Radiative Capture Reactions in Lattice Effective Field Theory

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Nuclear Reactions from Lattice QCD, INT March 12, 2013

Outline

- Motivation
- Continuum EFT for reactions
- Lattice EFT for reactions

Reaction theory:

- Reaction theory for nuclear experiments, e.g. FRIB
- Nuclear astrophysics where data might be lacking
- Reactions are more fun than static properties

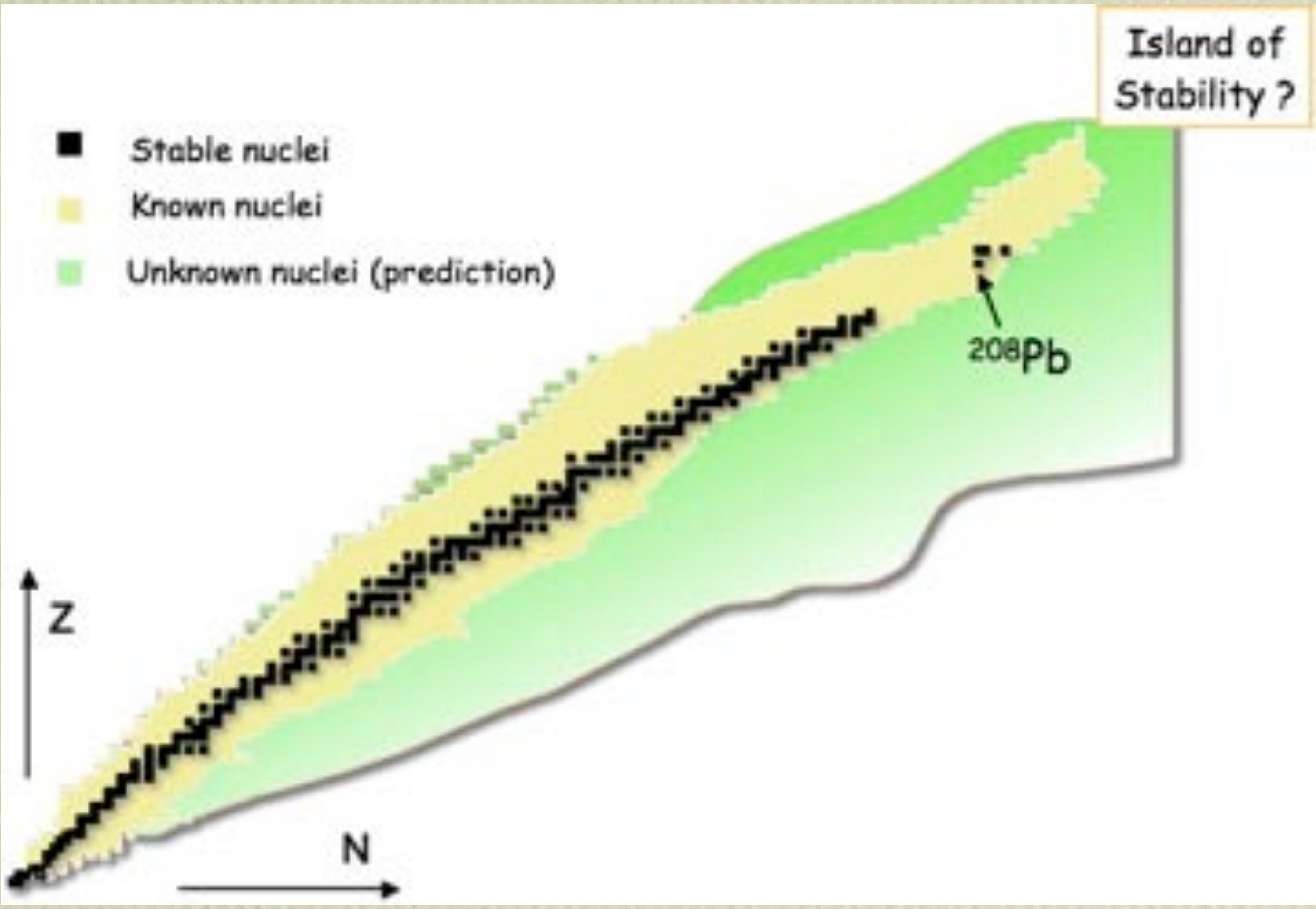
Examples: $d(p,\gamma)^3\mathrm{He},{}^7\,\mathrm{Be}(p,\gamma)^8\mathrm{B},{}^{14}\,\mathrm{C}(n,\gamma)^{15}\mathrm{C},\cdots,$

the list goes on

Only halo systems for now ...

Halo systems

- Characterized small neutron/proton separation energy. Large size.
- Interesting three-body physics: All-bound, Tango, Samba (^{12}Be), Borromean (^{11}Li)
- Exotic physics near the drip line



Some details on $^{7}\mathrm{Li}(n,\gamma)^{8}\mathrm{Li}$

- Isospin mirror systems $^{7}\mathrm{Li}(n,\gamma)^{8}\mathrm{Li} \leftrightarrow {}^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$

- Inhomogeneous BBN

Whats the theoretical error?

EFT

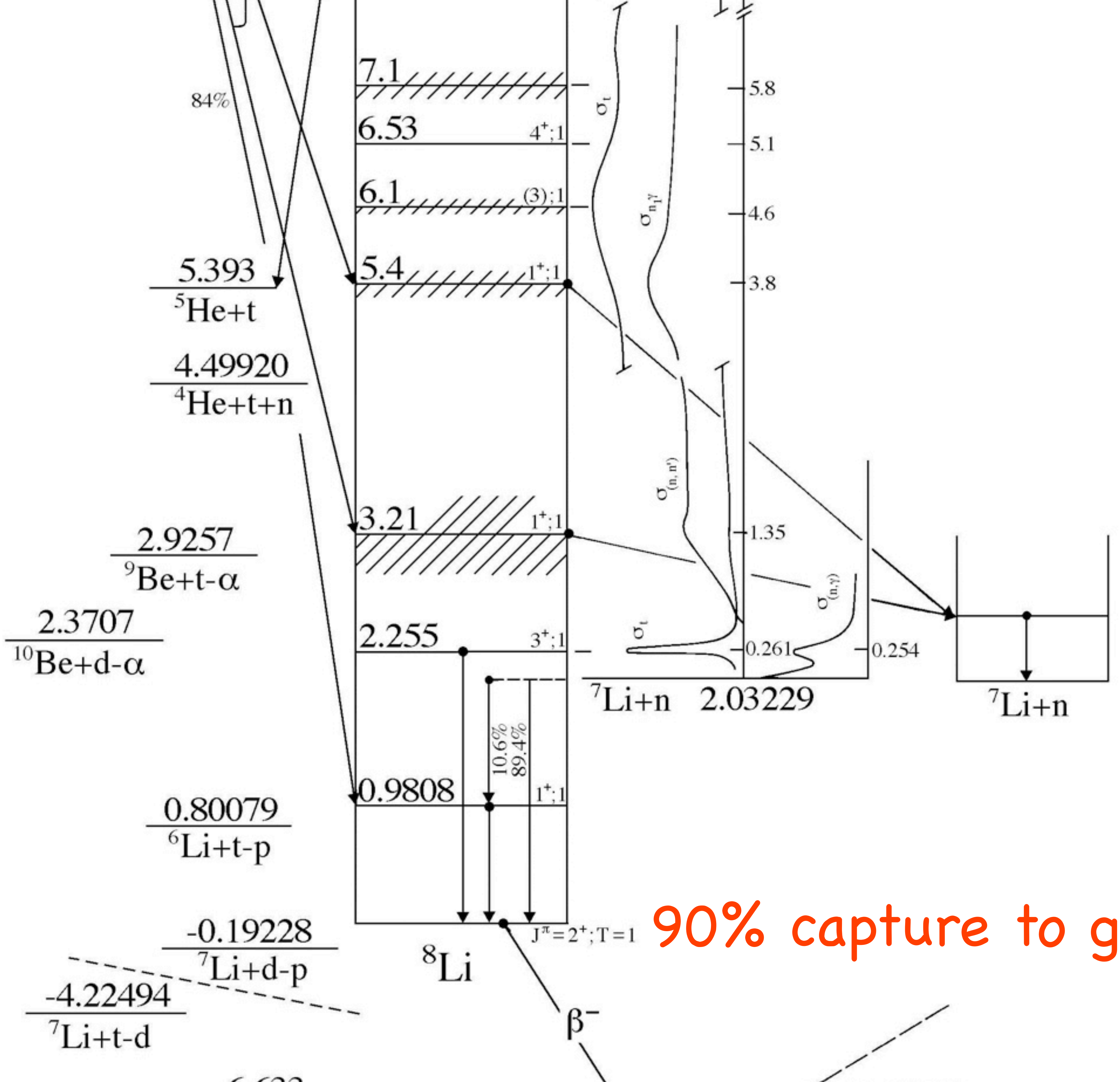
- Identify degrees of freedom

$$\mathcal{L} = c_0\, \mathrm{O}^{(0)} + c_1\, \mathrm{O}^{(1)} + c_2\, \mathrm{O}^{(2)} + \cdots$$

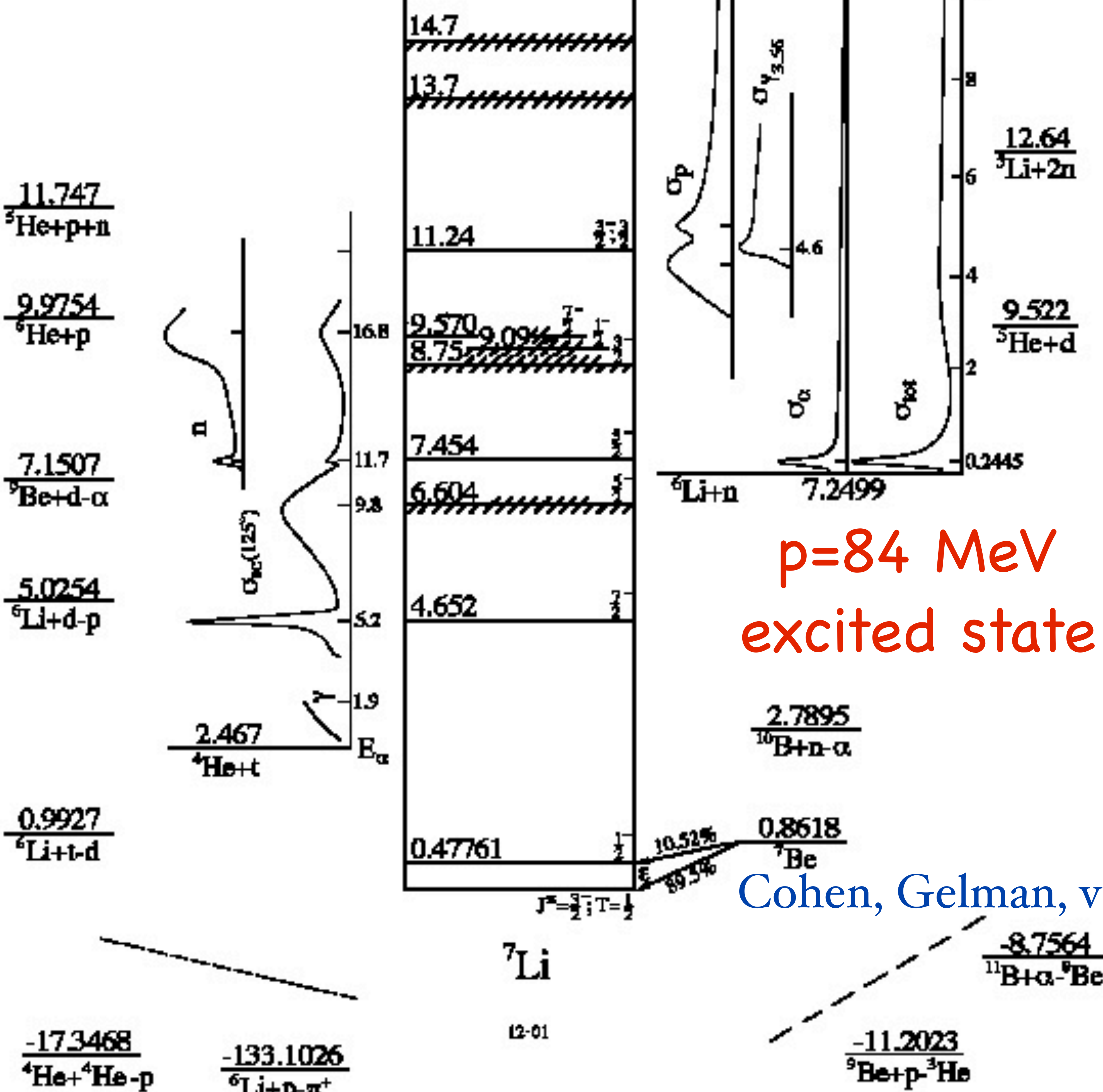
Hide UV ignorance   IR explicit

- Determine c_n from data (elastic, inelastic)
- EFT   ERE + currents + relativity

Not just Ward identity



90% capture to gs



p=84 MeV
excited state

Cohen, Gelman, van Kolck '04

Look at E1 transition

-- Initial state: s-wave

single operator fitted to scattering length a

-- Ground state: p-wave

p-wave needs two operators (a_V, r_1)

We also include the excited state and the p-wave 3+ resonance (M1 transition)

$$i\mathcal{A}(p) = \frac{2\pi}{\mu}\frac{i}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip}$$

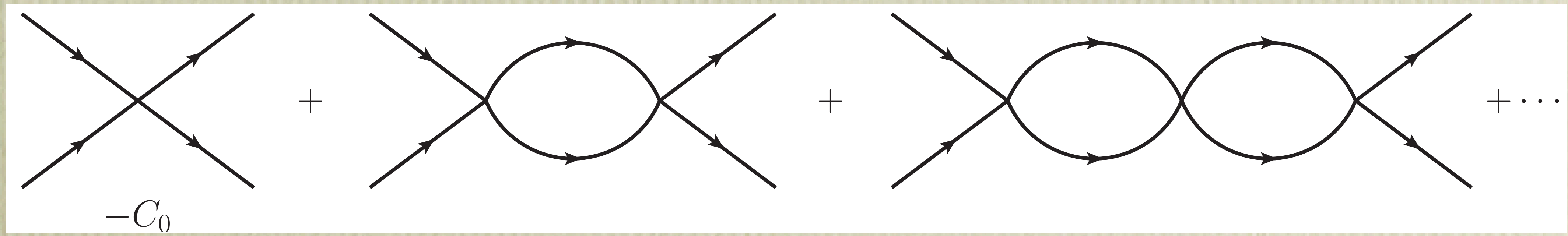
--- Natural case $a, r \sim 1/\Lambda << 1/p$

expand in small p, EFT perturbative

--- Large scattering length $a >> 1/\Lambda$

$$i\mathcal{A}(p) \approx -\frac{2\pi}{\mu}\frac{i}{1/a + ip}\left[1 + \frac{1}{2}\frac{rp^2}{1/a + ip} + \cdots\right]$$

EFT non-perturbative



$$i\mathcal{A}(p) = \frac{-i}{\frac{1}{C_0} + i\frac{\mu}{2\pi}p} \Rightarrow C_0 = \frac{2\pi a}{\mu}$$

Weinberg ’90
Bedaque, van Kolck ’97
Kaplan, Savage, Wise ’98

p-wave $$iA(p) = \frac{2\pi}{\mu} \frac{ip^2}{-\frac{1}{a_V} + \frac{r_1}{2}p^2 + \cdots - ip^3}$$

Shallow systems

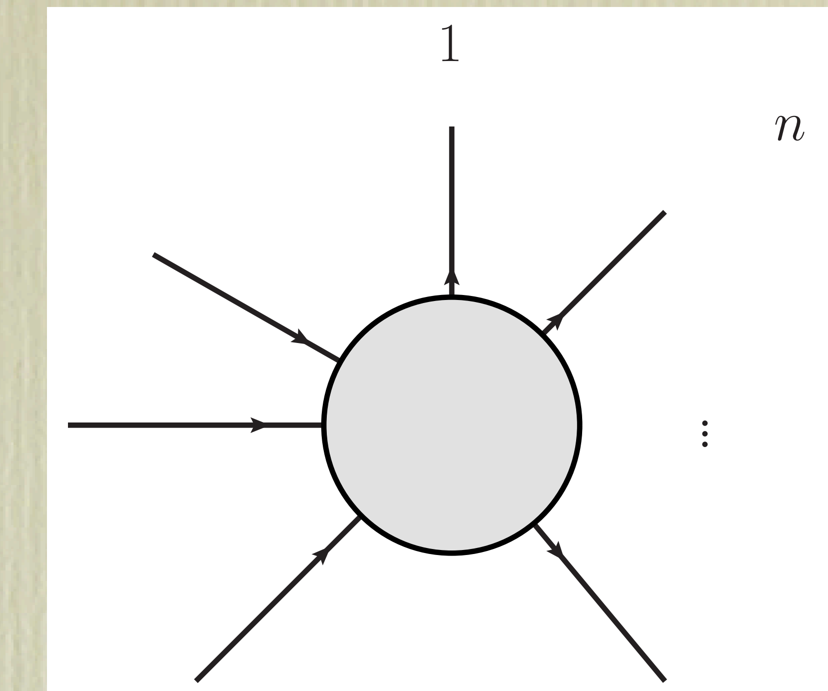
2 fine tuning Bertulani, Hammer, van Kolck '02

1 fine tuning Bedaque, Hammer, van Kolck '03

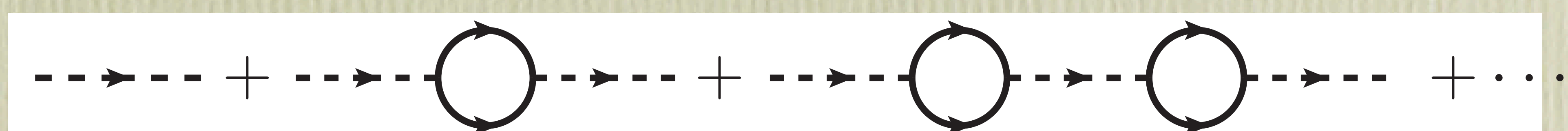
Requires two non-perturbative operators at LO

Residues and poles
n-point function

LSZ reduction: $G^{(n)} \sim \prod_{i=1}^{n} \sqrt{Z_i}$



Consider a scalar theory



$$= \frac{Z}{p_0^2 - p^2 - m^2}$$

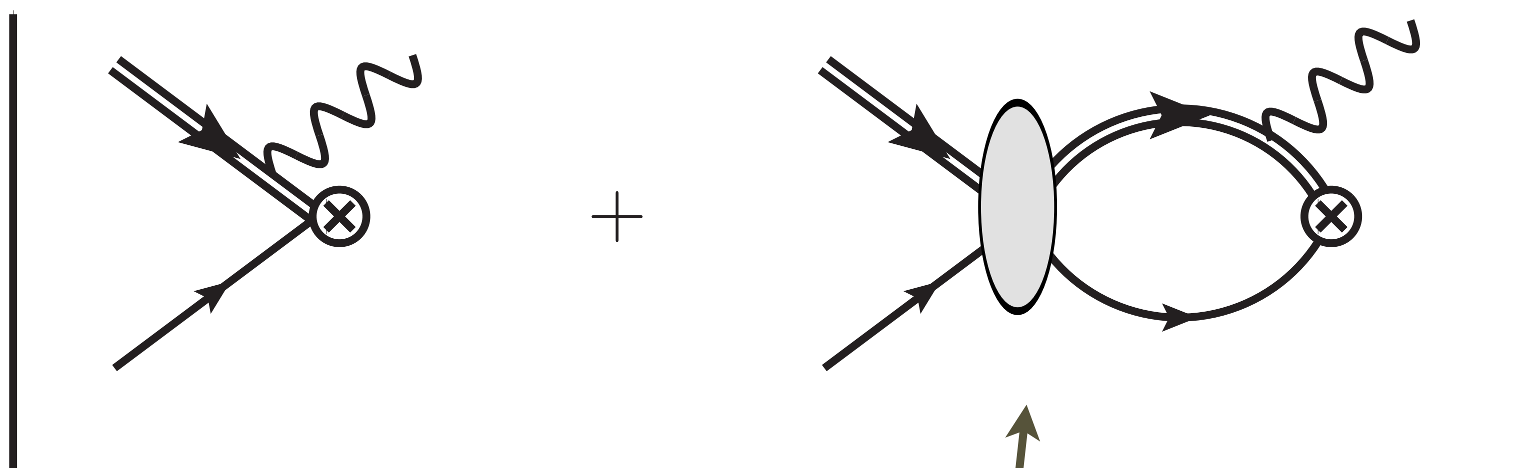
For deuteron: $$\frac{Z_d}{p_0 - \frac{p^2}{4M} + \frac{\gamma^2}{M}}, \quad Z_d = \frac{8\pi\gamma}{M^2}\frac{1}{1-\rho\gamma}$$

$$\psi_d(r) = \sqrt{\frac{\gamma}{2\pi}\frac{1}{1-\rho\gamma}}\,\frac{e^{-r\gamma}}{r} = \sqrt{\frac{\mu^2}{4\pi^2}Z_d}\,\frac{e^{-r\gamma}}{r}$$

Capture Cross Section

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{32\pi s}\frac{k}{p}|\Gamma|^2$$

$$\rightarrow |\Gamma| =$$

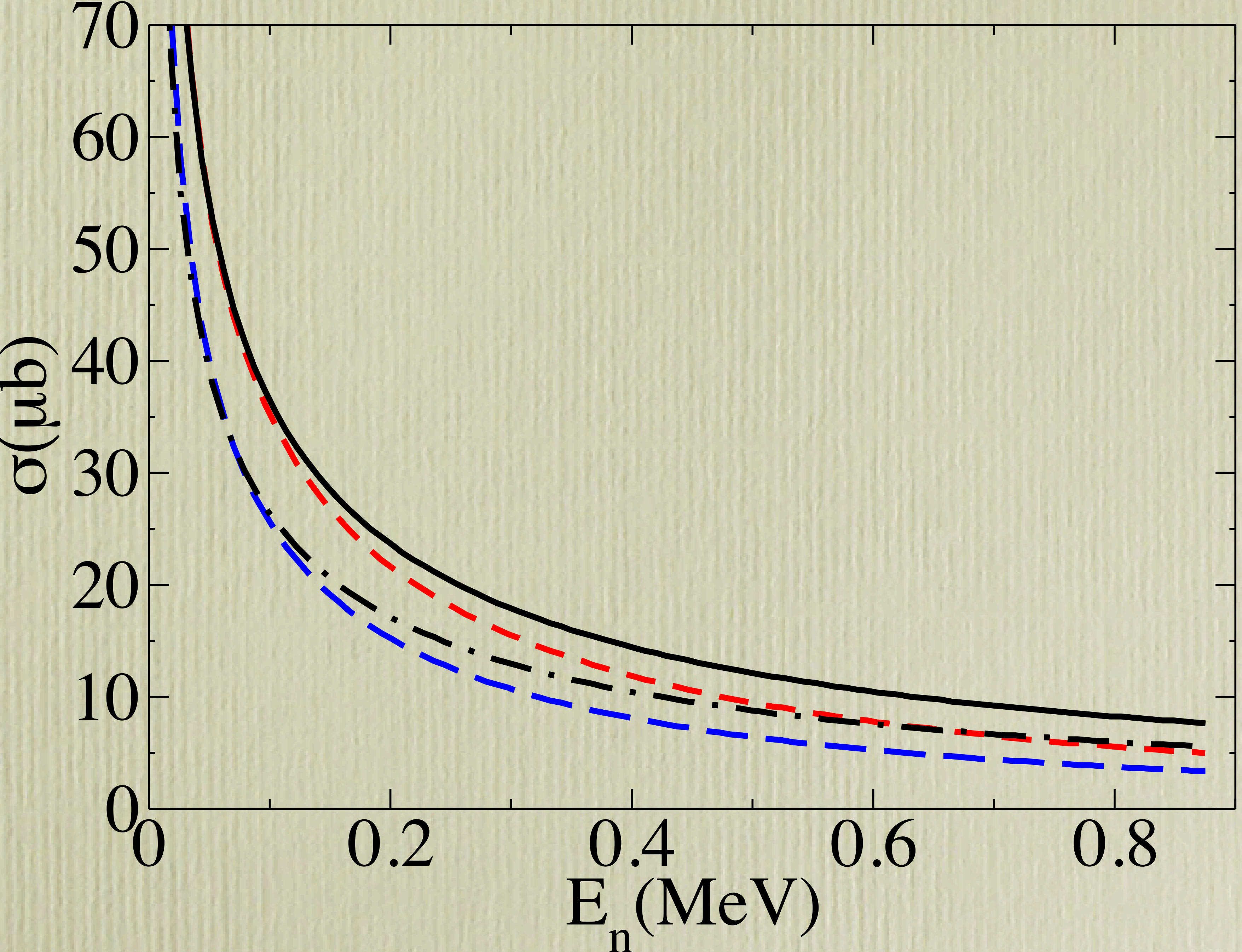


gives 1/p dependence

Analytic result, depends on $\sqrt{Z} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{1+3\gamma/r_1}}$

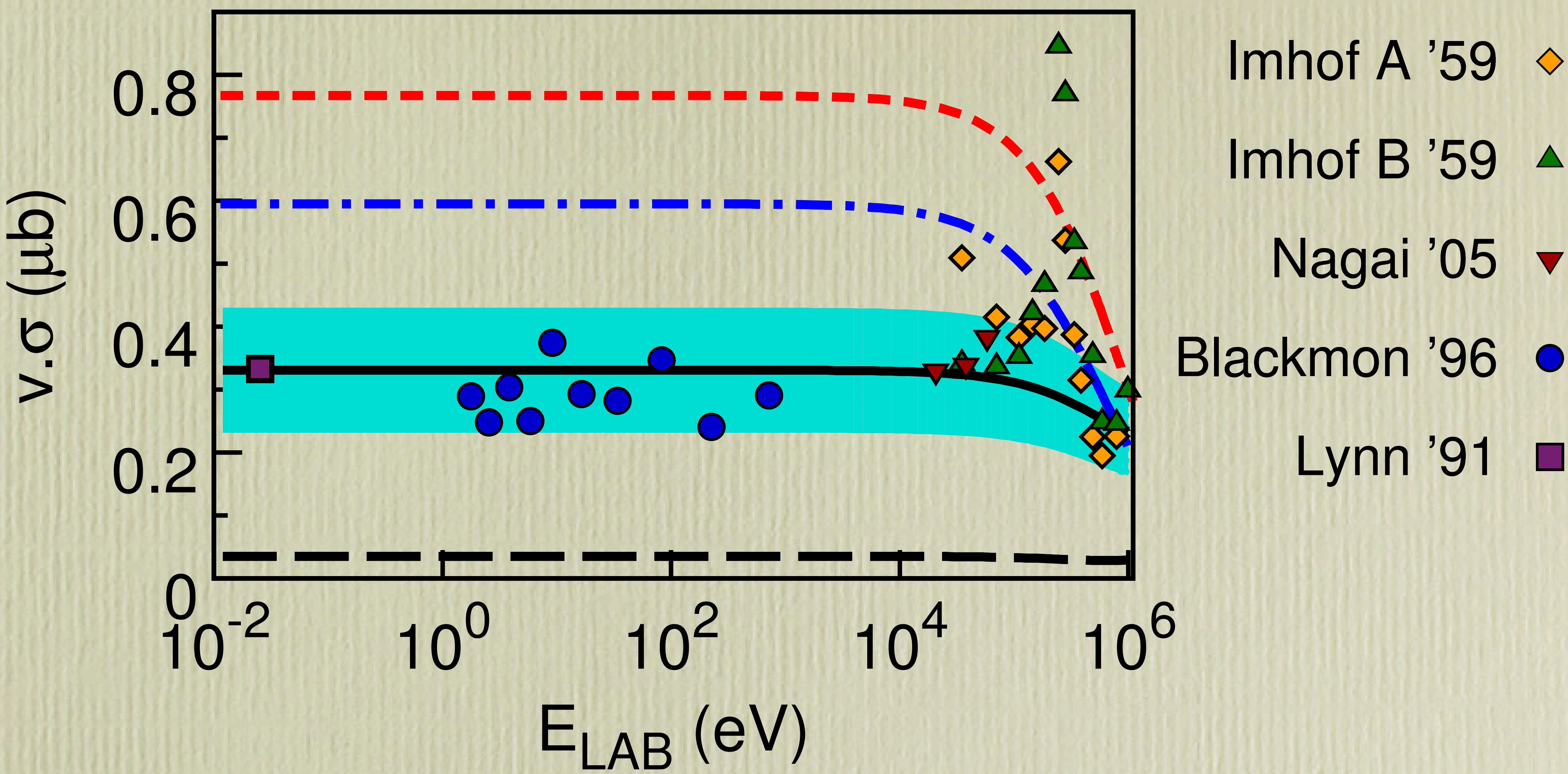
Need r1 at leading order

"Effective range" contribution



Red: Tombrello
Blue: Davids-Typel
Black: EFT

Rupak, Higa; PRL 106, 222501 (2011)



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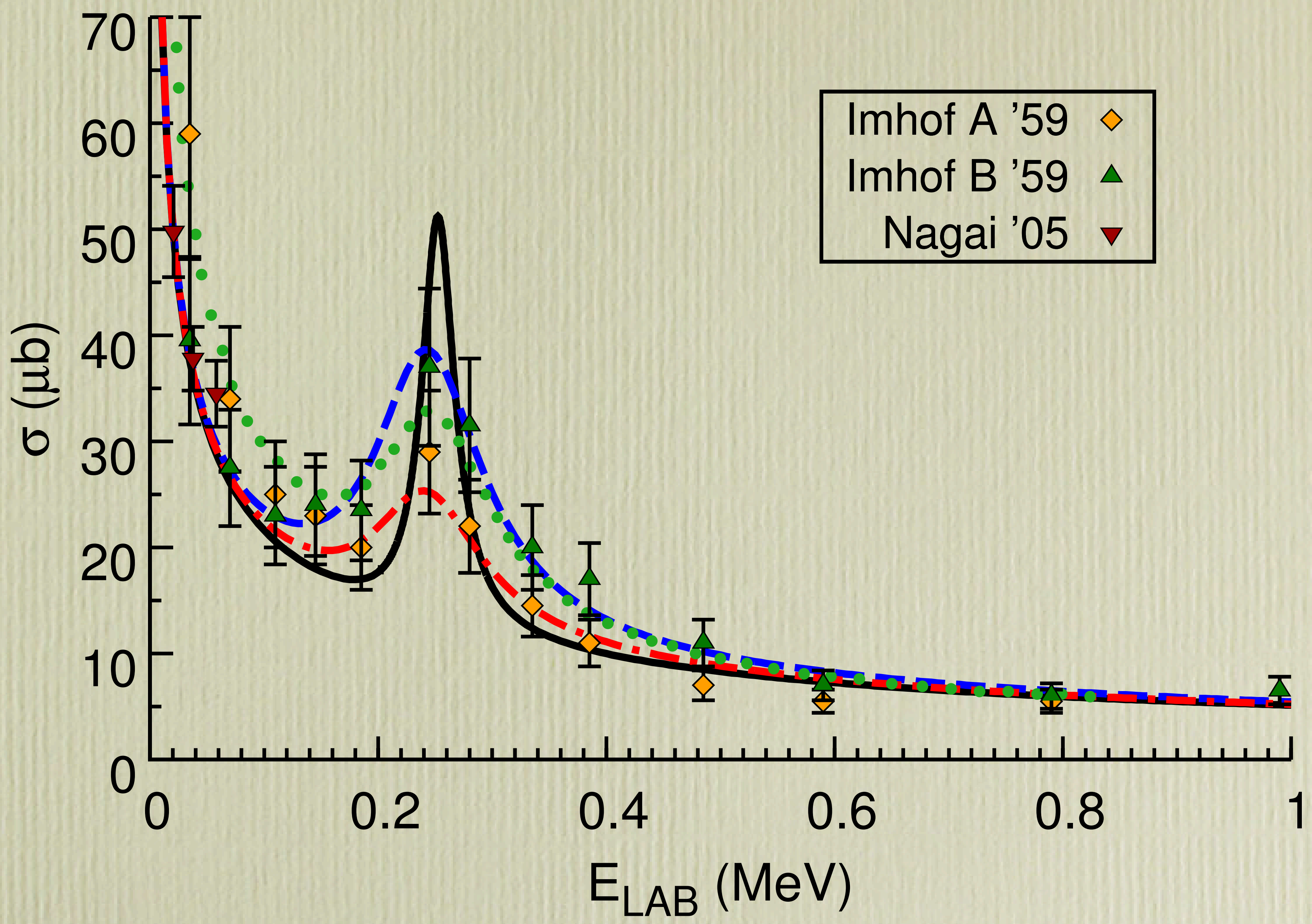
Fernando, Higa, Rupak; EPJA 48, 24 (2012)



Fernando, Higa, Rupak; EPJA 48, 24 (2012)

Lessons learned

--- Tuning potential to reproduce bound state energy is not sufficient to get the wave function renormalization constant.

--- In the strong sector directly applies to $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$

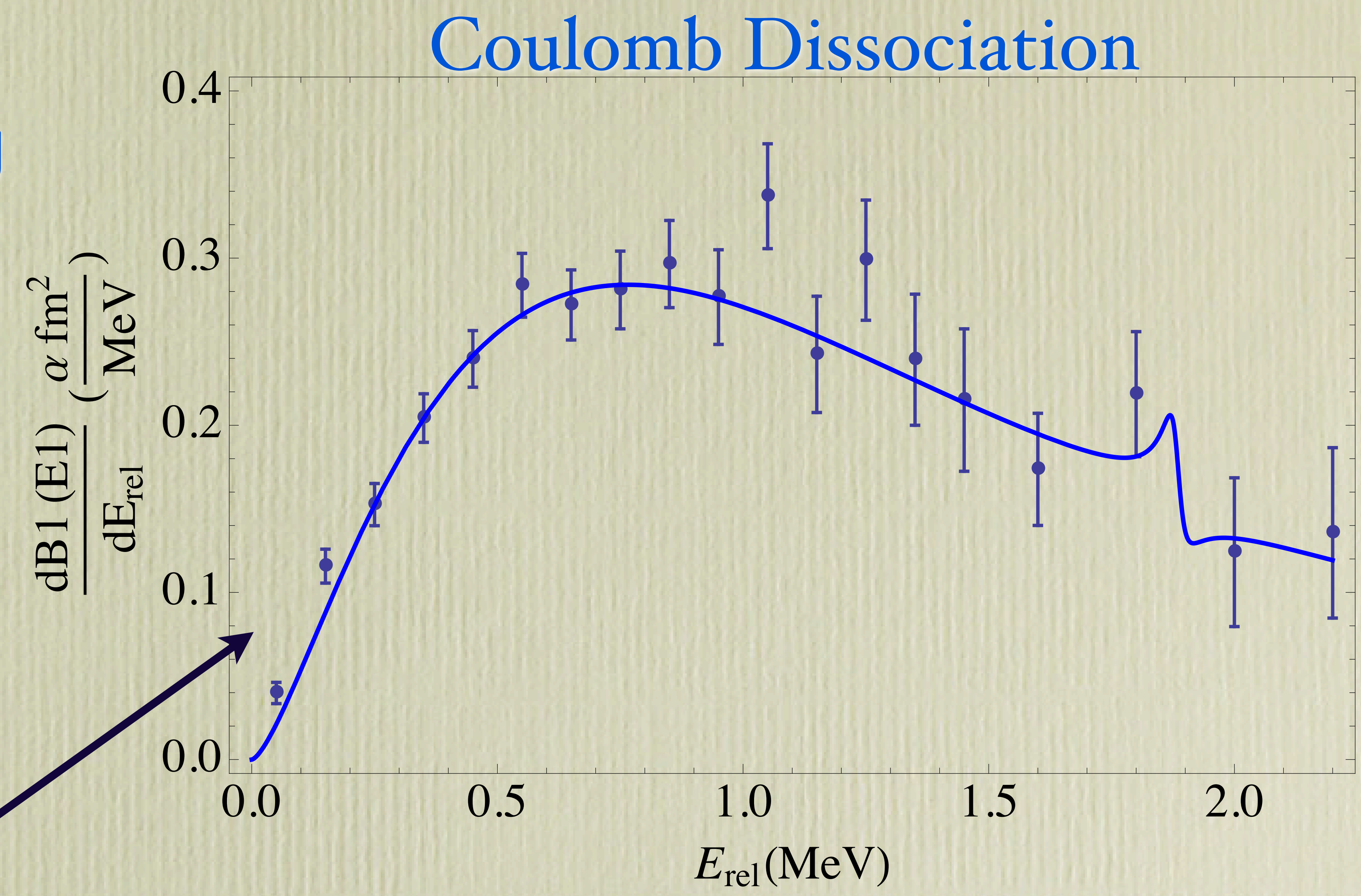
General problem : How to constrain low-energy nuclear theory?

Neutron Capture/Coulomb Dissociation on Carbon-14

Power counting

$$a_1 = -n_1/Q^3$$
$$r_1 = 2n_2Q$$

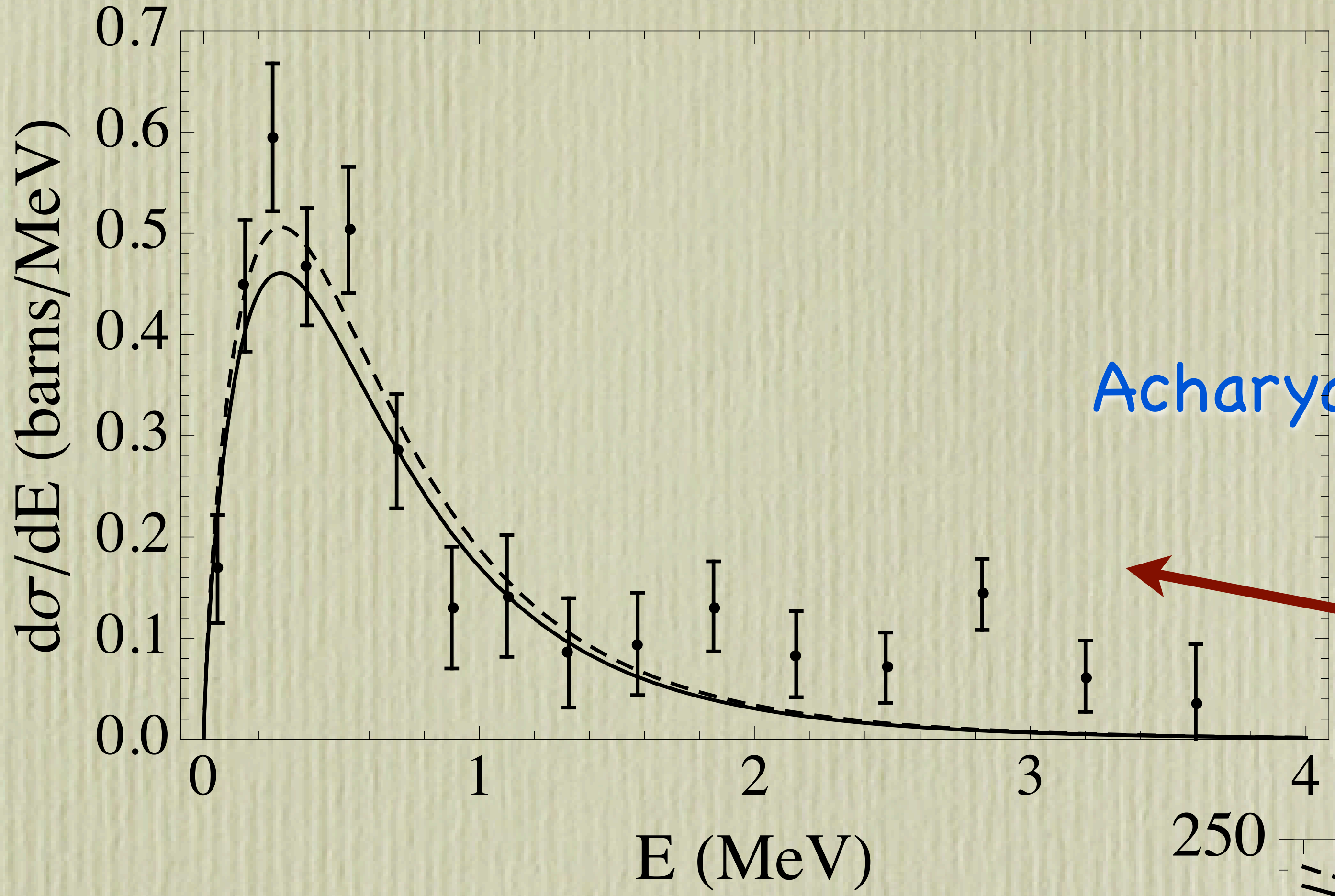
n1=0.7, n2=1, Q=40 MeV



Rupak, Fernando, Vaghani, PRC 86, 044608 (2012)

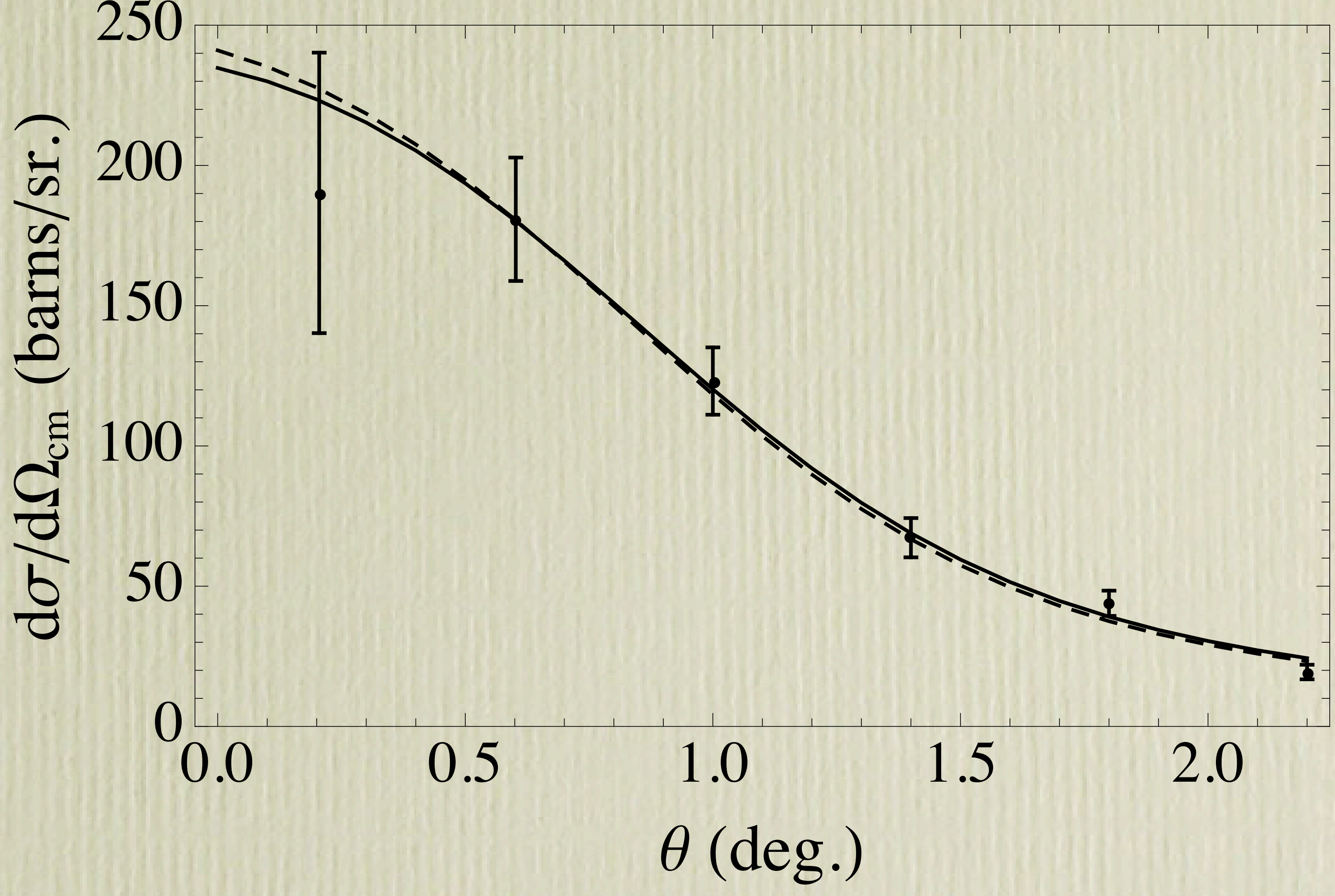
Data: T. Nakamura et al., PRC, 79, 035805 (2009)

Coulomb Dissociation of Carbon-19



Acharya & Phillips, arxiv:1302.4762

(a, r) fitted



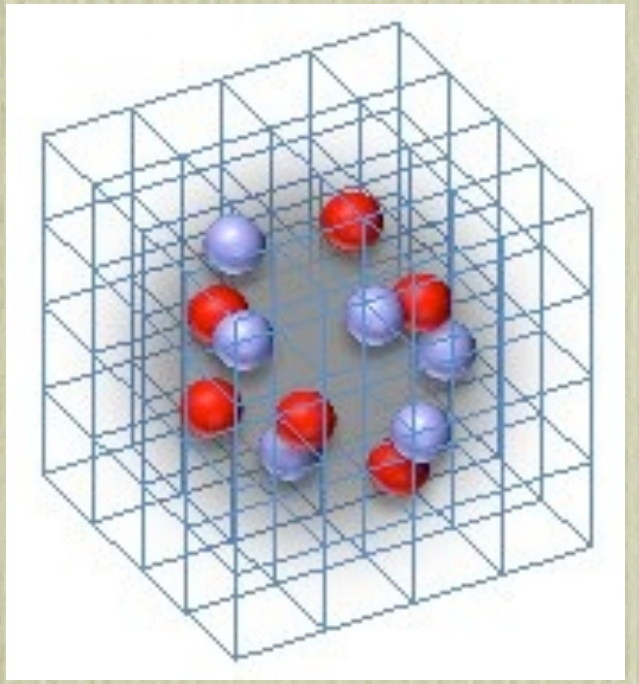
Lattice EFT for Halo Nuclei

- Interested in $a(b,\gamma)c$

- Need interaction between clusters

- Calculate capture with cluster interaction. Many possibilities --- traditional methods, continuum EFT, lattice method

Nuclear Lattice Effective Field Theory collaboration



Evgeny Epelbaum,
Hermann Krebs,
Timo Lahde
Dean Lee
Ulf-G. Meissner

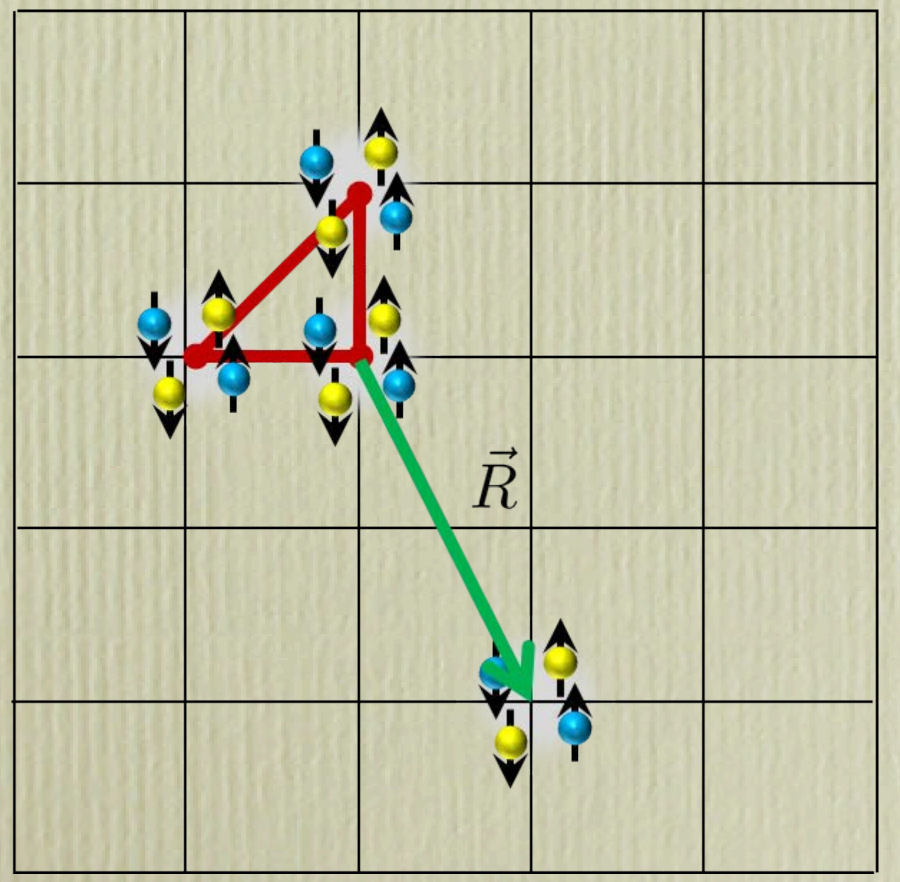
Adiabatic Hamiltonian

Microscopic Hamiltonian $L^{3(A-1)}$

Adiabatic Hamiltonian for the clusters L^3

-- acts on the cluster c.m. and spins

Blume, Greene 2000



Lee, Pine, Rupak

1D toy atom-dimer problem

Microscopic Hamiltonian: -2.130490, -2.130490, 0.1189620, 0.1189620, ...

Adiabatic Hamiltonian: -2.130505, -2.130493, 0.1189604, 0.1189781, ...

That was in 2012, Can do better now

n-d scattering in quartet channel

Microscopic Hamiltonian: 7.152, 23.37, 23.37, 23.37, 29.61, 29.61, 40.34, ...

Adiabatic Hamiltonian: 7.166, 23.42, 23.42, 23.42, 29.74, 29.74, 40.49, ...

Warm up $p(n,\gamma)d$

Write $\langle\psi_B|O_{\rm EM}|\psi_i\rangle$ using retarded Green's function

$$\mathcal{M}(\epsilon) = \left(\frac{p^2}{M} - E - i\epsilon\right)\sum_{\boldsymbol{x},\boldsymbol{y}}\psi_B^*(\boldsymbol{y})\langle\boldsymbol{y}|\frac{1}{E-\hat{H}_s+i\epsilon}|\boldsymbol{x}\rangle e^{i\boldsymbol{p}\cdot\boldsymbol{x}}$$

Exact analytic continuum result

$$\mathcal{M}_C(\epsilon) = \frac{1}{p^2+\gamma^2} - \frac{1}{(1/a+ip_\epsilon)(\gamma-ip_\epsilon)}, \quad p_\epsilon = \sqrt{p^2+iM\epsilon}$$

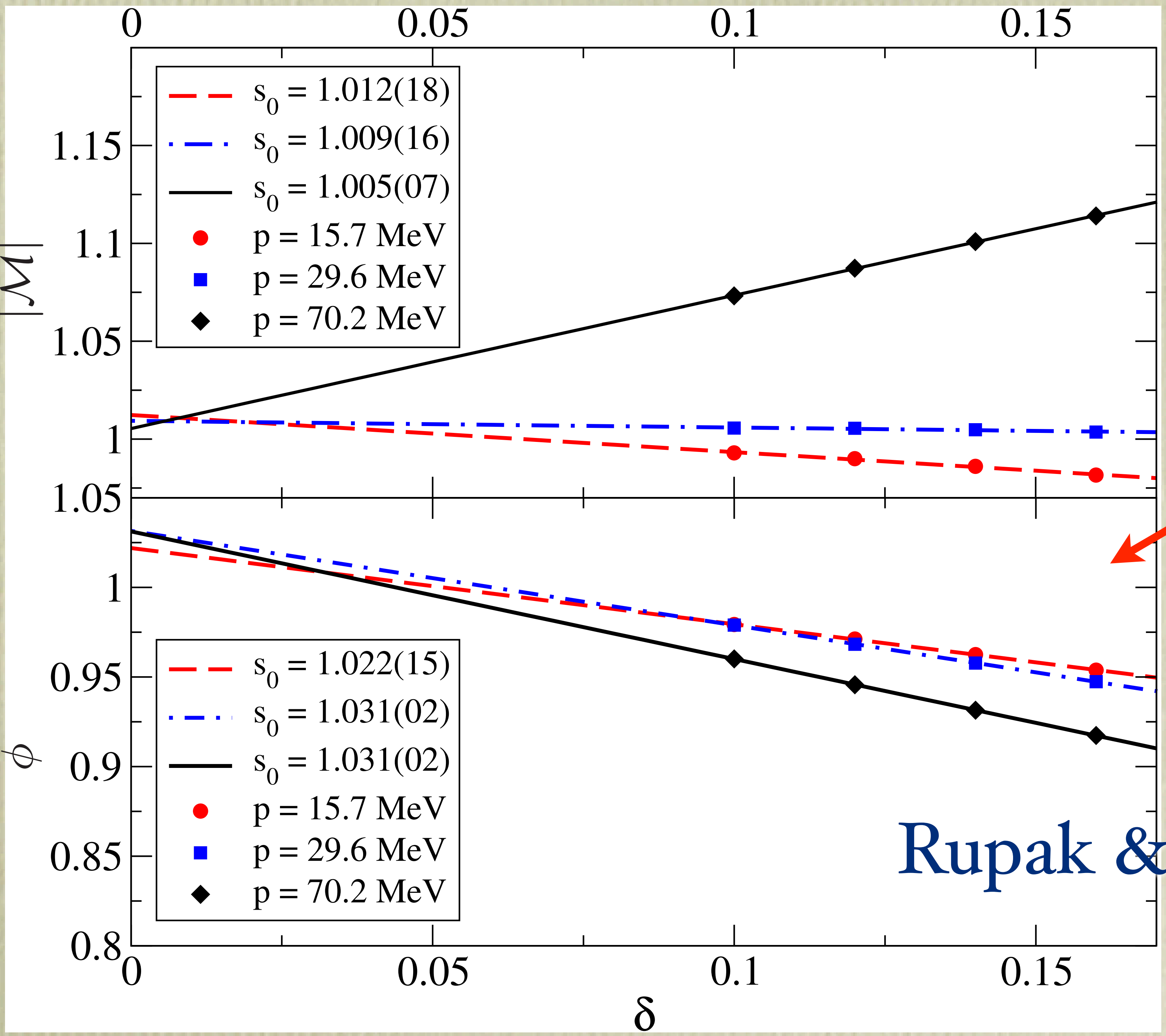
When $\epsilon\to 0^+$, $\mathcal{M}_C$ reduces to known M1 result

Rupak, 2000

Lee & Rupak

Lattice EFT results

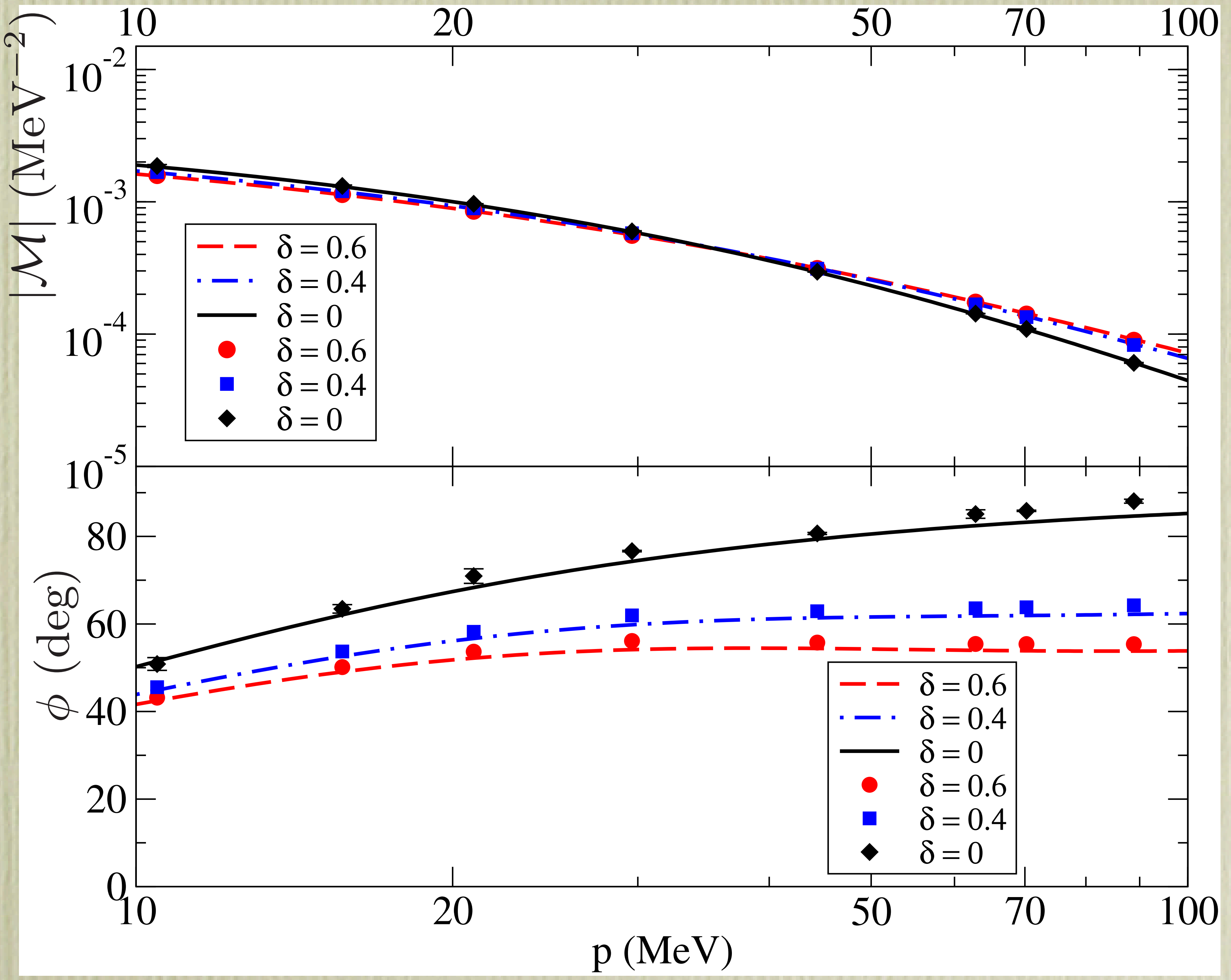
Magnitude, argument normalized to continuum



$$\delta = \epsilon M / p^2$$

Rupak & Lee, arXiv:1302.4158

Continuum extrapolation



Rupak & Lee, arXiv:1302.4158

Lattice QCD to lattice EFT

- Constrain Hamiltonian in elastic channels
- Electroweak currents --- Detmold, Savage 2004
  1. Fit pionless EFT at unphysical pion mass.
  2. Match observables in pionless and chiral EFT.
  3. Extrapolate to physical pion mass.
- Coulomb effect (in discussion)

Conclusions

- Capture reactions $^7\mathrm{Li}(n,\gamma)^8\mathrm{Li}$, $^{14}\mathrm{C}(n,\gamma)^{15}\mathrm{C}$, etc.
- In progress $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$
- Capture reactions in lattice EFT

Thank you