle \; .
$$



or

Single-nucleon current

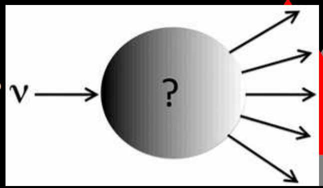
Electromagnetic current

$$j^\mu(\mathbf{p}', \mathbf{p}) = \overline{u}(\mathbf{p}') \left[2F_1 \gamma^\mu + i \frac{F_2}{m_N} \sigma^{\mu\nu} Q_\nu \right] u(\mathbf{p})$$

Weak CC current $j^\mu = j_V^\mu - j_A^\mu$.

$$j_V^\mu(\mathbf{p}', \mathbf{p}) = \overline{u}(\mathbf{p}') \left[2F_1^V \gamma^\mu + i \frac{F_2^V}{m_N} \sigma^{\mu\nu} Q_\nu \right] u(\mathbf{p})$$ ← Vector

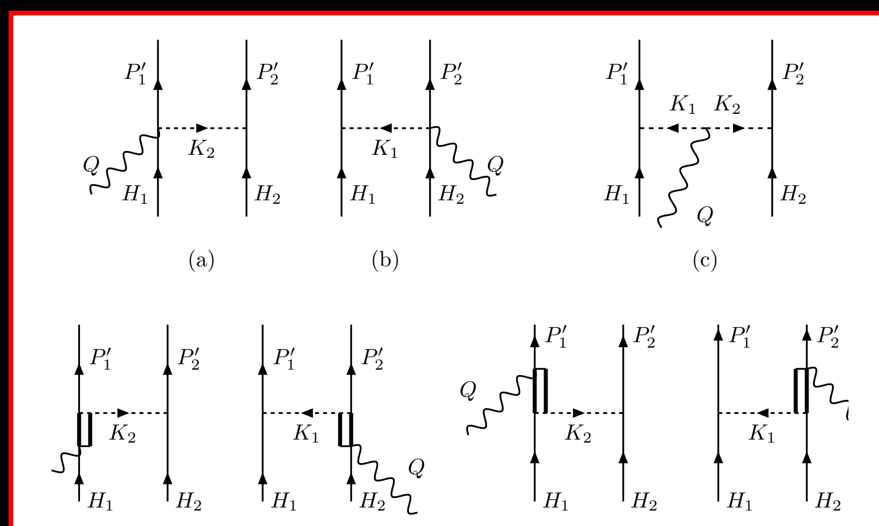
$$j_A^\mu(\mathbf{p}', \mathbf{p}) = \overline{u}(\mathbf{p}') \left[G_A \gamma^\mu + G_P \frac{Q^\mu}{2m_N} \right] \gamma^5 u(\mathbf{p})$$ ← Axial-Vector

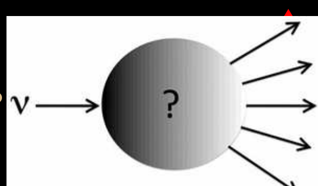


2 Meson-Exchange Currents (MEC)

Two-particle two-hole Meson Exchange Currents (MEC)

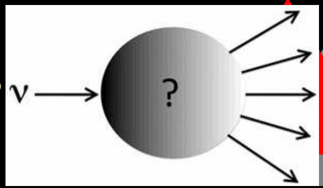
- Relativistic Fermi Gas two-nucleon emission channel
- Added to the SuSA results
- A. De Pace *et al.* NPA 726, 303 (2003)
- J.E. Amaro *et al.* PRC 82, 0444601 (2010)





Relativistic 2p-2h model in electron scattering

- A. De Pace, M. Nardi, W.M. Alberico, T.W. Donnelly, A. Molinari
  The 2p2h electromagnetic response in the quasielastic peak and beyond
  Nuclear Physics A 726 (2003)
- J.E. Amaro, C. Maieron, M.B. Barbaro, J.A. Caballero, and T.W. Donnellly
  Pionic Correlations and Meson-exchange currents in two-particle emission induced by electron sccattering.
  Physical Review C 82, 0444601 (2010)



Multi-nucleon emission in electron scattering

Inclusive electron scattering cross section

$$\frac{d\sigma}{d\Omega'_e d\omega} = \sigma_M \left[v_L R_L(q,\omega) + v_T R_T(q,\omega) \right] \,, \tag{1}$$

Longitudinal $R_L(q,\omega)$ and transverse $R_T(q,\omega)$ response functions

$$R_L = W^{00} \tag{2}$$

$$R_T = W^{11} + W^{22} \,, \tag{3}$$

Hadronic tensor

$$W^{\mu\nu} = \sum_f \langle f | J^\mu(Q) | i \rangle^* \langle f | J^\nu(Q) | i \rangle \delta(E_i + \omega - E_f) \tag{4}$$

$J^\mu(Q)$ is the nuclear current operator.

Relativistic Fermi gas (RFG)

- Initial state $|i\rangle = |F\rangle$,
- Sum over final states

$$\sum_f = \sum_{1p-1h} + \sum_{2p-2h} + \sum_{otherchannels}$$

- 2p-2h channel final states

$$|f\rangle = |2p-2h\rangle = |\mathbf{p}'_1 s'_1, \mathbf{p}'_2 s'_2, \mathbf{h}_1^{-1} s_1, \mathbf{h}_2^{-1} s_2\rangle$$

- Particle momenta $\mathbf{p}'_i \Longrightarrow$ Pauli blocking $p'_i > k_F$
  Four-momenta $P'_i = (E'_i, \mathbf{p}'_i)$,
- Hole momenta $\mathbf{h}_i \Longrightarrow h_i < k_F$.
  Four-momenta $H_i = (E_i, \mathbf{h}_i)$
- Spin indices: s'_i and s_i.

2p-2h Response Functions

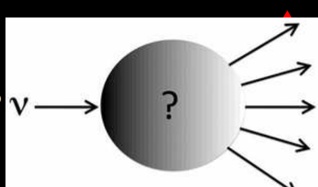
Proton (PP), Neutron (NN) and proton-neutron (PN) emission

$$R_K = R_K(PP) + R_K(NN) + R_K(PN)\,. \tag{5}$$

Example: L response for the PP channel

$$R_L(PP) = \frac{1}{4} \sum_{\mathbf{p}'_1 s'_1} \sum_{\mathbf{p}'_2 s'_2} \sum_{\mathbf{h}_1 s_1} \sum_{\mathbf{h}_2 s_2} \left| \langle \mathbf{p}'_1 \mathbf{p}'_2 \mathbf{h}_1^{-1} \mathbf{h}_2^{-1} | J^0(Q) | F \rangle \right|^2 \times \delta(E'_1 + E'_2 - \omega - E_1 - E_2)$$

Factor $\frac{1}{4}$ to avoid double counting under interchange $1' \leftrightarrow 2'$ and $1 \leftrightarrow 2$.



Many-body matrix elements

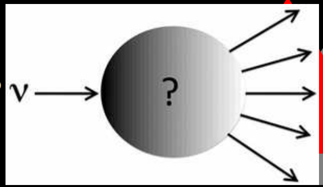
Two-body operator: direct minus exchange part of the two-body current matrix element

$$\langle \mathbf{p}'_1 \mathbf{p}'_2 \mathbf{h}_1^{-1} \mathbf{h}_2^{-1} | J^\mu | F \rangle = \langle \mathbf{p}'_1 \mathbf{p}'_2 | J^\mu | \mathbf{h}_1 \mathbf{h}_2 \rangle - \langle \mathbf{p}'_1 \mathbf{p}'_2 | J^\mu | \mathbf{h}_2 \mathbf{h}_1 \rangle \, ,$$

Two-body current function $j^\mu(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2)$:

$$\langle \mathbf{p}'_1 \mathbf{p}'_2 | J^\mu | \mathbf{h}_1 \mathbf{h}_2 \rangle = (2\pi)^3 \delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q}) \times \frac{m^2}{V^2 (E_1 E_2 E'_1 E'_2)^{1/2}} j^\mu(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{h}_1, \mathbf{h}_2). \quad (6)$$

$V = 3\pi^2 \mathcal{N} / k_F^3$.

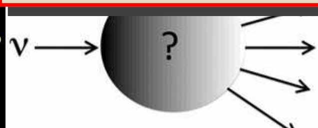


Integral over p_2'

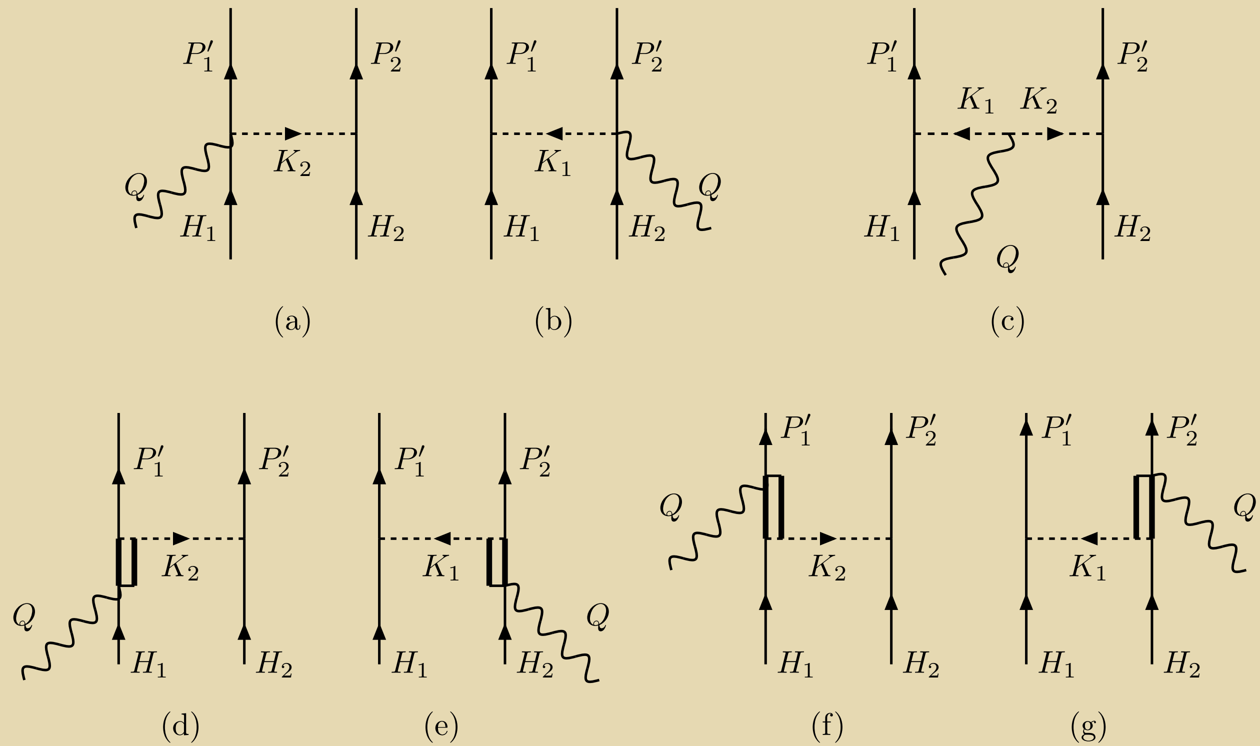
$$R_L(PP) = \frac{V}{4} \sum_{s_1' s_2' s_1 s_2} \int \frac{d^3p_1'}{(2\pi)^3} \frac{d^3h_1}{(2\pi)^3} \frac{d^3h_2}{(2\pi)^3}$$
$$\times \frac{m^4}{E_1 E_2 E_1' E_2'} \left| j^0(\mathbf{p}_1', \mathbf{p}_2', \mathbf{h}_1, \mathbf{h}_2)_A \right|^2$$
$$\times\, \delta(E_1' + E_2' - \omega - E_1 - E_2)\theta(p_2' - k_F)\,, \qquad (7)$$

Momentum conservation: $\mathbf{p}_2' = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q} - \mathbf{p}_1'$
Integration limits:
$h_1, h_2 < k_F$ and $p_1' > k_F$.
Anti-symmetrized current function

$$j^\mu(1', 2', 1, 2)_A \equiv j^\mu(1', 2', 1, 2) - j^\mu(1', 2', 2, 1)$$



Meson-Exchange Currents



Feynman diagrams:
Seagull (a,b), pionic (c), and Δ current (d-g)
Pionic four-momenta $K_i^\mu = P_i'^\mu - H_i^\mu$

Seagull and pionic

Seagull:

$$j_s^\mu(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_1, \mathbf{p}_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab}\overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1) \times \frac{F_1^V}{K_1^2 - m_\pi^2}\overline{u}(\mathbf{p}_2')\tau_b\gamma_5\gamma^\mu u(\mathbf{p}_2) + (1 \leftrightarrow 2)\,. \quad (8)$$

Pion in flight:

$$j_p^\mu(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_1, \mathbf{p}_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab}\frac{F_\pi(K_1 - K_2)^\mu}{(K_1^2 - m_\pi^2)(K_2^2 - m_\pi^2)} \times\overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1)\overline{u}(\mathbf{p}_2')\tau_b\gamma_5 \not{K}_2 u(\mathbf{p}_2)\,. \quad (9)$$

F_1^V and F_π: the electromagnetic form factors
pion-nucleon coupling constant: $f^2/4\pi = 0.08$.

Δ *Current*

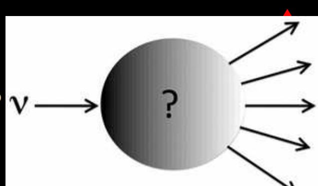
$$
j^\mu_\Delta(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2) = \frac{f_{\pi N \Delta} f}{m_\pi^2} \frac{1}{K_2^2 - m_\pi^2} \overline{u}(\mathbf{p}'_1) T^\mu_a(1) u(\mathbf{p}_1)
$$

$$
\times \overline{u}(\mathbf{p}'_2) \tau_a \gamma_5 \not{K}_2 u(\mathbf{p}_2) + (1 \leftrightarrow 2)\,. \qquad (10)
$$

$T^\mu_a(1)$ is related to the pion electroproduction amplitude

$$
T^\mu_a(1) = K_{2,\alpha} \Theta^{\alpha\beta} G^\Delta_{\beta\rho}(H_1 + Q) S^{\rho\mu}_f(H_1) T_a T_3^\dagger
$$

$$
+ T_3 T_a^\dagger S^{\mu\rho}_b(P'_1) G^\Delta_{\rho\beta}(P'_1 - Q) \Theta^{\beta\alpha} K_{2,\alpha}\,. \qquad (11)
$$



$\triangle$ *electromagnetic tensor*

Forward

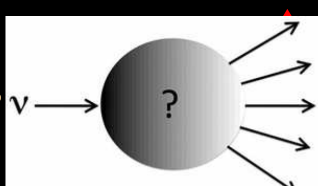
$$
\begin{aligned}
S_f^{\rho\mu}(H_1) &= \Theta^{\rho\mu}\left[g_1 \not{Q} - g_2 H_1\cdot Q + g_3 Q^2\right]\gamma_5 \\
&- \Theta^{\rho\nu} Q_\nu \left[g_1\gamma^\mu - g_2 H_1^\mu + g_3 Q^\mu\right]\gamma_5
\end{aligned}
\qquad (12)
$$

Backward

$$
\begin{aligned}
S_b^{\rho\mu}(P_1') &= \gamma_5\left[g_1 \not{Q} - g_2 P_1'\cdot Q - g_3 Q^2\right]\Theta^{\mu\rho} \\
&- \gamma_5\left[g_1\gamma^\mu - g_2 P_1'^{\mu} - g_3 Q^\mu\right] Q_\nu \Theta^{\nu\rho}\,.
\end{aligned}
\qquad (13)
$$

The tensor $\Theta_{\mu\nu}$

$$
\Theta_{\mu\nu} = g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\,. \qquad (14)
$$

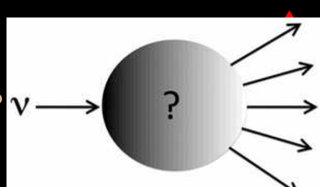


Δ *propagator*

Rarita-Schwinger tensor

$$G^{\Delta}_{\beta\rho}(P) = -\frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2} \times \left[g_{\beta\rho} - \frac{1}{3}\gamma_\beta\gamma_\rho - \frac{2}{3}\frac{P_\beta P_\rho}{m_\Delta^2} - \frac{\gamma_\beta P_\rho - \gamma_\rho P_\beta}{3m_\Delta} \right] . \quad (15)$$

Δ width: $m_\Delta \to m_\Delta + \frac{i}{2}\Gamma(P)$ in the denominator of the propagator to account for the Δ decay probability



Integration of the energy delta function

9-D integral for the 2p-2h response functions

$$\int d^3p'_1 d^3h_1 d^3h_2 \delta(E_1 + E_2 + \omega - E'_1 - E'_2) f(\mathbf{h}_1, \mathbf{h}_2, \mathbf{p}'_1, \mathbf{p}'_2) \,, \quad (16)$$

Momentum conservation $\mathbf{p}'_2 = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q} - \mathbf{p}'_1$.
We integrate over the momentum p'_1 using the delta function:

- For fixed $\mathbf{h}_1, \mathbf{h}_2$, θ'_1, ϕ'_1
- Change variables $p'_1 \to E' = E'_1 + E'_2$.
- compute the Jacobian of the transformation

$$dp'_1 = \frac{dE'}{\left| \frac{p'_1}{E'_1} - \frac{\mathbf{p}'_2 \cdot \mathbf{p}'_1}{E'_2 p'_1} \right|} \,, \quad (17)$$

Momentum of the final nucleon

- Compute p'_1 for fixed angles θ'_1, ϕ'_1, by solving the energy conservation equation.
- Second degree equation with two solutions

$$p'_1 = \frac{a}{b}\left(v \pm v_0 \sqrt{1 - \frac{bm_N^2}{a^2}}\right) , \quad (18)$$

where

$$a = \frac{1}{2}p'^2 \qquad b = E'^2 - p'^2 \cos^2 \beta'_1 \quad (19)$$

$$v_0 = E' \qquad v = p' \cos \beta'_1 \, , \quad (20)$$

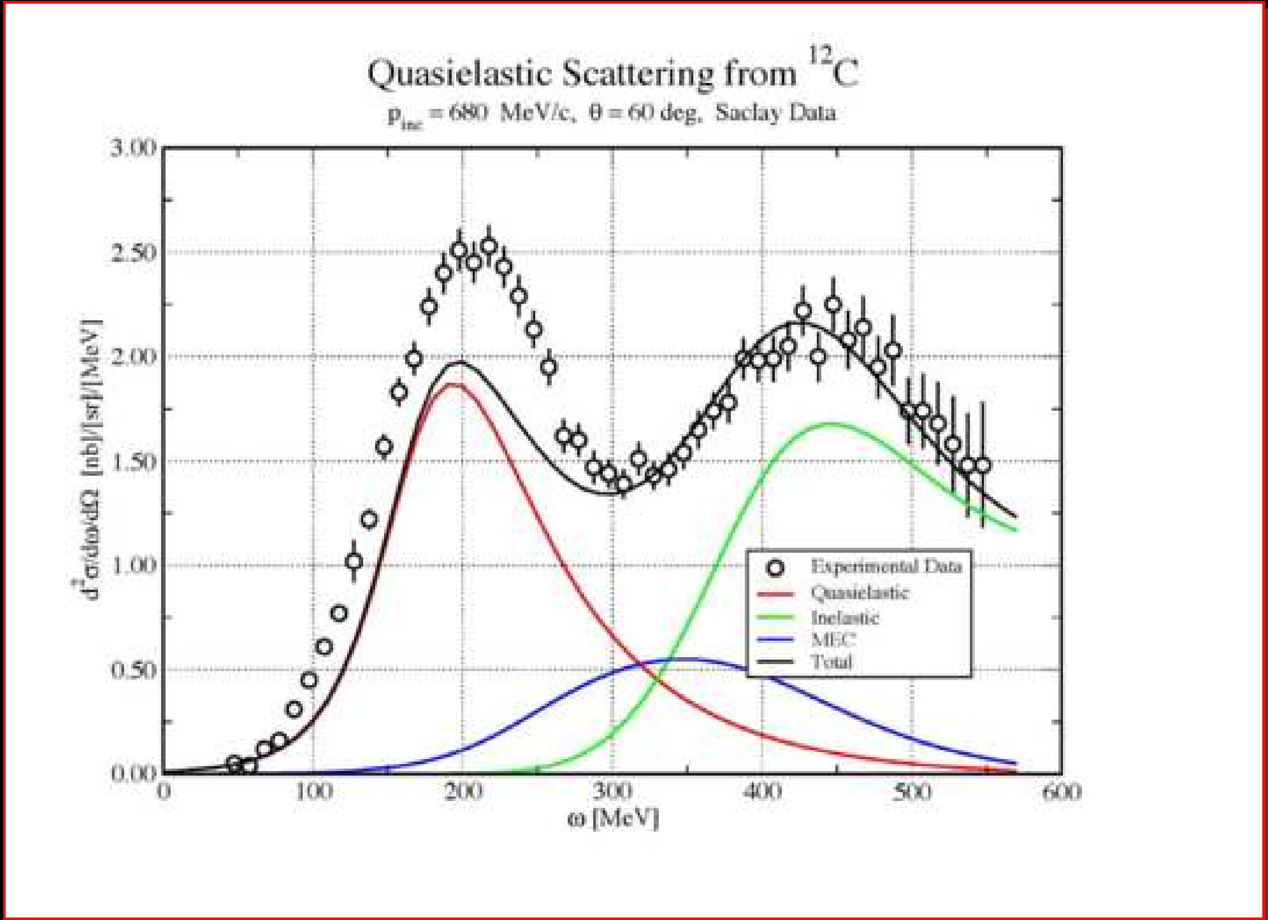
Final total energy: $E' = E_1 + E_2 + \omega$
Final total momentum: $\mathbf{p}' = \mathbf{h}_1 + \mathbf{h}_2 + \mathbf{q}$
$\beta'_1 =$ angle between $\mathbf{p}'_1$ and $\mathbf{p}'$.

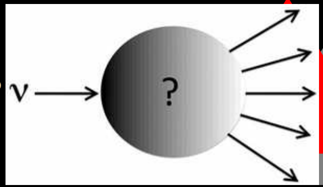
Results for 2p-2h responses

- Compute transverse response functions in the 2p-2h channel.
- Relativistic Fermi gas
- 7D integrals $\int d^3h_1 d^3h_2 d\theta'_1$
- We choose $\phi'_1 = 0$ and multiply by 2π.
- p'_1 is fixed from energy conservation

(e, e') *results with MEC 2p-2h*

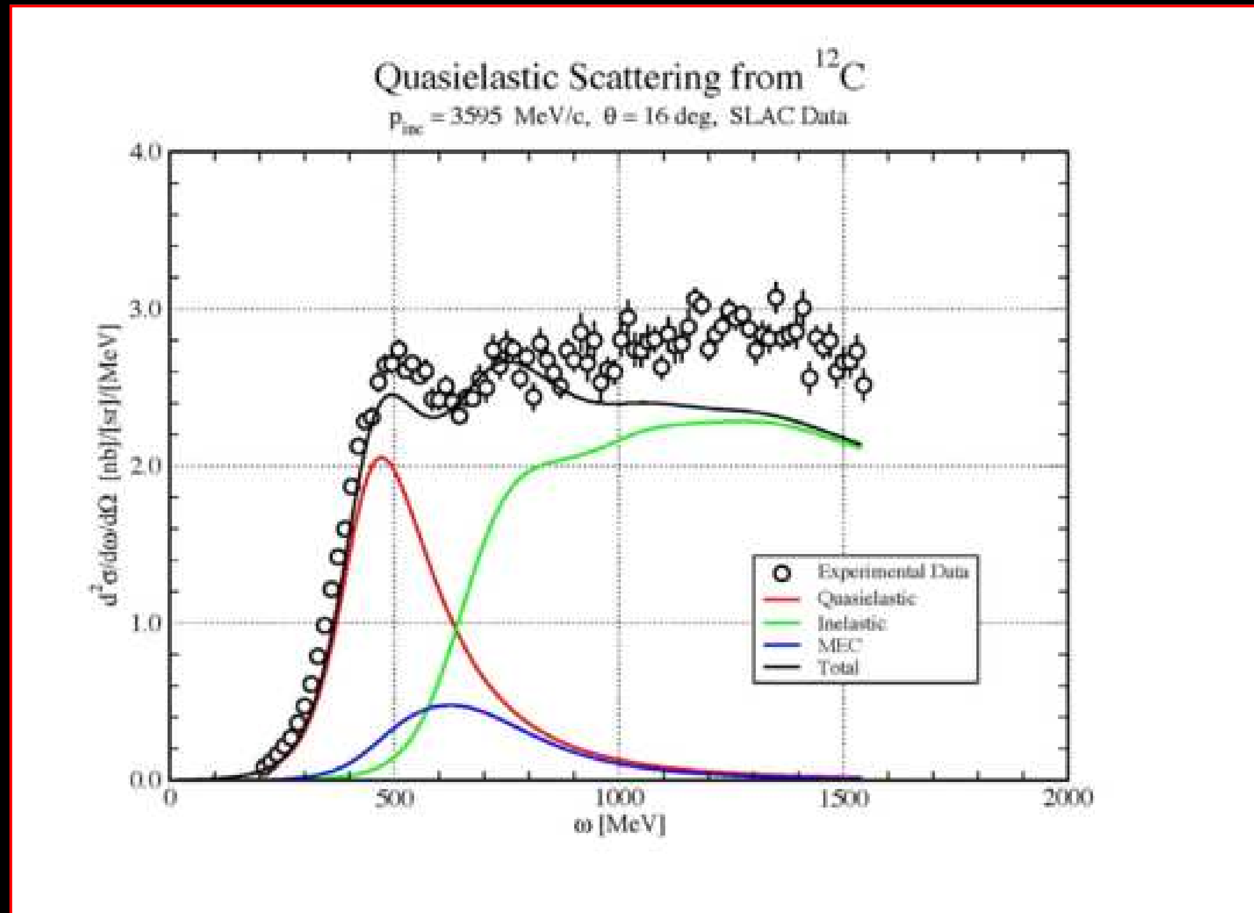
- Quasielastic + MEC + Inelastic cross section
- $\epsilon_e = 680$ MeV
- $\theta = 60^o$
- Data from Saclay

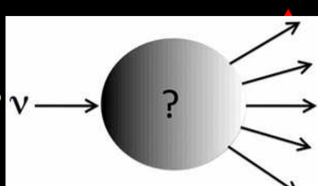




(e, e') *results with MEC 2p-2h*

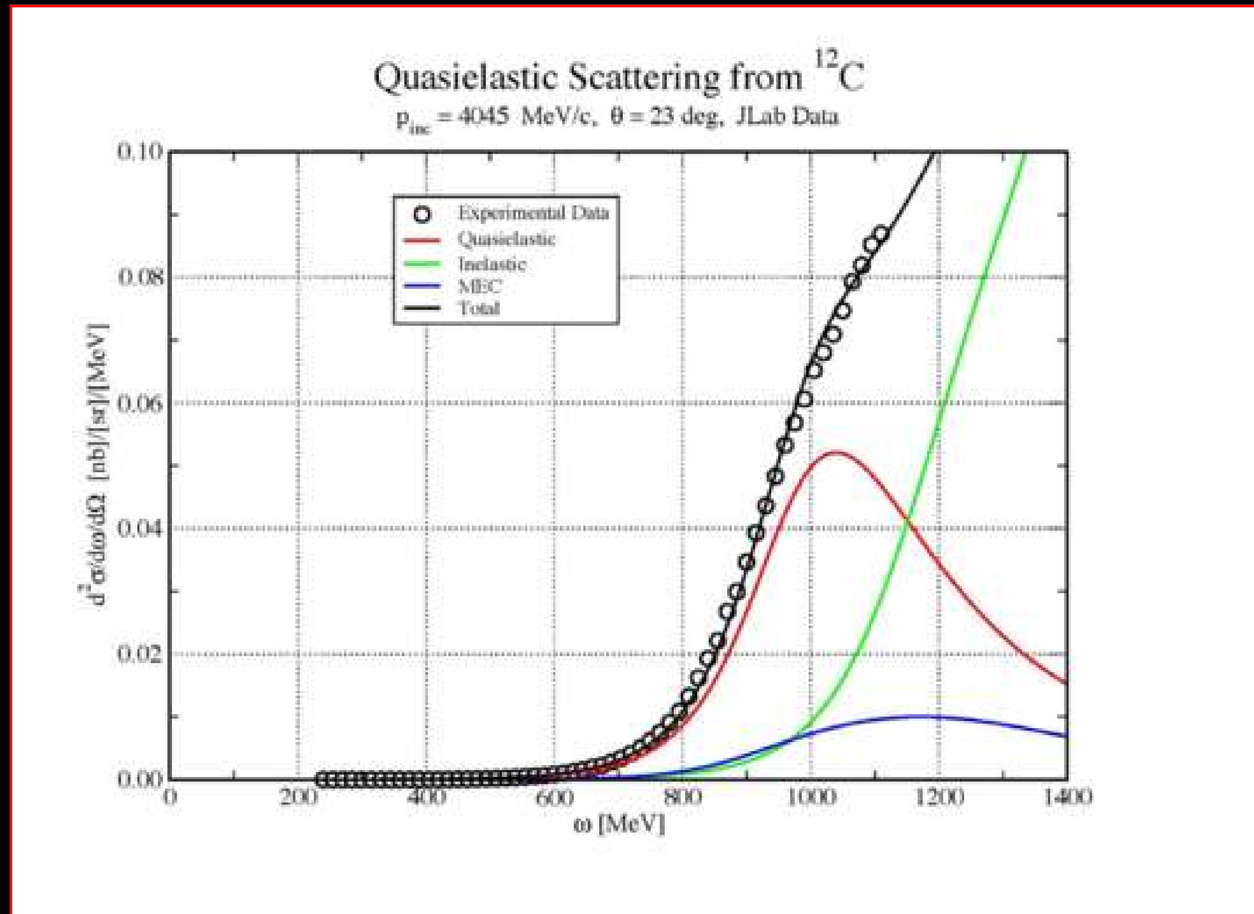
- Quasielastic + MEC + Inelastic cross section
- $\epsilon_e = 3595$ MeV
- $\theta = 16^o$
- Data from SLAC

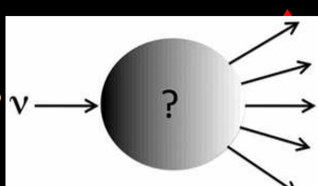




(e, e') *results with MEC 2p-2h*

- Quasielastic + MEC + Inelastic cross section
- $\epsilon_e = 4045$ MeV
- $\theta = 60^o$
- Data JLab

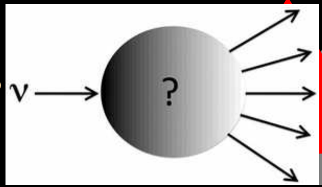




4 SuSA+MEC results for the MiniBooNE QE cross section

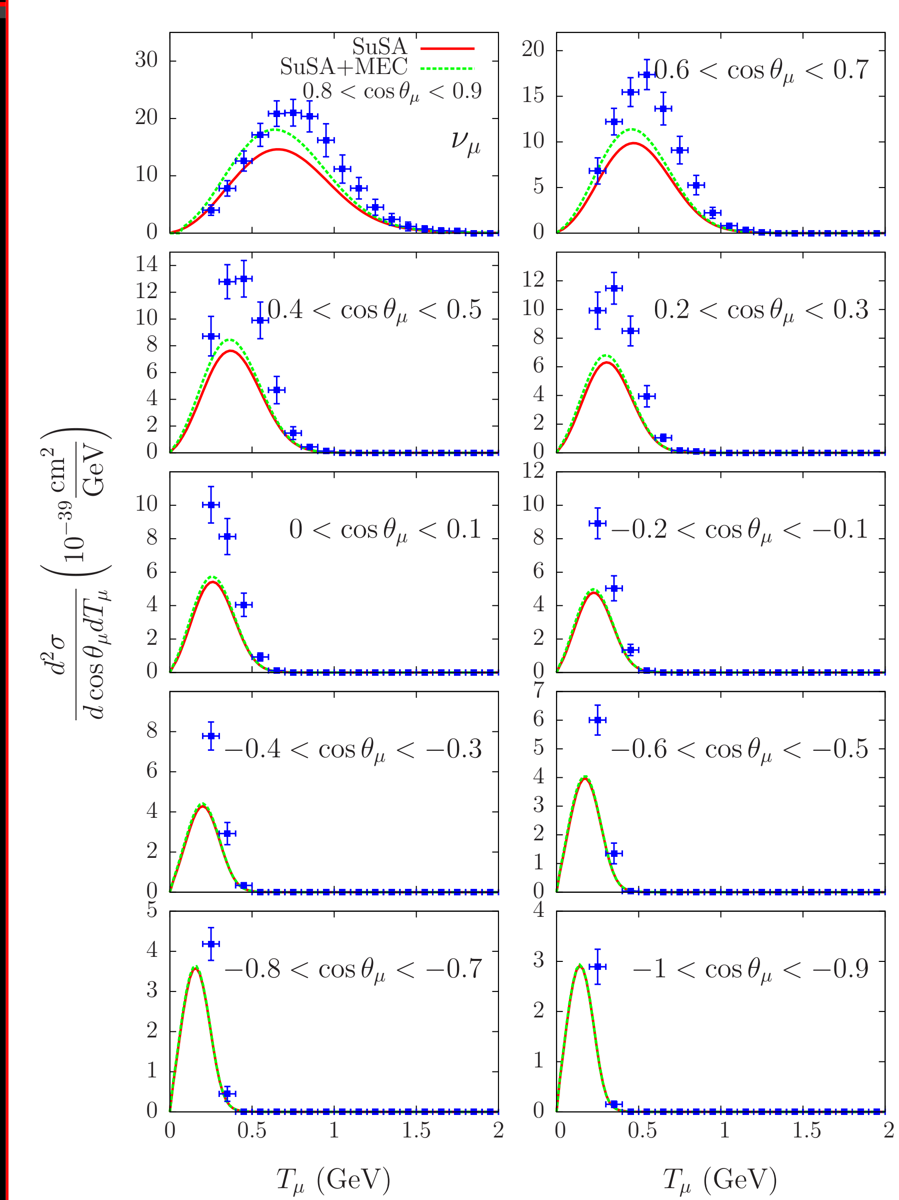
J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, C.F. Williamson, Physics Letters B (2011)

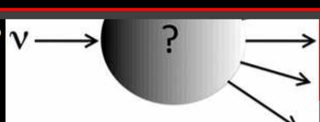
- Double differential neutrino cross sections from ^{12}C
- Integrated over the neutrino flux
- Contribution of vector meson-exchange currents in the 2p-2h sector



Neutrino results with MEC 2p-2h

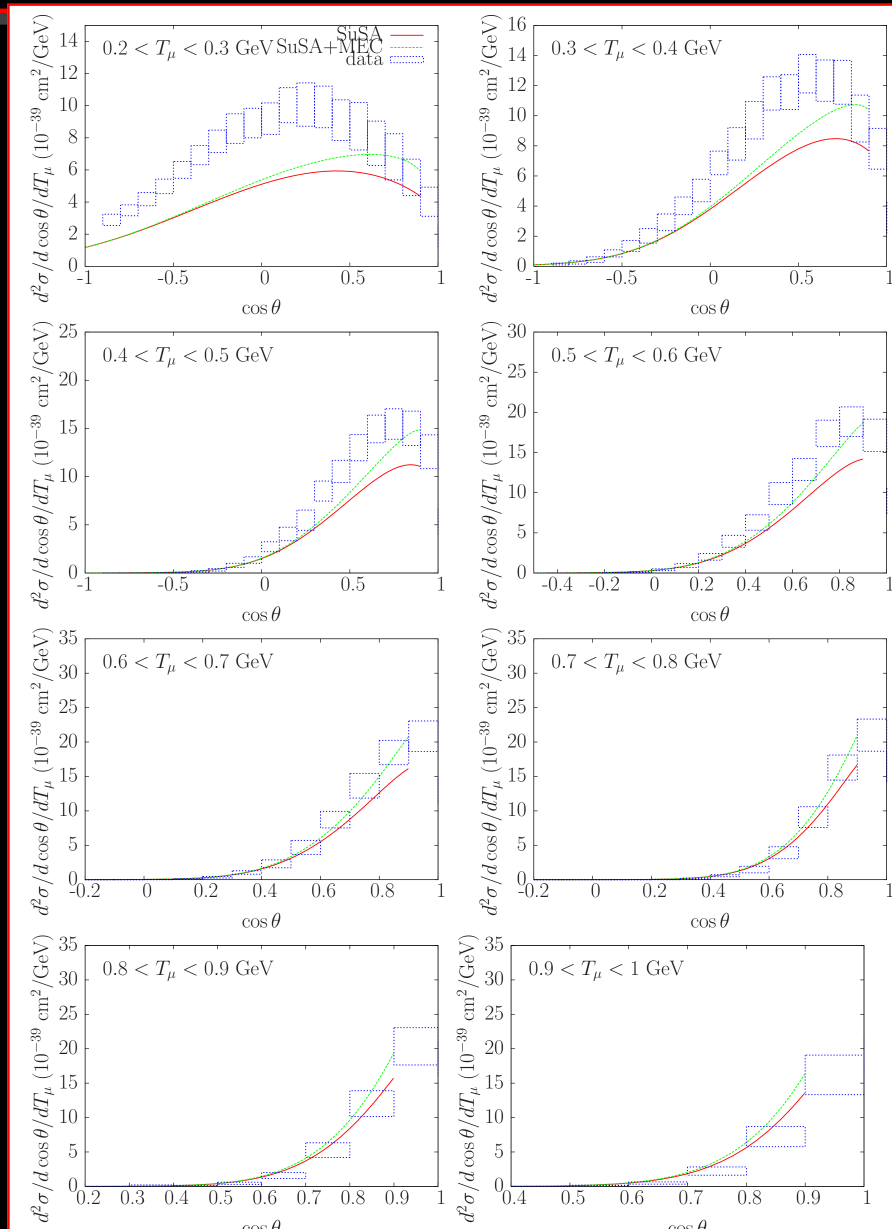
- Calculations from Amaro, Barbaro, Caballero, Donnelly, Williamson, PLB 696 (2011) 151.
- The MEC increase the cross section less than 10%
- Data from A.A. Aguilar-Arevalo *et al.*, (MiniBooNE Collaboration), PRD 81, 092005 (2010)

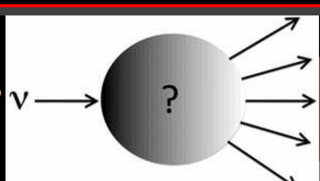




Neutrino results. Angle projection

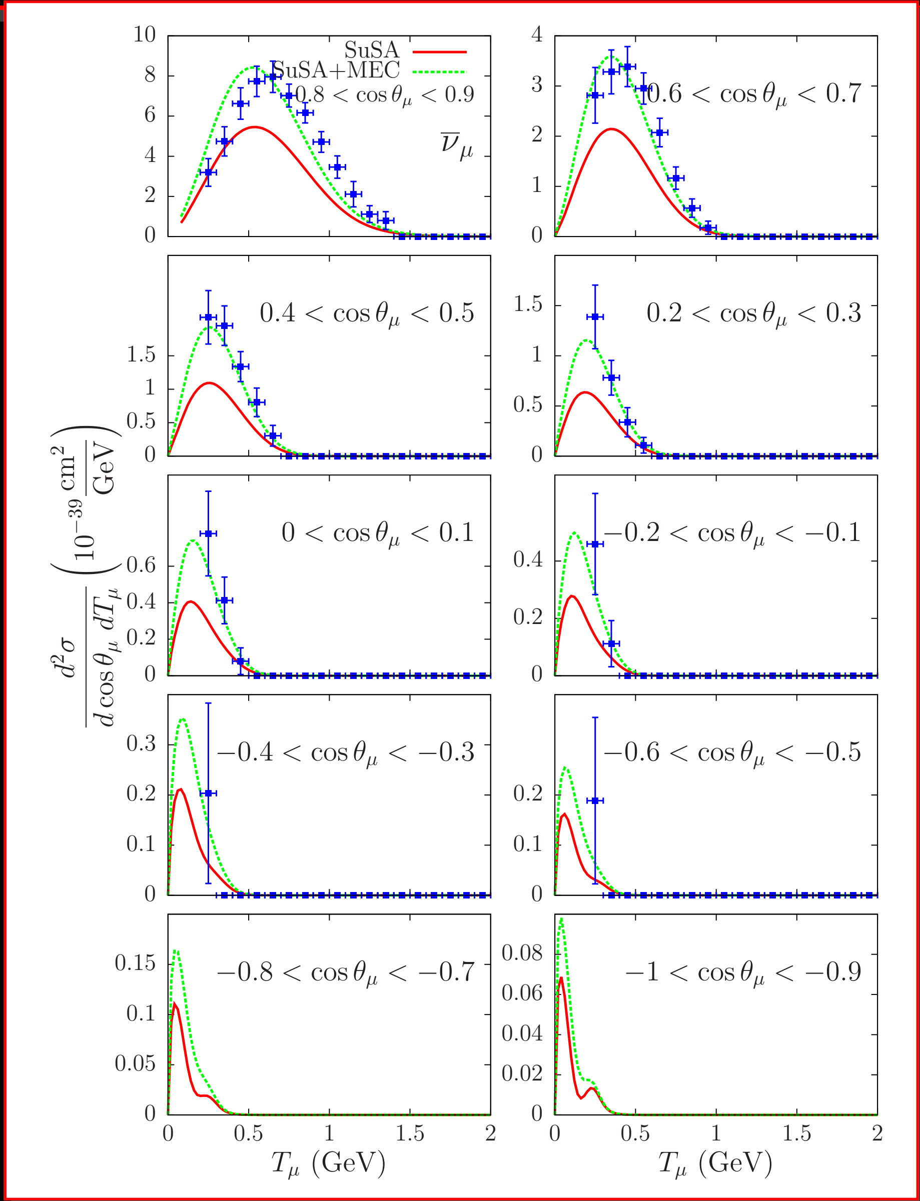
- Calculations from Amaro, Barbaro, Caballero, Donnelly, Williamson, PLB 696 (2011) 151.
- The MEC tend to increase the cross section about 5-10%
- Data from Aguilar-Arevalo et. al. (MiniBooNE Collaboration)

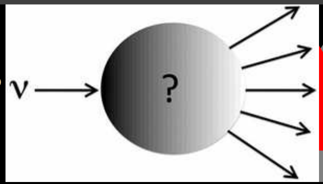




Antineutrino results with MEC 2p-2h

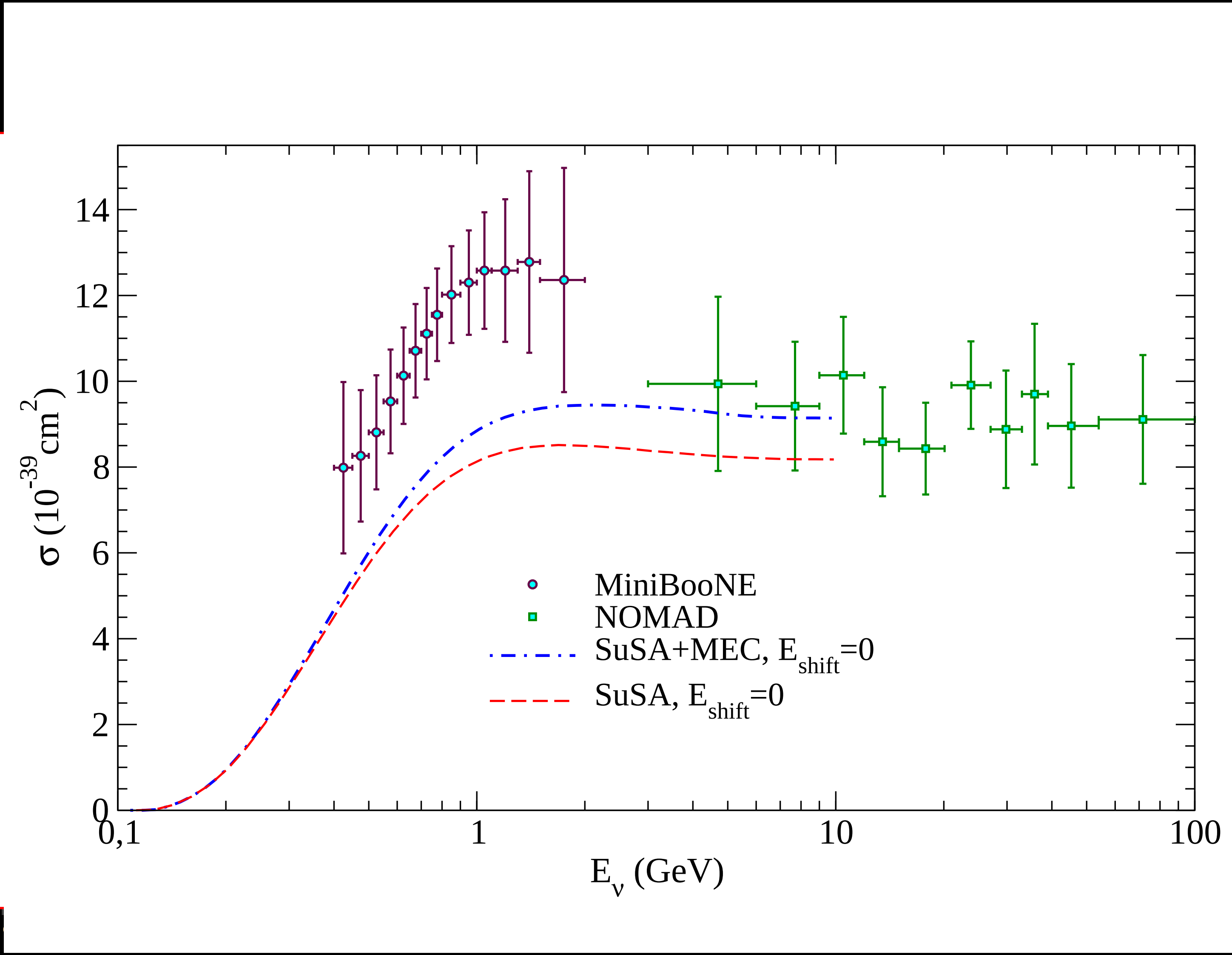
- Calculations from Amaro, Barbaro, Caballero, Donnelly, PRL 108 (2012).
- The MEC tend to increase the cross section more than for neutrinos.
- Data from Aguilar-Arevalo et. al. (MiniBooNE Collaboration) PRD 88 (2013)

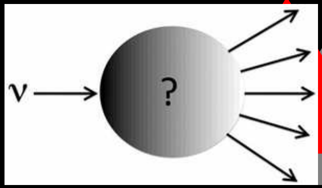




ν Total cross section. New results

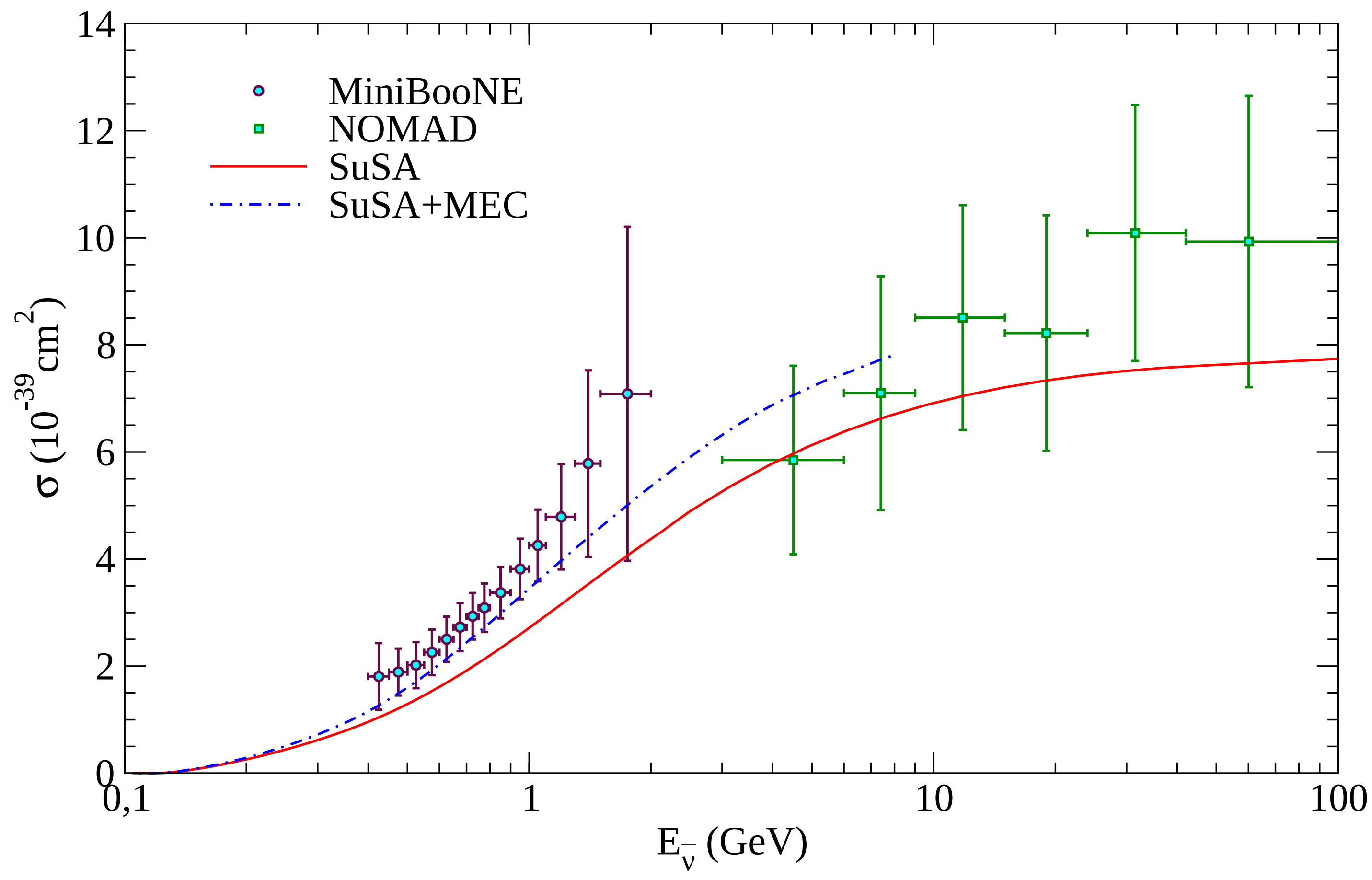
- Total cross section versus neutrino energy.
- SuSA with and without MEC compared to RFG

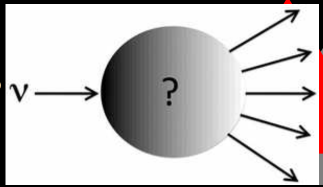




$\bar{\nu}$ Total cross section. New results

- Total cross section versus antineutrino energy.
- SuSA with and without MEC compared to RFG



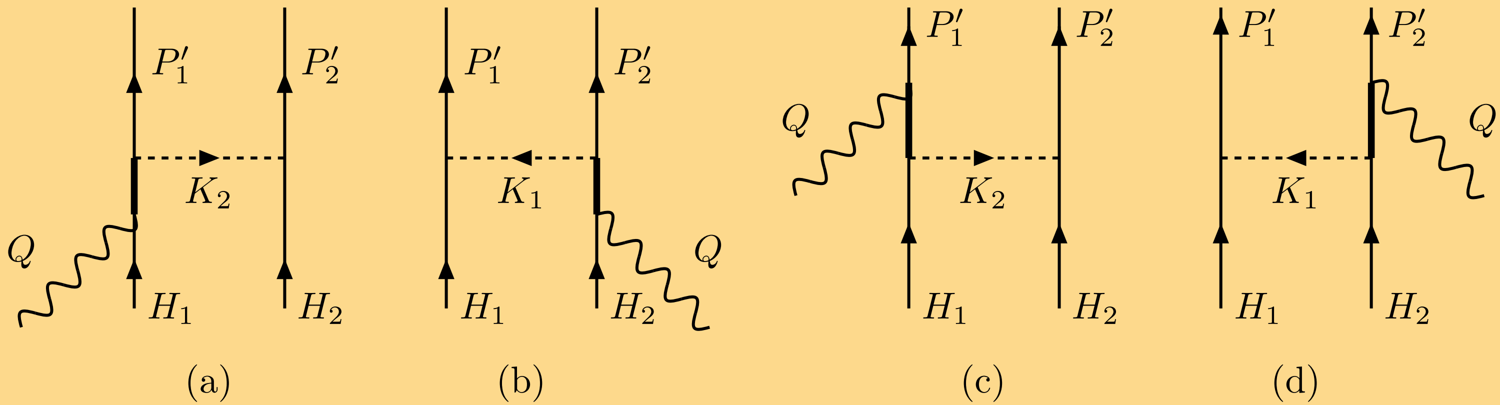


4. MEC open problems

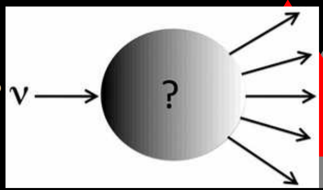
1. Correlation current and gauge invariance. Regularization of the correlation response function.
2. MEC in the 1p-1h channel: large effects for high momentum transfer?

Problem 1: Correlation currents

Feynman diagrams



- forward (a,b)
- backward (c-d)
- Pionic correlations only



Correlation current

$$j^{\mu}_{cor}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2) = \frac{f^2}{m_\pi^2} \overline{u}(\mathbf{p}'_1)\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1) \frac{1}{K_1^2 - m_\pi^2}$$
$$\times \overline{u}(\mathbf{p}'_2) \left[\tau_a\gamma_5 \not{K}_1 S_F(P_2 + Q)\Gamma^\mu(Q) + \Gamma^\mu(Q) S_F(P'_2 - Q)\tau_a\gamma_5 \not{K}_1\right] u(\mathbf{p}_2)$$
$$+ (1 \leftrightarrow 2)\,, \quad (21)$$

where $S_F(P)$ is the Feynman propagator for the nucleon

$$S_F(P) = \frac{\not{P} + m}{P^2 - m^2 + i\epsilon} \quad (22)$$

and $\Gamma^\mu(Q)$ is the electromagnetic nucleon vertex,

$$\Gamma^\mu(Q) = F_1\gamma^\mu + \frac{i}{2m} F_2 \sigma^{\mu\nu} Q_\nu\,. \quad (23)$$

Divergences of correlation response function

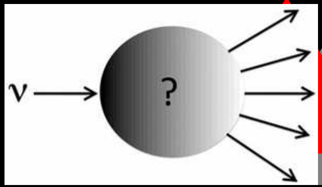
The response functions using the correlation current in are divergent in the Fermi gas.
Two sources for this divergence:

1. Double pole of the propagator when taking the square of the current.

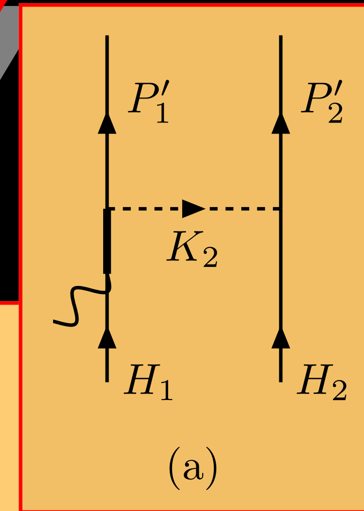
$$O(1/\epsilon)$$

2. Logarithmic divergence of principal values near the RFG boundary of the quasielastic peak,

$$O(\log \epsilon)$$



Poles in the correlation current



(a)

Forward diagram (a)

$$j^{\mu} = \frac{l^{\mu}}{E_1 + \omega - E_{\mathbf{h}_1+\mathbf{q}} + i\epsilon},$$

$E_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$ on-shell energy. In the limit $\epsilon \to 0$. there is a pole for

$$E_{\mathbf{h}_1+\mathbf{q}} = E_1 + \omega \tag{25}$$

Quasielastic condition for emission of an on-shell nucleon with four-momentum $H_1 + Q$.

$$\cos\theta_1 = \frac{Q^2 + 2E_1\omega}{2h_1 q}. \tag{26}$$

The pole is always reached in the quasielastic region where $-1 < \cos\theta_1 < 1$.

Off-shell integration variable

Integral $\int d^3\mathbf{h}_1$:
Change the variable θ_1 to a new variable x_1

$$x_1 \equiv E_1 + \omega - E_{\mathbf{h}_1+\mathbf{q}}$$

$x_1 = 0$ iff the intermediate nucleon is on-shell.
The current matrix elements can be written as

$$f(x_1) = \frac{\varphi(x_1)}{x_1 + i\epsilon} + g(x_1)\,, \tag{27}$$

The functions $f(x_1)$ and $g(x_1)$ are finite for $x_1 = 0$.
The current appears squared in the response function $\Rightarrow$

$$|f(x_1)|^2 = \frac{|\varphi(x_1)|^2}{x_1^2 + \epsilon^2} + |g(x_1)|^2 + 2\mathrm{Re}\,\frac{\varphi^*(x_1)g(x_1)}{x_1 - i\epsilon}\,. \tag{28}$$

Generalized Plemeli relation

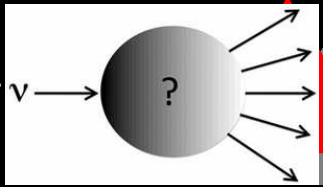
To deal with the single pole the Plemeli relation is used

$$\frac{1}{x+i\epsilon} = \mathcal{P}\,\frac{1}{x} - i\pi\delta(x)\,. \tag{29}$$

A similar relation for the double pole term?
Add and subtract $|\varphi(0)|^2/(x_1^2+\epsilon^2)$.

$$\int_{-a}^{b} \frac{\psi(x)-\psi(0)}{x^2+\epsilon^2}dx \to \mathcal{P}\,\int_{-a}^{b} \frac{\psi(x)-\psi(0)}{x^2}dx \tag{30}$$

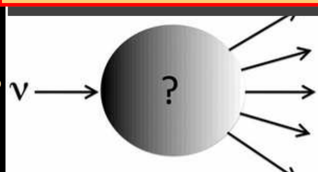
$$\int_{-a}^{b} \frac{\psi(0)}{x^2+\epsilon^2}dx = \frac{1}{\epsilon}\left[\tan^{-1}\frac{b}{\epsilon} + \tan^{-1}\frac{a}{\epsilon}\right]\psi(0) \sim \frac{\pi}{\epsilon}\psi(0)\,. \tag{31}$$



Square of the correlation current

$$
\begin{aligned}
|f(x_1)|^2 &= \mathcal{P}\,\frac{|\varphi(x_1)|^2 - |\varphi(0)|^2}{x_1^2} + |g(x_1)|^2 \\
&+ 2\mathcal{P}\,\frac{\mathrm{Re}\;\varphi^*(x_1)g(x_1)}{x_1} - 2\pi \mathrm{Im}\;\varphi^*(0)g(0)\delta(x_1) \\
&+ \frac{|\varphi(0)|^2}{\epsilon}\pi\delta(x_1) \\
&= O\left(\frac{1}{\epsilon}\right) \longrightarrow_{\epsilon\to 0} \infty
\end{aligned}
$$

Dominant contribution to the response function for $\epsilon \to 0$.
The $O(1/\epsilon)$ term does not contribute for $x_1 \neq 0$ (outside the quasielastic-peak region)

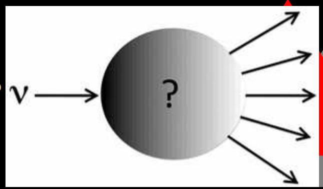


Divergence of the principal values

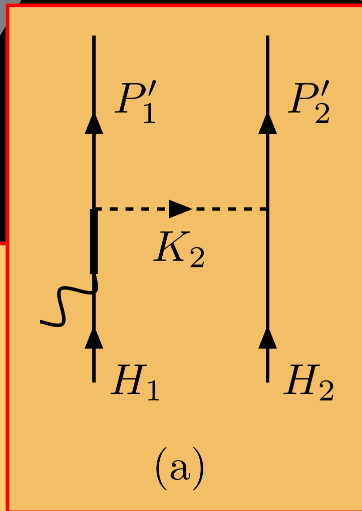
The principal values diverge in the particular case in which one of the limits of integration is zero.

$$\mathcal{P}\int_{-a}^{b}\frac{\psi(x)}{x}dx=\int_{-a}^{b}\frac{\psi(x)-\psi(0)}{x}dx+\frac{1}{2}\psi(0)\ln\frac{b^2+\epsilon^2}{a^2+\epsilon^2}$$

- Gives a $\ln\epsilon$ term if a or b is zero.
- That situation occurs throughout the quasielastic region
- In particular at the boundary of the quasielastic peak.
- One expects an additional divergence $\sim O(\ln\epsilon)$.



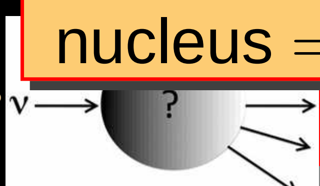
Meaning of the $1/\epsilon$ *term*



Term $\frac{|\varphi(0)|^2}{\epsilon}\pi\delta(x_1)$

- Correlation diagram when the intermediate nucleon is on shell =
- probability of a 1p-1h electroexcitation $times$ probability of quasielastic nucleon scattering.
- The interaction probability is proportional to the interaction time $T \Rightarrow$
- the probability of the re-scattering process is proportional to $T^2 \Rightarrow$
- The cross section is proportional to T.

In an infinite system the intermediate nucleon never leaves the nucleus $\Rightarrow T \to \infty$.



Finite systems

- In a finite nucleus one expects no divergence
- A high-energy nucleon will leave the nucleus in a finite time $\Rightarrow$ The interaction time T is finite.

Relation between ϵ and T:
ϵ as a regularization parameter in the Fourier transform of the time step function

$$\int_{-T/2}^{T/2} dt\, \mathrm{e}^{i(p_0-E_{\mathbf{p}})t}\theta(t) = \frac{i}{p_0 - E_{\mathbf{p}} + i\epsilon}\,, \tag{32}$$

For a real particle, $p_0 - E_{\mathbf{p}} = 0$,

$$\frac{T}{2} = \frac{1}{\epsilon}\,. \tag{33}$$

Estimation of ϵ

Regularization using a finite value for ϵ

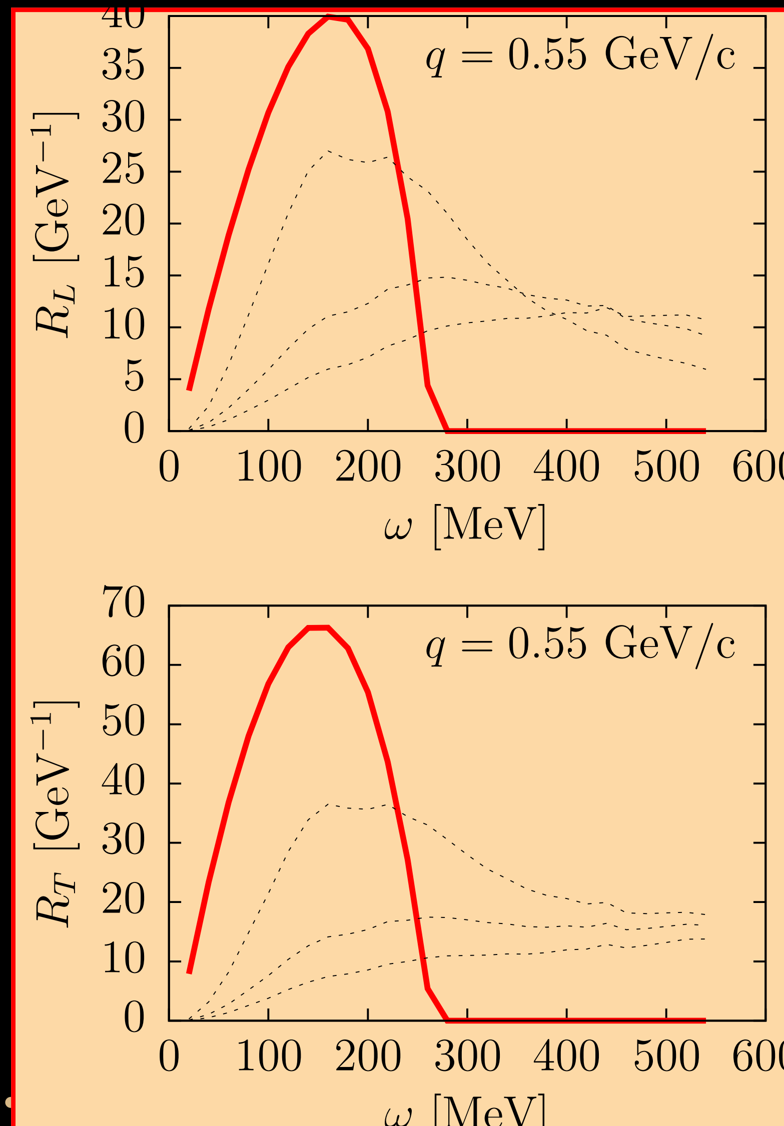
- $1/\epsilon$ accounts for the finite propagation time of a high-energy nucleon in a nucleus before leaving it.
- Estimation of ϵ for a nucleus such as ^{12}C,
- Assuming the speed of light for the nucleon
- It has to cross a distance equal to the nuclear radius $R \sim 2$ fm in a time $T \sim R/c$

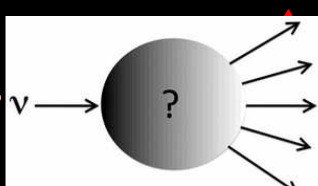
$$\epsilon \simeq \frac{2\hbar}{T} \simeq \frac{2\hbar c}{R} \simeq \frac{400}{2}\mathrm{MeV} \simeq 200\,\mathrm{MeV}\,. \tag{34}$$

- Nucleon width for nuclear inelastic interaction in nuclear matter: $\Gamma \sim 10$ MeV
- We study the dependence of our results upon ϵ as a parameter

Correlation responses

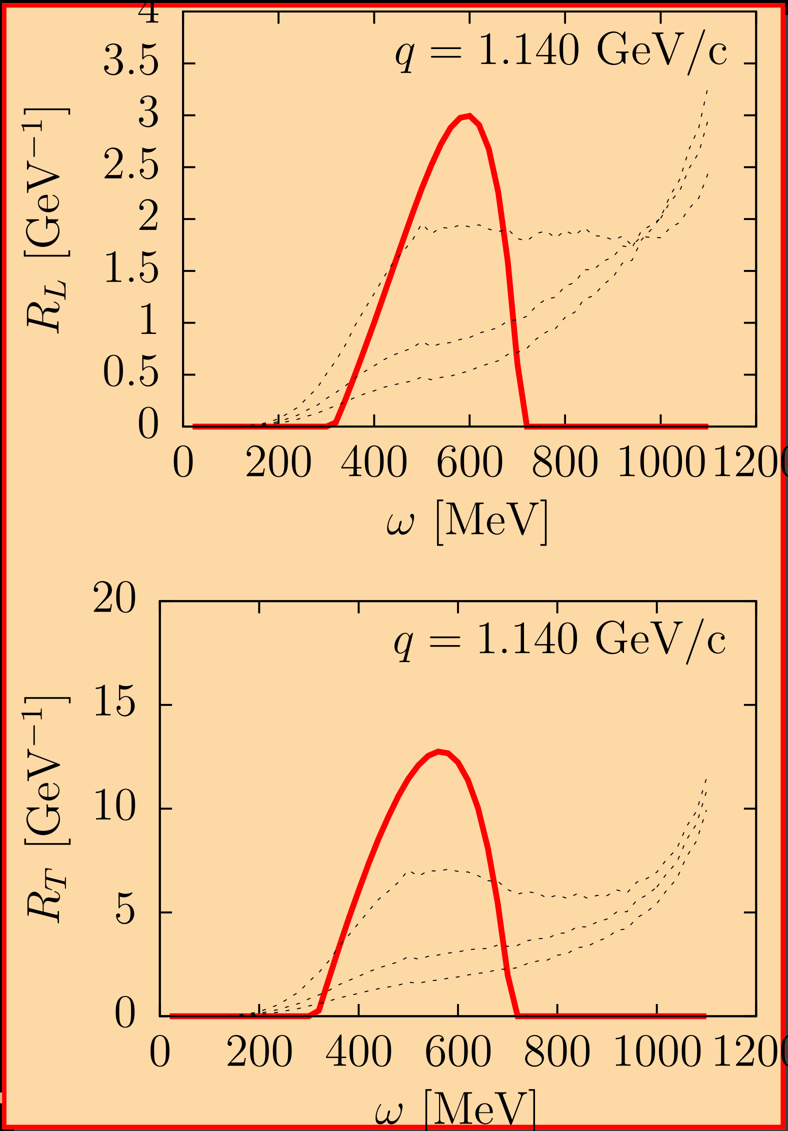
2p-2h correlation
L and T responses
^{56}Fe
$q = 550$ MeV/c.
With dotted lines from up to down, $\epsilon = 100, 200, 300$ MeV.
Solid lines: RFG one-body responses.

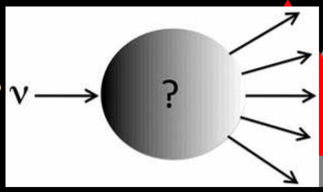




Correlation responses

$q = 1140$ MeV/c
With dotted lines from up to down, $\epsilon = 100, 200, 300$ MeV.

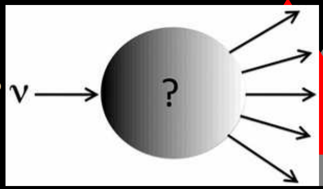




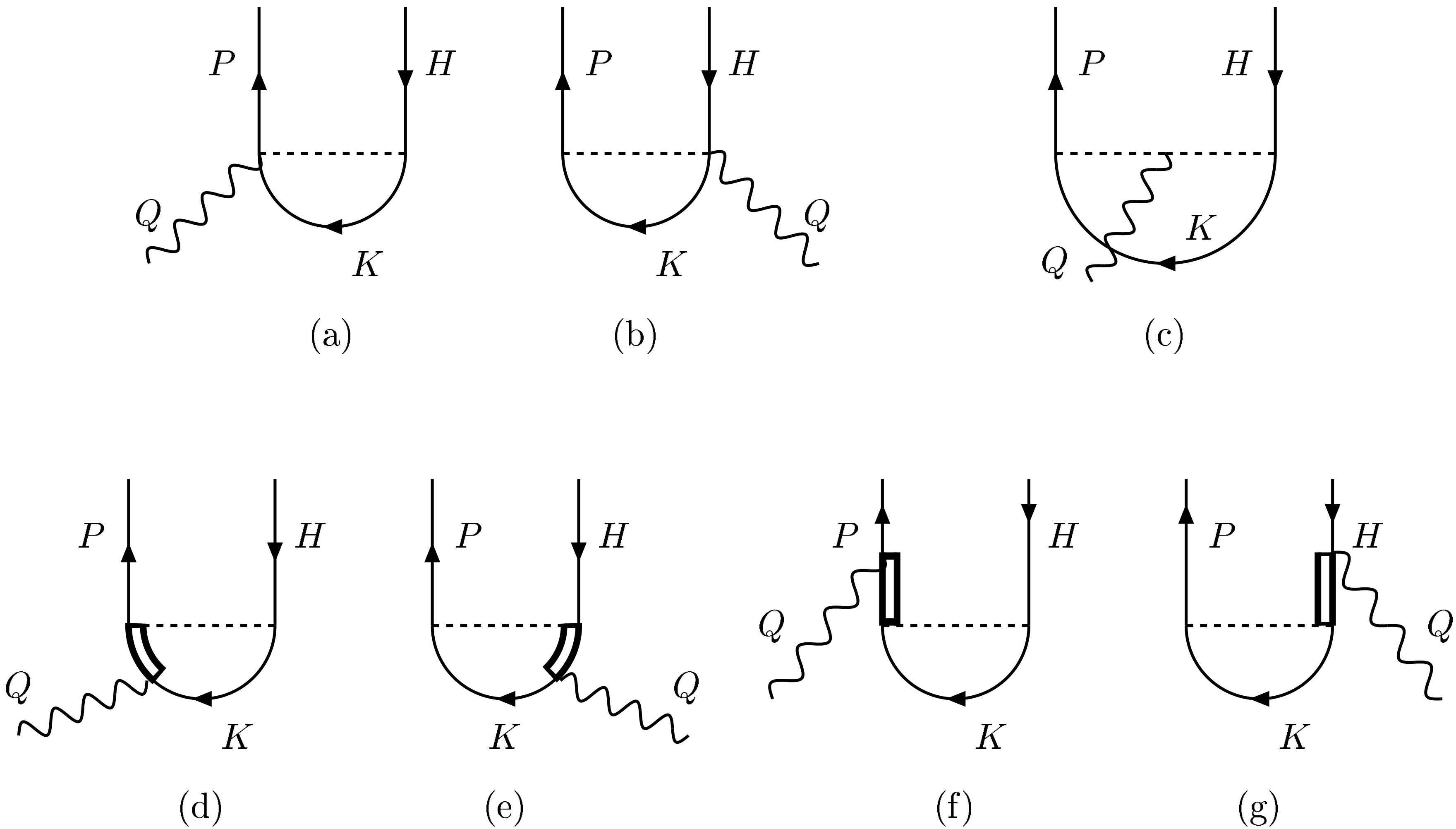
Work needed

Meson-exchange currents for high momentum transfer

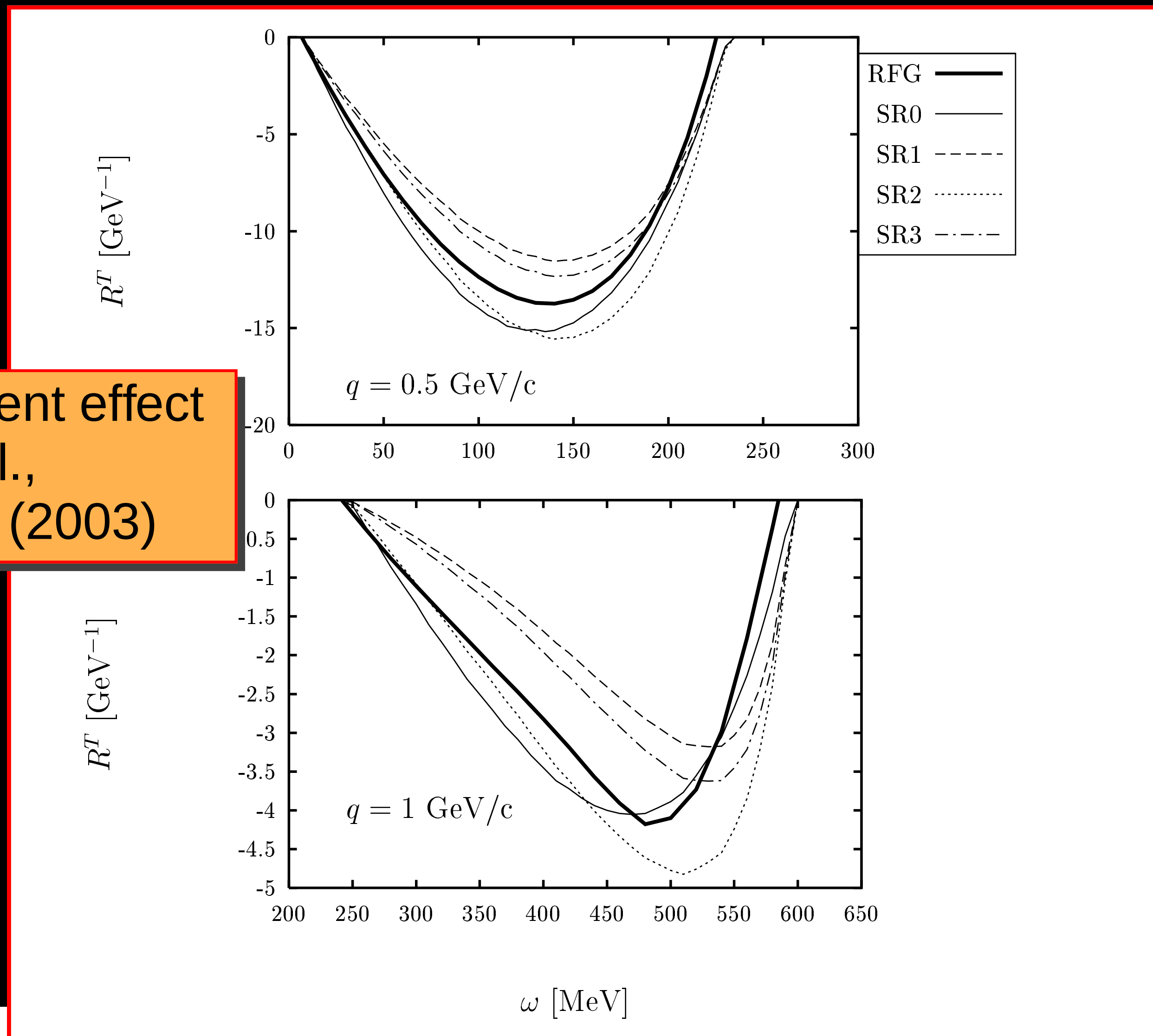
- Calculation of the 2p-2h correlation responses in a finite nucleus where there is no double pole.
  Work in progress in the semi-relativistic shell model in PW approximation.



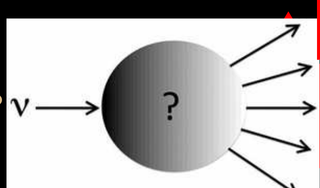
Problem 2: 1p-1h MEC contribution for high q



Semi-relativistic expansion of MEC



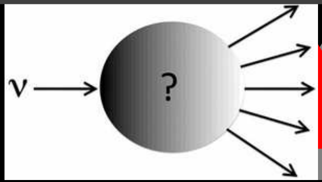
Example: Δ-current effect
From Amaro et al.,
PRC 68, 014604 (2003)

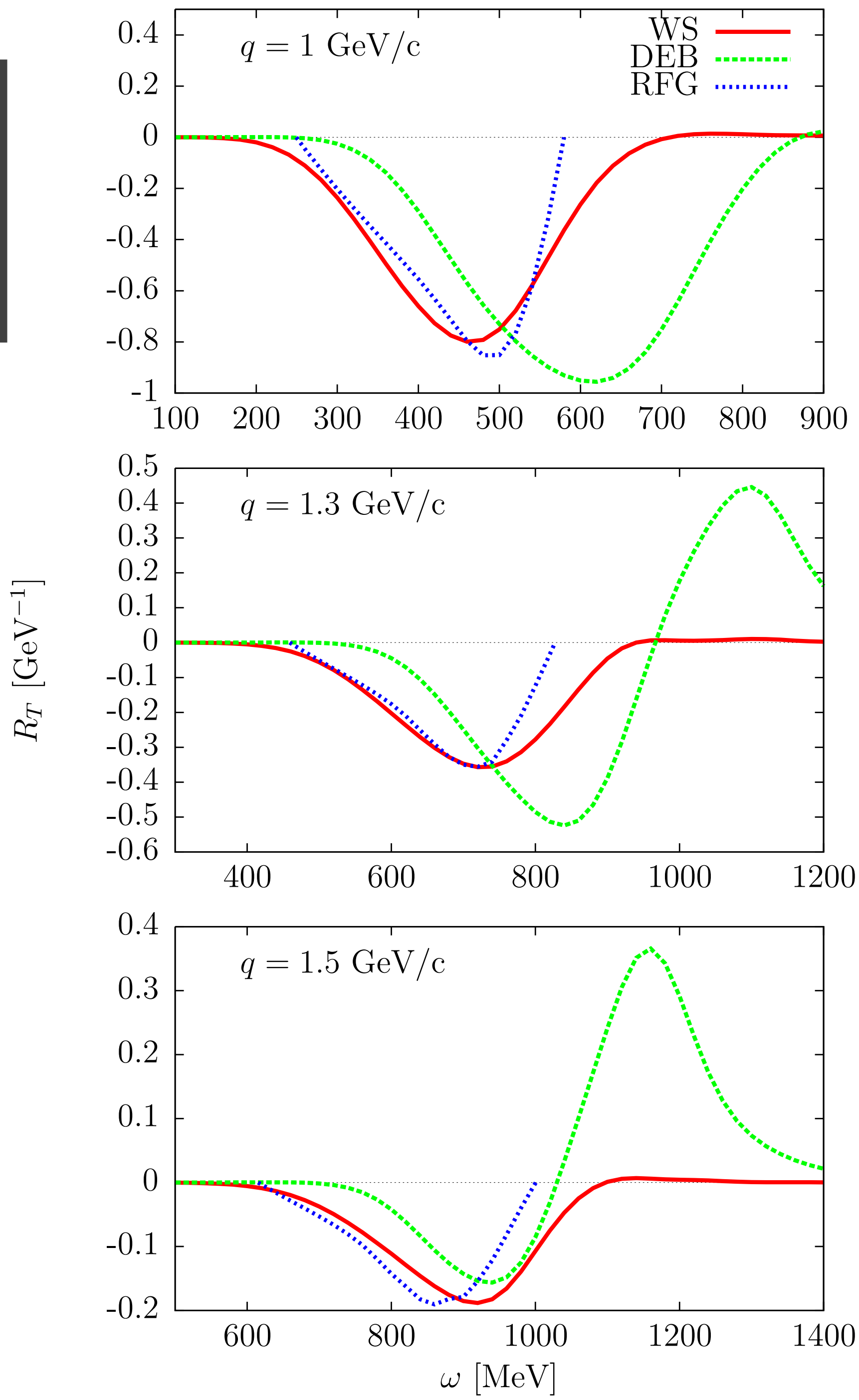


OB-Δ interference Comparison with the RFG

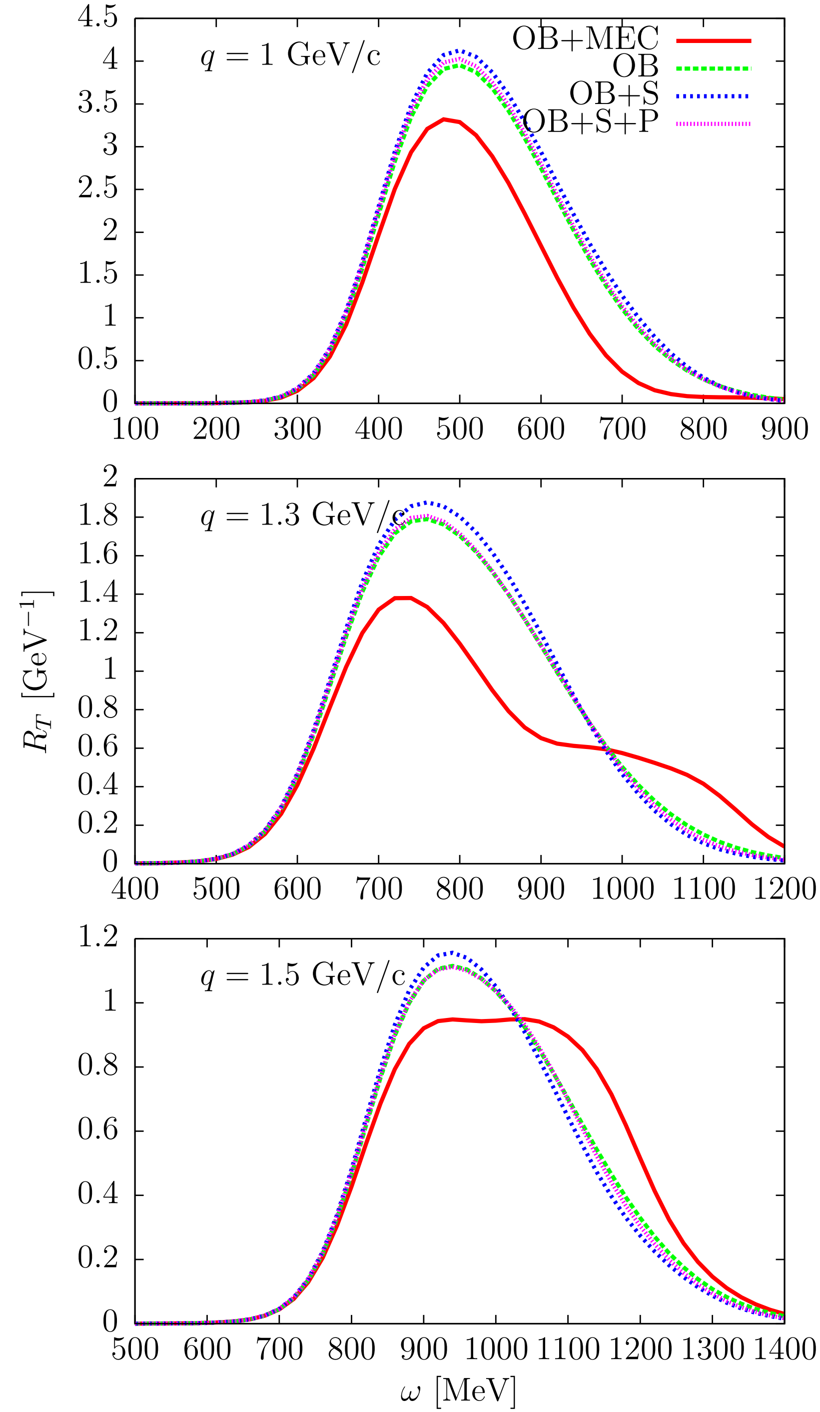
Δ contribution to R_T.
WS: Woods-Saxon potential
DEB: Dirac-Equation Based potential (from Relativistic Mean Field)

J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnely, C. Maieron, J.M. Udias, "Meson-exchange currents and final-state interactions in quasielastic electron scattering at high momentum transfers.
PRC 81 (2010) 014606



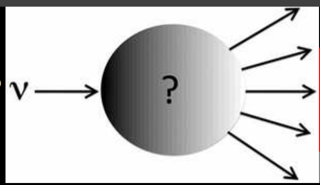


total MEC effect with DEB potential ^{12}C(e, e') Scaling is broken for high q and ω



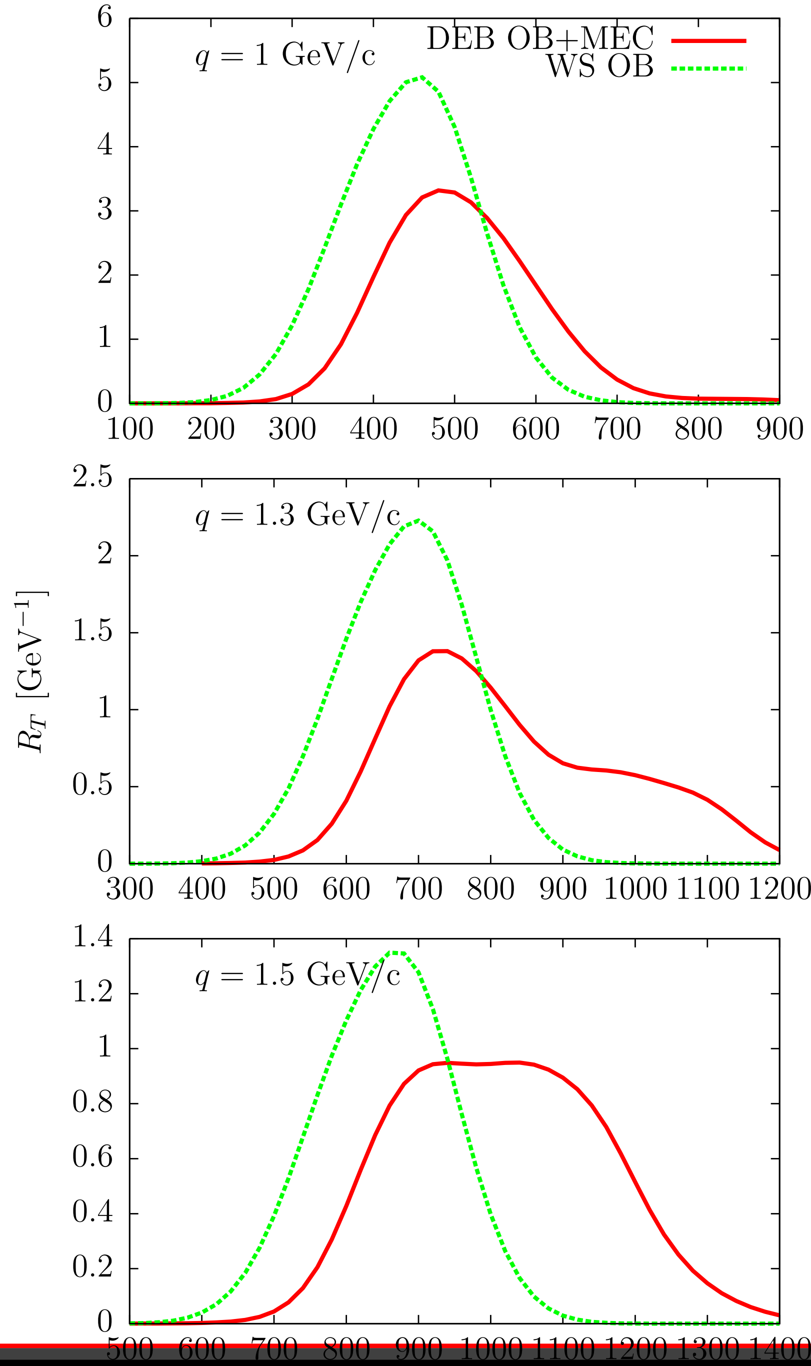
Transverse response

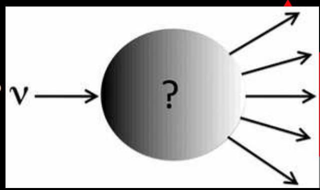
J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnely, C. Maieron, J.M. Udias, "Meson-exchange currents and final-state interactions in quasielastic electron scattering at high momentum transfers. PRC 81 (2010) 014606



total DEB+MEC effect compared with Woods-Saxon OB results ${}^{12}\mathrm{C}(e,e')$

Transverse response





Semi-relativistic MEC

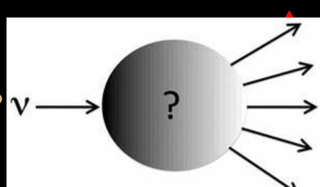
$$\vec{J}_\Delta = \frac{i}{\sqrt{1+\tau}}\frac{2}{9}\frac{G_1}{2m_N}\frac{f_{\pi N\Delta}f}{m_\pi^2}\frac{1}{m_\Delta - m_N}\frac{\vec{k}_2\cdot\vec{\sigma}^{(2)}}{m_\pi^2 - K_2^2}$$

$$\cdot\left\{4\tau_3^{(2)}\vec{k}_2 - [\vec{\tau}^{(1)}\times\vec{\tau}^{(2)}]_z\vec{\sigma}^{(1)}\times\vec{k}_2\right\}\times\vec{q}$$

$$+(1\leftrightarrow 2)$$

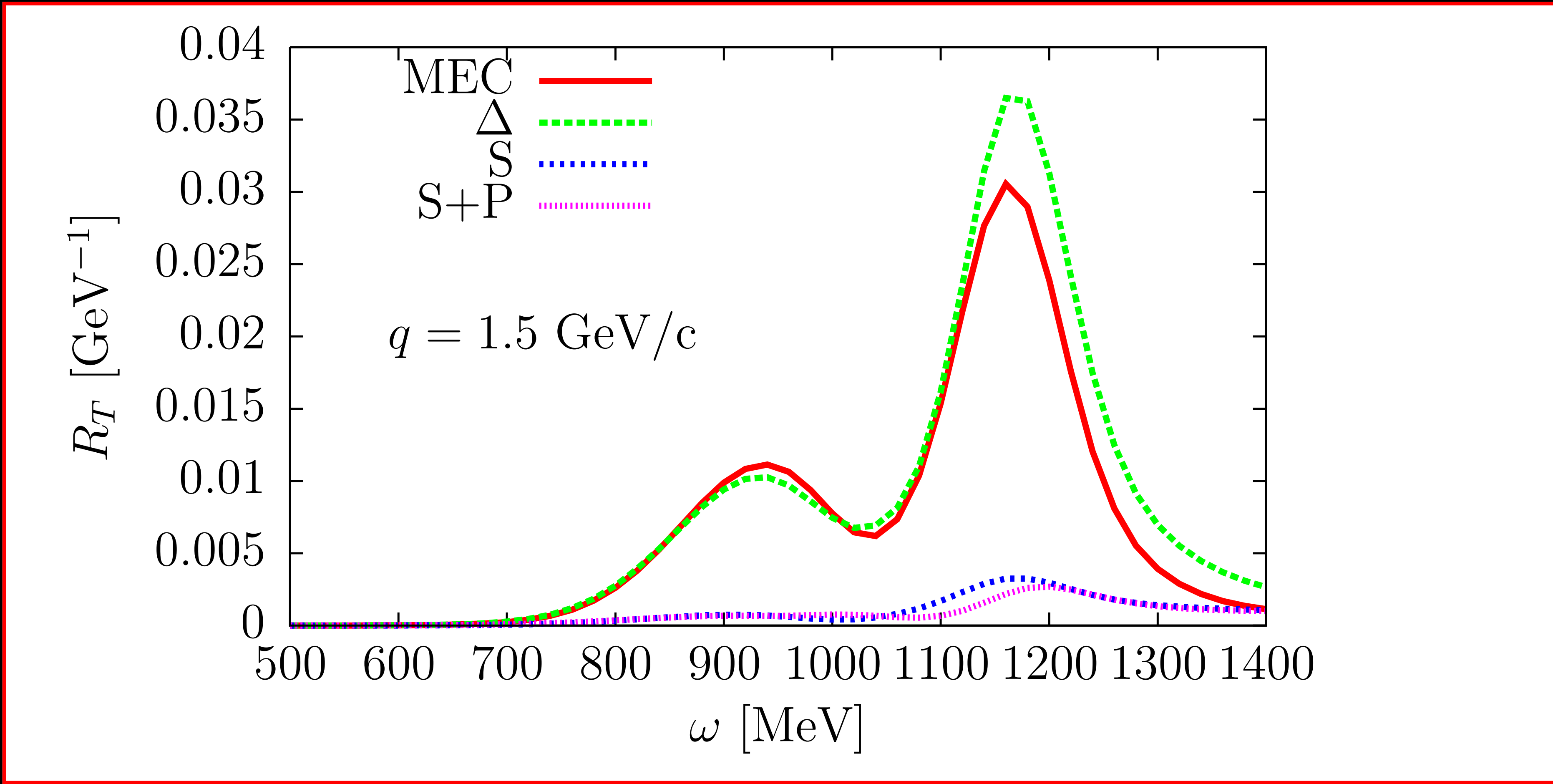
$$\vec{J}_{Seagull} = -\frac{i}{\sqrt{1+\tau}}\frac{f^2}{m_\pi^2}F_1^V\frac{\vec{k}_2\cdot\vec{\sigma}^{(2)}}{m_\pi^2 - K_2^2}$$

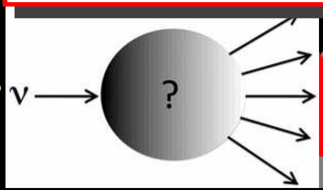
$$\times[\vec{\tau}^{(1)}\times\vec{\tau}^{(2)}]_z\vec{\sigma}^{(1)} + (1\leftrightarrow 2)$$



DEB Results with MEC only.

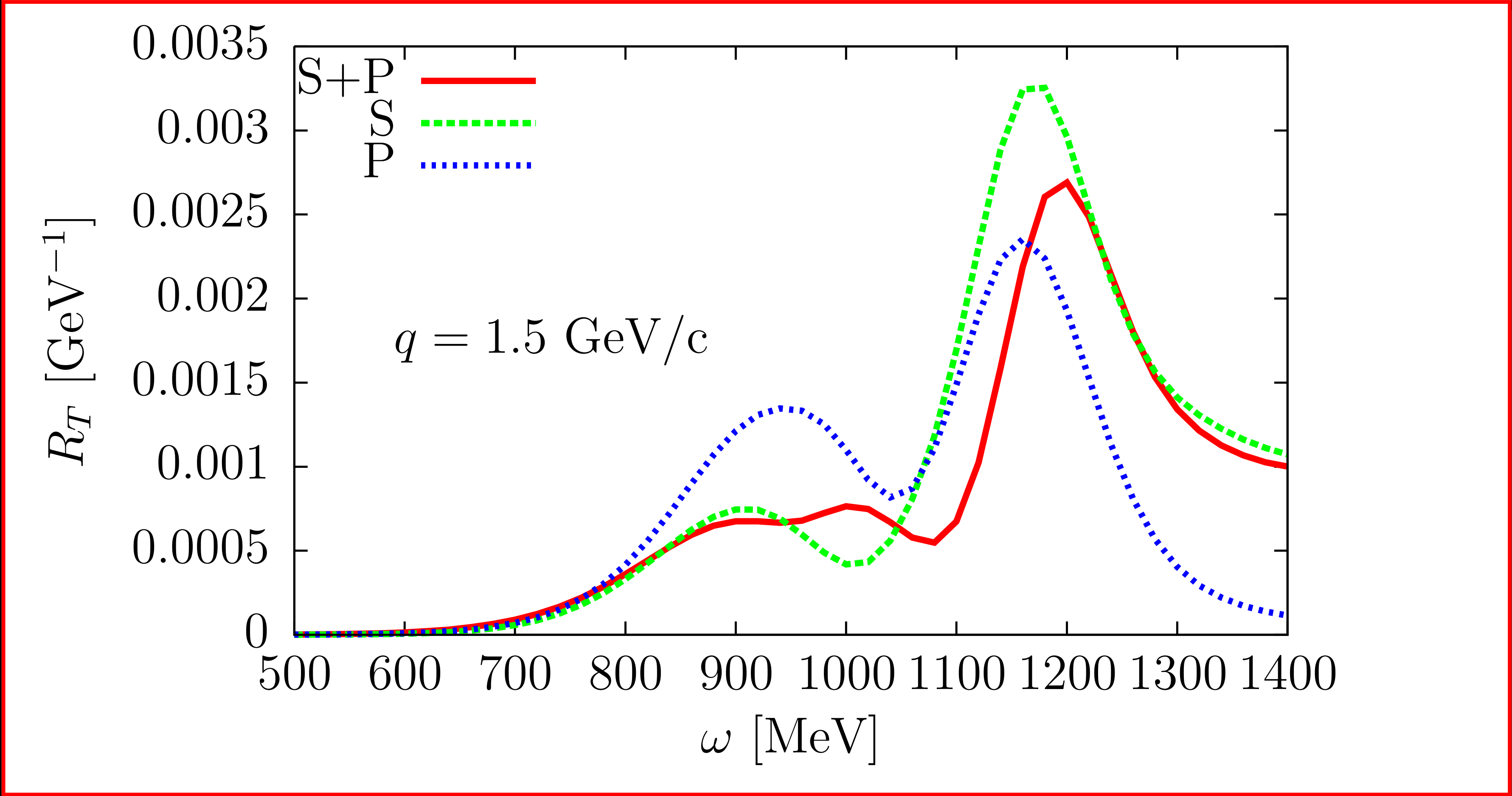
Without OB current

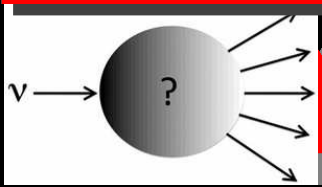




DEB Results with MEC only.

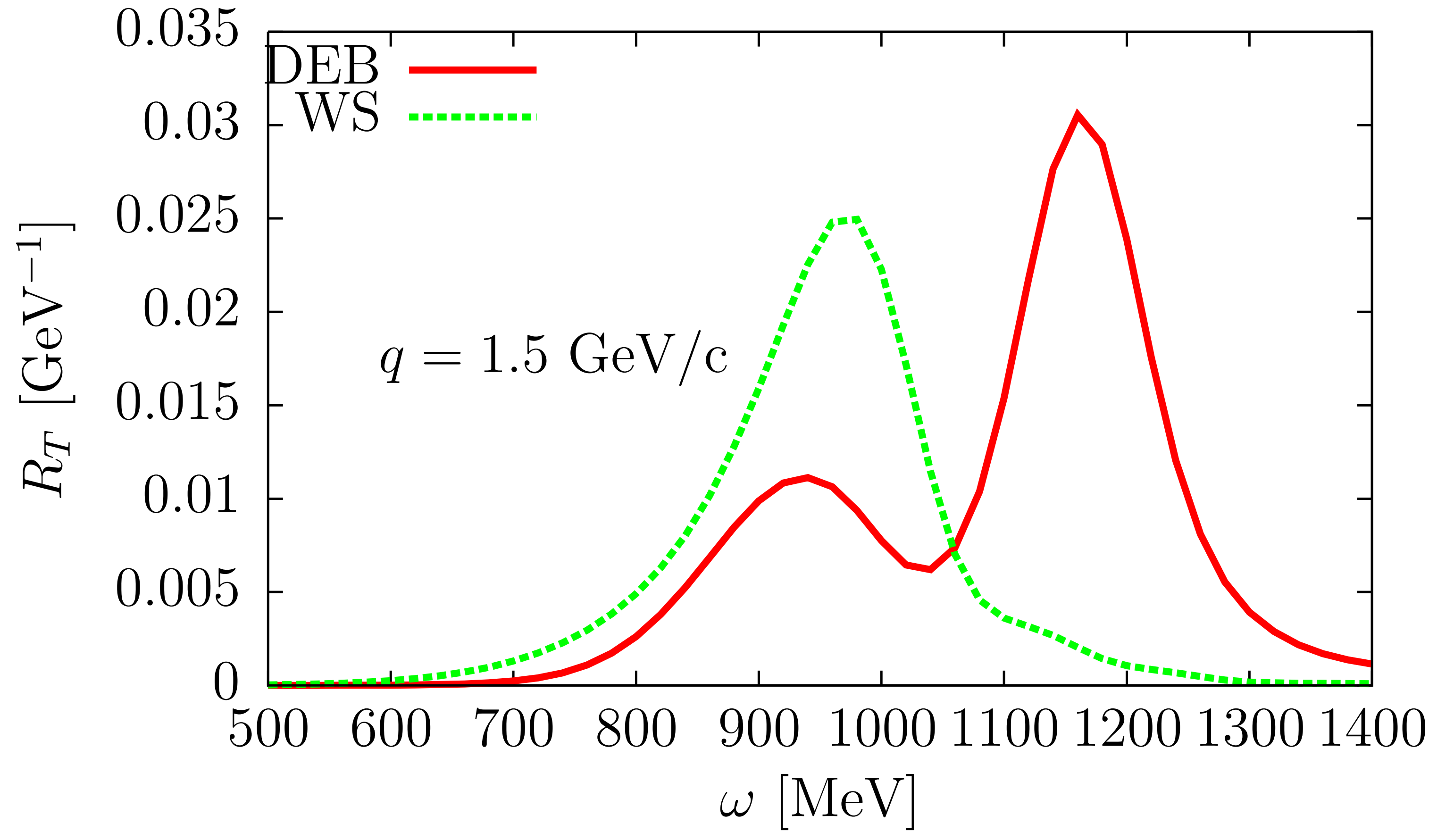
Seagull and pionic

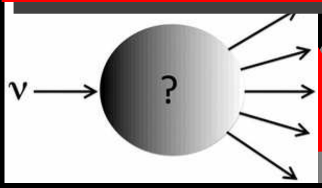




Results with MEC only.

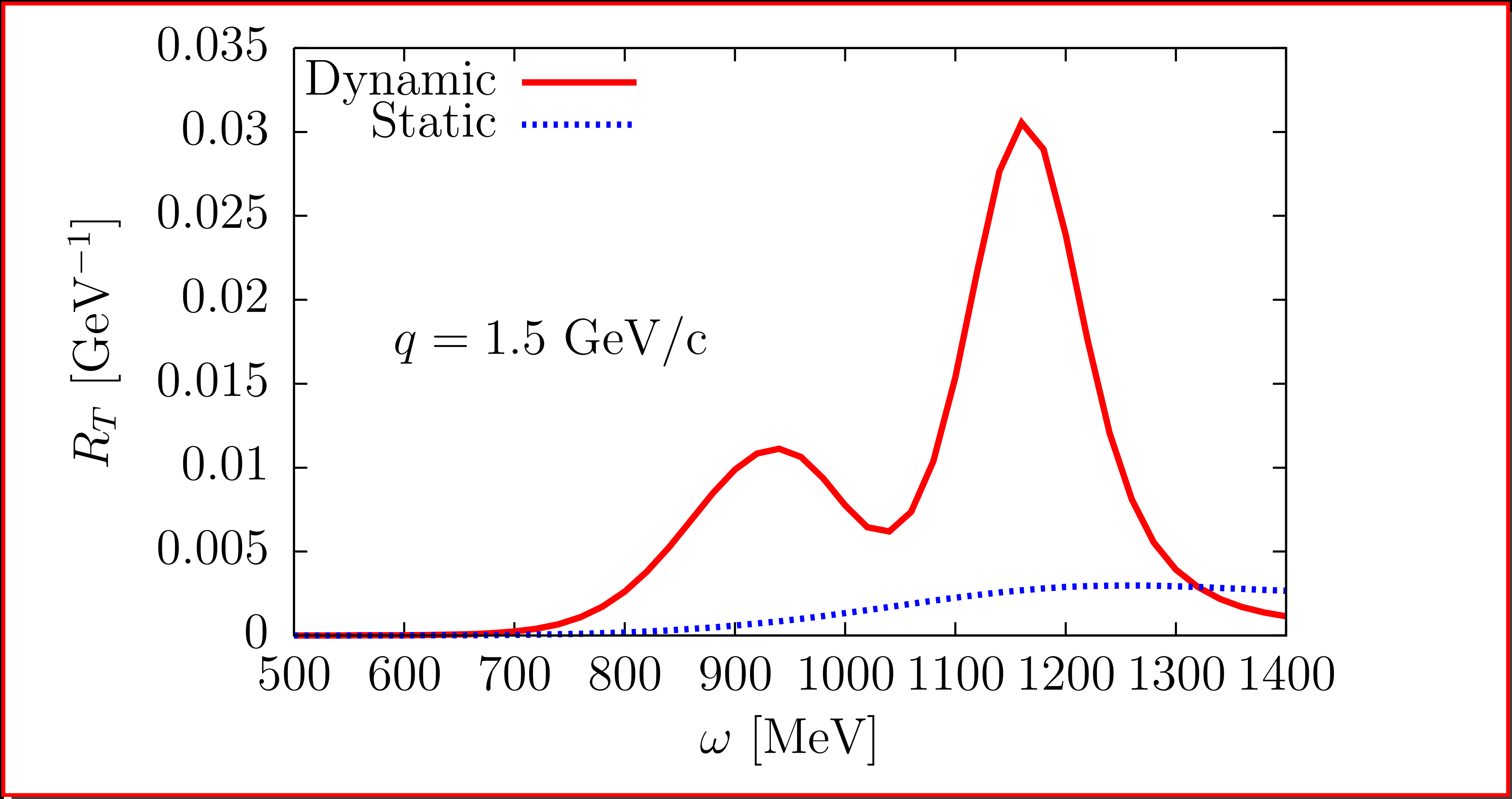
DEB and WS potentials

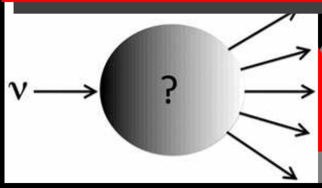




DEB Results with MEC only.

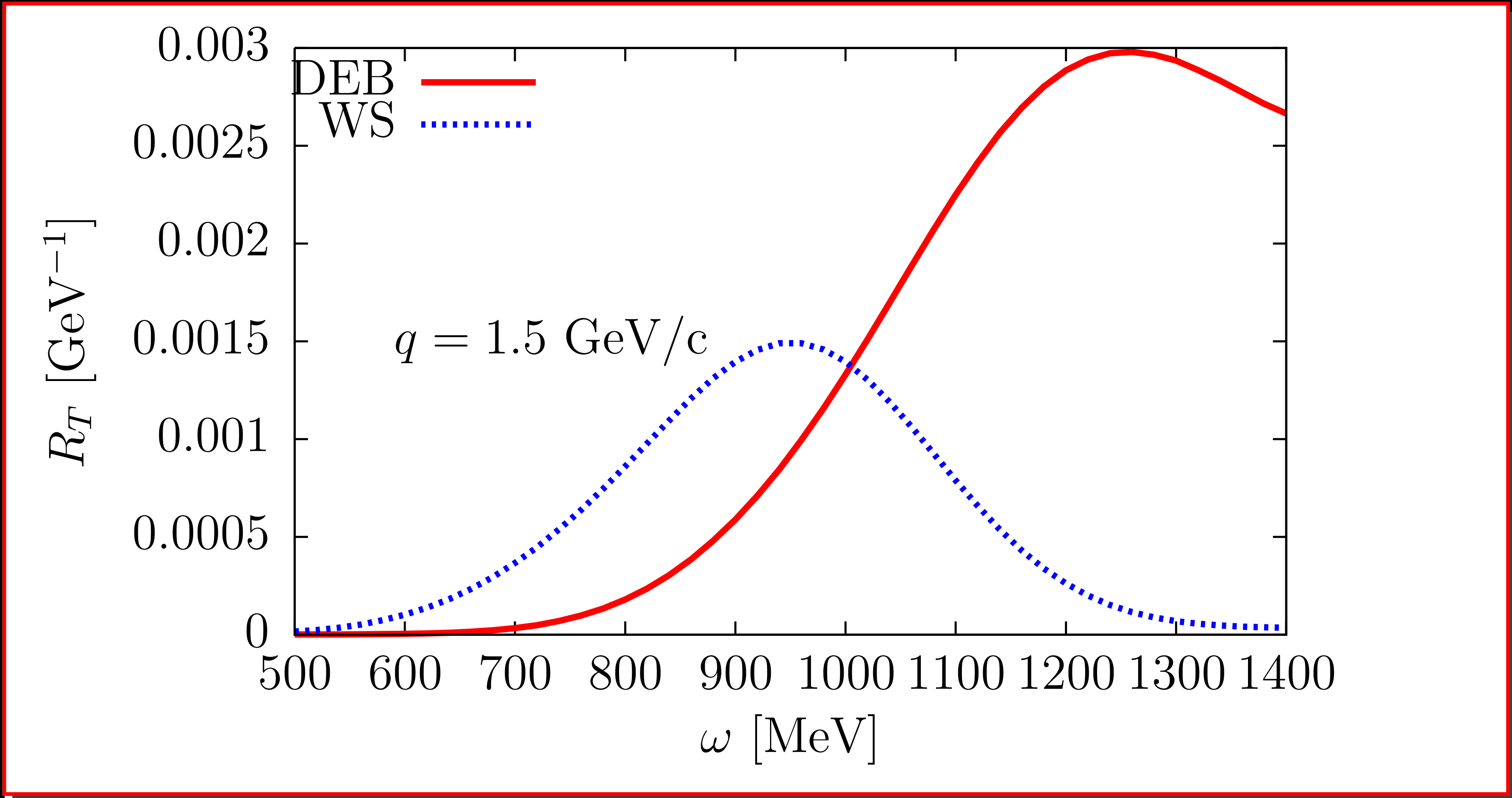
Pion propagator:

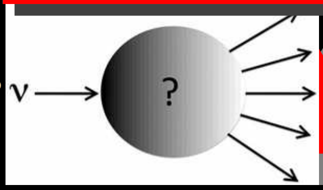




DEB effects over MEC only.

Static Pion propagator:

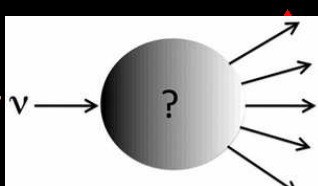




Work needed

Meson-exchange currents for high momentum transfer

- Adition of fully relativistic 1p-1h MEC effects to the Relativistic Mean Field of Amaro, Barbaro, Caballero, Donnelly, and Udias, Physical Review D 84 (2011) 033004



MEC work in progress

1. MEC in the 2p-2h channel: Improvements in the 7D phase-space integral for high momentum transfer (work in progress)
2. Add the Axial MEC contribution to the SuSA model (work in progress).
3. ....

THANK YOU