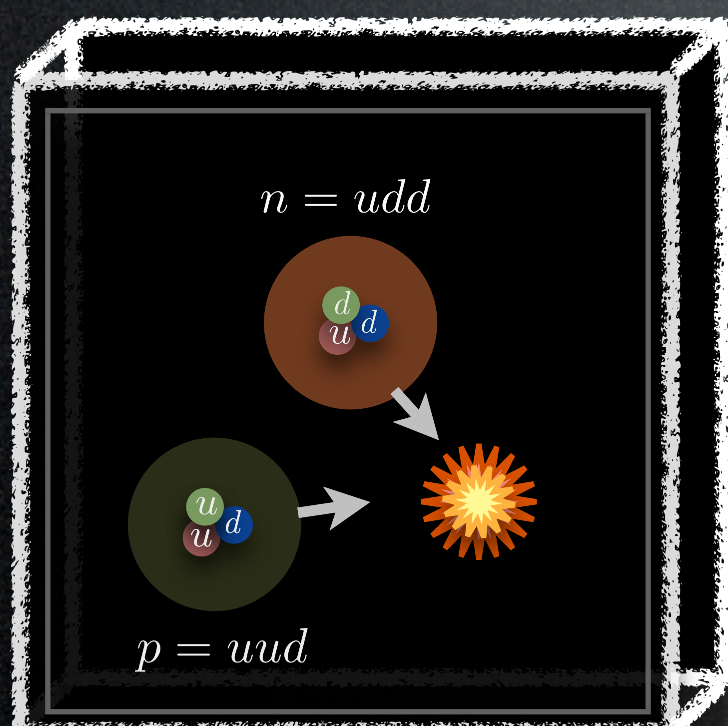
INT-13-53W
March 12, 2012

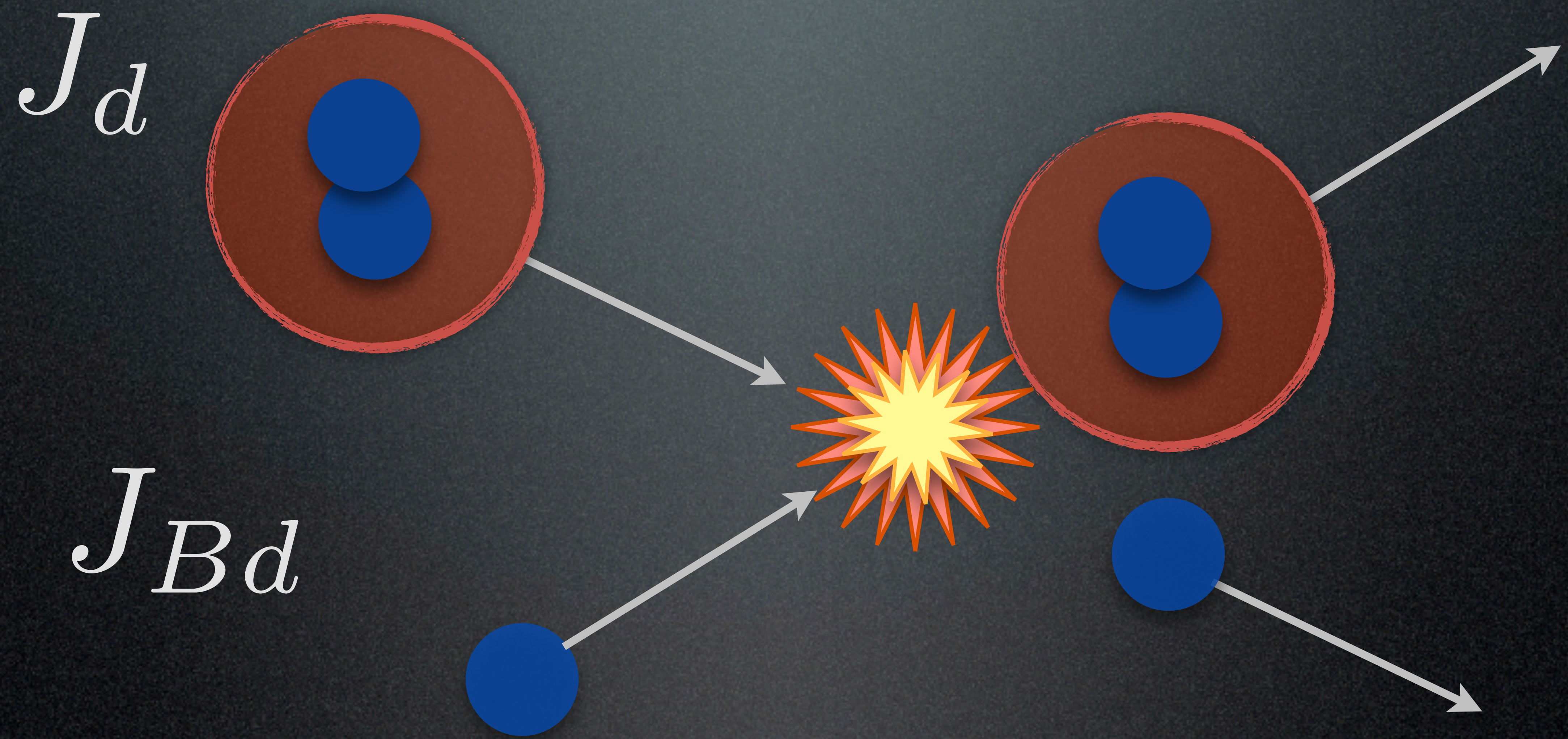
Nuclear physics in a box

the devil is in the details

Raúl Briceño
(in collaboration with Zohreh Davoudi)

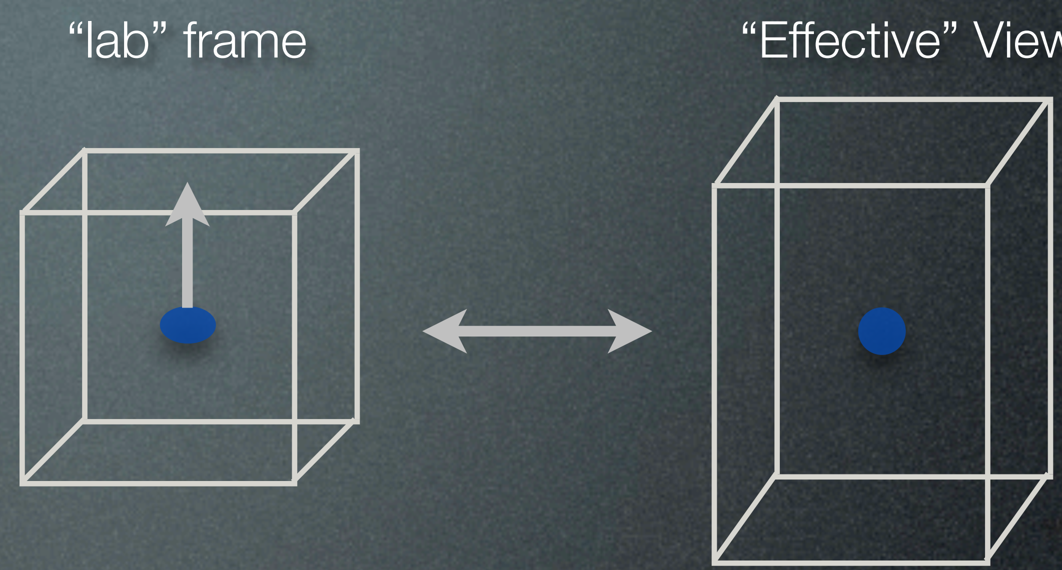


Three-Body Sector: Partial Waves & Mixing

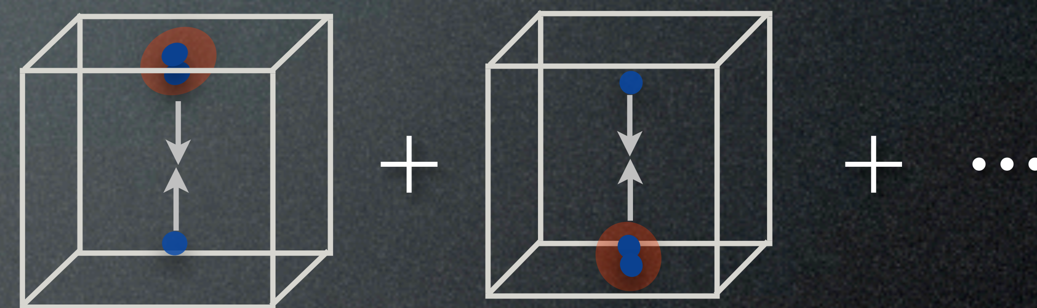


Boosts

Symmetry is reduced:



Boson-diboson CM:

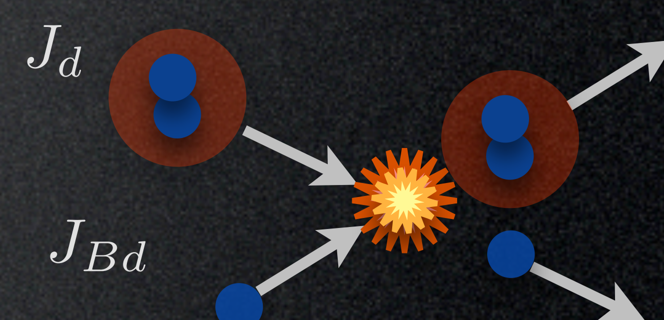


- diboson is boosted: $J_d = \{0, 2, 4, \ldots\}$
- Bd is unboosted: $J_{Bd} = \{0, 4, 6, \ldots\}$

Must generalize dimer formalism

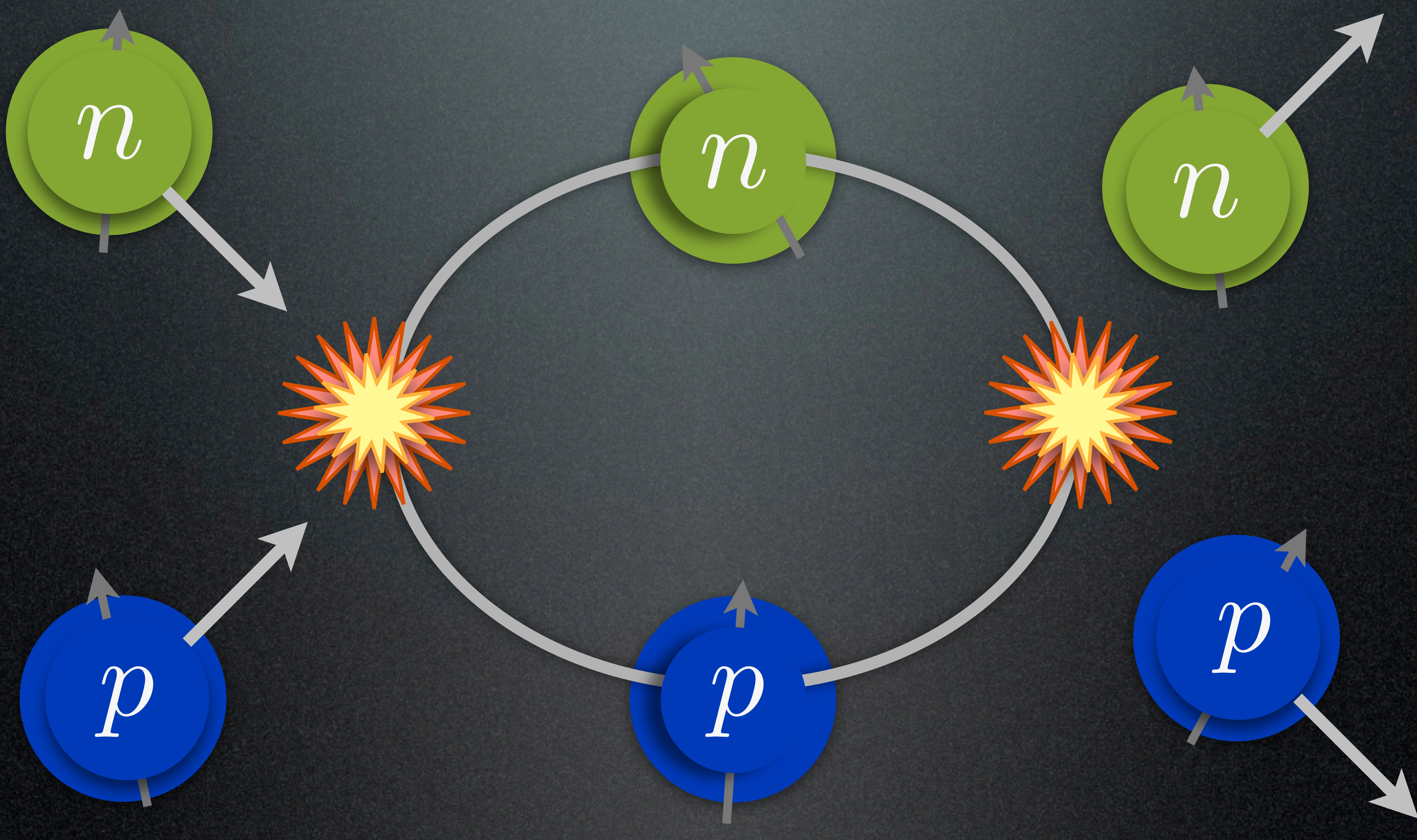
Boson-diboson Boosted:

- $J_{Bd} = \{0, 1, 2, \ldots\}$



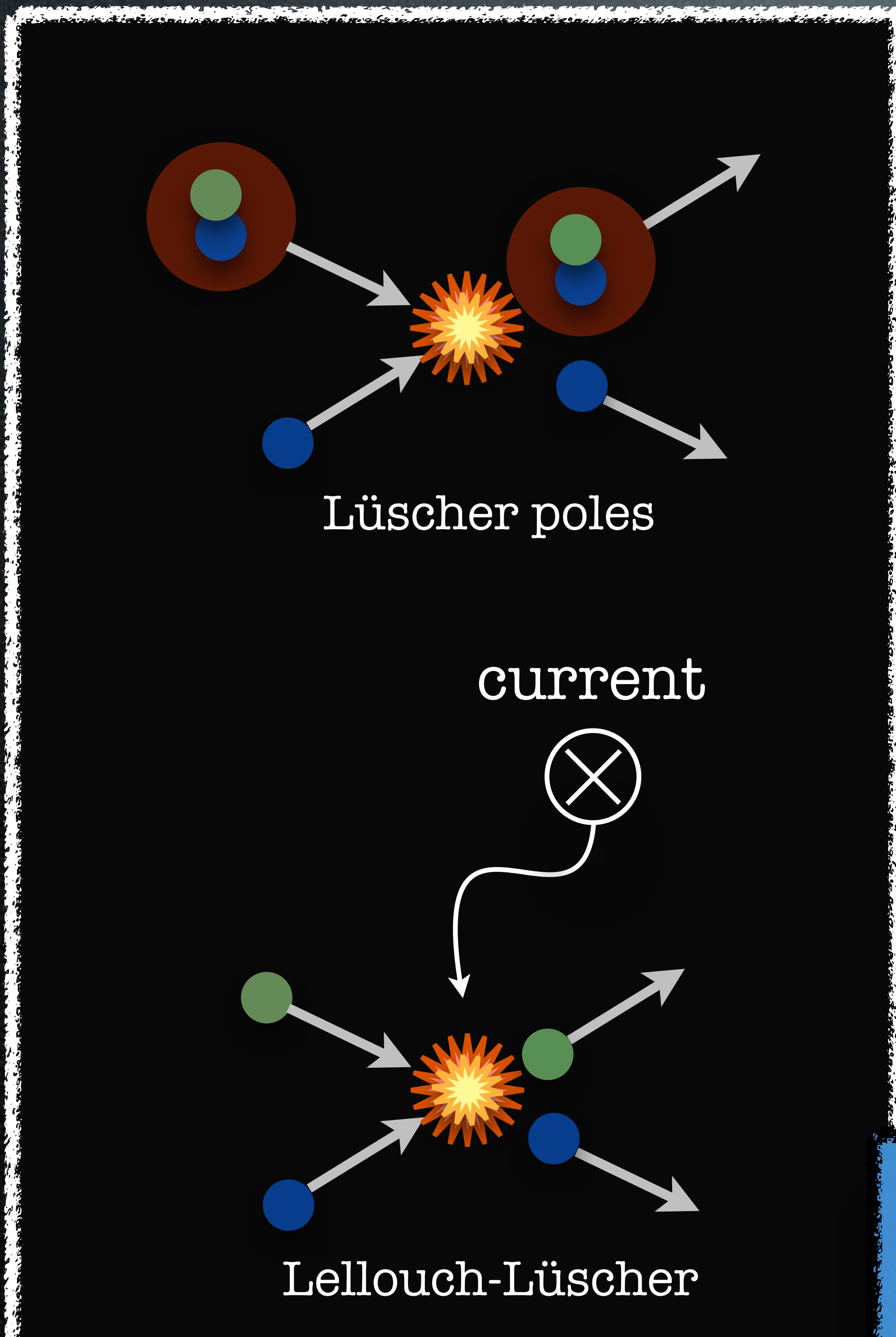
Bour et al. (2011), Davoudi & Savage (2011), Fu (2012): Boosted two-particle system with different masses

NN-Scattering

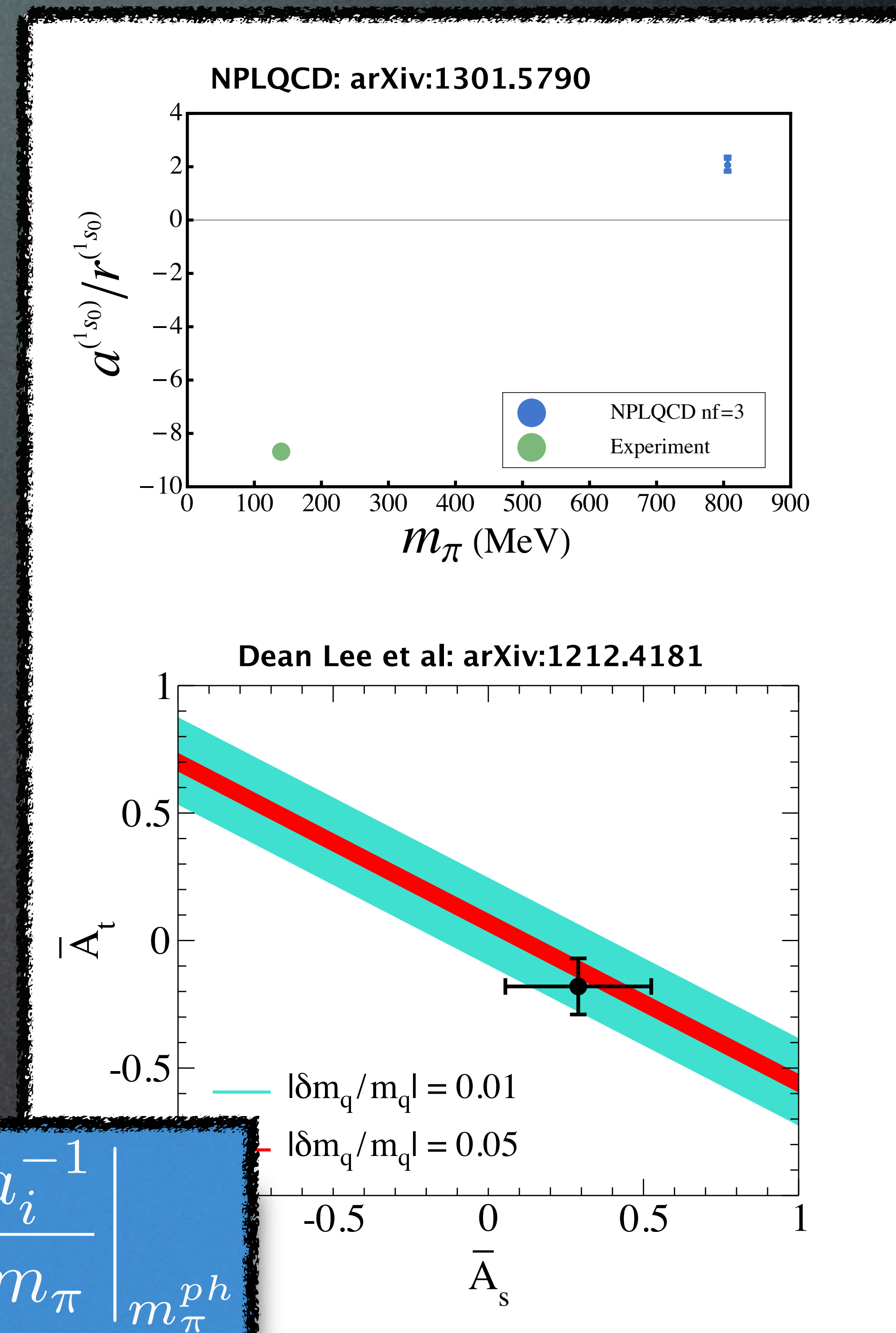


Why NN?

Pragmatic issues



Issues of Insight

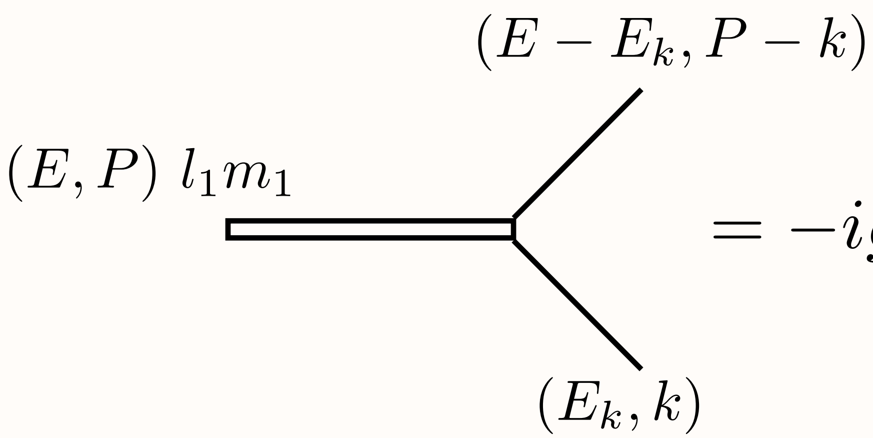


$$\bar{A}_i \equiv \left.\frac{\partial a_i^{-1}}{\partial m_\pi}\right|_{m_\pi^{ph}}$$

Dimer Formalism: Scalar Sector

spherical dimer

$$= \frac{-i\delta_{l_1 l_2}\delta_{m_1 m_2}}{E - \frac{\mathbf{P}^2}{4m} - \Delta_{l_1} + \sum_{n=1} c_{l_1,n}\left(E - \frac{\mathbf{P}^2}{4m}\right)^{n+1}}$$

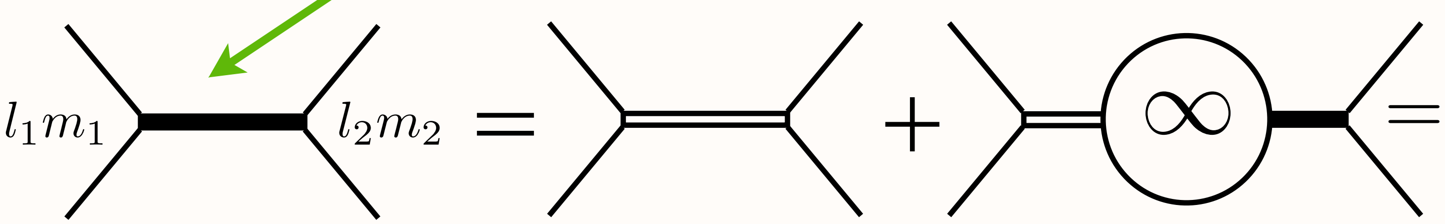


$$= -i g_{2,l_1}\sqrt{4\pi}\, Y^*_{l_1 m_1}(k^*)\ k^{*l_1}\ \delta_{1,(-1)^{l_1}}$$

$k^* \equiv k - P/2$

Galilean invariance

fully dressed dimer



$$= \frac{i\delta_{l_1 l_2}\delta_{m_1 m_2}\ k^{*2l_1}}{k^{*2l_1+1}\cot\delta_d^{(l_1)} - i k^{*2l_1+1}}$$

ERE: $k^{*2l+1}\cot\delta_d^{(l)} = -\frac{1}{a_l} + \frac{r_l k^{*2}}{2} + \sum_{n=2}^{\infty}\frac{\rho_{n,l}}{2n!}\ k^{*2n}$

Finite volume result agrees with Rummukainen & Gottlieb, and Kim, Sharpe & Sachrajda

Nuclear Sector: Infinite Volume

Complicated by physical mixing

[e.g. positive parity, iso-singlet sector, J<4]

$$\mathcal{M}^{\infty,\pi=+}_{S=1,I=0} = \begin{pmatrix} \mathcal{M}^{\infty}_{J=1} & 0 & 0 \\ 0 & \mathcal{M}^{\infty}_{J=2} & 0 \\ 0 & 0 & \mathcal{M}^{\infty}_{J=3} \end{pmatrix}$$

~~25 x 25~~

18 x 18

$$[\mathcal{M}^{\infty}_{J=1}]_{m_j m_{j'}} = \begin{pmatrix} \mathcal{M}^{S}_{1m_j;1m_{j'}} & \mathcal{M}^{SD}_{1m_j;1m_{j'}} \\ \mathcal{M}^{DS}_{1m_j;1m_{j'}} & \mathcal{M}^{D}_{1m_j;1m_{j'}} \end{pmatrix}$$

$$[\mathcal{M}^{\infty}_{J=2}]_{m_j m_{j'}} = \mathcal{M}^{D}_{1m_j;1m_{j'}}$$

$$[\mathcal{M}^{\infty}_{J=3}]_{m_j m_{j'}} = \begin{pmatrix} \mathcal{M}^{D}_{1m_j;1m_{j'}} & \mathcal{M}^{DG}_{1m_j;1m_{j'}} \\ \mathcal{M}^{DG}_{1m_j;1m_{j'}} & \mathcal{M}^{G}_{1m_j;1m_{j'}} \end{pmatrix}$$

Rich structure MUST be reflected in the dimer formalism

Dimer Formalism: Nuclear Sector

$$d^{lS}_{\nu_J \nu_I, P} = \sum_{m_l, m_s} \langle \nu_J | \nu_l \nu_S \rangle \; d_{\nu_l \nu_S \nu_I, P}$$

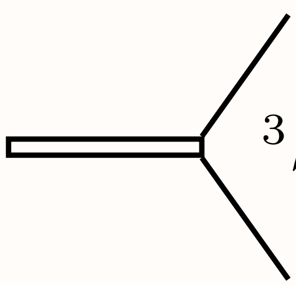
Dimer operator with (J,I) quantum numbers

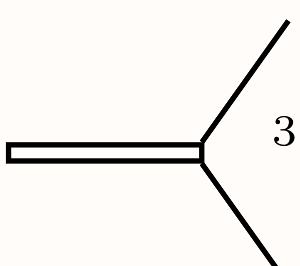
mixes different (l,S) states

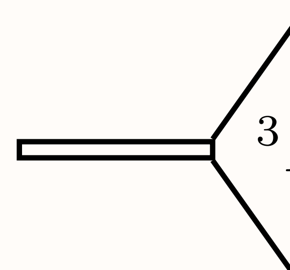
e.g. $^3S_1 \longleftrightarrow {}^3D_1$

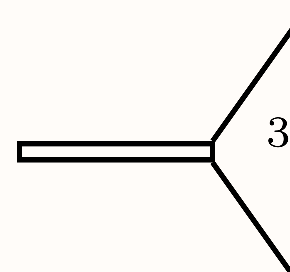
convenient notation: $|\nu_X\rangle \equiv |X m_X\rangle$

Dimer operator in the (l,S,I) - basis

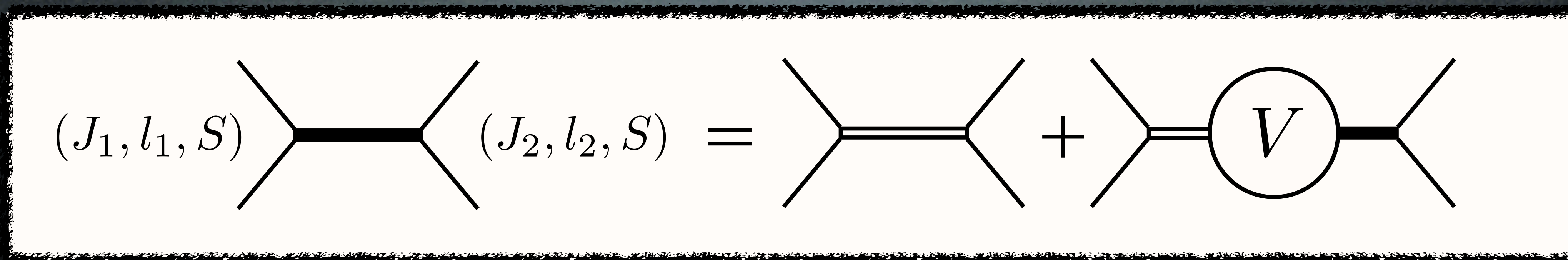
$^3S_1$  $^3S_1 \sim g^{01}_{10} \langle \nu_0 \nu_S | \nu_1 \rangle$

$^3D_1$  $^3S_1 \sim \alpha_{10} \langle \nu_0 \nu_S | \nu_1 \rangle$

$^3S_1$  $^3D_1 \sim \alpha_{10} Y_{2m_2}(\hat{k}^*) k^{*2} \langle \nu_2 \nu_S | \nu_1 \rangle$

$^3D_1$  $^3D_1 \sim g^{21}_{10} Y_{2m_2}(\hat{k}^*) k^{*2} \langle \nu_2 \nu_S | \nu_1 \rangle$

Quantization Condition



independent of dimer formalism

Holds for all partial waves, parity & boosts

$$\det\left(\left(\mathcal{M}^{\infty}\right)^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

Scattering amplitude
Diagonal in J_2-basis
mixes l states

Kinematic function of (L, E_L)
Mixes angular momentum

Quantization Condition

sanity check

$$\det\left(\left(\mathcal{M}^{\infty}\right)^{-1} + \delta\mathcal{G}^{V}\right) = 0$$

$$i\left[\delta\mathcal{G}^{V}\right]_{\nu_J l;\nu_{J'} l';SI} = -im\frac{P^{*l'+l}}{4\pi}\sum_{\substack{m_S,\\ m_l,m_{l'}}}\langle\nu_l\nu_S|\nu_J\rangle\langle\nu_{J'}|\nu_{l'}\nu_S\rangle\delta_{(-1)^{l+1-I-S},1}\left[F^P(P^{*2}) - iP^*\right]_{\nu_l,\nu_{l'}}$$

S = 0 limit

Agreement with
Gottlieb & Rummukainen
Kim, Sachrajda, Sharpe

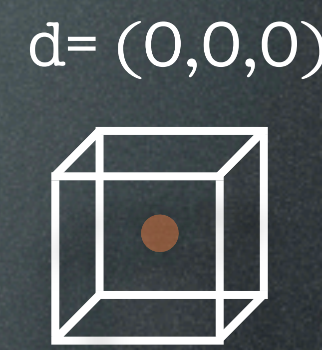
S = 1/2 limit

Agreement with Göckeler et al.

P=0 case by N. Ishizuka, proceeding

Boosts & Symmetry (I)

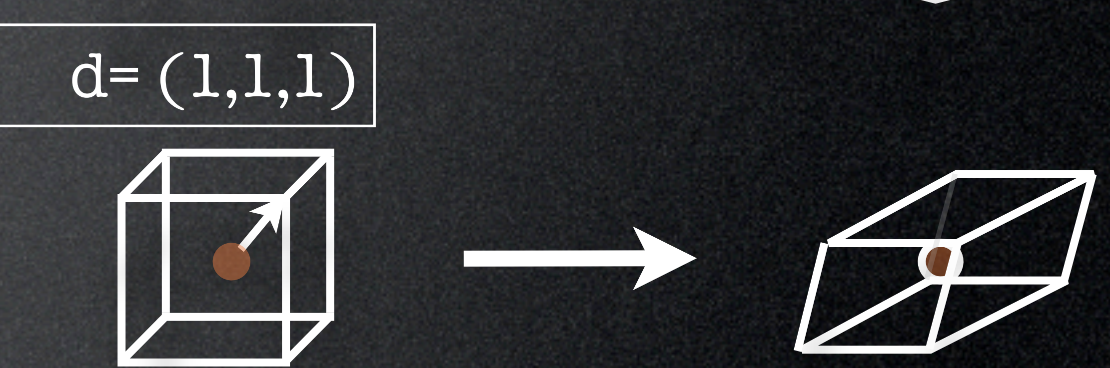
- O_h : Octahedral



- D_{4h} : Tetragonal

- D_{2h} : Orthorhombic

- D_{3h} : Trigonal



Boosts & Symmetry (II)

$$c^P_{\nu_{l''}}(x) = \left[\frac{1}{L^3} \sum_{\mathbf{k}} - \mathcal{P} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \right] \frac{\sqrt{4\pi} Y_{\nu_{l''}}(\hat{k}^*) k^{*l''}}{k^{*2} - x}$$

- Rotations: $c^P_{lm_l} = \sum_{m_{l'} = \{-l, l\}} \mathcal{D}^{(l)}_{m_l, m'_l}(R\chi)\, c^P_{lm'_l}$
- Parity + Isospin limit: $c^P_{lm_l} = (-1)^l c^P_{lm_l}$

e.g. 24 relations for O_h

- Examples [J < 4]:
  - O_h: $c^P_{00},\ c^P_{44} = \sqrt{\frac{5}{14}} c^P_{40},\ \text{else } c^P_{lm_l} = 0$
  - D_{2h}: $c^P_{lm_l} = 0, \quad \text{if } l \text{ or } m_l \text{ are odd}$

    $$c^P_{lm_l} = (-1)^{m_l/2}\ c^P_{l-m_l}$$

Lüscher (1990)

Cubic NN-Propagator

Evaluate determinant of $\left[\left(\mathcal{M}^{\infty}\right)^{-1}+\delta\mathcal{G}^{V}\right]_{\mathcal{O}_h}$

SD

$\left(\mathcal{M}^{\infty}\right)^{-1}$

$c_{lm} \sim Z_{lm}$

$$\begin{pmatrix}
b2 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z1 & 0 & 0 & 0 & z2 \\
0 & m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z3 & 0 & 0 & 0 \\
0 & 0 & m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & z1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & b3 & 0 & 0 & 0 & z5 & 0 & z6 & 0 & 0 & 0 & z6 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & 0 & 0 & 0 & z8 & 0 & 0 & 0 & z9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & -z9 & 0 & 0 & 0 & -z8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & z5 & 0 & 0 & 0 & b3 & 0 & -z6 & 0 & 0 & 0 & -z6 & 0 \\
0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & -z9 & 0 & b6 & 0 & 0 & 0 & z4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & z6 & 0 & 0 & 0 & -z6 & 0 & b7 & 0 & 0 & 0 & z10 & 0 \\
0 & 0 & 0 & z1 & 0 & 0 & 0 & z8 & 0 & 0 & 0 & 0 & 0 & b8 & 0 & 0 & 0 & z4 \\
0 & 0 & 0 & 0 & z3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & z1 & 0 & 0 & 0 & -z8 & 0 & z4 & 0 & 0 & 0 & b8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & z6 & 0 & 0 & 0 & -z6 & 0 & z10 & 0 & 0 & 0 & b7 & 0 \\
0 & 0 & 0 & z2 & 0 & 0 & 0 & z9 & 0 & 0 & 0 & 0 & 0 & z4 & 0 & 0 & 0 & b60
\end{pmatrix}$$

Cubic NN-Propagator

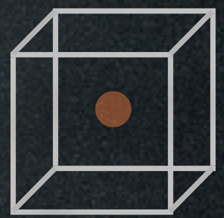
Unitary transformation: $S\left[\delta G^V\right]S^\dagger$

Diagonal blocks are diagonalized:

à la mode de Tom & Martin (2011)

$$
\begin{pmatrix}
\mathrm{b1} & 0 & 0 & \mathrm{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathrm{b1} & 0 & 0 & \mathrm{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathrm{b1} & 0 & 0 & \mathrm{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathrm{m1} & 0 & 0 & \mathrm{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{z1} & 0 & 0 \\
0 & \mathrm{m1} & 0 & 0 & \mathrm{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{z1} \\
0 & 0 & \mathrm{m1} & 0 & 0 & \mathrm{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{z1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b4} & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b4} & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b4} & 0 & 0 & -\mathrm{z2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 & 0 & 0 & \mathrm{b7} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{z2} & 0 & 0 & \mathrm{b7} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 & 0 & 0 & \mathrm{b7} & 0 & 0 & 0 \\
0 & 0 & 0 & -\mathrm{z1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b8} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mathrm{z1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b8} & 0 \\
0 & 0 & 0 & 0 & \mathrm{z1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b8}
\end{pmatrix}
$$

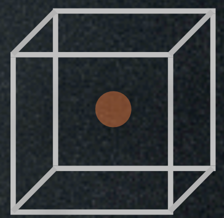
A_2-Irrep



$$\frac{1}{\mathcal{M}^{D}_{J=3}} + \frac{imp}{4\pi} = mc_{00} - 4\sqrt{\frac{2}{35}}\,\frac{mc_{44}}{p^4}$$

$$\begin{pmatrix}
b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z1 & 0 & 0 \\
0 & m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z1 \\
0 & 0 & m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & -z2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & 0 & 0 & b7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z2 & 0 & 0 & b7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & 0 & 0 & b7 & 0 & 0 & 0 \\
0 & 0 & 0 & -z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b8 & 0 \\
0 & 0 & 0 & 0 & z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b8
\end{pmatrix}$$

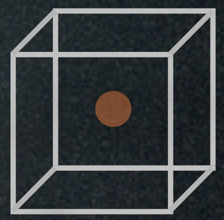
E-Irrep



$$\left(\frac{1}{\mathcal{M}^{D}_{J=2}}+\frac{imp}{4\pi}-mc_{00}+4\sqrt{\frac{2}{35}\frac{mc_{44}}{p^4}}\right)^{②}=0$$

$$\begin{pmatrix}
b1&0&0&m1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&b1&0&0&m1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&b1&0&0&m1&0&0&0&0&0&0&0&0&0&0&0&0\\
m1&0&0&b2&0&0&0&0&0&0&0&0&0&0&0&-z1&0&0\\
0&m1&0&0&b2&0&0&0&0&0&0&0&0&0&0&0&0&z1\\
0&0&m1&0&0&b2&0&0&0&0&0&0&0&0&0&0&-z1&0\\
0&0&0&0&0&0&b4&0&0&0&0&0&z2&0&0&0&0&0\\
0&0&0&0&0&0&0&b5&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&b4&0&0&0&0&0&z2&0&0&0\\
0&0&0&0&0&0&0&0&0&b5&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&b4&0&0&-z2&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&b6&0&0&0&0&0&0\\
0&0&0&0&0&0&z2&0&0&0&0&0&b7&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-z2&0&0&b7&0&0&0&0\\
0&0&0&0&0&0&0&0&z2&0&0&0&0&0&b7&0&0&0\\
0&0&0&-z1&0&0&0&0&0&0&0&0&0&0&0&b8&0&0\\
0&0&0&0&0&-z1&0&0&0&0&0&0&0&0&0&0&b8&0\\
0&0&0&0&z1&0&0&0&0&0&0&0&0&0&0&0&0&b8
\end{pmatrix}$$

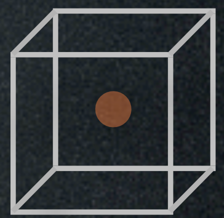
T_2-Irrep



$$\det\begin{pmatrix} \frac{1}{\mathcal{M}^D_{J=2}} + \frac{imp}{4\pi} - mc_{00} - \frac{8}{3}\sqrt{\frac{2}{35}}\frac{mc_{44}}{p^4} & \frac{4}{3}\sqrt{\frac{5}{7}}\frac{mc_{44}}{p^4} \\ \frac{4}{3}\sqrt{\frac{5}{7}}\frac{mc_{44}}{p^4} & \frac{1}{\mathcal{M}^D_{J=3}} + \frac{imp}{4\pi} - mc_{00} + \frac{2}{3}\sqrt{\frac{2}{35}}\frac{mc_{44}}{p^4} \end{pmatrix}^{3} = 0$$

$$\begin{pmatrix}
b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b1 & 0 & 0 & m1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z1 & 0 & 0 \\
0 & m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z1 \\
0 & 0 & m1 & 0 & 0 & b2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b4 & 0 & 0 & -z2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & 0 & 0 & b7 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z2 & 0 & 0 & b7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & z2 & 0 & 0 & 0 & 0 & 0 & b7 & 0 & 0 & 0 \\
0 & 0 & 0 & -z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b8 & 0 \\
0 & 0 & 0 & 0 & z1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b8
\end{pmatrix}$$

T_1-Irrep

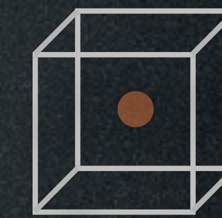


3

$$\det\begin{pmatrix} \frac{\mathcal{M}^{D}_{J=1}}{\det \mathcal{M}_{J=1}} + \frac{imp}{4\pi} - mc_{00} & \frac{\mathcal{M}^{SD}_{J=1}}{-\det \mathcal{M}_{J=1}} & 0 \\ \frac{\mathcal{M}^{SD}_{J=1}}{-\det \mathcal{M}_{J=1}} & \frac{\mathcal{M}^{S}_{J=1}}{\det \mathcal{M}_{J=1}} + \frac{imp}{4\pi} - mc_{00} & 4\sqrt{\frac{3}{35}}\frac{mc_{44}}{p^4} \\ 0 & 4\sqrt{\frac{3}{35}}\frac{mc_{44}}{p^4} & \frac{1}{\mathcal{M}^{D}_{J=3}} + \frac{imp}{4\pi} - mc_{00} - \frac{2}{3}\sqrt{\frac{2}{35}}\frac{mc_{44}}{p^4} \end{pmatrix} = 0$$

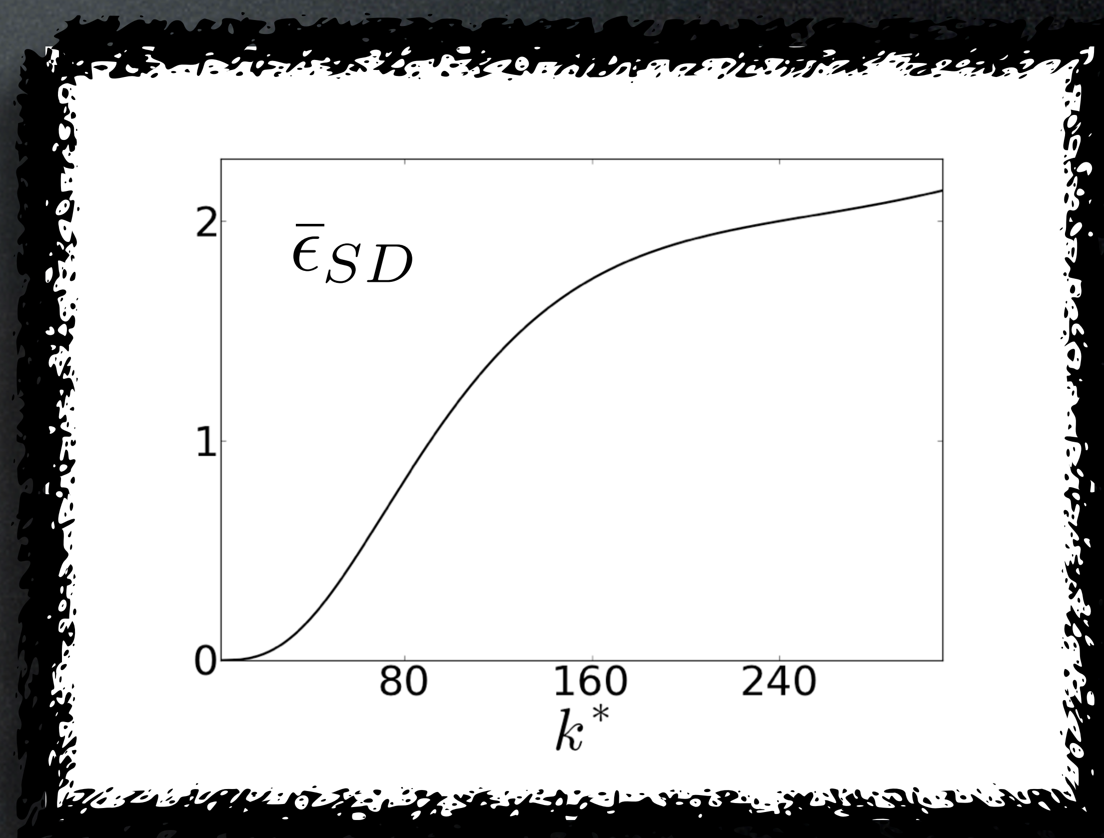
$$\begin{pmatrix}
\mathrm{b1} & 0 & 0 & \mathrm{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mathrm{b1} & 0 & 0 & \mathrm{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathrm{b1} & 0 & 0 & \mathrm{m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathrm{m1} & 0 & 0 & \mathrm{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{z1} & 0 & 0 \\
0 & \mathrm{m1} & 0 & 0 & \mathrm{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{z1} \\
0 & 0 & \mathrm{m1} & 0 & 0 & \mathrm{b2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{z1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b4} & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b4} & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b4} & 0 & 0 & -\mathrm{z2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 & 0 & 0 & \mathrm{b7} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{z2} & 0 & 0 & \mathrm{b7} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{z2} & 0 & 0 & 0 & 0 & 0 & \mathrm{b7} & 0 & 0 & 0 \\
0 & 0 & 0 & -\mathrm{z1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b8} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mathrm{z1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b8} & 0 \\
0 & 0 & 0 & 0 & \mathrm{z1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathrm{b8}
\end{pmatrix}$$

T_1-Irrep

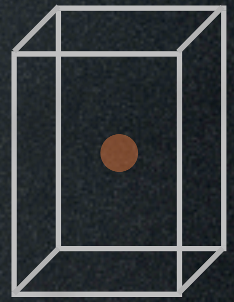


$$\det\left[\begin{pmatrix} \frac{\mathcal{M}^{D}_{J=1}}{\det\mathcal{M}_{J=1}}+\frac{imp}{4\pi}-mc_{00} & \frac{\mathcal{M}^{SD}_{J=1}}{-\det\mathcal{M}_{J=1}} & 0 \\ \frac{\mathcal{M}^{SD}_{J=1}}{-\det\mathcal{M}_{J=1}} & \frac{\mathcal{M}^{S}_{J=1}}{\det\mathcal{M}_{J=1}}+\frac{imp}{4\pi}-mc_{00} & 4\sqrt{\frac{3}{35}}\frac{mc_{44}}{p^4} \\ 0 & 4\sqrt{\frac{3}{35}}\frac{mc_{44}}{p^4} & \frac{1}{\mathcal{M}^{D}_{J=3}}+\frac{imp}{4\pi}-mc_{00}-\frac{2}{3}\sqrt{\frac{2}{35}}\frac{mc_{44}}{p^4} \end{pmatrix}\right]=0$$

- Free states split
- In CM the SD-mixing $\sim\sin(2\bar{\epsilon}_{SD})$
- Nearly uncoupled at physical point
- Unphysical m_π ?



Tetragonal



- Five irreps
- Two irreps with S-wave: A_2, E
- Quantization condition for A_2-irrep:

$$\det\begin{pmatrix} -\frac{2c_{20}m}{\sqrt{5}p^2}+\frac{ipm}{4\pi}-mc_{00}+\frac{2\mathcal{M}^S_{J=1}+2\sqrt{2}\mathcal{M}^{SD}_{J=1}+\mathcal{M}^D_{J=1}}{3\det\mathcal{M}_{J=1}} & -\frac{-\sqrt{2}\mathcal{M}^S_{J=1}+\mathcal{M}^{SD}_{J=1}+\sqrt{2}\mathcal{M}^D_{J=1}}{3\det\mathcal{M}_{J=1}} & \frac{9c_{20}m}{7\sqrt{5}p^2}+\frac{4c_{40}m}{7p^4} \\ -\frac{-\sqrt{2}\mathcal{M}^S_{J=1}+\mathcal{M}^{SD}_{J=1}+\sqrt{2}\mathcal{M}^D_{J=1}}{3\det\mathcal{M}_{J=1}} & \frac{2c_{20}m}{\sqrt{5}p^2}+\frac{ipm}{4\pi}-mc_{00}+\frac{\mathcal{M}^S_{J=1}-2\sqrt{2}\mathcal{M}^{SD}_{J=1}+2\mathcal{M}^D_{J=1}}{3\det\mathcal{M}_{J=1}} & \frac{2\sqrt{2}c_{40}m}{7p^4}-\frac{6\sqrt{\frac{2}{5}}c_{20}m}{7p^2} \\ \frac{9c_{20}m}{7\sqrt{5}p^2}+\frac{4c_{40}m}{7p^4} & \frac{2\sqrt{2}c_{40}m}{7p^4}-\frac{6\sqrt{\frac{2}{5}}c_{20}m}{7p^2} & -\frac{8c_{20}m}{7\sqrt{5}p^2}-\frac{2c_{40}m}{7p^4}+\frac{ipm}{4\pi}-mc_{00}+\frac{1}{\mathcal{M}^D_{J=3}} \end{pmatrix}=0$$

Physical mixing vs. **Symmetry reduction**

- E-irrep is 5D: mixes J=1, 2, & 3

Conclusion

- Generalization of dimer formalism
  - Scalar/Nuclear sectors
- Quantization condition for NN in a box
  - Arbitrary partial wave, parity, & boost
- Positive parity irreps for J<4
  - Boosts: (0,0,0), (0,0,1), (1,1,0), (1,1,1)
- Negative parity sector under way
- Good interporlating operators needed!

Thanks!

- Martin, David, INT
- Speakers
- Participants