Extensions of the HAL QCD approach to inelastic and multi-particle scatterings in lattice QCD

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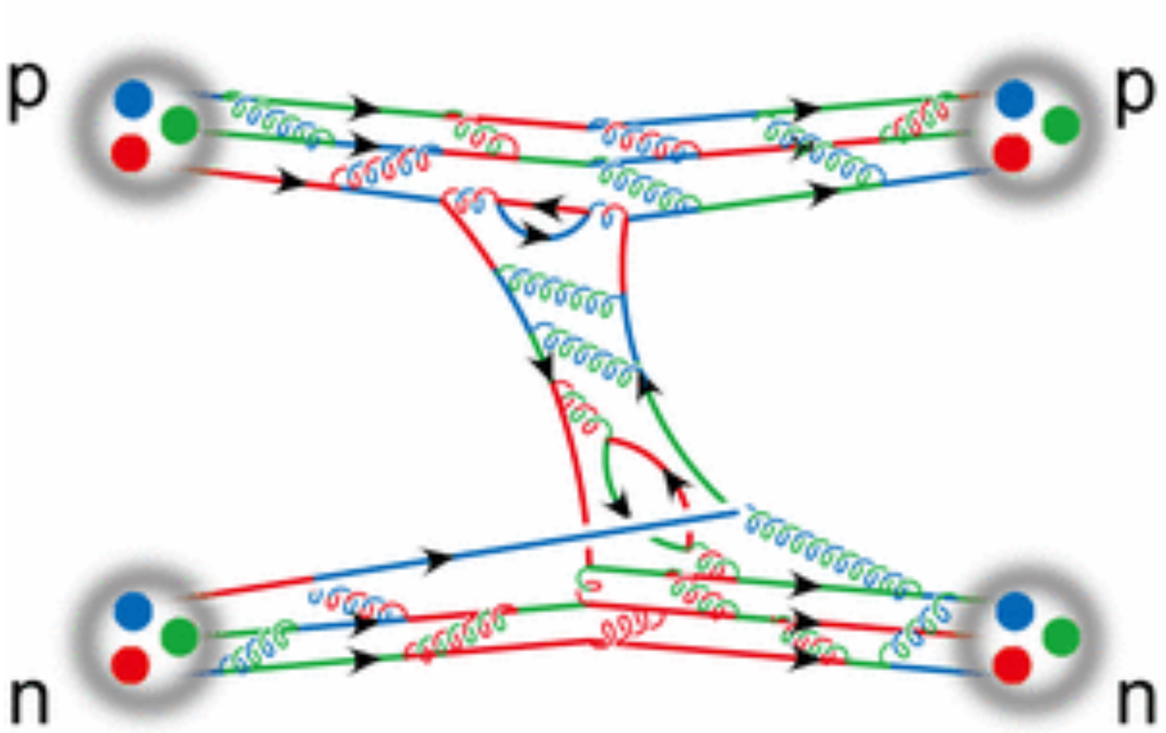
1. Introduction

HAL QCD approach to Nuclear Force



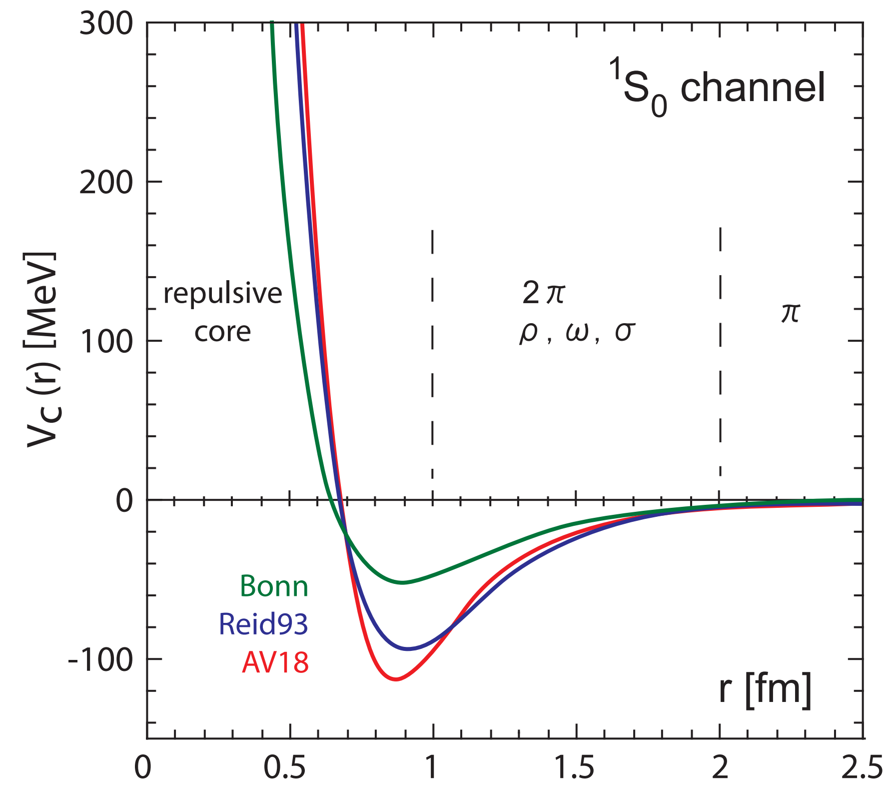
Potentials in QCD ?

What are “potentials” (quantum mechanical objects) in quantum field theories such as QCD ?

“Potentials” themselves can NOT be directly measured. cf. running coupling in QCD

scheme dependent, ambiguities in inelastic region

experimental data of scattering phase shifts → potentials, but not unique



“Potentials” are still useful tools to extract observables such as scattering phase shift.

↓

One may adopt a convenient definition of potentials as long as they reproduce correct physics of QCD.

HAL QCD strategy

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

Step 1

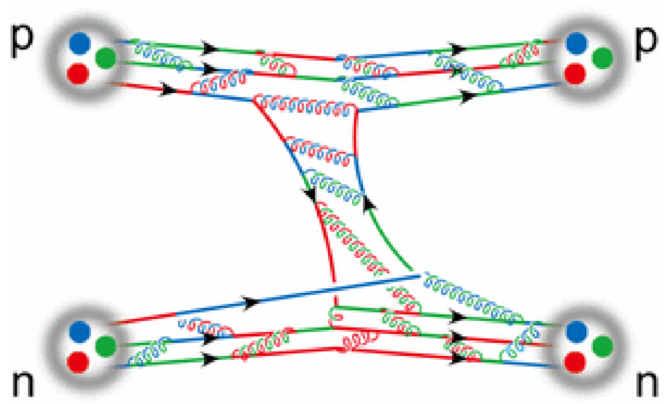
define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

Spin model: Balog et al., 1999/2001

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle \qquad W_k = 2\sqrt{\mathbf{k}^2 + m_N^2}$$

energy

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator



Key Property 1

Lin et al., 2001; CP-PACS, 2004/2005

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})$$

$r = |\mathbf{r}| \to \infty$

$\delta_l(k)$ scattering phase shift (phase of the S-matrix by unitarity) in QCD !

How can we extract it ?

cf. Luescher's finite volume method

Step 2

define non-local but energy-independent “potential” as

$$\left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

$\mu = m_N/2$

reduced mass

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

non-local potential

Key Property 2

A non-local but energy-independent potential can be constructed as

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\rm th}} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

inner product

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

$\varphi_{\mathbf{k}}$ is linearly independent.

For $^\forall W_{\mathbf{p}} < W_{\rm th} = 2m_N + m_\pi$ (threshold energy)

$$\int d^3y\, U(\mathbf{x},\mathbf{y})\phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(x) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \eta_{\mathbf{k}',\mathbf{p}} = \left[\epsilon_p - H_0\right] \varphi_{\mathbf{p}}(x)$$

Note 1: Potential satisfying this is not unique.

Note2: Non-relativistic approximation is NOT used. We just take the equal-time frame.

Step 3

expand the non-local potential in terms of derivative as $U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\sigma_{\mathbf{1}}\cdot\mathbf{x})(\sigma_{\mathbf{2}}\cdot\mathbf{x}) - (\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}})$

$V_A(\mathbf{x})$ local and energy independent coefficient function
(cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

Step 4

extract the local potential. At LO, for example, we simply have

$$V_{\rm LO}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

Step 5

solve the Schroedinger Eq. in the infinite volume with this potential.

→ phase shifts and binding energy below inelastic threshold

(We can calculate the phase shift at all angular momentum.)

$\delta_L(k)$ exact by construction

$\delta_L(p \neq k)$ approximated one by the derivative expansion

expansion parameter

$$\frac{W_p - W_k}{W_{\rm th} - 2m_N} \simeq \frac{\Delta E_p}{m_\pi}$$

We can check a size of errors at LO of the expansion.
We can improve results by extracting higher order terms in the expansion.

2. Results from lattice QCD

Ishii *et al.* (HALQCD), PLB712(2012) 437.

Extraction of NBS wave function

NBS wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

Potential

$$[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

4-pt Correlation function

source for NN

$$F(\mathbf{r}, t-t_0) = \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \overline{\mathcal{J}}(t_0) | 0 \rangle$$

complete set for NN

$$\begin{aligned} F(\mathbf{r}, t-t_0) &= \langle 0 | T\{N(\mathbf{x}+\mathbf{r},t) N(\mathbf{x},t)\} \sum_{n,s_1,s_2} |2N, W_n, s_1, s_2\rangle \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(t_0) | 0 \rangle + \cdots \\ &= \sum_{n,s_1,s_2} A_{n,s_1,s_2} \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(0) | 0 \rangle. \end{aligned}$$

ground state saturation at large t

$$\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})$$

NBS wave function

This is a standard method in lattice QCD and was employed for our first calculation.

Improved method

Ishii *et al.* (HALQCD), PLB712(2012) 437

normalized 4-pt Correlation function $R(\mathbf{r},t) \equiv F(\mathbf{r},t)/(e^{-m_N t})^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(\mathbf{r},t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t)$$

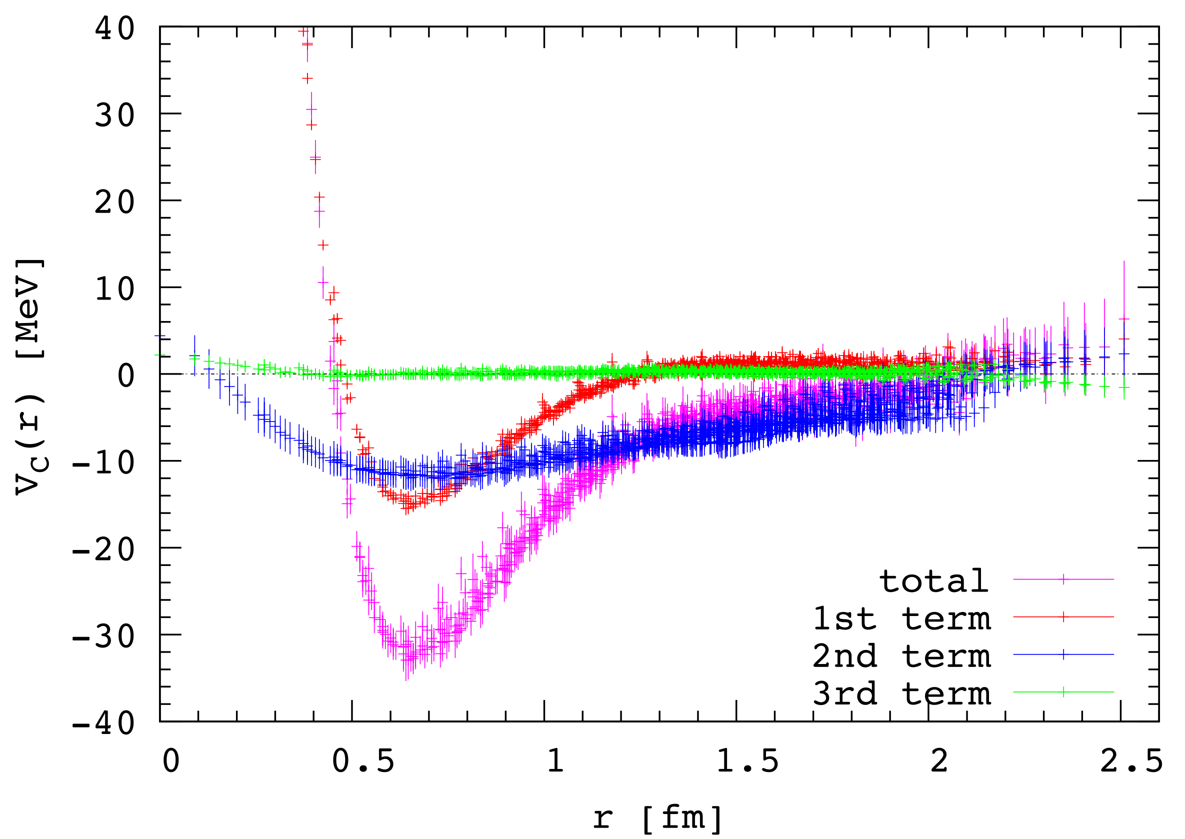
energy-independent **potential**

Leading Order

$$\left\{ -H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) = V_C(\mathbf{r}) R(\mathbf{r},t) + \cdots$$

1st **2nd** **3rd** **total**

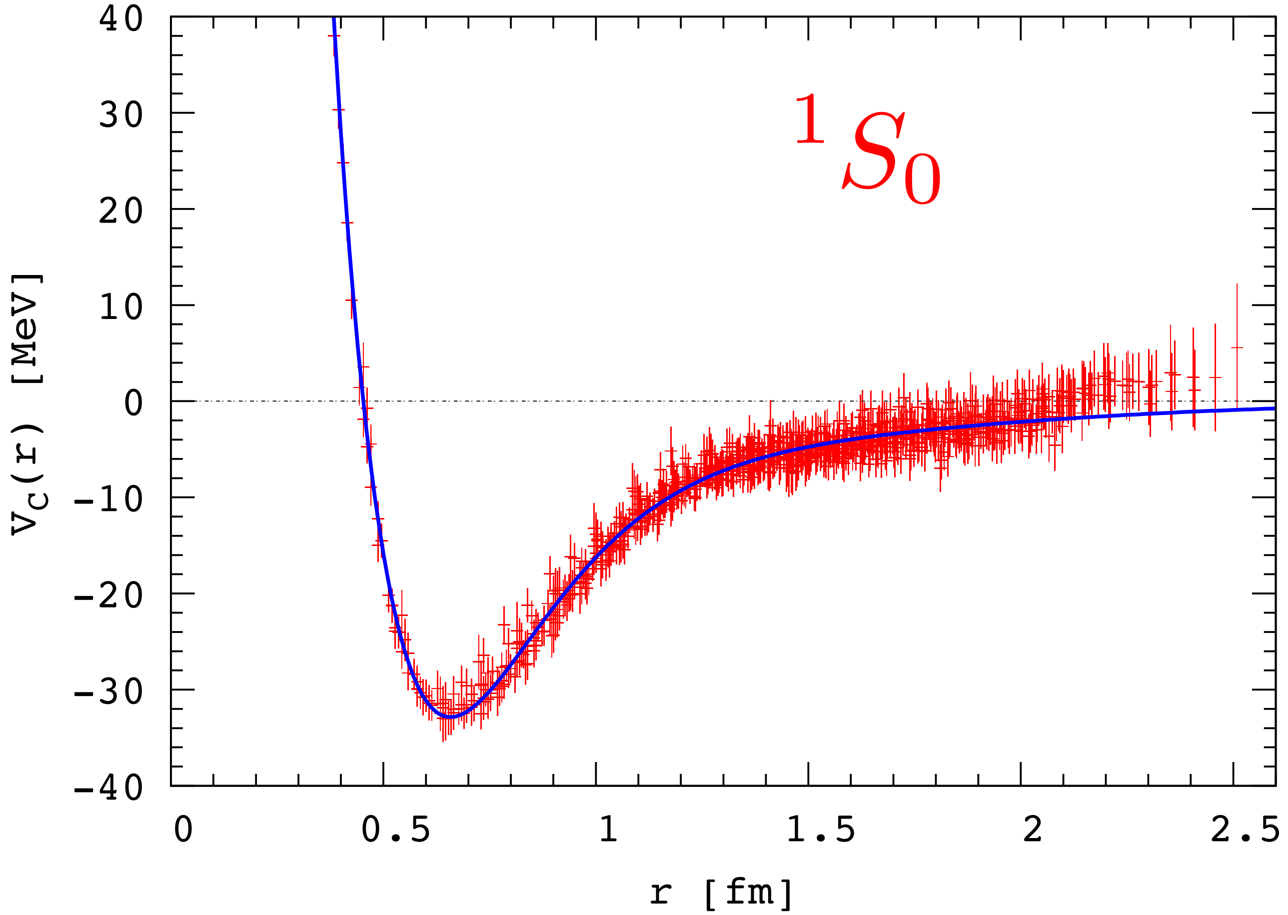
3rd term(relativistic correction) is negligible.

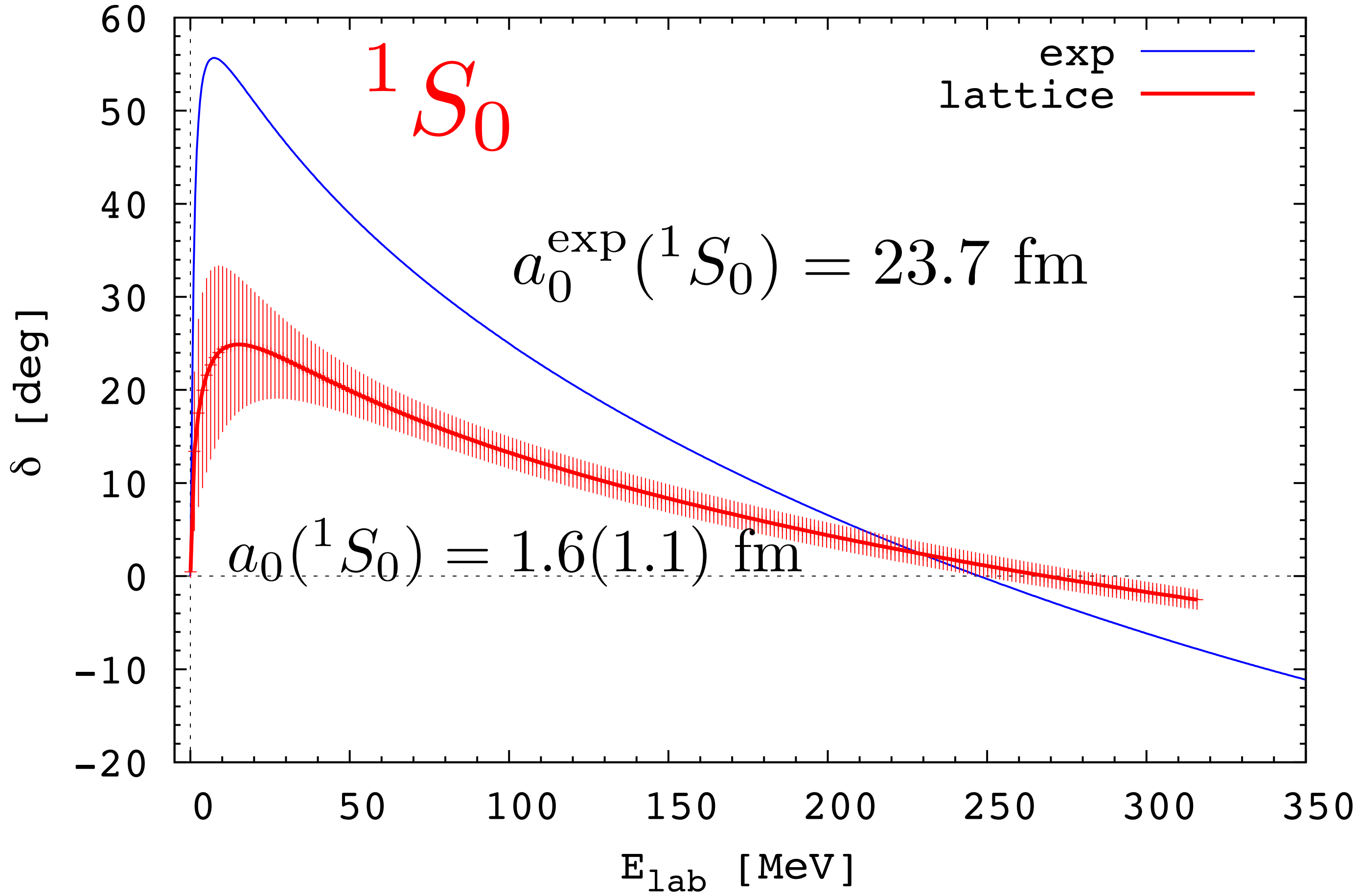


Ground state saturation is no more required. (advantage over finite volume method.)

2+1 flavor QCD, spin-singlet potential $m_\pi \simeq 700$ MeV a=0.09fm, L=2.9fm

NN potential → phase shift





Qualitative features of NN potential are reproduced.

(1)attractions at medium and long distances
(2)repulsion at short distance(repulsive core)

It has a reasonable shape.
The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

In order to extend the HAL QCD method to inelastic and/or multi-particle scatterings, we have to show

Key Property 1

Asymptotic behaviors of NBS wave functions for more than 2 particles

Key Property 2

Existence of energy independent potentials above inelastic thresholds

3. NBS wave functions for multi-particles

Key Property 1

Sinya Aoki, et al., arXiv.1303.2210 [hep-lat].

For simplicity,
(1) we consider scalar particles with “flavors”
(2) we assume no bound state exists.

Unitarity constraint

$$T^\dagger - T = iT^\dagger T.$$

parametrization

$${}_0\langle [\boldsymbol{p}^A]_n | T | [\boldsymbol{p}^B]_n \rangle_0 \;\equiv\; \delta(E^A - E^B)\delta^{(3)}(\boldsymbol{P}^A - \boldsymbol{P}^B) T([\boldsymbol{q}^A]_n, [\boldsymbol{q}^B]_n)$$

(modified) Jacobi coordinates and momenta

$$\boldsymbol{r}_k = \sqrt{\frac{k}{k+1}} \times \boldsymbol{r}^J_k, \qquad \boldsymbol{q}_k = \sqrt{\frac{k+1}{k}} \times \boldsymbol{q}^J_k$$

$$\boldsymbol{r}^J_k = \frac{1}{k}\sum_{i=1}^{k} \boldsymbol{x}_i - \boldsymbol{x}_{k+1}, \quad \boldsymbol{q}^J_k = \frac{k}{k+1}\left(\frac{1}{k}\sum_{i=1}^{k} \boldsymbol{p}_i - \boldsymbol{p}_{k+1}\right),$$

$$\begin{aligned} T([\boldsymbol{q}^A]_n, [\boldsymbol{q}^B]_n) \;&\equiv\; T(\boldsymbol{Q}_A, \boldsymbol{Q}_B) \\ &= \sum_{[L],[K]} T_{[L][K]}(Q_A, Q_B) Y_{[L]}(\Omega_{\boldsymbol{Q}_A}) \overline{Y_{[K]}(\Omega_{\boldsymbol{Q}_B})} \end{aligned}$$

$$\boldsymbol{Q}_X = (\boldsymbol{q}^X{}_1, \boldsymbol{q}^X{}_2, \cdots, \boldsymbol{q}^X{}_{n-1})$$

momentum in D=3(n-1) dim.

hyper-spherical harmonic function

$$\hat{L}^2 Y_{[L]}(\Omega_{\boldsymbol{s}}) \;=\; L(L+D-2) Y_{[L]}(\Omega_{\boldsymbol{s}})$$

solution to the unitarity constraint

with non-relativistic approximation

$$T_{[L][K]}(Q,Q) \;=\; \sum_{[N]} U_{[L][N]}(Q) T_{[N]}(Q) U^{\dagger}_{[N][K]}(Q),$$

↑

$$T_{[L]}(Q) = -\frac{2n^{3/2}}{mQ^{3n-5}} e^{i\delta_{[L]}(Q)} \sin\delta_{[L]}(Q),$$

"phase shift" $\delta_{[L]}(Q)$

Lippmann-Schwinger equation in QFT

$$|\alpha\rangle_{\rm in} = |\alpha\rangle_0 + \int d\beta \frac{|\beta\rangle_0 T_{\beta\alpha}}{E_\alpha - E_\beta + i\varepsilon},$$

$$T_{\beta\alpha} = {}_0\langle\beta|V|\alpha\rangle_{\rm in},$$

off-shell

$${}_0\langle\beta|T|\alpha\rangle_0 = 2\pi\delta(E_\alpha - E_\beta)T_{\alpha\beta}.$$

on-shell   off-shell

$$(H_0 + V)|\alpha\rangle_{\rm in} = E_\alpha|\alpha\rangle_{\rm in},$$ full

$$H_0|\alpha\rangle_0 = E_\alpha|\alpha\rangle_0.$$ free

NBS wave functions

n-scalar fields with different flavors

$$\Psi_\alpha^n([\boldsymbol{x}]) = {}_{\rm in}\langle 0|\varphi^n([\boldsymbol{x}],0)|\alpha\rangle_{\rm in},$$

$$\varphi^n([\boldsymbol{x}],t) = T\{\prod_{i=1}^n \varphi_i(\boldsymbol{x}_i,t)\},$$

$$\Psi_\alpha^n([\boldsymbol{x}]) = \frac{1}{Z_\alpha}{}_0\langle 0|\varphi^n([\boldsymbol{x}],0)|\alpha\rangle_0 + \int d\beta \frac{1}{Z_\beta}\frac{{}_0\langle 0|\varphi^n([\boldsymbol{x}],0)|\beta\rangle_0 T_{\beta\alpha}}{E_\alpha - E_\beta + i\varepsilon}.$$

$${}_0\langle 0|\varphi^n([\boldsymbol{x}],0)|[\boldsymbol{k}]_n\rangle_0 = \left(\frac{1}{\sqrt{(2\pi)^3}}\right)^n \prod_{i=1}^n \frac{1}{\sqrt{2E_{k_i}}} e^{i\boldsymbol{k}_i\boldsymbol{x}_i}$$

D-dimensional hyper-coordinates

$$\Psi^n(\boldsymbol{R},\boldsymbol{Q}_A) = C\left[e^{i\boldsymbol{Q}_A\cdot R} + \frac{2m}{2\pi n^{3/2}}\int d^DQ\,\frac{e^{i\boldsymbol{Q}\cdot R}}{Q_A^2 - Q^2 + i\varepsilon}T(\boldsymbol{Q},\boldsymbol{Q}_A)\right]$$

Expansion in terms of hyper-spherical harmonic function

$$e^{i\boldsymbol{Q}\cdot\boldsymbol{R}} = (D-2)!!\frac{2\pi^{D/2}}{\Gamma(D/2)}\sum_{[L]} i^L\, \underline{j_L^D(QR)}\, Y_{[L]}(\Omega_{\boldsymbol{R}})\,\overline{Y_{[L]}(\Omega_{\boldsymbol{Q}})},$$

hyper-spherical Bessel function

$$\Psi^n(\boldsymbol{R},\boldsymbol{Q}_A) = \sum_{[L],[K]}\Psi^n_{[L],[K]}(R,Q_A)Y_{[L]}(\Omega_{\boldsymbol{R}})\overline{Y_{[K]}(\Omega_{\boldsymbol{Q}_A})},$$

Asymptotic behavior of NBS wave functions

$R \to \infty$

$$\Psi^n_{[L],[K]}(R,Q_A) \simeq Ci^L\frac{(2\pi)^{D/2}}{(Q_AR)^{\frac{D-1}{2}}}\sum_{[N]}U_{[L][N]}(Q_A)e^{i\delta_{[N]}(Q_A)}U^\dagger_{[N][K]}(Q_A)$$

$$\times\sqrt{\frac{2}{\pi}}\;\underline{\sin\left(Q_AR - \Delta_L + \delta_{[N]}(Q_A)\right)}$$

$$\Delta_L = \frac{2L_D - 1}{4}\pi.$$

scattering wave with “phase shift” !

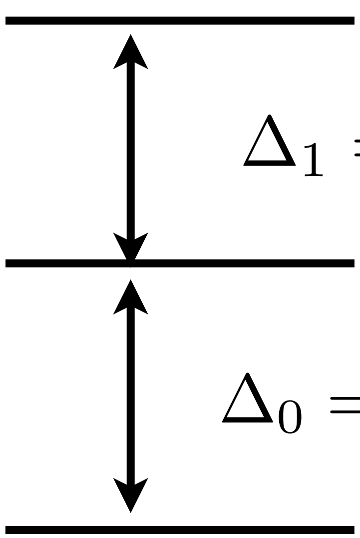
4. Energy-independent potential above inelastic thresholds

Key Property 2

Sinya Aoki, et al., Phys. Rev. D (in press).

Let us consider $NN \to NN, NN\pi$

energy $W \in \Delta_1$



$W_{\rm th}^2 = 2m_N + 2m_\pi$

$\Delta_1 = [W_{\rm th}^1, W_{\rm th}^2)$

$W_{\rm th}^1 = 2m_N + m_\pi$

$\Delta_0 = [W_{\rm th}^0, W_{\rm th}^1)$

$W_{\rm th}^0 = 2m_N$

2 operators

$N(x)N(y)$

$N(x)N(y)\pi(z)$

$\times$

2 states

$|NN, W, c_0\rangle$

$|NN+\pi, W, c_1\rangle$

other quantum numbers

4 NBS wave functions

$$
\begin{aligned}
Z_N \varphi^{00}_{W,c_0}(\boldsymbol{x}_0) &= \langle 0 | T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\}|NN, W, c_0\rangle_{\rm in}, \\
Z_N Z_\pi^{1/2} \varphi^{10}_{W,c_0}(\boldsymbol{x}_0,\boldsymbol{x}_1) &= \langle 0 | T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\pi(\boldsymbol{x}+\boldsymbol{x}_1,0)\}|NN, W, c_0\rangle_{\rm in}, \\
Z_N \varphi^{01}_{W,c_1}(\boldsymbol{x}_0) &= \langle 0 | T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\}|NN+\pi, W, c_1\rangle_{\rm in}, \\
Z_N Z_\pi^{1/2} \varphi^{11}_{W,c_1}(\boldsymbol{x}_0,\boldsymbol{x}_1) &= \langle 0 | T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\pi(\boldsymbol{x}+\boldsymbol{x}_1,0)\}|NN+\pi, W, c_1\rangle_{\rm in},
\end{aligned}
$$

$\varphi^{ij}_{W,c_j}([\mathbf{x}]_i)$ $i(j)$: number of π's in the operator(state) $[\boldsymbol{x}]_0 = \boldsymbol{x}_0$ $[\boldsymbol{x}]_1 = \boldsymbol{x}_0, \boldsymbol{x}_1$

coupled channel equation

$$(E_W^k - H_0^k)\varphi_{W,c_i}^{ki} = \sum_{l=0,1} \int \prod_{n=0}^{l} d^3 y_n\, U^{kl}([\boldsymbol{x}]_k, [\boldsymbol{y}]_l)\varphi_{W,c_i}^{li}([\boldsymbol{y}]_l), \quad k, i \in (0,1)$$

exists ?

$$E_W^n = \frac{\boldsymbol{p}_1^2}{2m_N} + \frac{\boldsymbol{p}_2^2}{2m_N} + \sum_{i=1}^{n} \frac{\boldsymbol{k}_i^2}{2m_\pi}$$

kinetic energy

non-relativistic approx. for n=1

$$W = \sqrt{m_N^2 + \boldsymbol{p}_1^2} + \sqrt{m_N^2 + \boldsymbol{p}_2^2} + \sum_{i=1}^{n} \sqrt{m_\pi^2 + \boldsymbol{k}_i^2}$$

total energy

$$\boldsymbol{p}_1 + \boldsymbol{p}_2 + \sum_{i=1}^{n} \boldsymbol{k}_i = 0.$$

Proof of existence for U

Define a vector of NBS wave functions as

$$\varphi_{W,c_i}^{i} \equiv \left(\varphi_{W,c_i}^{0i}([\boldsymbol{x}]_0), \varphi_{W,c_i}^{1i}([\boldsymbol{x}]_1)\right)^T, \quad i = 0, 1, \qquad W \in \Delta_1$$

$$\varphi_{W,c_0}^{0} \equiv \left(\varphi_{W,c_0}^{00}([\boldsymbol{x}]_0), \varphi_{W,c_0}^{10}([\boldsymbol{x}]_1)\right)^T, \qquad W \in \Delta_0$$

Norm kernel

$$\mathcal{N}^{ij}_{W_1 c_i, W_2 d_j} = \left(\varphi^i_{W_1, c_i}, \varphi^j_{W_2, d_j}\right) \equiv \sum_{k=0,1} \int \prod_{l=0}^{k} d^3 x_l \, \overline{\varphi^{ki}_{W_1, c_i}([\boldsymbol{x}]_k)} \varphi^{kj}_{W_2, d_j}([\boldsymbol{x}]_k).$$

Inverse

$$\sum_{W \in \Delta_0 + \Delta_1} \sum_{h \in I(W), e_h} (\mathcal{N}^{-1})^{ih}_{W_1 c_i, W e_h} \mathcal{N}^{hj}_{W e_h, W_2 d_j} = \delta^{ij} \delta_{W_1, W_2} \delta_{c_i, d_j}$$

Structure

$$\mathcal{N} = \begin{pmatrix} \mathcal{N}^{00}(\Delta_0, \Delta_0), & \mathcal{N}^{00}(\Delta_0, \Delta_1), & \mathcal{N}^{01}(\Delta_0, \Delta_1) \\ \mathcal{N}^{00}(\Delta_1, \Delta_0), & \mathcal{N}^{00}(\Delta_1, \Delta_1), & \mathcal{N}^{01}(\Delta_1, \Delta_1) \\ \mathcal{N}^{10}\underline{(\Delta_1, \Delta_0)}, & \mathcal{N}^{10}(\Delta_1, \Delta_1), & \mathcal{N}^{\underline{11}}(\Delta_1, \Delta_1) \end{pmatrix}$$

energy state

bra, ket

$$\langle [\boldsymbol{x}]_k | \varphi^i_{W, c_i} \rangle = \varphi^{ki}_{W, c_i}([\boldsymbol{x}]_k),$$

$$\langle \psi^i_{W, c_i} | [\boldsymbol{x}]_k \rangle = \sum_{W_1 \in \Delta_0 \cup \Delta_1} \sum_{j \in I(W_1), d_j} (\mathcal{N}^{-1})^{ij}_{W c_i, W_1 d_j} \overline{\varphi^{kj}_{W_1, d_j}([\boldsymbol{x}]_k)}$$

orthogonality

$$\langle \psi^i_{W_1, c_i} | \varphi^j_{W_2, d_j} \rangle = \sum_{k=0,1} \int \prod_{l=0}^{k} d^3 x_l \, \langle \psi^i_{W_1, c_i} | [\boldsymbol{x}]_k \rangle \langle [\boldsymbol{x}]_k | \varphi^j_{W_2, d_j} \rangle = (\mathcal{N}^{-1} \cdot \mathcal{N})^{ij}_{W_1 c_i, W_2 d_j}$$

$$= \delta^{ij} \delta_{W_1, W_2} \delta_{c_i, d_j}.$$

Abstract operators

$$\langle [\boldsymbol{x}]_k | (E_W - H_0) | [\boldsymbol{y}]_l \rangle \equiv \delta_{kl} (E_W^k - H_0^k) \prod_{n=0}^{k} \delta^{(3)}(\boldsymbol{x}_n - \boldsymbol{y}_n)$$

$$\langle [\boldsymbol{x}]_k | U | [\boldsymbol{y}]_l \rangle \equiv U^{kl}([\boldsymbol{x}]_k, [\boldsymbol{y}]_l),$$

Abstract coupled channel equation

$$(E_W - H_0) | \varphi^i_{W,c_i} \rangle = U | \varphi^i_{W,c_i} \rangle .$$

construction of non-local coupled channel potential

$$U = \sum_{W \in \Delta_0 \cup \Delta_1} \sum_{i \in I(W)} \sum_{c_i} (E_W - H_0) | \varphi^i_{W,c_i} \rangle \langle \psi^i_{W,c_i} |,$$

$$\because \quad U | \varphi^i_{W,c_i} \rangle = \sum_{W_1 \in \Delta_0 \cup \Delta_1} \sum_{j \in I(W_1)} \sum_{d_j} (E_W - H_0) | \varphi^j_{W_1,d_j} \rangle \langle \psi^j_{W_1,d_j} | \varphi^i_{W,c_i} \rangle = (E_W - H_0) | \varphi^i_{W,c_i} \rangle$$

Energy independent (coupled channel) potential exists above the inelastic threshold.

Hermiticity

$$U^{ij}_{W_1 c_i, W_2 d_j} \equiv \langle \varphi^i_{W_1,c_i} | U | \varphi^j_{W_2,d_j} \rangle = \langle \varphi^i_{W_1,c_i} | (E_{W_2} - H_0) | \varphi^j_{W_2,d_j} \rangle,$$

$$(U^\dagger)^{ij}_{W_1 c_i, W_2 d_j} = \overline{\langle \varphi^j_{W_2,d_j} | (E_{W_1} - H_0) | \varphi^i_{W_1,c_i} \rangle} = \langle \varphi^i_{W_1,c_i} | (E_{W_1} - H_0) | \varphi^j_{W_2,d_j} \rangle.$$

$$\text{effectively Hermite for } E_{W_1} = E_{W_2}$$

The construction of U can easily be generalized to $NN + n\pi \longrightarrow NN + k\pi$

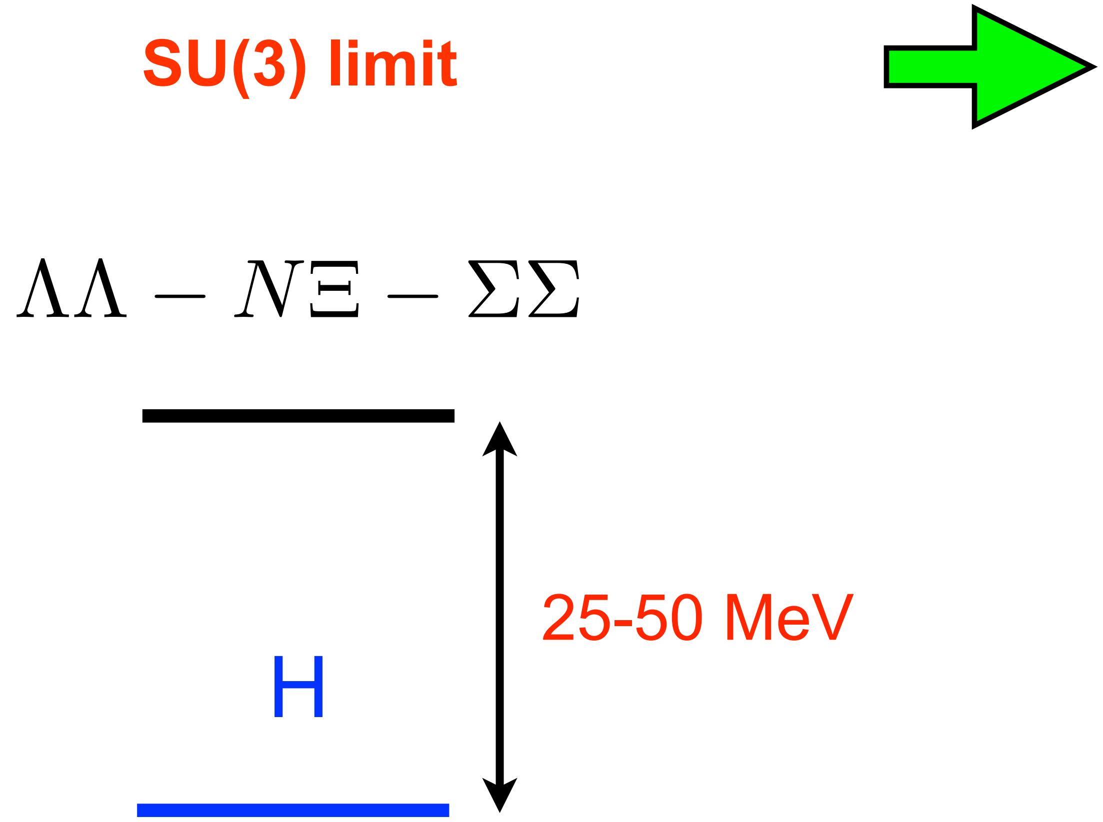
or to $\Lambda\Lambda \longrightarrow \Lambda\Lambda, N\Xi, \Sigma\Sigma$

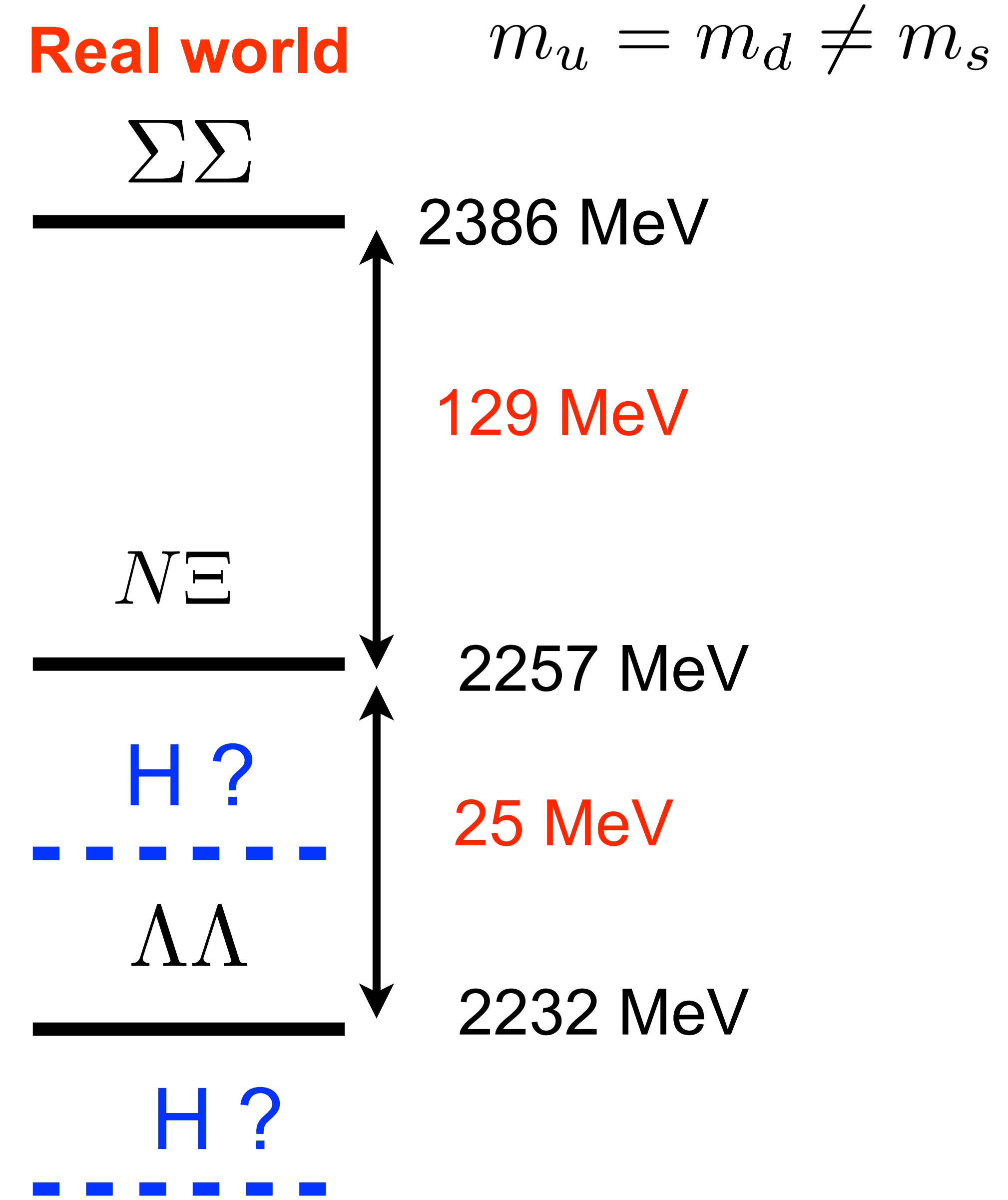
5. Related results

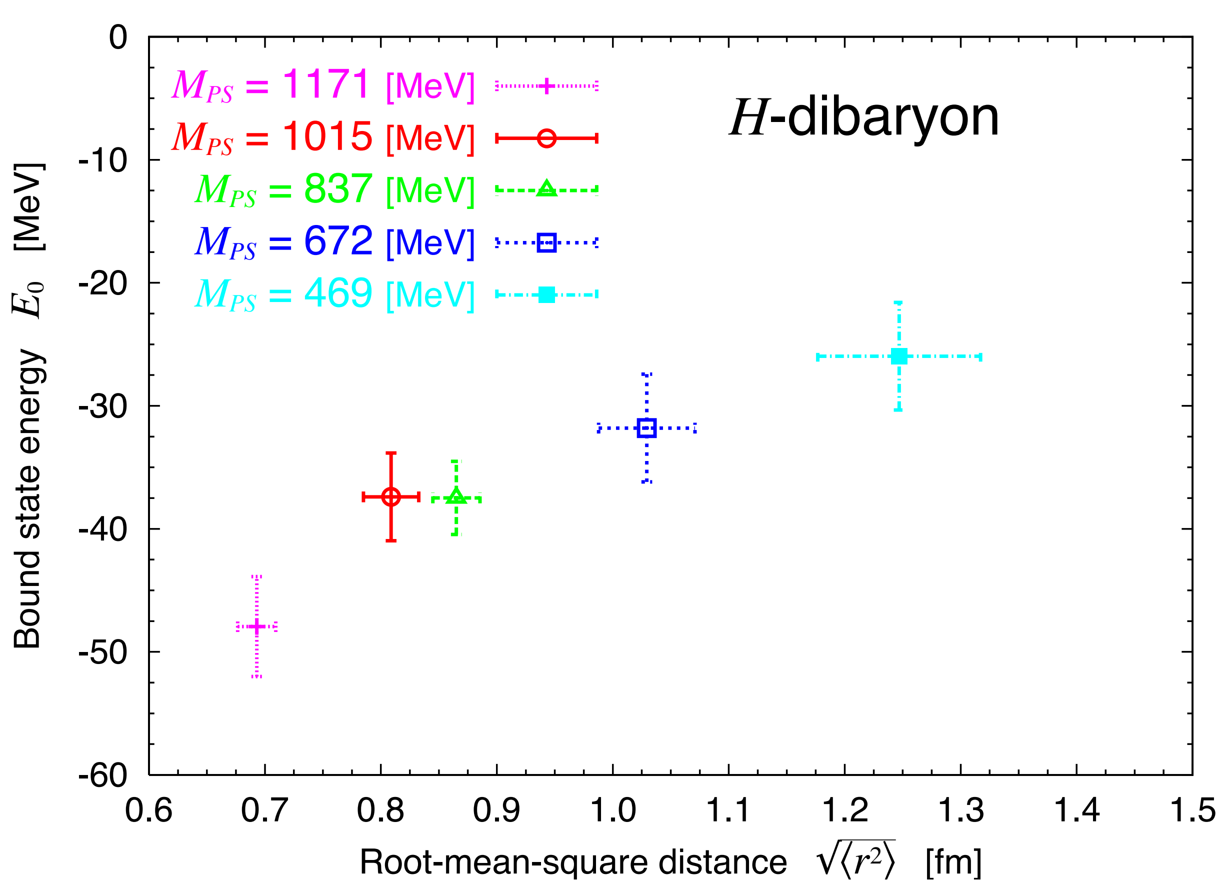
Kenji Sasaki, et al. (HAL QCD), in preparation

Takumi Doi et al. (HAL QCD), PTP 127 (2012) 723

H-dibaryon with the flavor SU(3) breaking



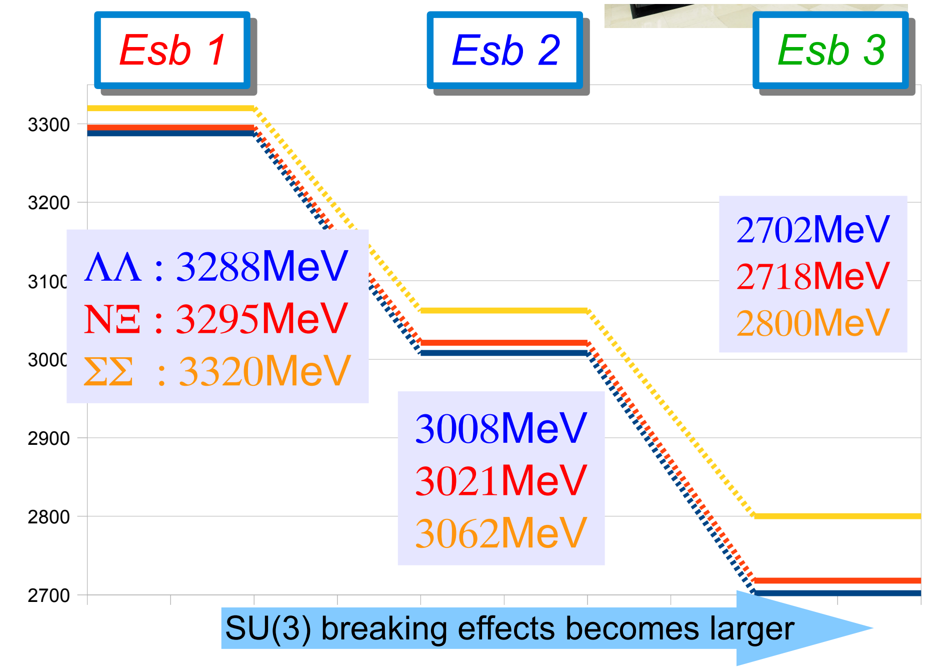




In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

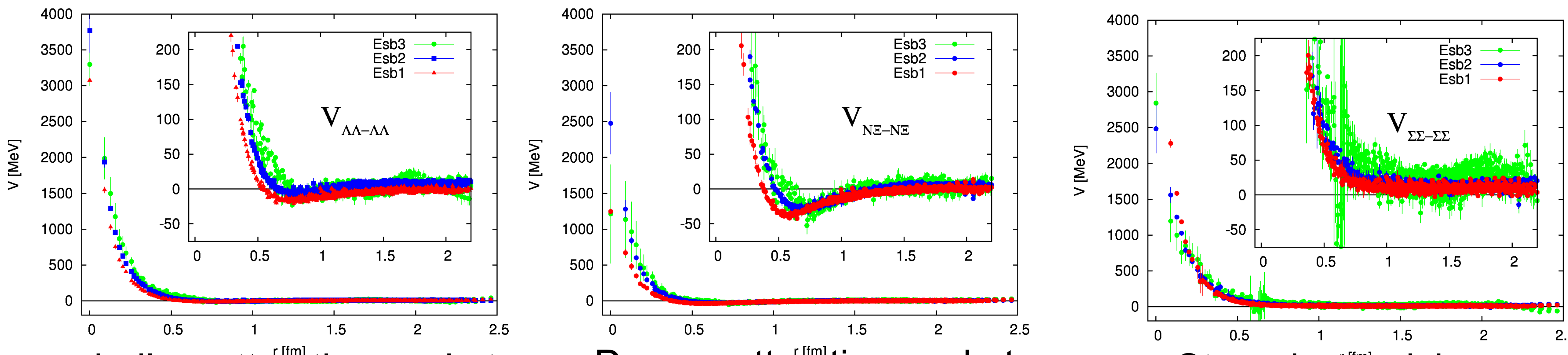
u,d quark masses lighter

Gauge ensembles



coupled channel 3x3 potentials

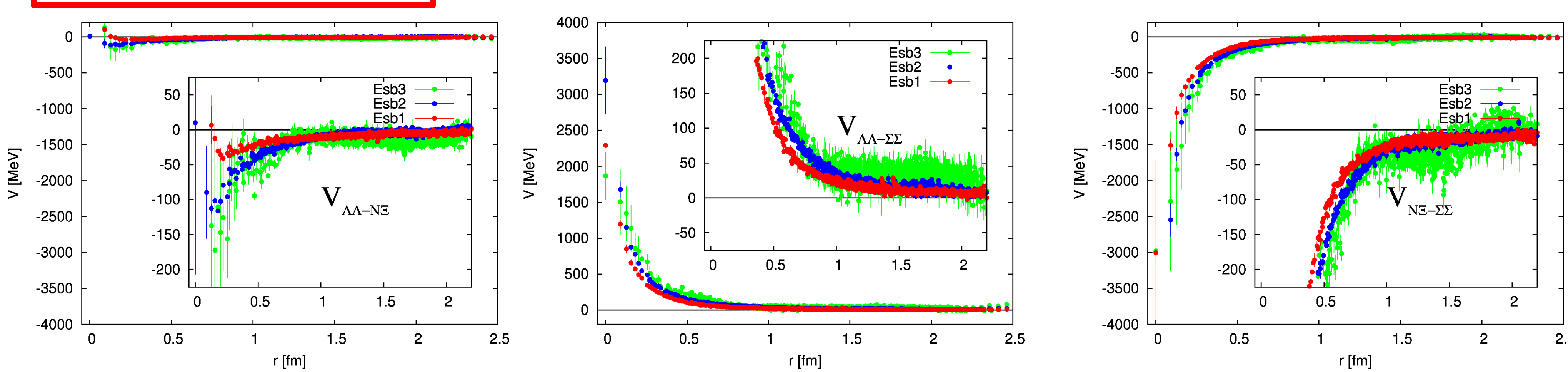
Diagonal elements



shallow attractive pocket | Deeper attractive pocket | Strongly repulsive

All channels have repulsive core

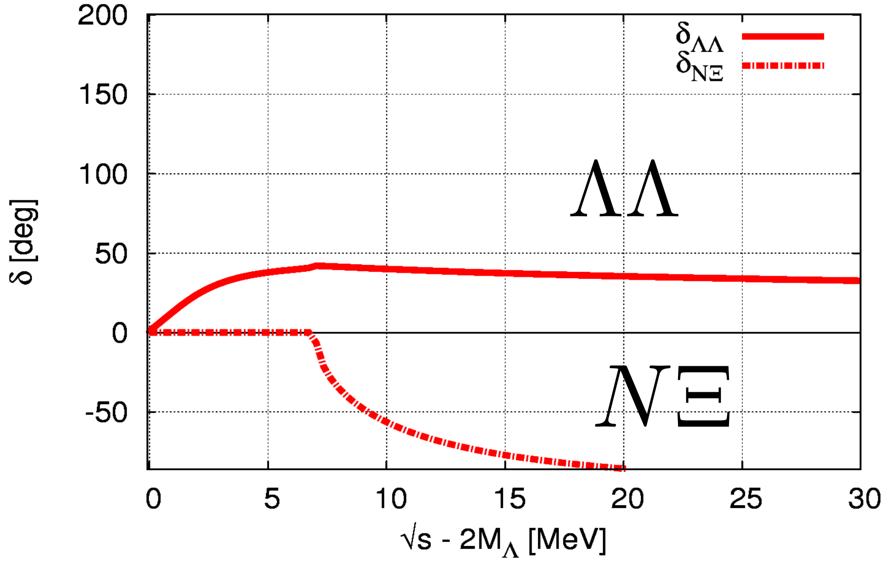
Off-diagonal elements



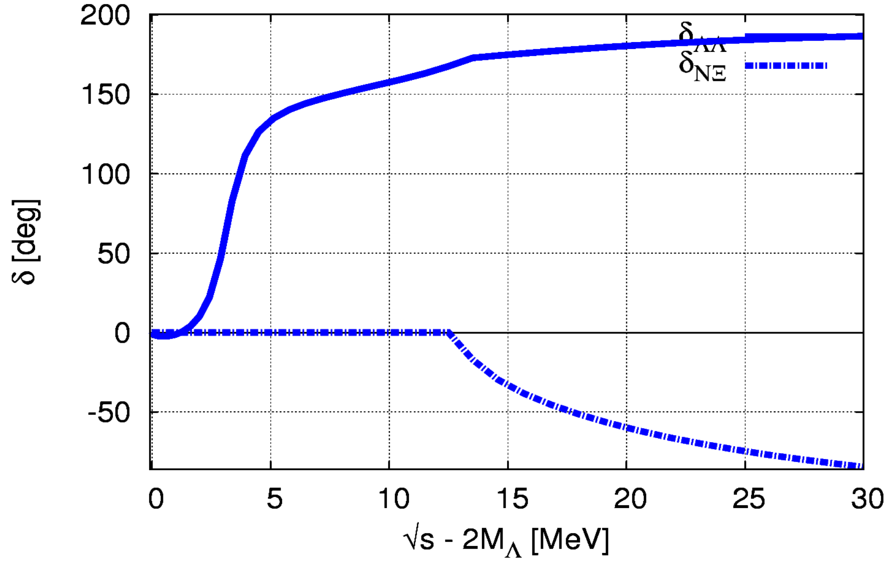
$\Lambda\Lambda$ and $N\Xi$ phase shift

Preliminary !

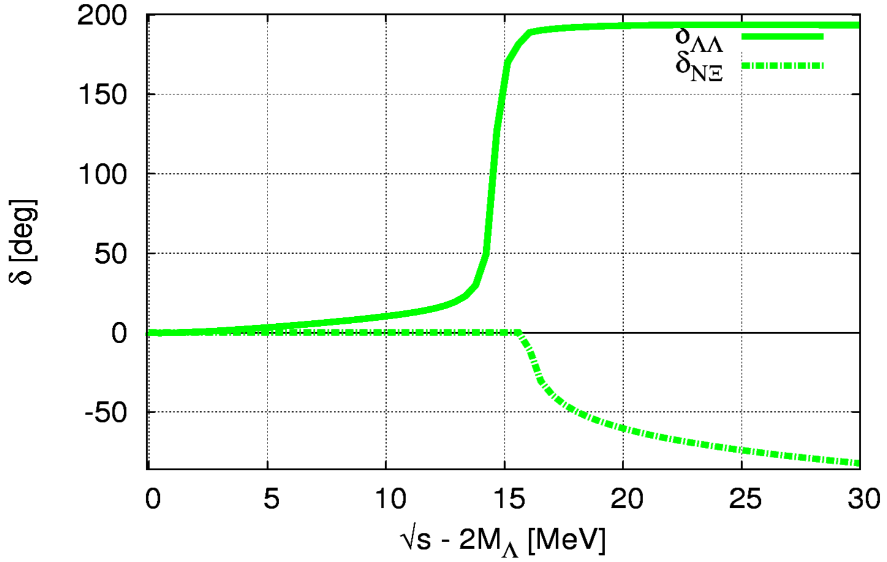
Esb1 : mπ= 701 MeV



Esb2 : mπ= 570 MeV



Esb3 : mπ= 411 MeV



Bound H-dibaryon

Resonance H

Resonance H

This suggests H-dibaryon becomes resonance at physical point.

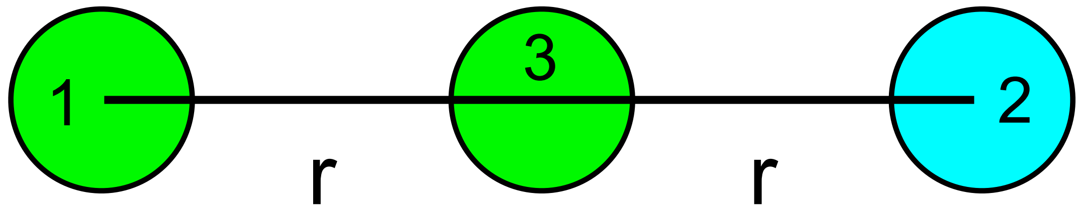
Three nucleon force (TNF)

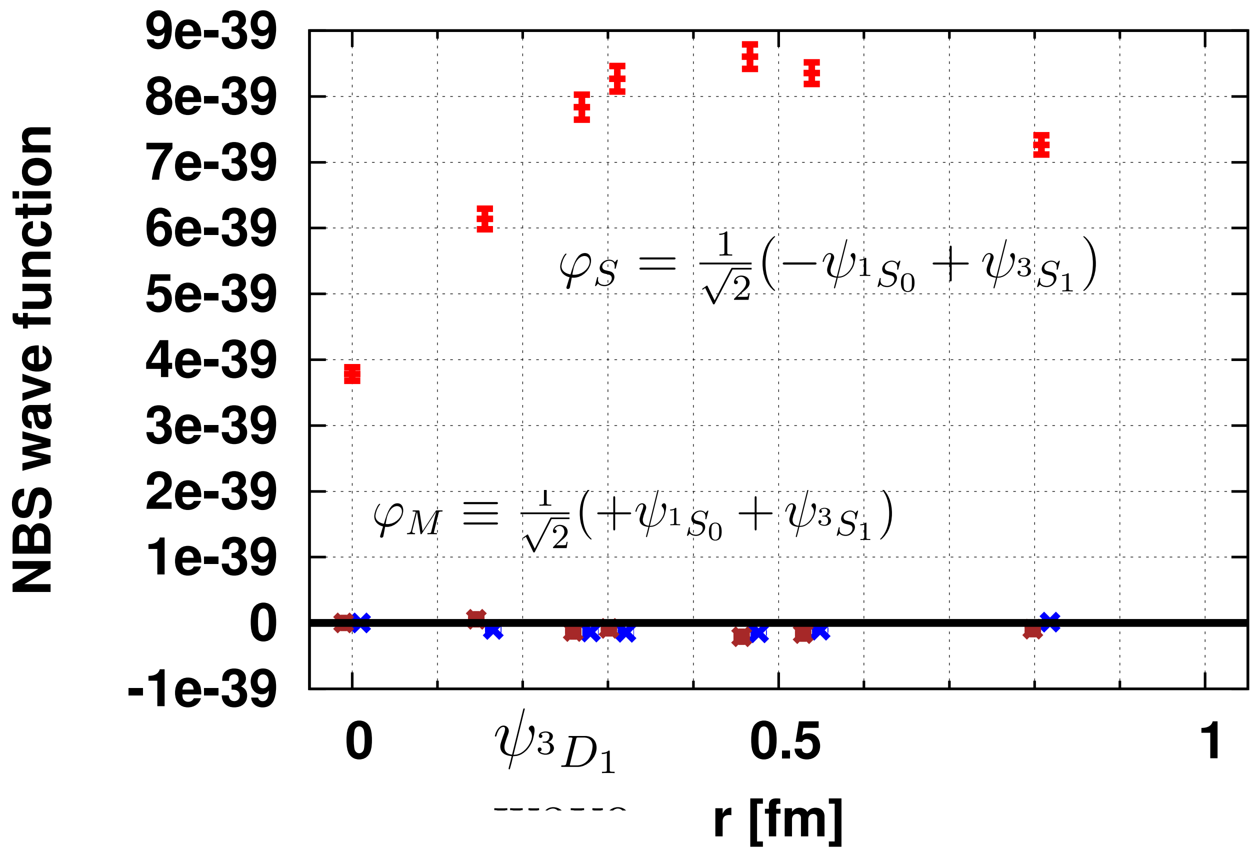
Doi et al. (HAL QCD), PTP 127 (2012) 723

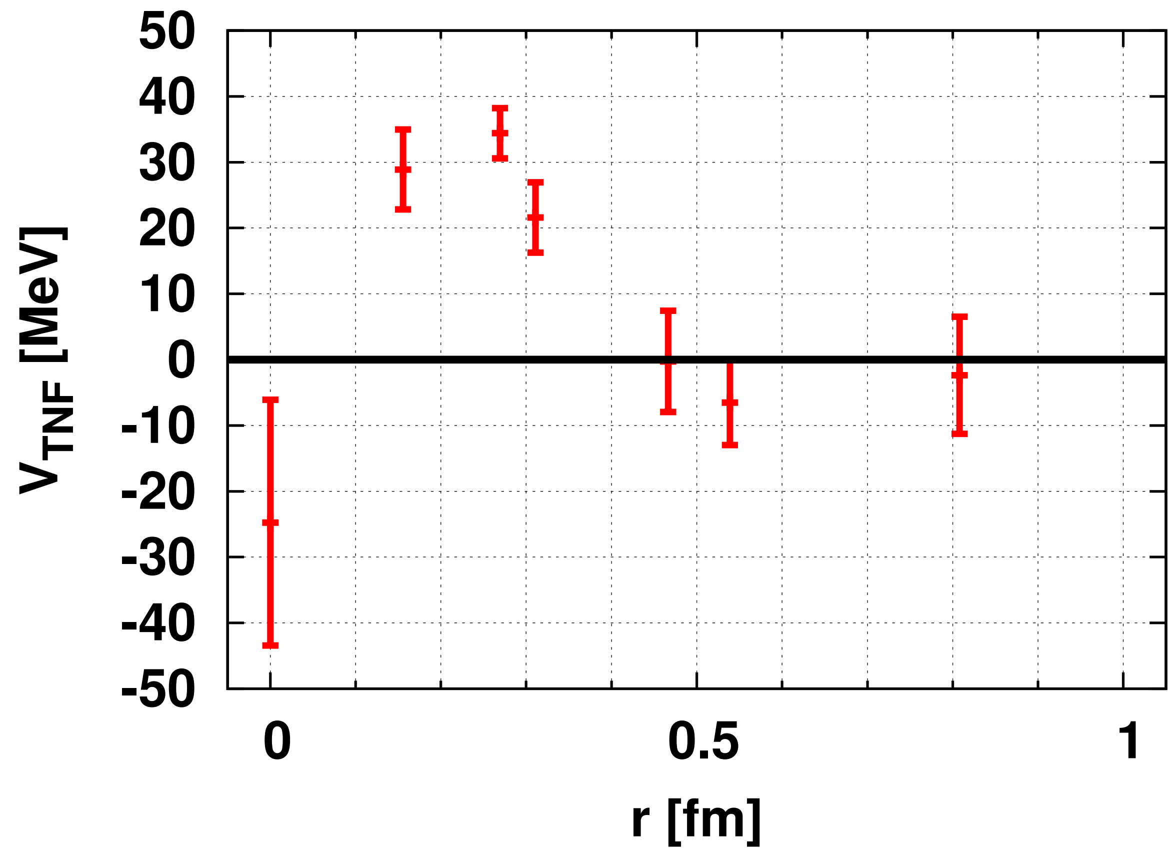
(1,2) pair $^1S_0,\ ^3S_1,\ ^3D_1$ S-wave only

$\mathrm{Triton}(I=1/2,\ J^P=1/2^+)$

Linear setup







scalar/isoscalar TNF is observed at short distance.

further study is needed to confirm this result.

Analysis by OPE (operator product expansion) in QCD predicts
universal short distance repulsions in TNF.

Aoki, Balog and Weisz, NJP14(2012)043046

6. Conclusion

- HAL QCD approach is shown to be a promising method to extract hadronic interactions in lattice QCD.
  - ground state saturation is not required.
  - Calculate potential in lattice QCD on a finite box.
  - Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
  - bound-state/resonance/scattering
- Extensions of the HAL QCD method to inelastic/multi-particle scatterings
  - Asymptotic behavior of the NBS wave functions
  - Existence of non-local but energy-independent coupled channel potentials
  - some preliminary results
- Future problems: Nuclear reactions ? Your inputs are important !

Thank you !

Backup slides

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Numerical check in quenched QCD

$m_\pi \simeq 0.53$ GeV

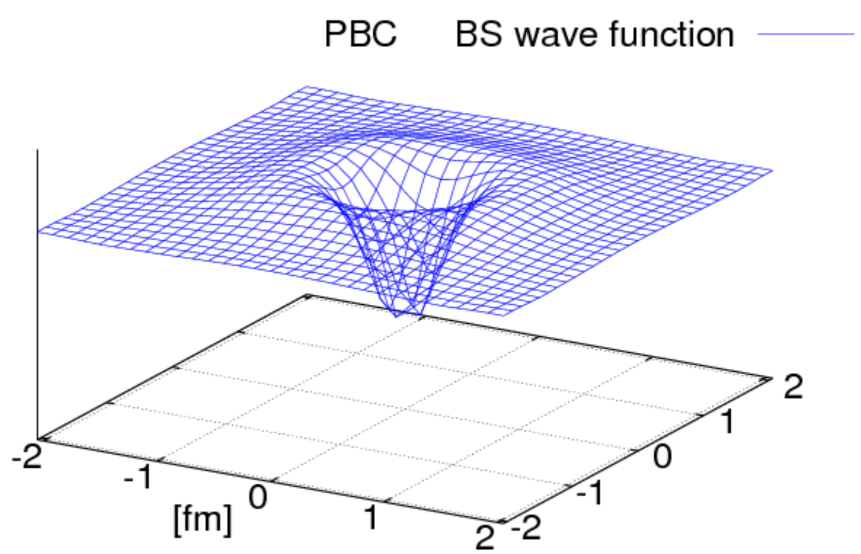
a=0.137fm, L=4.0 fm

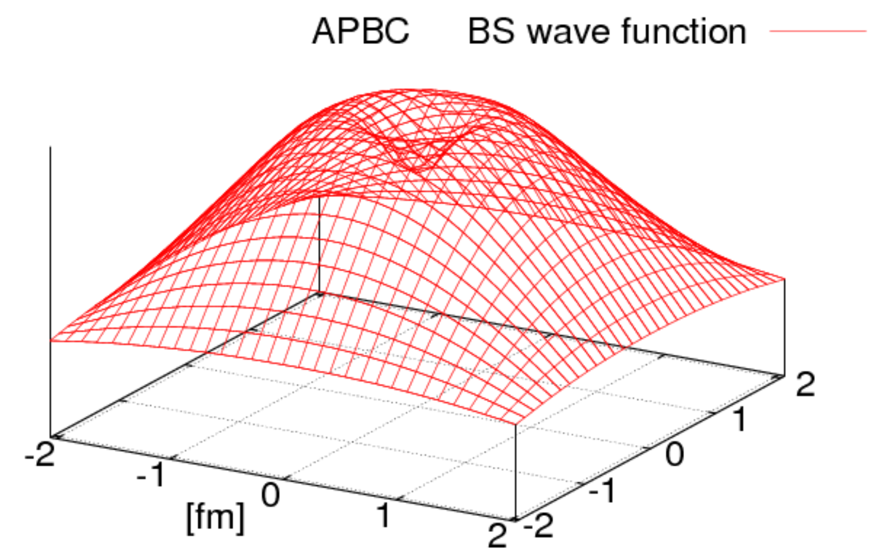
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

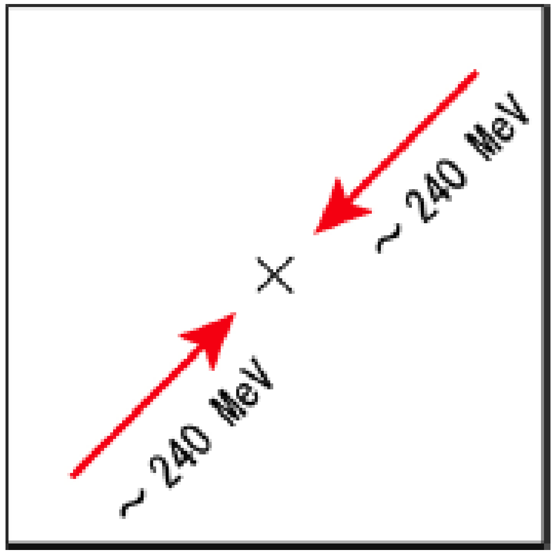
PTP 125 (2011)1225.

- PBC (E∼0 MeV)
- APBC (E∼46 MeV)

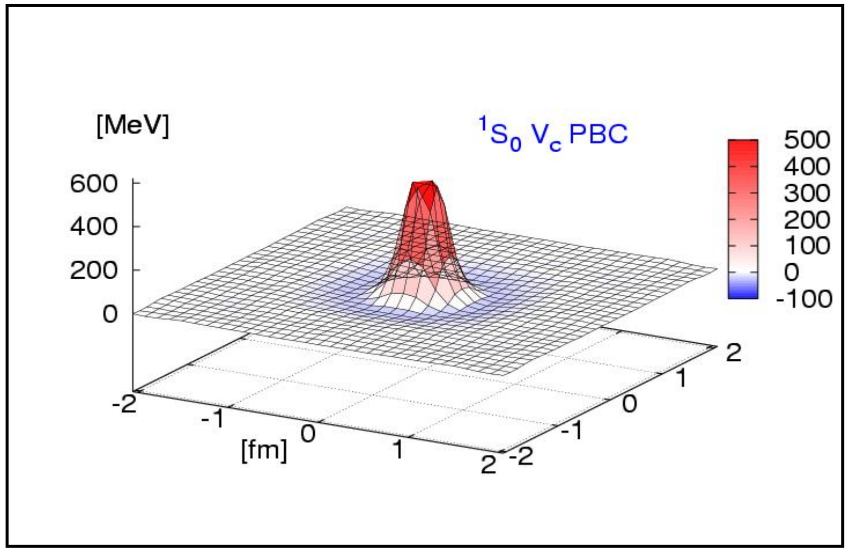
NBS wave functions

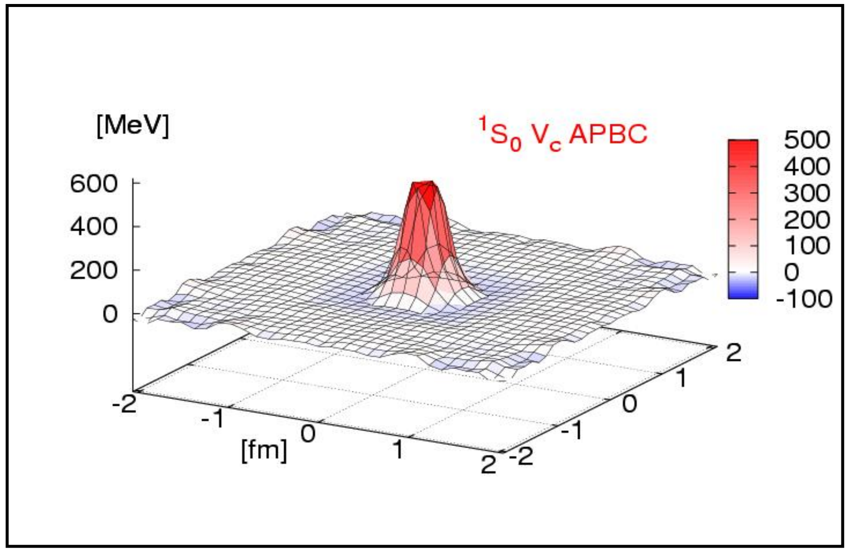


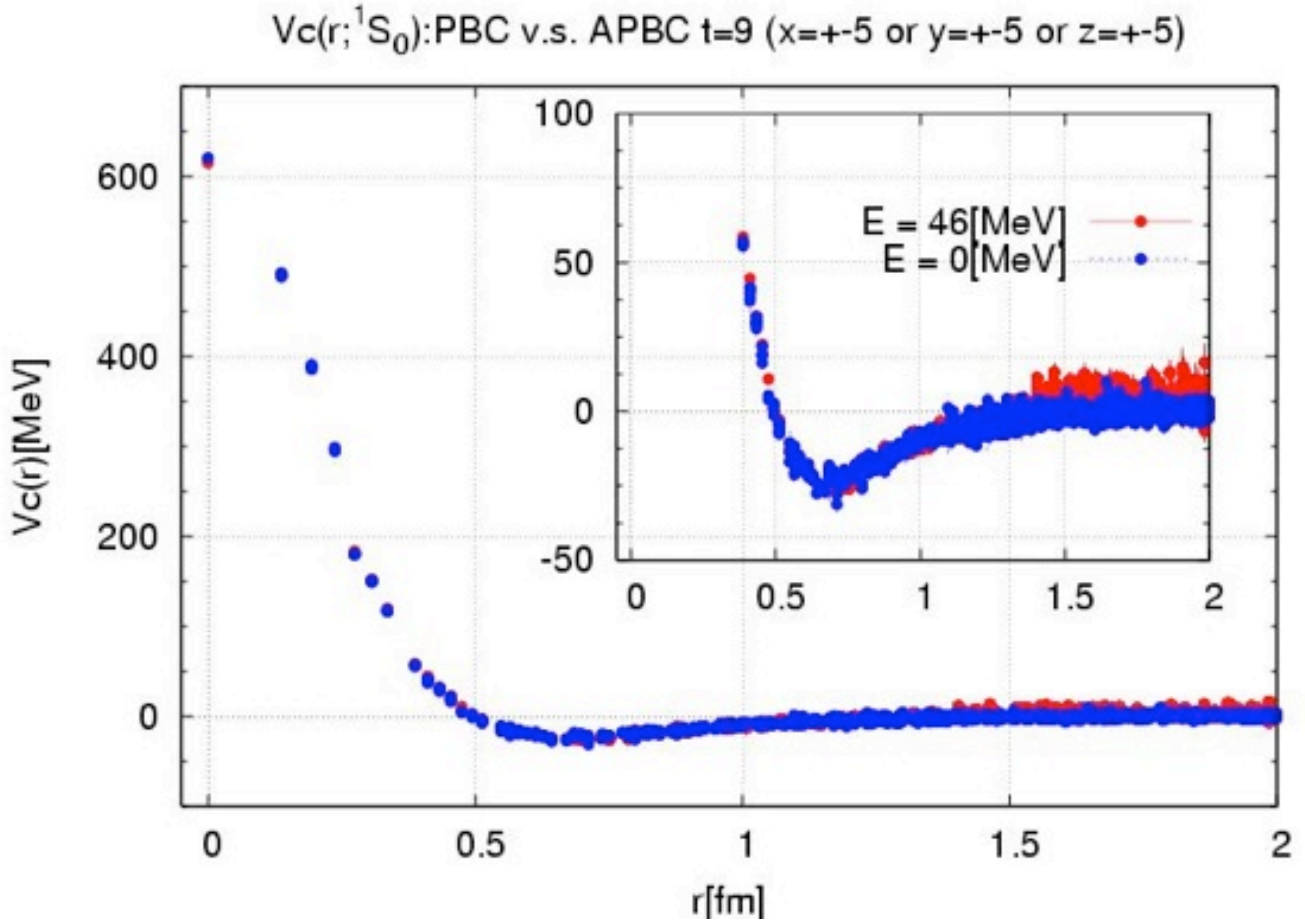


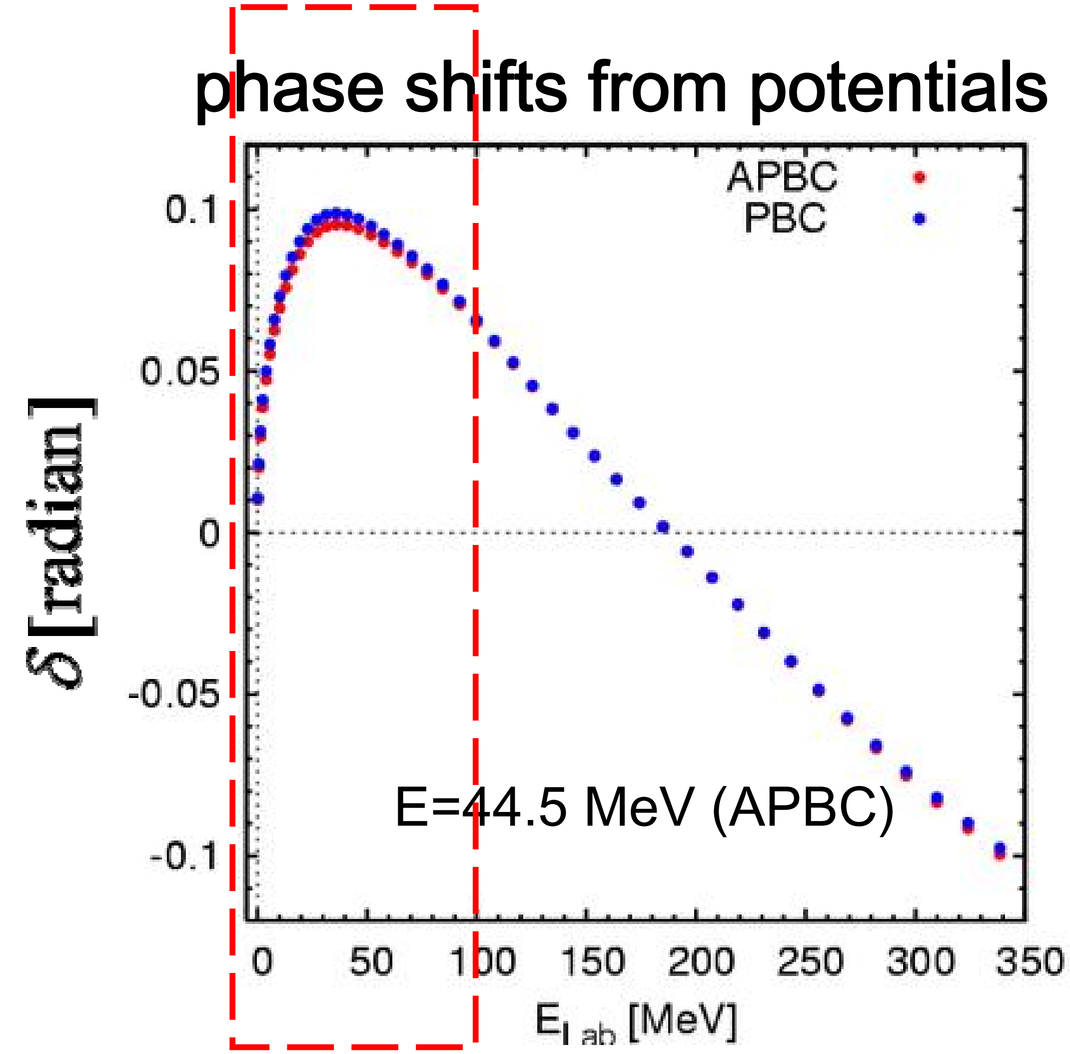


potentials









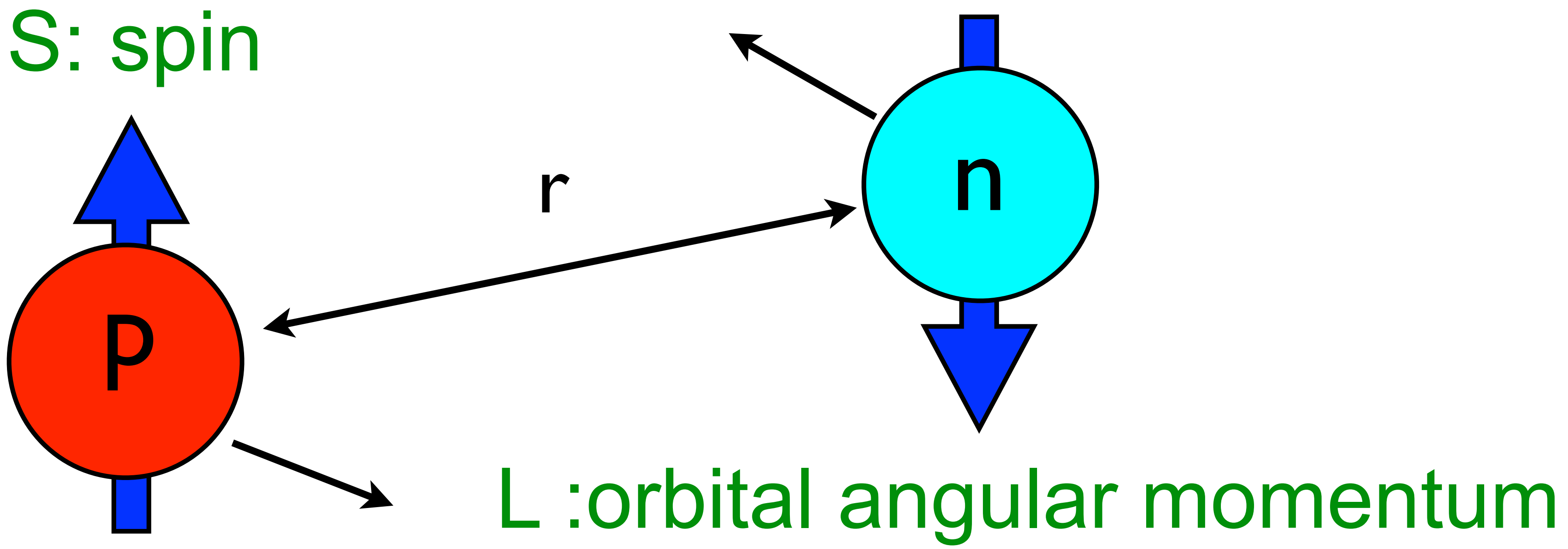
Higher order terms turn out to be very small at low energy in HAL QCD scheme.

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

4. More on nuclear force



Consider L=0, P(parity)=+

spin $\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$

$^{2S+1}L_J$

$\uparrow\uparrow$

$\uparrow\downarrow + \downarrow\uparrow \qquad \uparrow\downarrow - \downarrow\uparrow$

$\downarrow\downarrow$

→ $^3S_1 \qquad ^1S_0$

Tensor potential

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

$$(H_0 + V_C(r) + V_T(r)S_{12})\psi(\mathbf{r}; 1^+) = E\psi(\mathbf{r}; 1^+)$$

J=1, S=1

mixing between ${}^3S_1$ and ${}^3D_1$ through the tensor force

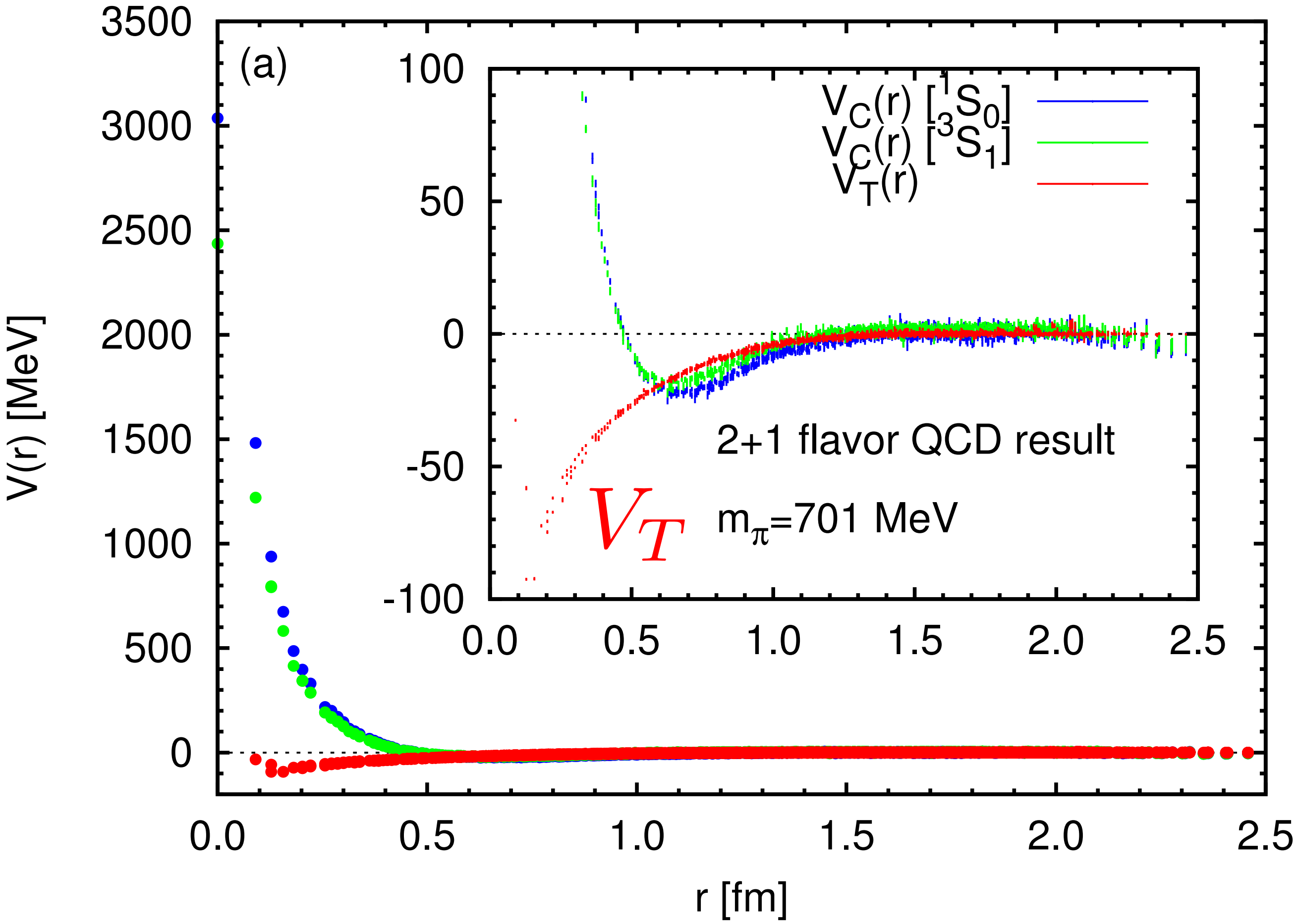
$$\psi(\mathbf{r}; 1^+) = \underbrace{\mathcal{P}\psi(\mathbf{r}; 1^+)}_{{}^3S_1} + \underbrace{\mathcal{Q}\psi(\mathbf{r}; 1^+)}_{{}^3D_1}$$

"projection" to L=0 "projection" to L=2

$$\begin{aligned} H_0[\mathcal{P}\psi](\mathbf{r}) + V_C(r)[\mathcal{P}\psi](\mathbf{r}) + V_T(r)[\mathcal{P}S_{12}\psi](\mathbf{r}) &= E[\mathcal{P}\psi](\mathbf{r}) \\ H_0[\mathcal{Q}\psi](\mathbf{r}) + V_C(r)[\mathcal{Q}\psi](\mathbf{r}) + V_T(r)[\mathcal{Q}S_{12}\psi](\mathbf{r}) &= E[\mathcal{Q}\psi](\mathbf{r}) \end{aligned}$$

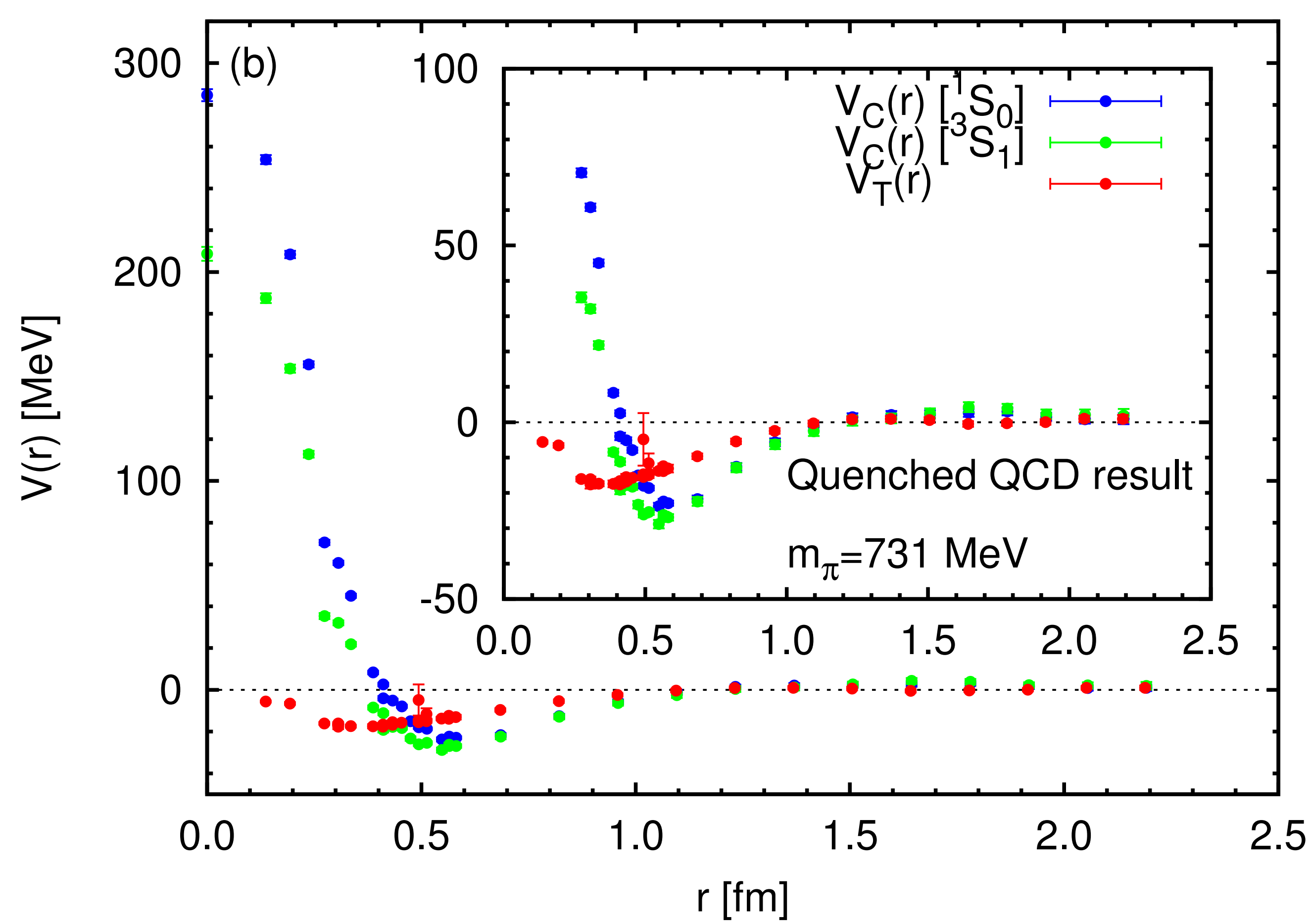
Potentials

full QCD



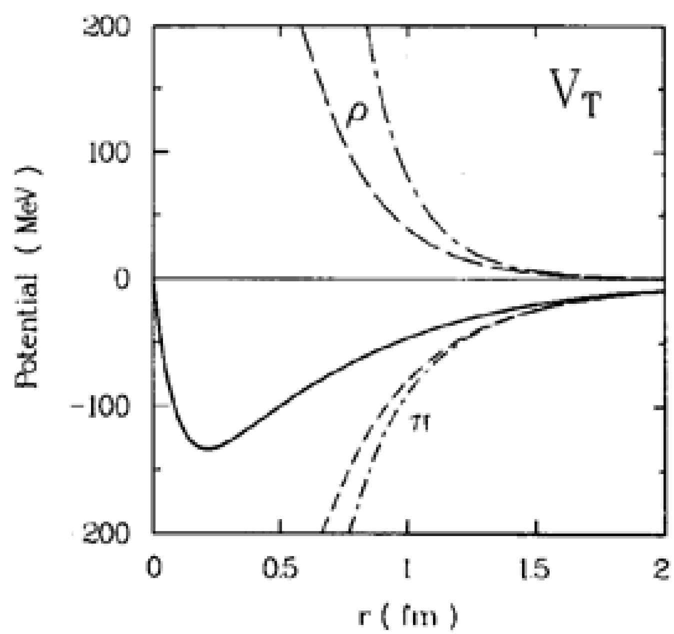
$a \simeq 0.091$ fm $\quad L \simeq 2.9$ fm

quenched QCD



$a \simeq 0.137$ fm $\quad L \simeq 4.4$ fm

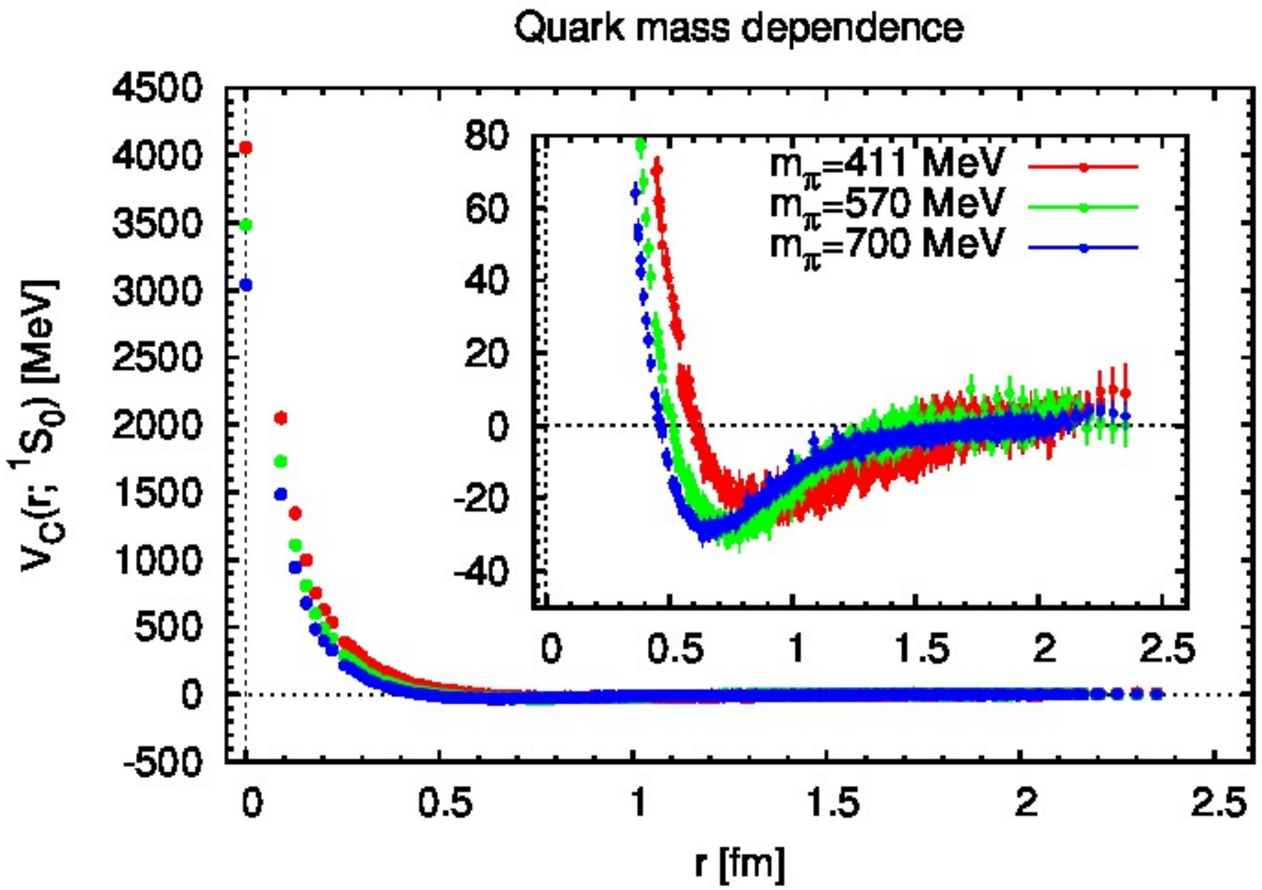
- no repulsive core in the tensor potential.
- the tensor potential is enhanced in full QCD

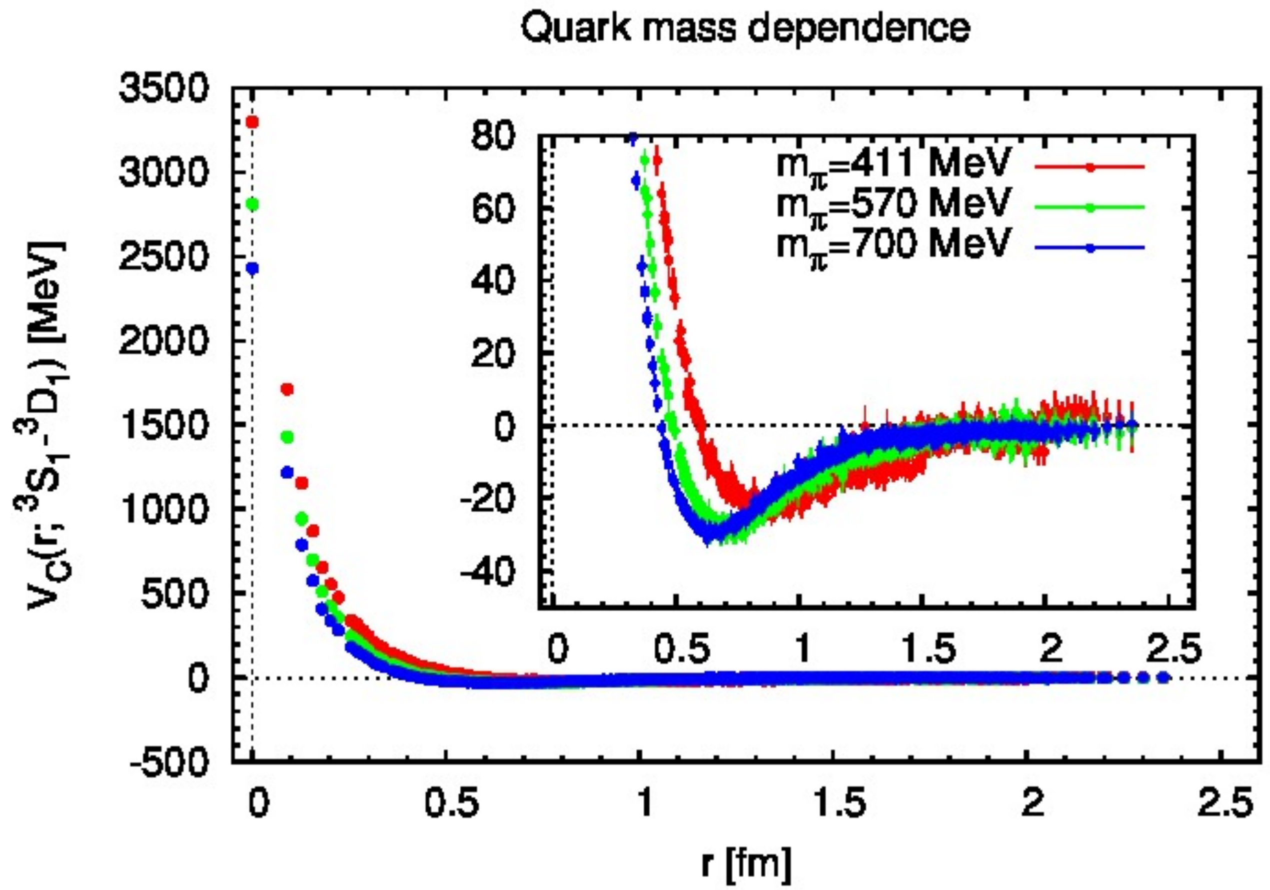


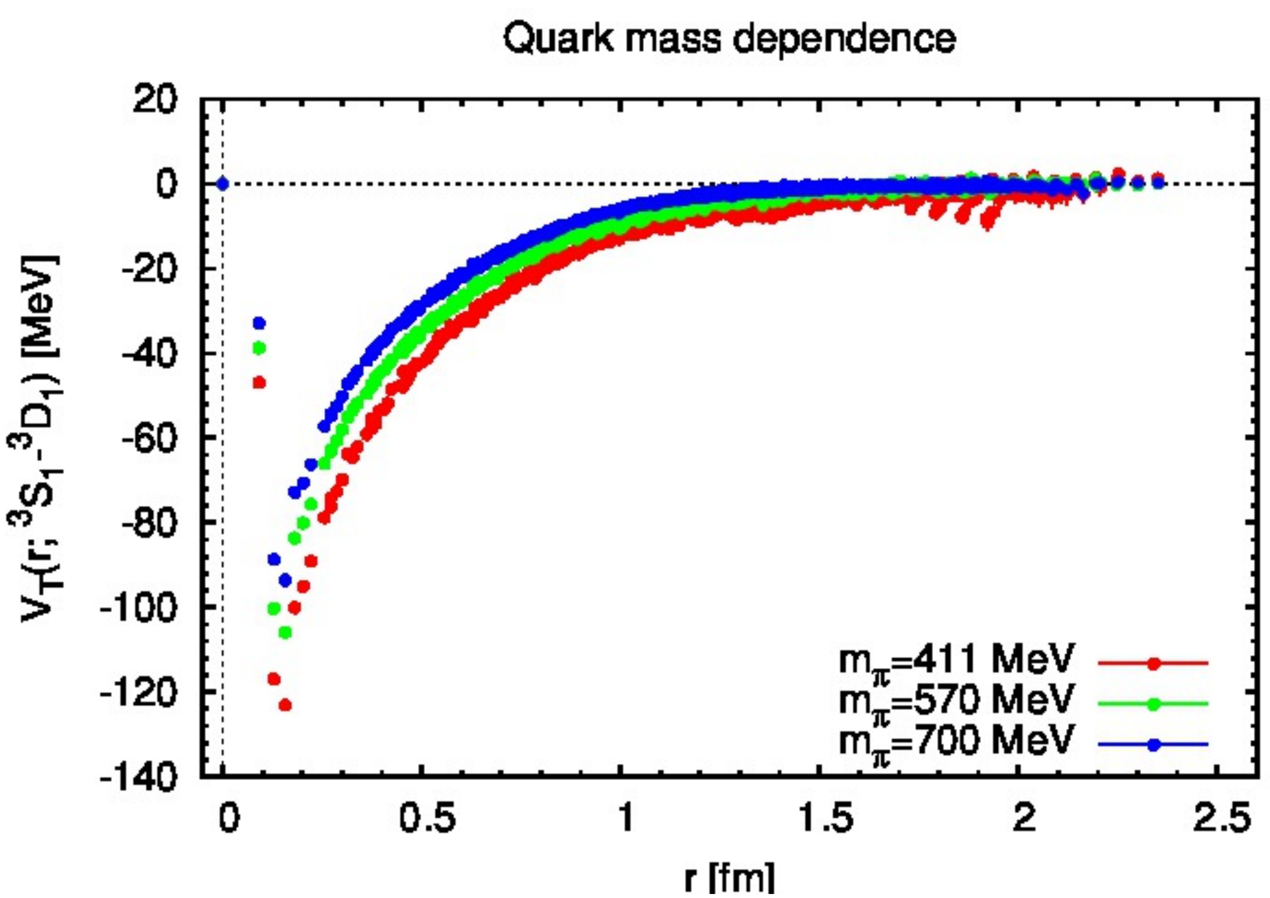
from R.Machleidt, Adv.Nucl.Phys.**19**

Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

Quark mass dependence (full QCD)







- the tensor potential increases as the pion mass decreases.
  - manifestation of one-pion-exchange ?
- both repulsive core and attractive pocket are also grow as the pion mass decreases.

Potentials for the negative parity sector

$$V_{NN}^{(I)}(\vec{r},\vec{\nabla}) = \underbrace{V_0^{(I)}(r) + V_\sigma^{(I)}(r)\cdot(\vec{\sigma}_1\cdot\vec{\sigma}_2)} + V_{\mathrm{T}}^{(I)}(r)\cdot S_{12} + V_{\mathrm{LS}}^{(I)}(r)\cdot\vec{L}\cdot\vec{S} + O(\nabla^2)$$

LO NLO

LO $V_{\mathrm{C}}(r) \equiv V_0(r) + V_\sigma(r)\cdot(\vec{\sigma}_1\cdot\vec{\sigma}_2)$

$$= \begin{cases} V_0(r) - 3V_\sigma(r) & \text{for S=0} \\ V_0(r) + V_\sigma(r) & \text{for S=1} \end{cases}$$

S=0,P=+ (I=1)	S=1,P=+ (I=0)	S=0,P=– (I=0)	S=1,P=– (I=1)
$V_{\mathrm{C}}(r)$	$V_{\mathrm{C}}(r), V_{\mathrm{T}}(r), V_{\mathrm{LS}}(r)$	$V_{\mathrm{C}}(r)$	$V_{\mathrm{C}}(r), V_{\mathrm{T}}(r), V_{\mathrm{LS}}(r)$

$^{2S+1}L_J$

- S=1 channel: $^3P_0,\ ^3P_1,\ ^3P_2{-}^3F_2$
  - Central & tensor forces in LO
  - Spin-orbit force in NLO

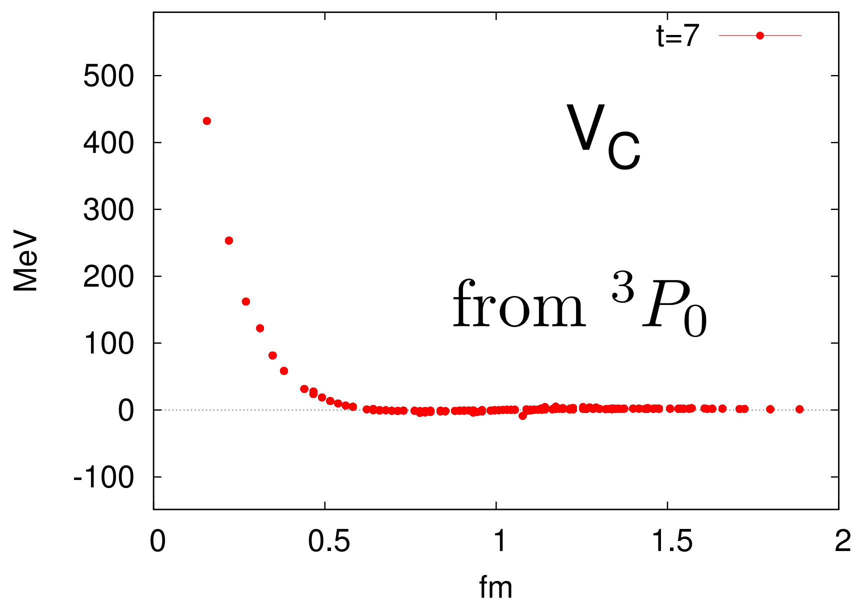
Preliminary Results (full QCD)

Murano et al. (HAL QCD), lat2012

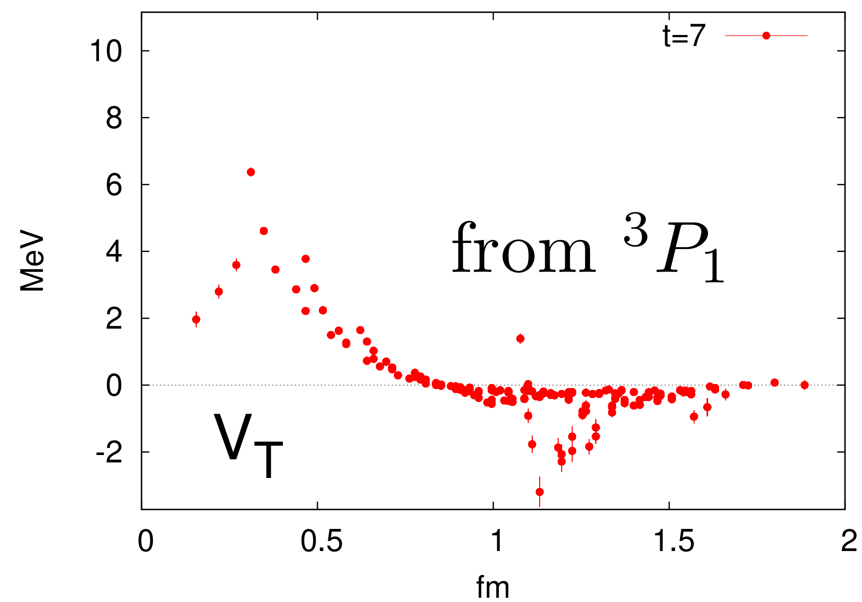
2-flavor QCD, a=0.16 fm

$m_\pi \simeq 1.1$ GeV

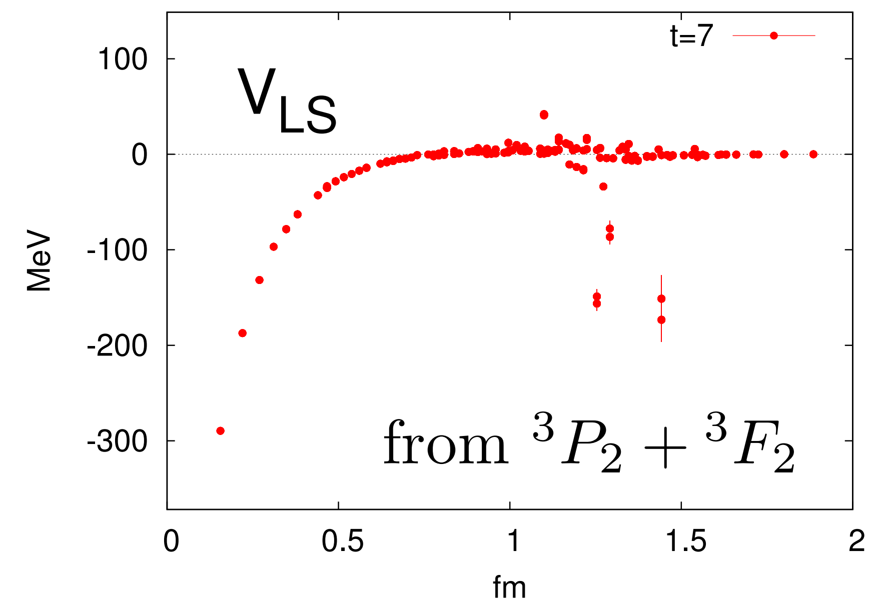
LO



LO



NLO



Very weak !