Transport Theory and Short Range Correlations

Based on work with
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Motivation and Content

- Transport equations as *the* tool to describe incoherent reactions
- Off-shell properties and spectral functions in transport theory
- Application: nucleon spectral functions, comparison with many-body theory
- Connection with EFT NN potentials????

Transport Equation

- Kadanoff-Baym equation for space-time development of one particle spectral phase space density *F* after gradient expansion:

$$\mathcal{D}F(x,p) + \mathrm{tr}\left\{\mathrm{Re}\tilde{S}^{\mathrm{ret}}(x,p), -\mathrm{i}\tilde{\Sigma}^{<}(x,p)\right\}_{\mathrm{pb}} = C(x,p).$$

F = spectral phase-space density: $F(x,p) = -2f(x,p)\mathrm{tr}[\mathrm{Im}\,(\tilde{S}^{\mathrm{ret}}(x,p))\gamma^0],$

$$\mathcal{D}F = \{p_0 - H, F\}_{\mathrm{pb}} \quad \text{with } H = E^*(x,p) - \mathrm{Re}\tilde{\Sigma}_V^0(x,p).$$

C = collision term, couples particles

trace term: responsible for off-shell transport

BM Simplification

Problem: ‚backflow‘ term does not directly depend on *F*

Boterman-Malfliet simplification for equilibrium:

$$\tilde{\Sigma}^{<}_{\mathrm{eq}}(x,p) = \mathrm{i}\Gamma_{\mathrm{eq}}(x,p) f_{\mathrm{eq}}(x,p),$$
$$\tilde{\Sigma}^{>}_{\mathrm{eq}}(x,p) = -\mathrm{i}\Gamma_{\mathrm{eq}}(x,p)[1 - f_{\mathrm{eq}}(x,p)]$$

$$\mathcal{D}F(x,p) - \mathrm{tr}\left\{\Gamma f, \mathrm{Re}\,\tilde{S}^{\mathrm{ret}}(x,p)\right\}_{\mathrm{pb}} = C(x,p).$$

Correction terms are of higher order in gradients

Spectral Function

$$A(x,p) := \frac{1}{g}\mathrm{tr}[\hat{A}(x,p)\gamma^0] = -\frac{1}{g\pi}\mathrm{tr}[\mathrm{Im}\,(\tilde{S}^{\mathrm{ret}}(x,p))\gamma^0],$$

$$F(x,p) = 2\pi g f(x,p) A(x,p).$$

„Spectral Phase Space Density“

= Product of phase-space density *f* and spectral function *A*

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Transport Equation

- Kadanoff-Baym (or BUU) equation

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \cdot \vec{\nabla}_x - \vec{\nabla}_x V \cdot \vec{\nabla}_p + \text{KB terms}\right) g^< = -i\Sigma^> g^< + i\Sigma^< g^>$$

- LHS: drift term
- RHS: collision term = - loss + gain terms

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GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

- **GiBUU : Theory and Event Generator**
  based on an approx. solution of Kadanoff-Baym equations

- Physics content (and code available): **Phys. Rept. 512 (2012) 1**
  http://theorie.physik.uni-giessen.de/GiBUU/

- **GiBUU** describes (within the same unified theory and code)
  - heavy ion reactions, particle production and flow
  - pion and proton induced reactions
  - low and high energy photon and electron induced reactions
  - neutrino induced reactions

……..using the same physics input! And the same code!

Theoretical Basis: GiBUU

Time evolution of spectral phase space density (for $i = N, \Delta, \pi, \rho, \ldots$) given by KB equation in Botermans-Malfliet form:

$$\left[\left(1 - \frac{\partial H}{\partial p_0}\right)\frac{\partial}{\partial t} + \frac{\partial H}{\partial p}\frac{\partial}{\partial x} - \frac{\partial H}{\partial x}\frac{\partial}{\partial p} + \frac{\partial H}{\partial t}\frac{\partial}{\partial p_0}\right] F_i(x,p) = C[F_i(x,p), F_j(x,p)]$$

Hamiltonian *H* includes off-shell propagation correction

8D-Spectral phase space density

Collision term

Off shell transport of collision-broadened hadrons included with proper asymptotic free spectral functions

Practical Basis: GiBUU

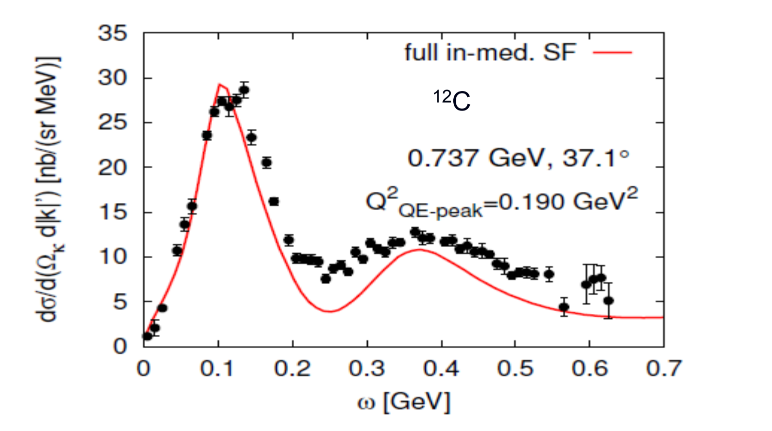
- one transport equation for each particle species (61 baryons, 21 mesons)
- coupled through the potential in *H* and the collision integral *C*
- W < 2.5 GeV: Cross sections from resonance model (PDG and MAID couplings), consistent with electronuclear physics
- W > 2.5 GeV: particle production through string fragmentation (PYTHIA)

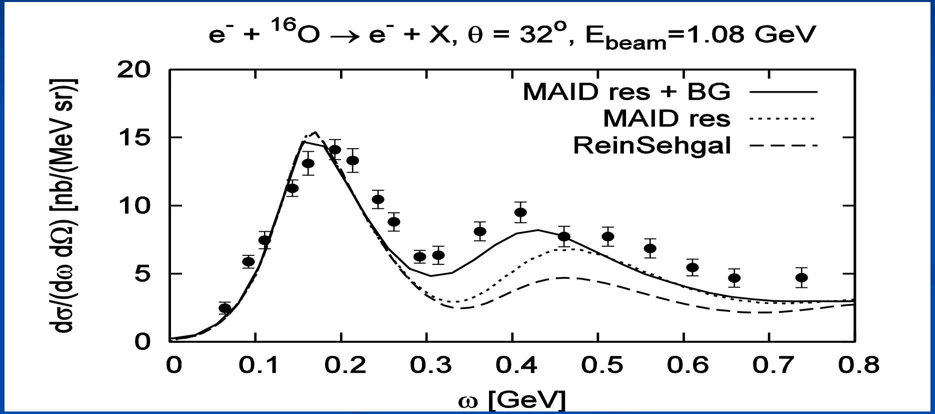
- **GiBUU**: widely tested with various hadronic and em reactions, **NO TUNING**

GiBUU Ingredients: ISI

- In-medium corrected primary interaction cross sections, boosted to restframe of moving bound nucleon in local Fermigas
- Includes spectral functions for baryons and mesons (binding + collision broadening)
- *Hadronic* couplings for FSI taken from PDG
- *Vector* couplings taken from electro-production (MAID)
- *Axial* couplings modeled with PCAC

Electrons as Benchmark for GiBUU

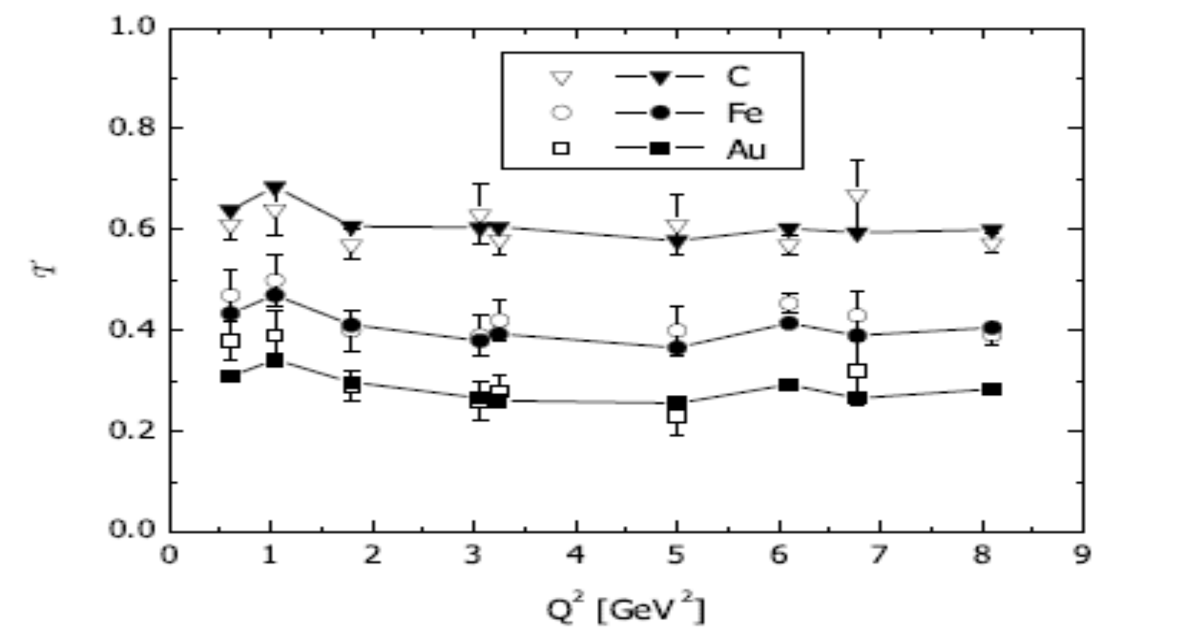




No free parameters!
no 2p-2h, contributes
in dip region and under Δ

Rein-Sehgal does not work for electrons!
Why should it work for neutrinos?

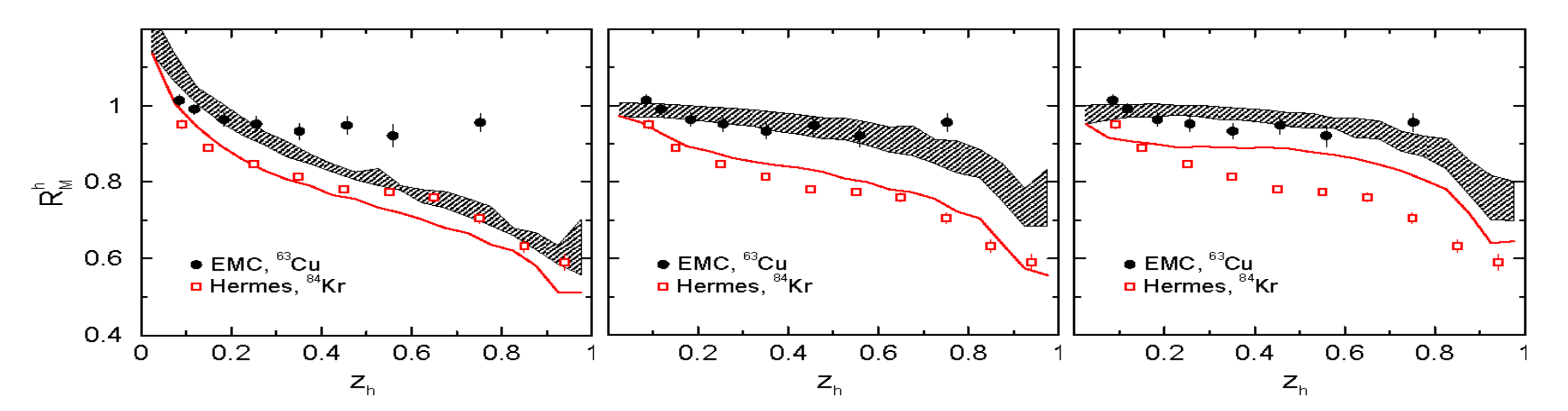
Validation: Proton Transparency



J. Lehr, U. Mosel Nucl.Phys.A699: 324-327,2002.

Free cross sections used, form. times have no effect, driven by NN X-section

Attenuation: EMC and HERMES

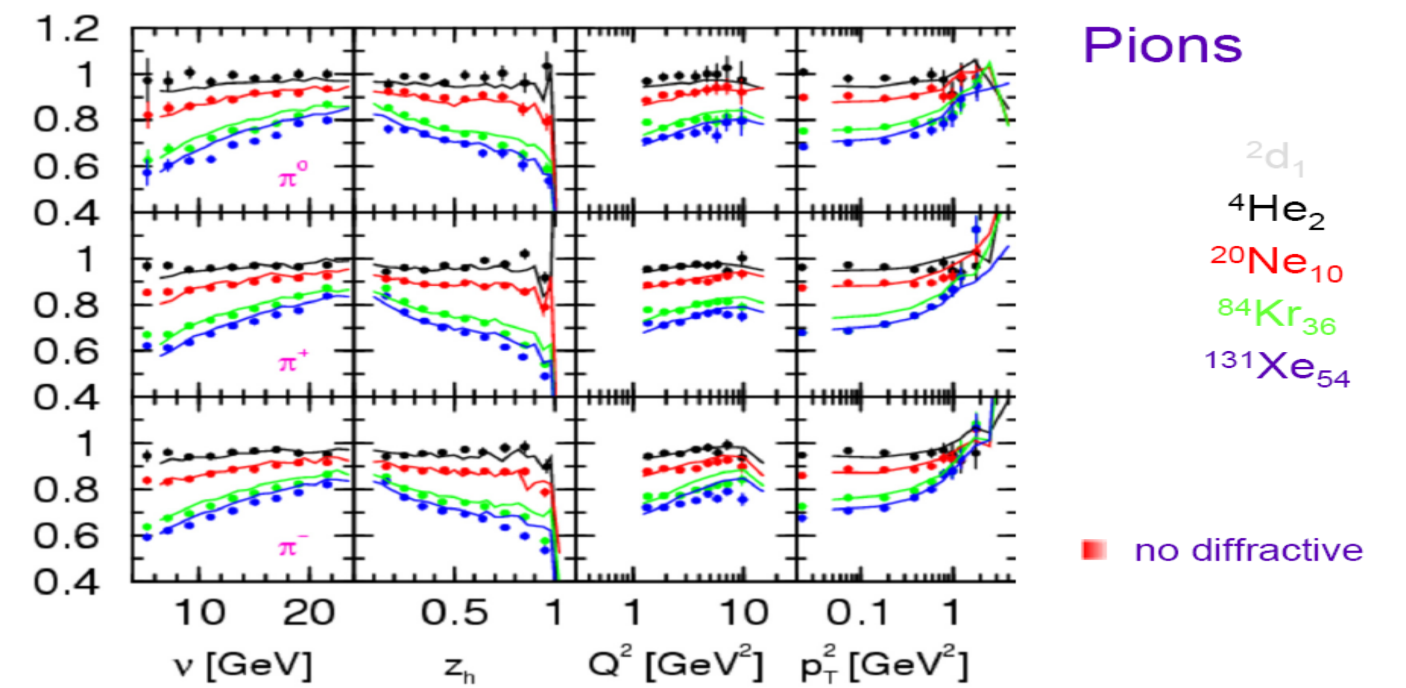


σ_{pre} = const (0.5) linear quadratic

Attenuation Data are sensitive to details of prehadronic interactions!

HERMES@27 GeV

Airapetian et al.



P_T Distribution well described!

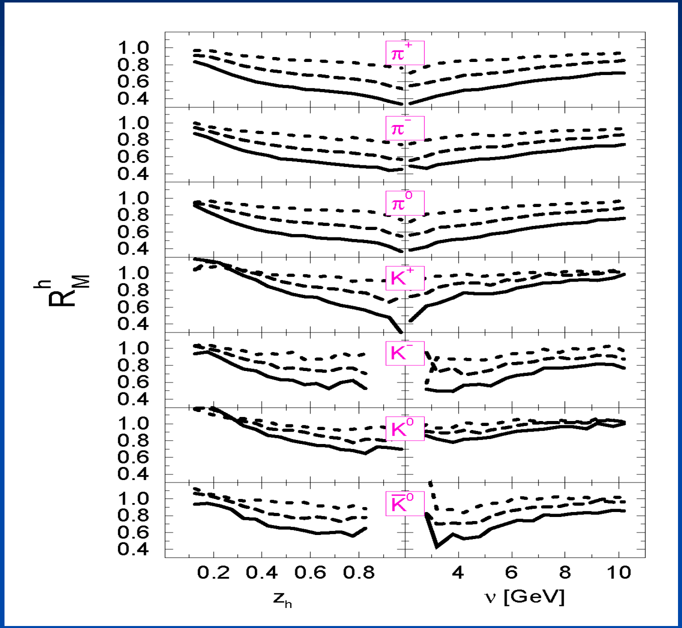
JLAB

CLAS acceptance corrected

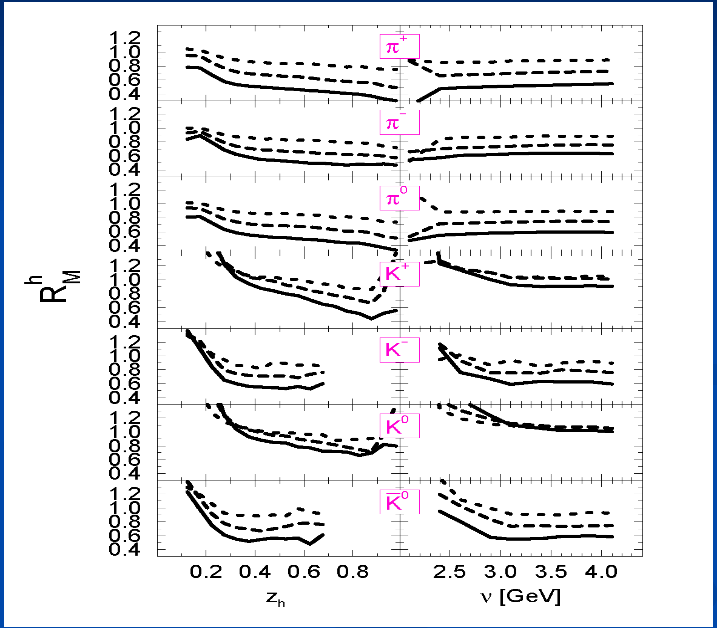
Prediction 2004

prelim data: Brooks et al, Hafidi et al

12 GeV



5 GeV



C
Fe
Pb

Gallmeister, Mosel, **Nucl.Phys.A801:68-79,2008.**

At 5 GeV strong nuclear effects: Fermi motion, overpopulation of low z

Now Collision terms in more detail

Single-Particle Spectral Function

- One particle Green's function:

for $t_1 > t_{1'}$:

$$g^{>}(1,1') \equiv -i\langle\Psi(1)\Psi^{\dagger}(1')\rangle = g(1,1')$$

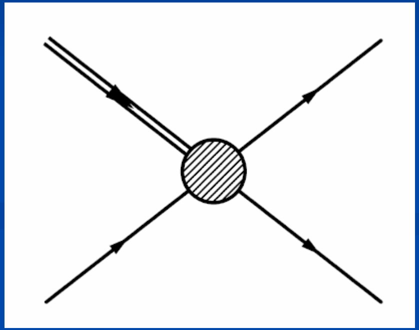
for $t_1 < t_{1'}$:

$$g^{<}(1,1') \equiv i\langle\Psi^{\dagger}(1')\Psi(1)\rangle = g(1,1')$$

- Interpretation:
  - $- i\, g^{<}(1,1)$ = particle density
  - $+i\, g^{>}(1,1)$ = hole density

Loss Terms

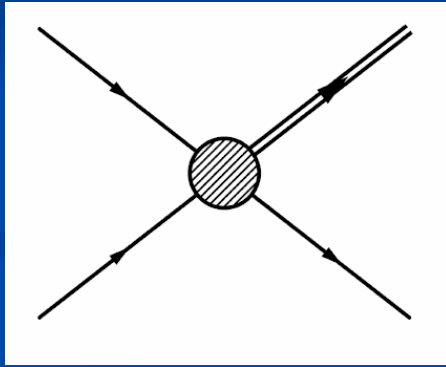
$$\Sigma^{>}(\omega,p) = g\int \frac{d^3p_2d\omega_2}{(2\pi)^4}\frac{d^3p_3d\omega_3}{(2\pi)^4}\frac{d^3p_4d\omega_4}{(2\pi)^4}$$
$$(2\pi)^4\delta^4(p+p_2-p_3-p_4)$$
$$|\mathcal{M}|^2$$
$$\times\, g^{<}(\omega_2,p_2)g^{>}(\omega_3,p_3)g^{>}(\omega_4,p_4)$$



Collision rate of particle in Born approximation: 2p1h
Propagators are dressed

Gain Terms

$$\Sigma^<(\omega,p) = g\int \frac{d^3p_2 d\omega_2}{(2\pi)^4}\frac{d^3p_3 d\omega_3}{(2\pi)^4}\frac{d^3p_4 d\omega_4}{(2\pi)^4}
(2\pi)^4\delta^4(p+p_2-p_3-p_4)
|\mathcal{M}|^2
\times g^>(\omega_2,p_2)g^<(\omega_3,p_3)g^<(\omega_4,p_4)$$



Collision rate of hole in Born approximation: 1p2h
Propagators are dressed

Equilibrium Nuclear Matter

Botermans-Malfliet relations:

$$\Sigma^{>}(\omega,\vec{p}) = -i\Gamma(\omega,\vec{p})(1 - f(\omega,\vec{p})$$
$$\Sigma^{<}(\omega,\vec{p}) = i\Gamma(\omega,\vec{p})f(\omega)$$

with

$$\Gamma(\omega,p) = 2\,\mathrm{Im}\,\Sigma(\omega,p) = i(\Sigma^{>}(\omega,p) - \Sigma^{<}(\omega,p))$$

Spectral Function

Define spectral function:

$$\mathcal{A}(\omega, p) = i\left[g^{>}(\omega, p) - g^{<}(\omega, p)\right]$$

Rewrite Green's functions in terms of *A:*

$$-ig^{<}(\omega, p) = f(\omega)\mathcal{A}(\omega, p) \qquad ig^{>}(\omega, p) = (1 - f(\omega))\mathcal{A}(\omega, p)$$

$$\Rightarrow f(\omega) = \text{average occupation number of a mode with energy } \omega$$

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Equilibrium Nuclear Matter

- *In equilibrium* collision term in KB equation vanishes:

$$-\Sigma^{>} g^{<} + \Sigma^{<} g^{>} = 0$$

Equilibrium Nuclear Matter

- Phase space distribution in nuclear matter for *T = 0:*
  $f(\omega) = \Theta(\omega_F - \omega)$
- $\omega < \omega_F$ $\Sigma^{>}(\omega,p) \;=\; 0, \quad \Gamma(\omega,p) = -i\Sigma^{<}(\omega,p)$
- $\omega > \omega_F$ $\Sigma^{<}(\omega,p) \;=\; 0, \quad \Gamma(\omega,p) = i\Sigma^{>}(\omega,p)$

$$\Rightarrow \Gamma(\omega_F, p) = 0$$

Spectral Function

$$a(\omega,p) = i\left(g^{>}(\omega,p) - g^{<}(\omega,p)\right)$$

becomes with

$$\Gamma(\omega,p) = 2\,\mathrm{Im}\,\Sigma(\omega,p) = i(\Sigma^{>}(\omega,p) - \Sigma^{<}(\omega,p))$$

Spectral function in Breit-Wigner form

$$a(\omega,p) = \frac{\Gamma(\omega,p)}{(\omega - \frac{p^2}{2m_N} - \mathrm{Re}\Sigma(\omega,p))^2 + \frac{1}{4}\Gamma^2(\omega,p)}$$

50 years Kadanoff & Baym

L.P. Kadanoff & G. Baym
Quantum Statistical Mechanics, 1962:

> [*These equations*] represent a horribly complex set of integral equations for a. To get detailed numerical answers, it is necessary to solve these equations. The best we can do, ... , is to use some iteration procedure.

Selfonsistent spectral function

$$\Gamma(\omega,p) = 2\,\mathrm{Im}\,\Sigma(\omega,p) = i(\Sigma^{>}(\omega,p) - \Sigma^{<}(\omega,p))$$

$$\Sigma^{>}(\omega,p) = -4i\frac{\overline{|\mathcal{M}|^2}}{(2\pi)^6}\int d\omega_3 \int d\omega_2 \int dp_3\, p_3^2 \int dp_2\, p_2^2 \frac{d\cos\vartheta_2}{p_{tot}p_3}\int dp_4\, p_4$$
$$\times\, a(\omega_2,p_2) f(\omega_2,p_2) a(\omega_3,p_3)(1-f(\omega_3,p_3)) a(\omega_4,p_4)(1-f(\omega_4,p_4))$$

$$\Sigma^{<}(\omega,p) = 4i\frac{\overline{|\mathcal{M}|^2}}{(2\pi)^6}\int d\omega_3 \int d\omega_2 \int dp_3\, p_3^2 \int dp_2\, p_2^2 \int \frac{d\cos\vartheta_2}{p_{tot}p_3}\int dp_4\, p_4$$
$$\times\, a(\omega_2,p_2)(1-f(\omega_2,p_2)) a(\omega_3,p_3) f(\omega_3,p_3) a(\omega_4,p_4) f(\omega_4,p_4)$$

$$a(\omega,p) = \frac{\Gamma(\omega,p)}{(\omega - \frac{p^2}{2m_N} - \mathrm{Re}\Sigma(\omega,p))^2 + \frac{1}{4}\Gamma^2(\omega,p)}$$

Selfconsistency Problem

$$\Gamma \to a \to \Sigma^{<>} \to \Gamma$$

Includes np-mh correlations in selfconsistent spectral functions

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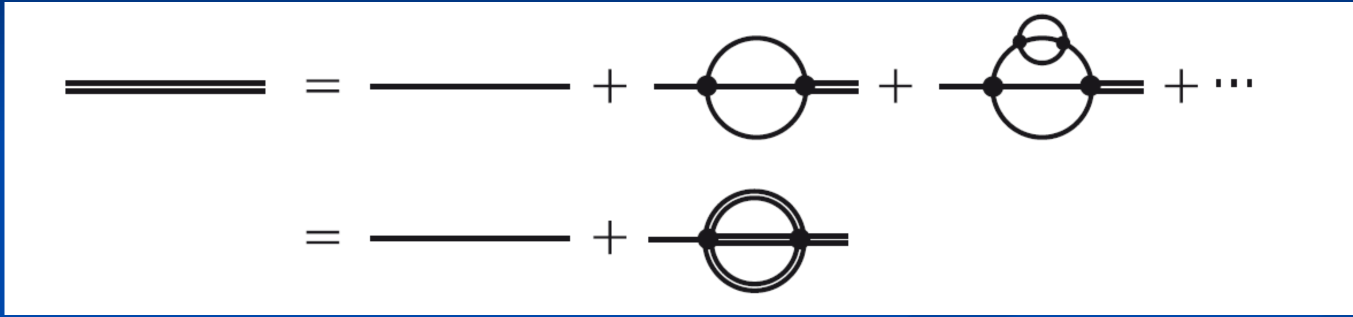
Simplified Collision term: Short-range Correlation

- First: Assume extreme short range interaction → M = const
  assume constant real potential

- Later: improve model for M

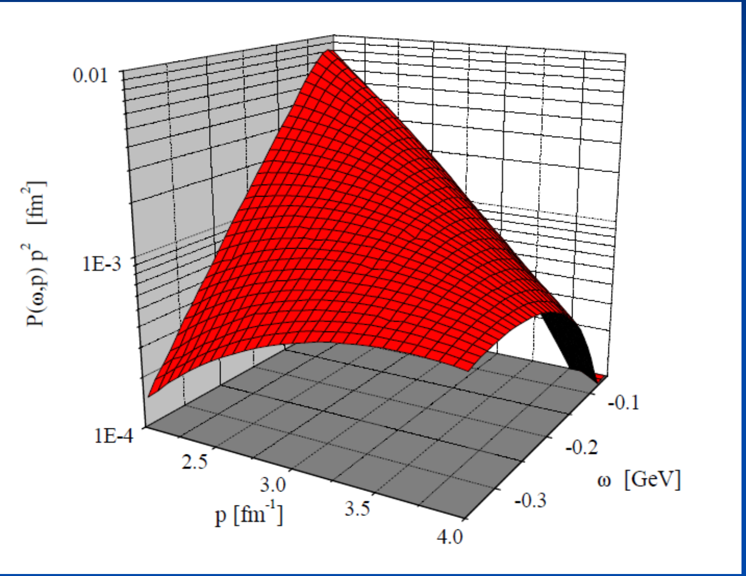
Selfconsistent particle propagator



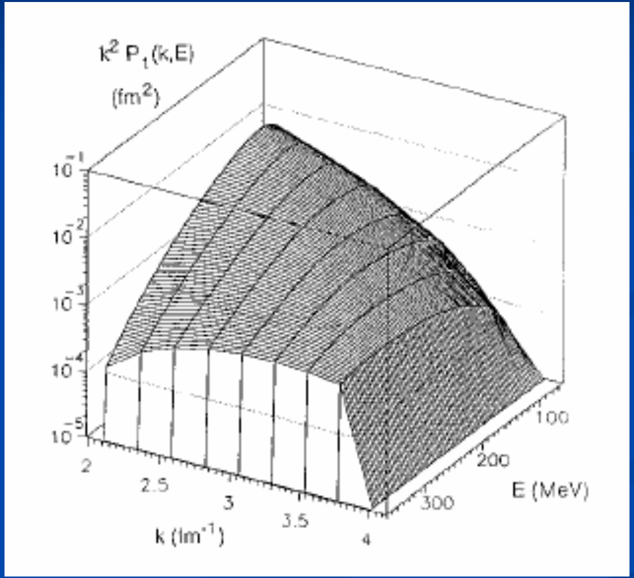
Dyson equation for propagator resummed

Nucleon Spectral Functions

M = 207 MeV fm^3



Lehr et al, 2000
Strength *M* obtained by fit to Benhar results



Benhar et al, 1992

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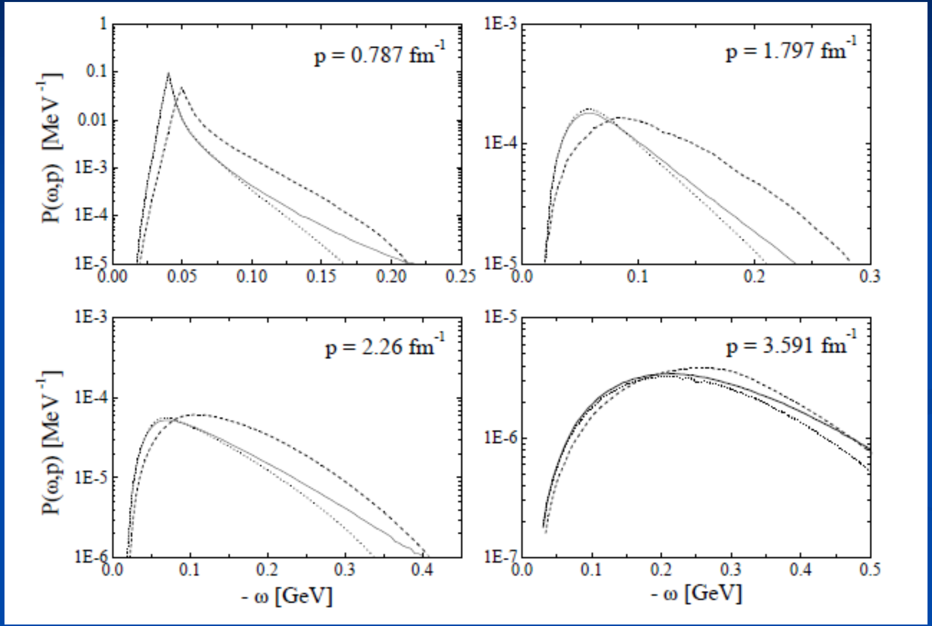
Spectral Functions

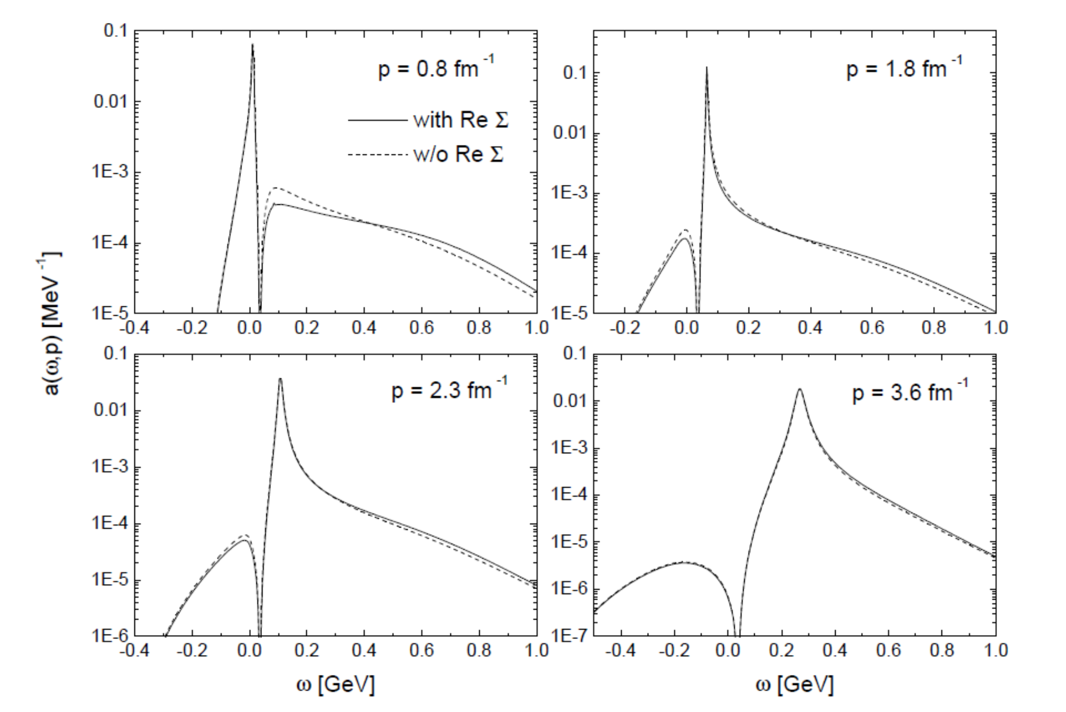


Dashed:
Benhar et al

Strength *M* fitted to Benhar's Results (dashed)

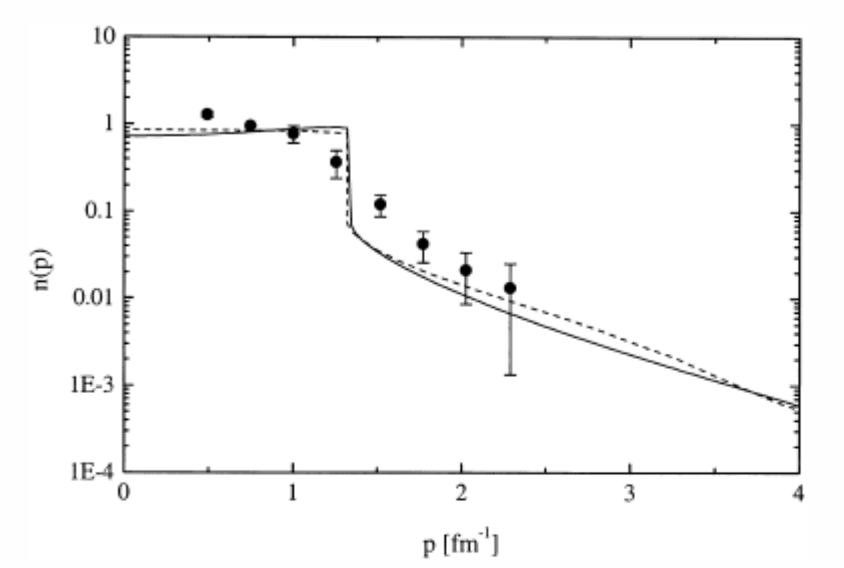
Reasonable agreement shows that SF shape is determined by phase-space

Nucleon spectral functions



Hole and particle spectral functions

Momentum distribution



Data from Benhar-Sick analysis

SRC in Born approximation

G.F. Bertsch and P. Danielewicz
Physics Letters B367 (1996) 55:

As the ground-state correlations have been studied over the last few years using an extensive apparatus of many-body theory, an impression might have been gained of an inherent complexity contrasting with the simple considerations made here [*i.e. Born approximation collision integrals with constant matrix elements*]. To counter such impression, we compare our estimate with the results of many-body nuclear matter calculations of Benhar et al, who used a realistic interaction.

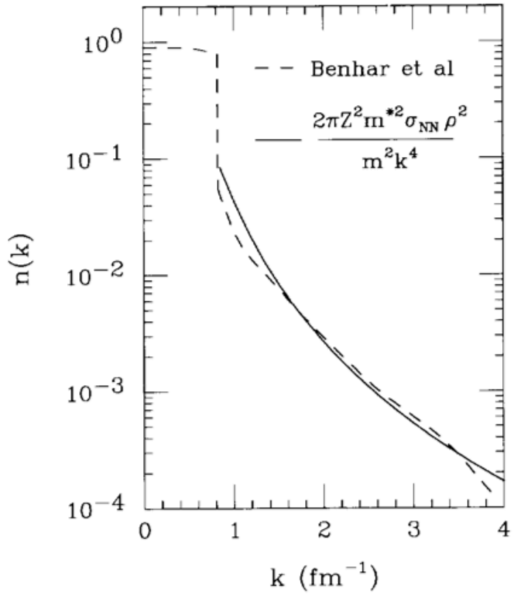
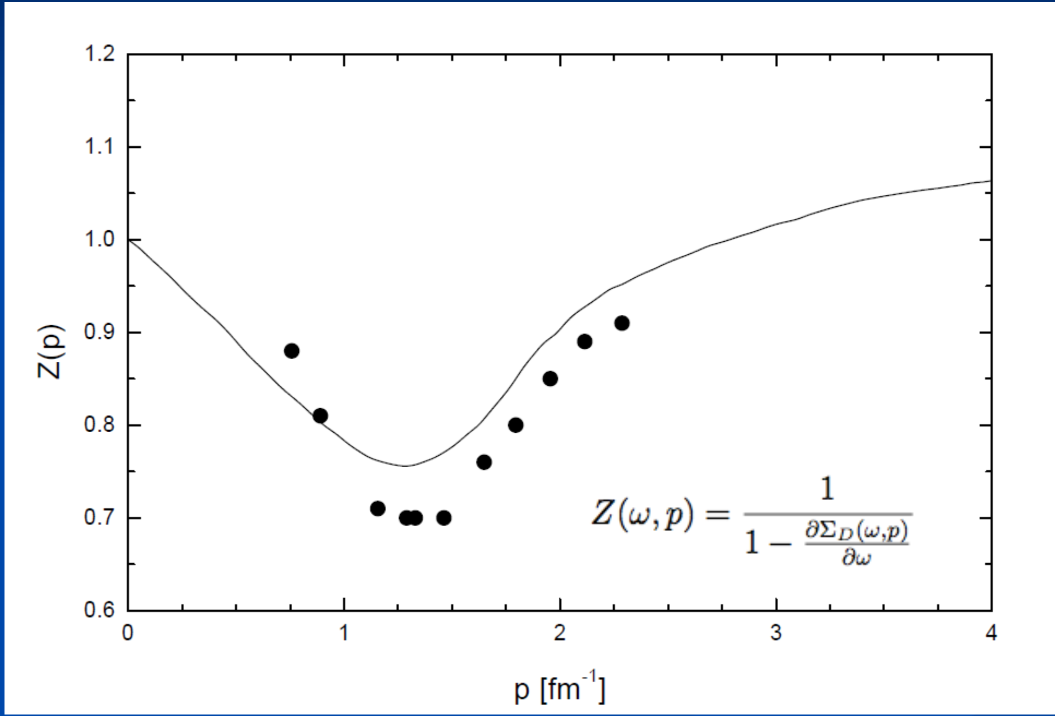


Fig. 2. Momentum occupation in nuclear matter at $\rho = \rho_0/4$. The solid line is our result, Eq. (9), applicable for $k > k_F$. The dashed line is the result of Ref. [9], including the region for $k < k_F$ as well as correlation contribution for $k > k_F$.

Nucleon spectroscopic factor



Dots: Benhar
Curve: Transport,

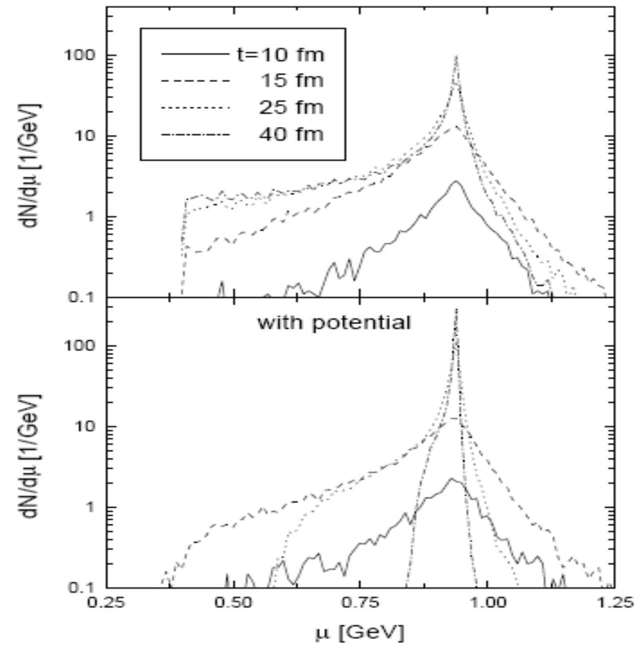
at on-shell point

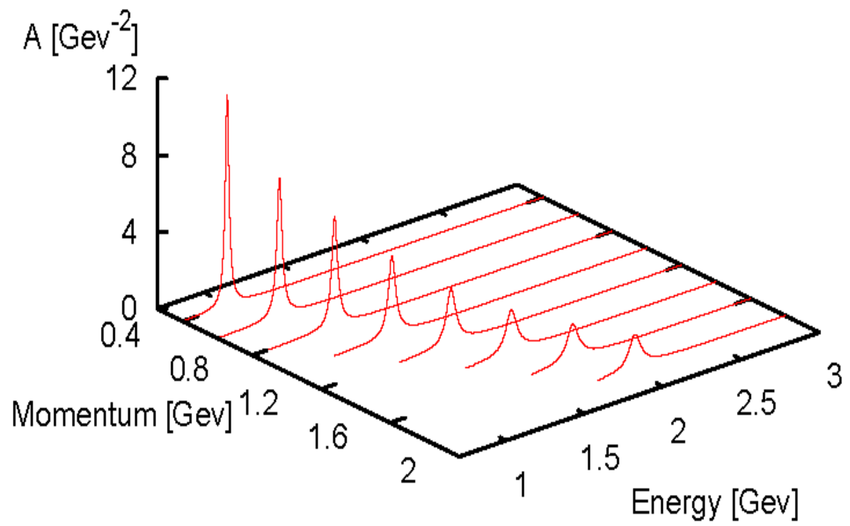
Off-shell transport

Hole Spectral Function

Spectral Function $\Gamma = \rho\ \sigma\ v$

Effenberger





Pb + Pb, 1.5 A GeV

Improve collision strength *M*

- So far: collision strength was fitted
- Now: obtain collision strength as short-range part of nuclear energy-density functional

Potentials from LM Theory

- Improve modeling of matrixelement *M:*
  Use as guideline Landau-Migdal Theory:

$$V^{qq'} = f^{qq'} + g'^{qq'} \vec{\sigma_1} \vec{\sigma_2}$$

- Calculate spin singlet contrib *f* and spin triplet contrib *g* from Skyrme (or any other) energy-density functional

Potentials from LM Theory

- Obtain short-range part of energy-density functional by removing pion interaction long range part:

$$E(\rho) = E_s(\rho) + E_\pi(\rho),$$

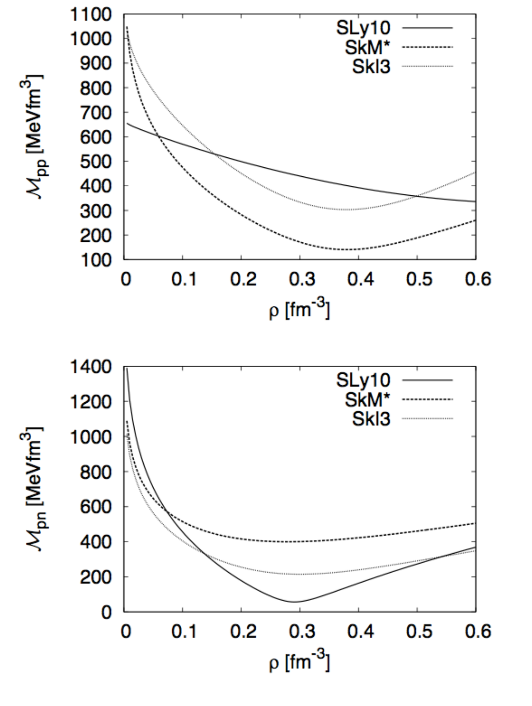
$$E_\pi(\rho) = \frac{1}{2} \sum_{qq'=p,n} \sum_{S,T=0,1} \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3} \int \frac{\mathrm{d}^3 k_2}{(2\pi)^3} \Theta(k_{F_q} - k_1) \Theta(k_{F_{q'}} - k_2) \times V_{ST}^{(\pi)}(\vec{k}_1, \vec{k}_2) \langle (\vec{\sigma}_1 \cdot \vec{\sigma}_2)^S (\vec{\tau}_1 \cdot \vec{\tau}_2)^T \rangle$$

Potentials from LM Theory

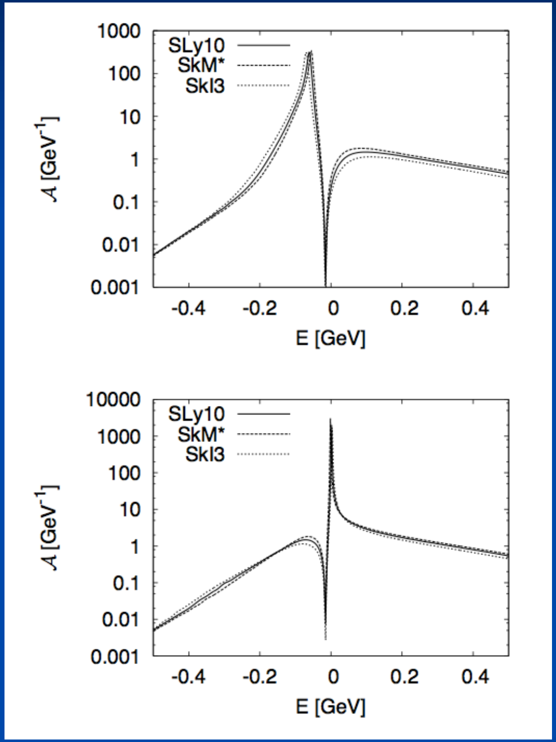
- Migdal parameters obtained by 2nd functional derivative of E-ρ functional E_s

Model *M* with Landau-Migdal theory

- $$\mathcal{M}_{qq'} = \frac{1}{2}\sqrt{(f^s_{qq'})^2 + 3(g^s_{qq'})^2}.$$



SF in symmetric nuclear matter

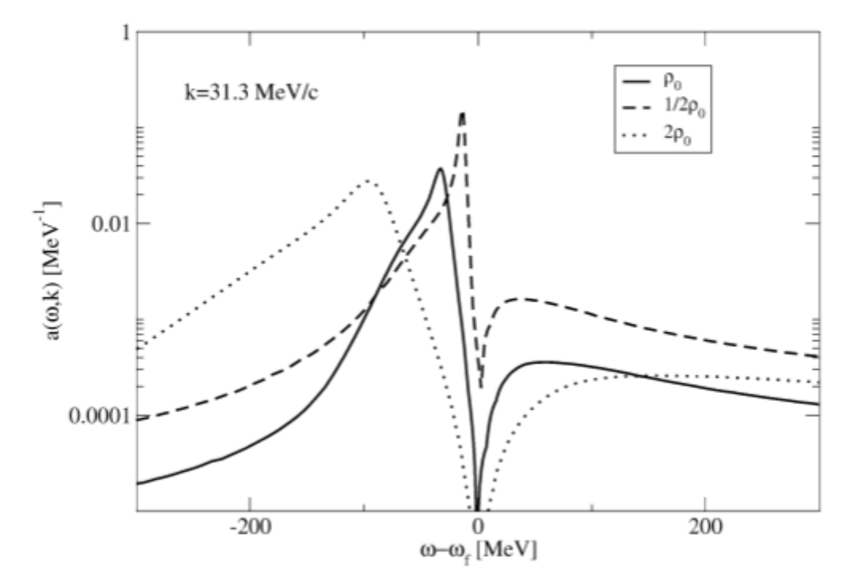


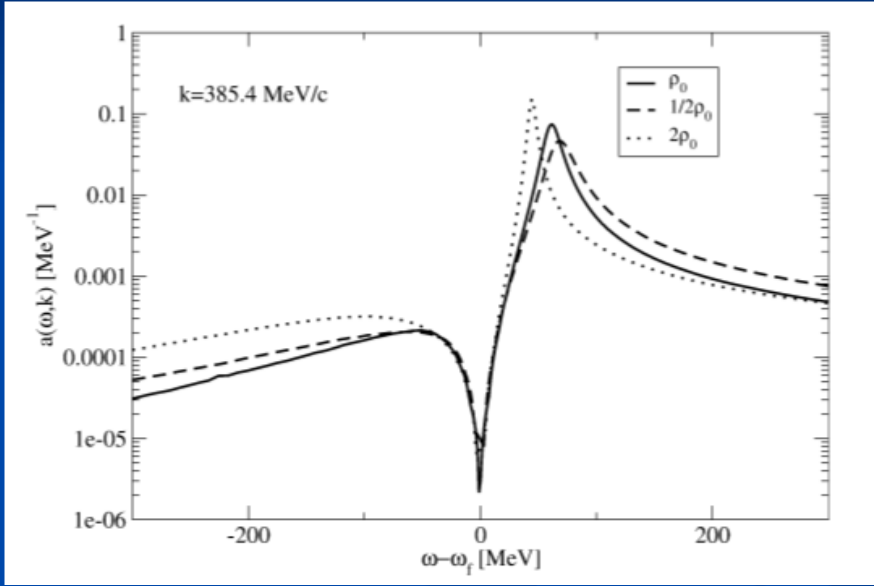
p = 0.1 GeV

Strength derived from eff. NN interaction at Fermi-Surface

p = 0.3 GeV

Density Dependence





EFT Potentials

- EFT NN potentials describe low-momentum ($p \leq p_F$) interactions
- EFT NN potentials need (many) free parameters for contact interactions
- Contact interactions can be used to calculate nucleon spectral functions
- Info on spectral functions can (should) be used to constrain contact interaction parameters

EFT NN Potentials

- EFT potentials contain src (contact terms):
  - 24 contact terms (parameters) in N^3LO, contribute to low partial waves ($l \leq 2$), in addition regulators
- Propose to fit these these parameters under constraint of empirical spectral functions

SRC in EFT

- EFT contact terms in LO, NLO, N^3LO

$$\frac{1}{m_\omega^2 + Q^2} \approx \frac{1}{m_\omega^2}\left(1 - \frac{Q^2}{m_\omega^2} + \frac{Q^4}{m_\omega^4} - + \cdots\right),$$

Skyrme-like terms

In LO: $$V_{\text{ct}}^{(0)}(\vec{p}',\vec{p}) = C_S + C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,,$$

Same structure as for LM force

NN EFT and Migdal force

- LO contact Lagrangian and Migdal force identical.
- LO contact terms can be chosen such that they produce realistic spectral functions.

Summary

- Transport theory describes (semi)inclusive reactions
- Transport theory can describe off-shell transport
- SRC in transport theory generate selfconsistently nucleon spectral functions
- Contact interactions in EFT NN potentials should be determined with constraints from spectral functions

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