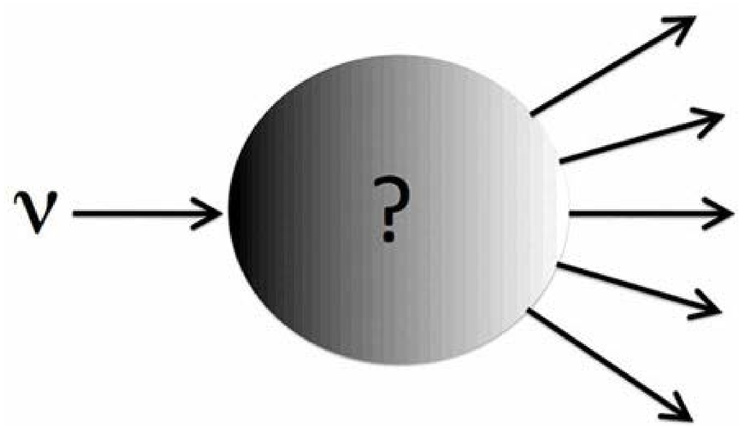
INT December 3-13, 2013

Neutrino-Nucleus Interactions for Current and Next Generation Neutrino Oscillation Experiments



Theory of resonance production and decay

Luis Alvarez-Ruso

IFIC, Valencia

Introduction

- Resonance properties
  - Originally from π N $\rightarrow$ π N
    - Quantum numbers
    - Breit-Wigner mass, width, branching ratios
    - Cleaner: pole position and residues
  - γ N $\rightarrow$ π N, γ^* N $\rightarrow$ π N (MAMI, JLab,...)
    - Electromagnetic properties: helicity amplitudes

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$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \left| \epsilon_\mu^+ J^\mu_{\mathrm{EM}} \right| N, J_z = -1/2 \right\rangle \zeta$$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 3/2 \left| \epsilon_\mu^+ J^\mu_{\mathrm{EM}} \right| N, J_z = 1/2 \right\rangle \zeta$$

$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \left\langle R, J_z = 1/2 \left| \epsilon_\mu^0 J^\mu_{\mathrm{EM}} \right| N, J_z = 1/2 \right\rangle \zeta$$

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- Resonance properties
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    - Electromagnetic properties: helicity amplitudes

- Goals:
  - Obtain a precise knowledge of the nucleon excitation spectrum
  - Compare to quark models
  - Missing resonances; decoupled from πN?
  - Compare to lattice QCD

Introduction

- Partial Wave Analyses (in a nutshell):
  - Key to resonance properties
  - Require high-quality data sets
  - Theoretical models:
    - Relativistic (many of them)
    - Cross-symmetric (few of them)
    - Gauge invariant (γ N $\rightarrow$ N' X)
    - Non resonant part
      - Fulfilling chiral symmetry constrains (close to threshold)
      - Phenomenological
    - Resonant part: Breit-Wigner parametrizations (mostly)
    - (Approximately) unitary
      - Bethe-Salpeter equation in coupled channels solved (ideally)
      - Background and resonances independently unitarized
      - K-matrix
      - Dynamical models

Introduction

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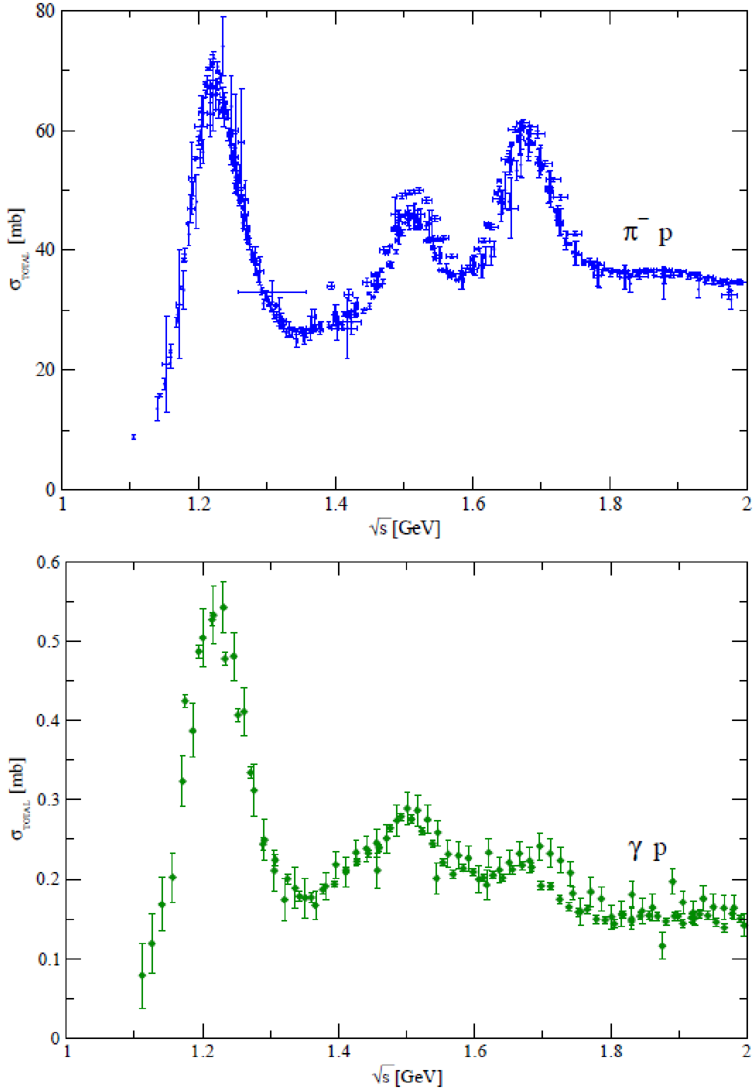

From PDG database

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Introduction

- Partial Wave Analyses (PWA):
  - Key to resonance properties
  - Karlsruhe-Helsinki (KH80)
  - Carnegie Mellon-Berkeley (CMB80)
  - George Washington University (GWU/SAID)
  - Mainz (MAID)
  - Giessen
  - JLab, Excited Baryon Analysis Center (EBAC)
  - Bonn-Gatchina

MAID

- Unitary isobar model for $\gamma^* N \to N \pi$ Tiator et al., EPJ Special Topics 198 (2011)

$$T_{\gamma\pi}(W,Q^2) = T^B_{\gamma\pi}(W,Q^2) + T^R_{\gamma\pi}(W,Q^2)$$

- For each partial wave α :

$$T^{B,\alpha}_{\gamma\pi}(W,Q^2) = V^{B,\alpha}_{\gamma\pi}(W,Q^2)\,[1 + iT^{\alpha}_{\pi N}(W)]$$

$V^{B,\alpha}_{\gamma\pi}(W,Q^2) \leftarrow$ Born terms, phenomenological model

$T^{\alpha}_{\pi N}(W) \leftarrow$ πN elastic amplitude, from SAID

$$T^{R,\alpha}_{\gamma\pi} = -\bar{A}^R_\alpha(W,Q^2)\frac{f_{\gamma N}(W)\Gamma_{\rm tot}(W) f_{\pi N}(W)}{W^2 - M_R^2 + iM_R\Gamma_{\rm tot}(W)}e^{i\phi_R(W,Q^2)}$$

$f_{\pi N}(W) \leftarrow$ Breit-Wigner factor for resonance decay

$f_{\gamma N}(W) \leftarrow$ γNR vertex

$\phi_R(W,Q^2) \leftarrow$ adjusted to fulfill Watson theorem

$\bar{A}^R_\alpha(W,Q^2) \leftarrow$ Multipole amplitudes

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$\bar{\mathcal{A}}^R_\alpha(W,Q^2) \leftarrow$ Multipole amplitudes

- j=l + ½ :

$$\begin{aligned} A_{1/2} &= -\frac{1}{2}\left[(l+2)\bar{E}_{l+} + l\bar{M}_{l+}\right] \\ A_{3/2} &= \frac{1}{2}\sqrt{l(l+2)}(\bar{E}_{l+} - \bar{M}_{l+}) \\ S_{1/2} &= -\frac{l+1}{\sqrt{2}}\bar{S}_{l+} \end{aligned}$$

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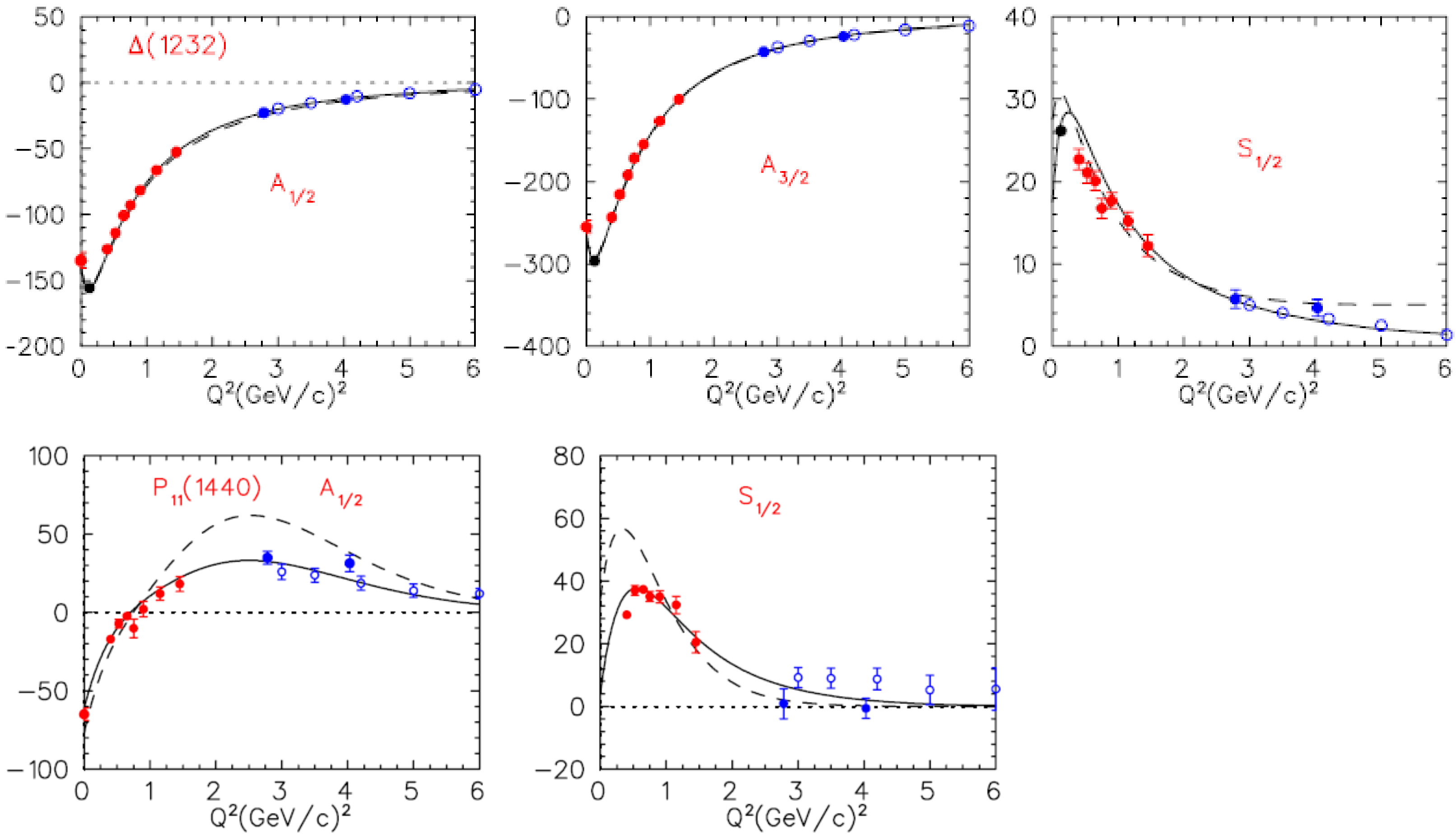
- j=l - ½ :

$$\begin{aligned} A_{1/2} &= \frac{1}{2}\left[(l+1)\bar{M}_{l-} - (l-1)\bar{E}_{l-}\right] \\ A_{3/2} &= -\frac{1}{2}\sqrt{(l-1)(l+1)}(\bar{E}_{l-} + \bar{M}_{l-}) \\ S_{1/2} &= -\frac{l}{\sqrt{2}}\bar{S}_{l-} \end{aligned}$$

MAID

- Transition N-R e.m. helicity amplitudes extracted for all 4-star resonaces with W < 1.8 GeV
- For example:

Tiator et al., EPJ Special Topics 198 (2011)



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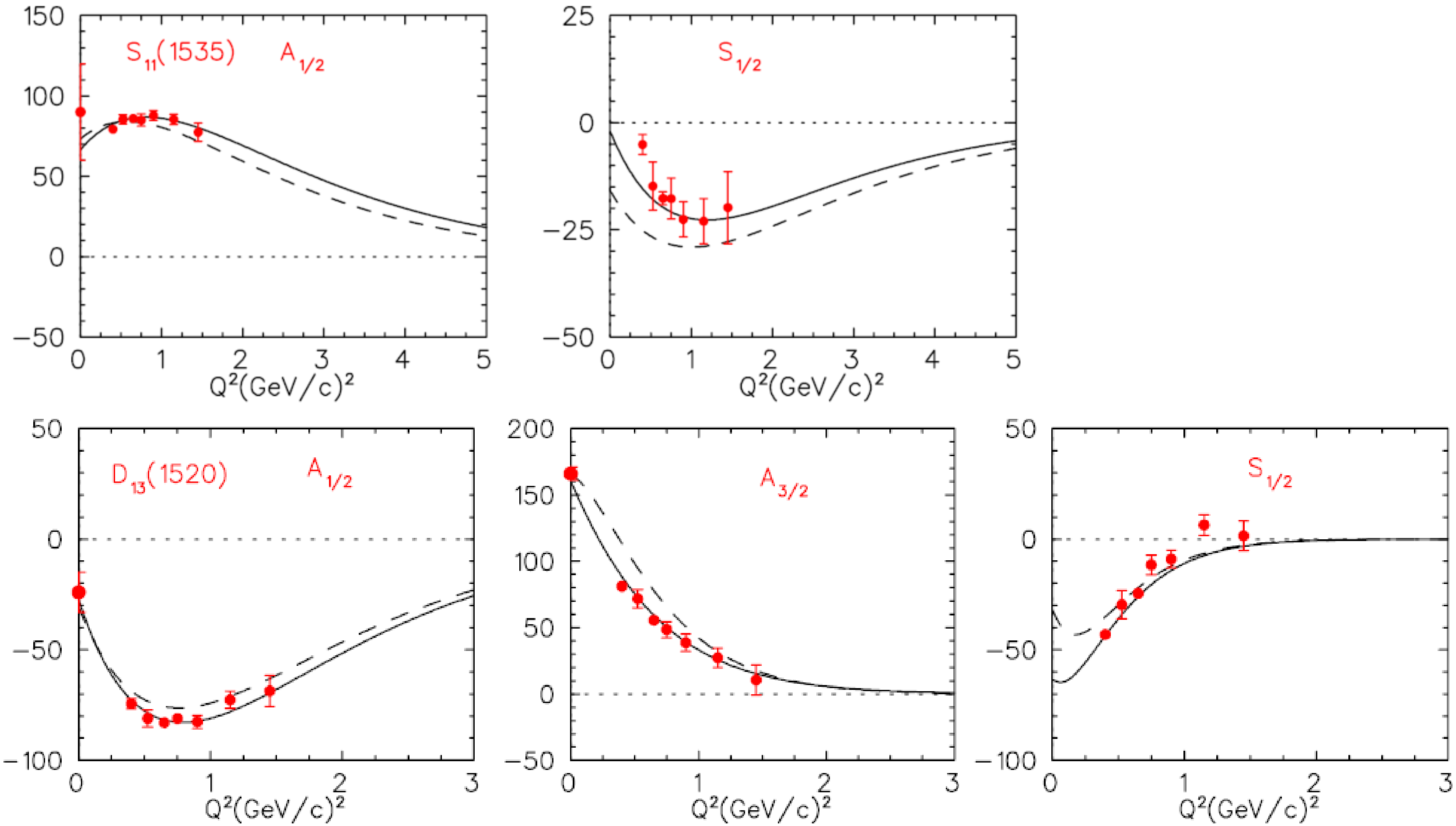


Weak Resonance excitation

- Resonances contribute to:
  - the inclusive $\nu_l \, N \to l \, X$ cross section
  - several exclusive channels: $\nu_l \, N \to l \, N' \, \pi$

$$\nu_l \, N \to l \, N' \, \gamma$$

$$\nu_l \, N \to l \, N' \, \eta$$

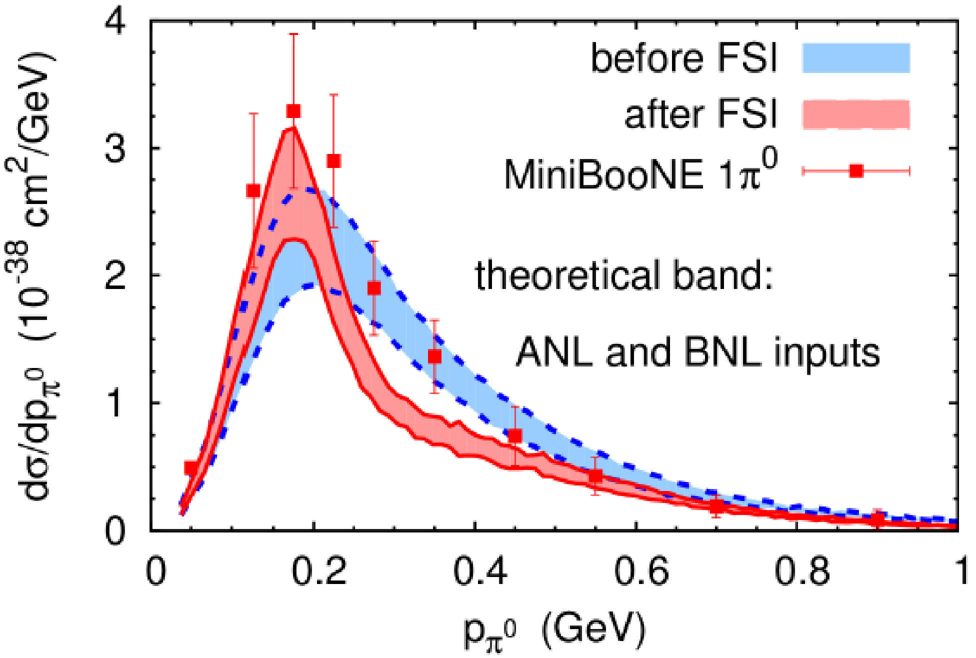
$$\nu_l \, N \to l \, \Lambda(\Sigma) \, \bar{K}$$

  - At $E_\nu \sim$ 1 GeV (MiniBooNE, SciBooNE, T2K,...) Δ(1232) is dominant

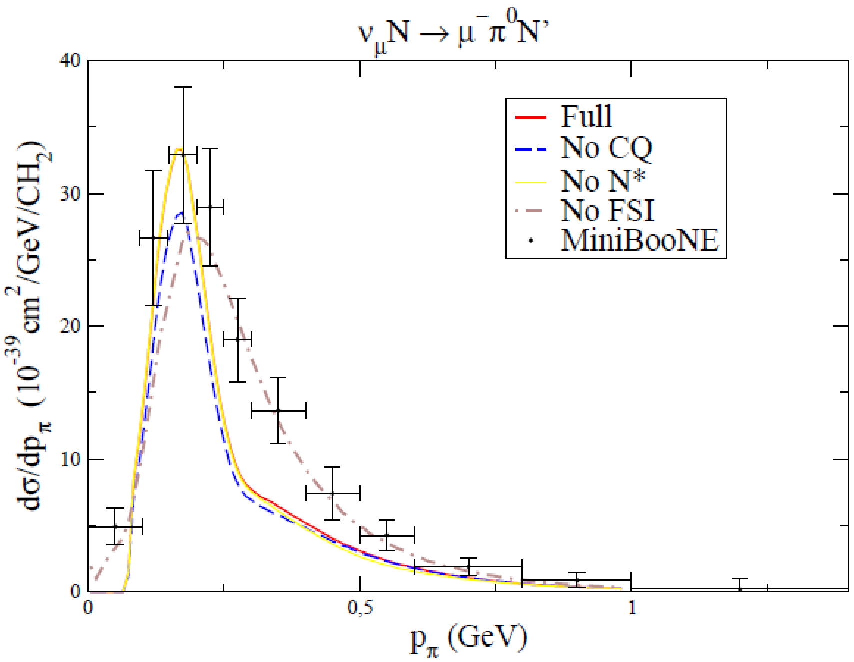
But

  - At E_ν>1 GeV (MINERνA) N* become important

π production in nuclei



Lalakulich@NuInt12

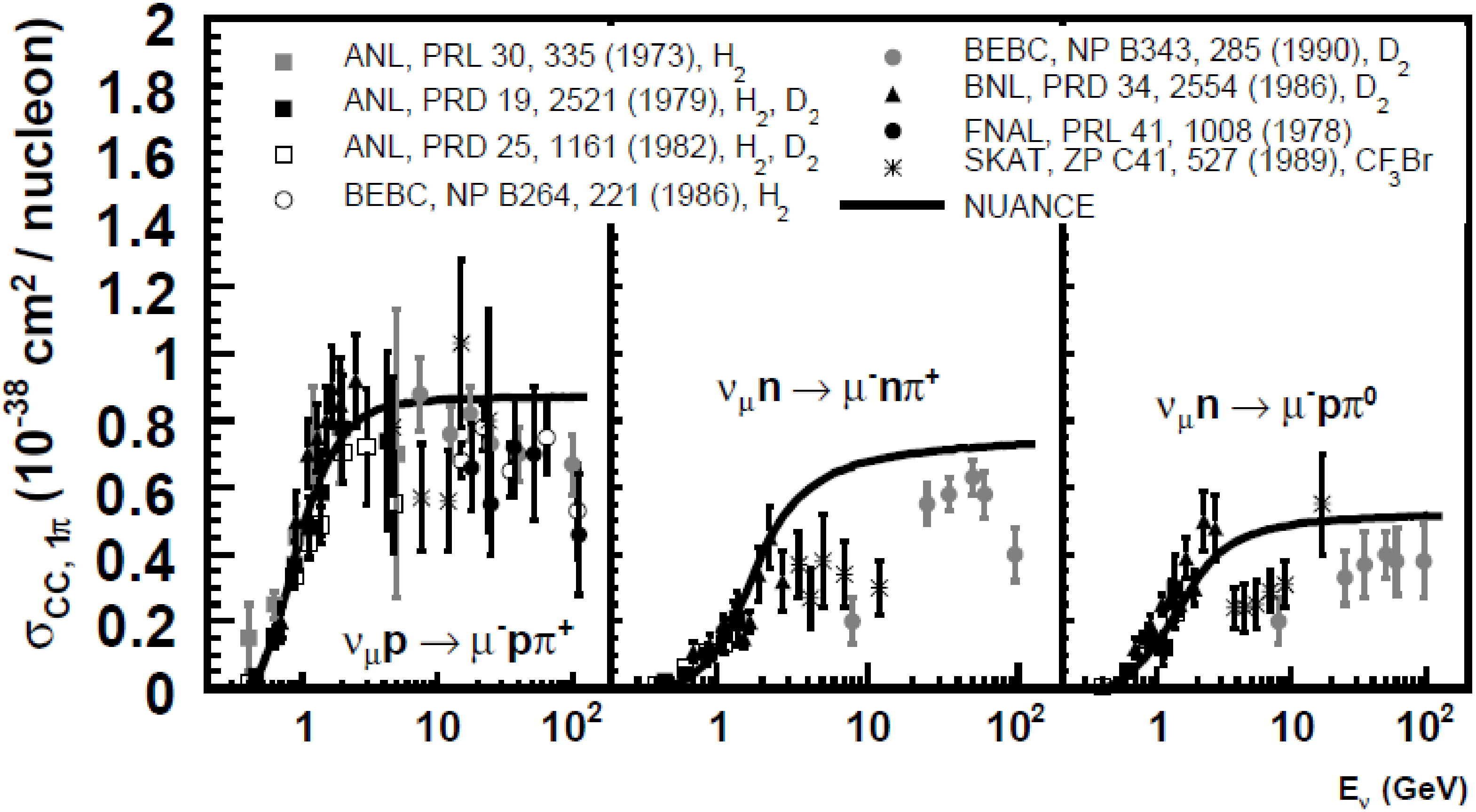


Hernandez@NuInt12

- Possible problems in:
  - π production model on the nucleon
  - medium modifications of amplitudes
  - FSI

π production in nuclei

Rodrigues@NuInt12 (backup)



- ANL/BNL data not particularly helpful...

Weak resonance excitation

- Is a PWA-like model needed for (anti)neutrino reactions in the resonance region?
  - There is a already one: Sato-Lee/EBAC

- Most models (GiBUU, GENIE, ...)
  - Single resonance excitation
  - Phenomenological/empirical backgrounds

  (+) simple, easy to apply to nuclear targets

  (-) wrong interferences / angular distributions (on nucleons)

  - Is this good enough?

Weak resonance excitation

- Is a PWA-like model needed for (anti)neutrino reactions in the resonance region?
  - There is a already one: Sato-Lee/EBAC

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  (+) simple, easy to apply to nuclear targets

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  - Is this good enough?
  - Yes... but available experimental information from πN, γN, $\gamma^* N$ should be taken into account

    LAR, Singh, Vicente-Vacas, PRC 57 (1998)
    Lalakulich, Paschos, Piranishvili, PRD 74 (2006)
    Leitner, Buss, LAR, Mosel, PRC 79 (2009)

Formalism

- CC N* excitation: $\nu_l(k)\, N(p) \to l^-(k')\, N^*(p')$

$$\frac{d\sigma}{dk_0' d\Omega'} = \frac{1}{32\pi^2} \frac{|\vec{k'}|}{k_0 M_N} \mathcal{A}(p') |\bar{\mathcal{M}}|^2$$ ← Inclusive cross section

$$\mathcal{A}(p') = \frac{M^*}{\pi} \frac{\Gamma(p')}{(p'^2 - M^{*2})^2 + M^{*2}\Gamma^2(p')}$$

$\Gamma(p')$ ← total momentum dependent width

$$\mathcal{M} = \frac{G_F \cos\theta_C}{\sqrt{2}} l^\alpha J_\alpha$$

$$l^\alpha = \bar{u}(k')\gamma^\alpha(1-\gamma_5)u(k)$$ ← leptonic current

$$J_\alpha = V_\alpha - A_\alpha$$ ← hadronic current

can be parametrized in terms of
N-N* transition form factors

Formalism

- CC N* excitation: $\nu_l(k)\, N(p) \to l^-(k')\, N^*(p')$

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← leptonic current

$$J_\alpha = V_\alpha - A_\alpha \qquad q^\alpha V_\alpha = 0$$

← CVC

Formalism

- Second resonance peak: N*(1440), N*(1520), N*(1535)
  - N*(1440) $J^P=1/2^+$

$$J_\alpha = \bar{u}(p') \left[\frac{F_1^V}{(2M_N)^2}(\not{q} q_\alpha - q^2 \gamma_\alpha) + i \frac{F_2^V}{2M_N} \sigma_{\alpha\beta} q^\beta - F_A \gamma_\alpha \gamma_5 - \frac{F_P}{M_N} \gamma_5 q_\alpha \right] u(p)$$

  - N*(1535) $J^P=1/2^-$

$$J_\alpha = \bar{u}(p') \left[\frac{F_1^V}{(2M_N)^2}(\not{q} q_\alpha - q^2 \gamma_\alpha)\gamma_5 + i \frac{F_2^V}{2M_N} \sigma_{\alpha\beta} q^\beta \gamma_5 - F_A \gamma_\alpha - \frac{F_P}{M_N} q_\alpha \right] u(p)$$

  - N*(1520) $J^P=3/2^-$

$$J_\alpha = \bar{u}^\mu(p') \left[\frac{C_3^V}{M_N}(g_{\alpha\mu} \not{q} - q_\alpha \gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu} q \cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu} q \cdot p - q_\alpha p_\mu) \right.$$
$$\left. + \left(\frac{C_3^A}{M_N}(g_{\alpha\mu} \not{q} - q_\alpha \gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu} q \cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2} q_\alpha q_\mu \right) \gamma_5 \right] u(p)$$

Formalism

- Vector and EM transition form factors

$\vec{V}^\alpha = \mathcal{V}^\alpha \frac{\vec{\tau}}{2} \leftarrow$ isovector current $\quad V_Y^\alpha = \mathcal{V}_Y^\alpha \frac{I}{2} \leftarrow$ hypercharge (isoscalar) current

$$\langle p^* | V_{\rm EM}^\alpha | p \rangle = \langle p^* | V_3^\alpha + \frac{1}{2} V_Y^\alpha | p \rangle = \frac{\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_p^\alpha$$

$$\langle n^* | V_{\rm EM}^\alpha | n \rangle = \langle n^* | V_3^\alpha + \frac{1}{2} V_Y^\alpha | n \rangle = \frac{-\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_n^\alpha$$

Then: $\langle p^* | V_{\rm CC}^\alpha | n \rangle = \langle p^* | V_1^\alpha + i V_2^\alpha | n \rangle = \mathcal{V}^\alpha = \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha$

$$\begin{aligned} \langle p^* | V_{\rm NC}^\alpha | p \rangle &= \langle p^* | (1 - 2\sin^2\theta_W) V_3^\alpha - \sin^2\theta_W V_Y^\alpha | p \rangle \\ &= \left(\frac{1}{2} - \sin^2\theta_W \right) \mathcal{V}^\alpha + \sin^2\theta_W \mathcal{V}_Y^\alpha \\ &= \left(\frac{1}{2} - 2\sin^2\theta_W \right) \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha \end{aligned} \tag{1}$$

- Vector CC and NC form factors can be expressed in terms of EM ones

Formalism

- Vector CC and NC form factors can be expressed in terms of EM ones
  - CC: $F^V_{1,2} = F^p_{1,2} - F^n_{1,2}$
  - NC: $\tilde{F}^{p(n)}_{1,2} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F^{p(n)}_{1,2} - F^{n(p)}_{1,2}$
  - The same applies for $C^V_{1,2,3}$
- Helicity amplitudes from π photo- and electro-production data

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \left| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = -1/2 \right\rangle \zeta$$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 3/2 \left| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = 1/2 \right\rangle \zeta$$

$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \left\langle R, J_z = 1/2 \left| \epsilon^0_\mu J^\mu_{\mathrm{EM}} \right| N, J_z = 1/2 \right\rangle \zeta$$

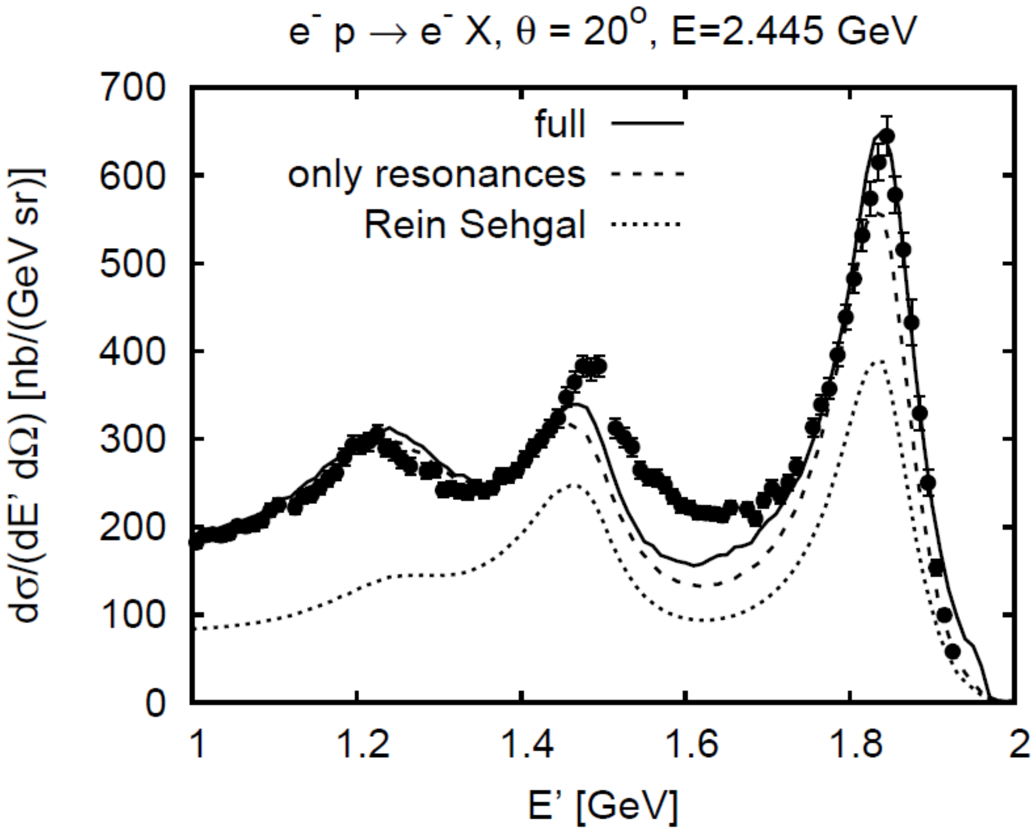
- Helicity amplitudes $\Rightarrow$ EM form factors

Resonances in ν generators

- Rein-Sehgal model: Rein, Sehgal, Ann. Phys. 133 (1981) 79.
  - Used by almost all MC generators
  - Relativistic quark model of Feynman-Kislinger-Ravndal with SU(6) spin-flavor symmetry
  - Helicity amplitudes for 18 baryon resonances
  - Lepton mass = 0
    - Corrections: Kuzmin et al., Mod. Phys. Lett. A19 (2004); Berger, Sehgal, PRD 76 (2007); Graczyk, Sobczyk, PRD 77 (2008)
    - Poor description of π electroproduction data on p

Resonances in ν generators

- Rein-Sehgal model: Rein, Sehgal, Ann. Phys. 133 (1981) 79.
  - Us
  - Re ... ʀavndal with SU(6)
    sp
  - He
  - Le ... Leitner et al., POS NUFACT08
    - ... ʀ004)
    - ... ı on p



Resonances in ν generators

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$$\frac{d\sigma}{dq^2 d\omega} \sim u^2 \left(|f_{-3}|^2 + |f_{-1}|^2\right) + v^2 \left(|f_{+3}|^2 + |f_{+1}|^2\right) + 2uv \frac{M_N^2}{M_R^2} \frac{\mathbf{q}^2}{(-q^2)} \left(|f_{0+}|^2 + |f_{0-}|^2\right)$$

$$f_{\pm 3} = \left\langle N, J_z = \mp 1/2 \left| \tfrac{1}{2M_R} \epsilon_\mu^\pm J_{\mathrm{EM}}^\mu \right| R, J_z = \mp 3/2 \right\rangle$$

$$f_{\pm 1} = \left\langle N, J_z = \pm 1/2 \left| \tfrac{1}{2M_R} \epsilon_\mu^\pm J_{\mathrm{EM}}^\mu \right| R, J_z = \mp 1/2 \right\rangle$$

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Helicity amplitudes

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Helicity amplitudes

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \left| \epsilon_\mu^+ J^\mu_{\mathrm{EM}} \right| N, J_z = -1/2 \right\rangle \zeta$$

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Helicity amplitudes

$$f_{\pm 1(\pm 3)} = -s \sqrt{\frac{M_N}{M_R}} \sqrt{\frac{k_R}{2\pi\alpha}} A_{1/2(3/2)}$$

$$f_{0\pm} = -s \frac{(-q^2)}{\mathbf{q}^2} \sqrt{\frac{M_N}{M_R}} \sqrt{\frac{k_R}{2\pi\alpha}} S_{1/2}$$

s $\leftarrow$ sign, depends on the resonance

Resonances in ν generators

- Rein-Sehgal model: Rein, Sehgal, Ann. Phys. 133 (1981) 79.
- It is possible to use helicity amplitudes from PWA without altering the structure of Rein-Sehgal model
- MAID helicity amplitudes ⇒ GENIE: LAR, S. Dytman
- First results: Steve's talk on Sunday
  - Agreement to (e,e') data improved
  - Discrepancies remain:
    - Mistakes
    - Ambiguity in the off-shell ($W \neq M_R$) dependence
    - Non-resonant background and interferences

Non-resonant background

- Specific for each excusive process
- Background terms interfere with the resonant contributions
- $\nu_l \, N \rightarrow l \, N' \, \pi$
- In Rein-Sehgal model: Rein, Sehgal, Ann. Phys. 133 (1981) 79.

“we have represented the background by a resonance amplitude of P11 character (like the nucleon), with the Breit-Wigner factor replaced by an adjustable constant. The corresponding cross section is added incoherently to the resonant cross section.”

- General principles:
  - CVC, PCAC
  - Threshold behavior dictated by chiral symmetry of QCD

Non-resonant background

- for MC generators
- Vector part: T. Leitner, O. Buss, LAR, U. Mosel, PRC 79 (2009)
  - Empirical $\gamma^* N \to N' \pi$ amplitudes:

$$\mathcal{V}^{\mu}_{\pi N} = \sum_{i=1}^{6} A_i^{EM} M_i^{\mu}$$

$$A_i^{EM} \leftarrow$$ Parametrized by MAID using (e,e') data

  - Weak amplitudes obtained by isospin rotations

$$A_i^{p\pi^+,\mathrm{CC}} = \sqrt{2} A_i^{n\pi^0,\mathrm{EM}} + A_i^{p\pi^-,\mathrm{EM}},$$
$$A_i^{n\pi^+,\mathrm{CC}} = \sqrt{2} A_i^{p\pi^0,\mathrm{EM}} - A_i^{p\pi^-,\mathrm{EM}},$$
$$A_i^{p\pi^0,\mathrm{CC}} = A_i^{p\pi^0,\mathrm{EM}} - A_i^{n\pi^0,\mathrm{EM}} - \sqrt{2} A_i^{p\pi^-,\mathrm{EM}}$$

  - After subtracting resonances $\Rightarrow$ background (+interference)

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$$\frac{d\sigma}{dk_0' d\Omega'} = \frac{1}{32\pi^2} \frac{|\vec{k}'|}{k_0 M_N} \mathcal{A}(p') |\bar{\mathcal{M}}|^2$$ ← Inclusive cross section

$$\mathcal{A}(p') = \frac{M^*}{\pi} \frac{\Gamma(p')}{(p'^2 - M^{*2})^2 + M^{*2}\Gamma^2(p')}$$

$\Gamma(p')$ ← total momentum dependent width

$$\mathcal{M} = \frac{G_F \cos\theta_C}{\sqrt{2}} l^\alpha J_\alpha$$

$$l^\alpha = \bar{u}(k')\gamma^\alpha(1-\gamma_5)u(k)$$ ← leptonic current

$$J_\alpha = V_\alpha - A_\alpha \qquad q^\alpha V_\alpha = 0$$ ← CVC

$$q^\alpha A_\alpha = i(m_u + m_d)\bar{q}_u \gamma_5 q_d$$ ← PCAC

Formalism

- Axial transition form factors
  - Poorly known (if at all...)
  - PCAC: $q^\alpha A_\alpha \approx 0$
  - π-pole dominance of the pseudoscalar form factor: F_P, C_6^A

- N*(1440) $J^P=1/2^+$

PCAC $\Rightarrow$ $F_P = -\frac{(M^* + M_N)M_N}{q^2 - m_\pi^2} F_A$

Using $\mathcal{L}_{N^* N\pi} = -\frac{g_{N^* N\pi}}{f_\pi} \bar{N}^* \gamma_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N$

$g_{N^* N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$

$f_\pi \leftarrow \pi$ decay constant

π-pole dominance $\Rightarrow F_P = -2 g_{N^* N\pi} F(q^2) \frac{(M^* + M_N)M_N}{q^2 - m_\pi^2}$ $\quad F(0) = 1$

Therefore $F_A(0) = 2 g_{N^* N\pi}$ $\leftarrow$ Goldberger-Treiman relation

Educated guess: $F_A(q^2) = F_A(0) \left(1 - \frac{q^2}{{M_A}^2}\right)^{-2}$ $\quad M_A = 1 \text{ GeV}$

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- Axial transition form factors
  - Poorly known (if at all...)
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- N*(1535) $J^P=1/2^-$

PCAC $\Rightarrow$ $$F_P = -\frac{(M^* - M_N)M_N}{q^2 - m_\pi^2} F_A$$

Using $$\mathcal{L}_{N^* N\pi} = -\frac{g_{N^* N\pi}}{f_\pi} \bar{N}^* \gamma_\mu (\partial^\mu \vec{\pi}) \vec{\tau} N$$

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Formalism

- Axial transition form factors
  - Poorly known (if at all…)
  - PCAC: $q^\alpha A_\alpha \approx 0$
  - π-pole dominance of the pseudoscalar form factor: F_P, C_6^A

- N*(1520) $J^P=3/2^-$

PCAC $\Rightarrow$ $C_6^A = -\dfrac{M_N^2}{q^2 - m_\pi^2} C_5^A$

Using $\mathcal{L}_{N^* N\pi} = -\dfrac{g_{N^* N\pi}}{f_\pi} \bar{N}^*_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N$

$g_{N^* N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$

f_π $\leftarrow$ π decay constant

π-pole dominance $\Rightarrow$ $C_6^A = 2 g_{N^* N\pi} F(q^2) \dfrac{(M^* - M_N) M_N}{q^2 - m_\pi^2}$ $\quad F(0) = 1$

Therefore $C_5^A(0) = -2 g_{N^* N\pi}$ $\leftarrow$ Goldberger-Treiman relation

Educated guess: $C_5^A(q^2) = C_5^A(0) \left(1 - \dfrac{q^2}{{M_A}^2}\right)^{-2}$ $M_A = 1$ GeV $\quad C_3^4 = C_4^A = 0$

Non-resonant background

- for MC generators
- Axial part:
  - PCAC + π-pole dominance of the pseudoscalar current
  - Axial current at $Q^2=0$ can be obtained from $\pi N \to \pi N$
    Kamano et al, PRD 86, but also Rein, Sehgal, PCAC Cohπ
  - There are several partial wave analyses of $\pi N \to \pi N$
  - After subtracting resonances $\Rightarrow$ background (+interference) at $Q^2=0$
  - At $Q^2 > 0$ there is no experimental information except ANL and BNL

Weak η production

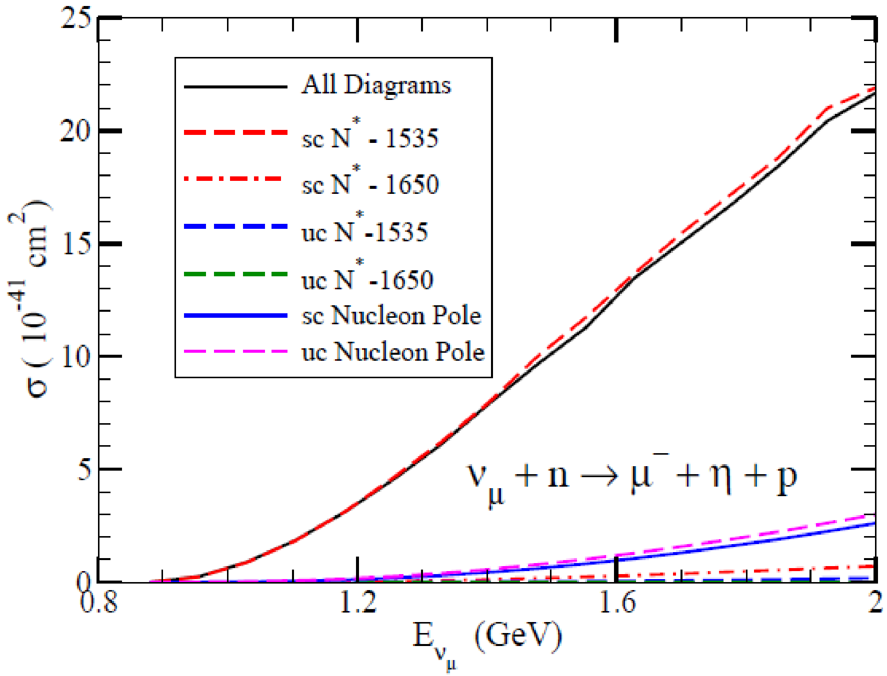
- LAR, M. Sajjad Athar, M. Rafi Alam, M. J. Vicente Vacas
- $\nu_l \, N \rightarrow l \, N' \, \eta$
- Background (from atmosferic ν) for proton decay searches: $p \rightarrow l^+ \, \eta$

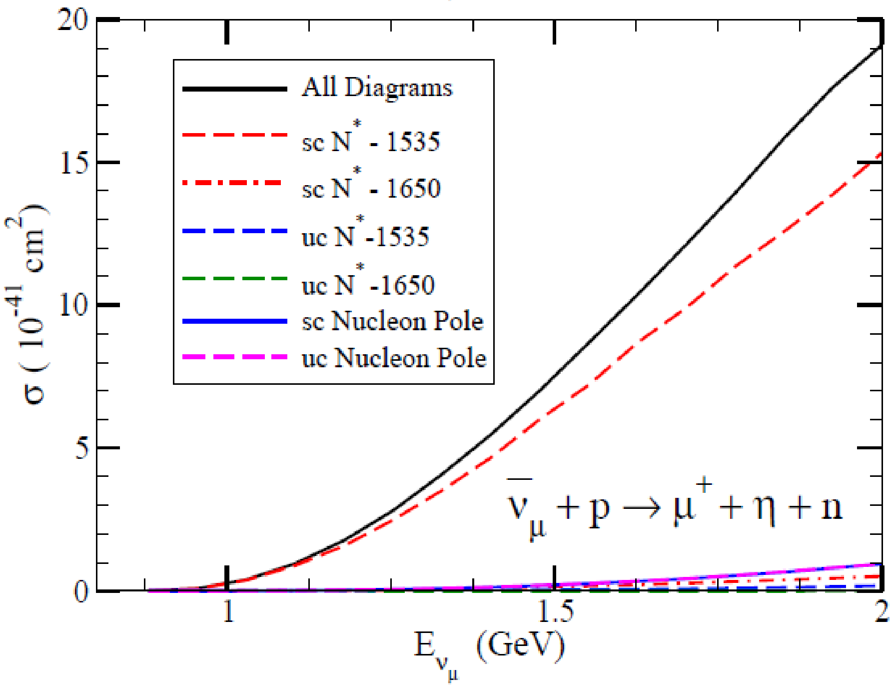
  D. Wall et al. PRD 62 (2000)
- A second class π-pole mechanism could be observed (forward)

  N. Dombey, PR 174 (1968)
- Sensitive to the N*(1535) (axial) properties
- Contributes to improvement of MC simulations in ν experiments

Weak η production

- LAR, M. Sajjad Athar, M. Rafi Alam, M. J. Vicente Vacas
- $\nu_l \, N \to l \, N' \, \eta$
- Ingredients: s,u-channel nucleon pole, N*(1535), N*(1650)
- Results:





- The N*(1535) excitation is dominant
- Small cross section but large enought to be measured at MINERνA

Questions

- Do we need a (drastically) better weak resonance production model?
- Which is the (best) way to take state-of-the-art pheno into account?
- How to deal with the non-resonant background (+interference)?
- How to deal with the RES -> DIS transition?
- Are there going to be new ν-nucleon measurements in the (near) future?
- Will Minerνa help (at the nucleon level)?
- Can we get useful info from PV (e,e') experiments?
- How to avoid the donkey effect?

The donkey effect