Extensions of the HAL QCD approach to inelastic and multi-particle scatterings in lattice QCD

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HAL QCD Collaboration

INT Workshop INT-15-53W "Nuclear Reactions from Lattice QCD" Institute for Nuclear Theory, University of Washington , Seattle, USA, March 11-12, 2013

1. Introduction

HAL QCD approach to Nuclear Force

Potentials in QCD ?

What are "potentials" (quantum mechanical objects) in quantum field theories such as QCD ?

HAL QCD strategy

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89.

define (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

Spin model: Balog et al., 1999/2001

$$
\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x} + \mathbf{r}, 0)N(\mathbf{x}, 0)|NN, W_k \rangle \qquad w_k = 2\sqrt{\mathbf{k}^2 + m_N^2}
$$

energy
energy

 $N(x) = \varepsilon_{abc}q^a(x)q^b(x)q^c(x)$: local operator

Key Property 1

Lin et al., 2001; CP-PACS, 2004/2005

$$
\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})
$$

$$
r = |\mathbf{r}| \to \infty
$$

 $\delta_l(k)$ scattering phase shift (phase of the S-matrix by unitarity) in QCD !

How can we extract it ?

cf. Luescher's finite volume method

define non-local but energy-independent "potential" as

$$
[\epsilon_k - H_0] \varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3 y \, \frac{U(\mathbf{x}, \mathbf{y}) \varphi_{\mathbf{k}}(\mathbf{y})}{\text{non-local potential}}
$$

$$
\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}
$$

A non-local but energy-independent potential can be constructed as

inner product

 $\mu = m_N/2$

reduced mass

$$
U(\mathbf{x}, \mathbf{y}) = \sum_{\mathbf{k}, \mathbf{k'}}^{W_k, W_{k'} \leq W_{\text{th}}} [\epsilon_k - H_0] \, \varphi_{\mathbf{k}}(\mathbf{x}) \eta_{\mathbf{k}, \mathbf{k'}}^{-1} \varphi_{\mathbf{k'}}^{\dagger}(\mathbf{y}) \qquad \qquad \eta_{\mathbf{k}, \mathbf{k'}}^{-1} : \text{inverse of } \eta_{\mathbf{k}, \mathbf{k'}} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k'}})
$$

$$
\varphi_{\mathbf{k}} \text{ is linearly independent.}
$$

For $\forall W_{\mathbf{p}} \leq W_{\text{th}} = 2m_N + m_{\pi}$ (threshold energy)

$$
\int d^3y U(\mathbf{x}, \mathbf{y}) \phi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k}, \mathbf{k'}} [\epsilon_k - H_0] \, \varphi_{\mathbf{k}}(x) \eta_{\mathbf{k}, \mathbf{k'}}^{-1} \eta_{\mathbf{k'}, \mathbf{p}} = [\epsilon_p - H_0] \, \varphi_{\mathbf{p}}(x)
$$

Note 1: Potential satisfying this is not unique.

Note2: Non-relativistic approximation is NOT used. We just take the equal-time frame.

expand the non-local potential in terms of derivative as $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla)\delta^3(\mathbf{x} - \mathbf{y})$

$$
V(\mathbf{x}, \nabla) = V_0(r) + V_\sigma(r)(\sigma_1 \cdot \sigma_2) + V_T(r)S_{12} + V_{LS}(r)\mathbf{L} \cdot \mathbf{S} + O(\nabla^2)
$$

\nLO
\nLO
\nLO

$$
\text{tensor operator} \qquad S_{12} = \frac{3}{r^2} (\sigma_1 \cdot \mathbf{x}) (\sigma_2 \cdot \mathbf{x}) - (\sigma_1 \cdot \sigma_2)
$$

local and energy independent coefficient function (cf. Low Energy Constants(LOC) in Chiral Perturbation Theory)

 $V_A(\mathbf{x})$

extract the local potential. At LO, for example, we simply have

$$
V_{\rm LO}(\mathbf{x}) = \frac{[\epsilon_k - H_0] \varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}
$$

 $\mathbf{Step\ 5}$ solve the Schroedinger Eq. in the infinite volume with this potential.

phase shifts and binding energy below inelastic threshold

(We can calculate the phase shift at all angular momentum.)

- $\delta_L(k)$ exact by construction
- $\delta_L(p\neq k)$ approximated one by the derivative expansion

expansion parameter

$$
\frac{W_p - W_k}{W_{\text{th}} - 2m_N} \simeq \frac{\Delta E_p}{m_\pi}
$$

We can check a size of errors at LO of the expansion. We can improve results by extracting higher order terms in the expansion.

2. Results from lattice QCD

Ishii *et al.* (HALQCD), PLB712(2012) 437.

Extraction of NBS wave function uniquely defined from it. This contradicts the fact discussed above that the fact discussed above that the pot
In this contradicts the potential is not an analyze that the potential is not an analyze that the potential is observable and therefore is not unique. This argument shows that the criticism of Ref. [18, 24, 25] is not unique. The criticism of Ref. [18, 25] is not unique. The criticism of Ref. [18, 25] is not unique shows that the c 3 Lattice formulation for the formulation for the formulation for the formulation $\overline{\mathsf{E}}$

NBS wave function *Potential* **4-pt Correlation function** $\varphi_{\mathbf k}(\mathbf r) = \langle 0 |$ In this section, we discuss the extraction, we discuss the NBS wave function from lattice $\mathcal{L}(\mathcal{L})$ the purpose of the correlation function on the correlation function on the lattice definition of the lattice d $F(\mathbf{r}, t - t_0) = \langle 0|T\{N(\mathbf{x} + \mathbf{r}, t)N(\mathbf{x}, t)\}\mathcal{J}(t_0)|0\rangle$ $\text{complete set for NN} \atop \text{for } (t, \ell) \in \text{Cov}(M) \times \text{Cov}($ $\begin{array}{lll} F({\bf r},t-t_0) & = & \langle 0|T\{N({\bf x}+{\bf r},t)N({\bf x},t)\} & \sum & |2N,W_n,s_1,s_2\rangle \langle 2N,W_n,s_1\rangle \end{array}$ $= \sum_{n, s_1, s_2} A_{n, s_1, s_2}$ $A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2 \rangle$ and the source for NN the source of the s \overline{a} wave function In this section, we define the extraction function \int which is a simulation from lattice \int simulations. For \int simulations. Fo this purpose, we consider the correlation function function $\mathcal{L}_{\mathbf{q}}$, $\mathcal{L}_{\mathbf{q}}$ later. By inserting the complete set and considering the baryon number conservation, we have n,s_1,s_2 $|2N,W_n, s_1, s_2\rangle\langle 2N,W_n, s_1, s_2|\mathcal{J}(t_0)|0\rangle$ $=$ \sum n,s_1,s_2 $= \sum A_{n,s_1,s_2} \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)}, \quad A_{n,s_1,s_2} = \langle 2N, W_n, s_1, s_2 | \overline{\mathcal{J}}(0) | 0 \rangle.$ complete set for NN **INBS wave function of this section of the extraction from lattice** \blacksquare **Potential** $\left(\begin{array}{cc} \begin{array}{cc} \end{array} & \begin{array}{c} \end{array} & \begin{array}{c} \end{array} & \begin{array}{c} \end{array} & \begin{$ $\sum_{i=1}^{n}$ 4-pt Correlation function source for NN $L(u + h) = \ln \left(\frac{1}{2} \left(\frac{1}{2} \right) - \frac{1}{2} \left(\frac{1}{2} \right) \right)$ $n \geq 1$ $= \frac{10|I|^{10} (A+1, t)^{10} (A, t)}{n, s_1, s_2}$ $=$ $\frac{21V, V(n, s_1, s_2)/21V, V(n, s_1, s_2)/210(0)}{100}$. For a large time separation to that the separation of the top of the $\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0|N(\mathbf{x} + \mathbf{r}, 0)N(\mathbf{x}, 0)|NN, W_k\rangle$ $d^3y U({\bf x},{\bf y})\varphi_{\bf k}({\bf y})$ $+ \cdot \cdot \cdot$

ground state saturation at large t ground state saturation at large t

$$
\lim_{(t-t_0)\to\infty} F(\mathbf{r}, t-t_0) = A_0 \varphi^{W_0}(\mathbf{r}) e^{-W_0(t-t_0)} + O(e^{-W_{n\neq 0}(t-t_0)})
$$

where W₀ is assumed to be the lowest energy of NN states. Since the source dependent term A0 is just the sour where W₀ is assumed to be the lowest energy of NN states. Since the source dependent term A0 is just the sour **NBS wave function**

This is a standard method in lattice QCD and was employed for our first calculation. This is a standard method in lattice QCD and was employed for our first calculation.

Ishii *et al.* (HALQCD), PLB712(2012) 437

Improved method

normalized 4-pt Correlation function $E^{2} = \sum A_{n} \varphi^{W_{n}}(\mathbf{r}) e^{-\Delta W_{n}t}$ \boldsymbol{n} $\Delta W_n = W_n - 2m_N =$ ${\bf k}_n^2$ m_N $-\frac{(\Delta W_n)^2}{4m_N}$ $4m_N$ $-\frac{\partial}{\partial z}$ ∂t $R(\mathbf{r},t)=\left\{H_0+U-\frac{1}{4m}\right\}$ $4m_N$ ∂^2 ∂t^2 $\sum_{i=1}^{n}$ $R(\mathbf{r},t)$ **potential Example 20 and 20 arrival contract and all energy-independent Leading Order** -40 -30 -20 -10 0 10 20 30 40 0 0.5 1 1.5 2 2.5 $V_C(\texttt{r})$ [MeV] $_{\rm C}$ (r) [MeV] r [fm] total 1st term 2nd term 3rd term $\left\{-H_0 - \frac{\partial}{\partial x}\right\}$ $\frac{\partial}{\partial t}$ + 1 $4m_N$ ∂^2 ∂t^2 $R(\mathbf{r},t) = \int d^3r' U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) = V_C(\mathbf{r})R(\mathbf{r},t) + \cdots$ **1st 2nd 3rd total** 3rd term(relativistic correction) is negligible. **energy-independent**

Ground state saturation is no more required. (advantage over finite volume method.)

Qualitative features of NN potential are reproduced.

(1)attractions at medium and long distances (2)repulsion at short distance(repulsive core)

It has a reasonable shape. The strength is weaker due to the heavier quark mass.

Need calculations at physical quark mass.

In order to extend the HAL QCD method to inelastic and/or multi-particle scatterings, we have to show

Key Property 1

Asymptotic behaviors of NBS wave functions for more than 2 particles

Key Property 2

Existence of energy independent potentials above inelastic thresholds

3. NBS wave functions for multi-particles

Key Property 1

Sinya Aoki, et al., arXiv.1303.2210 [hep-lat].

For simplicity,

(1) we consider scalar particles with "flavors"

(2) we assume no bound state exists.

as a sympath in terms of \blacksquare which in the phase shifts of phase shifts are constraint \blacksquare The unitarity of S-matrix implies the unit and mixing angles of the n-particle scattering. Conclusions and discussions are given in the n-particle scatter
Conclusions are given in the n-particle scattering. Conclusions are given in the n-particle scattering and dis For general n case, we introduce the non-relativistic approximation for the energy in the energy

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$$
T^{\dagger} - T = iT^{\dagger}T.
$$

Defining parametrization

$$
{}_0\langle [\boldsymbol{p}^A]_n|T|[\boldsymbol{p}^B]_n\rangle_0\ \equiv\ \delta(E^A-E^B)\delta^{(3)}(\boldsymbol{P}^A-\boldsymbol{P}^B)T([\boldsymbol{q}^A]_n,[\boldsymbol{q}^B]_n)
$$

(modified) Jacobi coordinates and momenta

The parties of the process of the p ² , ··· , p^X (modified) Jacobi coordinates and momenta and momenta are denoted by x_i , y_i , y Jacobi coordinates and corresponding momenta as \mathcal{L}_{m} (modified) Jacobi coordinates and momenta

(modified) Jacob coordinates and momenta

\n
$$
\boldsymbol{r}_k = \sqrt{\frac{k}{k+1}} \times \boldsymbol{r}_k^J, \qquad \boldsymbol{q}_k = \sqrt{\frac{k+1}{k}} \times \boldsymbol{q}_k^J \qquad \qquad \boldsymbol{r}_k^J = \frac{1}{k} \sum_{i=1}^k \boldsymbol{x}_i - \boldsymbol{x}_{k+1}, \quad \boldsymbol{q}_k^J = \frac{k}{k+1} \left(\frac{1}{k} \sum_{i=1}^k \boldsymbol{p}_i - \boldsymbol{p}_{k+1} \right),
$$

$$
\rightarrow
$$

$$
T([qA]n, [qB]n) \equiv T(QA, QB)
$$

=
$$
\sum_{[L],[K]} T_{[L][K]}(Q_A, Q_B) Y_{[L]}(\Omega_{Q_A}) \overline{Y_{[K]}(\Omega_{Q_B})}
$$

 $\left[\boldsymbol{q}^{X}\right]_{1},\boldsymbol{q}^{X}{}_{2},$ ${\bm Q}_X = ({\bm q}^X{}_1, {\bm q}^X{}_2, \cdots, {\bm q}^X{}_{n-1})$ mon $S_{(n-1)}$ momentum in D=3(n-1) dim. $\bm{Q}_X = (\bm{q}^X{}_1, \bm{q}^X{}_2, \cdots, \bm{q}^X{}_{n-1})$ momentum in D=3(n-1) dim. where Q

hyper-spherical harmonic function as a state of the unit of the written as a state of the written as a state o dimensions. With the non-relativistic approximation and orthogonal property, the unitarity dimensions satisfies

$$
\hat{L}^2 Y_{[L]}(\Omega_{\bm{s}}) = L(L+D-2)Y_{[L]}(\Omega_{\bm{s}})
$$

solution to the unitarity constraint $\frac{1}{2}$ where $\frac{1}{2}$ is used. By diagonalizing the unitary matrix $\frac{1}{2}$

$$
T_{[L][K]}(Q,Q) = \sum_{[N]} U_{[L][N]}(Q) T_{[N]}(Q) U_{[N][K]}^{\dagger}(Q),
$$

$$
T_{[L]}(Q) = -\frac{2n^{3/2}}{mQ^{3n-5}} e^{i\delta_{[L]}(Q)} \sin \delta_{[L]}(Q),
$$

"where c is if",

 p and contribution p in the standard p in the standard Jacobian standard J \mathbf{P} is a contracted of this section. Unfortunately, a relation of the phase shifts in the phase "phase shift" $\quad \delta_{[L]}(Q)$

Lippmann-Schwinger equation in QFT ward, however, to extend their derivations to extend their derivations to multi-particle systems. Instead, we u Lippmann-Schwinger equation[32], If we define S = 1 − it we obtain the set of the set of
If we obtain the set of the set o Lippmann-Schwinger equation in QFT From the Lippmann-Schwinger equation in QFT

$$
|\alpha\rangle_{\rm in} = |\alpha\rangle_{0} + \int d\beta \frac{|\beta\rangle_{0} T_{\beta\alpha}}{E_{\alpha} - E_{\beta} + i\varepsilon}, \qquad T_{\beta\alpha} = \frac{0}{\beta|V|\alpha\rangle_{\rm in}}, \qquad \frac{\frac{0}{\beta|T|\alpha\rangle_{0}}}{\text{off-shell}} = 2\pi\delta(E_{\alpha} - E_{\beta})T_{\alpha\beta}.
$$

$$
(H_0 + V)|\alpha\rangle_{\rm in} = E_{\alpha}|\alpha\rangle_{\rm in}, \quad \text{full}
$$

$$
H_0|\alpha\rangle_0 = E_{\alpha}|\alpha\rangle_0. \quad \text{free}
$$

which is found to be a powerful tool to study multi-particle systems. We assume in this system in this case of α

NBS wave functions NBS wave functions <u>where</u>

extions where $\frac{1}{2}$ is the normalization from the unit of $\frac{1}{2}$

$$
\Psi_{\alpha}^{n}([\boldsymbol{x}]) = \t{_{in}} \langle 0|\varphi^{n}([\boldsymbol{x}],0)|\alpha\rangle_{\text{in}}, \qquad \qquad \varphi^{n}([\boldsymbol{x}],t) = T\{\prod_{i=1}^{n}\varphi_{i}(\boldsymbol{x}_{i},t)\},
$$

$$
\rightarrow
$$

 $\frac{1}{2}$

 $W^n(\alpha) = (0, \alpha^n(\alpha)$

$$
\Psi_{\alpha}^{n}([\boldsymbol{x}]) = \frac{1}{Z_{\alpha}} \delta_{0} \langle 0 | \varphi^{n}([\boldsymbol{x}], 0) | \alpha \rangle_{0} + \int d\beta \frac{1}{Z_{\beta}} \frac{\delta_{0} \langle 0 | \varphi^{n}([\boldsymbol{x}], 0) | \beta \rangle_{0} T_{\beta \alpha}}{E_{\alpha} - E_{\beta} + i\varepsilon}.
$$
\n
$$
\int_{\delta_{0} \langle 0 | \varphi^{n}([\boldsymbol{x}], 0) | [\boldsymbol{k}]_{n} \rangle_{0}} = \left(\frac{1}{\sqrt{(2\pi)^{3}}} \right)^{n} \prod_{i=1}^{n} \frac{1}{\sqrt{2E_{k_{i}}}} e^{i\boldsymbol{k}_{i}\boldsymbol{x}_{i}}
$$

D-dimensional hyper-coordinates $\overline{\text{max}}$ D-dimer isional nyper-coord $\mathsf S$

with LD <mark>+ D−2</mark>

$$
\Psi^{n}(\boldsymbol{R},\boldsymbol{Q}_{A})\,=\,C\left[e^{i\boldsymbol{Q}_{A}\cdot\boldsymbol{R}}+\frac{2m}{2\pi n^{3/2}}\int d^{D}Q\,\frac{e^{i\boldsymbol{Q}\cdot\boldsymbol{R}}}{Q_{A}^{2}-Q^{2}+i\varepsilon}T(\boldsymbol{Q},\boldsymbol{Q}_{A})\right]
$$

of hyper-spherical h **imonic function**
The second seco Expansion in terms of hyper-spherical harmonic function ^A − Q² + iε In \mathcal{S} is defined by the set of \mathcal{S} h vper-spherical ham bherical harm anic function and the state of the state condition in terms of hyper-spherical narmonic function and expansion and eigenvalues ϵ

with equal of the control of the control

$$
e^{i\mathbf{Q}\cdot\mathbf{R}} = (D-2)!! \frac{2\pi^{D/2}}{\Gamma(D/2)} \sum_{[L]} i^L j^D_L(QR) Y_{[L]}(\Omega_R) \overline{Y_{[L]}(\Omega_Q)},
$$

2)
2 D−44 D−44

hyper-spherical Bessel function

$$
\Psi^n(\boldsymbol{R}, \boldsymbol{Q}_A) \ = \ \sum_{[L], [K]} \Psi^n_{[L], [K]}(R, Q_A) Y_{[L]}(\Omega_{\boldsymbol{R}}) \overline{Y_{[K]}(\Omega_{\boldsymbol{Q}_A})},
$$

² and the Bessel function of the first kind, J^L^D (x). <u>Asympto</u> j
L (x) = Γ(D−2) = Γ(D−2) : behavior of $\mathcal C$ $\overline{}$!!! " Asymptotic behavior of NBS wave functions and \overline{P} and \overline{P} are Bessel functions of the first and second kinds, respectively.

 $R\rightarrow\infty$

$$
\Psi_{[L],[K]}^{n}(R,Q_{A}) \simeq C i^{L} \frac{(2\pi)^{D/2}}{(Q_{A}R)^{\frac{D-1}{2}}} \sum_{[N]} U_{[L][N]}(Q_{A}) e^{i\delta_{[N]}(Q_{A})} U_{[N][K]}^{\dagger}(Q_{A})
$$

$$
\times \sqrt{\frac{2}{\pi}} \frac{\sin (Q_{A}R - \Delta_{L} + \delta_{[N]}(Q_{A}))}{\Delta_{L} = \frac{2L_{D} - 1}{4}\pi.
$$

scattering wave with "ph $\overline{}$ with price e shift" ! Scalightly wave with phase shift. scattering wave with "phase shift" !

4. Energy-independent potential above inelastic thresholds

Key Property 2

Sinya Aoki, et al., Phys. Rev. D (in press).

Let us consider $NN \rightarrow NN, NN\pi$ T_{H} consider $N N \sim N N N_{\text{tr}}$

$$
W_{\text{th}}^2 = 2m_N + 2m_\pi
$$

\n
$$
\Delta_1 = [W_{\text{th}}^1, W_{\text{th}}^2)
$$

\n
$$
W_{\text{th}}^1 = 2m_N + m_\pi
$$

\n
$$
W_{\text{th}}^1 = 2m_N + m_\pi
$$

\n
$$
N(x)N(y)
$$

\n
$$
\Delta_0 = [W_{\text{th}}^0, W_{\text{th}}^1)
$$

\n
$$
W_{\text{th}}^0 = 2m_N
$$

\n
$$
W_{\text{th
$$

4 NBS wave functions

$$
Z_N \varphi_{W,c_0}^{00}(\boldsymbol{x}_0) = \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\}|NN,W,c_0\rangle_{\text{in}},
$$

\n
$$
Z_N Z_{\pi}^{1/2} \varphi_{W,c_0}^{10}(\boldsymbol{x}_0,\boldsymbol{x}_1) = \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\pi(\boldsymbol{x}+\boldsymbol{x}_1,0)\}|NN,W,c_0\rangle_{\text{in}},
$$

\n
$$
Z_N \varphi_{W,c_1}^{01}(\boldsymbol{x}_0) = \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\}|NN+\pi,W,c_1\rangle_{\text{in}},
$$

\n
$$
Z_N Z_{\pi}^{1/2} \varphi_{W,c_1}^{11}(\boldsymbol{x}_0,\boldsymbol{x}_1) = \langle 0|T\{N(\boldsymbol{x},0)N(\boldsymbol{x}+\boldsymbol{x}_0,0)\pi(\boldsymbol{x}+\boldsymbol{x}_1,0)\}|NN+\pi,W,c_1\rangle_{\text{in}},
$$

 ω^{ij} ([x]) $\omega^{(i)}$ where $\epsilon \nabla^{i}$ in the energies for ω [m] ω = 1 $\sum_{j=1}^{n}$ \mathbf{r} , where \mathbf{r} and \mathbf{r} are renormalized nucleon and pionized nucleon and pionize $\varphi^{ij}_{\rm W}$ $W_{N,c_j}([\mathbf{x}]_i)$ *i*(*j*): number of π 's in the operator(state) $[\mathbf{x}]_0 = \mathbf{x}_0$ $[\mathbf{x}]_1 = \mathbf{x}_0, \mathbf{x}_1$. $\lbrack \bm{x}]_0 = \bm{x}_0 \quad \lbrack \bm{x} \rbrack_1 = \bm{x}_0, \bm{x}_1.$

coupled channel equation We consider the consideration the coupled channel Schrücken in NN α NN α given by the set of the olds for the NN scattering in the center of mass system. In this report we only consider production productions whose n-th threshold energy is given by Wing and Wing a

$$
(E_W^k - H_0^k)\varphi_{W,c_i}^{ki} = \sum_{l=0,1} \int \prod_{n=0}^l d^3y_n \, U^{kl}([\boldsymbol{x}]_k, [\boldsymbol{y}]_l) \varphi_{W,c_i}^{li}([\boldsymbol{y}]_l), \quad k, i \in (0,1)
$$

exists ? \sim the total energy W, the kinetic energy \sim α vioto α

olds for the NN scattering in the NN scattering in the center of mass system. In this report we only consider \sim

 P roof of existence for Π n ≥ 1 cannot be determined from W in general, we restrict our considerations in this paper. We restrict our co
Determined from W in this paper we restrict our considerations in this paper we restrict our considerations in Proof of existence for U is important to construct potentials from the Schröden and Engineerust potentials from the Schröden and Engineerust potentials from the Schröden and Engineerust potentials from the Schröden and Eng For this purpose, we define vectors from the second vectors from the second vectors from the second vectors at
Define vectors from the second vectors from the second vectors at W ⊆ ∆1 as well as well as well as well as we

Define a vector of NBS wave functions as $\mathcal{L} = \mathcal{L} \times \mathcal{L}$ cannot be determined from W in general, we restrict our considerations in this paper. r_{c} and r_{c} where non-relativistic, so that we can write we can write we can write we can write write write we can write writ Define a vector of NBS wave functions as to show that a W-independent 2 × 2 potential matrix Ukl exists.
The W-independent 2 × 2 potential matrix Ukl exists. The W-independent 2 × 2 potential matrix Ukl exists. The For the defined vector of this wave functions as Define a Vector of NBS wave functions as Defined at W ⊆ ∠ 2 as

$$
\begin{array}{lcl} \varphi_{W,c_i}^i \,\equiv \,\left(\varphi_{W,c_i}^{0i}([{\bm x}]_0), \varphi_{W,c_i}^{1i}([{\bm x}]_1) \right)^T, \quad i=0,1, \qquad & W \in \Delta_1 \\ \\ \varphi_{W,c_0}^{0} \,\equiv \,\left(\varphi_{W,c_0}^{00}([{\bm x}]_0), \varphi_{W,c_0}^{10}([{\bm x}]_1) \right)^T, \qquad & W \in \Delta_0 \end{array}
$$

Norm kernel As in the elastic case, we introduce the norm kernel in the space spanned by ϕⁱ and in Equal series I(W) α is a form where I(W) α and I(W) α and I(W) α and W α and W α . Otherwise, we will also an analyze α and I(W) α and I Here is j run over different ranges depending on values of W1, W2 such that i ∈ I(W1) such that i ∈ I(W1) such t

$$
\mathcal{N}^{ij}_{W_1c_i,W_2d_j} \ = \ \Big(\varphi^i_{W_1,c_i},\varphi^j_{W_2,d_j}\Big) \equiv \sum_{k=0,1} \int \prod_{l=0}^k d^3x_l\, \overline{\varphi^{ki}_{W_1,c_i}([\bm x]_k)} \varphi^{kj}_{W_2,d_j}([\bm x]_k).
$$

Here indices i, j run over different ranges depending on values of W1, W² such that i ∈ I(W1) Inverse $\frac{1}{2}$ long as $\frac{1}{2}$

$$
\sum_{W\in \Delta_0+\Delta_1}\sum_{h\in I(W),\,e_h}(\mathcal{N}^{-1})^{ih}_{W_1c_i,We_h}\mathcal{N}^{hj}_{We_h,W_2d_j}\;=\;\delta^{ij}\delta_{W_1,W_2}\delta_{c_i,d_j}
$$

Structure: whas a following structure:

$$
\mathcal{N} = \begin{pmatrix} \mathcal{N}^{00}(\Delta_0, \Delta_0), \ \mathcal{N}^{00}(\Delta_0, \Delta_1), \ \mathcal{N}^{01}(\Delta_0, \Delta_1) \\ \mathcal{N}^{00}(\Delta_1, \Delta_0), \ \mathcal{N}^{00}(\Delta_1, \Delta_1), \ \mathcal{N}^{01}(\Delta_1, \Delta_1) \\ \mathcal{N}^{10}(\Delta_1, \Delta_0), \ \mathcal{N}^{10}(\Delta_1, \Delta_1), \ \mathcal{N}^{11}(\Delta_1, \Delta_1) \end{pmatrix}
$$

energy state

bra, ket

$$
\begin{array}{ll}\text{bra, ket} & \langle [\bm x]_k | \varphi^i_{W,c_i} \rangle \; = \; \varphi^{ki}_{W,c_i}([\bm x]_k), \\ & \langle \psi^i_{W,c_i} | [\bm x]_k \rangle \; = \; \sum_{W_1 \in \Delta_0 \cup \Delta_1} \sum_{j \in I(W_1), d_j} (\mathcal{N}^{-1})^{ij}_{Wc_i, W_1d_j} \overline{\varphi^{kj}_{W_1,d_j}([\bm x]_k)} \\ & \text{orthogonality} & \langle \psi^i_{W_1,c_i} | \varphi^j_{W_2,d_j} \rangle \; = \; \sum_{k=0,1} \int \prod_{l=0}^k d^3x_l \, \langle \psi^i_{W_1,c_i} | [\bm x]_k \rangle \langle [\bm x]_k | \varphi^j_{W_2,d_j} \rangle = (\mathcal{N}^{-1} \cdot \mathcal{N})^{ij}_{W_1c_i, W_2d_j} \\ & \; = \; \delta^{ij} \delta_{W_1,W_2} \delta_{c_i,d_j}. \end{array}
$$

Abstract operators extending operators and U such that \mathcal{A} Introducing operators EW , H0 and U such that EW , H0 and U such that EW , H0 and U such that EW , H0 and U su

$$
\langle [\boldsymbol{x}]_k | (E_W - H_0) | [\boldsymbol{y}]_l \rangle \equiv \delta_{kl} (E_W^k - H_0^k) \prod_{n=0}^k \delta^{(3)} (\boldsymbol{x}_n - \boldsymbol{y}_n)
$$

$$
\langle [\boldsymbol{x}]_k | U | [\boldsymbol{y}]_l \rangle \equiv U^{kl} ([\boldsymbol{x}]_k, [\boldsymbol{y}]_l),
$$

Abstract coupled channel equation (15) can be compact to compact coupled channel equation the coupled channel Schr¨odinger equation (15) can be compactly written as³ Abstract coupled channel equation (15) can be compact complex as a set of the compact of the compact

W1∈∆0∪∆¹

$$
(E_W - H_0)|\varphi^i_{W,c_i}\rangle = U|\varphi^i_{W,c_i}\rangle.
$$

!

!

 \sim is in the above equation of non-local satisfies the above equation \sim construction of non-local coupled channel potential ————————————————————
∩annel potential

U = 1

je
Ige

$$
U = \sum_{W \in \Delta_0 \cup \Delta_1} \sum_{i \in I(W)} \sum_{c_i} (E_W - H_0) |\varphi^i_{W, c_i} \rangle \langle \psi^i_{W, c_i}|,
$$

 \bigcap $\sqrt{2}$ $\ddot{\bullet}$

$$
U|\varphi_{W,c_i}^i\rangle = \sum_{W_1 \in \Delta_0 \cup \Delta_1} \sum_{j \in I(W_1)} \sum_{d_j} (E_W - H_0)|\varphi_{W_1,d_j}^j\rangle\langle \psi_{W_1,d_j}^j|\varphi_{W,c_i}^i\rangle = (E_W - H_0)|\varphi_{W,c_i}^i\rangle
$$

for example, one can use eq. (18) instead of eq. (18) instead of eq. (18) instead of eq. (18) for α instead of eq. (17) for α $\overline{}$, so that the resulting potential potenti $\frac{1}{\sqrt{1-\frac{1}{2}}}\left\vert \frac{1}{\sqrt{1-\frac{1}{2}}}\right\vert$ \vert Energy independent (coupled channel) potential exists above the inelastic threshold. \vert An energy-independent potential matrix U independent potential matrix U independent potential matrix U is not
U independent potential matrix U is not unique since, and unique since, and unique since, and unique since, an for example, one can use the cannot contain the instead of the contact of the solar the instant of the results
May independent (coupled channel) potential evists above the inelastic threshold from eq. (28) different from the one with eq. (28) differs from the one with eq. (18). Energy independent (coupled channel) potential exists above the inelastic threshold.

Hermiticity Finally let us consider the Hermiticity of U. A matrix element of U. A matrix elemen

$$
U^{ij}_{W_1c_i,W_2d_j} \equiv \langle \varphi^i_{W_1,c_i} | U | \varphi^j_{W_2,d_j} \rangle = \langle \varphi^i_{W_1,c_i} | (E_{W_2} - H_0) | \varphi^j_{W_2,d_j} \rangle,
$$

$$
(U^{\dagger})^{ij}_{W_1c_i,W_2d_j} \; = \; \overline{\langle \varphi^j_{W_2,d_j}|(E_{W_1}-H_0)|\varphi^i_{W_1,c_i}\rangle} = \langle \varphi^i_{W_1,c_i}|(E_{W_1}-H_0)|\varphi^j_{W_2,d_j}\rangle.
$$

 \mathbb{C}^1 effectively Hermite for $E_{W_1} = E_{W_2}$

cases, where the total energy satisfies W \sim 1. As discussed before, the validity of validity of validity of The construction of U can easily be generalized to $NN + n\pi \rightarrow NN + k\pi$ $\Lambda \Lambda \rightarrow \Lambda \Lambda, N \Xi, \Sigma \Sigma,$

5. Related results

Kenji Sasaki, et al. (HAL QCD), in preparation

Takumi Doi et al. (HAL QCD), PTP 127 (2012) 723

u,d quark masses lighter

Kenji Sasaki (*University of Tsukuba*) for HAL QCD collaboration *S0 S0* coupled channel 3x3 potentials

 $\Lambda\Lambda$ and $N\Xi$ phase shift **Preliminary**! ΛΛΛΛ*and and* ΝΞΝΞ *phase shifts phase shifts phase shifts phase shifts*

Bound H-dibaryon **Resonance H** Resonance H Bound H-dibaryon $\sum_{i=1}^{n}$

This suggests H-dibaryon becomes resonance at physical point. We can see the capture of the clear resonance at physical point.

The H-dibaryon resonance energy is close to \mathbb{R}^n is close to \mathbb{R}^n is close to \mathbb{R}^n threshold...

scalar/isoscalar TNF is observ (Right) shows reduced the scalar TNF, where the results is not the region of \mathbb{R}^n

further study is needed to confirm this result. short-range TNF is phenomenologically required to explain the saturation density of nuclear matter,

> $p = \frac{1}{2}$ Analysis by OPE (operator product expansion) in QCD predicts $\bm{\delta}$ ort-distance repulsions in TNF is a contraction of the saturation and Weisz NJP14(2012)043046 8.2 Meson-baryon interactions universal short distance repulsions in TNF Aoki, Balog and Weisz, NJP14(2012)043046 s_{max} is subsection, the potential method to the method to the method to the meson-baryon system are meson-baryon system are β

Aoki, Balog and Weisz, NJP14(2012)043046 etc., this is very encouraging result. Of course, further study is necessary to confirm this result, e.g., the

6. Conclusion

- HAL QCD approach is shown to be a promising method to extract hadronic interactions in lattice QCD.
	- ground state saturation is not required.
	- Calculate potential in lattice QCD on a finite box.
	- Calculate phase shift by solving (coupled channel) Shroedinger equation in infinite volume.
	- bound-state/resonance/scattering
- Extensions of the HAL QCD method to inelastic/multi-particle scatterings
	- Asymptotic behavior of the NBS wave functions
	- Existence of non-local but energy-independent coupled channel potentials
	- some preliminary results
- Future problems: Nuclear reactions ? Your inputs are important !

Thank you !

Backup slides

Convergence of velocity expansion

If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

Need to be checked at lighter pion mass in 2+1 flavor QCD.

Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

4. More on nuclear force

Tensor potential

Aoki, Hatsuda, Ishii, PTP 123 (2010)89 arXiv:0909.5585

$$
(H_0 + V_C(r) + V_T(r)S_{12})\psi(\mathbf{r}; 1^+) = E\psi(\mathbf{r}; 1^+)
$$

 $J=1, S=1$

mixing between 3S_1 and 3D_1 through the tensor force

$$
\psi(\mathbf{r};1^+) = \mathcal{P}\psi(\mathbf{r};1^+) + \mathcal{Q}\psi(\mathbf{r};1^+)
$$

"projection" to L=0 "projection" to L=2

 $H_0[\mathcal{P}\psi](\mathbf{r}) + V_C(r)[\mathcal{P}\psi](\mathbf{r}) + V_T(r)[\mathcal{P}S_{12}\psi](\mathbf{r}) = E[\mathcal{P}\psi](\mathbf{r})$ $H_0[\mathcal{Q}\psi](\mathbf{r}) + [V_C(r)[\mathcal{Q}\psi](\mathbf{r}) + [V_T(r)][\mathcal{Q}S_{12}\psi](\mathbf{r}) = E[\mathcal{Q}\psi](\mathbf{r})$

Potentials

full QCD quenched QCD

- no repulsive core in the tensor potential.
- the tensor potential is enhanced in full QCD

- With(decreasing(m(pion),(mass decreases. • the tensor potential increases as the pion
	- manifestation of one-pion-exchange?
- ϵ epulsive core and attractive row as the pion mass decre $\frac{1}{2}$ repulsive ears and off $\frac{1}{2}$ • both repulsive core and attractive pocket are 3 grow as the pion mass ded also grow as the pion mass decreases.

Potentials for the negative parity sector $R_{\text{obs}}(t)$ nuclear $\mathbf{f}_{\text{obs}}(t)$ and $\mathbf{f}_{\text{obs}}(t)$ $\overline{}$ Ē *r*, $\overline{\mathbf{C}}$ <u>*<u>v</u> v i* /*i* $\frac{1}{2}$ </u> C *r*, $\overline{\mathbf{K}}$ ∇)δ (<u>or the negative parity sector</u>

)

$$
V_{NN}^{(I)}(\vec{r}, \vec{\nabla}) = V_0^{(I)}(r) + V_{\sigma}^{(I)}(r) \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_{\text{T}}^{(I)}(r) \cdot S_{12} + V_{\text{LS}}^{(I)}(r) \cdot \vec{L} \cdot \vec{S} + O(\nabla^2)
$$

LO
LO

$$
V_{\text{C}}(r) = V_0(r) + V_{\sigma}(r) \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2)
$$

$$
= \begin{cases} V_0(r) - 3V_{\sigma}(r) & \text{for } S=0 \\ V_0(r) + V_{\sigma}(r) & \text{for } S=1 \end{cases}
$$

 $2S+1$ _L_J

UNN (*I*)

- S=1 channel: ${}^{3}P_{0}$, ${}^{3}P_{1}$, 3
	- Central & tensor forces in LO
	- Spin-orbit force in NLO

Fig. 7. Potentials in order from 3PD very weak !

