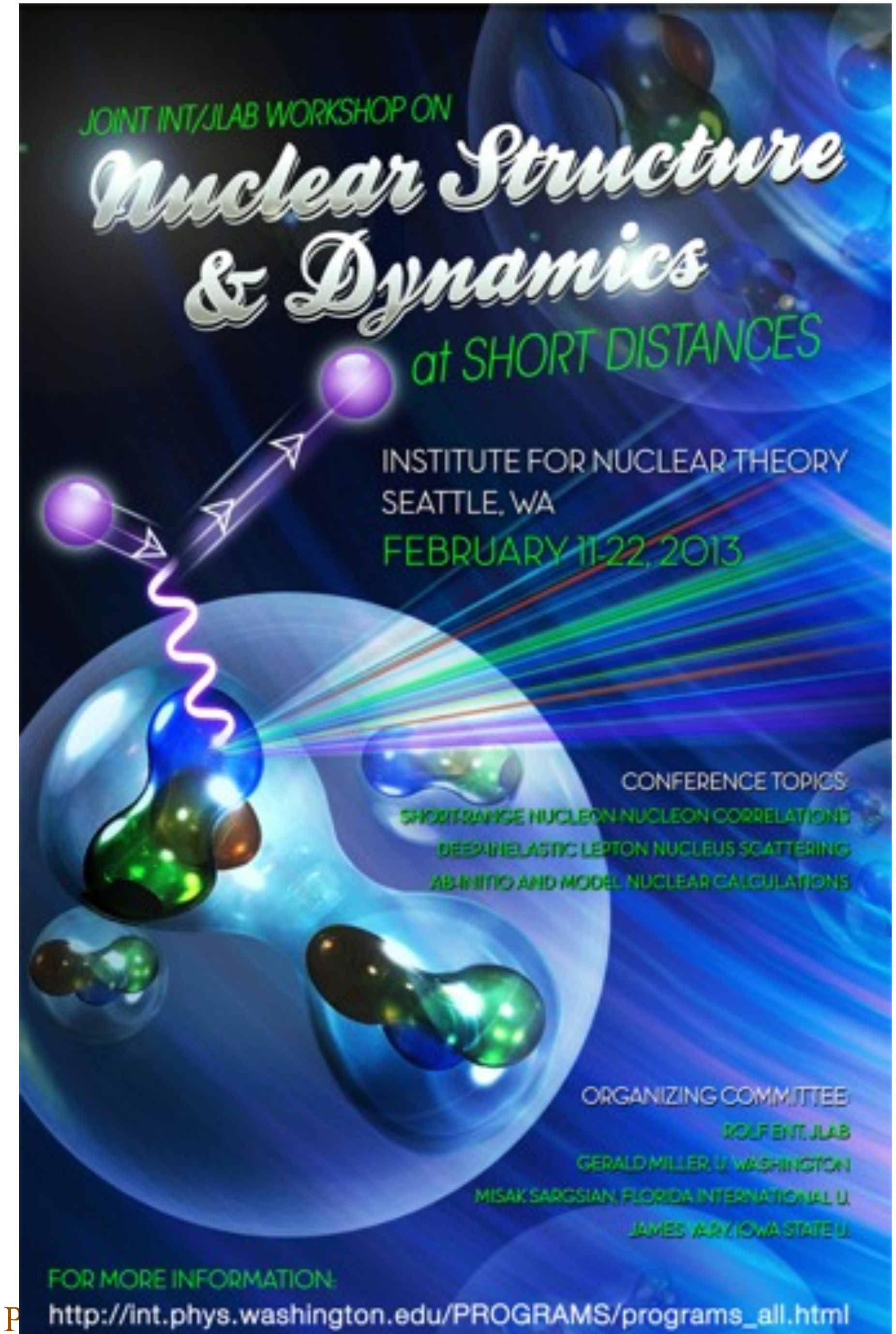
Measuring transverse size with virtual photons



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Work done with Samu Kurki

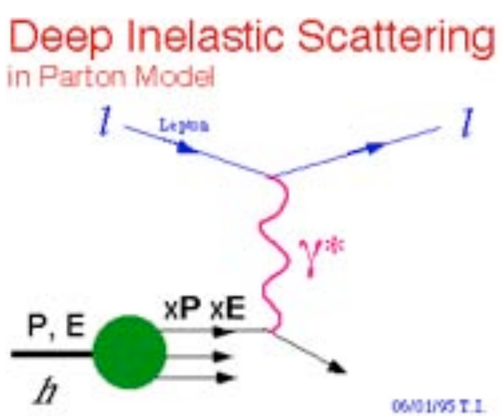
arXiv:0911.3011
arXiv:1101.4810

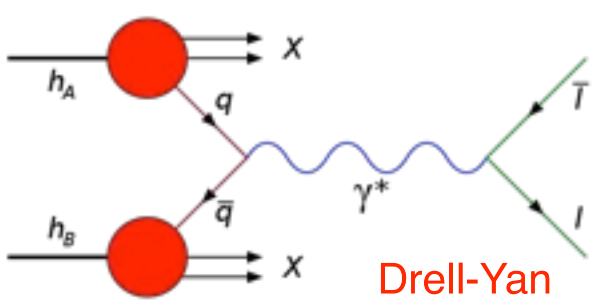
How to determine the size of the interaction region in electro-production from the dependence on the photon virtuality.

Photons are useful probes of strong dynamics at any Q^2

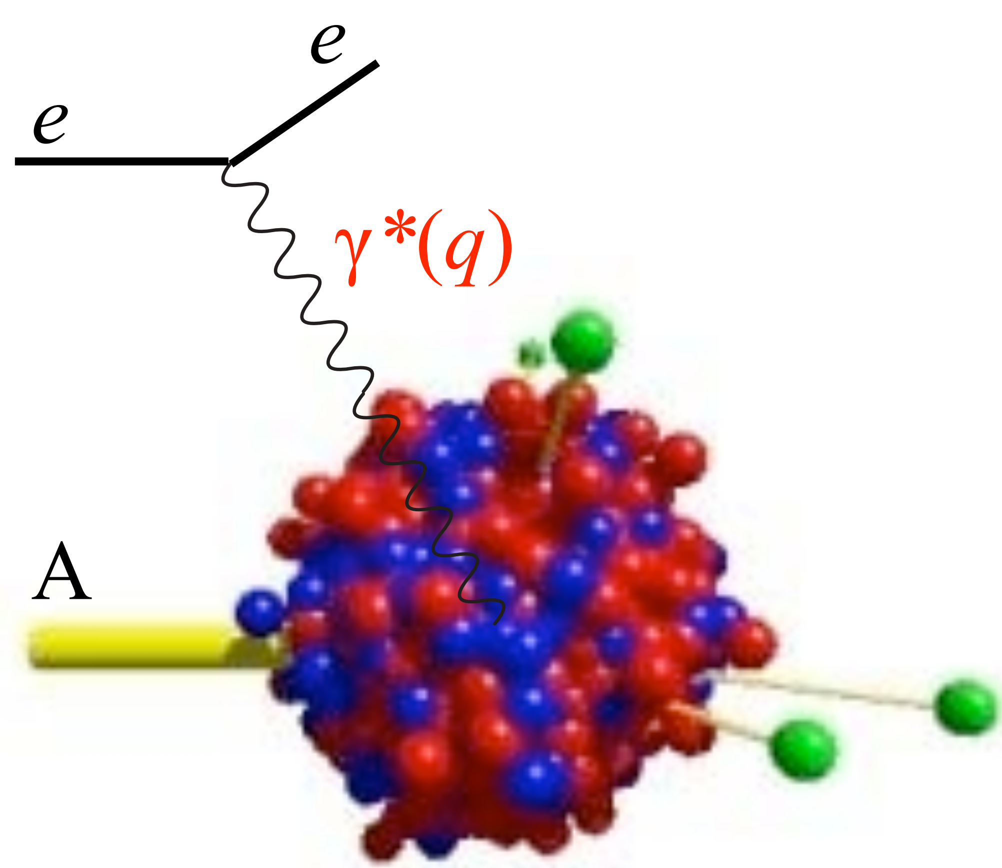
Scattering from pointlike sources at LT in the Bj limit

$$\sigma(q^2 \to \infty) \propto 1/q^2$$

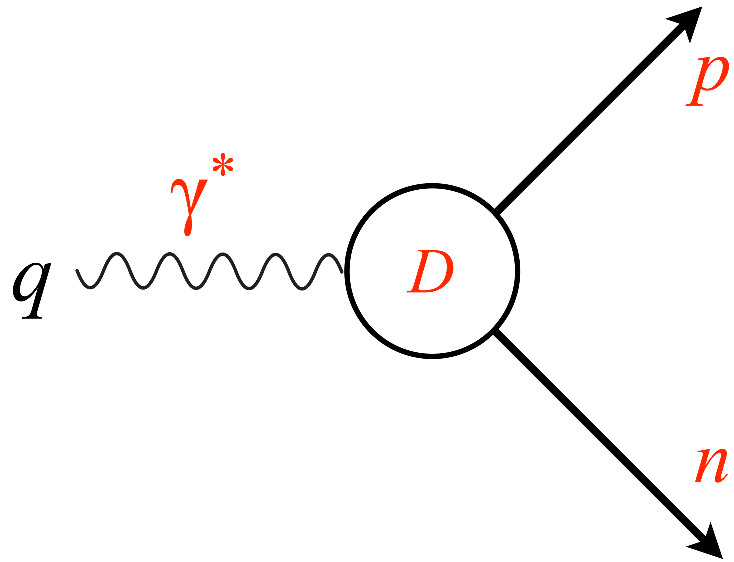




The q^2 dependence reflects the effective size of the interaction region



Example: Deuteron photodisintegration

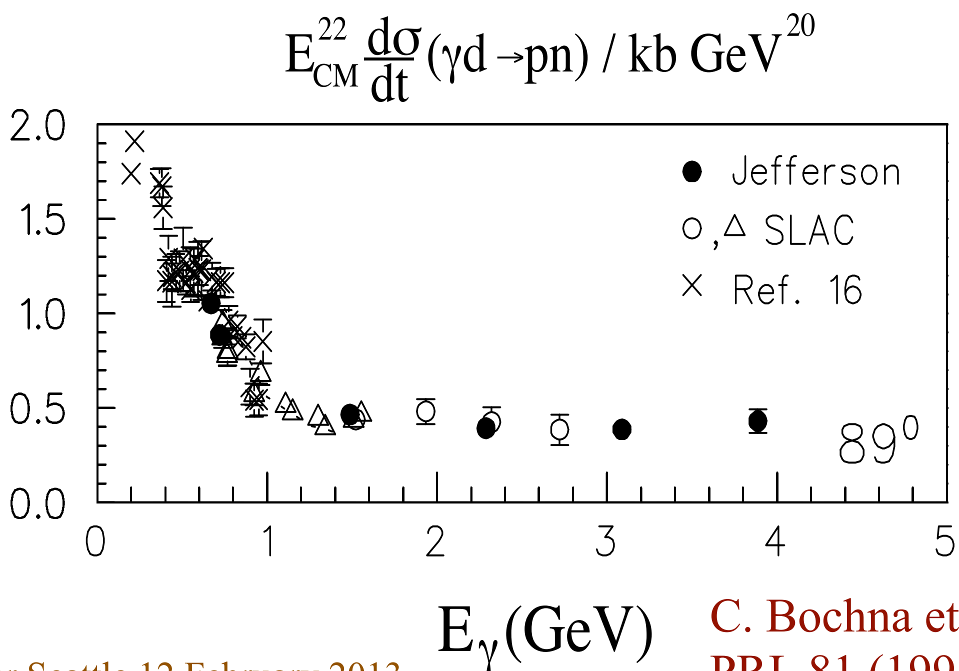


The 90° break-up cross section at $q^2=0$ agrees with dimensional scaling for $E_\gamma > 1$ GeV.

Does this mean that only compact configurations of the deuteron, with $R < 0.2$ fm, contribute to this process?

If so, expect no q^2-dependence for $q^2 < 1\ \text{GeV}^2$.

This can be formulated more precisely through a Fourier transform to coordinate space.



C. Bochna et al, PRL 81 (1998) 4576

Relativistic effects on photon resolution

From DIS, recall that resolution of photon with $q^2 = -Q^2$ is different in the longitudinal and transverse directions:

$$\Delta r_{\parallel} \sim \frac{1}{Q}\frac{\nu}{Q} = \frac{1}{2m\,x_{Bj}} \qquad\qquad \Delta r_{\perp} \sim \frac{1}{Q}$$

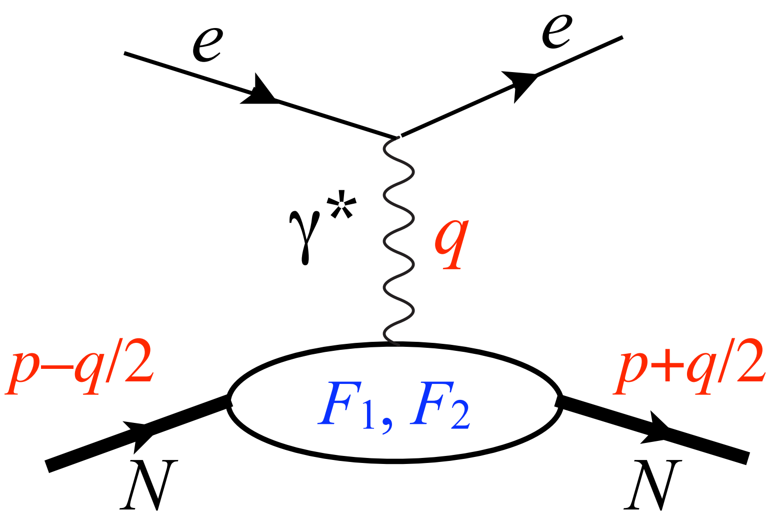
Hence we can accurately measure only the transverse size (impact parameter $\boldsymbol{b}$):

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \langle p(p_1)\, n(p_2)|J^{\mu}(q)|D(p)\rangle$$

- $\boldsymbol{q}$ is frame dependent: Is there a preferred frame?
- $p_1+p_2 = p+q$ depends on q: What is $p_1(q)$, $p_2(q)$?
- Does the result depend on μ ?

Recall: Nucleon Form Factors

Using Lorentz and gauge invariance, the scattering amplitude is expressed in terms of the Dirac F_1 and Pauli F_2 form factors, which depend on $Q^2 = -q^2$



$$A^{\mu}_{\lambda\lambda'} = \langle p + \tfrac{1}{2}q, \lambda' | J^{\mu}(0) | p - \tfrac{1}{2}q, \lambda \rangle$$

$$= \bar{u}(p + \tfrac{1}{2}q, \lambda') \left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i}{2m}\sigma^{\mu\nu}q_{\nu} \right] u(p - \tfrac{1}{2}q, \lambda)$$

The photon scatters from quarks, which are ultrarelativistic.

- How can we measure the quark postions with resolution $\Delta b \sim 1/Q$, when the photon itself is not moving faster than the quarks?

Boosting to the Infinite Momentum Frame

The photon probes a hadron at an instant of $x^+ = t+z$, not at an instant in t

The Light Front ≈ Infinite Momentum Frame

Quark motion in the transverse direction slows down in the IMF: $v_\perp = \dfrac{p_\perp}{xE_h}$

A hadron state of momentum $P^+ = P^0 + P^3$ defined at given $x^+ = x^0 + x^3$ can be expanded in terms its quark and gluon Fock states as

$$|P^+, \boldsymbol{P}_\perp, \lambda\rangle_{x^+=0} = \sum_{n,\lambda_i} \prod_{i=1}^{n} \Big[\int_0^1 \frac{dx_i}{\sqrt{x_i}} \int \frac{d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

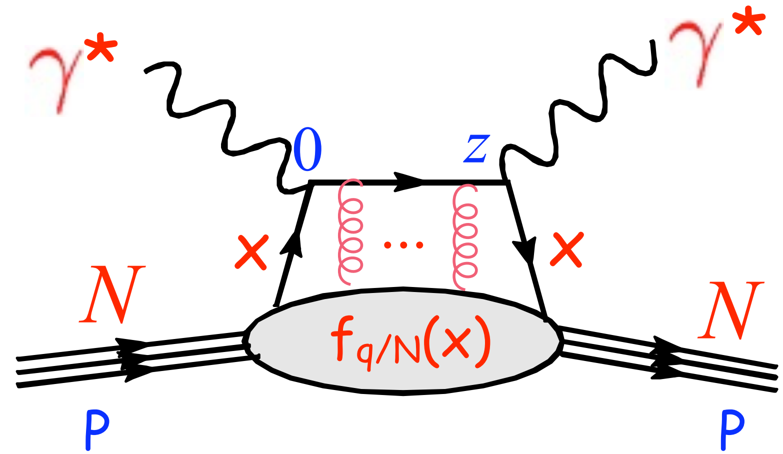
$$\times \psi_n(x_i, \boldsymbol{k}_i, \lambda_i)\, |n;\, x_i P^+, x_i \boldsymbol{P}_\perp + \boldsymbol{k}_i, \lambda_i\rangle_{x^+=0}$$

where the LF wave functions $\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)$ are independent of the hadron momentum P^+, $P_\perp$.

Note: The partons carry fractions x_i of the hadron momentum P, like in non-relativistic physics with $m_i/M \to x_i$.

Inclusive Deep Inelastic Scattering (DIS)

In the DIS cross section the photon vertices of the amplitude and amplitude* are separated by a light-like distance z: $z^+, z_\perp \to 0$; $z^- \sim 1/(2mx_{Bj})$.



The parton distributions can be expressed in terms of LF wave functions:

$$f_{q/N}(x) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \Big[\int \frac{dx_i\, d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1-\sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \delta(x-x_k) |\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)|^2$$

Notes: – The parton distribution is defined in the Bj limit ($Q^2 \to \infty$)

– The above expression is approximate, since rescattering of the struck parton (described by the Wilson line) is neglected.

The Generalized Parton Distributions: GPD's

The GPD's are non-forward matrix elements of the PDF operator:

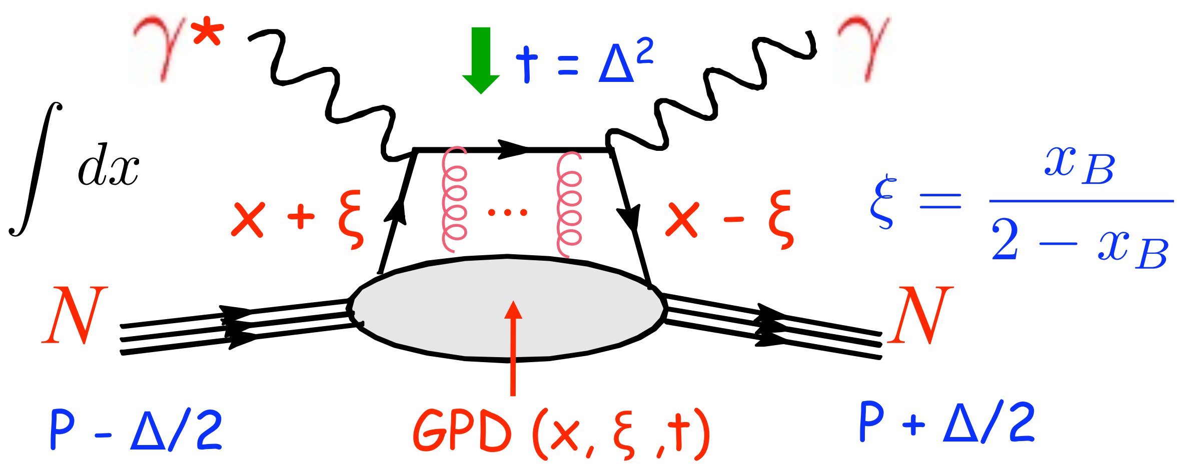
$$\frac{1}{8\pi}\int dr^- e^{imxr^-/2}\langle P+\tfrac{1}{2}\Delta|\bar{q}(-\tfrac{1}{2}r)\gamma^+ W[\tfrac{1}{2}r^-,-\tfrac{1}{2}r^-]q(\tfrac{1}{2}r)|P-\tfrac{1}{2}\Delta\rangle_{r^+=r_\perp=0}$$

$$=\frac{1}{2P^+}\bar{u}(P+\tfrac{1}{2}\Delta)\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)i\sigma^{+\nu}\frac{\Delta_\nu}{2m}\right]u(P-\tfrac{1}{2}\Delta)$$

The GPD amplitudes can be accessed experimentally through the Deeply Virtual Compton Scattering cross section at leading twist: $Q^2 \to \infty$.

DVCS: $e\,N \to e' + \gamma + N$

Through $\Delta_\perp$, the GPD's contain information about the parton distributions in transverse space.



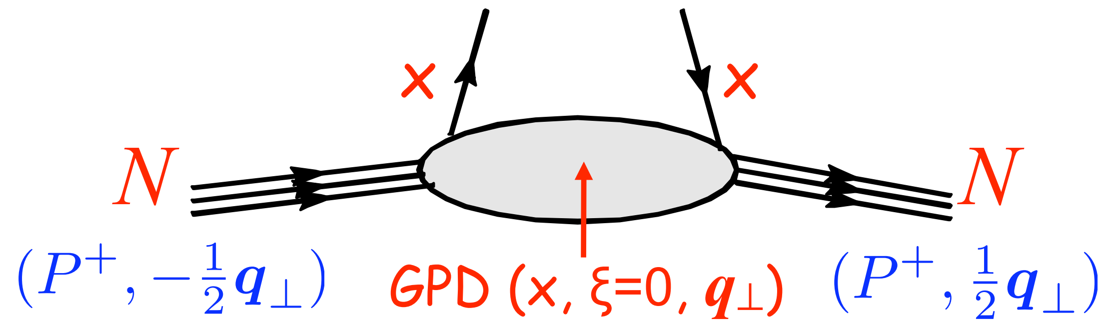
Impact parameter distributions via the GPD's

Extrapolating the GPD to $\xi = 0$ and Fourier transforming it *wrt.* $\boldsymbol{q}_\perp$

Soper (1977)
Burkardt (2000)
Diehl (2002)

$$f_{q/N}(x,\boldsymbol{b}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \int \frac{dz^-}{8\pi} e^{ixP^+z^-/2}$$
$$\times \quad \langle P^+, \tfrac{1}{2}\boldsymbol{q}, \lambda | \bar{\mathrm{q}}(0^+, -\tfrac{1}{2}z^-, \boldsymbol{0}_\perp)\gamma^+ \mathrm{q}(0^+, \tfrac{1}{2}z^-, \boldsymbol{0}_\perp) | P^+, -\tfrac{1}{2}\boldsymbol{q}, \lambda\rangle$$

the GPD can be expressed in terms of LF wf's with the struck quark at transverse position $\boldsymbol{b}$ (still ignoring the Wilson line):



$$f_{q/N}(x,\boldsymbol{b}) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \left[\int dx_i \int 4\pi d^2\boldsymbol{b}_i\right] \delta\Big(1-\sum_i x_i\Big) \frac{1}{4\pi} \delta^2\Big(\sum_i x_i \boldsymbol{b}_i\Big)$$
$$\times \quad \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\delta(x-x_k)\,|\psi_n^\lambda(x_i,\boldsymbol{b}_i,\lambda_i)|^2$$

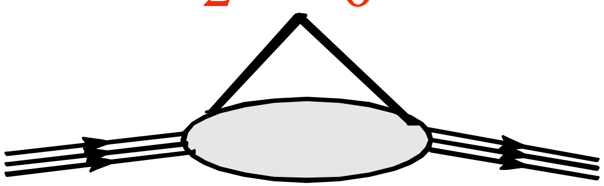
"Center of momentum" at the origin

Fourier-transformed wave function

Relation of GPD's to Form Factors

When GPD's are integrated over x the GPD reduces to a form factor, since

$$\int_{-\infty}^{\infty} dx \exp(ixP^+z^-/2) \propto \delta(z^-)$$



ensures that the photon vertices coalesce.
The GPD's vanish for $|x| > 1$, hence the relations reduce to

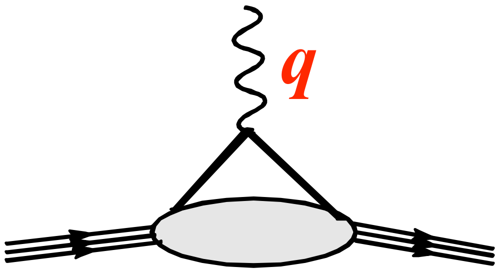
$$\int_{-1}^{1} dx H^q(x,\xi,t) = F_1^q(t) \qquad \text{Dirac}$$

$$\int_{-1}^{1} dx E^q(x,\xi,t) = F_2^q(t) \qquad \text{Pauli}$$

This gives constraints on GPD models and a great experimental simplification:
Form factors are easy to measure (compared to GPD's!).

Impact parameter picture of GPD's inherited by FF's

Fourier transforms of form factors give
charge densities in impact parameter space:



$$\rho_0(\boldsymbol{b}) = \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

$$= \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n} \int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big] \delta(1 - \sum_i x_i) \frac{1}{4\pi} \delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$

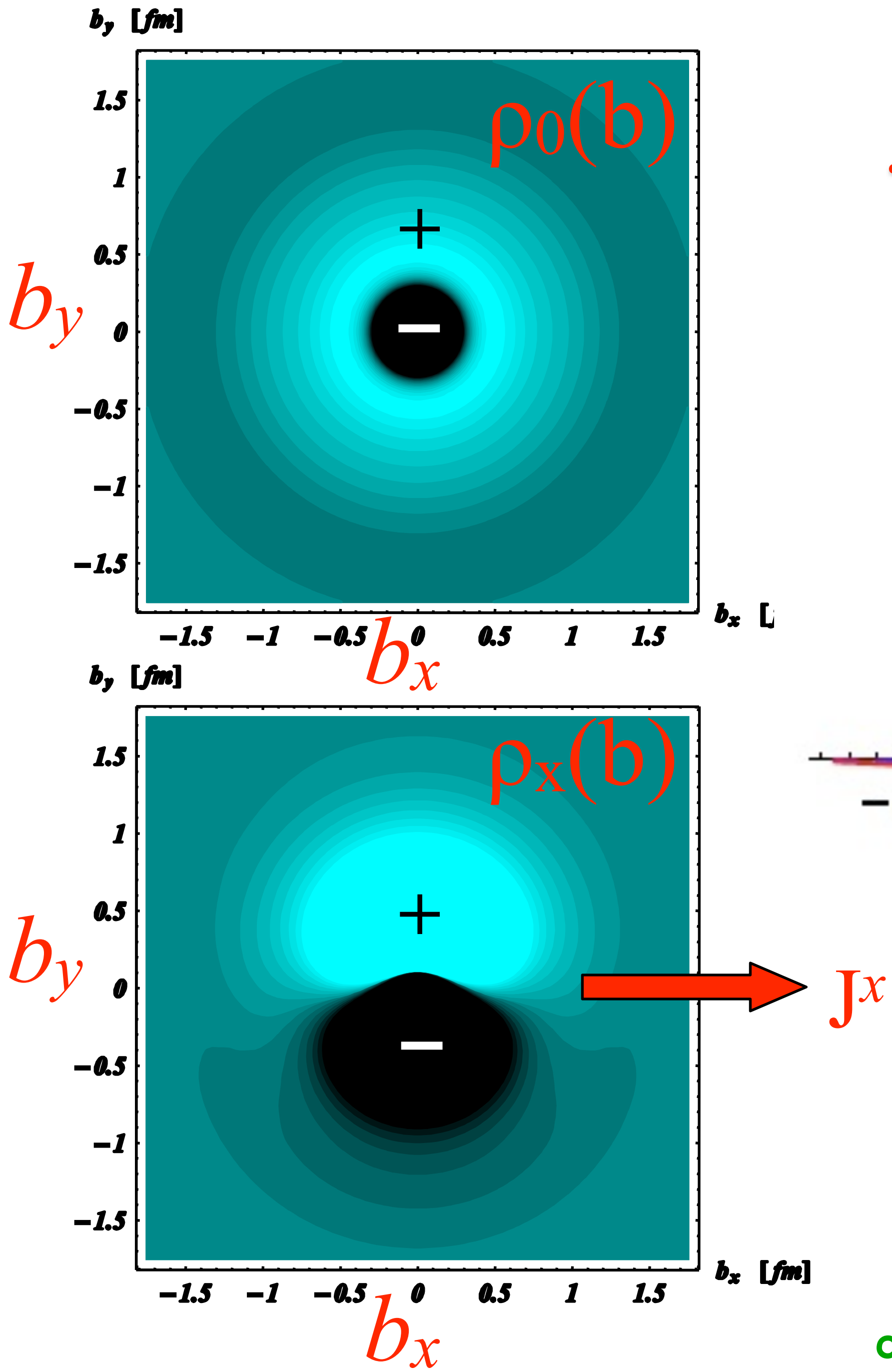
$$\times\; \delta^{(2)}(\boldsymbol{b} - \boldsymbol{b}_k)\, |\psi_n^\lambda(x_i, \boldsymbol{b}_i, \lambda_i)|^2$$

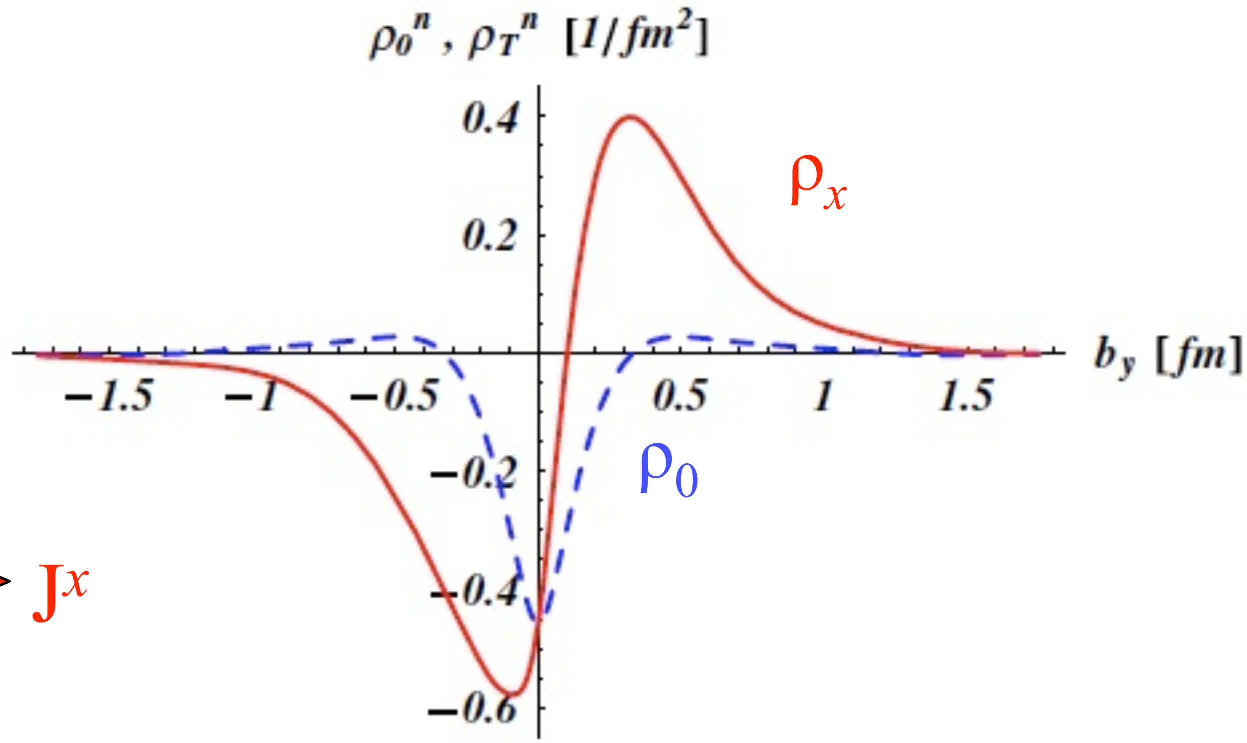
No more Wilson line: Fock expansion is “exact”

No more “leading twist”: Resolution in $b \sim 1/\boldsymbol{q}_{max}$

Using measured form factors, find the

empirical quark transverse densities in neutron





Miller (2007)
Carlson and Vanderhaeghen (2008)

data : Bradford, Bodek, Budd, Arrington (2006)

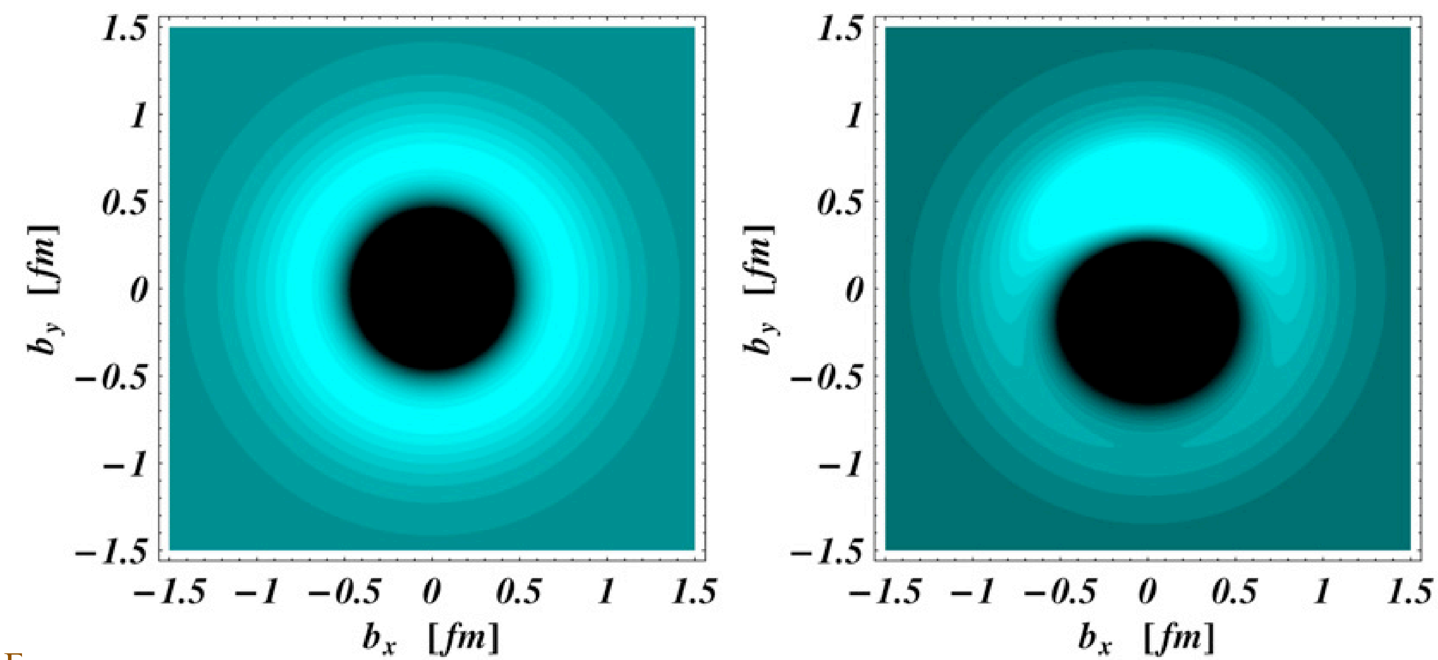
Generalization to transition Form Factors

In the case of transition form factors, the density is no longer positive definite but the charge distribution is still interesting:

"It is found that the transition from the proton to its first radially excited state is dominated by up quarks in a central region of around 0.5 fm and by down quarks in an outer band which extends up to about 1 fm."

Tiator and Vanderhaeghen (2009)

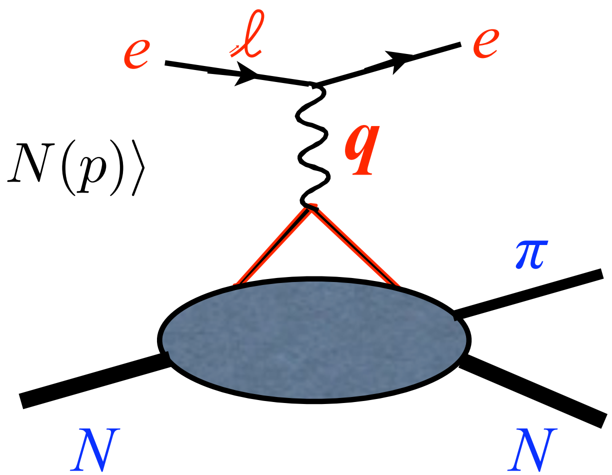
$$\gamma^* N \to P_{11}(1440)$$



Generalization to any γ* transition

PH and S. Kurki
arXiv:1101.4810

$$\mathcal{M}(\ell N \to \ell' f) = -e^2 \bar{u}(\ell')\gamma_\mu u(\ell)\frac{1}{q^2}\int d^4x e^{-iq\cdot x}\langle f|J^\mu(x)|N(p)\rangle$$



Need to identify J^+ current contribution for

LF Fock expansion: E.g.: $\ell^- \to \infty$ at fixed q

$$J^+(x) = e_q\,\bar{\mathrm{q}}(x)\gamma^+\mathrm{q}(x) = 2e_q\,\mathrm{q}_+^\dagger(x)\mathrm{q}_+(x)$$

$$\mathrm{q}_+(x) = \tfrac{1}{4}\,\gamma^-\gamma^+\,\mathrm{q}(x)$$

$$\mathrm{q}_+(0^+, x^-, \boldsymbol{x}) = \int \frac{dk^+}{k^+}\theta(k^+)\Big[b(k^+,\boldsymbol{x})u_+(k^+)e^{-i\frac{1}{2}k^+x^-} + d^\dagger(k^+,\boldsymbol{x})v_+(k^+)e^{i\frac{1}{2}k^+x^-}\Big]$$

where the LF spinors satisfy: $u_+^\dagger(k^+,\lambda')u_+(k^+,\lambda) = k^+\delta_{\lambda'\lambda}$

Fourier transform to impact parameter: $|p^+,\boldsymbol{p}\rangle = 4\pi\int d^2\boldsymbol{b}\,e^{i\boldsymbol{p}\cdot\boldsymbol{b}}|p^+,\boldsymbol{b}\rangle$

Expand into LF Fock states:

$$|p^+,\boldsymbol{b}\rangle = \frac{1}{4\pi}\sum_n\Big[\prod_{i=1}^n\int_0^1\frac{dx_i}{\sqrt{x_i}}\int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\boldsymbol{b}-\sum_i x_i\boldsymbol{b}_i)$$

$$\times\ \psi_n(x_i,\boldsymbol{b}_i-\boldsymbol{b})\prod^n b^\dagger(x_ip^+,\boldsymbol{b}_i)d^\dagger(\ )a^\dagger(\ )|0\rangle$$

$\Rightarrow$

$$\frac{1}{2p^+}\langle f(p^+,\boldsymbol{b}_f)|J^+(0)|N(p^+,\boldsymbol{b}_N)\rangle \equiv \frac{1}{(4\pi)^2}\delta^2(\boldsymbol{b}_f-\boldsymbol{b}_N)\mathcal{A}_{fN}(-\boldsymbol{b}_N)$$

where

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi}\sum_n\Big[\prod_{i=1}^n\int_0^1 dx_i\int 4\pi d^2\boldsymbol{b}_i\Big]\delta(1-\sum_i x_i)\delta^2(\sum_i x_i\boldsymbol{b}_i)$$

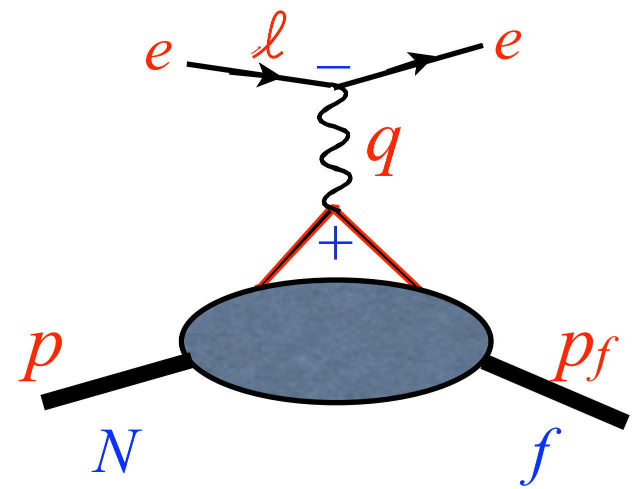
$$\times\ \psi_n^{f\,*}(x_i,\boldsymbol{b}_i)\psi_n^N(x_i,\boldsymbol{b}_i)\sum_k e_k\delta^2(\boldsymbol{b}_k-\boldsymbol{b})$$

is diagonal in Fock states n in frames where $q^+ = 0$ ($\Rightarrow$ no pair production)

FT of γ* matrix element in momentum space

In the frame:

$$\begin{aligned} p &= (p^+, p^-, -\tfrac{1}{2}\boldsymbol{q}) \\ q &= (0^+, q^-, \boldsymbol{q}) \\ p_f &= (p^+, p^- + q^-, \tfrac{1}{2}\boldsymbol{q}) \end{aligned}$$



we have

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle = \mathcal{A}_{fN}(\boldsymbol{b})$$

where $\mathcal{A}_{fN}(\boldsymbol{b})$ is given by the previous overlap of Fock amplitudes, which are universal features of N and f.

The ***b***-distribution may be studied as a function of the final state f, providing information about the transverse size of the contributing Fock states.

When f consists of several hadrons their relative momenta must be consistent with the LF Fock expansion at all $p_f = q + p$

Example: f = π (p_1) N(p_2)

In order to conform with the Lorentz covariance of LF states, at any p_f :

$$|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle \equiv \int_0^1 \frac{dx}{\sqrt{x(1-x)}} \int \frac{d^2\boldsymbol{k}}{16\pi^3} \Psi^f(x,\boldsymbol{k}) |\pi(p_1)N(p_2)\rangle$$

where $\Psi^f(x,\boldsymbol{k})$ is a freely chosen function of the relative variables x, $\boldsymbol{k}$:

$$p_1^+ = xp_f^+ \qquad \boldsymbol{p}_1 = x\boldsymbol{p}_f + \boldsymbol{k}$$

$$p_2^+ = (1-x)p_f^+ \qquad \boldsymbol{p}_2 = (1-x)\boldsymbol{p}_f - \boldsymbol{k}$$

With x, $\boldsymbol{k}$ being independent of p_f, this defines the pion and nucleon momenta p_1, p_2 at all photon momenta q.

The $|\pi N(p_f^+, \boldsymbol{p}_f; \Psi^f)\rangle$ state has an LF Fock expansion of standard form, in terms of the pion and nucleon Fock amplitudes.

Illustration (1): $\gamma^* + \mu \to \mu + \gamma$

The QED matrix element $\mathcal{A}^{\mu\gamma}_{\lambda_1,\lambda_2} = \dfrac{1}{2p^+}\langle \mu(p_1,\lambda_1)\gamma(p_2,\lambda_2)|J^+(0)|\mu(p,\lambda=\tfrac{1}{2})\rangle$

expressed in terms of the relative variables x, $\boldsymbol{k}$ is:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{q};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2} - \frac{\boldsymbol{e}_-\cdot(\boldsymbol{k}-(1-x)\boldsymbol{q})}{(1-x)^2m^2+(\boldsymbol{k}-(1-x)\boldsymbol{q})^2}\right]$$

where $\boldsymbol{e}_\lambda\cdot\boldsymbol{k} = -\lambda e^{i\lambda\phi_k}|\boldsymbol{k}|/\sqrt{2}$. The Fourier transform gives:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2}\delta^2(\boldsymbol{b}) - \frac{i}{2\sqrt{2}\pi}\frac{m\,e^{-i\phi_b}}{1-x}K_1(mb)\exp\left(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right)\right]$$

In the first term the γ^* interacts with the initial muon, which by definition is at b = 0. The second term reflects the distribution of the final muon in transverse space.

This expression agrees exactly with the wave function overlap formula.

Illustration (2): $\gamma^{*}+\mu \to \mu+\gamma$

Choosing $\Psi(x',\boldsymbol{k}) = \delta(x'-x)\sqrt{x(1-x)}\,\exp\Big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}'_\mu}{1-x}\Big)$

corresponds to fixing the impact parameter b_μ' of the final muon. Then

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b};x,\boldsymbol{b}'_\mu) = \sqrt{x(1-x)}\,\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\boldsymbol{b}'_\mu)\Big[-\delta^{(2)}(\boldsymbol{b})+\delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}'_\mu)\Big]$$

which again conforms with the general overlap expression of LF Fock state wave functions.

Fourier transform of the cross section

The $\gamma^*+N \to f$ amplitudes have dynamical phases (resonances,...).
$\Rightarrow$ Calculating their Fourier transforms requires a partial wave analysis.

However, one can Fourier transform the measured cross section itself.

Then the $\boldsymbol{b}$-distribution reflects the difference between the impact parameters of the photon vertex in the amplitude and its complex conjugate:

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left| \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle \right|^2 = \int d^2\boldsymbol{b}_q\, \mathcal{A}_{fN}(\boldsymbol{b}_q)\, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b})$$

For $|f\rangle = |\pi(p_1)N(p_2)\rangle$, parametrized with the relative variables x and $\boldsymbol{k}$,

$$\mathcal{S}_{fN}(\boldsymbol{b};x,\boldsymbol{k}) = \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\, \boldsymbol{q}^4\, \frac{d\sigma(\ell N \to \ell'\pi N)}{d^2\boldsymbol{q}\, dx\, d^2\boldsymbol{k}}$$

$$= \frac{\alpha^2}{4\pi^3} \frac{1}{x(1-x)} \int d^2\boldsymbol{b}_q\, \mathcal{A}_{fN}(\boldsymbol{b}_q;x,\boldsymbol{k})\, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b};x,\boldsymbol{k})$$

Illustration (3): $\sigma(\gamma^* + \mu \rightarrow \mu + \gamma)$

For the QED example considered above the Fourier transform of the cross section can be done analytically:

$$\mathcal{S}^{\mu\gamma}(\boldsymbol{b};x,\boldsymbol{k}) = 4e^2x\Bigg\{\frac{\boldsymbol{k}^2/2}{[(1-x)^2m^2+\boldsymbol{k}^2]^2}\delta^{(2)}(\boldsymbol{b}) \;-\; \frac{|\boldsymbol{k}|\cos(\phi_b-\phi_k)}{(1-x)^2m^2+\boldsymbol{k}^2}\,\frac{im}{2\pi}\,\frac{\exp\big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\big)}{1-x}K_1(mb)$$

$$+\;\frac{1}{4\pi}\,\frac{\exp\big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\big)}{(1-x)^2}\Big[K_0(mb)-\tfrac{1}{2}mb\,K_1(mb)\Big]\Bigg\}$$

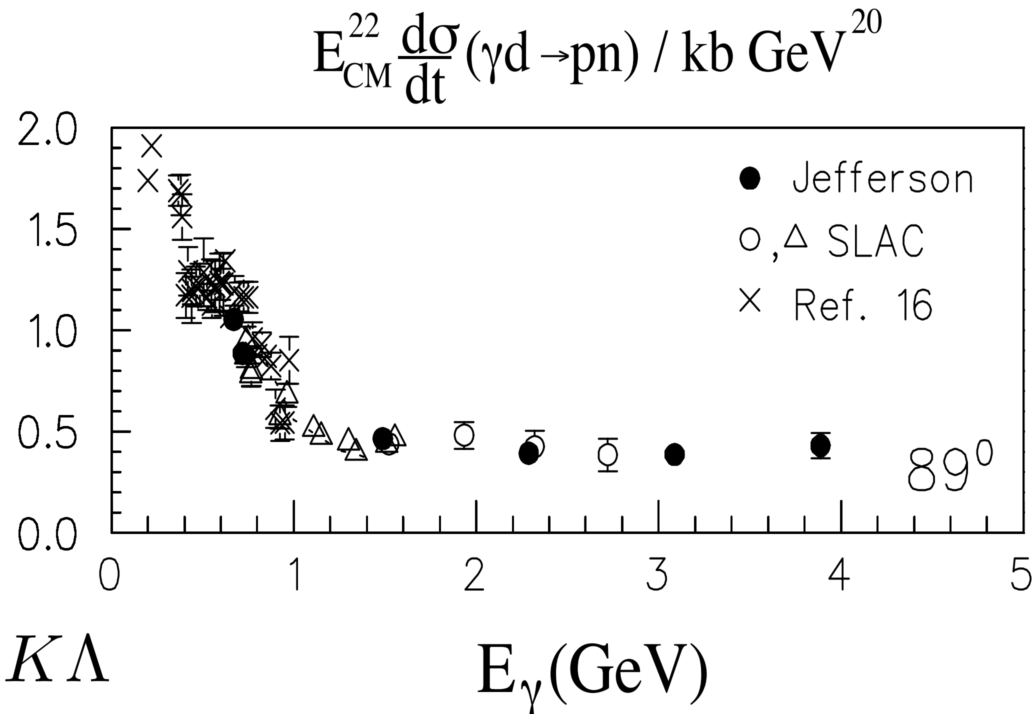
The 3 terms correspond to 2, 1 and 0 of the γ^* interactions occurring on the initial muon.

The imaginary part arises from an angular correlation between ***b*** and ***k*** .

Remarks

In $\gamma^* N \to \pi N$, expect the b-distribution to narrow with the relative transverse momentum k between the π and the N.

$\sigma(\gamma D \to pn) \propto E^{-22}$ at large angles, suggesting compact states. A measurement of the q^2-dependence would allow a direct measurement of the transverse size.



In heavy quark production: $\gamma^* N \to K\Lambda$
$\gamma^* N \to D\Lambda_c$

the b-distribution should narrow with the quark mass if the photon couples directly to the heavy quarks.

One may compare the b-distribution in ordinary and diffractive events.

Summary (1)

Intuitively, the ***q*** -dependence of a virtual photon interaction gives information about the charge distribution in space.

The target is illuminated "instantaneously" only when the charge carriers are non-relativistic. This is the case in electron microscopy.

Quarks move inside hadrons with ≈ velocity of light.

The photon phase is constant at fixed Light-Front time $x^+ = t + z$

In the IMF ≈ LF formulation, transverse quark velocities are non-relativistic

2-dim. FT's of form factors describe charge densities in transverse space

Unlike pdf's, no "leading twist" limit is implied.

The resolution in impact parameter is expected to be $\Delta b \sim 1/Q_{max}$

Summary (2)

The formulation can be generalized to transition form factors $\gamma^* N \to N^*$ and to any (multi-hadron) final (and initial) state: $\gamma^* \mathrm{A} \to f$

FT of the cross section $\sigma(\gamma^* N \to f)$ gives the distribution in the transverse distance $\boldsymbol{b}$ between the photon vertex in $\mathrm{T}(\gamma^* N \to f)$ and $[\mathrm{T}(\gamma^* N \to f)]^*$