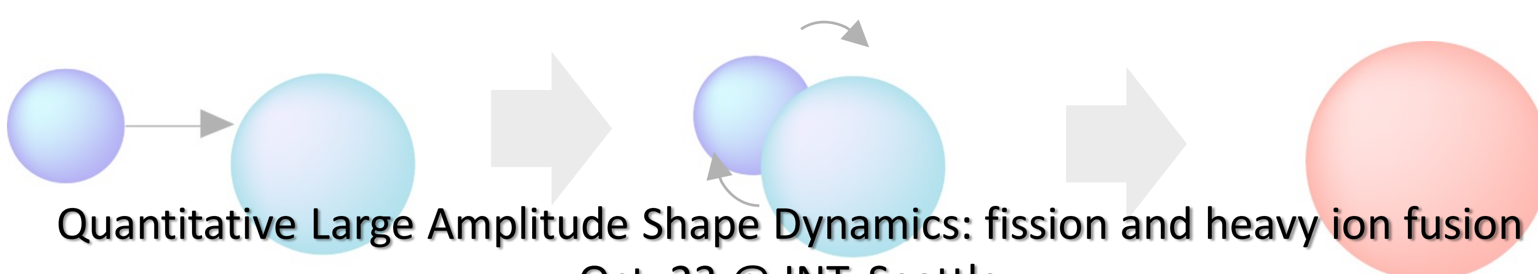
Macroscopic properties in low-energy nuclear reactions by microscopic TDDFT

Kouhei Washiyama (RIKEN, Nishina Center, Japan)

Collaboration with Denis Lacroix (GANIL), Sakir Ayik (Tennesse Tech.)

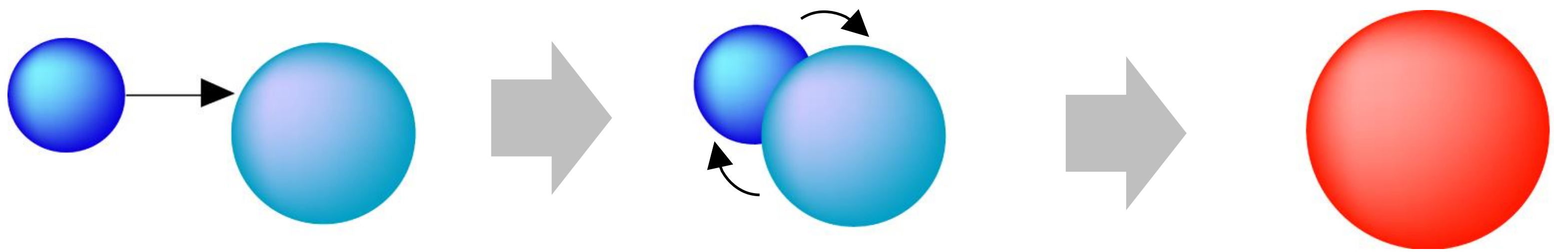
Key word: fusion, TDDFT(TDHF), nucleus-nucleus potential, energy dissipation, fusion hindrance



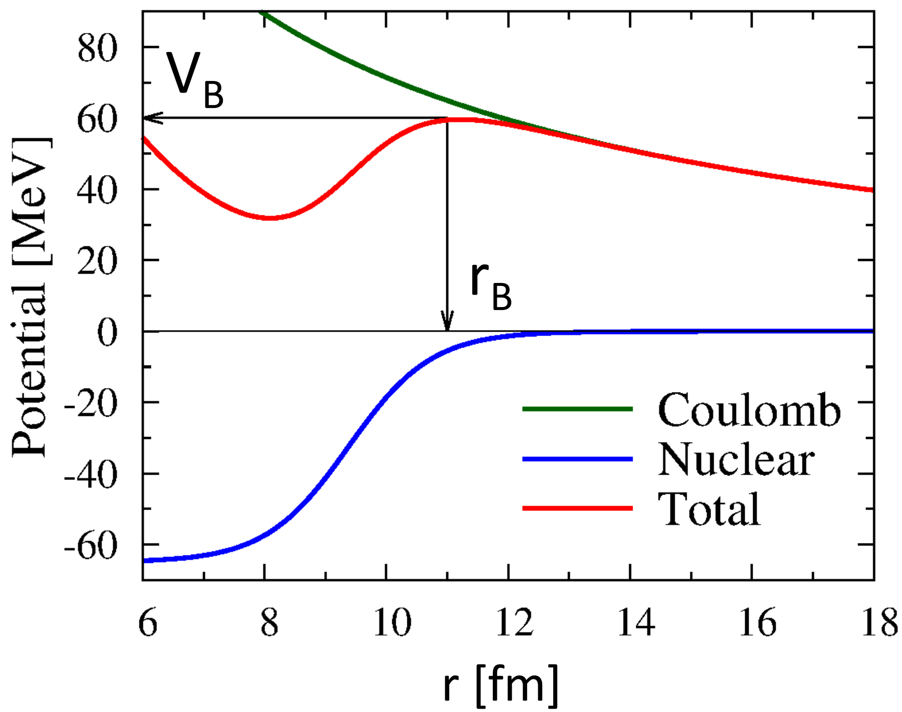
Quantitative Large Amplitude Shape Dynamics: fission and heavy ion fusion
Oct. 23 @ INT, Seattle

Nucleus-nucleus potential

- Low-energy nuclear reactions E/A < 10 MeV



$^{16}O + ^{154}Sm$



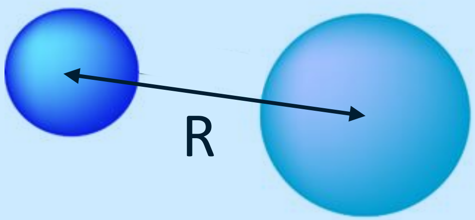
- Woods-Saxon
- Proximity
- Double-folding

→ Coupled-channels method

- Dasgupta et al., Ann. Rev. Nuc. Par. Sci. (1998)
- Balantekin, Takigawa, Rev. Mod. Phys. (1998)
- Hagino, Takigawa, PTP (2012)

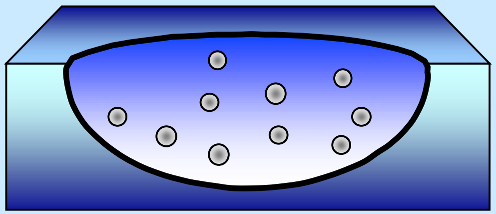
Macroscopic, Microscopic picture

Macroscopic



- Collective degrees of freedom

Microscopic



- Single-particle degrees of freedom

Potential: V(R)
Energy dissipation

Aim

Study the property of macroscopic quantities by TDHF

Time-dependent Hartree-Fock (TDHF)

$$i\hbar\frac{\partial}{\partial t}\phi_i(\boldsymbol{r},t) = \hat{h}[\rho(\boldsymbol{r},t)]\phi_i(\boldsymbol{r},t)$$

h: self-consistent single-particle Hamiltonian

(static case $\hat{h}[\rho]\phi_i = \epsilon_i\phi_i$)

- Bonche et al., PRC 13 (1976)
- Flocard et al., PRC 17 (1978)
- Kim et al., J. Phys. G 23 (1997)
- Simenel et al., PRL 86 (2001)
- Nakatsukasa, Yabana, PRC 71 (2005)
- Umar, Oberacker, PRC 73 (2006)
- Maruhn et al., PRC 74 (2006)

- Self-consistent description for both **static** and **dynamical** properties
- Single-particle: **Quantum**, Collective motion: **Classical**
- Low energy: mean-field approximation

◆ 3D-TDHF code by P. Bonche (Kim, Otsuka, Bonche, J. Phys. G23(1997)1267)

Input:

✓ Skyrme SLy4d parameterization

✓ 3D, dx = 0.8 fm, dt = 0.45 fm/c

Energy density functional

- Skyrme energy density functional

$$\mathcal{E}_{\rm Sk} \equiv \int d^3r \sum_{t=0,1} \mathcal{H}_t(\boldsymbol{r}) \qquad \rho_{t=0} = \rho_n + \rho_p \qquad \rho_{t=1} = \rho_n - \rho_p$$

$$\mathcal{H}_t = C_t^\rho \rho_t^2 + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^\tau \rho_t \tau_t + C_t^{\nabla\cdot J} \rho_t \nabla \cdot \boldsymbol{J}_t - C_t^T \sum_{\mu,\nu=x,y,z} J_{t,\mu\nu} J_{t,\mu\nu} \cdots$$
$$+ C_t^s \boldsymbol{s}_t^2 + C_t^{\Delta s} \boldsymbol{s}_t \cdot \Delta \boldsymbol{s}_t - C_t^\tau \boldsymbol{j}_t^2 + C_t^{\nabla\cdot J} \boldsymbol{s}_t \cdot \nabla \times \boldsymbol{j}_t + C_t^T \boldsymbol{s}_t \cdot \boldsymbol{T}_t + \cdots$$

$$\rho_q(\boldsymbol{r}) = \rho_q(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}, \quad \tau_q(\boldsymbol{r}) = \nabla\cdot\nabla'\rho_q(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}, \quad J_{q,\mu\nu}(\boldsymbol{r}) = -\frac{i}{2}(\nabla_\mu - \nabla'_\mu) s_{q,\nu}(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}$$
$$\boldsymbol{s}_q(\boldsymbol{r}) = \boldsymbol{s}_q(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}, \quad \boldsymbol{T}_q(\boldsymbol{r}) = \nabla\cdot\nabla'\boldsymbol{s}_q(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}, \quad \boldsymbol{j}_q(\boldsymbol{r}) = -\frac{i}{2}(\nabla - \nabla')\rho_q(\boldsymbol{r},\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}$$

$$\rho_q(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma=\pm1} \rho_q(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma) = \sum_{\sigma=\pm1}\sum_k n_k \phi_k(\boldsymbol{r}\sigma q)\phi_k^*(\boldsymbol{r}'\sigma q)$$
$$\boldsymbol{s}_q(\boldsymbol{r},\boldsymbol{r}') = \sum_{\sigma,\sigma'=\pm1} \rho_q(\boldsymbol{r}\sigma,\boldsymbol{r}'\sigma')\langle\sigma'|\hat{\boldsymbol{\sigma}}|\sigma\rangle$$

Bender, Heenen, Reinhard, Rev. Mod. Phys. 75 (2003) 121

Numerical method for solving TDHF

$$i\hbar\frac{d\phi_i}{dt} = \hat{h}[\rho(t)]\phi_i(t)$$

$$\phi_i(t+\Delta t) = \exp\left\{-\frac{i}{\hbar}\int_t^{t+\Delta t} ds\ \hat{h}[\rho(s)]\right\}\phi_i(t)$$

$$\simeq \exp\left\{-i\frac{\Delta t}{\hbar}\hat{h}[\rho(t+\frac{\Delta t}{2})]\right\}\phi_i(t)$$

$$\{\phi_i^{(n)}\}$$

$$\rho^{(n)}$$

$$\hat{h}^{(n)} = \hat{h}[\rho^{(n)}]$$

$$\tilde{\phi}_i^{(n+1)} = \exp[-i\frac{\Delta t}{\hbar}\hat{h}^{(n)}]\ \phi_i^{(n)}$$

$$\tilde{\rho}^{(n+1)}$$

$$\rho^{(n+\frac{1}{2})} = \frac{1}{2}[\rho^{(n)} + \tilde{\rho}^{(n+1)}]$$

$$\hat{h}^{(n+\frac{1}{2})} = \hat{h}[\rho^{(n+\frac{1}{2})}]$$

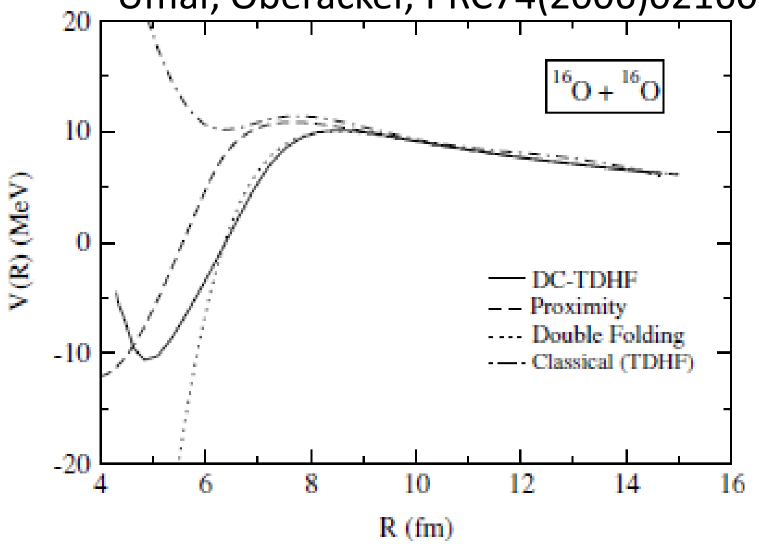
$$\phi_i^{(n+1)} = \exp[-i\frac{\Delta t}{\hbar}\hat{h}^{(n+\frac{1}{2})}]\ \phi_i^{(n)}$$

$$\{\phi_i^{(n+1)}\}$$

Recent applications of TDHF

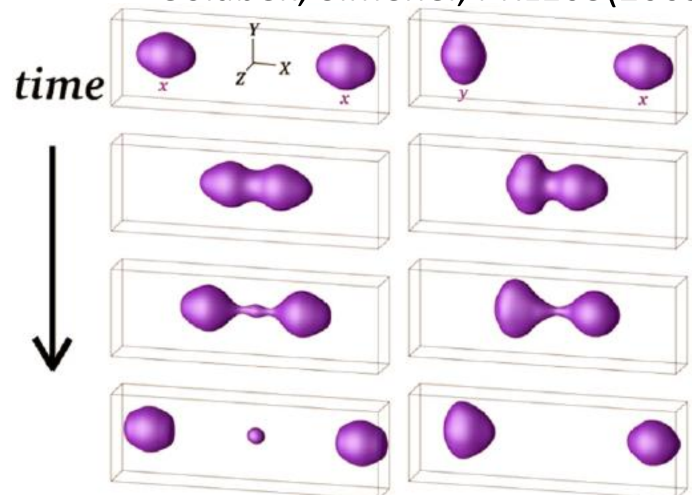
- Density-constrained TDHF

  Umar, Oberacker, PRC74(2006)021601



- $^{238}U + ^{238}U$ central collision

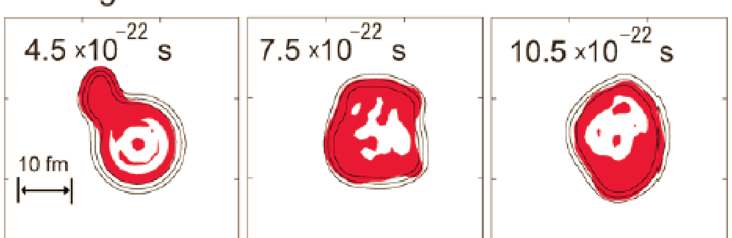
  Golabek, Simenel, PRL103(2009)042701



- Charge equilibration

  Iwata et al., PRL104 (2010) 252501

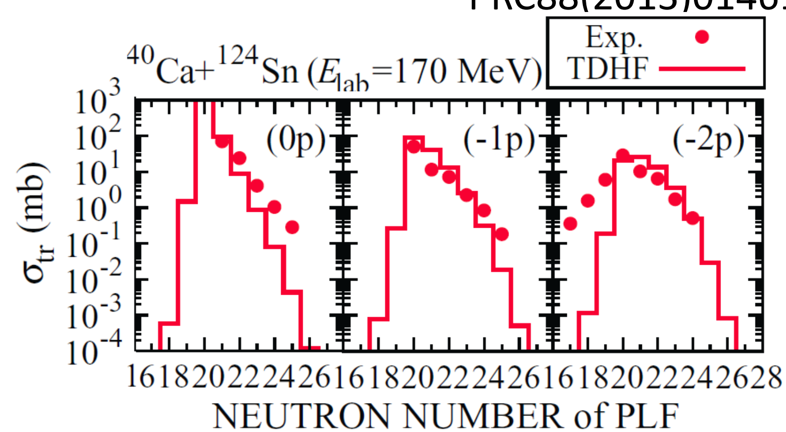
$^{24}Mg + ^{208}Pb$



$$\frac{E_{CE,lab}}{A} = \frac{\hbar^2(3\pi^2\rho_{min})^{2/3}}{2m} + \frac{e^2 Z_1 Z_2}{4\pi\epsilon_0 r_0} \frac{A_1 + A_2}{A_1 A_2 (A_1^{1/3} + A_2^{1/3})}$$

- Transfer reaction

  Sekizawa, Yabana, PRC88(2013)014614

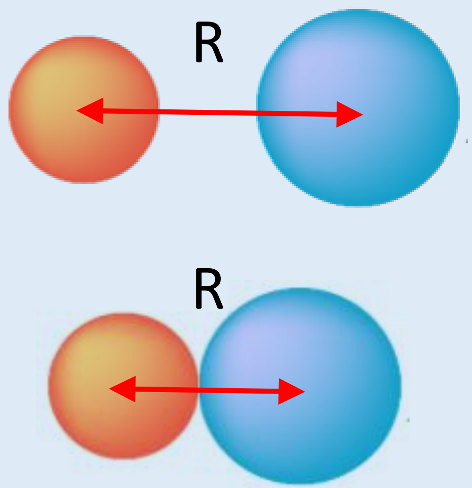


Frozen density approximation: Sudden

$$V_{\mathrm{FD}}(R) = \mathcal{E}[\rho_{P+T}](R) - \mathcal{E}[\rho_P] - \mathcal{E}[\rho_T]$$

$$\rho_{P+T} = \rho_P + \rho_T$$

Denisov, Norenberg, EPJA15(2002)375



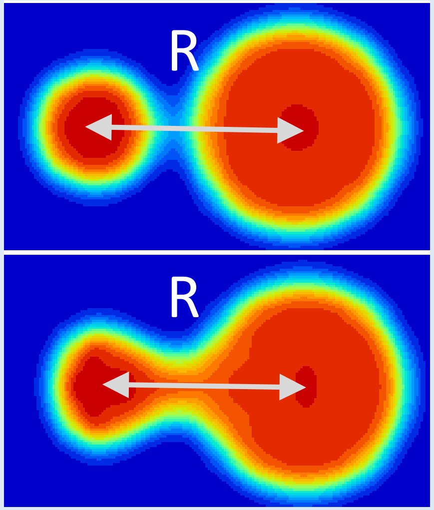
- Approximate that densities are *frozen* to be their ground state densities during time evolution

Density-constrained TDHF: "Adiabatic"

$$V_{\mathrm{DC}}(R) = \mathcal{E}_{\mathrm{DC}}[\rho_{\mathrm{TDHF}}](R) - \mathcal{E}[\rho_P] - \mathcal{E}[\rho_T]$$

with minimization $\left\langle H - \int d^3r \lambda(r)\hat{\rho}(r) \right\rangle_{\rho = \rho_{\mathrm{TDHF}}(t)}$

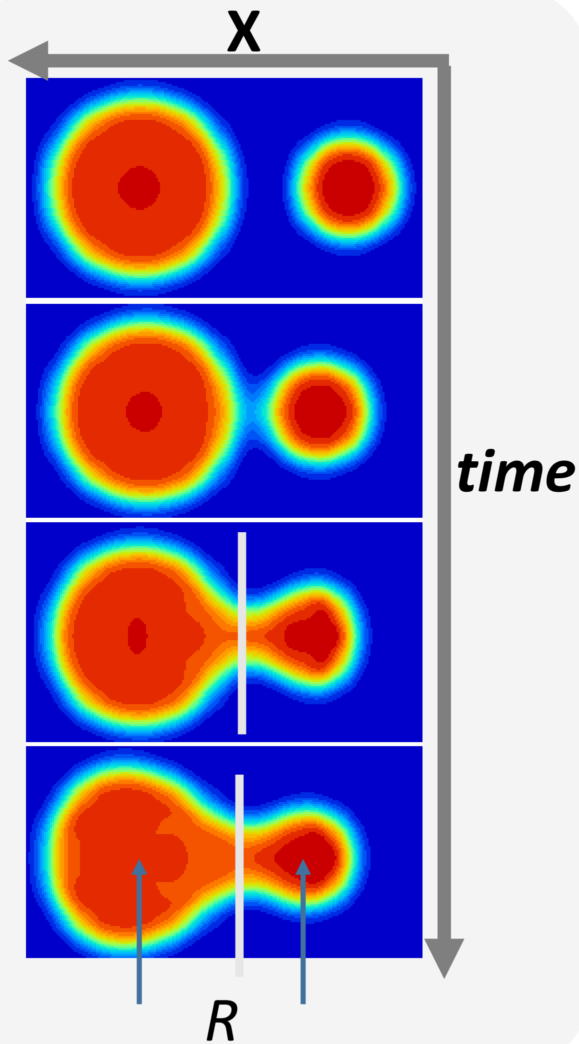
Umar, Oberacker, PRC74(2006)021601



- Energy is obtained by minimization with a constraint on the density from TDHF time evolution

Method: Extract potential and dissipation

Washiyama, Lacroix, PRC78 (2008) 024610



1. $\rho(t) \rightarrow R(t),\ P(t),\ \frac{dR}{dt},\ \frac{dP}{dt}$

2. $\frac{dP}{dt} = -\frac{dV(R)}{dR} - \gamma(R)\frac{dR}{dt}$

3. $\frac{dP}{dt},\ \frac{dR}{dt} \Longrightarrow \gamma(R),\ V(R)$

Central collisions
Mass

R: Relative distance

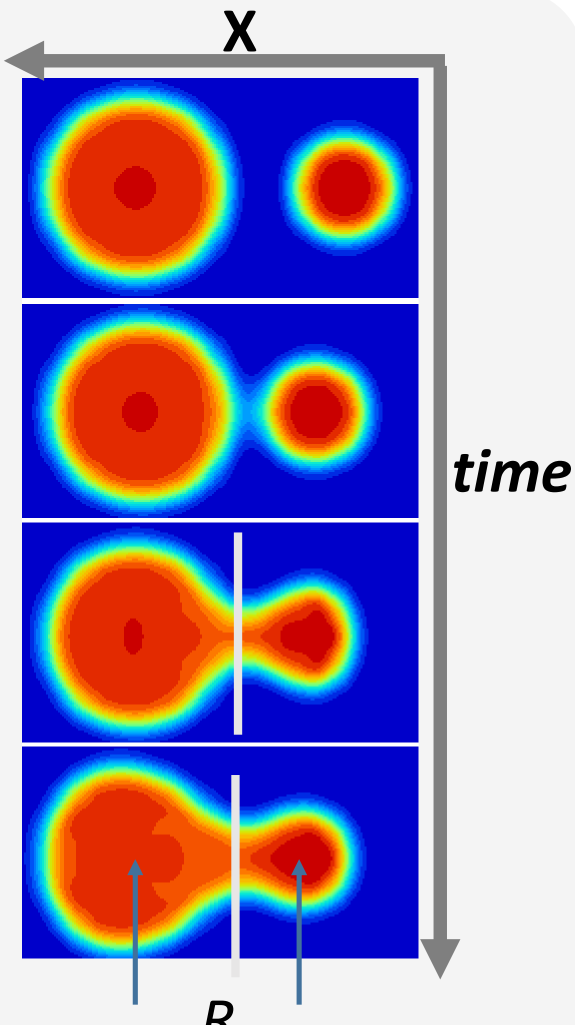
P: Momentum

V: Potential

γ: Friction coefficient

Method: Extract potential and dissipation

Washiyama, Lacroix, PRC78 (2008) 024610



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3. $\frac{dP}{dt},\ \frac{dR}{dt} \Longrightarrow \gamma(R),\ V(R)$

Central collisions
Mass

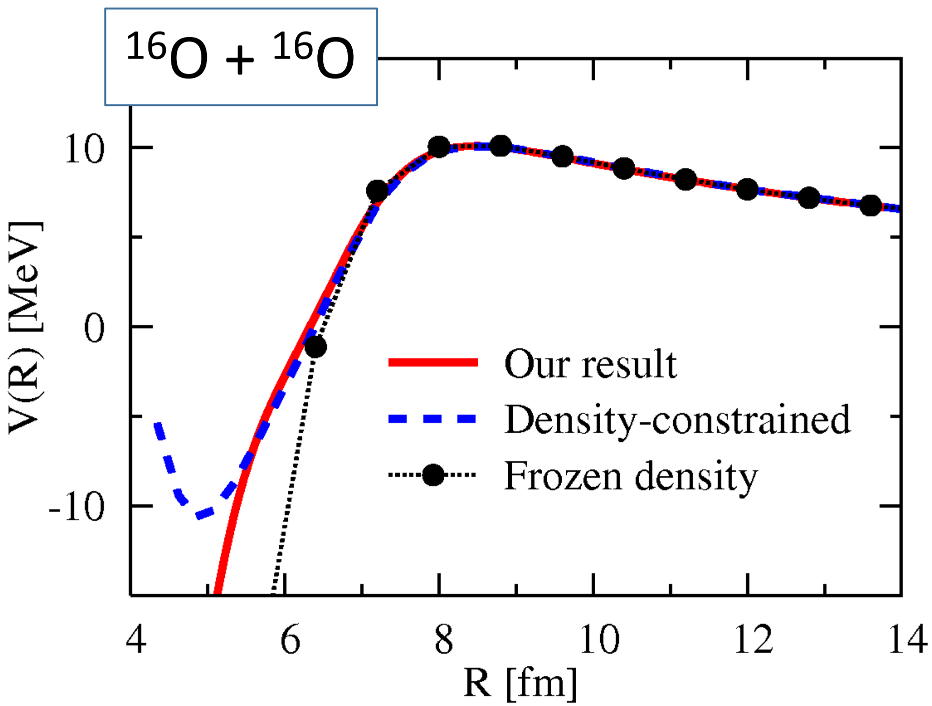
- Two trajectories (R_1, P_1) , (R_2, P_2) at slightly different energies (E_1, E_2)

$$\gamma(R) = \frac{\frac{dP_1}{dt} - \frac{dP_2}{dt}}{\frac{dR_1}{dt} - \frac{dR_2}{dt}}\Bigg|_{R_1, R_2 = R}$$

Result: Comparison of potentials

Washiyama, Lacroix, PRC78 (2008) 024610

$$\frac{dP}{dt} = -\frac{dV(R)}{dR} - \gamma(R)\frac{dR}{dt}$$



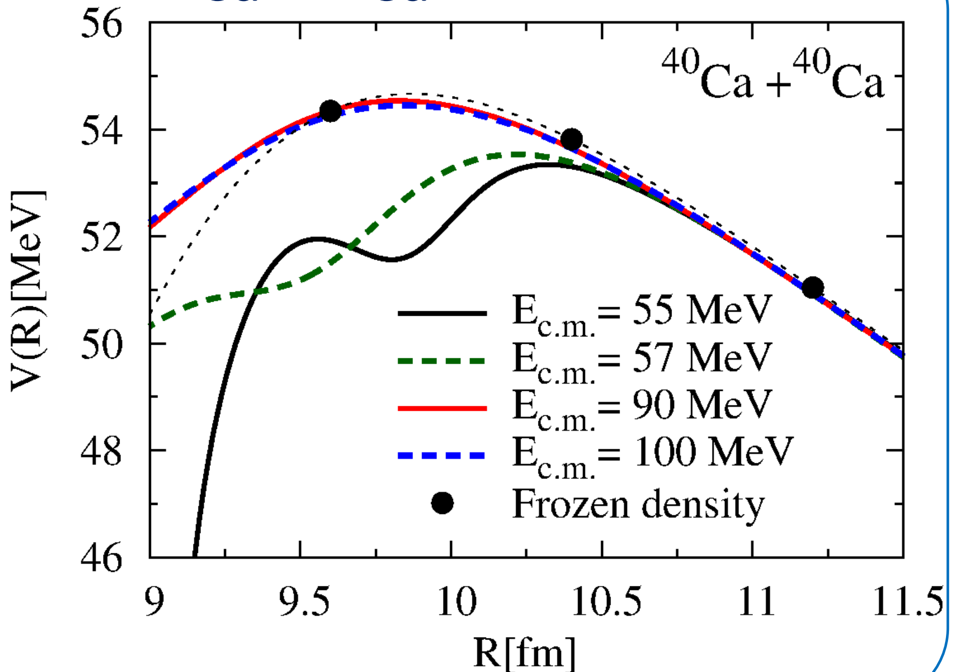
- Agree with each model
- Validation of our model

Energy dependence of potential

Washiyama, Lacroix, PRC78 (2008) 024610

$$\frac{dP}{dt} = -\frac{dV(R)}{dR} - \gamma(R)\frac{dR}{dt}$$

$^{40}Ca + ^{40}Ca$



- High E_{cm} = Frozen density

Energy dependence of potential

Washiyama, Lacroix, PRC78 (2008) 024610

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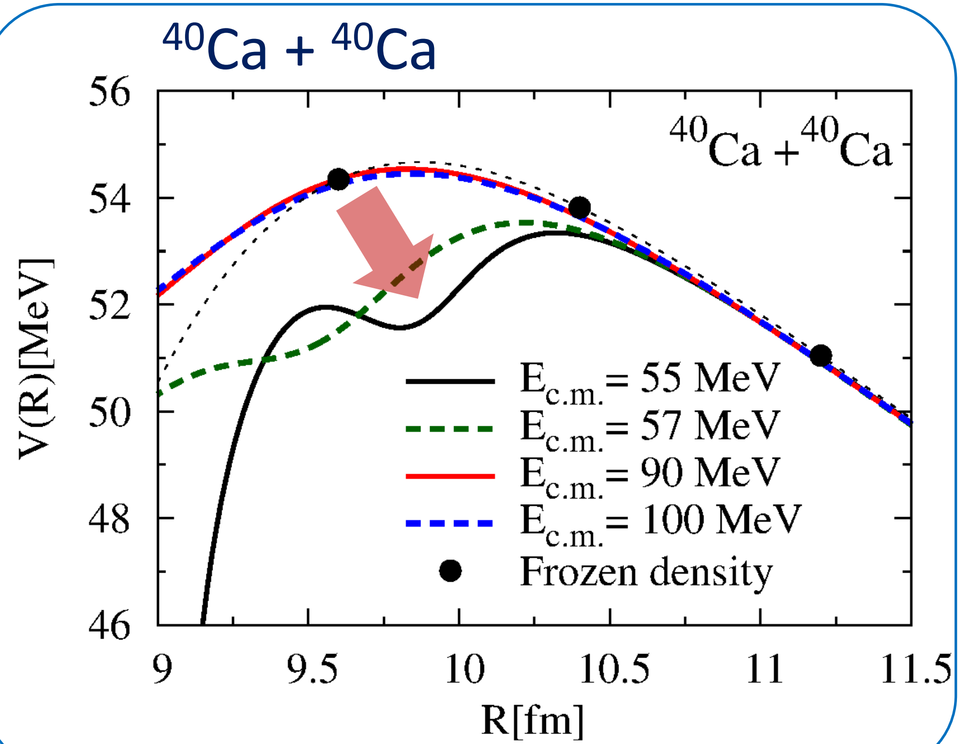
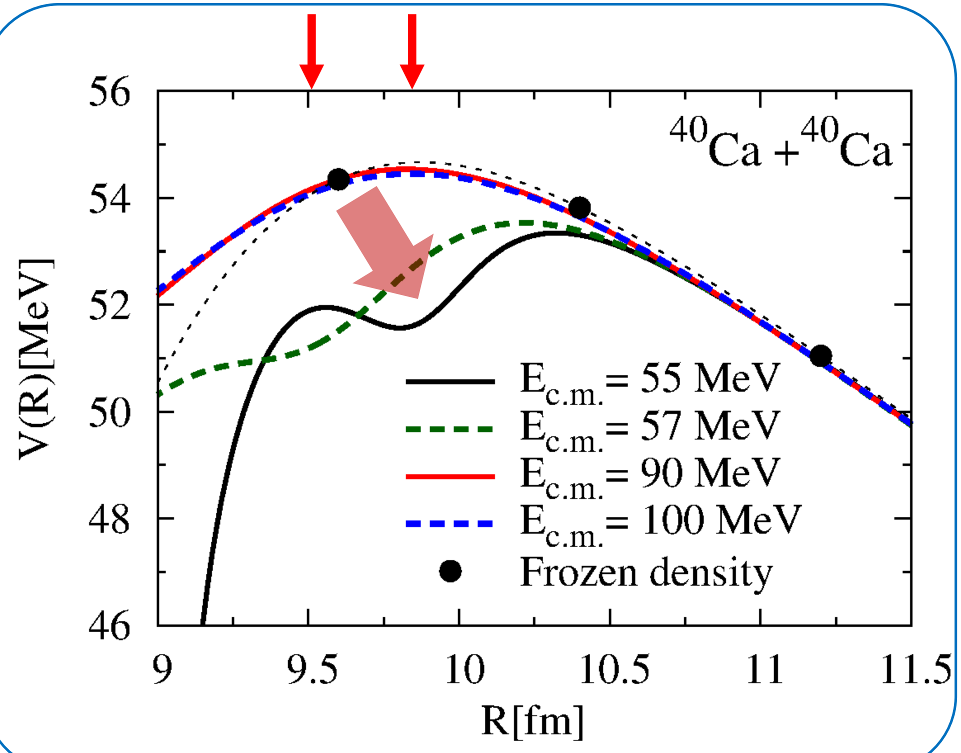


- High E_{cm} = Frozen density
- Decrease of the barrier: from high E_{cm} to low E_{cm}

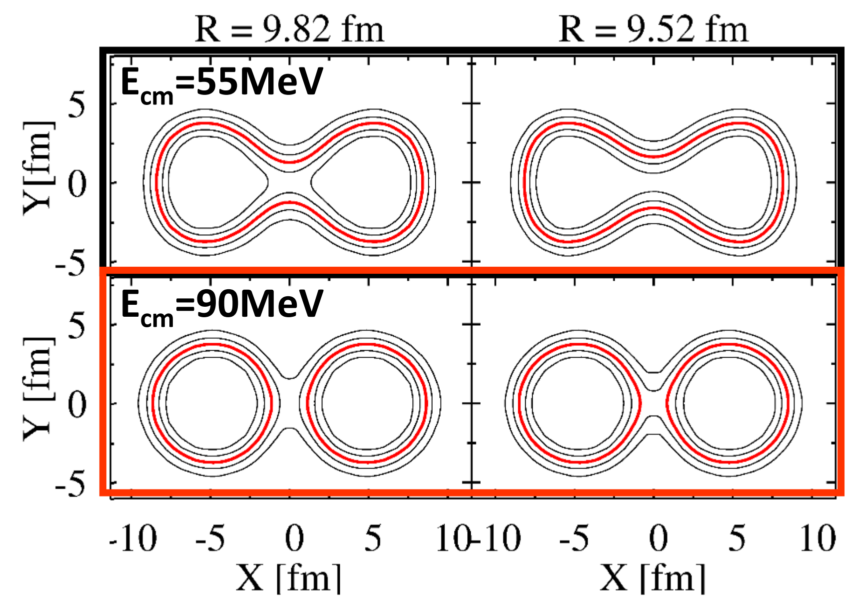
Energy dependence of potential

Washiyama, Lacroix, PRC78 (2008) 024610

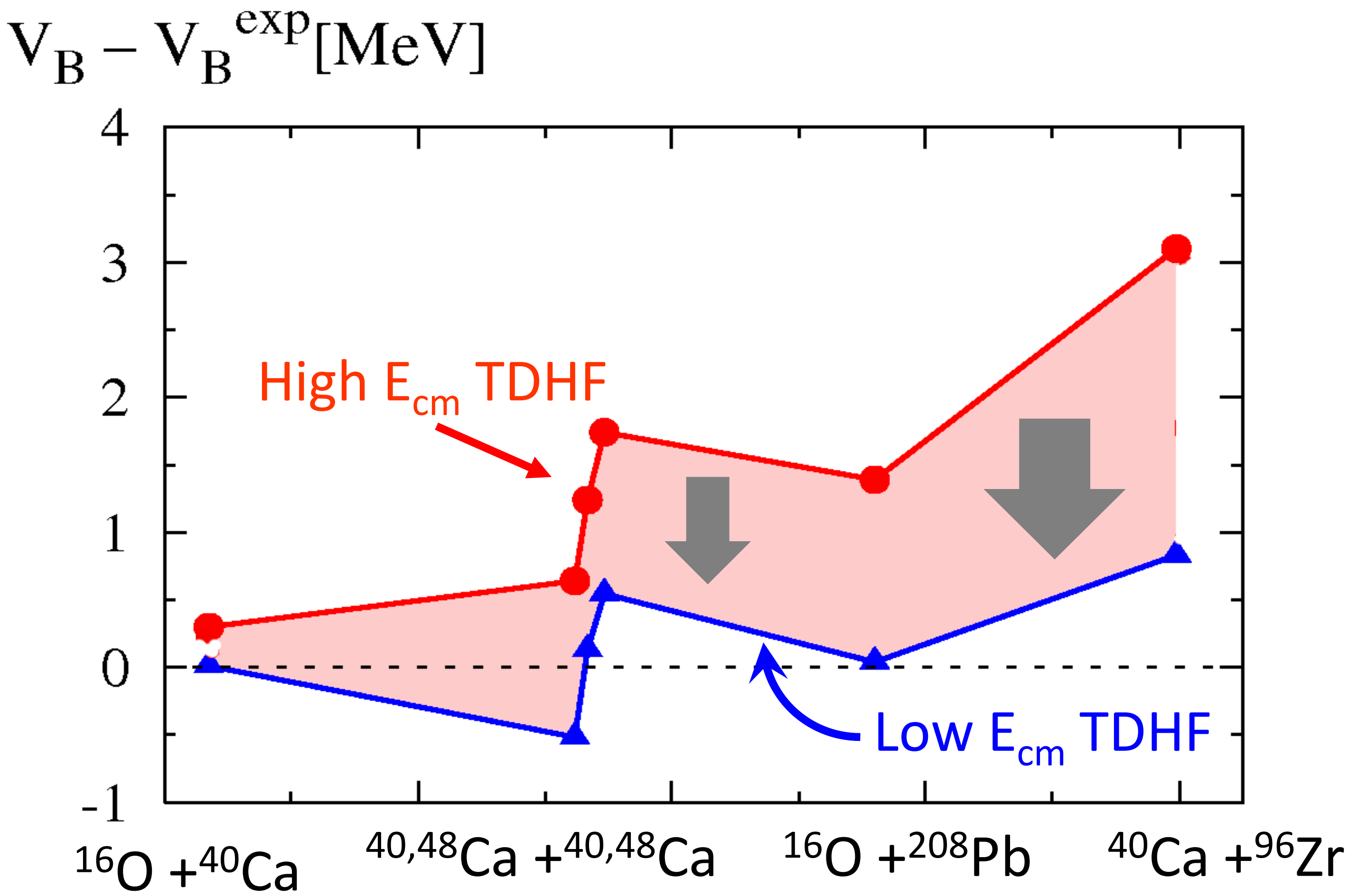
$$\frac{dP}{dt} = -\frac{dV(R)}{dR} - \gamma(R)\frac{dR}{dt}$$



Density distribution



Systematics of potentials



Energy dependence of friction coefficient

$$\frac{dP}{dt} = -\frac{dV(R)}{dR} - \gamma(R)\frac{dR}{dt}$$

γ: friction coefficient

$E_{cm} = P^2/2\mu + V + E_{dissipation}$

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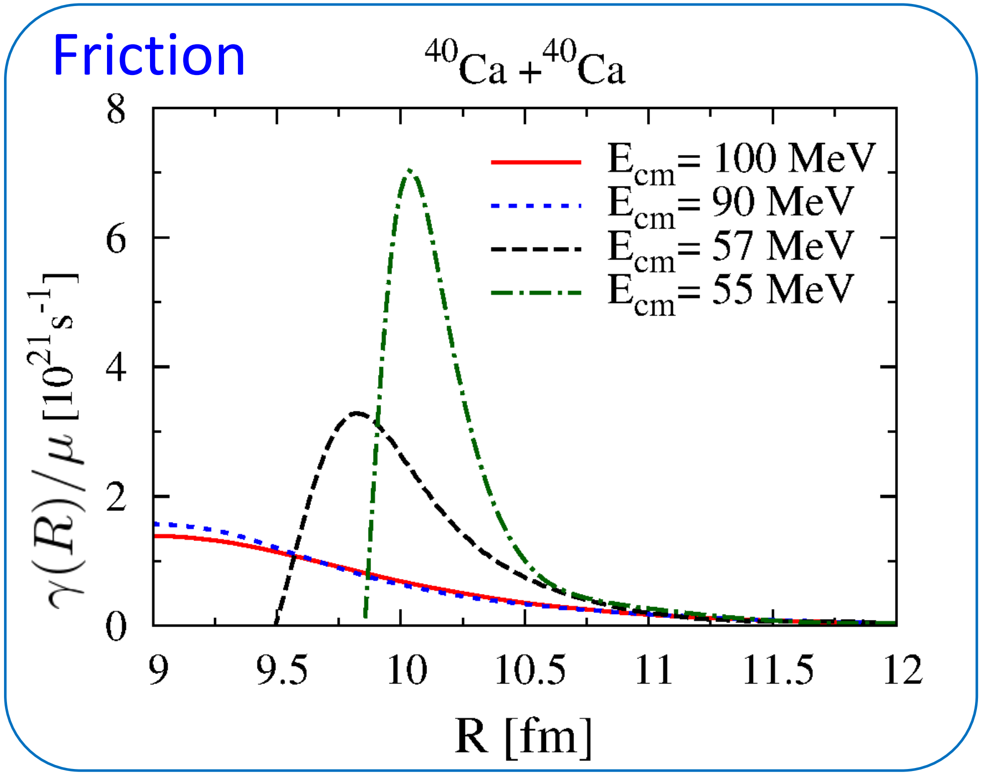

Washiyama, Lacroix, Ayik, PRC79(2009)024609

Energy dependence of friction coefficient

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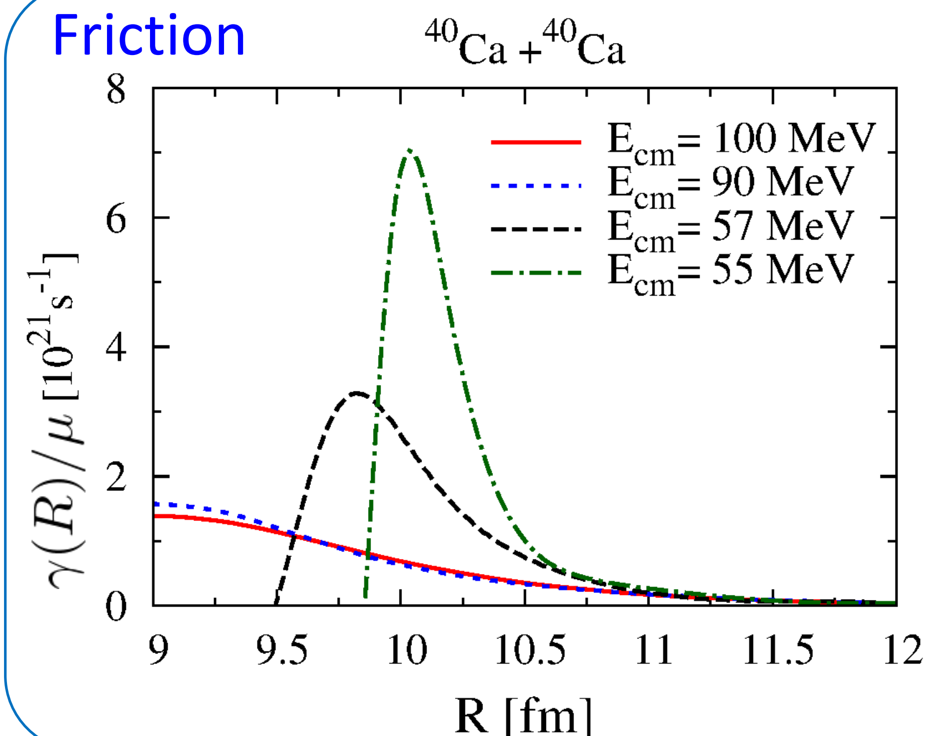
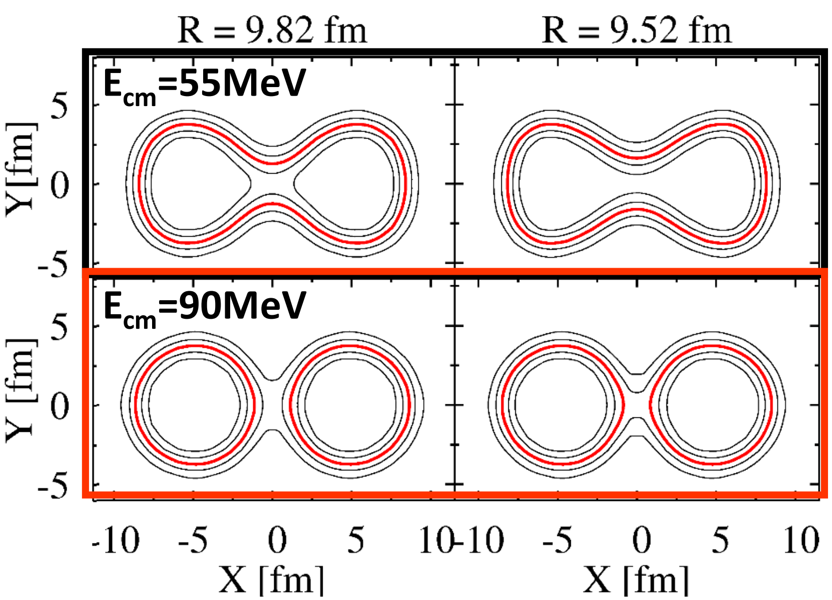
Friction



Density distribution



Washiyama, Lacroix, Ayik, PRC79(2009)024609

Energy dependence of friction coefficient

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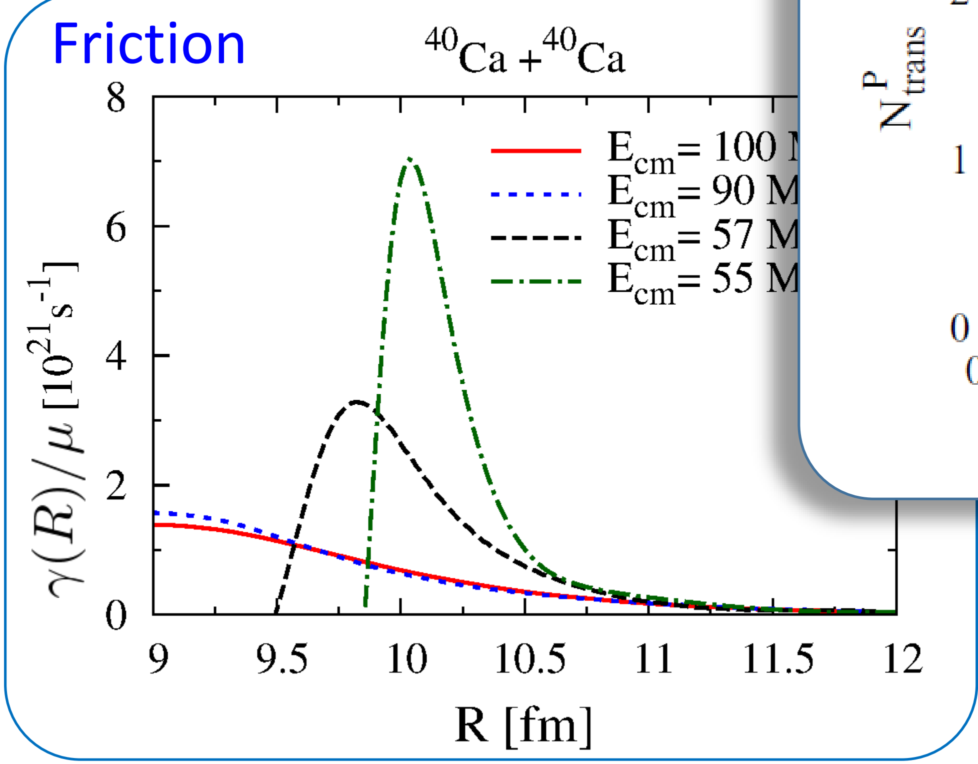
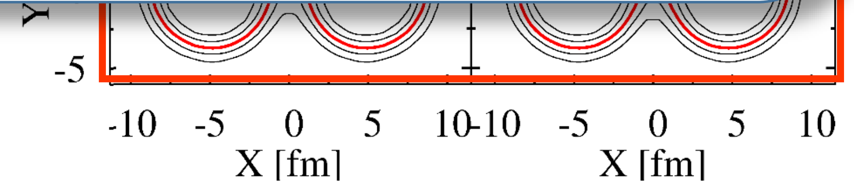
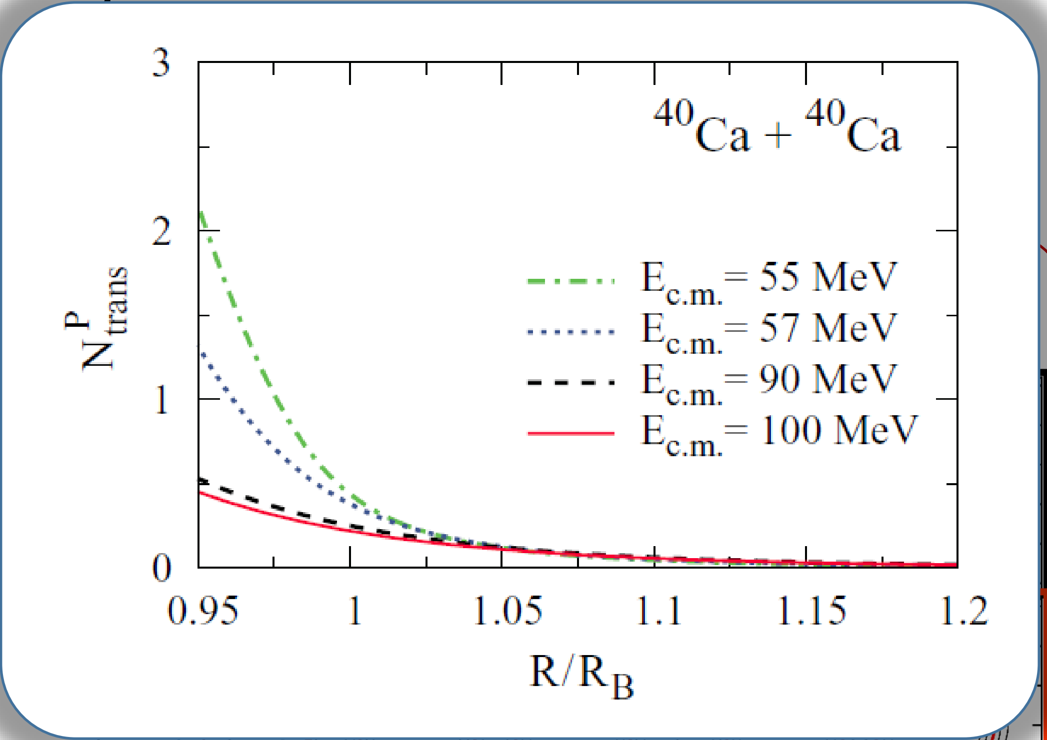
$E_{cm} = P^2/2\mu + V + E_{dissip}$

γ: friction coefficient







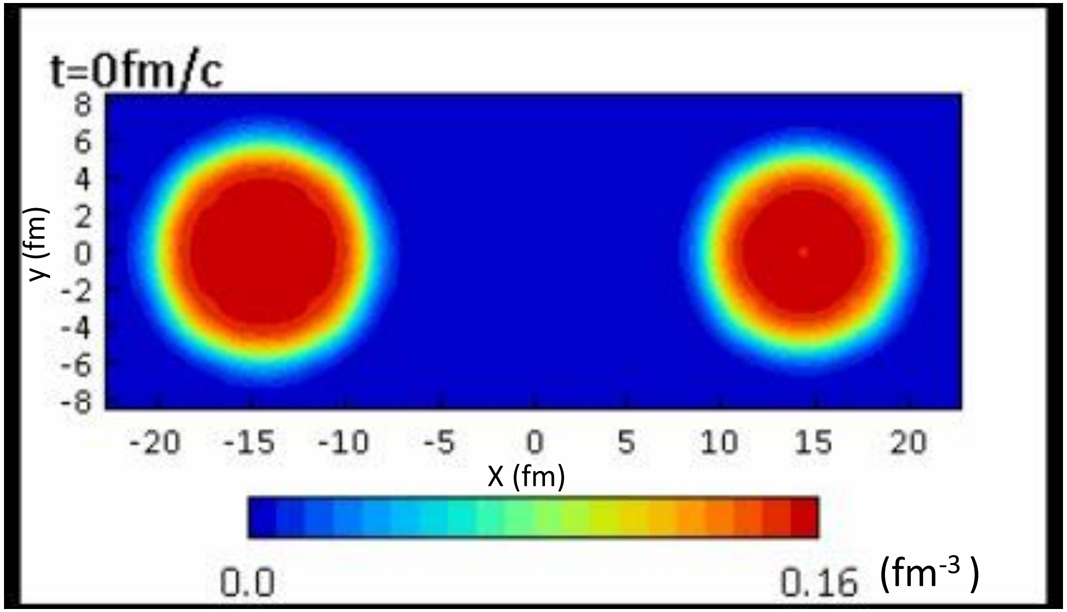
Washiyama, Lacroix, Ayik, PRC79(2009)024609

How about heavy systems ?

Heavy: Charge product > 1600 - 1800

$^{124}Sn + ^{96}Zr$, E_{cm} = 228 MeV,

Charge product = 50 x 40 = 2000



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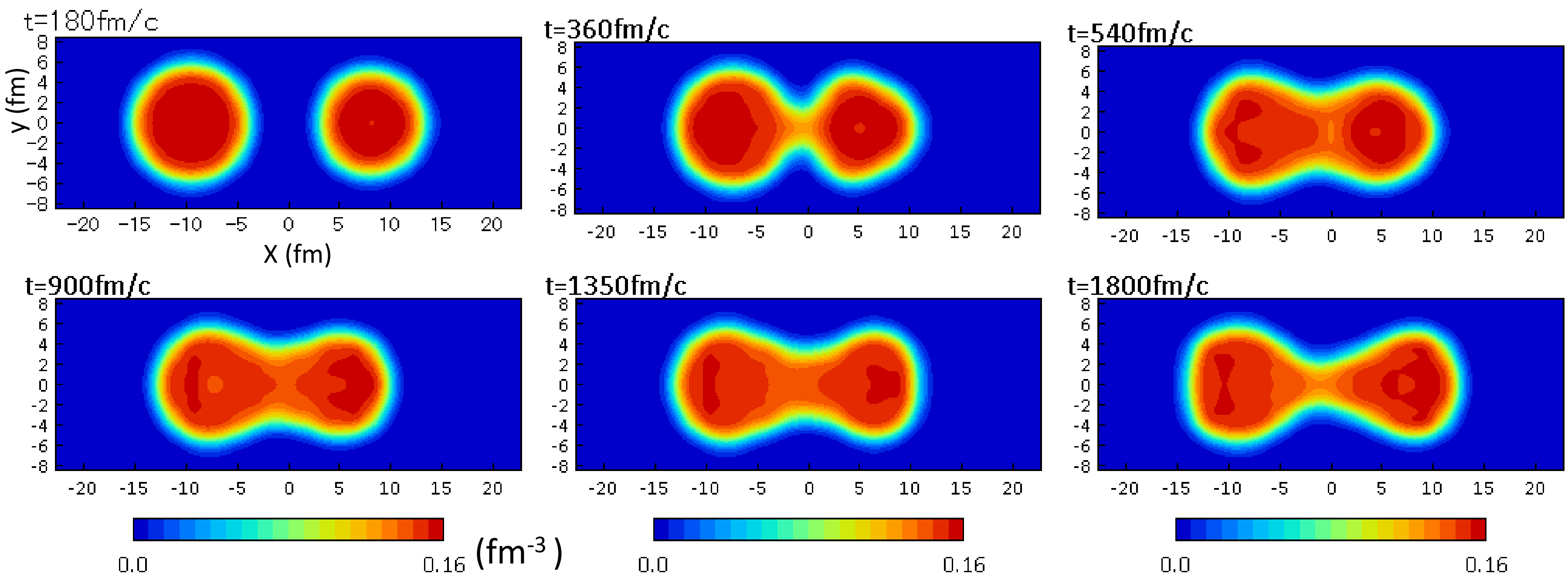
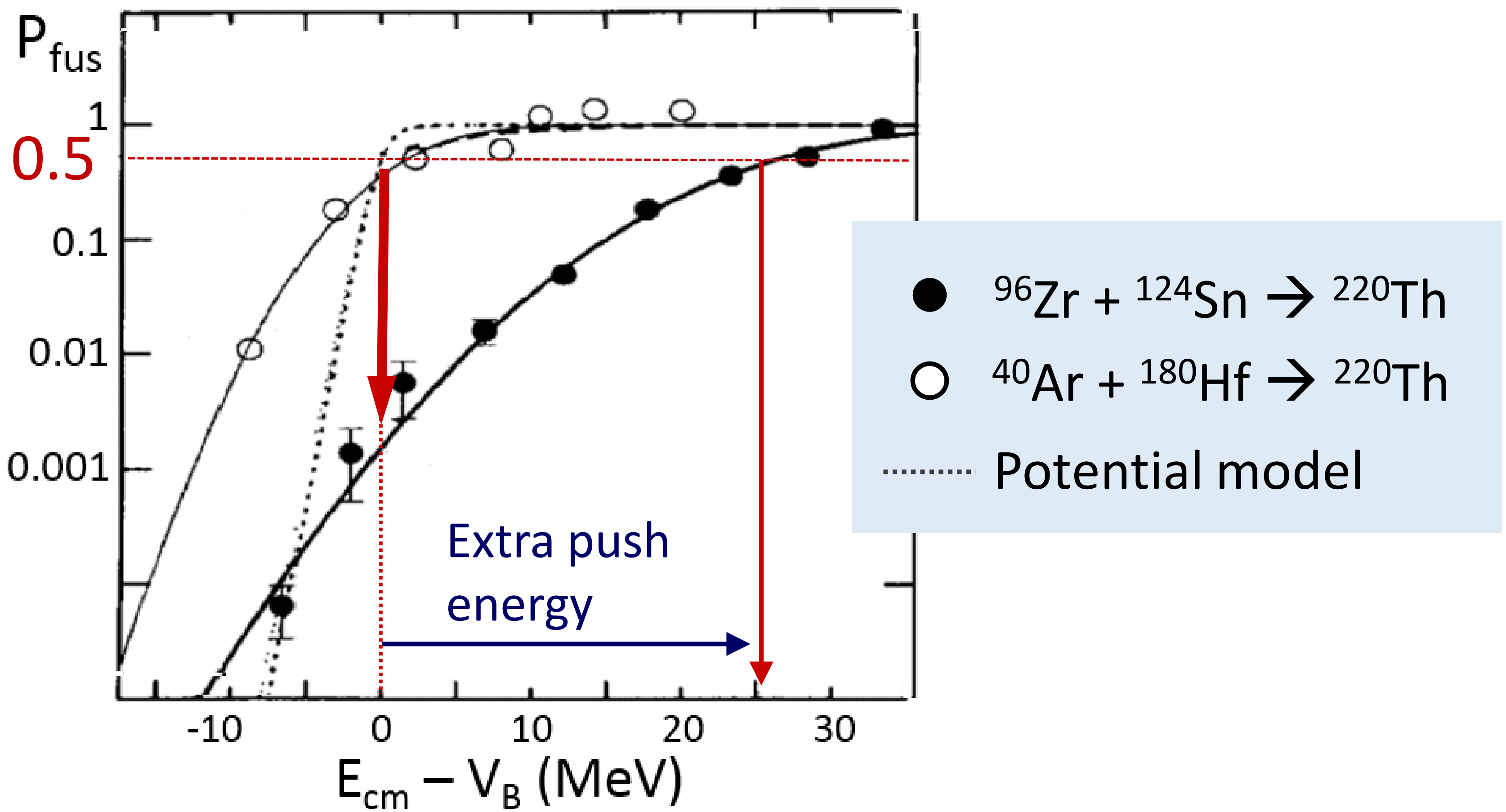


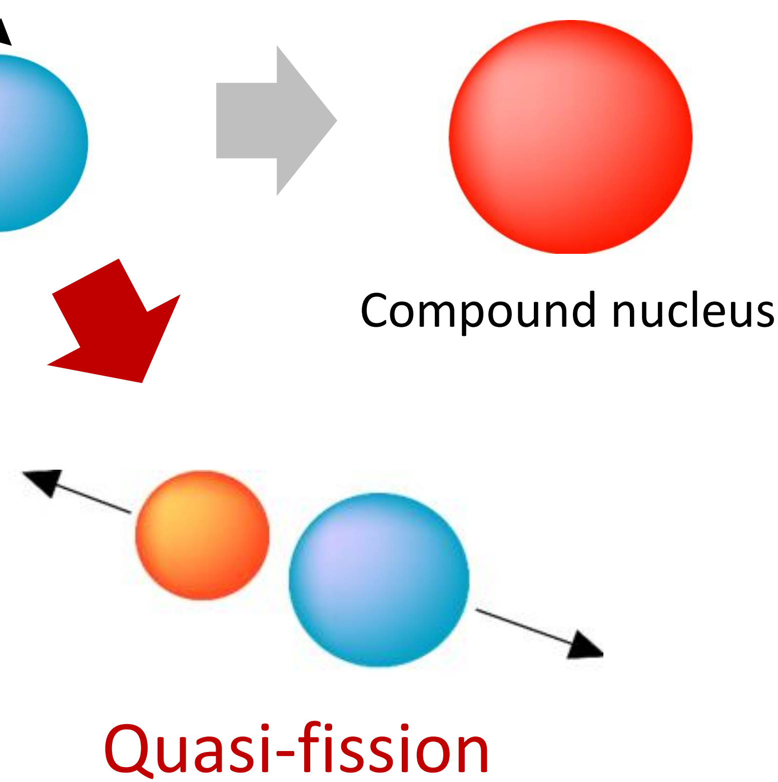
Fusion hindrance in heavy systems

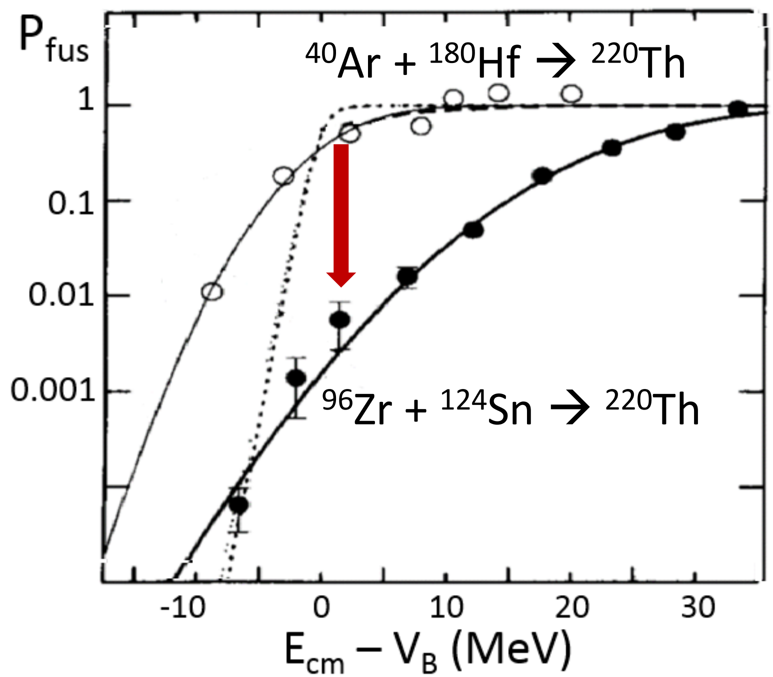
- Fusion probability decreases in $Z_P Z_T$ > 1600 systems

C.-C. Sahm et al., NPA441 (1985) 316



Fusion hindrance in heavy systems

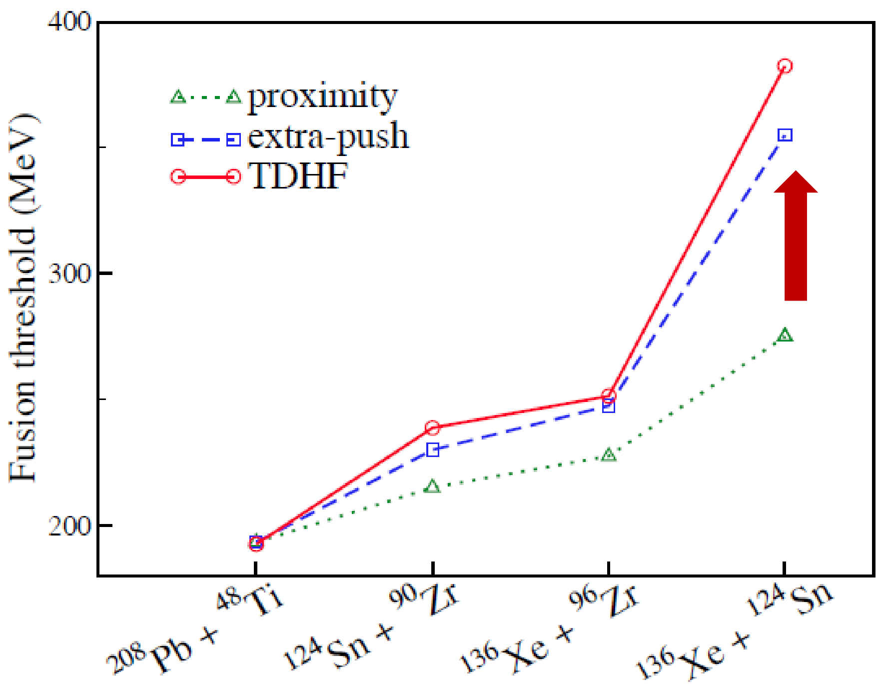




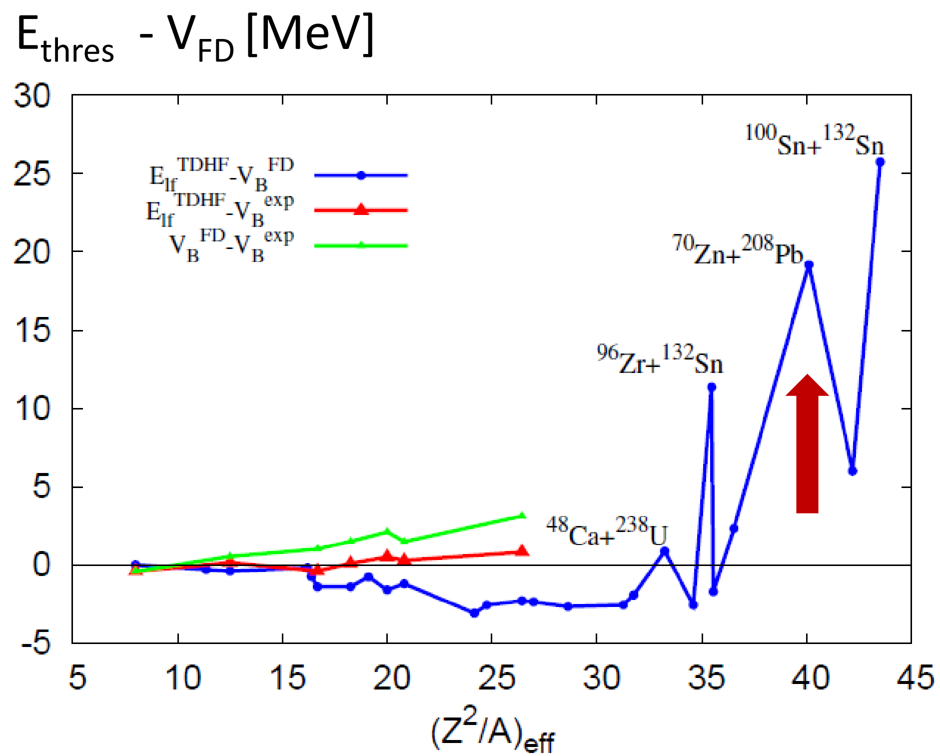
Analysis with Langevin equation

Aritomo et al., PRC85 (2012) 044614

Fusion threshold energy from TDHF



Simenel et al., J.Phys.Conf.Ser. 420 (2013) 012113

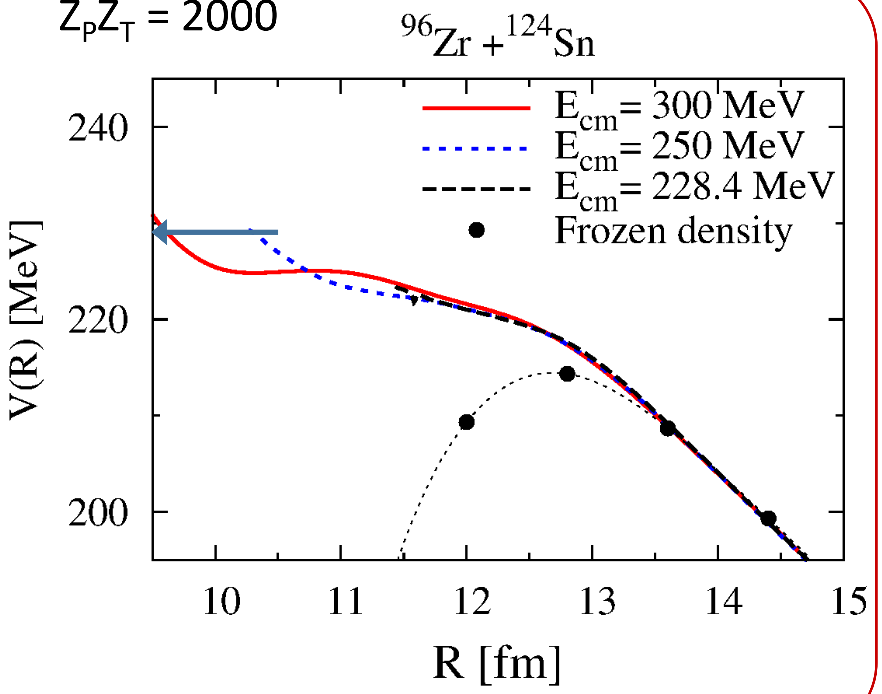


Guo, Nakatsukasa, EPJ.Conf. 38 (2012) 09003

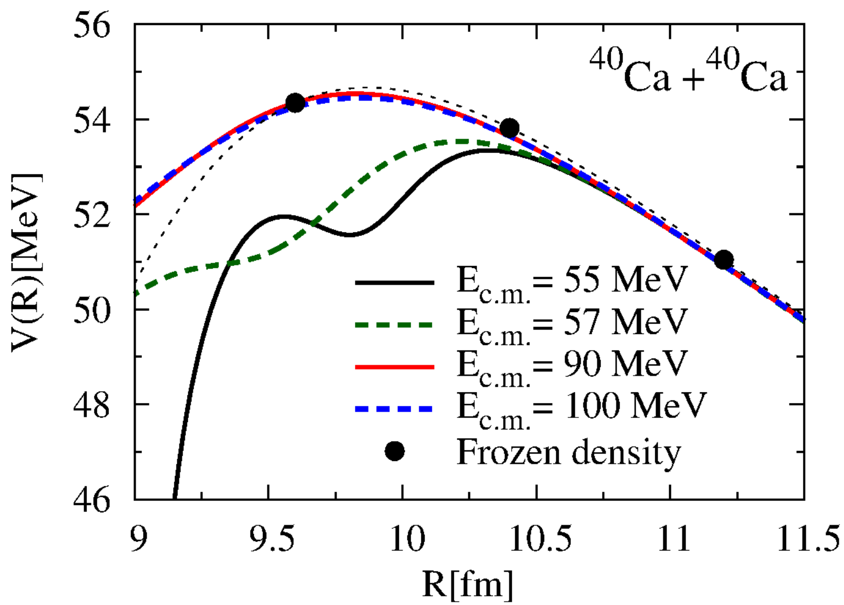
Results: Comparison of potentials

Heavy system vs. Light system

$Z_P Z_T = 2000$



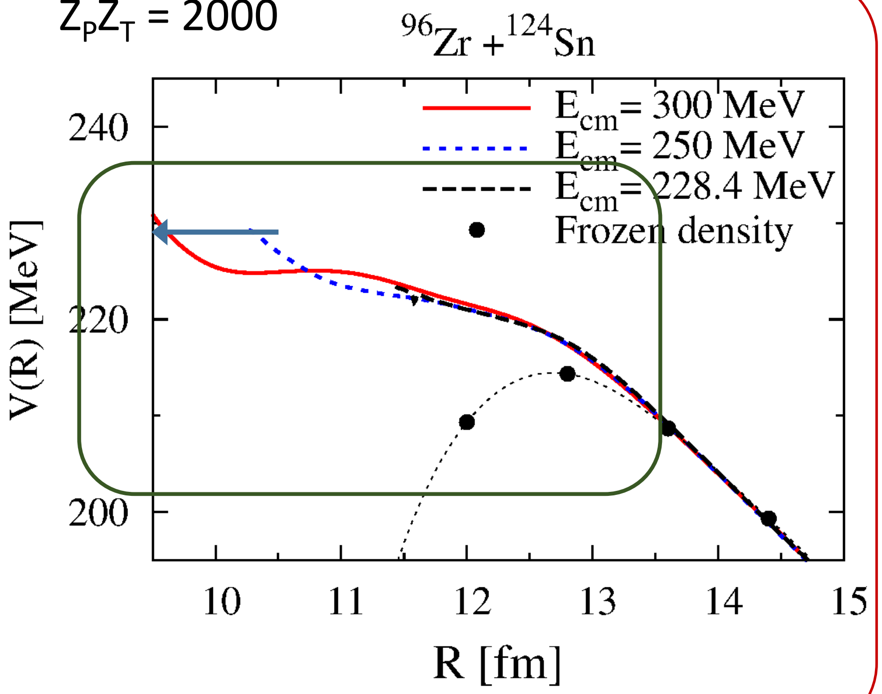
$Z_P Z_T = 400$



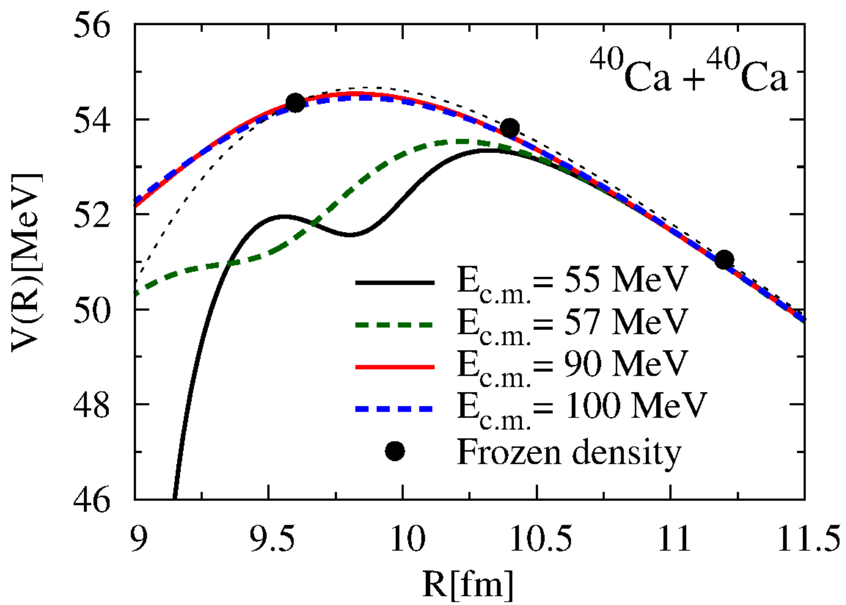
Results: Comparison of potentials

Heavy system vs. Light system

$Z_P Z_T$ = 2000



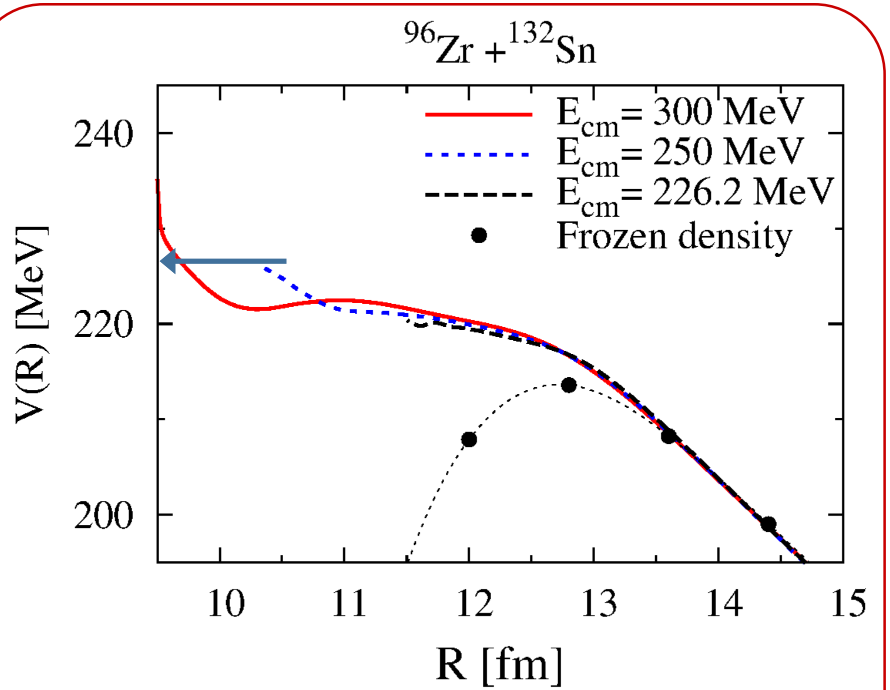
$Z_P Z_T$ = 400



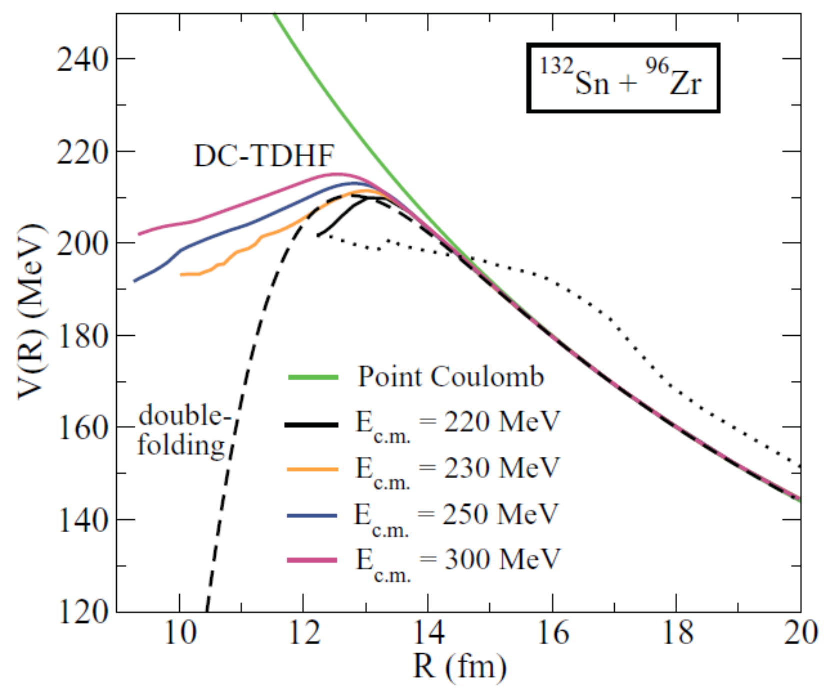
- Vanish the potential barrier
- Energy dependence is less around the “barrier”

Results: Comparison of potentials

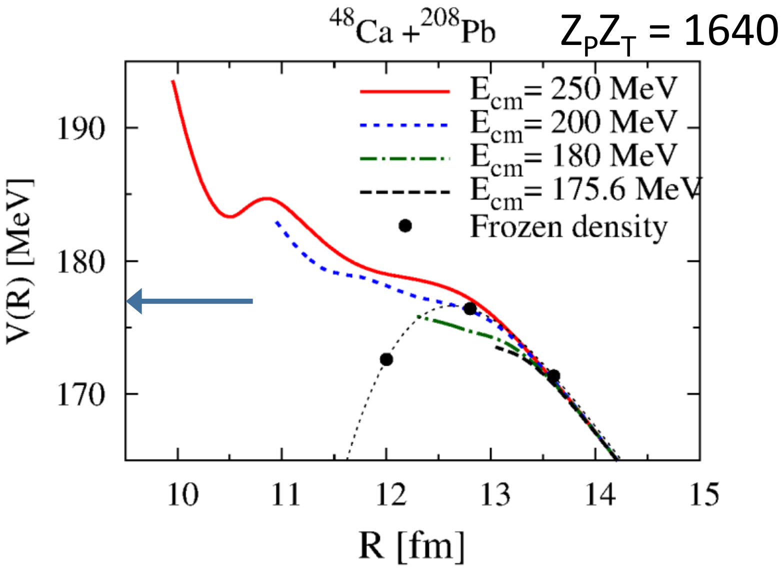
Our model v.s. Density-constrained TDHF

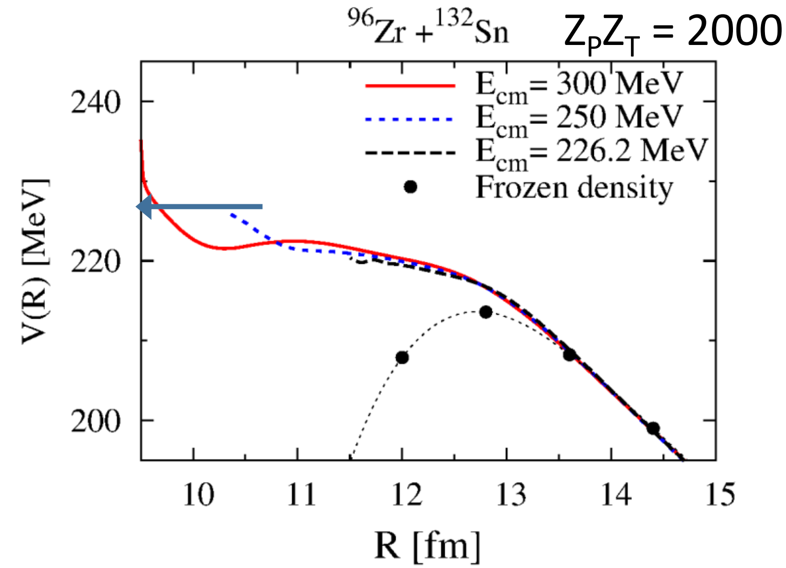


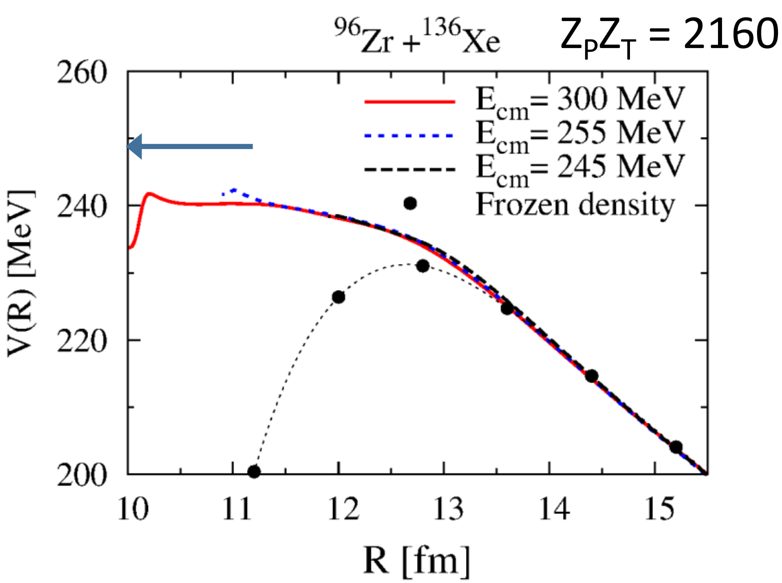
Oberacker et al., PRC 82 (2010) 034603

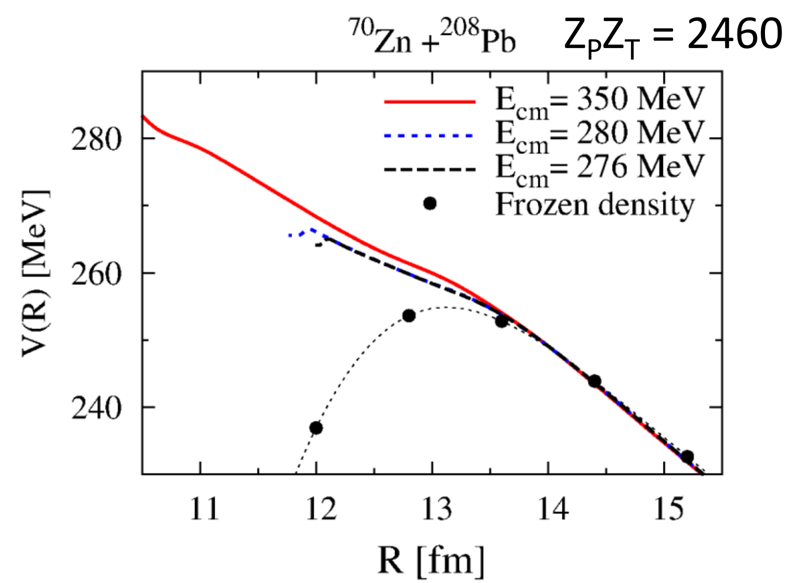


Potentials of heavy systems



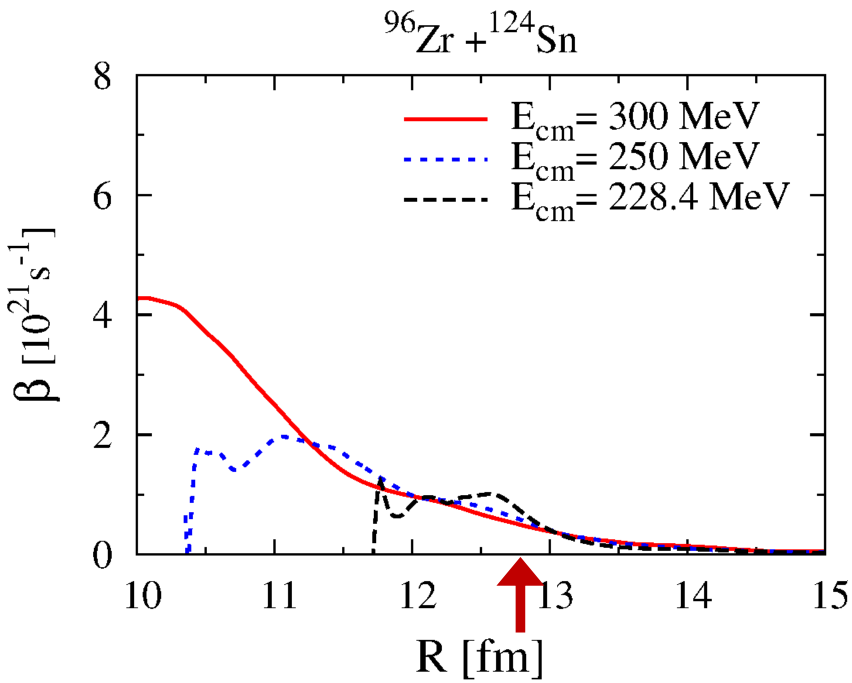


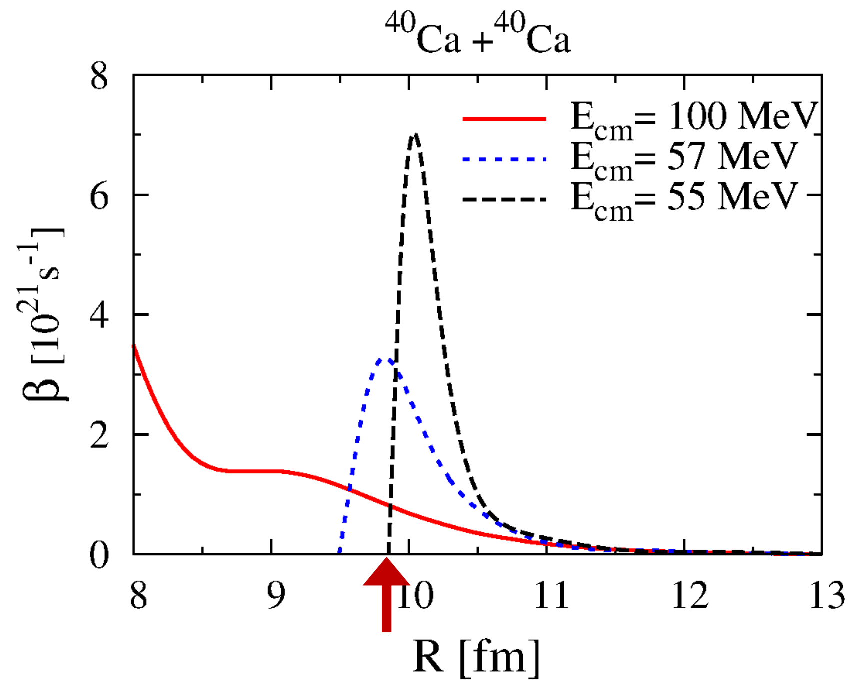




Property of friction $\frac{dP}{dt} = -\frac{dV(R)}{dR} - \gamma(R)\frac{dR}{dt}$

Heavy system vs. Light system



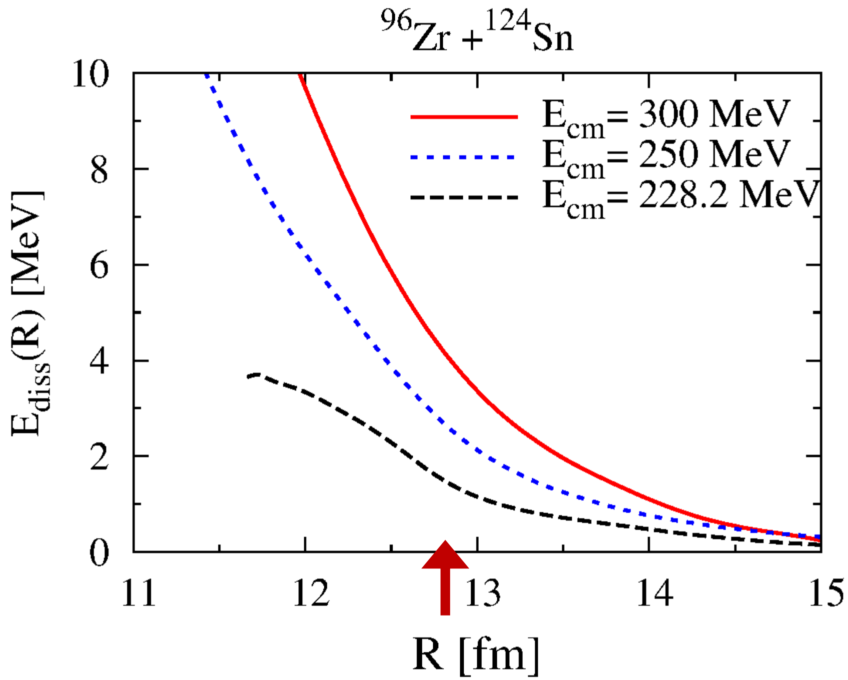


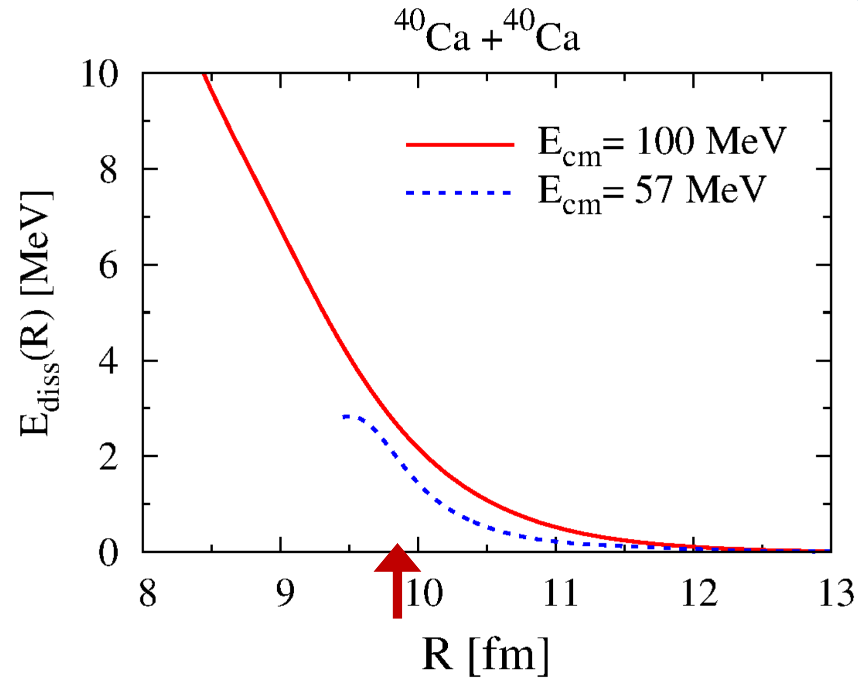
- Same order of magnitude
- Energy dependence is less

Dissipation energy

$$E_{\rm diss}(R(t)) = \int_0^t dt'\ \gamma(R(t'))\left(\frac{dR}{dt}\right)^2$$

Heavy system vs. Light system

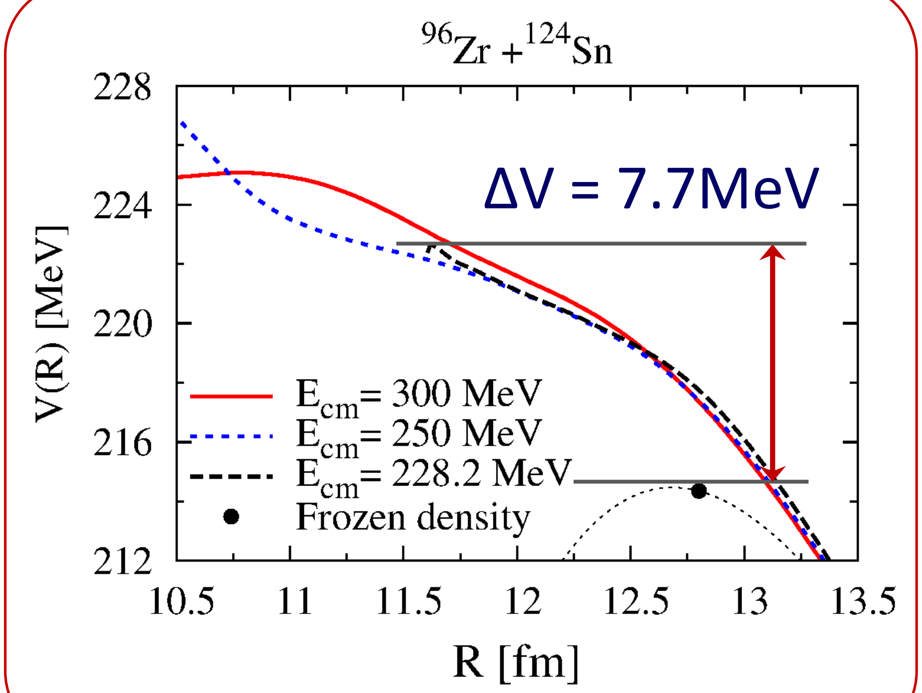




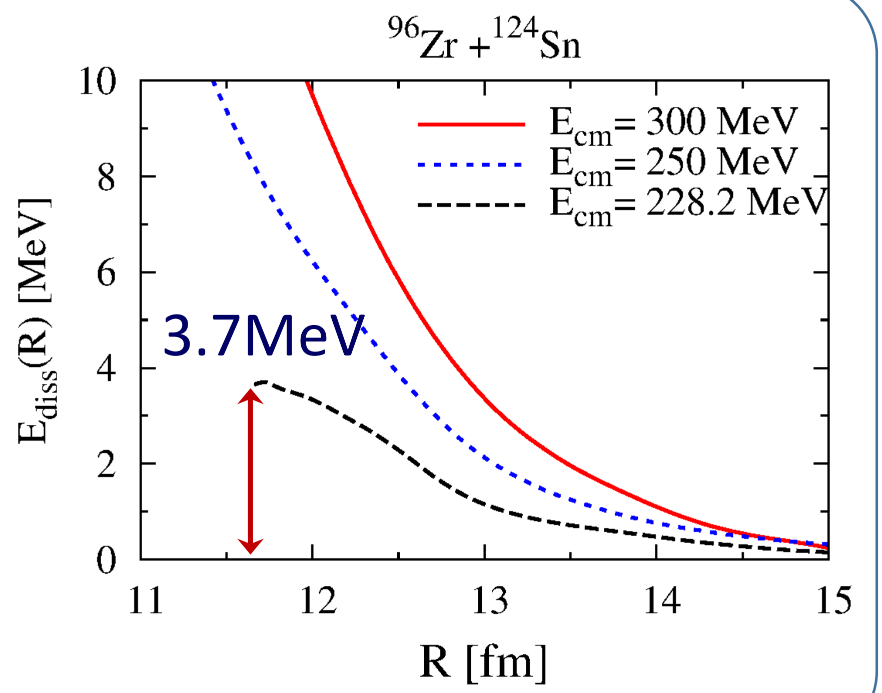
- Same order of magnitude

Origin of fusion hindrance?

Potential



Dissipated energy



- $E_{thres} - V_{FD}$ = 228.4 – 214.4 = 14 MeV
- $\Delta V + E_{diss}$ = 7.7 + 3.7 = 11.4 MeV at R_{stop}

→ A part of the origin comes from more inside of the “barrier”

Summary

- Macroscopic reduction from TDHF in low-energy reactions
- Nucleus-nucleus potential and energy dissipation are extracted
- Energy dependence
- Fusion hindrance
- Change the property of the barrier