Short-range correlations studied with unitarily transformed interactions and operators

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GSI theory

HELMHOLTZ GEMEINSCHAFT

Motivation

Short-range correlations and high-momentum components in wave functions

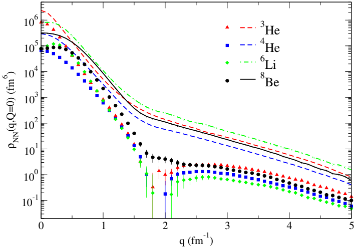
Schiavilla, Wiringa, Pieper, Carlson, PRL **98**, 132501 (2007)

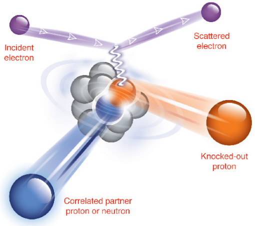
Wiringa, Schiavilla, Pieper, Carlson, PRC **85**, 021001(R) (2008)

Interaction dependence – AV18 versus N3LO

SRC, high-momenta and unitary transformations

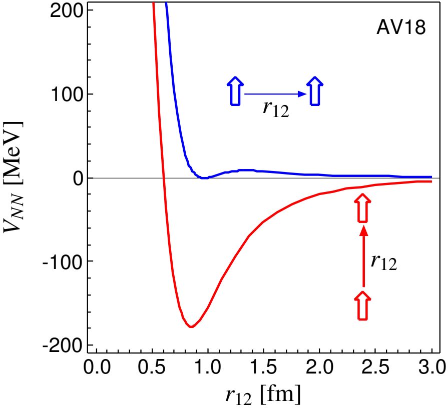
Anderson, Bogner, Furnstahl, Perry, PRC **82**, 054001 (2010)





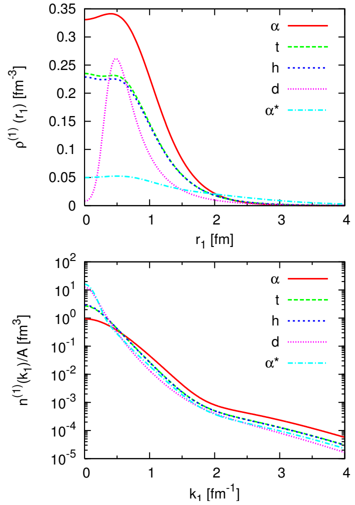
Short-range and tensor correlations

T=0



- strong repulsive core: nucleons can not get closer than ≈ 0.5 fm→ **central correlations**
- strong dependence on the orientation of the spins due to the tensor force → **tensor correlations**
- the nuclear force will induce **strong short-range correlations** in the nuclear wave function

One-body densities for *A*=2,3,4 nuclei



Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

$$\rho^{(1)}(\boldsymbol{r}_1) = \langle\Psi|\sum_{i=1}^{A}\delta^3(\hat{\boldsymbol{r}}_i - \boldsymbol{r}_1)|\Psi\rangle$$

$$n^{(1)}(\boldsymbol{k}_1) = \langle\Psi|\sum_{i=1}^{A}\delta^3(\hat{\boldsymbol{k}}_i - \boldsymbol{k}_1)|\Psi\rangle$$

- one-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8’ interaction
- coordinate space densities reflect different sizes and densities of ^{2}H, ^{3}H, ^{3}He, ^{4}He and the 0_2^+ state in ^{4}He
- similar high-momentum tails in the one-body momentum distributions

Definition: Two-body densities

of pairs in given spin-, isospin channels

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \langle\Psi|\sum_{i<j}^{A} \hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{r}}_i-\boldsymbol{r}_1)\delta^3(\hat{\boldsymbol{r}}_j-\boldsymbol{r}_2)\,|\Psi\rangle$$

$$n^{(2)}_{SM_S,TM_T}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \langle\Psi|\sum_{i<j}^{A} \hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{k}}_i-\boldsymbol{k}_1)\delta^3(\hat{\boldsymbol{k}}_j-\boldsymbol{k}_2)\,|\Psi\rangle$$

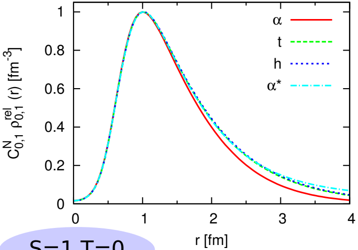
integrated over center-of-mass position $\boldsymbol{R}=\frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2)$ or the total momentum of the nucleon pair $\boldsymbol{K}=\boldsymbol{k}_1+\boldsymbol{k}_2$ of the nucleon:

$$\rho^{\text{rel}}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Psi|\sum_{i<j}^{A} \hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{r}}_i-\hat{\boldsymbol{r}}_j-\boldsymbol{r})\,|\Psi\rangle$$

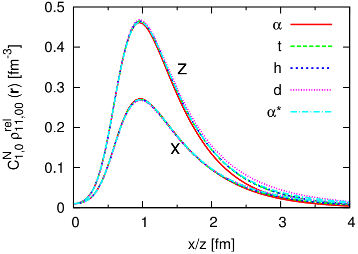
$$n^{\text{rel}}_{SM_S,TM_T}(\boldsymbol{k}) = \langle\Psi|\sum_{i<j}^{A} \hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3\big(\frac{1}{2}(\hat{\boldsymbol{k}}_i-\hat{\boldsymbol{k}}_j)-\boldsymbol{k}\big)\,|\Psi\rangle$$

Two-body densities in coordinate space for A=2,3,4

S=0,T=1



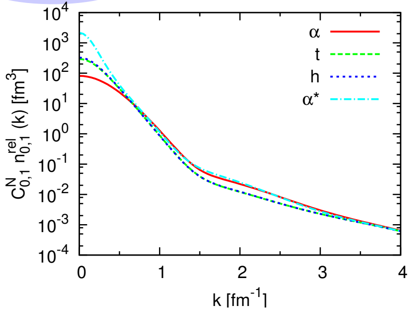
S=1,T=0



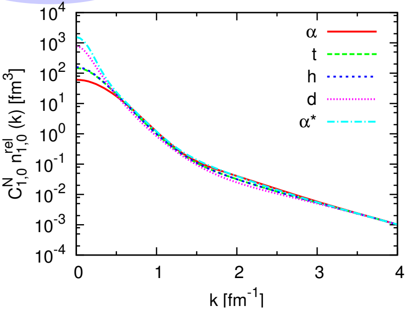
- two-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- → normalize two-body density in coordinate space at r=1.0 fm
- → normalized two-body densities in coordinate space are identical at short distances for all nuclei
- also true for angular dependence in the tensor channel

Two-body densities in momentum space for A=2,3,4

S=0,T=1



S=1,T=0



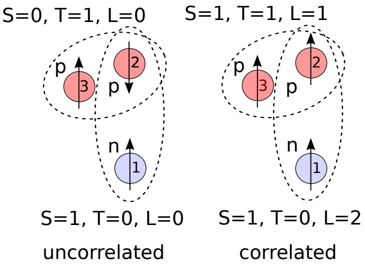
- use **normalization factors fixed in coordinate space**
- → two-body densities in momentum space identical for very high momenta $k > 3\mathrm{fm}^{-1}$
- moderate nucleus dependence in high momentum region $1.5\mathrm{fm}^{-1} < k < 3\mathrm{fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

Two-body densities reflect many-body correlations

count the number of pairs in the (ST) channels.

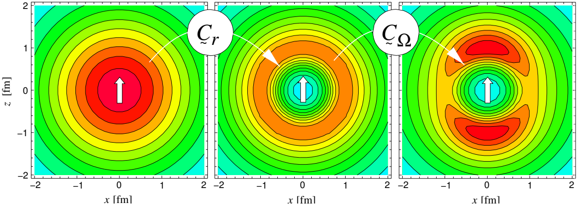
state\\(ST)	(10)	(01)	(11)	(00)
d	1	–	–	–
t	1.490	1.361	0.139	0.010
h	1.489	1.361	0.139	0.011
α	2.992	2.572	0.428	0.008
α^*	2.966	2.714	0.286	0.034



- occupation in (ST)=(10) almost exactly as in IPM
- (ST)=(01) significantly depopulated in favor of (ST)=(11)
- → three-body correlations induced by the two-body tensor force: depopulation of (ST)=(01) channel is the price one has to pay for getting the full binding from the tensor force

Unitary Correlation Operator Method

$n^{\mathrm{rel}}_{S,T}(\boldsymbol{r}) \quad S=1, M_S=1, T=0$

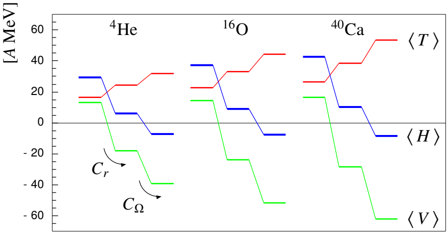


central correlator $\hat{C}_r$ shifts density out of the repulsive core

tensor correlator $\hat{C}_\Omega$ aligns density with spin orientation

both central and tensor correlations are essential for binding

$0\hbar\omega$ Harmonic Oscillator



Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 51 (2010)

Similarity Renormalization Group

Flow equation

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right]_-$$

Unitary transformation of Hamiltonian and other operators

$$\hat{H}_\alpha = \hat{U}_\alpha^\dagger \hat{H} \hat{U}_\alpha, \qquad \hat{B}_\alpha = \hat{U}_\alpha^\dagger \hat{B} \hat{U}_\alpha$$

Flow equation for $\hat{U}_\alpha$

$$\frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

Metagenerator

$$\hat{\eta}_\alpha = (2\mu)^2 \left[\hat{T}_{\text{int}}, \hat{H}_\alpha\right]_- = 2\mu \left[\hat{\boldsymbol{k}}^2, \hat{H}_\alpha\right]_-$$

Similarity Renormalization Group

Outline of calculation

simultaneous SRG evolution for transformed Hamiltonian and transformation matrix on the two-body level

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right]_- , \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

Solve many-body problem with SRG transformed Hamiltonian in the NCSM

$$\hat{H}_\alpha \left|\Psi_\alpha\right\rangle = E_\alpha \left|\Psi_\alpha\right\rangle$$

Calculate expectation values of “bare” and “effective” two-body density operators

$$\rho_{\text{bare}} = \left\langle\Psi_\alpha\right| \hat{\rho} \left|\Psi_\alpha\right\rangle, \qquad \rho_{\text{effective}} = \left\langle\Psi_\alpha\right| \hat{U}^\dagger_\alpha \hat{\rho} \hat{U}_\alpha \left|\Psi_\alpha\right\rangle$$

→ **Check for convergence of NCSM calculations and α-dependence**

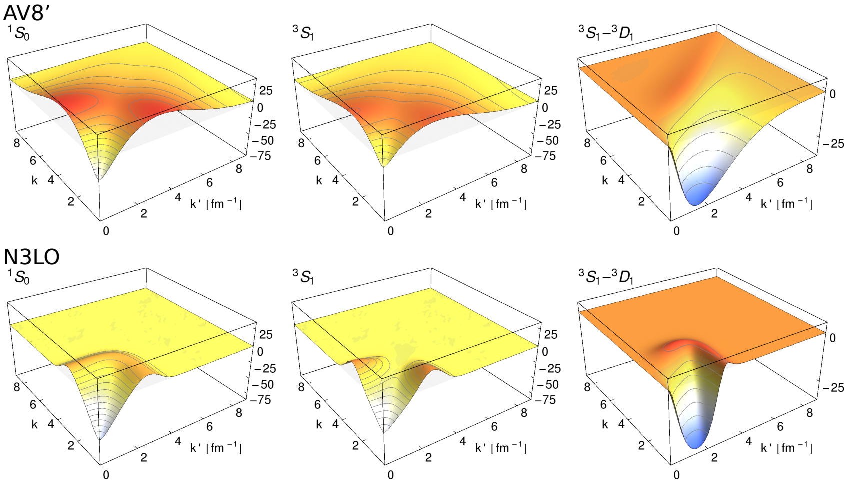
Similarity Renormalization Group

Implementation Details

- SRG evolution for $\hat{H}_\alpha$ and $\hat{U}_\alpha$ in momentum space $k_{\mathrm{max}} = 15\mathrm{fm}^{-1}$
- Operators only depend on relative coordinates and not on the center-of-mass of the pairs
- (SRG transformed) momentum space matrix elements are expanded in HO basis
- *jj*-coupled matrix elements are calculated using the Talmi-Moshinski procedure
- a slight modification is needed if we look at the two-body densities also as a function of pair momentum

Similarity Renormalization Group

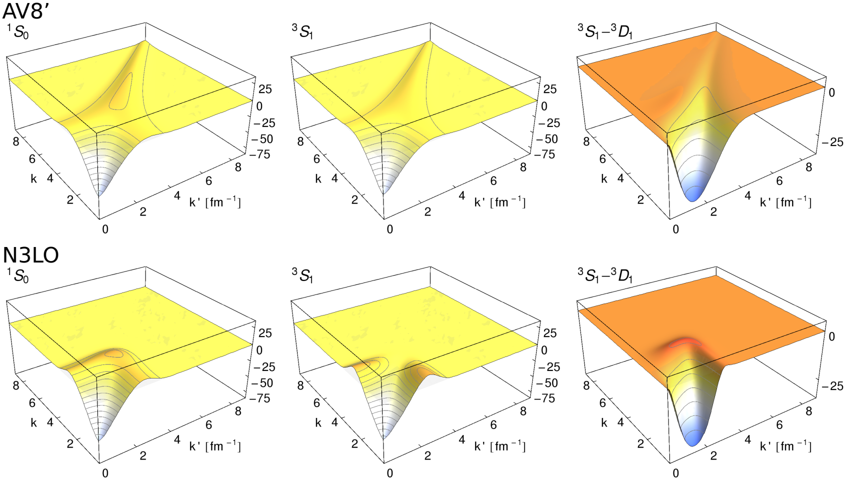
Hamiltonian Flow



$\alpha = 0.00$ (bare)

Similarity Renormalization Group

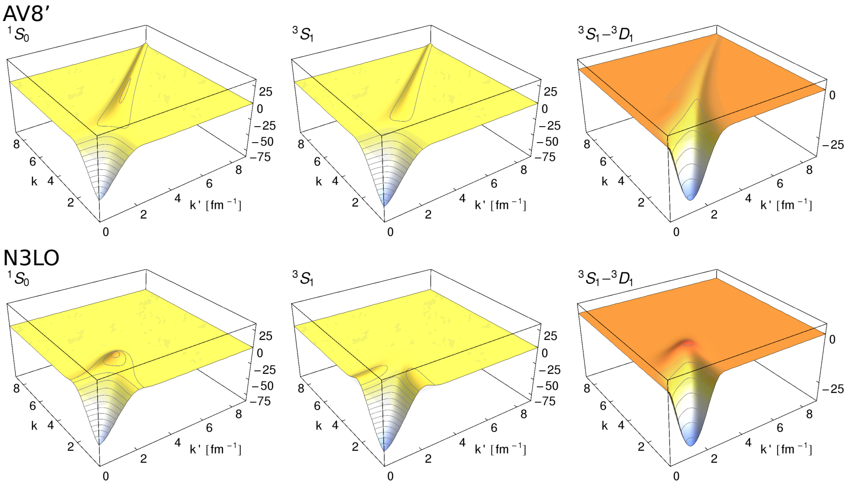
Hamiltonian Flow



$\alpha = 0.01\text{fm}^4$

Similarity Renormalization Group

Hamiltonian Flow

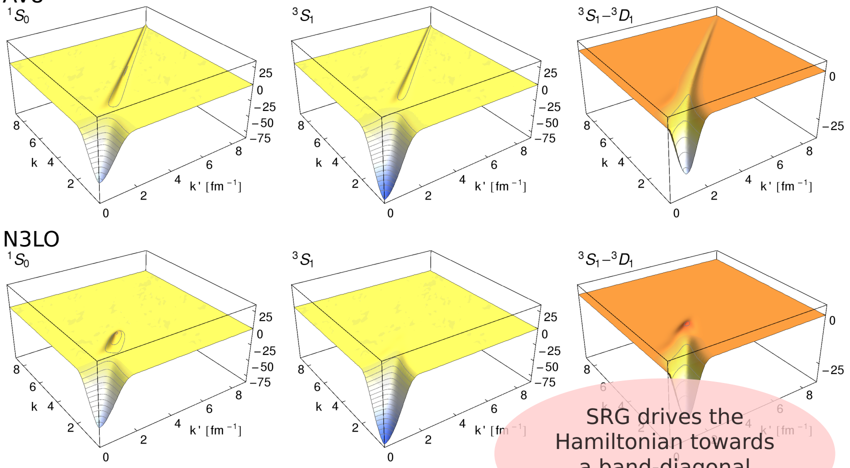


$\alpha = 0.04\text{fm}^4$

Similarity Renormalization Group

Hamiltonian Flow

AV8'



$\alpha = 0.20 \text{fm}^4$

SRG drives the Hamiltonian towards a band-diagonal structure

^{4}He Results

^{4}He advantages

- exact two-body densities available for AV8' interaction
- "bare" N3LO can be converged in NCSM

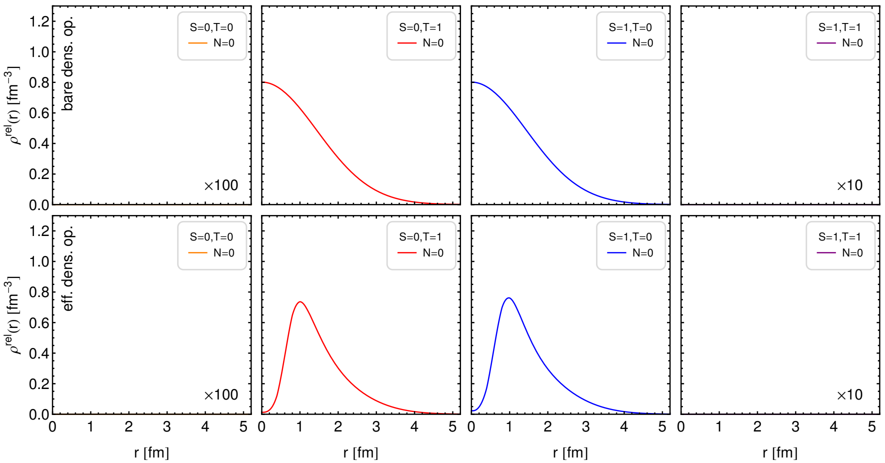
Objectives

- Compare AV8' and N3LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.01, 0.04, 0.20\text{fm}^4$ ($\Lambda = 3.16, 2.24, 1.50\text{fm}^{-1}$)
- Can we see many-body effects ?

Convergence with the model space size

AV8' – $\alpha = 0.04\,\mathrm{fm}^4$

Two-body Density in Coordinate Space



$\hbar\Omega = 20\mathrm{MeV}$ corresponds to roughly the size of ^{4}He

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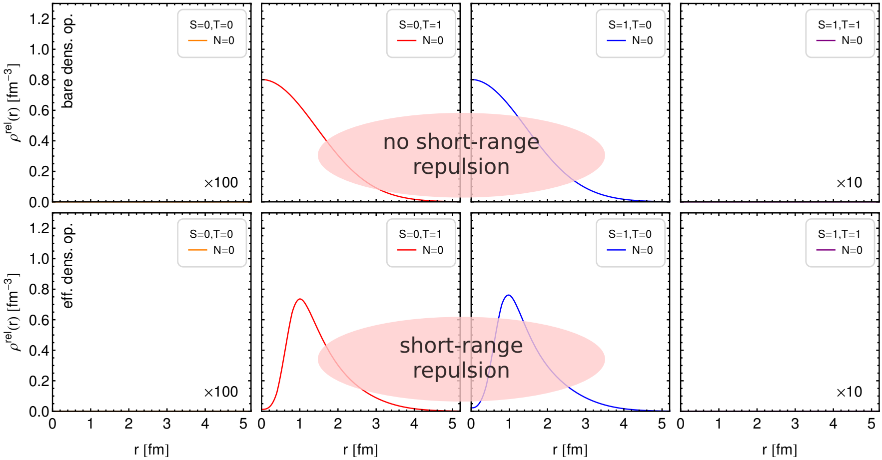

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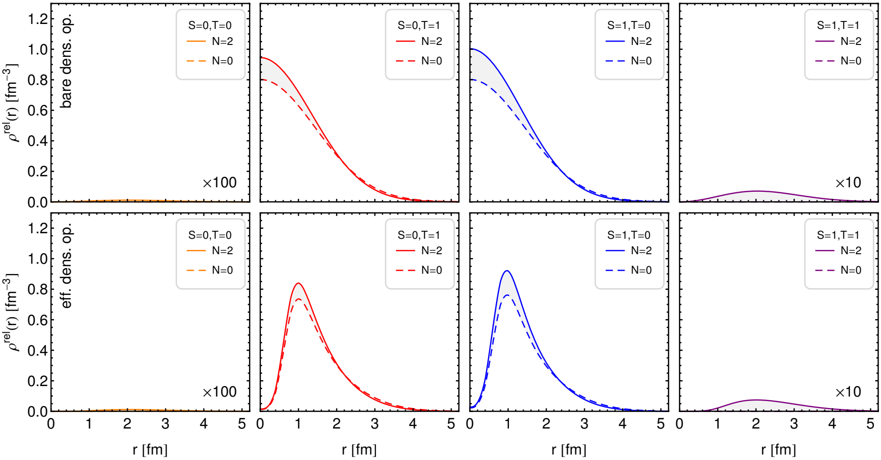

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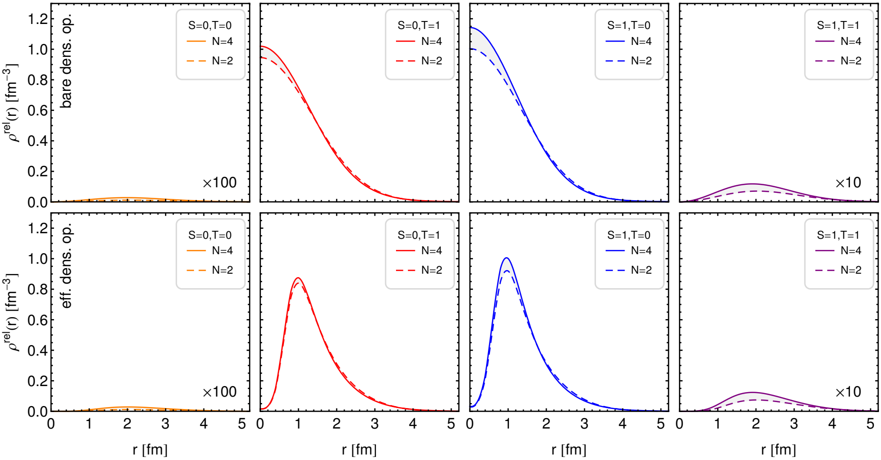

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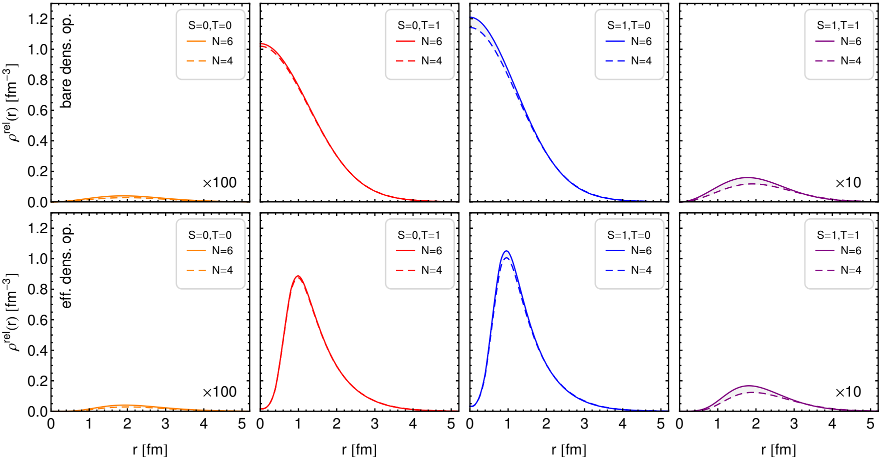

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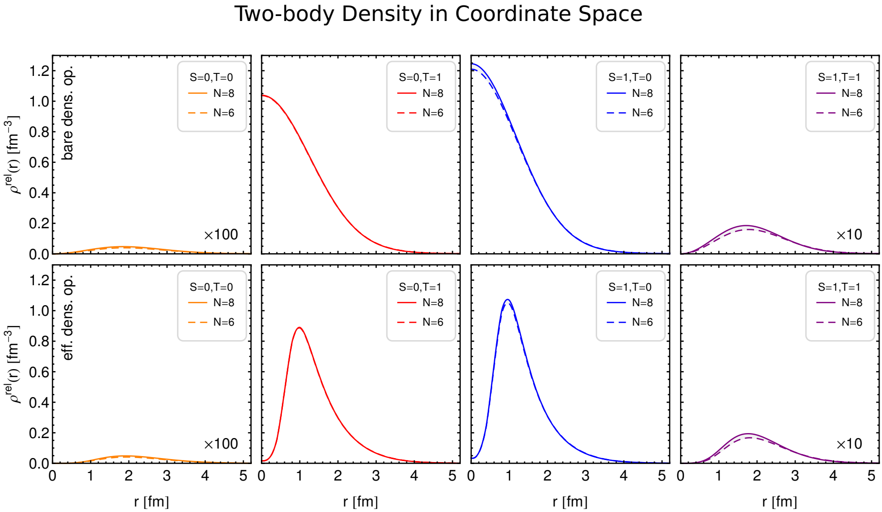

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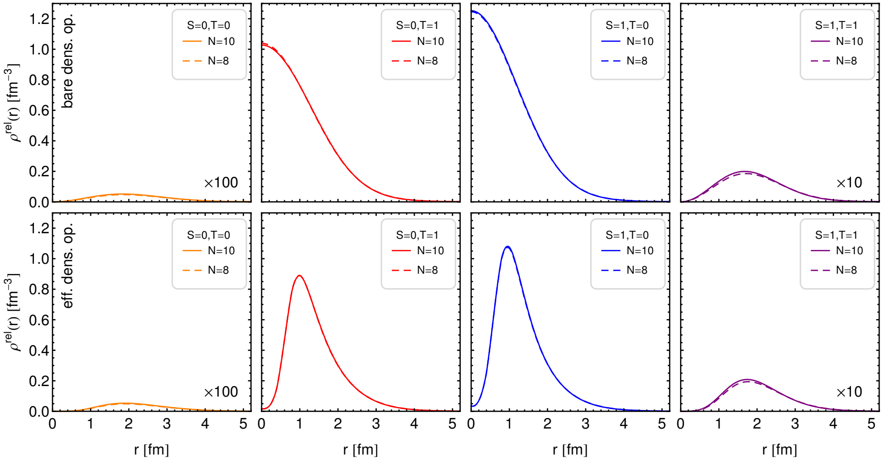

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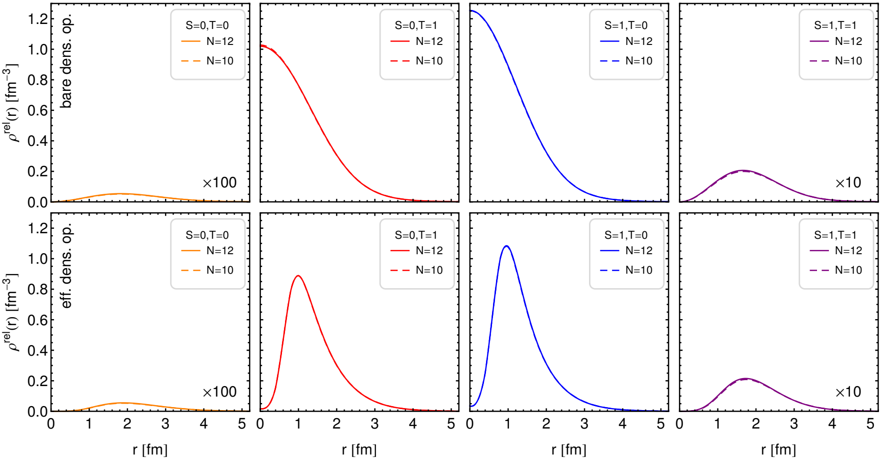

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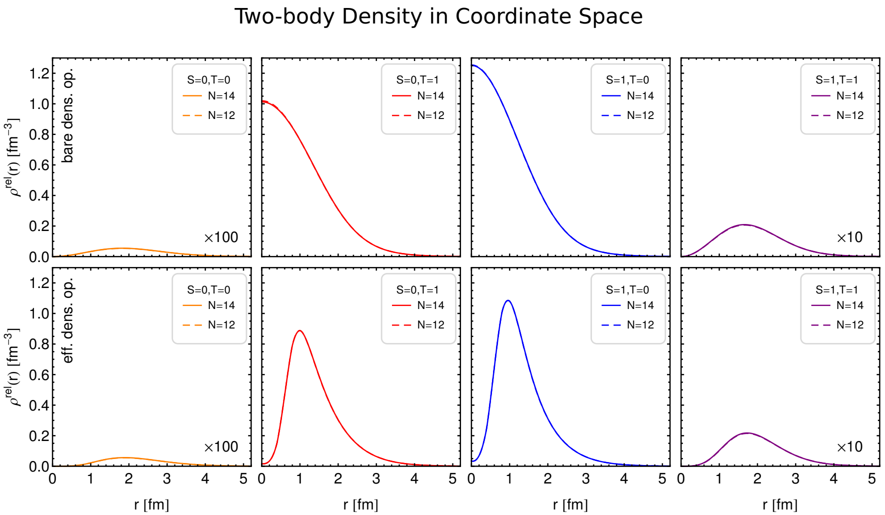

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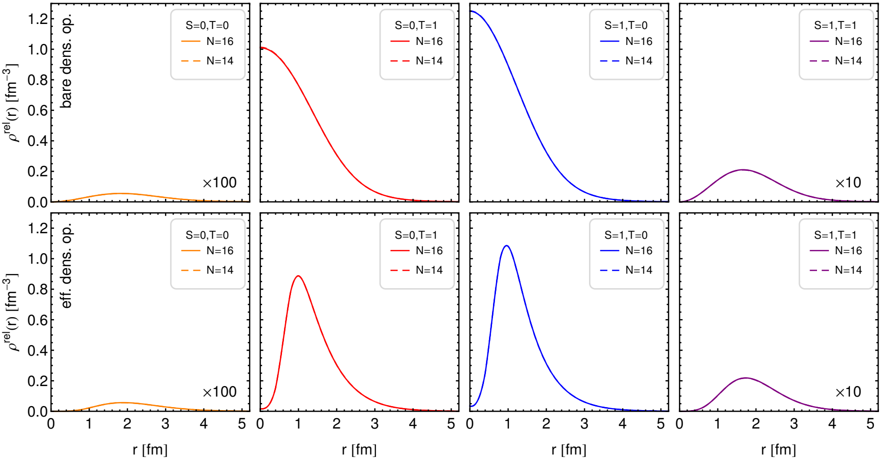

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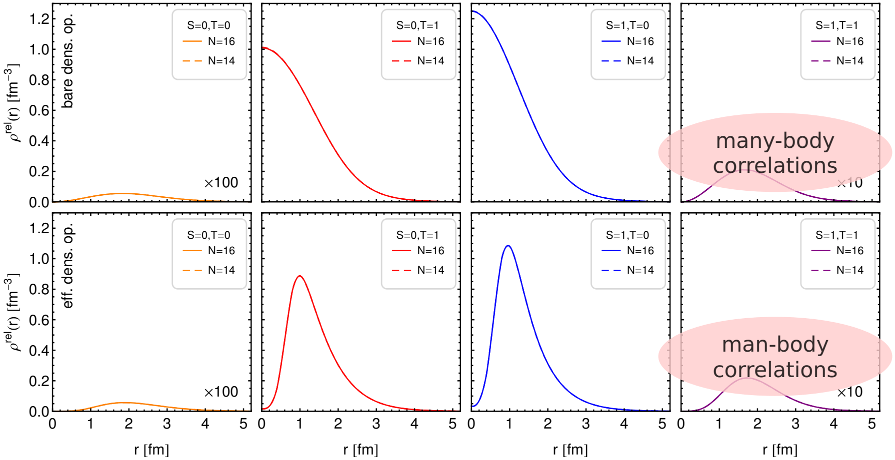

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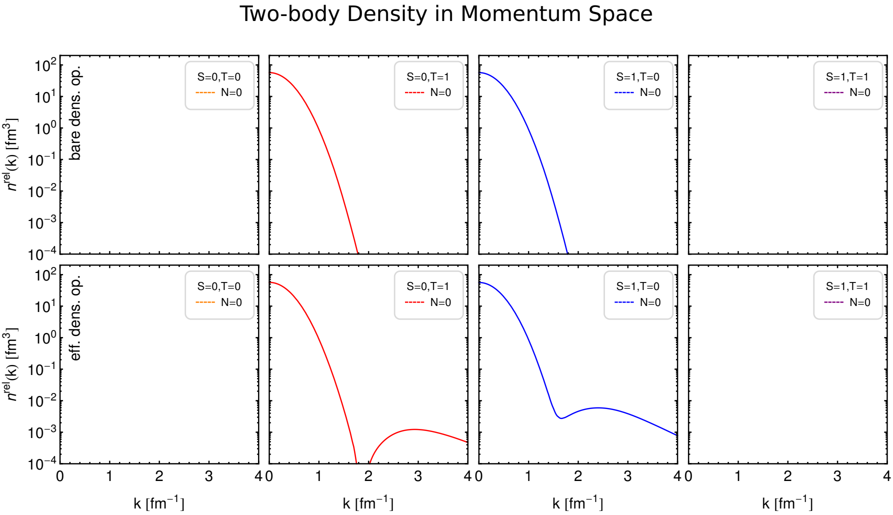

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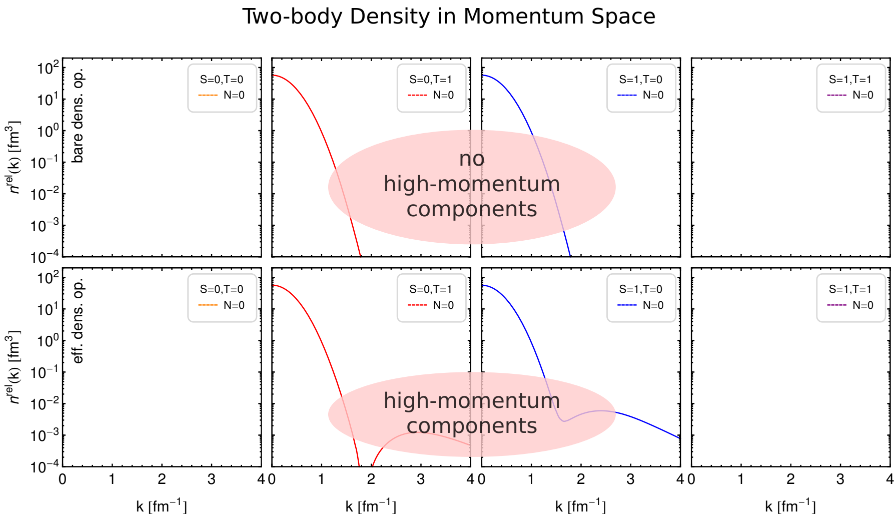

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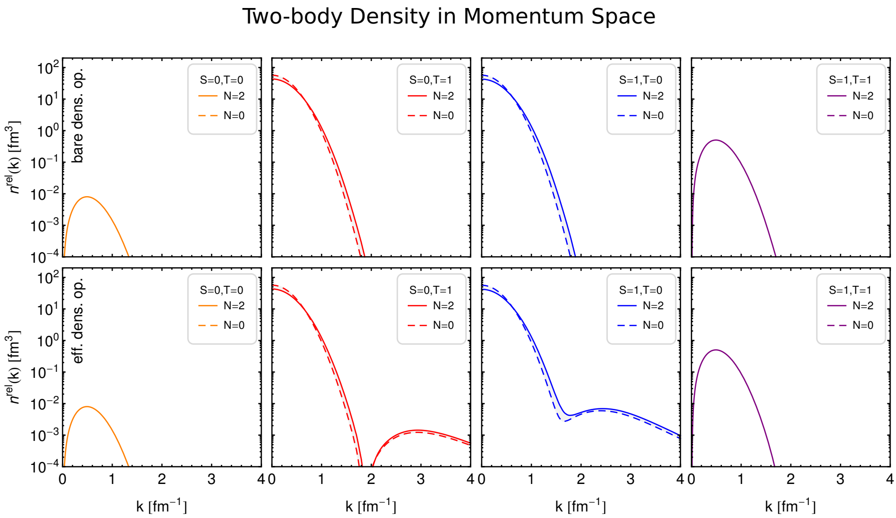

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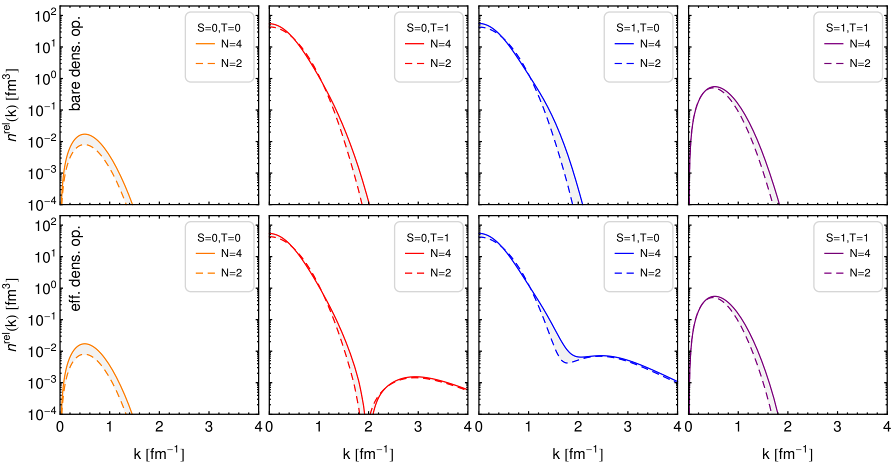
Two-body Density in Momentum Space



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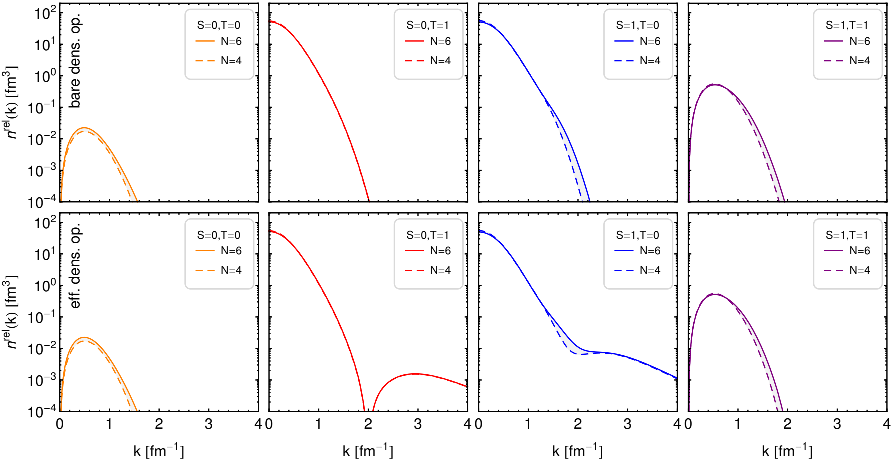

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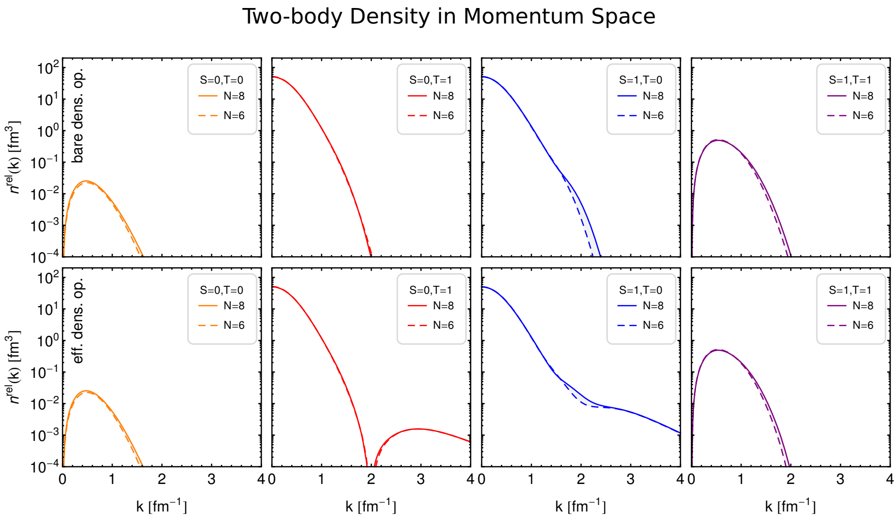

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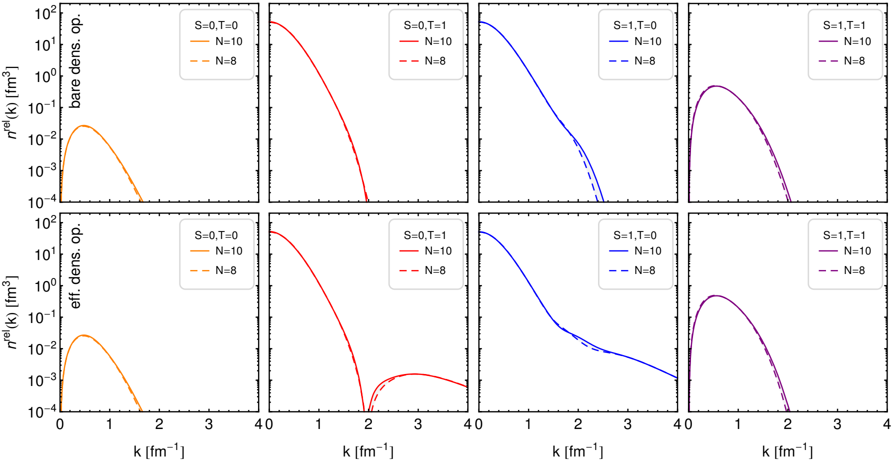

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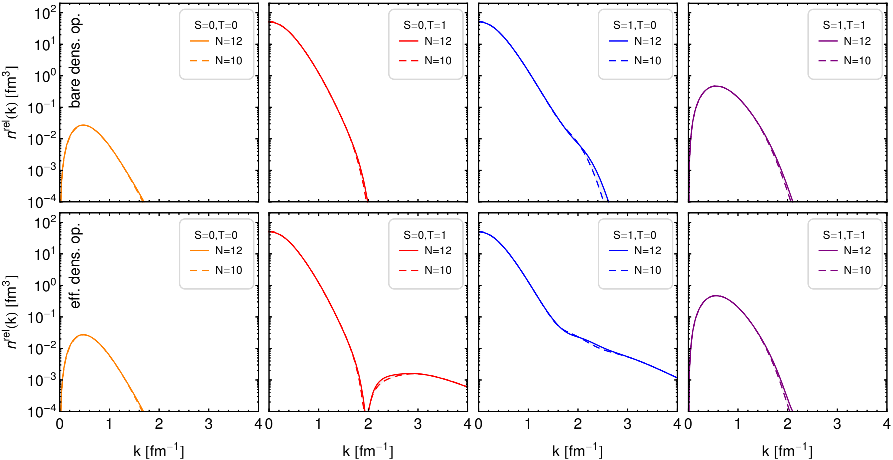

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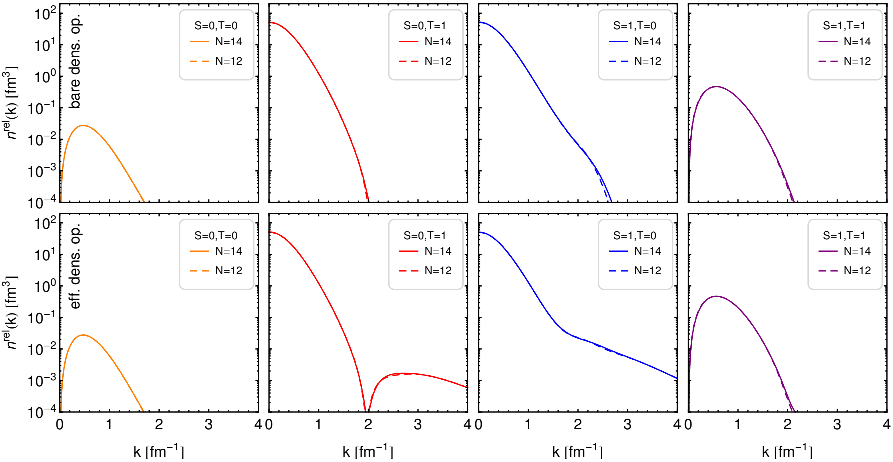

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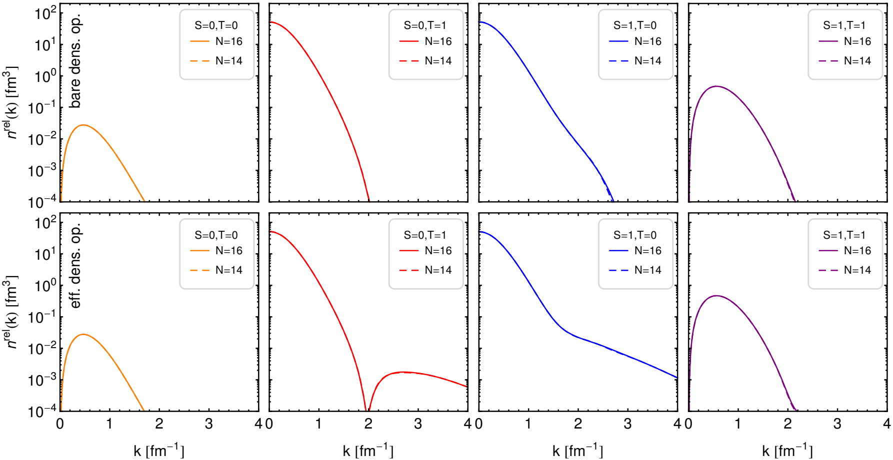

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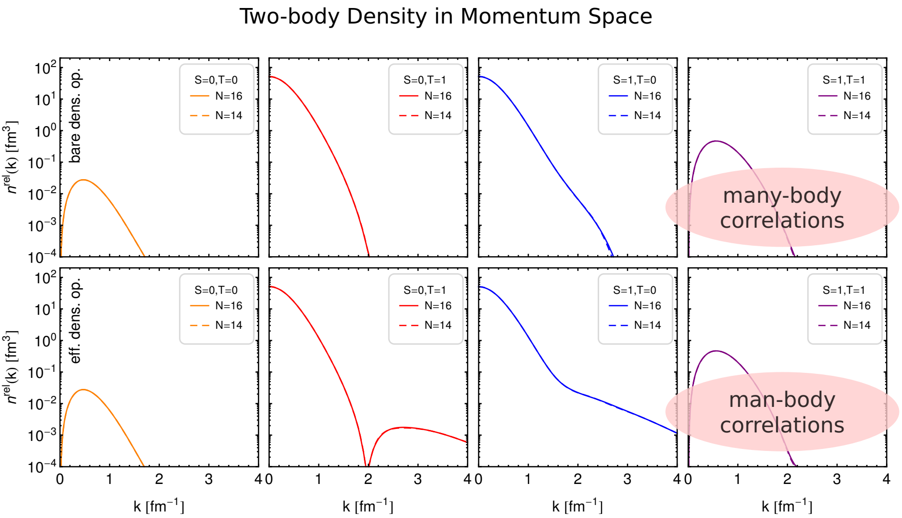

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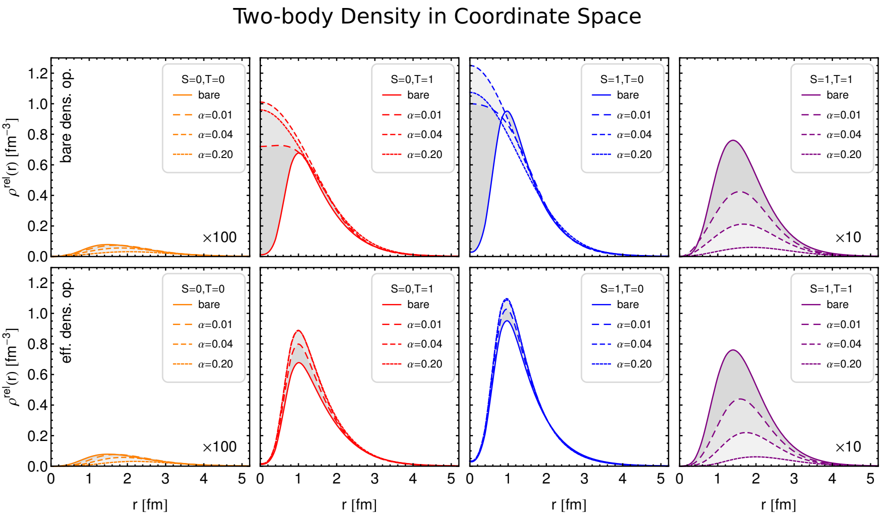
Flow dependence

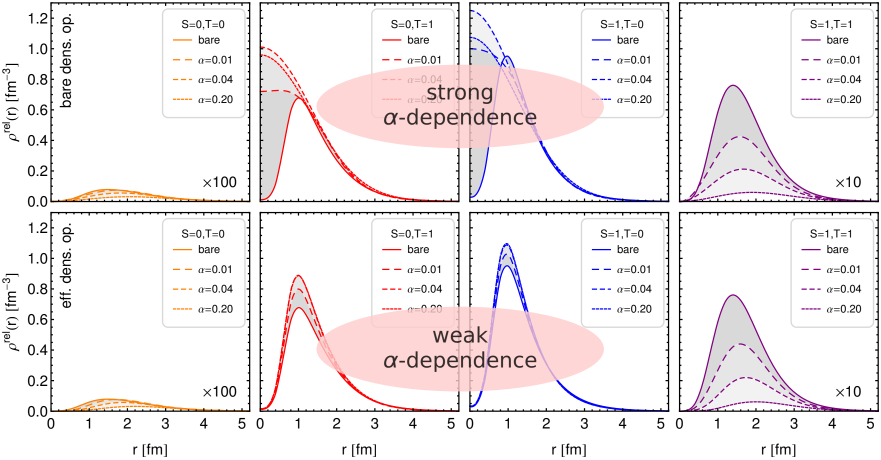
AV8' Interaction



Flow dependence

AV8' Interaction

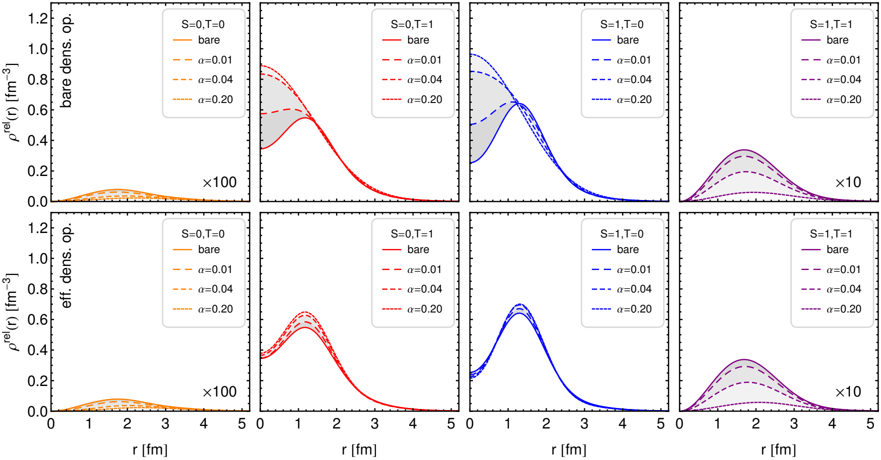
Two-body Density in Coordinate Space



Flow dependence

N3LO Interaction

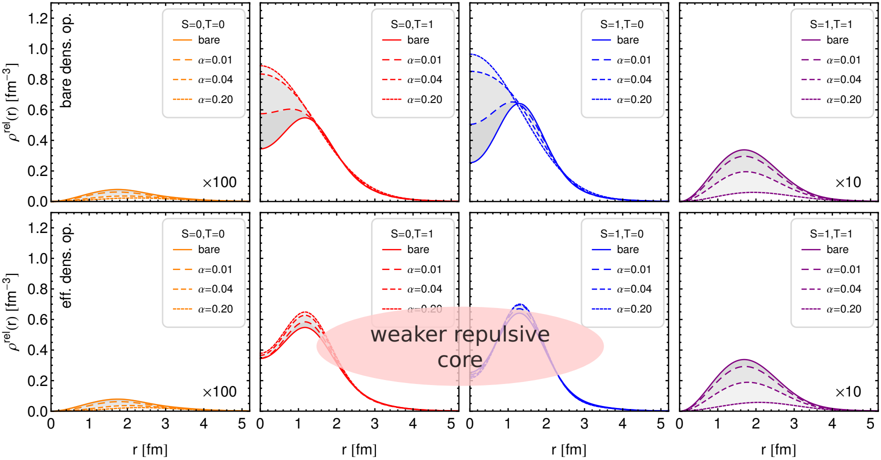
Two-body Density in Coordinate Space



Flow dependence

N3LO Interaction

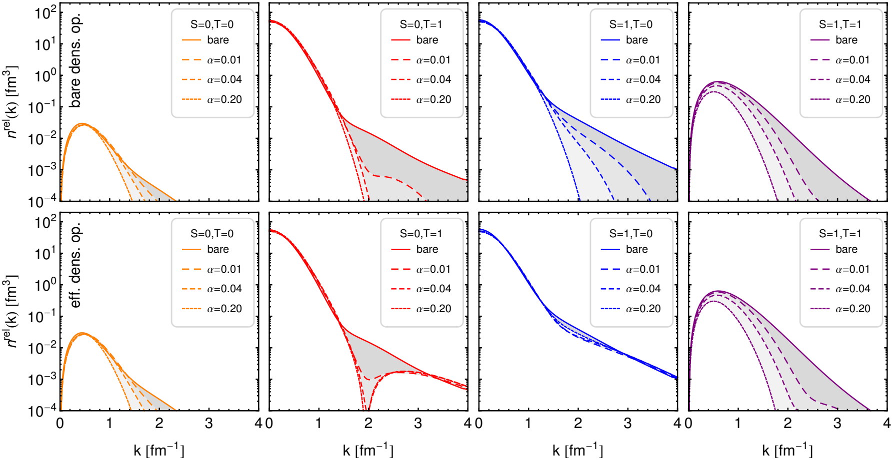
Two-body Density in Coordinate Space



Flow dependence

AV8' Interaction

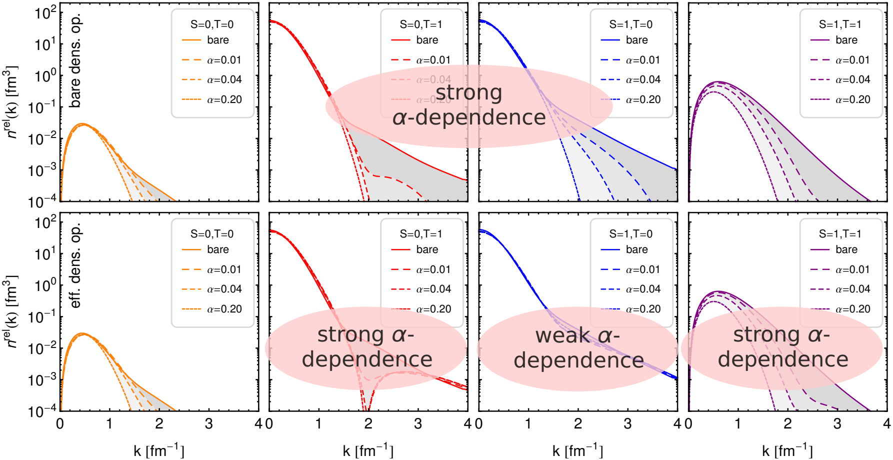
Two-body Density in Momentum Space



Flow dependence

AV8' Interaction

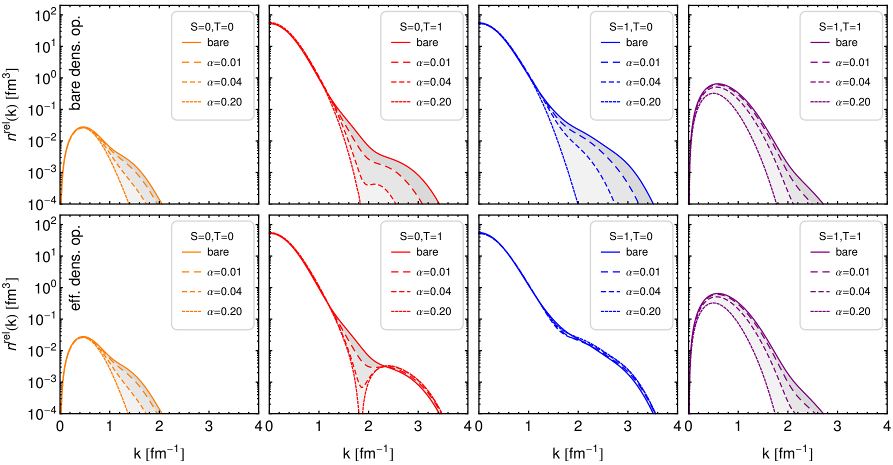
Two-body Density in Momentum Space



Flow dependence

N3LO Interaction

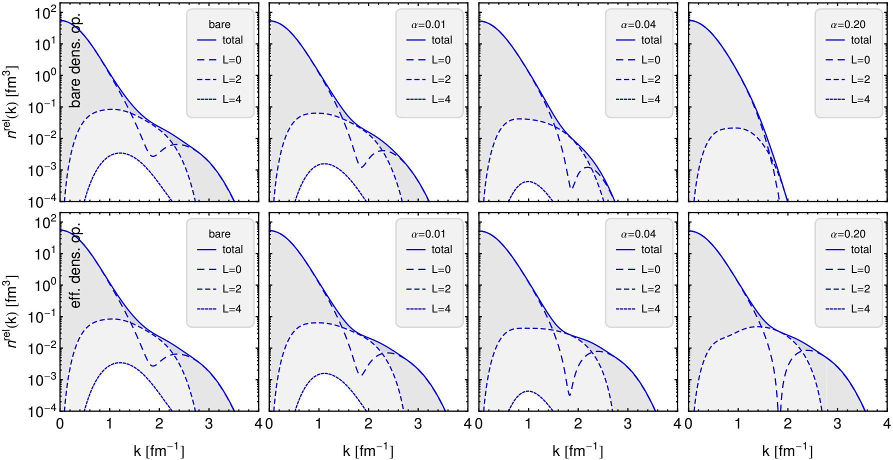
Two-body Density in Momentum Space



Contributions from different angular momenta

N3LO Interaction

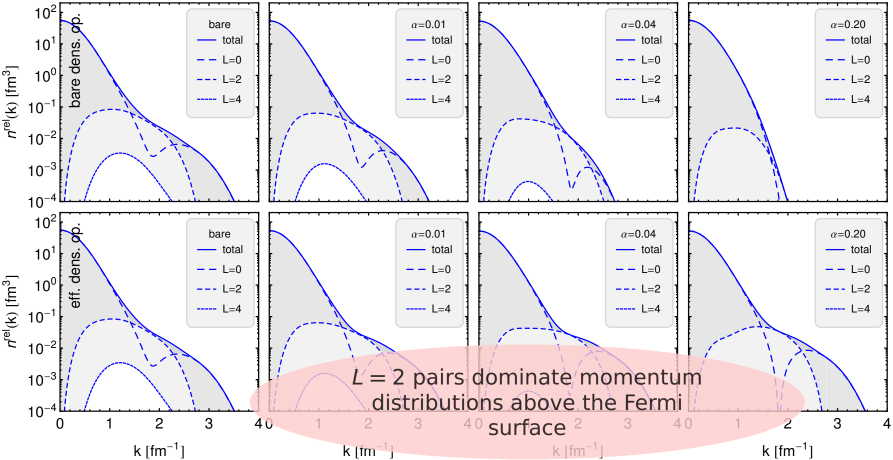
Two-body Density in Momentum Space $S = 1, T = 0$



Contributions from different angular momenta

N3LO Interaction

Two-body Density in Momentum Space $S = 1, T = 0$

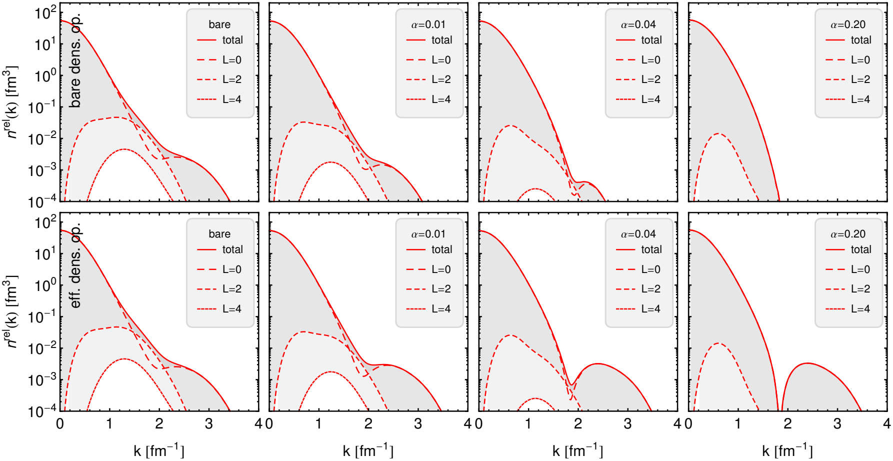


$L = 2$ pairs dominate momentum distributions above the Fermi surface

Contributions from different angular momenta

N3LO Interaction

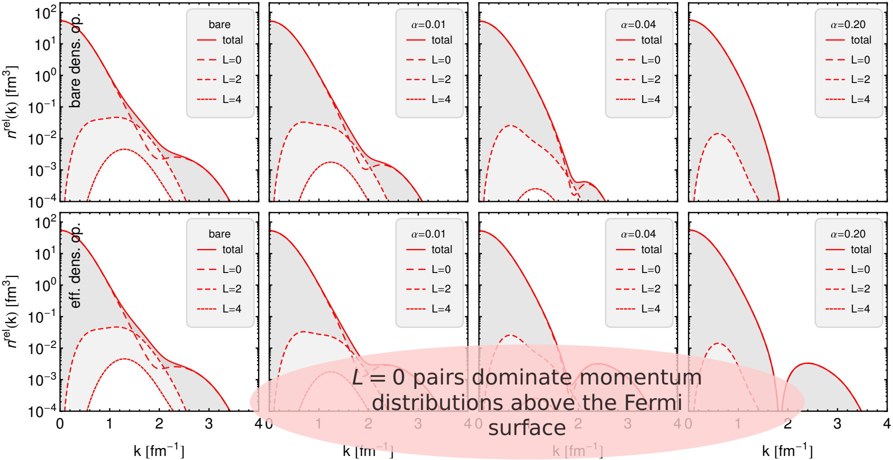
Two-body Density in Momentum Space $S = 0, T = 1$



Contributions from different angular momenta

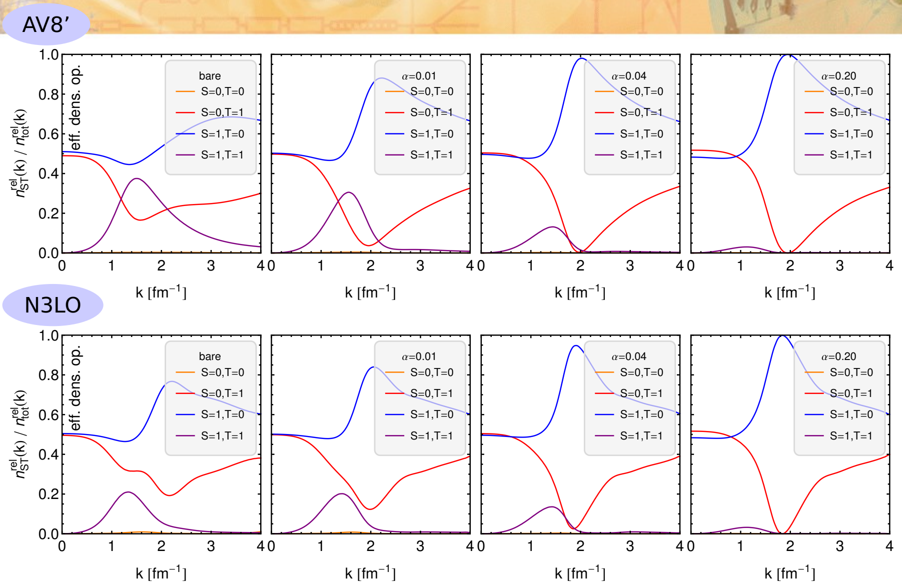
N3LO Interaction

Two-body Density in Momentum Space $S = 0, T = 1$



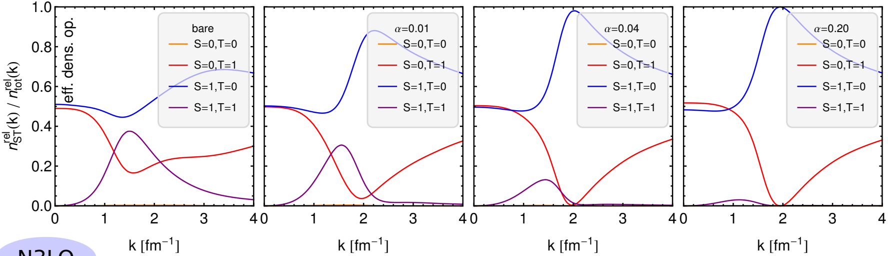
$L = 0$ pairs dominate momentum distributions above the Fermi surface

Relative contributions of ST channels

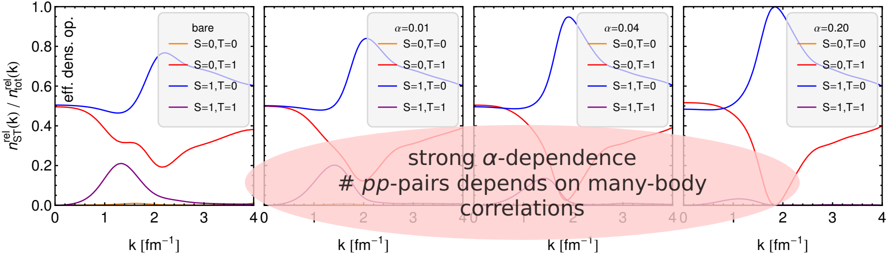


Relative contributions of ST channels

AV8’



N3LO

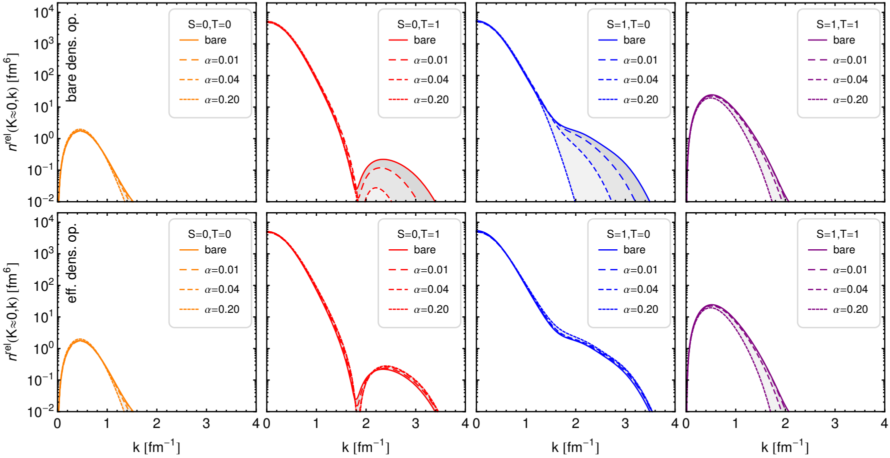


strong α-dependence
pp-pairs depends on many-body correlations

Pair momentum ≈ 0

N3LO Interaction

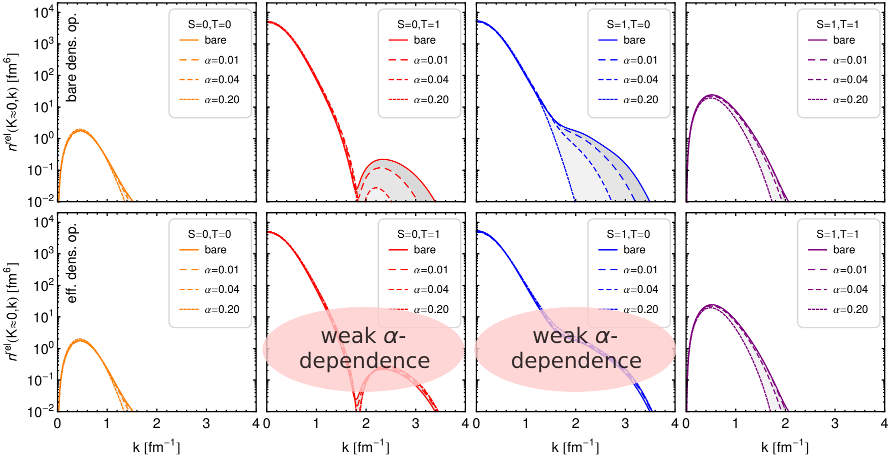
Two-body Density in Momentum Space



Pair momentum ≈ 0

N3LO Interaction

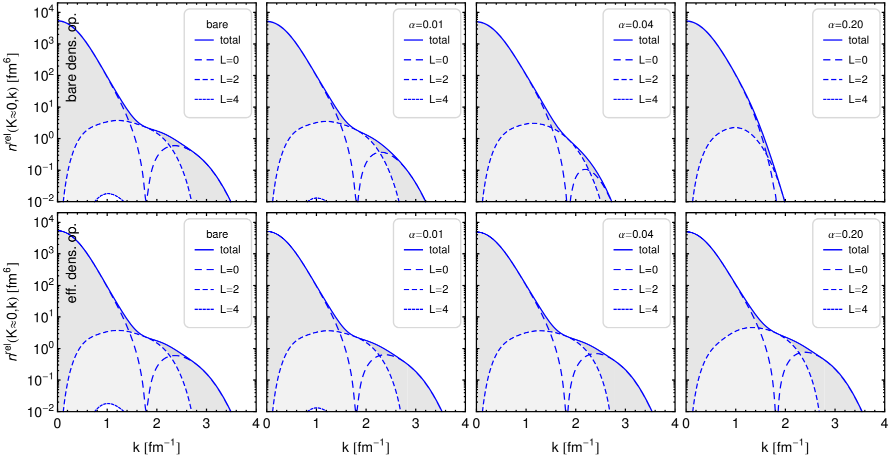
Two-body Density in Momentum Space



Pair momentum ≈ 0

N3LO Interaction

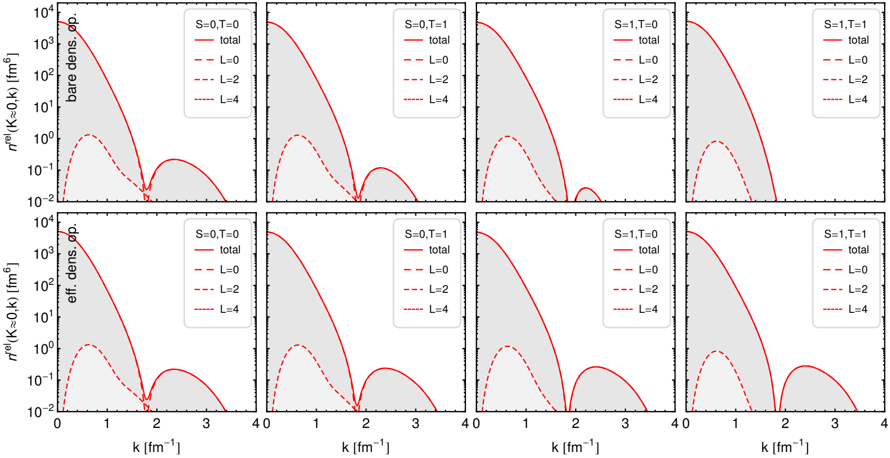
Two-body Density in Momentum Space $S = 1, T = 0$



Pair momentum ≈ 0

N3LO Interaction

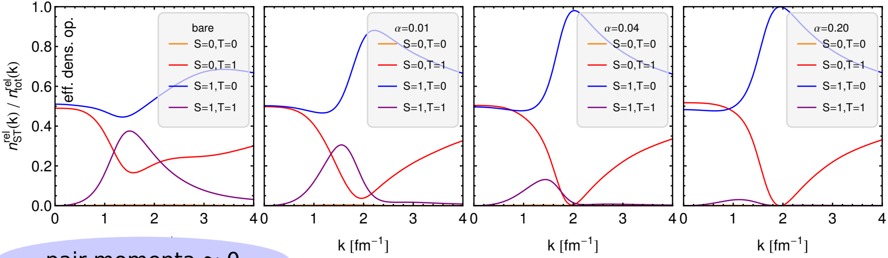
Two-body Density in Momentum Space $S = 0, T = 1$



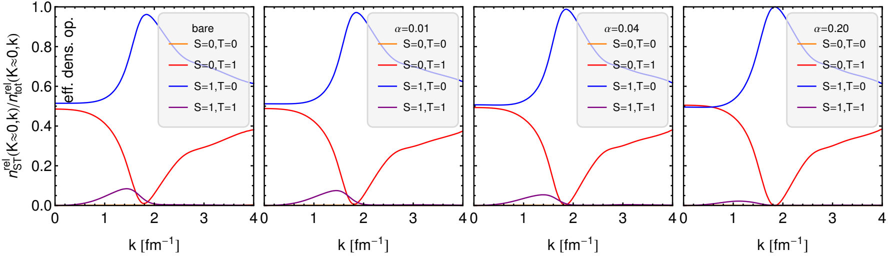
Relative contributions of ST channels

N3LO interaction

all pair momenta



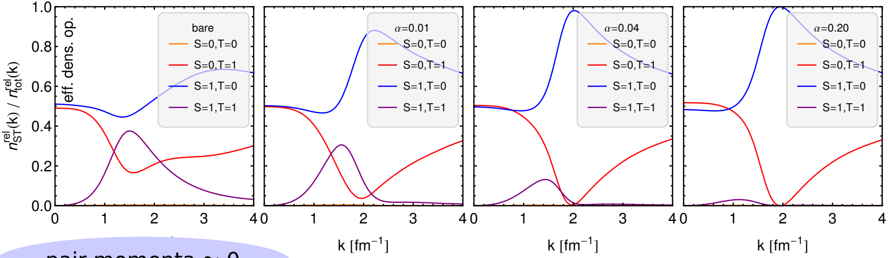
pair momenta ≈ 0



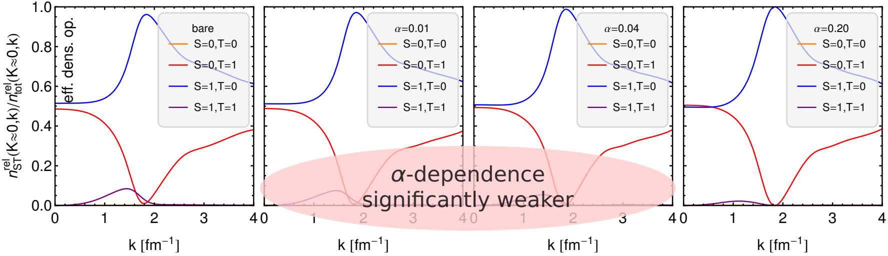
Relative contributions of ST channels

N3LO interaction

all pair momenta



pair momenta ≈ 0



α-dependence significantly weaker

^{4}He,^{6}Li,^{10}B,^{12}C Results

Calculation

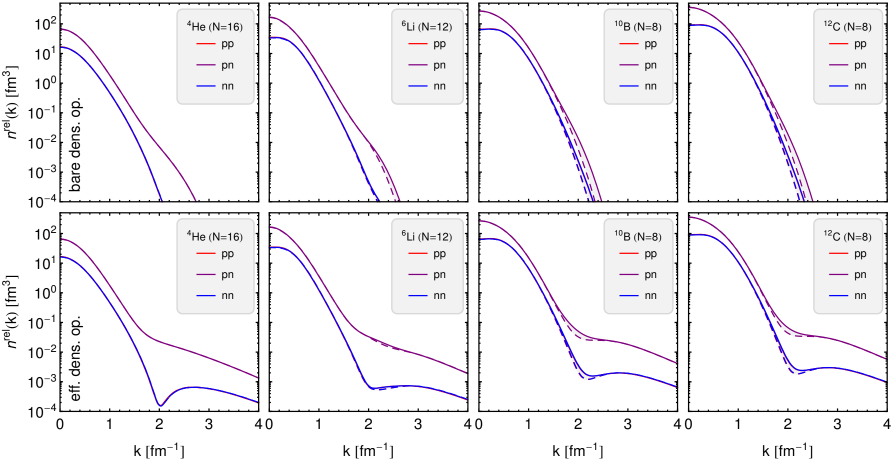
- “bare” AV18 and N3LO can not be converged
- NCSM convergence only for larger flow parameters

Objectives

- Compare AV18 and N3LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.04, 0.20 \mathrm{fm}^4$ ($\Lambda = 2.24, 1.50 \mathrm{fm}^{-1}$)
- What is different from ^{4}He ?

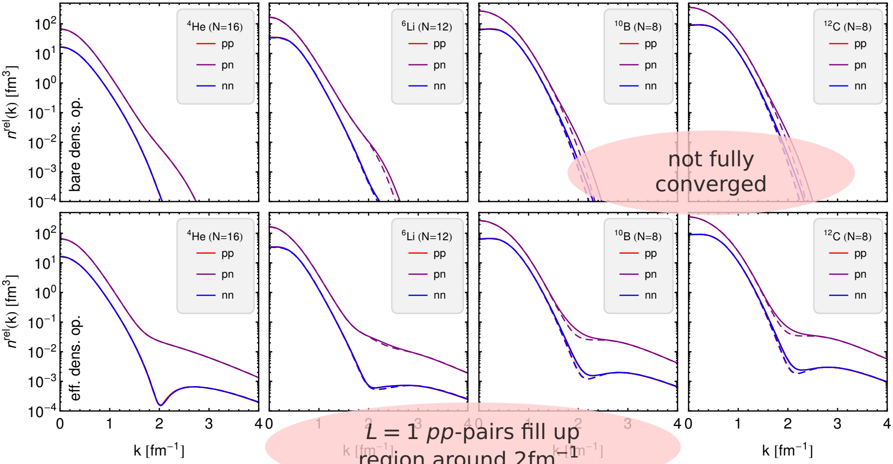
AV18, $\alpha = 0.04\,\mathrm{fm}^4$

Two-body Density in Momentum Space



AV18, $\alpha = 0.04\mathrm{fm}^4$

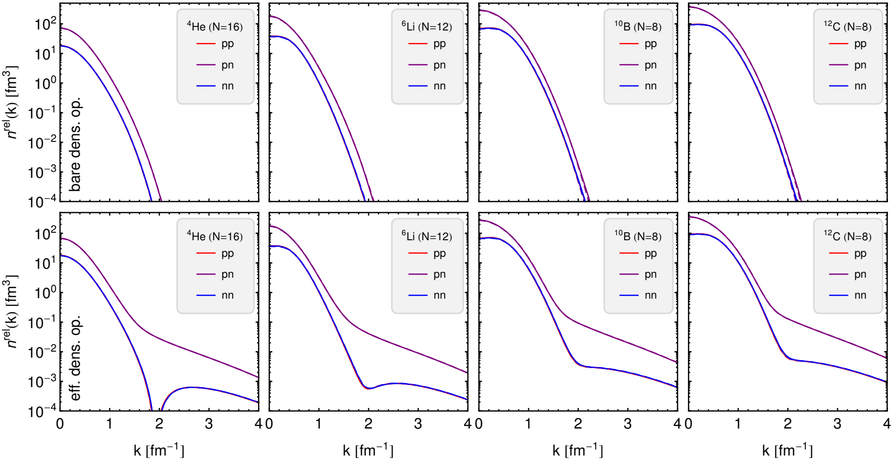
Two-body Density in Momentum Space



$L = 1$ pp-pairs fill up region around $2\mathrm{fm}^{-1}$.

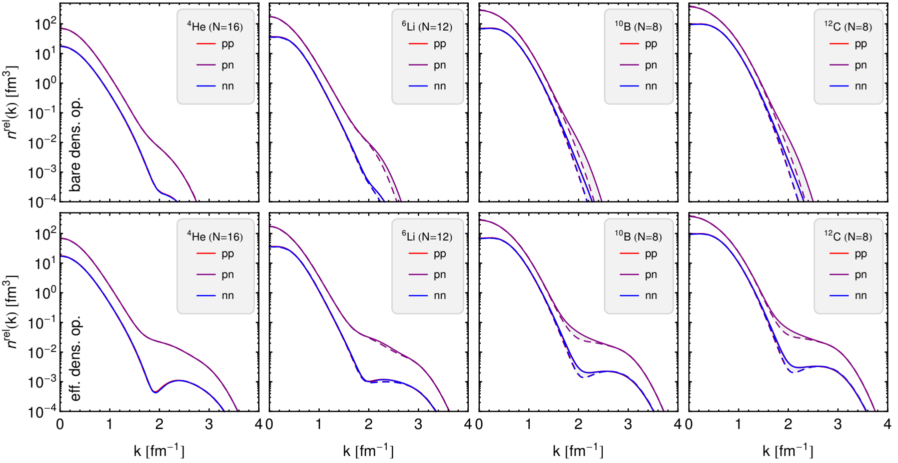
AV18, $\alpha = 0.20\,\mathrm{fm}^4$

Two-body Density in Momentum Space



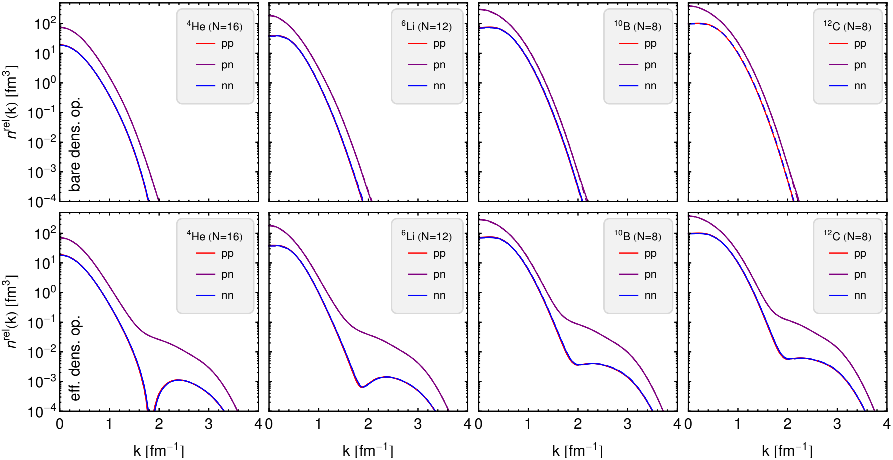
N3LO, $\alpha = 0.04 \text{fm}^4$

Two-body Density in Momentum Space



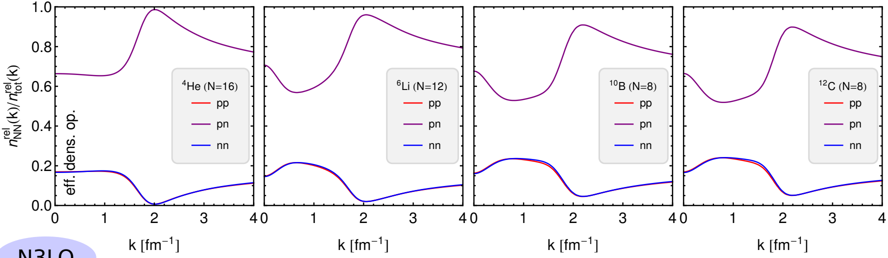
N3LO, $\alpha = 0.20\text{fm}^4$

Two-body Density in Momentum Space

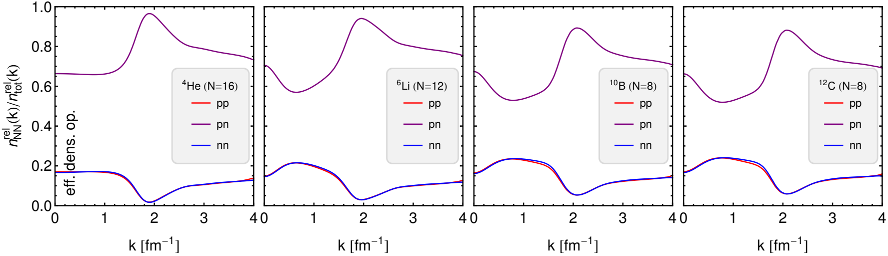


pp, pn, nn contributions

AV18

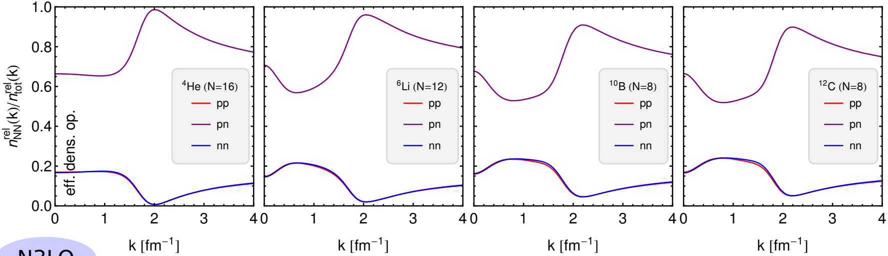


N3LO

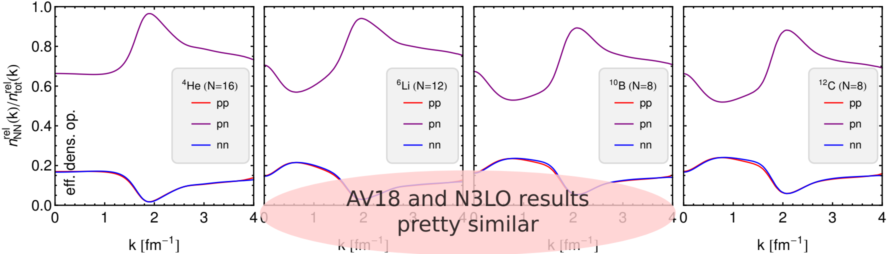


pp, pn, nn contributions

AV18



N3LO



AV18 and N3LO results pretty similar

Summary

Similarity Renormalization Group

- SRG evolved Hamiltonian and transformation matrix
- “bare” and “effective” density operators

^{4}He Two-body densities

- AV8’ and N3LO interactions
- short-range and high-momentum components described by effective operators
- high-momentum components above the Fermi momentum dominated by $L = 2$ pairs
- weak α-dependence in the $S = 1, T = 0$ channel
- strong α-dependence in the $S = 0, T = 1$ channel due to many-body correlations
- AV8’ and N3LO interaction results differ mainly in the $S = 0, T = 1$ channel due to different many-body correlations

^{4}He,^{6}Li,^{10}B,^{12}C Two-body densities

- $T = 1$ pairs with $L = 1$ fill up the momentum distribution above the Fermi momentum
- less sensitivity to many-body correlations
- AV18 and N3LO provide very similar results