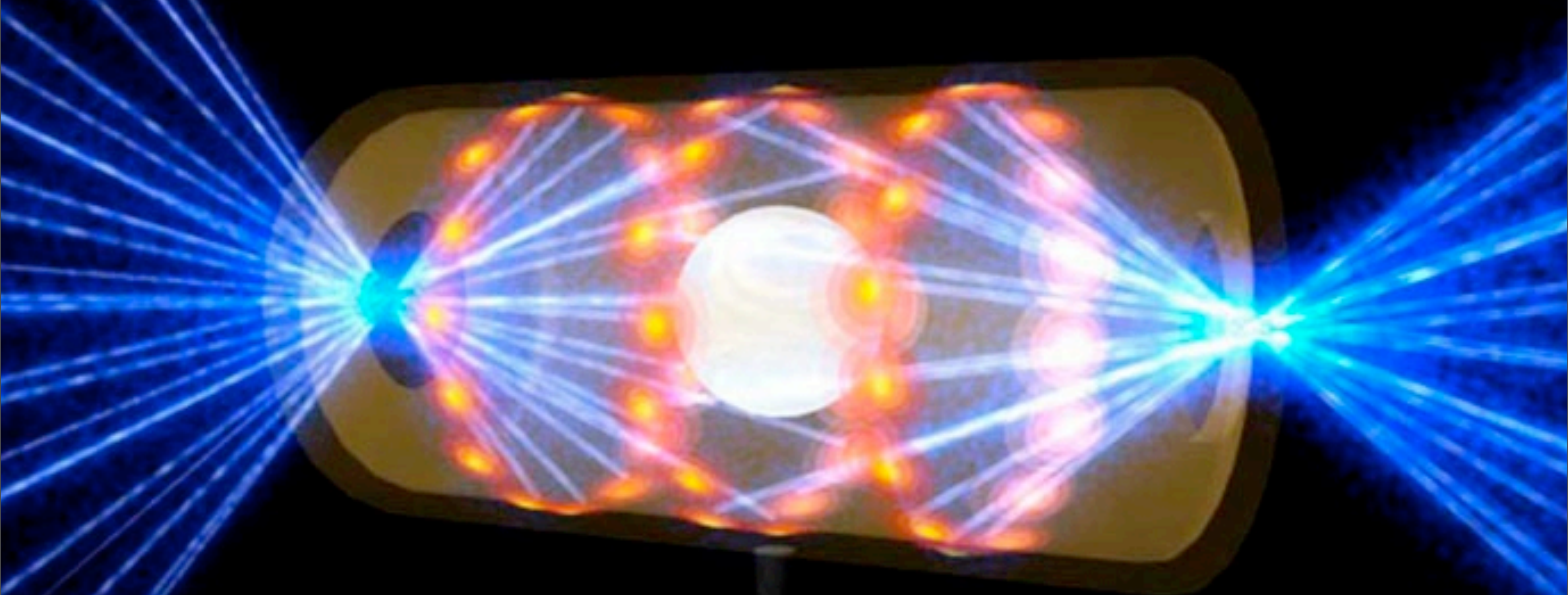
INT-13-53W
March 2013

Three-particle scattering amplitudes from a finite volume formalism*

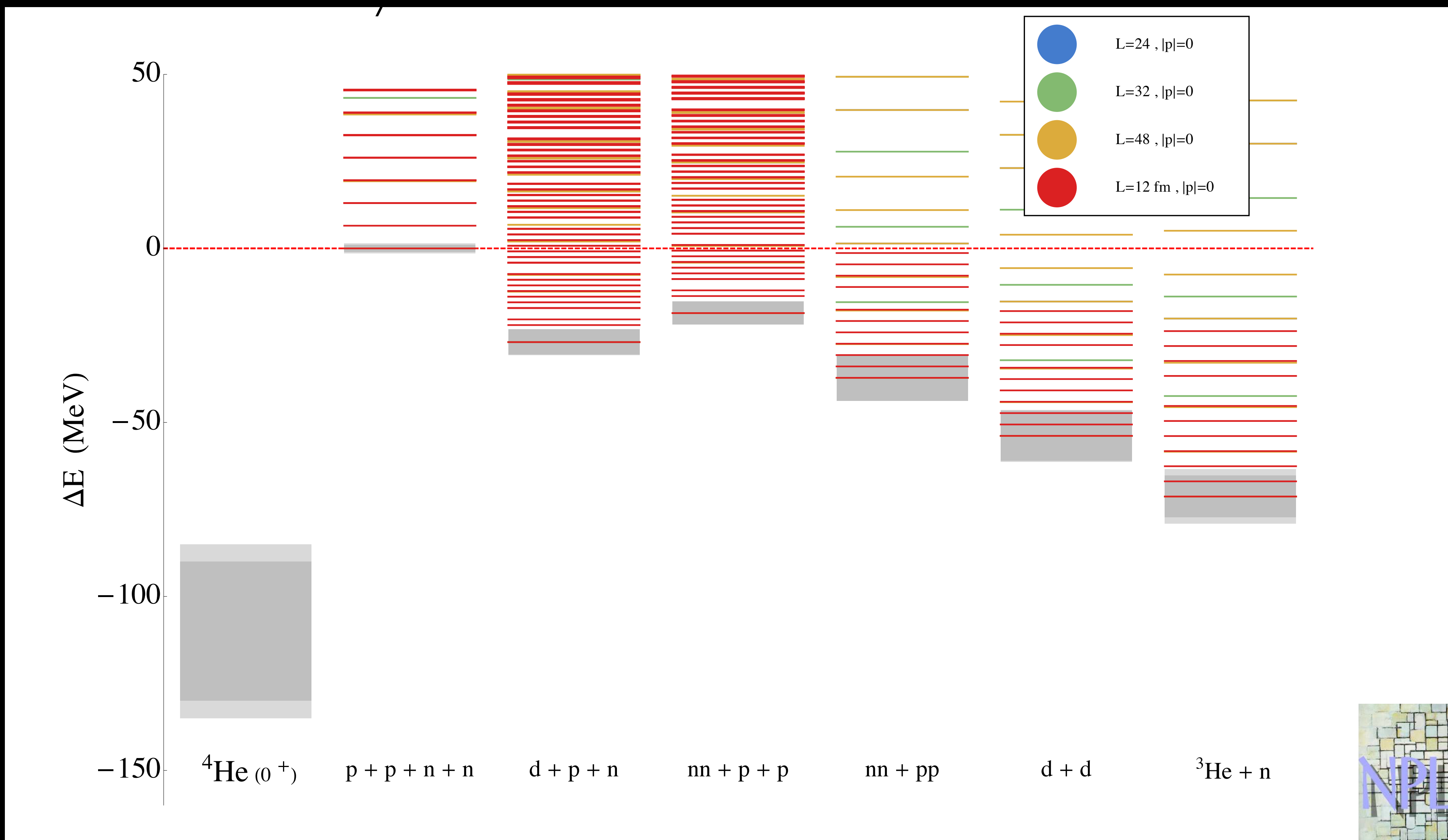


Zohreh Davoudi
University of Washington

*Raul Briceno, ZD, arXiv: 1212.3398

Why a finite volume formalism?

Lattice QCD: From spectrum to physical observables?



Beane, et. al. (2012).

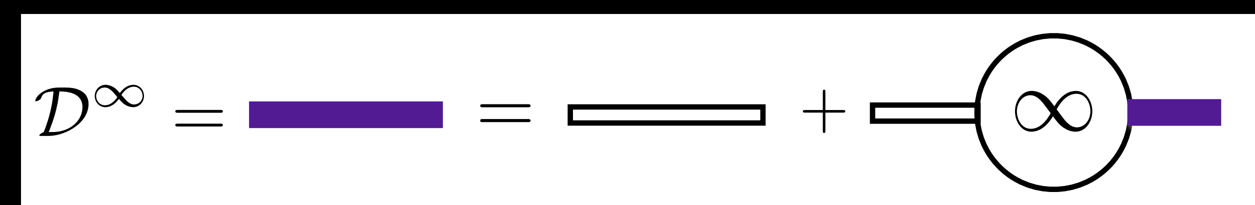
Resonance spectroscopy- coupled multi-particle channels

Dimer formalism and Luscher formula

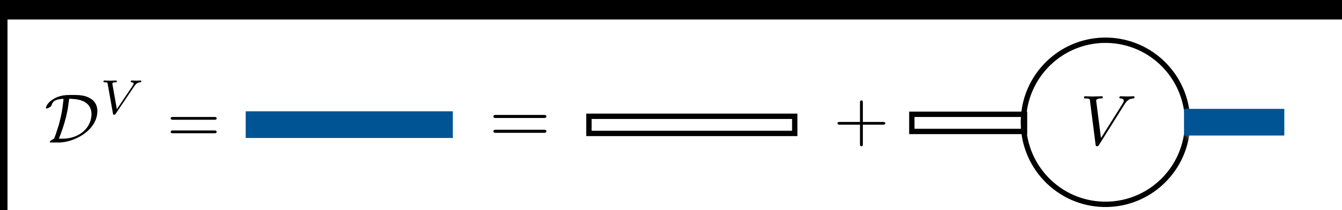
A NR EFT approach

$$\mathcal{L} = \phi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m} \right) \phi - d^\dagger \left(i\partial_0 + \frac{\nabla^2}{4m} - \Delta \right) d^\dagger - \frac{g_2}{2}(d\phi^2 + \text{h.c.}) + \ldots$$

Eliminate in favor of physical observables: a, r



$$i\mathcal{D}^\infty(E, \mathbf{q}) = \frac{-imr/2}{\bar{q}\cot\delta_d - i\bar{q} + i\epsilon}$$



$$i\mathcal{D}^V(E, \mathbf{q}) = \frac{-imr/2}{\bar{q}\cot\delta_d - 4\pi\, c^q_{00}(\bar{q}^2 + i\epsilon) + i\epsilon}$$

The spectrum in FV can be written in a model-independent way

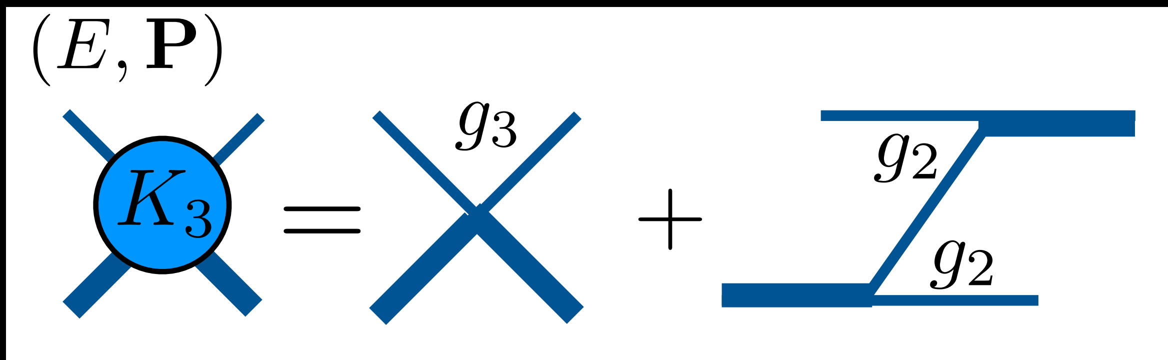
$$c^q_{00}(x) = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \mathcal{P} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \right] k^{*l} \frac{\sqrt{4\pi} Y_{00}(\hat{k}^*)}{k^{*2} - x}$$

$$\mathbf{k}^* = \mathbf{k} - \mathbf{q}/2$$

Luscher (1986, 1991). Rummukainen, Gottlieb, (1995). Kim, Sachrajda, Sharpe (2005). Beane, et al (2005).

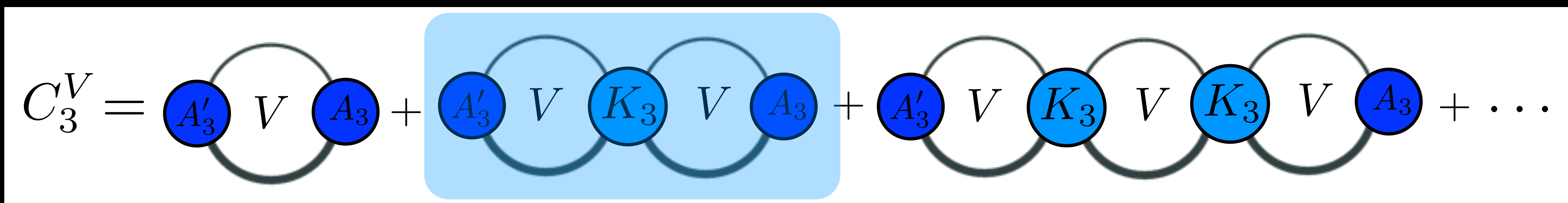
Monday, March 11, 2013

Three-body correlation functions with the dimer field



Kinematic region below four-particle threshold

Expand the correlation function in powers of kernel



$$\frac{1}{L^6}\sum_{\mathbf{q}_1,\mathbf{q}_2} A_3\left(\mathbf{q}_1\right) i\mathcal{D}^V\left(E-\frac{q_1^2}{2m},|\mathbf{P}-\mathbf{q}_1|\right) iK_3(\mathbf{q}_1,\mathbf{q}_2;\mathbf{P},E)\, i\mathcal{D}^V\left(E-\frac{q_2^2}{2m},|\mathbf{P}-\mathbf{q}_2|\right) A_3'\left(\mathbf{q}_2\right)$$

$$-\int \frac{d^3\mathbf{q}_1}{(2\pi)^3}\frac{d^3\mathbf{q}_2}{(2\pi)^3} \quad ?$$

$$iK_3(\mathbf{q}_1,\mathbf{q}_2;\mathbf{P},E) \equiv -ig_3 - \frac{ig_2^2}{E-\frac{\mathbf{q}_1^2}{2m}-\frac{\mathbf{q}_2^2}{2m}-\frac{(\mathbf{P}-\mathbf{q}_1-\mathbf{q}_2)^2}{2m}+i\epsilon}$$

The poles of three-body kernel cancel with zero's of full FV dimer propagator!

Identifying the on-shell states

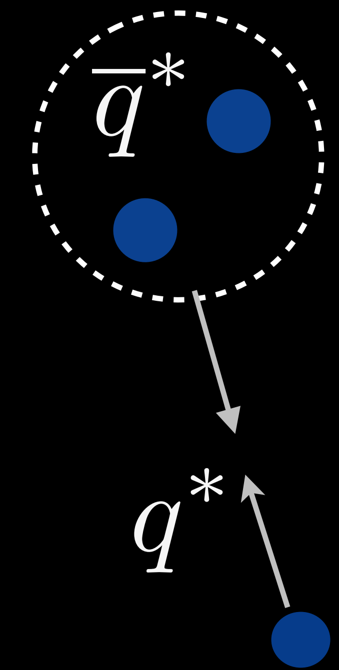
Only Luscher poles matter

(I) $$\overline{q}^*_\kappa \cot\delta_d = 4\pi\, c_{00}^{(\frac{2\mathbf{P}}{3}-q^*_\kappa)}(\overline{q}^2_\kappa)$$

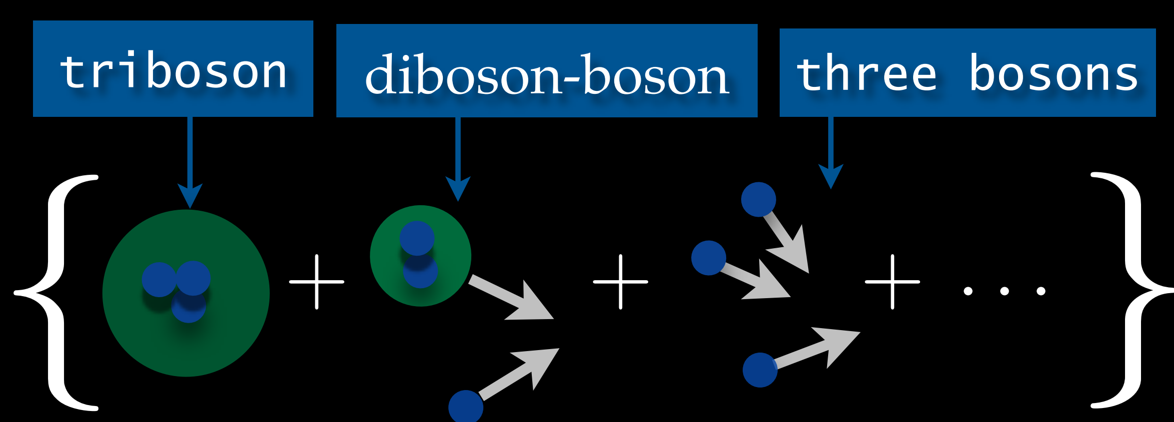
$$\overline{q}^{*2}_\kappa = mE^* - \frac{3}{4}q^{*2}_\kappa$$

COUPLED-CHANNELS

In CM frame



Three-particle states



Power-law corrections

$$\{\overline{q}^*_\kappa, q^*_\kappa\} = \left\{\left(\overline{q}^*_0, \sqrt{\frac{4}{3}(mE^* - \overline{q}^{*2}_0)}\right), \left(\overline{q}^{*2}_1, \sqrt{\frac{4}{3}(mE^* - \overline{q}^*_1)}\right), \ldots, \left(\overline{q}^*_{N_{E^*}}, \sqrt{\frac{4}{3}(mE^* - \overline{q}^{*2}_{N_{E^*}})}\right)\right\}$$

Off-shell states

$$mE^* < \frac{3}{4}q^{*2}_\kappa$$

Exponential corrections

Three-particle quantization conditions

(II) $$\mathrm{Det}(1+\tilde{\mathcal{M}}_{\mathrm{V}}^{\infty}\delta\tilde{\mathcal{G}}^{\mathrm{V}}) \equiv \mathrm{det}_{\mathrm{oc}}[\mathrm{det}_{\mathrm{pw}}(1+\tilde{\mathcal{M}}_{\mathrm{V}}^{\infty}\delta\tilde{\mathcal{G}}^{\mathrm{V}})] = 0$$

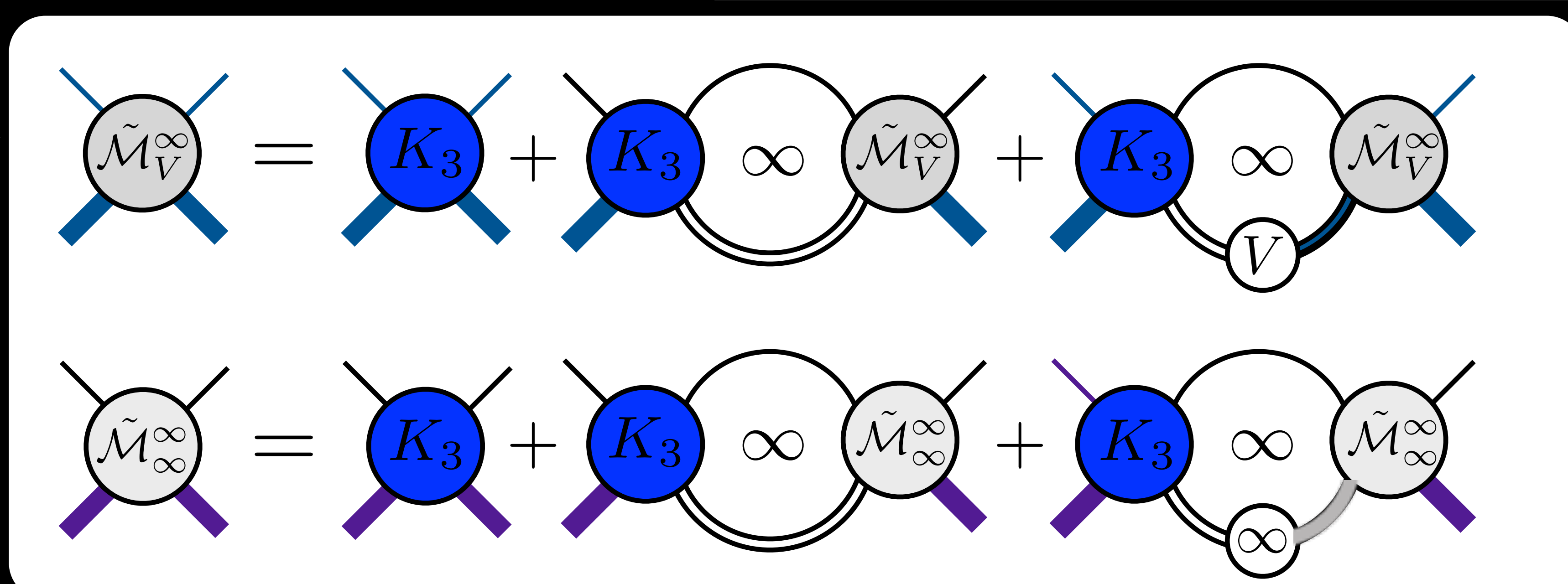
Determinant over open kinematic channels

Determinant over partial-wave channels of boson-dimer state

$$\tilde{\mathcal{M}}_{V}^{\infty}(\mathbf{p},\mathbf{k};\mathbf{P},E) = \tilde{\mathcal{M}}_{\infty}^{\infty}(\mathbf{p},\mathbf{k};\mathbf{P},E) - \int \frac{d^3q}{(2\pi)^3}\tilde{\mathcal{M}}_{\infty}^{\infty}(\mathbf{p},\mathbf{q};\mathbf{P},E)\,\delta\mathcal{D}^{V}\left(E-\frac{q^2}{2m},|\mathbf{P}-\mathbf{q}|\right)\tilde{\mathcal{M}}_{V}^{\infty}(\mathbf{q},\mathbf{k};\mathbf{P},E)$$

Diagonal in angular momentum

Mixes the three particle states



Briceno, ZD (2012).

Bound-state particle scattering- recovering Luscher

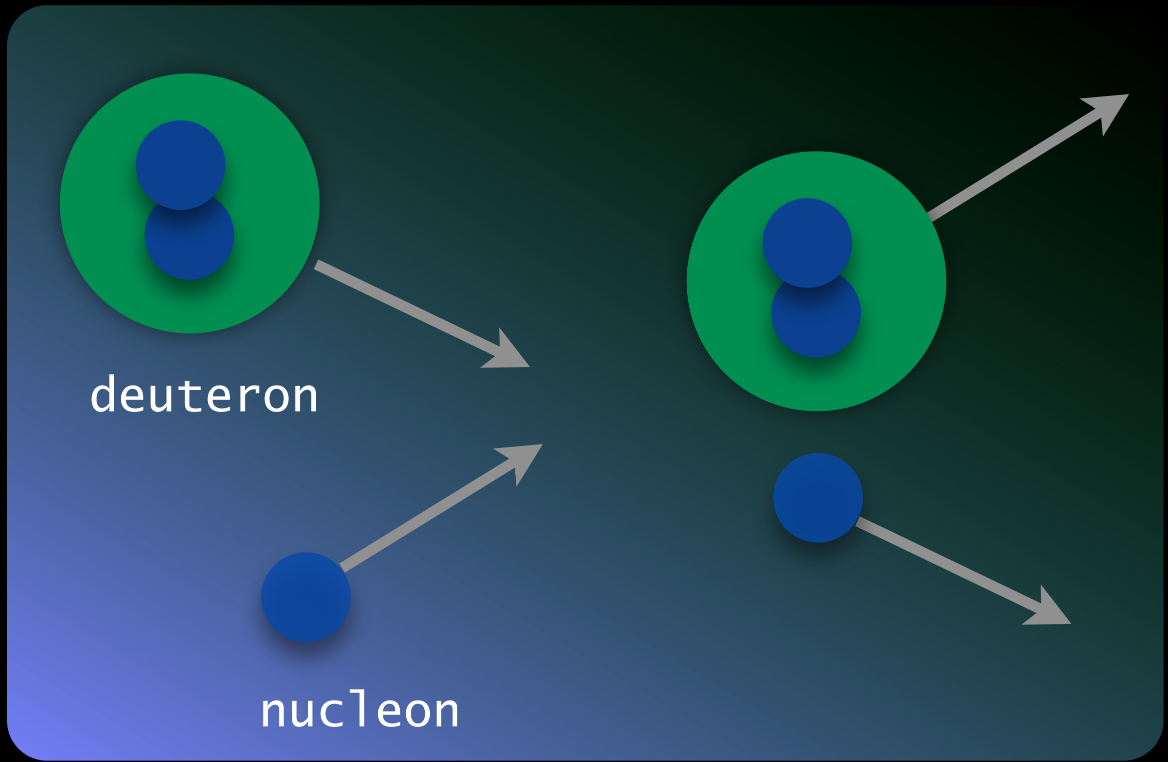
S-wave boson-diboson elastic scattering amplitude

$$\mathcal{M}_{Bd} = \frac{3\pi}{m}\frac{1}{q_0^* \cot\delta_{Bd} - iq_0^*}$$

$$\tilde{\mathcal{M}}_V^\infty \text{ vs. } \tilde{\mathcal{M}}_\infty^\infty \equiv \mathcal{M}_{Bd} \text{ ?}$$

Key: Diboson is a compact object in sufficiently large volumes

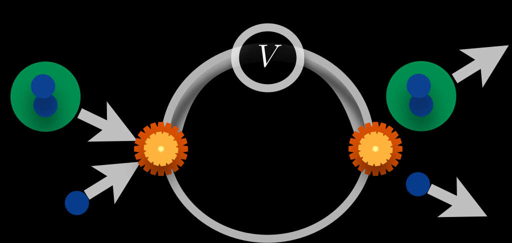
$$\underline{q}_0^* = i\gamma_d + \mathcal{O}(e^{-\gamma_d L}/L)$$

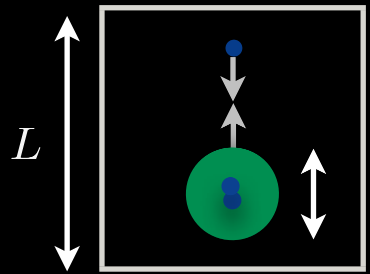


$$q_0^* \cot\delta_{Bd} = 4\pi\ c_{00}^P(q_0^*) + \eta\frac{e^{-\gamma_d L}}{L}$$

A coefficient that needs to be fit to data

$$q_0^* = \sqrt{\frac{4}{3}(mE^* - \bar{q}_0^{*2})}$$





$\sim \frac{1}{\gamma_d}$

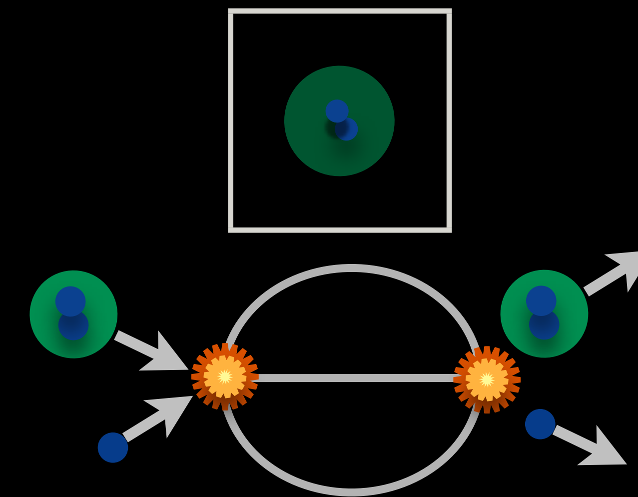
Diboson infinite volume binding momentum

Bound-state particle scattering-recovering Luscher

- Other sources of systematics to the Luscher approximation

NLO correction due to size of diboson $\mathcal{O}\left(e^{-\sqrt{2}\gamma_d L}/L\right)$

First off-shell state ignored $\mathcal{O}\left(\frac{e^{-\sqrt{\frac{4}{3}(q_1^{*2}-mE^*)}L}}{L}\right)$

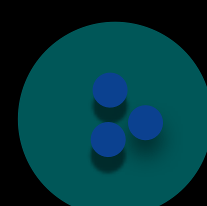


Partial-wave mixing, S-wave dimer?

$$(J_d, J_{Bd}) = \{(0,0),(2,0),(4,0),(0,4),(2,4),(2,6),\ldots\}$$
$$(J_d, J_{Bd}) = \{(0,0),(0,1),(2,0),(2,1),(0,2),(2,2),\ldots\}$$

See Raul's talk!

- Triton binding energy?

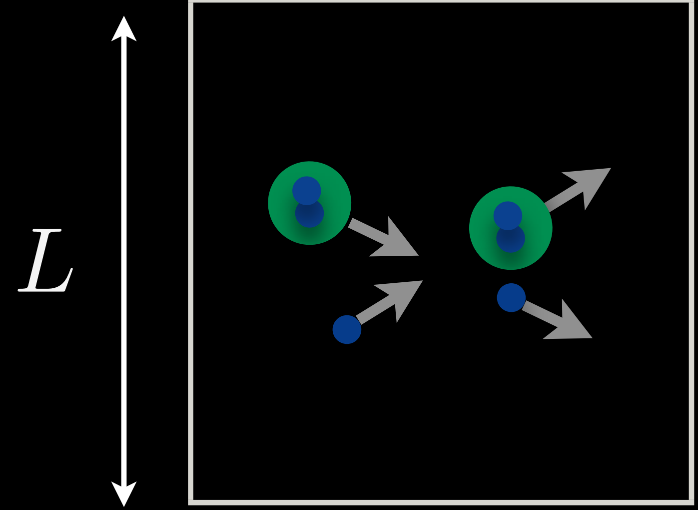


$$\gamma_{Bd} + q^*_{Bd}\cot\delta_{Bd}\big|_{q^{*2}_{Bd}=-\gamma^2_{Bd}} = \mathcal{O}(e^{-\gamma_{Bd}L})$$

How big must the volumes be?

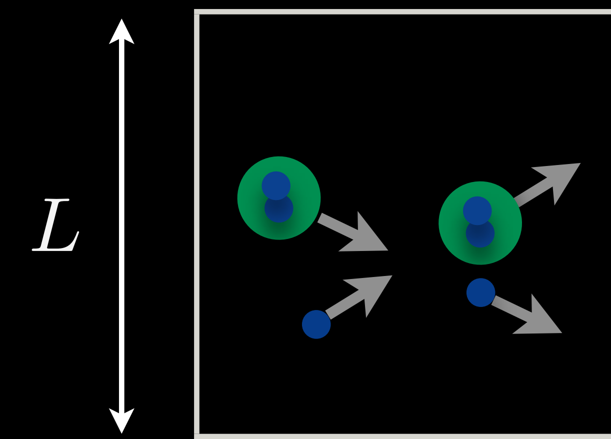
A few percent determination of phase-shifts for dN scattering?

Physical pion mass



$L \gtrsim 17$ fm

Presumably there exists a trick to eliminate LO corrections!



$L \gtrsim 12$ fm

Two-body evidence:

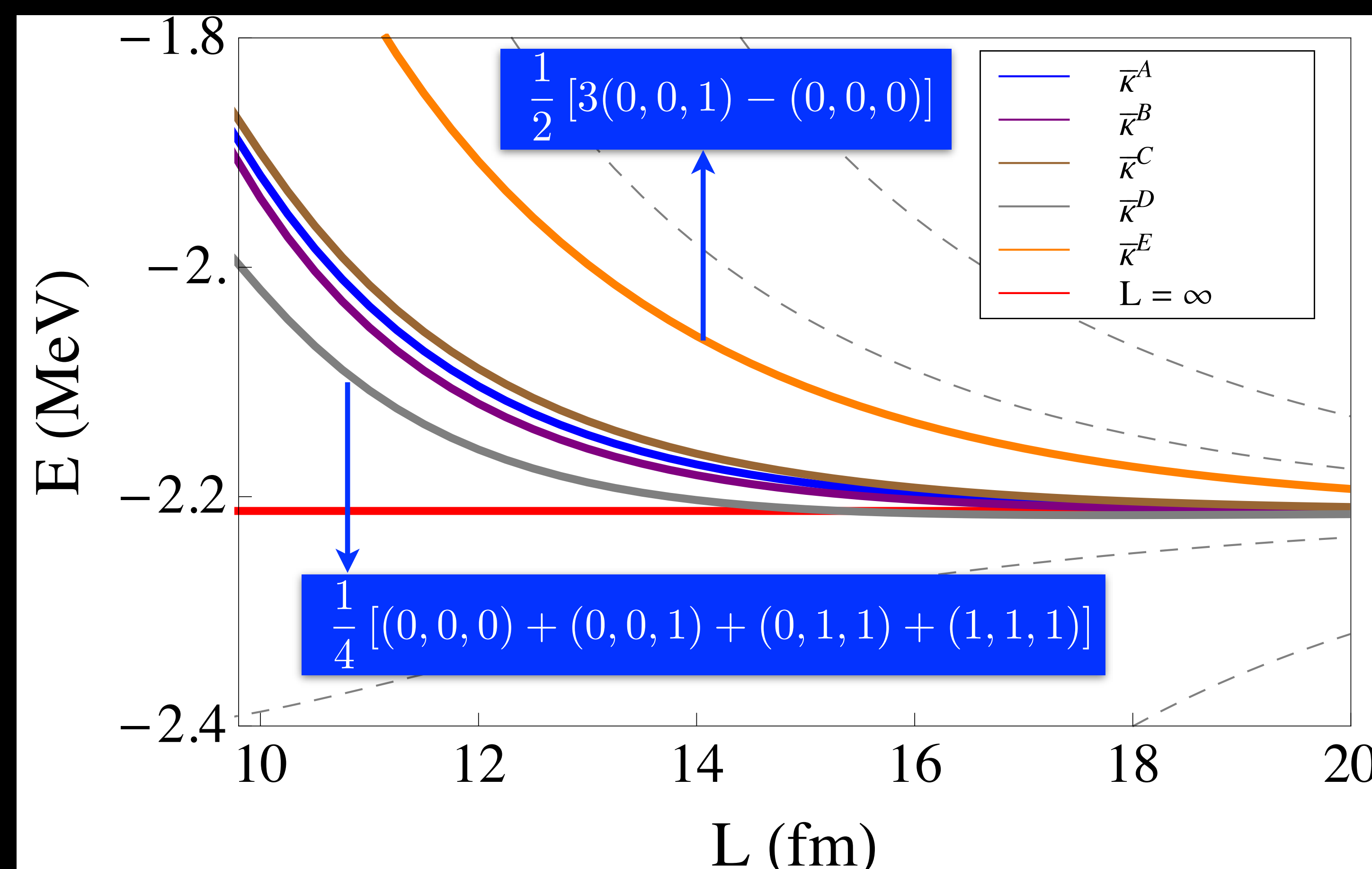
Do the calculation with different boosts

Form suitable linear combinations

Three-body problem?

Requires more extensive numerical work!

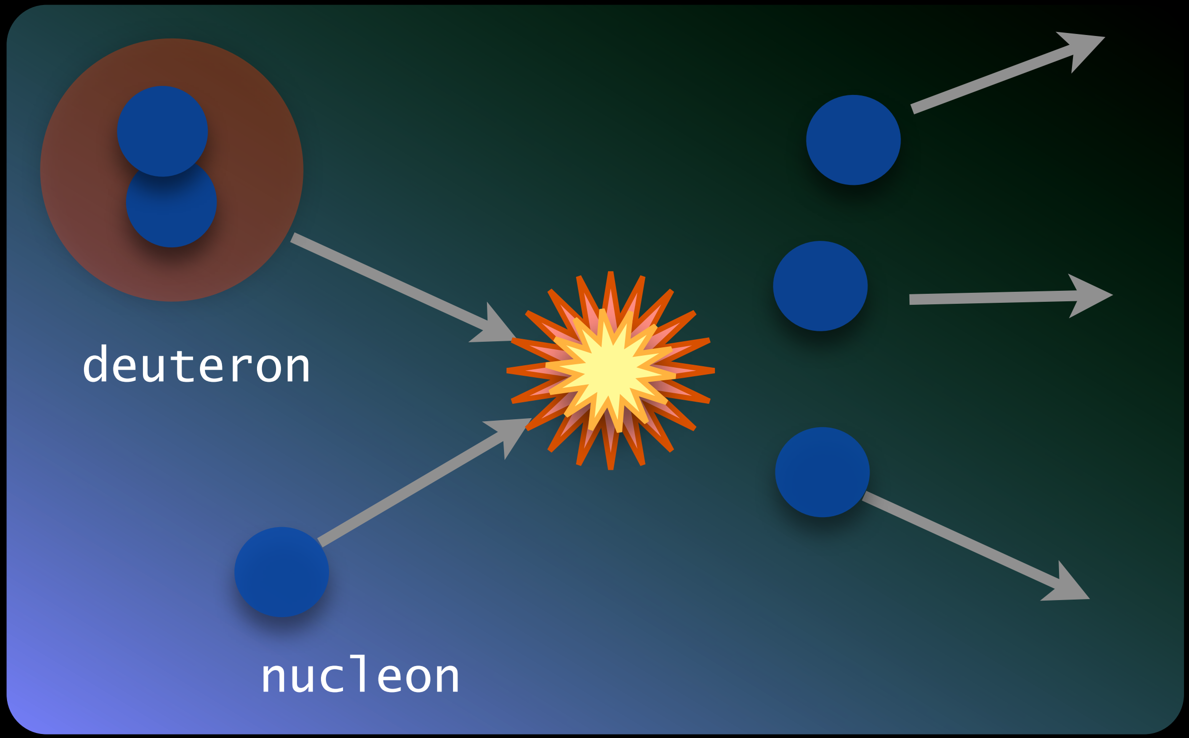
ZD, Savage (2011).



Recombination and breakup processes?

Just above the threshold

$$\left\{\left(\overset{\boxed{1}}{\bar{q}_0^*, \sqrt{\frac{4}{3}(mE^* - \bar{q}_0^{*2})}}\right), \left(\overset{\boxed{2}}{\bar{q}_1^{*2}, \sqrt{\frac{4}{3}(mE^* - \bar{q}_1^*)}}\right)\right\}$$



A coupled-channels problem

$$(1 + \tilde{\mathcal{M}}^{\infty}_{V,Bd-Bd}\,\delta\tilde{\mathcal{G}}^{V}_{Bd})(1 + \tilde{\mathcal{M}}^{\infty}_{V,BBB-BBB}\,\delta\tilde{\mathcal{G}}^{V}_{BBB}) = |\tilde{\mathcal{M}}^{\infty}_{V,Bd-BBB}|^2\,\delta\tilde{\mathcal{G}}^{V}_{Bd}\,\delta\tilde{\mathcal{G}}^{V}_{BBB}$$

Relate to physical scattering amplitudes through an integral equation

NO ALGEBRAIC APPROXIMATION ABOVE BREAKUP

Hansen, Sharpe (2012); Briceno, ZD (2012).

Briceno, ZD (2012).

Implementation

(I) Determine two-body scattering parameters well

Obtain the Luscher poles as a function of two-particle boost momentum and energy $\overline{q}^*_\kappa$

(II) Calculate three-body spectrum well

$$E^*, \overline{q}^*_\kappa \rightarrow q^*_\kappa$$

(III) Numerically solve the quantization condition for three-body kernel.

Summary and conclusion

- The spectrum of three bosons in FV is related to three-particle scattering amplitudes through an integral equation.
- In theories with two-body bound states a simple Luscher formula exists. The phase shifts should be extrapolated to infinite volume limit.
- The problem is in general a coupled-channel problem - more than one kinematic channels contributiong
- Sources of systematics include the presence of nearby off-shell states + FV partial-wave mixing
- Implementation is an extensive numerical effort.

Future work and open questions

- Nuclear sector? Generalized dimer formalism? See Raul's Talk.
- Study volume dependence of Triton and dN scattering numerically
- Three-body problem without dimer? See Max's talk.
- To determine scattering amplitudes without any reference to the EFT?
- Breakup recombination processes? Coupled-channel analysis

THANKS !