Monte Carlo simulations for $(e, e'pp)$ reactions

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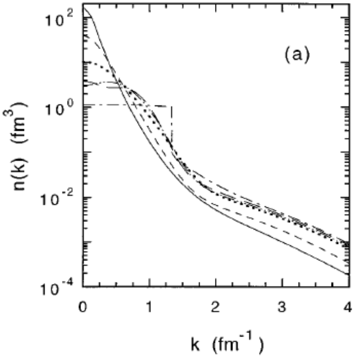
INT Workshop “Nuclear Structure and Dynamics at Short Distances”

i) `PRC84, 031302(R) (2011)`
ii) `PRC86, 044619 (2012)`
iii) `arXiv:1210.6175`

Short Range Correlations in Nuclei



C. Ciofi degli Atti et al. PRC53 4 (1996)

Shell Model (SM) gives a fine description of nuclear structure but it is unable to account for short (intermediate) ranged correlations due to strong interactions between nucleons. A reflection of these correlations is the high-momentum tail:

- for $300 < k < 600\,\mathrm{MeV}$: universal and generally assumed to be generated via tensor component of *NN* force.
- for $k > 600\,\mathrm{MeV}$: generated by central repulsive core.

Short Range Correlations in Nuclei

How to include central repulsive core and tensor component of *NN* force in SM?

- Or can high momentum components be generated from mean field wave functions Bogner and Roscher PRC 86, 064304
- A time-honored method to account for the effect of correlations (classical and quantum systems): correlation functions
- Realistic wave functions $\mid \overline{\Psi} \rangle$ after applying a many-body correlation operator to Slater determinant $\mid \Psi \rangle$

$$\mid \overline{\Psi} \rangle = \frac{1}{\sqrt{\langle \Psi \mid \widehat{\mathcal{G}}^{\dagger} \widehat{\mathcal{G}} \mid \Psi \rangle}} \widehat{\mathcal{G}} \mid \Psi \rangle .$$

- The $\widehat{\mathcal{G}}$ reflects the full complexity of the NN force but is dominated by the central and tensor correlations

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left[\prod_{i<j=1}^{A} \left(1 - g_c(r_{ij}) + f_{t\tau}(r_{ij}) \widehat{S_{ij}} \vec{\tau}_i . \vec{\tau}_j \right) \right]$$

Short Range Correlations in Nuclei

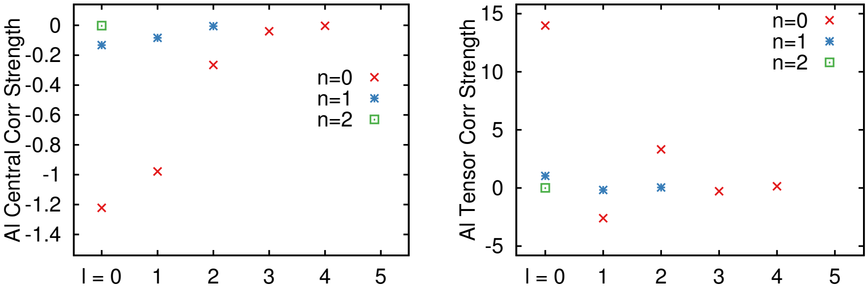
Which SM nucleon pairs are most susceptible to short-range *NN* force?

- Closest configurations in coordinate space.
- High momentum tail at $300 < k < 600\,\mathrm{MeV}$:
  the pairs mostly susceptible to tensor component of *NN* force
- High momentum tail at $k > 600\,\mathrm{MeV}$:
  pairs mostly susceptible to central repulsive core.

Short Range Correlations in Nuclei

Which SM nucleon pairs are most susceptible to short-range *NN* force?

Figure: Comparison of the correlation strength (overlap of correlation function and wave function) of all relative pair configurations (n, l) in Al.



Short Range Correlations in Nuclei

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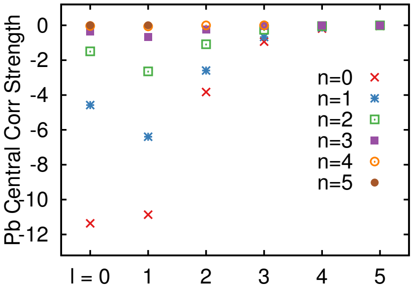
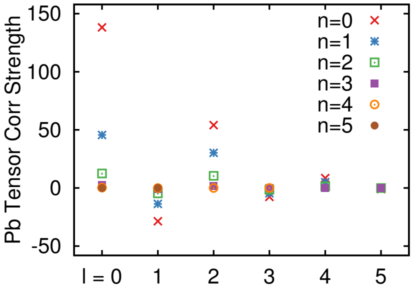
Figure: Comparison of the correlation strength (overlap of correlation function and ave function) of all relative pair configurations (n, l) in Pb.





Short Range Correlations in Nuclei

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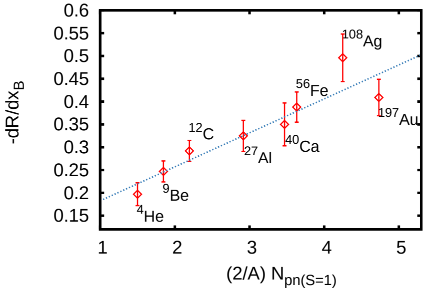
Above arguments suggest dominance of pairs with relative quantum numbers $n = 0,\ l = 0$.

SRC versus EMC effect

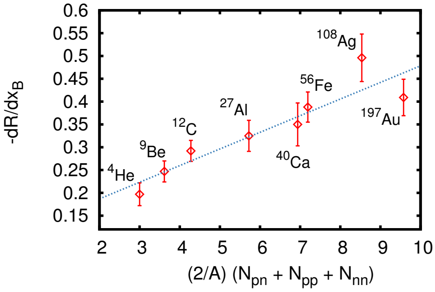
EMC effect versus number of $n = 0$, $l = 0$ np$(S = 1)$ pairs

pn pairs prone to tensor correlation



EMC effect versus number of $n = 0$, $l = 0$ pairs

pairs prone to central correlation

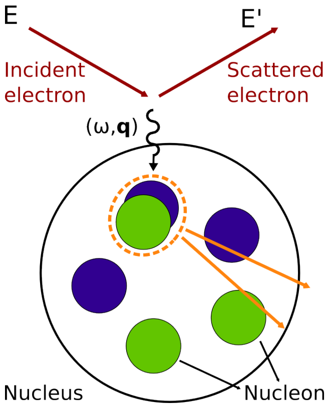


arXiv:1210.6175

Linear relation between $-\frac{\mathrm{d}R}{\mathrm{d}x_B}$ (size of EMC effect) and the predicted number of SRC susceptible pairs

$(e, e'pp)$

How can we study the small dense structures of nuclei?



$$\omega = E' - E$$

$$Q^2 = -q_\mu q^\mu = q^2 - \omega^2$$

$$x_B = \frac{Q^2}{2m\omega}$$

$$Q^2 \geq 1.4\,\mathrm{GeV}^2$$

$$1 \leq x_B \leq A$$

Hard process that has the resolving power to probe the partonic (nucleon) structure of the nucleus

$(e, e'pp)$: Cross Sections

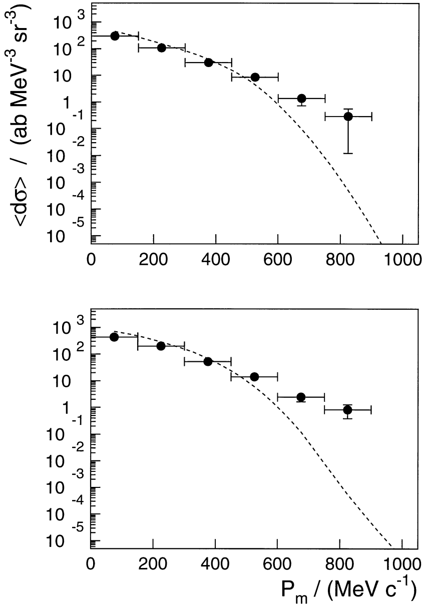
- The corresponding cross sections do NOT scale according to $K\sigma_{eN}S(\vec{p}_m, E_m)$ (reflects one-body dynamics)
- Factorized cross section for 2N knockout

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e, e'pp) = E_1 p_1 E_2 p_2 f_{rec}^{-1}$$
$$\times \sigma_{epp}(k_+, k_-, q) F_{h_1,h_2}(P)$$

J. Ryckebusch PLB383 1 (1996)

- Factorization requires relative $l = 0$ states and plane waves!
- $F_{h_1,h_2}(P)$: Probability to find a diproton in a relative $l = 0$ state and a c.m. momentum P
- $\sigma_{epp}(k_+, k_-, q)$: Probability to have an electromagnetic interaction with a dinucleon with relative momentum $k_\pm$

What do $^{12}C(e, e'pp)$ measurements tell us?



- Theory prediction (dashed) used factorized $A(e, e'pp)$ model
- $^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)

- Up to **P = 0.5 GeV** c.m. motion of correlated pairs in ^{12}C is mean-field like
- Data agree with the factorization in terms of $F(P)$ (relative S states!).

MC simulation for $(e, e'pp)$

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e, e'pp) \propto \sigma_{epp}\left(k_+, k_-, q\right) F_{h_1,h_2}(P)$$

- $F(P)$, is the c.m. distribution of $l = 0$ pairs.
- Does observed c.m. distributions agree with the statement that only $l = 0$ pairs should be included?
- And what is their A dependence?
- What is the effect of kinematics and cuts of the experiment?
- Can the A dependence of the $(e, e'pp)$ cross section ratios give us information on the relative quantum number of the pp pairs.

Two-body momentum distribution

Two-body momentum distribution $P_2(\vec{k},\vec{P})$

$$P_2\left(\vec{k},\vec{P}\right) = \frac{1}{(2\pi)^6}\int \mathrm{d}\vec{r}\int \mathrm{d}\vec{r}\,'\int \mathrm{d}\vec{R}\int \mathrm{d}\vec{R}\,' \times e^{\imath\vec{k}\cdot(\vec{r}-\vec{r}\,')}e^{\imath\vec{P}\cdot(\vec{R}-\vec{R}\,')}\rho_2(\vec{r}\,',\vec{R}\,';\vec{r},\vec{R}),$$

where $\rho_2(\vec{r}\,',\vec{R}\,';\vec{r},\vec{R})$ is the non-diagonal two-body density (TBD) matrix

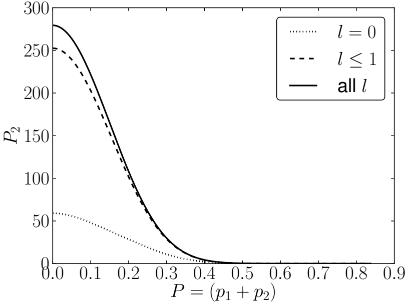
$$\rho_2(\vec{r}\,',\vec{R}\,';\vec{r},\vec{R}) = \int \{\mathrm{d}\vec{r}_{3-N}\}\bar{\Psi}_A^*(\vec{r}_1\,',\vec{r}_2\,',\vec{r}_3,\ldots,\vec{r}_A)\bar{\Psi}_A(\vec{r}_1,\vec{r}_2,\vec{r}_3,\ldots,\vec{r}_A).$$

Two-body c.m. momentum distribution $P_2(P)$

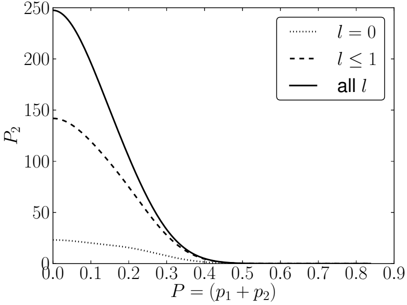
$$P_2(P) = \int \mathrm{d}\Omega_P\int \mathrm{d}\vec{k}\;\; P_2(\vec{k},\vec{P}) = \int \mathrm{d}k\;\; k^2\frac{4}{\pi}\sum_{lm_l}\sum_{LM_L} n_2^{lm_lLM_L}(k,P)$$

The quantity $n_2^{lm_lLM_L}(k,P)k^2\mathrm{d}kP^2\mathrm{d}P$ is related to the probability of finding a nucleon pair with qauntum numbers lm_lLM_L , a relative momentum in the interval $[k,k+\mathrm{d}k]$ and a c.m. momentum in $[P,P+\mathrm{d}P]$.

$P_2^{pp}(P|l)$ for various nuclei



^{12}C



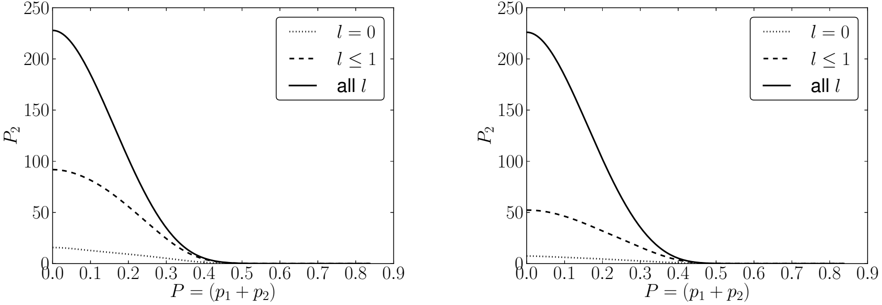
^{27}Al

- Conditional momentum distribution

$$P_2^{pp}(P|l=\lambda) = \int \mathrm{d}k \; k^2 \frac{4}{\pi} \sum_{LM_L} \sum_{m_\lambda} n_2^{\lambda m_\lambda LM_L}(k,P)$$

- Width $l = 0$ is larger than width of total two-body c.m. distribution
- For increasing A, $l = 0$ is smaller fraction of the total two-body c.m. distribution.

$P_2^{pp}(P|l)$ for various nuclei



^{56}Fe ^{208}Pb

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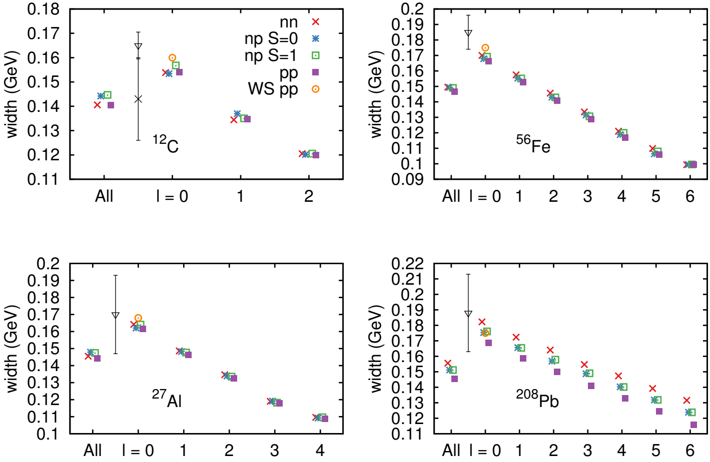


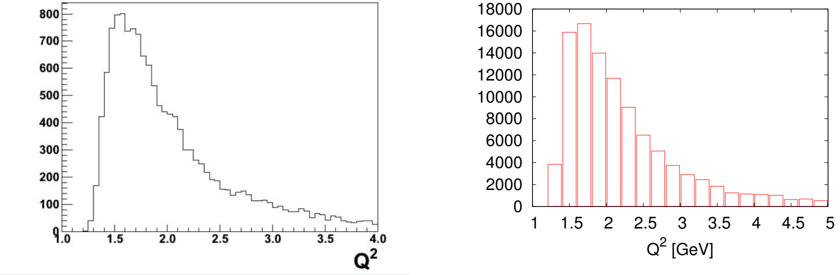
Figure: Width of $P_2(P_x|l)$ in HO and WS basis. The black triangles are preliminary experimental results from the CLAS data-mining group. The black cross is the experimental result from Tang et. al. (PRL 90, 042301).

MC simulation for $(e, e'pp)$

$(e, e'pp)$ simulations with the calculated pp c.m. distribution $P_2(P)$ using Monte Carlo simulations in order to take the experimentally selected phase space into account.

1. Input the observed $x_B - Q^2$ distribution determined by detector geometries and acceptances (phase space selection)
2. Simulate c.m. momentum $\vec{P}$ from calculated distribution of $l = 0$ pairs.
3. Simulate an event and check its feasibility from energy-momentum conservation of the two-proton knockout reaction.
4. Apply leading proton selection criteria
   - $0.62 < \frac{|\vec{p_f}|}{|\vec{q}|} < 0.92$
   - $p_m > 300\,\mathrm{MeV}$
5. Compare simulated distributions with experimental distributions

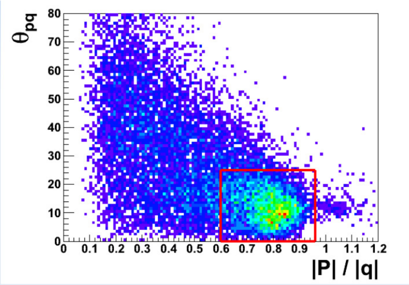
MC simulation for $(e, e'pp)$



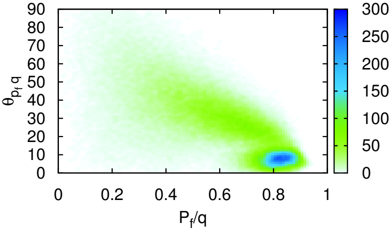
(a) Experiment (b) Simulation

Figure: Q^2 distribution of the $^{12}C(e, e'pp)$ events after applying kinematic and leading proton cuts. The left panel is copied from O. Hen's presentation at ECT* meeting. The right panel is the result of our simulations with 100000 events. Obviously, the simulated Q^2 distribution agrees with the measured one.

MC simulation for $(e, e'pp)$



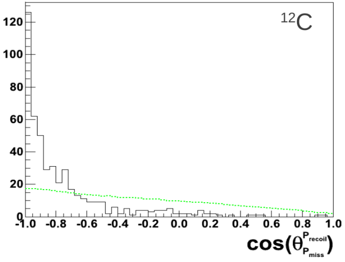
(a) Experiment



(b) Simulation

Figure: $\theta_{p_f,q}$-$\frac{|\vec{p}_f|}{|\vec{q}|}$ distribution before applying the leading proton selection criteria in the analysis of the experiment. This selection criteria ($0.62 < \frac{|\vec{p}_f|}{|\vec{q}|} < 0.92$ and $\Theta_{p_f,q} < 25^\circ$) are marked with a red rectangle in the left panel. Obviously, experimental and simulated distributions are similar.

MC simulation for $(e, e'pp)$



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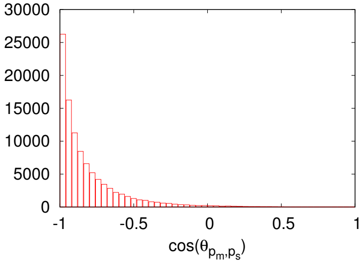
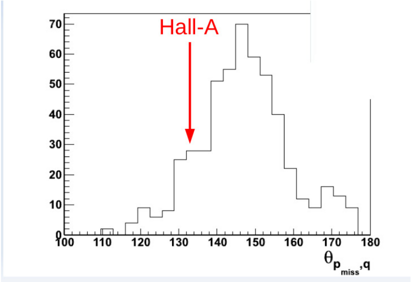


(b) Simulation

Figure: The distribution of the events as a function of the angle between the momentum of the recoil proton ($\vec{p}_s$) and the missing momentum ($\vec{p}_m$).

MC simulation for $(e, e'pp)$



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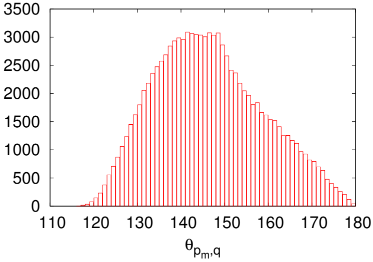
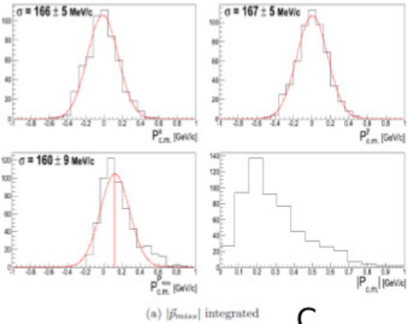


(b) Simulation

Figure: The distribution of the events as a function of the angle between the momentum transfer $\vec{q}$ and the missing momentum ($\vec{p}_m$) in ^{12}C. Again there is a close similarity between the simulations and the experimentally obtained distribution.

MC simulation for $(e, e'pp)$



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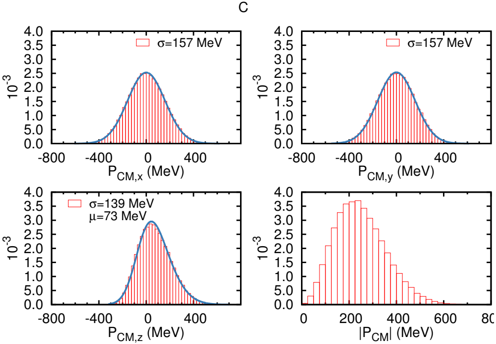
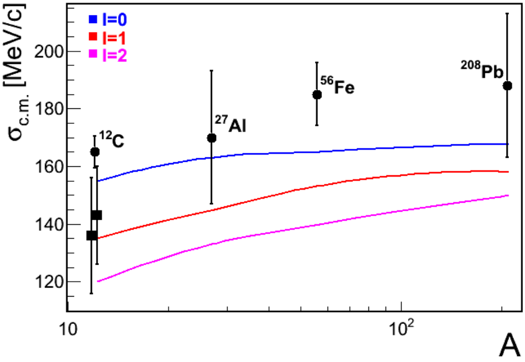


We can reconstruct the shift along the direction of p_m ($\propto$ the z-axis).
The two-body c.m. distribution after simulation is slightly wider than the initial one.

$A(e, e'pp)$: c.m. width

- Data is preliminary (courtesy of O. Hen and E. Piasetzky)
- Analysis of exclusive $A(e, e'pp)$ for ^{12}C, ^{27}Al, ^{56}Fe and ^{208} Pb
- σ_{cm} Guassian widths from fit to measured c.m. distributions
- Theory lines: Momentum analysis of uncorrected calculated HO c.m. distributions for $l = 0, 1, 2$.

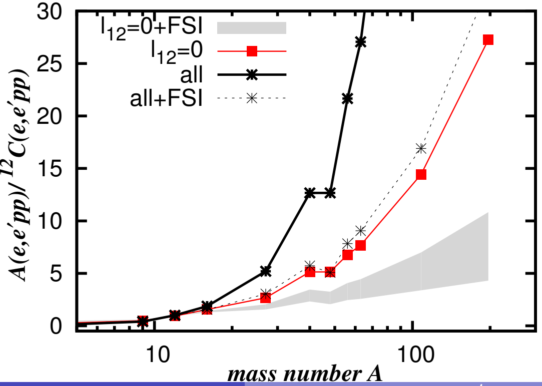


- Effect of FSI under study

Mass dependence of the $A(e, e'pp)$ cross sections

$$\frac{A(e,e'pp)}{^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left(^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$

Prediction: The A dependence is soft



Preliminary data (courtesy of O. Hen and E. Piasetzky) compatible with absorption on $n = 0$, $l = 0$ pairs!

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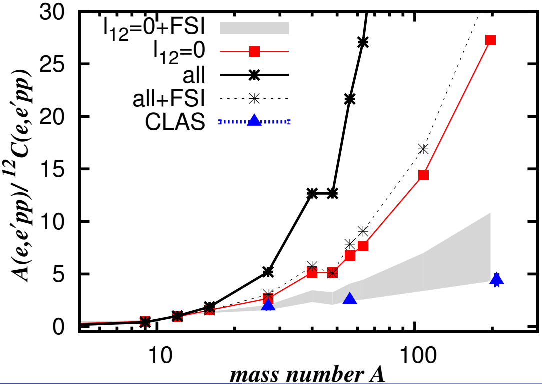

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Summary

- SRC-prone pairs are in a relative state of $n = 0$, $l = 0$. (relative S state)
- For the exclusive $A(e, e'pp)$ this has important implifications
  - A factorized model can be used for the cross section.
  - The A dependence is soft.
- The extracted widths and cross section ratios are compatible with $l = 0$, $n = 0$.
- The soft A dependence of the cross section ratios agree with the number of $l = 0$, $n = 0$ pairs.
- The $l = 0$ dominance is a great asset for further theory-experiment comparison.

Thank you!