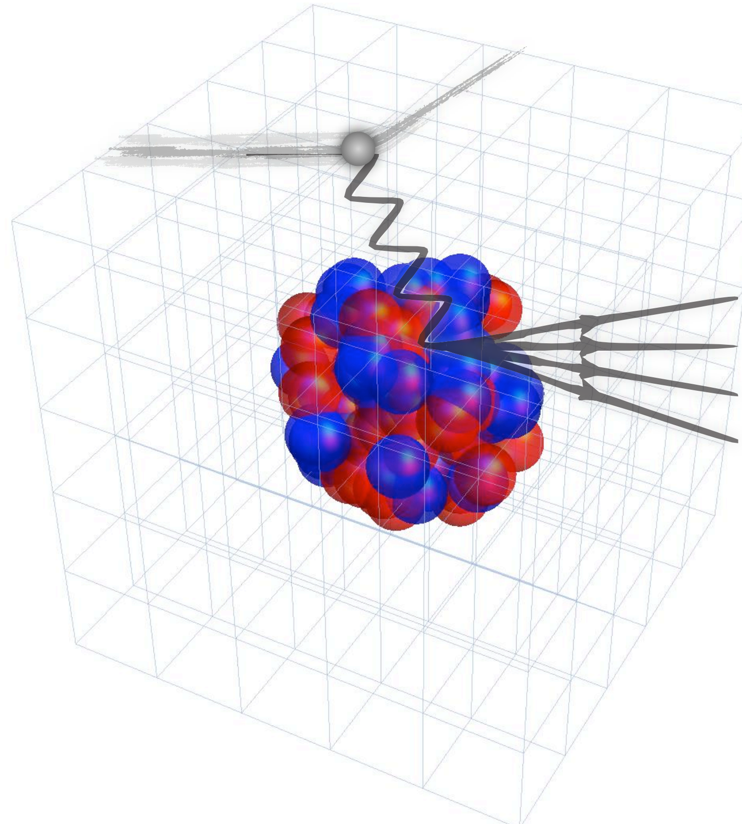
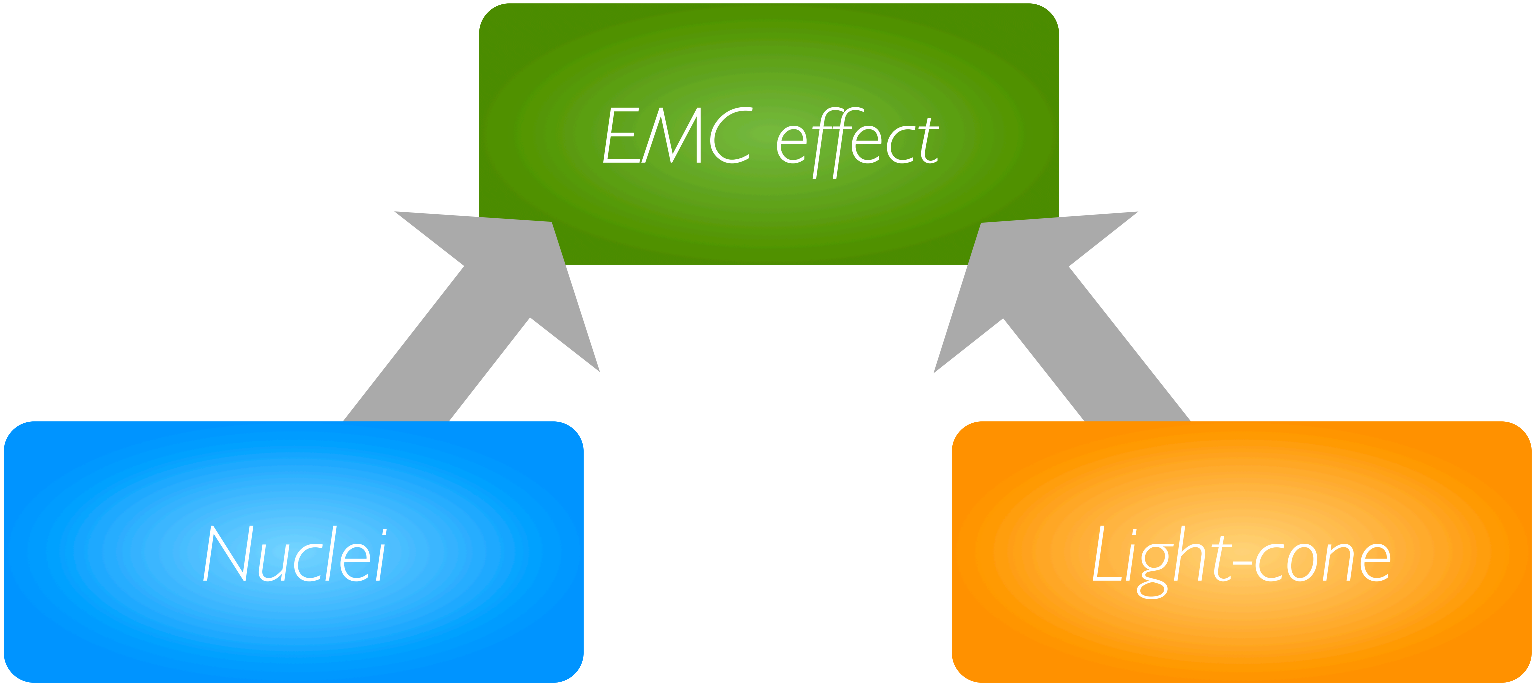


# *QCD, Nuclei, and the EMC effect*

William Detmold

Massachusetts Institute of Technology

INT workshop on *Nuclear Structure and Dynamics at Short Distances*, Feb 12th 2013

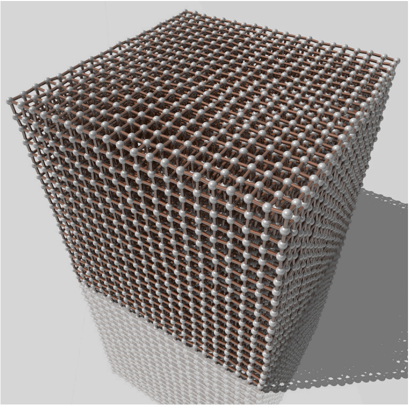


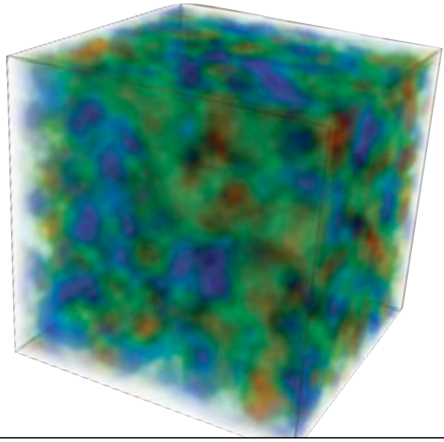

- Nuclei and light-cone physics are *both* challenging in lattice QCD
- Demonstrating such an iconic effect from QCD would be a major achievement
- Can it be done?

# *Nuclei*

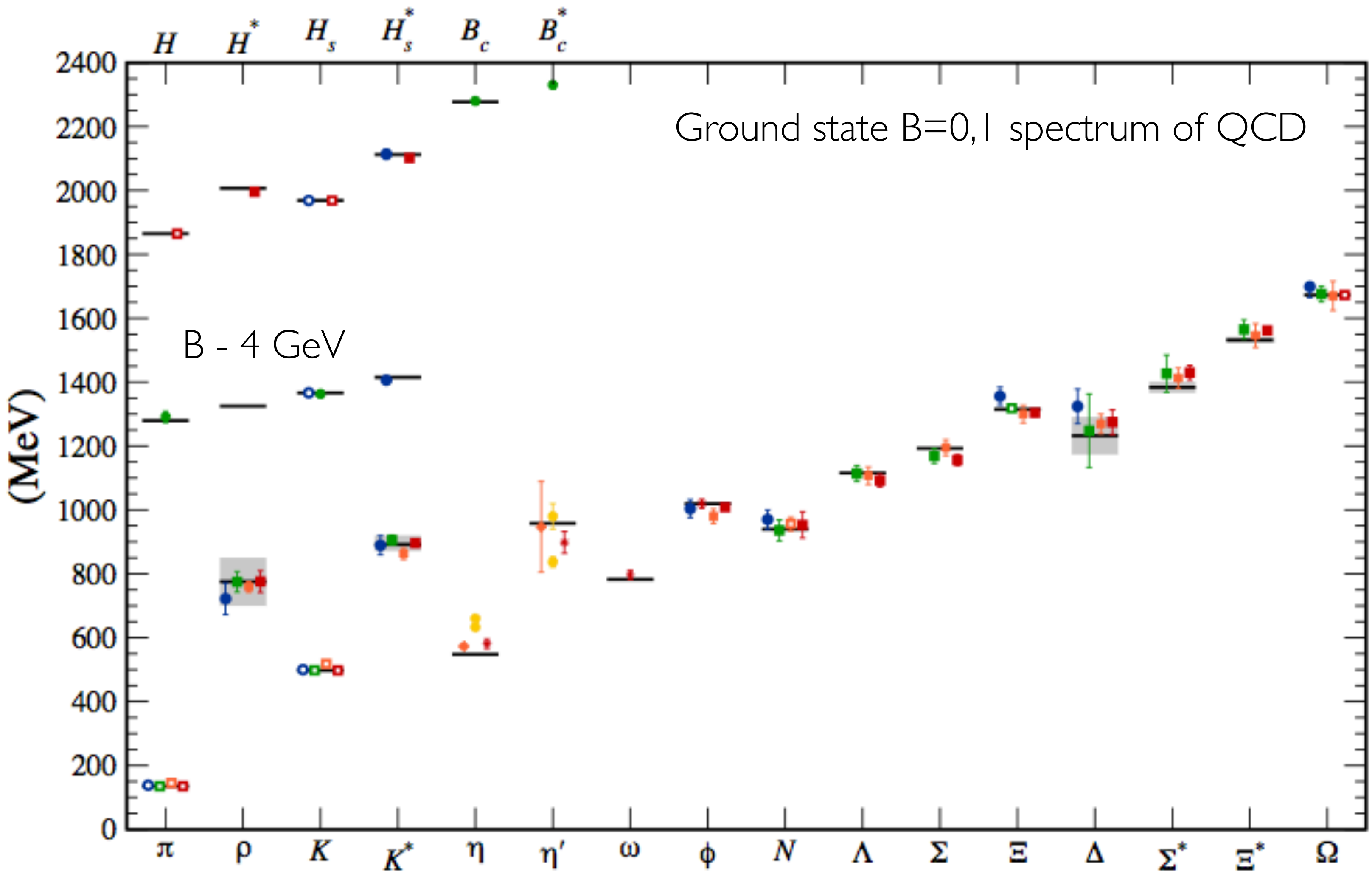
# *Quantum chromodynamics*

- Lattice QCD: quarks and gluons
  - Formulate problem as functional integral over gluonic degrees of freedom on $R_4$
  - Discretise and compactify system
  - Integrate via importance sampling (average over important gluon cfgs)
  - Undo the harm done in previous steps
- Major computational challenge ...





# QCD: meson/baryon spectrum




[A Kronfeld, 1209.3468]

points correspond to different sets of calculations

# QCD matrix elements

**Nucleon Structure from Lattice QCD Using a Nearly Physical Pion Mass**

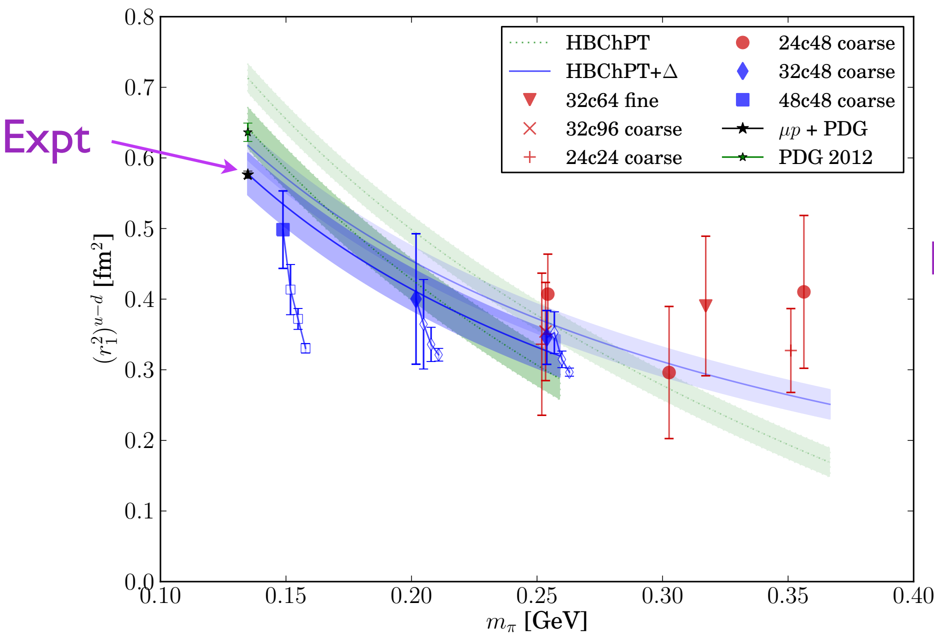
J. R. Green,[1] M. Engelhardt,[2] S. Krieg,[3] J. W. Negele,[1] A. V. Pochinsky,[1] and S. N. Syritsyn[4,*]

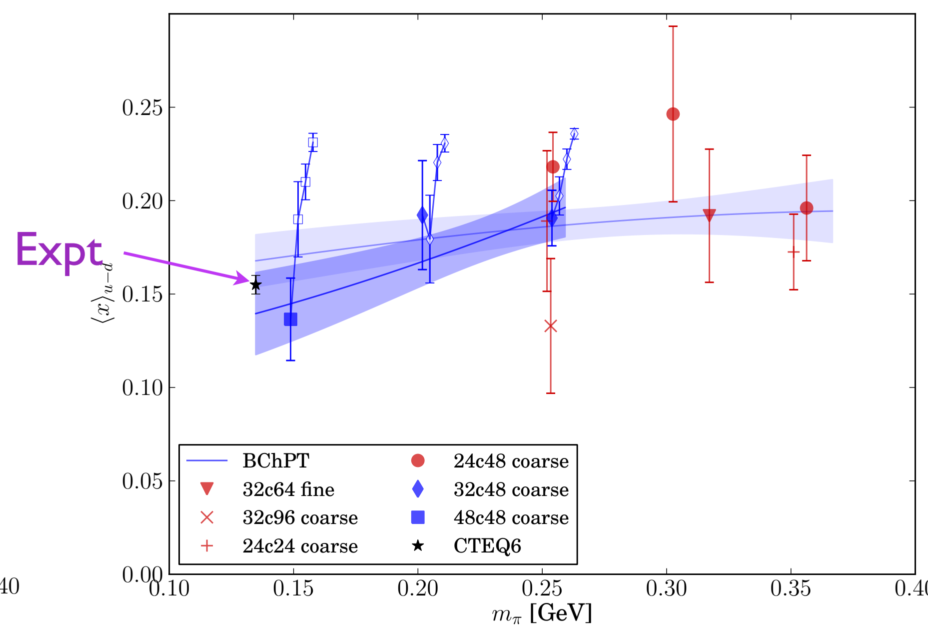
[1] *Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2] *Physics Department, New Mexico State University, Las Cruces, New Mexico 88003, USA*
[3] *Bergische Universität Wuppertal, D-42119 Wuppertal, Germany and IAS, Jülich Supercomputing Centre, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[4] *Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

(Dated: September 11, 2012)

[1209.1687]

We report the first lattice QCD calculation using the almost physical pion mass $m_\pi = 149$ MeV that agrees with experiment for four fundamental isovector observables characterizing the gross structure of the nucleon: the Dirac and Pauli radii, the magnetic moment, and the quark momentum fraction. The key to this success is excluding the contributions of excited states. An analogous calculation of the nucleon axial charge governing beta decay fails to agree with experiment, and we discuss possible sources of error.



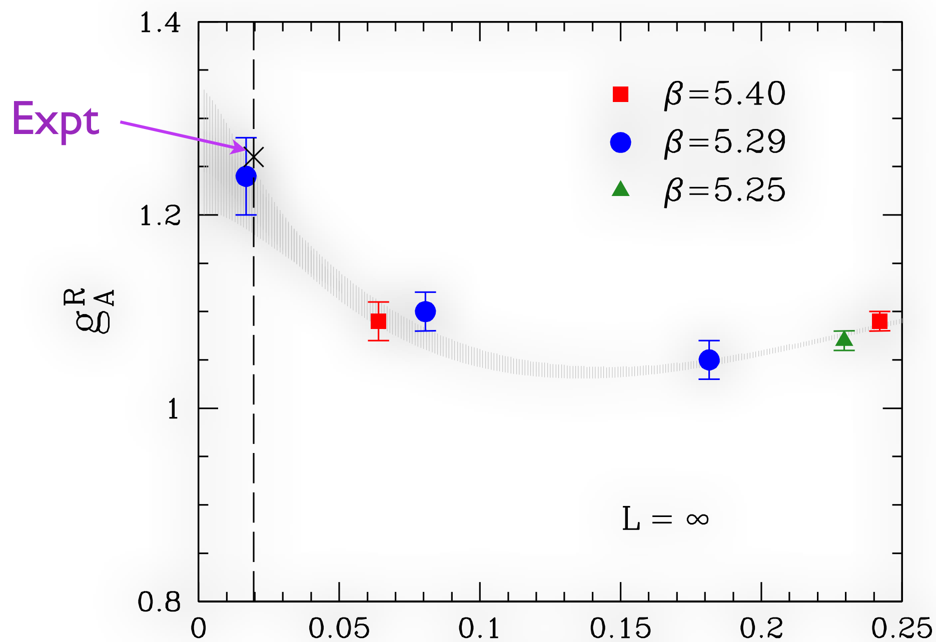
# QCD matrix elements

[1302.2233]

**Nucleon axial charge and pion decay constant from two-flavor lattice QCD**

R. Horsley[1], Y. Nakamura[2], A. Nobile[3], P.E.L. Rakow[4], G. Schierholz[5] and J.M. Zanotti[6]

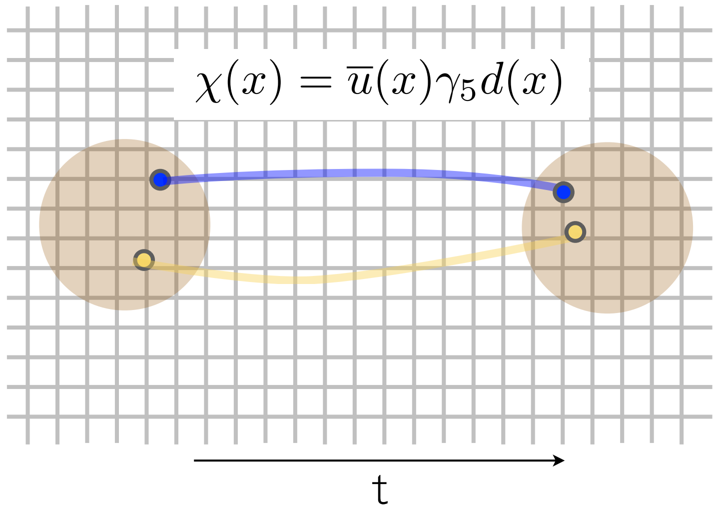
The axial charge of the nucleon $g_A$ and the pion decay constant $f_\pi$ are computed in two-flavor lattice QCD. The simulations are carried out on lattices of various volumes and lattice spacings. Results are reported for pion masses as low as $m_\pi = 130\,\text{MeV}$. The volume dependence of $g_A$ and $f_\pi$ can be understood quantitatively in terms of lattice ChPT. At the physical pion mass we find $g_A = 1.24(4)$ and $f_\pi = 89 \pm 1.1 \pm 1.8\,\text{MeV}$, using $r_0 = 0.50(1)\,\text{fm}$ to set the scale, in good agreement with experiment. As a by-product we obtain the low-energy constant $\bar{l}_4 = 4.2(1)$.



# QCD Spectroscopy

- Measure correlator (χ = object with q# of hadron)

$$C_2(t) = \sum_{\mathbf{x}} \langle 0|\chi(\mathbf{x},t)\overline{\chi}(\mathbf{0},0)|0\rangle$$




- Unitarity: $\sum_n |n\rangle\langle n| = 1$

$$= \sum_{\mathbf{x}} \sum_n \langle 0|\chi(\mathbf{x},t)|n\rangle\langle n|\overline{\chi}(\mathbf{0},0)|0\rangle$$

- Hamiltonian evolution

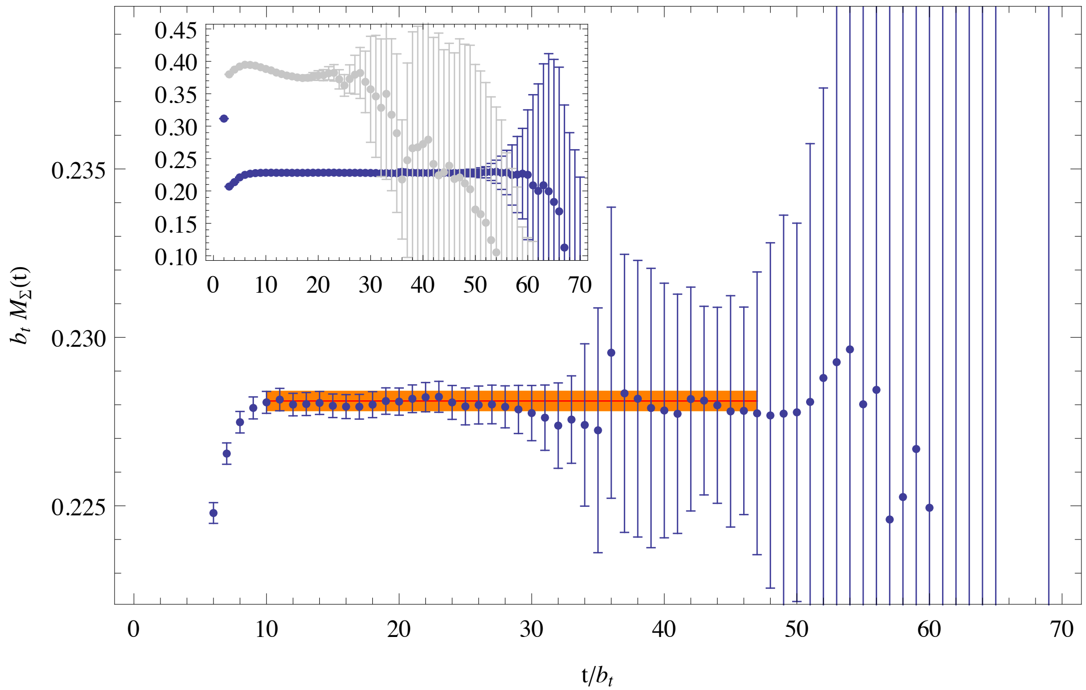
$$= \sum_{\mathbf{x}} \sum_n e^{-E_n t} e^{i\mathbf{p}_n\cdot\mathbf{x}} \langle 0|\chi(\mathbf{0},0)|n\rangle\langle n|\overline{\chi}(\mathbf{0},0)|0\rangle$$

- Long times only ground state survives

$$\xrightarrow{t\to\infty} e^{-E_0(\mathbf{0})t}|\langle \mathbf{0};0|\overline{\chi}(\mathbf{x_0},t)|0\rangle|^2 = Z\, e^{-E_0(\mathbf{0})t}$$

# Effective mass

- Construct $M(t) = \ln\left[C_2(t)/C_2(t+1)\right] \xrightarrow{t\to\infty} M$
  - Plateau corresponds to energy of ground state



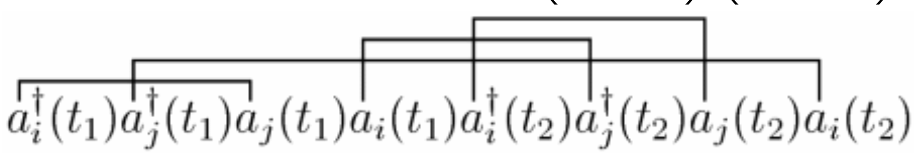

- Fancier techniques able to resolve multiple eigenstates

# *Nuclei: an (exponentially hard)$^2$ problem*

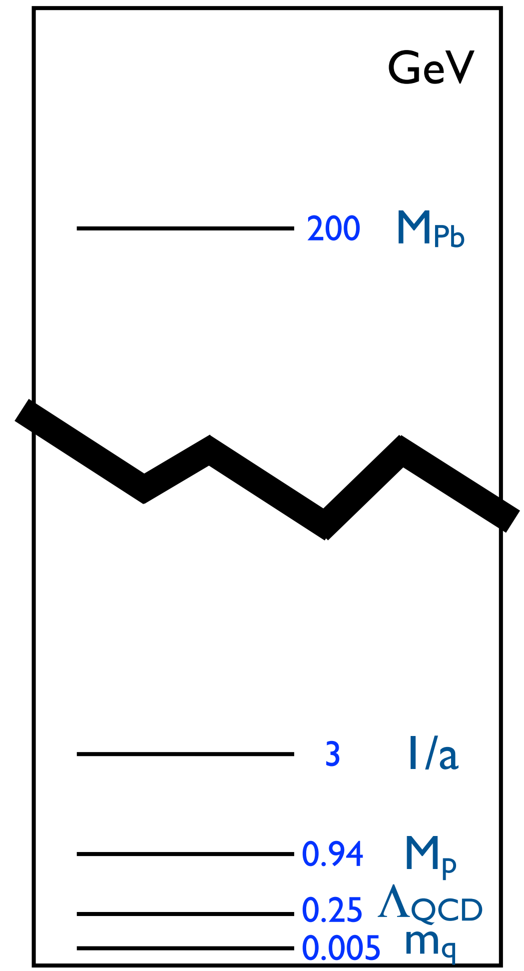
- Nuclear spectroscopy?

$$\langle 0|T q_1(t)\ldots q_{624}(t)\overline{q}_1(0)\ldots\overline{q}_{624}(0)|0\rangle$$
$$\xrightarrow{t\to\infty} \#\exp(-M_{Pb}t)$$

- Complexity: number of Wick contractions = (A+Z)!(2A-Z)!




- Dynamical range of scales (numerical precision)
- Small energy splittings
- Importance sampling: statistical noise exponentially increases with A



# *The trouble with baryons*

- Importance sampling of QCD functional integrals
  ➤ correlators determined stochastically

- Proton

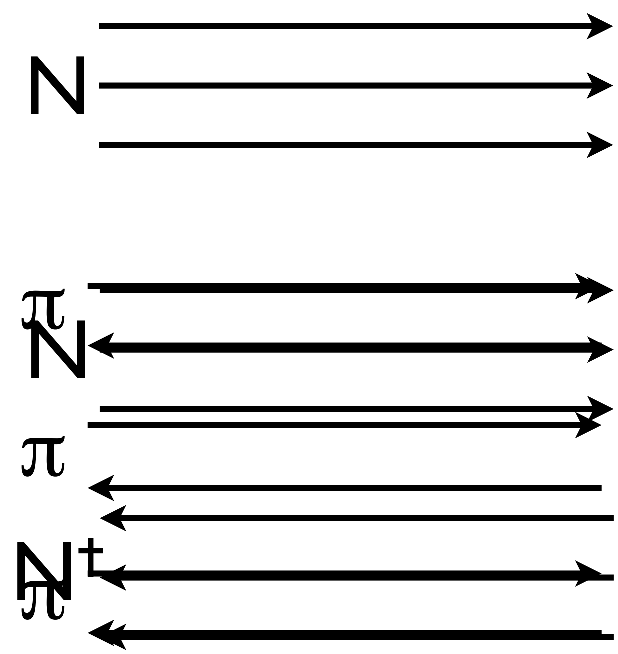
  signal $\sim \langle C \rangle \sim \exp[-M_N t]$

  - Variance determined by

$$\sigma^2(C) = \langle C C^\dagger \rangle - |\langle C \rangle|^2$$

$$\text{noise} \sim \sqrt{\langle C C^\dagger \rangle} \sim \exp[-3/2 M_\pi t]$$

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-(M_N - 3/2 m_\pi) t\right]$$



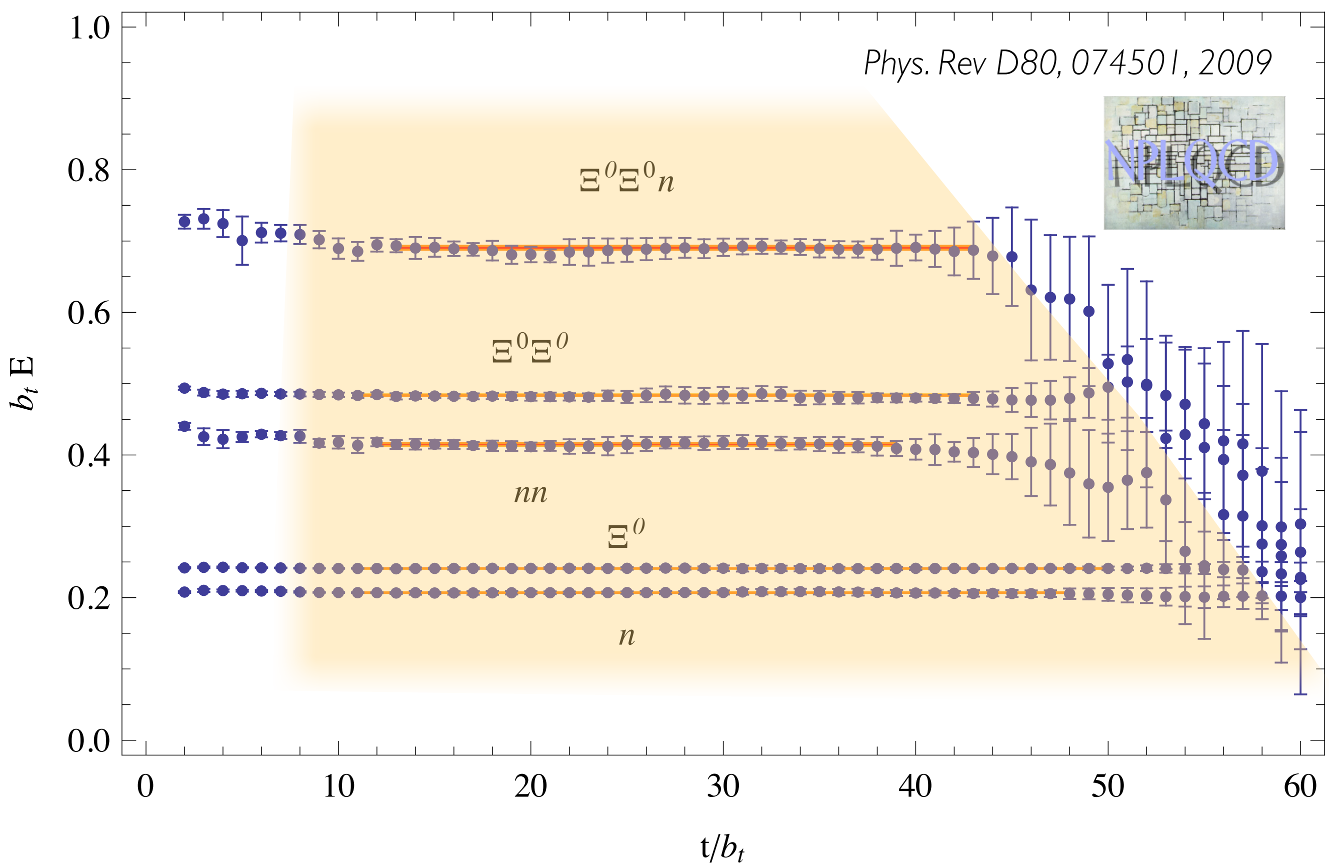

- For nucleus A:

$$\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi) t\right]$$

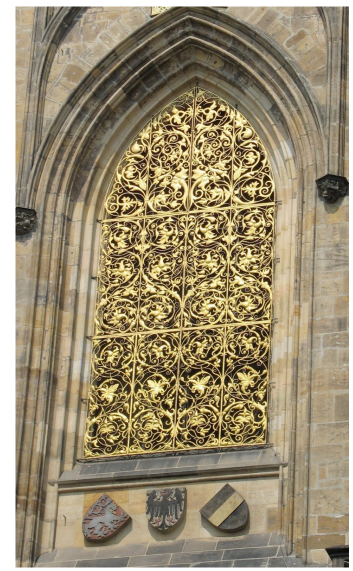
[Lepage '89]

# No? trouble with baryons

High statistics study using anisotropic lattices (fine temporal resolution)




@ $m_\pi$ = 390 MeV

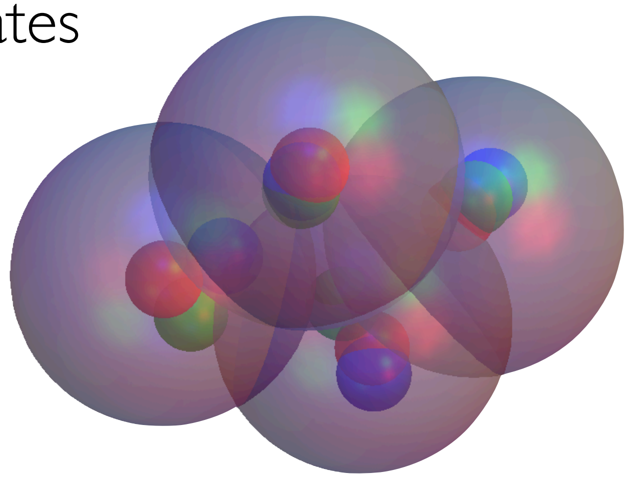


Golden window of time-slices where signal/noise const

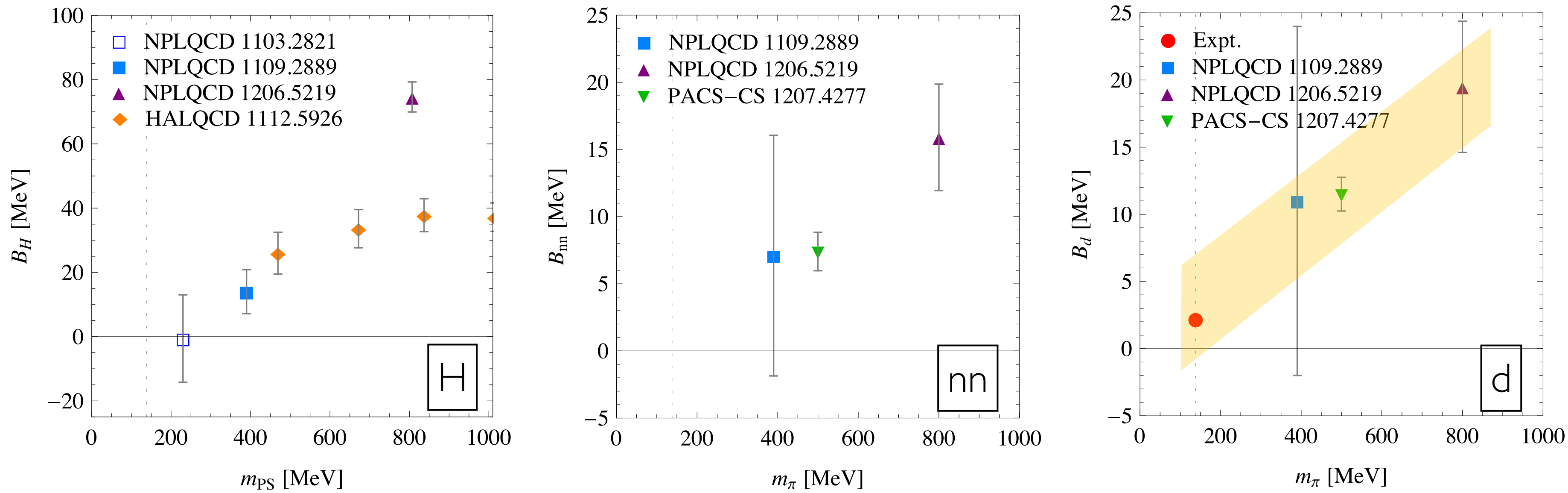
Interpolator choice can be used to suppress noise

# *Multi-baryon systems*

- Scattering/bound systems
    - Focus on (strong interaction) bound states
- Dibaryons : H, deuteron, ΞΞ
- $^3$H, $^4$He and hypernuclei: $^4$He$_\Lambda$, $^4$He$_{\Lambda\Lambda}$ ,...
- Correlators for significantly larger A
- Caveat: at unphysical quark masses and no electroweak interactions



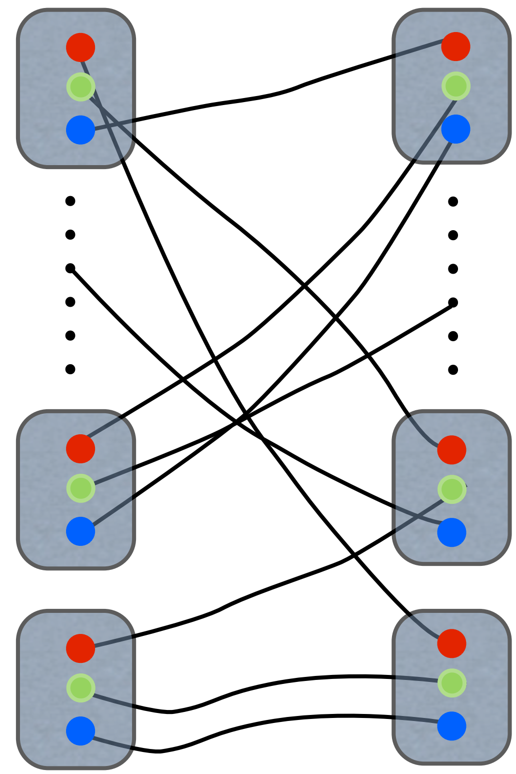
# Dibaryons



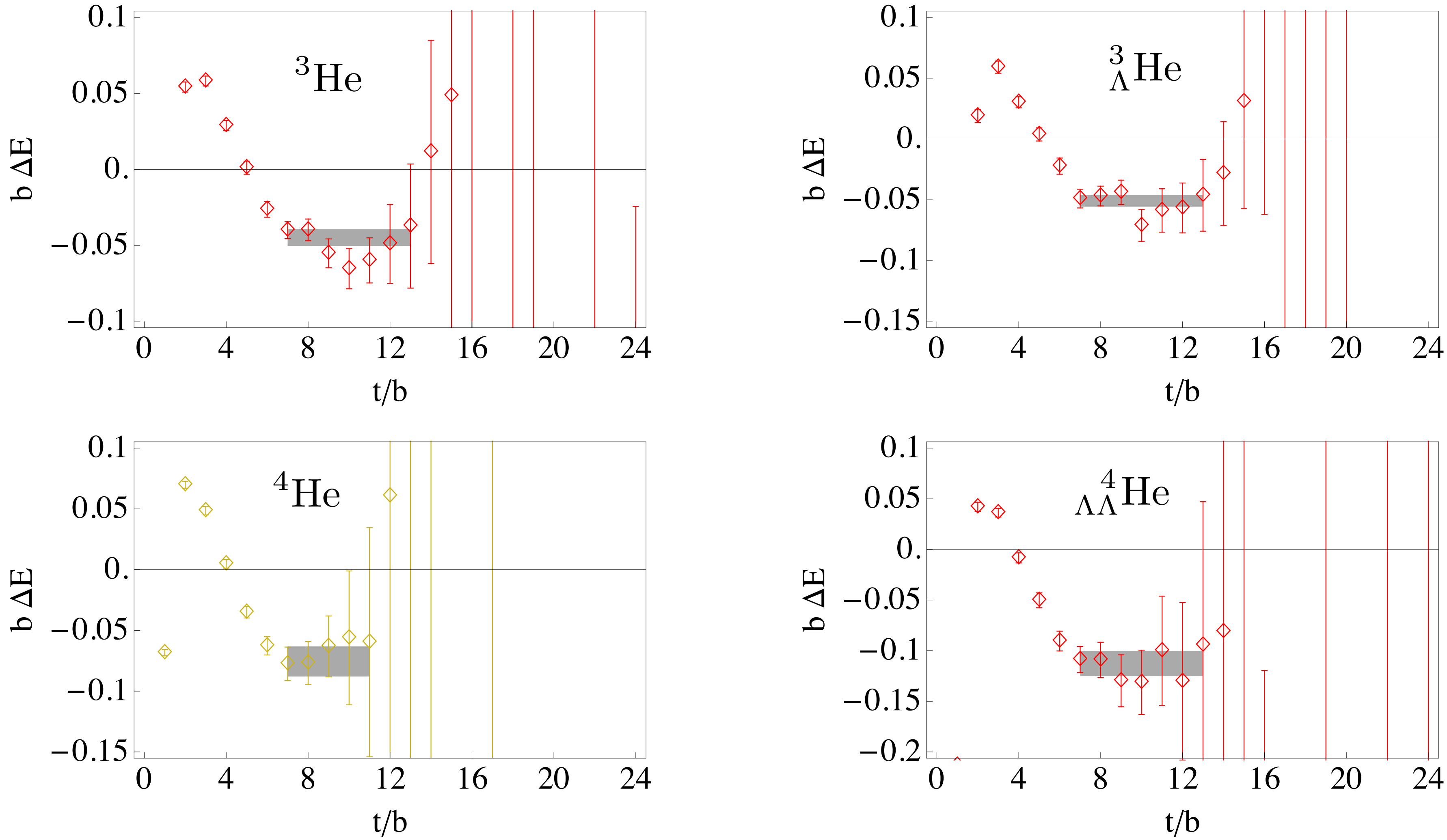

- H dibaryon, di-neutron and deuteron
- More exotic channels also considered ($\Xi\Xi$ and $\Omega\Omega$)
- Clearly more work needed at lighter masses

# *Many baryon systems*

- Many baryon correlator construction is messy and expensive
  - Techniques learnt in many-pion studies
    [WD & M. Savage; WD,, K Orginos, Z. Shi]
  - New tricks
    [T. Doi & M. Endres.; WD, K Orginos]
- Enables study of few (and many) baryon systems
- NPLQCD collaboration study
  - Unphysical SU(3) symmetric world @ $m_s^{phys}$
  - Multiple big volumes, single lattice spacing



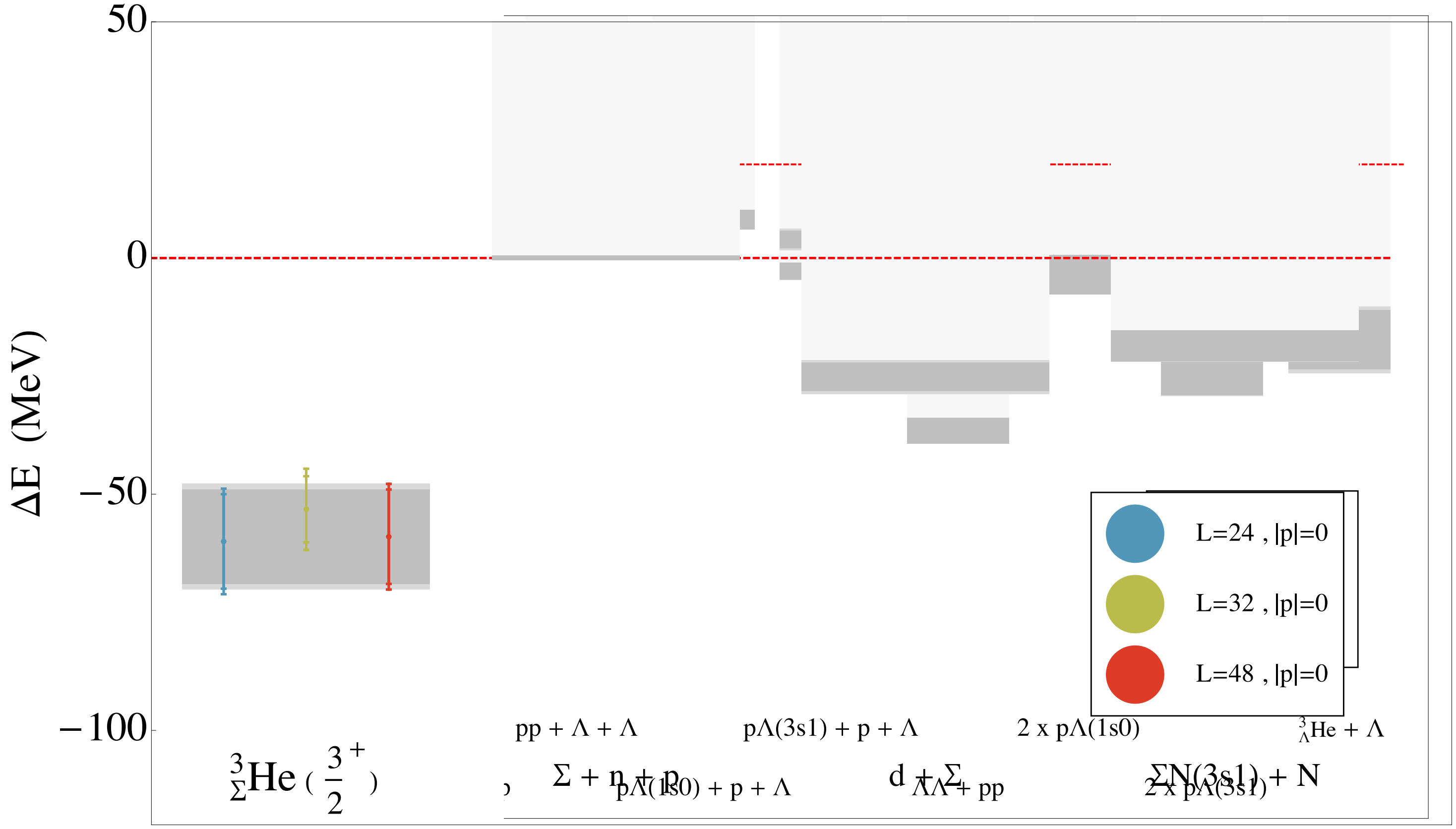
# *Nuclei (A=3,4)*




NPLQCD arXiv:1206.5219

# Nuclei (A=3,4)

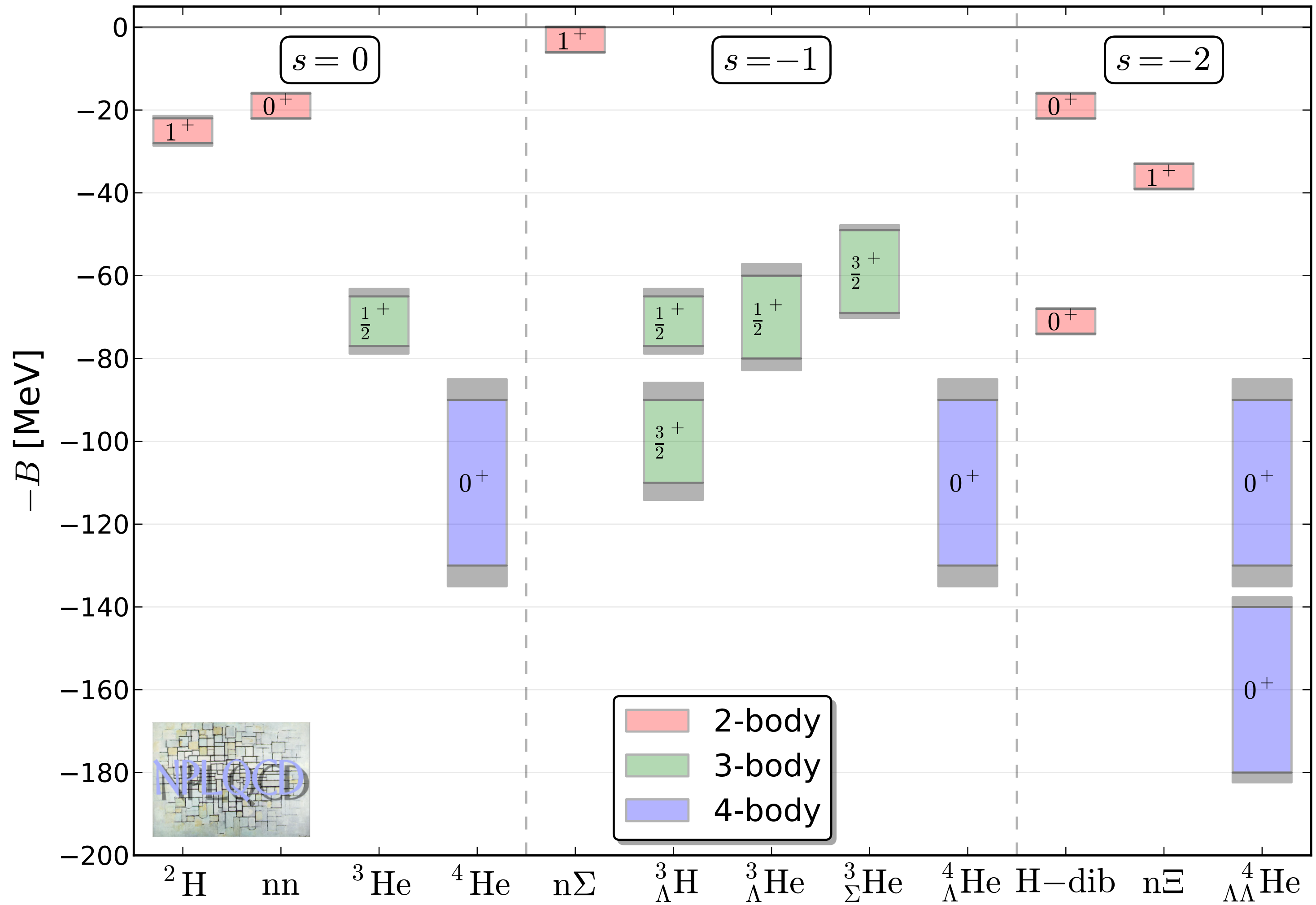
- Empirically investigate volume dependence
- Need to ask if this is a 2+1 or 3+1 or 2+2 etc scattering state




NPLQCD arXiv:1206.5219

# *Nuclei (A=2,3,4)*

NPLQCD arXiv:1206.5219

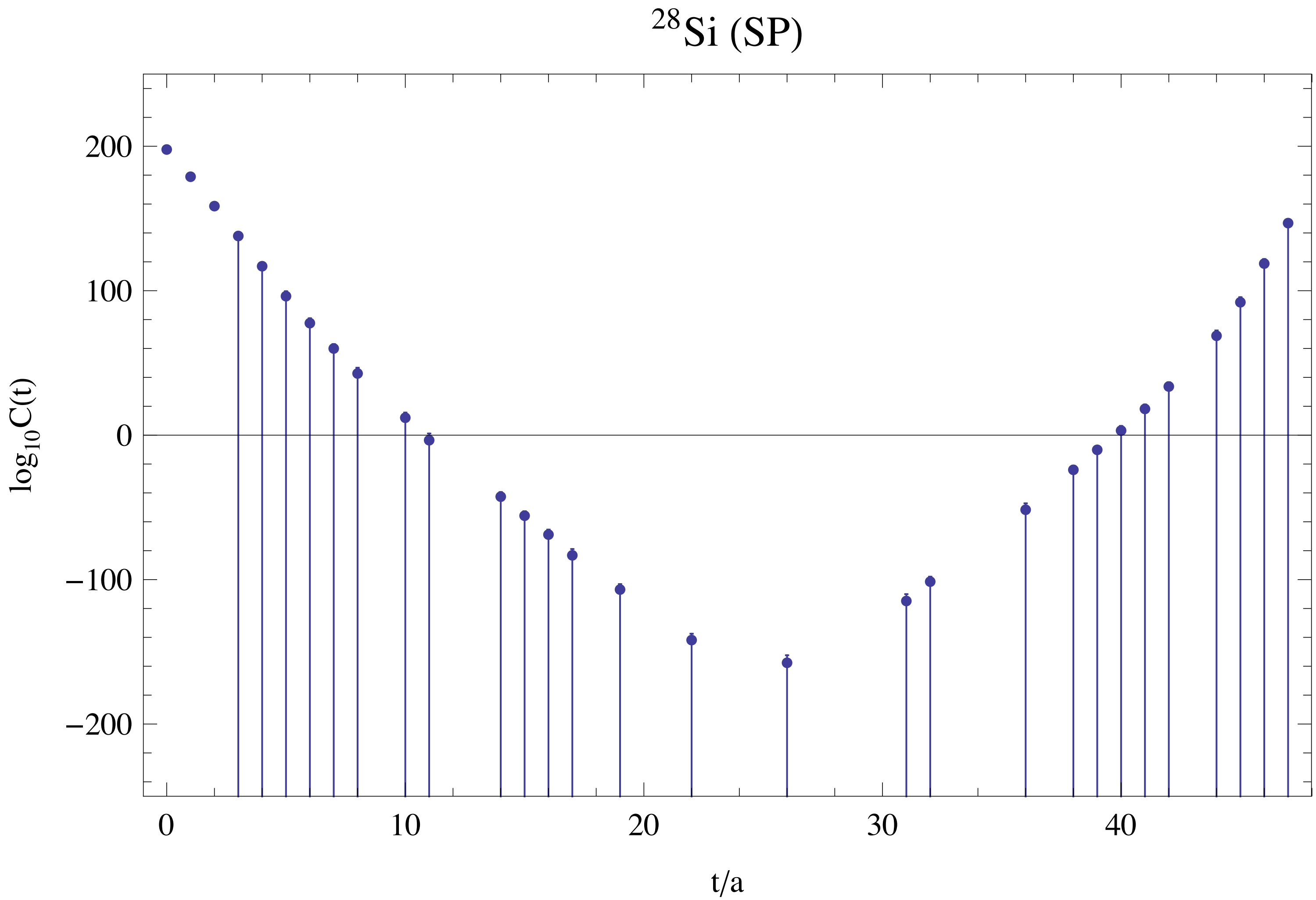



NB: SU(3) symmetry leads to unphysical degeneracies

see also Yamazaki et al. 1207.4277

# *Nuclei (A=4,...)*

Quark-quark determinant based contraction method




(low statistics, single volume)

WD, Kostas Orginos, 1207.1452

# Phase shifts

- Measurement of multiple energy levels allows extraction of phase shifts

- Ex: $^3S_1$ phase shift at $m_\pi$=800 MeV

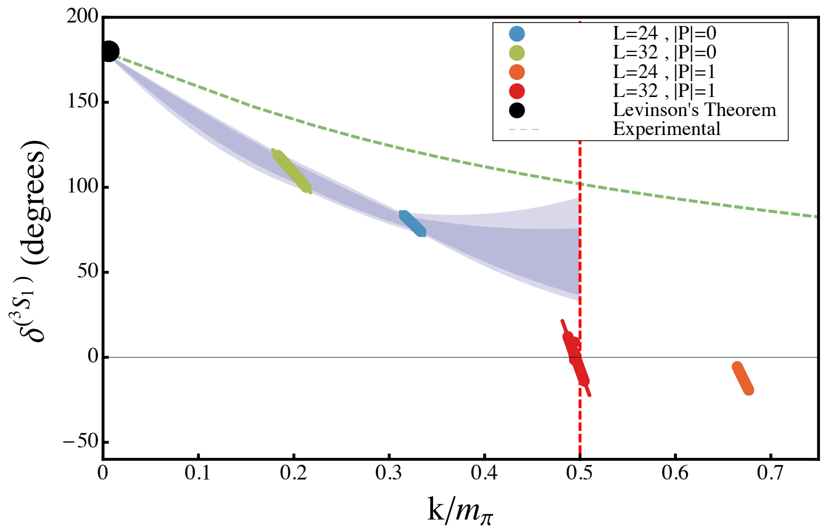
$$a^{(^3S_1)} = 1.82^{+0.14+0.17}_{-0.13-0.12}\ \text{fm} \qquad r^{(^3S_1)} = 0.906^{+0.068+0.068}_{-0.075-0.084}\ \text{fm}$$

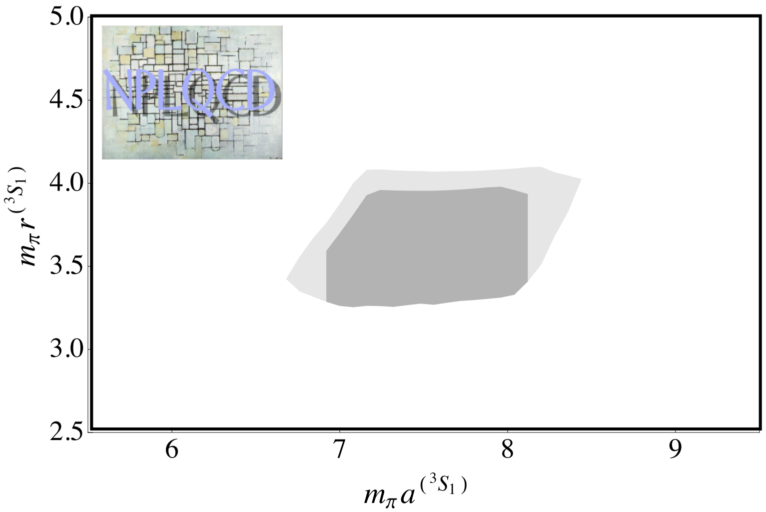
$$a^{(^3S_1)}/r^{(^3S_1)} = 2.06^{+0.22+0.25}_{-0.18-0.19}$$

$$a^{(^1S_0)}/r^{(^1S_0)} = 2.02^{+0.23+0.29}_{-0.19-0.18}$$

- *c.f.* fine-tuning of NN at physical mass

- Wigner SU(4) symmetry




[NPLQCD 1301.5790]

*Light-cone*

# *Partonic structure in $R_4$*

- Lattice QCD necessarily in Euclidean space
- DIS probes light-cone distributions $q_H(x)$
- OPE to the rescue
  Mellin moments of PDFs defined by
  forward matrix elements of local operators

$$\langle x^n \rangle_H = \int_{-1}^{1} dx\, x^n q_H(x)$$

$$\langle H | \overline{\psi} \gamma^{\{\mu_0} D^{\mu_1} \dots D^{\mu_n\}} | H \rangle = p^{\{\mu_0} \dots p^{\mu_n\}} \langle x^n \rangle_H$$

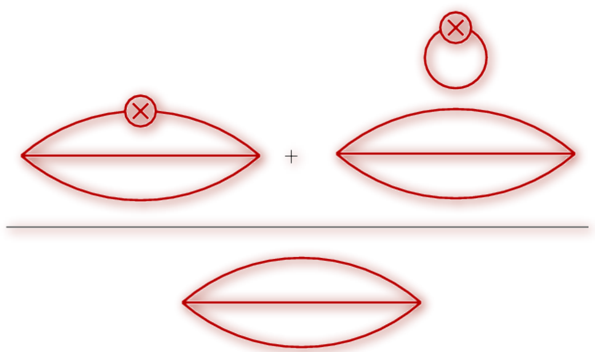
- Local matrix elements can be determined from Euclidean calculations
- NB: renormalisation scale dependent

# Partonic structure in LQCD

- PDF moments intensively studied in QCD using 3-pt functions

$$C_2(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi_H(0)\chi_H^\dagger(\mathbf{x},t)|0\rangle$$

$$C_3(t,\mathbf{p}) = \sum_{\mathbf{y},\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \langle 0|\chi_H(0)\mathcal{O}(\mathbf{y},\tau)\chi_H^\dagger(\mathbf{x},t)|0\rangle$$



$$R = \frac{C_3(t,\mathbf{p})}{C_2(t,\mathbf{p})} \xrightarrow{t\to\infty} \langle H|\mathcal{O}|H\rangle$$

- Most studies for nucleon, but also pion, rho, ...
- So far limited to low moments by reduced lattice symmetry
- Some ideas for how to go further

# The problem at high $x_{Bj}$

- LQCD necessarily formulated on a discrete geometry, typically 4d hypercube: O(4)→H(4)

$$H(4) = \{(a, \pi) | a \in \mathbb{Z}_2^4, \, \pi \in S_4\}$$

- Operators classified by H(4) quantum numbers
  - Finite number of irreducible representations
- Operator mixing:

$$\mathcal{O}_i^{cont} = Z_i \mathcal{O}_i^{lat} + \sum_j Z_{ij} \mathcal{O}_j^{lat}$$

Sum over all operators with right quantum numbers

- Allows (forces) mixing with operators of lower dimension
  - Coefficients scale with inverse powers of lattice cutoff
    Taking the continuum limit is difficult

# Nice example

- Continuum operator $\mathcal{O}_{\mu\nu} = \overline{q}\gamma_{\{\mu}D_{\nu\}}q$ belongs to

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right) \otimes \left(\tfrac{1}{2},\tfrac{1}{2}\right) = (0,0) \oplus [(1,0) \oplus (0,1)] \oplus (1,1)$$

- Hypercubic decomposition

$$\mathbf{4}_1 \otimes \mathbf{4}_1 = \mathbf{1}_1 \oplus \mathbf{3}_1 \oplus \mathbf{6}_1 \oplus \mathbf{6}_3$$

- Lattice operators (symmetric traceless):

$$\mathcal{O}_{14} + \mathcal{O}_{41}, \qquad \mathcal{O}_{44} - \frac{1}{3}\left(\mathcal{O}_{11} + \mathcal{O}_{22} + \mathcal{O}_{33}\right)$$

- Have same continuum limit ($6_3$ requires **p**$\neq$0)

- No operators of lower dimension (☺)

# Not-so-nice example

- Continuum operator $\mathcal{O}_{\{\mu\nu\rho\}} = \overline{q}\gamma_{\{\mu}D_\nu D_{\rho\}}q$ lives in

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right) = 4\cdot\left(\tfrac{1}{2},\tfrac{1}{2}\right)\oplus 2\cdot\left(\tfrac{3}{2},\tfrac{1}{2}\right)\oplus 2\cdot\left(\tfrac{1}{2},\tfrac{3}{2}\right)\oplus\left(\tfrac{3}{2},\tfrac{3}{2}\right)$$

- Hypercubic decomposition

$$\mathbf{4}_1\otimes\mathbf{4}_1\otimes\mathbf{4}_1 = 4\cdot\mathbf{4}_1\oplus\mathbf{4}_2\oplus\mathbf{4}_4\oplus 3\cdot\mathbf{8}_1\oplus 2\cdot\mathbf{8}_2$$

- Lattice operators:

$$\mathcal{O}_{111},\qquad \mathcal{O}_{\{123\}},\qquad \mathcal{O}_{\{441\}} - \tfrac{1}{2}\left(\mathcal{O}_{\{221\}} + \mathcal{O}_{\{331\}}\right)$$

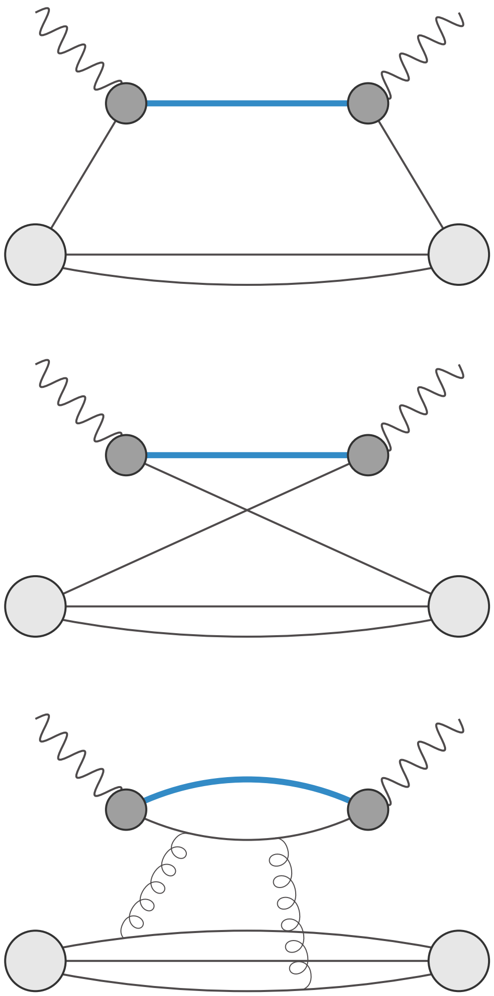
- Same continuum limit but $\mathcal{O}_{111}$ mixes with $\overline{q}\gamma_1 q \in \mathbf{4_1}$ and the coefficient absorbs the missing dimensions (☹)
  - Always the case for all n > 4 operators

# Euclidean Compton tensor

[WD, CJD Lin, Phys.Rev. D73 (2006) 014501]

- Directly study Euclidean space Compton tensor and extract moments via OPE
- Use fictitious heavy quark to connect currents
  - Mass suppresses higher twists
  - Integrated out in OPE, gives usual PDF moments
- Needs very fine lattice spacing: $a^{-1} > 5$ GeV

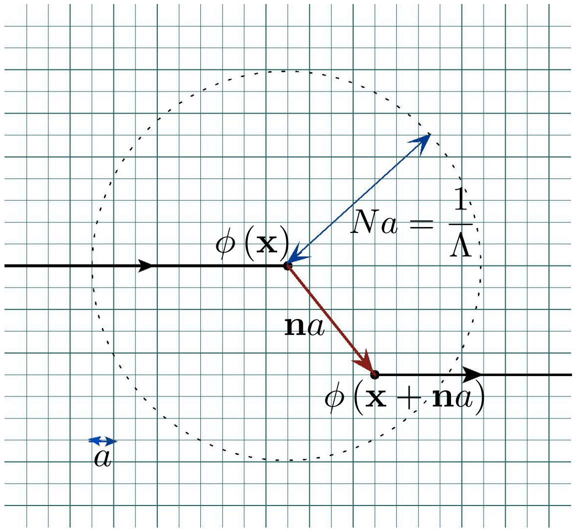
$$\Lambda_{\mathrm{QCD}} \ll |Q|,\, m_\Psi \ll a^{-1}$$

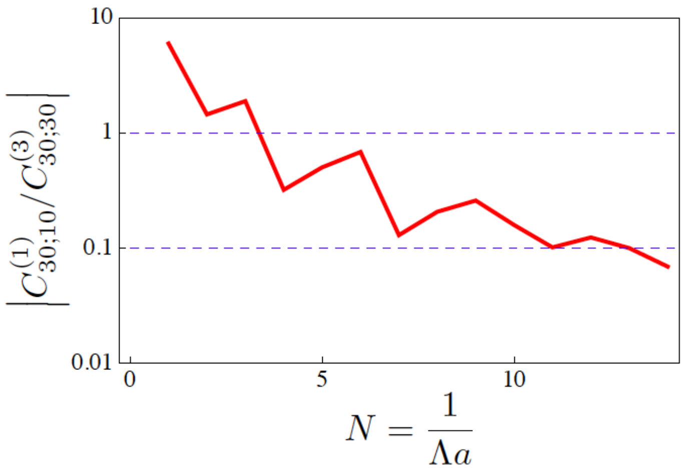


# Lorentz symmetry restoration

[Z Davoudi & M Savage, Phys.Rev. D86 (2012) 054505]

- Physically: a lattice spacing of 100 $GeV^{-1}$ should not stop study of PDF moments at a scale of 2 GeV
- Higher moments: more sophisticated operators small compared to the resolution scale (large multiplicities of irreps)
- Demonstrated to work at one-loop in QCD
- Also requires fine lattice spacing

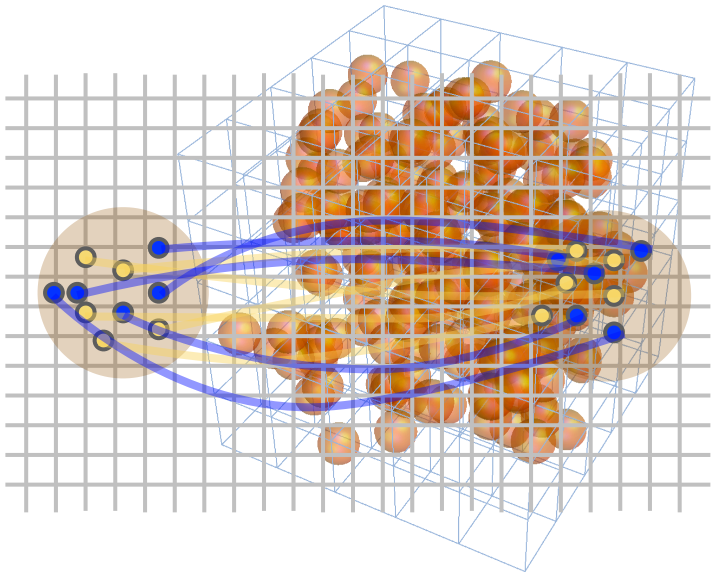


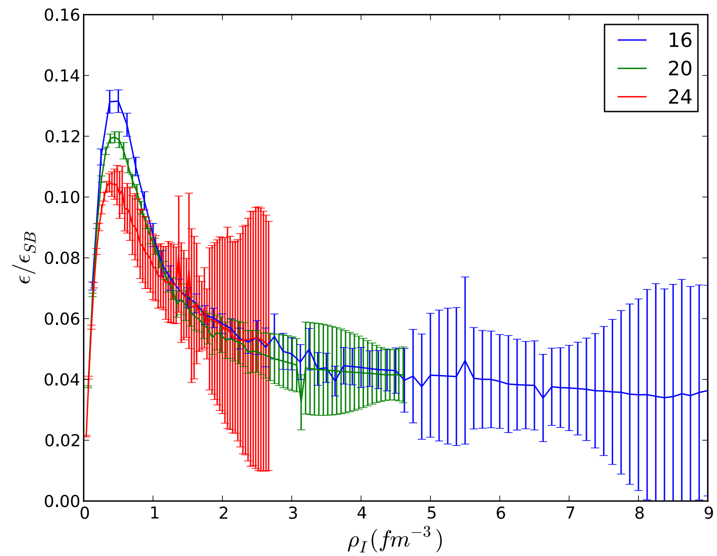
*EMC effect*

# *Many pions – a precursor to nuclei*

- Pions as a testing ground
  - Systems of up to $I_z$=72:
  - Similar many-body problems, but constant noise
  - Contractions satisfy recursion
    [WD & M Savage; Z Shi & WD]
- Systems interesting in their own right
  - Use to extract 2 & 3 body interactions
  - Canonical approach to QCD with an effective isospin chemical potential
  - Explore pion BEC and crossover to BCS



Energy density vs Stefan-Boltzmann



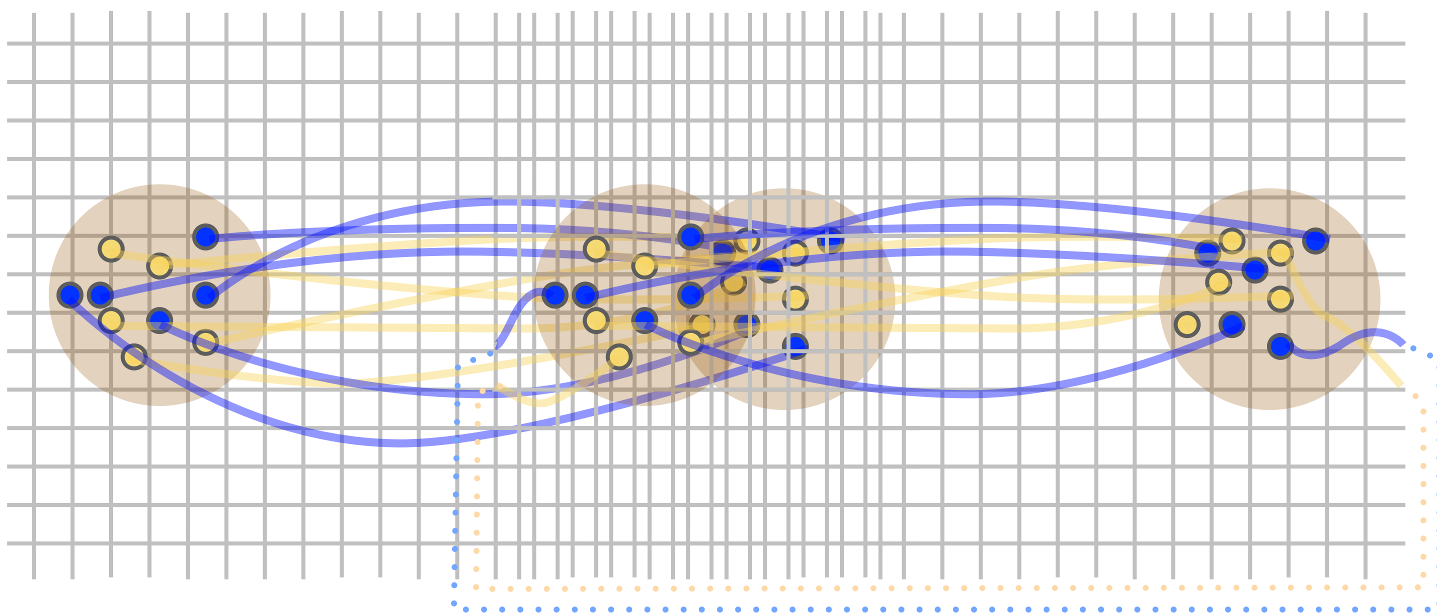

[WD, Shi, Orginos 1205.4224]

# *Many meson correlator*

- Now an $n$ $\pi^+$ correlator ($m_u$=$m_d$)

$$C^{(n)}(t) = \left\langle 0 \left| \left[ \sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0) \right]^n \right| 0 \right\rangle$$

$$\xrightarrow{t \text{ large}} \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} A_{n,m} e^{-(E_m+E_{n-m})T/2} \cosh\left((E_m - E_{n-m})(t-T/2)\right)$$



# Many meson correlator

- Now an *n* $\boldsymbol{\pi}^+$ correlator ($m_u$=$m_d$)

$$C^{(n)}(t) = \left\langle 0 \left| \left[ \sum_{\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x},t)\bar{u}\gamma_5 d(\mathbf{0},0) \right]^n \right| 0 \right\rangle$$

$$\xrightarrow{t \text{ large}} \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} A_{n,m} e^{-(E_m+E_{n-m})T/2} \cosh\left((E_m - E_{n-m})(t - T/2)\right)$$

- $n!^2$ Wick contractions: $(12!)^2 \sim 10^{17}$

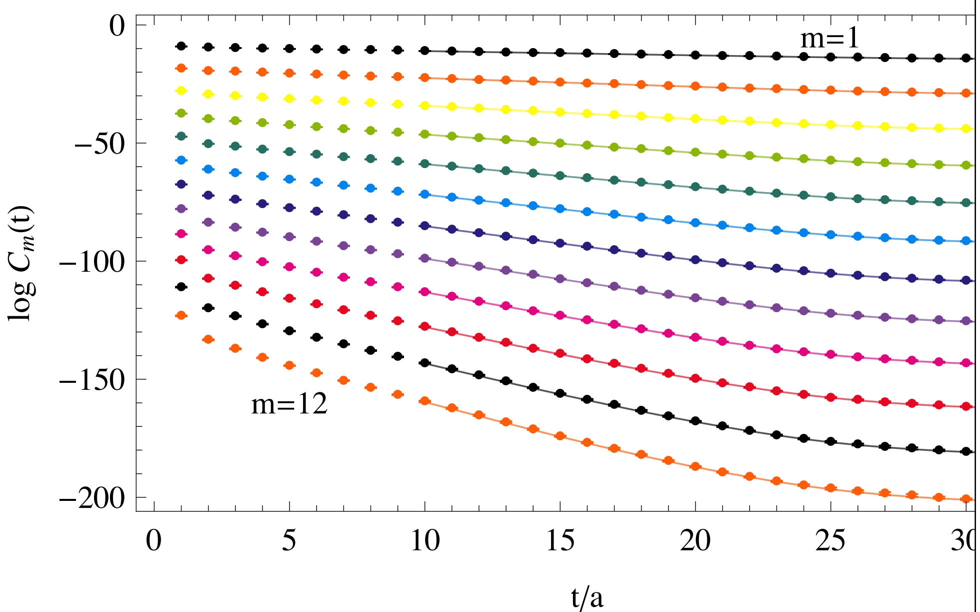
$$C_3(t) = \mathrm{tr}\left[\Pi\right]^3 - 3\ \mathrm{tr}\left[\Pi\right] \mathrm{tr}\left[\Pi^2\right] + 2\ \mathrm{tr}\left[\Pi^3\right]$$

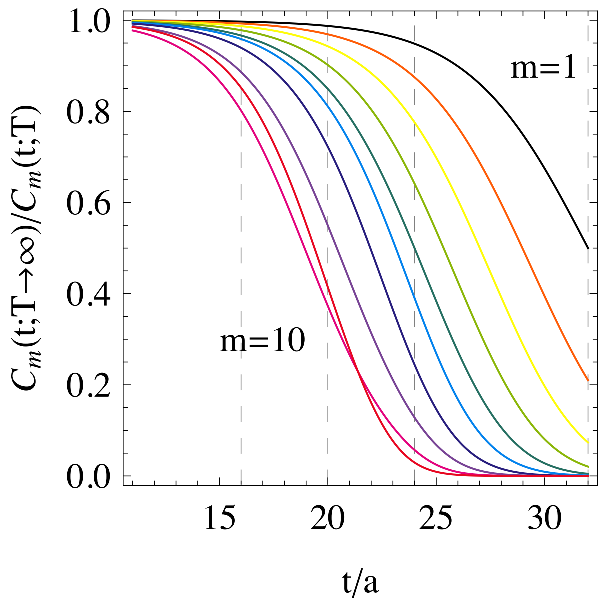
$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$


- Maximal isospin: only a single quark propagator

# *n-meson energies*

- Correlators: $\log[C_n(t)]$
  - Finite T effects  very important
- DWF on MILC gauge configurations
  - $m_\pi$ ~ 291, 318, 352, 358, 491 MeV
  - L=2.5 fm, a=0.12 fm
  - also L=3.5 fm and a=0.09 fm



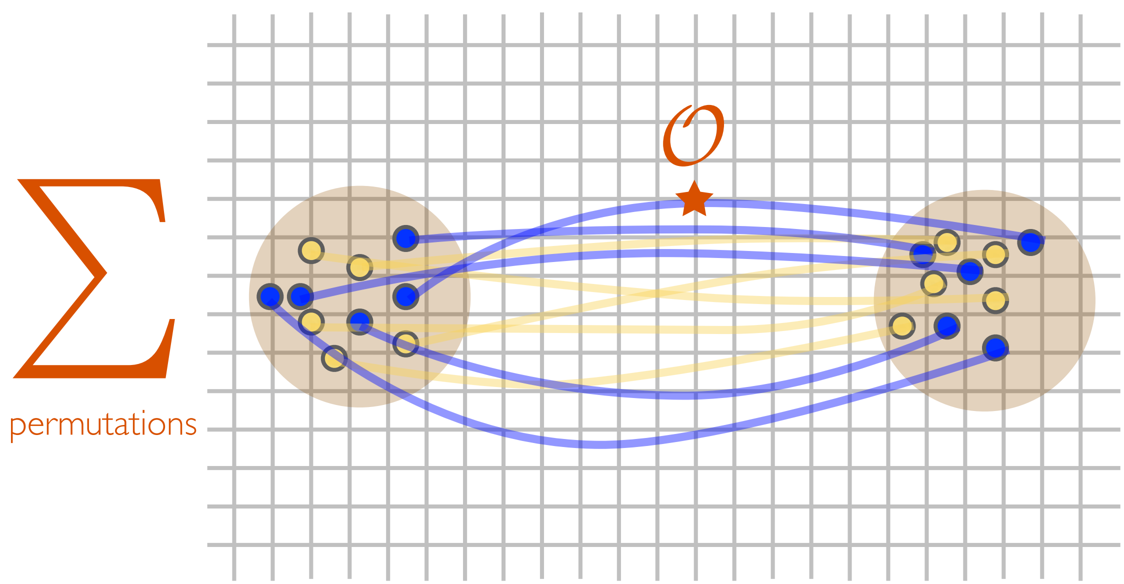
# *Many meson 3-point correlator*

- $n\ \boldsymbol{\pi}^+$ 3-point correlator

$$C_3^{(n)}(t;\tau) = \left\langle 0 \left| \left[ \sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0) \right]^n \sum_{\mathbf{y}} \mathcal{O}(\mathbf{y},\tau) \right| 0 \right\rangle$$

$$\xrightarrow{t\gg\tau\gg 0} A\ e^{-E_n t}\langle n\pi|\mathcal{O}|n\pi\rangle + \ldots$$

Excitations and thermal effects



# Many meson 3-point correlator

- $n\ \boldsymbol{\pi}^+$ 3-point correlator

$$C_3^{(n)}(t;\tau) = \left\langle 0 \left| \left[ \sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(\mathbf{0},0) \right]^n \sum_{\mathbf{y}} \mathcal{O}(\mathbf{y},\tau) \right| 0 \right\rangle$$

$$\xrightarrow{t\gg\tau\gg 0} A\ e^{-E_n t}\langle n\pi|\mathcal{O}|n\pi\rangle + \ldots$$

Excitations and thermal effects

- Contractions performed by treating the struck meson as a separate species

Colour/Dirac structure of operator

$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0), \qquad \tilde{\Pi}_\tau =_{\mathbf{x},\mathbf{y}} \gamma_5 S(\mathbf{x},t;\mathbf{y},\tau)\Gamma_{\mathcal{O}} S(\mathbf{y},\tau;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$

- System looks like (n-1) pions + 1 "kaon"
  - Can be written as products of traces of two matrices

[WD & B Smigielski, arXiv:1103.4362]

# Double ratio

- Define ratio to extract matrix elements

$$R^{(n)}(t,\tau) = \frac{C_3^{(n)}(t;\tau)}{C_2^{(n)}(t)} \xrightarrow{t\gg\tau} \frac{1}{E_{n\pi}}\langle n\ \pi^+|\mathcal{O}^{44}|n\ \pi^+\rangle$$

- Double ratio

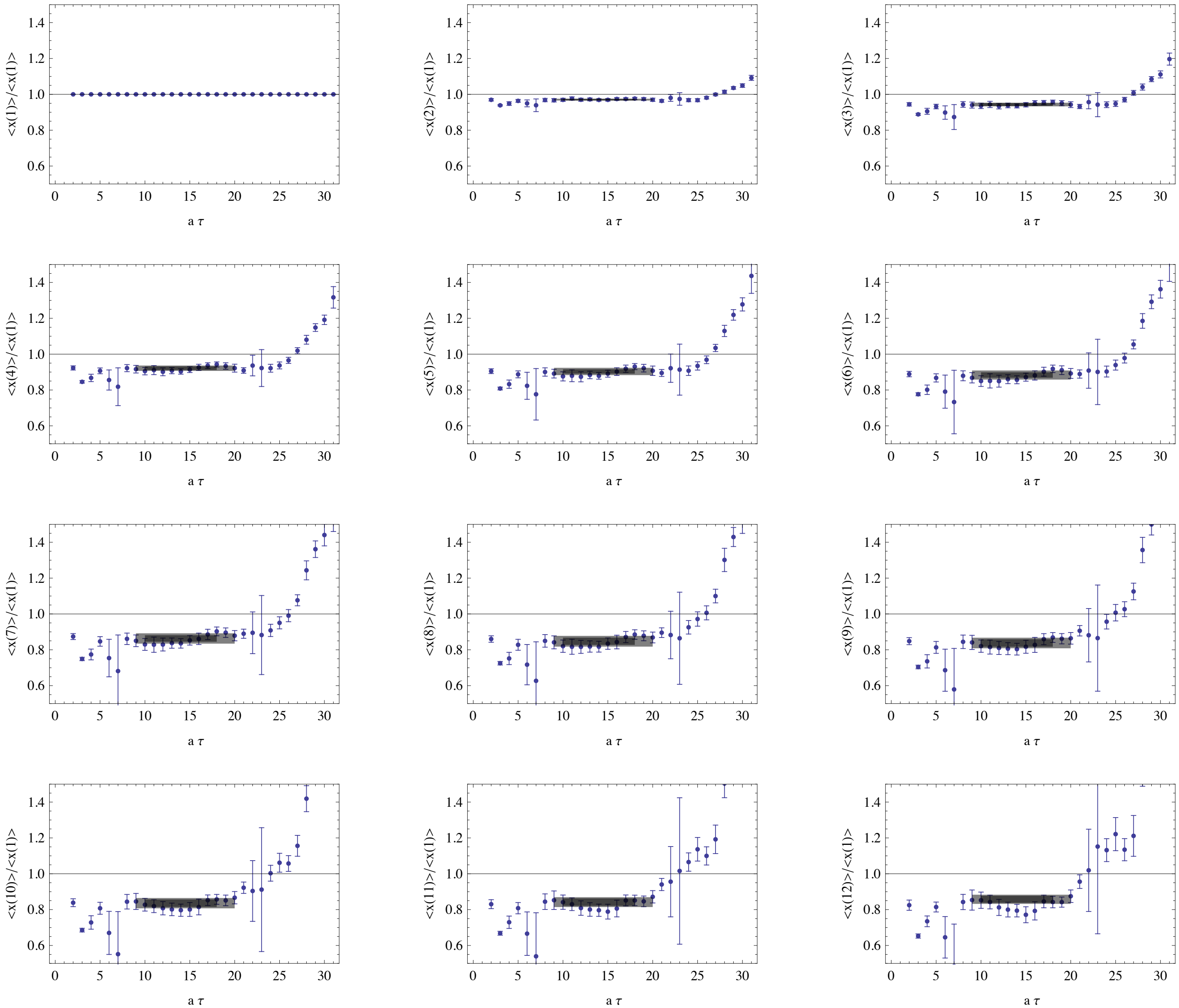
$$\frac{R^{(n)}(t,\tau)}{R^{(1)}(t,\tau)} \longrightarrow \frac{m_\pi\ \langle n\ \pi^+|\mathcal{O}^{44}|n\ \pi^+\rangle}{E_{n\pi}\ \langle \pi^+|\mathcal{O}^{44}|\pi^+\rangle} \longrightarrow \frac{E_{n\pi}\ \langle x\rangle_{n\pi^+}}{m_\pi \langle x\rangle_{\pi^+}}$$

  - No need to renormalise operator!

- Allows investigation of ratio of moments

- Focus on momentum fraction: $\mathcal{O}^{44}$

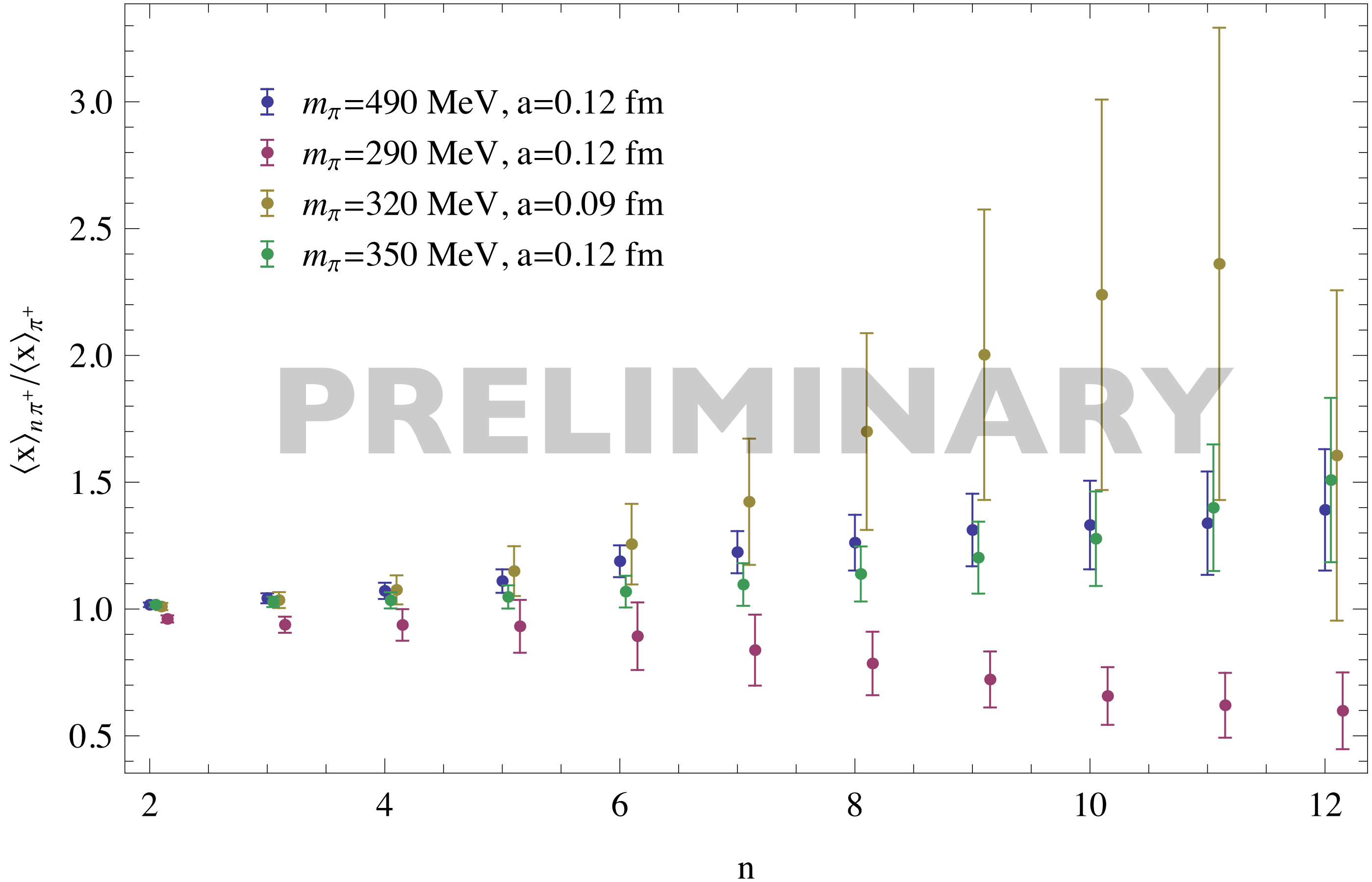
# Double ratio



DWF on MILC
$m_\pi = 350$ MeV
a=0.12 fm, $20^3$x64

# *Pionic EMC effect*

- LC momentum fraction carried by quarks in a pion in a dense medium *c.f.* in free space




Caveat: Volume effects not completely sorted out

# Outlook

- Light nuclei can be calculated from QCD
  - Work needed to get to physical masses
- Simple nuclear matrix elements are harder but not that much harder than spectroscopy
  - High moments of PDFs: new ideas need testing
- Ultimately I hope we can study PDFs of nuclei in QCD
  - which might take a few years ...

[FIN]

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