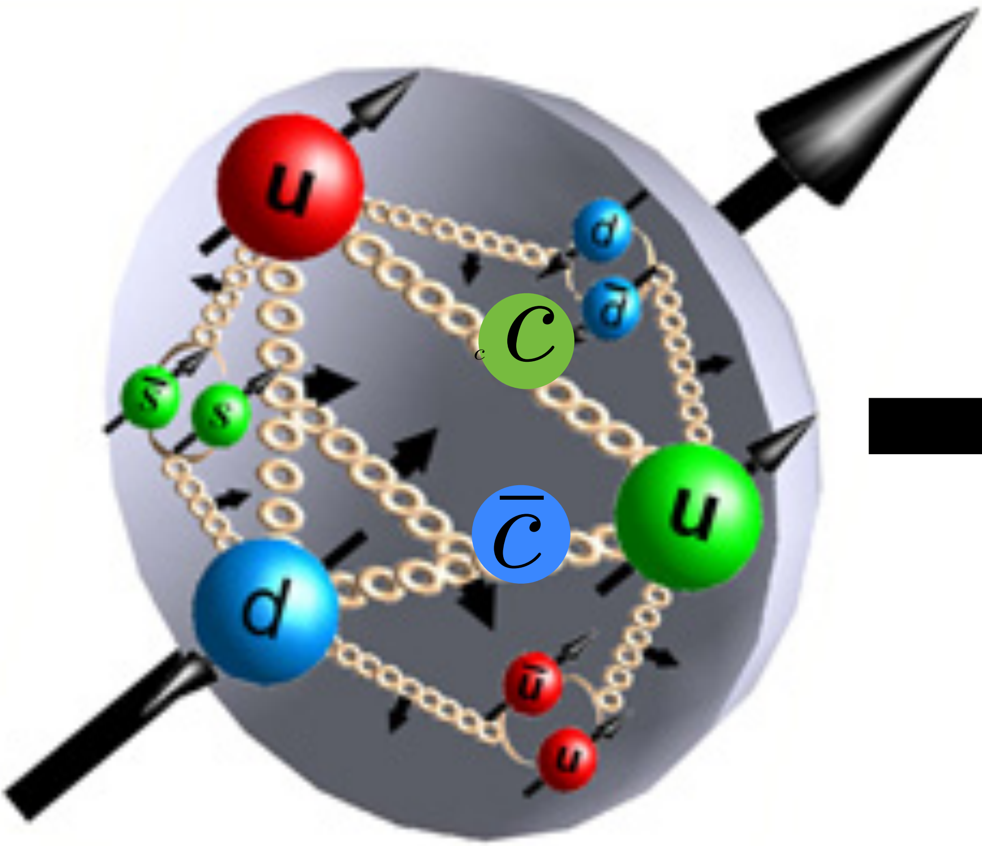
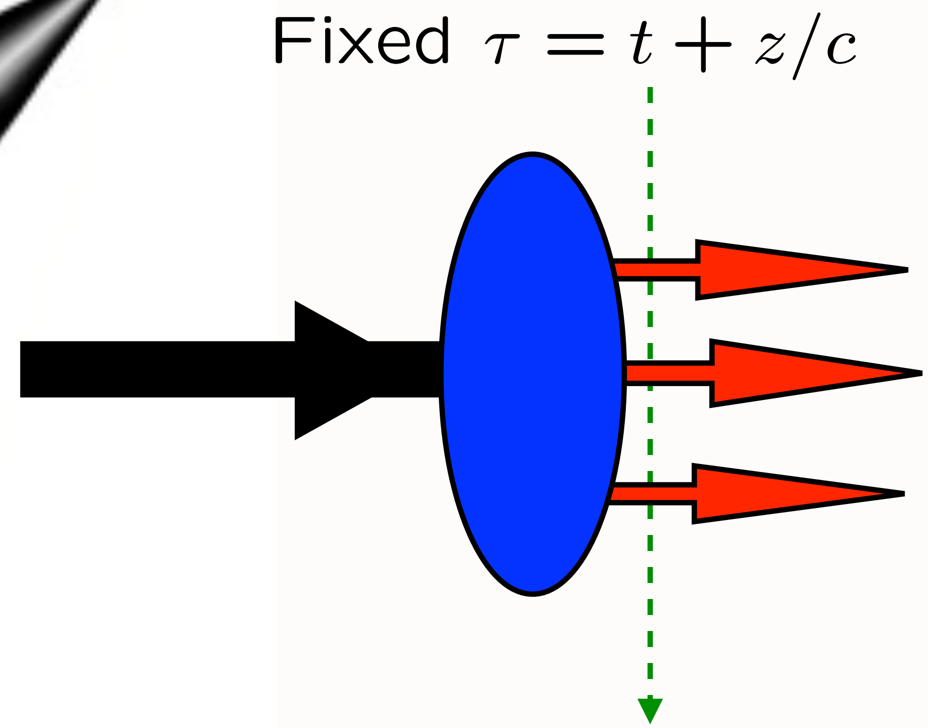
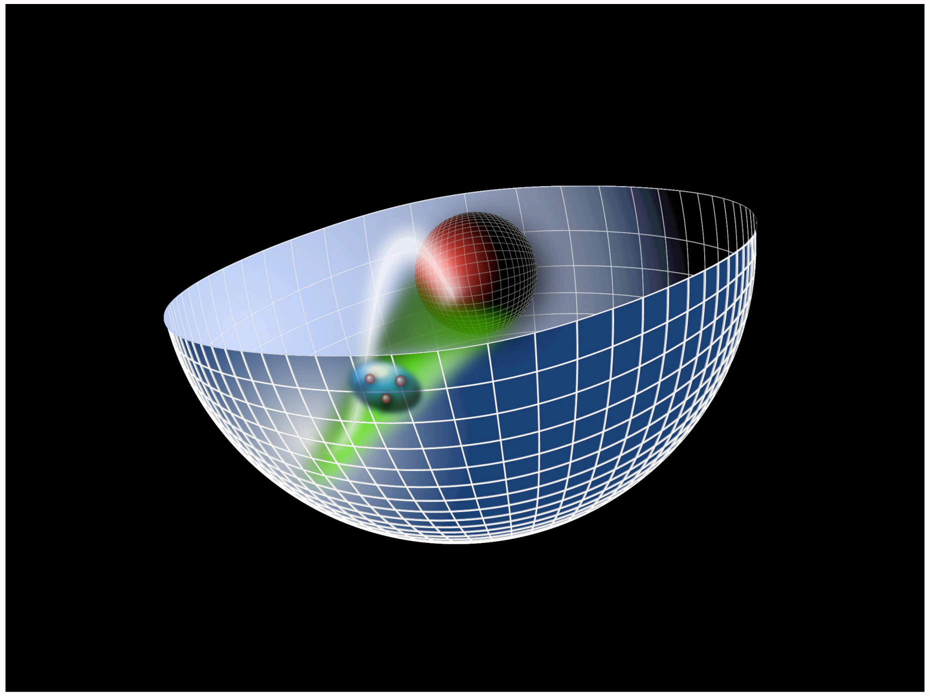
# Light-Front Hadronic and Nuclear Physics





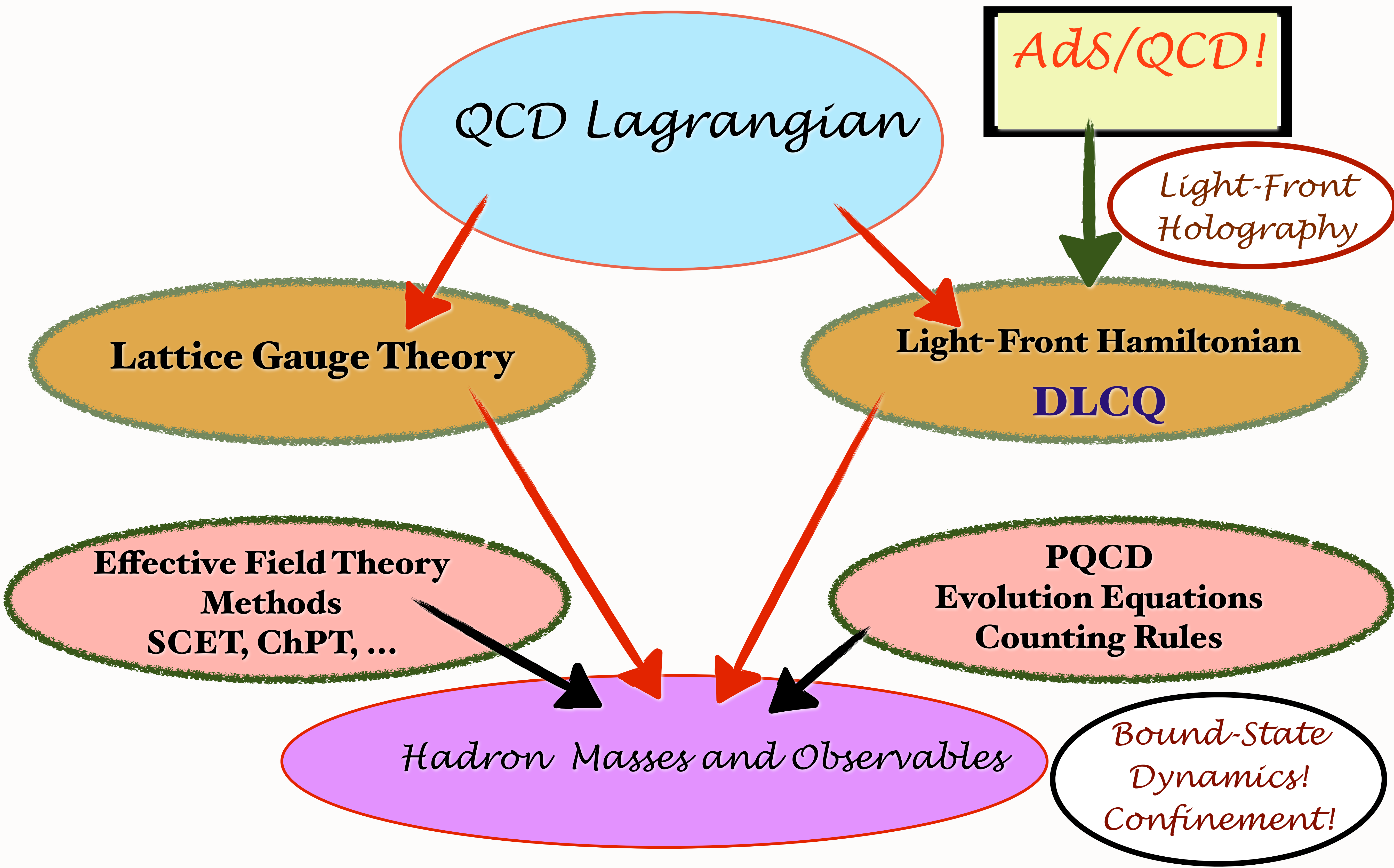



Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

**February 12, 2013**

# Predict Hadron Properties from First Principles!



# *Goal: an analytic first approximation to QCD*

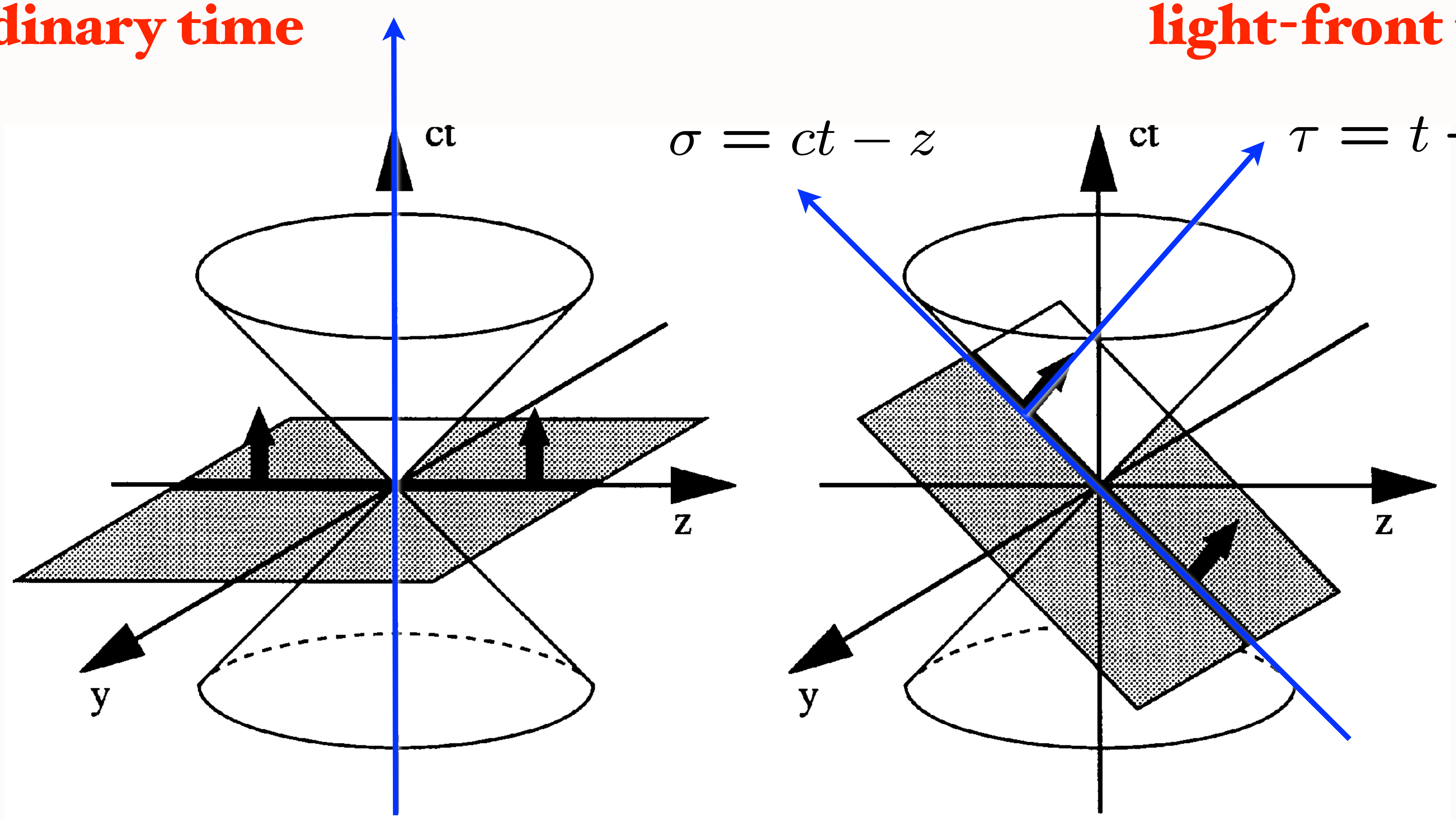
- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Hadronic Observables, Constituent Counting Rules
- Insights into QCD Condensates
- Systematically improvable
- Eliminate scale ambiguities

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

# Dirac's Amazing Idea: The Front Form

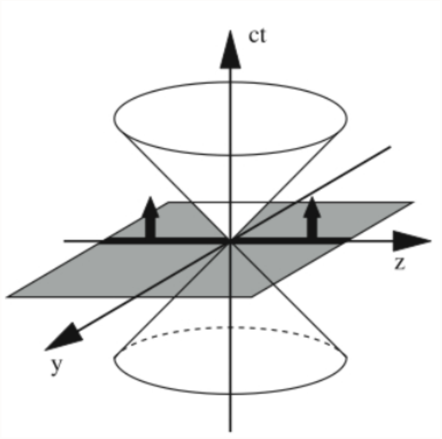
**Evolve in ordinary time**

**Evolve in light-front time!**




**Instant Form** **Front Form**

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one




- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

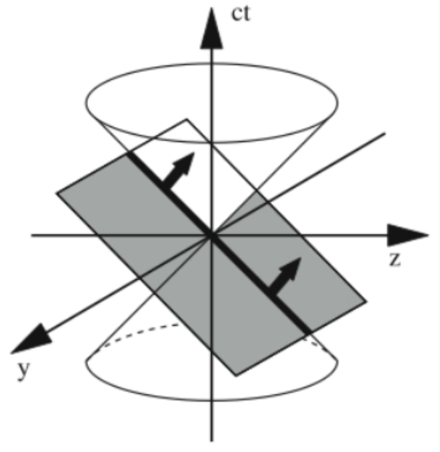
$x^+ = x^0 + x^3$ light-front time

$x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ > 0)$

$k^- = k^0 - k^3$ light-front energy

$$\tau = t + z/c$$




$$k \cdot x = \tfrac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$


**Quantum chromodynamics and other field theories on the light cone.**
Stanley J. Brodsky (SLAC), Hans-Christian Pauli (Heidelberg, Max Planck Inst.),
Stephen S. Pinsky (Ohio State U.). SLAC-PUB-7484, MPIH-V1-1997. Apr 1997. 203 pp.
Published in **Phys.Rept. 301 (1998) 299-486**
e-Print: **hep-ph/9705477**

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

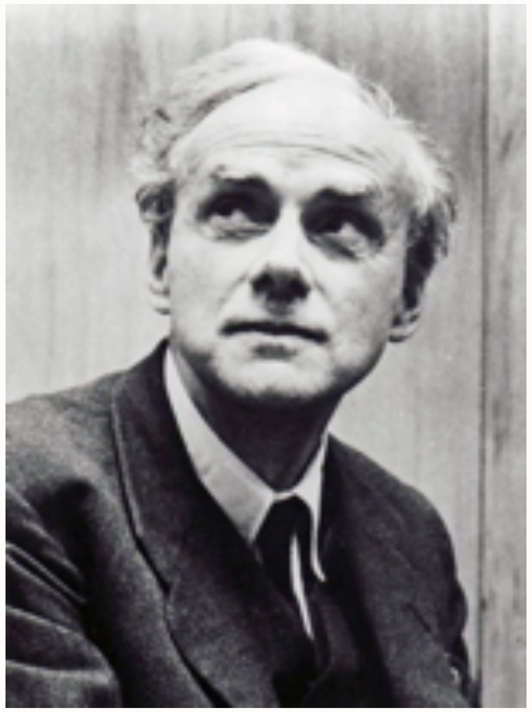
Eigenstate -- independent of $\tau$

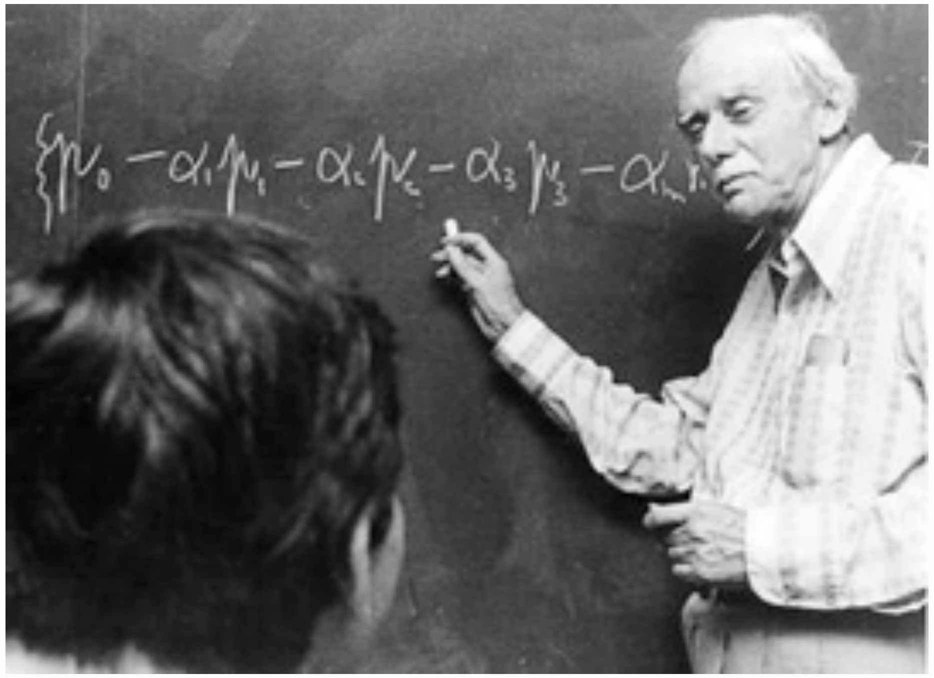
$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

**P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)**





*"Working with a front is a process that is unfamiliar to physicists.*

*But still I feel that the mathematical simplification that it introduces is all-important.*

*I consider the method to be promising and have recently been making an extensive study of it.*

*It offers new opportunities, while the familiar instant form seems to be played out " - P.A.M. Dirac (1977)*

**Light-Front + RG: Wilson and Glazek**

# Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

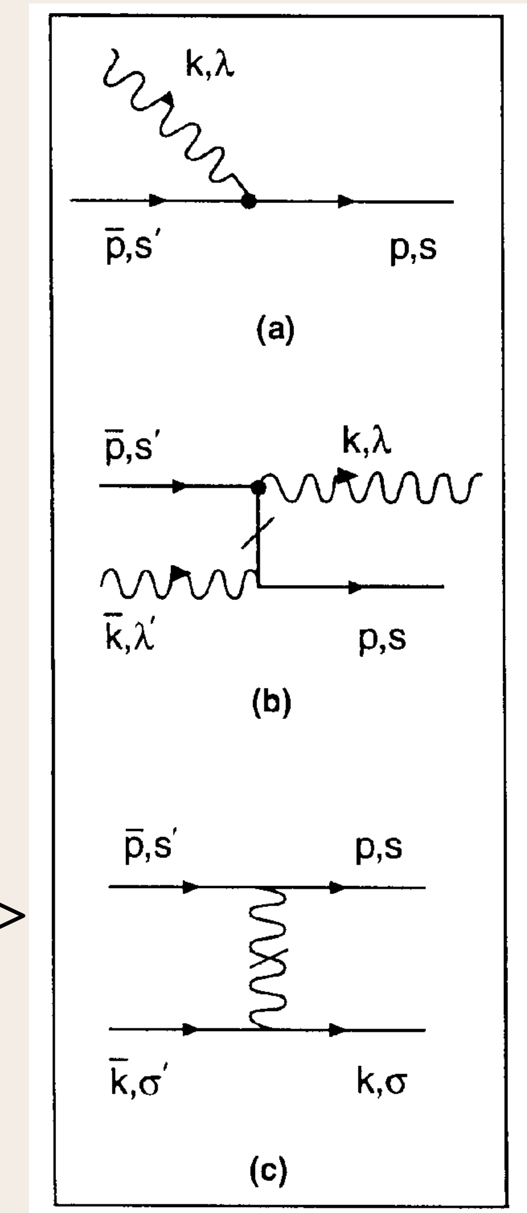
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

$H^{int}_{LF}$: Matrix in Fock Space

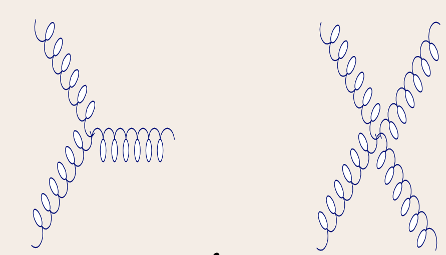
$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$




Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

**LFWFs: Off-shell in P- and invariant mass**



$H^{int}_{LF}$

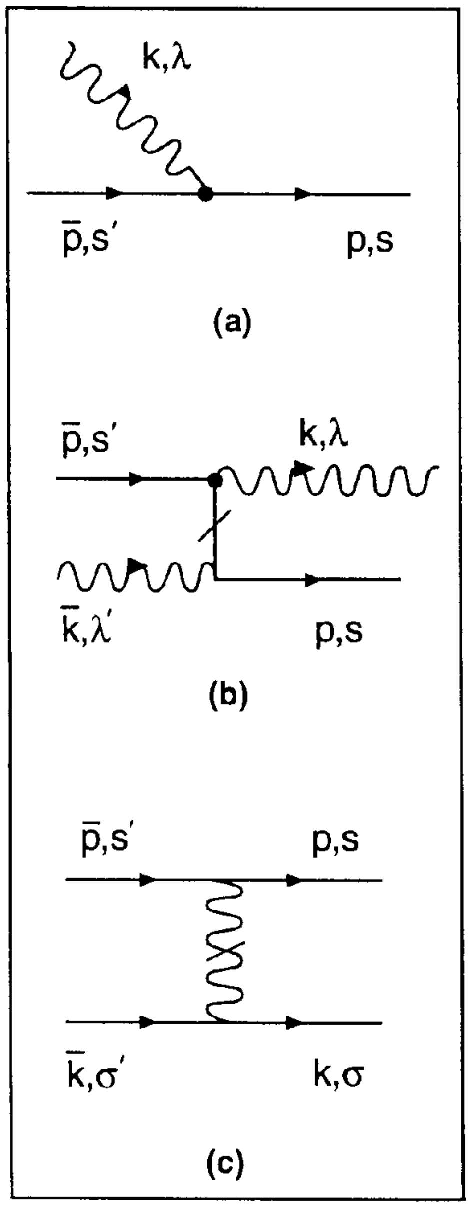
*Light-Front QCD*

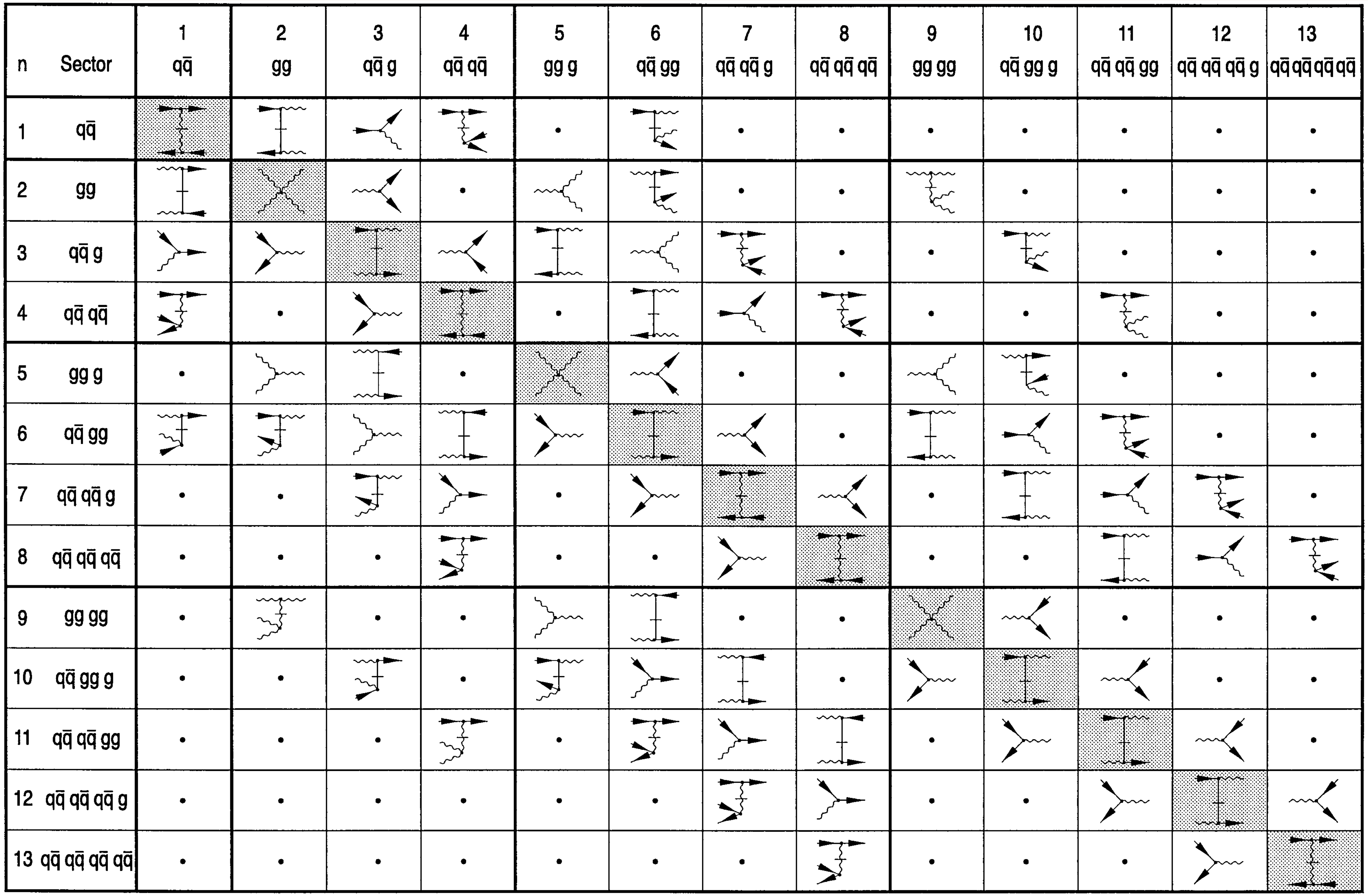
*Heisenberg Equation*

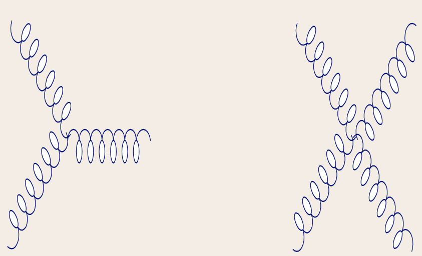
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\;|\Psi_h\rangle$$

*DLCQ: Solve QCD(1+1) for any quark mass and flavors*

**Hornbostel, Pauli, sjb**



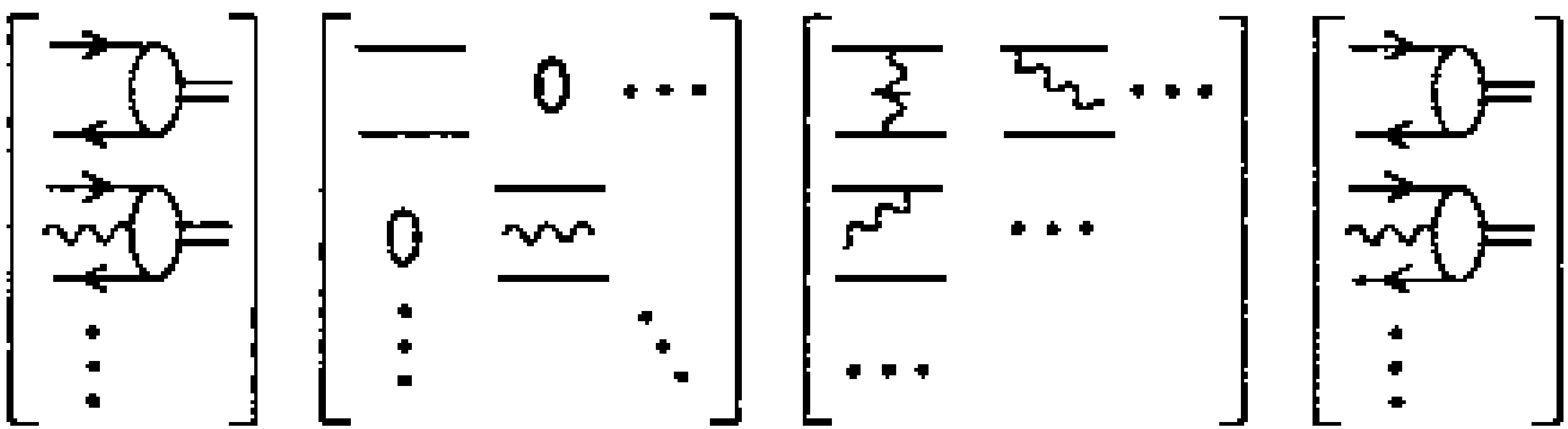
G.P. Lepage, sjb

# LIGHT-FRONT MATRIX EQUATION

## Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar q/\pi} \\ \psi_{q\bar q g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar q|\, V \,|q\bar q\rangle & \langle q\bar q|\, V \,|q\bar q g\rangle & \cdots \\ \langle q\bar q g|\, V \,|q\bar q\rangle & \langle q\bar q g|\, V \,|q\bar q g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar q/\pi} \\ \psi_{q\bar q g/\pi} \\ \vdots \end{bmatrix}$$

$A^+ = 0$



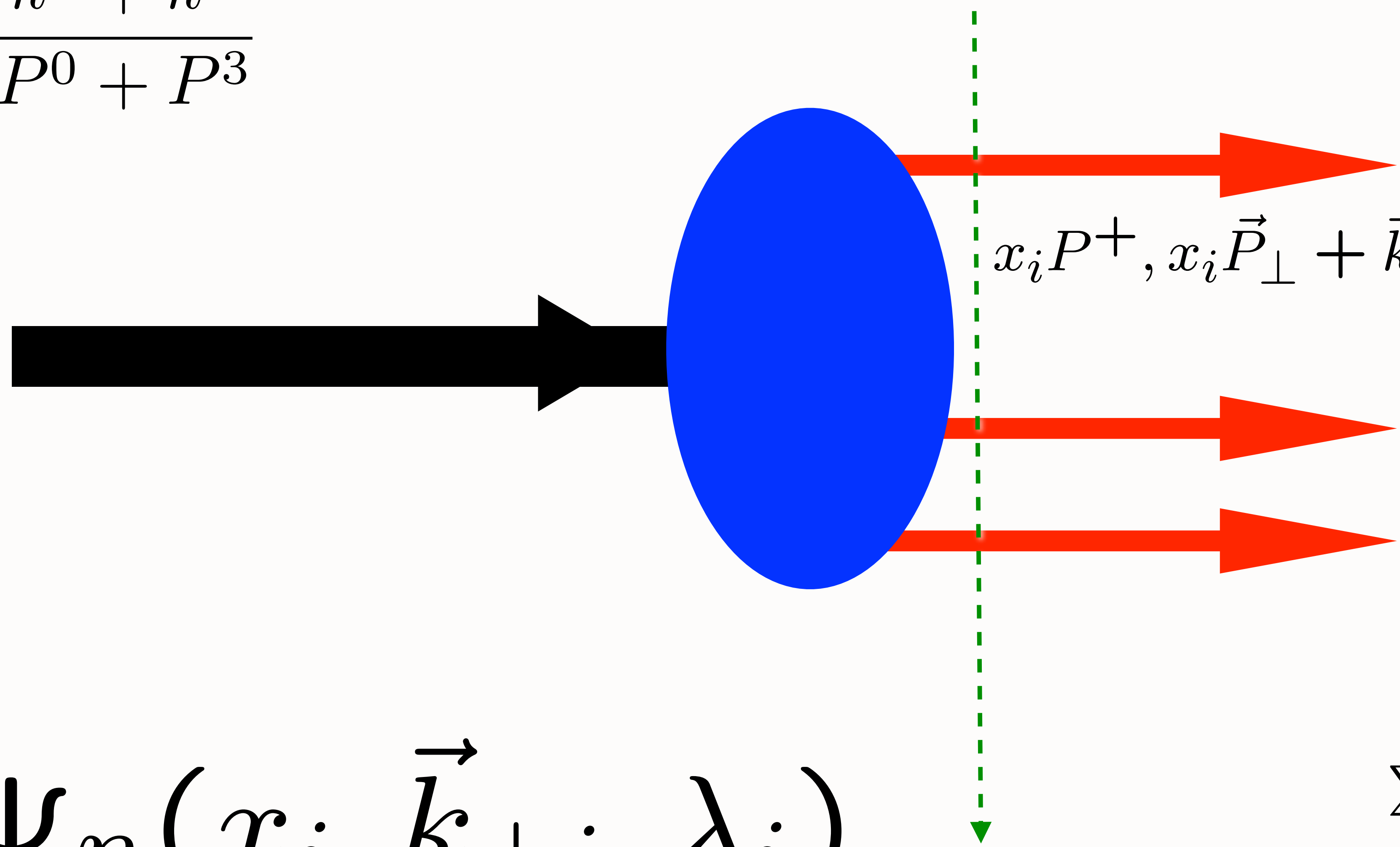

*Minkowski space; frame-independent; no fermion doubling; no ghosts*

- *Light-Front Vacuum = vacuum of free Hamiltonian!*

# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

*Invariant under boosts! Independent of $P^\mu$*

**Bethe-Salpeter WF integrated over k⁻**

# Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

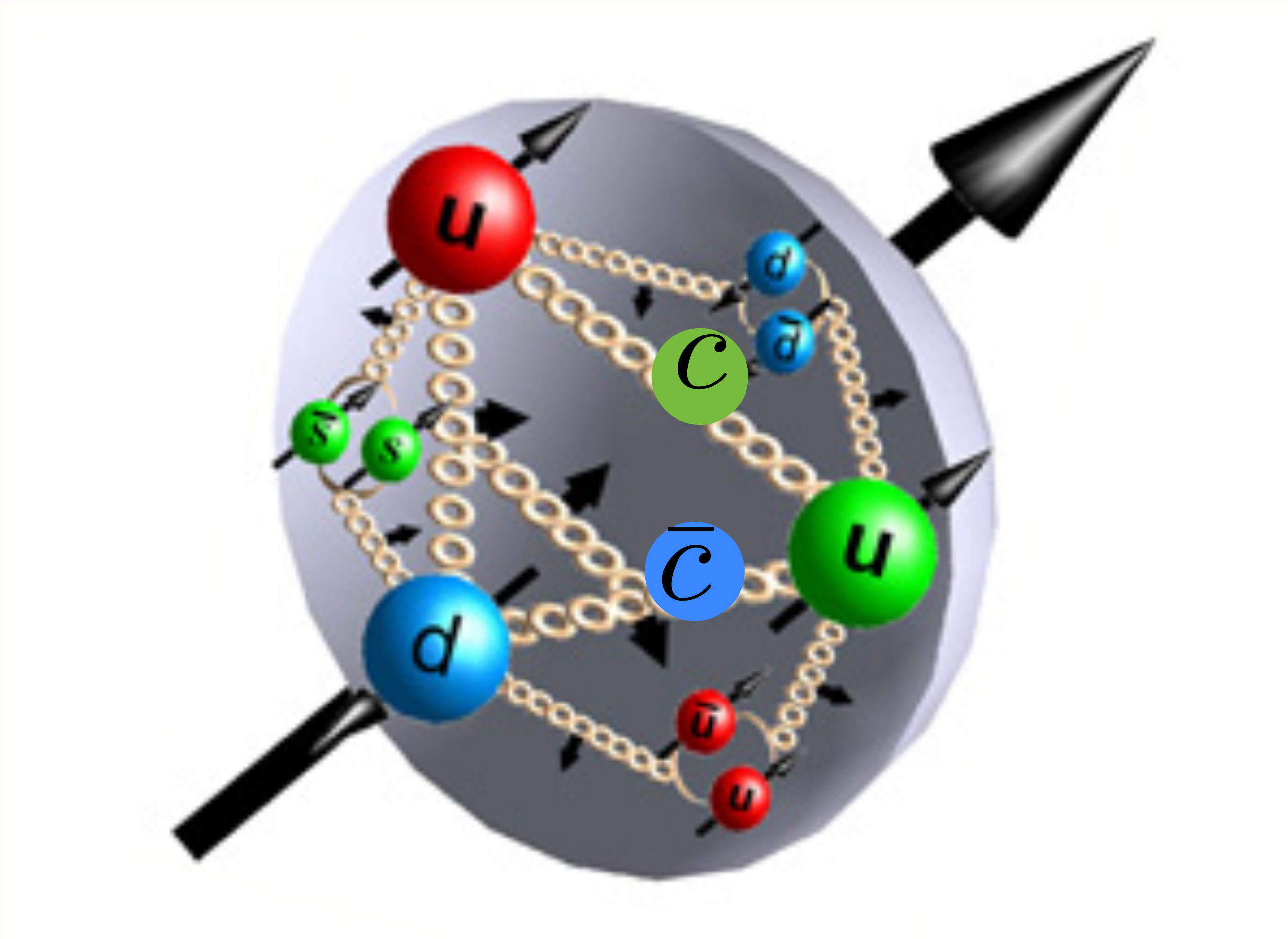
**Conserved**
**LF Fock-State by Fock-State**
**Every Vertex**

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

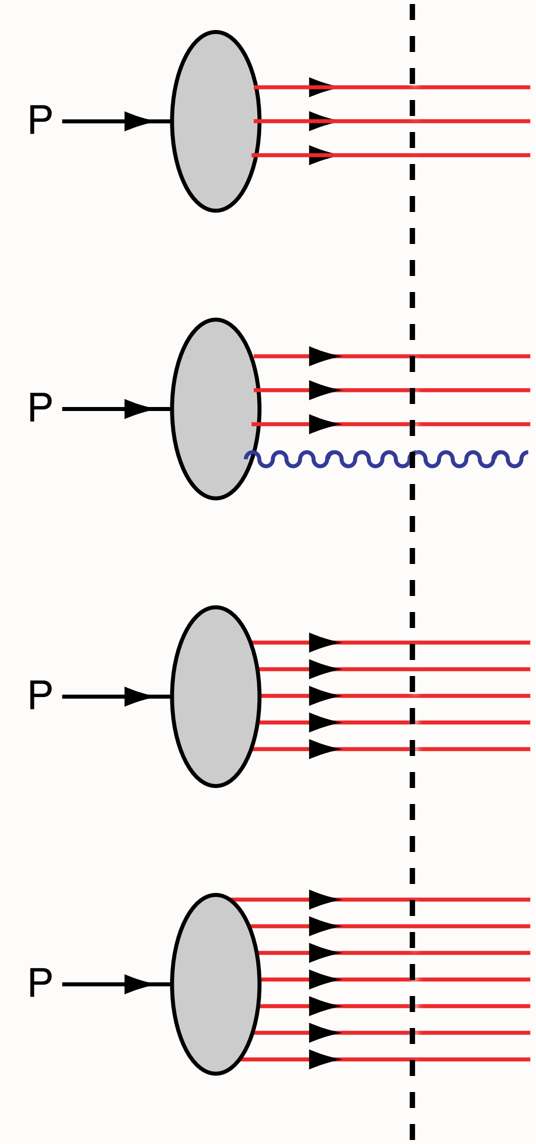
**n-1 orbital angular momenta**

*Parke-Taylor Amplitudes* **Stasto**

*Nonzero Anomalous Moment <--> Nonzero orbital angular momentum*
**Drell, sjb**




*Higher Fock States of the Proton*




Fixed LF time

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

*sum over states with n=3, 4, ...constituents*

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

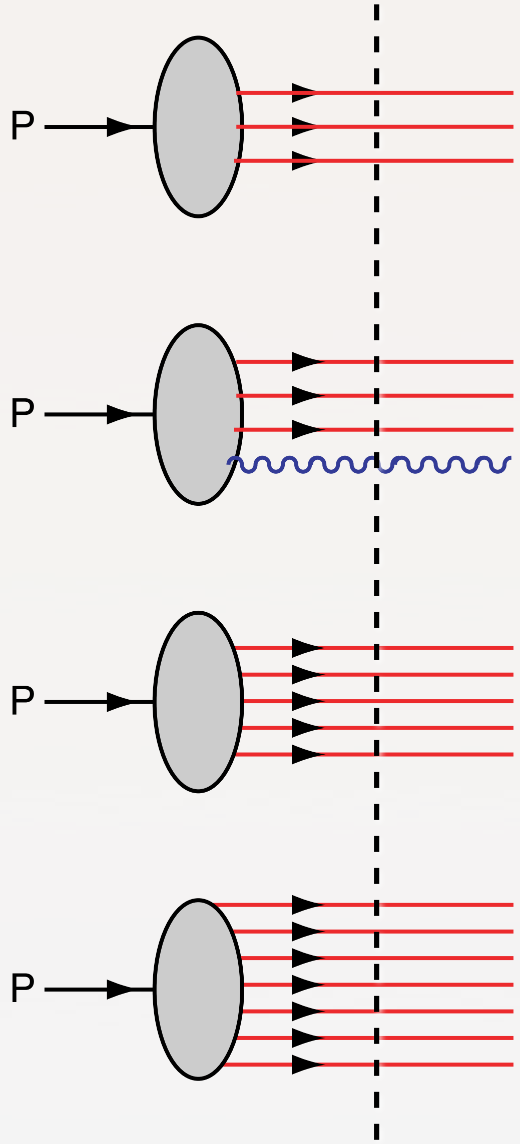
are boost invariant; they are independent of the hadron's energy and momentum $P^\mu$.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$




*Fixed LF time*

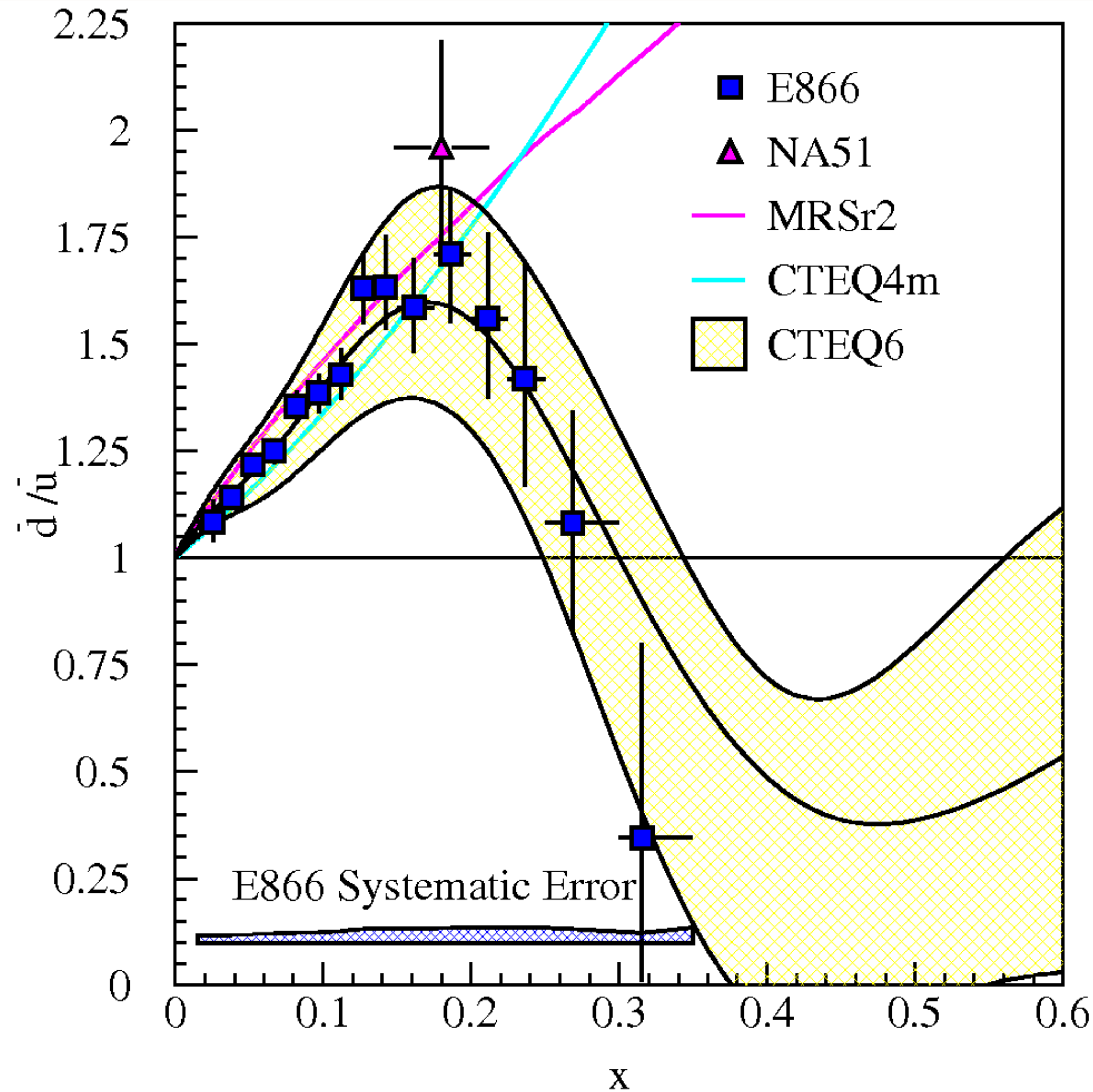
*Intrinsic heavy quarks*

***s(x), c(x), b(x) at high x !***

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

**Mueller: gluon Fock states** **BFKL Pomeron** *Hidden Color*

$\bar{d}(x)/\bar{u}(x)$ for $0.015 \le x \le 0.35$




- E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

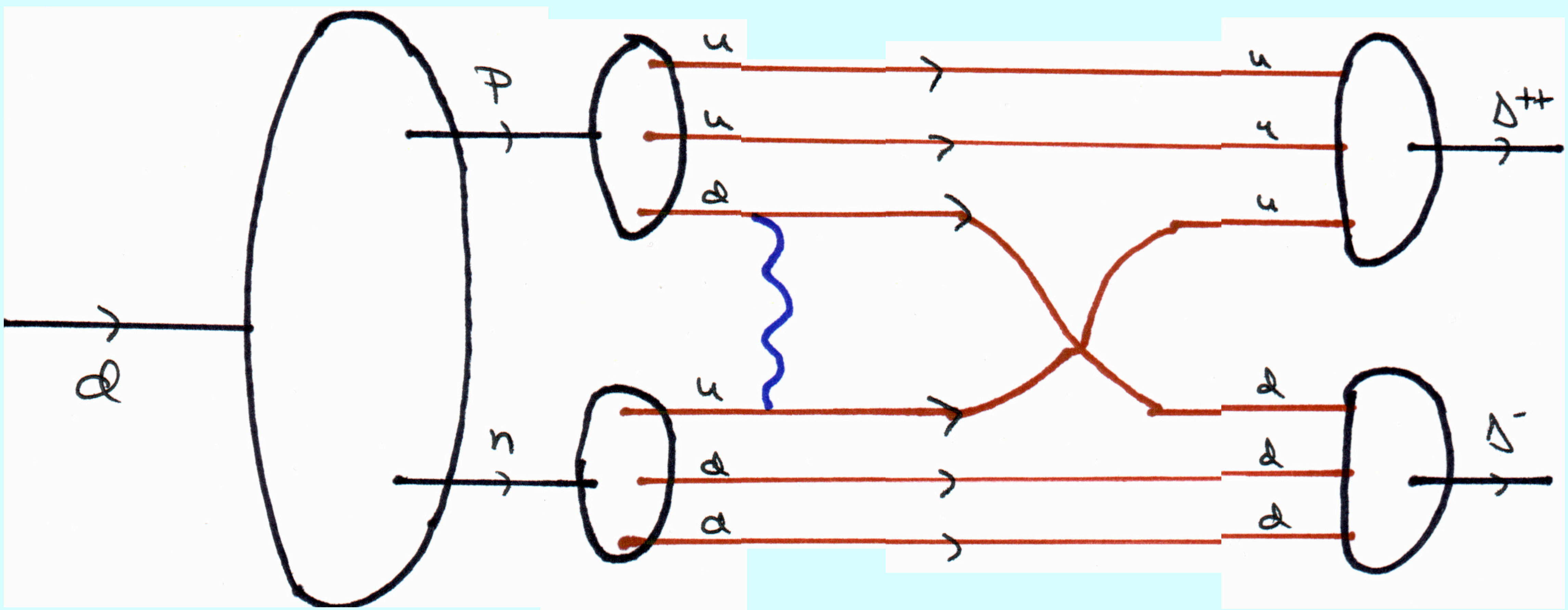
$$s(x) \neq \bar{s}(x)$$

*Intrinsic glue, sea, heavy quarks*

# Remarkable Features of Hadron Structure

- **Valence quark helicity represents less than half of the proton's spin and momentum**
- **Non-zero quark orbital angular momentum!**
- **Asymmetric sea:** $\bar{u}(x) \neq \bar{d}(x)$ **relation to meson cloud**
- **Non-symmetric strange and anti-strange sea** $\bar{s}(x) \neq s(x)$
- **Intrinsic charm and bottom at high x** $\Delta s(x) \neq \Delta \bar{s}(x)$
- **Hidden-Color Fock states of the Deuteron**

# Structure of Deuteron in QCD




Hidden Color Fock State

Delta-Delta Fock State

# QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Chertok, Lepage, Ji, sjb

Define "Reduced" Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N{}^2(Q^2/4)} .$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

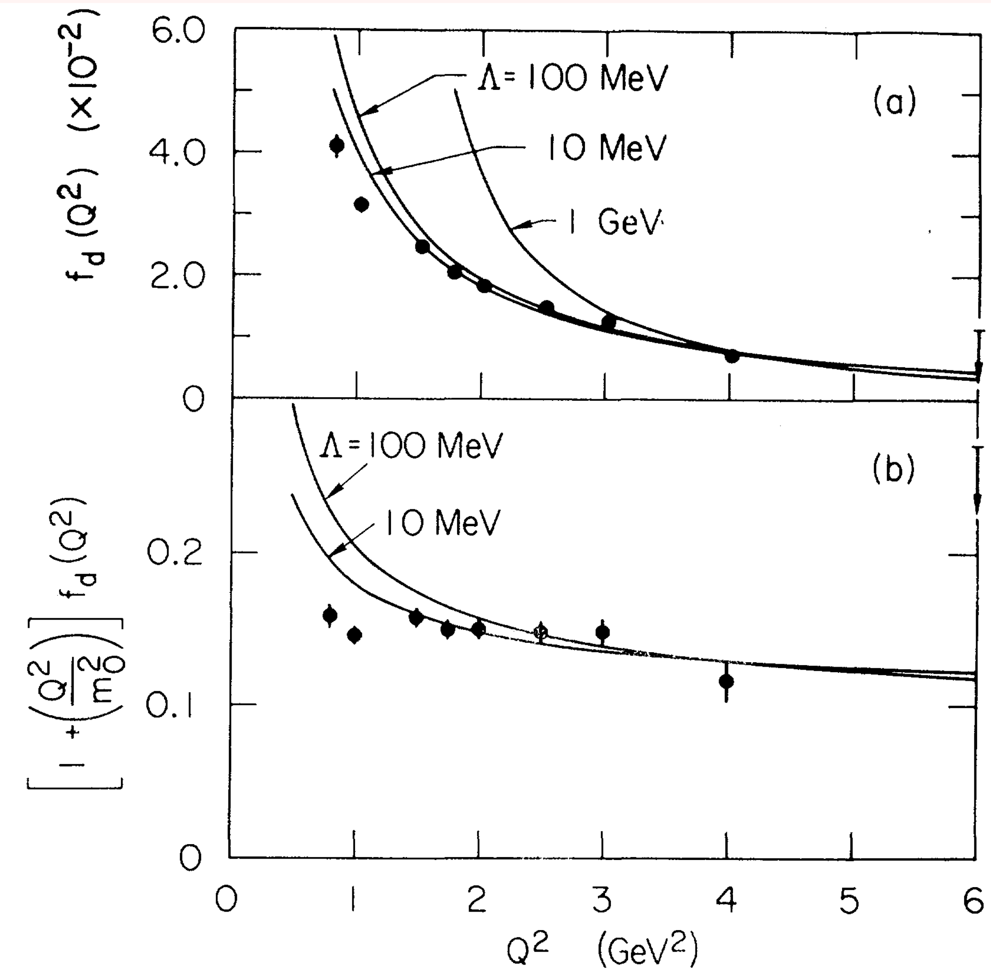
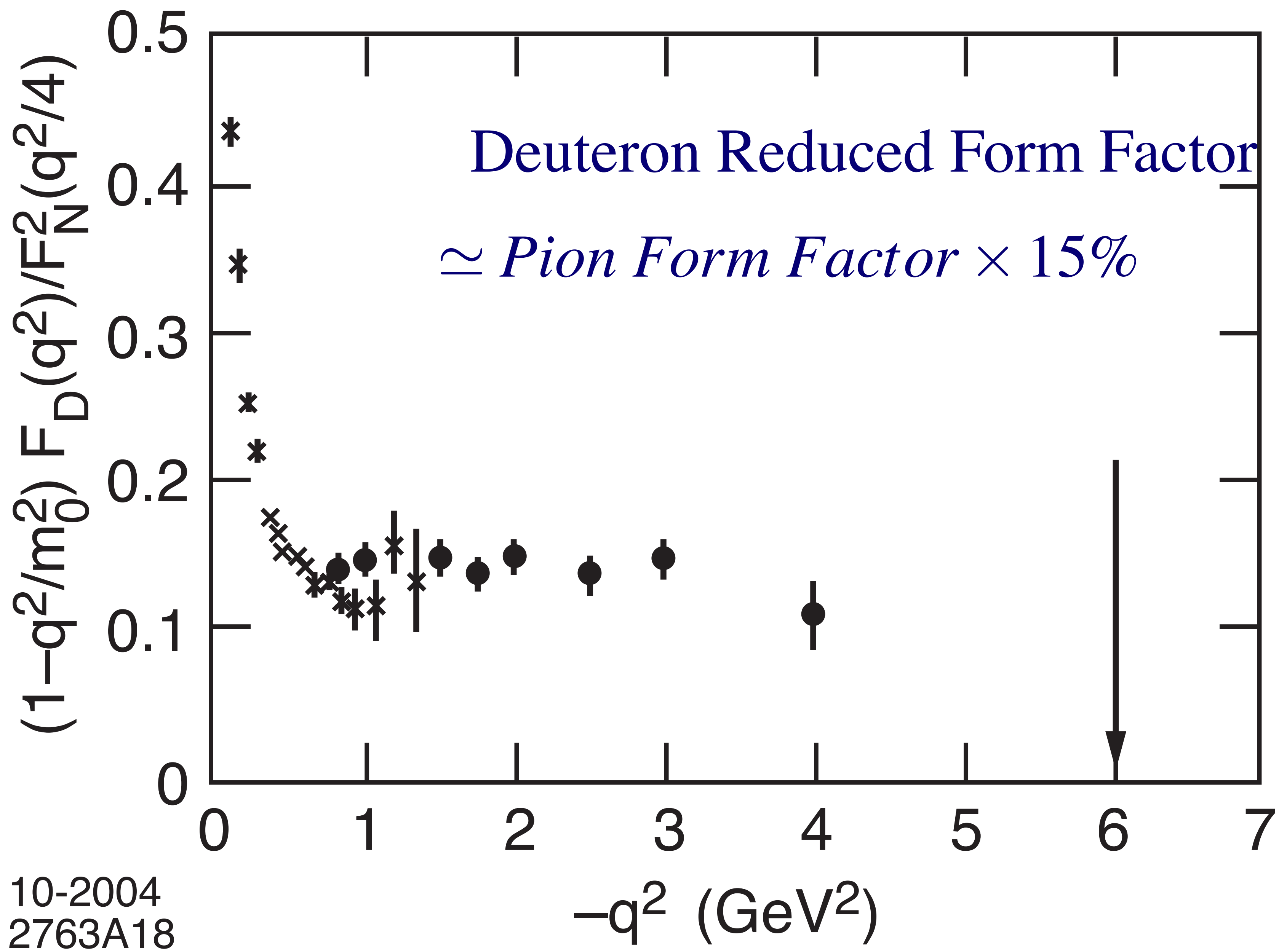



FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0{}^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0{}^2 = 0.28\ \mathrm{GeV}^2$ is used (Ref. 8).




Resembles pion form factor times 15%

- Indicates: ~ 15% Hidden Color in the Deuteron

# Scaling of deuteron FFs

**CCR in elastic scattering**

leading term: $FF \propto \left(Q^2\right)^{-n_d}$

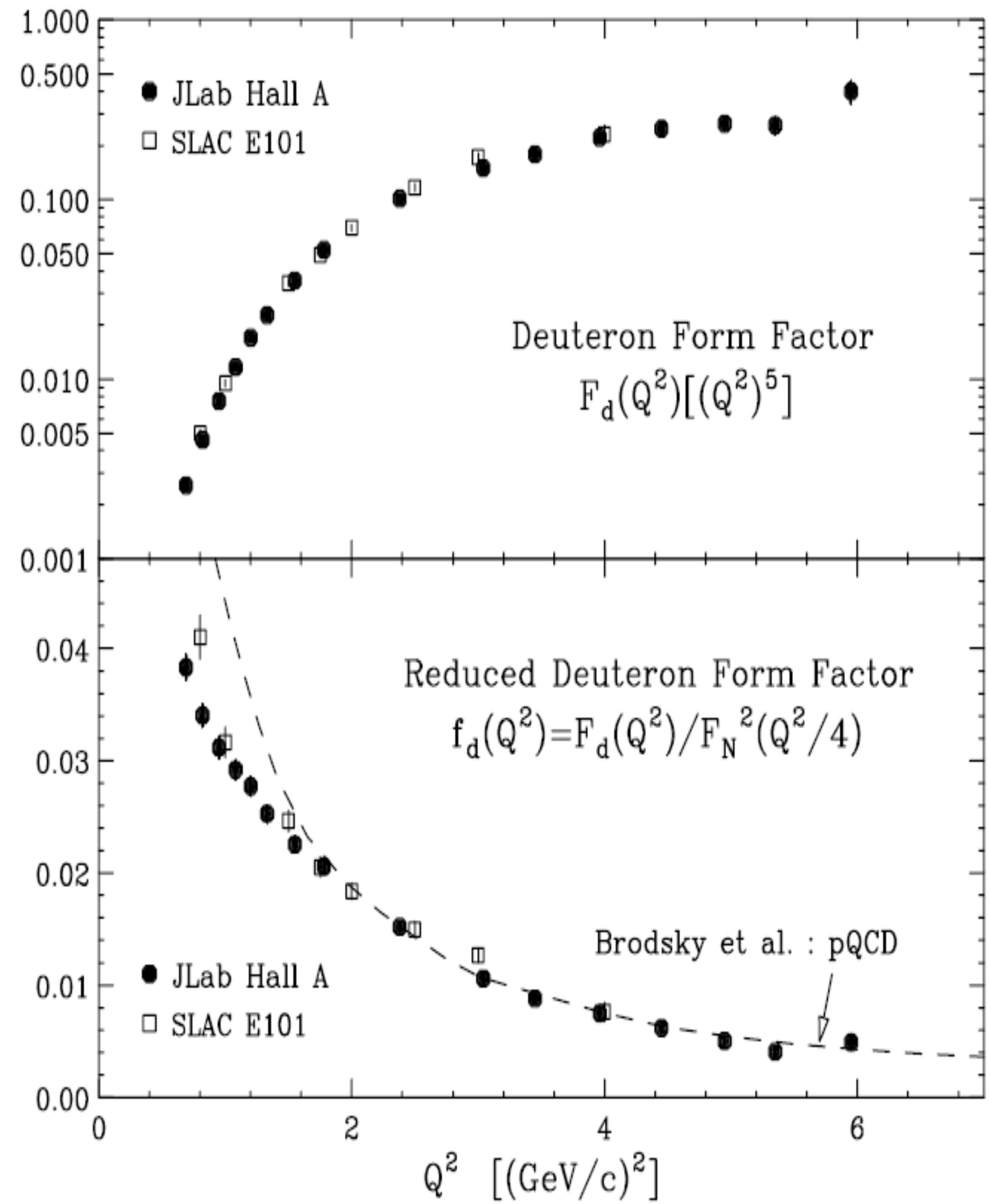
the "deuteron FF":

$$F_d \equiv \sqrt{A} \propto Q^{-10}$$

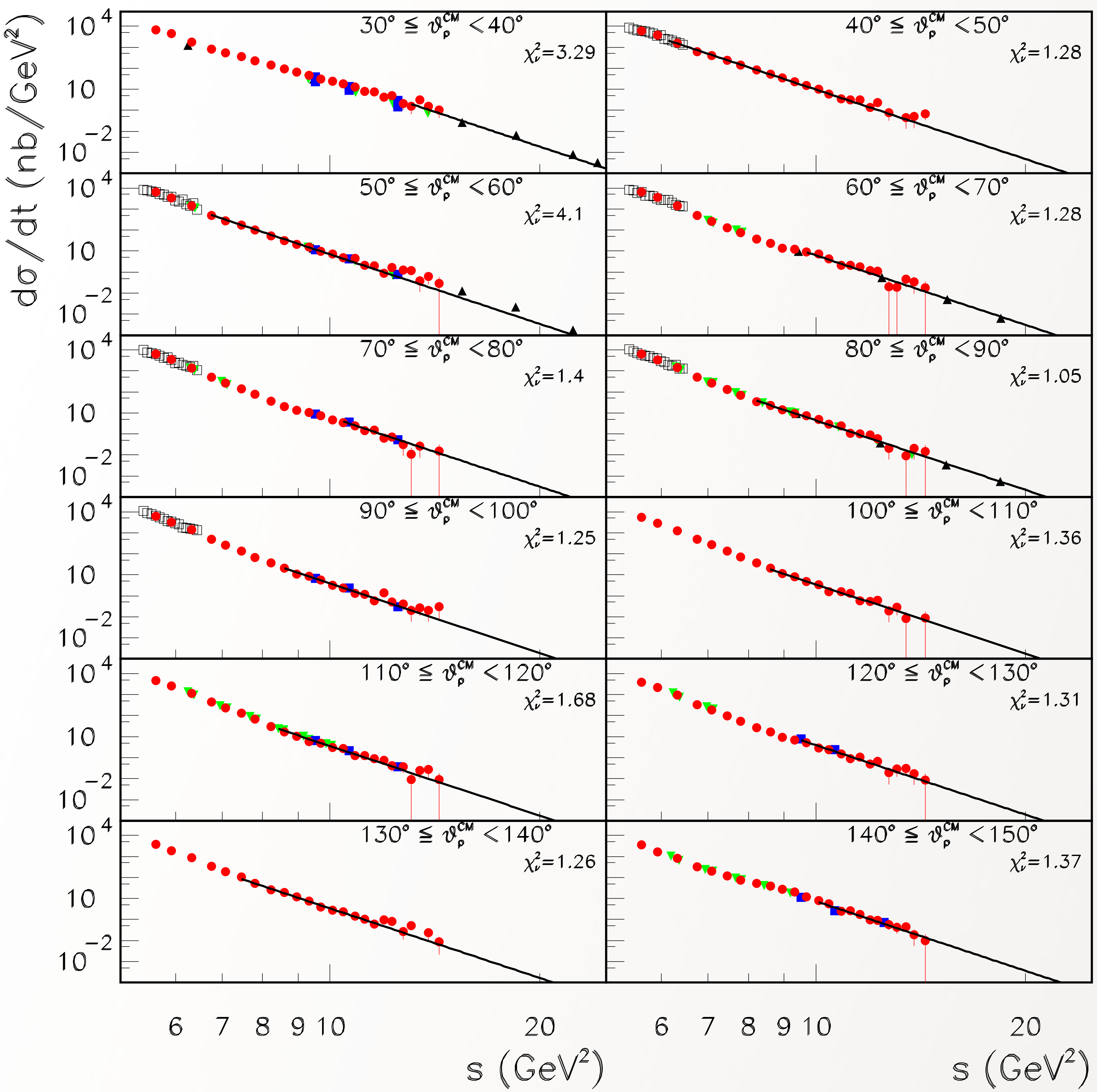
**For $Q^2$ above ≈4 $GeV^2$ data are consistent with CCR**

Super B: Measure deuteron pairs

Alexa et al., PRL 82,1374 (1999)



# Deuteron Photodisintegration & Dimensional Counting Rules




PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D)=$$
$$F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np)=F(\theta_{CM})$$

$n_{tot}-2=$
$(1 + 6 + 3 + 3) - 2 = 11$

- Remarkable Test of Quark Counting Rules

- Deuteron Photo-Disintegration $\gamma d \to np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$

- $$n_{tot} = 1 + 6 + 3 + 3 = 13$$

Scaling characteristic of
scale-invariant theory at short distances

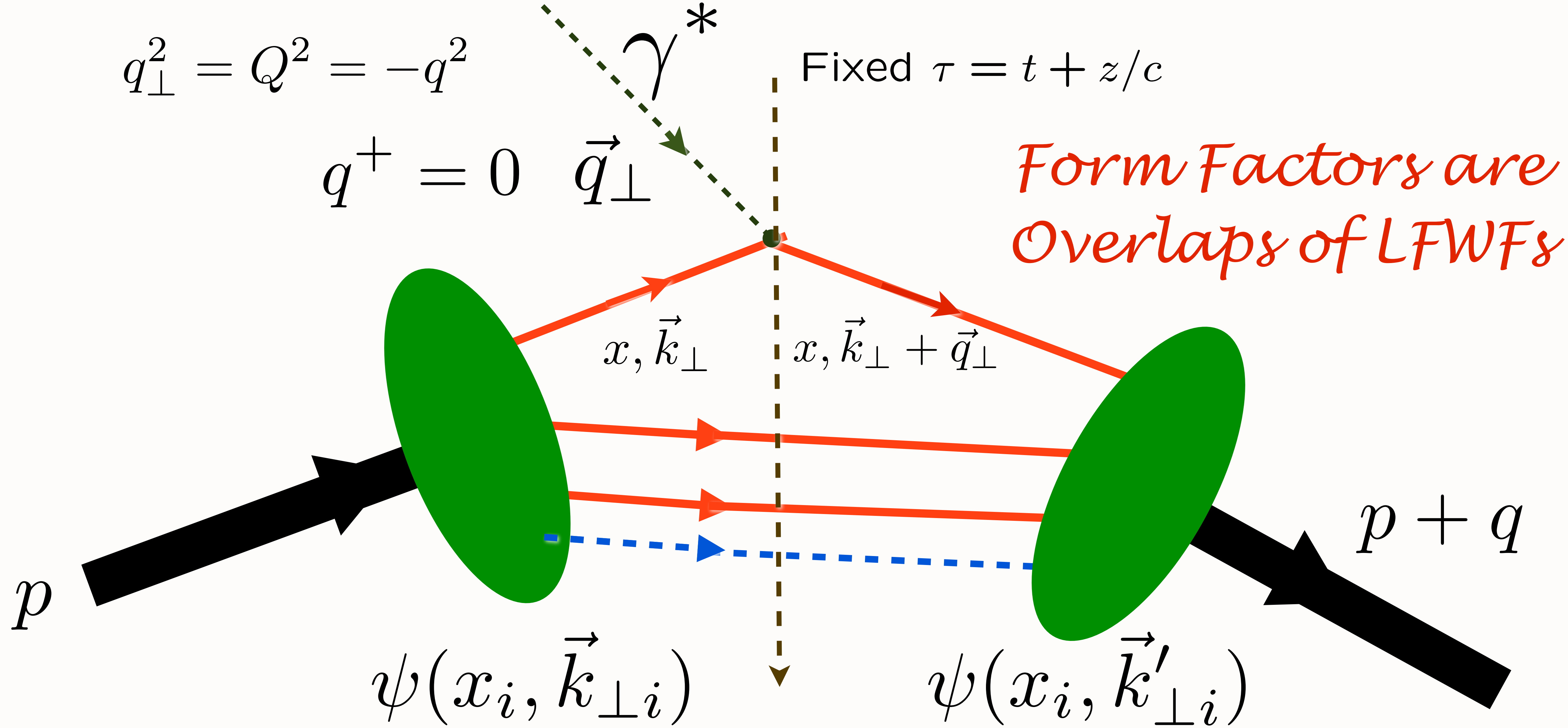
Conformal symmetry

Hidden color: $$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$$

at high $p_T$

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture




*struck* $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

*spectators* $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

**Drell &Yan, West**
**Exact LF formula**

# Exact LF Formula for Pauli Form Factor

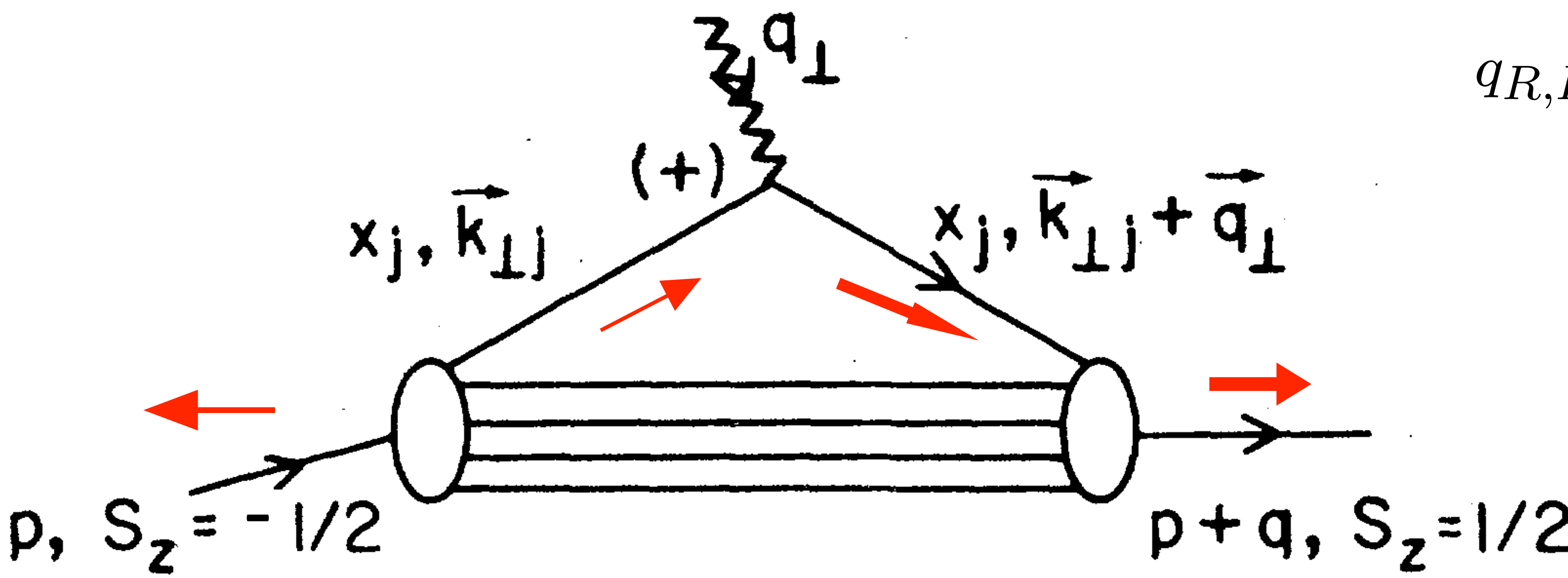
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \, \times$$

Drell, sjb

$$\left[ -\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

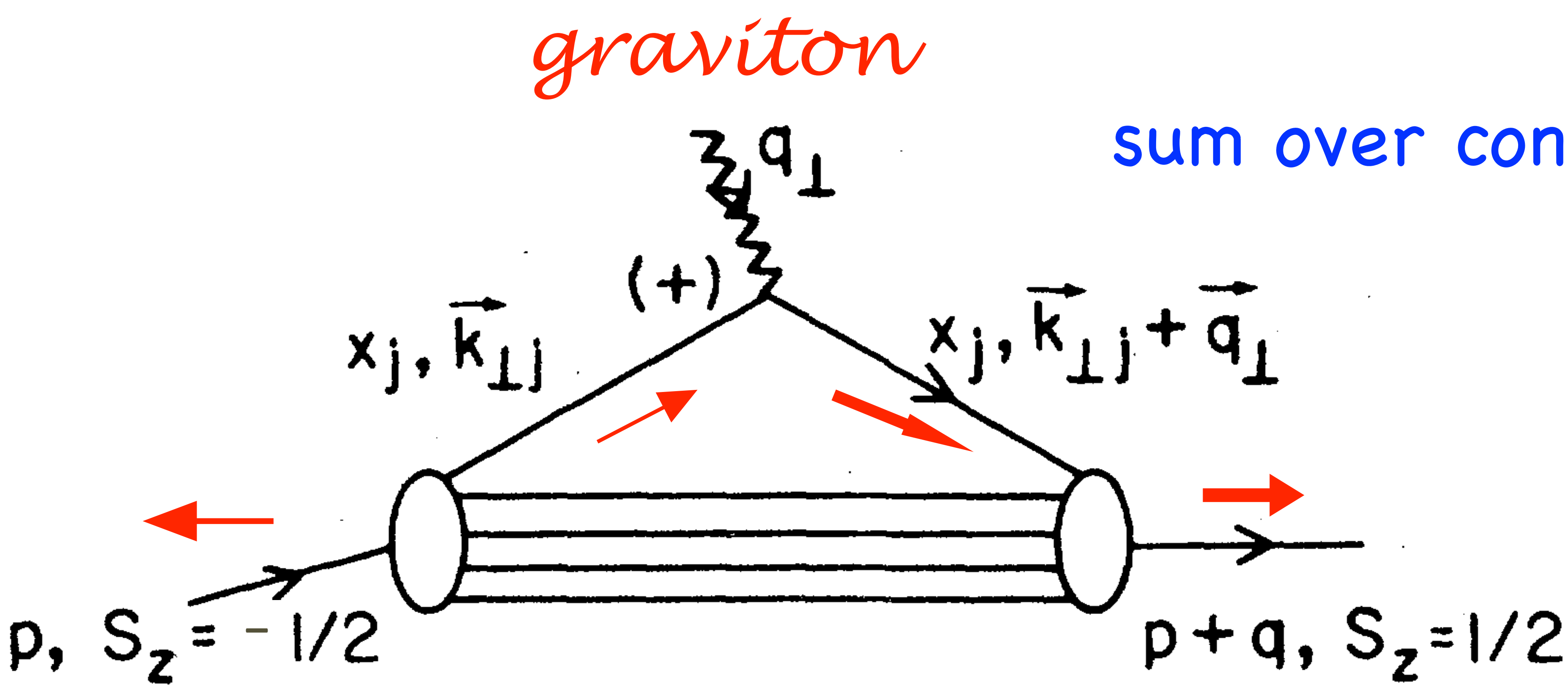



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

*Nonzero Proton Anomalous Moment -->*
*Nonzero orbital quark angular momentum*

# Anomalous gravitomagnetic moment B(0)

**Terayev, Okun, et al:** *B(0) Must vanish because of Equivalence Theorem*



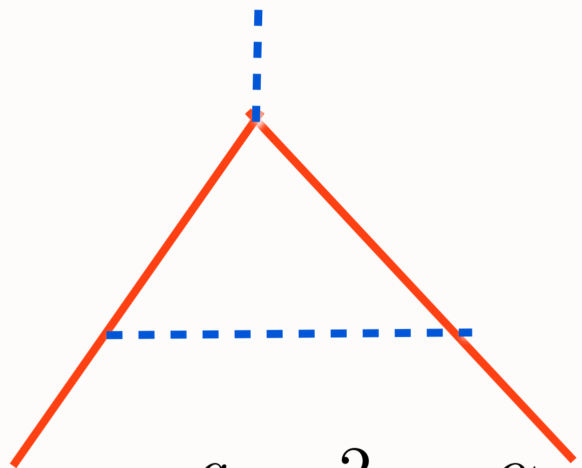

sum over constituents

**Hwang, Schmidt, sjb; Holstein et al**

B(0) = 0

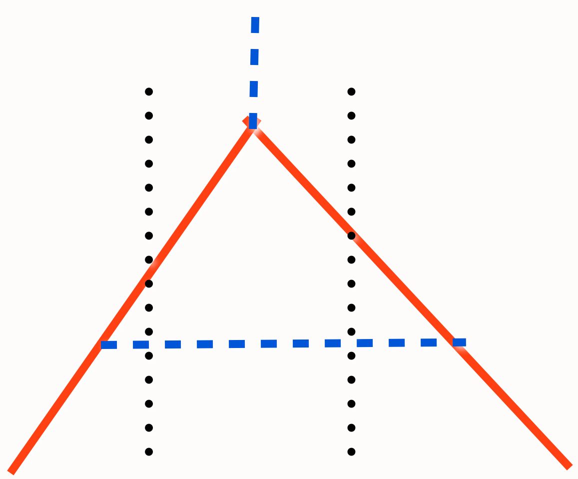
*Each Fock State*

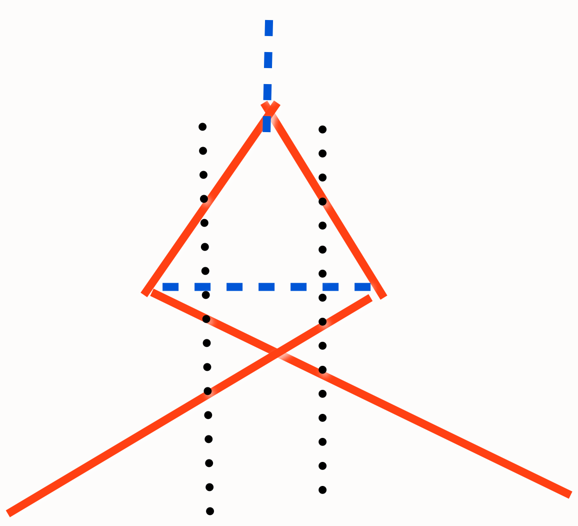
# Wick Theorem

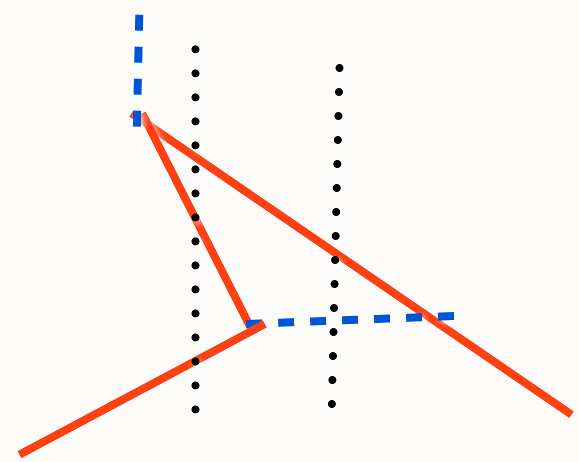


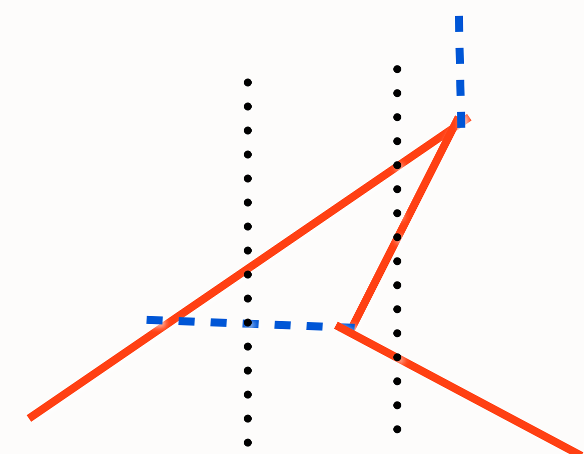
$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

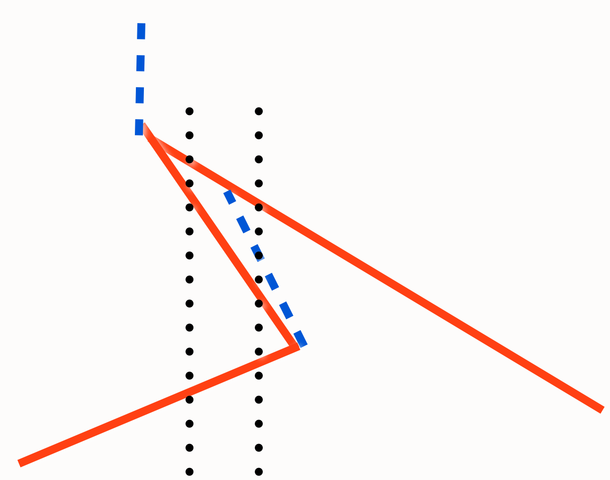
*Feynman diagram = sum n!*
*instant-form time-ordered diagrams*

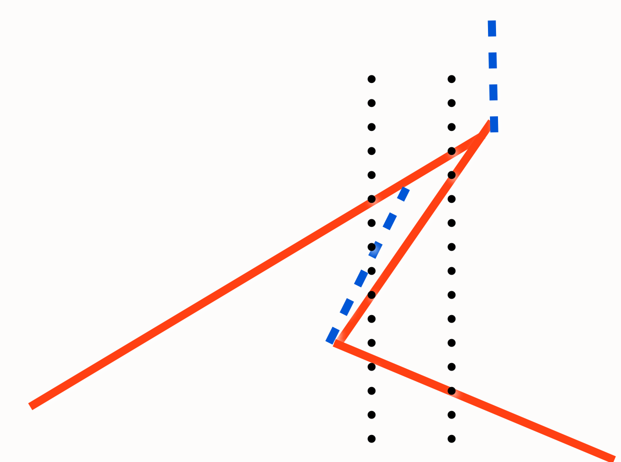






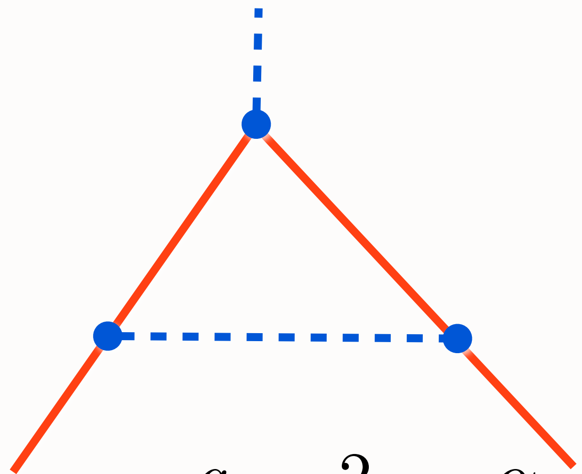




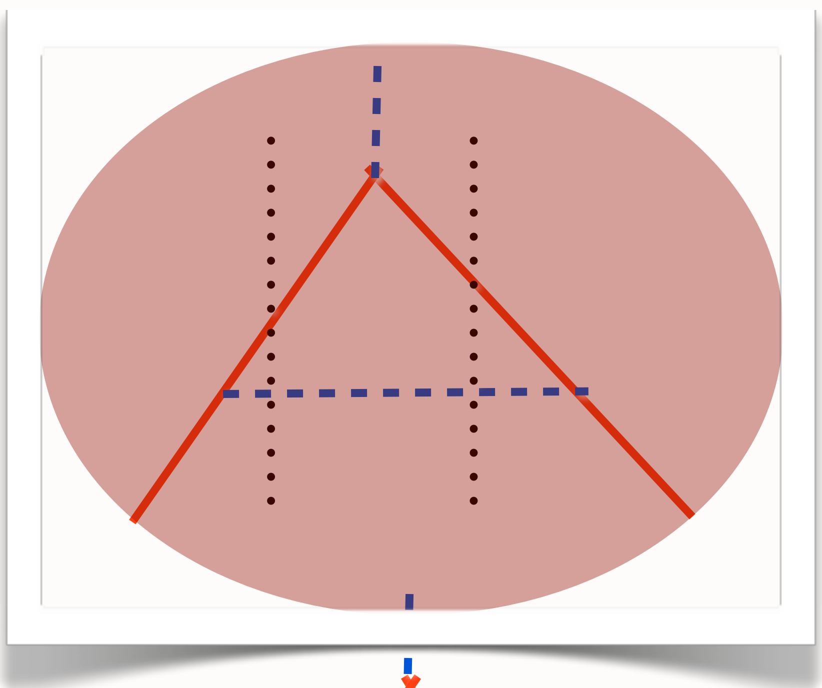


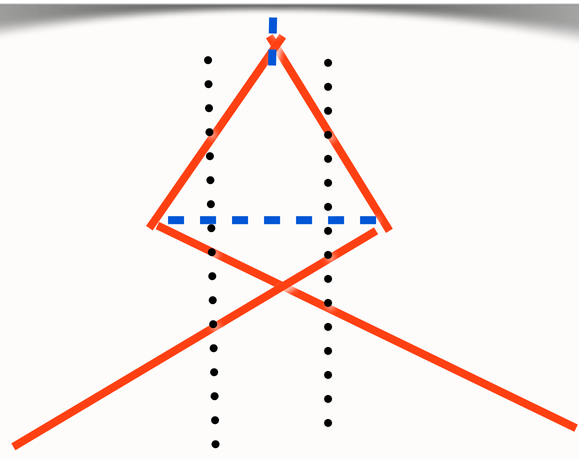
# Wick Theorem

*Feynman diagram =*
*single front-form time-ordered diagram!*

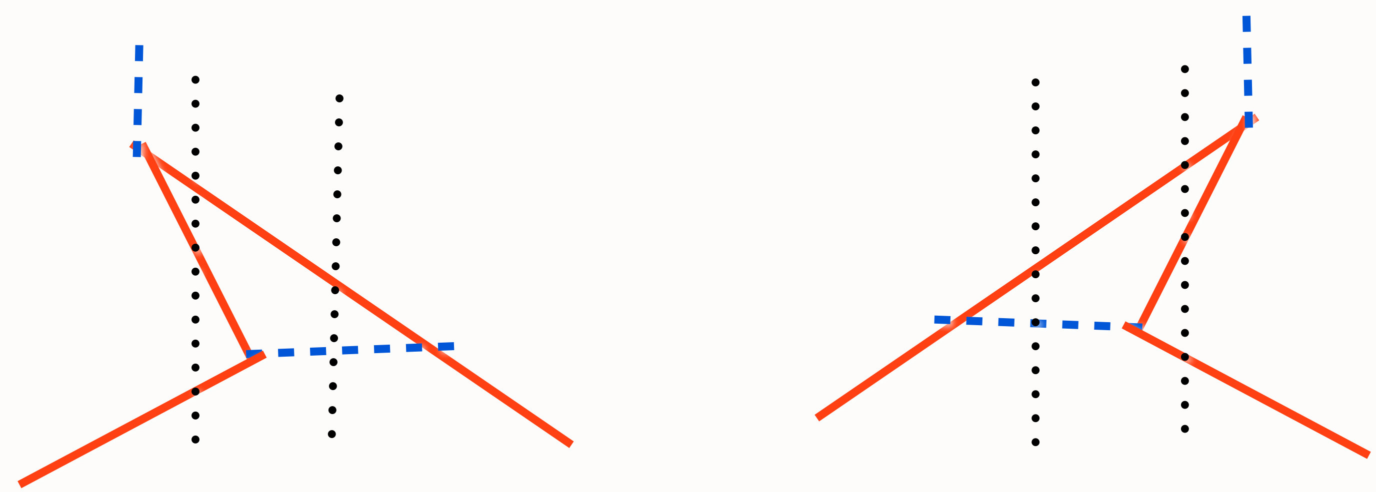


$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

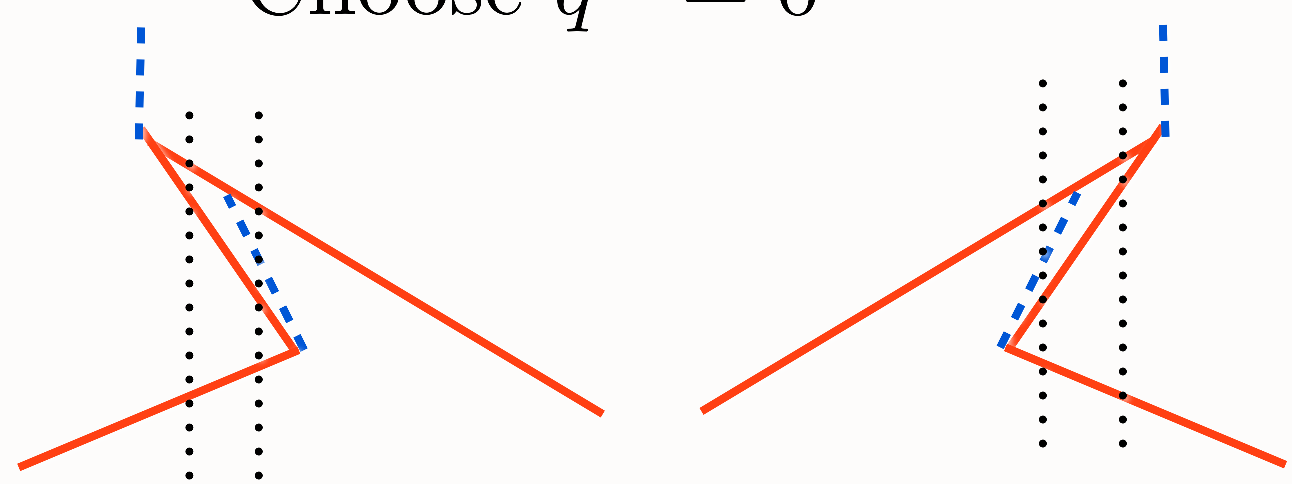




Also $P \to \infty$ observer frame (Weinberg)

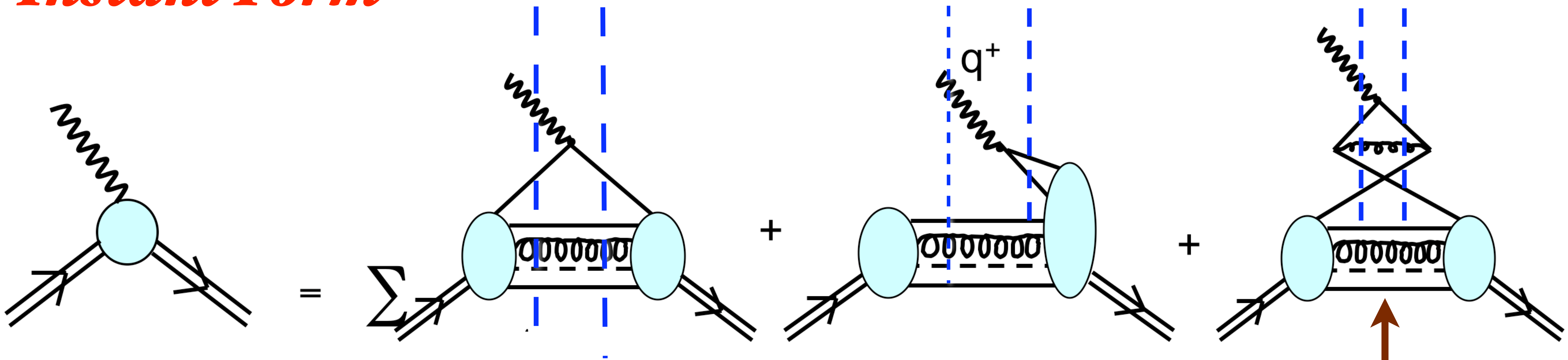


Choose $q^+ = 0$



# Calculation of Form Factors in Equal-Time Theory

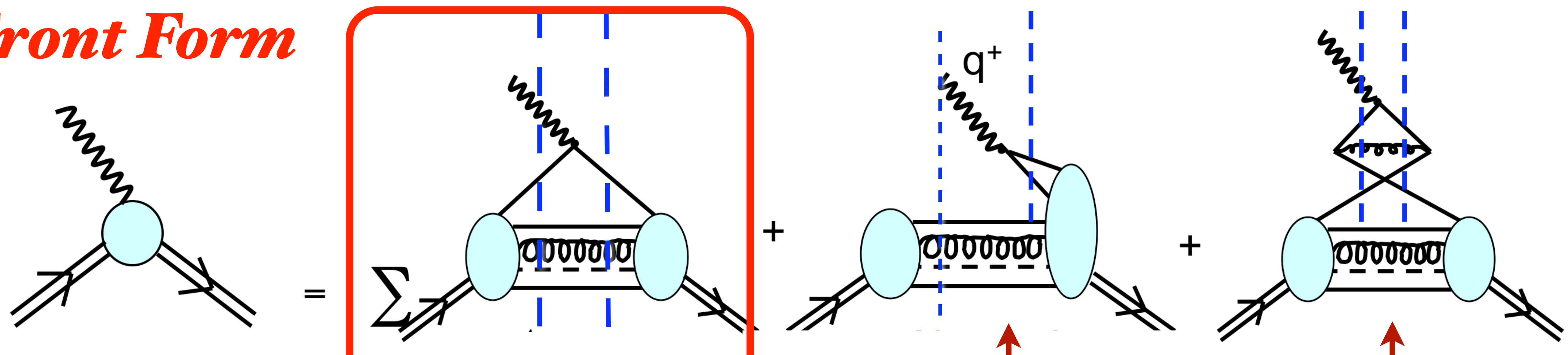
## Instant Form




**Need vacuum-induced currents**

# Calculation of Form Factors in Light-Front Theory

## Front Form




*Exact Answer!*

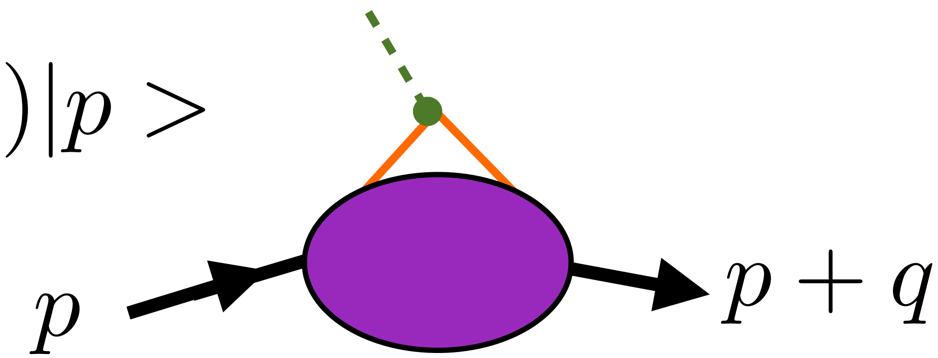
Absent for $q^+ = 0$

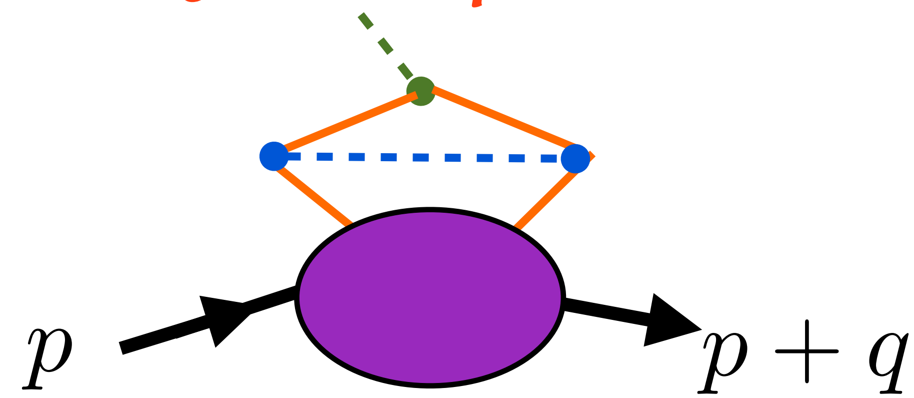
***zero !!***

*No vacuum graphs*

# Calculation of proton form factor in Instant Form

$< p + q | J^{\mu}(0) | p >$




- **Need to boost proton wavefunction from p to p +q: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**

# Light-Front vs. Instant Form

- **Light-Front Wavefunctions are frame-independent**
- **Boosting an instant-form wavefunctions dynamical problem -- extremely complicated even in QED**
- **Vacuum state is lowest energy eigenstate of Hamiltonian**
- **Light-Front Vacuum same as vacuum of free Hamiltonian**
- **Zero anomalous gravitomagnetic moment**
- **Instant-Form Vacuum infinitely complex even in QED**
- **n! time-ordered diagrams in Instant Form**
- **Causal commutators using LF time; cluster decomposition**

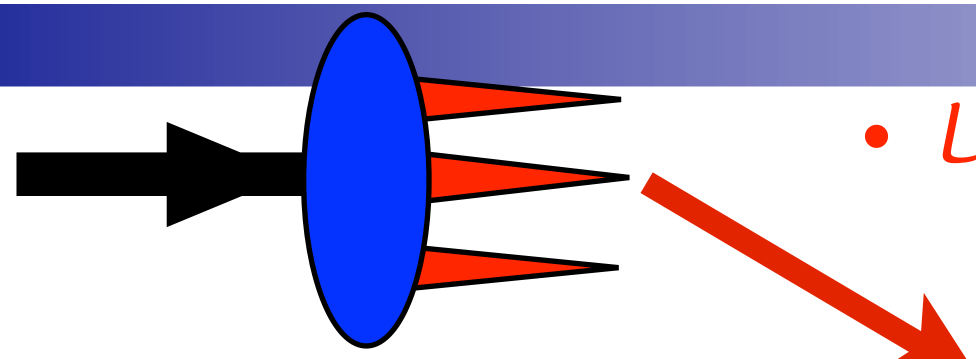
# Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

- *Light Front Wavefunctions:*



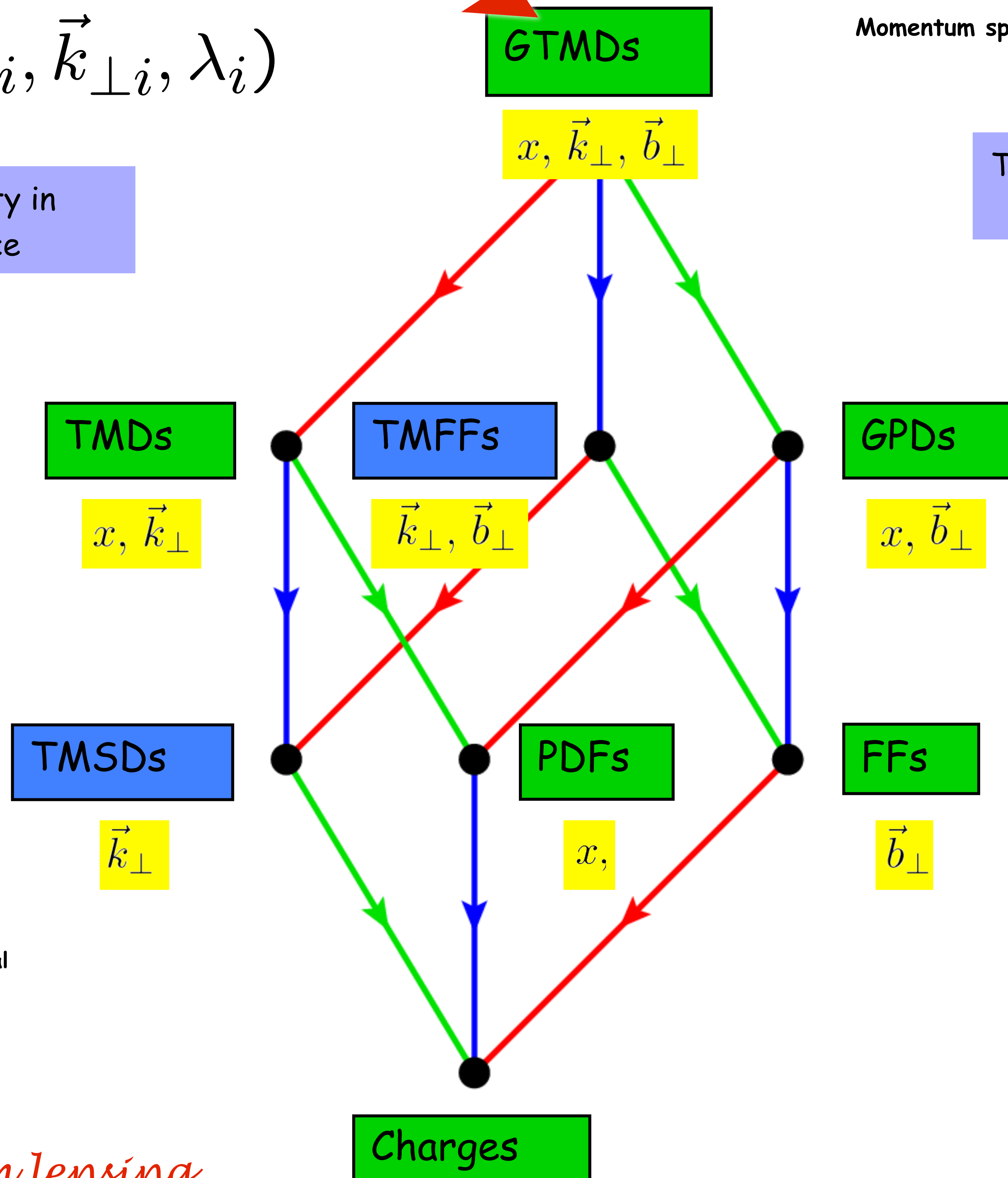
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

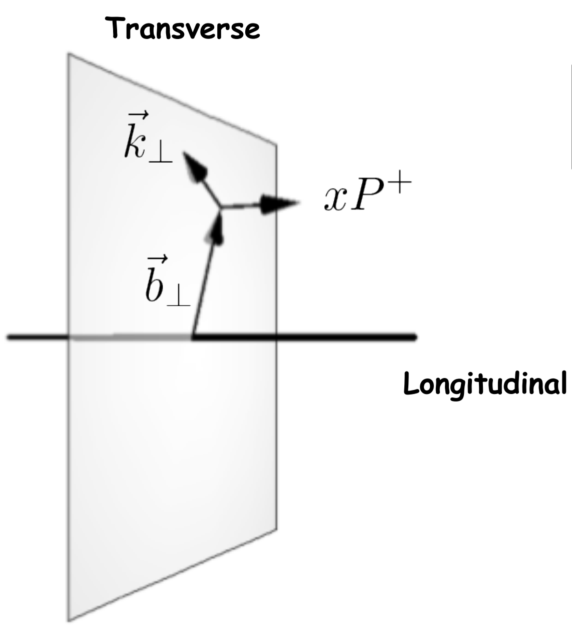
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

Transverse density in position space




*Sivers, T-odd from lensing*

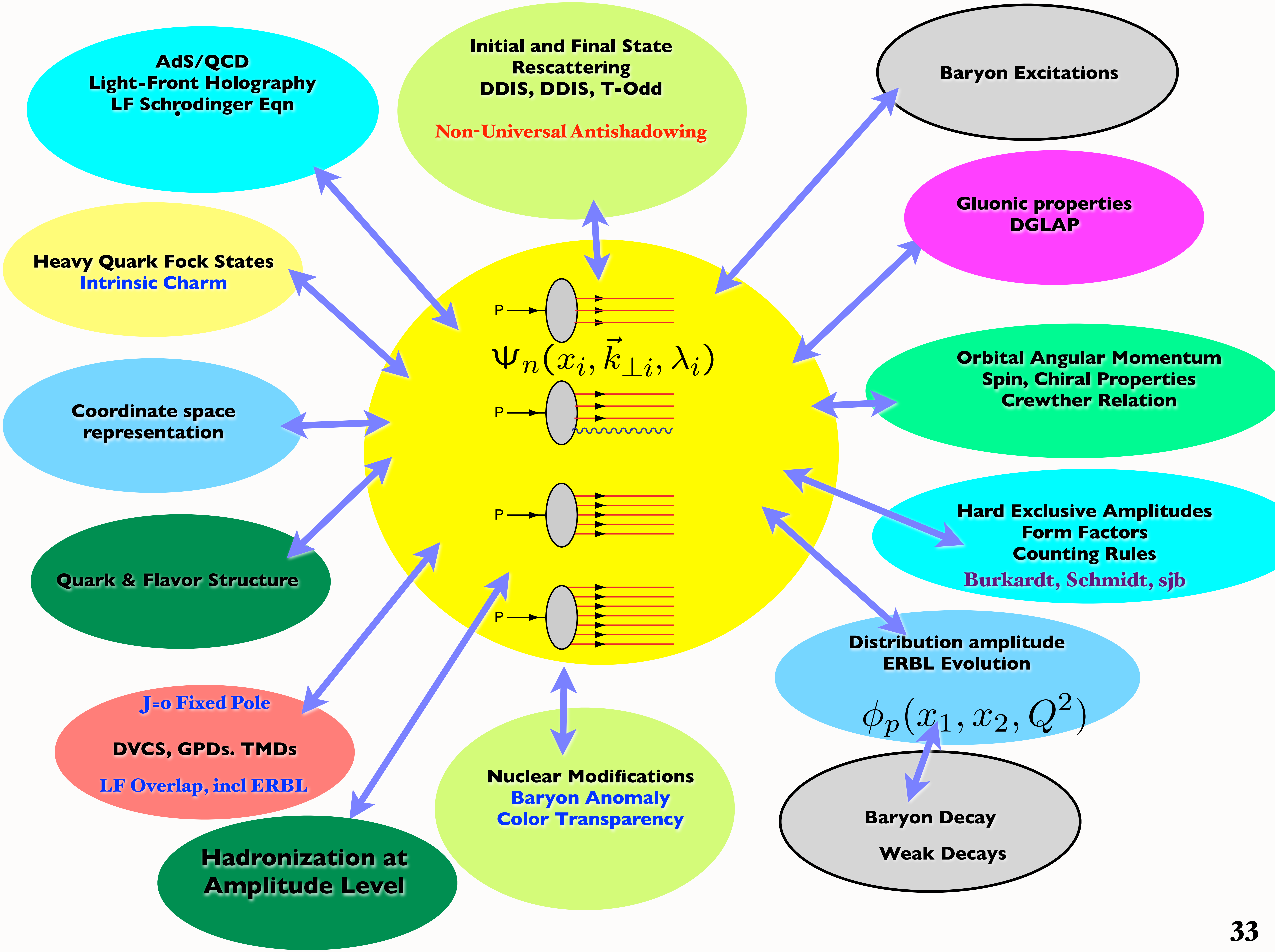
**Lorce**

$\int d^2 b_\perp$

$\int dx$

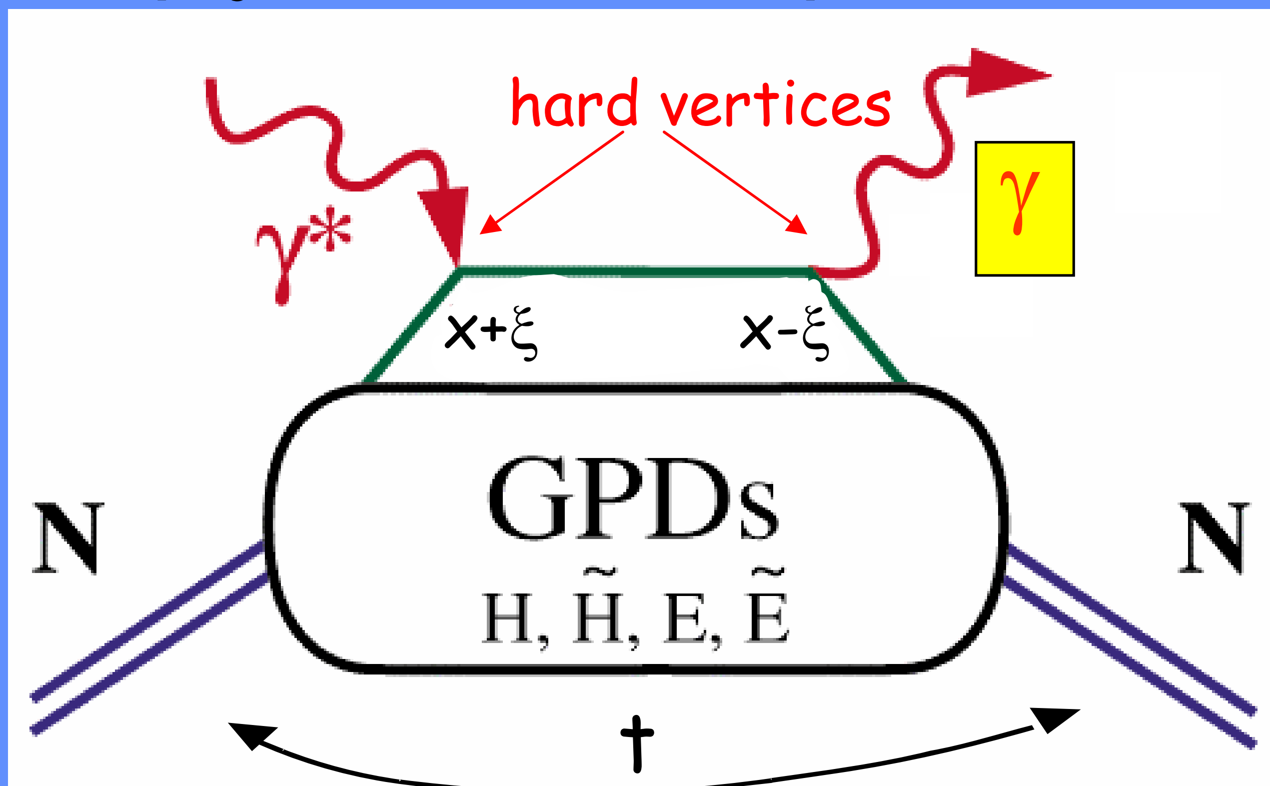
$\int d^2 k_\perp$

# QCD and the LF Hadron Wavefunctions



# GPDs & Deeply Virtual Exclusive Processes
## - New Insight into Nucleon Structure

### Deeply Virtual Compton Scattering (DVCS)




x – quark momentum fraction

ξ– longitudinal momentum transfer

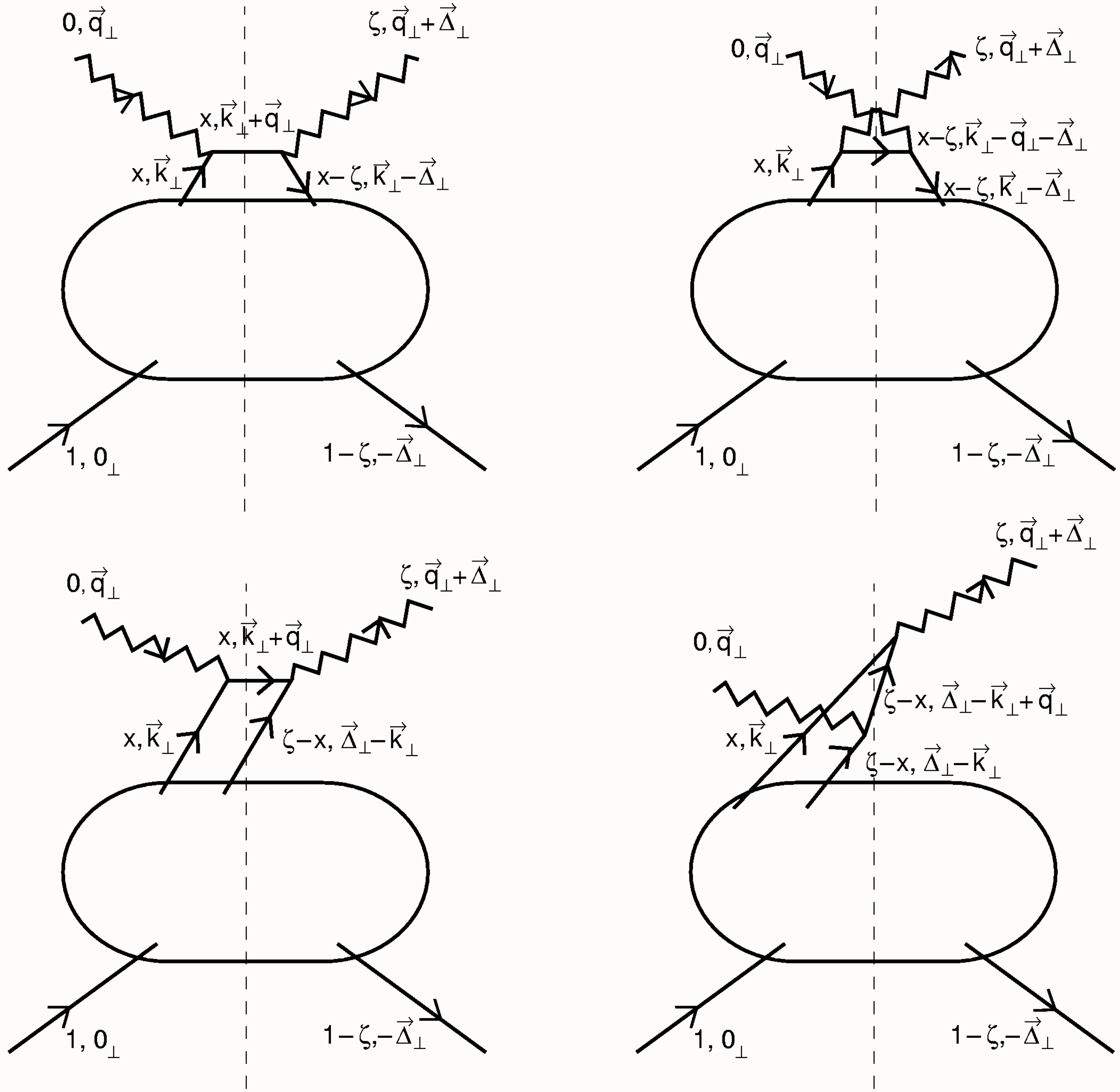
$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

H(x,ξ,t), E(x,ξ,t), . . . "Generalized Parton Distributions"

- Generalized Parton Distributions in gauge/gravity duals

  [Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]

  [Nishio and Watari, arXiv:1105.290]




# Light-cone wavefunction representation of deeply virtual Compton scattering ☆

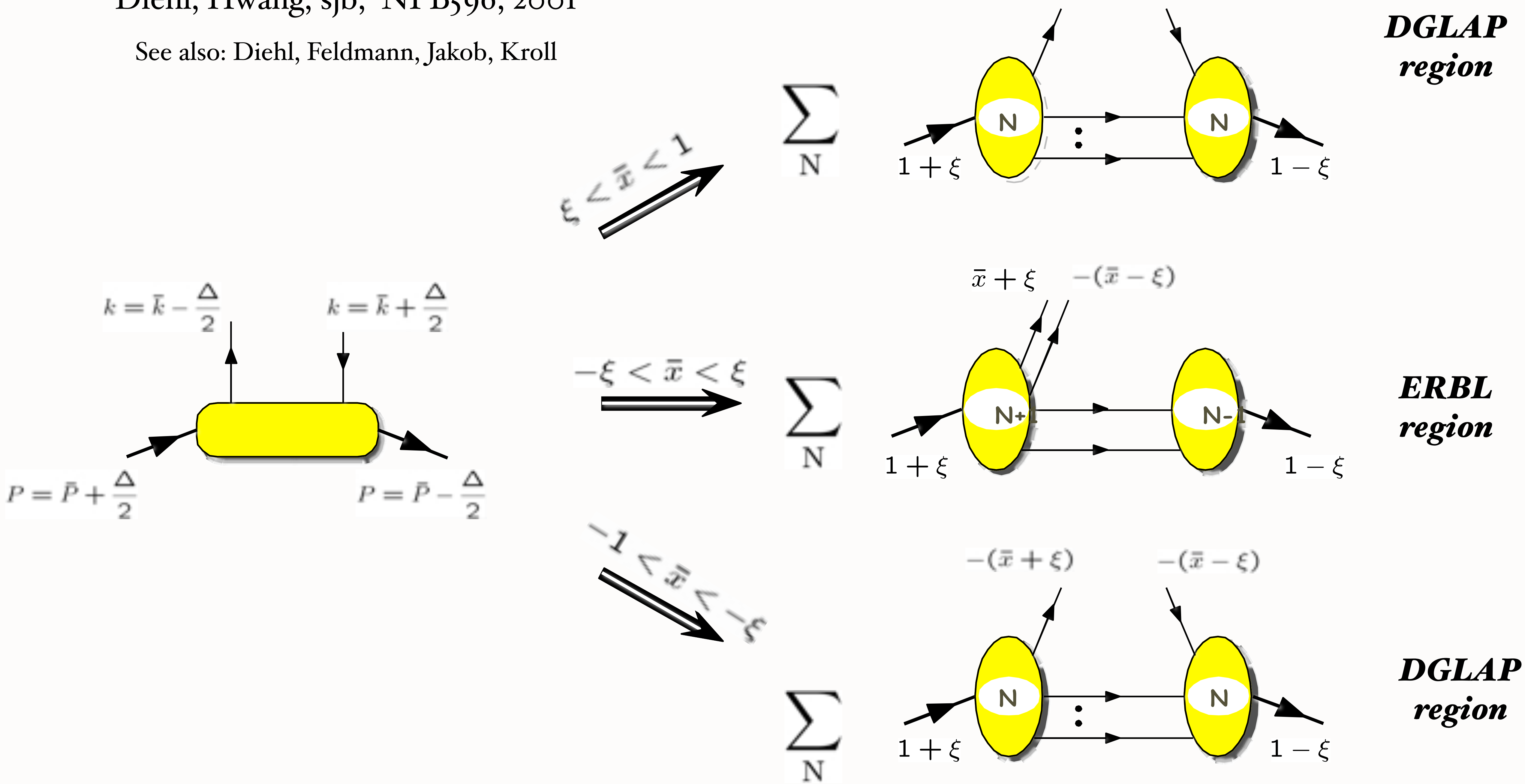
Stanley J. Brodsky [a], Markus Diehl [a,1], Dae Sung Hwang [b]

# Light-Front Wave Function Overlap Representation

## DVCS/GPD

Diehl, Hwang, sjb, NPB596, 2001

See also: Diehl, Feldmann, Jakob, Kroll




*Bakker & Ji*
*Lorce*

# Link to DIS and Elastic Form Factors

DIS at $\xi = t = 0$

$$H^q(x,0,0) = q(x), \quad -\bar{q}(-x)$$

$$\widetilde{H}^q(x,0,0) = \Delta q(x), \quad \Delta\bar{q}(-x)$$

Form factors (sum rules)

$$\int^1 dx \sum_q \left[H^q(x,\xi,t)\right] = F_1(t) \text{ Dirac f.f.}$$

$$\int^1 dx \sum_q \left[E^q(x,\xi,t)\right] = F_2(t) \text{ Pauli f.f.}$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = G_{A,q}(t), \quad \int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = G_{P,q}(t)$$

$$H^q, E^q, \widetilde{H}^q, \widetilde{E}^q(x,\xi,t)$$

Verified using LFWFs

Diehl, Hwang, sjb

**Quark angular momentum** (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x dx \left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

# Example of LFWF representation of GPDs (n => n)

Diehl, Hwang, sjb

$$\frac{1}{\sqrt{1-\zeta}} \frac{\Delta^1 - i\,\Delta^2}{2M} E_{(n\to n)}(x,\zeta,t)$$

$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1 - \sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$

$$\times\, \delta(x - x_1)\, \psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\downarrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

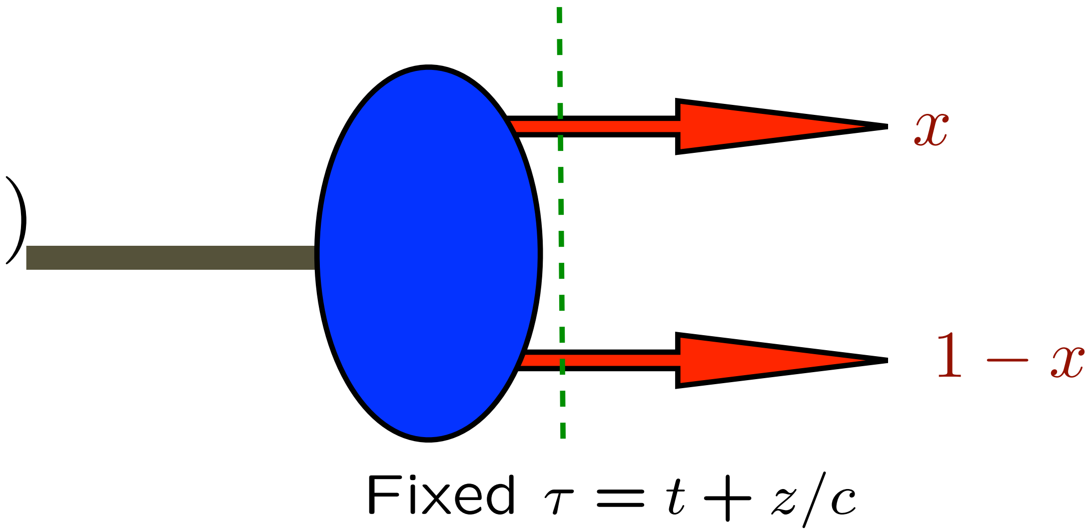
where the arguments of the final-state wavefunction are given by

$$x'_1 = \frac{x_1 - \zeta}{1-\zeta}, \qquad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{1-x_1}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the struck quark,}$$

$$x'_i = \frac{x_i}{1-\zeta}, \qquad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\zeta}\vec{\Delta}_\perp \quad \text{for the spectators } i = 2, \ldots, n.$$

# Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$




$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

***Lepage, sjb***

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

*Lepage, sjb*

*Efremov, Radyushkin*

- Evolution Equations from PQCD, OPE

*Sachrajda, Frishman Lepage, sjb*

- Conformal Expansions

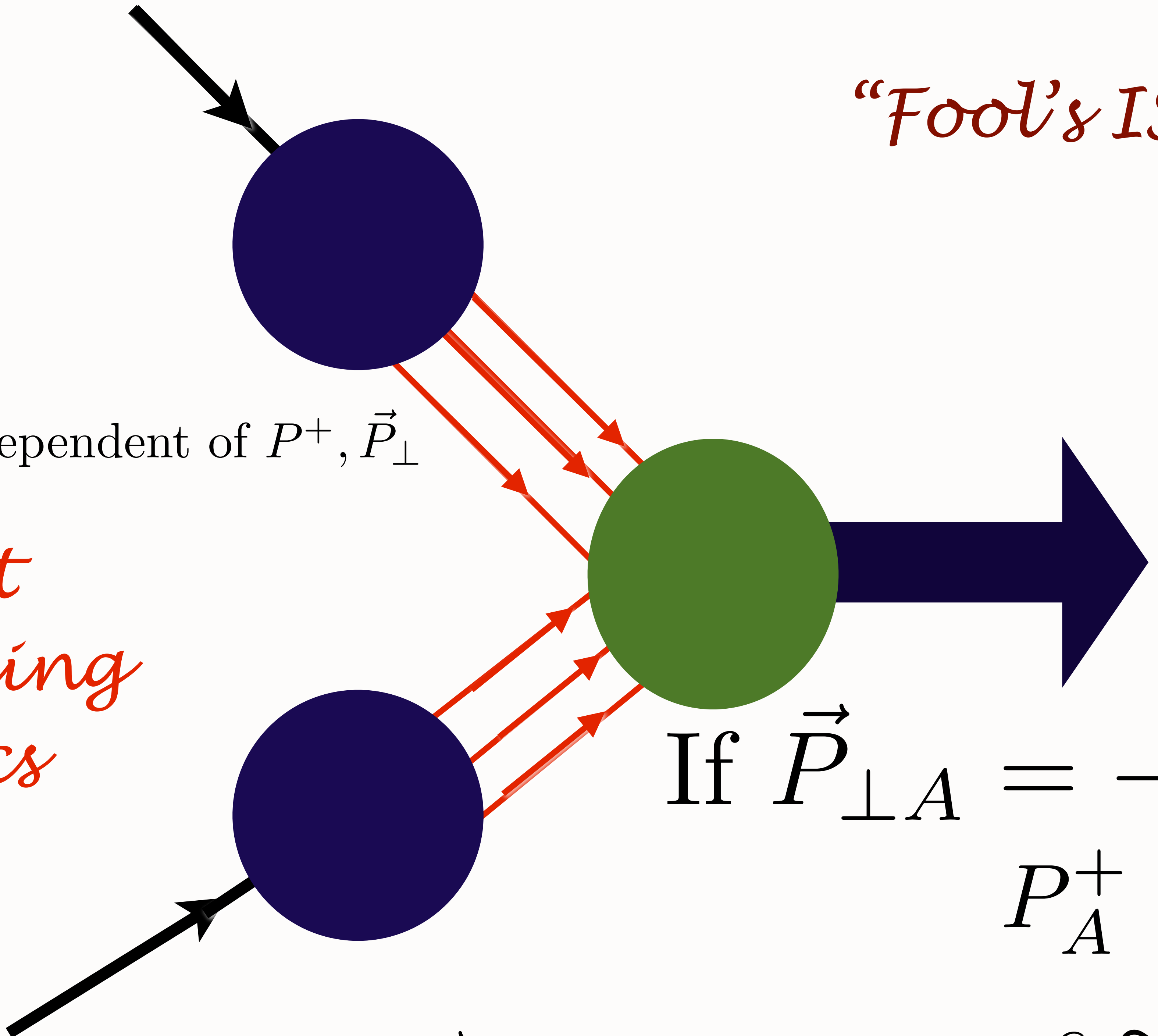
*Braun, Gardi*

- Compute from valence light-front wavefunction in light-cone gauge

$$P_A^\mu = (P_A^+, P_A^-, \vec{P}_{\perp A})$$

$$P^- = \frac{P_\perp^2 + M^2}{P^+}$$

"Fool's ISR Frame"



$\psi(x, k_\perp)$ independent of $P^+, \vec{P}_\perp$

Not colliding discs

If $\vec{P}_{\perp A} = -\vec{P}_{\perp B} = \vec{P}_\perp$

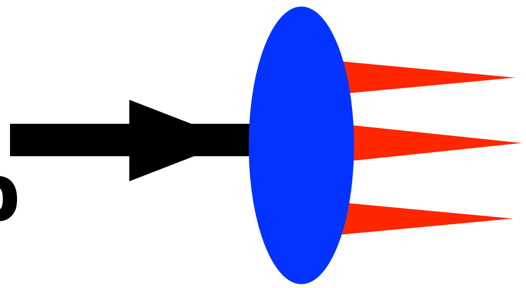
$$P_A^+ = P_B^+$$

$$s \simeq 4P_\perp^2$$

$$P_B^\mu = (P_B^+, P_B^-, \vec{P}_{\perp B})$$

$$s = (P_A + P_B)^2 = M_A^2 + M_B^2 + \frac{P_{\perp A}^2 + M_A^2}{P_A^+} P_B^+ + \frac{P_{\perp B}^2 + M_B^2}{P_B^+} P_A^+ - 2\vec{P}_{\perp A} \cdot \vec{P}_{\perp B}$$

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**

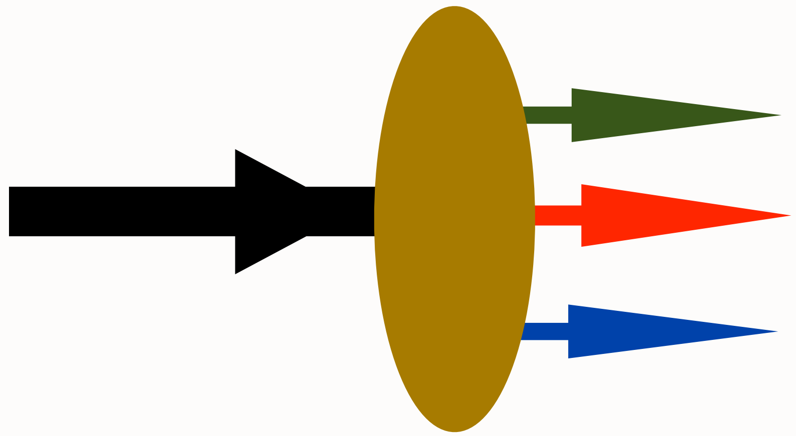
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**

- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, .... modulo `lensing' from ISIs, FSIs**

- **Cannot compute current matrix elements using instant or point form from eigensolutions alone -- need to include vacuum currents!**

- **Hadron Physics without LFWFs is like Biology without**

- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

# Solving nonperturbative QCD using the Front Form

*Hornbostel, Pauli, sjb*

- **Heisenberg: Diagonalize the QCD LF Hamiltonian**
- **DLCQ: Complete solutions QCD(1+1): any number of colors, flavors, quark masses**
- **AdS/QCD and Light-Front Holography: Soft-Wall Model predicts light-quark spectrum and dynamics** *de Teramond, sjb*
- **BFLQ: Use AdS/QCD orthonormal basis functions** *Vary, Maris. Zhao,et al.*
- **RGPEP: Systematically reduce off-diagonal elements; RG equations which evolve LFQCD in scale** *Glazek*
- **Reduce QCD to equation for LF valence state with effective potential** *Pauli*
- **Reduce QCD to one-dimensional LF Schrödinger Equation in radial coordinate conjugate to the invariant mass.**

*de Teramond, sjb*

- **Lippmann-Schwinger expansion in $\Delta U = U_{QCD} - U_{AdS}$** *Hiller-sjb*
- **Cluster expansion methods**

*Hiller-Chabysheva*

# *Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian*

**Pauli, Hornbostel, X. Zhao, Hiller, Chabysheva, sjb**

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  **J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,**
  **G.F. de Teramond, P. Sternberg, X. Zhao, E.G. Ng, C. Yang, sjb**

$$H_{QED}$$

*QED atoms: positronium and muonium*

$$(H_0 + H_{int}) \; |\Psi> = E \; |\Psi>$$

*Coupled Fock states*

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

*Effective two-particle equation*

**Includes Lamb Shift, quantum corrections**

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)] \; \psi(r) = E \; \psi(r)$$

*Spherical Basis* $r, \theta, \phi$

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

**Bohr Spectrum**

*Semiclassical first approximation to QED*

$$H_{QCD}^{LF}$$

*QCD Meson Spectrum*

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

*Coupled Fock states*

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

*Effective two-particle equation*

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

*Azimuthal Basis* $\zeta, \phi$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

*Confining AdS/QCD potential*

*Semiclassical first approximation to QCD*

# Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions.}$$

**Change variables** $(\vec{\zeta},\varphi), \; \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp :\quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta \frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

$$= \int d\zeta \, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

- Functional relation: $\frac{|\phi|^2}{\zeta} = \frac{2\pi}{x(1-x)}|\psi(x, \mathbf{b}_\perp)|^2$

- Invariant mass $\mathcal{M}^2$ in terms of LF mode $\phi$

$$\begin{aligned}
\mathcal{M}^2 &= \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)U(\zeta)\phi(\zeta) \\
&= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}\right)\phi(\zeta) + \int d\zeta\, \phi^*(\zeta)U(\zeta)\phi(\zeta)
\end{aligned}$$

where the interaction terms are summed up in the effective potential $U(\zeta)$ and the orbital angular momentum in $\nabla^2$ has the $SO(2)$ Casimir representation $SO(N) \sim S^{N-1}:\ L(L+N-2)$

$$-\frac{\partial^2}{\partial\varphi^2}|\phi\rangle = L^2|\phi\rangle$$

- LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is a LF wave equation for $\phi$

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta)}$$

- Effective light-front Schrödinger equation: relativistic, covariant and analytically tractable.

# Light-Front Schrödinger Equation

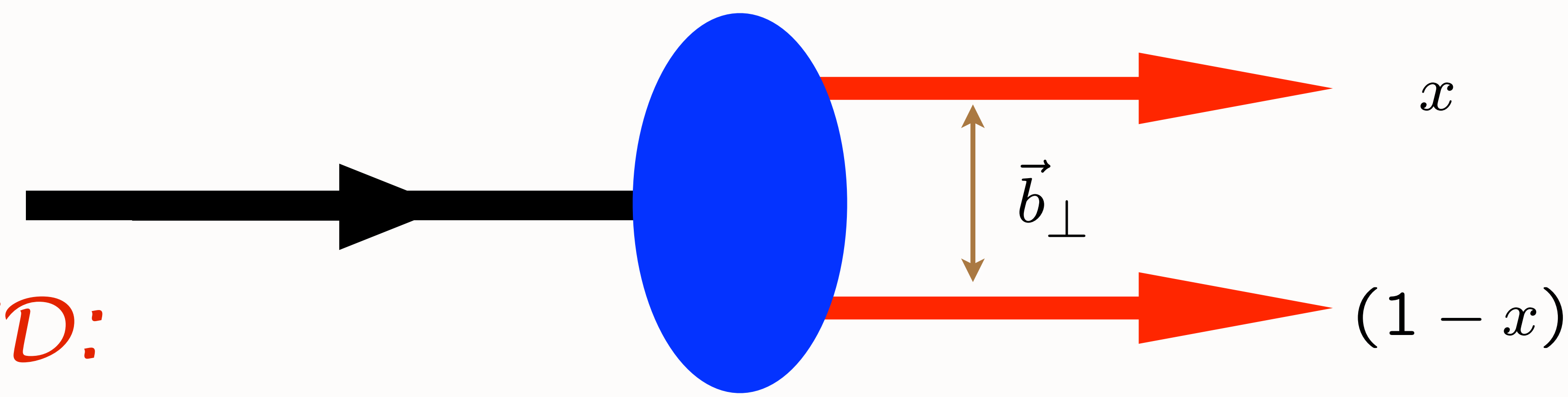
G. de Teramond, sjb

*Relativistic LF single-variable radial equation for QCD & QED*

*Frame Independent!*

$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

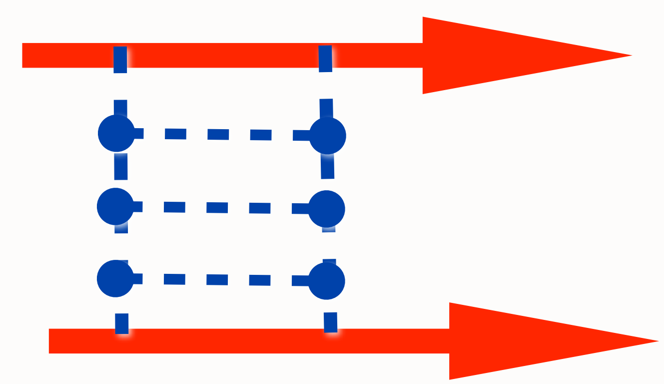
$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$




*AdS/QCD:*

**U is the exact QCD potential**
**Conjecture: 'H'-diagrams generate**

$$U(\zeta,S,L)=\kappa^2\zeta^2+\kappa^2(L+S-1/2)$$



# Light-Front: Universal Tool for atoms, nuclei, hadrons

- LFWFs are Frame Independent
- No colliding pancakes
- One-dimensional *Light-Front Schrödinger Equation*
- Precision QED; Atoms in flight
- Avoid dynamical boosts
- Avoid vacuum currents!
- Angular momentum conservation
- Hadronization at amplitude level

# Light-Front Schrödinger Equation

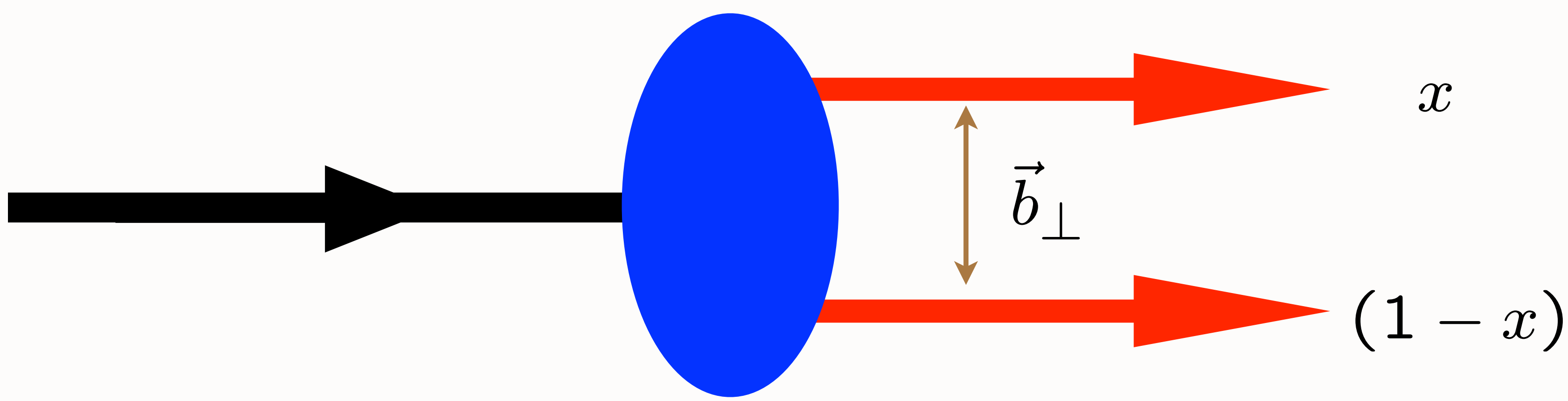
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

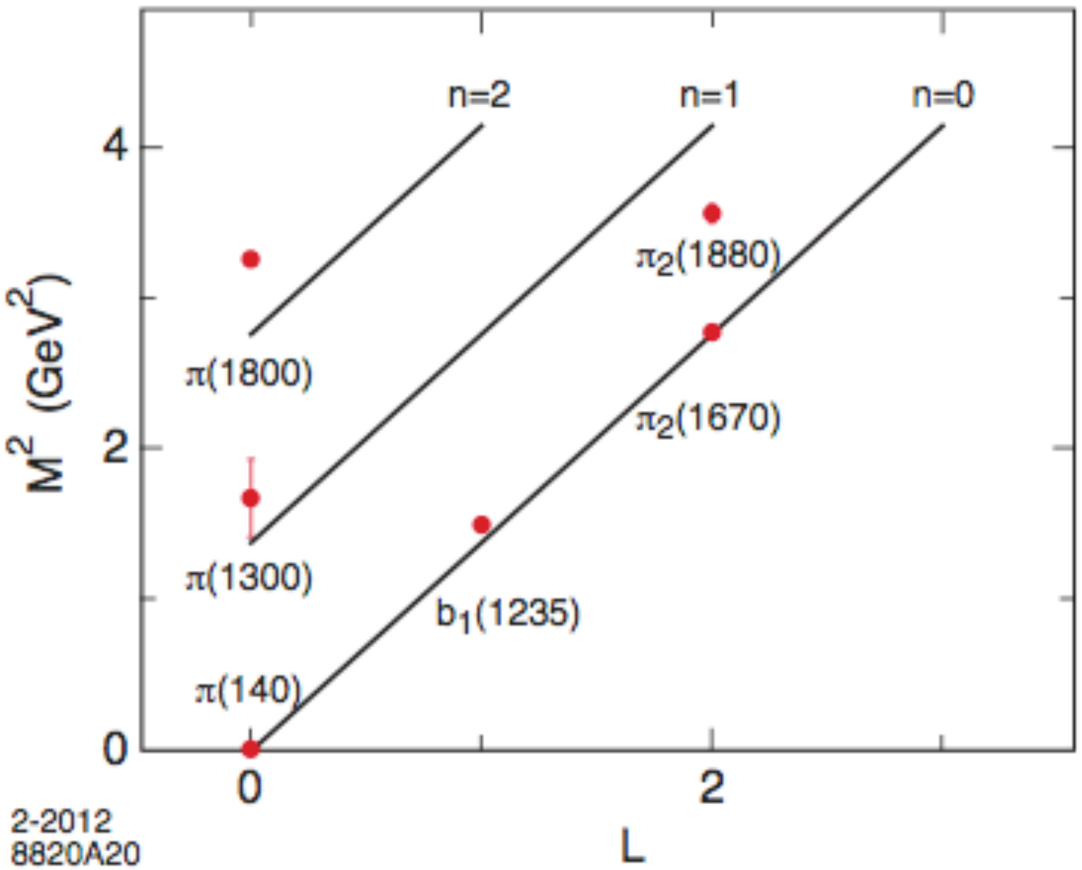


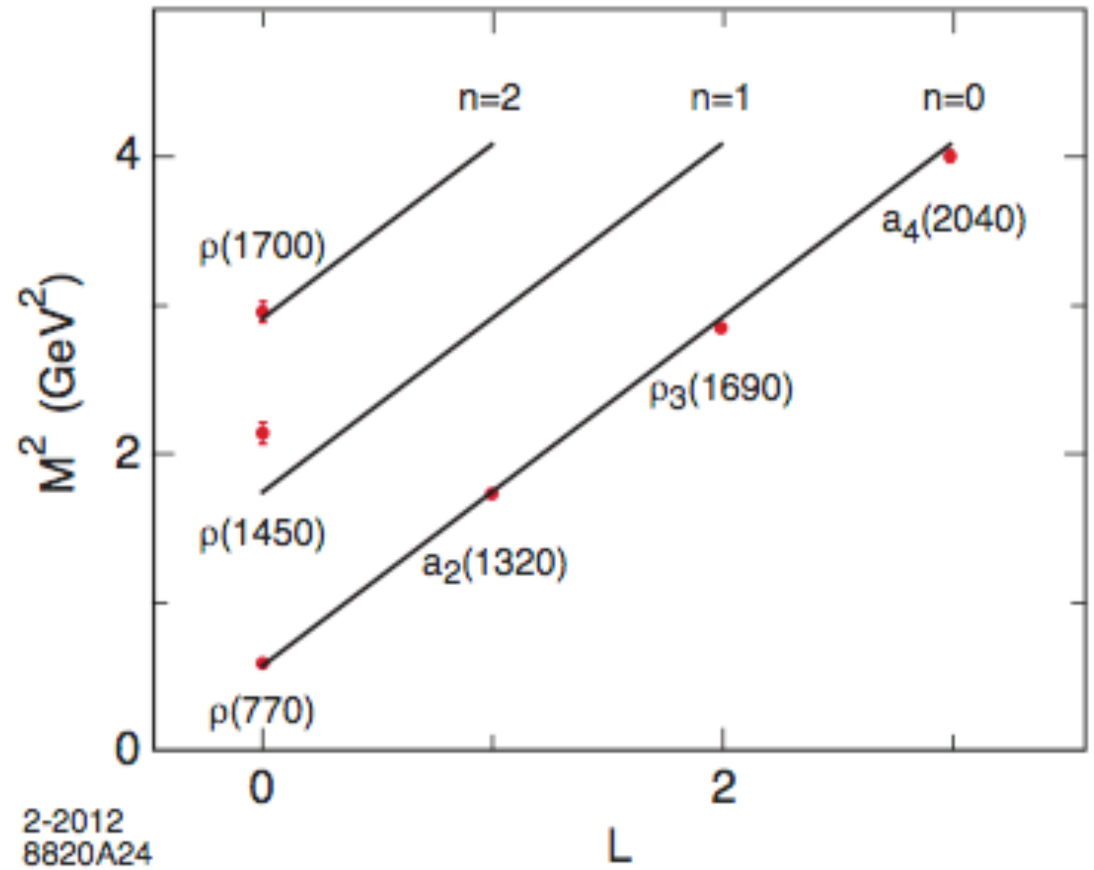

where the potential $U(\zeta^2, J, L, M^2)$ represents the contributions from higher Fock states. It is also the kernel for the forward scattering amplitude $q\bar{q} \to q\bar{q}$ at $s = M^2$. It has only "proper" contributions; i.e. it has no $q\bar{q}$ intermediate state. The potential can be constructed systematically using LF time-ordered perturbation theory. Thus the exact QCD theory has the identical form as the AdS theory, but with the quantum field-theoretic corrections due to the higher Fock states giving a general form for the potential. This provides a novel way to solve nonperturbative QCD. Complex eigenvalues for excited states n>0

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 (n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

***Same slope in n and L*** ***Massless pion***

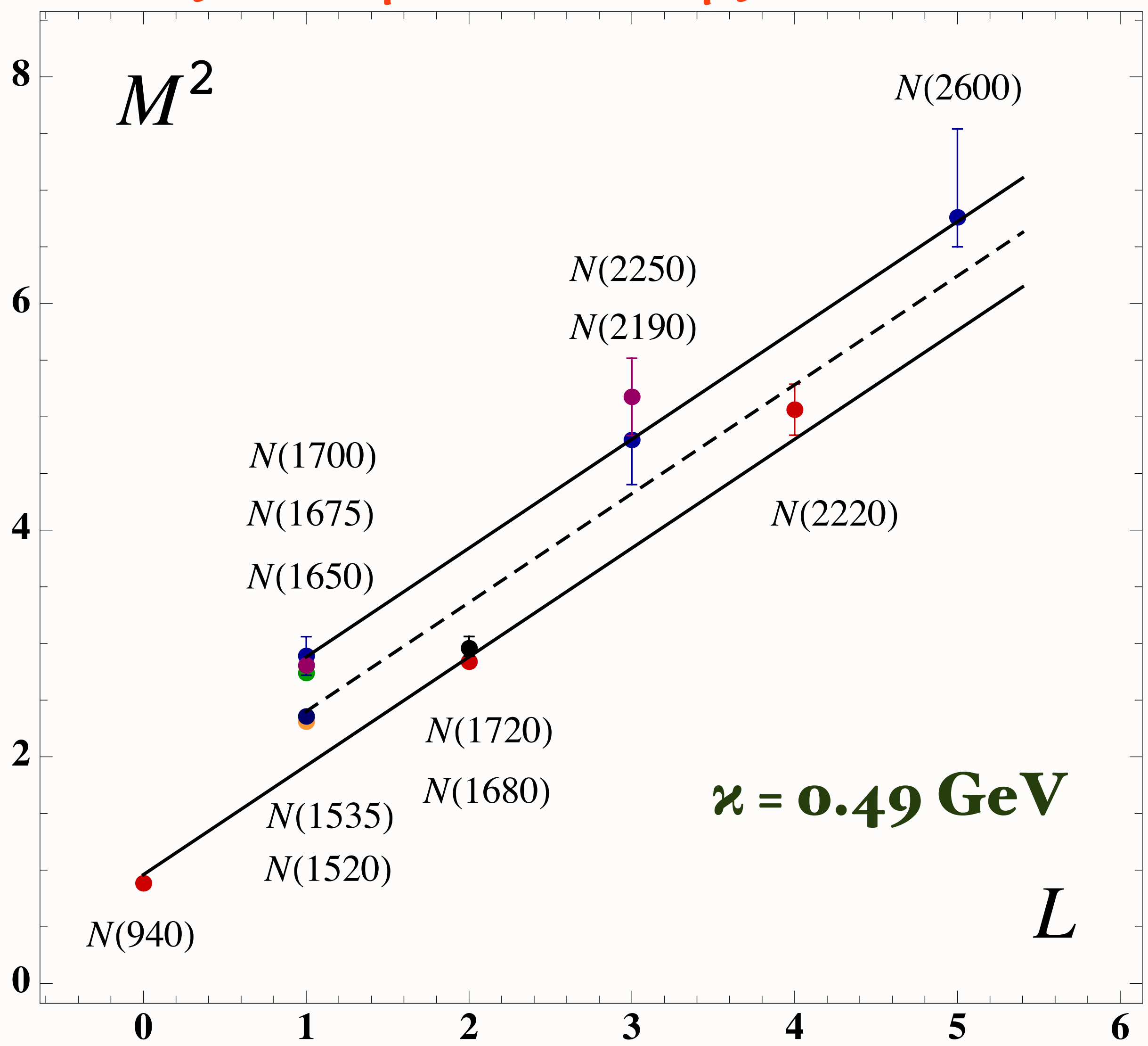


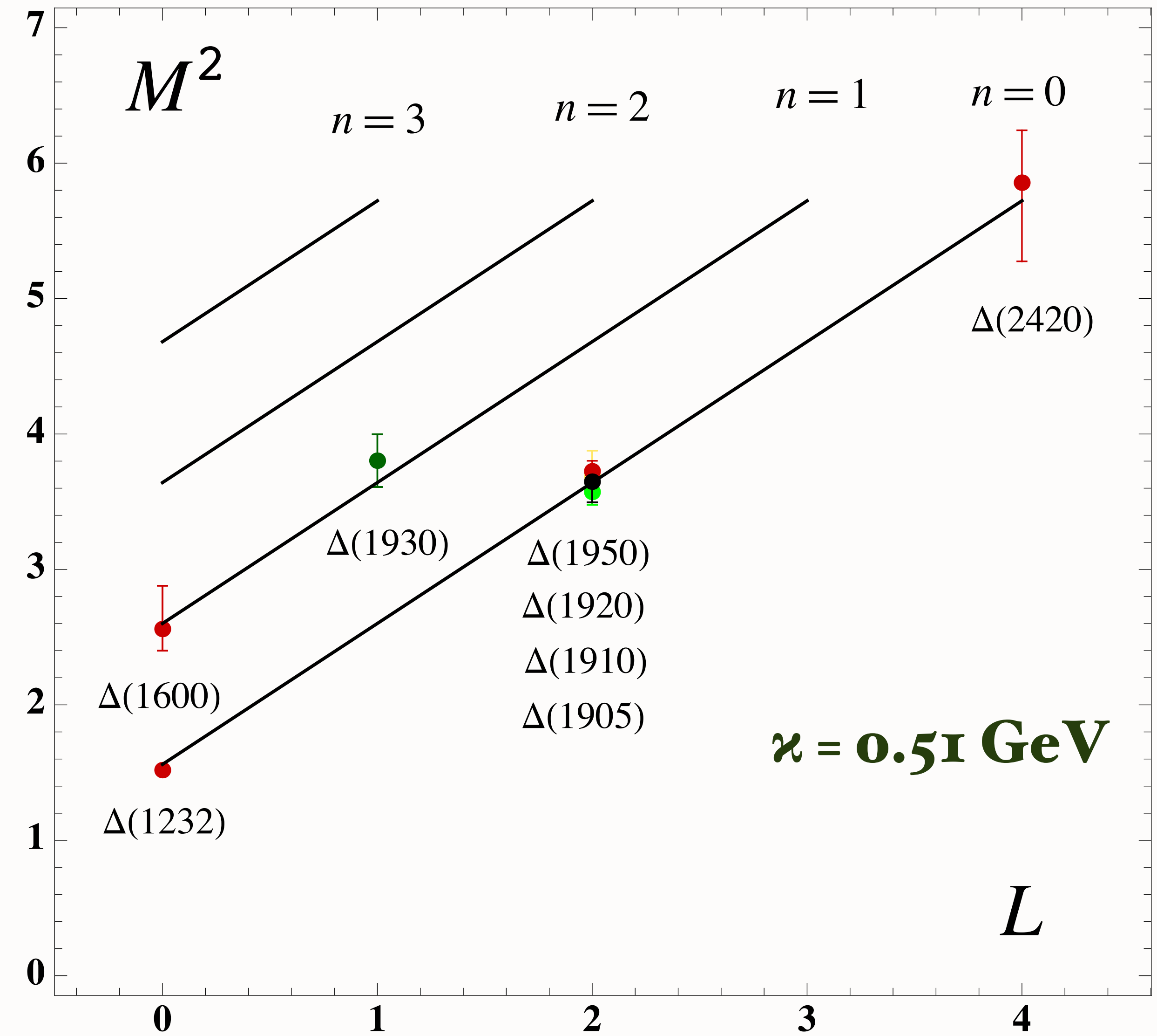




I=1 orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson $a$-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$




**de Teramond, sjb**

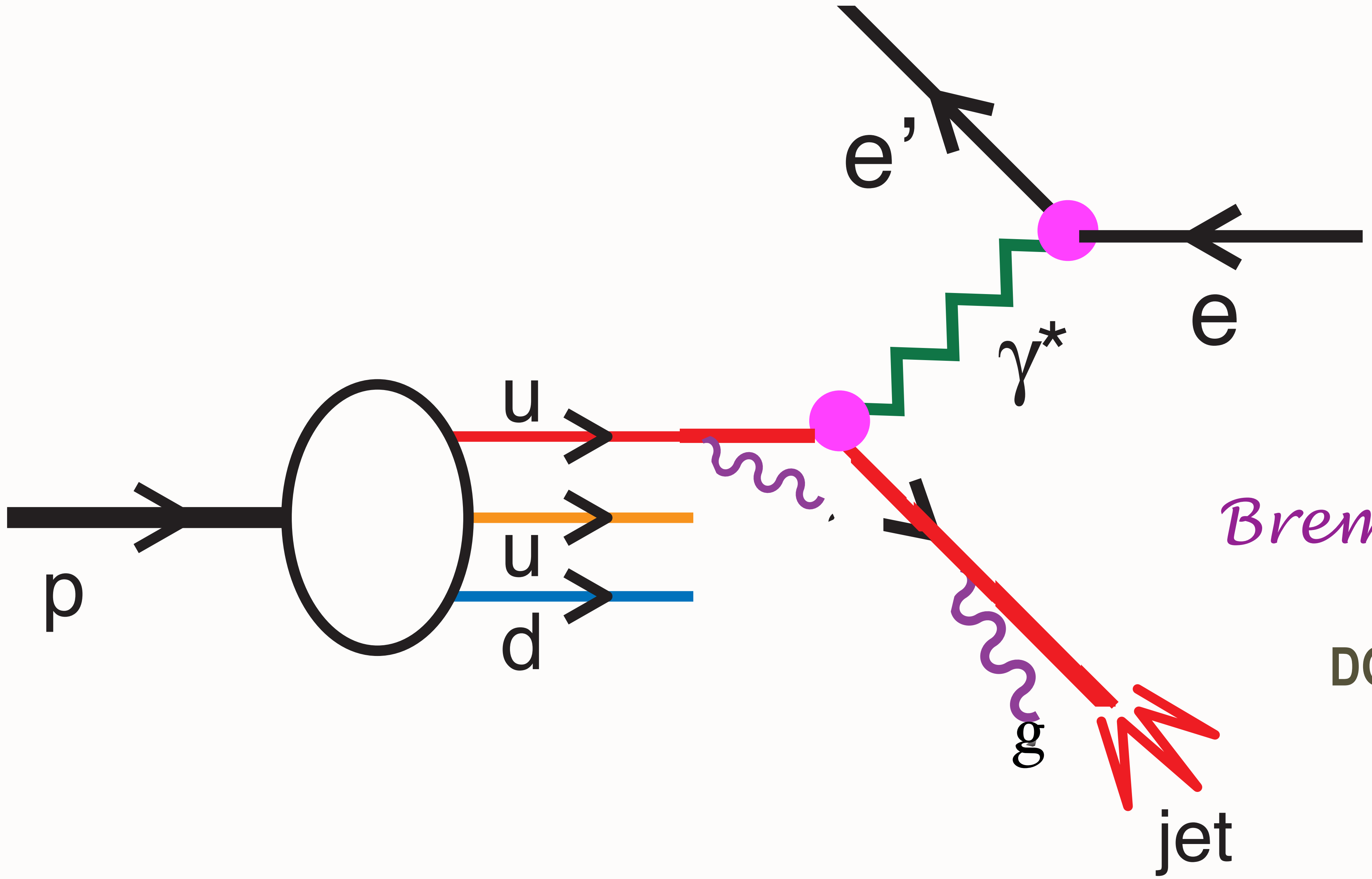
**All confirmed resonances from PDG 2012**

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$ **positive parity**

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$ **negative parity**

**See also Forkel, Beyer, Federico, Klempt**

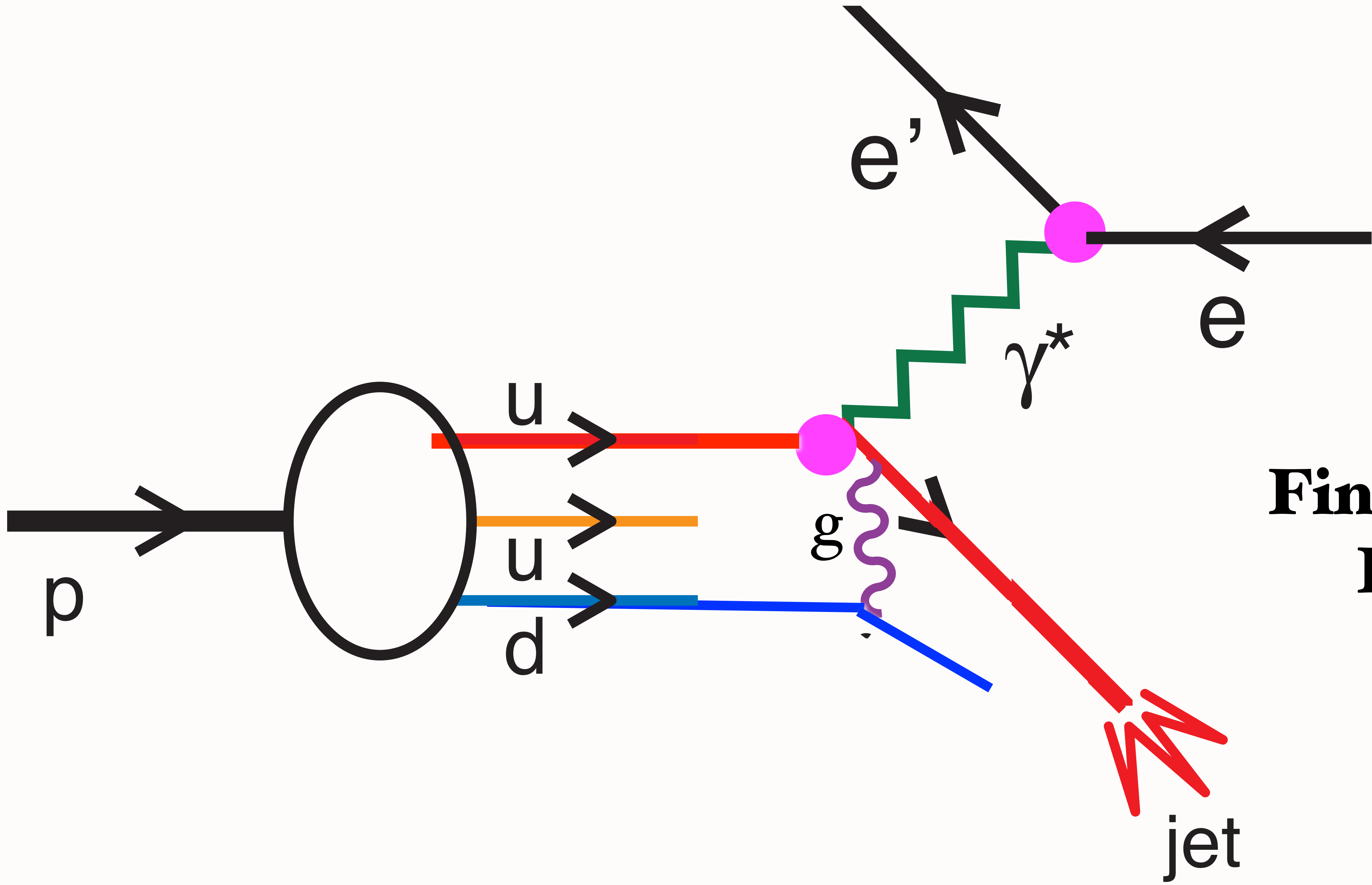
# Deep Inelastic Electron-Proton Scattering




Gluonic Bremmstrahlung

DGLAP Evolution

# Deep Inelastic Electron-Proton Scattering




**Final-State QCD Interaction**

*Conventional wisdom:*
*Final-state interactions of struck quark can be neglected*

*Single-spin asymmetries*

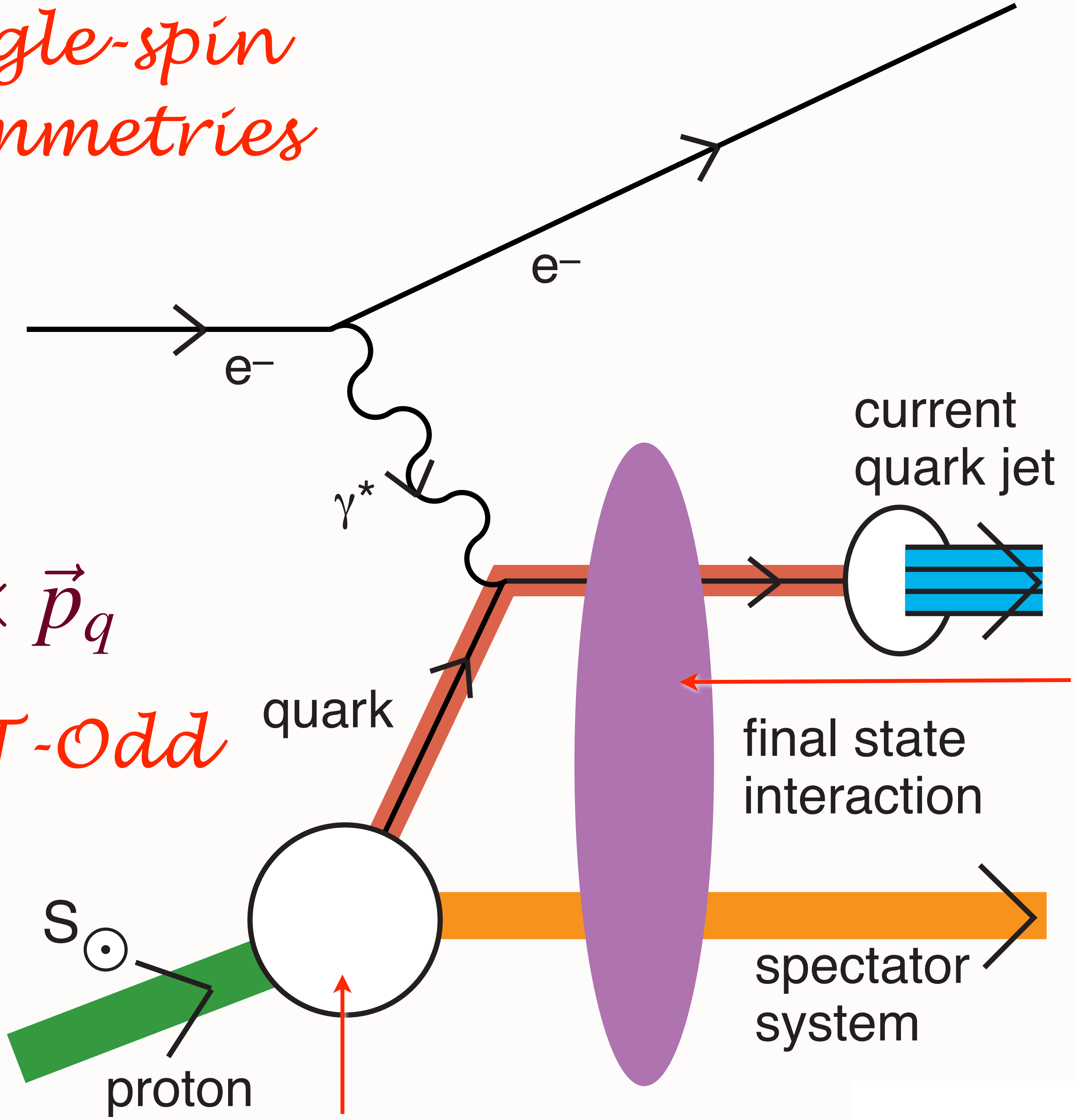
# Leading Twist Sivers Effect

**Hwang, Schmidt, sjb**

**Collins, Burkardt, Ji, Yuan. Pasquini, ...**

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

*Pseudo- T-Odd*




*QCD S- and P-Coulomb Phases --Wilson Line*

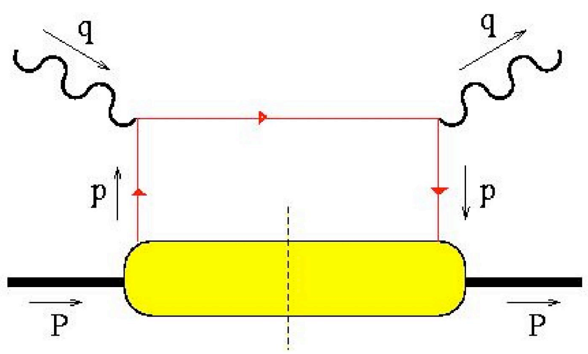
**"Lensing Effect"**

**QED: Lensing involves soft scales**

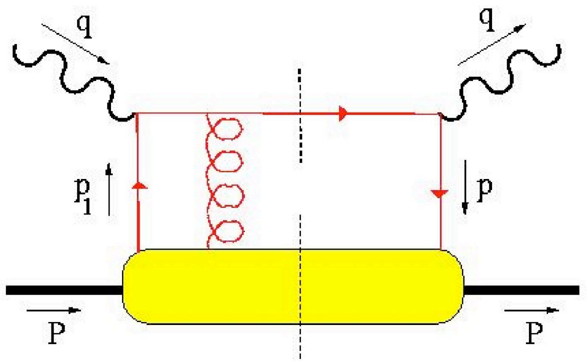
*Light-Front Wavefunction S and P- Waves!*

*Leading-Twist Rescattering Violates pQCD Factorization!*

*Sign reversal in DY!*



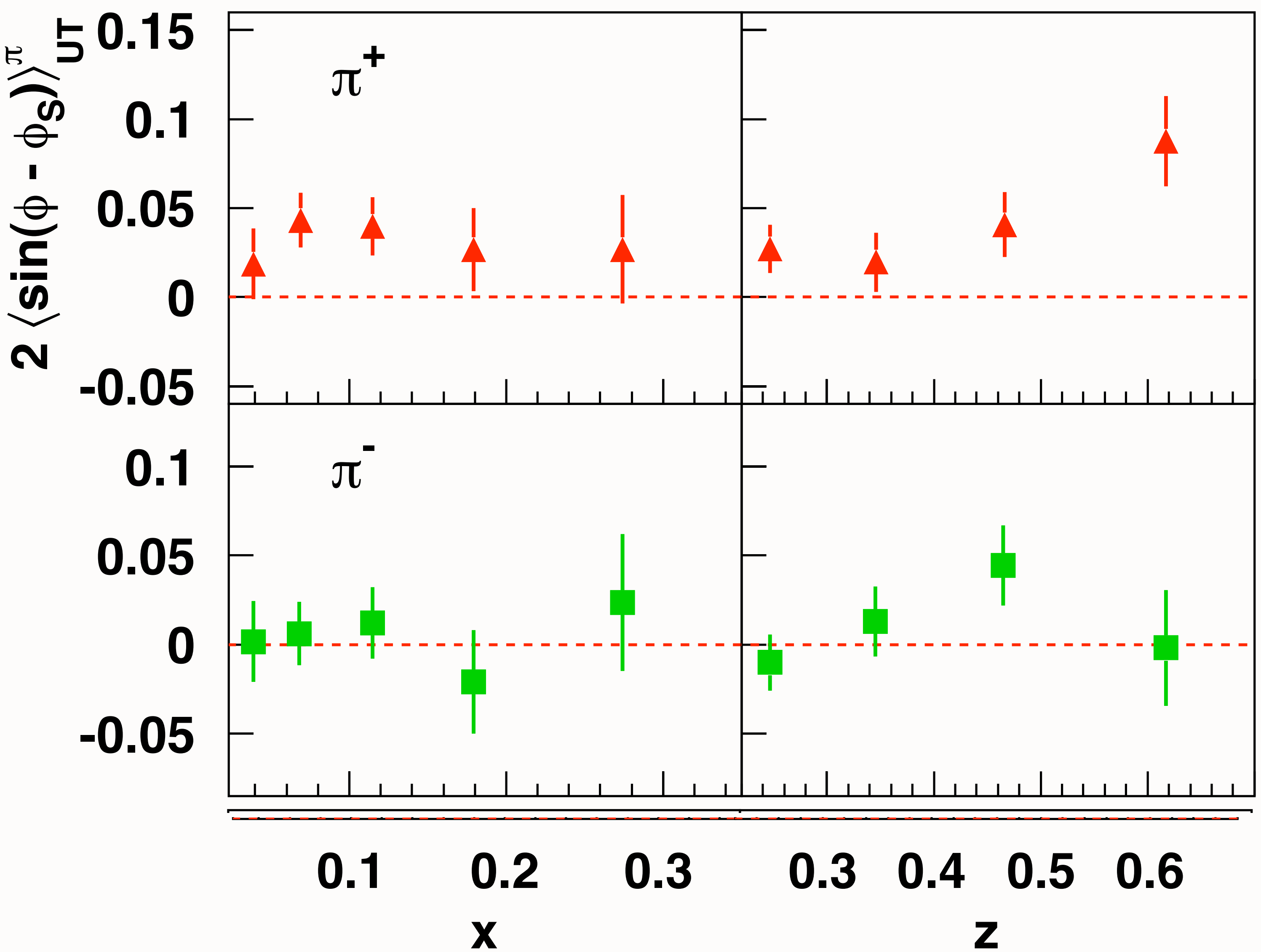

can interfere with




and produce a T-odd effect! (also need $L_z \neq 0$)

HERMES coll., A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.

## Sivers asymmetry from HERMES




- First evidence for non-zero Sivers function!
- ⇒ presence of non-zero **quark orbital angular momentum**!
- Positive for **$\pi^+$**... Consistent with zero for **$\pi^-$**...

**Gamberg: Hermes data compatible with BHS model**

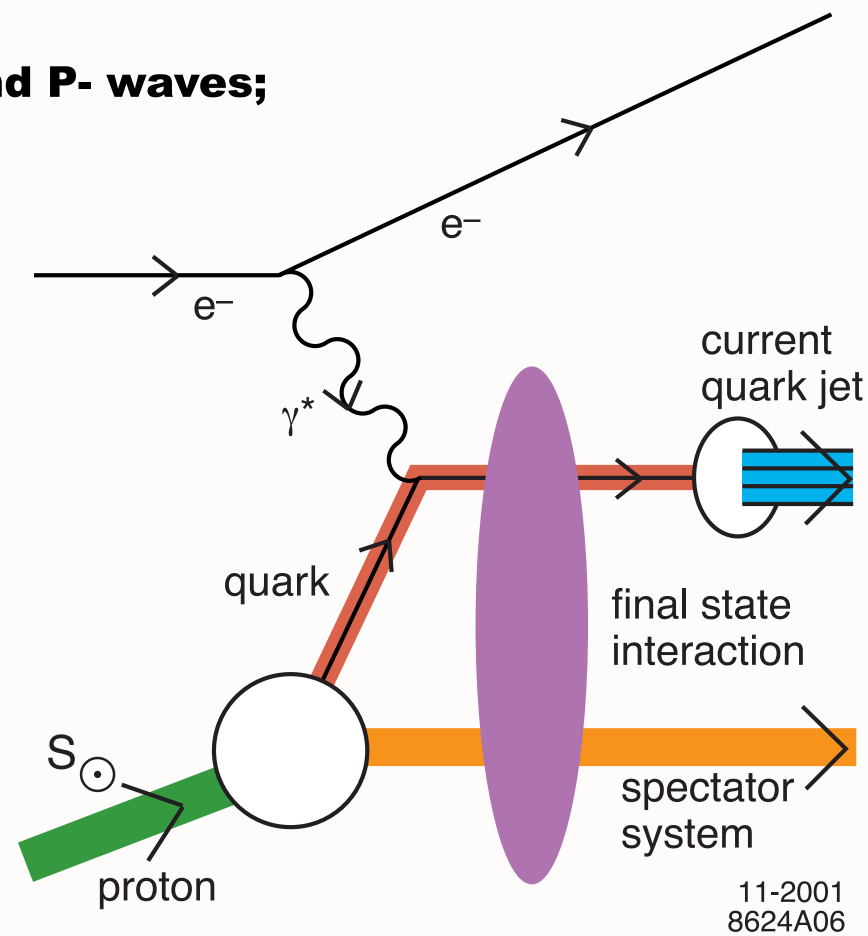
**Schmidt, Lu: Hermes charge pattern follow quark contributions to anomalous moment**

# *Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)*

**Hwang, Schmidt, sjb**
**Collins**

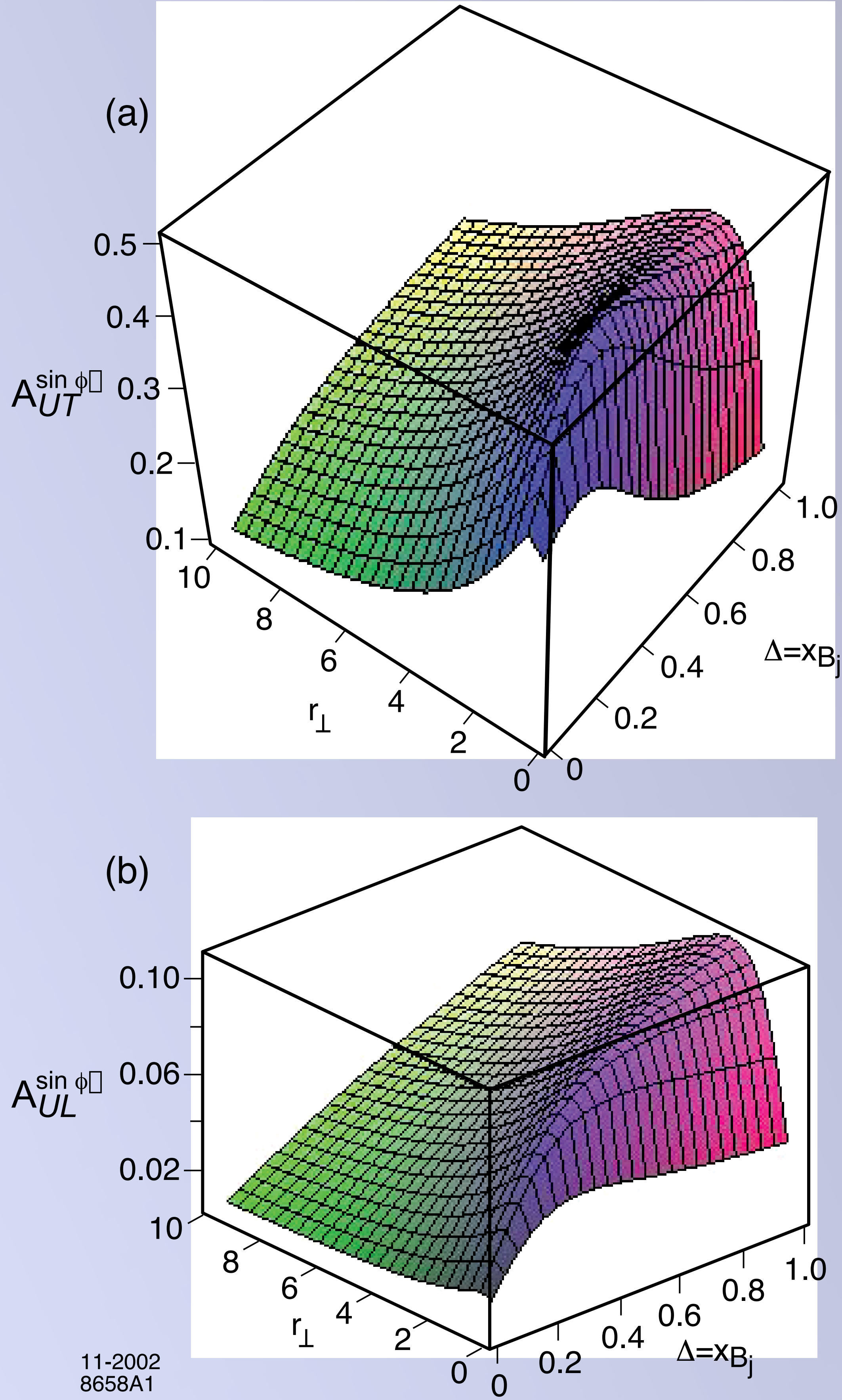
$$\mathbf{i}\ \vec{S} \cdot \vec{p}_{jet} \times \vec{q}$$

- **Leading-Twist Bjorken Scaling!**
- **Requires nonzero orbital angular momentum of quark**
- **Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves;**
- **Wilson line effect -- lc gauge prescription**
- **Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases**
- **QCD phase at soft scale!**
- **New window to QCD coupling and running gluon mass in the IR**
- **QED S and P Coulomb phases infinite -- difference of phases finite!**
- **Alternate: Retarded and Advanced Gauge: Augmented LFWFs**




**Pasquini, Xiao, Yuan, sjb**
**Mulders, Boer      Qiu, Sterman**

# *Prediction for Single-Spin Asymmetry*




**Hwang, Schmidt, sjb**

| Static | Dynamic |
|---|---|
| Square of Target LFWFs | Modified by Rescattering: ISI & FSI |
| No Wilson Line | Contains Wilson Line, Phases |
| Probability Distributions | No Probabilistic Interpretation |
| Process-Independent | Process-Dependent - From Collision |
| T-even Observables | T-Odd (Sivers, Boer-Mulders, etc.) |
| No Shadowing, Anti-Shadowing | Shadowing, Anti-Shadowing, Saturation |
| Sum Rules: Momentum and $J^z$ | Sum Rules Not Proven |
| DGLAP Evolution; mod. at large x | DGLAP Evolution |
| No Diffractive DIS | Hard Pomeron and Odderon Diffractive DIS |

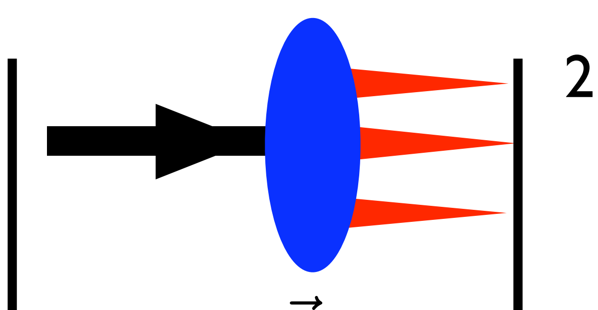
**Hwang, Schmidt, sjb,**

**Mulders, Boer**

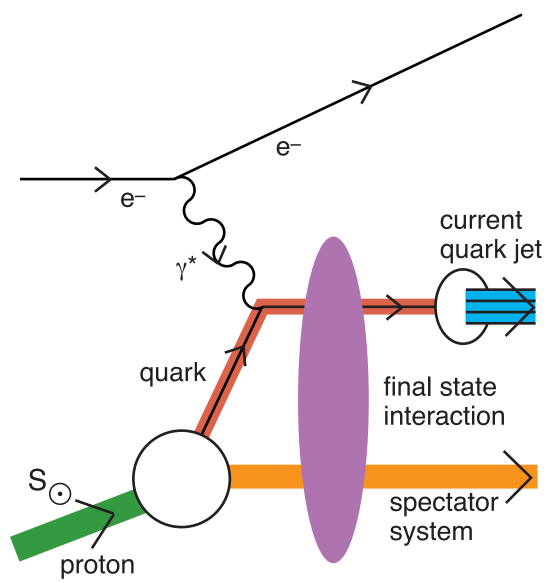
**Qiu, Sterman**

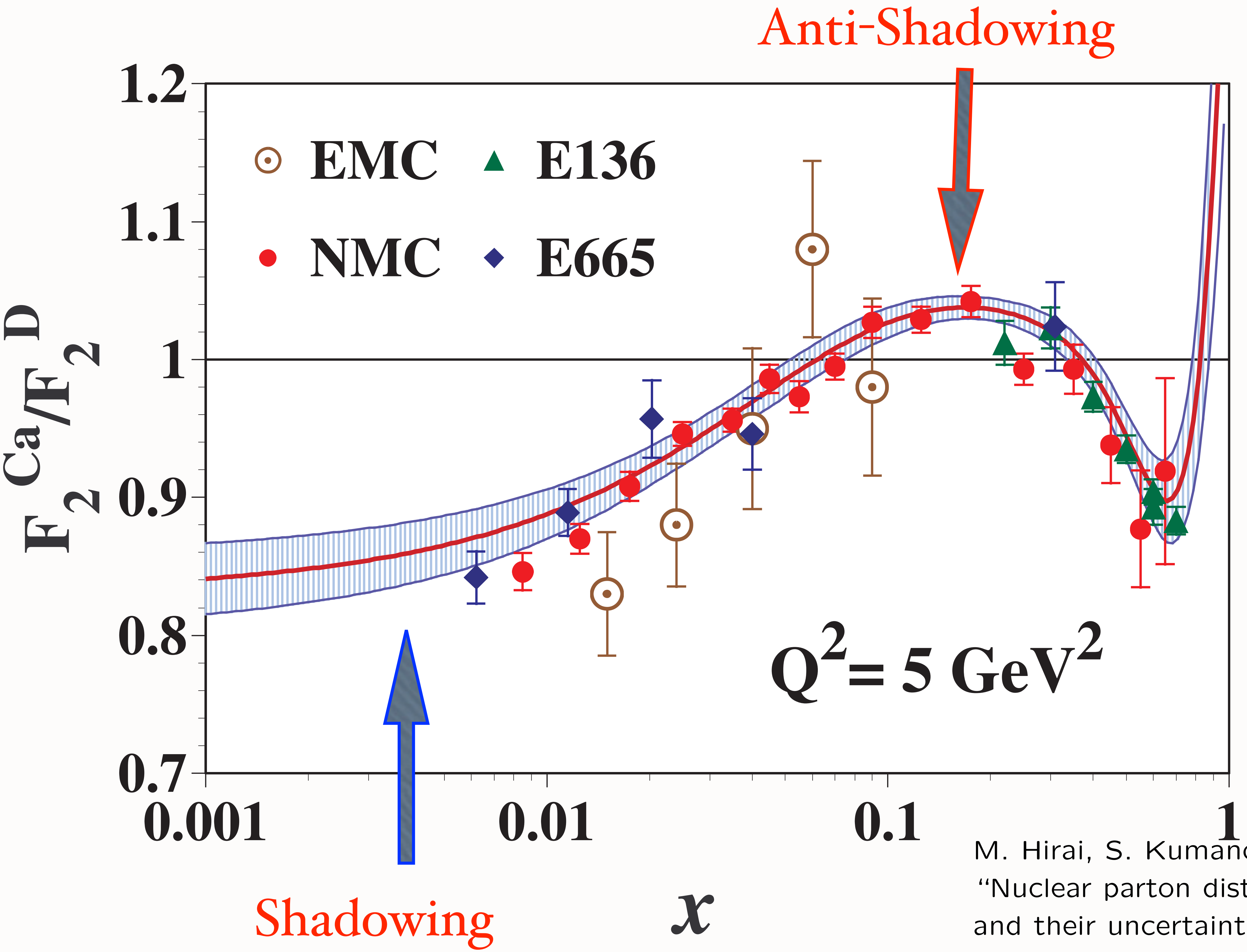
**Collins, Qiu**

**Pasquini, Xiao, Yuan, sjb**

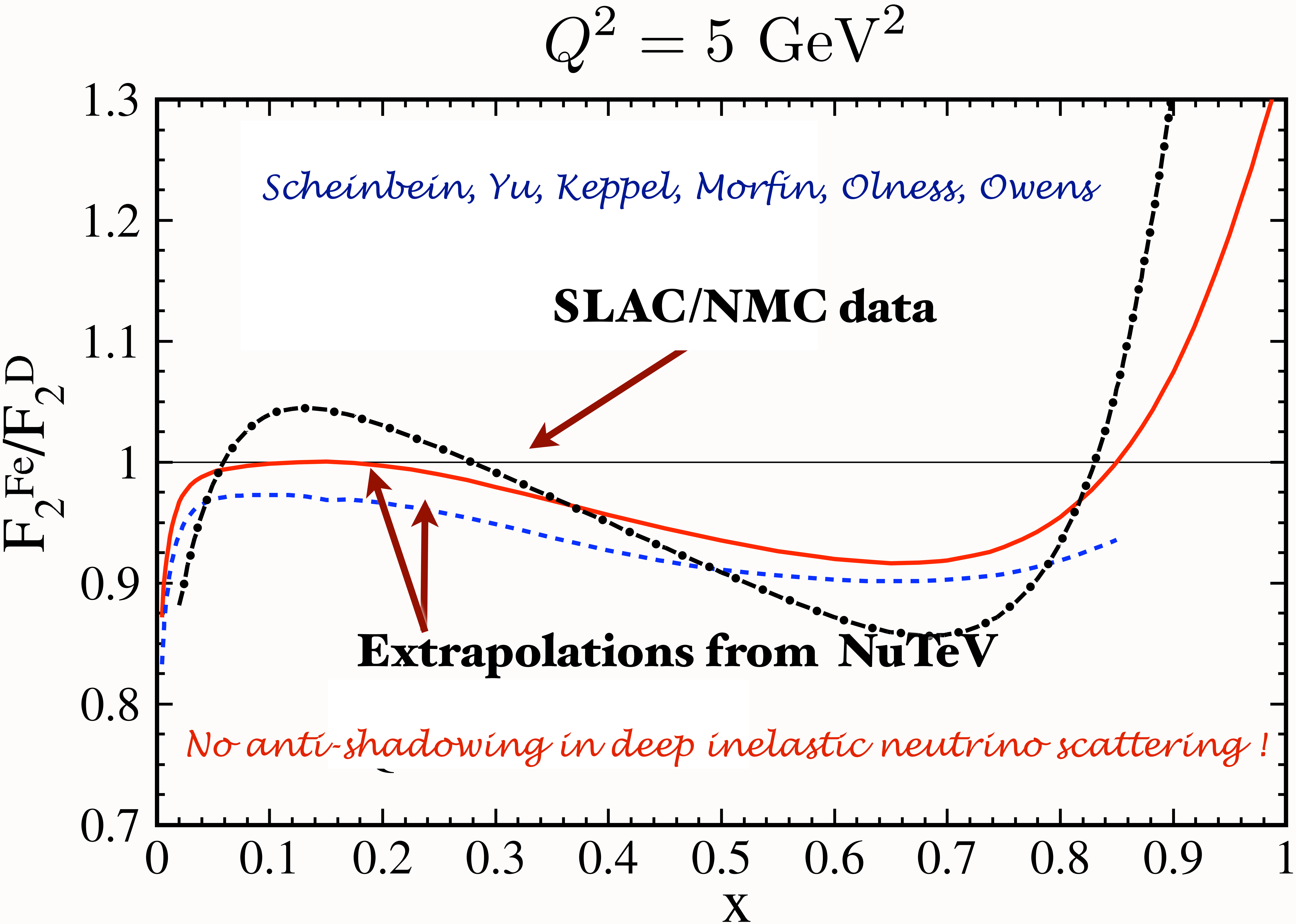



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$





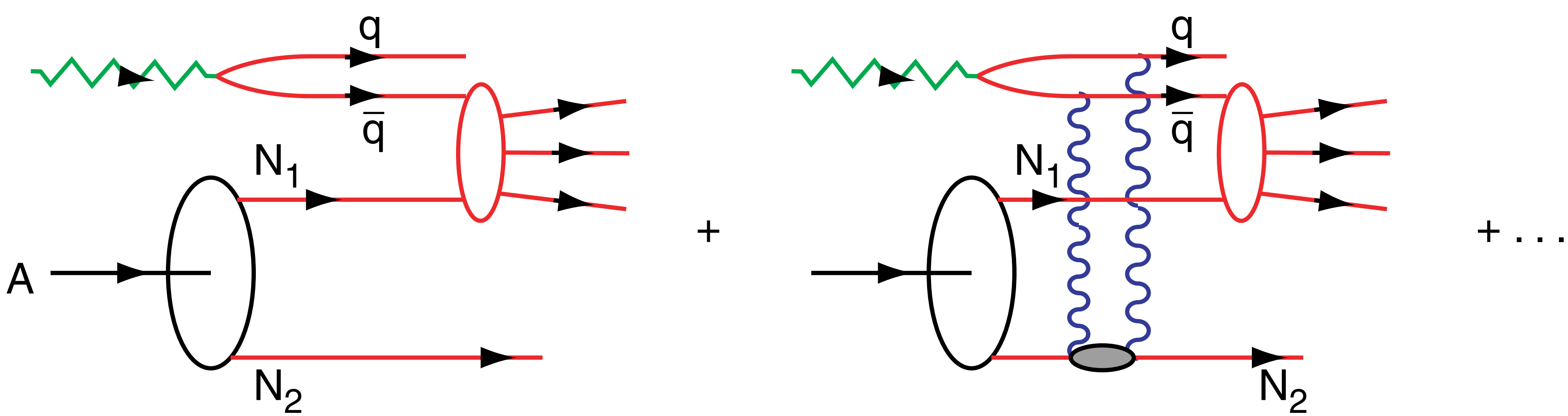

M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].



$Q^2 = 5\ \mathrm{GeV}^2$
Scheinbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
$F_2^{Fe}/F_2^{D}$
x
1.3
1.2
1.1
1
0.9
0.8
0.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Stodolsky
Pumplin, sjb
Gribov

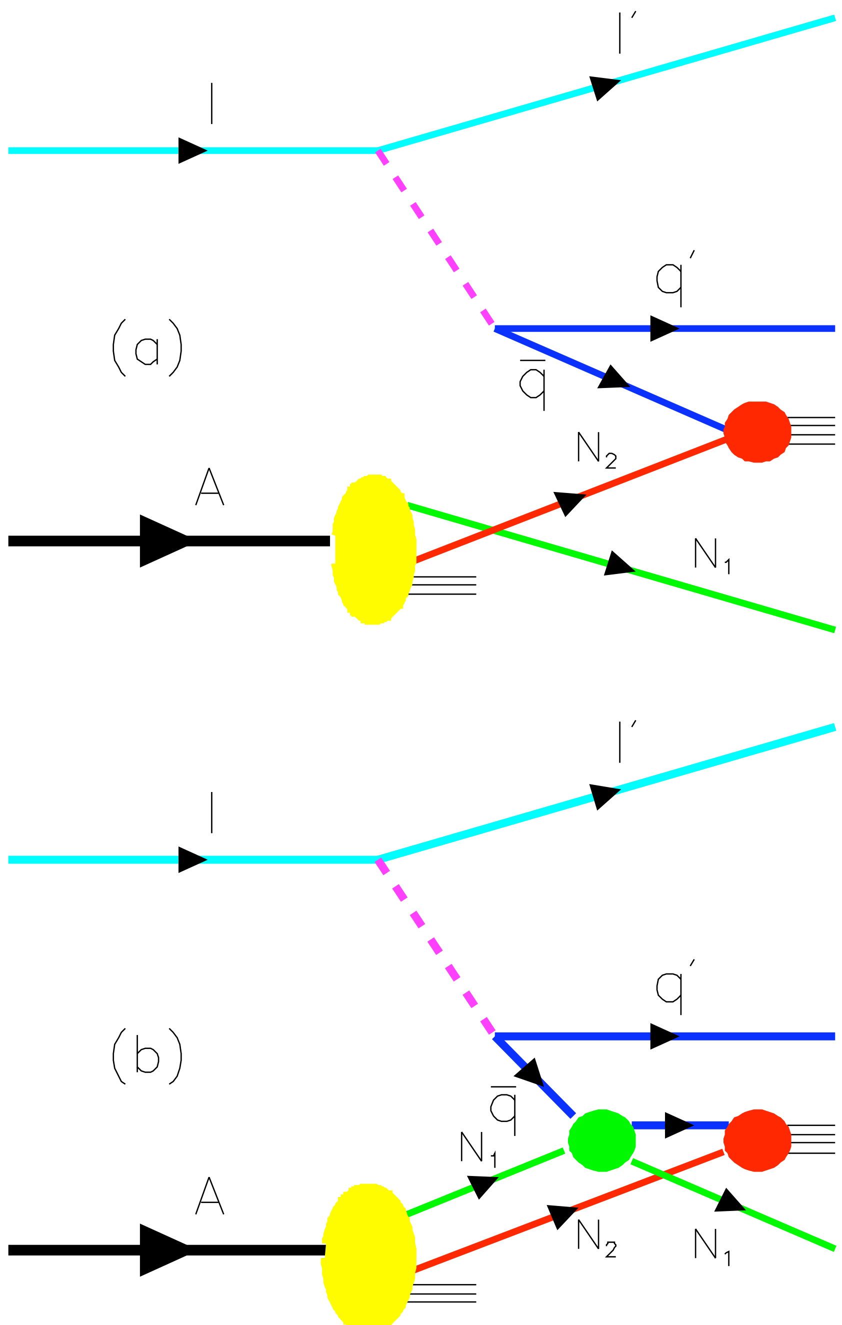
# Nuclear Shadowing in QCD




*Shadowing depends on understanding leading twist-diffraction in DIS*

**Nuclear Shadowing not included in nuclear LFWF !**

**Dynamical effect due to virtual photon interacting in nucleus**



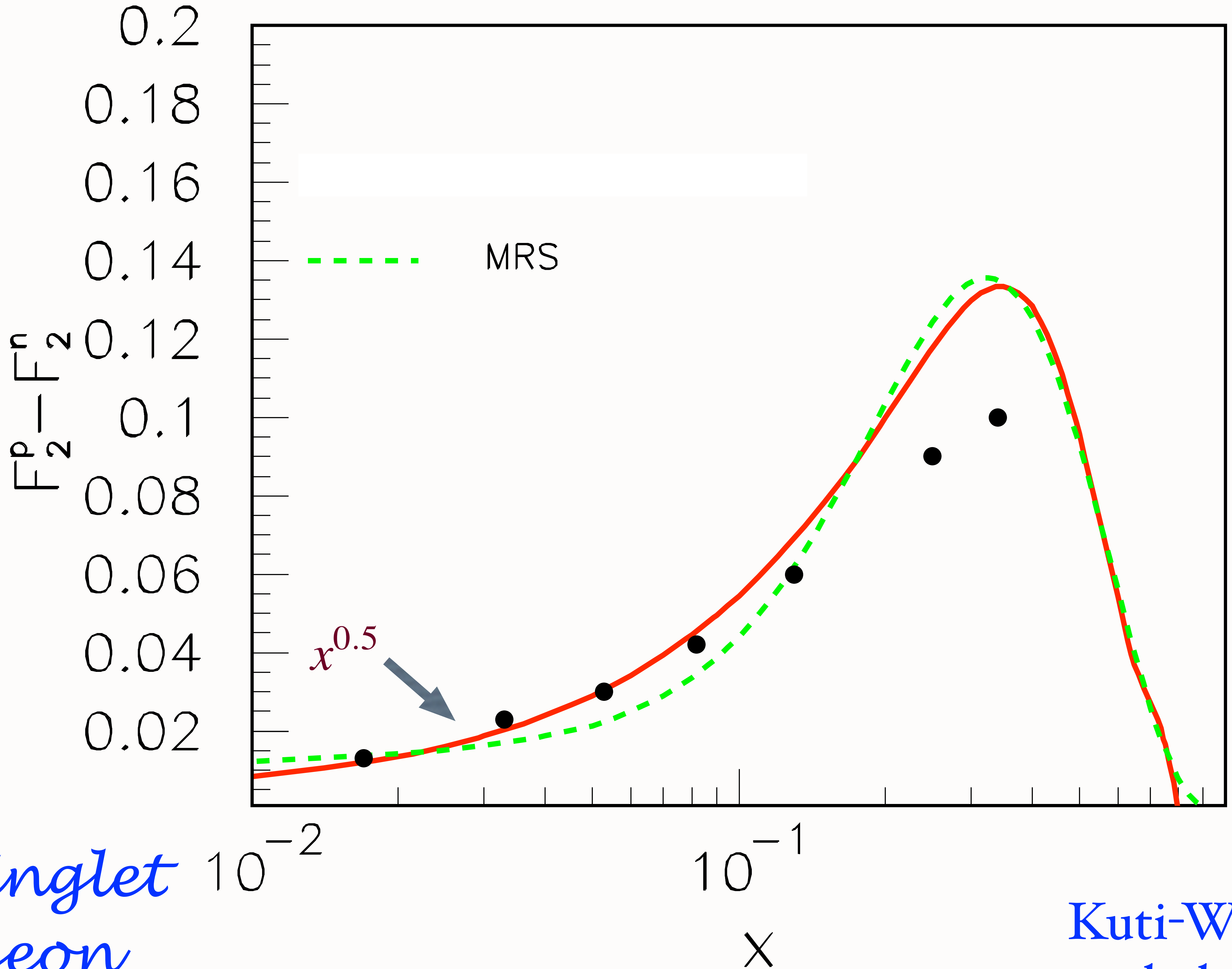

The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken $x_B$ :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

If the scattering on nucleon $N_1$ is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\overline{q}$ flux reaching $N_2$.

$\rightarrow$ Shadowing of the DIS nuclear structure functions.

# Observed HERA DDIS produces nuclear shadowing




Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

# Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1 - i) \times i = \frac{1}{\sqrt{2}}(i + 1)$$

Constructive Interference

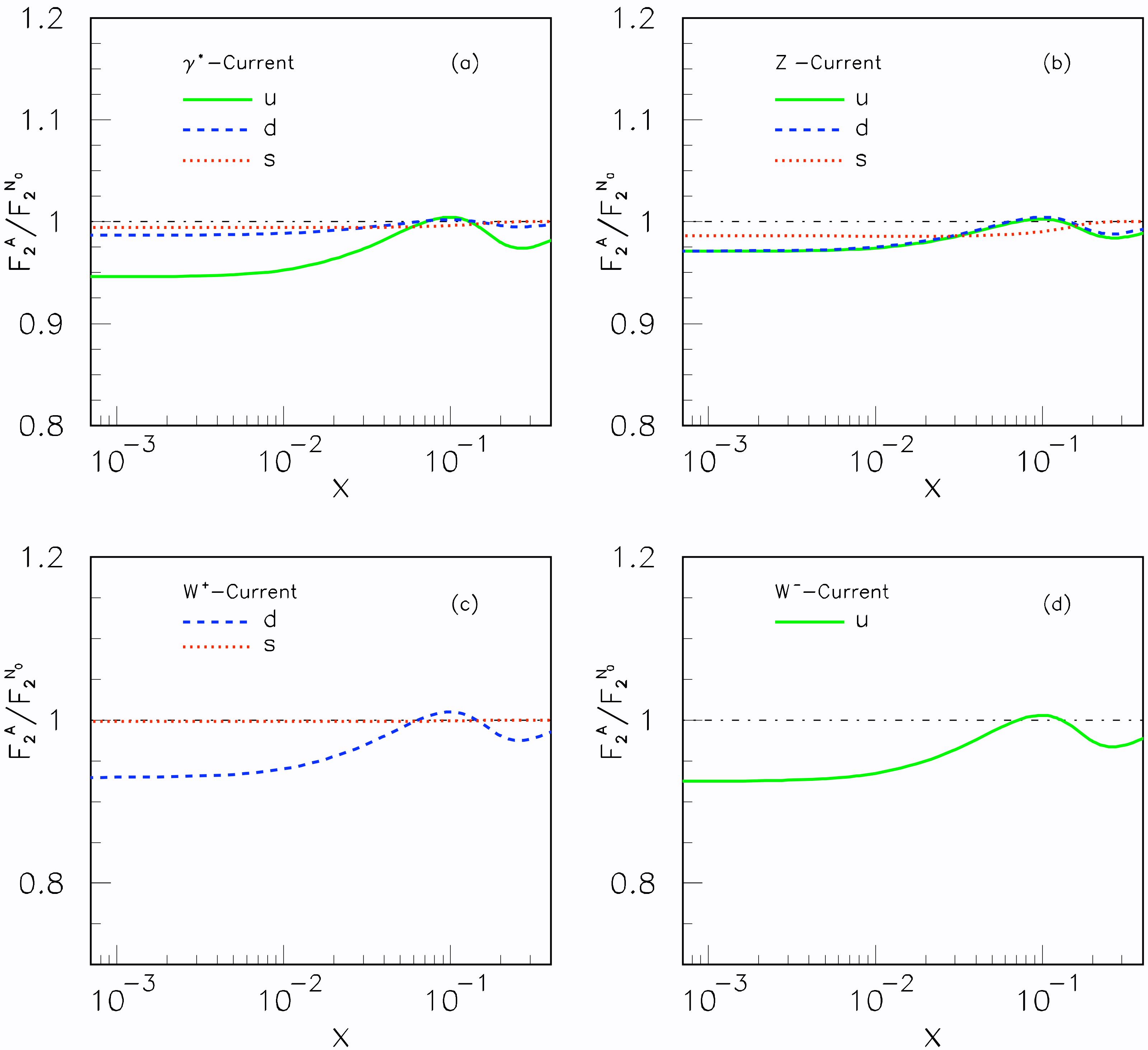
Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

*Critical test: Tagged Drell-Yan*

# Shadowing and Antishadowing of DIS Structure Functions

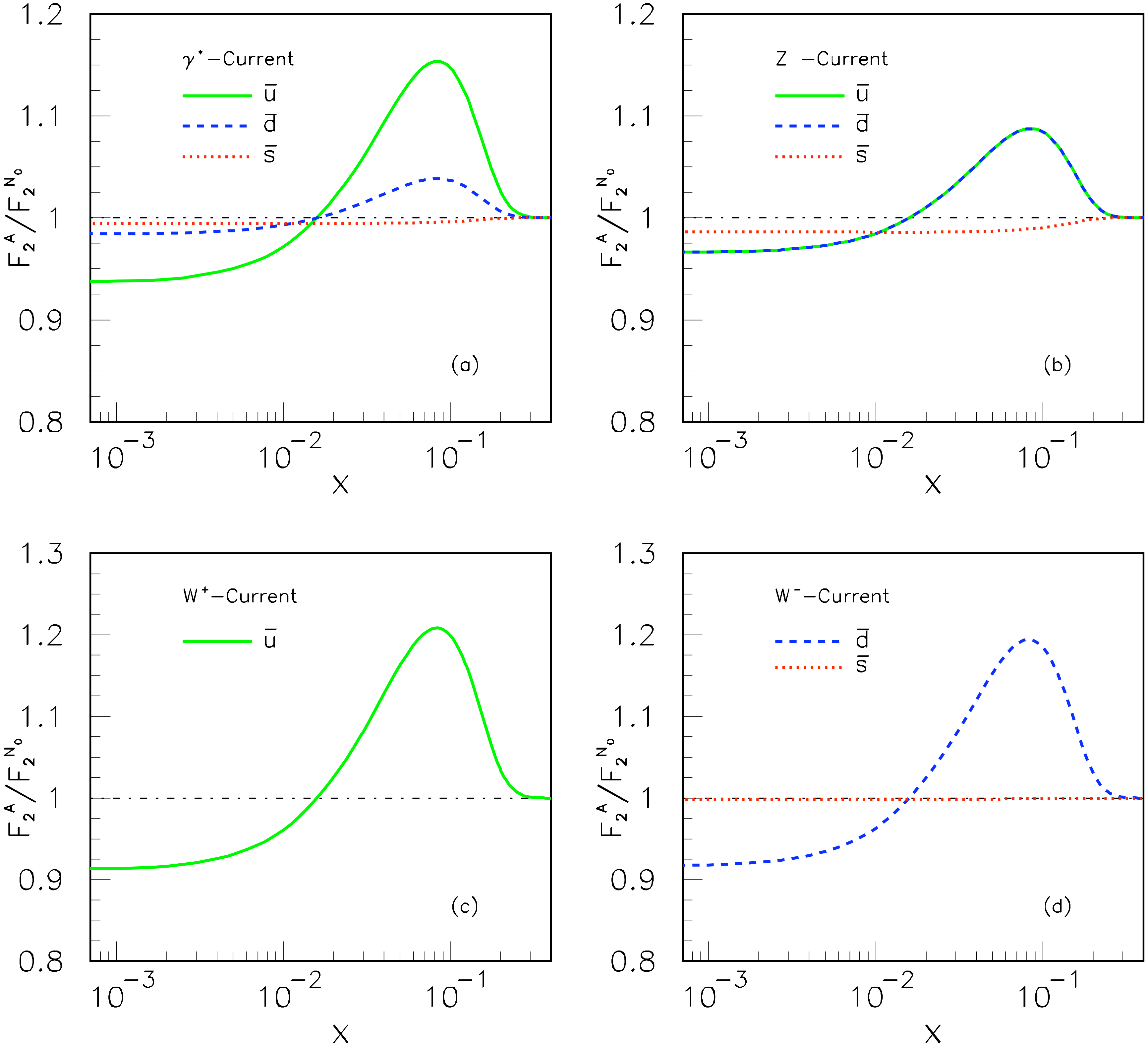



S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

**Modifies NuTeV extraction of**

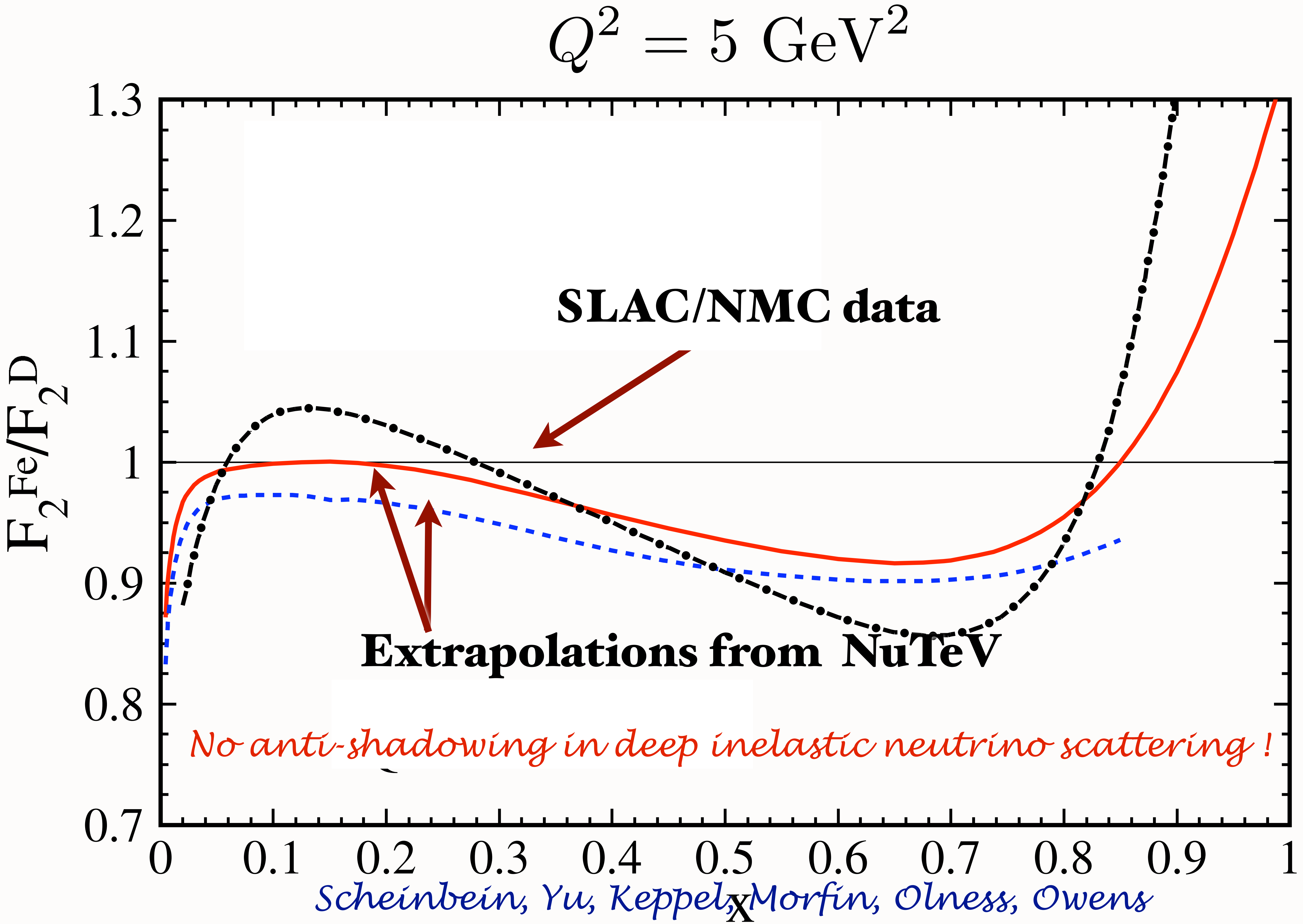
$$\sin^2\theta_W$$

**Test in flavor-tagged lepton-nucleus collisions**




Schmidt, Yang; sjb

*Nuclear Antishadowing not universal !*

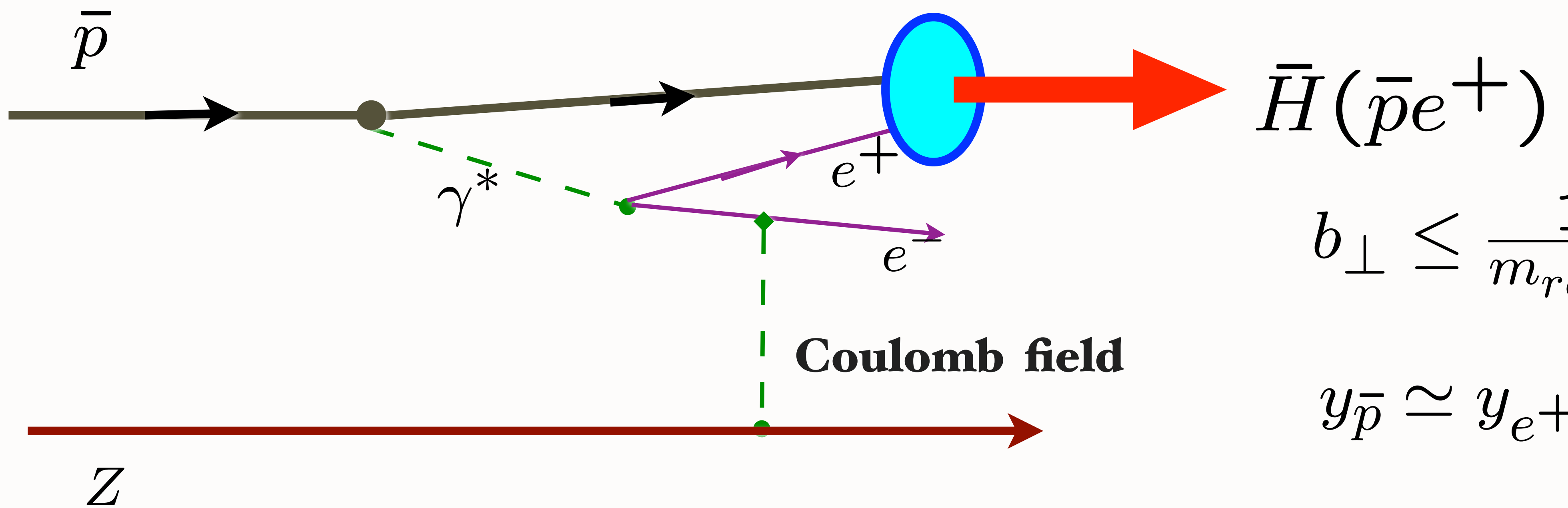



Scheinbein, Yu, Keppel, Morfin, Olness, Owens

# Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

**Munger, Schmidt, sjb**




$$b_\perp \leq \frac{1}{m_{red}\alpha}$$

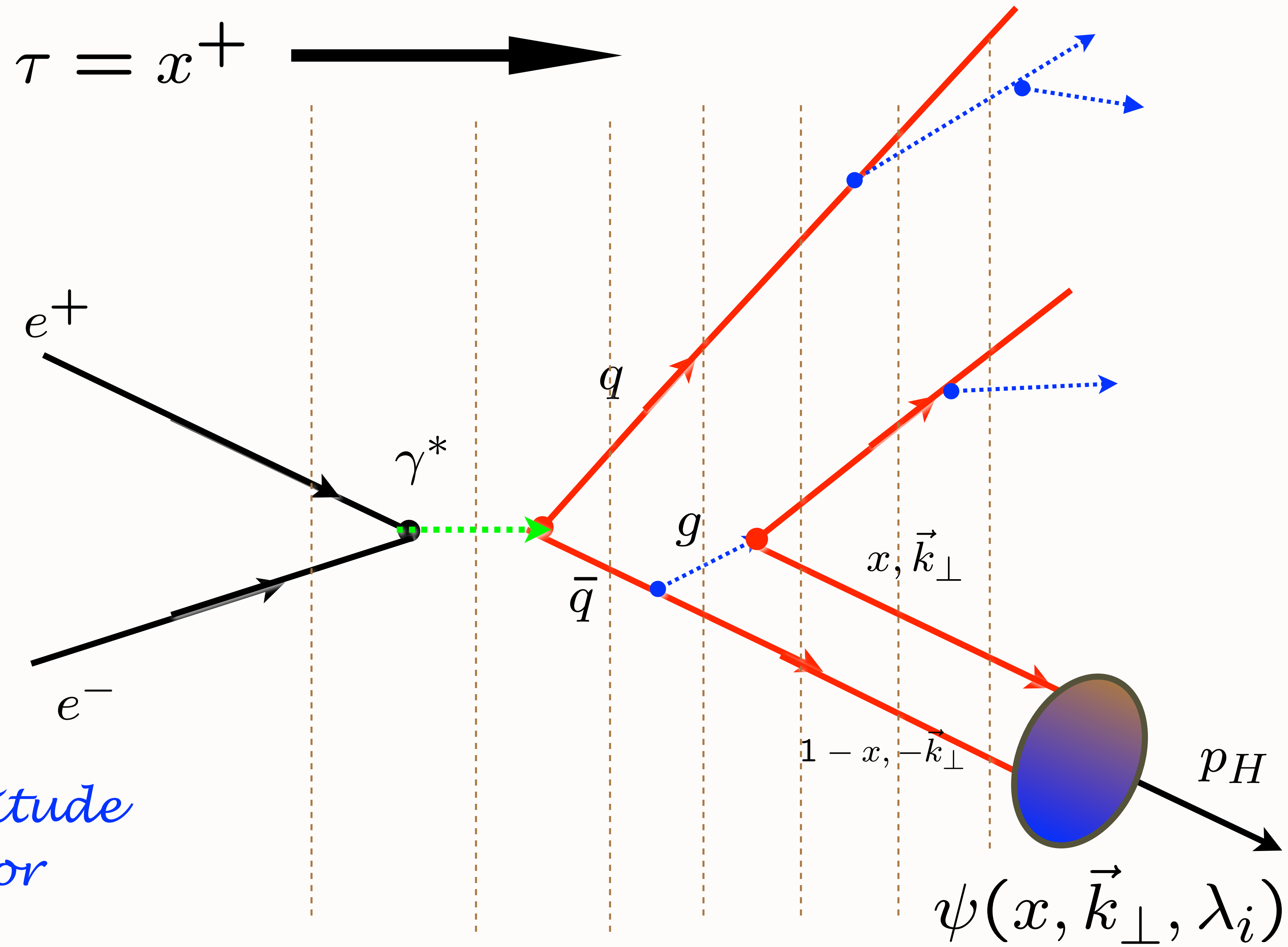
$$y_{\bar{p}} \simeq y_{e^+}$$

***Coalescence of off-shell co-moving positron and antiproton***

***Wavefunction maximal at small impact separation and equal rapidity***

"Hadronization" at the Amplitude Level

# Hadronization at the Amplitude Level



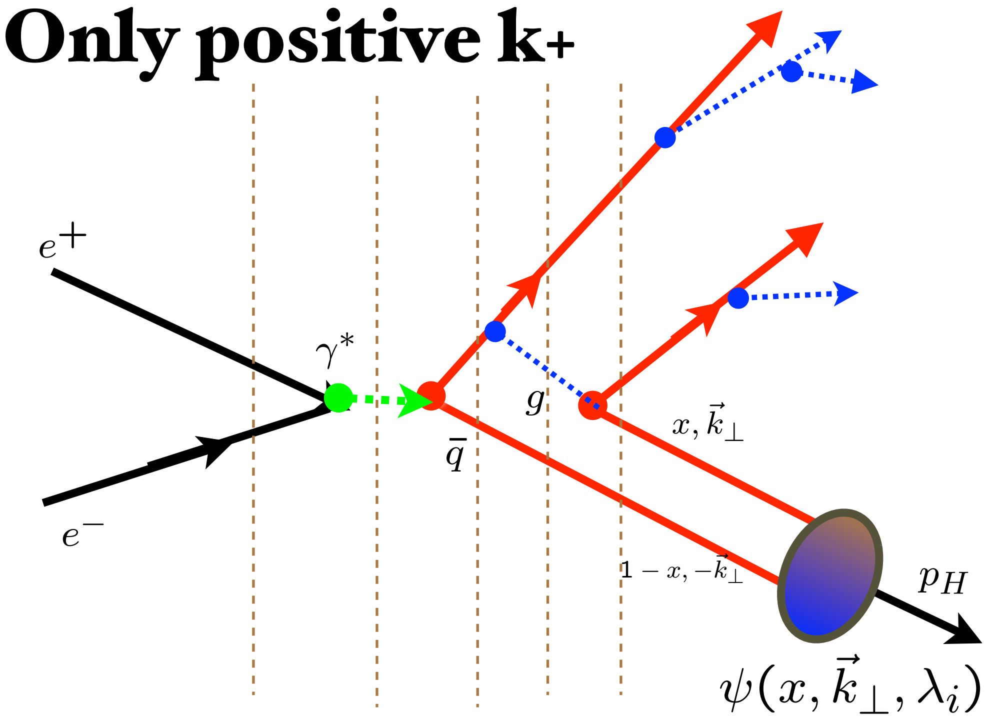

Event amplitude generator

**Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs**

# Off -Shell T-Matrix

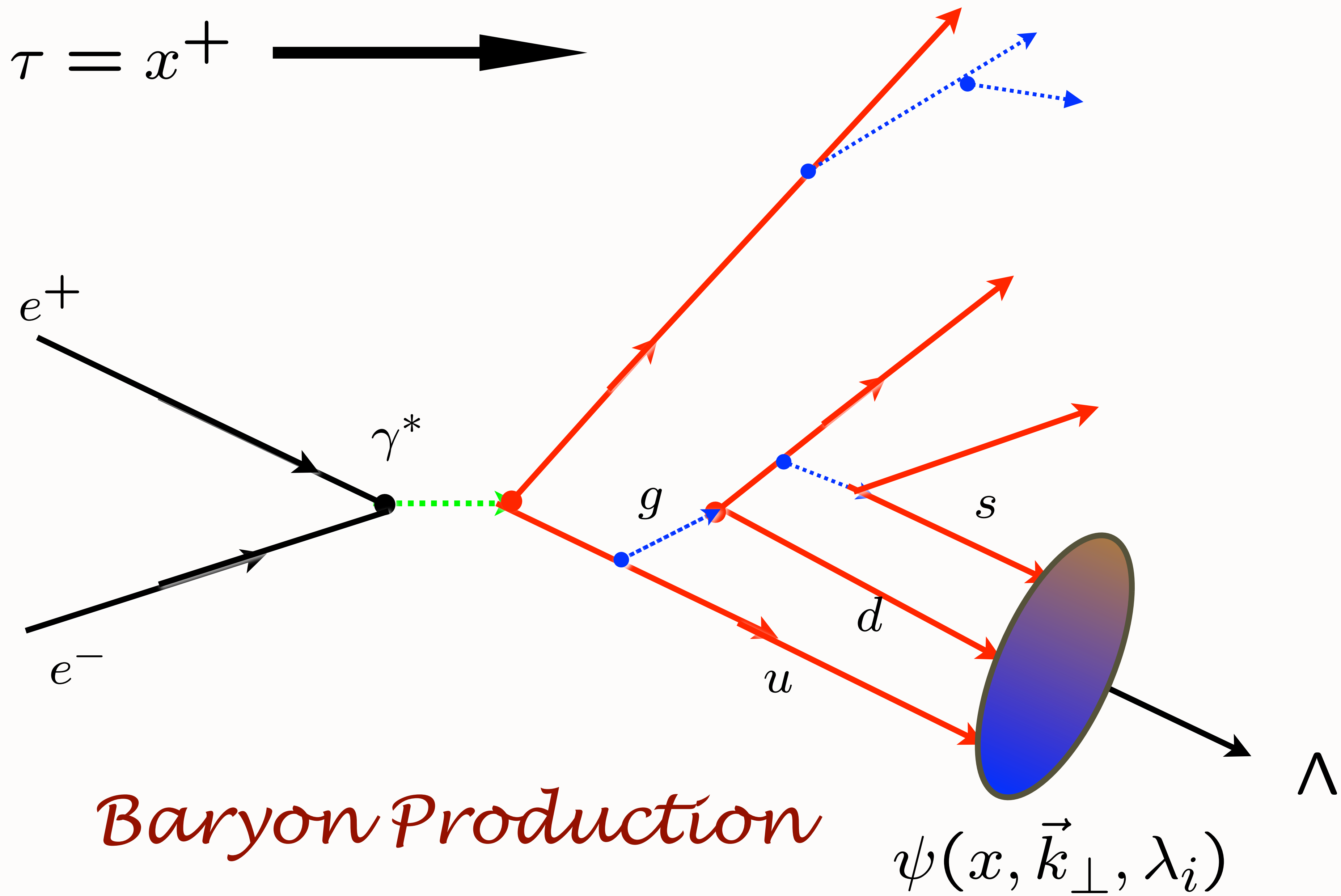
*Event amplitude generator*

- **Quarks and Gluons Off-Shell**
- **LFPth: Minimal Time-Ordering Diagrams-Only positive k+**
- **$J^z$ Conservation at every vertex**
- **Frame-Independent**
- **Cluster Decomposition** Chueng Ji, sjb
- **"History"-Numerator structure universal**
- **Renormalization- alternate denominators**
- **LFWF takes Off-shell to On-shell**
- **Tested in QED: g-2 to three loops**




Roskies, Suaya, sjb

# Hadronization at the Amplitude Level

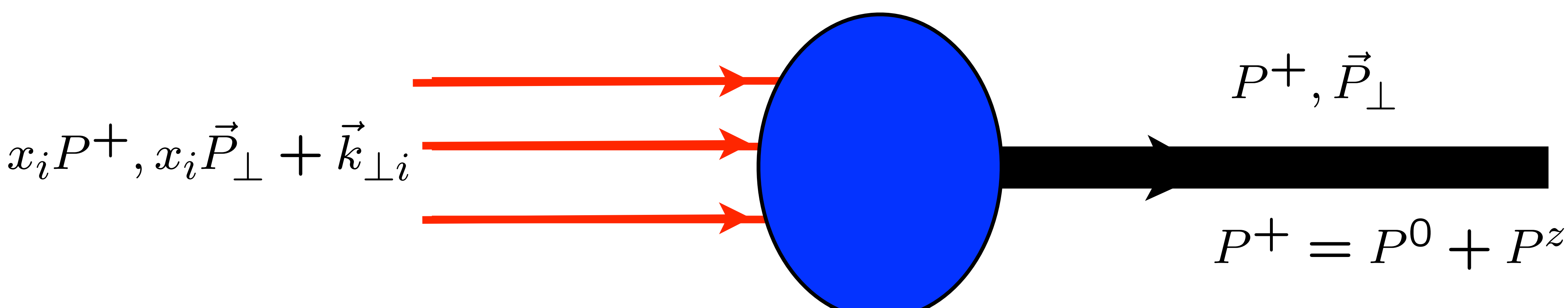



Baryon Production

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

# Features of LF T-Matrix Formalism
# "Event Amplitude Generator"

- Same principle as antihydrogen production: off-shell coalescence
- coalescence to hadron favored at equal rapidity, small transverse momenta
- leading heavy hadron production: D and B mesons produced at large z
- hadron helicity conservation if hadron LFWF has $L^z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin



# An analytic first approximation to QCD

## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable $\zeta$ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**

# Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of $P^\mu$

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

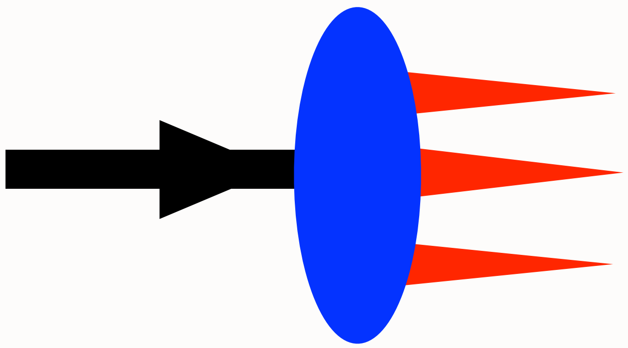
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

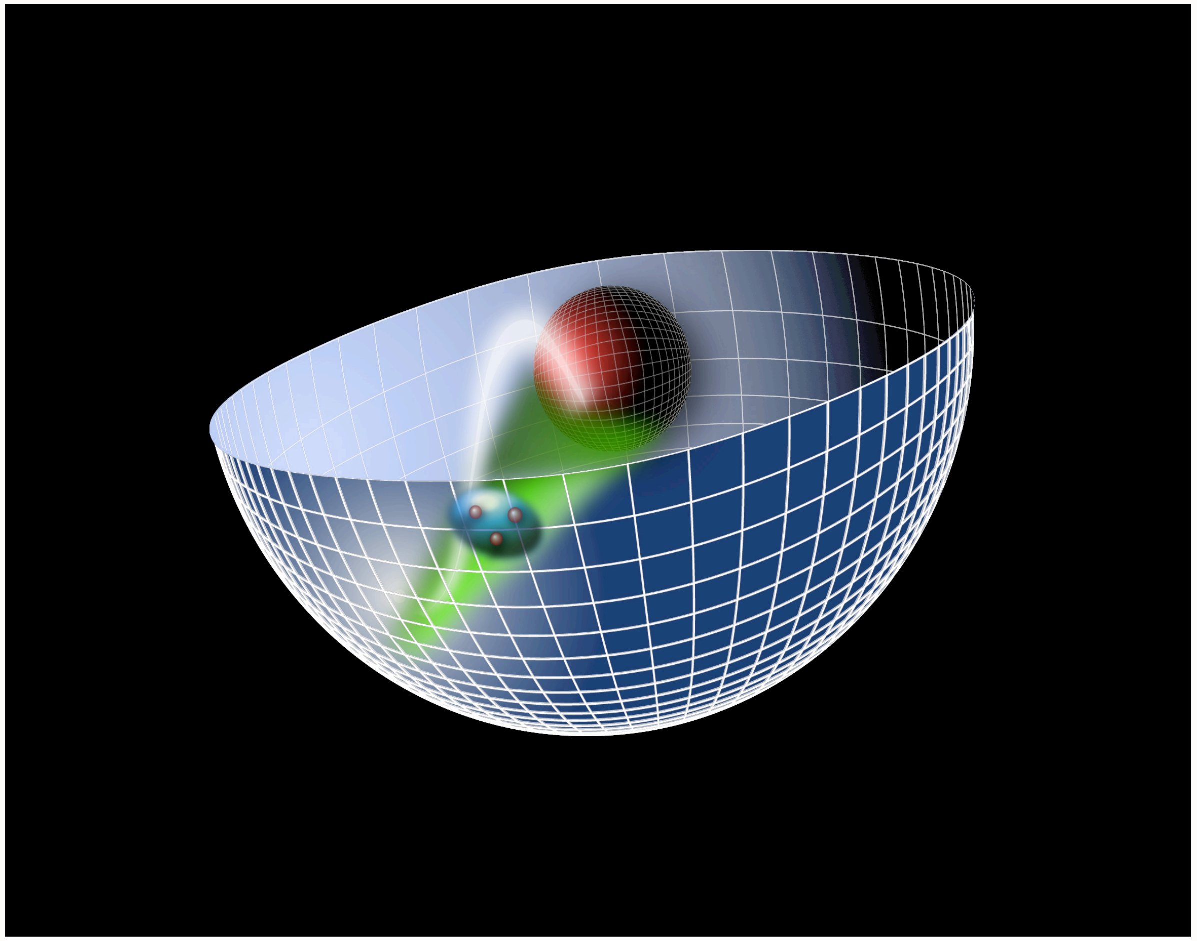
# Light-Front Holography and Non-Perturbative QCD

**Goal:**
**Use AdS/QCD duality to construct a first approximation to QCD**

*Hadron Spectrum*
*Light-Front Wavefunctions,*
*Running coupling in IR*

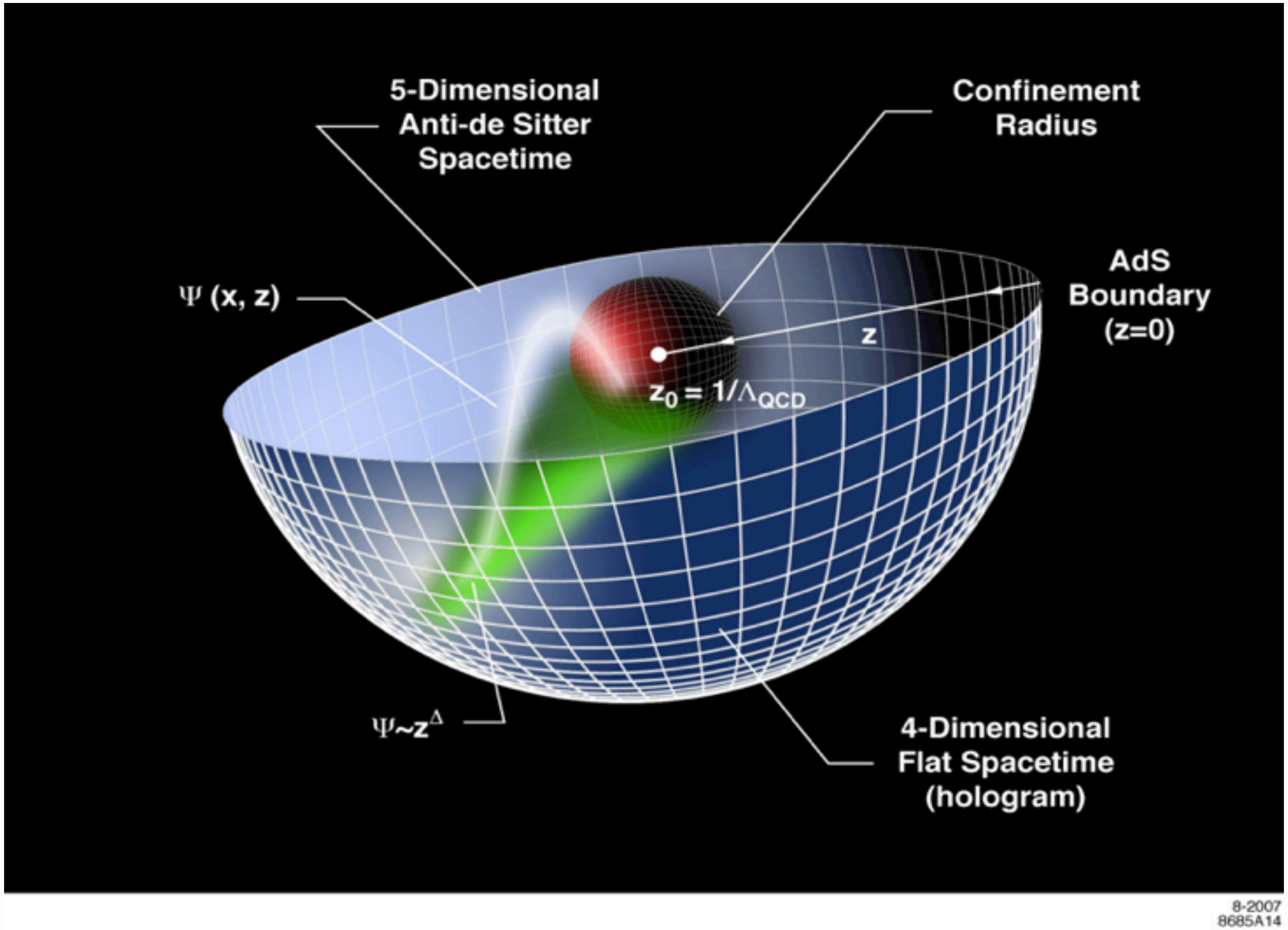


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



in collaboration with
Guy de Teramond

***Central problem for strongly-coupled gauge theories***




*Changes in physical length scale mapped to evolution in the 5th dimension z*

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

# Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

*invariant measure*

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

# Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$ .
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along $x^\mu$-coordinates , $\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$ :

$$\left[ z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

*AdS Schrodinger Equation for bound state of two scalar constituents:*

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2}\right]\phi(z) = \mathcal{M}^2\phi(z)$$

**L = $L^z$ : Light-Front orbital angular momentum**

*Derived from variation of Action in $AdS_5$*

*Hard wall model: truncated space*

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

# *Match fall-off at small z to conformal twist-dimension at short distances*

*twist*

$$\Delta = 2 + L$$

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z =$ $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

$$\Delta = 2 + L$$

*Twist = equivalent to dimensions of chiral superfields*

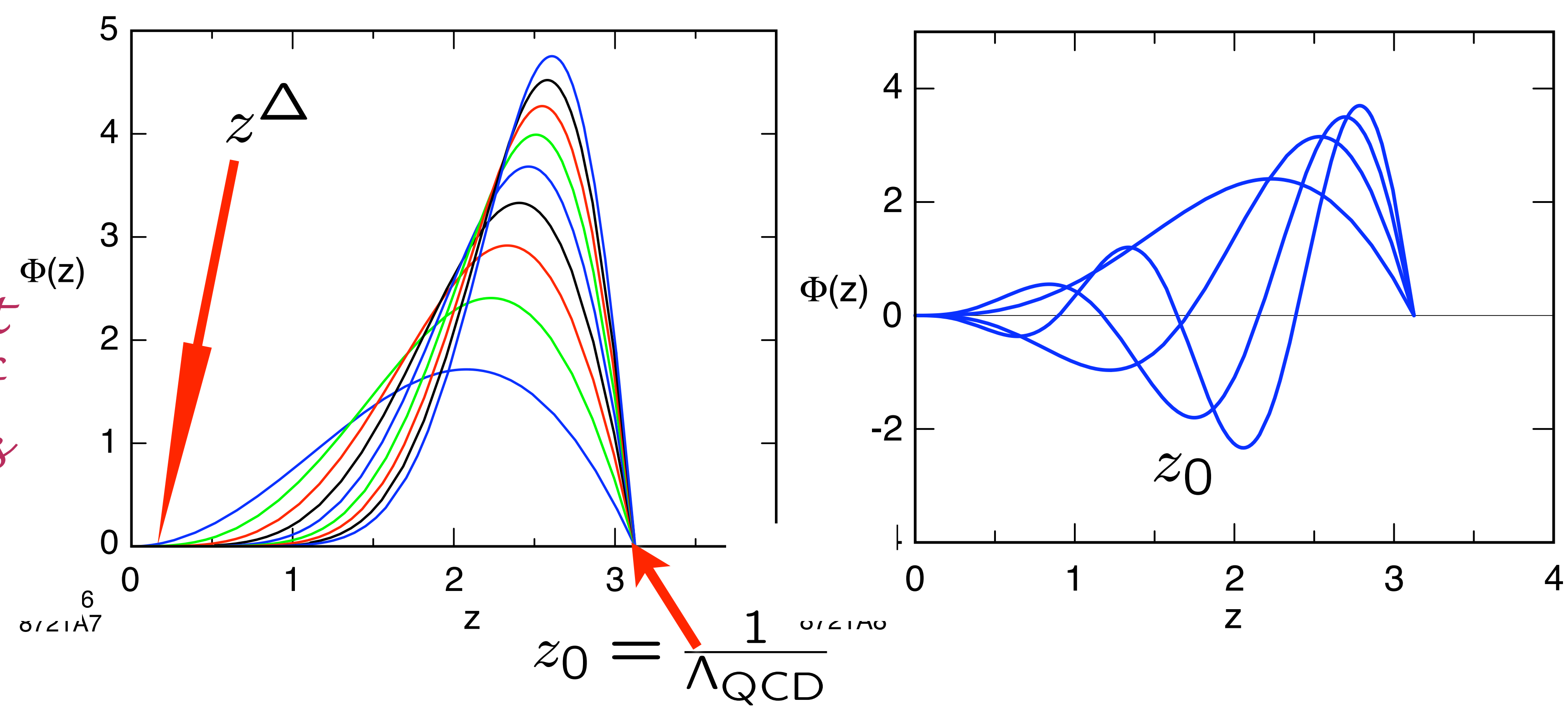



Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

**Identify hadron by its interpolating operator at z --> 0**

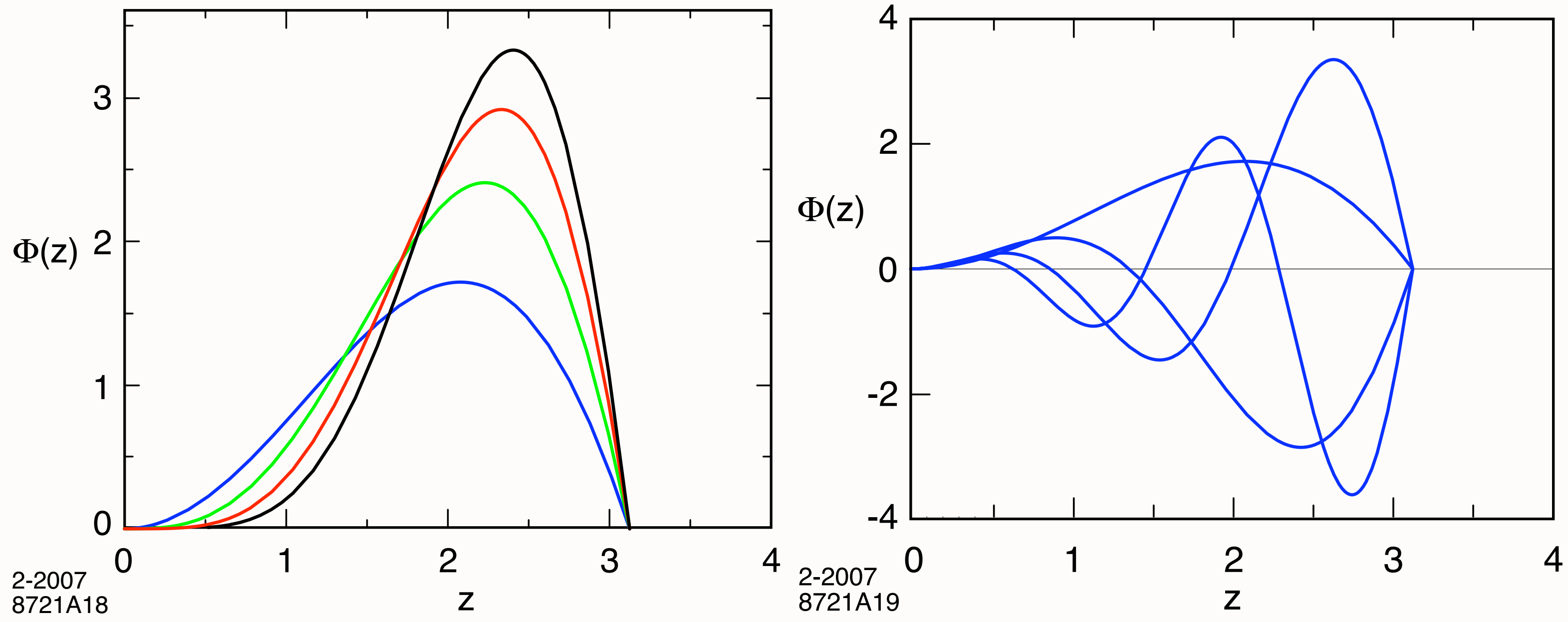
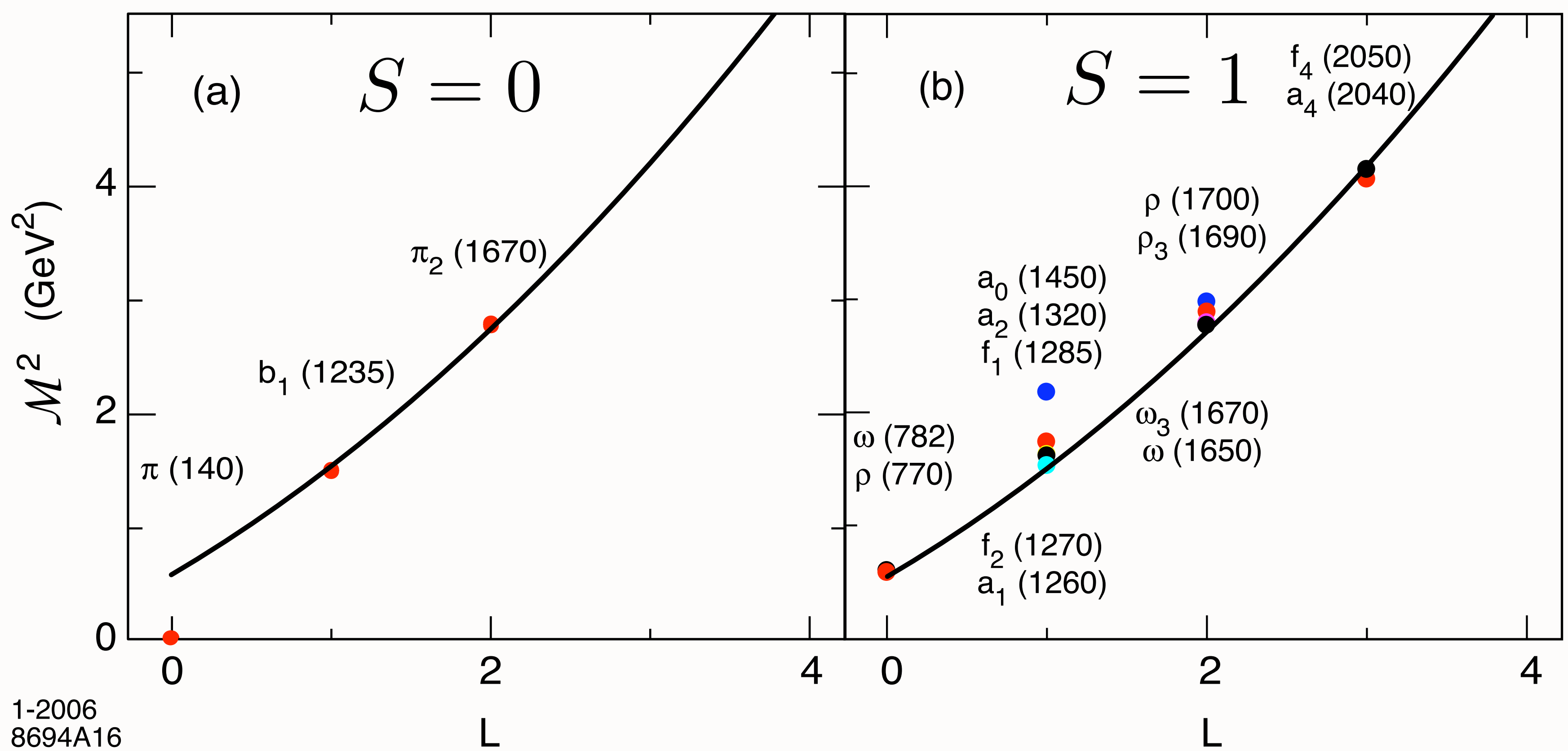



Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\rm QCD}$ = 0.32 GeV .




***Hard Wall***

$$m_\pi = m_\rho$$

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

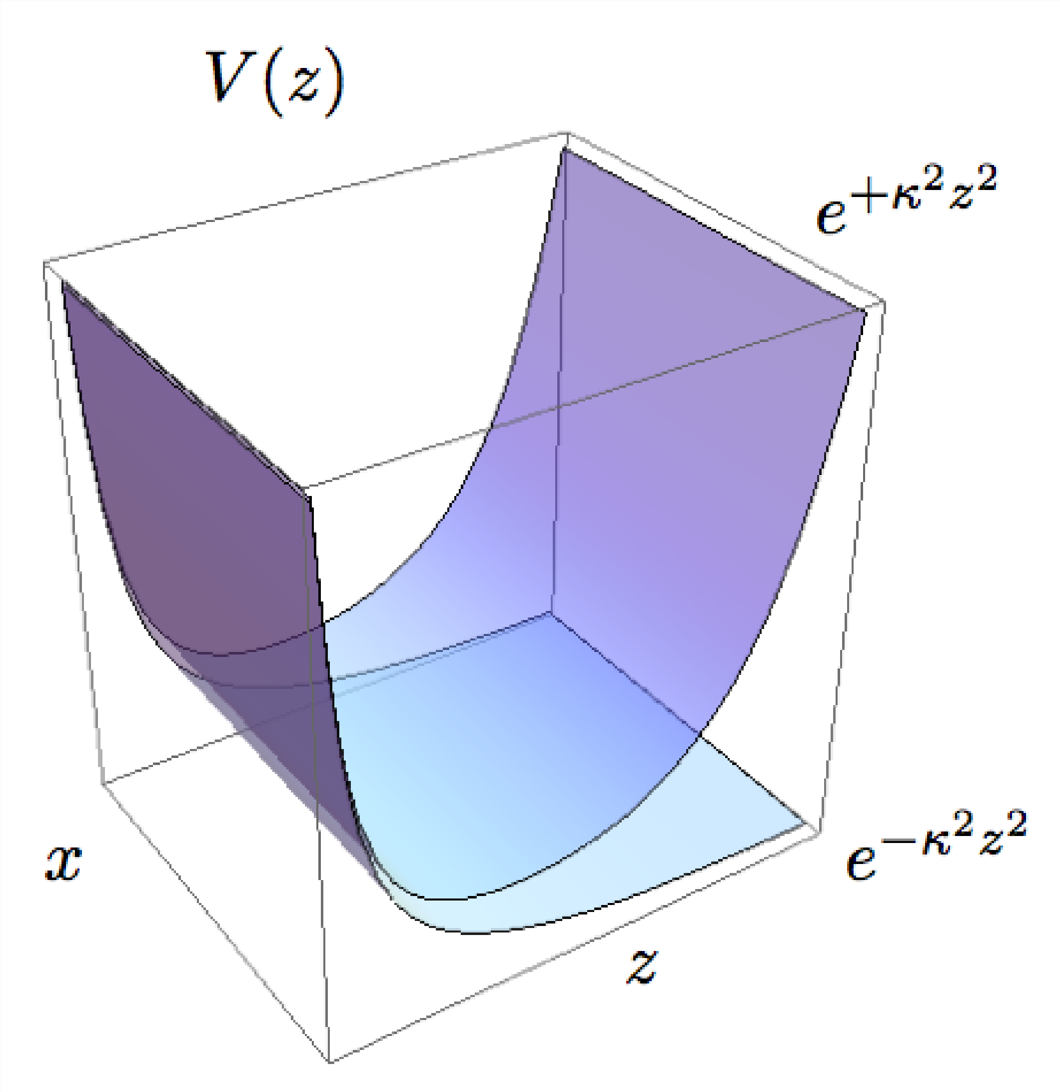
where $\varphi(z) \to 0$ at small $z$ for geometries which are asymptotically $AdS_5$

- Gravitational potential energy for object of mass $m$

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$

- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$




Klebanov and Maldacena

## Dual QCD Light-Front Wave Equation

$$\boxed{z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle}$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z) = \mathcal{M}^2\Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension $\tau$ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

de Teramond, Dosch, sjb

# General-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[z^2 \partial_z^2 - \left(3-2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2-J)^2 + L^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2}$$

- de Teramond, sjb

**Positive-sign dilaton**

# AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[ -\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action for Dilaton-Modified $AdS_5$*

***Identical to Light-Front Bound State Equation!***

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

# Uniqueness

de Teramond, Dosch, sjb
(preliminary)

- $\zeta^2$ confinement potential and dilaton profile unique!
- linear Regge trajectories in n and L: same slope
- massless pion in chiral limit
- derive from conformal invariance in time: invariant action for massless quarks
- Same principle, equation of motion as dAFF:
- Conformal Invariance in Quantum Mechanics

Vittorio de Alfaro (Turin U. & INFN, Turin), S. Fubini, G. Furlan (CERN). Jan 1976. 57 pp.
Published in **Nuovo Cim. A34 (1976) 569**

## QCD Meson Spectrum

$$H^{LF}_{QCD}$$

**Fixed Light-Front Time (Front form)**

Fixed $\tau = t + z/c$

$$(H^0_{LF} + H^I_{LF})|\Psi >= M^2|\Psi >$$

Coupled Light-Front Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis $\zeta, \phi$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

# Light-Front Schrödinger Equation

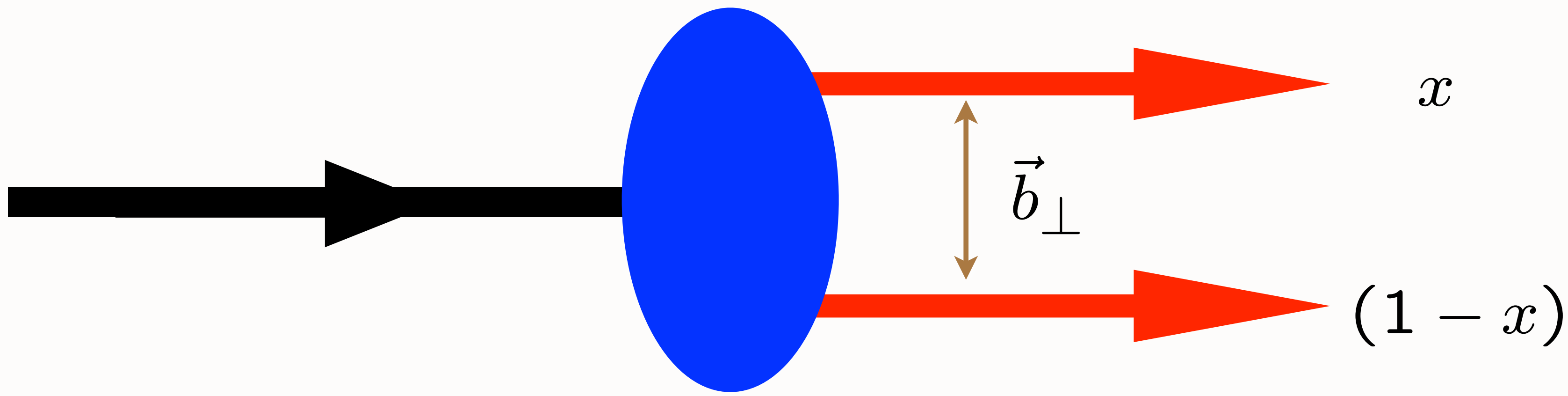
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$




AdS/QCD:

$$U(\zeta, S, L) = \kappa^2\zeta^2 + \kappa^2(L + S - 1/2)$$

Quark separation increases with L

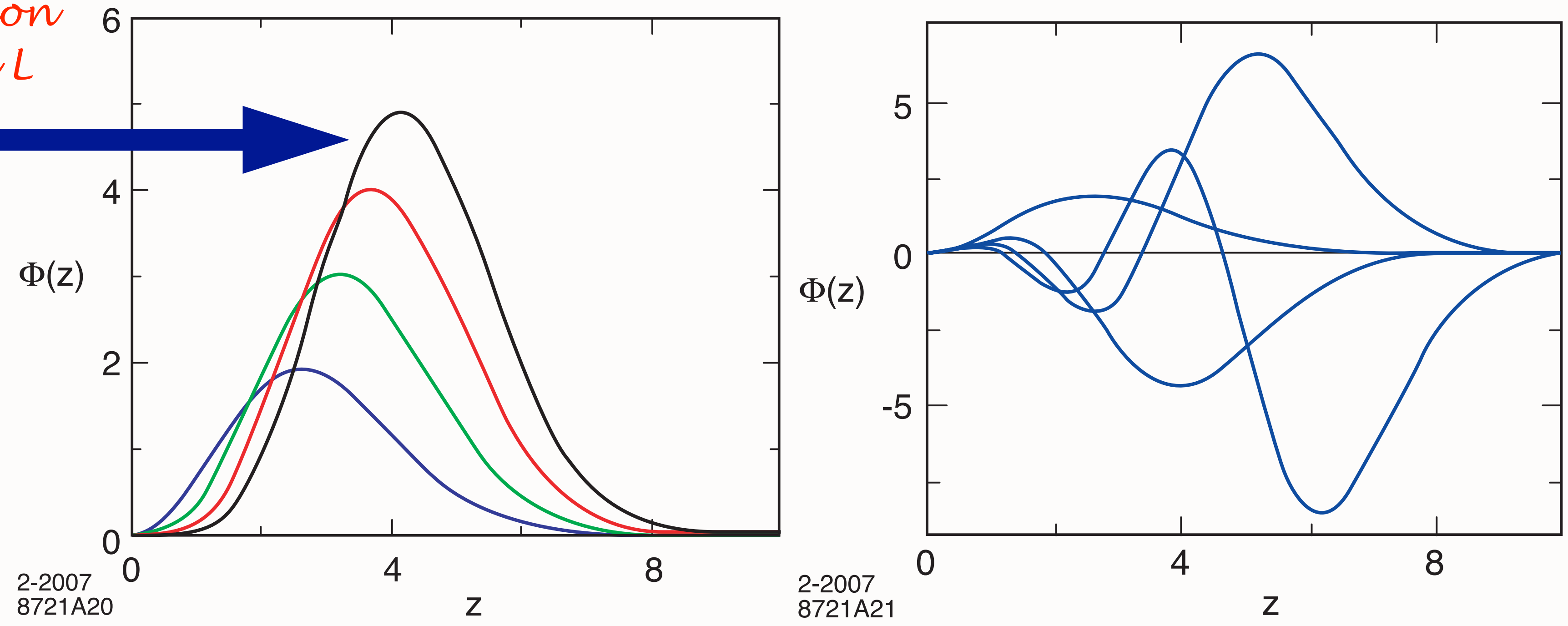
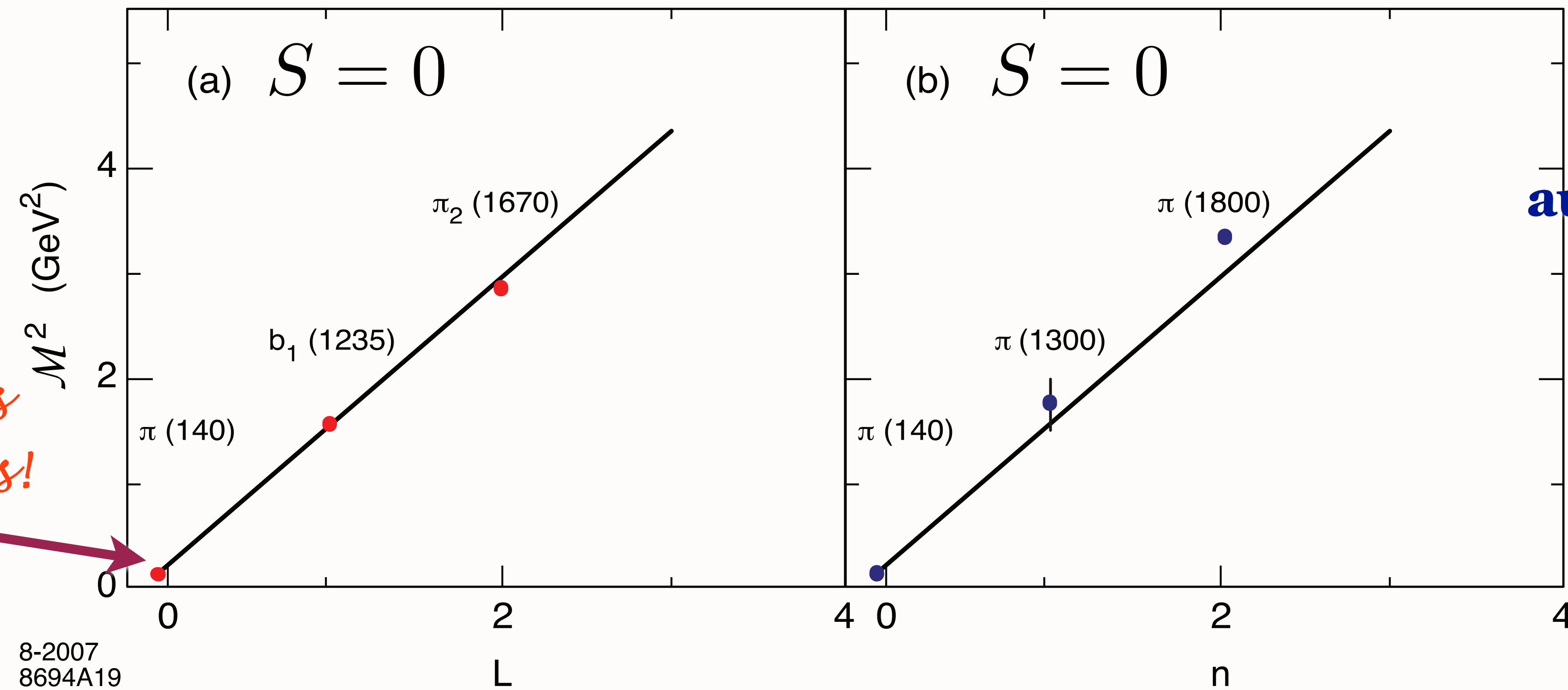



Fig: Orbital and radial AdS modes in the soft wall model for $\kappa$ = 0.6 GeV .

# Soft Wall Model

**Pion mass automatically zero!**



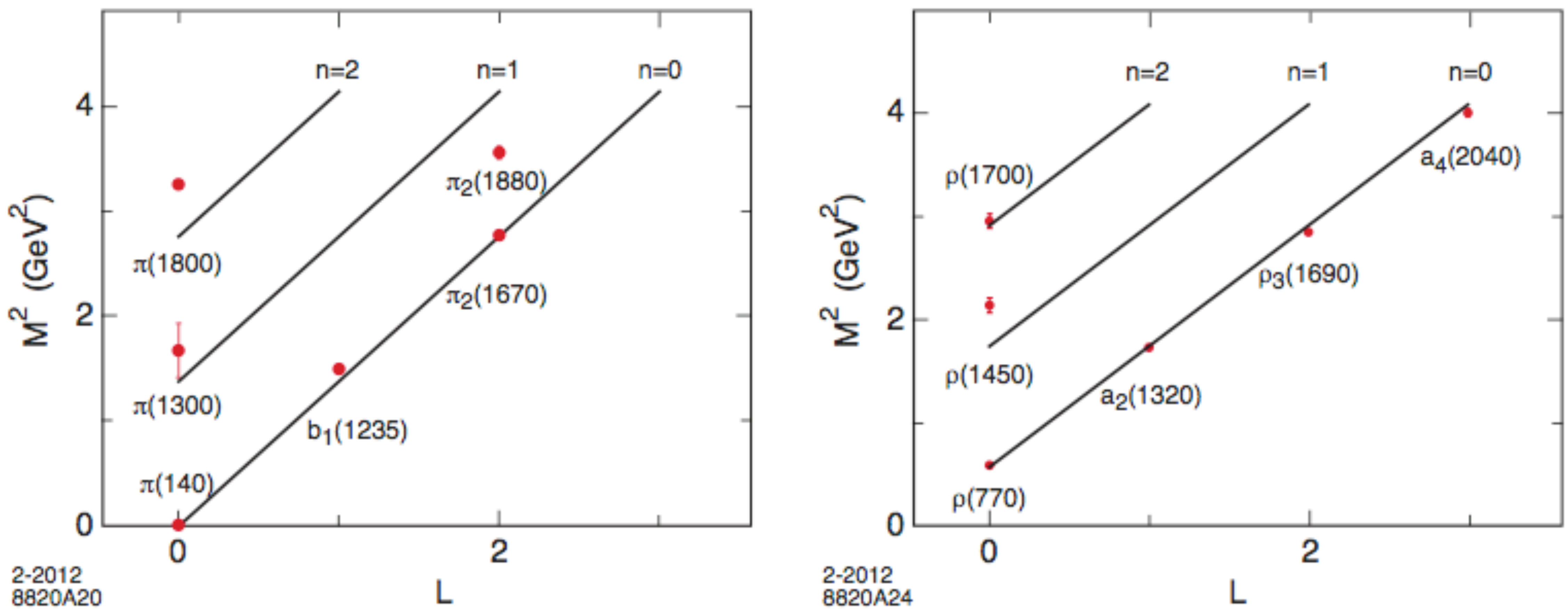

Pion has zero mass!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$




I=1 orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson $a$-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

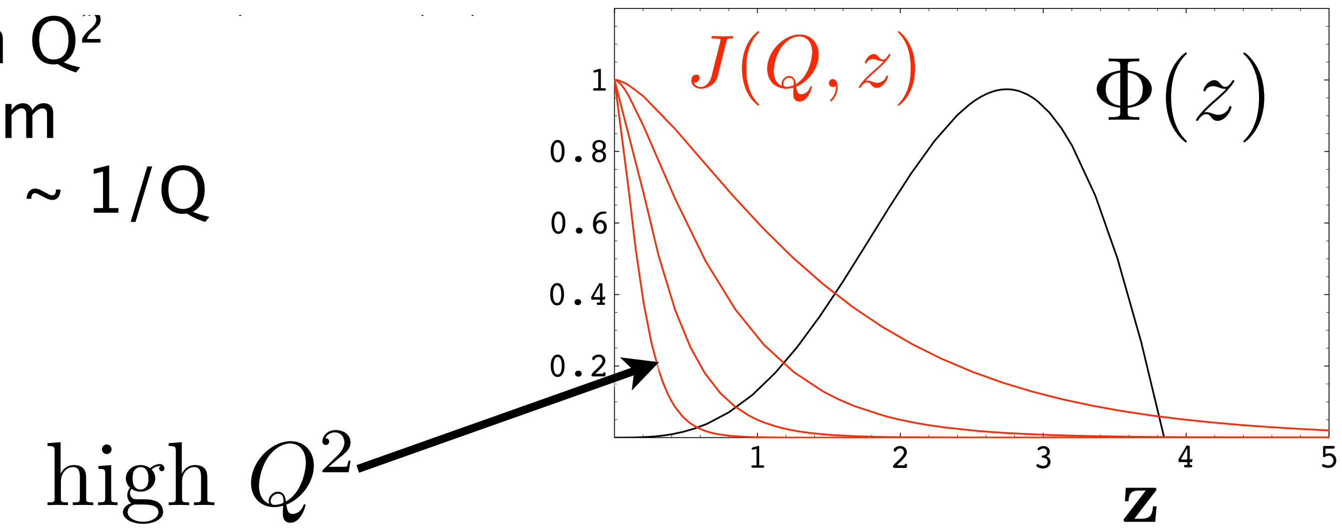
# Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High $Q^2$
from
small z ~ 1/Q




**Polchinski, Strassler**
**de Teramond, sjb**

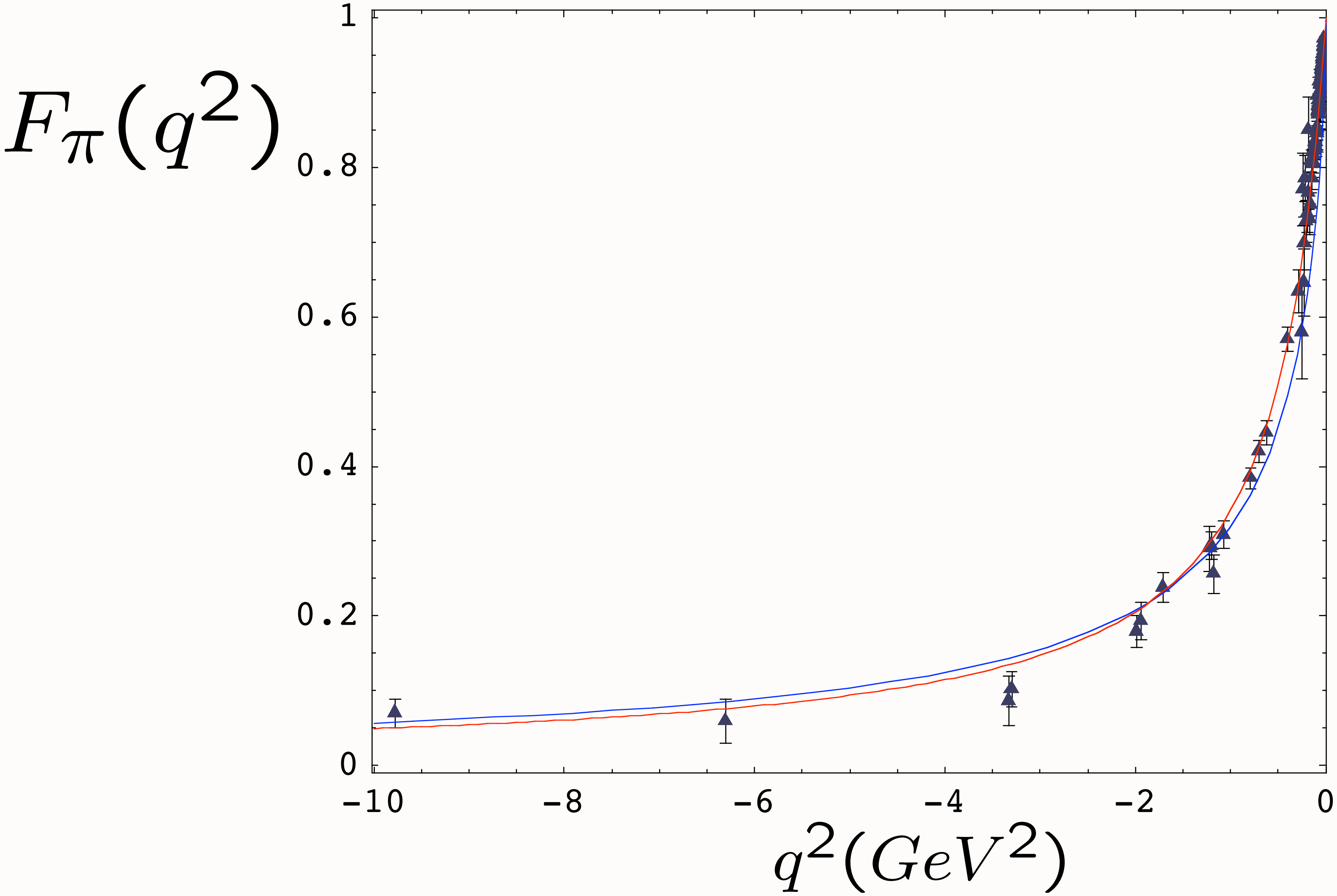
Consider a specific AdS mode $\Phi^{(n)}$ dual to an $n$ partonic Fock state $|n\rangle$. At small $z$, $\Phi$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

**Dimensional Quark Counting Rules:**
**General result from**
**AdS/CFT and Conformal Invariance**

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

# Spacelike pion form factor from AdS/CFT




**Data Compilation Baldini, Kloe and Volmer**

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant*

**de Teramond, sjb**
**See also: Radyushkin**

## Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary $Q$ using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$ !

*Identical to Polchinski-Strassler Convolution of AdS Amplitudes*

# Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) \left|\Phi_\pi(z)\right|^2 ,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x\, dx\, J_0\left(zQ\sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x\, dx \int \frac{dz}{z^3} J_0\left(zQ\sqrt{\frac{1-x}{x}}\right) \left|\Phi_\pi(z)\right|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary $Q$

$$\boxed{\left|\tilde\psi_{q\bar q/\pi}(x, \zeta)\right|^2 = \frac{R^3}{2\pi}\, x(1-x) \frac{\left|\Phi_\pi(\zeta)\right|^2}{\zeta^4},}$$

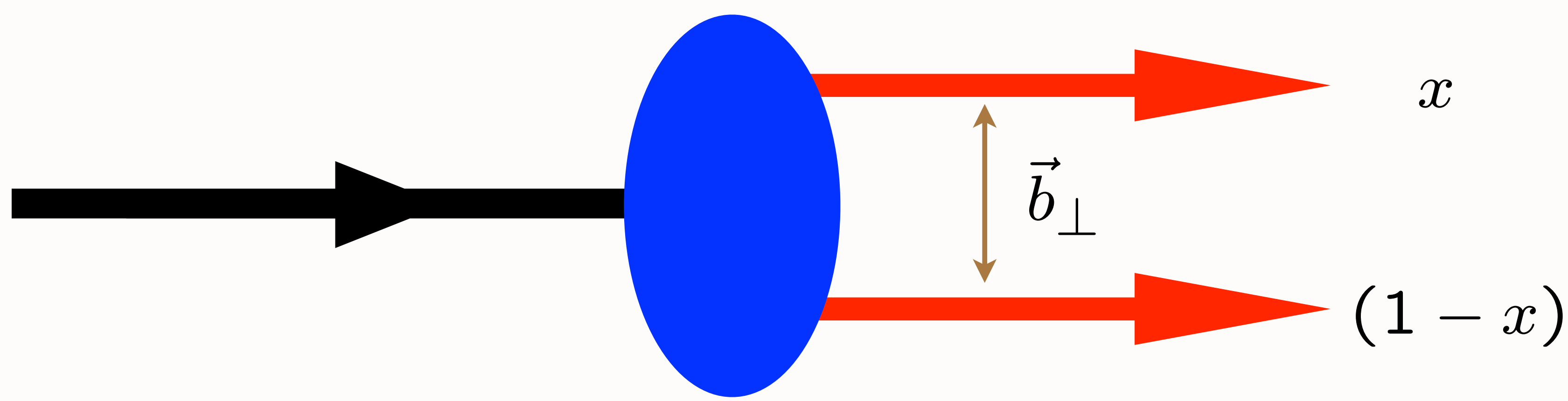
*Identical to LF Holography obtained from electromagnetic current*

de Teramond, sjb

LF(3+1) ⟷ $AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

*Light Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements*

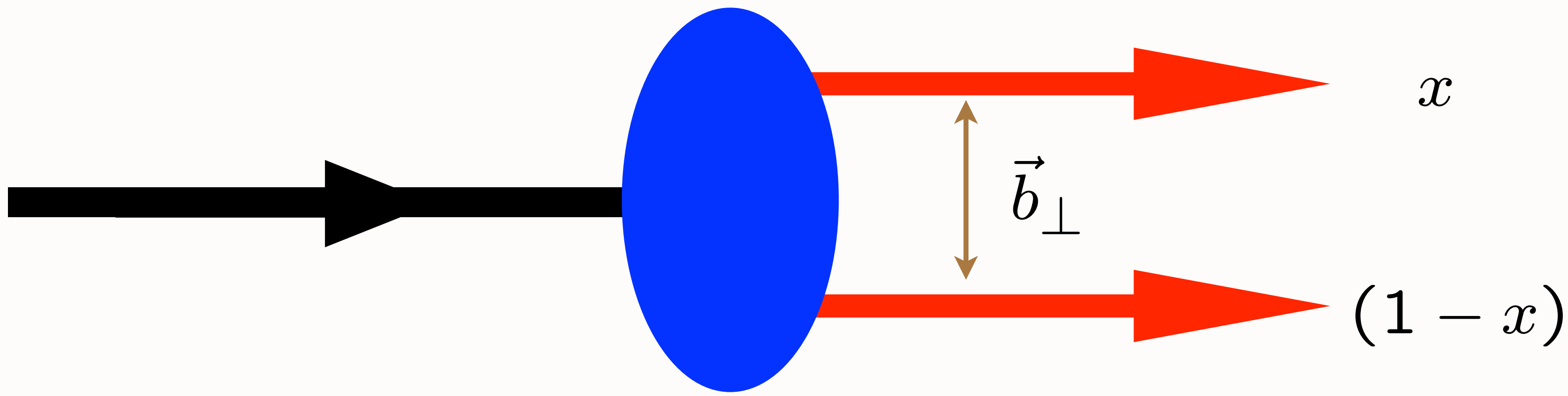
# Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$




$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

soft wall confining potential:

G. de Teramond, sjb

## Meson Spectrum in Soft Wall Model

- Linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]
- Dilaton profile $\varphi(z) = +\kappa^2 z^2$
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

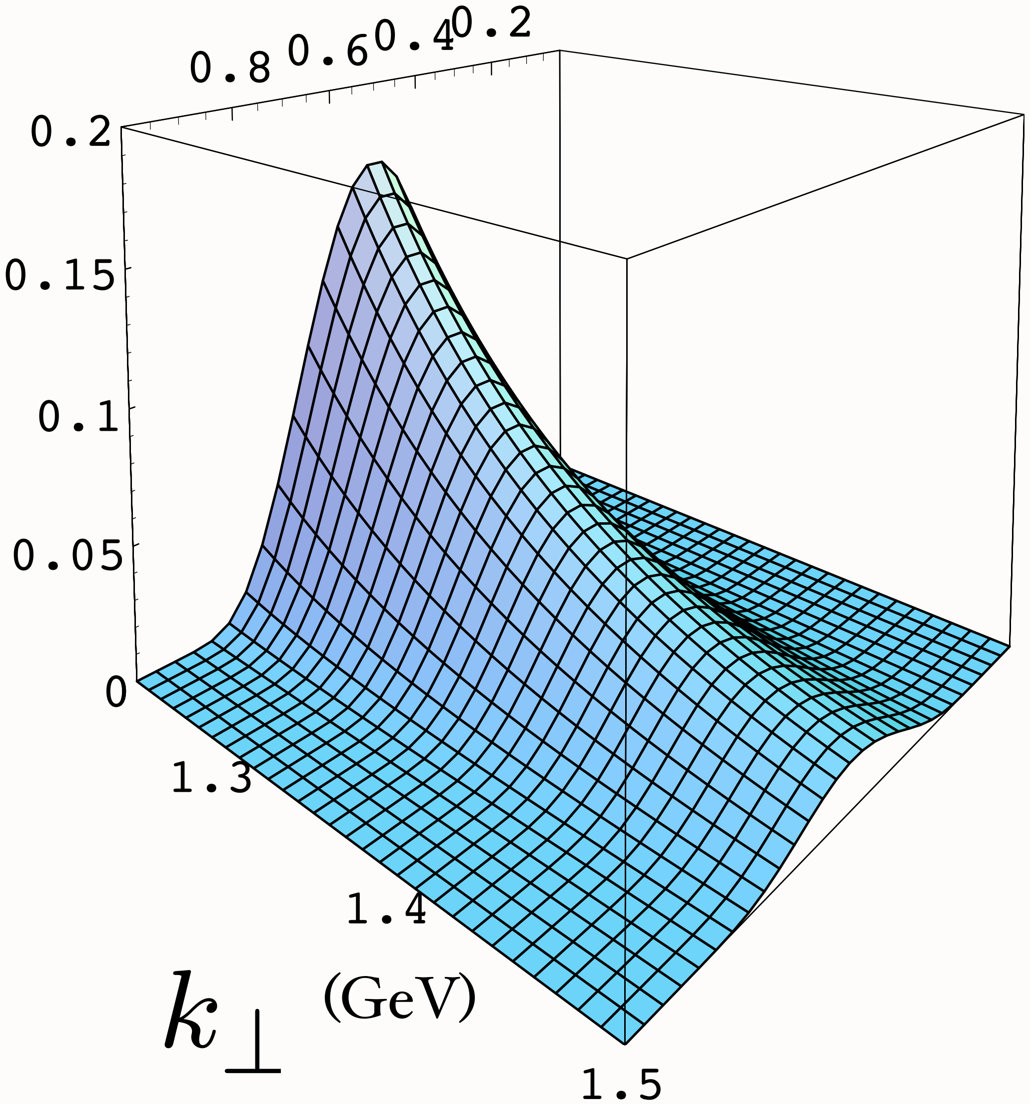
$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

# Prediction from AdS/CFT: Meson LFWF

$\psi_M(x, k_\perp^2)$




**de Teramond, sjb**

**"Soft Wall" model**

$\kappa = 0.375$ GeV

massless quarks

**Note coupling**
$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

*Connection of Confinement to TMDs*

# Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

# AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

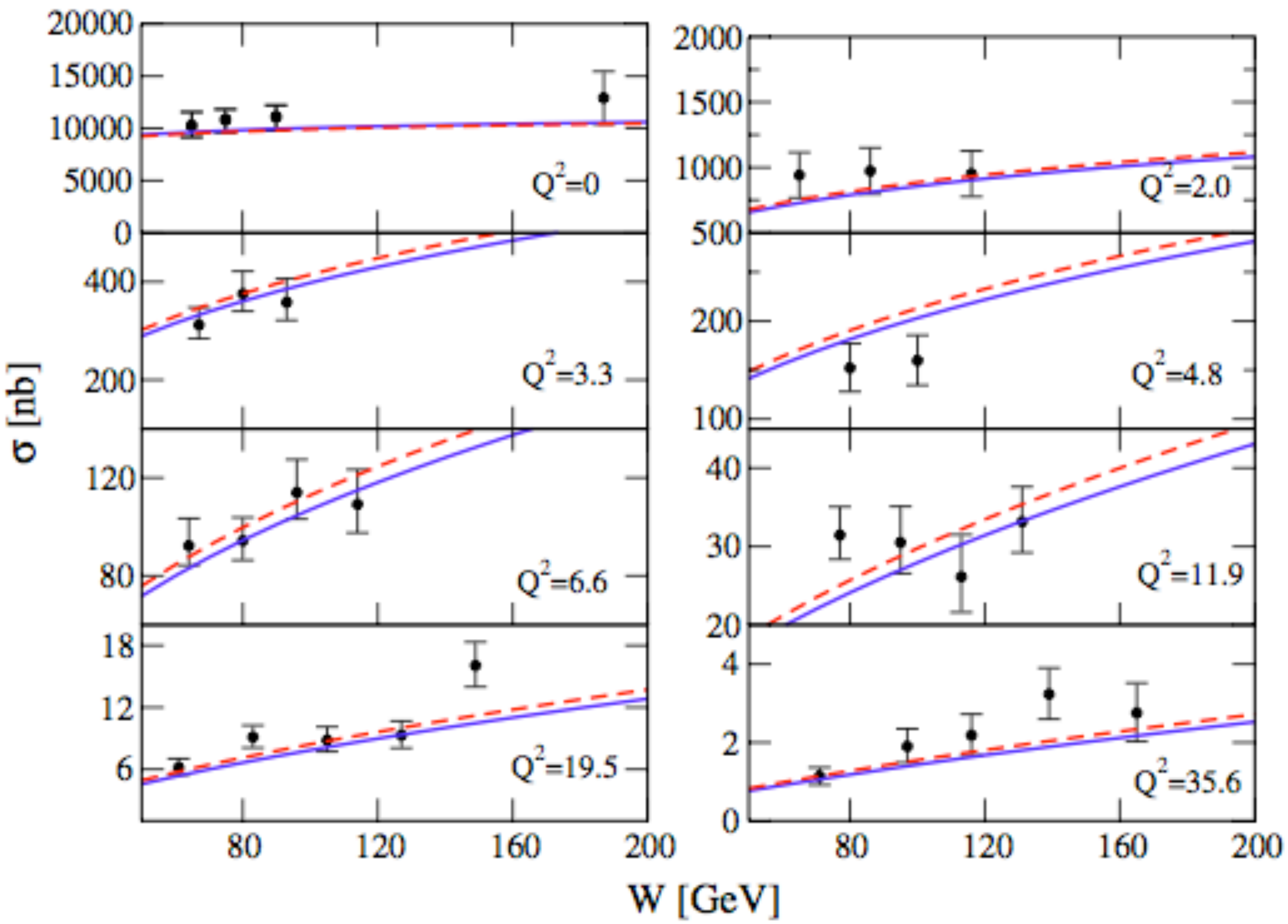
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive

$$\phi(x, \zeta) = \mathcal{N} \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} \exp\left(-\frac{\kappa^2 \zeta^2}{2}\right),$$

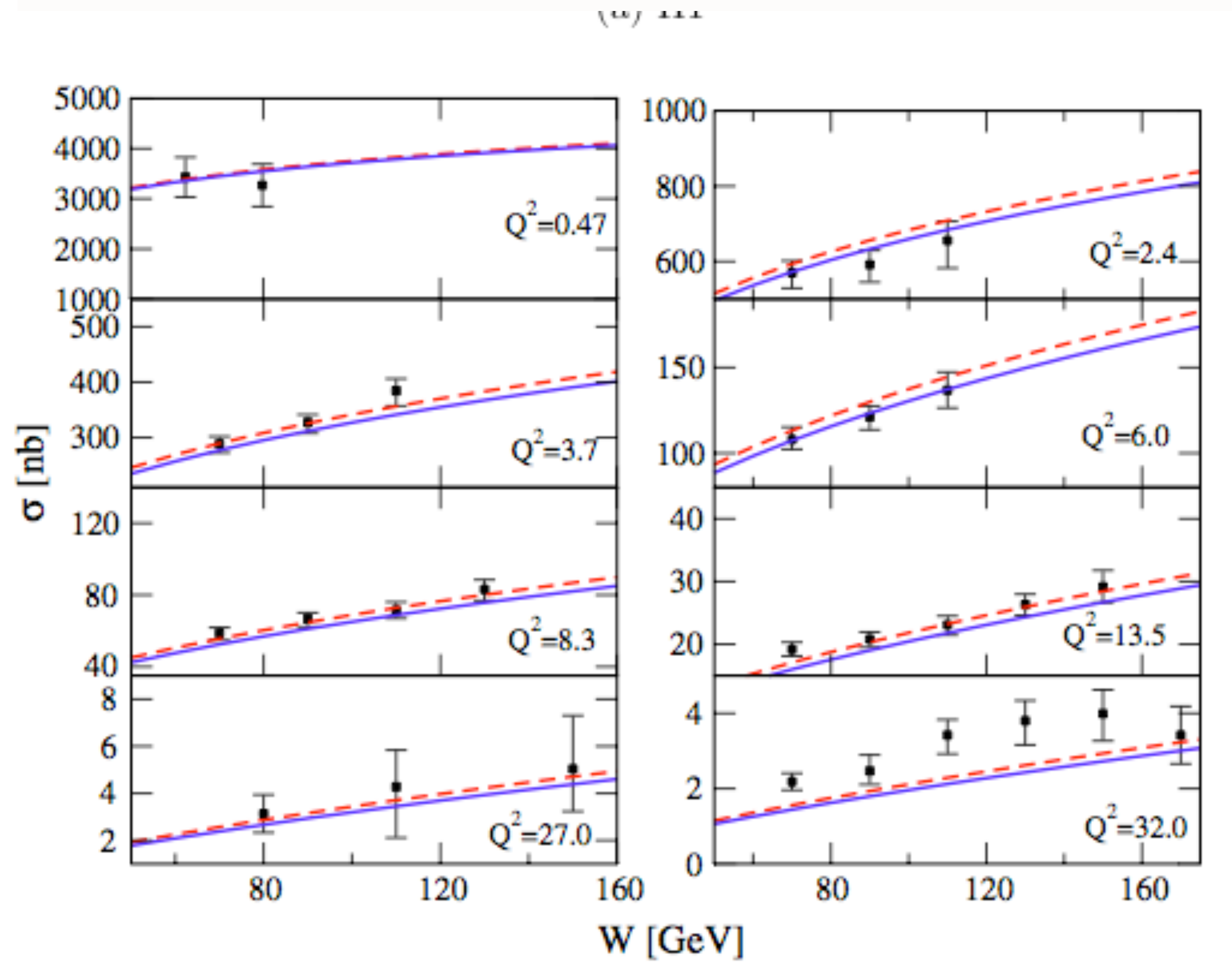
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

# AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction



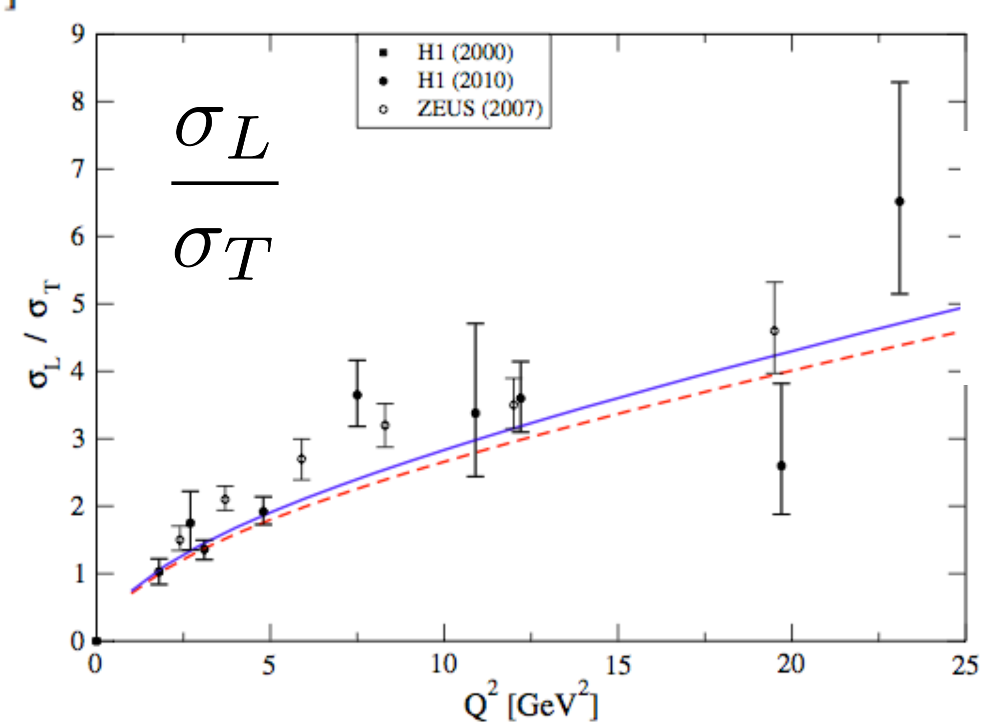

(a) H




(b) ZEUS

**J. R. Forshaw,**
**R. Sandapen**

$$\gamma^* p \to \rho^0 p'$$




$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

de Teramond, Dosch, sjb

# General-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

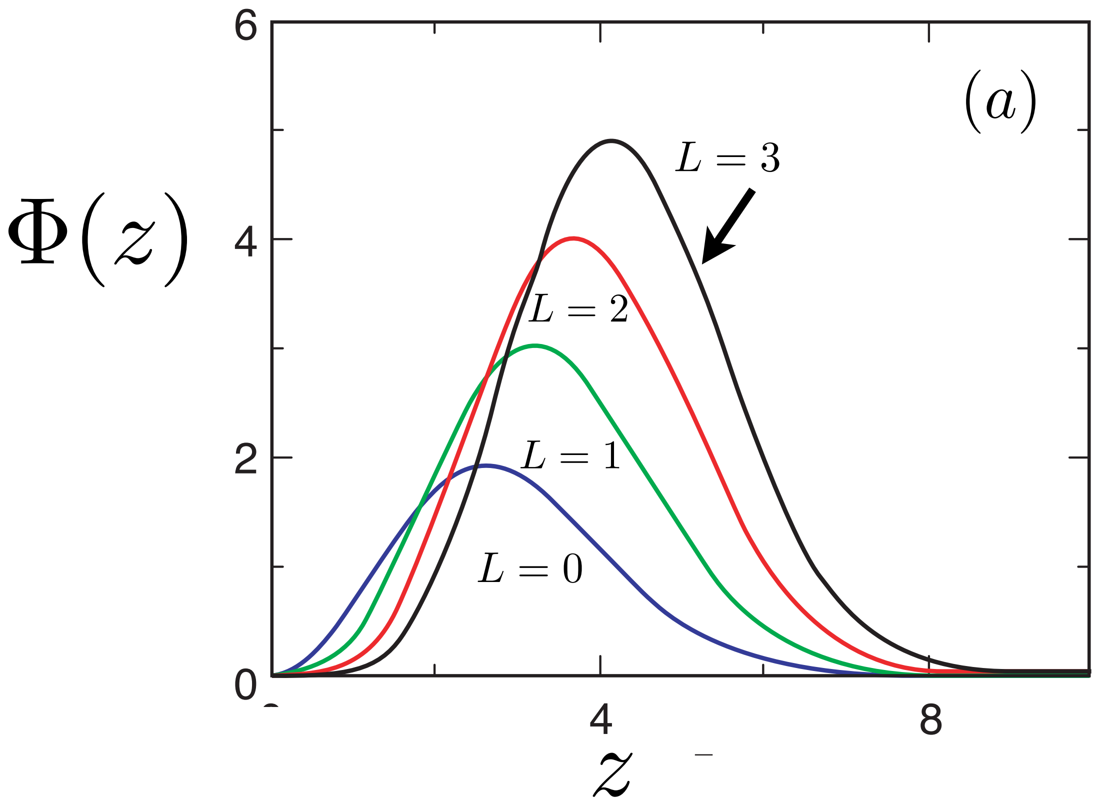
- Upon substitution $z \rightarrow \zeta$

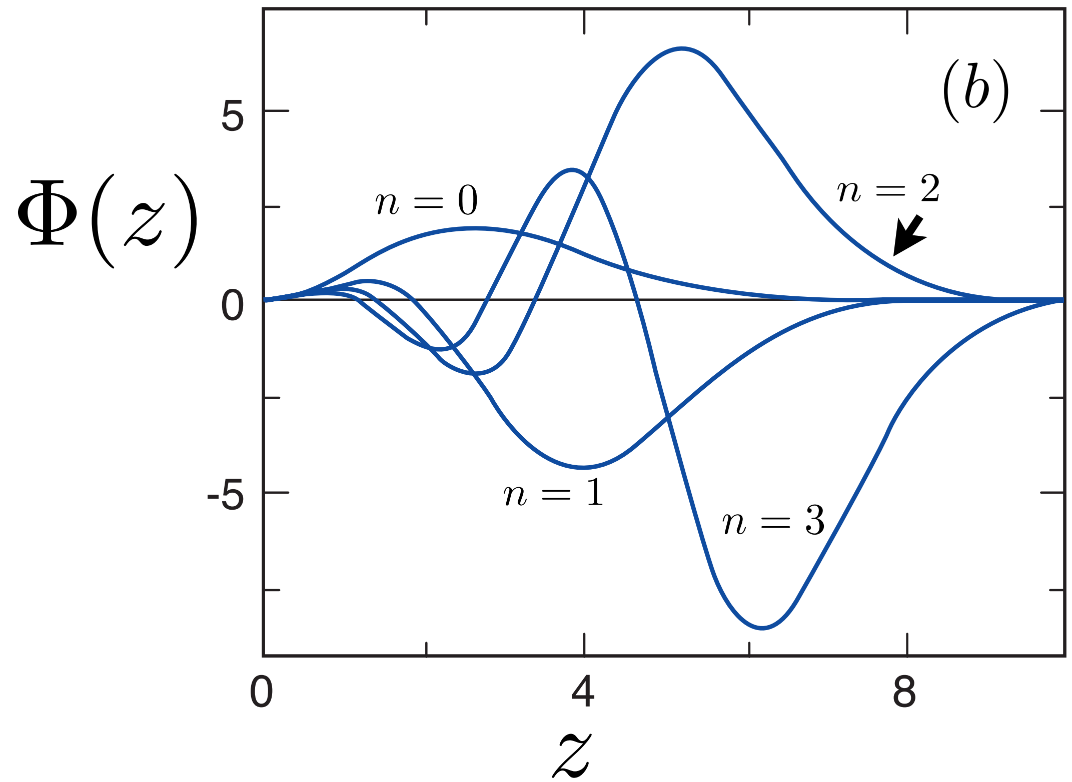
$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

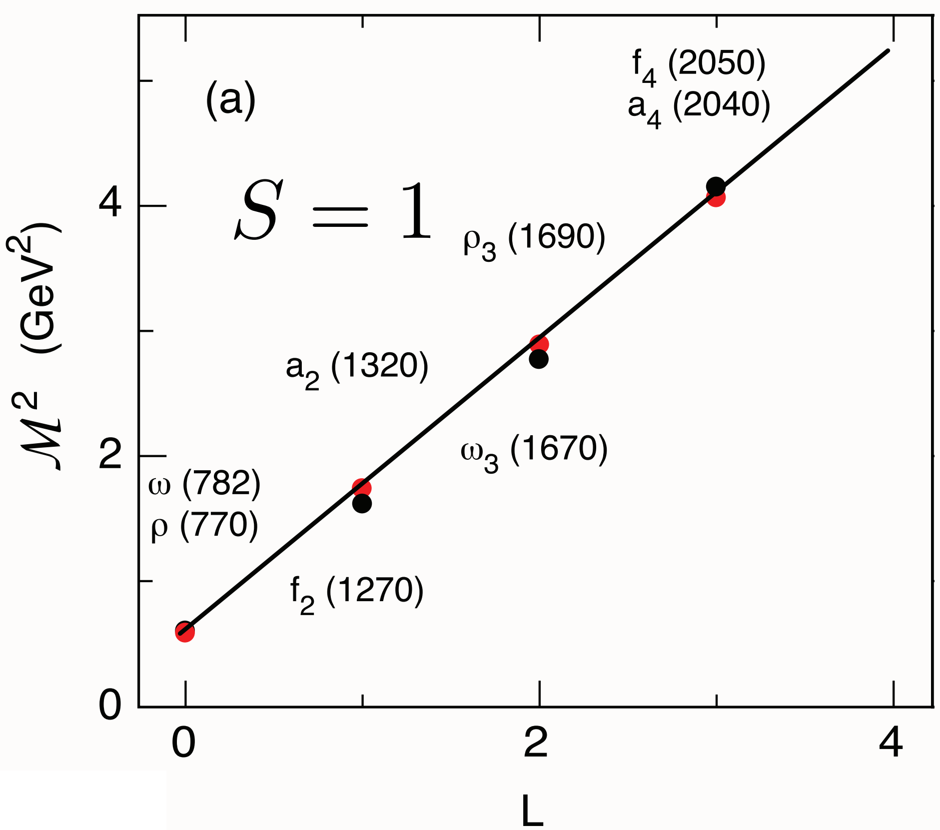
we find the LF wave equation

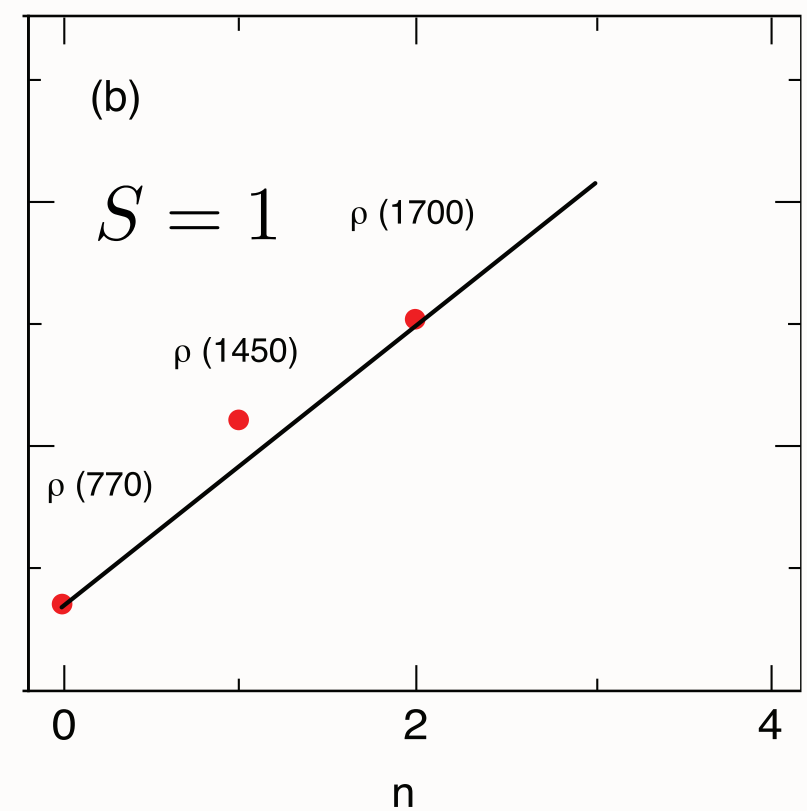
$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$


(a)
Φ(z)
L = 3
L = 2
L = 1
L = 0
0
2
4
6
4
8
z

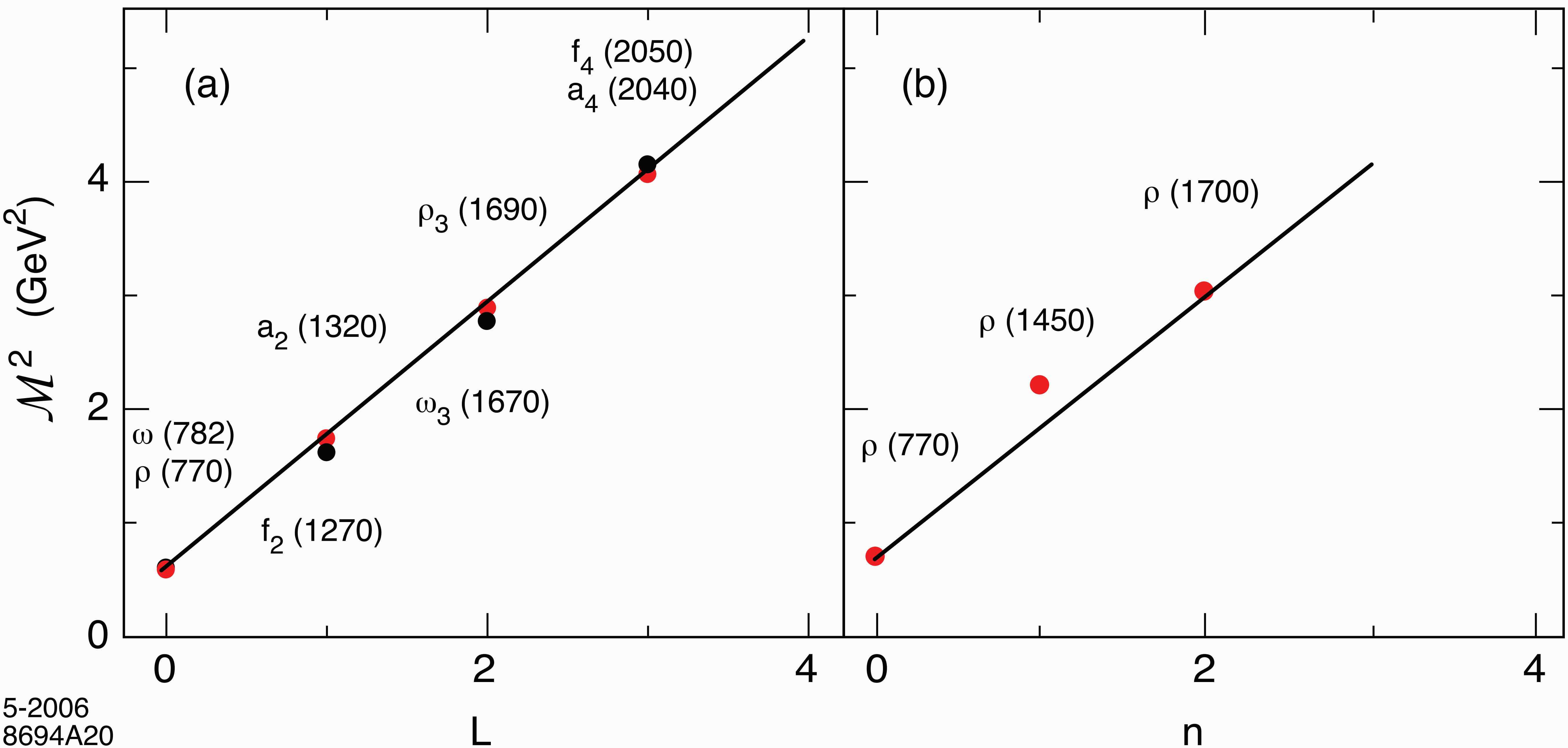

(b)
Φ(z)
n = 0
n = 1
n = 2
n = 3
5
0
-5
0
4
8
z


(a)
S = 1
f4 (2050)
a4 (2040)
ρ3 (1690)
a2 (1320)
ω3 (1670)
ω (782)
ρ (770)
f2 (1270)
M² (GeV²)
0
2
4
L


(b)
S = 1
ρ (1700)
ρ (1450)
ρ (770)
0
2
4
n

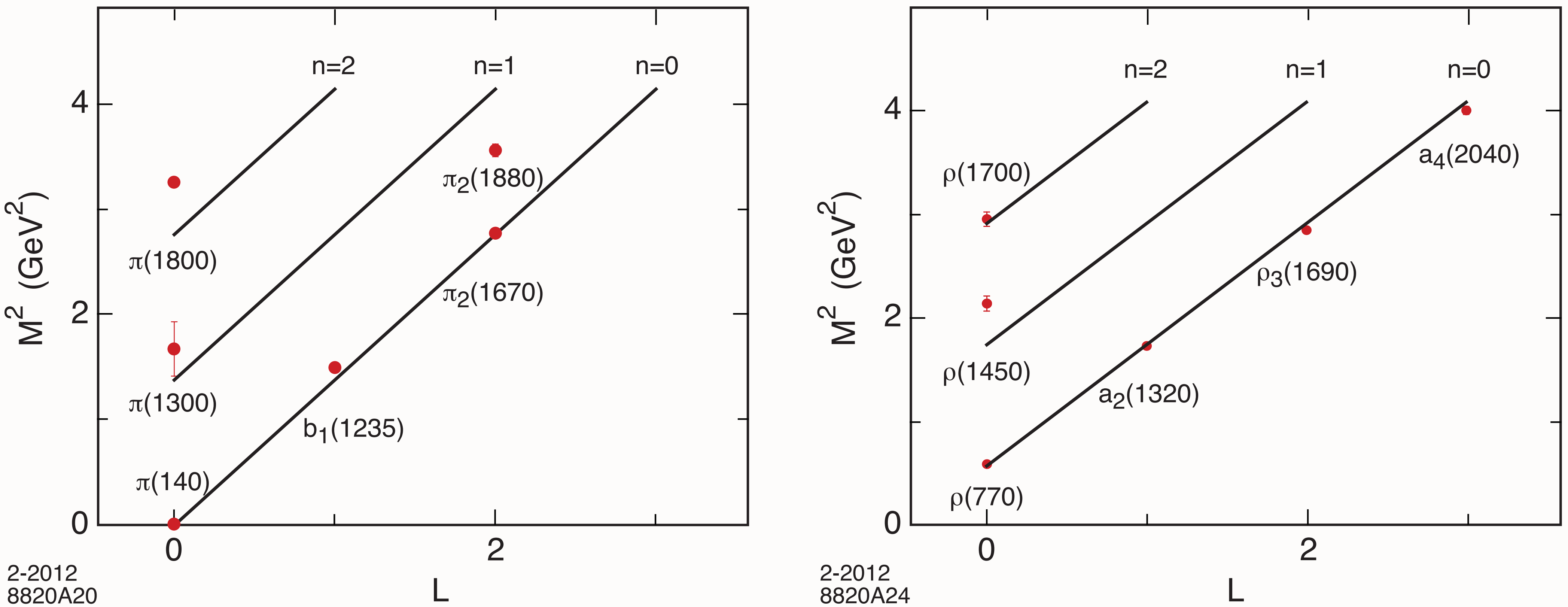
$$\mathcal{M}^2 = 2\kappa^2(2n + 2L + S).$$

$$S = 1$$



- $J = L + S, \ I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



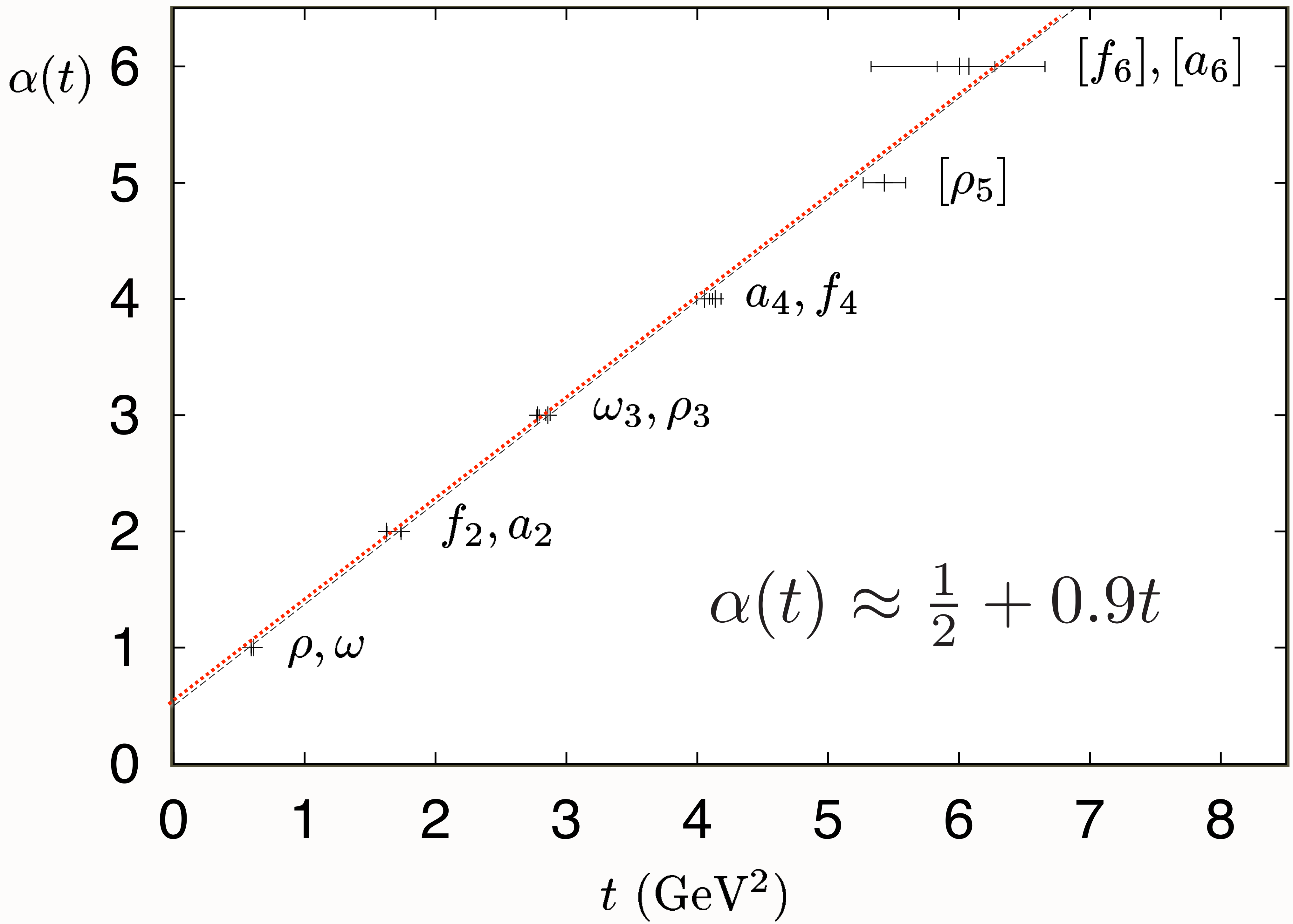

Orbital and radial excitations for the $\pi$ ($\kappa = 0.59$ GeV) and the $\rho$ I=1meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $L = 1$, $J = 0, 1, 2$, $I = 1$ vector meson $a$-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

- $J - L$ splitting in mesons and radial excitations are well described in soft-wall model

- Light scalar mesons P. Colangelo, F. De Fazio *et al.*, Phys. Rev. D **78**, 055009 (2008)]




*AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories*

# Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrodinger equation
- Massless pion ($m_q$ =0)
- Regge Trajectories: universal slope in n and L
- Valid for all integer J & S.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large Nc limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis

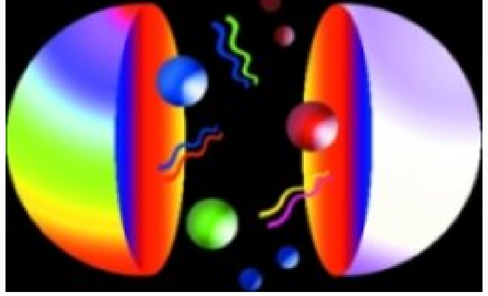
# AdS/QCD and Light-Front Holography

- AdS/QCD: Incorporates scale transformations characteristic of QCD with a single scale -- RGE
- Light-Front Holography; unique connection of $AdS_5$ to Front-Form
- Profound connection between gravity in 5th dimension and physical 3+1 space time at fixed LF time $\tau$
- Gives unique interpretation of $z$ in AdS to physical variable $\zeta$ in 3+1 space-time

# Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g}\, e^{\varphi(z)} \left( i\overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z) \overline{\Psi}\Psi - \mu \overline{\Psi}\Psi \right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x}\Psi(z)$

$$\left[ i\left( z\eta^{\ell m}\Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution $\left(\nu = \mu R - \frac{1}{2},\ \nu = L + 1\right)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$ positive parity

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from $\Psi$ by shifting dimensions
- Large $N_C$: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\,\Lambda_{QCD}$

# Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator $\Pi$

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right), \qquad \nu = L + 1$$

and its adjoint $\Pi^\dagger$, with commutation relations

Soft Wall

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

$$\nu = L + 1$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

## Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \longrightarrow \zeta$ and

$$\Psi_J(x,z) = e^{-iP\cdot x} z^2 \psi^J(z) u(P),$$

find LFWE for $d = 4$

$$\begin{aligned}
\frac{d}{d\zeta}\psi_+^J + \frac{\nu + \frac{1}{2}}{\zeta}\psi_+^J + U(\zeta)\psi_+^J &= \mathcal{M}\psi_-^J, \\
-\frac{d}{d\zeta}\psi_-^J + \frac{\nu + \frac{1}{2}}{\zeta}\psi_-^J + U(\zeta)\psi_-^J &= \mathcal{M}\psi_+^J,
\end{aligned}$$

where $U(\zeta) = \frac{R}{\zeta} V(\zeta)$

- Choose linear potential $U = \kappa^2 \zeta$
- Eigenfunctions

$$\psi_+^J(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2), \qquad \psi_-^J(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)$$

- Eigenvalues

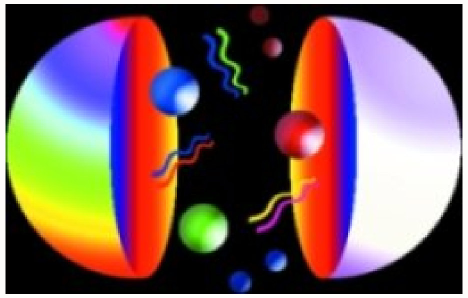
$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1), \quad \nu = L + 1 \quad (\tau = 3)$$

- Full $J - L$ degeneracy (different $J$ for same $L$) for baryons along given trajectory !

## Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

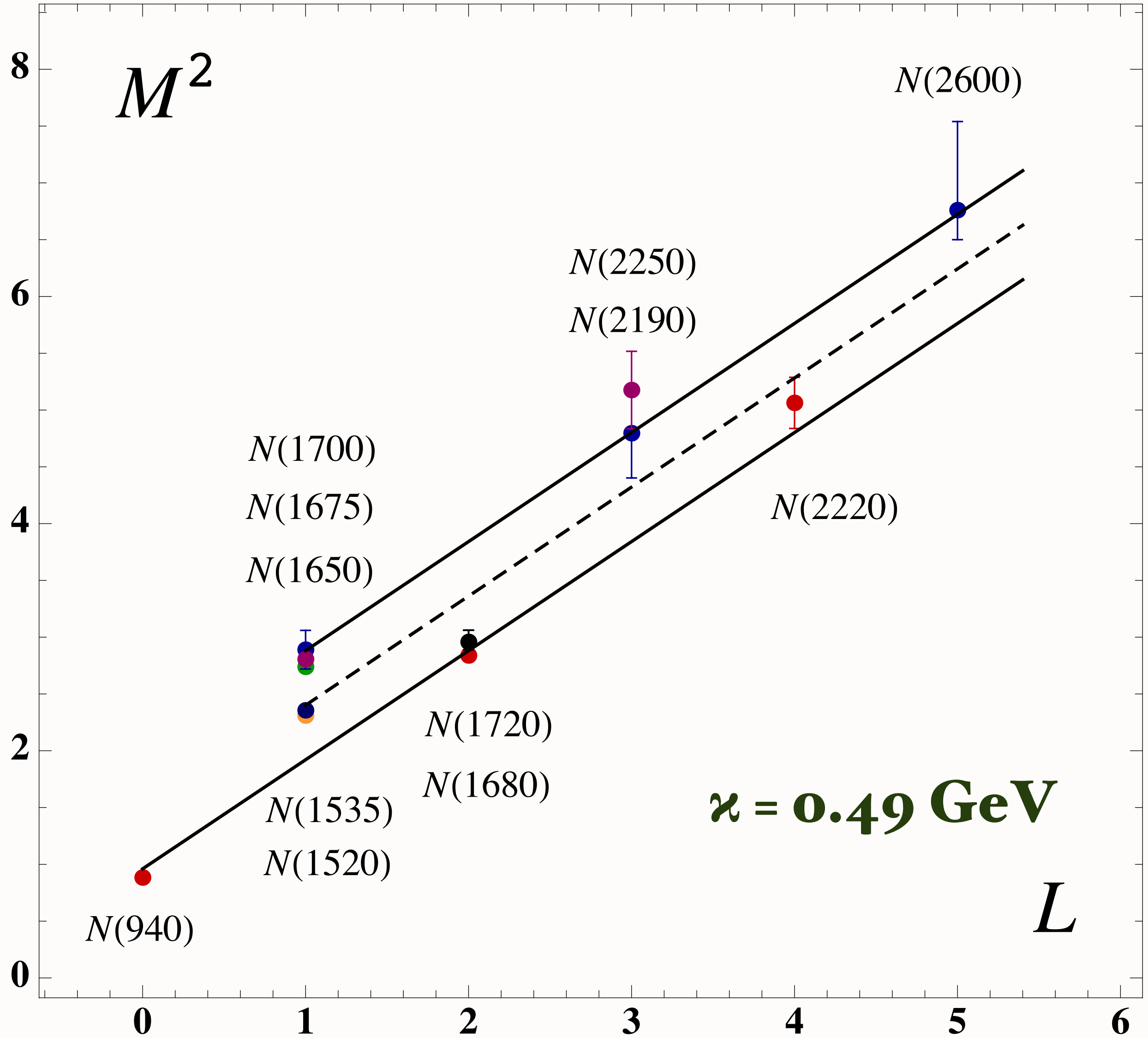
- "Chiral partners"

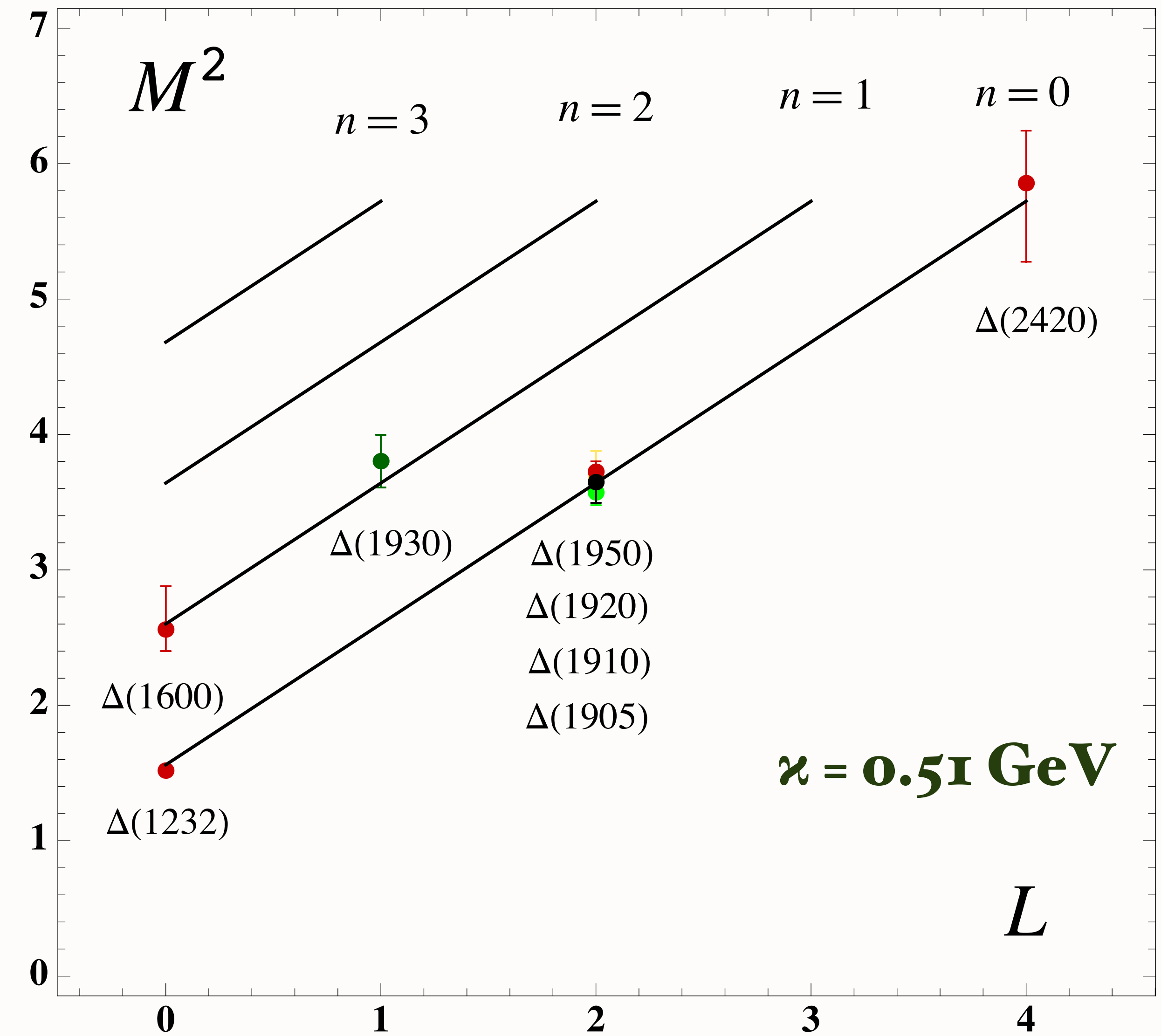
$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels $S$, $L$ and $n$ refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

| $SU(6)$ | $S$ | $L$ | $n$ | Baryon State |
|---|---|---|---|---|
| **56** | $\frac{1}{2}$ | 0 | 0 | $N\frac{1}{2}^+(940)$ |
| | $\frac{1}{2}$ | 0 | 1 | $N\frac{1}{2}^+(1440)$ |
| | $\frac{1}{2}$ | 0 | 2 | $N\frac{1}{2}^+(1710)$ |
| | $\frac{3}{2}$ | 0 | 0 | $\Delta\frac{3}{2}^+(1232)$ |
| | $\frac{3}{2}$ | 0 | 1 | $\Delta\frac{3}{2}^+(1600)$ |
| **70** | $\frac{1}{2}$ | 1 | 0 | $N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$ |
| | $\frac{3}{2}$ | 1 | 0 | $N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$ |
| | $\frac{3}{2}$ | 1 | 1 | $N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$ |
| | $\frac{1}{2}$ | 1 | 0 | $\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$ |
| **56** | $\frac{1}{2}$ | 2 | 0 | $N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$ |
| | $\frac{1}{2}$ | 2 | 1 | $N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$ |
| | $\frac{3}{2}$ | 2 | 0 | $\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$ |
| **70** | $\frac{1}{2}$ | 3 | 0 | $N\frac{5}{2}^-$ $N\frac{7}{2}^-$ |
| | $\frac{3}{2}$ | 3 | 0 | $N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$ |
| | $\frac{1}{2}$ | 3 | 0 | $\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$ |
| **56** | $\frac{1}{2}$ | 4 | 0 | $N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$ |
| | $\frac{3}{2}$ | 4 | 0 | $\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$ |
| **70** | $\frac{1}{2}$ | 5 | 0 | $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ |
| | $\frac{3}{2}$ | 5 | 0 | $N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$ |

**PDG 2012**



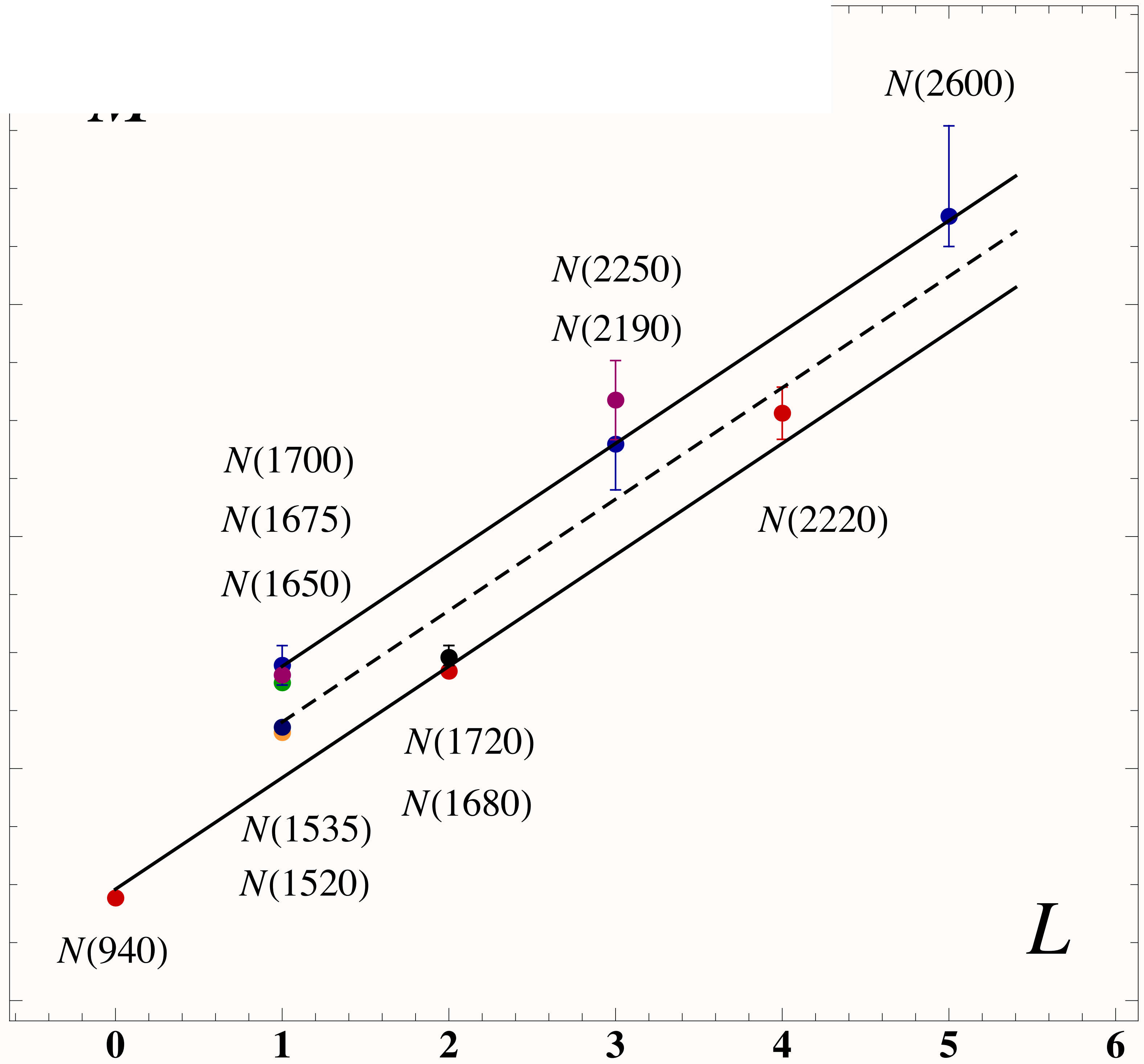

**de Teramond, sjb**

**All confirmed resonances from PDG 2012**

$$\mathcal{M}^{2\,(+)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{3}{4}\right),$$ positive parity

$$\mathcal{M}^{2\,(-)}_{n,L,S} = 4\kappa^2\left(n + L + \frac{S}{2} + \frac{5}{4}\right),$$ negative parity

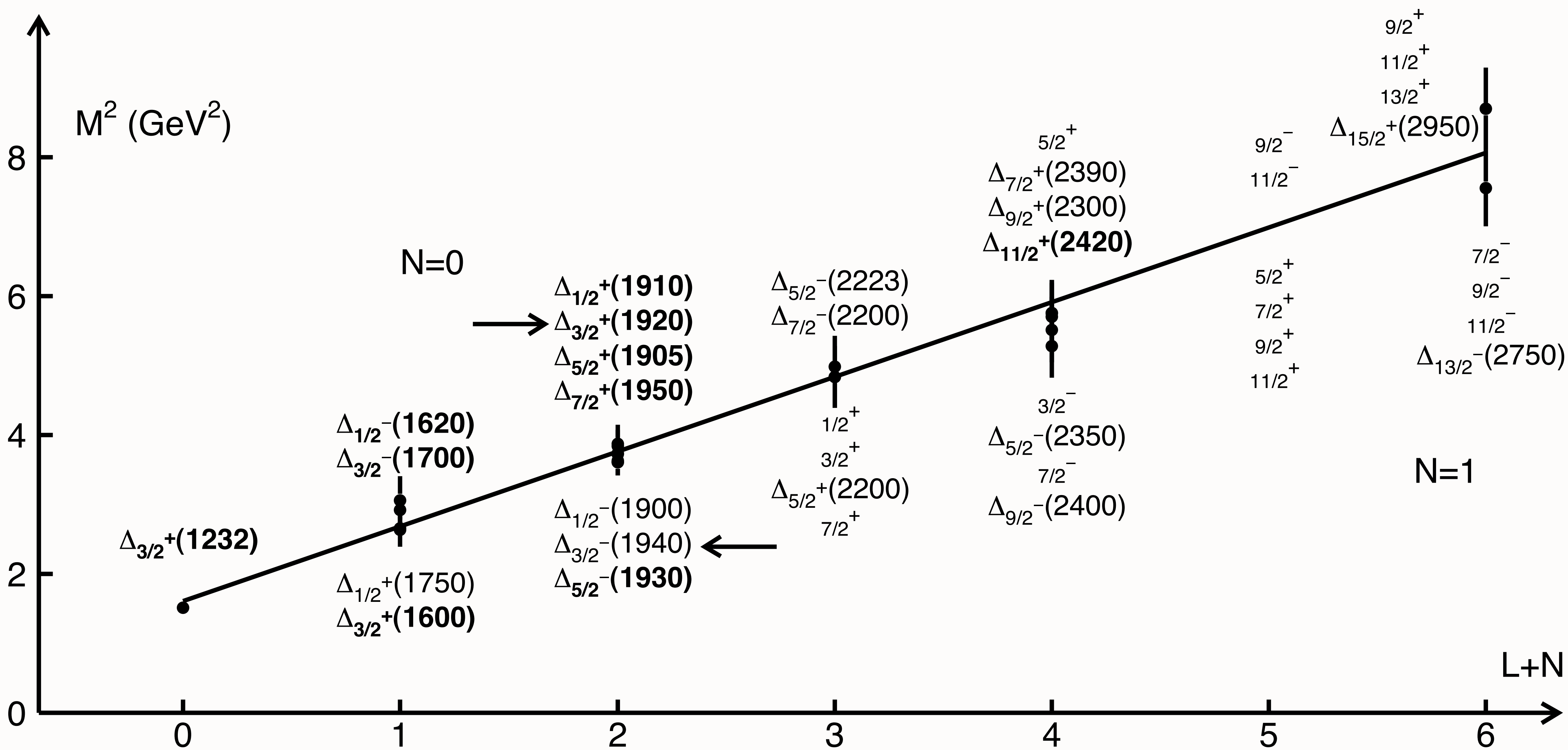
**See also Forkel, Beyer, Federico, Klempt**




**Data from PDG 2012**

**de Teramond, sjb**

**See also Forkel, Beyer, Federico,**




E. Klempt *et al.*: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, JHEP **0707** (2007) 077.
H. Forkel, M. Beyer and T. Frederico, Int. J. Mod. Phys. E **16** (2007) 2794.

# Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum.**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different $L^z$**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

# An analytic first approximation to QCD

## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable $\zeta$ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter** $\kappa$
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ Methods**

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

  where the effective charges $g_+$ and $g_-$ are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

## Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Soft Wall Model

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

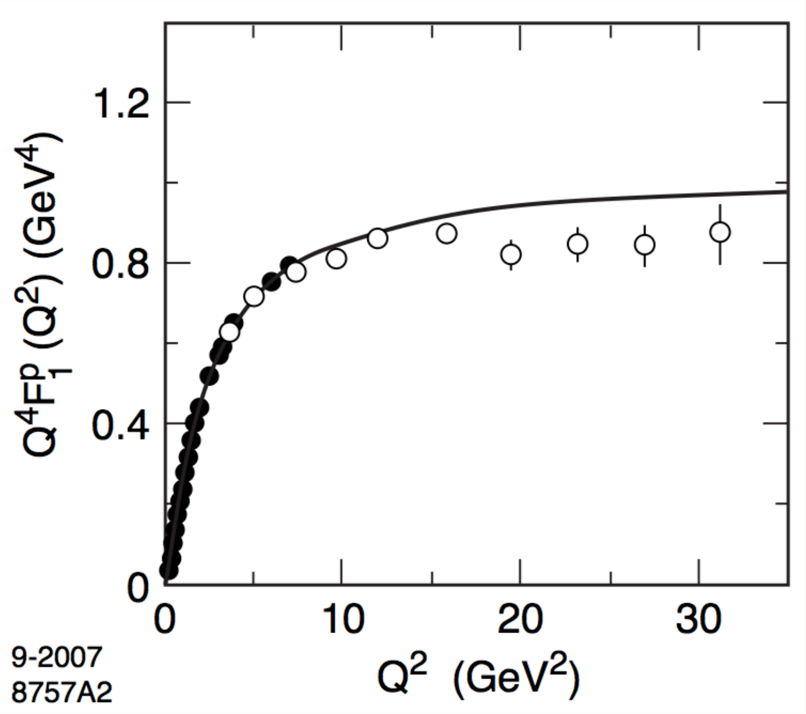
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

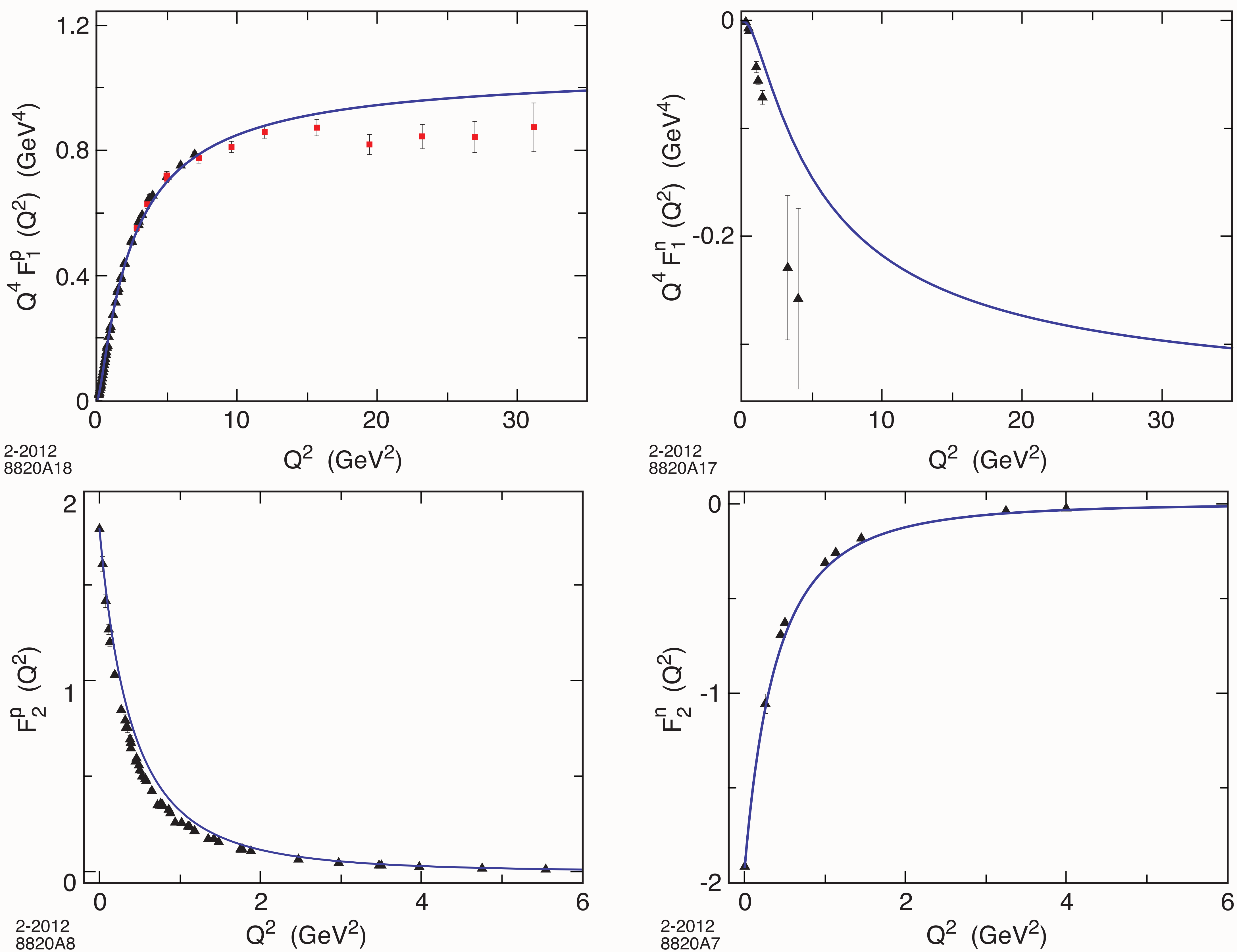
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n+1/2)$



Using $SU(6)$ flavor symmetry and normalization to static quantities



## Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_1{}^p_{N \to N^*}(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_1{}^p_{N \to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

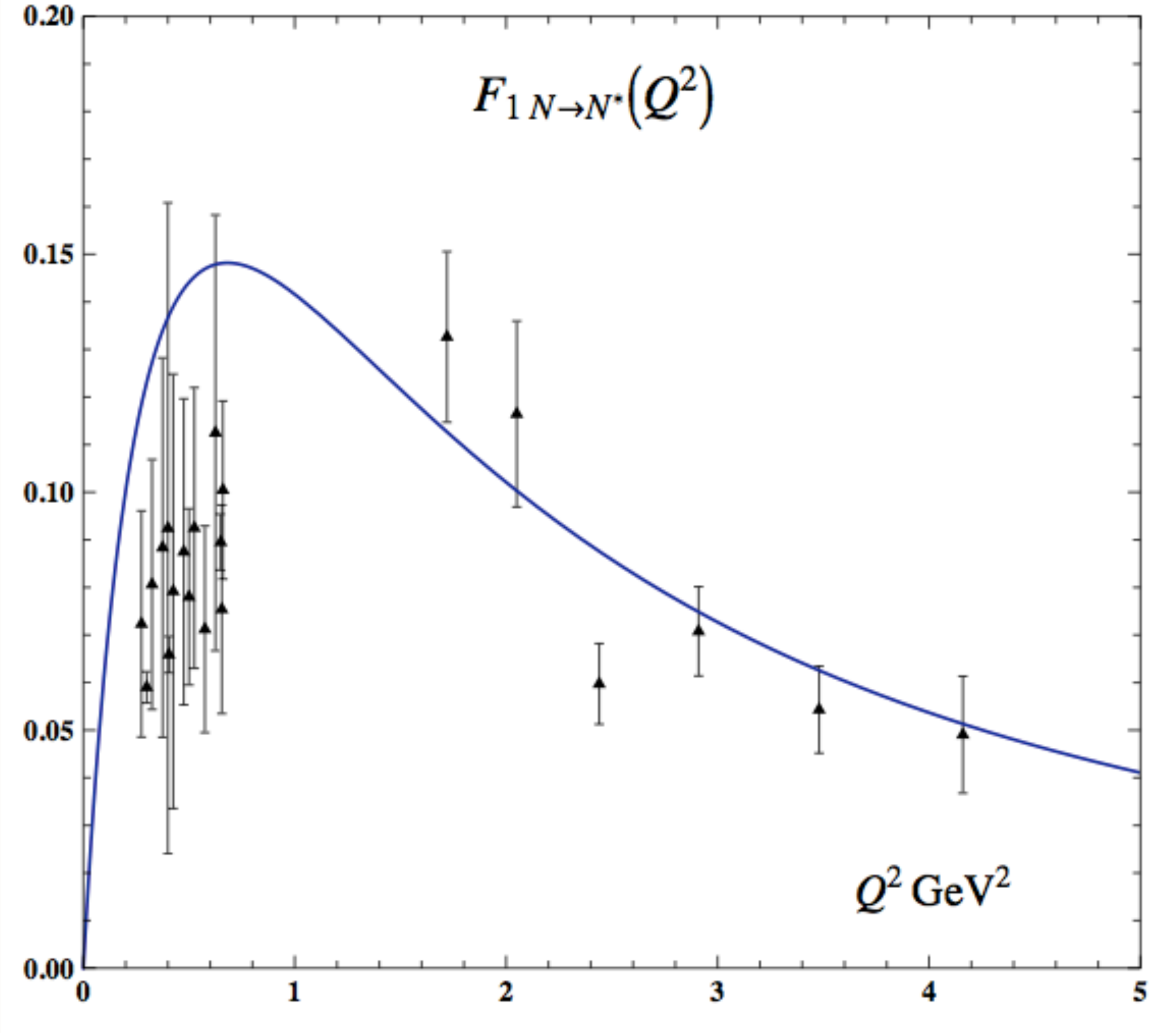
$$F_1{}^p_{N \to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n + 1/2)$

de Teramond, sjb

*Consistent with counting rule, twist 3*

$N(940) \to N^*(1440)$:   $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

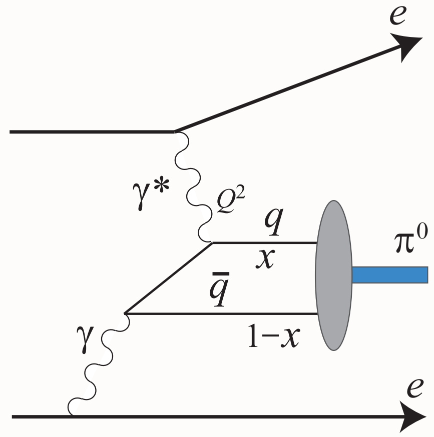



Data from I. Aznauryan, *et al.* CLAS (2009)

$$F_{1\,N\to N^*}^{p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^{\,2} \to 4\kappa^2(n+1/2)$

Cao, de Teramond, sjb




- Definition of $\pi - \gamma$ TFF from $\gamma^* \pi^0 \to \gamma$ vertex in the amplitude $e\pi \to e\gamma$

$$\Gamma^\mu = -ie^2 F_{\pi\gamma}(q^2)\epsilon_{\mu\nu\rho\sigma}(p_\pi)_\nu \epsilon_\rho(k) q_\sigma, \quad k^2 = 0$$

- Asymptotic value of pion TFF is determined by first principles in QCD:
  $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$ [Lepage and Brodsky (1980)]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)}\left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Find for $A_z \propto \Phi_\pi(z)/z$

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\pi} \int_0^\infty \frac{dz}{z}\, \Phi_\pi(z) V\left(Q^2, z\right)$$

  with normalization fixed by asymptotic QCD prediction

- $V(Q^2, z)$ bulk-to-boundary propagator of $\gamma^*$

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

$$\sim (2\pi)^4 \delta^{(4)} \left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

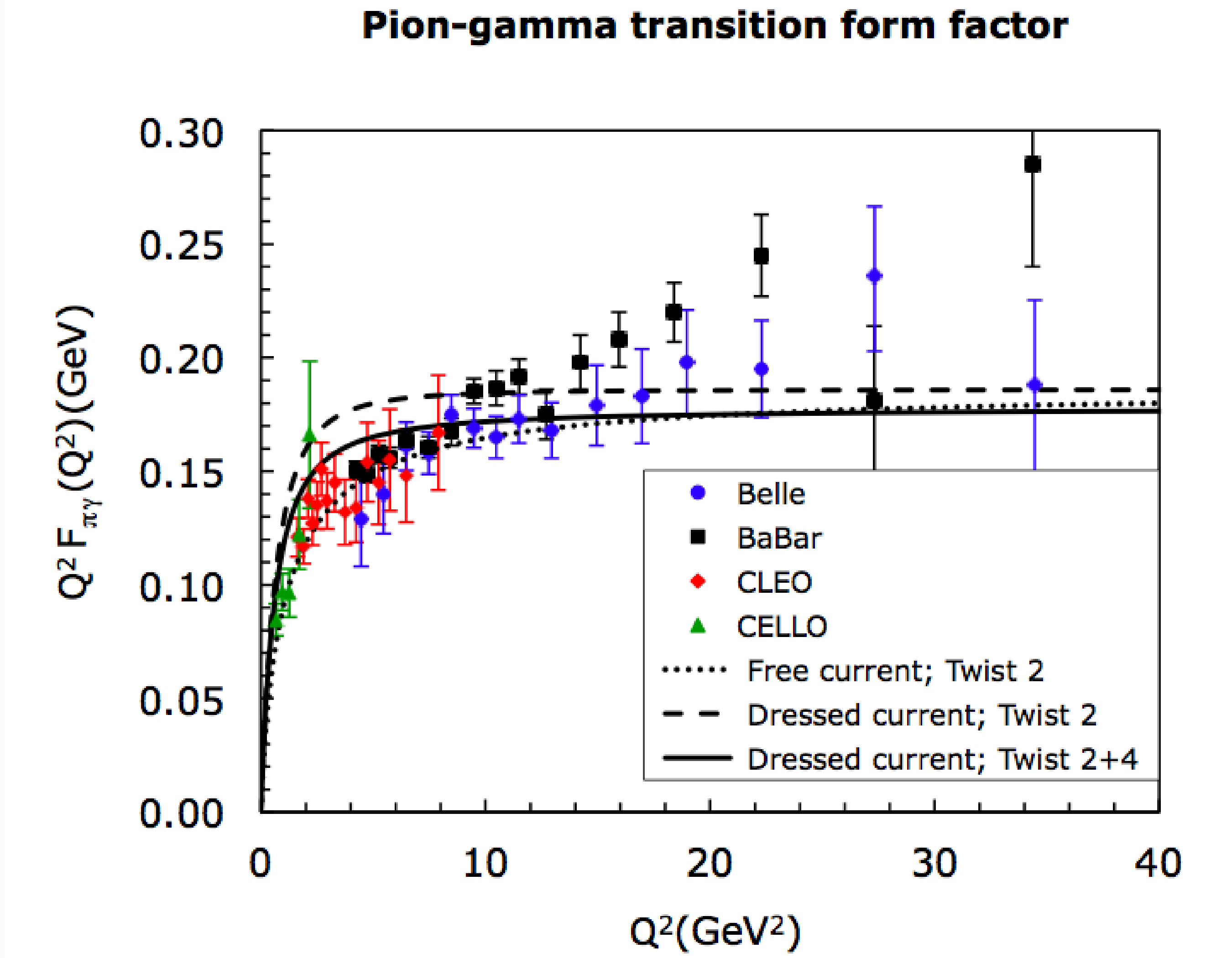
- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2 P_{q\bar q}}\, \kappa \, z^2 e^{-\kappa^2 z^2/2}, \quad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar q}$
- Find $\quad \left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\bar q}}\, \kappa/\sqrt{2}\pi\right)$

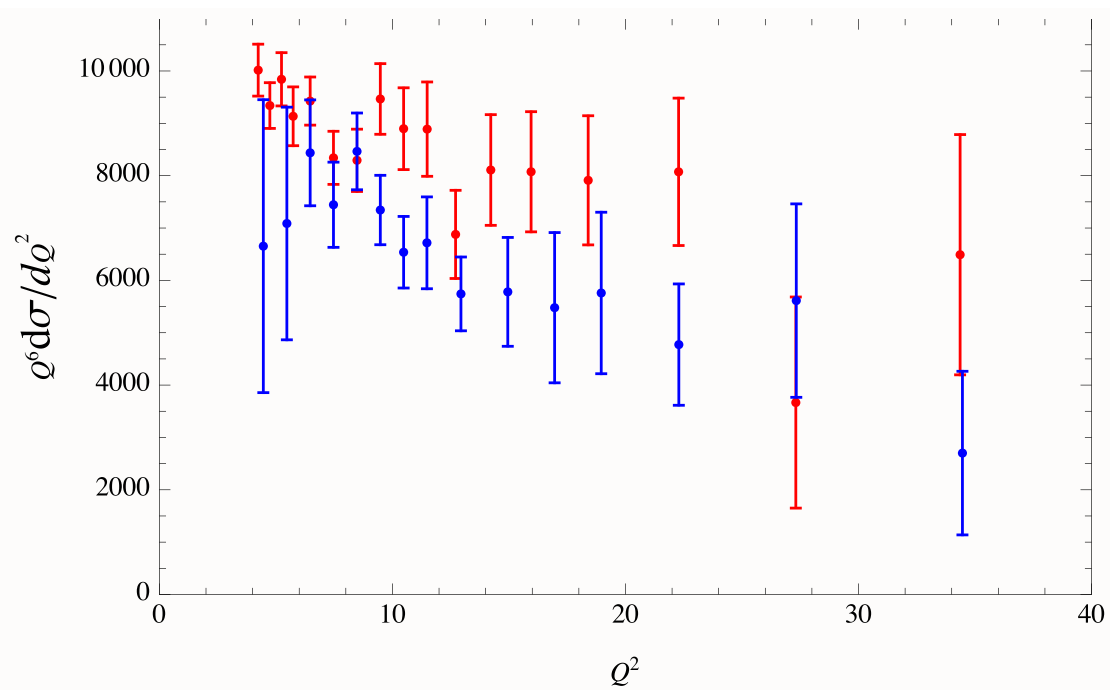
$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\bar q} Q^2 (1-x)/4\pi^2 f_\pi^2 \, x}\right]$$

normalized to the asymptotic DA $[P_{q\bar q} = 1 \to$ Musatov and Radyushkin (1997)]

- Large $Q^2$ TFF is identical to first principles asymptotic QCD result $\quad Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA

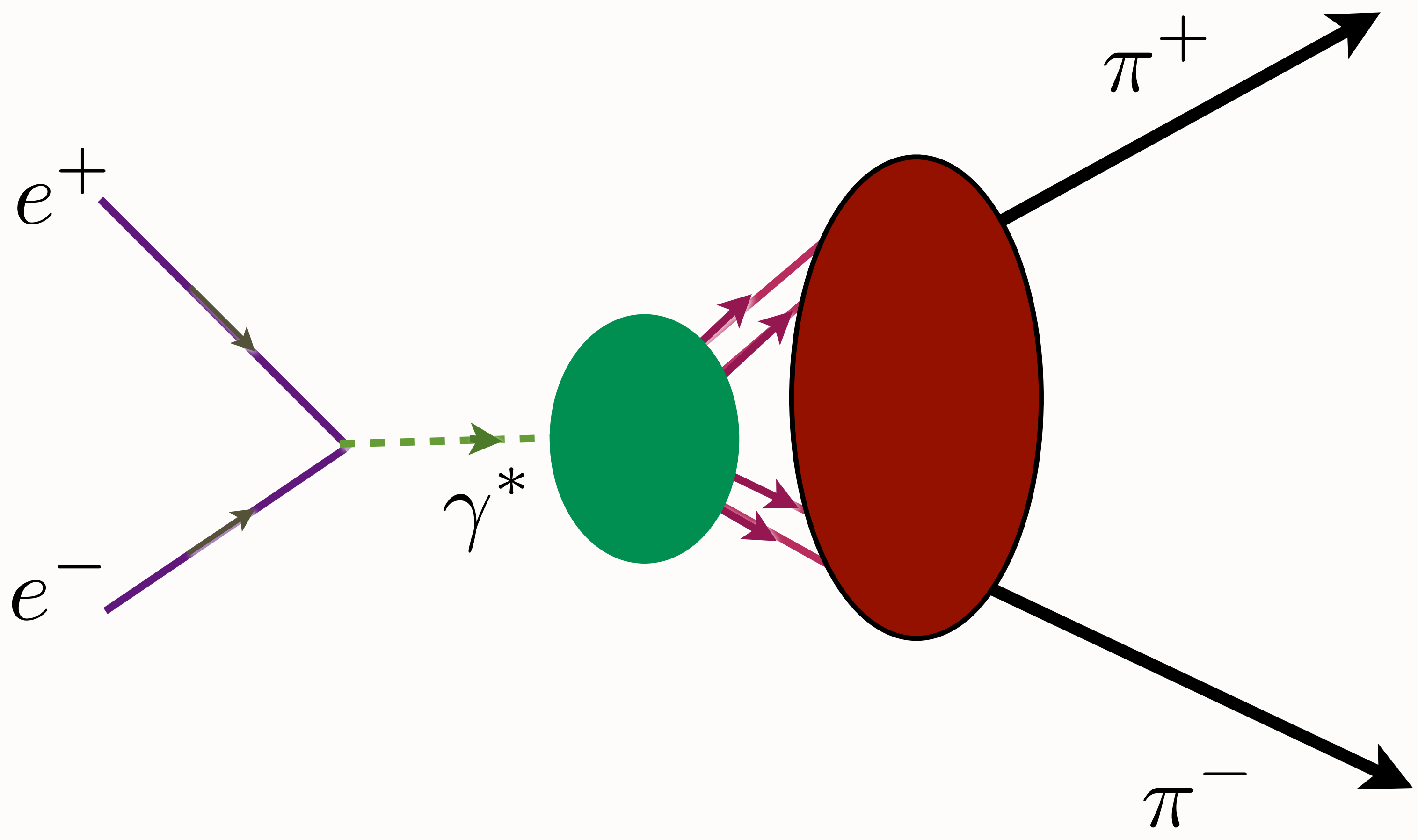
G.P. Lepage, sjb


Pion-gamma transition form factor
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$Q^2 F_{\pi\gamma}(Q^2)$(GeV)
0
10
20
30
40
$Q^2(GeV^2)$
Belle
BaBar
CLEO
CELLO
Free current; Twist 2
Dressed current; Twist 2
Dressed current; Twist 2+4


10 000
8000
6000
4000
2000
0
$Q^6 d\sigma/dQ^2$
0
10
20
30
40
$Q^2$

46

Dressed soft-wall current brings in higher Fock states and more vector meson poles



# Form Factors in AdS/QCD

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}, \quad N = 3,$$

$$\cdots$$

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}^2_{\rho^{N-2}}}\right)}, \quad N,$$

**Positive Dilaton Background** $\exp\left(+\kappa^2 z^2\right)$ $\quad \mathcal{M}_n^2 = 4\kappa^2\left(n + \frac{1}{2}\right)$

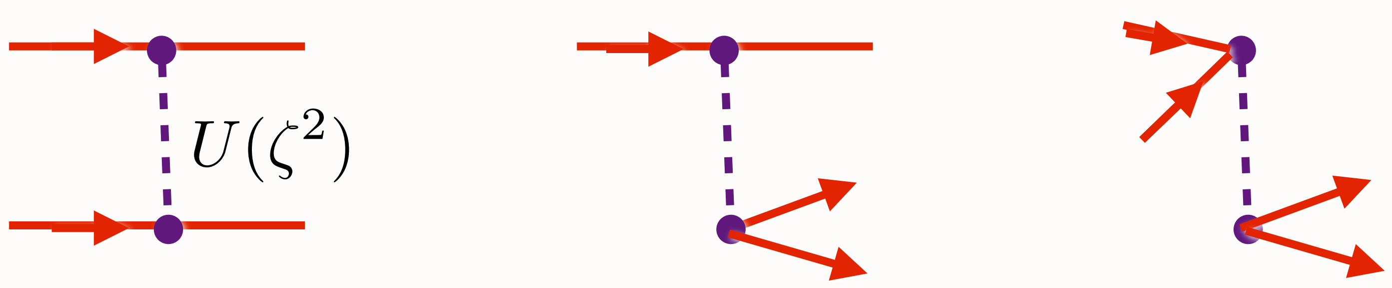
$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \qquad Q^2 \to \infty$$

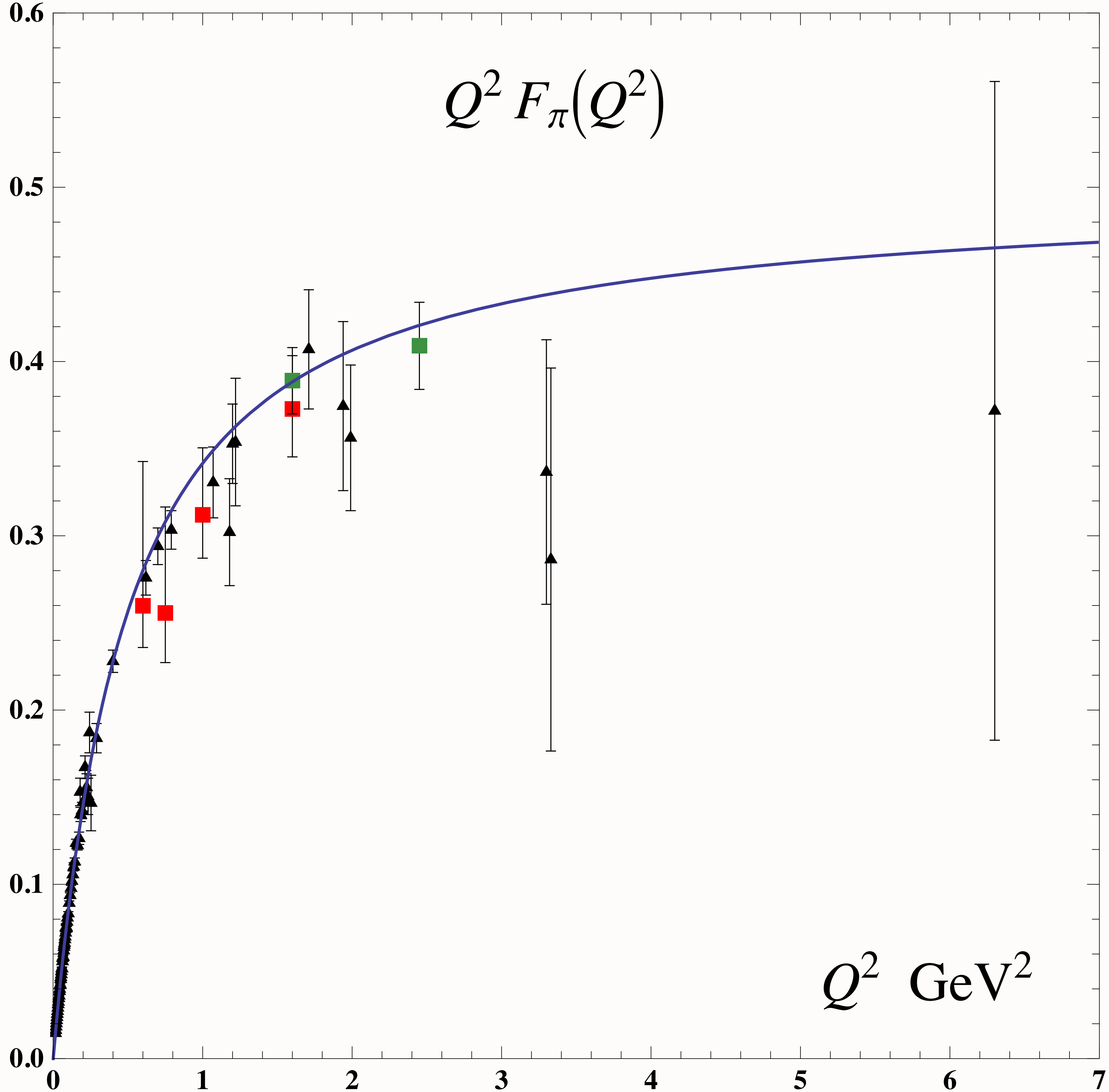
Constituent Counting

- **Exposed by timelike form factor through Heisenberg dressed current.**
- **Created by confining interaction**

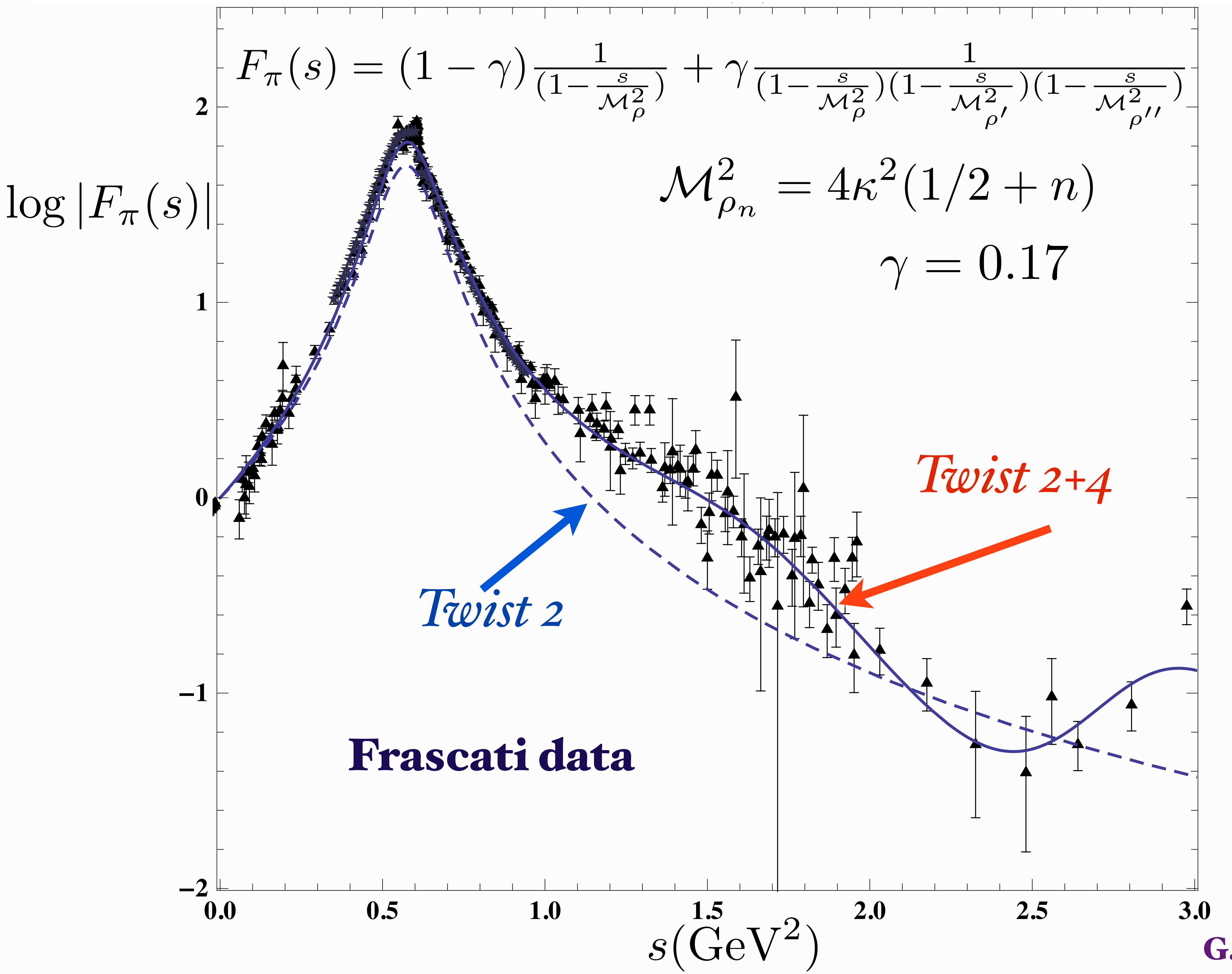
$$P^{-}_{\text{confinement}} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

- **Similar to QCD(1+1) in lcg**




$Q^2 F_\pi(Q^2)$
$Q^2$ GeV$^2$
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7

# Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



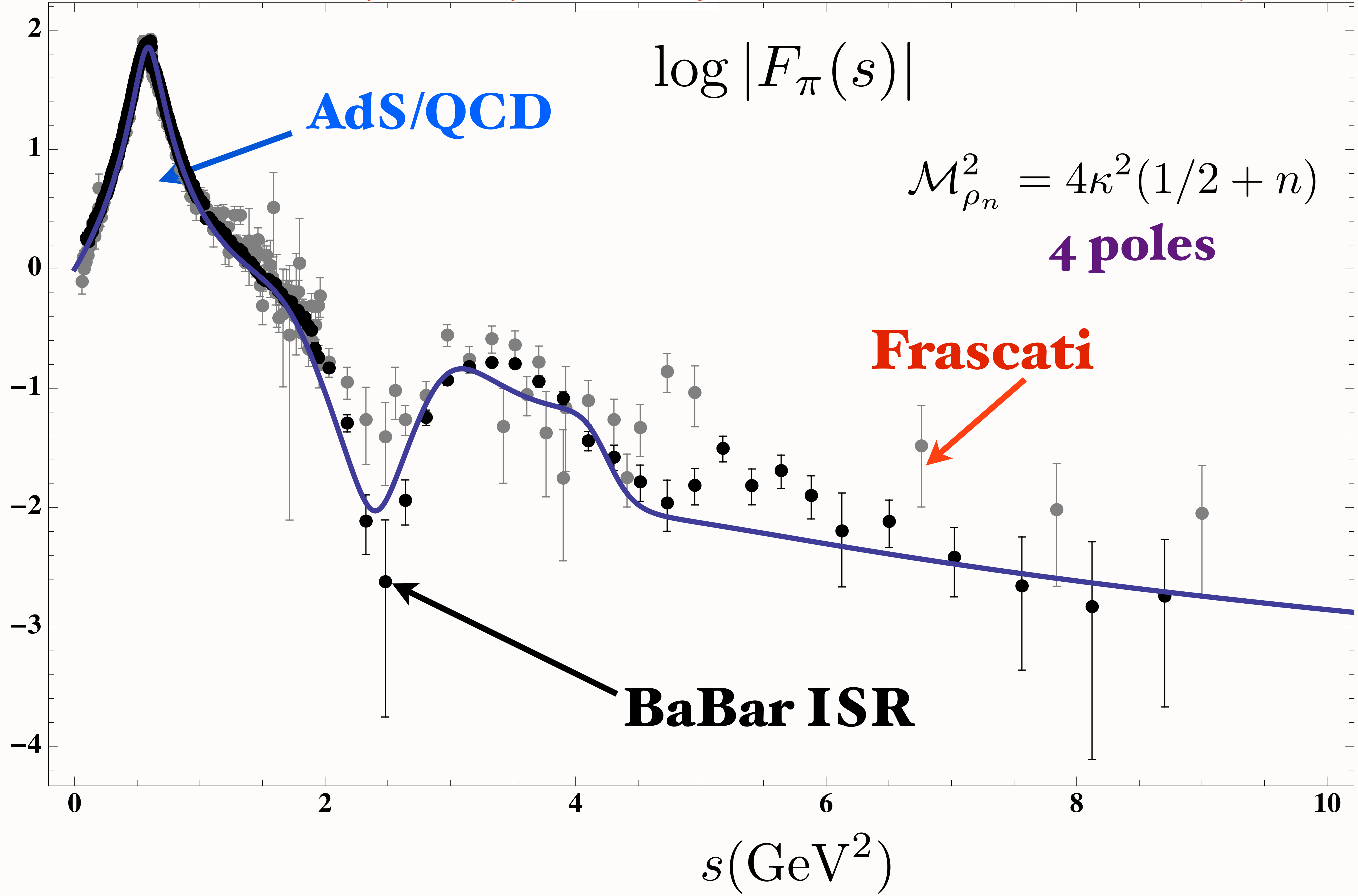

***Prescription for Timelike poles :***

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

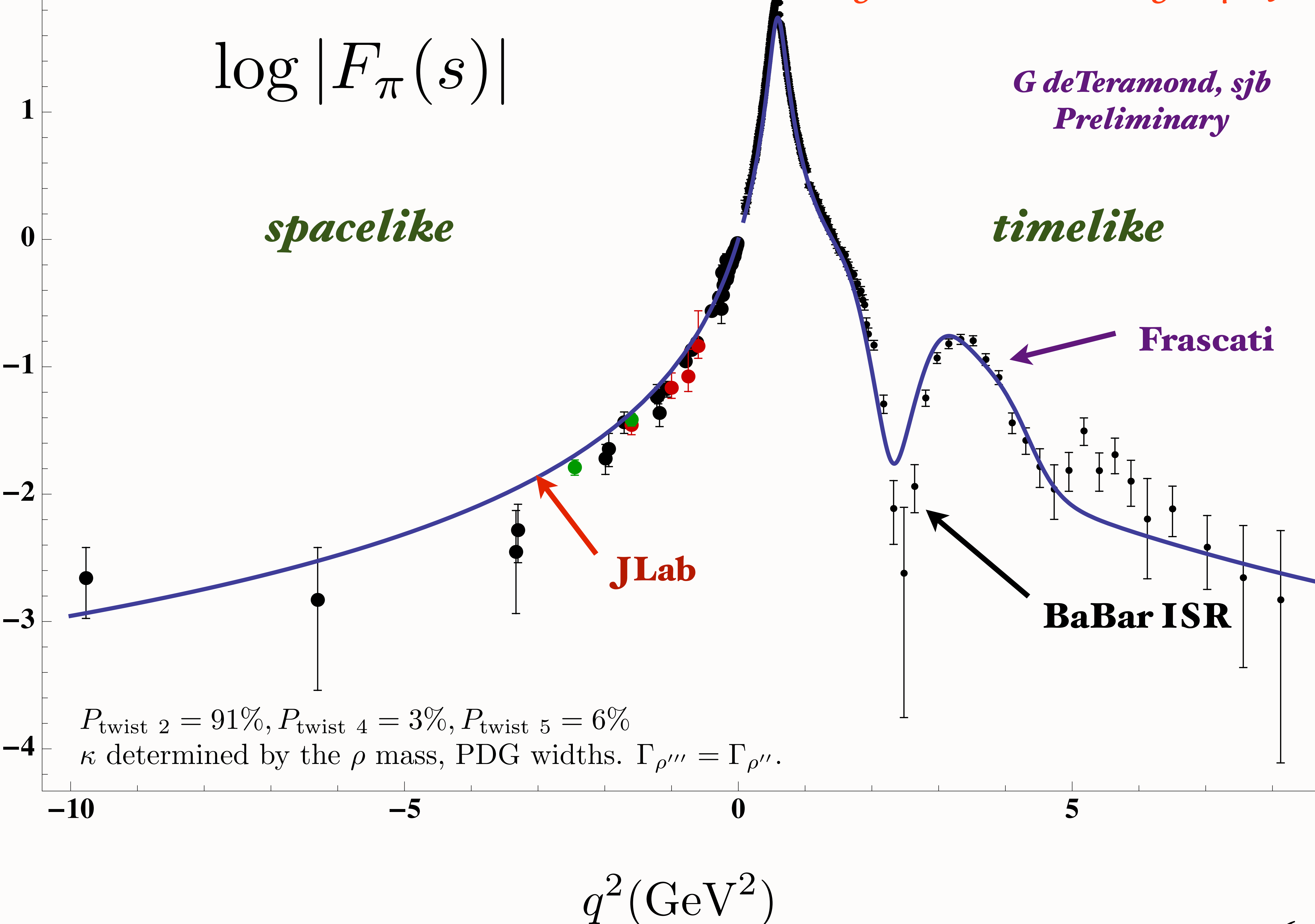
**14% four-quark probability**

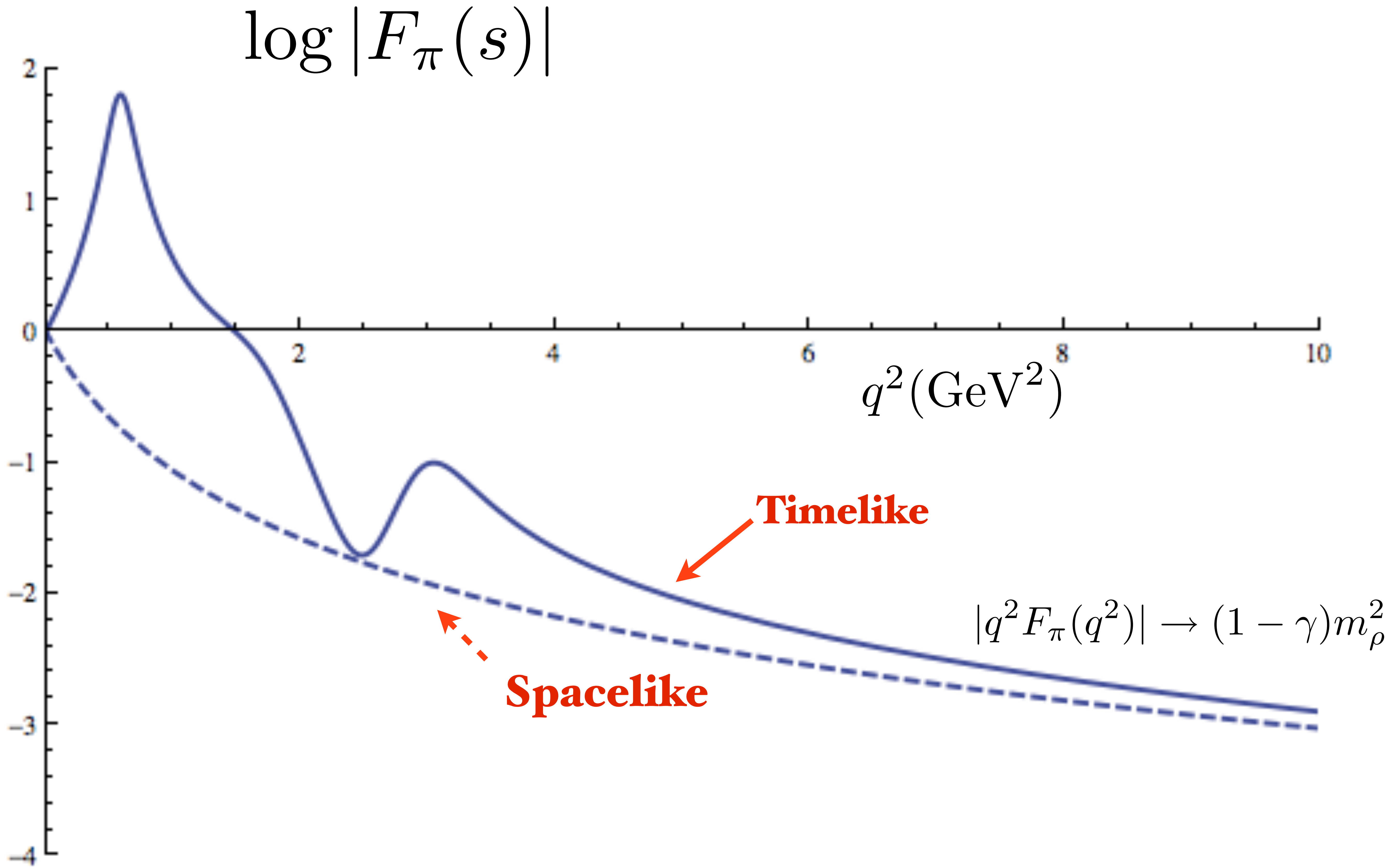
**G. de Teramond & sjb**

# Pion Timelike Form Factor (Includes Twist 2 to 5)



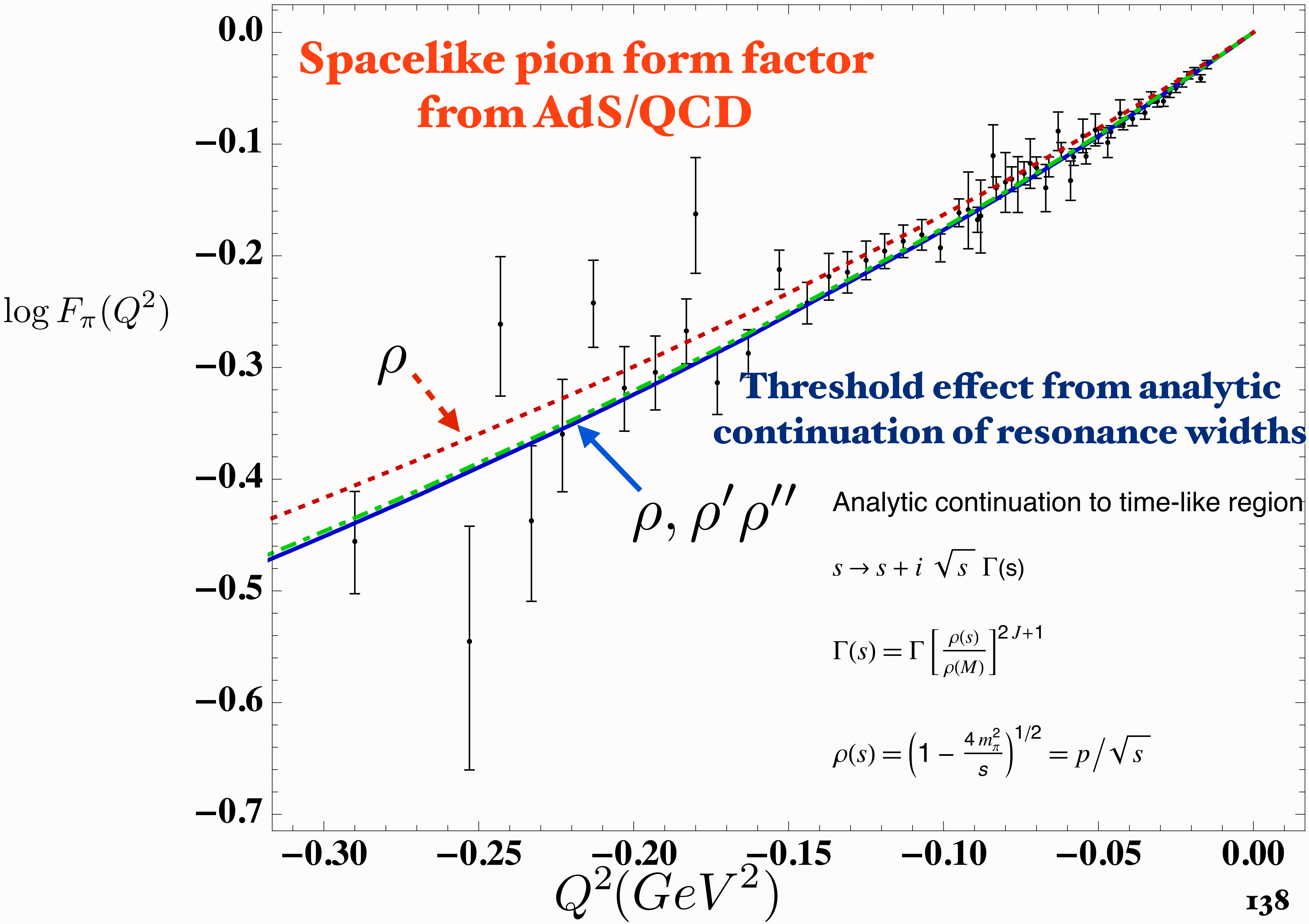
# Pion Form Factor from AdS/QCD and Light-Front Holography

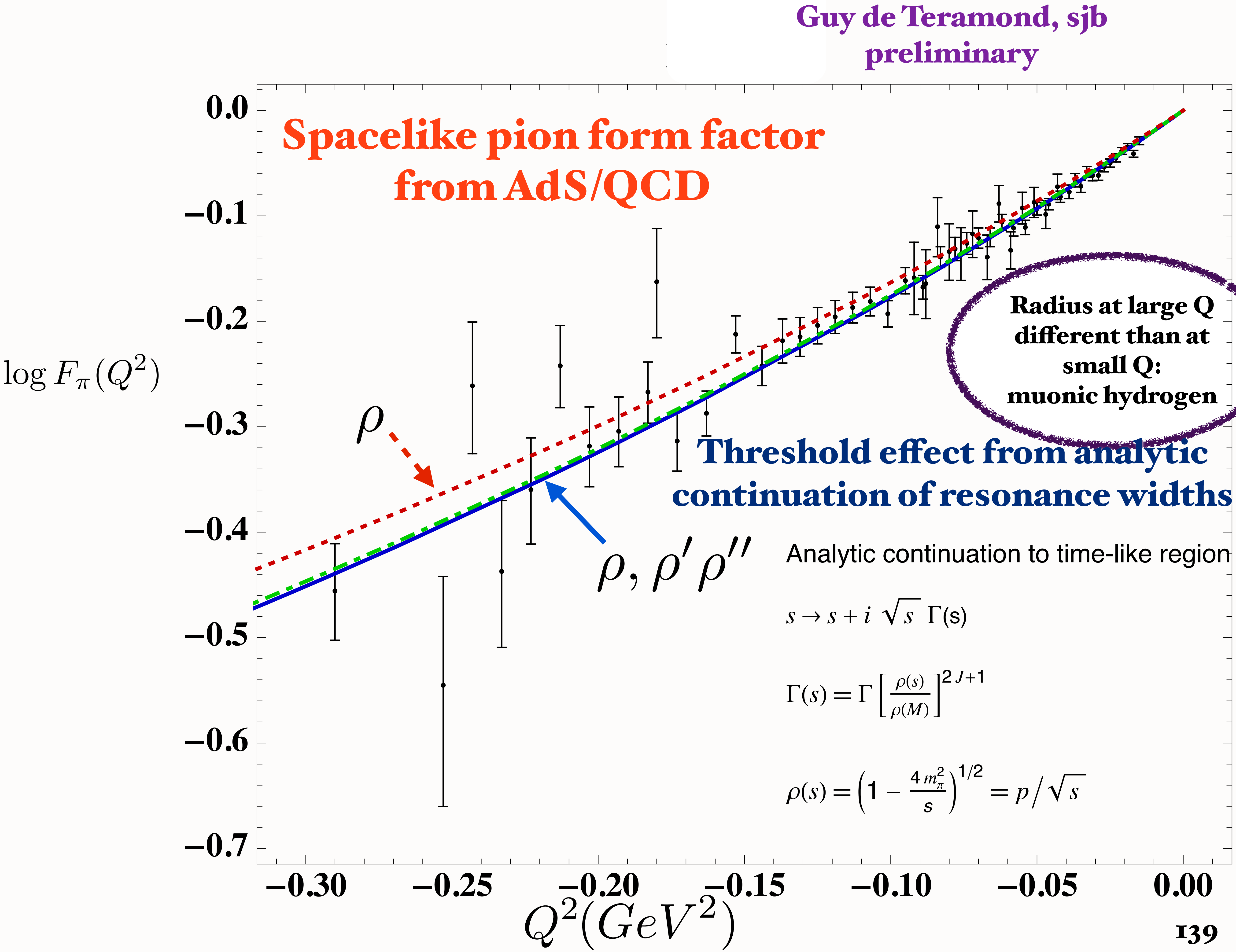




$\log|F_\pi(s)|$
2
1
0
−1
−2
−3
−4
2
4
6
8
10
$q^2(\text{GeV}^2)$
Timelike
Spacelike
$|q^2 F_\pi(q^2)| \rightarrow (1-\gamma)m_\rho^2$

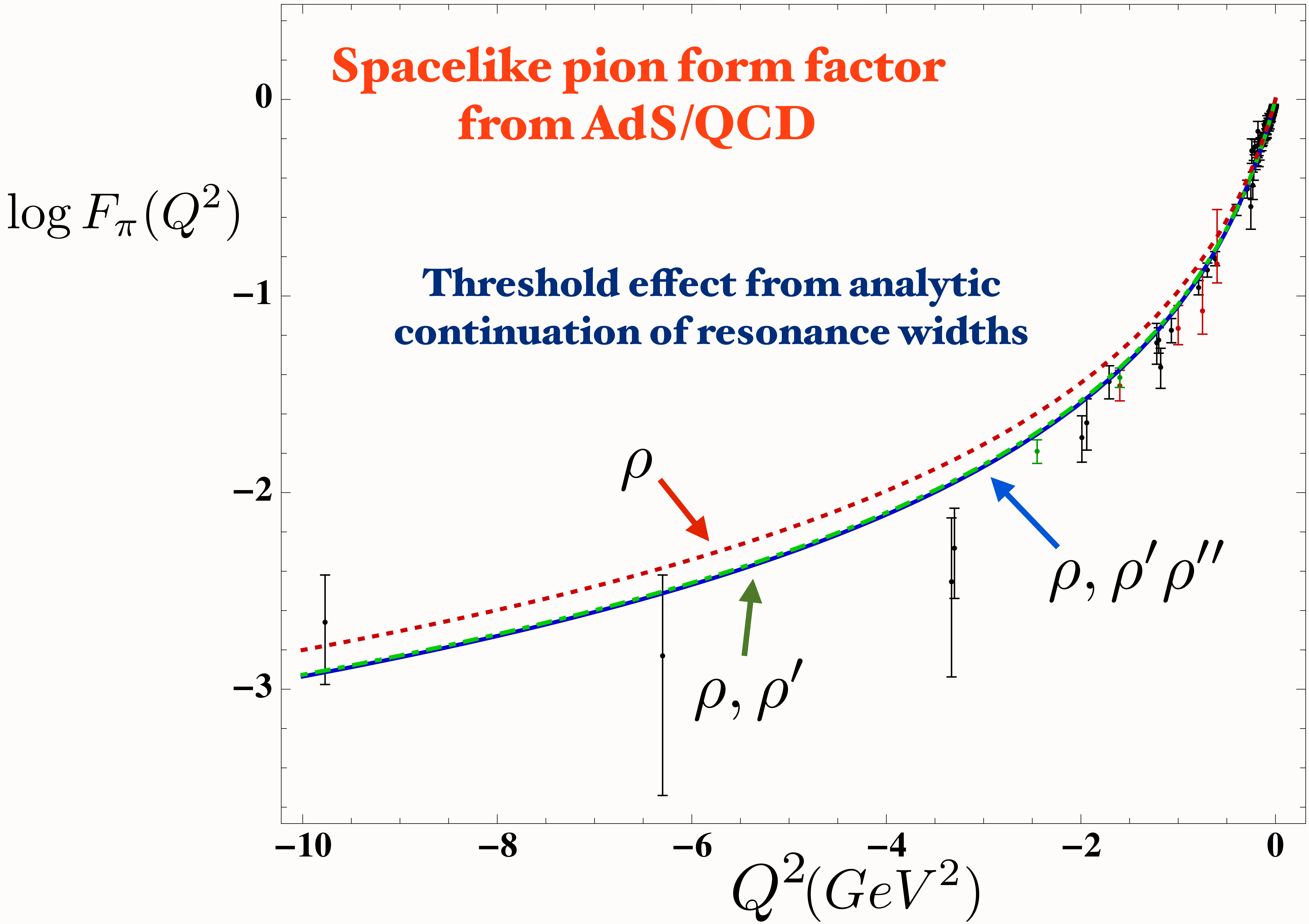
Guy de Teramond, sjb
preliminary





Guy de Teramond, sjb
preliminary
Spacelike pion form factor
from AdS/QCD
Radius at large Q
different than at
small Q:
muonic hydrogen
Threshold effect from analytic
continuation of resonance widths
$\rho$
$\rho, \rho' \rho''$
Analytic continuation to time-like region
$s \to s + i\ \sqrt{s}\ \Gamma(s)$
$\Gamma(s) = \Gamma \left[\frac{\rho(s)}{\rho(M)}\right]^{2J+1}$
$\rho(s) = \left(1 - \frac{4 m_\pi^2}{s}\right)^{1/2} = p/\sqrt{s}$
$\log F_\pi(Q^2)$
$Q^2 (GeV^2)$
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.30
−0.25
−0.20
−0.15
−0.10
−0.05
0.00

Guy de Teramond, sjb
preliminary

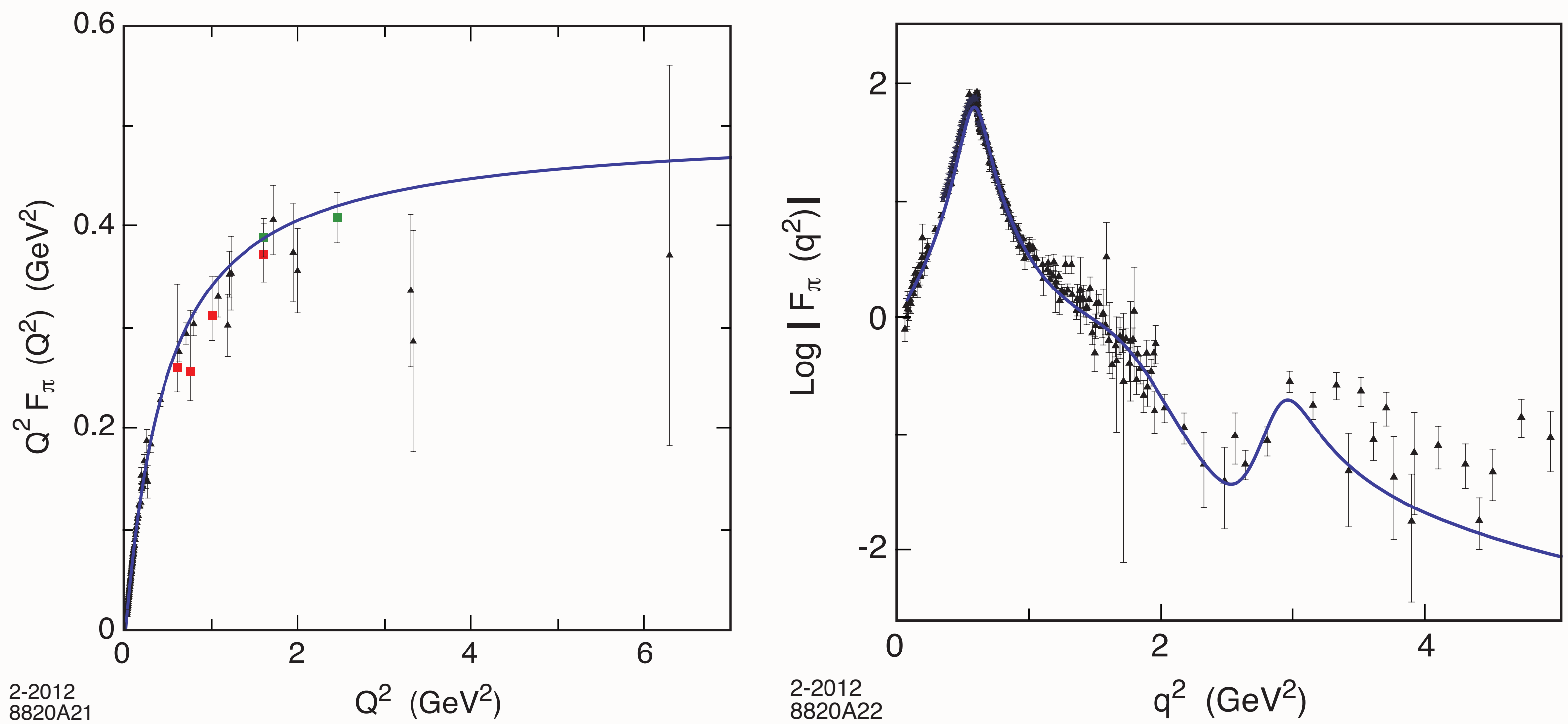


## Higher Fock Components in LF Holographic QCD

- Effective interaction leads to $qq \to qq$, $q\bar{q} \to q\bar{q}$ but also to $q \to qq\bar{q}$ and $\bar{q} \to \bar{q}q\bar{q}$
- Higher Fock states can have any number of extra $q\bar{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

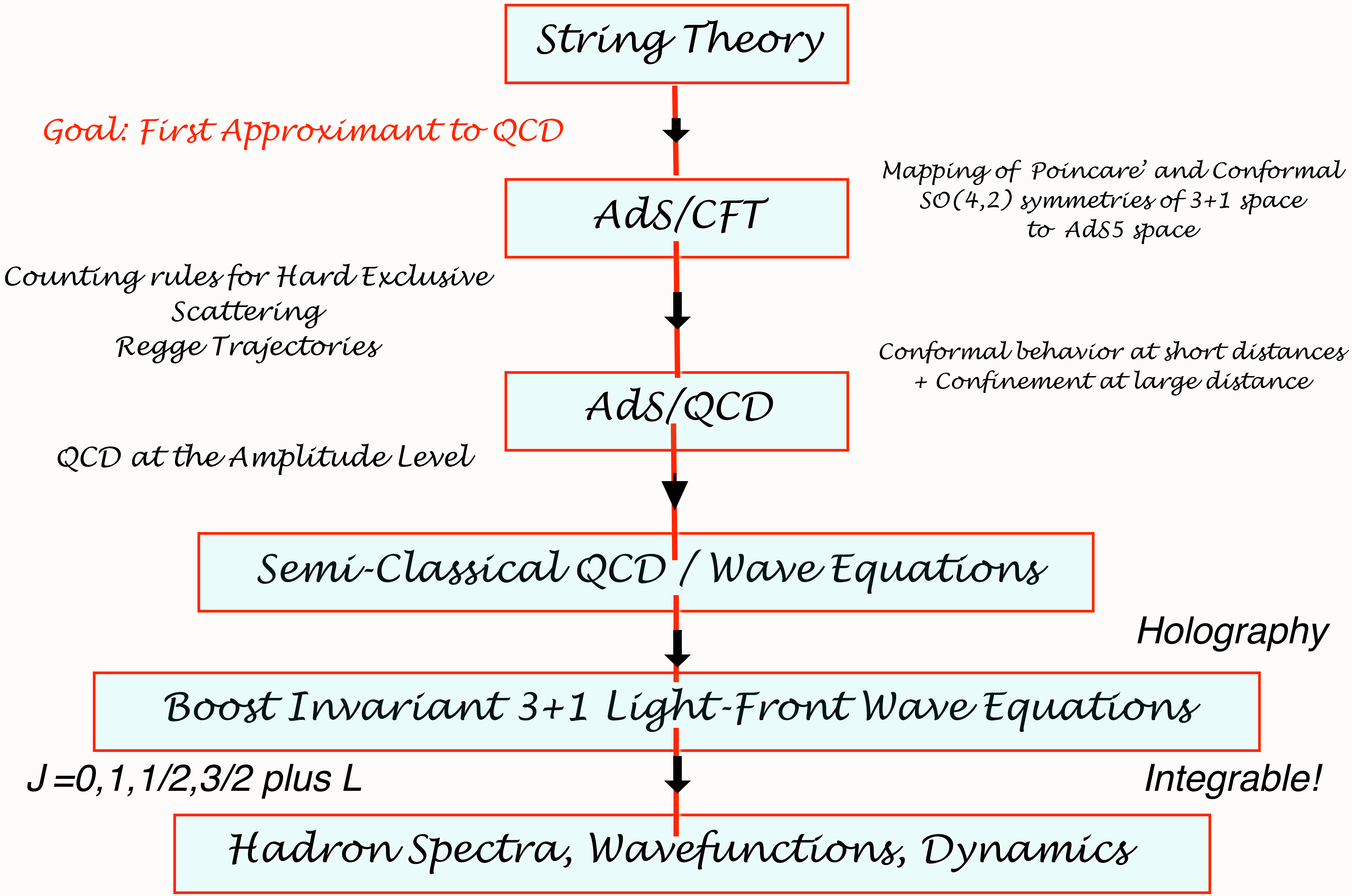
$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

- Modify form factor formula introducing finite width: $q^2 \to q^2 + \sqrt{2} i \mathcal{M}\Gamma$ $(P_{q\bar{q}q\bar{q}}$ = 13 %)

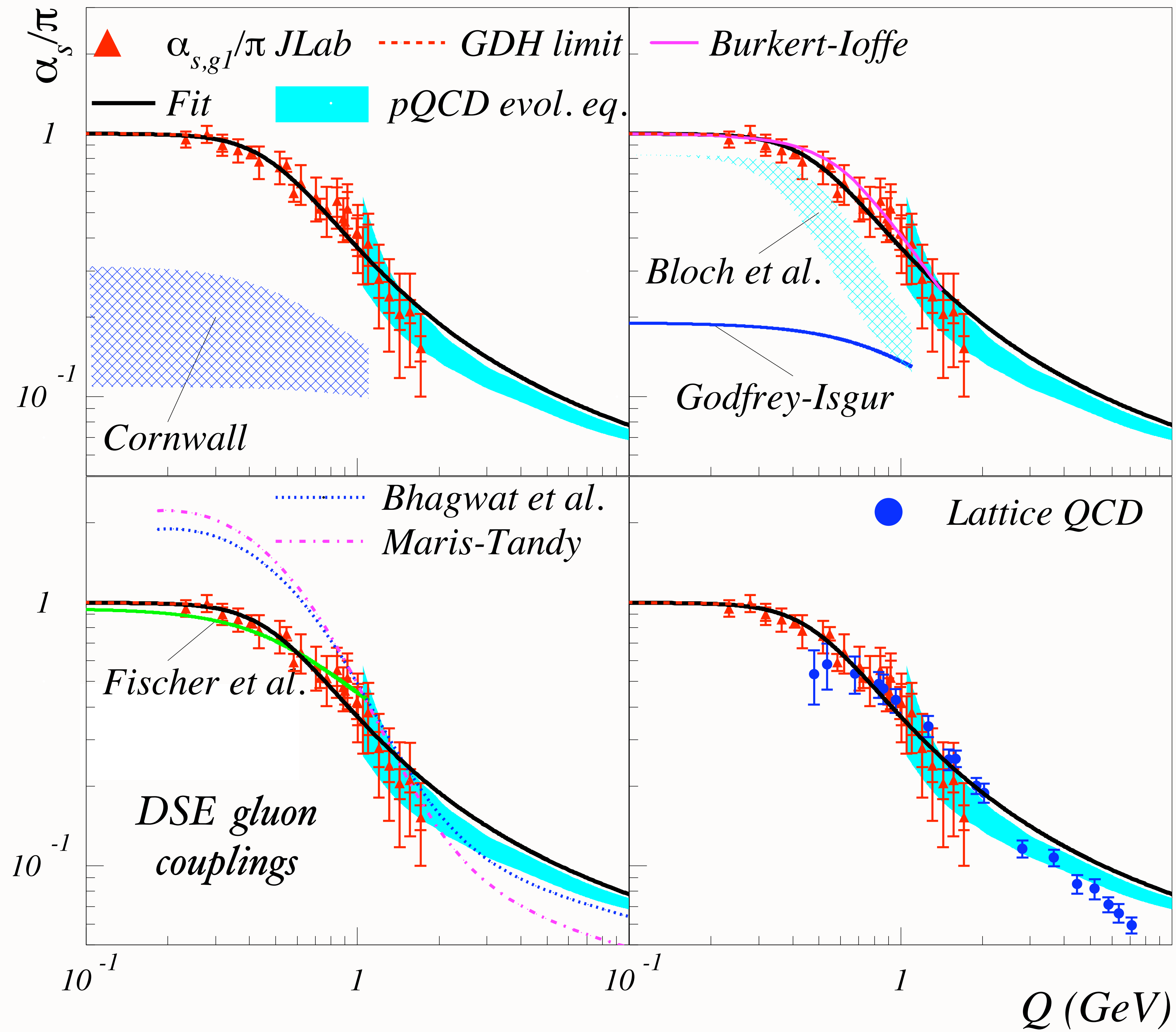


# Features of AdS/QCD LF Holography

- **Based on Conformal Scaling of Infrared QCD Fixed Point**
- **Conformal template: Use isometries of AdS5**
- **Interpolating operator of hadrons based on twist, superfield dimensions**
- **Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required**
- **Break Conformal symmetry with dilaton**
- **Dilaton introduces confinement -- positive exponent**
- **Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)**
- **Effective Charge from AdS/QCD at all scales**
- **Conformal Dimensional Counting Rules for Hard Exclusive Processes**


String Theory
Goal: First Approximant to QCD
AdS/CFT
Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Counting rules for Hard Exclusive Scattering
Regge Trajectories
AdS/QCD
Conformal behavior at short distances + Confinement at large distance
QCD at the Amplitude Level
Semi-Classical QCD / Wave Equations
Holography
Boost Invariant 3+1 Light-Front Wave Equations
J =0,1,1/2,3/2 plus L
Integrable!
Hadron Spectra, Wavefunctions, Dynamics

## Deur, Korsch, et al.



# Running Coupling from Modified AdS/QCD

**Deur, de Teramond, sjb**

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

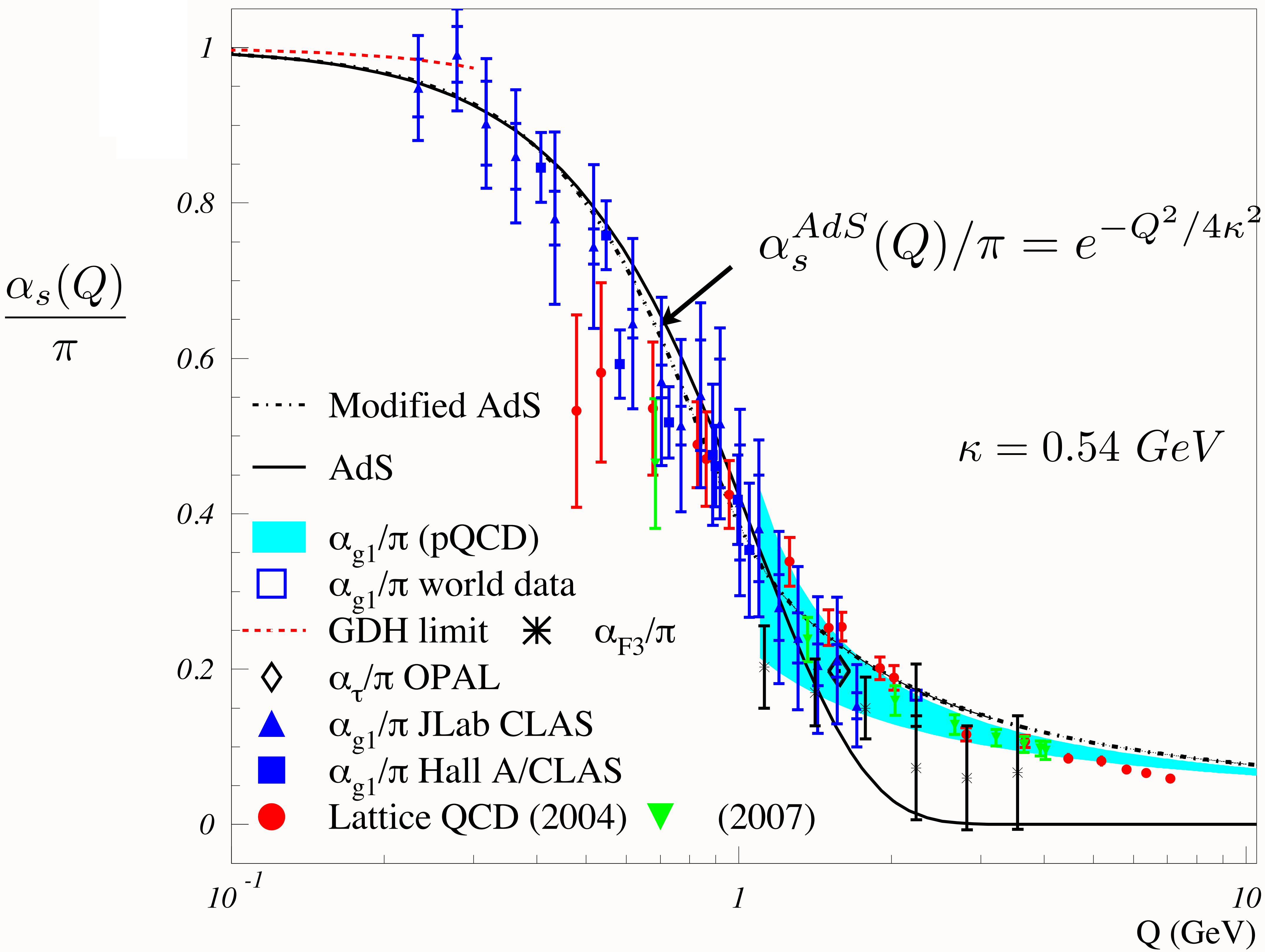
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

where the coupling $\alpha_s^{AdS}$ incorporates the non-conformal dynamics of confinement

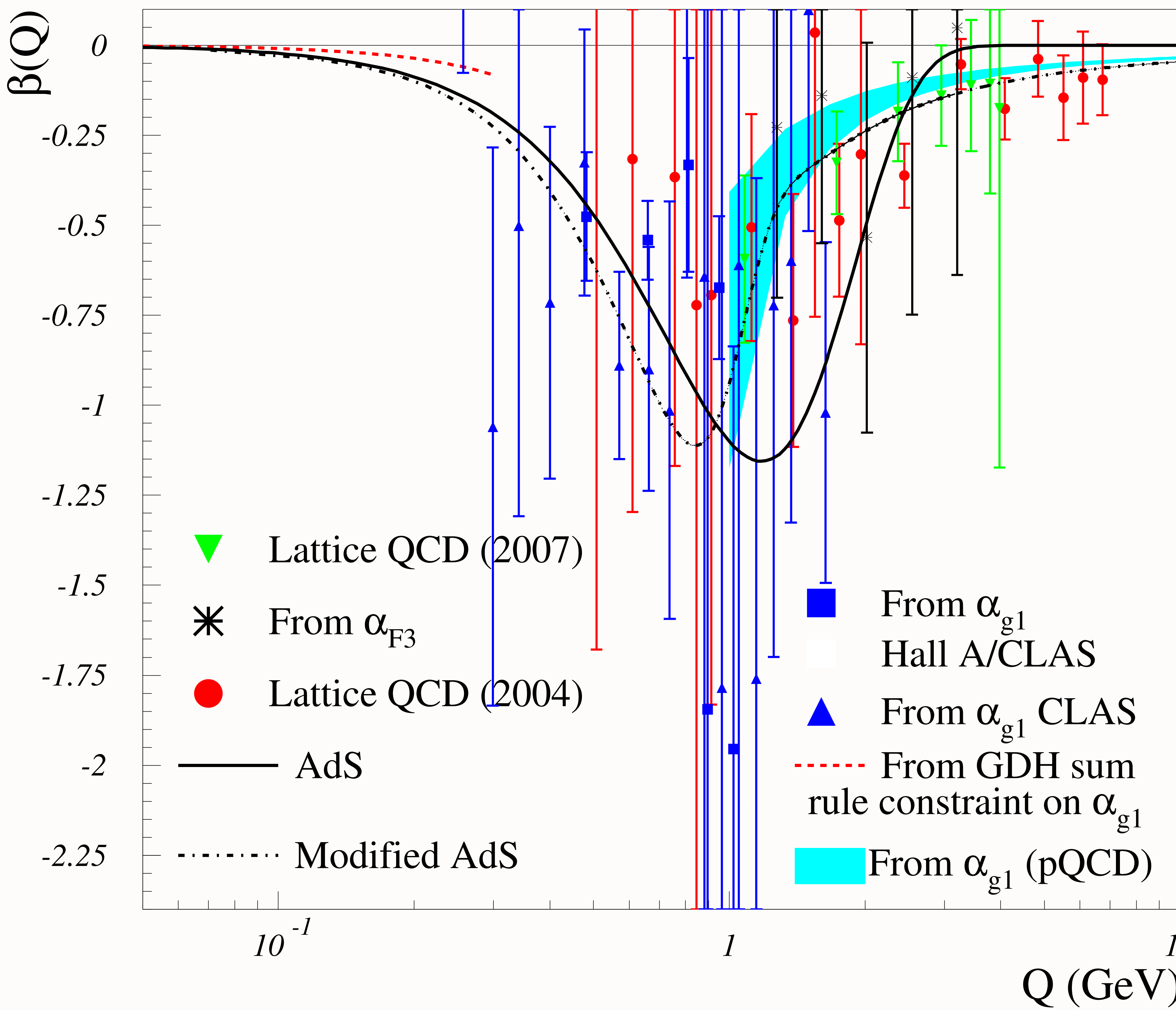
# Running Coupling from Light-Front Holography and AdS/QCD

## Analytic, defined at all scales, IR Fixed Point




**Deur, de Teramond, sjb**

$$\beta^{AdS}(Q^2) = \frac{d}{d\log Q^2}\alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$




Deur, de Teramond, sjb

# "One of the gravest puzzles of theoretical physics"

**DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA*
*Kavil Institute for Theoretical Physics, University of California,*
*Santa Barbara, CA 93106, USA*
*zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

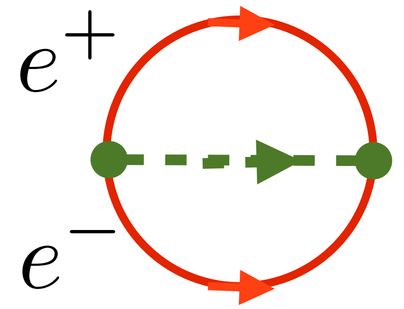
$$(\Omega_\Lambda)_{QCD} \propto < 0|q\bar{q}|0 >^4$$

*QCD Problem Solved if quark and gluon condensates reside within hadrons, not vacuum!*

**R. Shrock, sjb** Proc.Nat.Acad.Sci. 108 (2011) 45-50 "Condensates in Quantum Chromodynamics and the Cosmological Constant"

**C. Roberts, R. Shrock, P. Tandy, sjb** Phys.Rev. C82 (2010) 022201 "New Perspectives on the Quark Condensate"

# Instant Form Vacuum in QED




- Loop diagrams of all orders contribute

$$\Omega_\Lambda \sim 10^{120}$$

- Huge vacuum energy
- $$\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3} \sqrt{\vec{k}^2 + m^2}$$

Cutoff quad div at $M_{Planck}$

- :Normal order: prescription
- Divide S-matrix by disconnected vacuum diagrams
- Contrast: Light-Front Vacuum empty since plus momenta are positi and conserved:

$$k^+ = k^0 + k^3 > 0$$

# Light-Front vacuum can simulate empty universe

**Shrock, Tandy, Roberts, sjb**

- Independent of observer frame
- Causal
- Lowest invariant mass state M= 0.
- Trivial up to $k^+=0$ zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: In hadron condensates (Maris, Tandy Roberts)
- QED vacuum; no loops
- Zero cosmological constant

# Gell-Mann Oakes Renner Formula in QCD

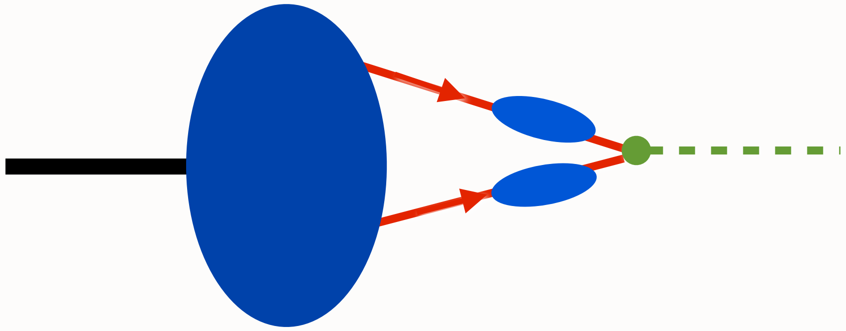
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

**current algebra: effective pion field**

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

**QCD: composite pion Bethe-Salpeter Eq.**

*vacuum condensate actually is an "in-hadron condensate"*

$\pi^-$ 

$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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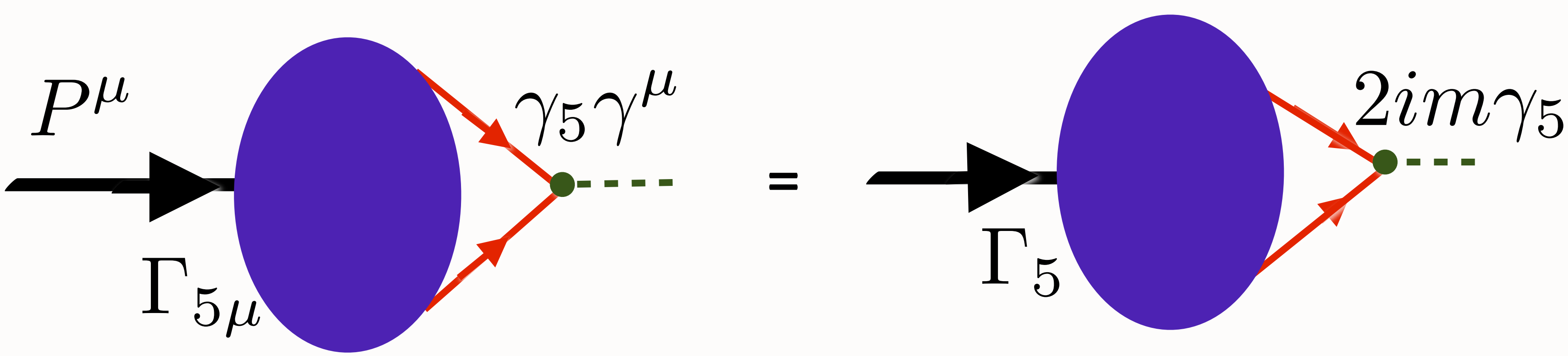
**R. Shrock, sjb**

**arXiv:0905.1151 [hep- th], Proc. Nat’l. Acad. Sci., (in press);**
**``Condensates in Quantum Chromodynamics and the Cosmological Constant.”**

# Ward-Takahashi Identity for axial current

$$P^{\mu}\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$




plus non-pole

Identify pion pole at $P^2 = m_\pi^2$

$$P^{\mu} < 0|\bar{q}\gamma_5\gamma^{\mu}q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u + m_d)\rho_\pi$$

PHYSICAL REVIEW C **82**, 022201(R) (2010)

# New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

[1]*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

[2]*Centre for Particle Physics Phenomenology: CP$^3$-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[4]*Department of Physics, Peking University, Beijing 100871, China*

[5]*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

[6]*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

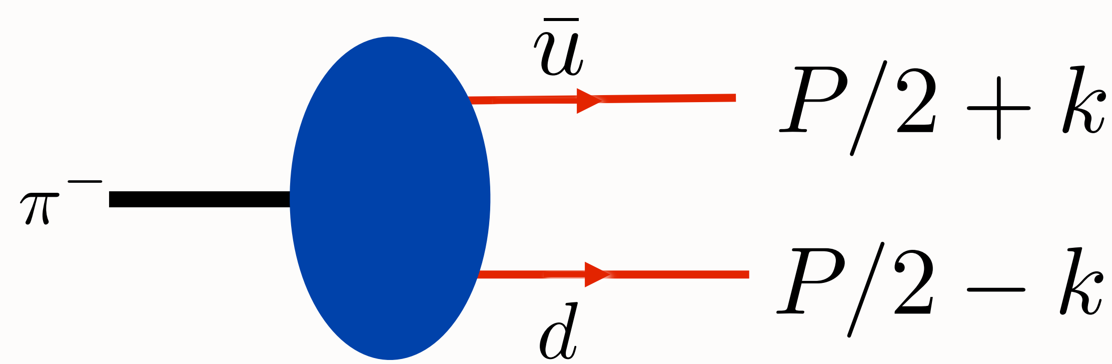
(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

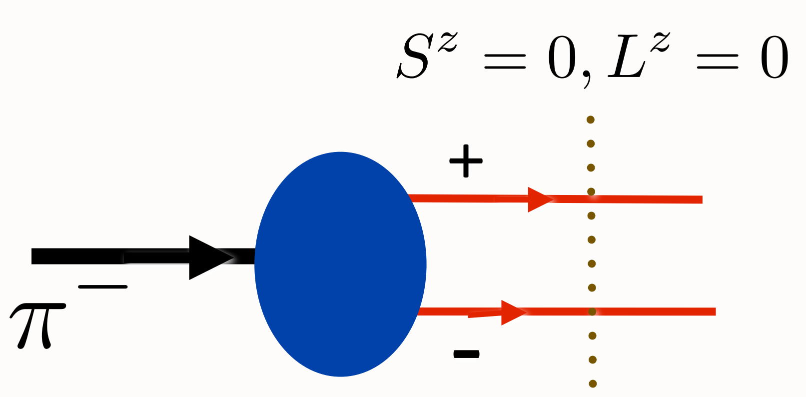
# General Form of Bethe-Salpeter Wavefunction

$$\Gamma_\pi(k;P) = i\gamma_5 E_\pi(k,P) + \gamma_5\gamma\cdot P F_\pi(k;P)$$
$$+\gamma_5\gamma\cdot k G_\pi(k;P) - \gamma_5\sigma_{\mu\nu}k^\mu P^\nu H_\pi(k;P)$$

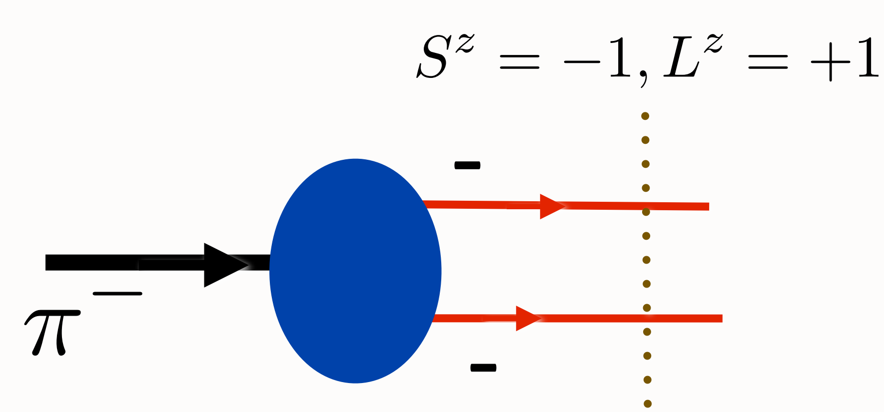
$\Gamma_\pi(k;P)$




Allows both $<0|\bar{q}\gamma_5\gamma_\mu q|\pi>$ and $<0|\bar{q}\gamma_5 q|\pi>$

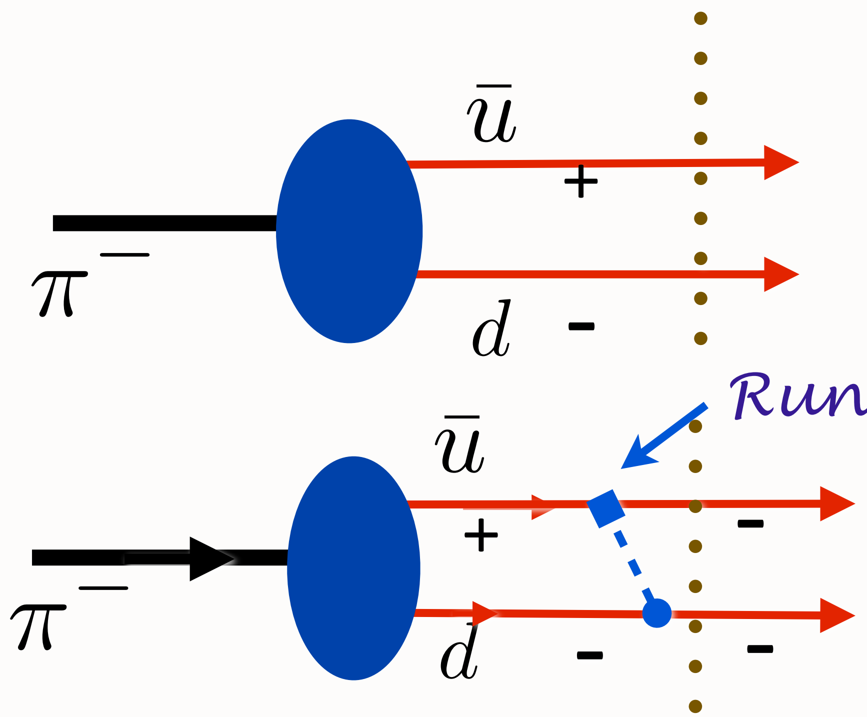



LFWFs



# Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$




**Couples to**

$$L^z = 0, S^z = 0 \quad < \pi|\bar\gamma^\mu q \gamma_5 q|0 > \sim f_\pi$$

*Running constituent mass at vertex*

$$L^z = +1, S^z = -1 \quad < \pi|\bar q \gamma_5 q|0 > \sim \rho_\pi$$

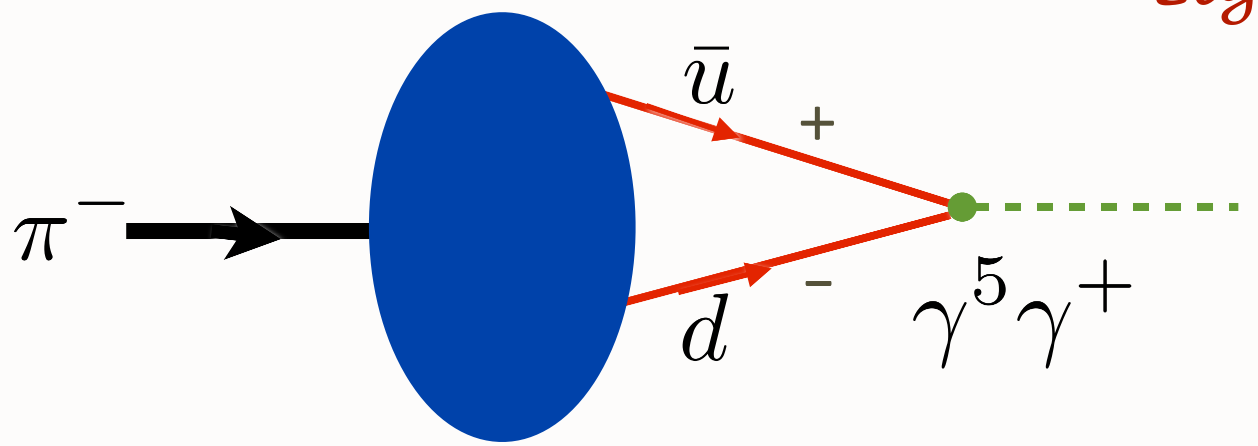
$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

***Angular Momentum Conservation***

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

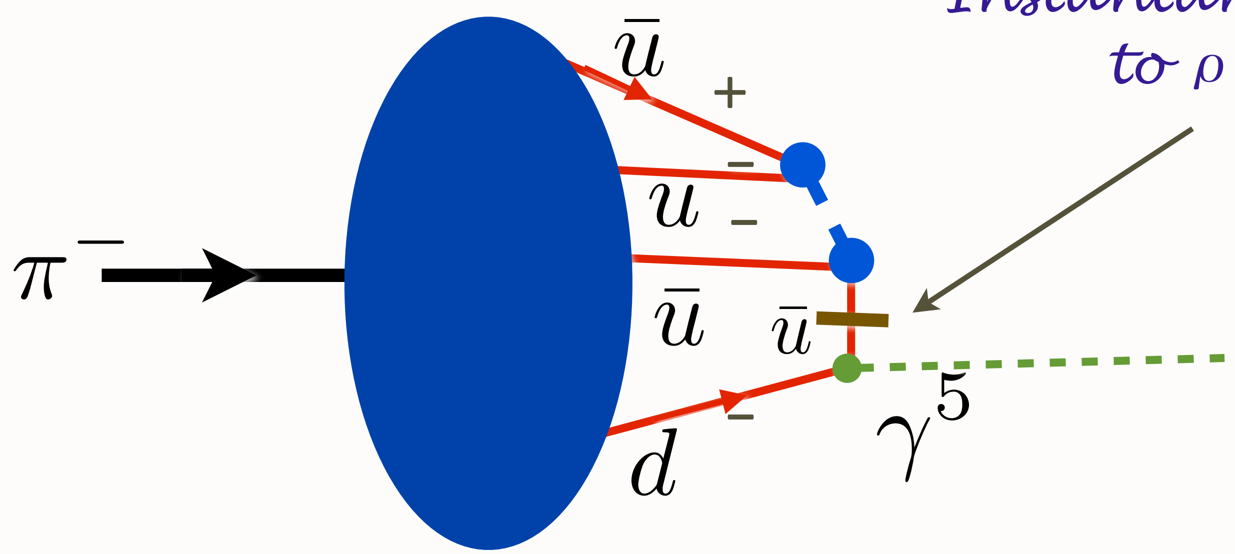
# Higher Light-Front Fock State of Pion Simulates DCSB

## Light Front Fock state Analysis

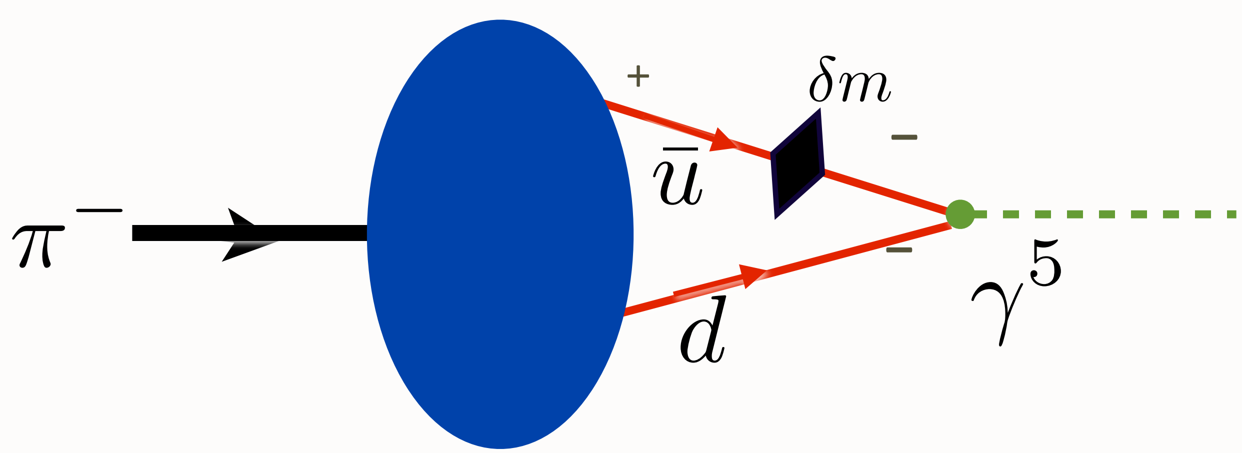



$$f_\pi P^+ =< 0|\bar{q}\gamma^5\gamma^+ q|\pi >$$

Instantaneous quark propagator contribution to $\rho_\pi$ derived from higher Fock state




$$i\rho_\pi =< 0|\bar{q}\gamma^5 q|\pi >$$




Higher Fock state acts like mass insertion

PHYSICAL REVIEW D VOLUME 9, NUMBER 2 15 JANUARY 1974

# Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon. Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame. A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.

*Light-Front Formalism*

# *Gell-Mann Oakes Renner Formula in QCD*

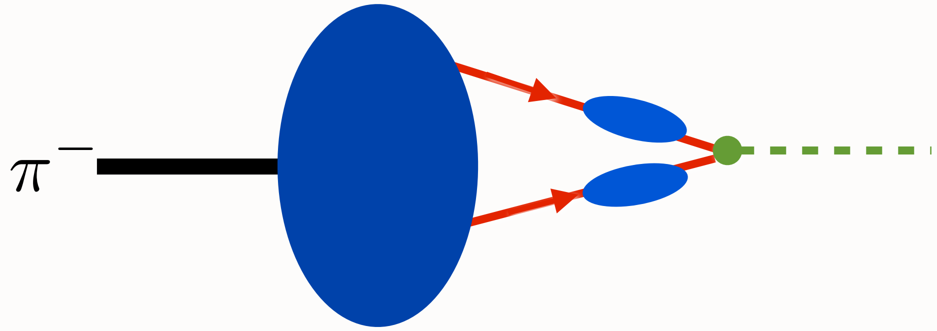
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

**current algebra:**
**effective pion field**

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

**QCD: composite pion**
**Bethe-Salpeter Eq.**

*vacuum condensate actually is an "in-hadron condensate"*



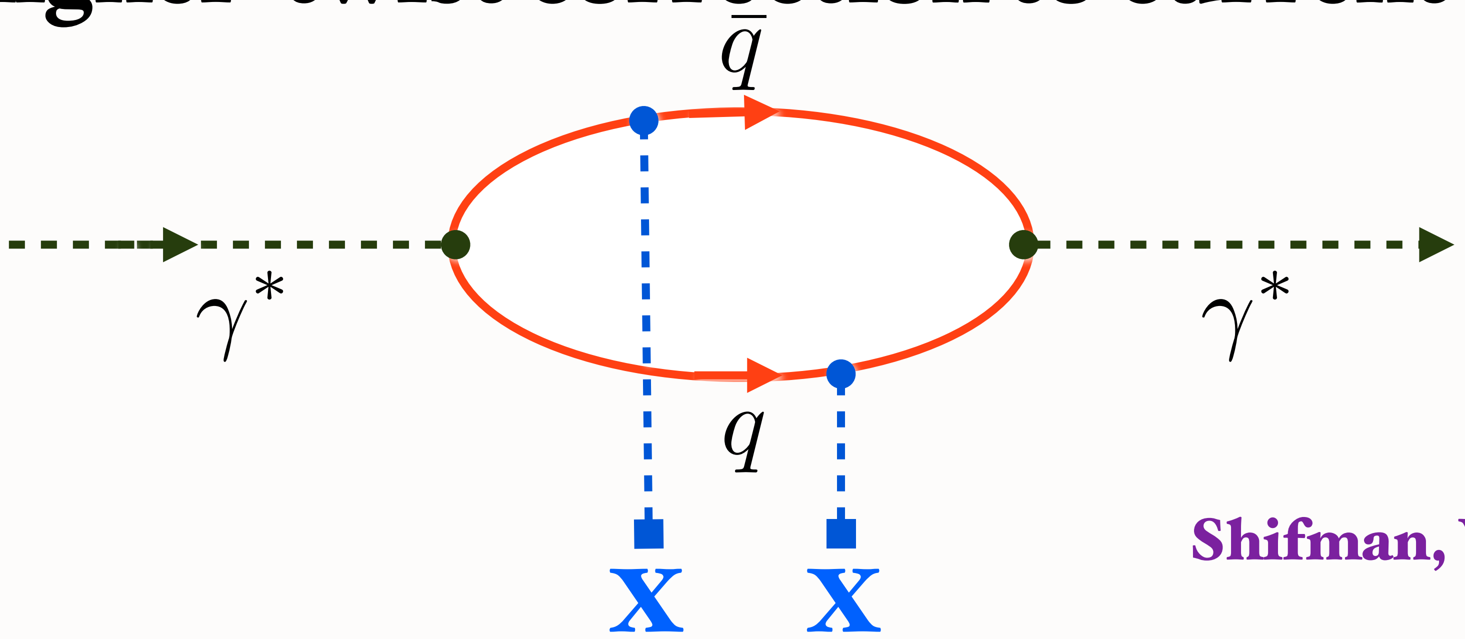

$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

# *Is there evidence for a gluon vacuum condensate?*

$$< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$$

**Look for higher-twist correction to current propagator**




**Shifman, Vainshtein, Zakharov**

$$e^+e^- \to X, \ \tau \text{ decay}, \ Q\bar{Q} \text{ phenomenology}$$

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi} \frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

# Determinations of the vacuum Gluon Condensate

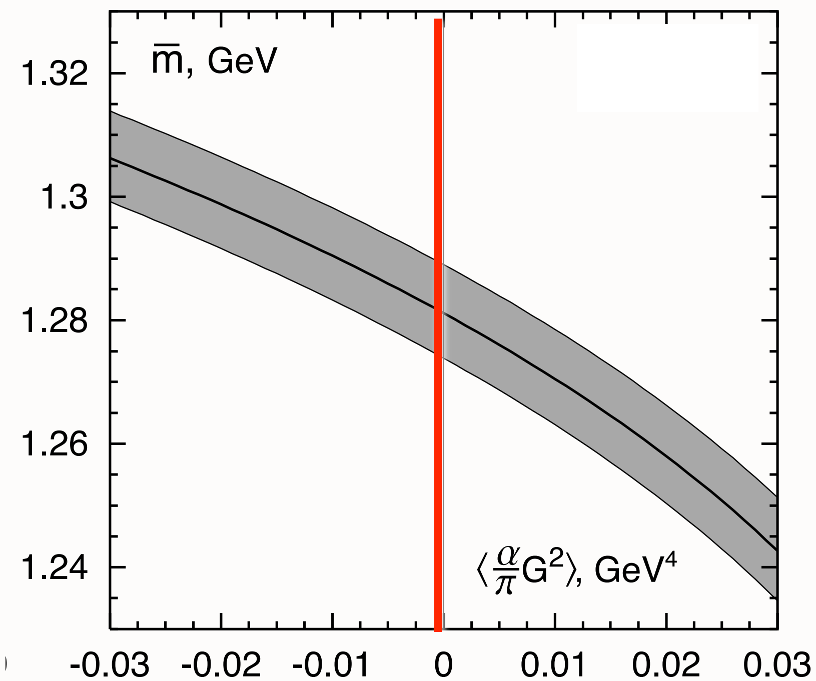
$$< 0|\frac{\alpha_s}{\pi}G^2|0 > [\mathrm{GeV}^4]$$

$-0.005 \pm 0.003$ from $\tau$ decay. Davier et al.

$+0.006 \pm 0.012$ from $\tau$ decay. Geshkenbein, Ioffe, Zyablyuk

$+0.009 \pm 0.007$ from charmonium sum rules

Ioffe, Zyablyuk

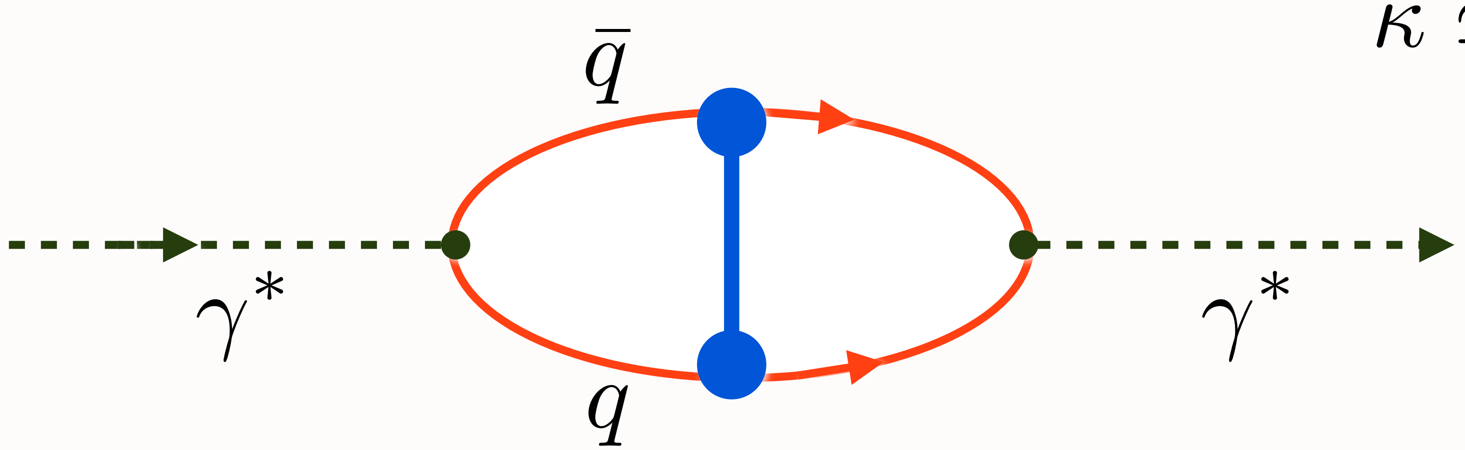



Consistent with zero vacuum condensate

*Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator*

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$$\kappa \simeq 0.5 \text{ GeV}$$




$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

*mimics dimension-4 gluon condensate* $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

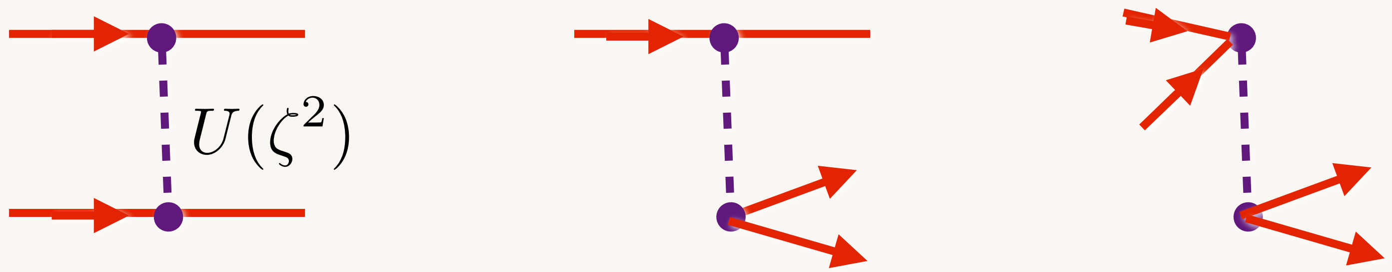
$e^+e^- \to X$, $\tau$ decay, $Q\bar{Q}$ phenomenology

# Higher Fock States

- Exposed by timelike form factor through dressed current.
- Created by confining interaction

$$P^-_{\text{confinement}} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

- Similar to QCD(1+1) in lcg




de Teramond, sjb

# Novel QCD Phenomena and Perspectives

- Hadroproduction at large transverse momentum does not derive exclusively from 2 to 2 scattering subprocesses: Baryon Anomaly at RHIC Sickles, sjb

- Color Transparency Mueller, sjb; Diffractive Di-Jets and Tri-jets Strikman et al

- Heavy quark distributions do not derive exclusively from DGLAP or gluon splitting -- component intrinsic to hadron wavefunction. Hoyer, et al

- Higgs production at large $x_F$ from intrinsic heavy quarks Kopeliovitch, Goldhaber, Schmidt, Soffer, sjb

- Initial and final-state interactions are not always power suppressed in a hard QCD reaction: Sivers Effect, Diffractive DIS, Breakdown of Lam Tung PQCD Relation Schmidt, Hwang, Hoyer, Boer, sjb; Collins

- LFWFS are universal, but measured nuclear parton distributions are not universal -- antishadowing is flavor dependent Schmidt, Yang, sjb

- Renormalization scale is not arbitrary; multiple scales, unambiguous at given order. Disentangle running coupling and conformal effects, Skeleton expansion: Gardi, Grunberg, Rathsman, sjb

- Quark and Gluon condensates reside within hadrons: Shrock, sjb

Bjorken, Kogut, Soper; Blankenbecler, Gunion, sjb;
Blankenbecler, Schmidt

## Crucial Test of Leading -Twist QCD:
## Scaling at fixed $x_T$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{cm})}{p_T^{n_{\rm eff}}} \qquad x_T = \frac{2p_T}{\sqrt{s}}$$

**Parton model: $n_{eff} = 4$**

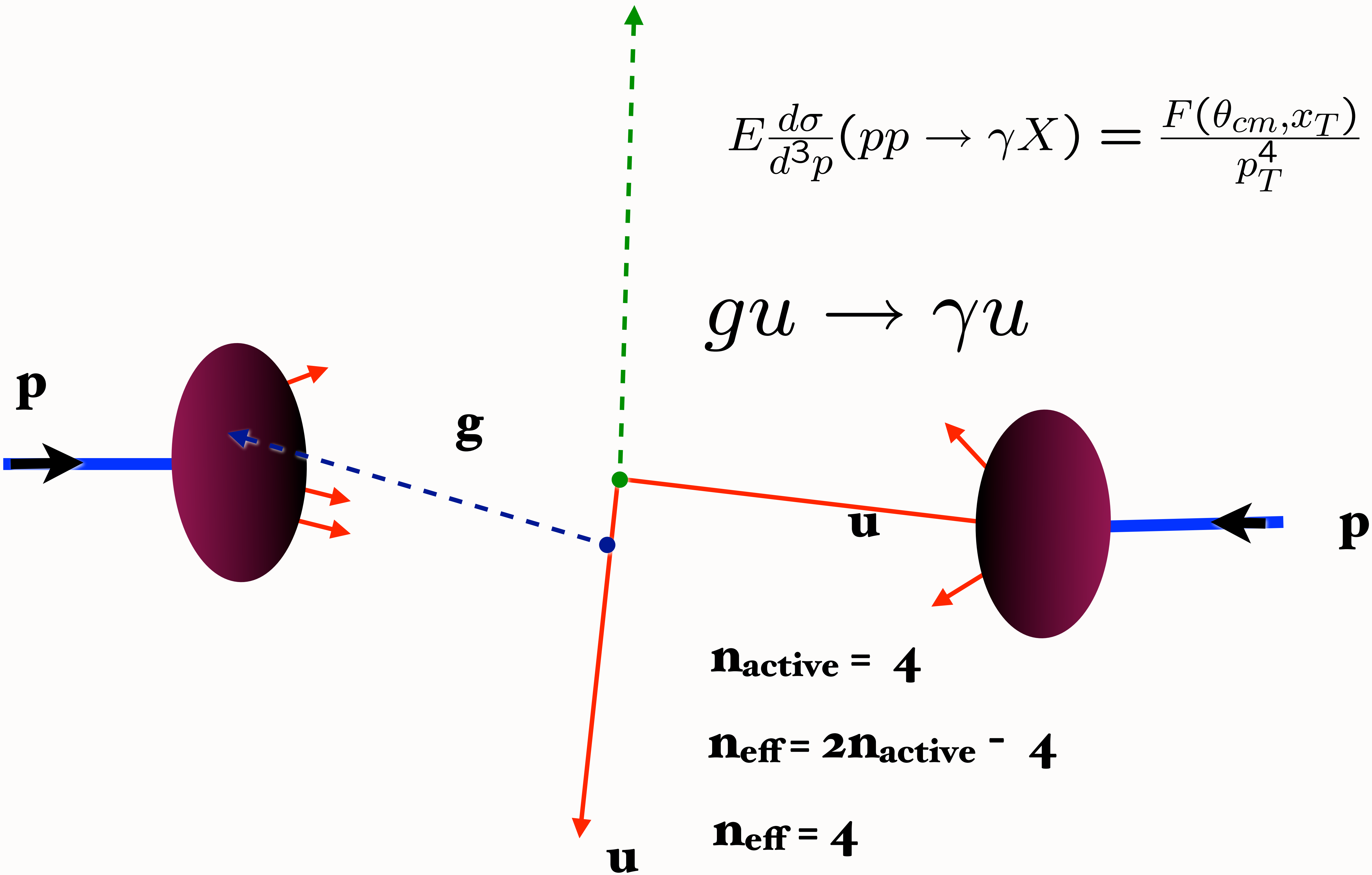
**As fundamental as Bjorken scaling in DIS**

**scaling law: $n_{eff} = 2\, n_{active} - 4$**

$$pp \rightarrow \gamma X$$

$$E\frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$$

$$gu \rightarrow \gamma u$$



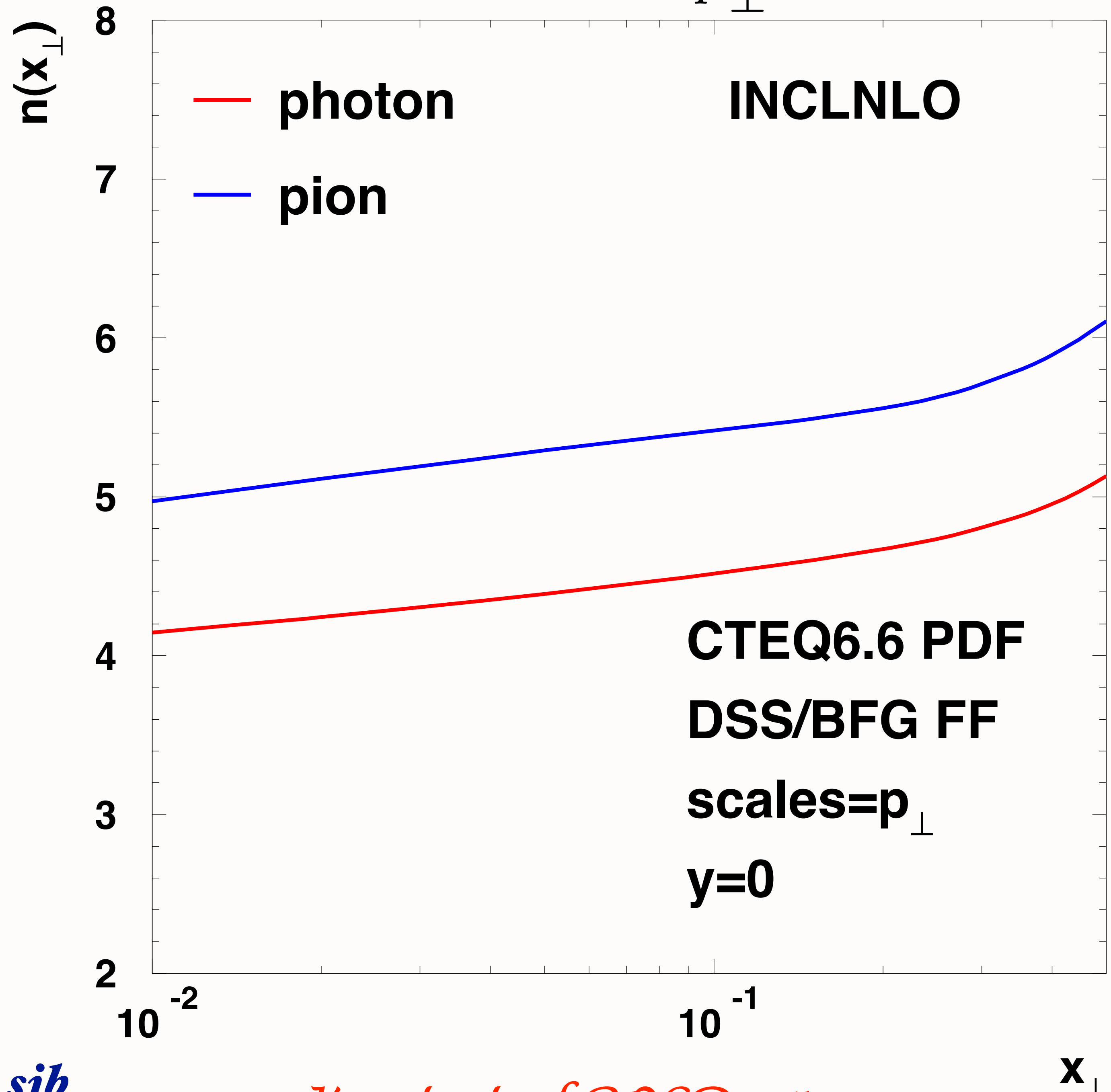

$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$

## QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

$$\frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp^{n(x_\perp)}}$$




$$pp \to \pi X$$

$$pp \to \gamma X$$

$$5 < p_\perp < 20\ GeV$$

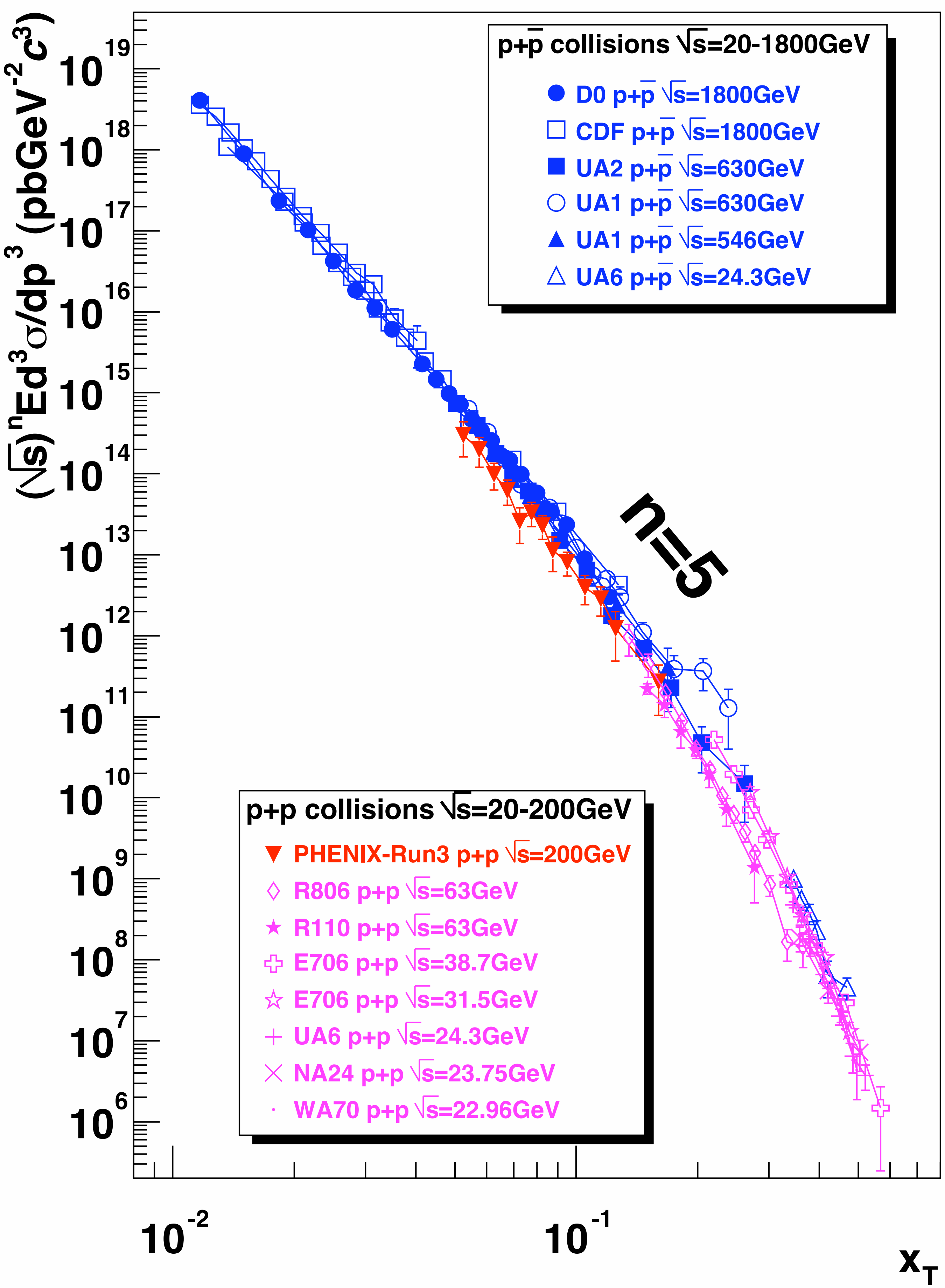
$$70\ GeV < \sqrt{s} < 4\ TeV$$

***Arleo, Hwang, Sickles, sjb***

***Pirner, Raufeisen, sjb***

Key test of PQCD: power-law fall-off at fixed $x_T$

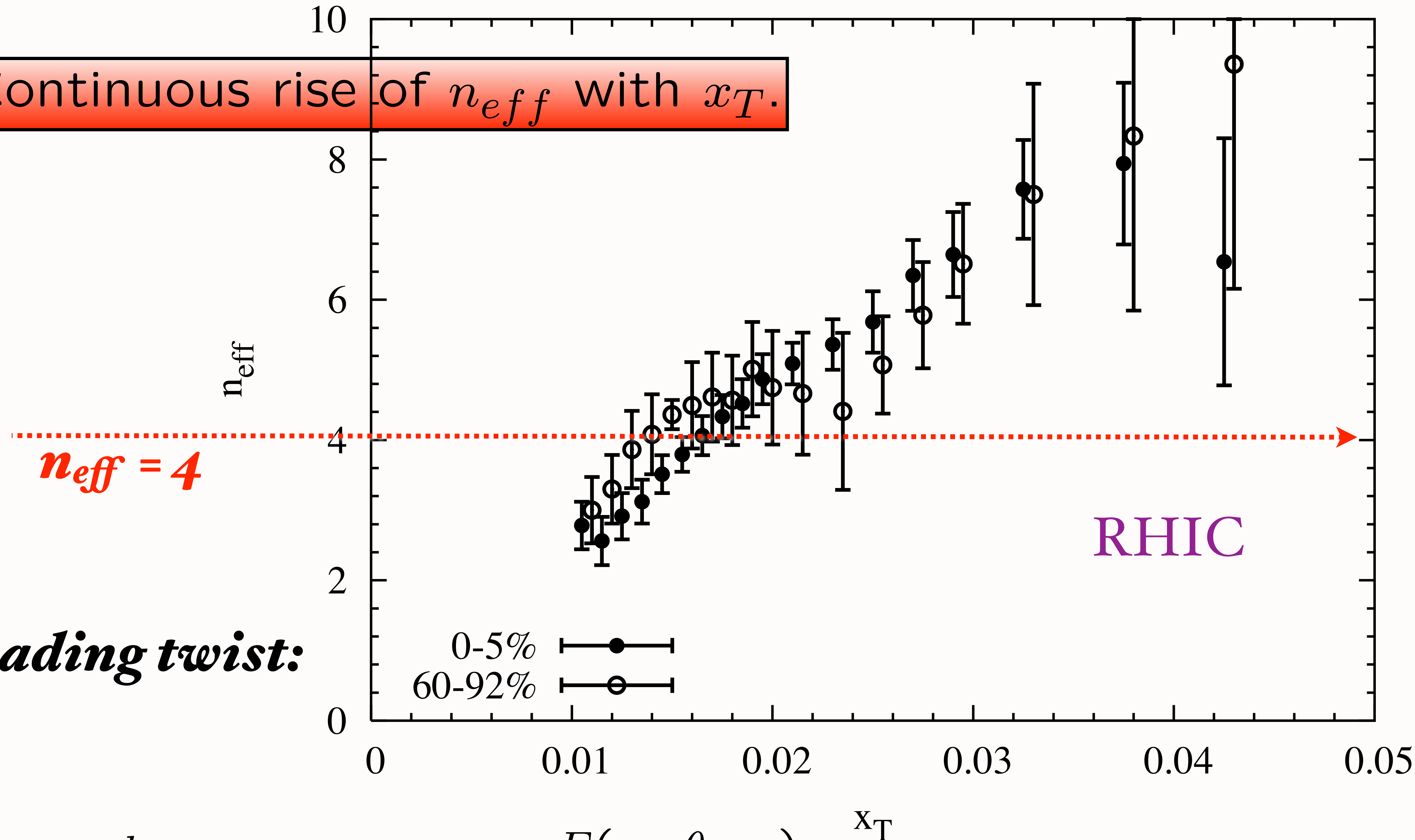
$\sqrt{s}^n E \frac{d\sigma}{d^3p}(pp \to \gamma X)$ at fixed $x_T$




**$x_T$-scaling of direct photon production: consistent with PQCD**

*Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available $p_T$ range. Shown are data for central $(0-5\%)$ and for peripheral $(60-90\%)$ collisions.*

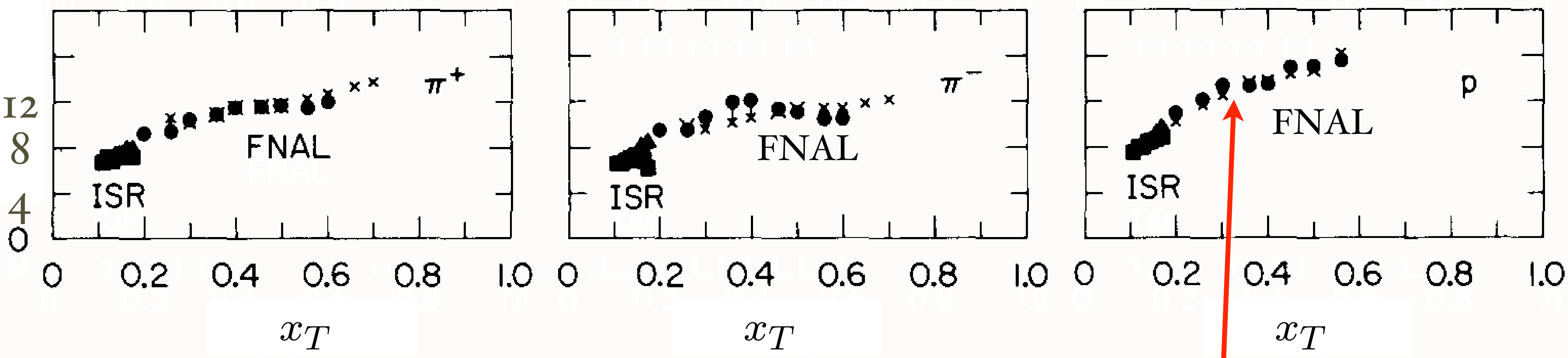
**Continuous rise of $n_{eff}$ with $x_T$.**

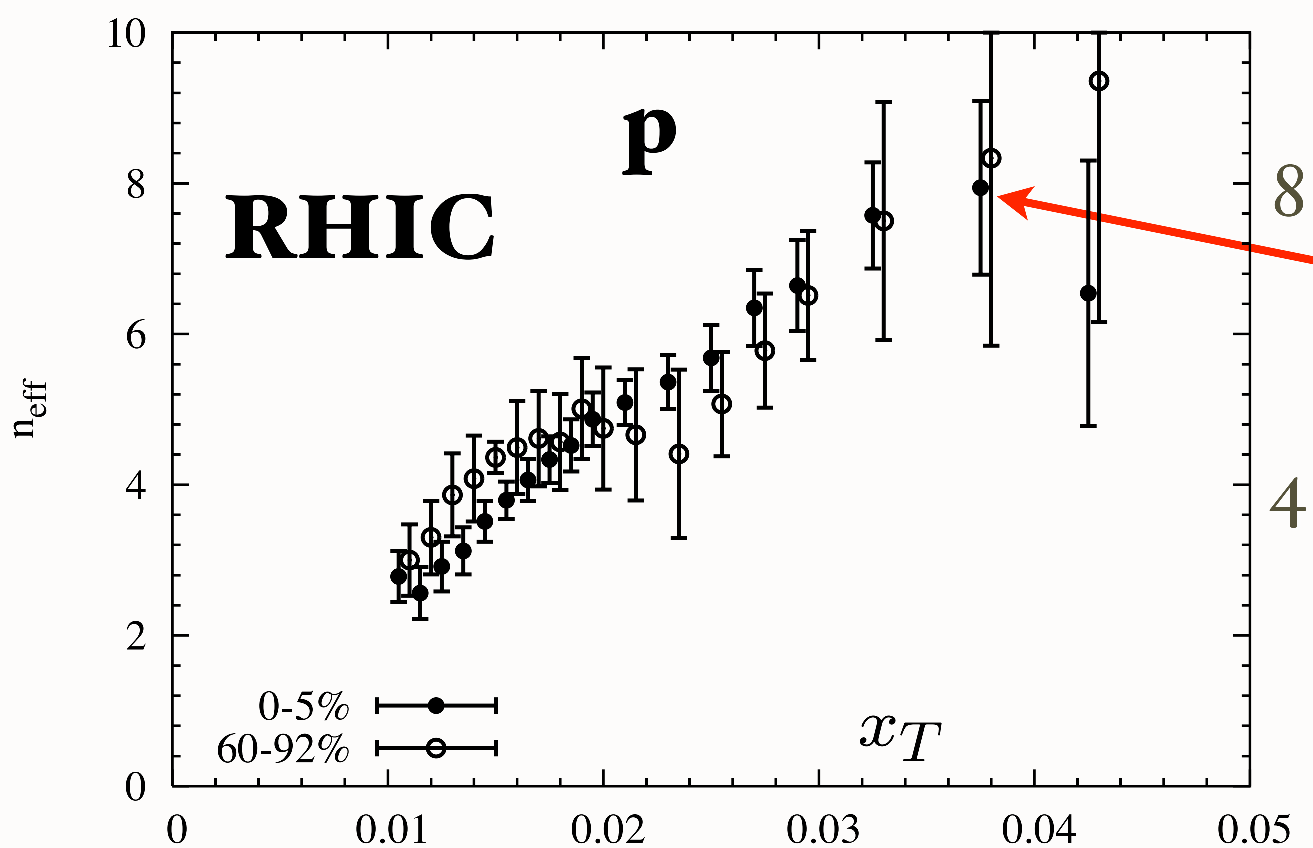



***Leading twist:***

$$E\frac{d\sigma}{d^3p}(pN \to pX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$

$$E\frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T,\theta_{CM})}{p_T^{n_{eff}}}$$

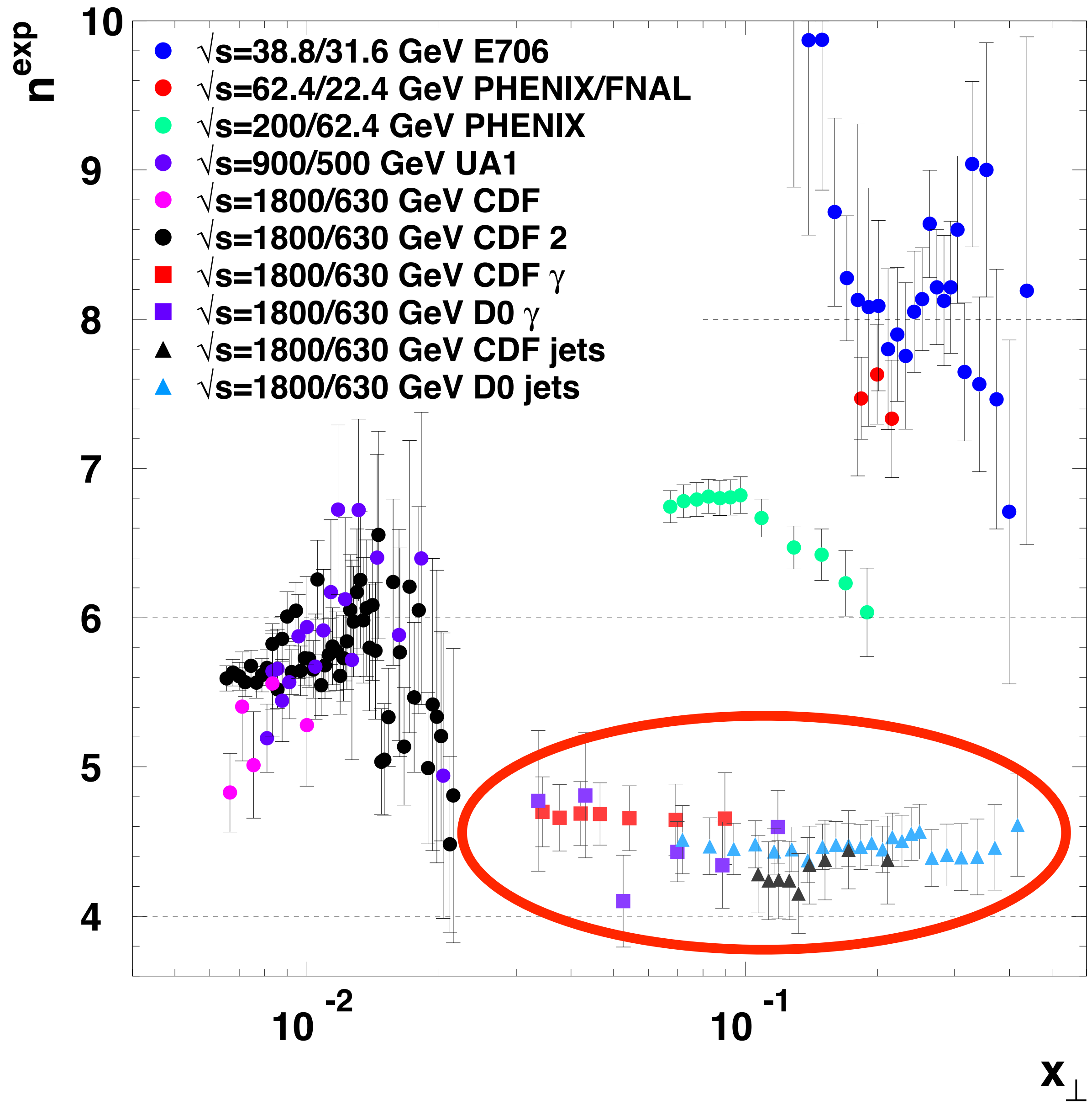



$$E\frac{d\sigma}{d^3p}(pp \rightarrow pX) = \frac{F(x_T,\theta_{CM})}{p_T^{12}}$$

$$E\frac{d\sigma}{d^3p}(pp \rightarrow pX) = \frac{F(x_T,\theta_{CM})}{p_T^{8}}$$

*Trend consistent with RHIC at small $x_T$*

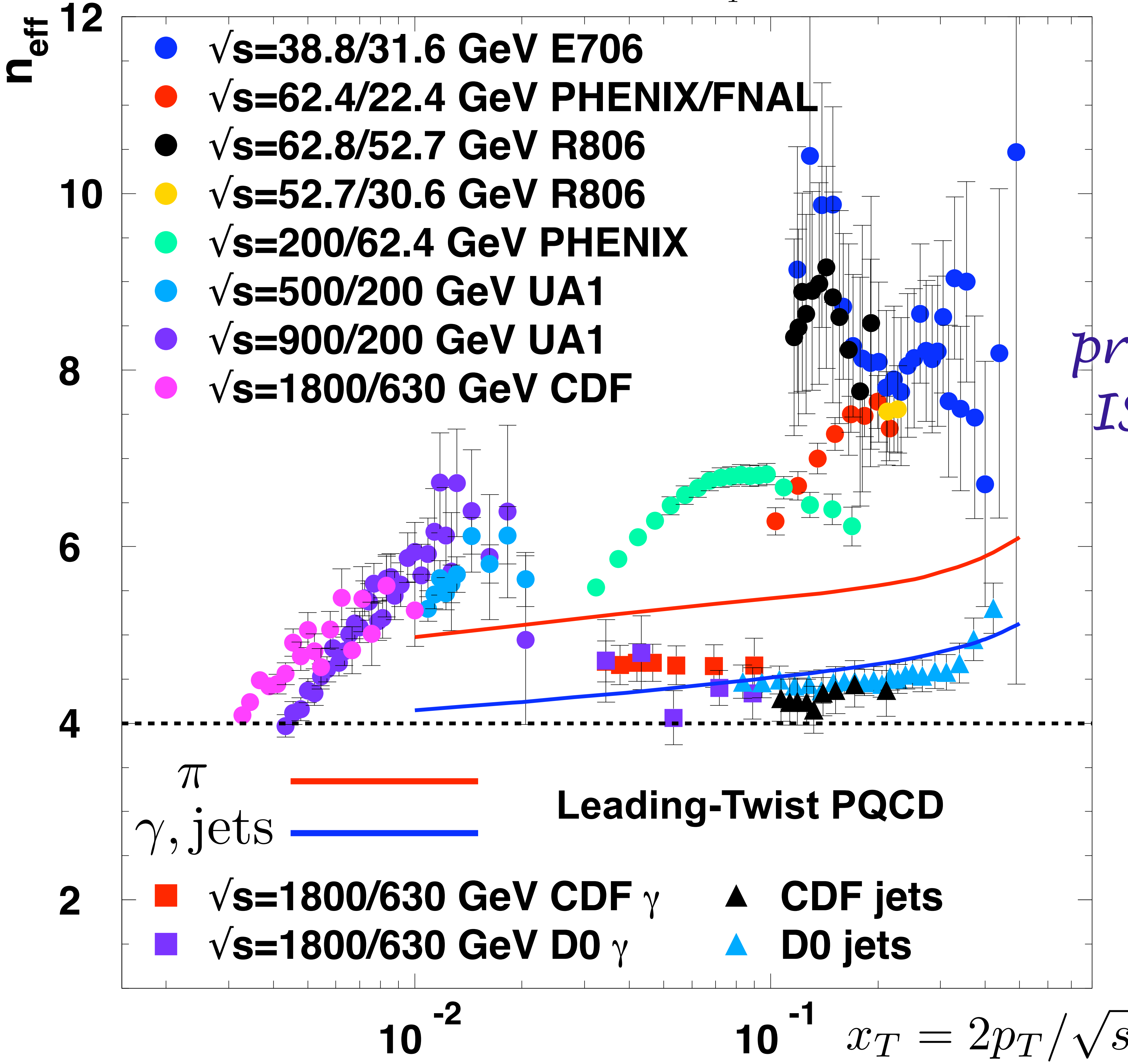



Photons and Jets agree with PQCD $x_T$ scaling

Hadrons do not!

- Significant increase of the hadron $n^{\mathrm{exp}}$ with $x_\perp$
  - $n^{\mathrm{exp}} \simeq 8$ at large $x_\perp$
- Huge contrast with photons and jets !
  - $n^{\mathrm{exp}}$ constant and slight above 4 at all $x_\perp$

$$E\frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{\rm eff}}}$$

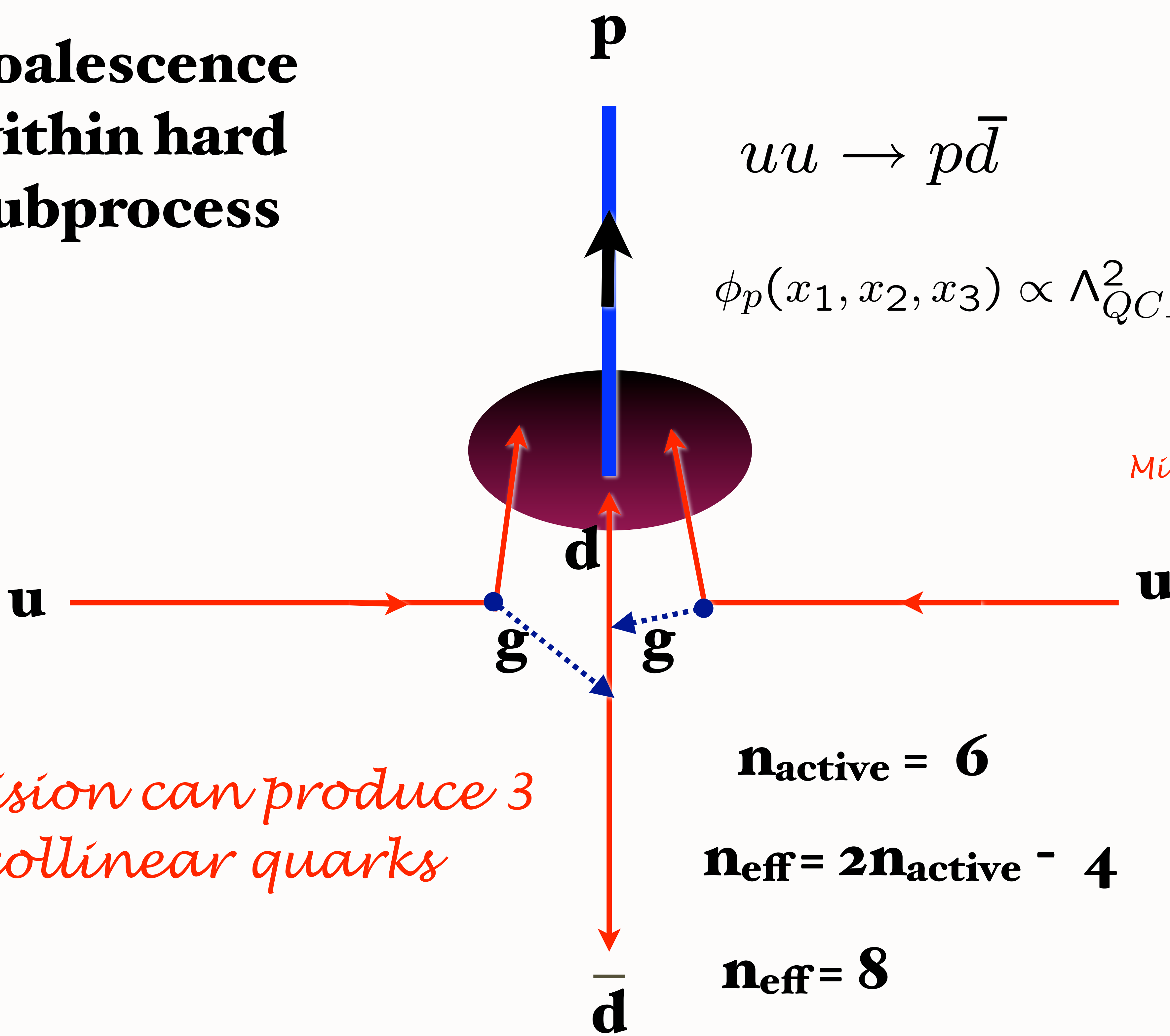



*Leading-twist prediction fails at ISR, FNAL, RHIC, CDF!*

***Arleo, Hwang, Sickles, sjb***

# *Baryon can be made directly within hard subprocess*

**Coalescence within hard subprocess**




$uu \to p\bar{d}$

$\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}$

**Bjorken**
**Blankenbecler, Gunion, sjb**
**Berger, sjb**
**Hoyer, et al: Semi-Exclusive**

**Sickles; sjb**

*Small color-singlet*
*Color Transparent*
*Minimal same-side energy*

*Explains Baryon anomaly*

$qq \to B\bar{q}$

**$n_{active}$ = 6**

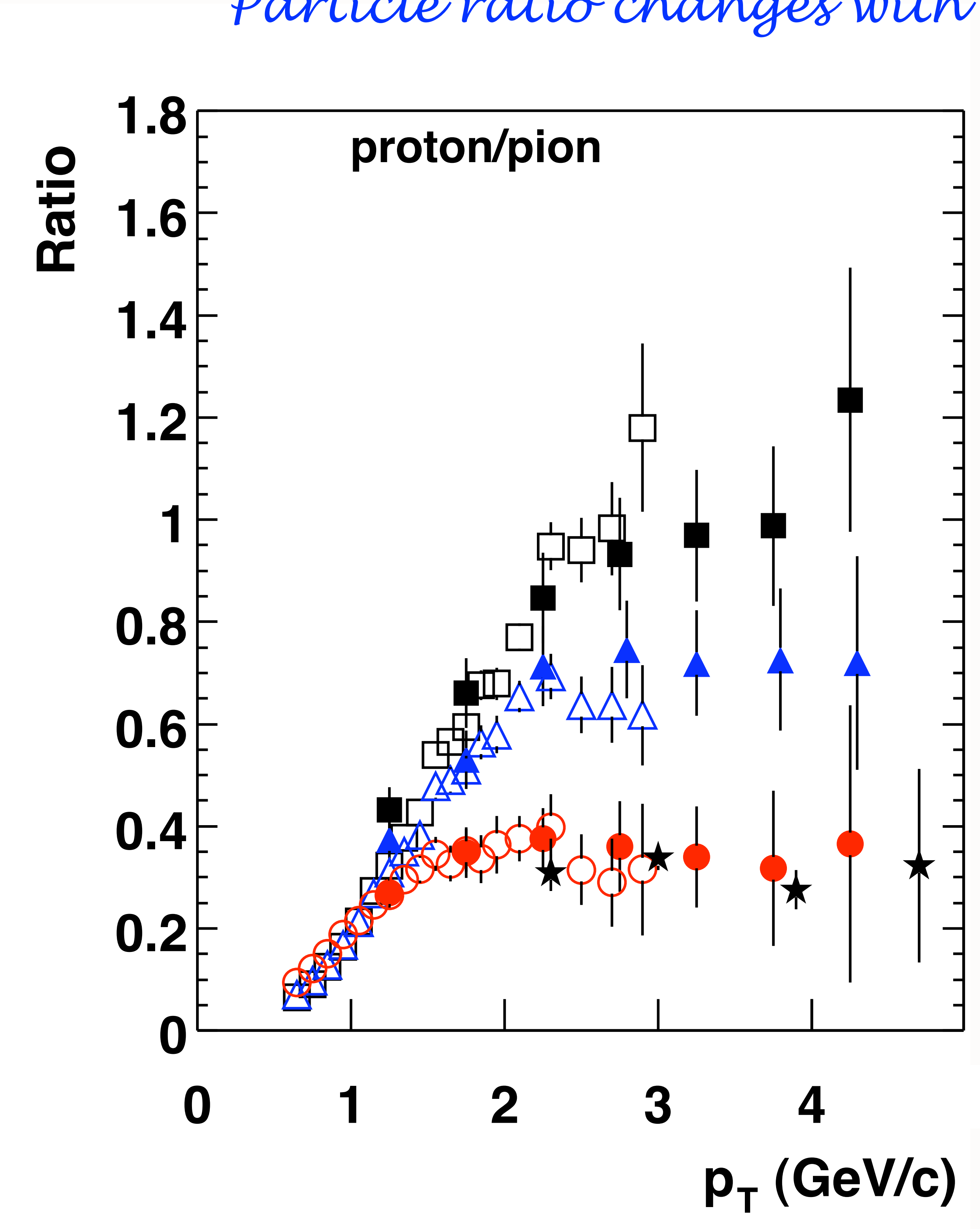
**$n_{eff} = 2n_{active} - 4$**

**$n_{eff}$ = 8**

*Collision can produce 3 collinear quarks*

S. S. Adler *et al.* PHENIX Collaboration *Phys. Rev. Lett.* **91**, 172301 (2003).

*Particle ratio changes with centrality!*




***Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process***

**Central**

**Peripheral**

*Tannenbaum:*

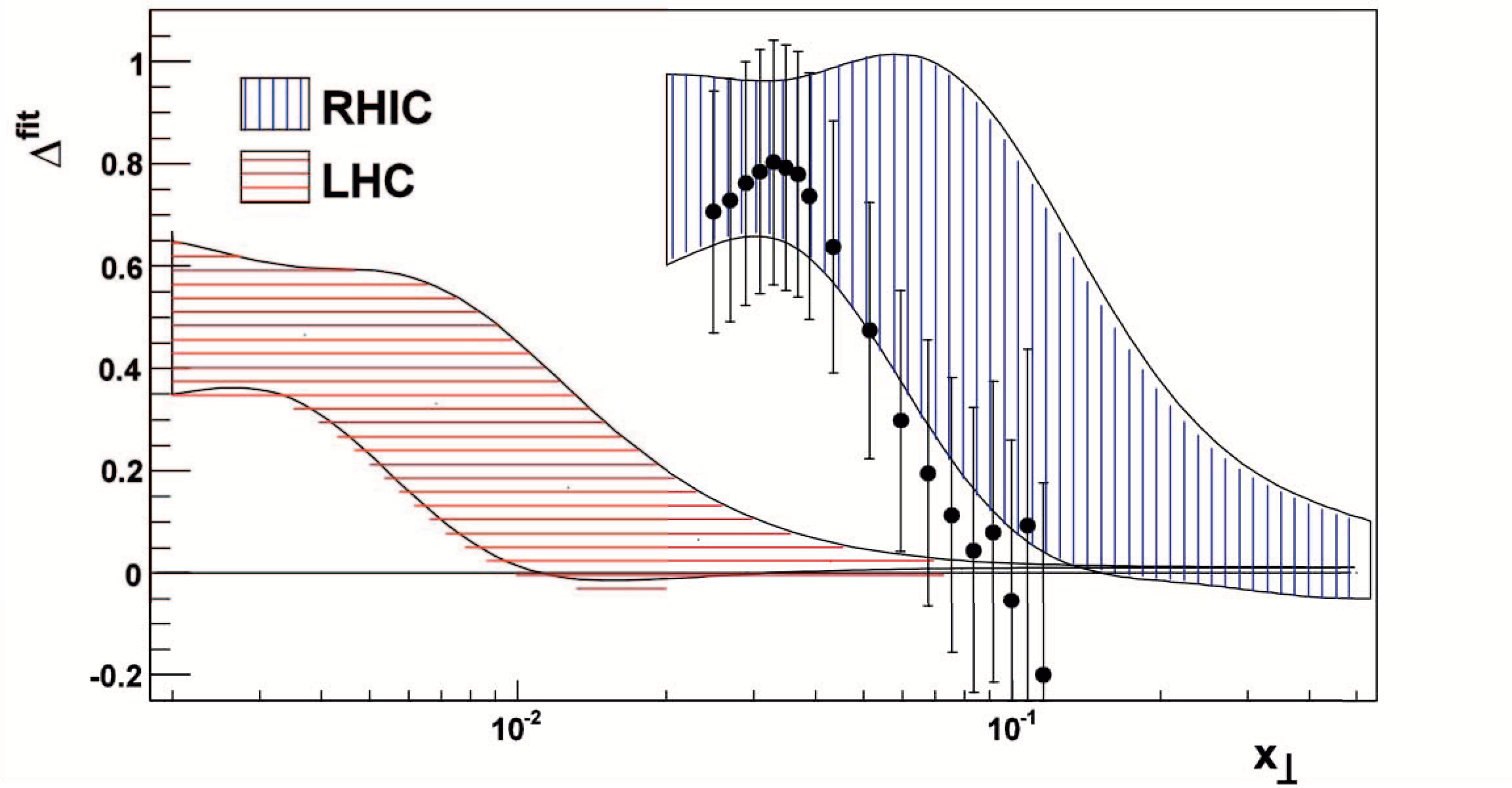
*Baryon Anomaly:*

# RHIC/LHC predictions

## PHENIX results

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

[ A. Bezilevsky, APS Meeting ]



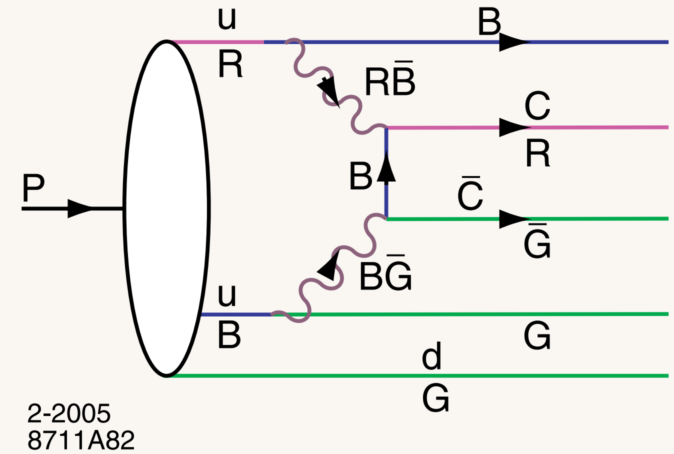

- Magnitude of $\Delta$ and its $x_\perp$-dependence consistent with predictions

$$\Delta = n_{expt} - n_{PQCD}$$

***Arleo, Hwang, Sickles, sjb***

Hoyer, Peterson, Sakai, sjb

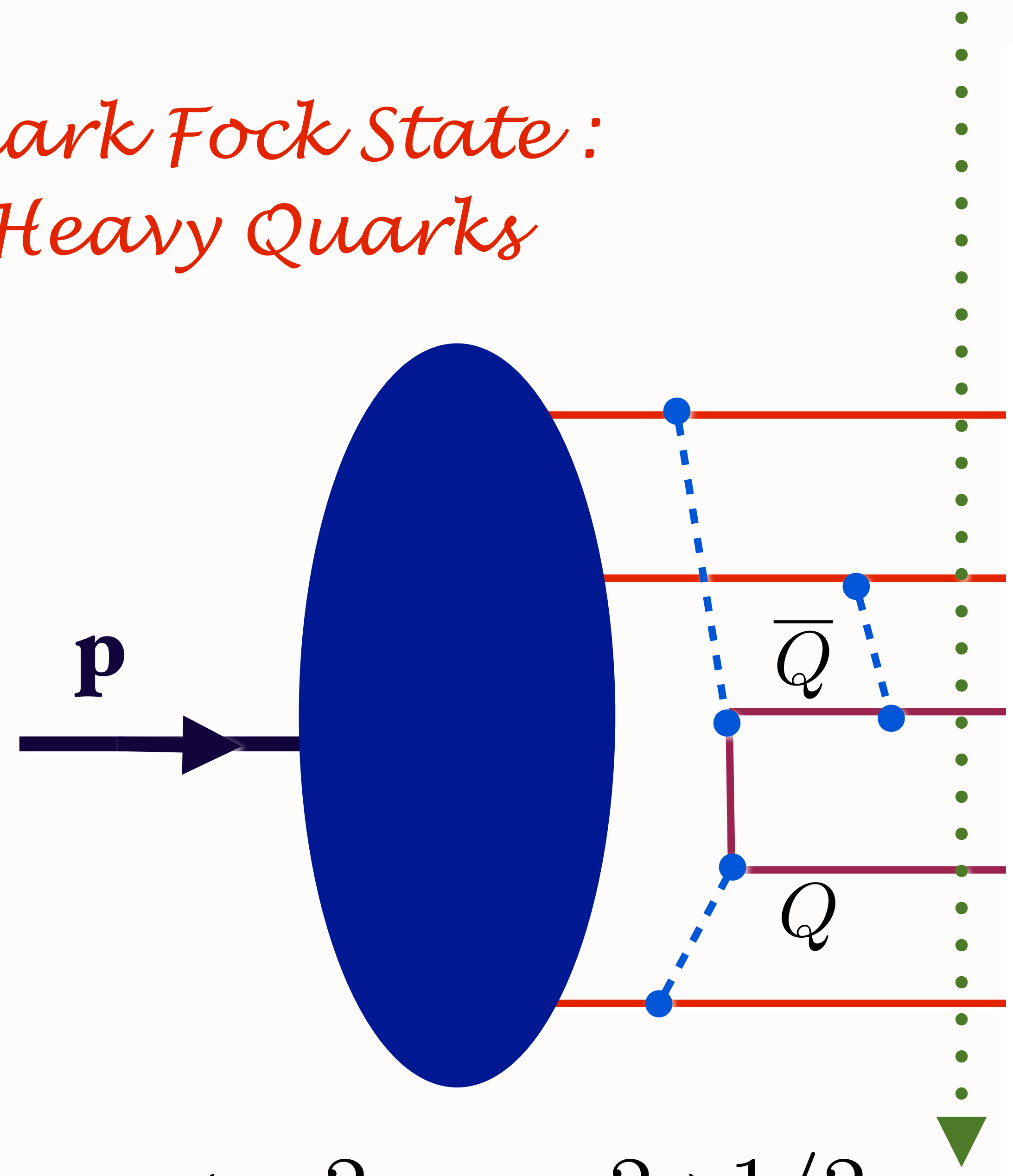
# Intrinsic Heavy-Quark Fock States




- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$ $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$ $P_{c\bar{c}/p} \simeq 1\%$
- Large Effect at high x
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

Fixed LF time

Proton 5-quark Fock State :
Intrinsic Heavy Quarks




QCD predicts Intrinsic Heavy Quarks at high x!

**Minimal off-shellness**

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

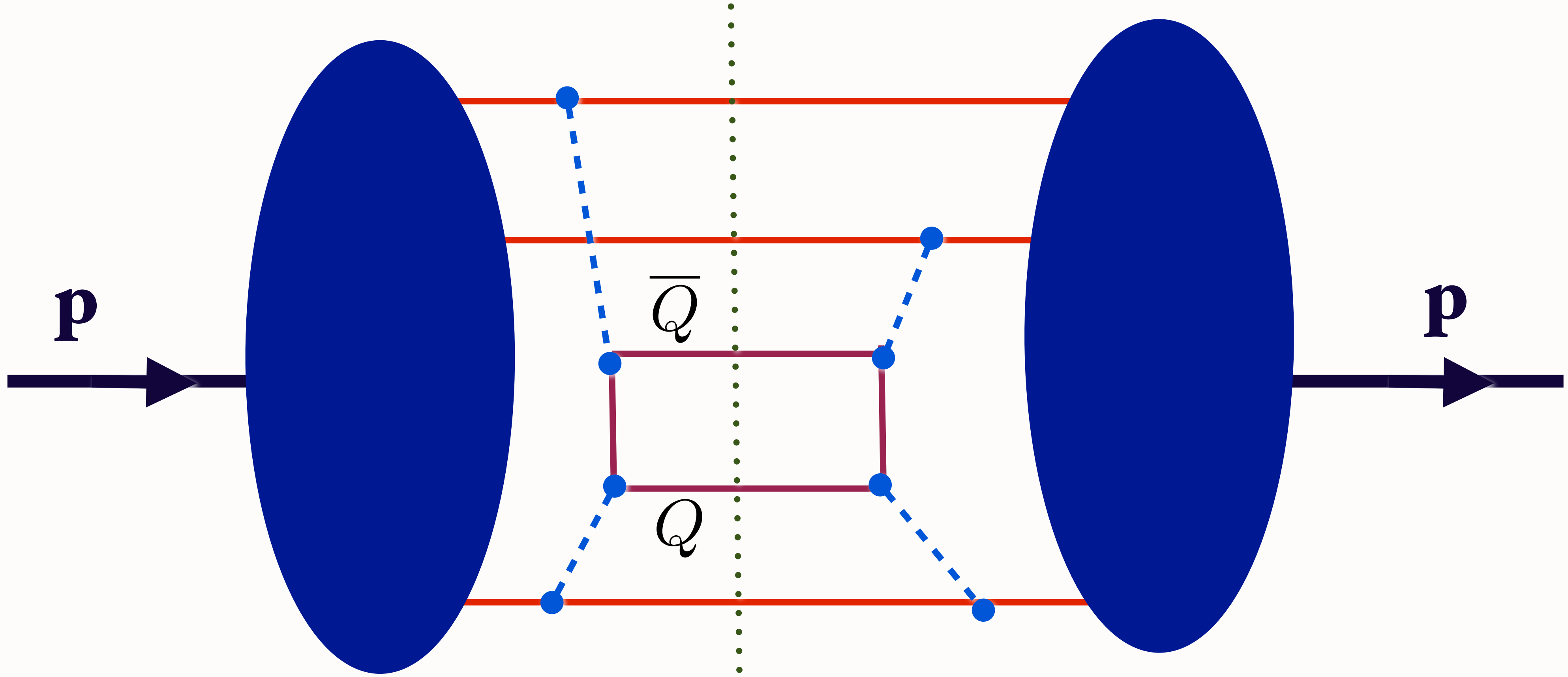
Probability (QED) $\propto \frac{1}{M_\ell^4}$ Probability (QCD) $\propto \frac{1}{M_Q^2}$

**Collins, Ellis, Gunion, Mueller, sjb**
**M. Polyakov**

Fixed LF time

Proton Self Energy
Intrinsic Heavy Quarks

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

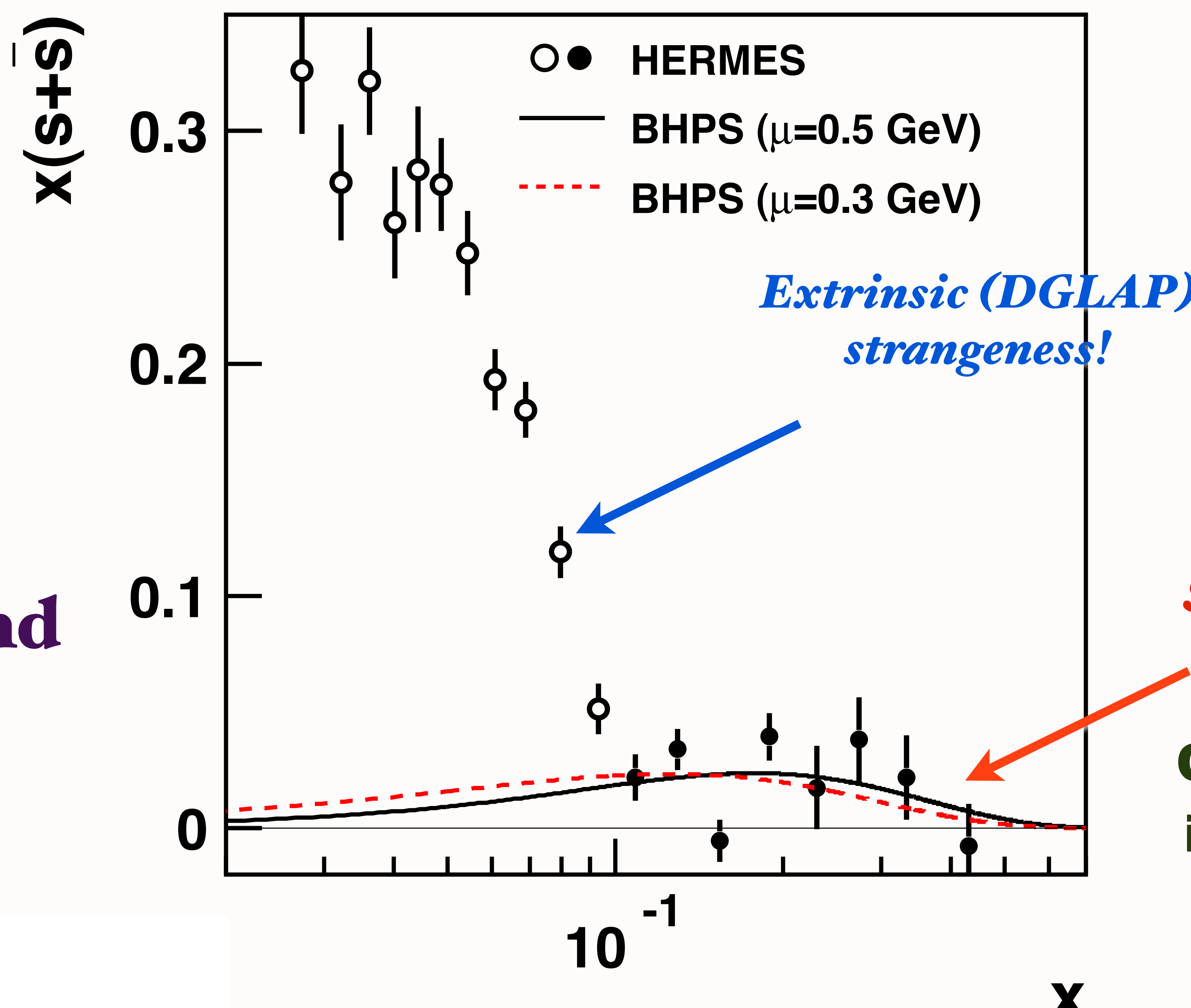



Probability (QED) $\propto \frac{1}{M_\ell^4}$

Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
M. Polyakov

# **HERMES: Two components to $s(x,Q^2)$!**




**Intrinsic strangeness!**

**W. C. Chang and J.-C. Peng**

arXiv:1105.2381

**Consistent with intrinsic charm data**

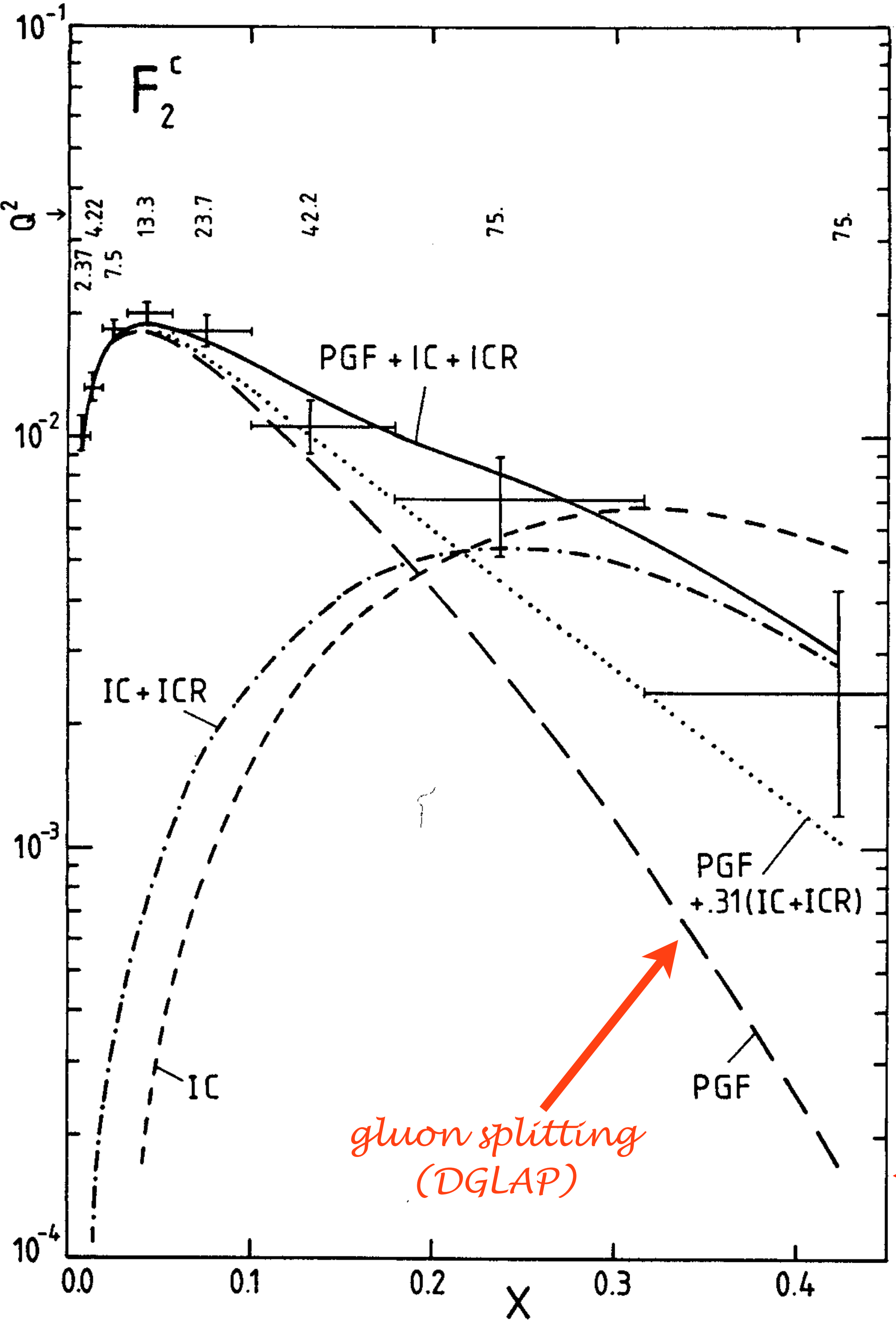
Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV$^2$ using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

QCD: $\frac{1}{M_Q^2}$ scaling

$$s(x,Q^2) = s(x,Q^2)_{\rm extrinsic} + s(x,Q^2)_{\rm intrinsic}$$

# Measurement of Charm Structure Function

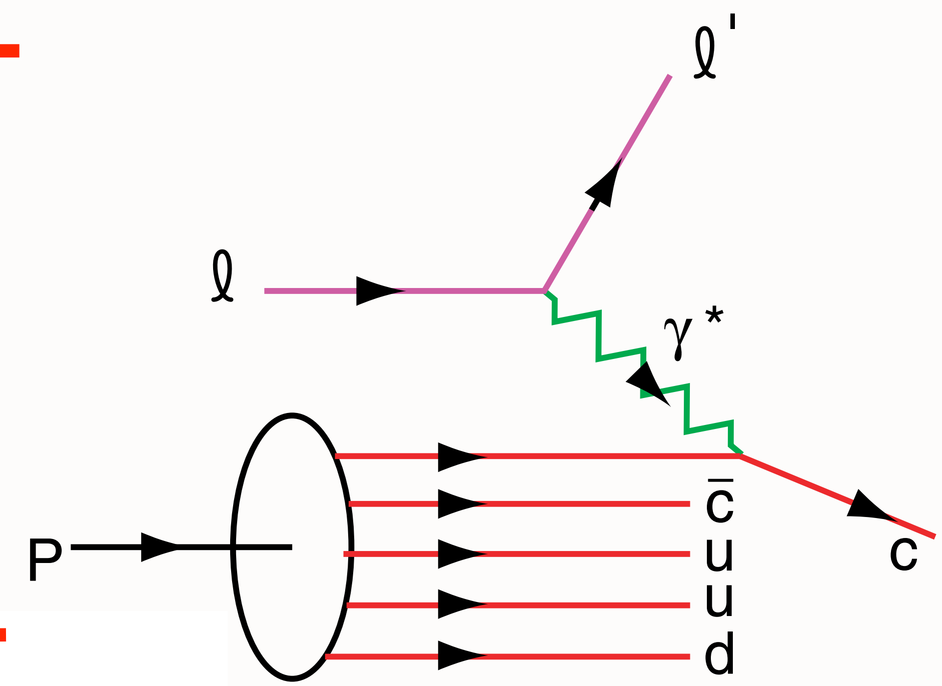
J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).




## First Evidence for Intrinsic Charm

**Hoyer, Peterson, Sakai, sjb**

*factor of 30 !*




***DGLAP / Photon-Gluon Fusion: factor of 30 too small***

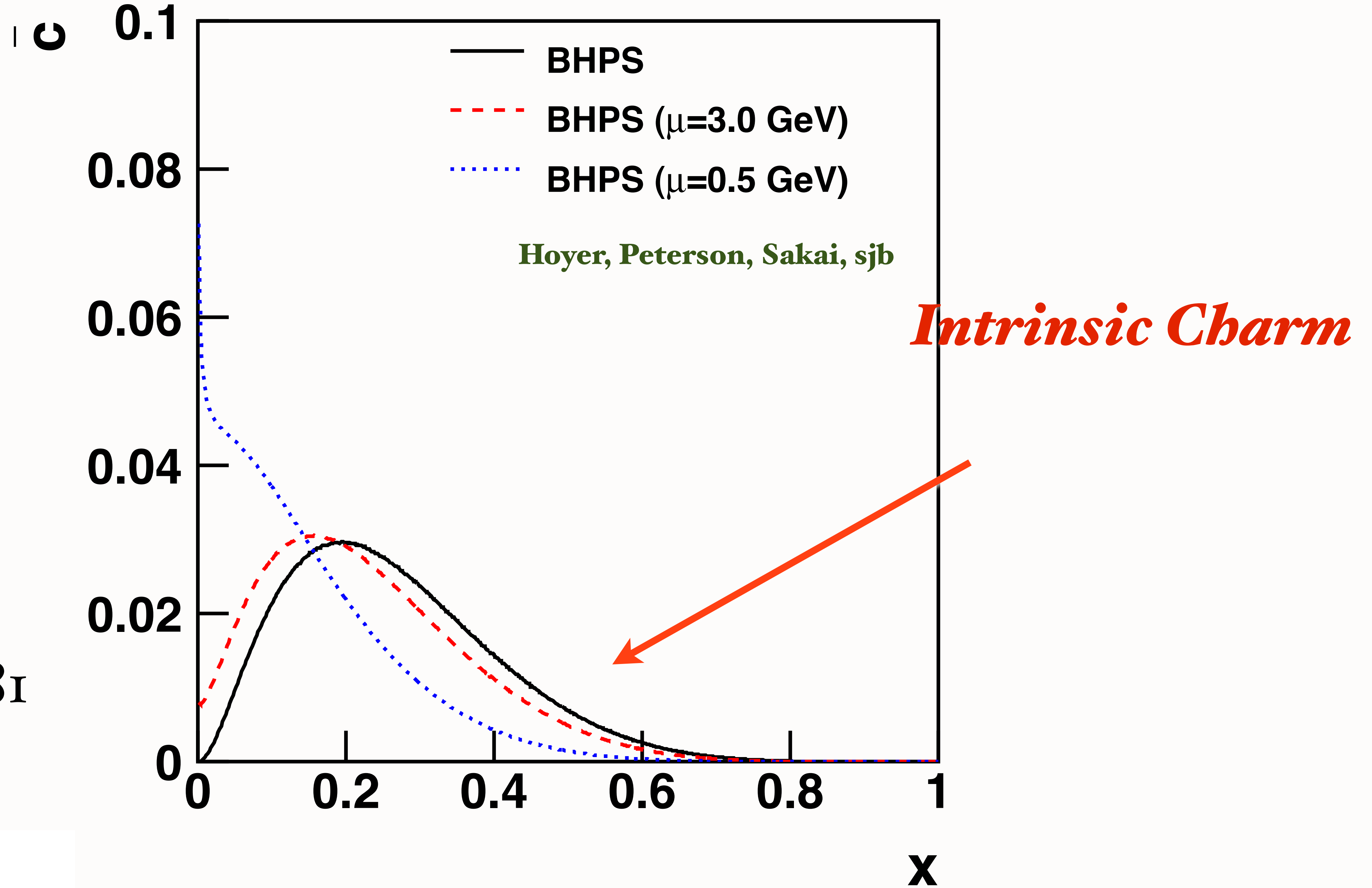
*Two Components (separate evolution):*

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

# QCD ($1/m_Q^2$) scaling: predict IC !

W. C. Chang and
J.-C. Peng

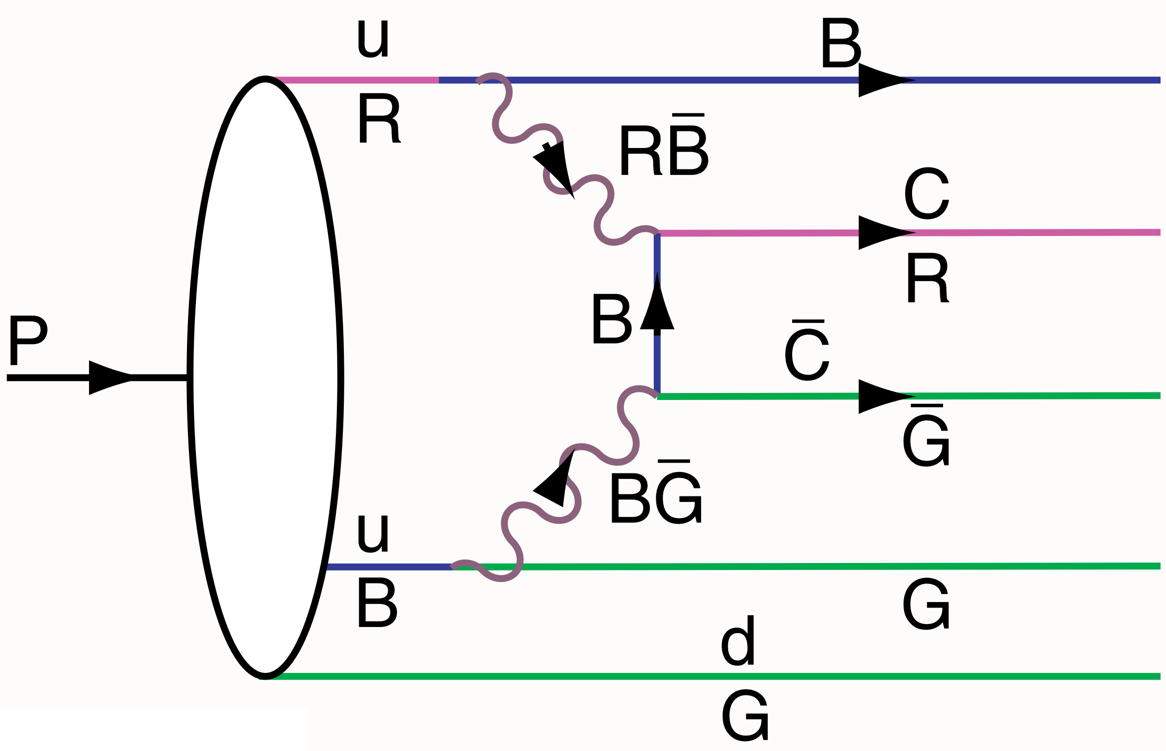
arXiv:1105.2381




Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV$^2$ using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

***Consistent with EMC***

Hoyer, Peterson, Sakai, sjb




$|uudcc>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positronium

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

$< p|\frac{G^3_{\mu\nu}}{m^2_Q}|p >$ vs. $< p|\frac{F^4_{\mu\nu}}{m^4_\ell}|p >$

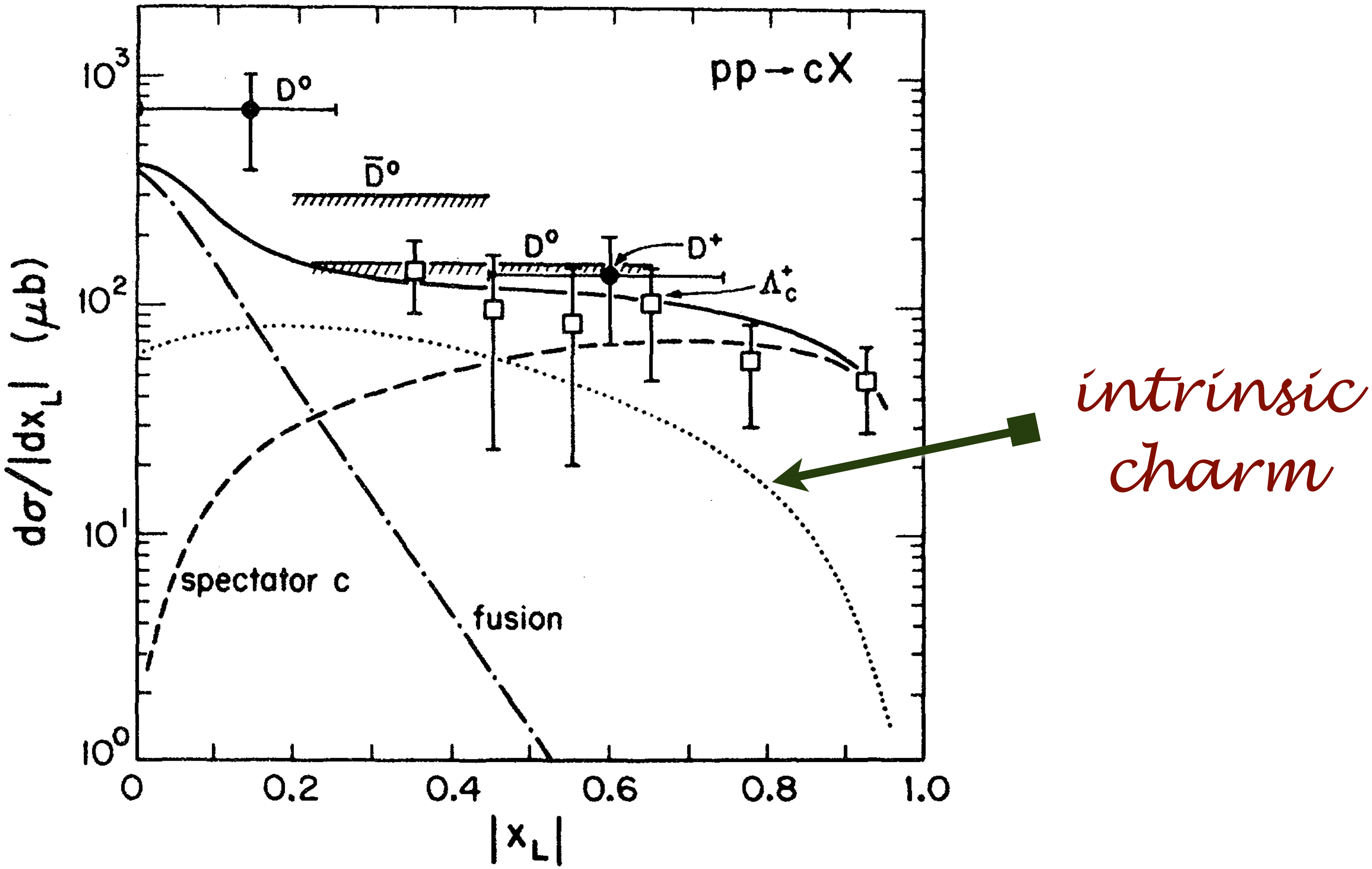
$cc$ in Color Octet

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

$$\hat{x}_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}}$$

*High x charm!* *Charm at Threshold*

**Action Principle: Minimum KE, maximal potential**




**Barger, Halzen, Keung**

*Evidence for charm at large x*

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV$^2$, $x = 0.42$

- High $x_F$ $pp \to J/\psi X$

- High $x_F$ $pp \to J/\psi J/\psi X$

- High $x_F$ $pp \to \Lambda_c X$

- High $x_F$ $pp \to \Lambda_b X$

C.H. Chang, J.P. Ma, C.F. Qiao and X.G.Wu,

- High $x_F$ $pp \to \Xi(ccd) X$ (SELEX)

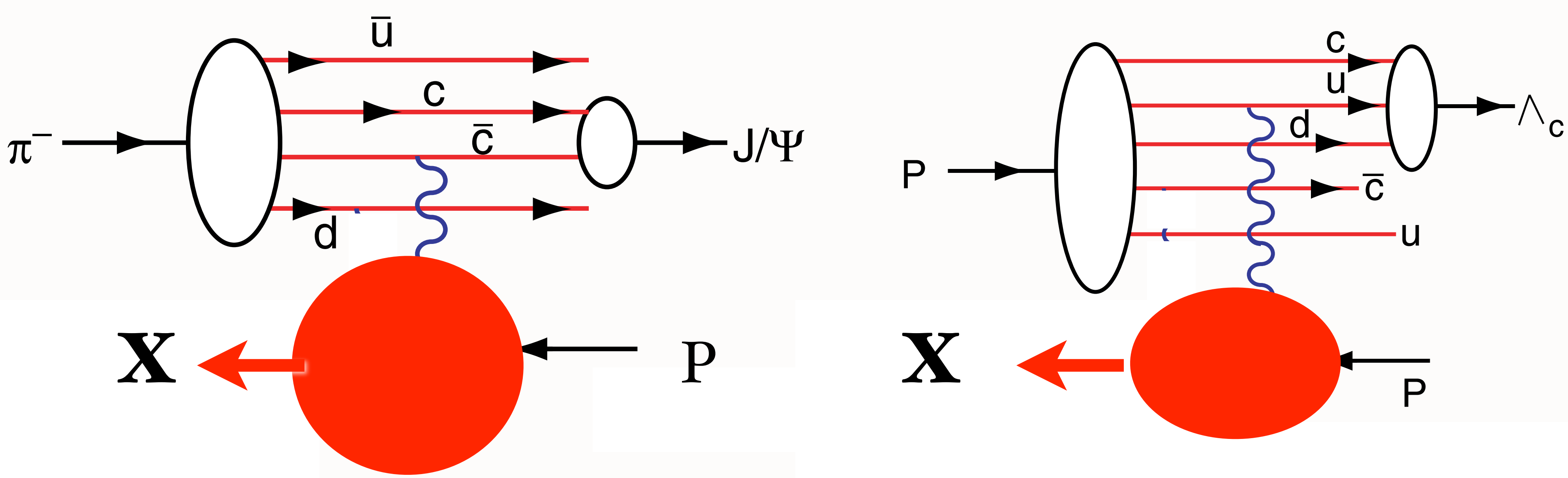
**Critical Measurements at threshold for JLab, PANDA**

**Interesting spin, charge asymmetry, threshold, spectator effects**

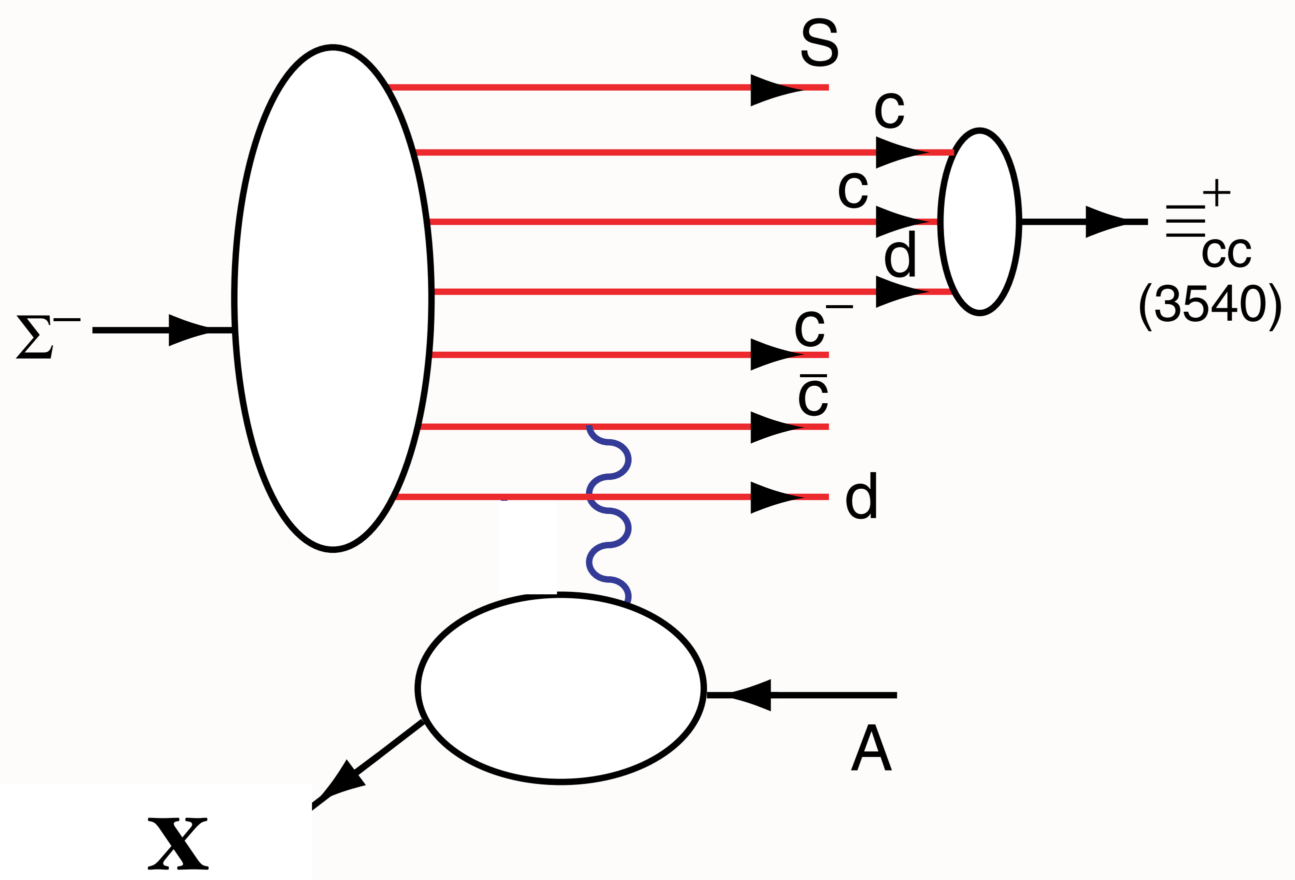
*Important corrections to B decays; Quarkonium decays*

Gardner, Karliner, sjb

# Leading Hadron Production from Intrinsic Charm




Coalescence of Comoving Charm and Valence Quarks
Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$

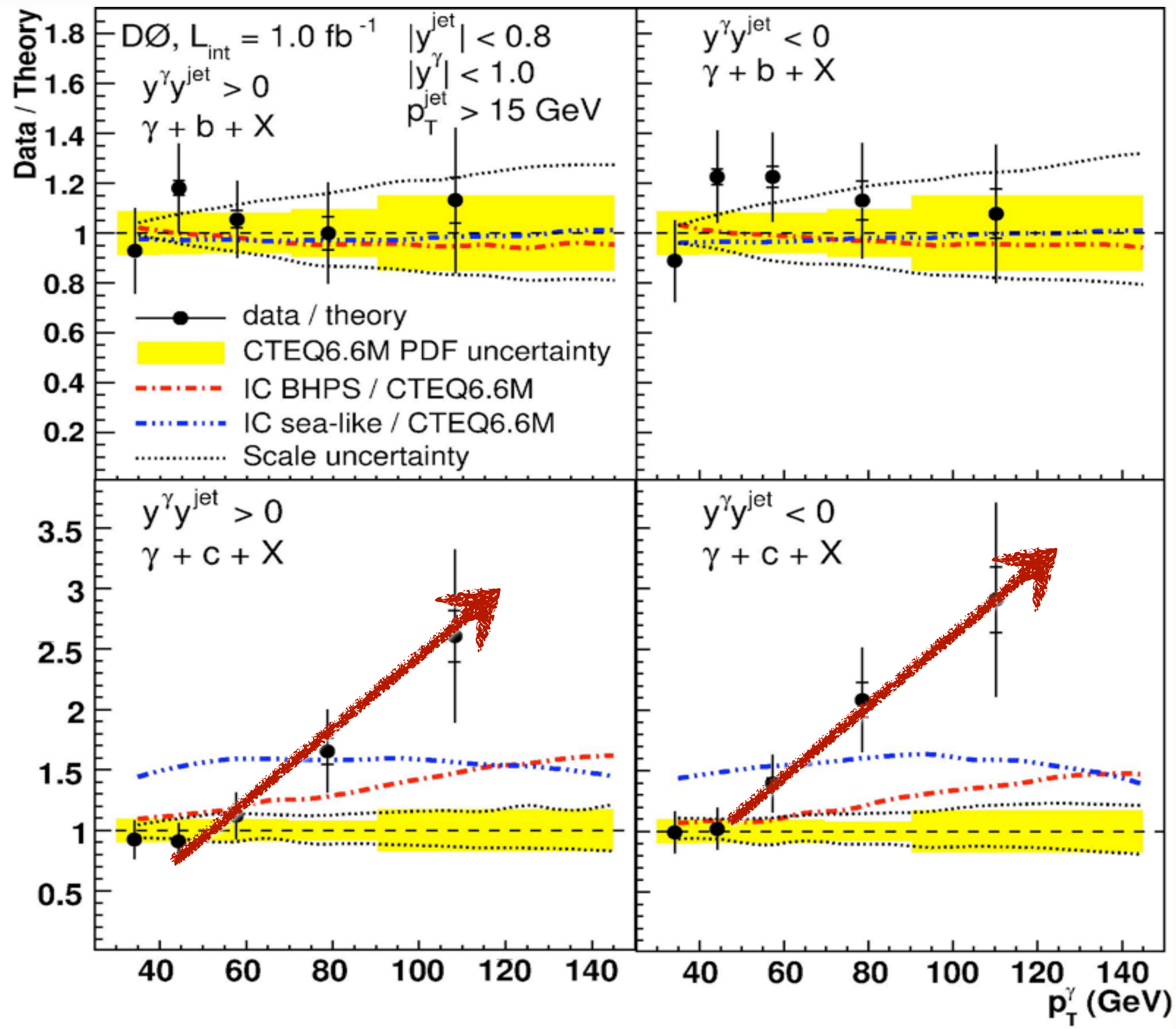



*Production of a Double-Charm Baryon*

**SELEX high $x_F$** $<x_F>= 0.33$

PRL 102, 192002 (2009) PHYSICAL REVIEW LETTERS week ending 15 MAY 2009

**Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV**



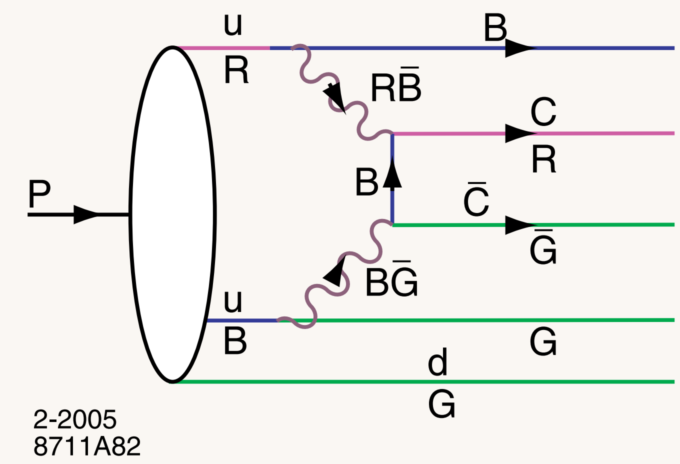

$$\frac{\Delta\sigma(\bar{p}p \to \gamma c X)}{\Delta\sigma(\bar{p}p \to \gamma b X)}$$

**Ratio insensitive to gluon PDF, scales**

**Signal for significant IC at x > 0.1 ?**

Hoyer, Peterson, Sakai, sjb
M. Polyakov, et. al

# Intrinsic Heavy-Quark Fock States

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- Large Effect at high x
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- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests (Gardener, Karliner, ..)

## *Do heavy quarks exist in the proton at high x?*

***Conventional wisdom: impossible!***

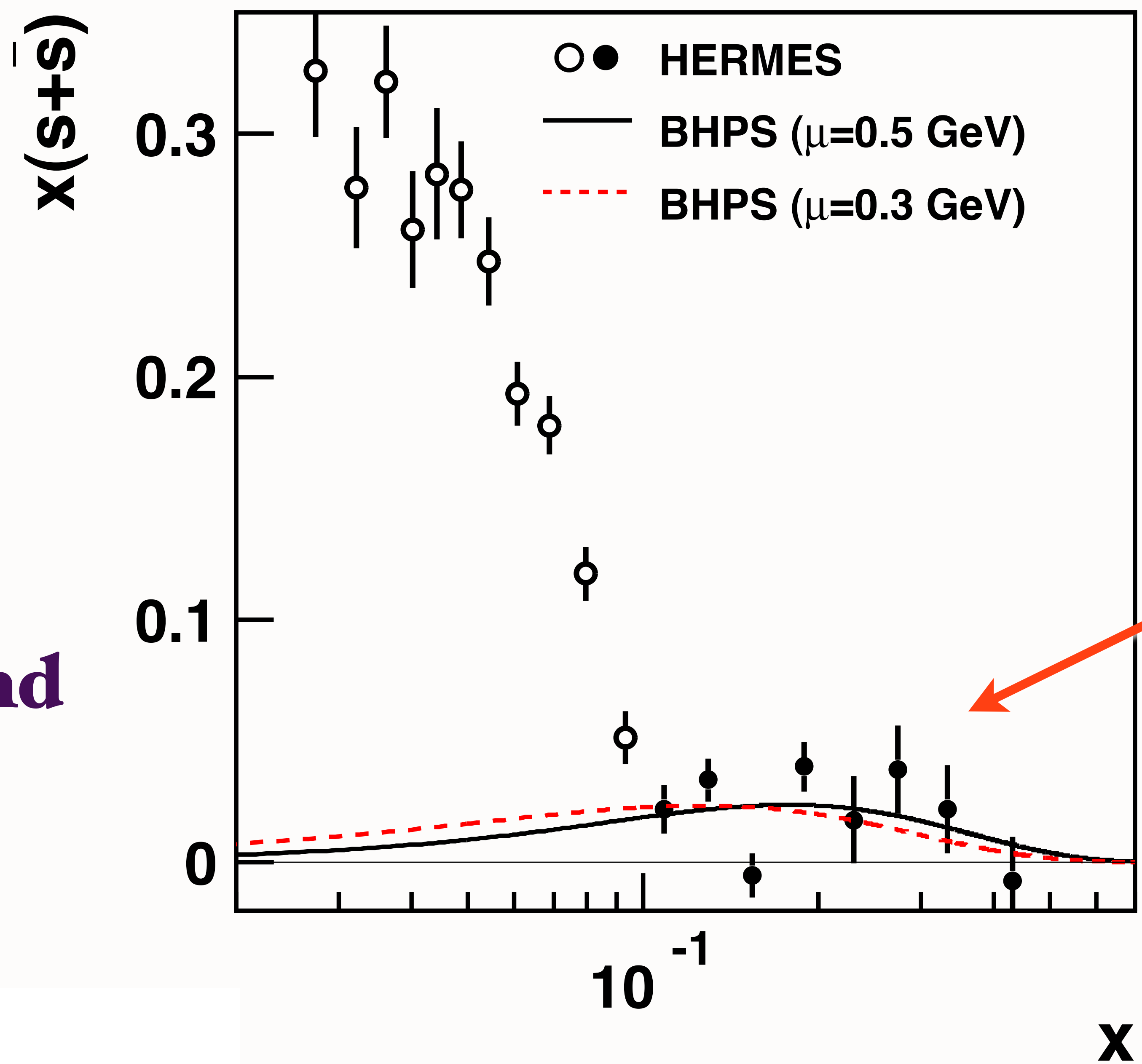
***Heavy quarks generated only at low x via DGLAP evolution from gluon splitting***

$$s(x, \mu_F^2) = c(x, \mu_F^2) = b(x, \mu_F^2) \equiv 0$$

$$\text{at starting scale } \mu_F^2$$

*Conventional wisdom is wrong even in QED!*

# HERMES: Two components to s(x,Q²)!




***Intrinsic strangeness!***

**W. C. Chang and J.-C. Peng**

arXiv:1105.2381

Comparison of the HERMES $x(s(x) + \bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV$^2$ using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x, Q^2) = s(x, Q^2)_{\text{extrinsic}} + s(x, Q^2)_{\text{intrinsic}}$$

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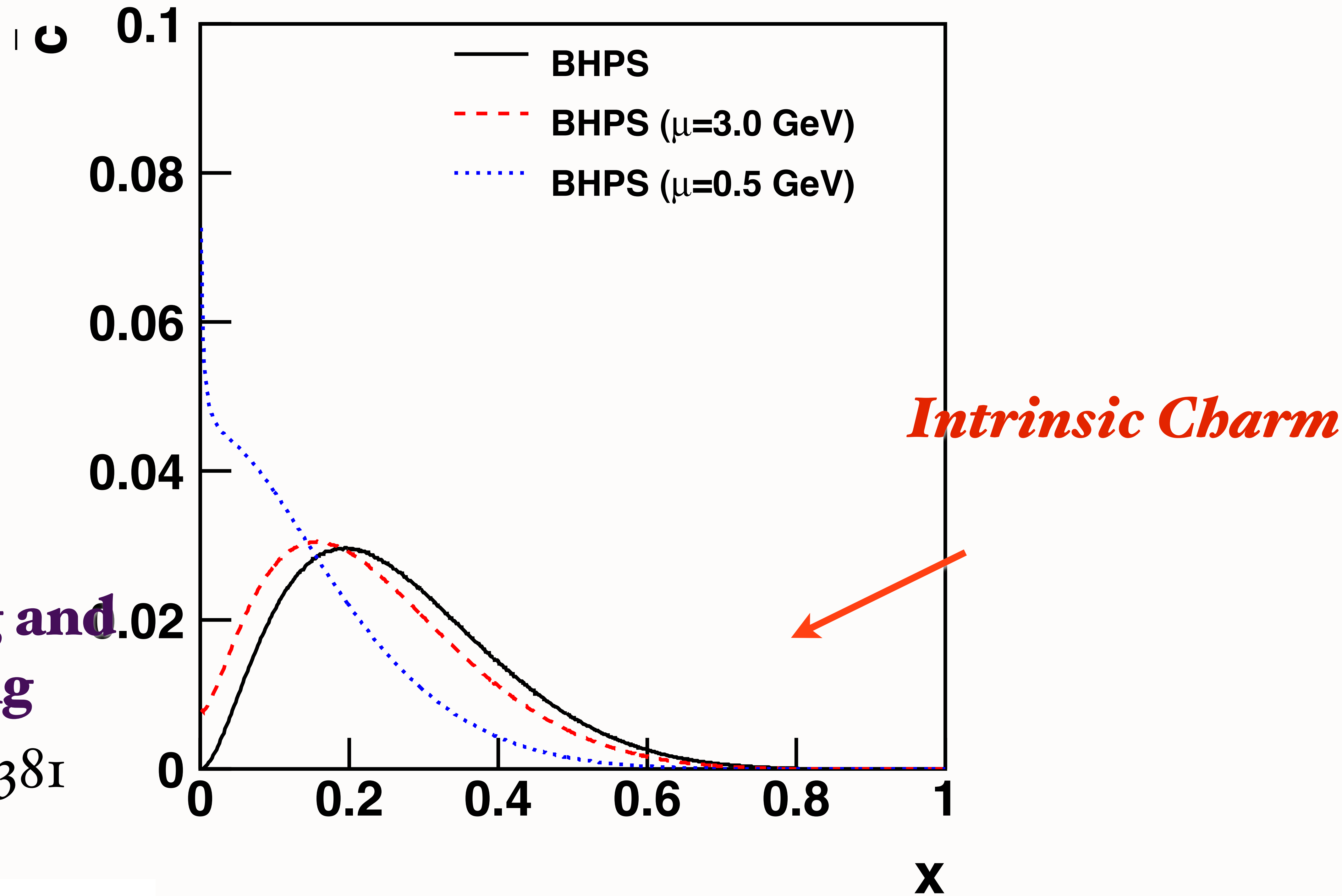
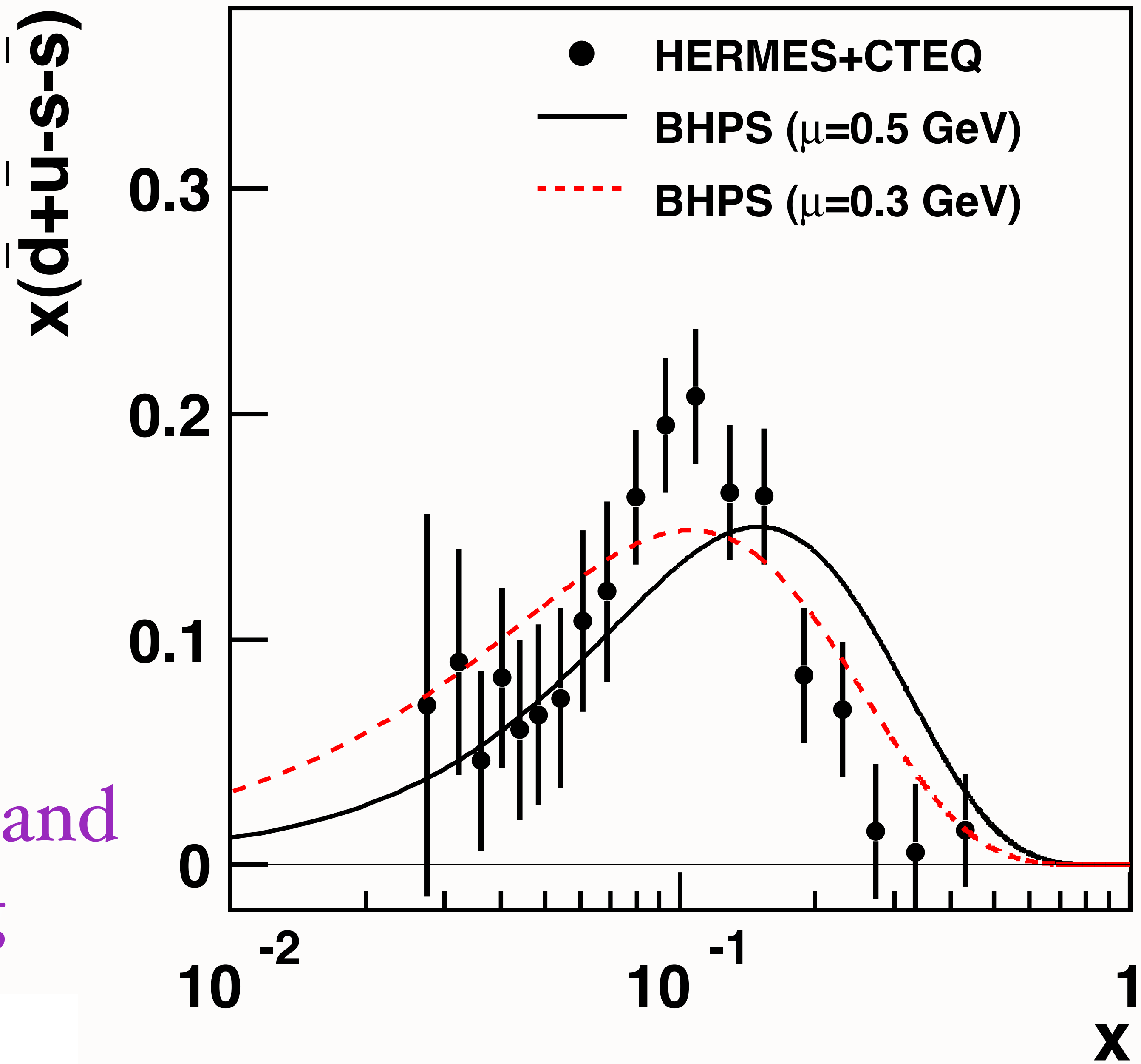



Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV$^2$ using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.




W. C. Chang and J.-C. Peng

Comparison of the $x(\bar{d}(x)+\bar{u}(x)-s(x)-\bar{s}(x))$ data with the calculations based on the BHPS model. The values of $x(s(x)+\bar{s}(x))$ are from the HERMES experiment [6], and those of $x(\bar{d}(x)+\bar{u}(x))$ are obtained from the PDF set CTEQ6.6 [11]. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV$^2$ using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalization of the calculations are adjusted to fit the data.

**W. C. Chang and J.-C. Peng**

arXiv:1105.2381

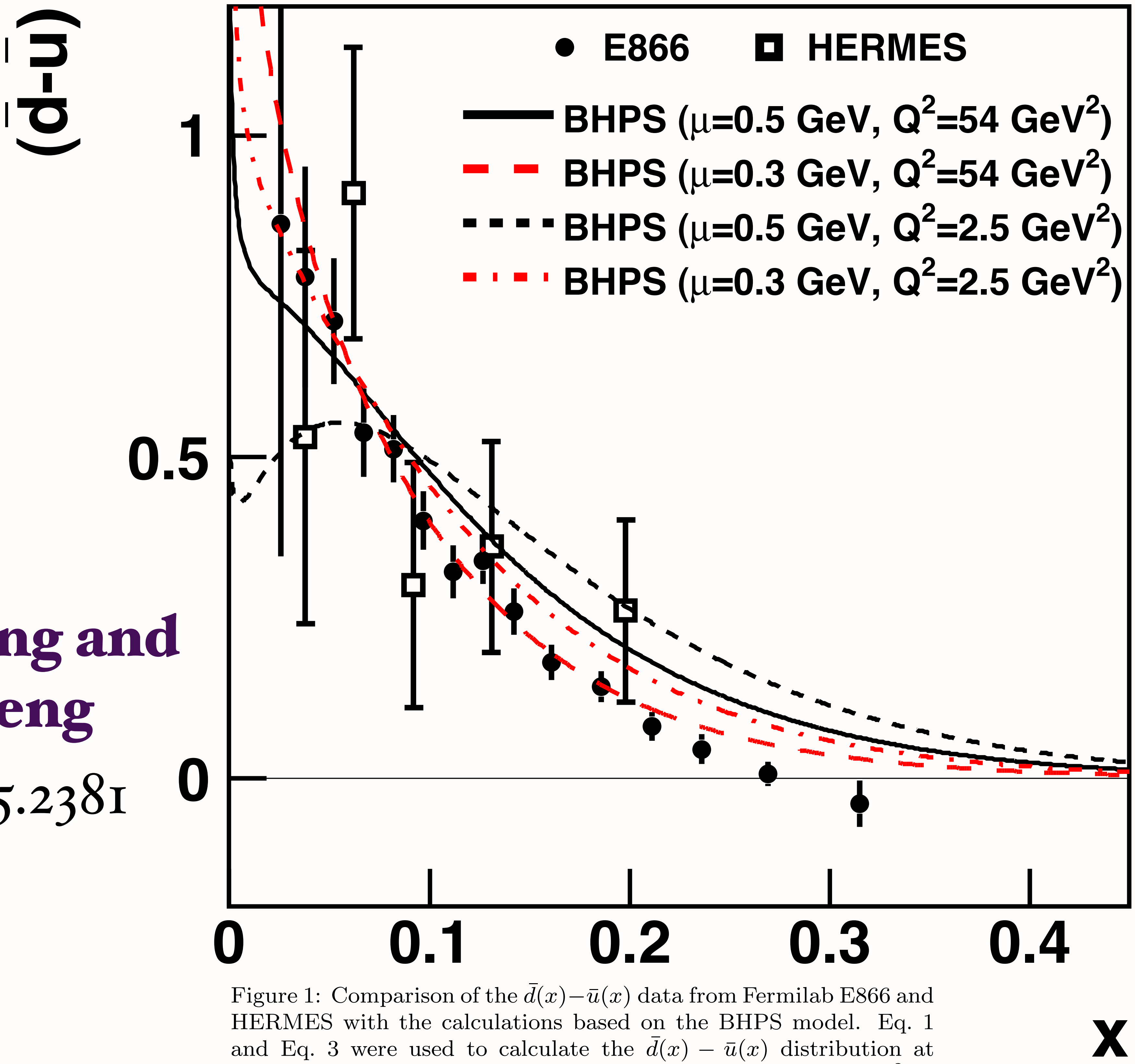
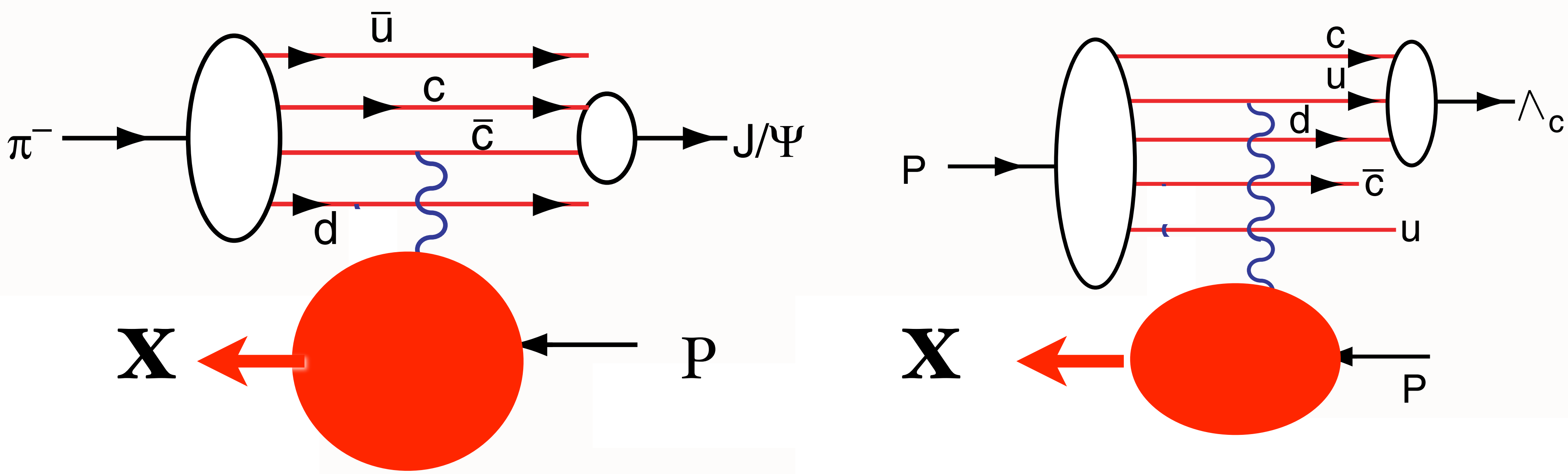



Figure 1: Comparison of the $\bar{d}(x) - \bar{u}(x)$ data from Fermilab E866 and HERMES with the calculations based on the BHPS model. Eq. 1 and Eq. 3 were used to calculate the $\bar{d}(x) - \bar{u}(x)$ distribution at the initial scale. The distribution was then evolved to the $Q^2$ of the experiments and shown as various curves. Two different initial scales, $\mu = 0.5$ and 0.3 GeV, were used for the E866 calculations in order to illustrate the dependence on the choice of the initial scale.

# Leading Hadron Production from Intrinsic Charm




Coalescence of Comoving Charm and Valence Quarks Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$

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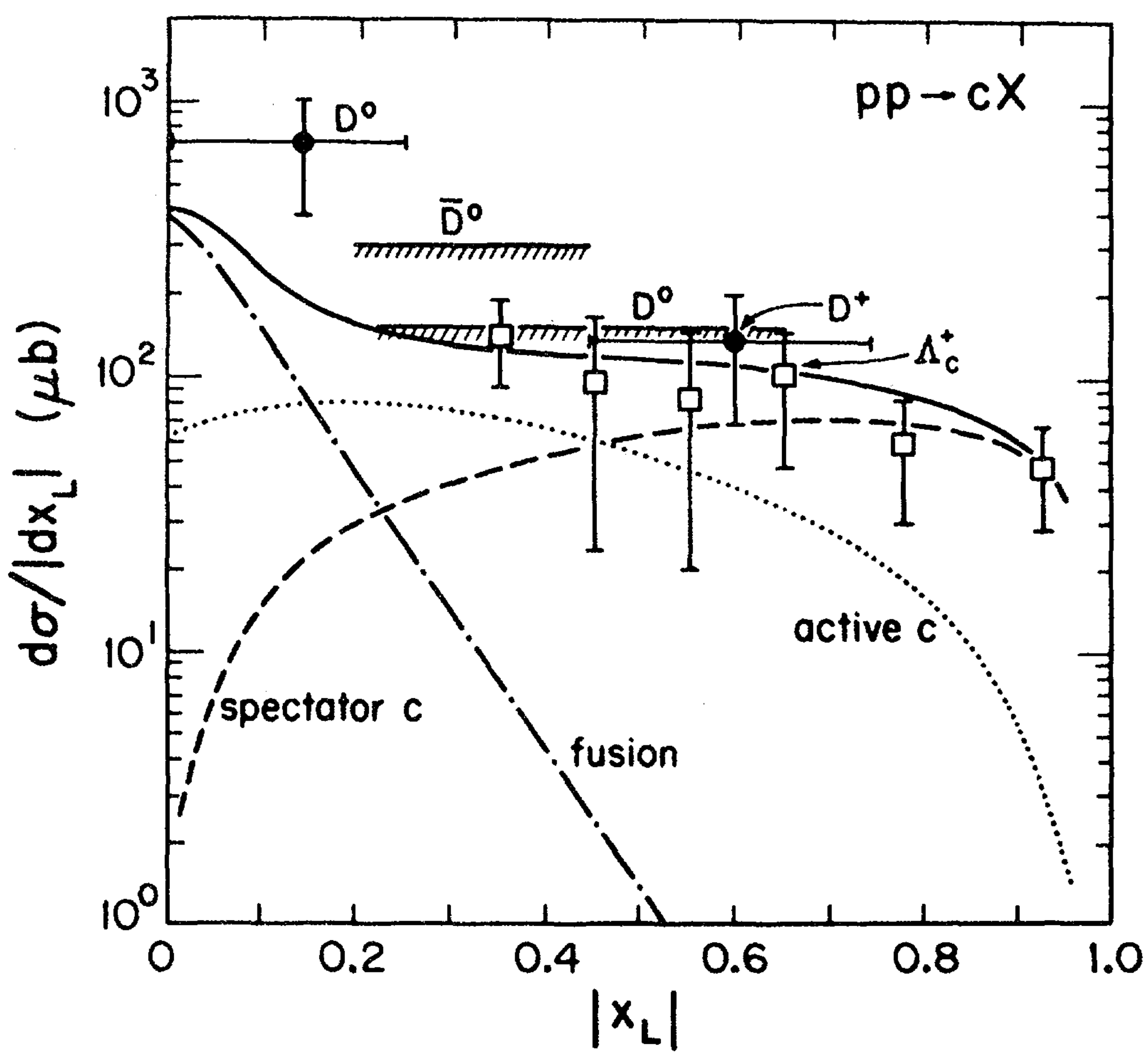
- High $x_F$ $pp \rightarrow \Lambda_c X$

- High $x_F$ $pp \rightarrow \Lambda_b X$

- High $x_F$ $pp \rightarrow \Xi(ccd) X$ (SELEX)

**IC Structure Function: Critical Measurement for EIC**

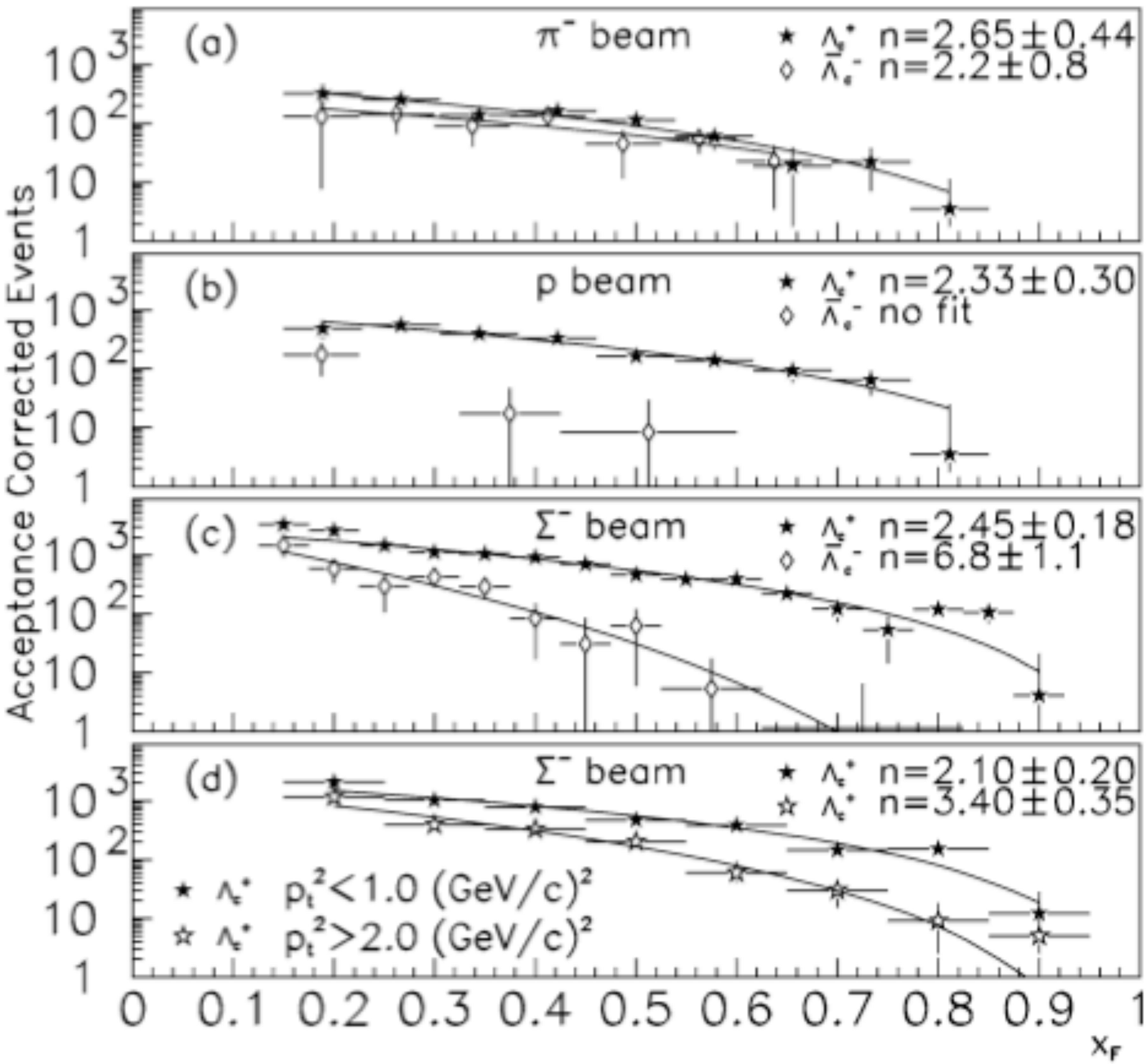
**Many interesting spin, charge asymmetry, spectator effects**



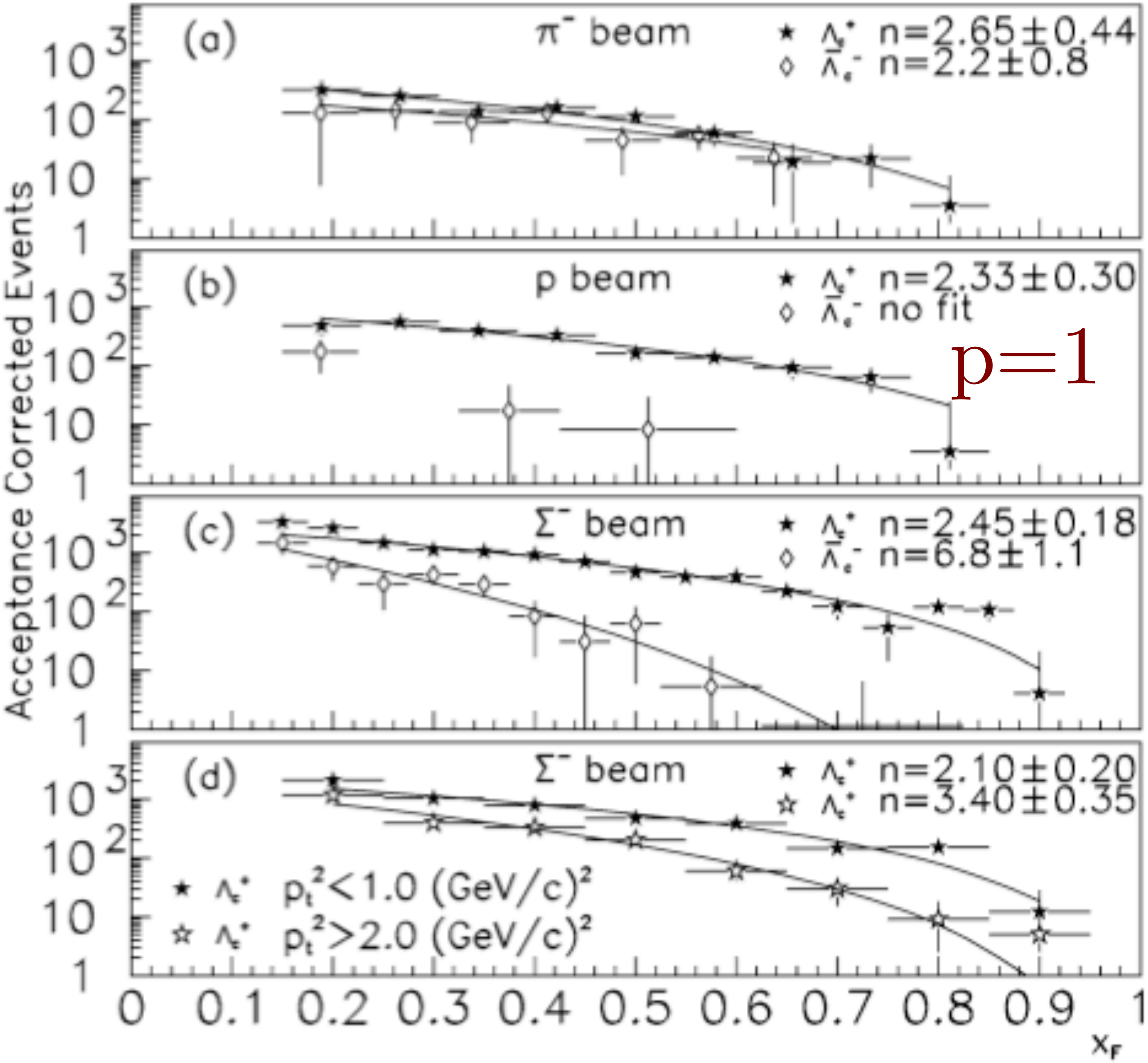

*Model similar to Intrinsic Charm*

V. D. Barger, F. Halzen and W. Y. Keung,
"The Central And Diffractive Components Of Charm Production,"
Phys. Rev. D 25, 112 (1982).

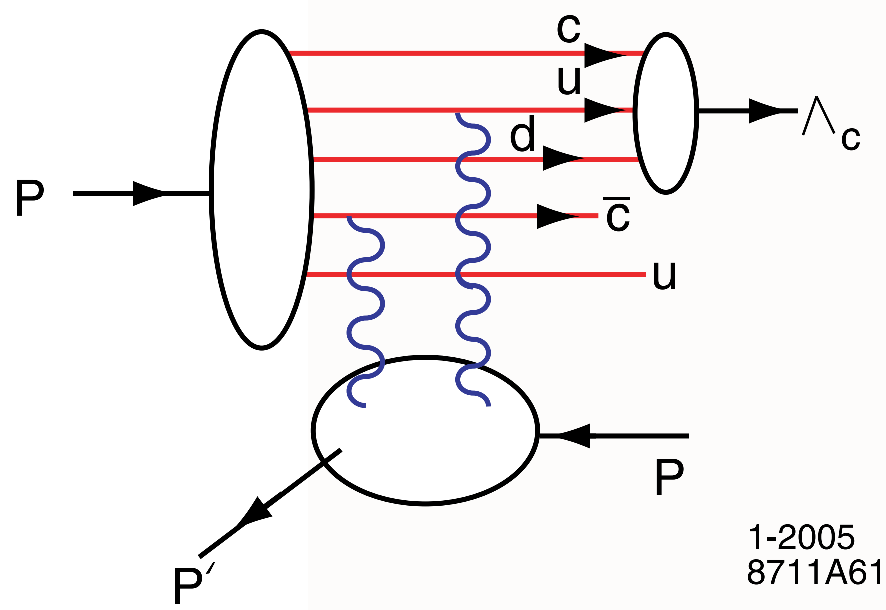
SELEX Collaboration / Physics Letters B 528 (2002) 49–57




***Large xF production close to the maximum allowed by phase space!***




p=1



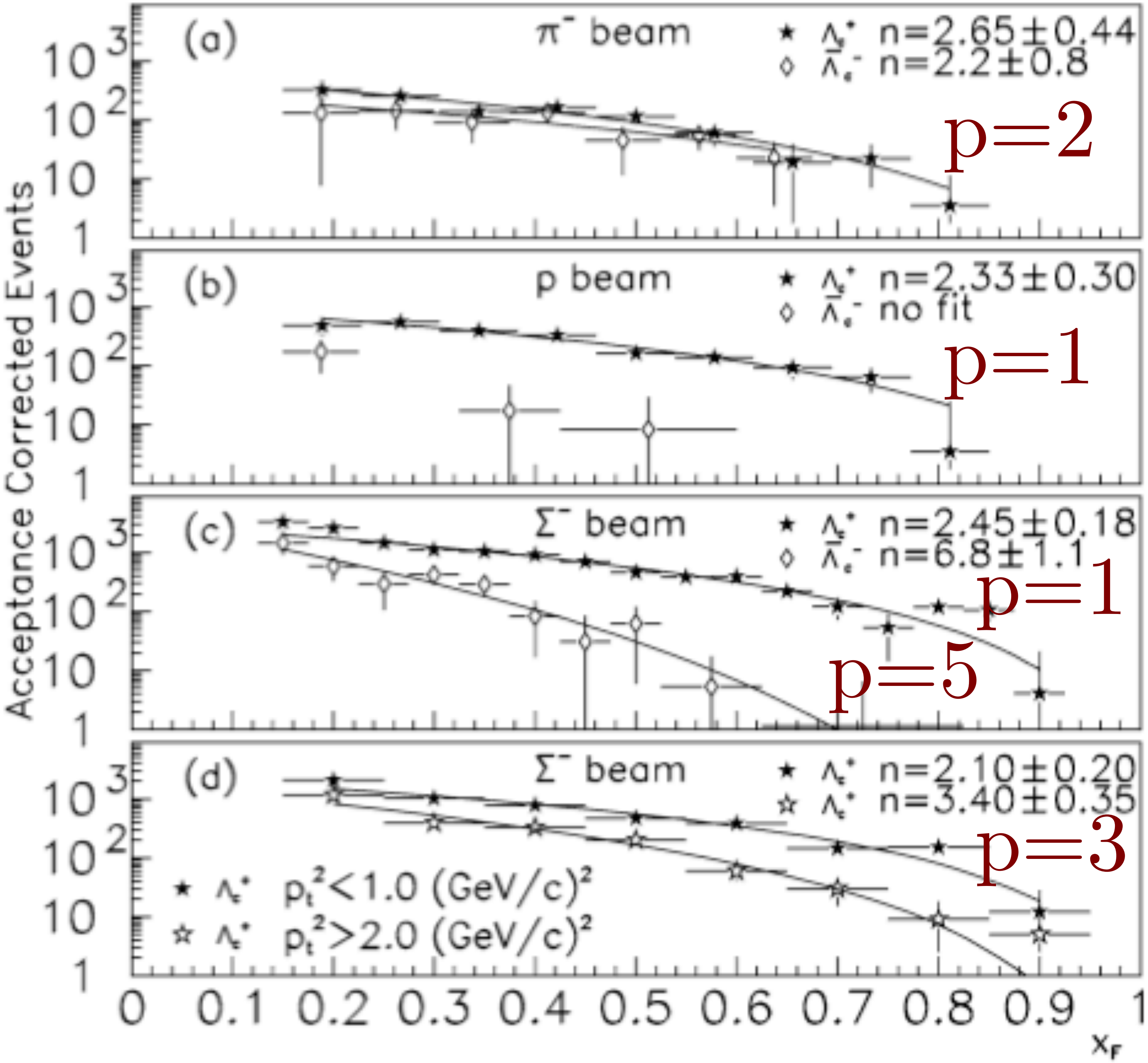

$p(uudc\bar{c})$

$\rightarrow \Lambda_c(cud)$

$n_s = 2$

**Phase space gives minimum power p**

$(1 - x_F)^p, p = n_s - 1$




$$(1-x_F)^p, p = n_s - 1$$

$\pi^-(d\bar{u}) \rightarrow \Lambda_c(cud)$, $\bar{\Lambda}_c(\bar{c}\bar{u}\bar{d})$

$$n_s = 2 + 1 = 3$$

p=2

$\Sigma^-(sdd) \rightarrow \Lambda_c(cud)$

$$n_s = 3 + 1 = 4$$

p=3

$p(uud)$, $\Sigma^-(sdd) \rightarrow \bar{\Lambda}_c(\bar{c}\bar{u}\bar{d})$

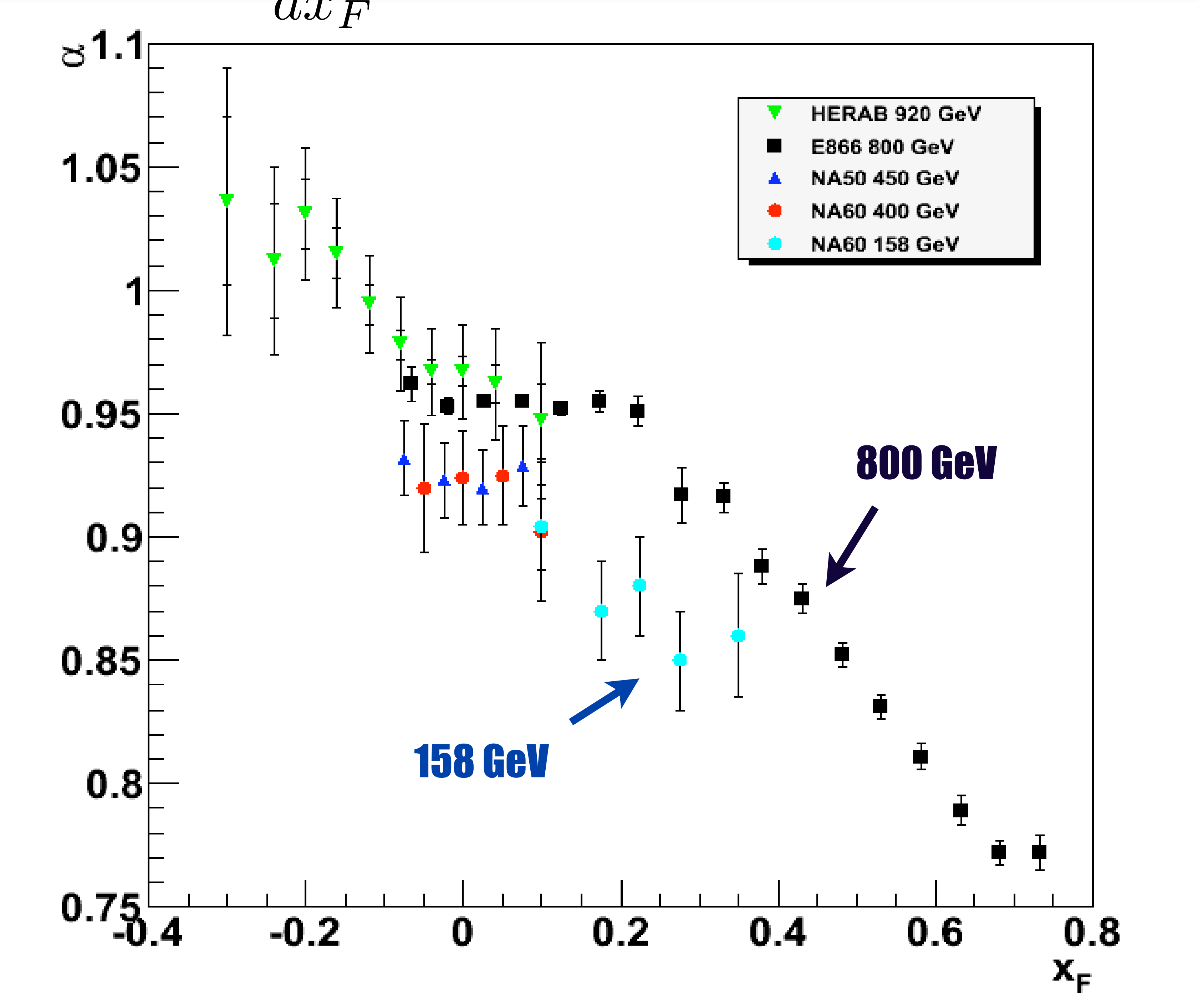
$$n_s = 4 + 2 = 6$$

p=5

**Phase space gives minimum power p**

# NA60 pA data @ 158GeV

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) \propto A^\alpha$$

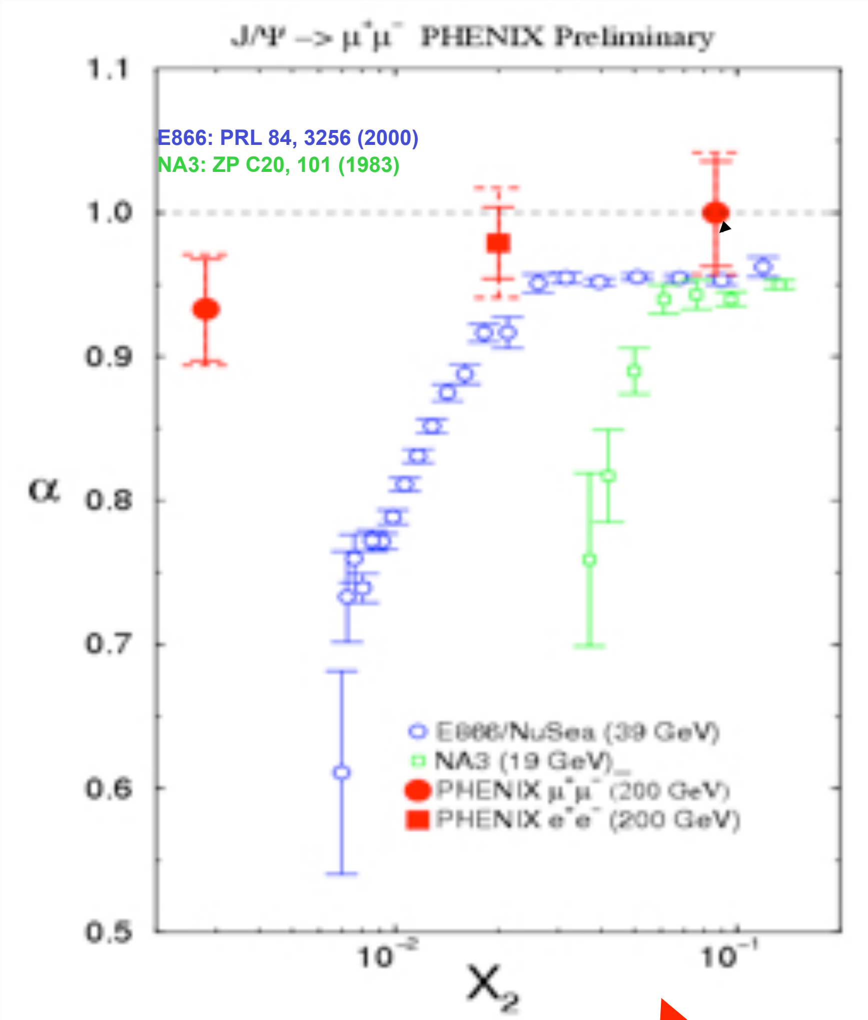



*Clear dependence on $x_F$ and beam energy*

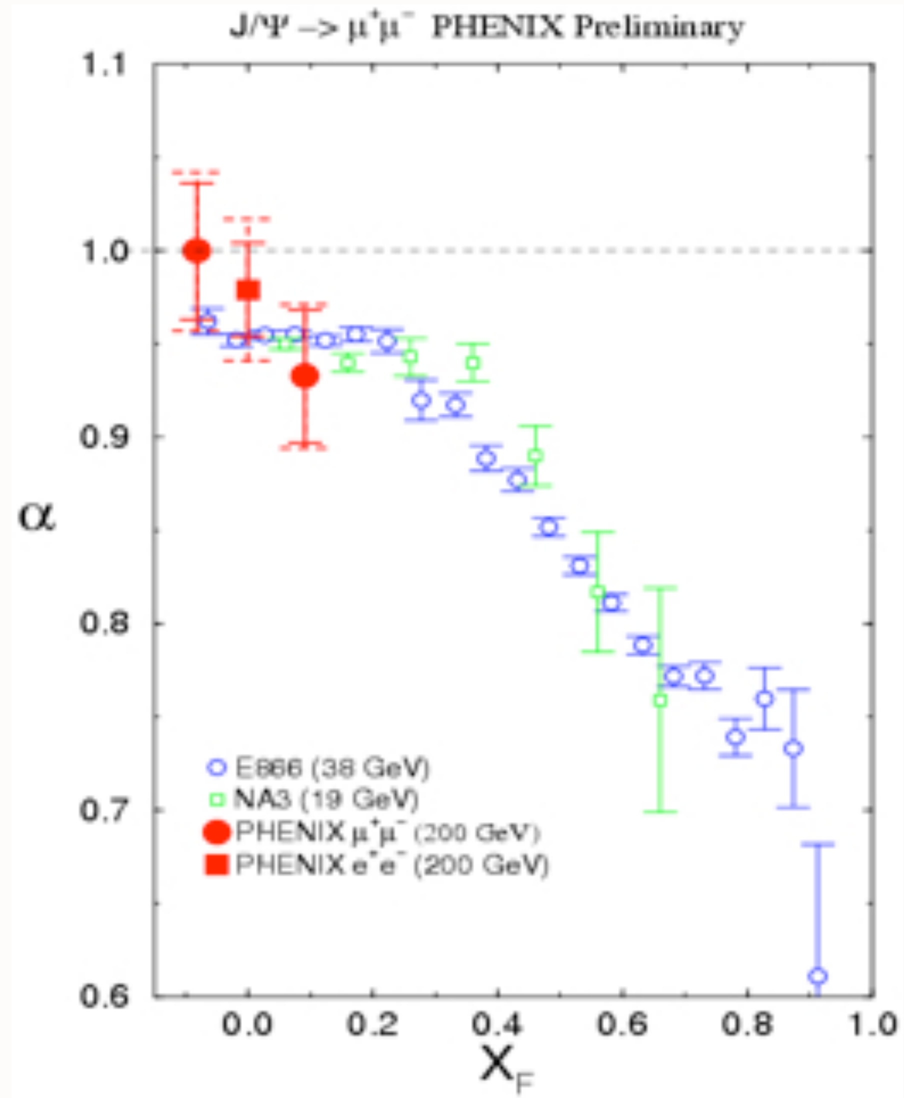
# J/ψ nuclear dependence vrs rapidity, $x_{Au}$, $x_F$

M.Leitch

PHENIX compared to lower energy measurements




Klein,Vogt, PRL 91:142301,2003
Kopeliovich, NP A696:669,2001




*Huge "absorption" effect*

*Violates PQCD factorization!*

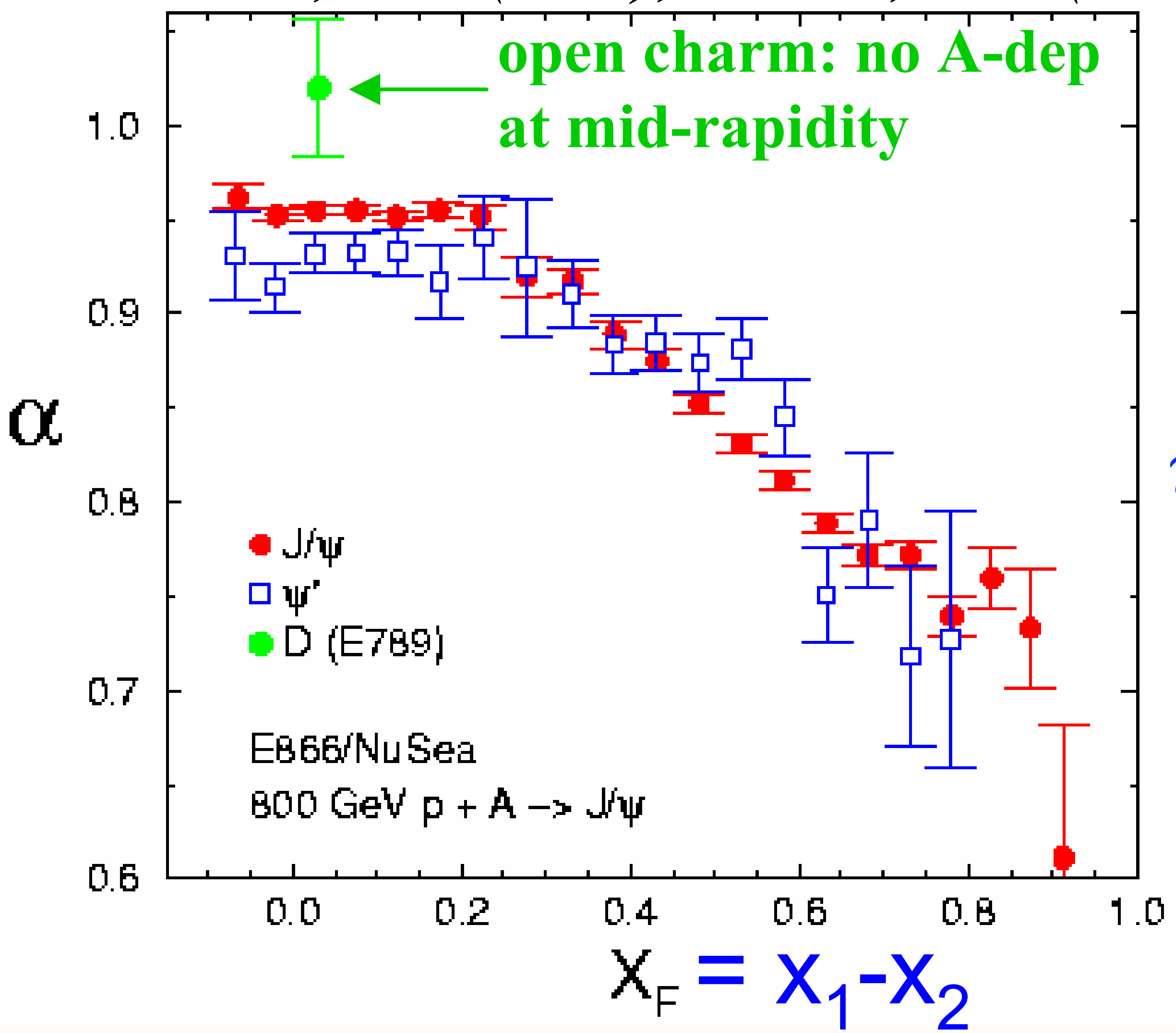
$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

Hoyer, Sukhatme, Vanttinen

**800 GeV p-A (FNAL)** $\sigma_A = \sigma_p * A^{\alpha}$

*PRL 84, 3256 (2000); PRL 72, 2542 (1994)*

M. Leitch




$$\frac{d\sigma}{dx_F}(pA \to J/\psi X)$$

*Remarkably Strong Nuclear Dependence for Fast Charmonium*

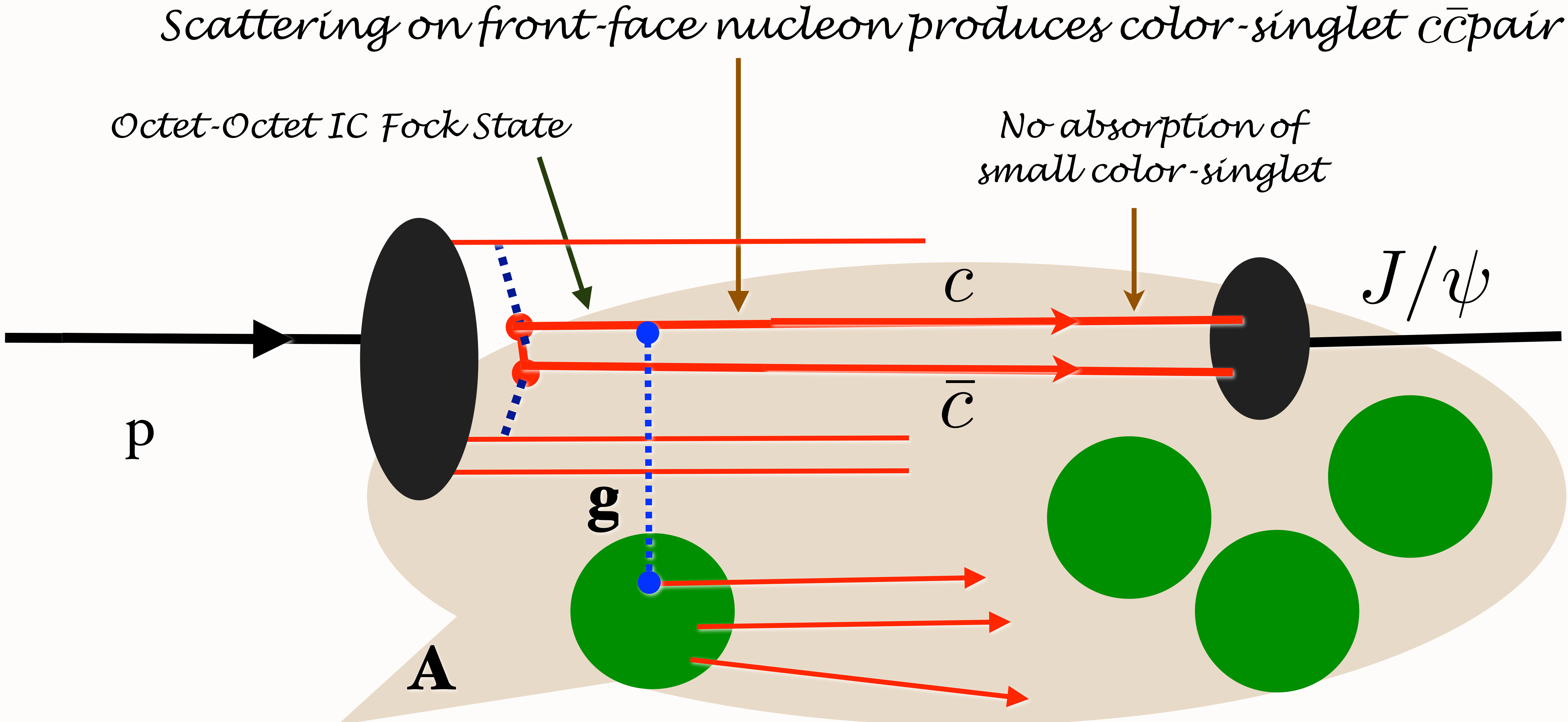
*Violation of PQCD Factorization!*

Violation of factorization in charm hadroproduction.
P. Hoyer, M. Vanttinen (Helsinki U.) , U. Sukhatme (Illinois U., Chicago) . HU-TFT-90-14, May 1990. 7pp.
Published in Phys.Lett.B246:217-220,1990

**IC Explains large excess of quarkonia at large $x_F$, A-dependence**

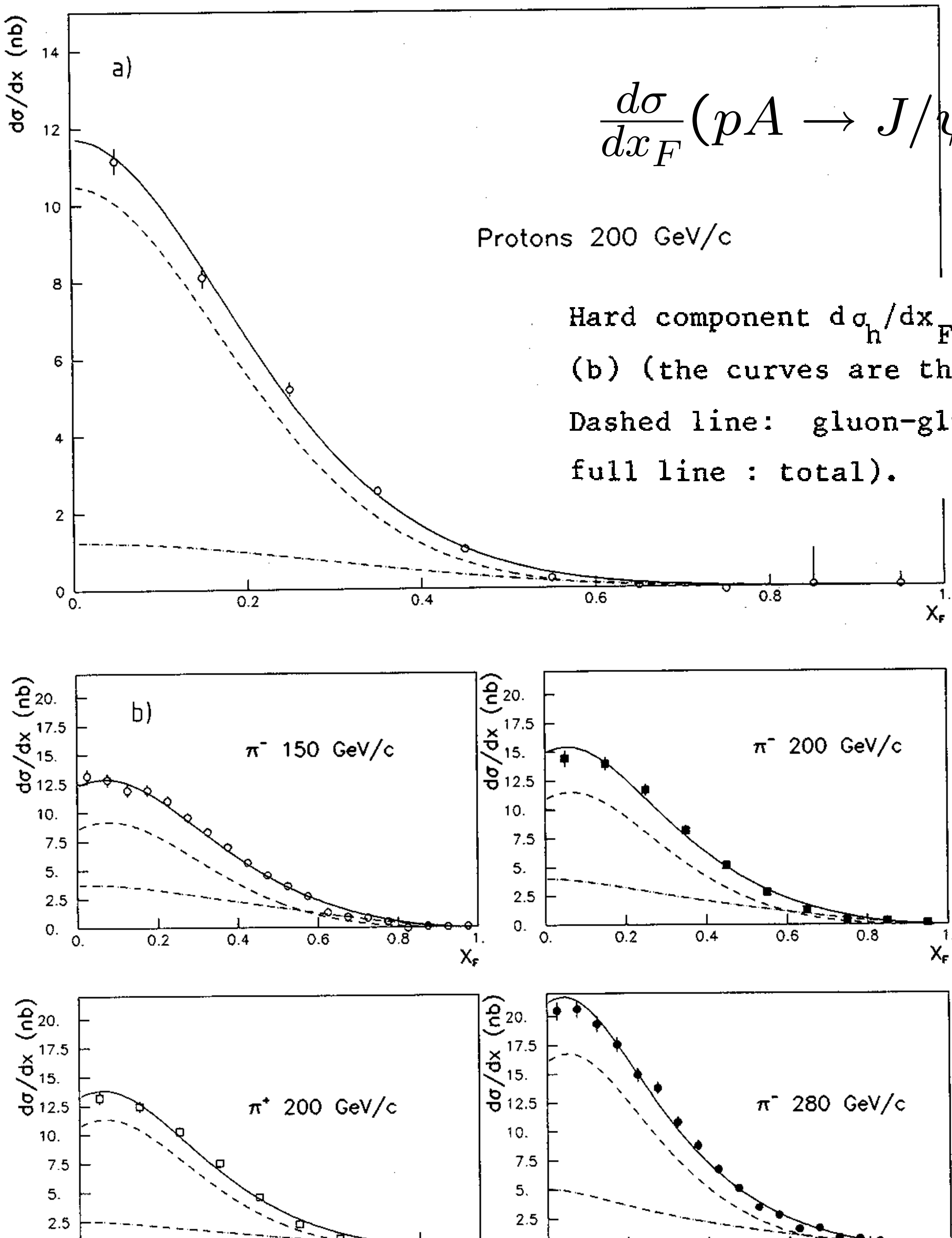
# Color-Opaque IC Fock state interacts on nuclear front surface

Kopeliovich, Schmidt, Soffer, sjb



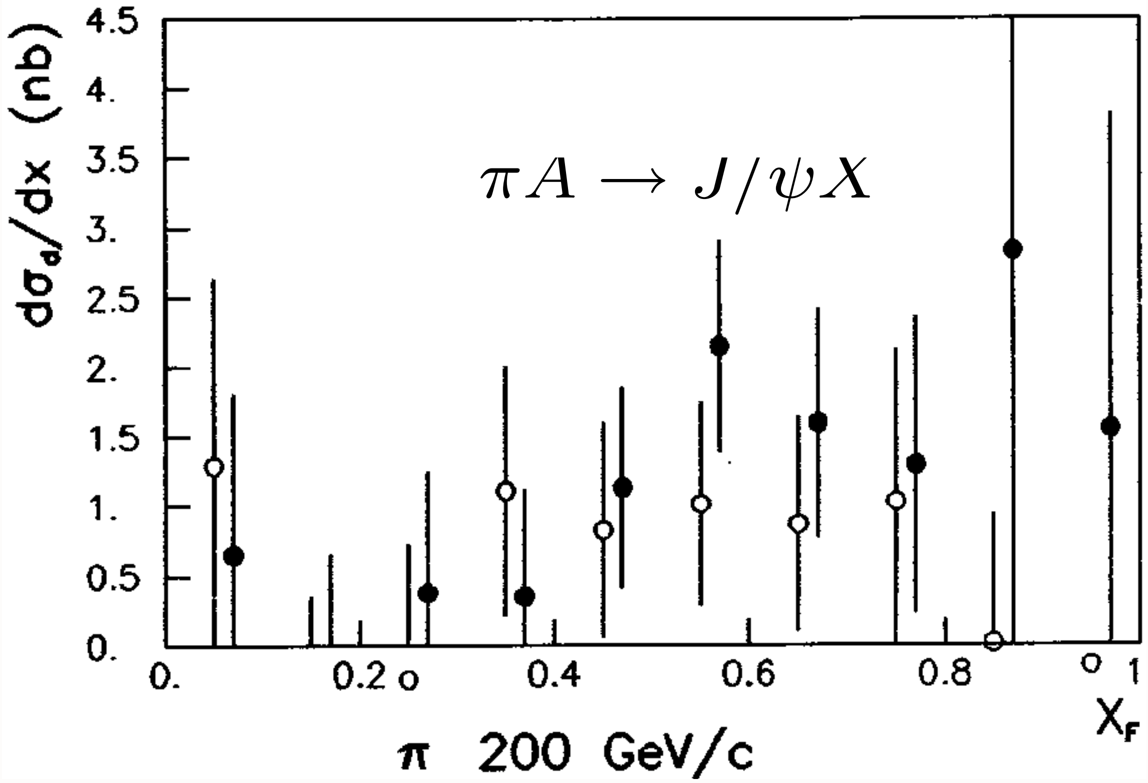

$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \to J/\psi X)$$

$$\frac{d\sigma}{dx_F}(pA \to J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$



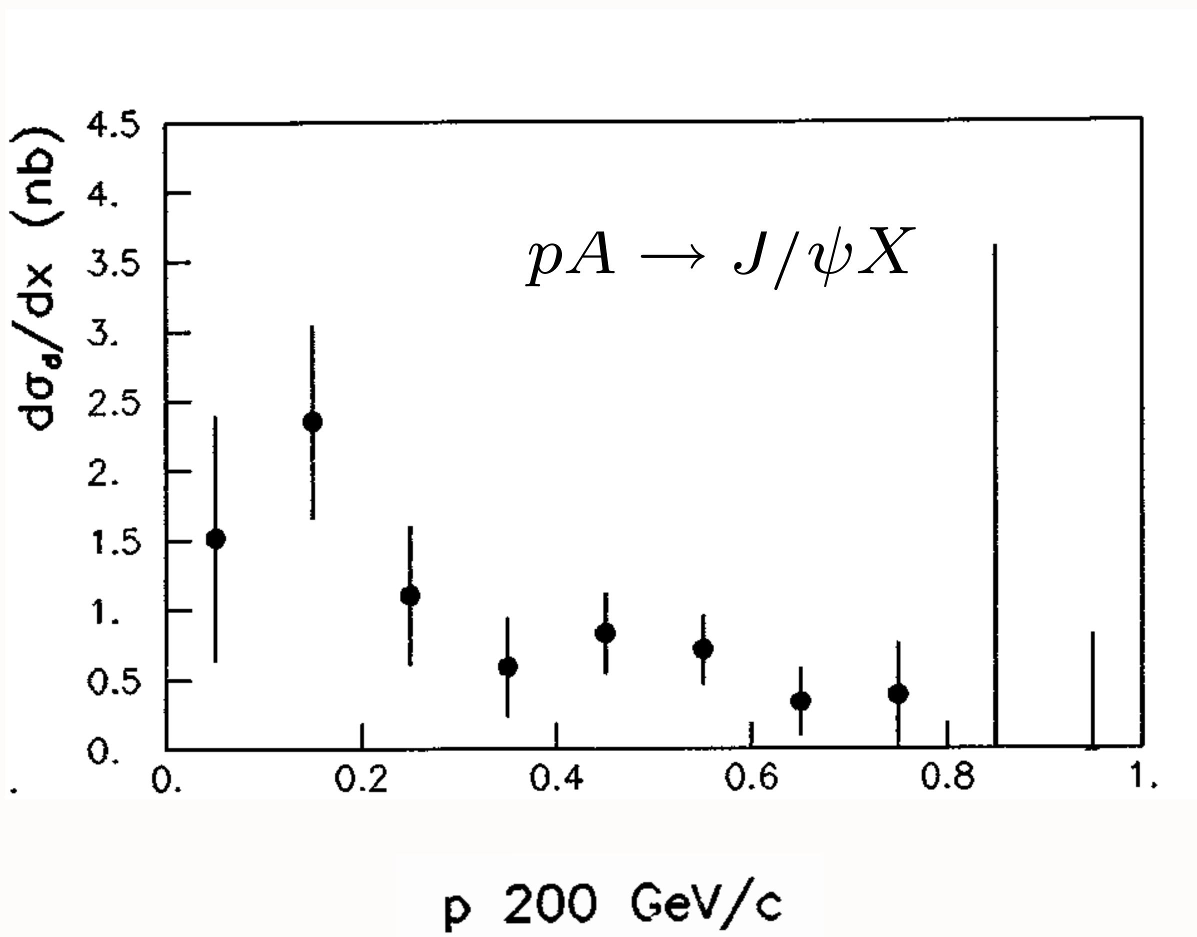

Hard component $d\sigma_h/dx_F$ for incident protons (a) and pions (b) (the curves are the result of the fit described in the text. Dashed line: gluon-gluon fusion; dash-dotted line : $q\bar{q}$ fusion; full line : total).

$A^1$ component consistent with sum of $gg$ and $\bar{q}q$ fusion




$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

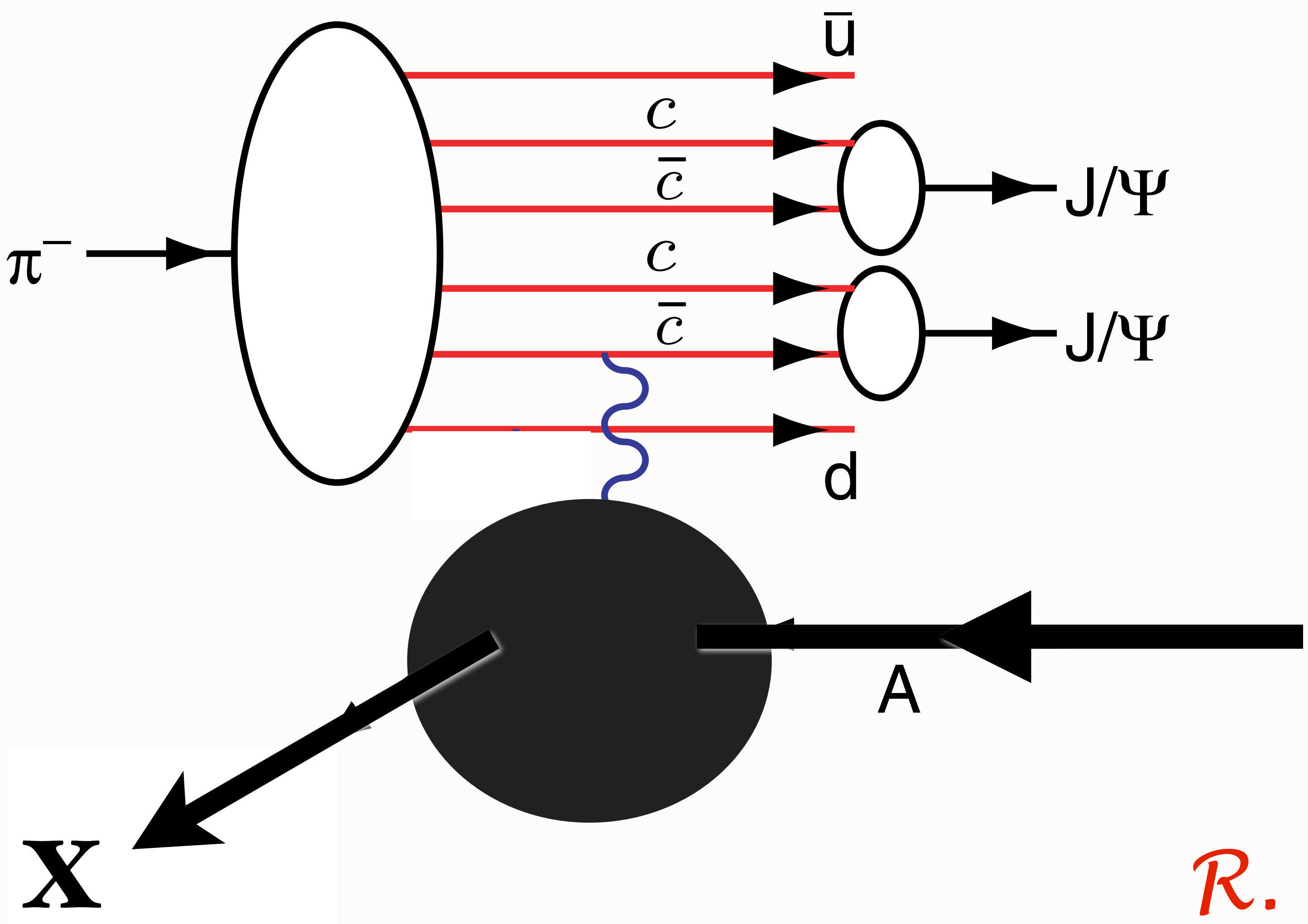
# $A^{2/3}$ component




**J. Badier et al, NA3**

**Excess beyond conventional PQCD subprocesses**

# Production of Two Quarkonia at High $x_F$




R. Vogt, sjb

# Excludes `color drag' model

All events have $x^F_{\psi\psi} > 0.4$ !

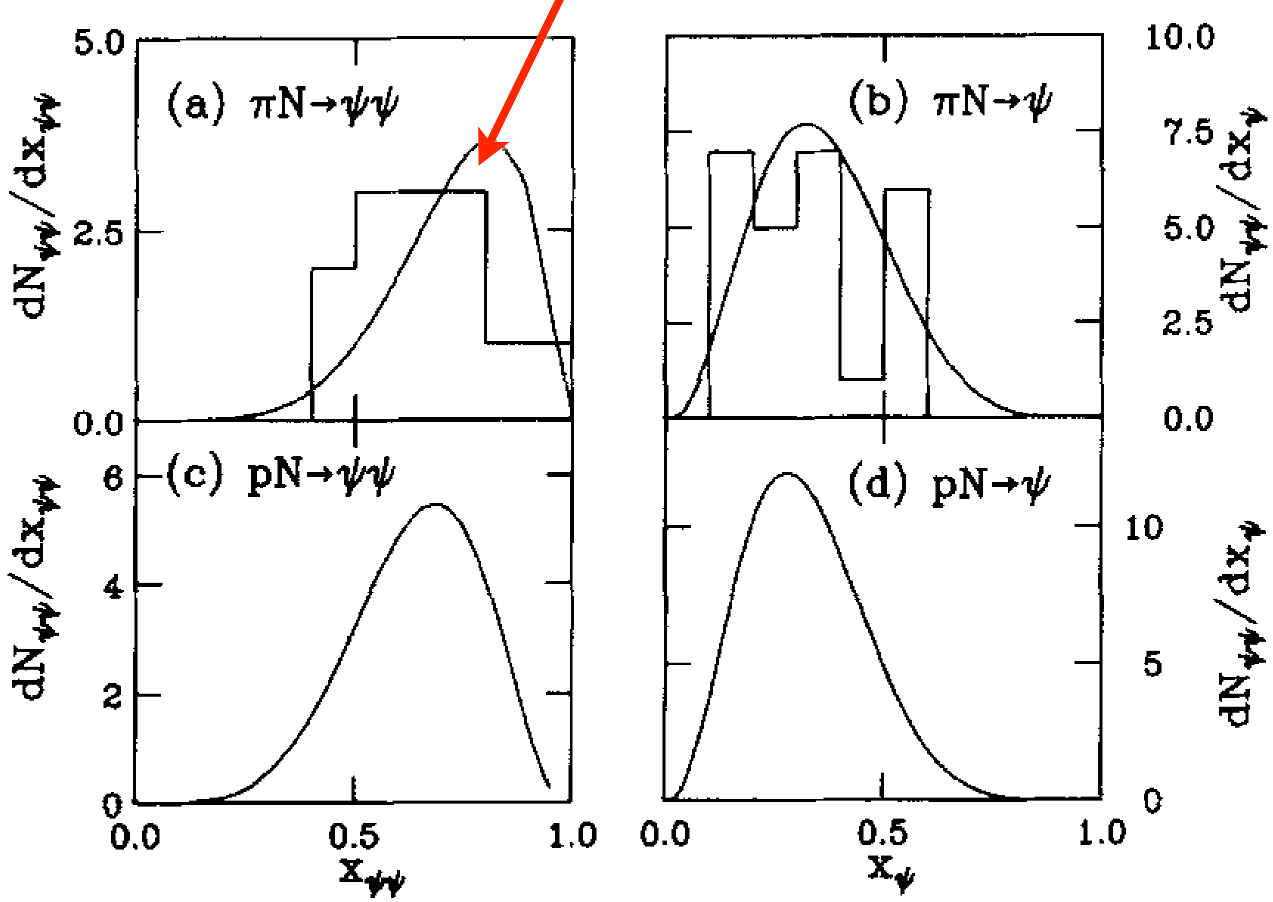



Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of $J/\psi$'s from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/$c$ [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single $J/\psi$'s is twice the number of pairs.

**NA3 Data**

$$\pi A \to J/\psi J/\psi X$$

R. Vogt, sjb

The probability distribution for a general $n$-particle intrinsic $c\bar{c}$ Fock state as a function of $x$ and $\boldsymbol{k}_T$ is written as

$$\frac{dP_{\rm ic}}{\prod_{i=1}^n dx_i d^2 k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n \boldsymbol{k}_{T,i})\delta(1-\sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$

- IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \to J/\psi X$ (Mueller, Gunion, Tang, SJB)

- Color Octet IC Explains $A^{2/3}$ behavior at high $x_F$ (NA3, Fermilab) (Kopeliovitch, Schmidt, Soffer, SJB)

*Color Opaqueness*

- IC Explains $J/\psi \to \rho\pi$ puzzle (Karliner, SJB)

- IC leads to new effects in $B$ decay (Gardner, SJB)

**Higgs production at $x_F$ = 0.8**

# QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary

# *Goal: an analytic first approximation to QCD*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **QCD Coupling at all scales**
- **Hadron Spectroscopy**
- **Light-Front Wavefunctions**
- **Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable**

**de Teramond, sjb**

# Silas Beane

- Bethe-Salpeter and LF: No vacuum matrix element from GMOR.
- Physical massive pion cannot be LF zero mode
- No conflict between IF and LF of one normal; orders
- AdS/QCD and LF Holography predict massless pion for mq =0.
- Beane assumes chiral perturbation theory for vacuum energy not QCD

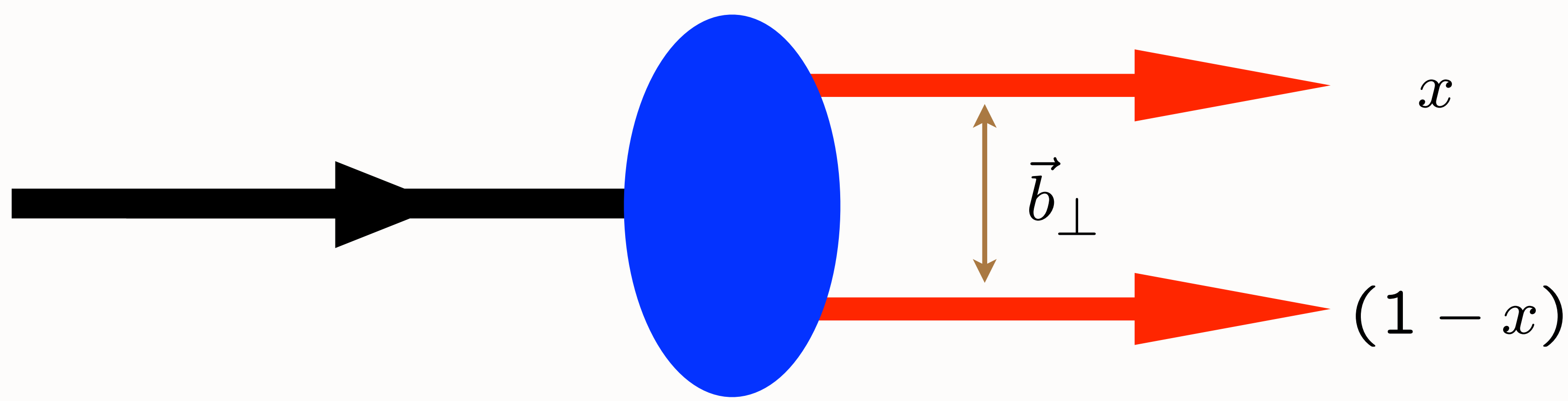
de Teramond, sjb

LF(3+1) ⟷ AdS$_5$

# Light-Front Holography

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

*Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors*

# Light-Front Schrödinger Equation

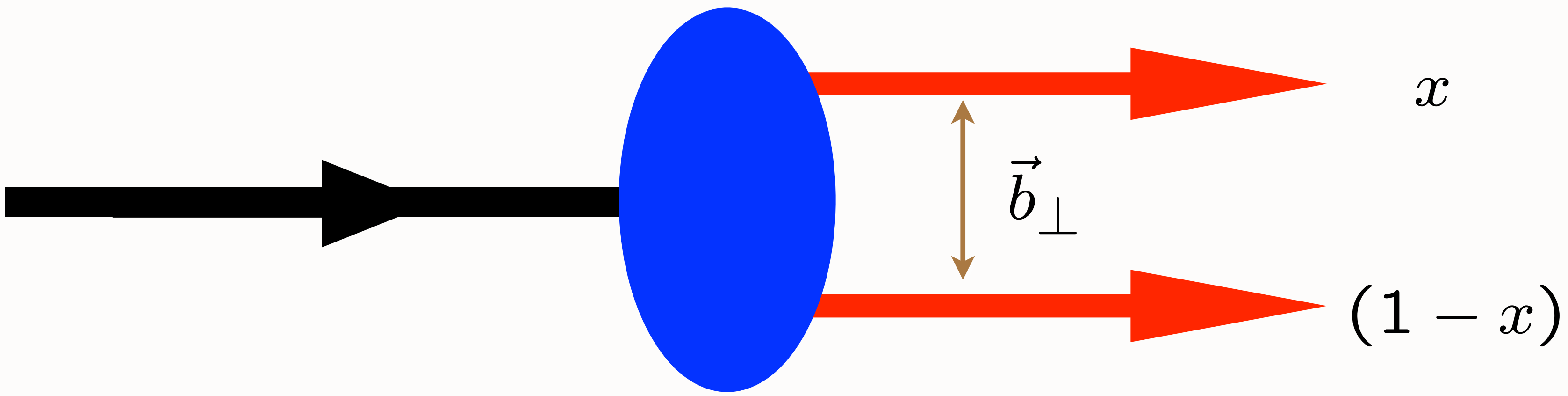
G. de Teramond, sjb

*Relativistic LF single-variable radial equation for QCD & QED*

*Frame Independent!*

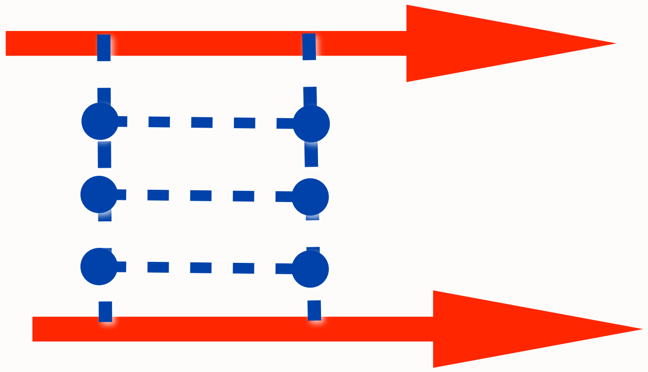
$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$




**U is the exact QCD potential**
**Conjecture: 'H'-diagrams generate**

$$U(\zeta,S,L)=\kappa^2\zeta^2+\kappa^2(L+S-1/2)$$



## LF Quantization
Bjorken, Kogut, Soper, Susskind

## LFWFs and Exclusive QCD:
Lepage and SJB, Efremov, Radyushkin

## RGE and LF Hamiltonians:
Glazek & Wilson

## DLCQ:
Hornbostel, Pauli, & SJB
Pinsky, Hiller

## Renormalization of $H_{LF}$
Hiller, Chabysheva, Pauli, Pinsky, McCartor, Suaya, sjb

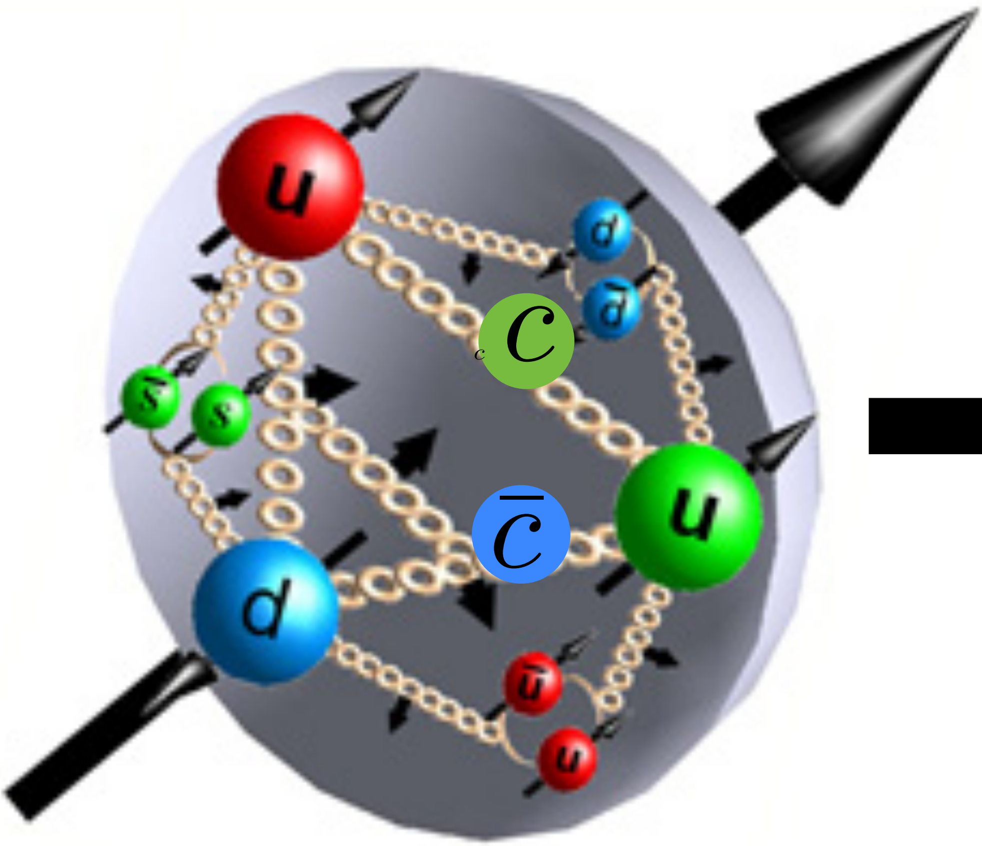
## Rotation Invariance, Regularization
Karmanov, Mathiot

## Zero-Modes: Standard Model
Srivastava, sjb

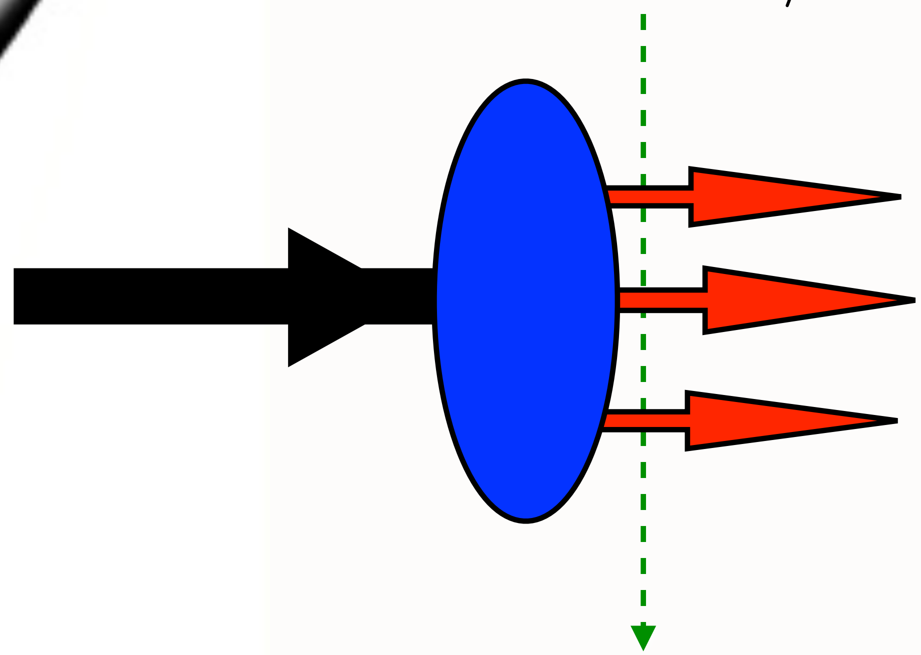
# Features of AdS/QCD LF Holography

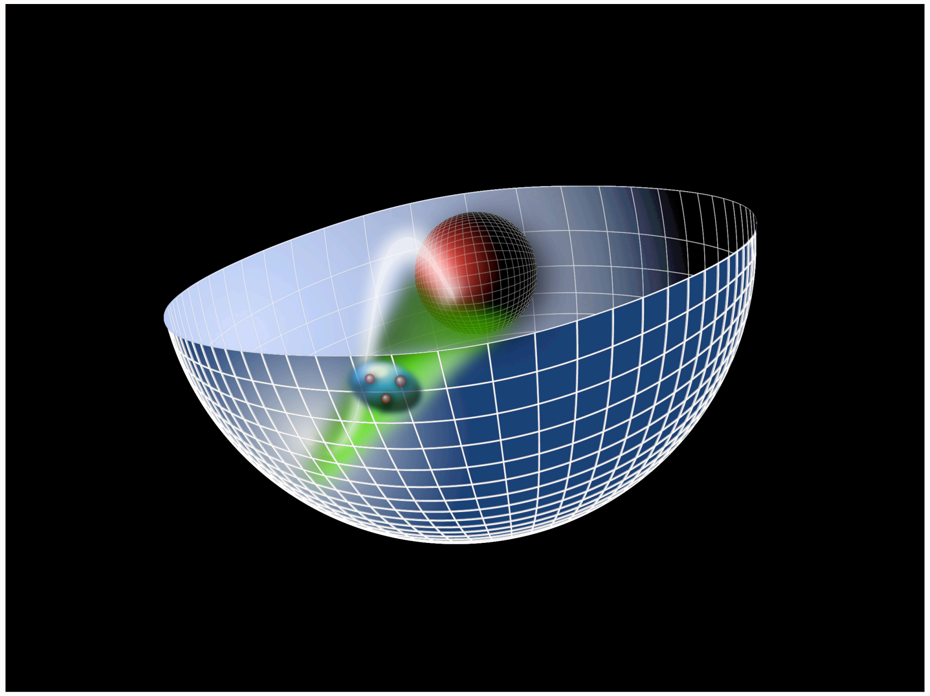
- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent for spacelike observables
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General 'classical' potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)

# Light-Front Hadronic and Nuclear Physics




Fixed $\tau = t + z/c$





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