Mass and Isospin Dependence of Short-Range Correlations

Maarten Vanhalst, Wim Cosyn, Jan Ryckebusch

Department of Physics and Astronomy, Ghent University

Nuclear Structure and Dynamics at Short Distances (INT-13-52W)

PRC84,031302(R); PRC86,044619; arXiv:1210.6175

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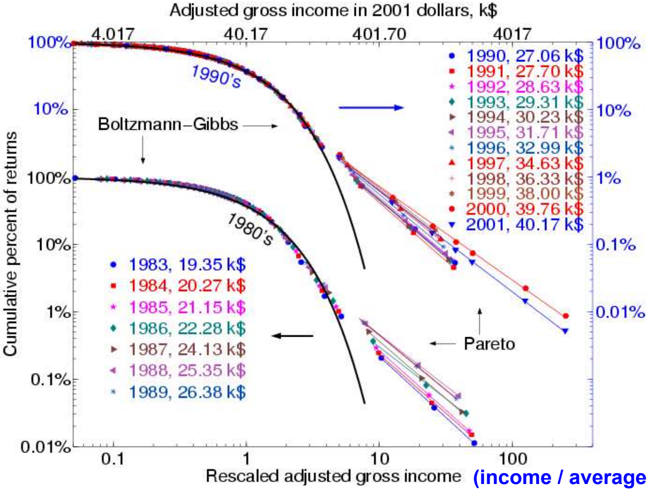
FACULTEIT WETENSCHAPPEN

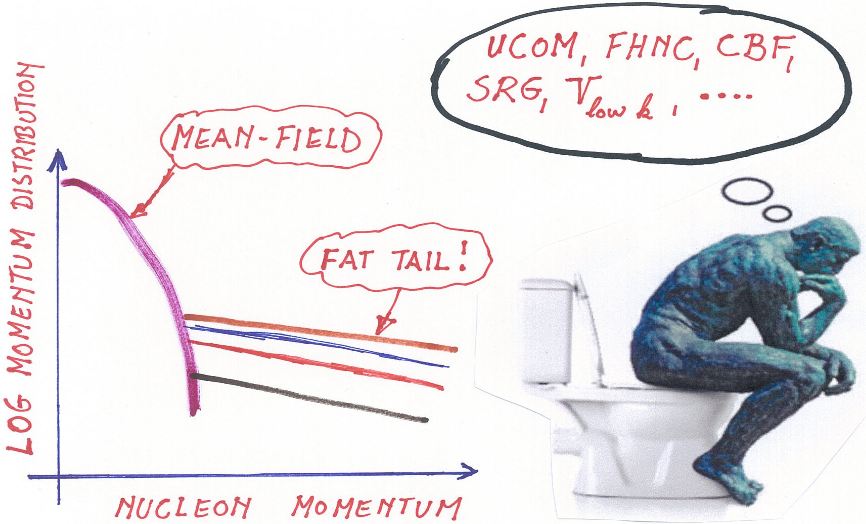
Income distribution in the USA, 1983-2001



(income / average income *T*)

The rescaled exponential part does not change,

but the power-law part changes significantly.


UCOM, FHNC, CBF,
SRG, V low k, ...
MEAN-FIELD
FAT TAIL!
LOG MOMENTUM DISTRIBUTION
NUCLEON MOMENTUM

STYLIZED FACTS OF NUCLEAR SRC?

1. How can one quantify
   - the number of 2N pairs prone to SRC?
   - the mass and isospin dependence of 2N SRC?
   - the number of 3N triples prone to SRC?
2. How to connect this knowledge to electron-scattering observables?
   - Inclusive $A(e,e')$ at $1.5 \lesssim x_B$ (2N)
   - Inclusive $A(e,e')$ at $2.2 \lesssim x_B$ (3N)
   - The magnitude of the EMC effect
   - Exclusive $A(e,e'pp)$

Electron scattering and nuclear SRC (I)

- Lots of nuclear-structure activity in computing
  1. **One-body momentum distribution $n_1(k)\,k^2dk$:** probability of finding a nucleon with momentum in $[k, k+dk]$
  2. **Two-body momentum distribution $n_2(k_{12}, P_{12})\,k_{12}^2dk_{12}P_{12}^2dP_{12}$:** combined probability of finding a pair with relative and c.m. momentum in $[k_{12}, k_{12}+dk_{12}]$ and $[P_{12}, P_{12}+dP_{12}]$
- The mean-field $^{(0)}$ and correlated $^{(1)}$ parts can be separated

$$
\begin{aligned}
n_1(k) &= n_1^{(0)}(k) + n_1^{(1)}(k) \\
n_2(k_{12}, P_{12}) &= n_2^{(0)}(k_{12}, P_{12}) + n_2^{(1)}(k_{12}, P_{12})
\end{aligned}
$$

- In practice: perturbative (cluster, virial) expansions are required to compute the $n_1^{(1)}(k)$ and $n_2^{(1)}(k_{12}, P_{12})$ for $A > 4$
- Nucleon-nucleon short-range correlations are highly “local” which naturally truncates the expansions (2N $\gg$ 3N)

Electron scattering and nuclear SRC (II)

- One-body and two-body momentum distributions are not directly observable and the obtained information on SRC is indirect.
- Need for an effective approximation scheme to link the electron-scattering data to SRC information!
- Unitary Correlation Operator Method (UCOM): correlations are dynamically generated by operating with $\widehat{\mathcal{G}}$ on IPM wave functions
- Realistic wave functions $|\,\overline{\Psi}\rangle$ after applying a many-body correlation operator to a Slater determinant $|\,\Psi\rangle$

$$|\,\overline{\Psi}\,\rangle = \frac{1}{\sqrt{\langle\,\Psi\,|\,\widehat{\mathcal{G}}^{\dagger}\widehat{\mathcal{G}}\,|\,\Psi\,\rangle}}\,\widehat{\mathcal{G}}\,|\,\Psi\,\rangle\ .$$

- The $\widehat{\mathcal{G}}$ reflects the full complexity of the NN force but is dominated by the central and tensor correlations

Electron scattering and nuclear SRC (III)

- Dominant contributions to nuclear correlation operator

$$\widehat{\mathcal{G}} \approx \widehat{\mathcal{S}} \left[\prod_{i<j=1}^{A} \left(1 - g_c\left(r_{ij}\right) + \widehat{t}(i,j) \right) \right] \qquad \left(\widehat{t}(i,j) = f_{t\tau}\left(r_{ij}\right) \widehat{S_{ij}} \vec{\tau}_i \cdot \vec{\tau}_j \right)$$

- A one-body operator $\widehat{\Omega} = \sum_{i=1}^{A} \widehat{\Omega}^{[1]}(i)$ recieves SRC corrections (NPA 672 (2000) 285)

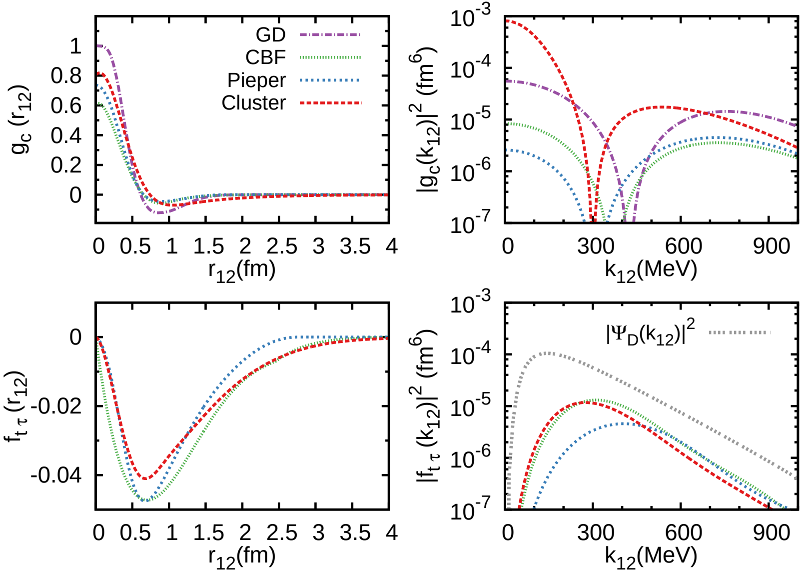
$$\widehat{\Omega}^{eff} = \widehat{\mathcal{G}}^{\dagger}\, \widehat{\Omega}\, \widehat{\mathcal{G}} \approx \widehat{\Omega} + \sum_{i<j=1}^{A} \left(\left[\widehat{\Omega}^{[1]}(i) + \widehat{\Omega}^{[1]}(j) \right] \times \left[-g_c(r_{ij}) + \widehat{t}\,(i,j) \right] + h.c. \right) .$$

Electron-nucleon coupling receives two-body, . . . contributions

Two-nucleon knockout $A(e,e'NN)$ is the hallmark of SRC

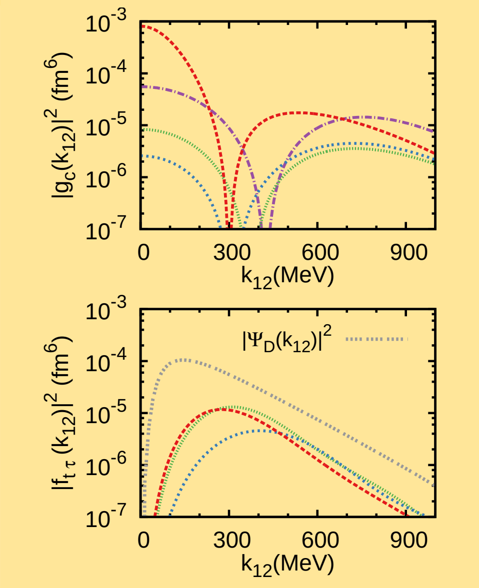
- Many models for $g_c\left(r_{ij}\right)$ and $f_{t\tau}\left(r_{ij}\right)$

Electron scattering and nuclear SRC (IV)



Adopted conventions: $k_{12} = \mid \vec{k}_1 - \vec{k}_2 \mid / \sqrt{2}$ and $r_{12} = \mid \vec{r}_1 - \vec{r}_2 \mid / \sqrt{2}$

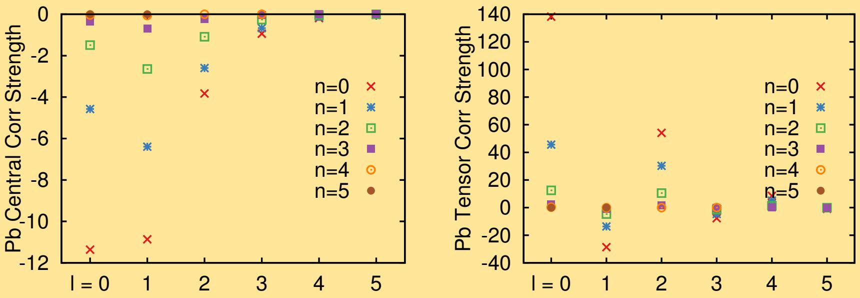
Electron scattering and nuclear SRC (V)



- very high relative pair momenta: central correlations
- moderate relative pair momenta: tensor correlations
- $|f_{t\tau}(k_{12})|^2 \sim |\Psi_D(k_{12})|^2$
- $|f_{t\tau}(k_{12})|^2$ is well constrained! (*D*-state deuteron wave function)
- the $g_C(k_{12})$ looks like the correlation function of a monoatomic classical liquid (reflects finite-size effects)
- the $g_c(k_{12})$ are ill constrained!

Correlated part of pn two-body momentum distribution

In two-nucleon cluster approximation: how much does each pair relative orbital configuration (nl) contribute to the correlated part of the two-body momentum distribution $n_2^{(1)}(k_{12}, P_{12})$?



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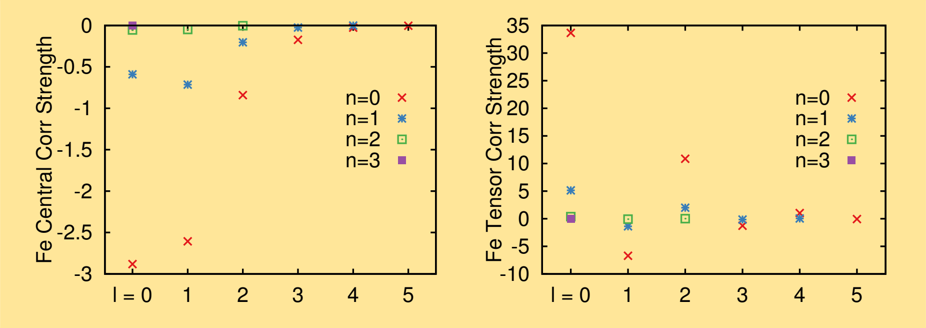
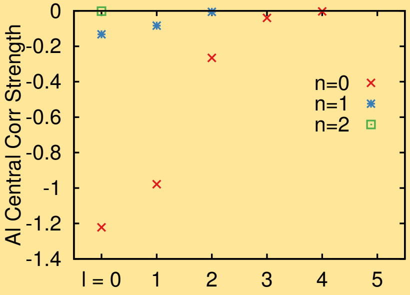
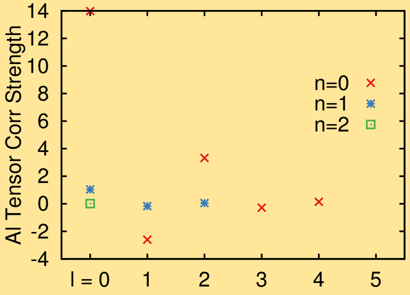


Correlated part of pn two-body momentum distribution





- integrated effect of the tensor correlations is larger by a factor of ≈ 10 compared to central correlations
- effect of tensor correlations strongest for pn pairs with $(n = 0, l = 0)$!
- effect of central correlations strongest for pairs with $(n = 0, l = 0)$!

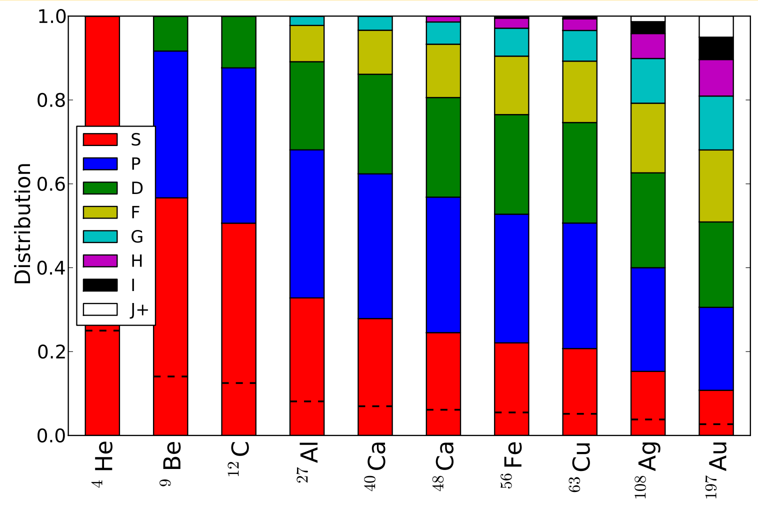
Quantifying 2N correlations (I)

- **Suggestion: significance of 2N correlations in $A(N,Z)$ is proportional to the number of relative S states**
- Requires transformation from $(\vec{r}_1, \vec{r}_2)$ to $\left(\vec{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \vec{R}_{12} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}}\right)$ which can be easily achieved in a HO basis ($\alpha_a = (n_a l_a j_a t_a)$)

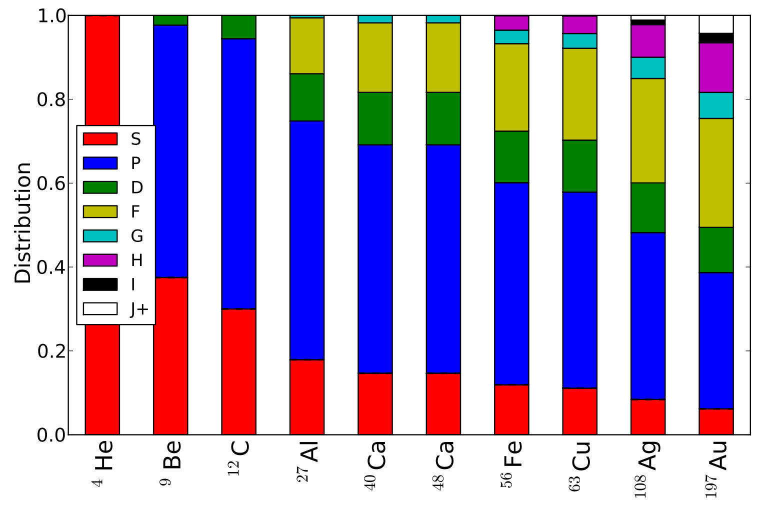
$$
\begin{aligned}
|\alpha_a \alpha_b; J_R M_R\rangle_{nas} = & \sum_{LM_L} \sum_{nl} \sum_{N\Lambda} \sum_{SM_S} \sum_{TM_T} \frac{1}{\sqrt{2\left(1+\delta_{\alpha_a \alpha_b}\right)}} \\
& \times \left[1 - (-1)^{l+S+T}\right] \\
& \times \mathcal{C}\left(\alpha_a \alpha_b J_R M_R; (nlN\Lambda) LM_L SM_S TM_T\right) \\
& \times \left| \left[nl\left(\vec{r}_{12}\right), N\Lambda\left(\vec{R}_{12}\right)\right] LM_L, \left(\frac{1}{2}\frac{1}{2}\right) SM_S, \left(\frac{1}{2}\frac{1}{2}\right) TM_T \right\rangle ,
\end{aligned}
$$

$\left| nl\left(\vec{r}_{12}\right)\right\rangle$ ($\left| N\Lambda\left(\vec{R}_{12}\right)\right\rangle$) is the relative (c.m.) pair wave function

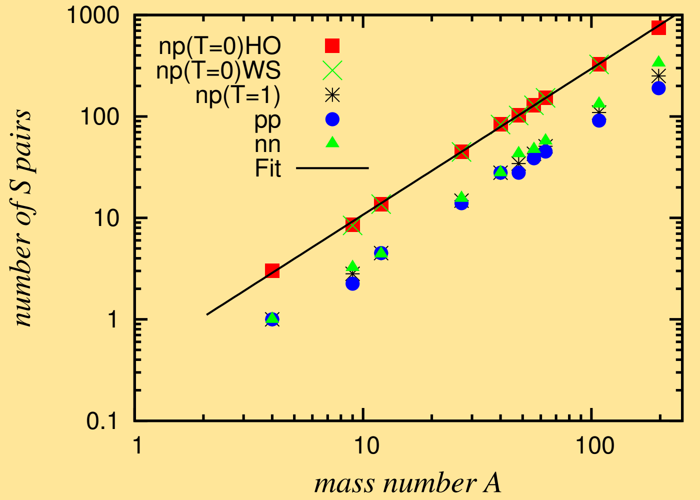
Relative orbital momentum (l_{12}) distribution for pn pairs



Relative orbital momentum l_{12} distribution for pp pairs

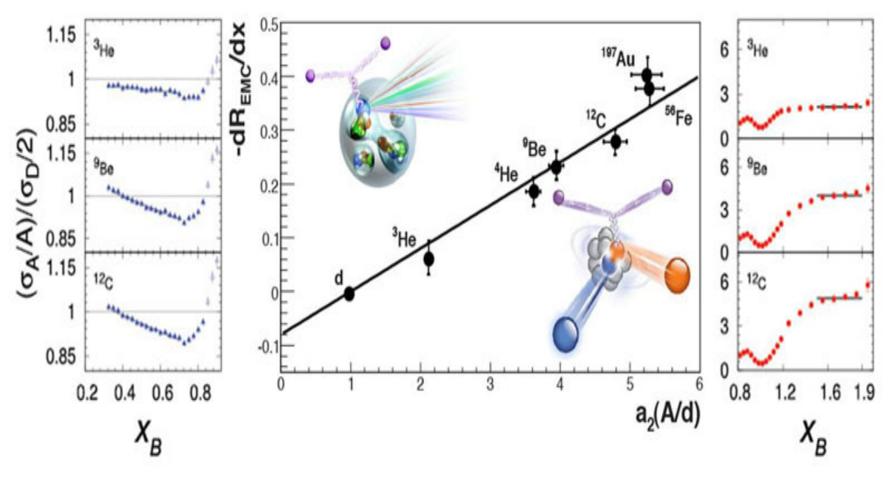


Number of pairs with $l_{12} = 0$ (SRC-prone pairs)



- Number of pairs prone to SRC: a robust power law $\sim A^{1.44\pm0.01}$.
- Strong isospin and spin dependence!
- Some asymmetry $N \neq Z$ effects!

Connection between stylized facts of SRC and observables?

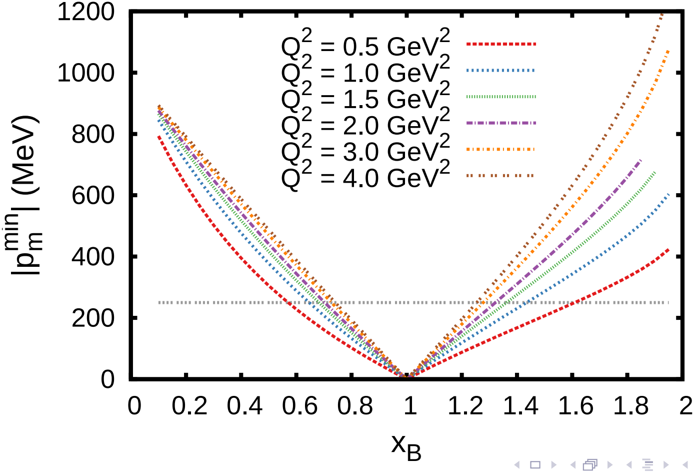


Connection between stylized facts of SRC and observables?

- Dispersion of "IPM" nucleons: low virtualities $v = p^{\mu} p_{\mu} - M_N^2$ (p^{μ} is initial momentum of the "active" nucleon)
- SRC induce spatio-temporal fluctuations from mean-field quantities
- Dispersion of "SRC" nucleons is two-nucleon like and accesses a much wider range in virtualities v
- The fluctuations induced by SRC can be probed with reactions which select large virtualities
- In $A(e, e')$ reactions one has $v \sim -p_m^2$ (missing momentum of the hit nucleon)
- Large virtualities in $A(e, e')$ for $1.5 \lesssim x_B$ and $0.3 \leq x_B \leq 0.7$ (EMC effect) and under those conditions the IPM part of the momentum distributions runs out of steam - small things (SRC) can become big!

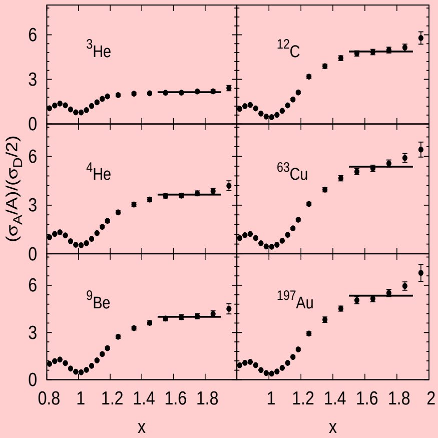
Virtuality of the hit nucleon in the deuteron

Relation between minimum missing momentum $\left|p_m^{min}\right|$ and $\left(x_B, Q^2\right)$ in deuteron



$A(e,e')$ for $1.5 \lesssim x_B$ and 2N SRCs (I)

Scaling of the $A(e,e')$ response to the deuteron one



Hall-C, PRL108,092502(2012)

- Quantify scaling behavior:

$$a_2\left(A/D\right) \equiv \frac{2}{A}\frac{\sigma^A\left(x_B,Q^2\right)}{\sigma^D\left(x_B,Q^2\right)} ,$$

- Assume that signal is dominated by the pn correlations!
- Assume that $\sigma_{epn}(Q^2,x_B) \approx \sigma_{eD}(Q^2,x_B)$
- Very naive counting (all pn pairs contribute): $a_2 \sim A$
- Suggestion: $a_2(A/D) \sim \frac{2}{A} N_{pn(S=1)}(A,Z)$ (number of deuteron-like pn pairs)

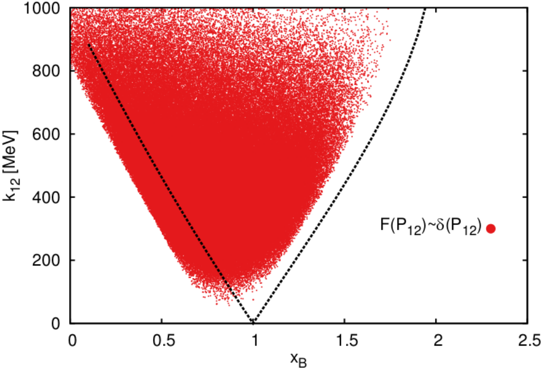
$A(e,e')$ for $1.5 \lesssim x_B$ and 2N SRCs (III)

Corrections to the ratio's of cross-section data which affect the extracted value of $a_2(A/D)$: unlike the deuteron

- $A-2$ fragment can be left with excitation energy
- Pairs have c.m. motion
- Final-state interactions on the ejected two nucleons (?)
- Contribution of the pp and nn correlations (small)

Talk of W. Cosyn (Feb.20)

Effect of $A-2$ excitation energy



MC simulations of breakup of 2N correlated pairs in ^{12}C for $\epsilon = 5.766$ GeV and $\langle Q^2 \rangle$=2.7 GeV^2

$A(e,e')$ for $1.5 \lesssim x_B$ and 2N SRCs (III)

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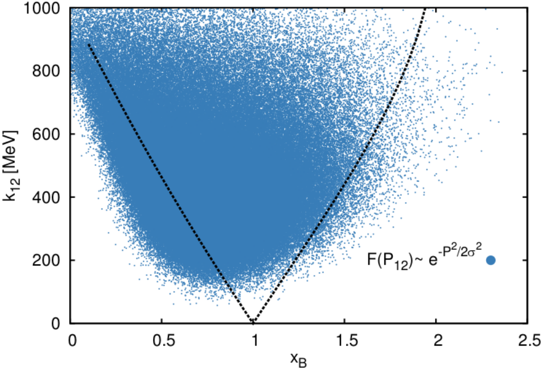
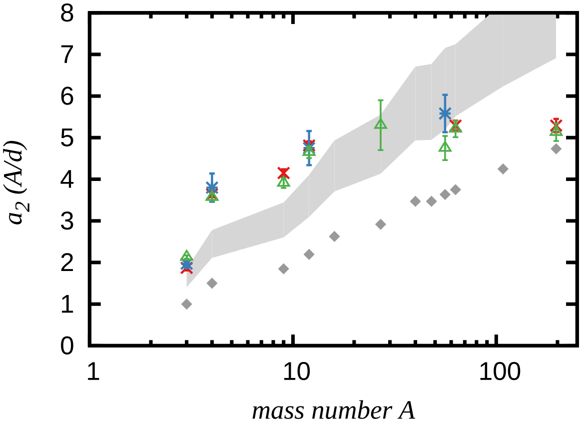
Talk of W. Cosyn (Feb.20)

Effect of c.m. motion of pn pairs



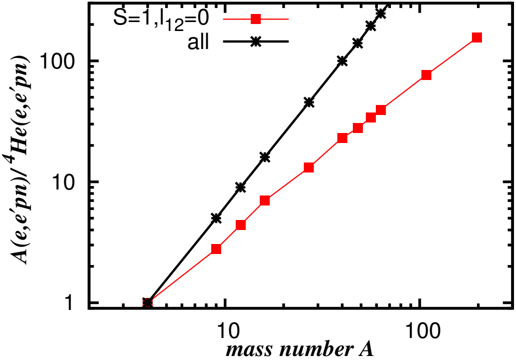
MC simulations of breakup of 2N correlated pairs in ^{12}C for $\epsilon = 5.766$ GeV and $\langle Q^2 \rangle$=2.7 GeV^2

Corrections for c.m. motion have been applied to the computed values of $a_2(A/D)$



- C.m. corrections from a Monte-Carlo based on $A(e,e') \sim \int d\vec{P}_{12} F^{pn}(P_{12})$
- Band is the estimated model uncertainty on the c.m. corrections!
- Role of FSI? (O. Benhar)

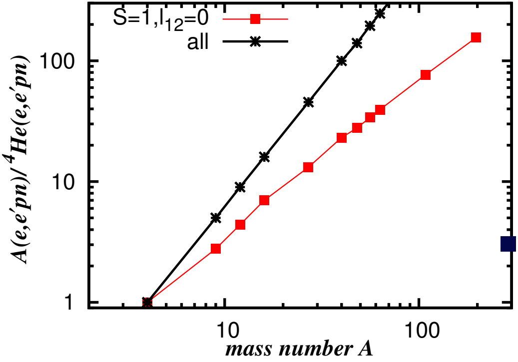
Mass dependence of the pn SRC



- One has

$$\frac{\sigma^{A}(e,e'pn)}{\sigma^{^4\text{He}}(e,e'pn)} \approx \frac{N_{pn(S=1)}(A)}{N_{pn(S=1)}\left(^4\text{He}\right)}$$

$N_{pn(S=1)}$: number of $(l_{12}=0, S=1)$ pn pairs



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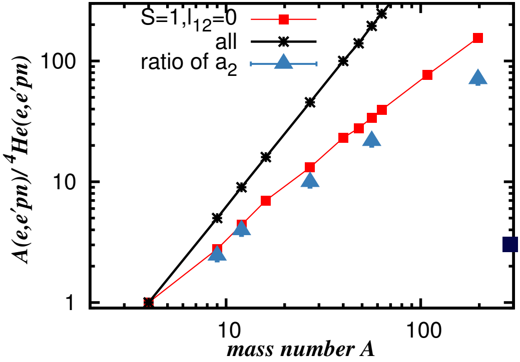
$N_{pn(S=1)}$: number of $(l_{12}=0, S=1)$ pn pairs

- At $1.5 \lesssim x_B$: dominance of pn tensor correlations in $A(e,e')$

$$\frac{\sigma^{A}(e,e')}{\sigma^{^4\mathrm{He}}(e,e')} \approx \frac{\sigma^{A}(e,e'pn)}{\sigma^{^4\mathrm{He}}(e,e'pn)} \approx \frac{A \times a_2(A)}{4 \times a_2\left(^4\mathrm{He}\right)}$$

Mass dependence of the pn SRC



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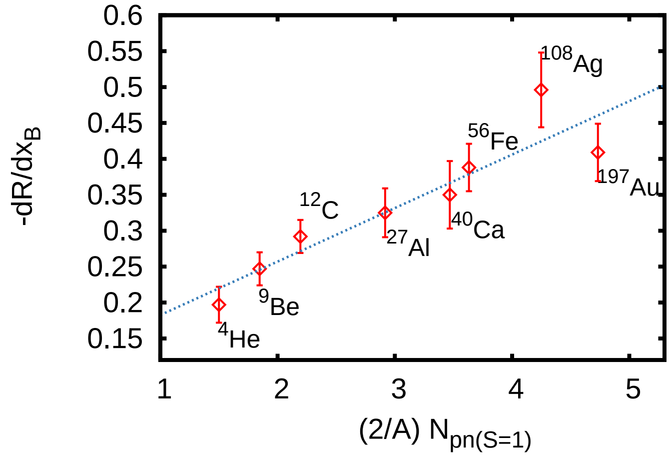
$$\frac{\sigma^A(e,e')}{\sigma^{^4\mathrm{He}}(e,e')} \approx \frac{\sigma^A(e,e'pn)}{\sigma^{^4\mathrm{He}}(e,e'pn)} \approx \frac{A \times a_2(A)}{4 \times a_2\left(^4\mathrm{He}\right)}$$

MASS DEPENDENCE OF THE "CORRELATED" STRENGTH IS MUCH SOFTER THAN NZ!

Magnitude of the EMC effect and 2N SRCs

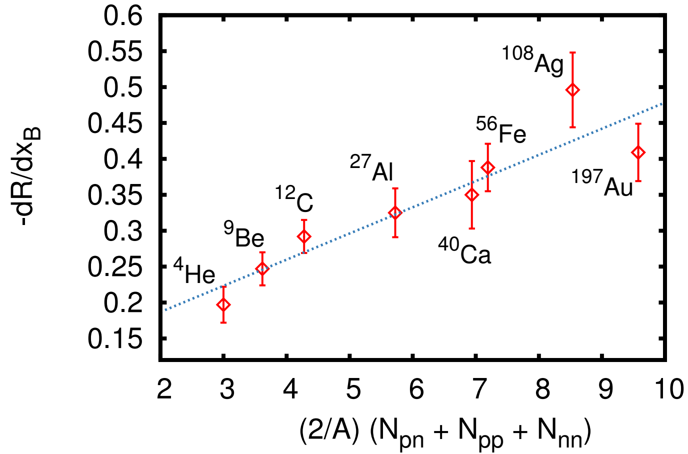
Magnitude of EMC effect ∼ number of SRC-prone pairs!



$\frac{2}{A}N_{pn(S=1)}(A,Z)$: per nucleon number of SRC pn pairs (measure for the magnitude of the pn SRC in ^{A}Z)

Magnitude of the EMC effect and 2N SRCs

Magnitude of EMC effect ∼ number of SRC-prone pairs!

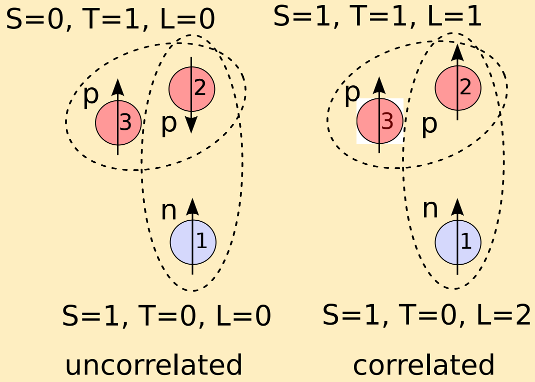


$\frac{2}{A}\left(N_{pn(S=1)} + N_{pn(S=0)} + N_{nn} + N_{pp}\right)$: per nucleon number of SRC NN pairs (measure for the magnitude of the NN SRC in ^{A}Z)

Quantifying 3N correlations? (I)

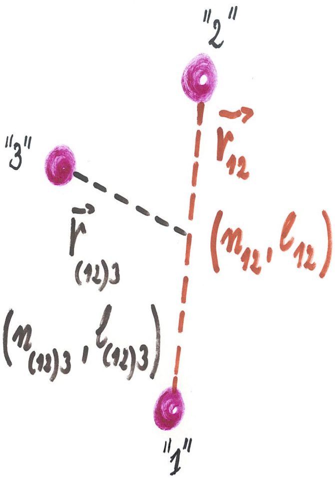
3N correlations induced by products of 2N correlation functions

$$\left(f_{t\tau}(r_{12})\widehat{S_{12}}\right)\left[f_{LS}(r_{23})\vec{L}_{23}\cdot\vec{S}_{23}\right]$$



Feldmeier,Horiuchi,Neff,Suzuki: PRC**84**,054003 (2011)

Quantifying 3N correlations? (II)



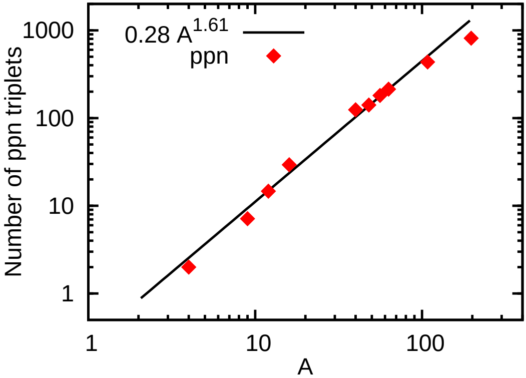
- seek for those wave-function components where all three nucleons are "close"
- requires transformation from $(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ to $\left(\vec{r}_{12}, \vec{r}_{(12)3}, \vec{R}_{123}\right)$ (Jacobi coordinates)

$$\vec{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{\sqrt{2}}, \quad \vec{R}_{12} = \frac{\vec{r}_1 + \vec{r}_2}{\sqrt{2}}$$

$$\vec{r}_{(12)3} = \frac{\vec{R}_{12} - \sqrt{2}\vec{r}_3}{\sqrt{3}},$$

$$\vec{R}_{123} = \frac{\sqrt{2}\vec{R}_{12} + \vec{r}_3}{\sqrt{3}},$$

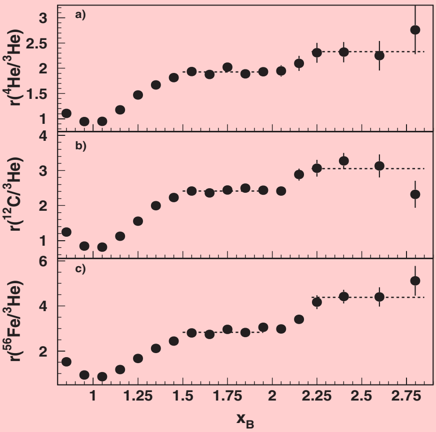
Mass dependence of number of ppn triples with $(l_{12} = 0, l_{(12)3} = 0)$



- Number of ppn triples prone to SRC effects $(l_{12} = 0, l_{(12)3} = 0)$: $N_{ppn}(A) = 0.28A^{1.61}$

$A(e,e')$ for $2.2 \lesssim x_B$ and 3N SRCs

Scaling of the $A(e,e')$ response to the ^{3}He one



Hall-B, PRL**96**, 082501

- Quantify scaling behavior:

$$a_3\left(A/^3He\right) \equiv \frac{3}{A}\frac{\sigma^A\left(x_B,Q^2\right)}{\sigma^{^3He}\left(x_B,Q^2\right)} ,$$

- Assume that signal is dominated by the ppn correlations!
- Assume that $\sigma_{epn}(Q^2,x_B) \approx \sigma_{e^3He}(Q^2,x_B)$
- Very naive counting (all ppn triples contribute): $a_3 \sim A^2$
- Suggestion: $a_3(A/^3He) \sim \frac{3}{A}N_{ppn}(A)$ (number of ppn triples with $l_{12}=0, l_{(12)3}=0$)

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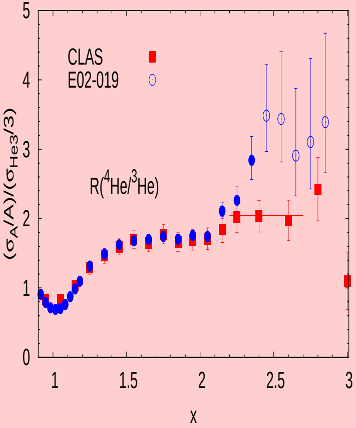
Scaling of the $A(e,e')$ response to the ^{3}He one



Hall-C, PRL**108**, 092502

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- Assume that signal is dominated by the ppn correlations!
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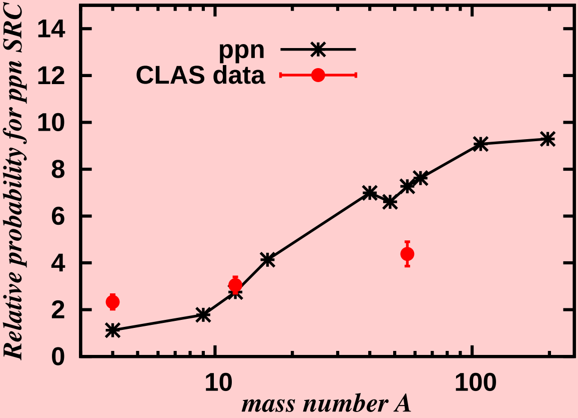
- Very naive counting (all ppn triples contribute): $a_3 \sim A^2$
- Suggestion: $a_3(A/^3He) \sim \frac{3}{A}N_{ppn}(A)$ (number of ppn triples with $l_{12}=0, l_{(12)3}=0$)

$A(e, e')$ for $2.2 \lesssim x_B$ and 3N SRCs (II)

$a_3(A/^3He)$ as a measure of the per-nucleon probability of ppn SRC relative to ^{3}He *(calculations are NOT corrected for c.m. motion, FSI, ...)*



Exclusive $A(e, e'pp)$ reactions

The fact that SRC-prone proton-proton pairs are mostly in a state with relative orbital momentum $l_{12} = 0$ has important consequences for the EXCLUSIVE $A(e, e'pp)$ cross sections (PLB 383,1 ('96))!!

1. **The $A(e, e'pp)$ cross sections factorizes according to**

$$\frac{d^8\sigma}{d\epsilon' d\Omega_{\epsilon'} d\Omega_1 d\Omega_2 dT_{p_2}}(e, e'pp) = E_1 p_1 E_2 p_2 f_{rec}^{-1} \times \sigma_{eN_1N_2}(k_+, k_-, q) F_{h_1,h_2}(P)$$

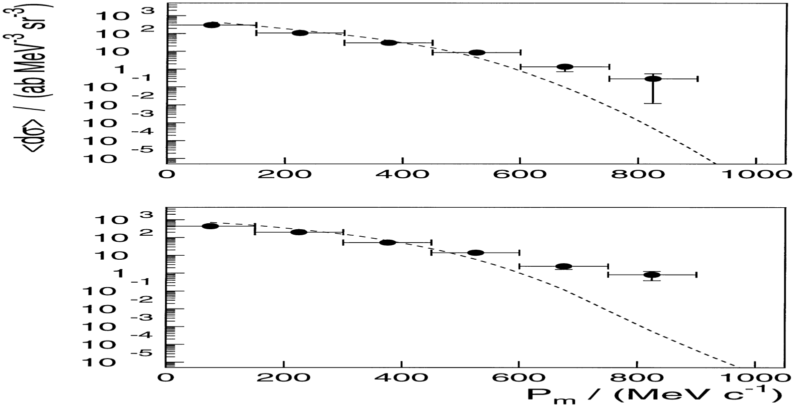
$F_{h_1,h_2}(P)$: probability to find a diproton with c.m. momentum P and relative orbital momentum $l_{12} = 0$!

2. **The A dependence of the $A(e, e'pp)$ cross sections is soft** (much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e, e'pp)}{{}^{12}\mathrm{C}(e, e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left({}^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e, e'p)}{T_{^{12}\mathrm{C}}(e, e'p)}\right)^{1-2}$$

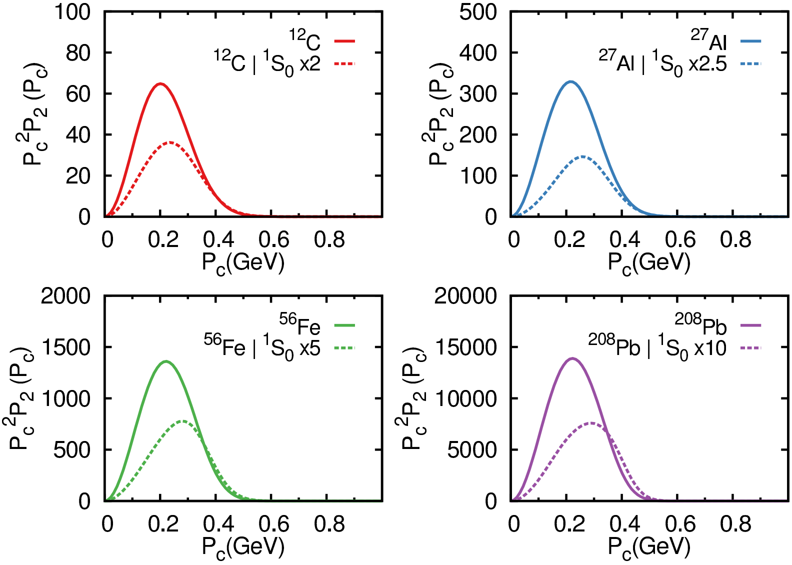
Factorization of the $A(e, e'pp)$ cross sections

$^{12}C(e, e'pp)$ @ MAMI (Mainz) (Physics Letters B **421** (1998) 71.)

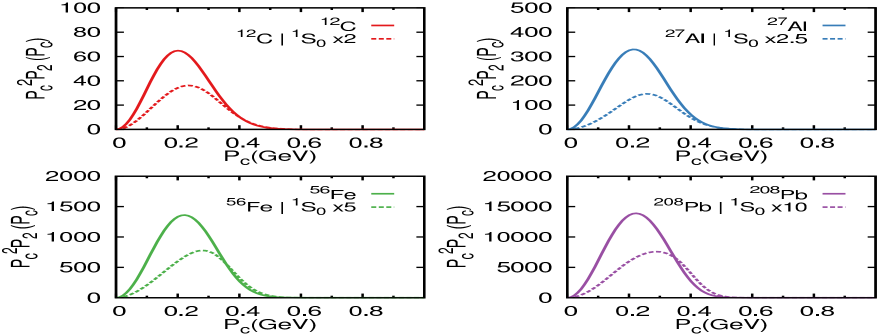


For $P \lesssim 0.5$ GeV c.m. motion of correlated pairs in ^{12}C is mean-field like $\left(\exp \frac{-P^2}{2\sigma^2_{c.m}}\right)$! Data prove factorization in terms of $F(P)$ (relative $l_{12} = 0$!).

The pp c.m. momentum distribution



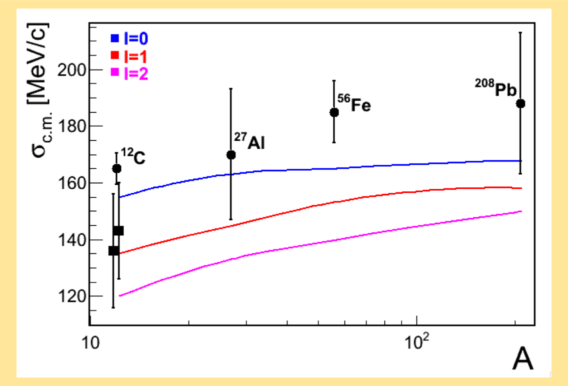
The pp c.m. momentum distribution



- **The c.m. momentum distributions carry information about the quantum numbers of the pairs**
- **The $A(e, e'pp)$ cross sections do not scale with A^2 but rather with A^{x} $^{(x<1.44)}$ (FSI corrections!)**

C.m. motion of correlated pp pairs

DATA IS PRELIMINARY! (COURTESY OF O. HEN AND E. PIASETZKY)



- analysis of exclusive $A(e, e'pp)$ for ^{12}C, ^{27}Al, ^{56}Fe, ^{208}Pb
- distribution of events against P is fairly Gaussian
- $\sigma_{c.m.}$: Gaussian widths from a fit to measured c.m. distributions
- theory lines: Gaussian fits to computed c.m. distributions for $I = 0, 1, 2$

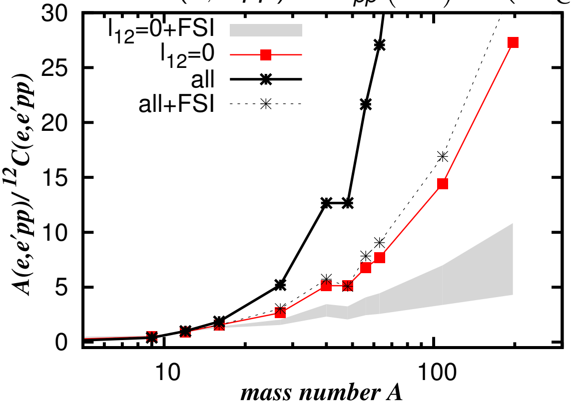
More on the $A(e, e'pp)$ results from CLAS Data Mining: O. Hen (Expt), M. Vanhalst (MC simulations) (Wed, Feb. 20)

Mass dependence of the $A(e, e'pp)$ cross sections

PREDICTION: A dependence of $A(e, e'pp)$ c.s. is soft

(much softer than predicted by naive $Z(Z-1)$ counting)

$$\frac{A(e,e'pp)}{{}^{12}\mathrm{C}(e,e'pp)} \approx \frac{N_{pp}(A)}{N_{pp}\left({}^{12}\mathrm{C}\right)} \times \left(\frac{T_A(e,e'p)}{T_{{}^{12}\mathrm{C}}(e,e'p)}\right)^{1-2}$$



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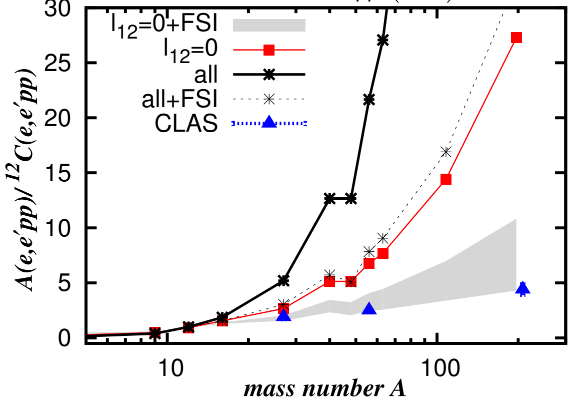


PRELIMINARY DATA (COURTESY OF O. HEN AND E. PIASETZKY) COMPATIBLE WITH ABSORPTION ON $l_{12}=0$ PAIRS!

CONCLUSIONS (I)

Stylized features of nuclear SRC: The mass dependence of the magnitude of the 2N and 3N correlations can be captured by some approximate principles

- The number of SRC-prone pairs in a nucleus $A(N,Z)$ is proportional with the number of pairs in a relative S state ($l_{12} = 0$)!
  (2N pairs are prone to correlations when they are "close")
- The number of SRC-prone pairs follows a robust power law for pp,pn,nn: $Constant \times A^{1.44 \pm 0.01}$
- Technique can be extended to count the number of SRC-prone nucleon triples in a nucleus
  (3N triples are prone to correlations when they are "close")

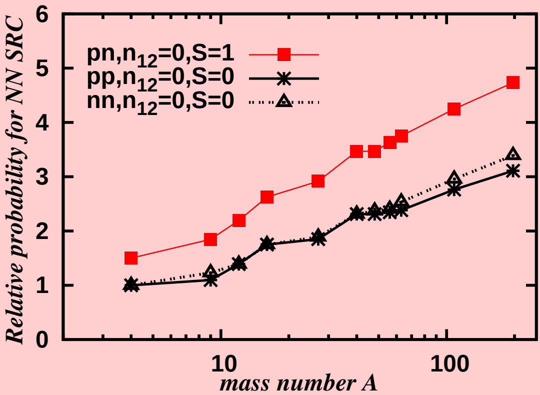
CONCLUSIONS (II)

Experimental evidence for this stylized features of nuclear SRC?

- Inclusive $A(e,e')$ at $1.5 \lesssim x_B$ (2N):
  The a_2 (A/D) can be predicted and these predictions are not inconsistent with trends and magnitude of the data (corrections for c.m. motion, FSI)
- Inclusive $A(e,e')$ at $2.2 \lesssim x_B$ (3N):
  Fair prediction for the a_3 $(A/^3\text{He})$
- The magnitude of the EMC effect:
  The $\frac{-dR_{EMC}}{dx_B}$ is proportional with the predicted number of SRC-prone pairs!
- Exclusive $A(e,e'pp)$:
  1. scaling behavior of c.s. ($\sim F(P)$): CONFIRMED!
  2. very soft mass dependence of c.s.: CONFIRMED!

Universality of SRC (I)

$a_2(A/D) = \frac{2}{A} N_{pn(S=1)}$ as a measure of the per-nucleon probability of pn SRC relative to the deuteron



Universality of SRC: $n_2^{(1)}(k_{12} \gg, P_{12} \approx 0) \approx N_{pn(S=1)} n^{(D)}(k_{12}) F^{pn}(P_{12}$


THANK YOU!