High-Resolution Probes of Low-Resolution Nuclei

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Ohio State University

"Nuclear Structure and Dynamics at Short Distances"

INT February, 2013

Context:

- "From low-momentum interactions to nuclear structure ,"
  S. Bogner, rjf, A. Schwenk, arXiv:1012.3381
- "Operator evolution via the SRG: The deuteron,"
  E. Anderson et al., arXiv:1008.1569
- "High-momentum tails from low-momentum effective theories,"
  S. Bogner, D. Roscher, arXiv:1208.1734

Outline

Nuclei at low resolution

Fate of high-momentum physics

Probing low-resolution nuclei at high momentum

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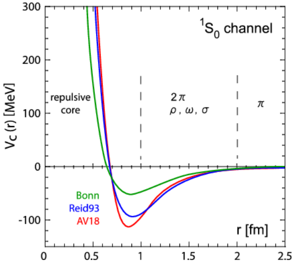
Nuclei at low resolution

Fate of high-momentum physics

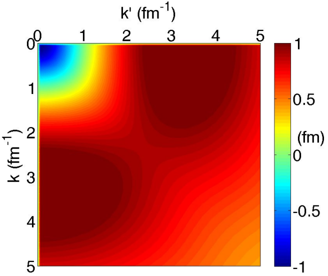
Probing low-resolution nuclei at high momentum

Uses of the renormalization group (RG) [cf. S. Weinberg]

- Improving perturbation theory; e.g., in QCD calculations
  - Mismatch of energy scales can generate large logarithms
  - Shift between couplings and loop integrals to reduce logs
- Identifying universality in critical phenomena
  - Filter out short-distance degrees of freedom
- Simplifying calculations of nuclear structure/reactions
  - Make nuclear physics look more like quantum chemistry!



AV18, Bonn, Reid93



$\langle k|V_{\rm AV18}|k'\rangle$

Coupling of low-k/high-k modes: non-perturbative, strong correlations, . . .

Remedy: Use RG to decouple modes $\Longrightarrow$ low resolution

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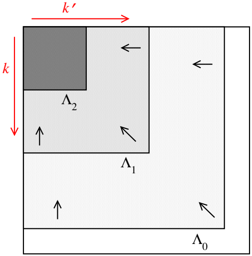
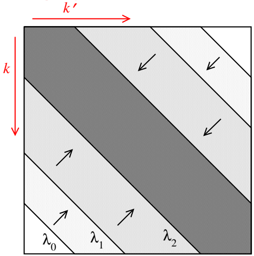


"$V_{\text{low }k}$"



Similarity RG

- $V_{\text{low }k}$: lower cutoff Λ_i in k, k' via $dT(k, k'; k^2)/d\Lambda = 0$
- SRG: drive H toward diagonal with flow equation

$$dH_s/ds = [[G_s, H_s], H_s]$$

Continuous unitary transforms (cf. running couplings)

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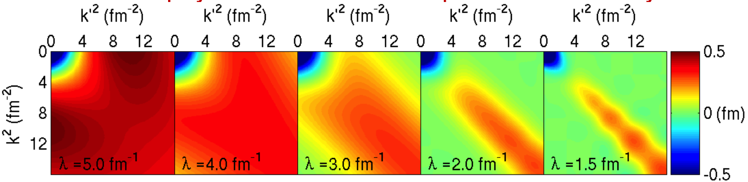
AV18:



- Decoupling naturally visualized in momentum space for $G_s = T$
  - Phase-shift equivalent! Width of diagonal given by $\lambda^2 = 1/\sqrt{s}$
  - What does this look like in coordinate space?

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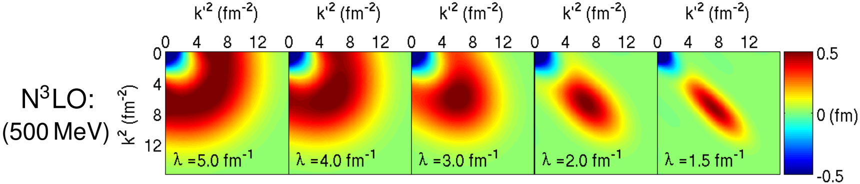

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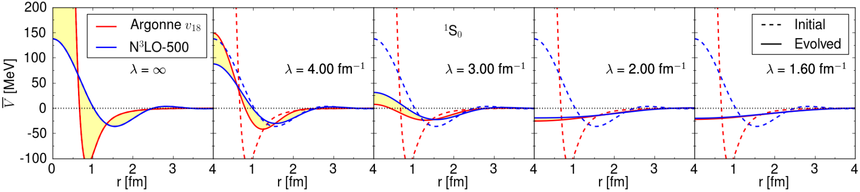
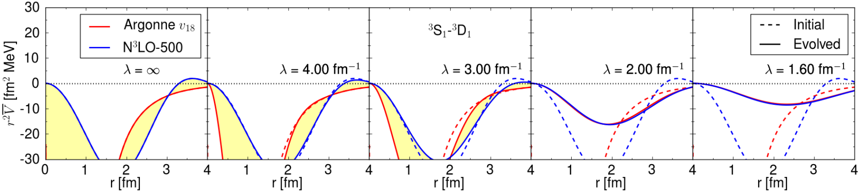
Local Projections [K. Wendt et al., PRC 86, 014003 (2012)]

- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]



- Tensor part (S-D mixing) [graphs from K. Wendt]



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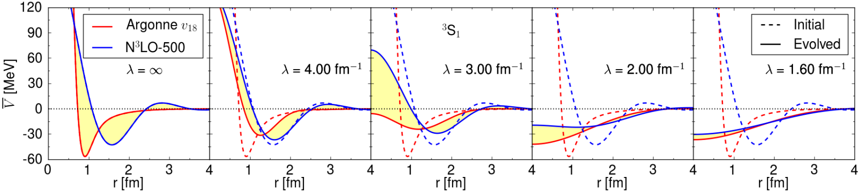

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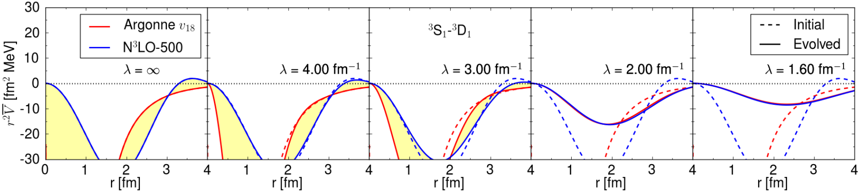
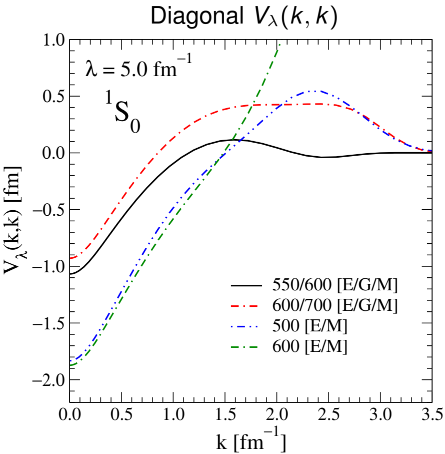
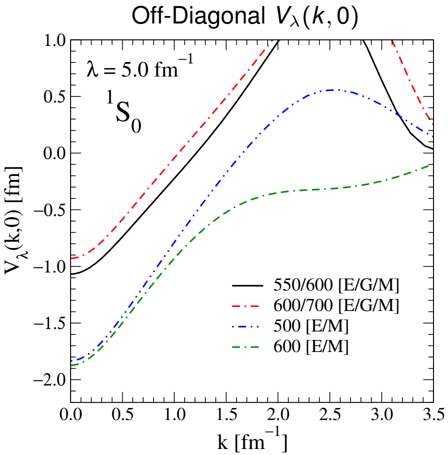
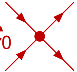


Run to lower λ via SRG $\Longrightarrow \approx$Universal low-k V_{NN}





- Similar pattern with phenomenological potentials (e.g., AV18)
- As resolution changes, shift high-k details to contact C_0



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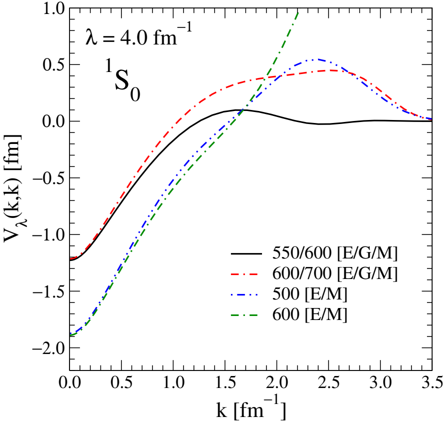
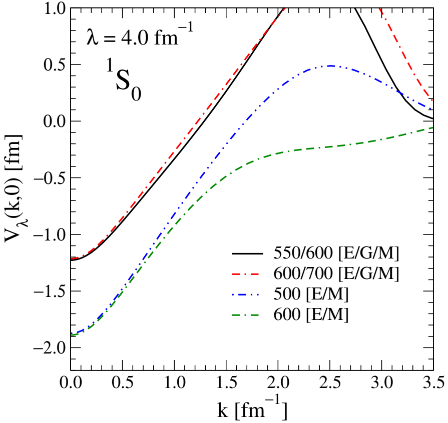
Diagonal $V_\lambda(k,k)$



Off-Diagonal $V_\lambda(k,0)$



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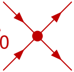

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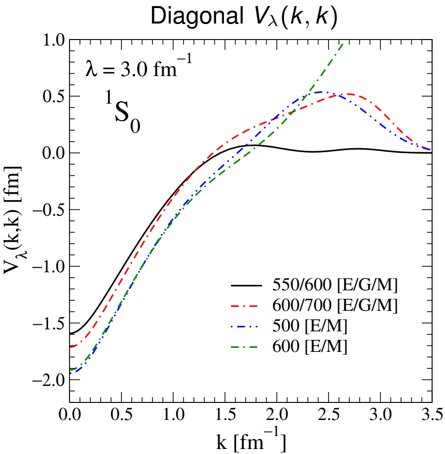

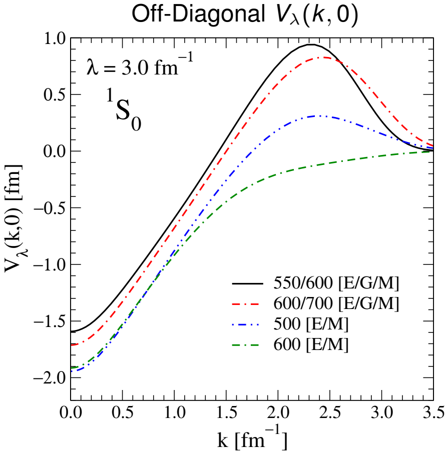

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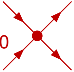

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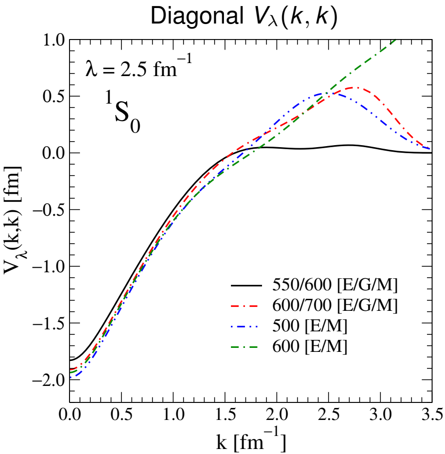

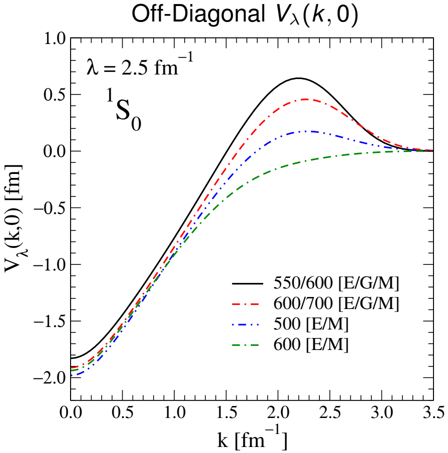

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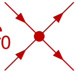

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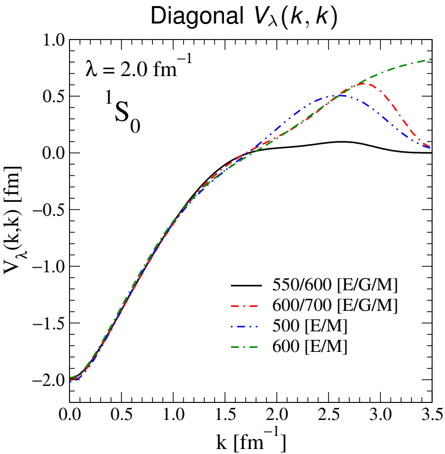

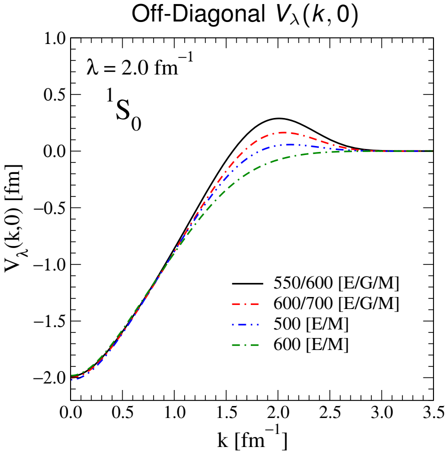

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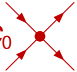

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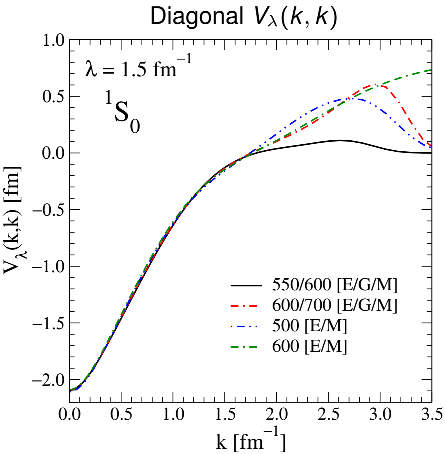

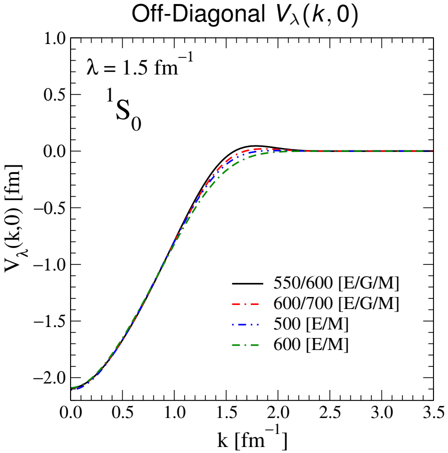

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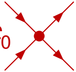
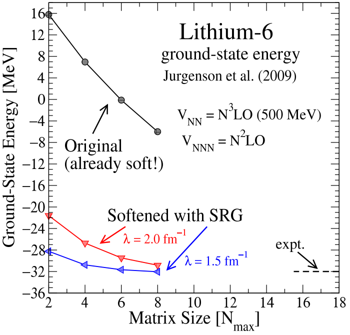


Nuclear structure natural with *low* momentum scale

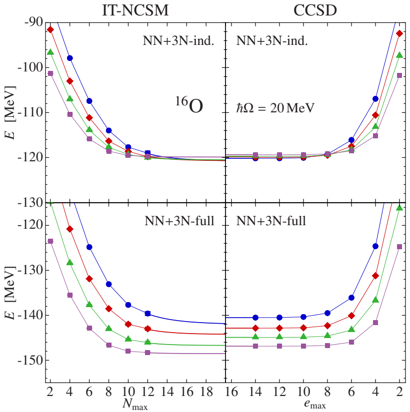
Softened potentials (SRG, $V_{\text{low } k}$, UCOM, ...) enhance convergence

- Convergence for no-core shell model (NCSM):



- (Already) soft chiral EFT potential and evolved (softened) SRG potentials, including NNN

- Softening allows importance truncation (IT) and converged coupled cluster (CCSD)

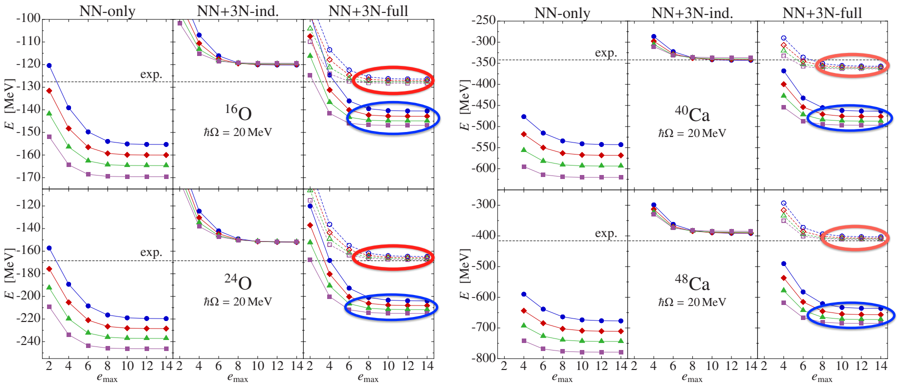


[Roth et al., arXiv:1112.0287]

Nuclear structure natural with *low* momentum scale

R. Roth et al. SRG-evolved N^3LO with NNN [arXiv:1112.0287]

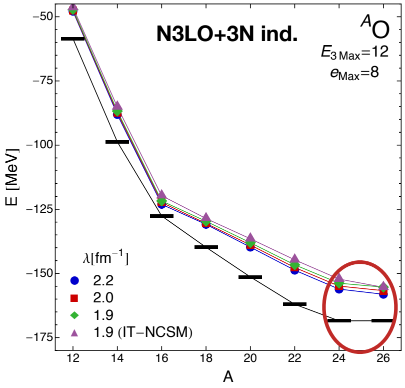
- Coupled cluster with interactions $H(\lambda)$: λ is a decoupling scale
  - *Only* when NNN-induced added to NN-only $\Longrightarrow$ λ independent
  - With initial NNN: predictions from fit only to $A = 3$ properties
- Open questions: red (400 MeV) works, blue (500 MeV) doesn't!

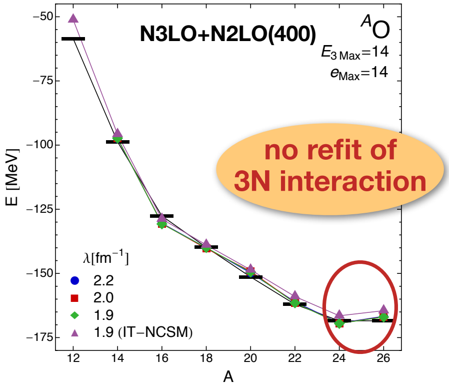


Nuclear structure natural with *low* momentum scale

In-medium Similarity Renormalization Group made possible

- E.g, IM-SRG for open-shell nuclei [H. Hergert et al., in preparation]
  - Start with SRG-evolved NN+NNN Hamiltonian
  - Evolve normal-ordered wrt reference state $\Longrightarrow$ decouple ph excitations
  - Direct application to isotope chains, shell-model $H_{\rm eff}$, . . .

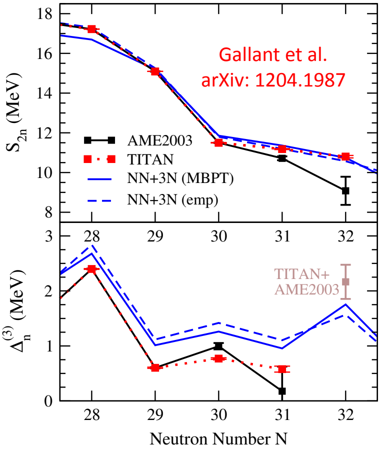




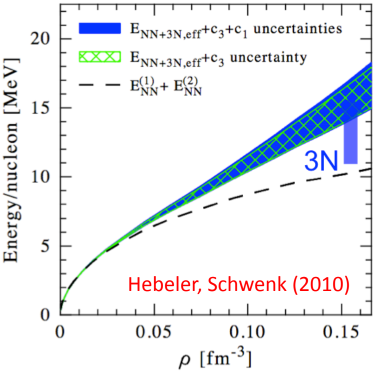
Nuclear structure natural with *low* momentum scale

Lowered scale enables many-body perturbation theory (MBPT)

- Quantitative prediction for Ca isotope S_{2n} trends (verified!)



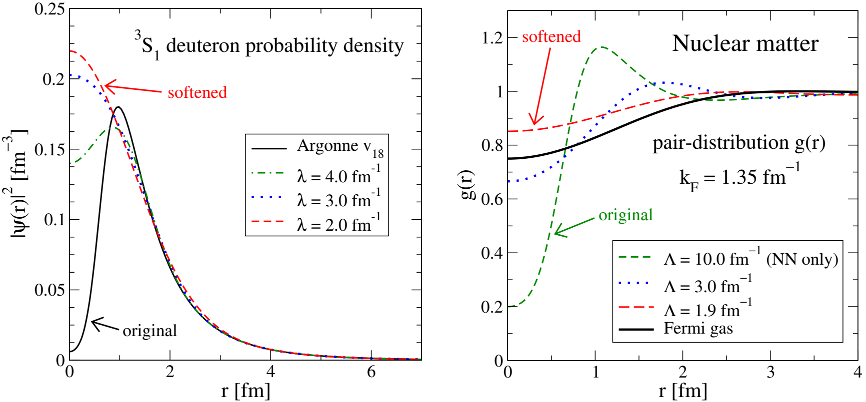
- Neutron/nuclear matter



- Constrain neutron stars: $R = 10$–14 km for $1.4\,M_{\rm sun}$ [Hebeler et al. (2010)]

Nuclear structure natural with *low* momentum scale

But soft potentials don't lead to short-range correlations (SRC)!



- Continuously transformed potential $\Longrightarrow$ variable SRC's in wfs!
- Therefore, it seems that SRC's are very resolution dependent

Outline

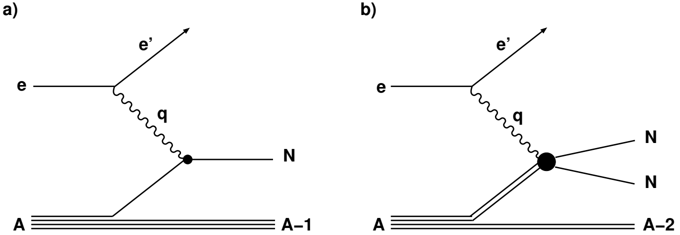
Nuclei at low resolution

Fate of high-momentum physics

Probing low-resolution nuclei at high momentum

Changing resolution shifts physics: Not unique!

- From D. Higinbotham, arXiv:1010.4433

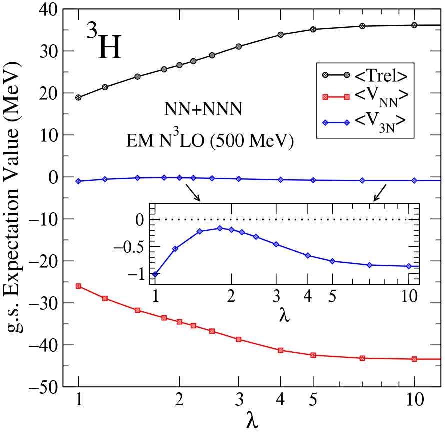


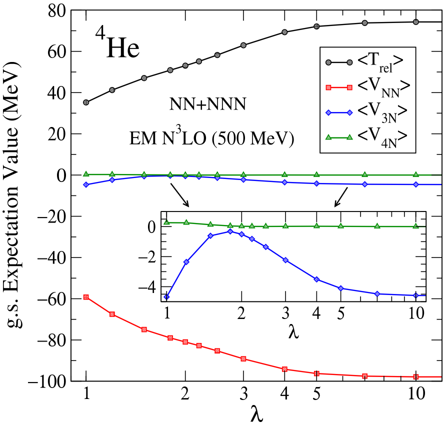
"The simple goal of short-range nucleon-nucleon correlation studies is to cleanly isolate diagram b) from a). Unfortunately, there are many other diagrams, including those with final-state interactions, that can produce the same final state as the diagram scientists would like to isolate. If one could find kinematics that were dominated by diagram b) it would finally allow electron scattering to provide new insights into the short-range part of the nucleon-nucleon potential."

- What is in the blob in b)? A one-body vertex and an SRC, or a two-body vertex? Depends on the resolution! (Also FSI+ will mix.)

Contributions to the ground-state energy

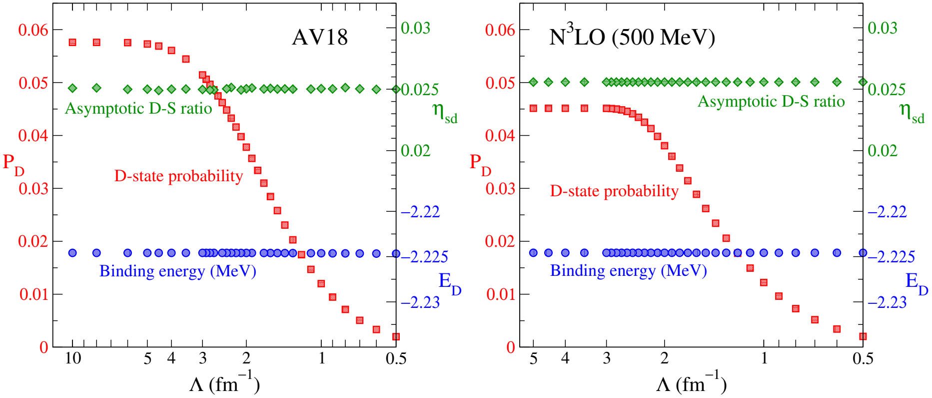
- Look at ground-state matrix elements of KE, NN, 3N, 4N





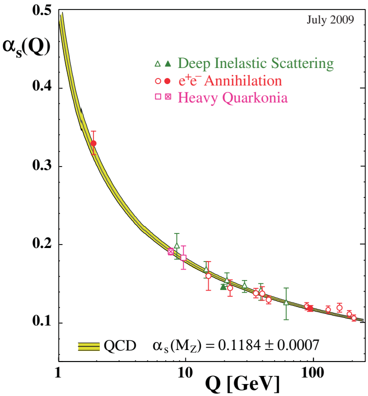
- Clear hierarchy, but also strong cancellations at NN level
- Kinetic energy is resolution dependent!

Deuteron scale-(in)dependent observables



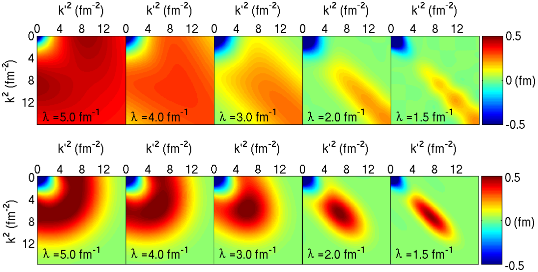
- $V_{\mathrm{low}\,k}$ RG transformations labeled by Λ (different V_Λ's)
  $\Longrightarrow$ soften interactions by lowering resolution (scale)
  $\Longrightarrow$ reduced short-range and tensor correlations
- Energy and asymptotic D-S ratio are unchanged (cf. ANC's)
- But D-state probability changes (cf. spectroscopic factors)

Running QCD $\alpha_s(Q^2)$ vs. running nuclear V_λ



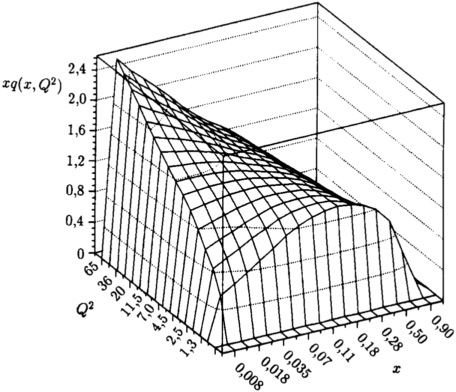
- The QCD coupling is *scale* dependent (cf. low-E QED): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})]^{-1}$
- The QCD coupling strength α_s is *scheme* dependent (e.g., "V" scheme used on lattice, or $\overline{\rm MS}$)

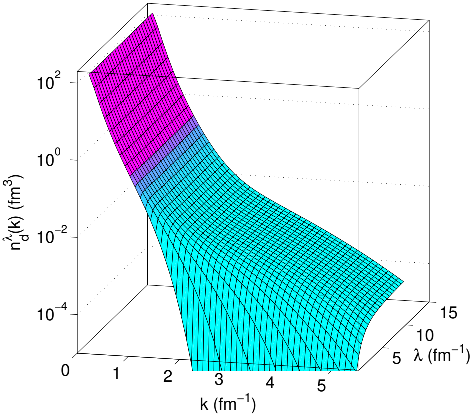
- Vary scale ("resolution") with RG
- Scale dependence: SRG (or $V_{\text{low }k}$) running of initial potential with λ (decoupling or separation scale)



- Scheme dependence: AV18 vs. N^{3}LO (plus associated 3NFs)
- But all are (NN) phase equivalent!
- Shift contributions between interaction and sums over intermediate states

Parton vs. nuclear momentum distributions

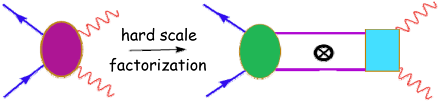




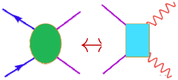
- The quark distribution $q(x, Q^2)$ is scheme *and* scale dependent
- $x\, q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval

- Deuteron momentum distribution is scheme *and* scale dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\text{fm}^{-1}$

Factorization: high-E QCD vs. low-E nuclear



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Scheme: parton distributions $\Longleftrightarrow$ Wilson coefficients

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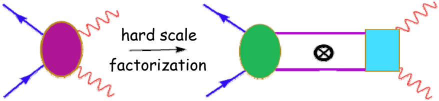

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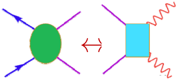

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- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section

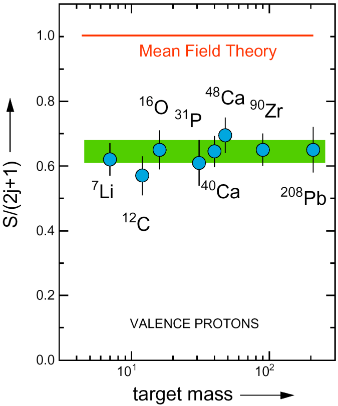
Structure model: spectroscopic factor

Reaction model: single-particle cross section

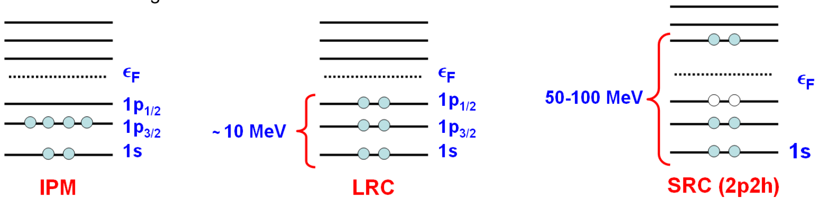
$$\sigma^{if} = \sum_{|J_f - J_i| \le j \le J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What does it mean to be *consistent* between structure and reaction models? Treat separately? No!
- How does scale/scheme dependence come in?
- What are the trade-offs? (Does simpler structure part always mean more complicated reaction part?)

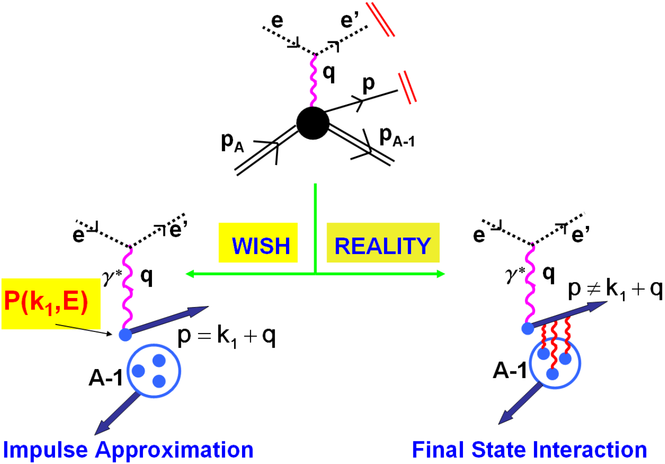
Scale/scheme dependence: spectroscopic factors



- Spectroscopic factors for valence protons have been extracted from $(e, e'p)$ experimental cross sections (e.g., Nikhef 1990's at left)
- Used as canonical evidence for "correlations", particularly short-range correlations (SRC's)
- But if SFs are scale/scheme dependent, how do we explain the cross section?

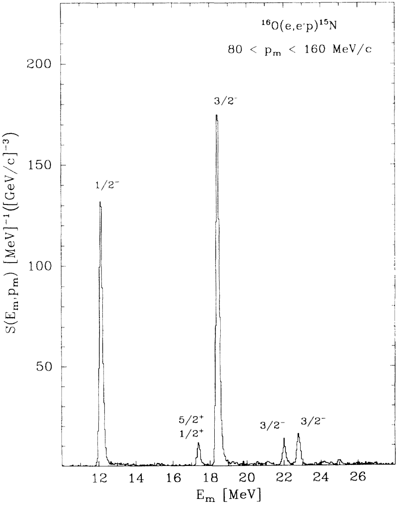


Standard story for $(e, e'p)$ [from C. Ciofi degli Atti]

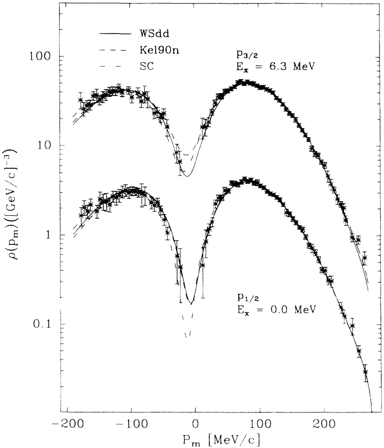


- In IA: "missing" momentum $p_m = k_1$ and energy $E_m = E$
- Choose E_m to select a discrete final state for range of p_m
- Can FSI be treated as *add-on* theoretical correction to IA?

(Assumed) factorization of $(e, e'p)$ cross section



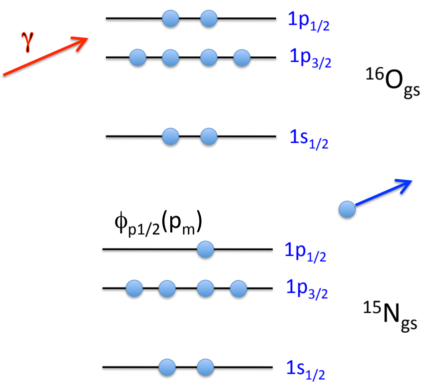
Missing energy spectrum for $^{16}O(e, e'p)^{15}N$ [Leuschner (1994)]

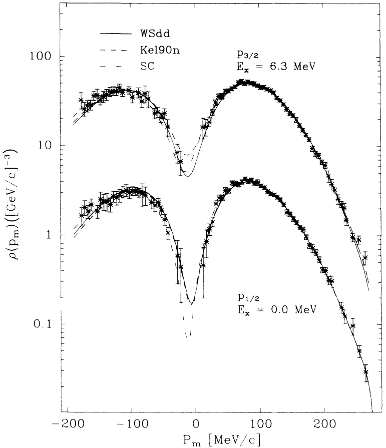


$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \propto |\phi_\alpha(\mathbf{p}_m)|^2$$

$\Longrightarrow p_{1/2}$ spectroscopic factor ≈ 0.63

(Assumed) factorization of $(e, e'p)$ cross section





- Knock out $p_{1/2}$ proton from ^{16}O to ^{15}N ground state in IPM
- Adjust s.p. well depth and radius to identify $\phi_\alpha(\mathbf{p}_m)$
- Final state interactions (FSI) added

$$\frac{d\sigma}{d\mathbf{p}'_e d\mathbf{p}'_N} = K\sigma_{ep} \times \rho(\mathbf{p}_m) \;\propto |\phi_\alpha(\mathbf{p}_m)|^2$$

$\Longrightarrow p_{1/2}$ spectroscopic factor ≈ 0.63

Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle\Psi_m|\widehat{O}|\Psi_n\rangle = \left(\langle\Psi_m|U^\dagger\right) U\widehat{O}U^\dagger \left(U|\Psi_n\rangle\right) = \langle\widetilde{\Psi}_m|\widetilde{O}|\widetilde{\Psi}_n\rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

But the matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

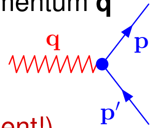
$$O_{\widetilde{m}\widetilde{n}} \equiv \langle\widetilde{\Psi}_m|O|\widetilde{\Psi}_n\rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle\Psi_n^{A-1}|a_\alpha|\Psi_0^A\rangle \text{ changes}$$

- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states. So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- [Field theory version: the equivalence principle says that only on-shell quantities can be measured. Field redefinitions change off-shell dependence only.]

Generic knockout reaction [e.g., Dickhoff/Van Neck text]

- Consider a scalar external probe that just transfers momentum $\mathbf{q}$

$$\rho(\mathbf{q}) = \rho_0 \sum_{j=1}^{A} e^{-i\mathbf{q}\cdot\mathbf{r}} \Longrightarrow \widehat{\rho}(\mathbf{q}) = \rho_0 \sum_{\mathbf{p},\mathbf{p}'} \langle \mathbf{p} | e^{-i\mathbf{q}\cdot\mathbf{r}} | \mathbf{p}' \rangle a^\dagger_{\mathbf{p}} a_{\mathbf{p}'}$$



  - First assumption: one-body operator (scale dependent!)

- Then the cross section from Fermi's golden rule is

$$d\sigma \sim \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | \widehat{\rho}(\mathbf{q}) | \Psi_i \rangle|^2$$

- Complication: ejected final particle A interacts on way out (FSI)

$$H_A = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} V(i,j) = H_{A-1} + \frac{p_A^2}{2m} + \sum_{i=1}^{A-1} V(i,A)$$

  - If we neglect this interaction $\Longrightarrow$ PW (no FSI)

$$|\Psi_i\rangle = |\Psi_0^A\rangle\,, \qquad |\Psi_f\rangle = a^\dagger_{\mathbf{p}} |\Psi_n^{A-1}\rangle \Longrightarrow \langle \Psi_f | = \langle \Psi_n^{A-1} | a_{\mathbf{p}}$$

$\Longrightarrow$ factorized knockout cross section $\propto$ hole spectral fcn:

$$d\sigma \sim \rho_0^2 \sum_n \delta(E_m - E_0^A + E_n^{A-1}) |\langle \Psi_n^{A-1} | a_{\mathbf{p}_m} | \Psi_0^A \rangle|^2 = \rho_0^2\, S_h(\mathbf{p}_m, E_m)$$

Now repeat with a unitary transformation $\widehat{U}$

- The cross section is *guaranteed* to be the same from $\widehat{U}^\dagger \widehat{U} = 1$

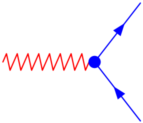
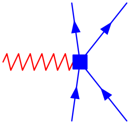
$$\begin{aligned} d\sigma &\sim \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | \widehat{\rho}(\mathbf{q}) | \Psi_i \rangle|^2 \\ &= \sum \delta(\omega + E_i - E_f) |\langle \Psi_f | (\widehat{U}^\dagger \widehat{U}) \widehat{\rho}(\mathbf{q}) (\widehat{U}^\dagger \widehat{U}) | \Psi_i \rangle|^2 \\ &= \sum \delta(\omega + E_i - E_f) |(\langle \Psi_f | \widehat{U}^\dagger)(\widehat{U} \widehat{\rho}(\mathbf{q}) \widehat{U}^\dagger)(\widehat{U} | \Psi_i \rangle)|^2 \end{aligned}$$

but the pieces are different now.

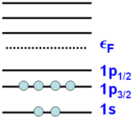
- Schematically, the SRG has $\widehat{U} = 1 + \frac{1}{2}(U - 1) a^\dagger a^\dagger a a + \cdots$
  - U is found by solving for the unitary transformation in the $A = 2$ system (this is the easy part!)
  - The $\cdots$'s represent higher-body operators
  - One-body operators ($\propto a^\dagger a$) gain many-body pieces (EFT: there are always many-body pieces at some level!)
  - Both initial and final states are modified (and therefore FSI)

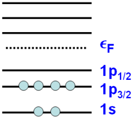
New pieces after the unitary transformation

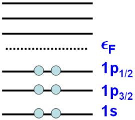
- The current is no longer just one-body (cf. EFT current):

$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger =$  $+ \alpha$  $+ \cdots$

- New correlations have appeared (or disappeared):

$\widehat{U}|\Psi_0^A\rangle = \widehat{U}$ 

$+ \cdots \Longrightarrow$ 

$+ \alpha$ 

$+ \cdots$

  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger|\Psi_n^{A-1}\rangle$
  - So the spectroscopic factors are modified

- Final state interactions are also modified by $\widehat{U}$
- Bottom line: the cross section is unchanged *only* if all pieces are included, with the same U: $H(\lambda)$, current operator, FSI, ...

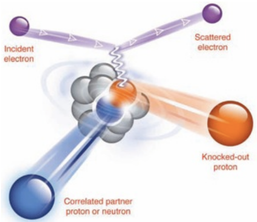
Outline

Nuclei at low resolution

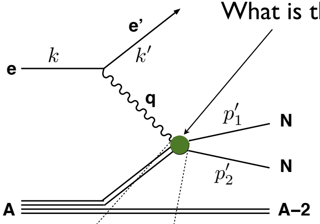
Fate of high-momentum physics

Probing low-resolution nuclei at high momentum

Looking for missing strength at large Q^2



Subedi et al., Science 320, 1476 (2008)



What is this vertex?

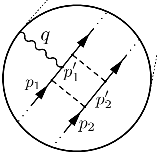
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

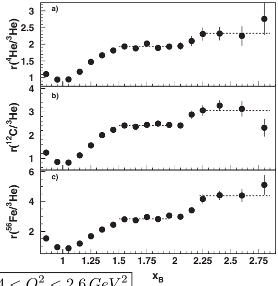
$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2m_N\nu}$$

Higinbotham, arXiv:1010.4433



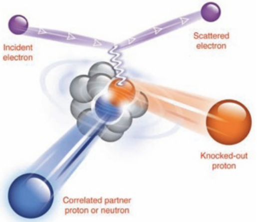
SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon



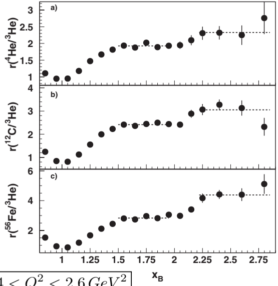
$1.4 < Q^2 < 2.6\,GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)

Looking for missing strength at large Q^2



Subedi et al., Science 320, 1476 (2008)



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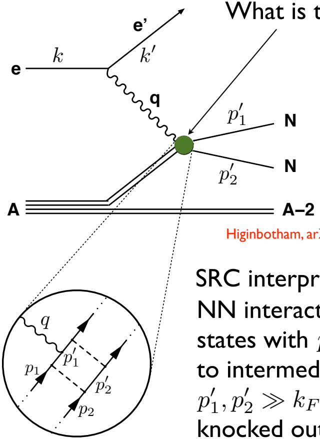

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---

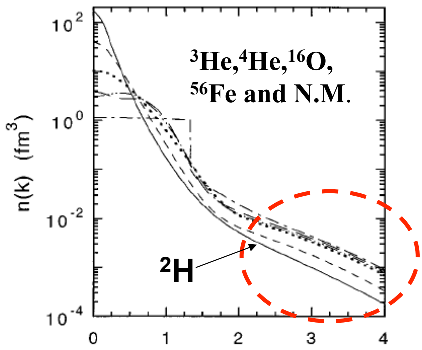
How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

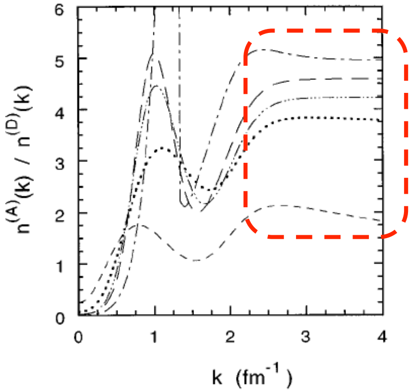
Deuteron-like scaling at high momenta from factorization

C. Ciofi and S. Simula, *Phys.Rev* **C53**, 1689(1996)

Momentum Distributions n(k)



Ratio to the Deuteron



n(k) at high Momentum regions are similar to it of the Deuteron

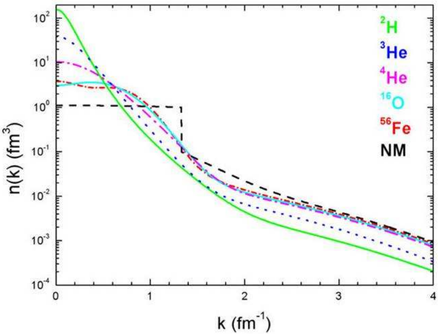
Almost Flat!

High resolution: Dominance of V_{NN} and SRCs (Frankfurt et al.)

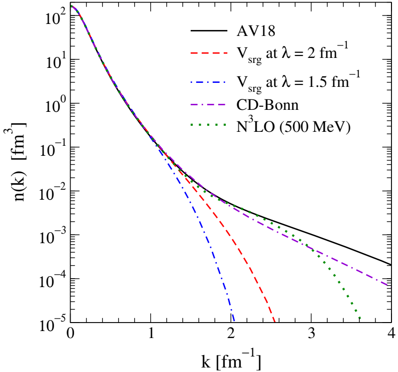
Lower resolution $\Longrightarrow$ lower separation scale $\Longrightarrow$ fall-off depends on V_λ

Changing the separation scale with RG evolution

- Conventional analysis has (implied) high momentum scale
  - Based on potentials like AV18 and one-body current operator



[From C. Ciofi degli Atti and S. Simula]



- With RG evolution, probability of high momentum decreases, but

$$n(k) \equiv \langle A|a^\dagger_{\mathbf{k}} a_{\mathbf{k}}|A\rangle = \left(\langle A|\widehat{U}^\dagger\right) \widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger \left(\widehat{U}|\Psi_n\rangle\right) = \langle\widetilde{A}|\widehat{U} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \widehat{U}^\dagger|\widetilde{A}\rangle$$

is unchanged! $|\widetilde{A}\rangle$ is easier to calculate, but is operator too hard?

Operator flow in practice [see arXiv:1008.1569]

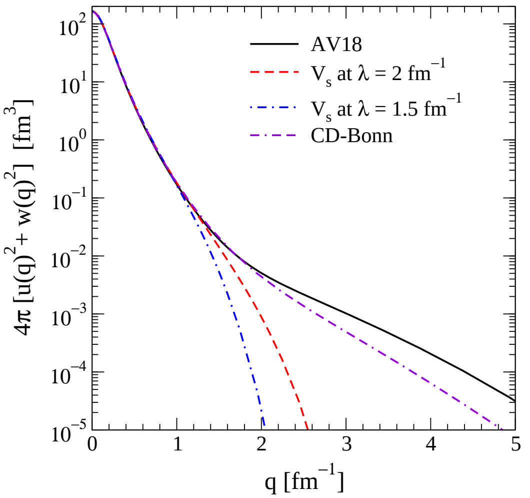
- Evolution with s of any operator O is given by:

$$O_s = U_s O U_s^\dagger$$

so O_s evolves via

$$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 0.34$ and $3.0\,\mathrm{fm}^{-1}$ in deuteron



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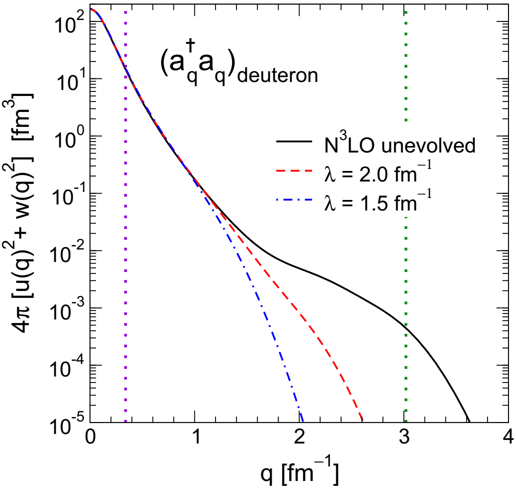
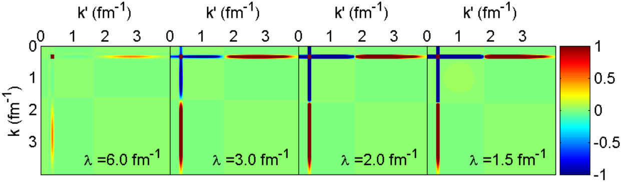
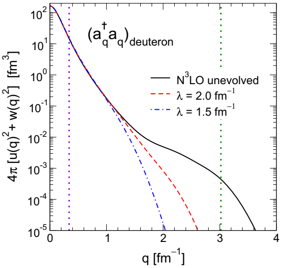


High and low momentum operators in deuteron

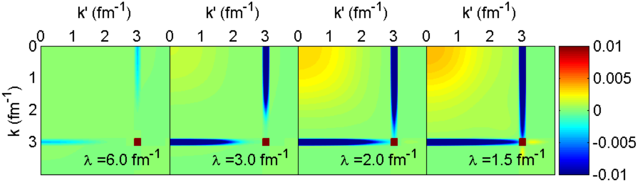
- **Integrand** of $\langle U a_q^\dagger a_q U^\dagger \rangle$ for $q = 0.34\,\text{fm}^{-1}$



- Momentum distribution



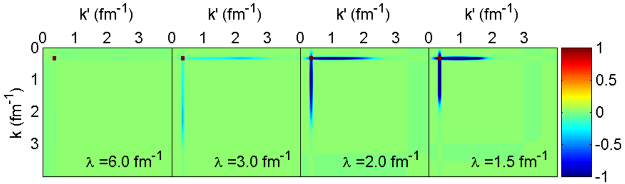
- **Integrand** for $q = 3.02\,\text{fm}^{-1}$



- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2 \rangle$, $\langle Q_d \rangle$, $\langle 1/r \rangle$ $\left\langle \frac{1}{r} \right\rangle$, $\langle G_C \rangle$, $\langle G_Q \rangle$, $\langle G_M \rangle$

High and low momentum operators in deuteron

- **Integrand** of $\langle\psi_d|\,(Ua_q^\dagger a_q U^\dagger)\,|\psi_d\rangle$ for $q = 0.34\,\text{fm}^{-1}$



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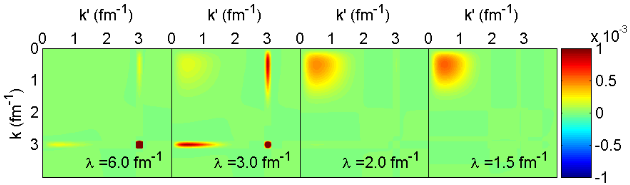

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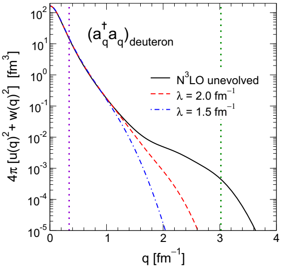

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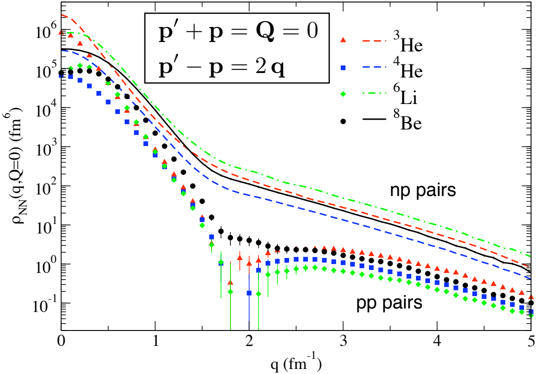
Perturbative calculations of pair densities [Anderson, Hebeler]

- Preliminary calculations of nucleon pair densities

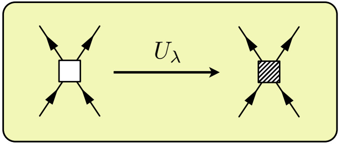


Schiavilla et al. PRL 98, 132501 (2007)

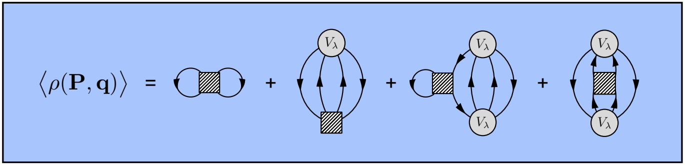
Perturbative calculations of pair densities [Anderson, Hebeler]

- Preliminary calculations of nucleon pair densities
  - leading induced operators only (two-body)

RG transformation of pair density operator (induced many-body terms neglected):

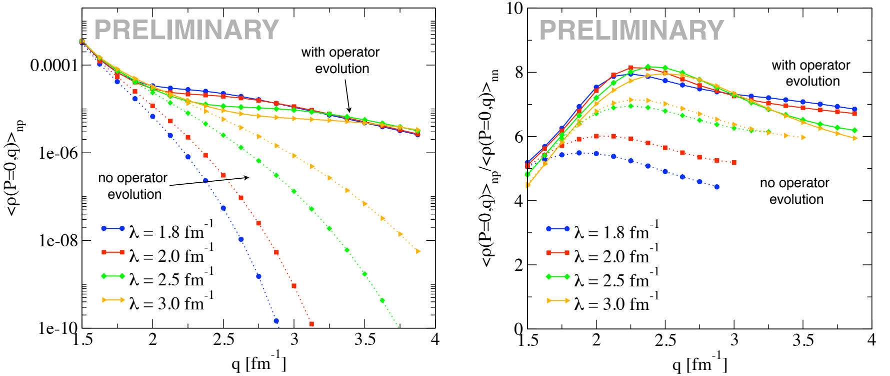


simple calculation of pair density at low resolution in nuclear matter:



Perturbative calculations of pair densities [Anderson, Hebeler]

- Preliminary calculations of nucleon pair densities
  - leading induced operators only (two-body)
  - left: operator evolution restores initial $\langle \rho(P = 0, q) \rangle_{np}$
  - right: ratio of *np* to *nn* $\Longrightarrow$ role of tensor



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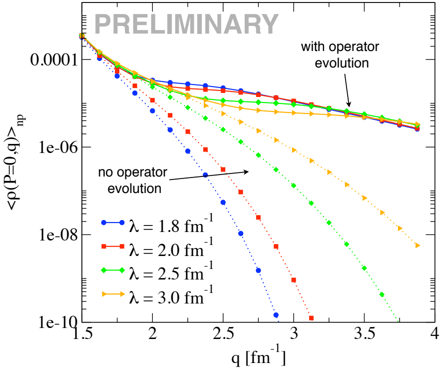
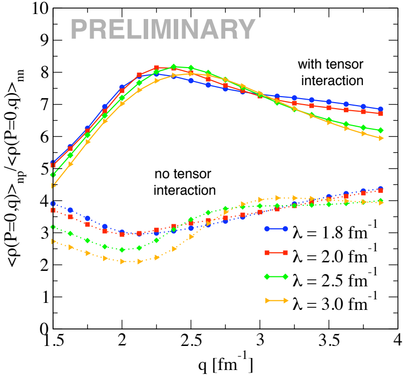




Factorization with SRG [Anderson et al., arXiv:1008.1569]

- Factorization: $U_\lambda(k,q) \to K_\lambda(k)Q_\lambda(q)$ when $k<\lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q)\int_0^\lambda p^2dp\, Z(\lambda)\Psi_\alpha^\lambda(p) + \eta^\lambda(q)\int_0^\lambda p^2dp\, p^2\, Z(\lambda)\,\Psi_\alpha^\lambda(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k)Q_\lambda(q)$

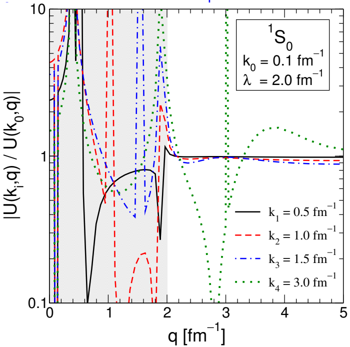
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}}\langle k|\psi_\alpha^\lambda\rangle\int^\lambda p^2dp\, Z(\lambda)\Psi_\alpha^\lambda(p)\Big]\gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \frac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



Factorization with SRG [Anderson et al., arXiv:1008.1569]

- Factorization: $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp\, Z(\lambda) \Psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp\, p^2\, Z(\lambda)\, \Psi_\alpha^\lambda(p) + \cdots$$

- Construct unitary transformation to get $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

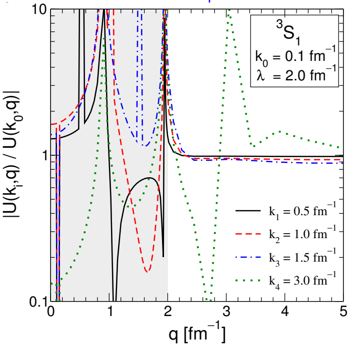
$$U_\lambda(k,q) = \sum_\alpha \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k|\psi_\alpha^\lambda\rangle \int^\lambda p^2 dp\, Z(\lambda)\Psi_\alpha^\lambda(p)\Big]\gamma^\lambda(q) + \cdots$$

- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \frac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!

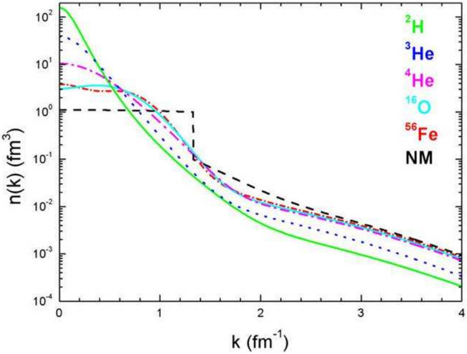


Nuclear scaling from factorization (schematic!)

- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$

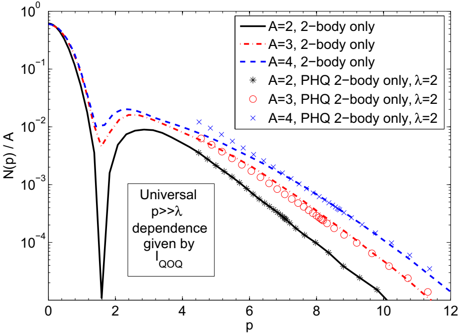
$$\frac{n_A(q)}{n_d(q)} = \frac{\langle \widetilde{A} | \widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger | \widetilde{A} \rangle}{\langle \widetilde{d} | \widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger | \widetilde{d} \rangle} = \frac{\langle \widetilde{A} | \int U_\lambda(k',q') \delta_{q'q} U^\dagger_\lambda(q,k) | \widetilde{A} \rangle}{\langle \widetilde{d} | \int U_\lambda(k',q') \delta_{q'q} U^\dagger_\lambda(q,k) | \widetilde{d} \rangle}$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

Test case: A bosons in toy 1D model



[Anderson et al., arXiv:1008.1569]

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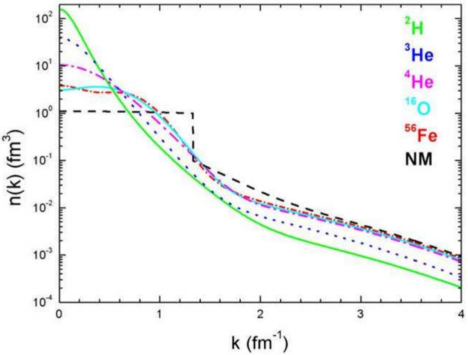

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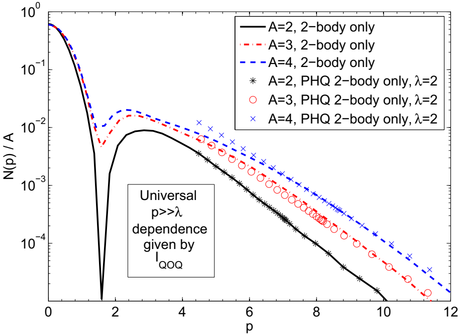

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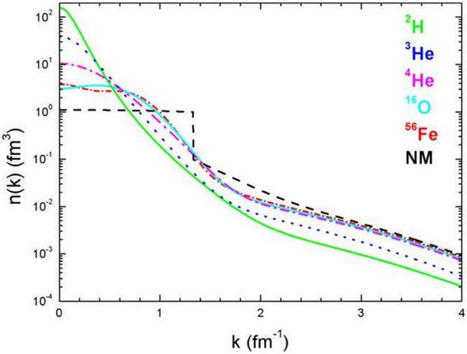
$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



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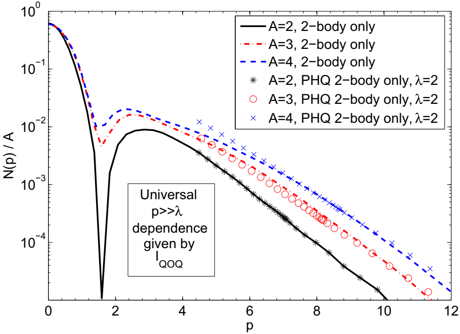


[Anderson et al., arXiv:1008.1569]

EMC effect from the EFT perspective

- Exploit scale separation between short- and long-distance physics
  - *Match* complete set of operator matrix elements (power count!)
  - Cf. needing a *model* of short-distance nucleon dynamics
  - Distinguish long-distance nuclear from nucleon physics
- EMC and effective field theory (examples)
  - "DVCS-dissociation of the deuteron and the EMC effect" [S.R. Beane and M.J. Savage, Nucl. Phys. A 761, 259 (2005)]

    "By constructing all the operators required to reproduce the matrix elements of the twist-2 operators in multi-nucleon systems, one sees that operators involving more than one nucleon are not forbidden by the symmetries of the strong interaction, and therefore must be present. While observation of the EMC effect twenty years ago may have been surprising to some, in fact, its absence would have been far more surprising."
  - "Universality of the EMC Effect" [J.-W. Chen and W. Detmold, Phys. Lett. B 625, 165 (2005)]

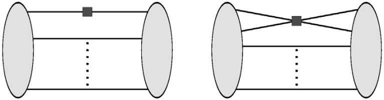
Dependence of EMC effect on A is long-distance physics!

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2 \, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

"The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics."

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators $\Longrightarrow$ parton dist. moments



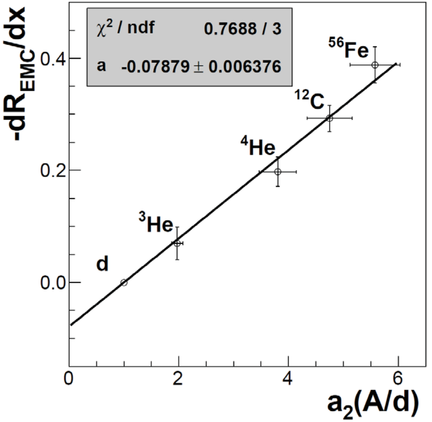
$$\Longrightarrow \langle x^n \rangle_q v^{\mu_0} \cdots v^{\mu_n} N^\dagger N [1 + \alpha_n N^\dagger N] + \cdots$$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A|(N^\dagger N)^2|A\rangle / A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited more?]

Scaling and EMC correlation via low resolution

- SRG factorization, e.g., $U_\lambda(k, q) \to K_\lambda(k) Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
  - Dependence on high-q independent of A $\Longrightarrow$ universal [cf. T. Neff]
  - A dependence from low-momentum matrix elements $\Longrightarrow$ calculate!
- EMC from EFT using OPE:
  - Isolate A dependence, which factorizes from x
  - EMC A dependence from long-distance matrix elements



L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

If same leading operators dominate, then linear A dependence of ratios?

Final comments and questions

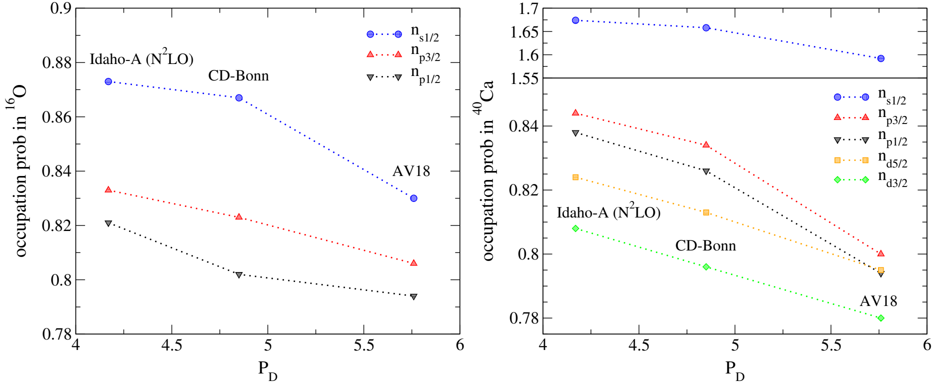
- Summary (and follow-up) points
  - Lower resolution $\Longrightarrow$ more natural nuclear structure
  - While scale and scheme-dependent observables can be (to good approximation) unambiguous for *some* systems, they are often (generally?) not for nuclei. Physics changes!
  - Scale/scheme includes *consistent* Hamiltonian and operators. How dangerous is it to treat experimental analysis in pieces?
  - Unitary transformations reveal *natural* scheme dependence
- Questions for which RG/EFT perspective + tools can help
  - Can we have controlled factorization at low energies?
  - How should one choose a scale/scheme?
  - What *is* the scheme-dependence of SF's and other quantities?
  - What are the roles of short-range/long-range correlations?
  - How do we match Hamiltonians and operators?
  - When is the assumption of one-body operators viable?
  - . . . and many more!

Some on-going calculations to address basic issues

- More general treatment of factorization [S. Bogner et al.]
- Deuteron electrodisintegration [S. More et al.]
  - No issues with three-body operators
  - Do full calculation with final state interactions (FSI)
  - Evolve with SRG, observe FSI/operator/wf contributions
- MBPT for operators: relative momentum distributions
- Quantitative scaling factors [E. Anderson, K. Hebeler]
  - Few-body directly; LDA from infinite mattter MBPT
- Many-body operators [E. Anderson, E. Jurgenson, K. Wendt]
  - Technology for evolution and embedding
  - Power counting investigations
- Variation of spectroscopic factors, single-particle quantities
  - T. Duguet, rjf, and G. Hagen

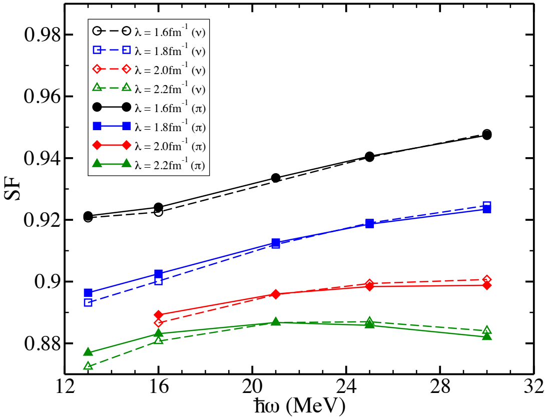
Correlation of P_D with spectroscopic factors?

Calculations from Gad and Muether, Phys. Rev. C **66**, 044361 (2002)



- Decrease in resolution (more non-local, reduced short-range tensor strength) $\Longrightarrow$ Increased occupation probability
- Are these calculations sufficiently complete/consistent?
- If so, is the correlation quantitatively predictable?

Scale dependence in coupled cluster calculations



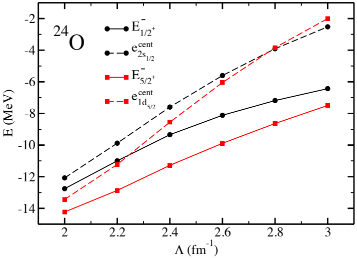
^{16}O spectroscopic factors (SFs) [From Ø. Jensen et al., PRC **82**, 014310 (2010)]

- SF increases as SRG resolution λ decreases from 2.2 to 1.6 fm^{-1}
- But significant $\hbar\omega$ dependence and no NNN
- Need to check that direct measurables are invariant

Wave functions become less correlated as Λ/λ decreases; how does the nature of reaction operators change?

See T. Duguet and G. Hagen, arXiv:1110.2468 for first steps

Resolution scale dependence [Slide courtesy of T. Duguet]



One-neutron removal in ^{24}O

- ⇝ E_ν^- and e_p^{cent} versus Λ_{RG}
- ⇝ $\Lambda_{\text{RG}} \in [2.0; 3.0]$ fm^{-1}

Non-observability of ESPEs

1. Scale dependence of E_ν^- from omitted induced forces and clusters
2. Intrinsic scale dependence of $e_p^{\text{cent}} \approx 6$ MeV for $\Lambda_{\text{RG}} \in [2.0, 3.0]$ fm^{-1}
3. Extracting *the* shell structure from $(E_k^\pm, \sigma_k^\pm)$ is an illusory objective
   - ⇝ One shell structure per (preferably low) resolution scale Λ_{RG}
   - ⇝ Using *consistent* structure and reaction models is mandatory
   - ⇝ Requires consistent many-body techniques and same $H(\Lambda_{\text{RG}})$

Thanks: collaborators and others at low resolution

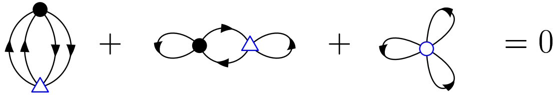
- Darmstadt: R. Roth, A. Schwenk
- ANL: L. Platter
- Iowa State: P. Maris, J. Vary
- Jülich: A. Nogga
- Michigan State: S. Bogner, A. Ekström
- LLNL: E. Jurgenson, N. Schunck
- Ohio State: E. Anderson, K. Hebeler, H. Hergert, S. More R. Perry, K. Wendt
- ORNL / UofT: G. Hagen, M. Kortelainen, W. Nazarewicz, T. Papenbrock
- TRIUMF: S. Bacca, P. Navratil
- Warsaw: S. Glazek
- many others in UNEDF and NUCLEI

Using EFT and field redefinitions as tools

- EFT: $\mathcal{L}_{\rm eft} = \psi^\dagger\big[i\frac{\partial}{\partial t} + \frac{\nabla^2}{2M}\big]\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2 - \frac{D_0}{6}(\psi^\dagger\psi)^3 + \ldots$
  - general short-range interactions, but not unique!
- Try simple field redefinition to check scheme dependence:

$$\psi \longrightarrow \psi + \alpha\frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi \qquad \alpha \sim \mathcal{O}(1) \Longrightarrow \text{“natural”} \Longrightarrow \text{estimate!}$$

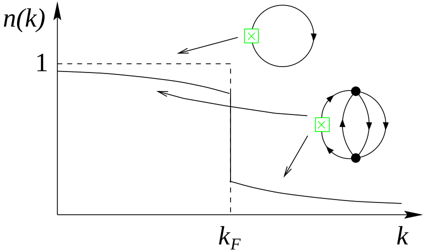
  - “new” vertices: 2–body off-shell $\triangle$, 3–body $\circ \propto \frac{8\pi\alpha}{\Lambda^3} C_0(\psi^\dagger\psi)^3$
  - asymptotic “on-shell” quantities (S-matrix elements) must be unchanged by redefinition
- Energy density is model (α) independent *if* all terms kept
  - sum of new terms is zero, so energy is unchanged



- What about momentum occupation number?

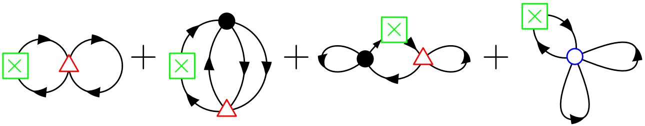
Occupation No. $\Longrightarrow$ Momentum Distribution

- Insert $a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \Longrightarrow \boxtimes$



- But nonzero contribution $\Delta n(k)$ from induced vertices:

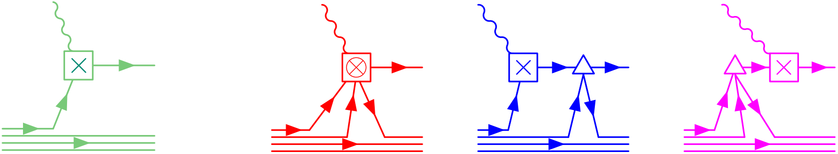
$\Delta n(k) =$



- There is no *preferred* definition for transformed operator
  - $\Longrightarrow$ only defined for specific convention
  - $\Longrightarrow$ momentum distributions for different scales differ

Analysis of (e,e'p) Experiments? [cf. (e,2e) on atoms]

- Suppose external source $J(x)$ coupled to fermions
  - EFT: need most general current coupled to $J(x)$ for all α
- Consider lowest order with simplest ($\alpha = 0$) current
  - if $\alpha = 0$, just impulse approximation $J\psi^\dagger\psi$



  - if $\alpha \neq 0$ [recall $\psi \longrightarrow \psi + \alpha\frac{4\pi}{\Lambda^3}(\psi^\dagger\psi)\psi$], then same cross section *only* if vertex contribution from modified operator *and* modified final (and initial) state interactions are included
- There are *always* contributions from all three at each order
  - sub-leading pieces are mixed by field redefinitions
    $\Longrightarrow$ isolating $J\psi^\dagger\psi$ is model dependent
  - How large is ambiguity? Set by natural size $\alpha \sim \mathcal{O}(1)$

What about long-range correlations?

- SF calculations with FRPA
- Chiral N^{3}LO Hamiltonian
  - Soft $\Longrightarrow$ small SRC
  - SRC contribution to SF changes dramatically with lower resolution
- Compare short-range correlations (SRC) to long-range correlations from particle-vibration coupling
- LRC $\gg$ SRC!!
- How scale/scheme dependent are long-range correlations?
- Additional microscopic calculations are needed!

C. Barbieri, PRL 103 (2009)

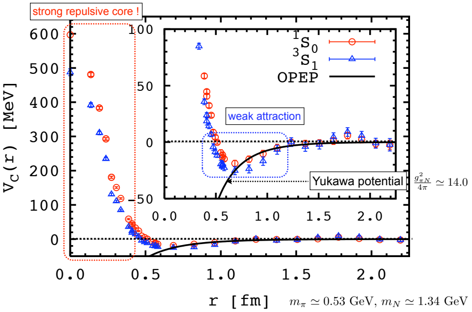
TABLE I. Spectroscopic factors (given as a fraction of the IPM) for valence orbits around ^{56}Ni. For the SC FRPA calculation in the large harmonic oscillator space, the values shown are obtained by including only SRC, SRC and LRC from particle-vibration couplings (full FRPA), and by SRC, particle-vibration couplings and extra correlations due to configuration mixing (FRPA + ΔZ_α). The last three columns give the results of SC FRPA and SM in the restricted $1p0f$ model space. The ΔZ_αs are the differences between the last two results and are taken as corrections for the SM correlations that are not already included in the FRPA formalism.

	10 osc. shells			Exp. [29]	$1p0f$ space		
	FRPA (SRC)	Full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
^{57}Cu:							
$\pi 1p_{1/2}$	0.96	0.66	0.62		0.80	0.76	-0.04
$\pi 0f_{5/2}$	0.96	0.60	0.58		0.80	0.78	-0.02
$\pi 1p_{3/2}$	0.96	0.67	0.65		0.81	0.79	-0.02
^{55}Co:							
$\pi 0f_{7/2}$	0.95	0.73	0.71		0.89	0.87	-0.02

Determining the nuclear potential from lattice QCD

[S. Aoki, *Hadron interactions in lattice QCD*, arXiv:1107.1284]

NN (effective) central potentials $m_\pi \simeq 0.53$ GeV $t - t_s = 6$



Bethe-Salpeter amplitude

$$\varphi_E(\vec{r}) = \langle 0|N(\vec{x},0)N(\vec{y},0)|2N,E\rangle$$

Nucleon fields

2N state with energy E

- define non-local $U(\mathbf{x},\mathbf{y})$

$$[E - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y})$$

- Expand $U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta(\mathbf{x}-\mathbf{y})$ to get AV18 form of local V

- Why not just calculate energy as function of separation $\Longrightarrow V(r)$?
  - Only works in heavy mass limit (e.g., works for B-mesons)
- But is this unique? No!
  - choice of nucleon interpolating field $\Longrightarrow$ different $V(\mathbf{x})$
  - choice of “wave function” smearing (changes overlap)

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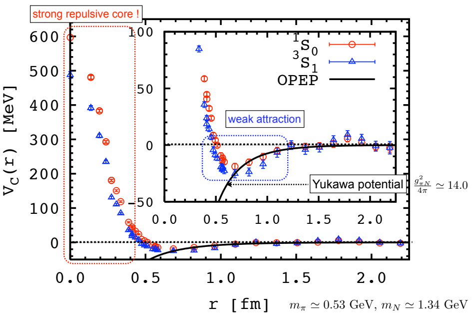
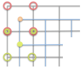
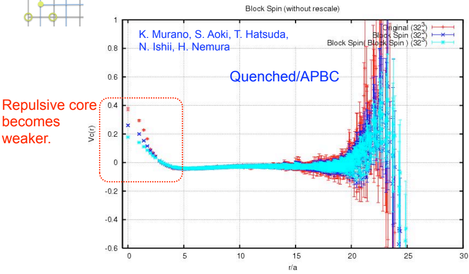


Smearing and potentials Preliminary



Wave function is smeared. → "smeared" potential



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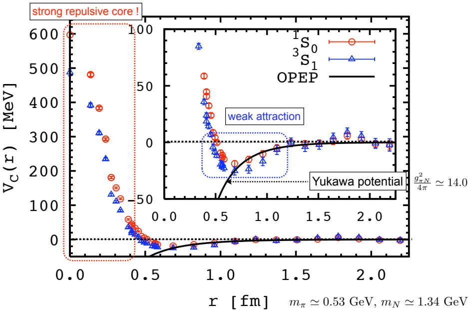
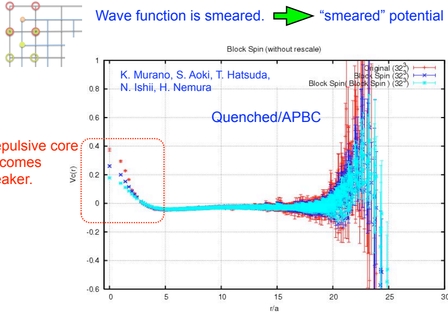


Smearing and potentials

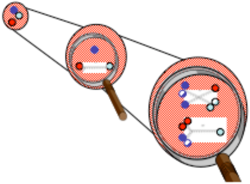
Preliminary

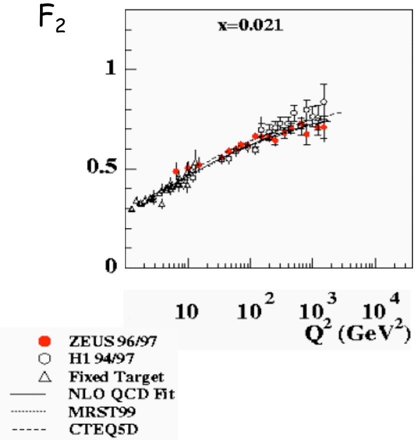
Wave function is smeared. ⟹ "smeared" potential



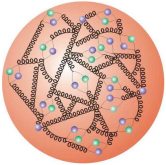
- "...the potential depends on the choice of nucleon operator..." which "...is considered to be a 'scheme' to define the potential."
- "Is such a scheme-dependent quantity useful? The answer to this question is probably 'yes', since the potential is useful to understand or describe the phenomena." $\Longrightarrow$ choose close to local

Parton distributions as paradigm [C. Keppel]



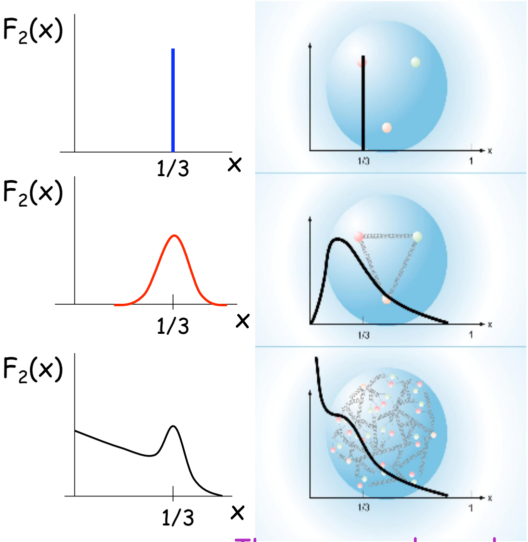


Higher the resolution (i.e. higher the Q^2) more low x partons we "see".



So what do we expect F_2 as a function of x at a fixed Q^2 to look like?

Parton distributions as paradigm [C. Keppel]



Three quarks with 1/3 of total proton momentum each.

Three quarks with some momentum smearing.

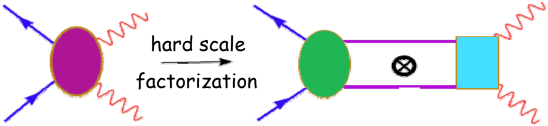
The three quarks radiate partons at low x.

....The answer depends on the Q^2!

Parton distributions as paradigm [Marco Stratman]

Factorization schemes

pictorial representation of factorization:



e.g. $F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \hat{F}_2^a(x, \frac{Q}{\mu_f})$

the separation between long- and short-distance physics is not unique

long-distance parton density

short-distance Wilson coefficient

1. **choice of** μ_f: defines borderline between long-/short-distance
2. **choice of scheme**: re-shuffling finite pieces

Parton distributions as paradigm [Marco Stratman]

Deep-inelastic scattering (DIS) according to pQCD

the physical structure fct. is independent of μ_f
(this will lead to the concept of renormalization group eqs.)

both, pdf's and the short-dist. coefficient depend on μ_f
(choice of μ_f: shifting terms between long- and short-distance parts)

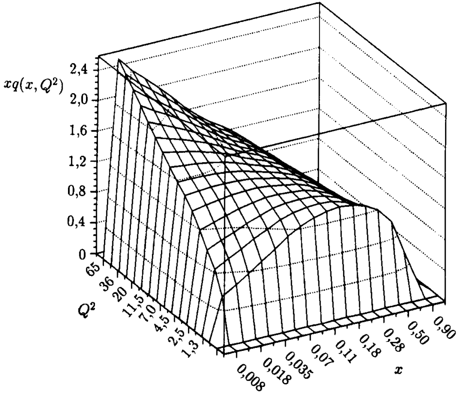
$$F_2(x,Q^2) = x \sum_{a=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_a(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s(\mu_r)}{2\pi}\left[P_{qq}\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})\left(\frac{x}{\xi}\right)\right]\right]$$

yet another scale: μ_r
due to the renormalization
of ultraviolet divergencies

short-distance "Wilson coefficient"

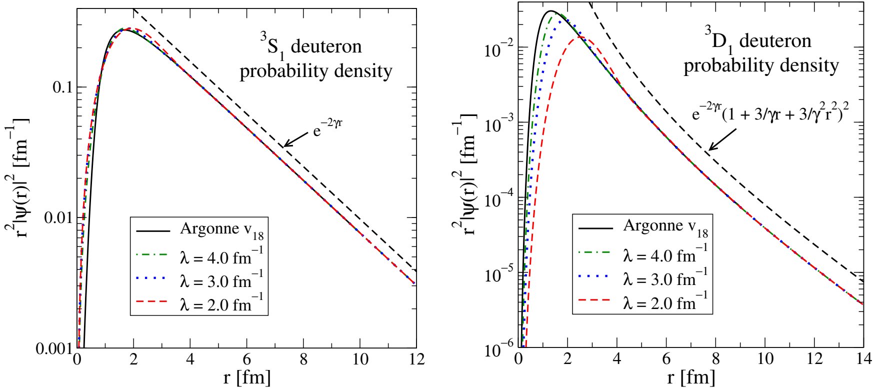
choice of the factorization scheme

Parton distributions as paradigm: Evolution



- The quark distribution $q(x, Q^2)$ is both scheme and scale dependent
- $x\, q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ can be related by well-controlled evolution equations

Why are ANC's different? Coordinate space

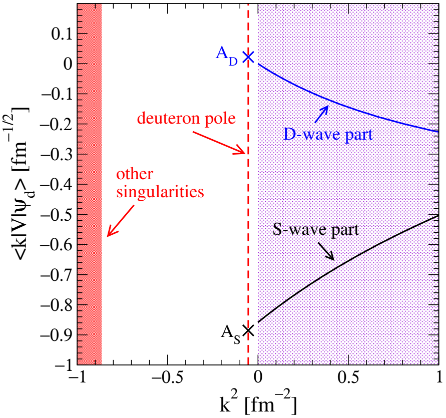


- ANC's, like phase shifts, are asymptotic properties $\Longrightarrow$ short-range unitary transformations do not alter them [e.g., see Mukhamedzhanov/Kadyrov, PRC **82** (2010)]
- In contrast, SF's rely on *interior* wave function overlap
- (Note difference in S-wave and D-wave ambiguities)

Why are ANC's different? Momentum space

[based on R.D. Amado, PRC **19** (1979)]

1. $$\frac{k^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle + \langle \mathbf{k}|V|\psi_n\rangle = -\frac{\gamma_n^2}{2\mu}\langle \mathbf{k}|\psi_n\rangle$$
   $$\Longrightarrow \langle \mathbf{k}|\psi_n\rangle = -\frac{2\mu\langle \mathbf{k}|V|\psi_n\rangle}{k^2+\gamma_n^2}$$
2. $$\langle \mathbf{r}|\psi_n\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}}\langle \mathbf{k}|\psi_n\rangle$$
   $$\xrightarrow{|\mathbf{r}|\to\infty} A_n e^{-\gamma_n r}/r$$
3. integral dominated by pole from 1.
4. extrapolate $\langle \mathbf{k}|V|\psi_n\rangle$ to $k^2 = -\gamma_n^2$



- Or, residue from extrapolating on-shell T-matrix to deuteron pole $\Longrightarrow$ invariant under unitary transformations
- Next vertex singularity at $-(\gamma + m_\pi)^2 \Longrightarrow$ same for FSI

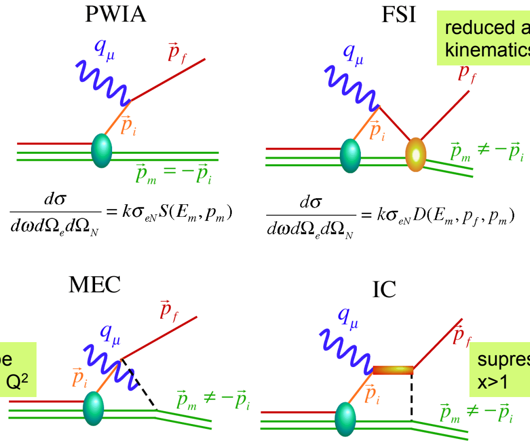
Questions about short-range correlations (SRCs)

- How should we interpret the universal features of SRCs for different nuclei?
- Can SRCs inform us about high density matter (e.g., the EOS or physics of neutron stars)?
- Are SRCs important for understanding low-energy nuclear structure?
- How can we understand the observed correlation between the A-dependence of the EMC slope and scaling factors from $x > 1$?
- How does one explain cross sections from (e, e'), $(e, e'p)$ and $(e, e'pN)$ experiments with soft interactions that have minimal SRCs?
- How should one interpret the high-momentum tails of momentum distributions in nuclei, which vary significantly with different Hamiltonians?
- How should one choose the factorization scale for these experiments?

Can we treat corrections independently? [Boeglin ECT*]

D(e,e' p) Reaction Mechanisms

reduced at certain kinematics ?



expected to be small at large Q^2

supressed for x>1

11/14/11 SRC IN NUCLEI AND HARD QCD PHENOMENA , Trento 2011 6

Answer: Mixtures are scale/scheme dependent (cf. 3NF)

Questions and some possible answers

How should one choose a scale/scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - (Near-) local potential: quantum Monte Carlo methods work
  - Low-k potential: many-body perturbation theory works, or to make microscopic connection to shell model
- Better interpretation or intuition $\Longrightarrow$ predictability
- Use range of scales to test calculations and physics
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguity

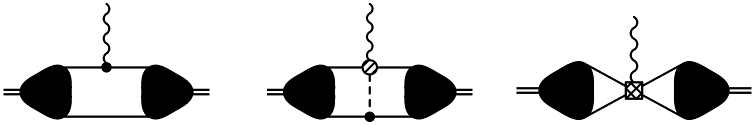
Can we (should we) use a reference Hamiltonian?

- That is, to allow us to make comparisons
- If so, which one? (Cleanest extraction from experiment?)
  - Can one "optimize" validity of impulse approximation?

More questions and some possible answers

How do we consistently match Hamiltonians and operators?

- Use EFT perspective
  - E.g., electromagnetic currents [D.R. Phillips, nucl-th/0503044]



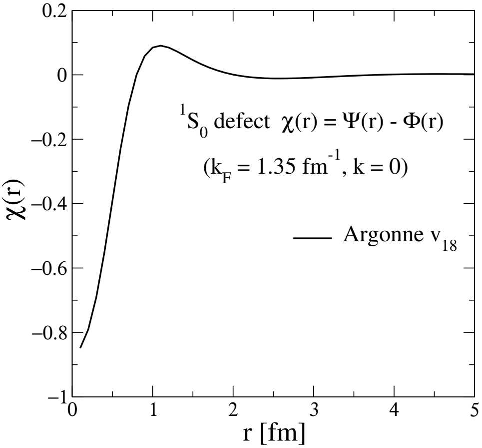
$O(e)$ $O(eP^3)$ $O(eP^5)$

  - Model independent because complete (up to some order)
  - Can identify consistent operator and interaction
  - Tells you when new info is required
- Use RG as tool to evolve consistent operators

Can EFT or RG help to construct optical potentials?

Two-Body Correlations at Nuclear Matter Density

- Defect wf $\chi(r)$ for particular kinematics ($k = 0$, $P_{\rm cm} = 0$)
- AV18: "Wound integral" provides expansion parameter



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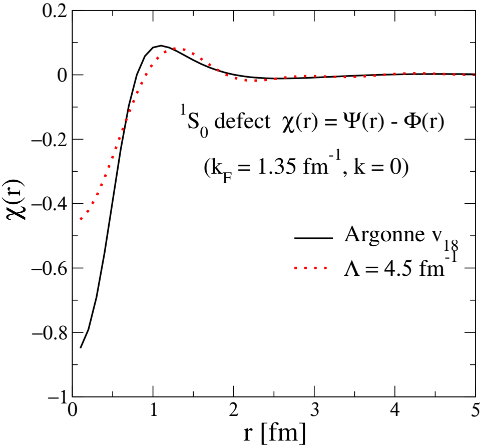

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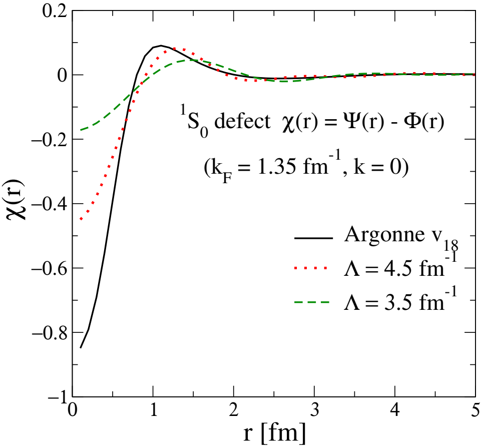

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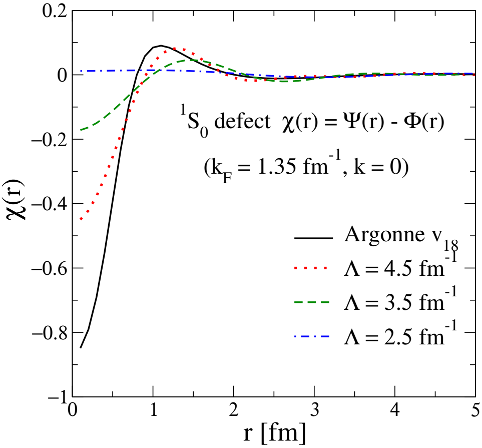

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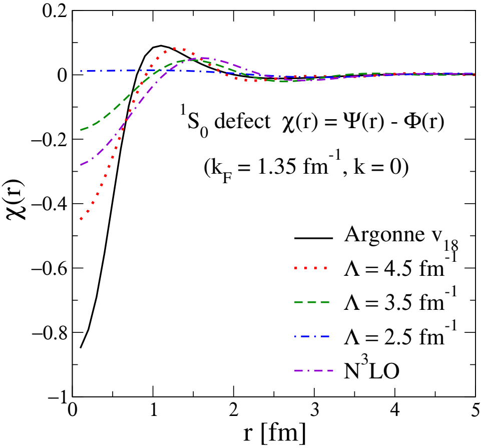
- Still a sizable wound for N^3LO



What parts of wf's can be extracted from experiment?

- Measurable: asymptotic (IR) properties like phase shifts, ANC's
- Not observables, but well-defined theoretically given a Hamiltonian: interior quantities like spectroscopic factors
  - These depend on the scale and the scheme
  - Extraction from experiment requires robust factorization of structure and reaction; only the combination is scale/scheme independent (e.g., cross sections) [What if weakly dependent?]

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  - Compare to nuclear case

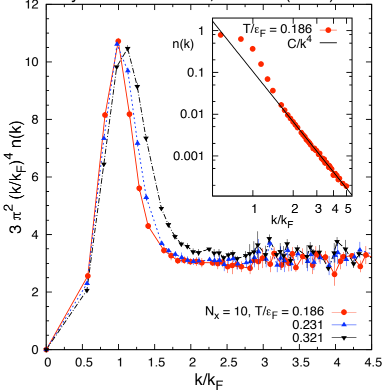
Unitary cold atoms: Is $n(k)$ observable?

- Tail of momentum distribution + contact [Tan; Braaten/Platter]

$$n(k) \xrightarrow{k\to\infty} \frac{C}{k^4}$$

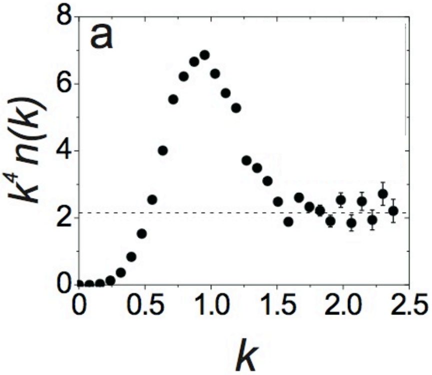
Theory (lattice)

J. E. Drut, T. A. Lähde, T. Ten
Phys. Rev. Lett. **106**, 205302 (2011)

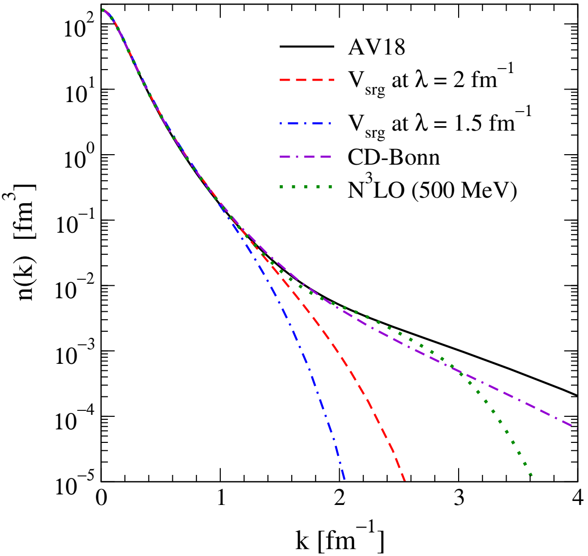


Experiment

J. T. Stewart et al
PRL **104**, 235301 (2010)



Is the tail of $n(k)$ for nuclei measurable? (cf. SRC's)



- E.g., extract from electron scattering?
- Scale- and scheme-dependent high-momentum tail!
- $n(k)$ from V_{SRG} has *no* high-momentum components!
- No region where $1/a_s \ll k \ll 1/R$ (cf. large k limit for unitary gas)

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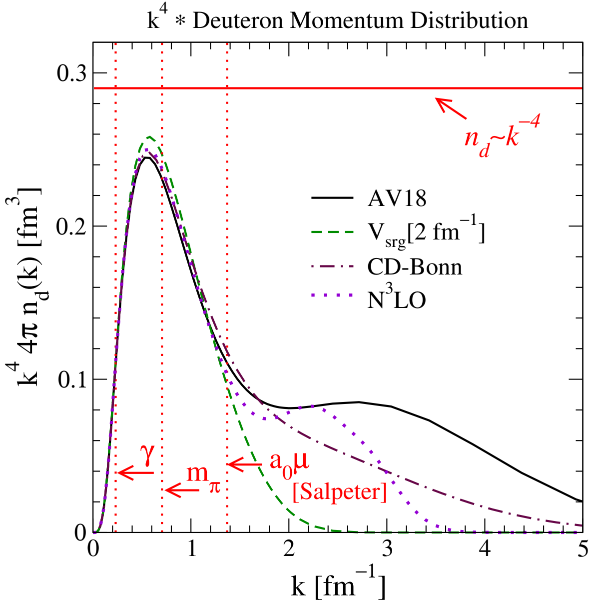

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- Short-range correlations (SRCs) depend on the Hamiltonian *and* the resolution scale (cf. parton distribution functions)
- So might expect Hamiltonian- and resolution-dependent but *A*-independent high-momentum tails of wave functions [T. Neff]
  - Universal extrapolation for different *A*, but $\lambda_{\rm SRG}$ dependent

High-momentum tails from low-momentum ET's

S.K. Bogner & D. Roscher [arXiv:1208.1734]

Generalization of **factorization** to arbitrary A-body systems at low-momentum:

$$n(q) \approx Z_\Lambda^2\, \gamma^2(\mathbf{q};\Lambda) \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^\Lambda_{\alpha,A} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}\, a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}\, a_{\frac{\mathbf{K}}{2}+k'}\, a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} | \psi^\Lambda_{\alpha,A} \rangle$$

– Can be shown for other operators

Example: Unitary Fermi gas

- Reproduction of contact Tan relation à la Braaten & Platter [2008]:

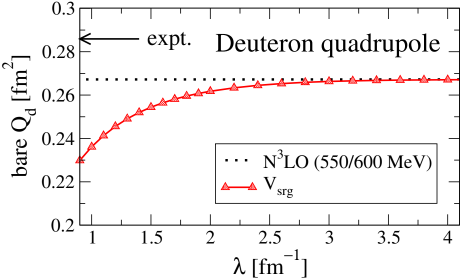
$$n(q) \approx \frac{Z_\Lambda^2\, g^2(\Lambda)}{q^4} \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}} \langle \psi^\Lambda_{\alpha,A} | a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}}\, a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}}\, a_{\frac{\mathbf{K}}{2}+k'}\, a_{\frac{\mathbf{K}}{2}-\mathbf{k}'} | \psi^\Lambda_{\alpha,A} \rangle = \frac{C(\Lambda_0)}{q^4}$$

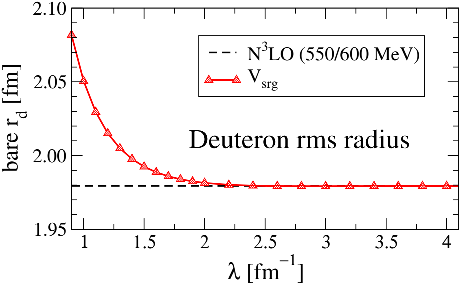
- Static structure factor

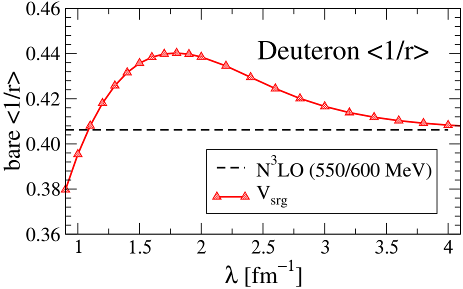
$$S_{\uparrow\downarrow}(q) \approx -\left(\frac{2}{q^2\, g(\Lambda)} + \frac{1}{8q} + \frac{\Lambda}{\pi^2 q^2}\right) Z_\Lambda^2\, C(\Lambda) \longrightarrow \left(\frac{1}{8q} - \frac{1}{2\pi a\, q^2}\right) C(\Lambda_0)$$

Long-distance physics largely unchanged

- Matrix elements dominated by long range run slowly for $\lambda \geqslant 2\,\mathrm{fm}^{-1}$
- Here: examples from deuteron (compressed scales)





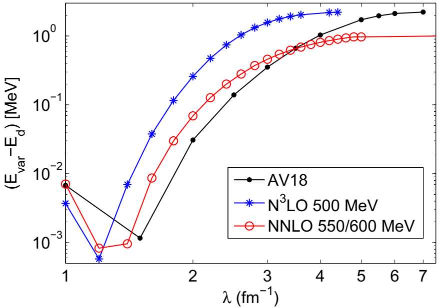


Variational calculations in the deuteron

- Test whether operators are fine-tuned
- Try a simple variational ansatz (from *k*-space S-eqn)

$$u(k) = \frac{1}{(k^2+\gamma^2)(k^2+\mu^2)} e^{-\left(\frac{k^2}{\lambda^2}\right)^2} \qquad w(k) = \frac{ak^2}{(k^2+\gamma^2)(k^2+\nu^2)^2} e^{-\left(\frac{k^2}{\lambda^2}\right)^2}$$

- error in energy for different starting potentials
- small λ works great!
- no fine-tuning



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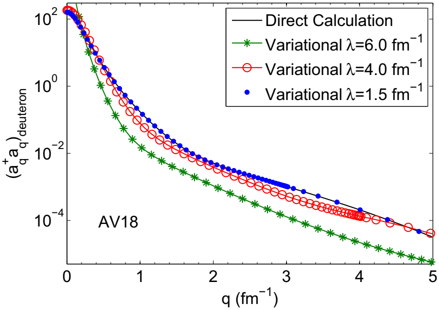
- momentum distribution for AV18 at several λ's
- small λ works great!
- no fine-tuning



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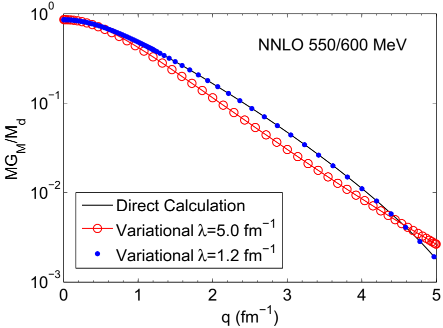
- form factor G_M for NNLO at two λ's vs. direct
- small λ works great!
- no fine-tuning



Decoupling and phase shifts

- Unevolved AV18 phase shifts (black solid line)
- Cutoff AV18 potential at $k = 2.2\,\text{fm}^{-1}$ (dotted blue) $\Longrightarrow$ fails for all but F wave
- Uncut evolved potential agrees perfectly for all energies
- Cutoff evolved potential agrees up to cutoff energy
- F-wave is already soft (π's) $\Longrightarrow$ already decoupled