Fluctuation-Dissipation Dynamics of Fission and Heavy-Ion Fusion Reactions

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INT13-3 Quantitative Large Amplitude Shape Dynamics: fission and heavy ion fusion

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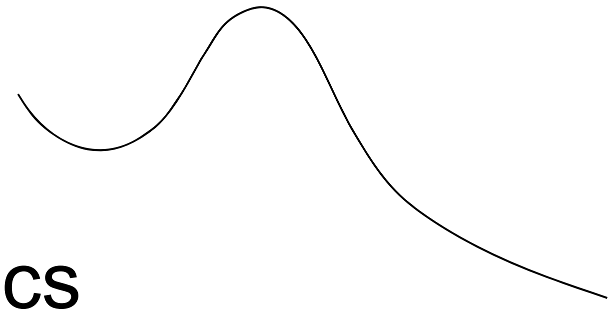
Theories for fission

- Transition state method
  - Statistical approach
    - Fission width
- Fluctuation-dissipation dynamics
  - Dynamical approach (Macroscopic)
    - Fission width
    - Saddle-to-scission dynamics
- Time-dependent mean field theory
  - Dynamical approach (Microscopic)
    - Saddle-to-scission dynamics



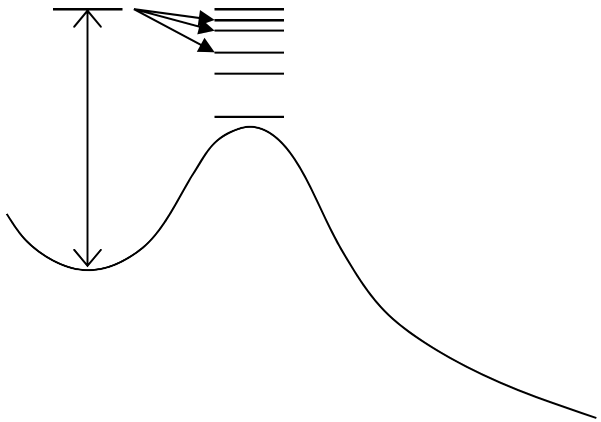
Transition state method

- Fission width

$$dE\int_0^{E-B_f}\frac{vdp}{2\pi\hbar}\rho_A^*(E-B_f-\varepsilon)=dE\frac{\Gamma_f}{\hbar}\rho_A(E)$$

# of decay per unit time

$$\Gamma_f=\frac{1}{2\pi\rho_A(E)}\int_0^{E-B_f}d\varepsilon\rho_A^*(E-B_f-\varepsilon)$$



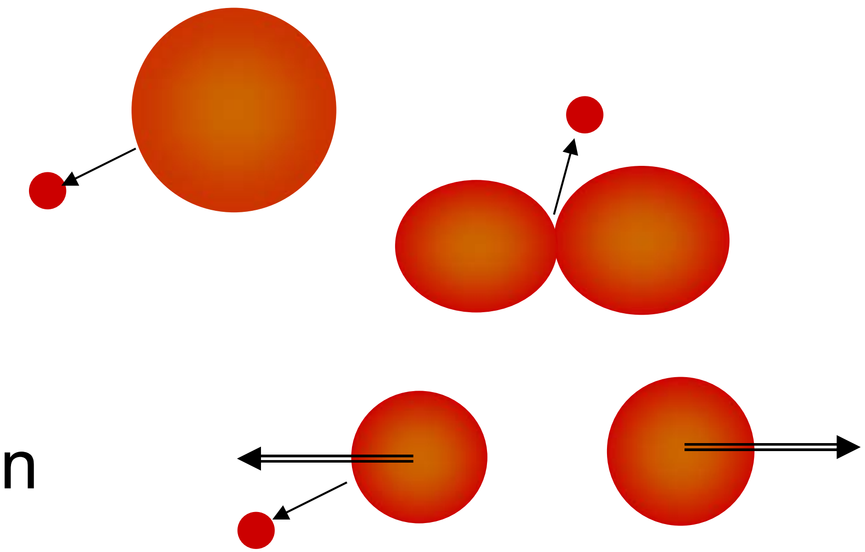
- Fermi gas level density

$$\rho(E^*)\propto\exp\left(2\sqrt{aE^*}\right)$$

$$\Gamma_f=\frac{T}{2\pi}\exp\left(-\frac{B_f}{T}\right)\qquad E^*=aT^2$$

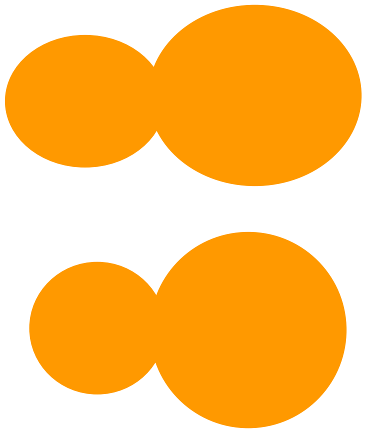
Key quantities in fission

- Fission rate
  - Height of fission barrier
- Particle emission
  - Neutron emission
    - Pre-scission neutrons
    - Scission neutrons
    - Post-scission neutrons
  - Charged particle emission
  - Gamma emission
- Fission fragments
  - Mass distribution
  - Total kinetic energy distribution



Mass & TKE distribution

- Mass distribution is essentially determined by shell correction energy
  - At saddle :position & height fission barrier
  - Fission valleys and ridges
  - Fragments :magic number
- TKE (total kinetic energy)
  - Mainly Coulomb repulsion
  - Scission configuration
    - Nature of nuclear dissipation
- Dynamical treatment is necessary

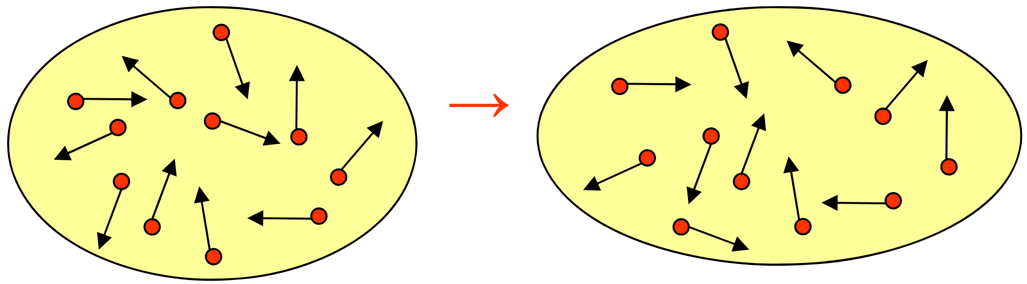


Fluctuation-dissipation dynamics

- Brownian picture
  - Macroscopic degree(s) of freedom interacting with microscopic degrees of freedom in thermal motion

    Dissipation (collective → nucleonic) ⇐ Friction

    Fluctuation (nucleonic → collective) ⇐ Random force



- Macroscopic degrees of freedom

  = Nuclear shape

  (elongation, deformation, neck, mass asymmetry etc.)

Two approaches

Langevin equation

$$\frac{dq}{dt} = \frac{p}{m} \qquad \frac{dp}{dt} = -\frac{\partial U}{\partial q} - \frac{\gamma}{m} p + \sqrt{\gamma T} R(t)$$

$$\langle R(t) \rangle = 0, \ \langle R(t_1) R(t_2) \rangle = 2\delta(t_1 - t_2)$$

Eq. of motion of a Brownian particle

Focker-Planck (Kramers) equation

$$\frac{\partial f}{\partial t} = -\frac{p}{m}\frac{\partial f}{\partial q} + \frac{\partial U}{\partial q}\frac{\partial f}{\partial p} + \frac{\gamma}{m}\frac{\partial}{\partial p}(pf) + \gamma T \frac{\partial^2 f}{\partial p^2}$$

Distribution of the Brownian particles

Fission width by Kramers theory

- Diffusion process over barrier

$$\frac{\partial f}{\partial t} = -\frac{p}{m}\frac{\partial f}{\partial q} + \frac{\partial U}{\partial q}\frac{\partial f}{\partial p} + \frac{\gamma}{m}\frac{\partial}{\partial p}(pf) + \gamma T\frac{\partial^2 f}{\partial p^2}$$

- U = parabola + inverse parabola
  - Analytic solution

$$\Gamma_f = K\frac{\hbar\omega}{2\pi}\exp\left(-\frac{B_f}{T}\right)$$

$$U = \begin{cases} \frac{1}{2}m\omega^2 q^2 \\ B_f - \frac{1}{2}m\omega_B^2(q-q_B)^2 \end{cases}$$

$$K = \frac{1}{2\omega_B}\left[\sqrt{\beta^2 + 4\omega_B^2} - \beta\right], \quad \beta = \frac{\gamma}{m}$$

Kramers factor

cf. BW
$$\Gamma_f = \frac{T}{2\pi}\exp\left(-\frac{B_f}{T}\right)$$

Transport coefficients

- Inertia mass tensor
  - Hydrodynamical mass
    - Werner-Wheeler approximation
  - Cranking mass
- Friction tensor
  - One-body friction
    - Wall and Window formula
    - Interaction of nucleons with nuclear surface (wall)
    - Exchanging nucleons through the neck window
  - Two-body viscosity
    - Energy loss by nucleon-nucleon collisions

TKE systematics

- Viola systematics
  (1985, V. E. Viola)

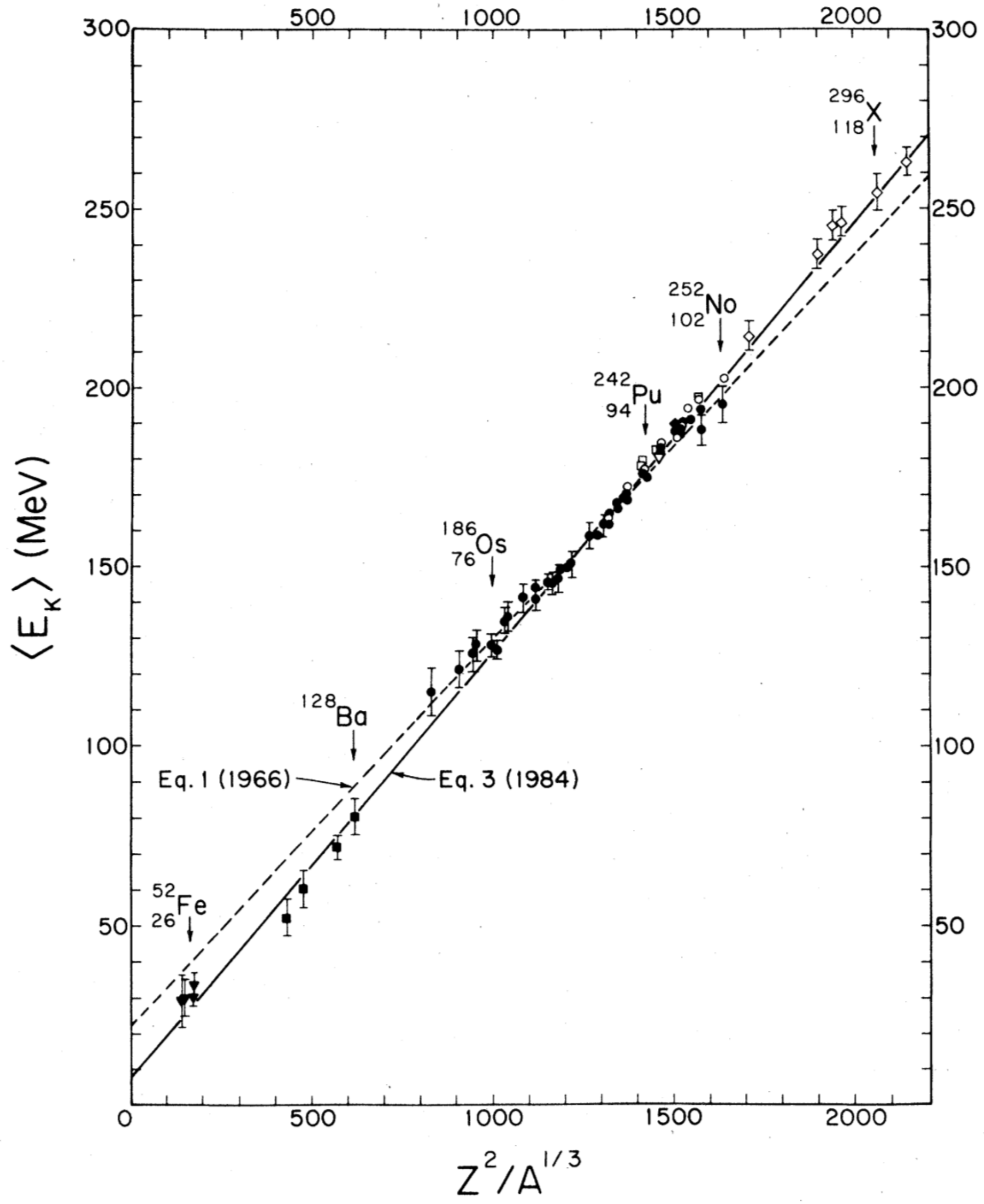
$$\langle E_K \rangle = 0.1071 Z^2/A^{1/3} + 22.2 \text{ MeV}$$

Data before 1966

$$\langle E_K \rangle = 0.1189 Z^2/A^{1/3} + 7.3 \text{ MeV}.$$

Data up to 1984

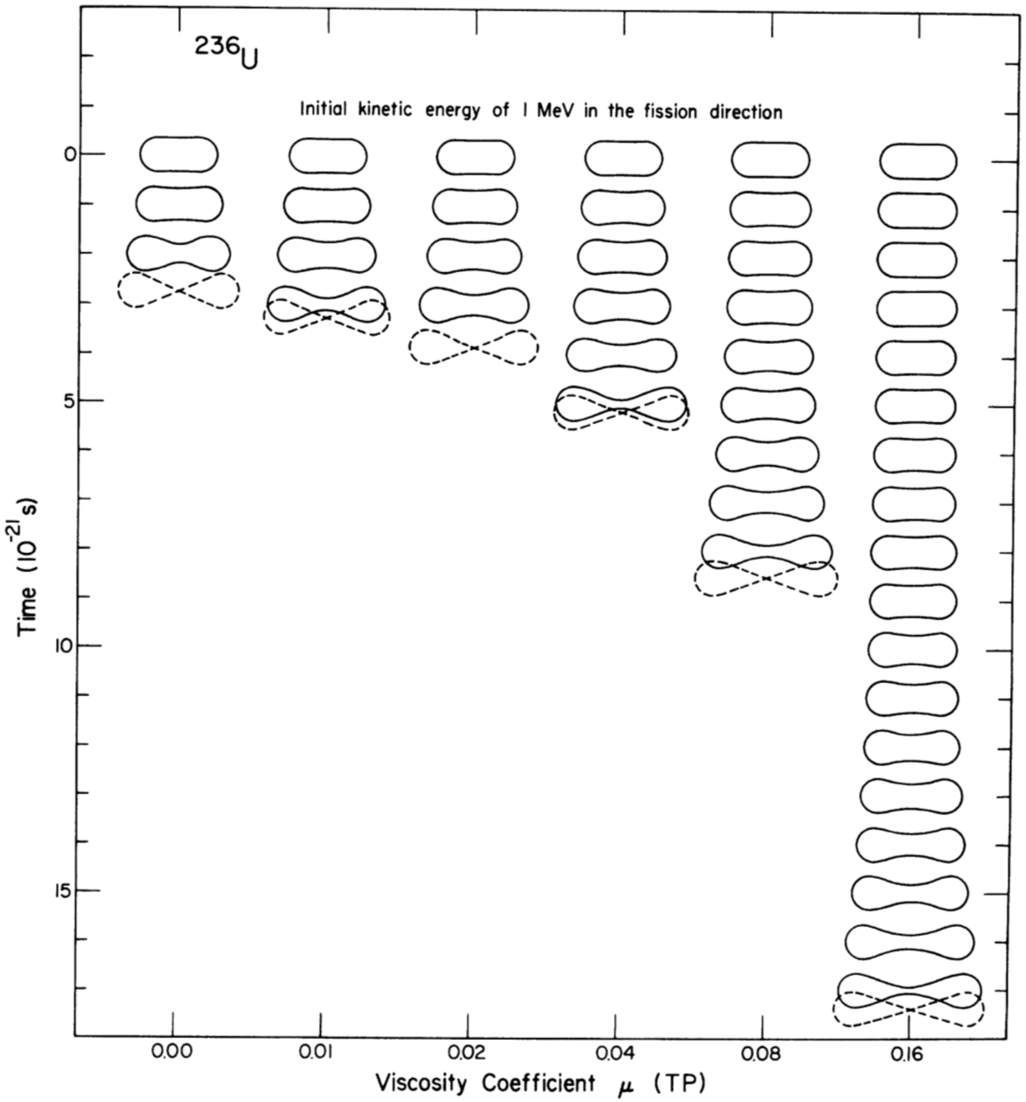
- Main contribution to TKE comes from Coulomb repulsion
- Measure of the fragment deformation



Saddle-to-scission dynamics

- Saddle-to-scission time (τ_{ssc}) depends on the dissipation
- Scission shape also depends on the dissipation
- Fragment TKE also depends on the dissipation

$$TKE = K_{pre} + V_C$$

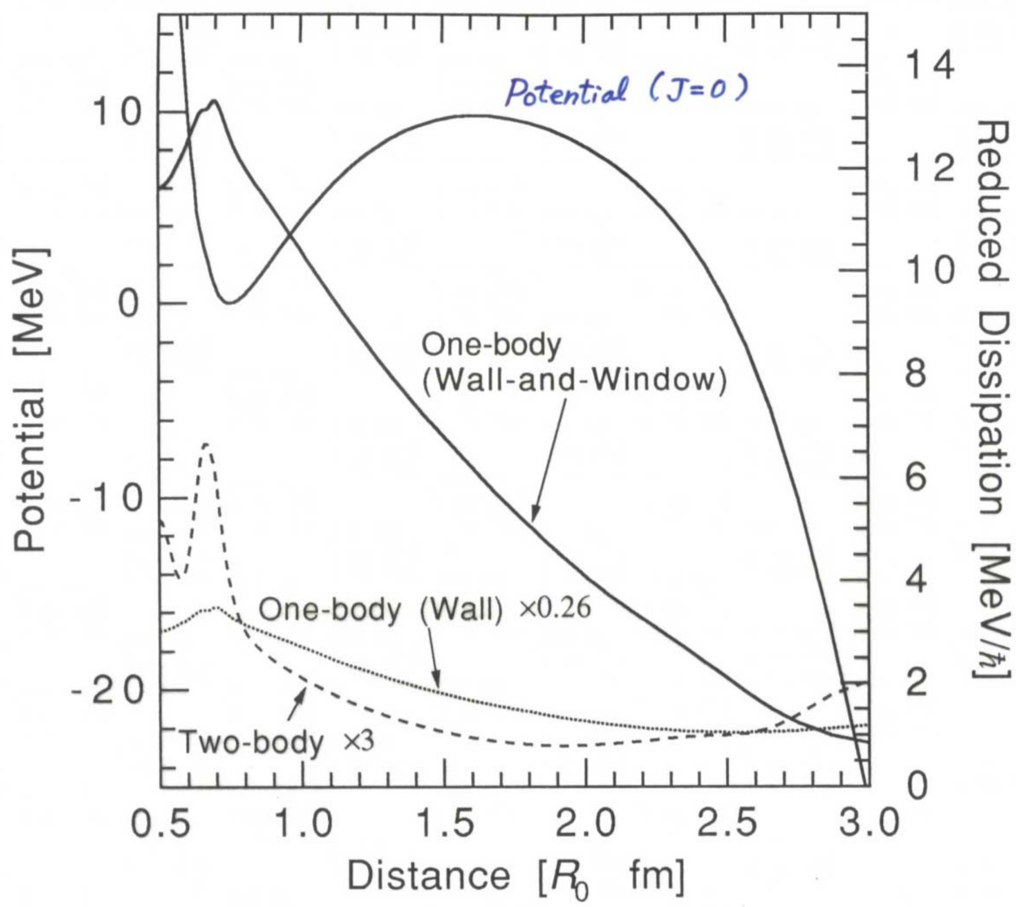


Coordinate dependence of friction

- Reduced dissipation

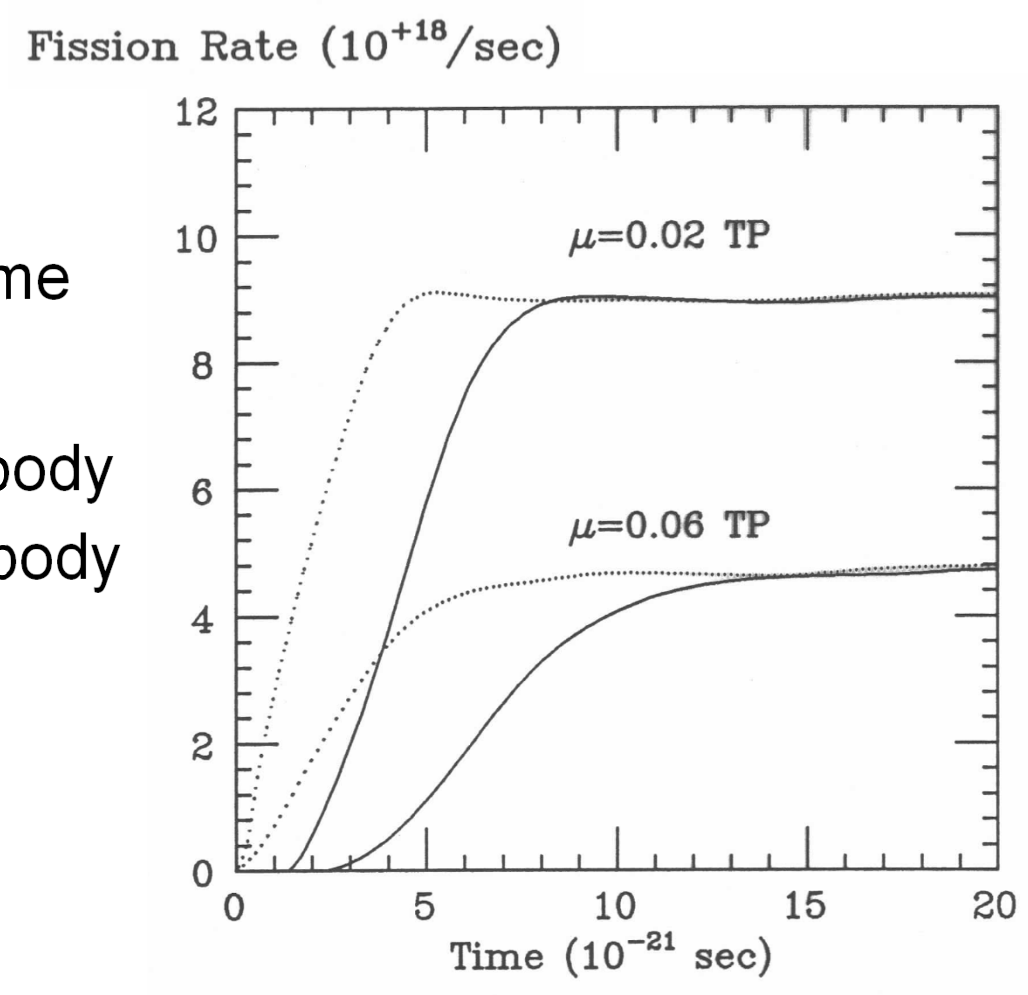
$$\beta = \gamma / m$$

- Different dependence on elongation
  - Wall-and-Window
  - Wall
  - Two-body



Time-dependent fission rate

- Delay of fission
  - $\tau_{delay} = \tau_{tr} + \tau_{ssc}$
  - Transient time
  - Saddle-to-scission time
- Typical time
  - $\tau_{tr}, \tau_{ssc} \approx 10^{-21}$ s : two-body
  - $\tau_{tr}, \tau_{ssc} \approx 10^{-20}$ s : one-body



Fission and nuclear dissipation

- Two types of dissipation tensors
  - One-body Wall-and-Window dissipation
  - Two-body hydrodynamical dissipation

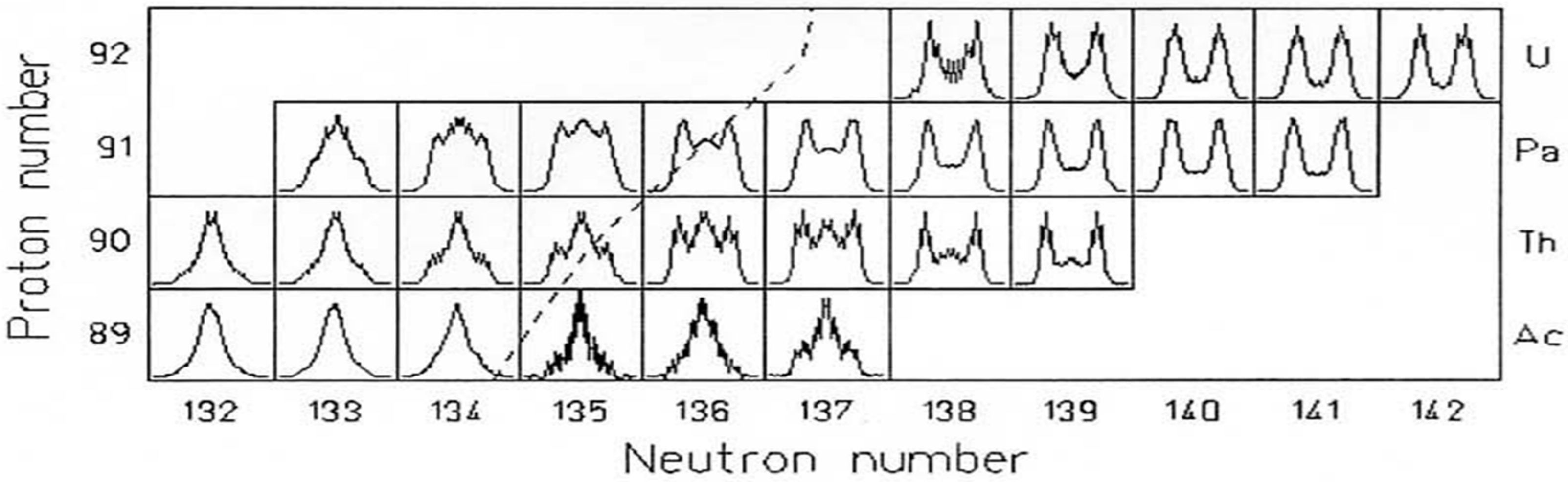
$$\tau_{delay} = \tau_{tr} + \tau_{ssc}$$

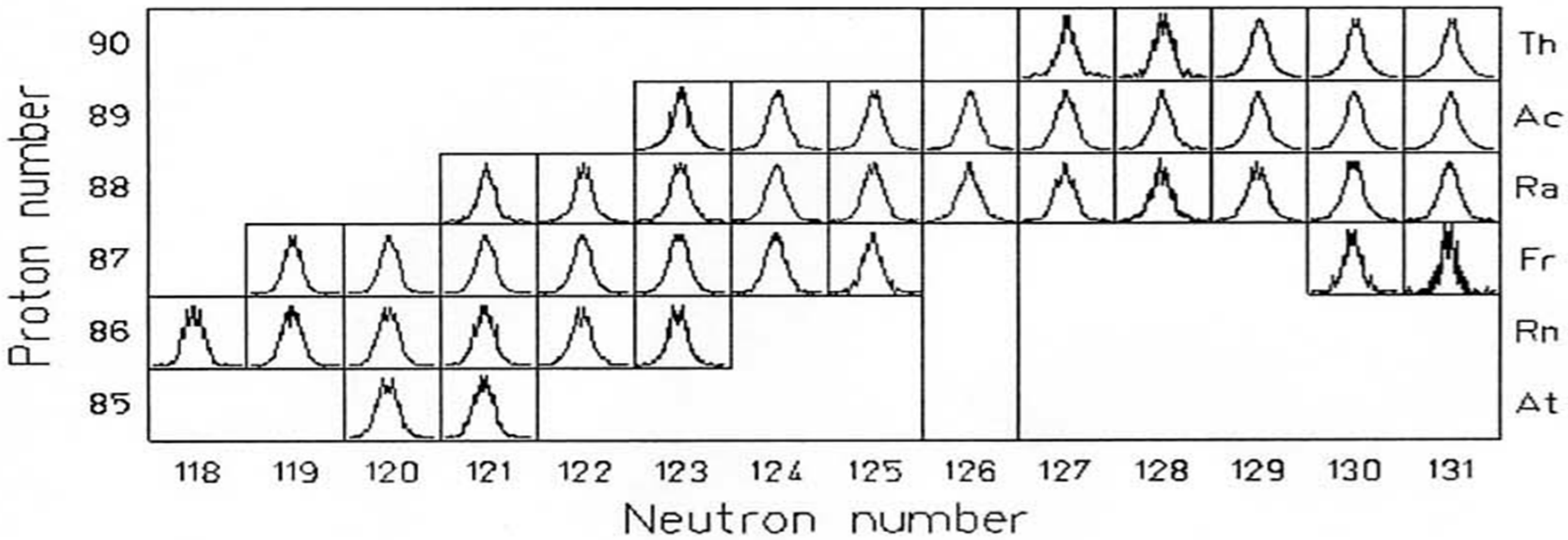
$$TKE = K_{pre} + V_C$$

80.7MeV		σ_{fus}	σ_{fiss}	σ_{er}	ν_{pre}	*TKE*
1-body		1150	790	360	2.93	135.1
2-body	0.02 TP	1150	928	222	2.06	124.9
	0.06 TP	1150	817	333	2.84	108.6
Exp.		1150	767	383	3.2±0.3	–

195.8MeV		σ_{fus}	σ_{fiss}	σ_{er}	ν_{pre}	*TKE*
1-body		1400	1244	156	7.33	137.0
2-body	0.02 TP	1400	1338	62	4.79	123.6
	0.06 TP	1400	1261	139	7.03	107.4
Exp.		1400	–	–	7.7±0.3	139

Fission modes





K. –H. Schmidt et al.

Shell correction energy

- Strutinsky method

$$\Delta E_{shell} = \sum_{i}^{N} \varepsilon_i - \widetilde{E}$$

$$N = \int^{\varepsilon_F} g(\varepsilon)\, d\varepsilon \ , \ \widetilde{E} = \int^{\varepsilon_F} g(\varepsilon)\varepsilon\, d\varepsilon$$

$$g(\varepsilon) = \sum_{i}^{N} \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(\varepsilon - \varepsilon_i)^2}{2\sigma^2} \right) \sum_{m} H_m \left(\frac{\varepsilon - \varepsilon_i}{\sigma} \right)$$

- Macro-microscopic approach

$$\widetilde{E} \to E_{Macro} \ , \ E = E_{Macro} + \Delta E_{shell}$$

- Damping of shell correction

$$\Delta E_{shell}(q, E_x) = \Delta E_{shell}(q, E_x = 0) \exp(-E_x / E_d)$$

- Essential for asymmetric fission

Extension to multi-dimension

Multi-dimensional Langevin equation

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j \qquad i, j, k = 1, \ldots, N$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

$$\langle R_i(t) \rangle = 0, \ \langle R_i(t_1) R_j(t_2) \rangle = 2\delta_{ij}\delta(t_1 - t_2) \qquad \sum_k g_{ik} g_{jk} = T\gamma_{ij}$$

$m_{ij}(q)$ Hydrodynamical inertial mass

$\gamma_{ij}(q)$ Wall-and-Window (one-body) friction

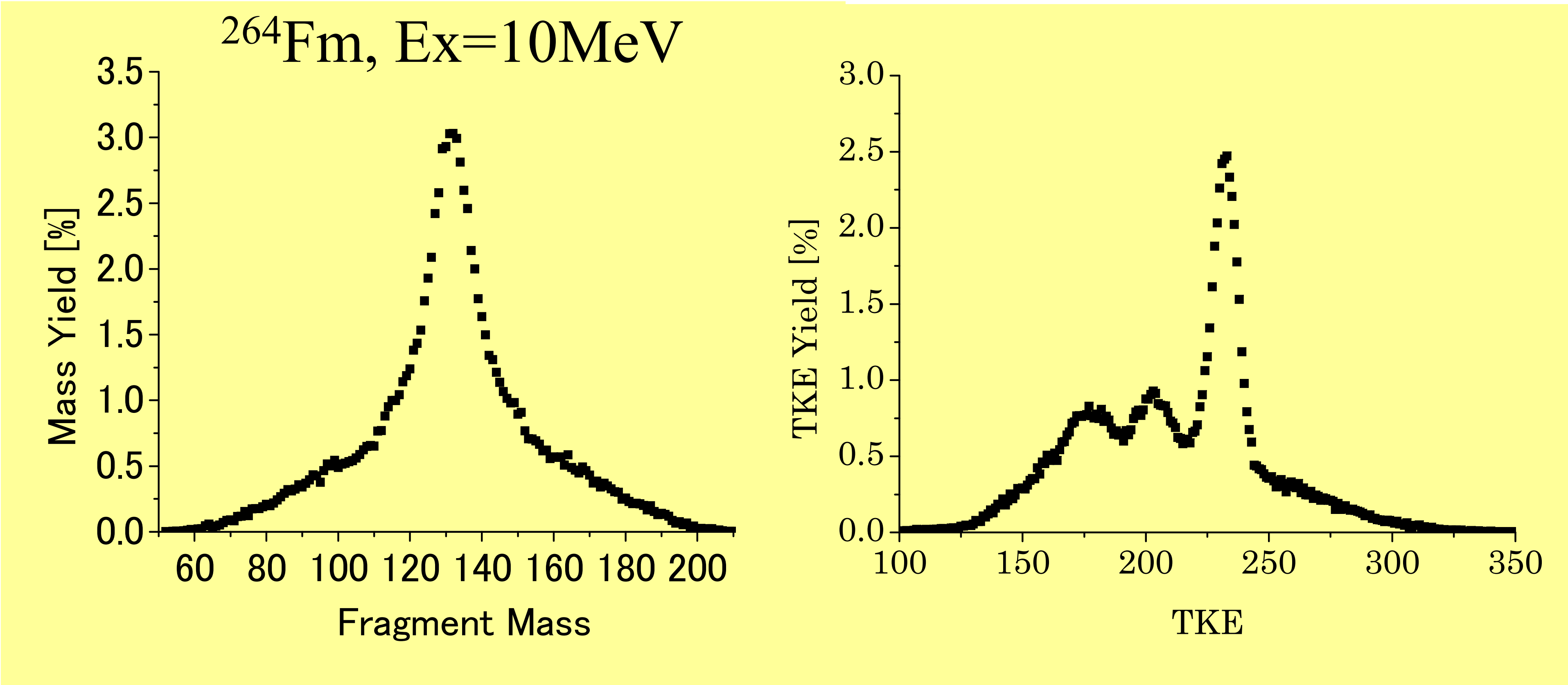
$V(q)$ Macro-microscopic potential

$\{q_i\}$: collective parameters
(elongation, fragment deformation, neck parameter, mass asymmetry)

TKE and mass distribution of fragments

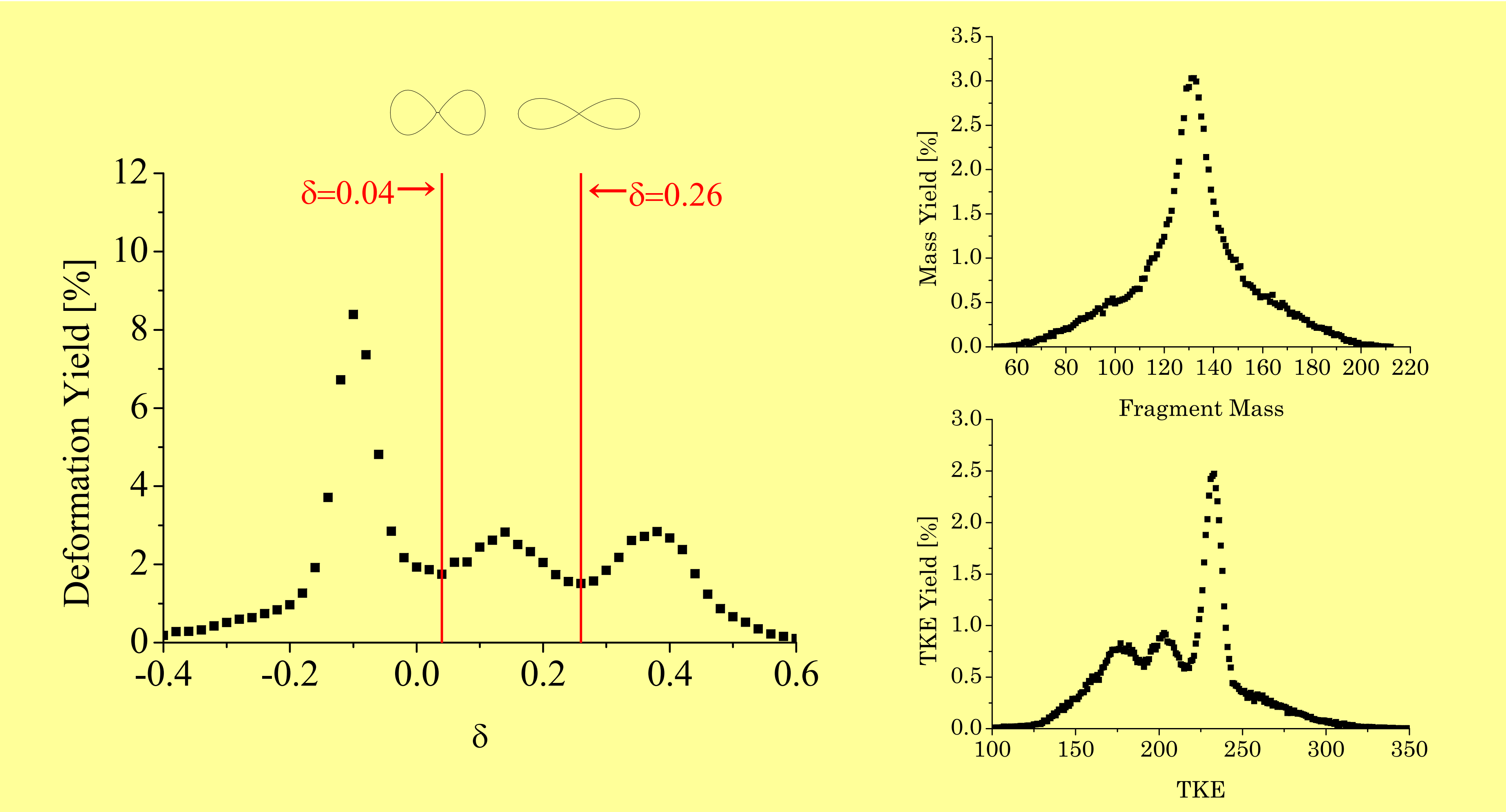
Three-dimensional Langevin calculation

elongation, fragment deformation, mass-asymmetry



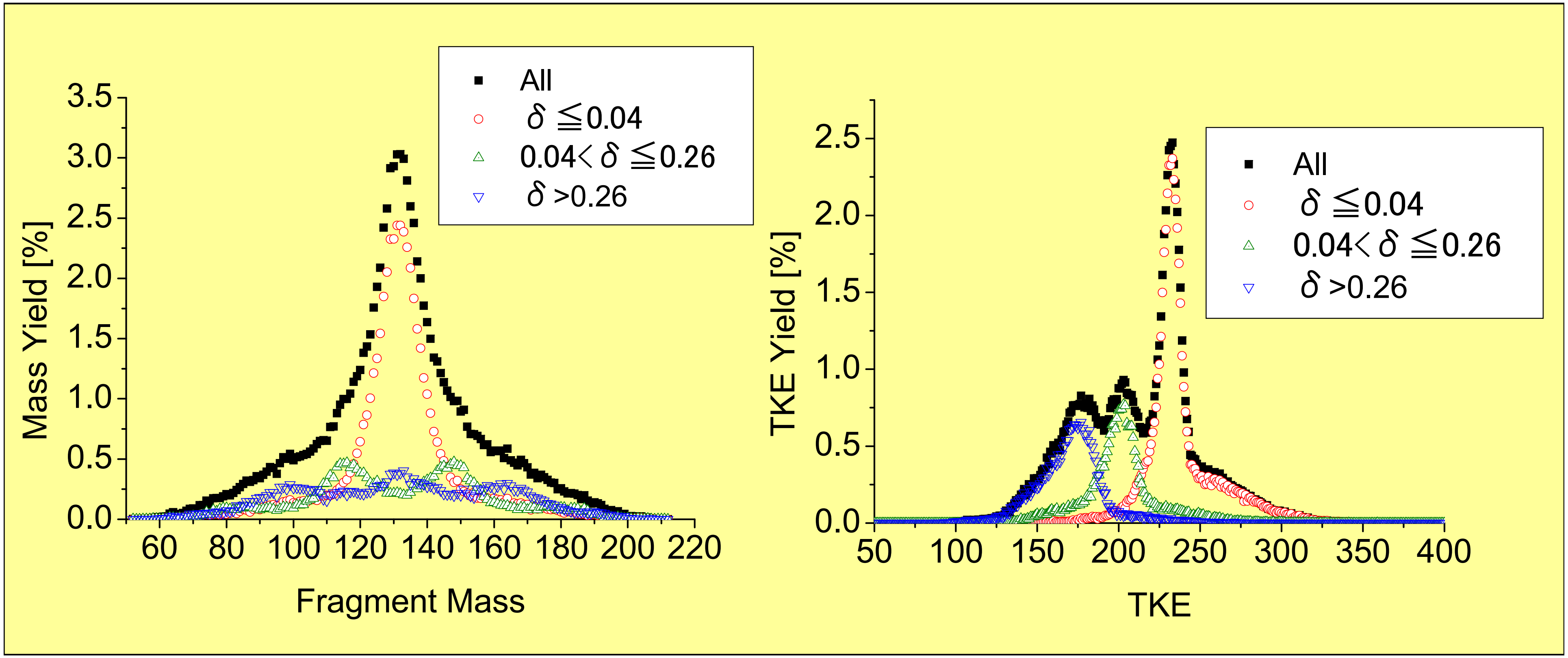
Numbers of the peaks are different in mass and TKE

Deformation of the fragments at scission



Fission modes

T. Asano et al, (J. Nucl. Radiochem. Sci. 5 (2004) 1)

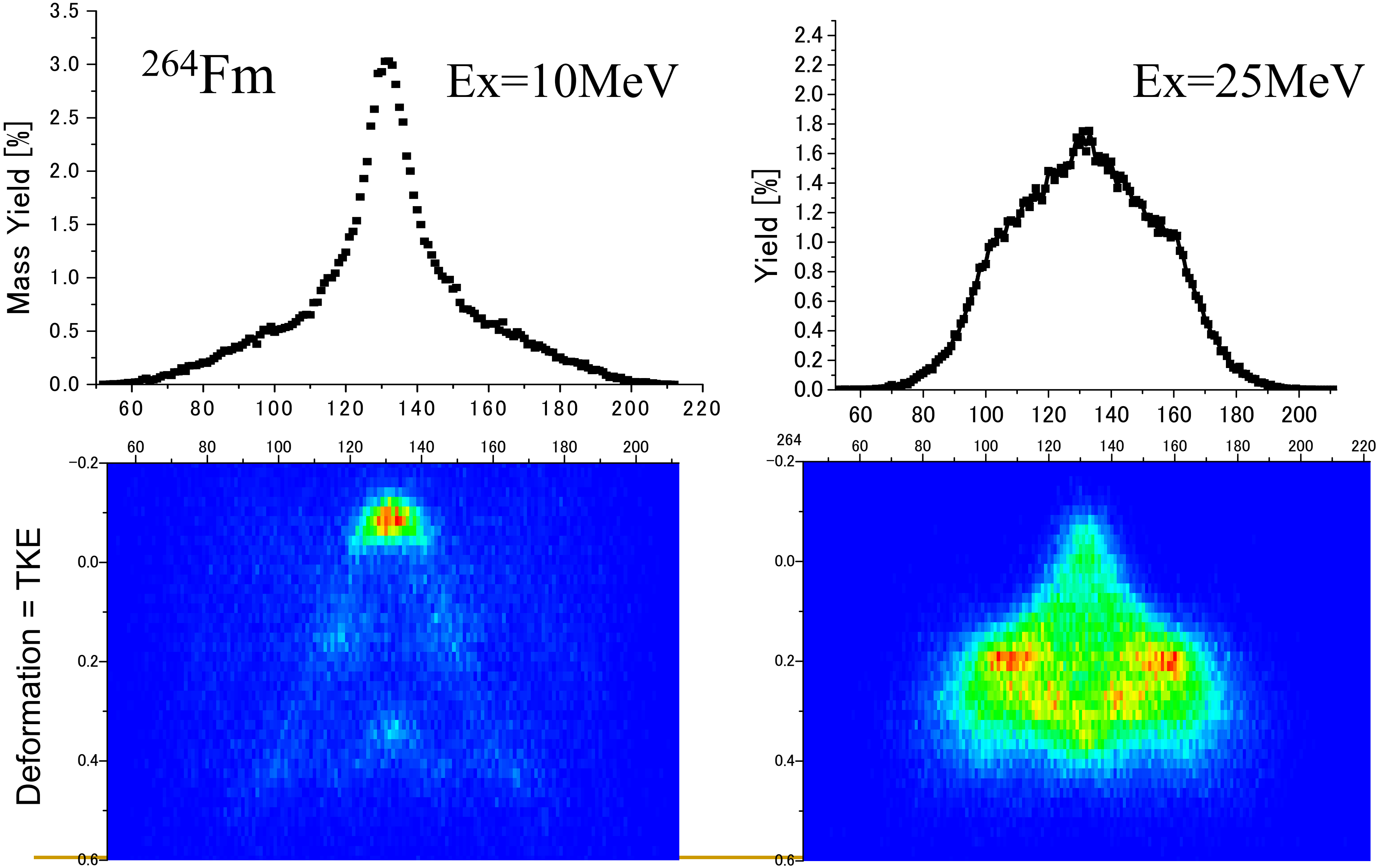


δ≦0.04 : Mass-symmetric & High TKE mode

0.04<δ≦0.26 : Mass-asymmetric & Medium TKE mode

δ>0.26 : Mass-symmetric ? & Low TKE mode

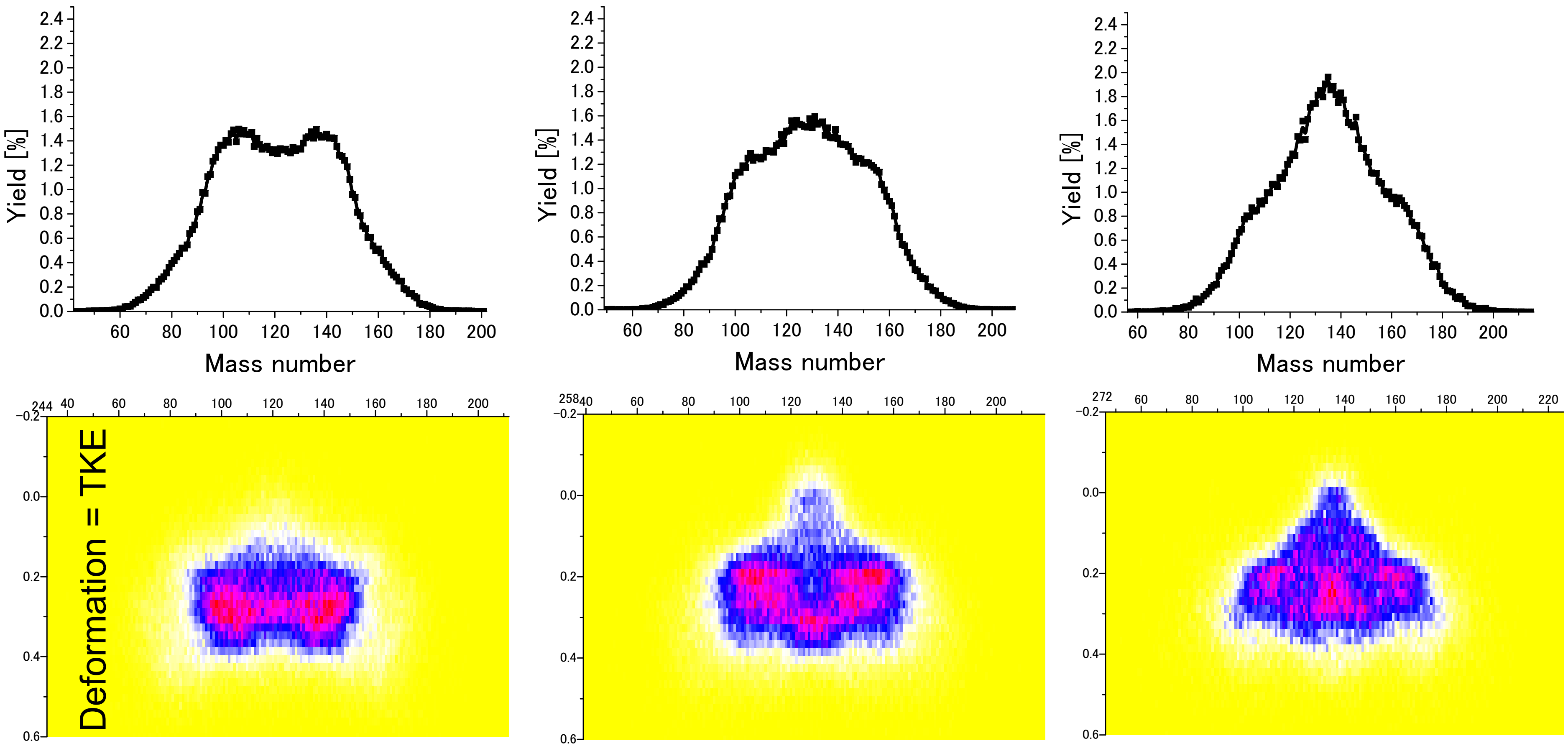
Energy dependence of mass distribution



Isotopic dependence of mass distribution

Ex=25MeV

^{244}Fm ^{258}Fm ^{272}Fm

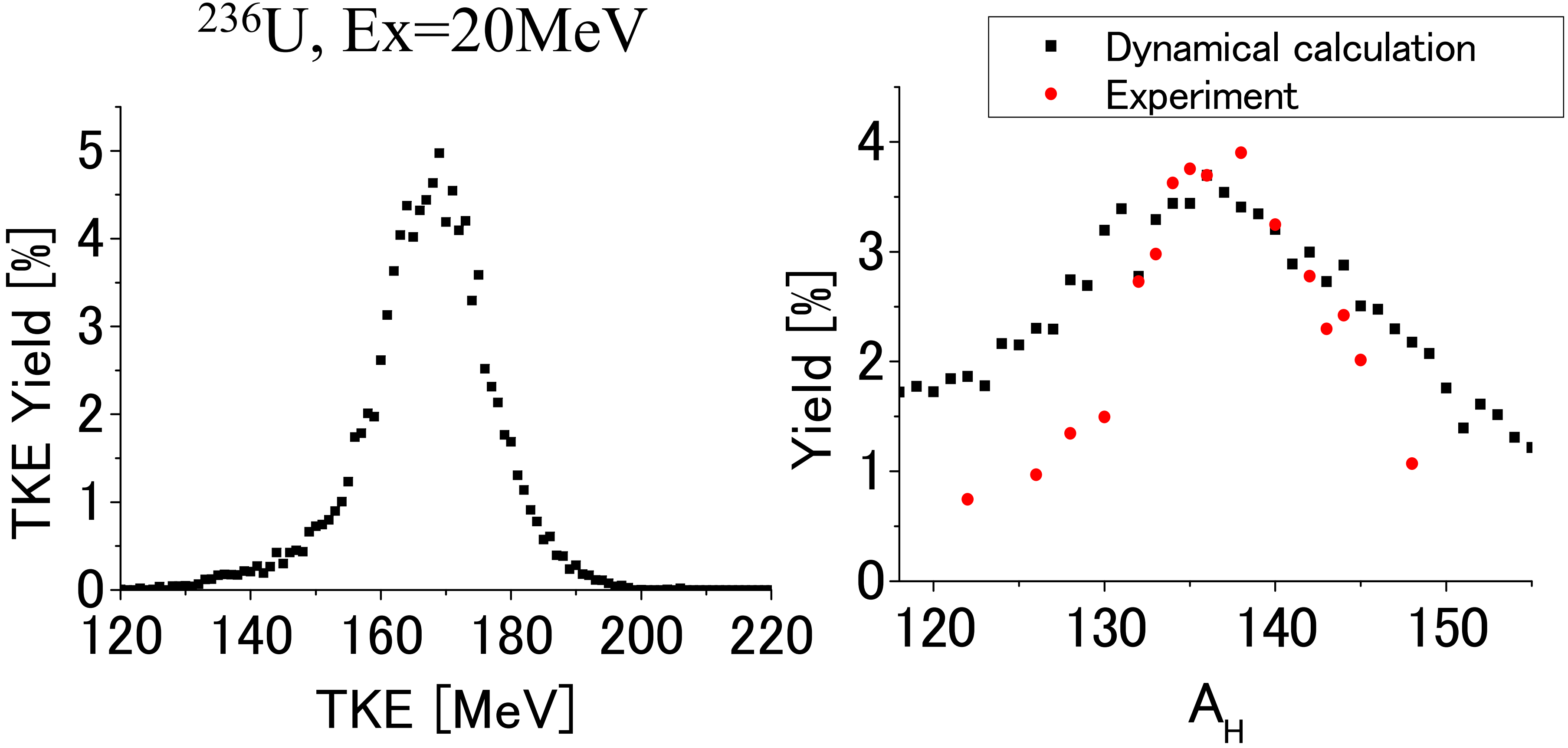


TKE and mass distribution of fragments

Three-dimensional Langevin calculation

elongation, fragment deformation, mass-asymmetry

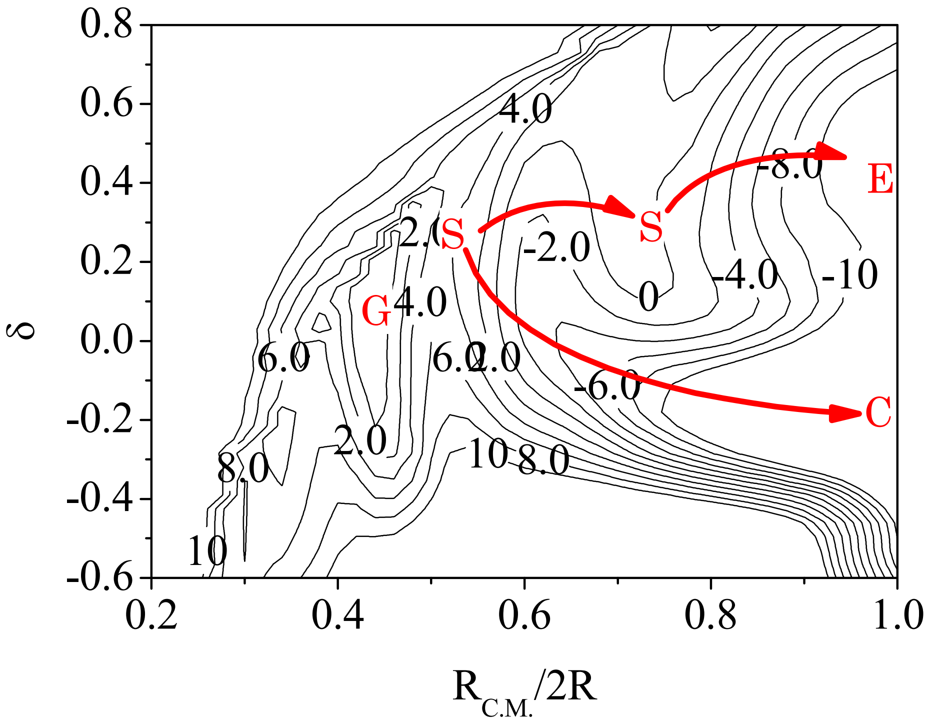
^{236}U, Ex=20MeV

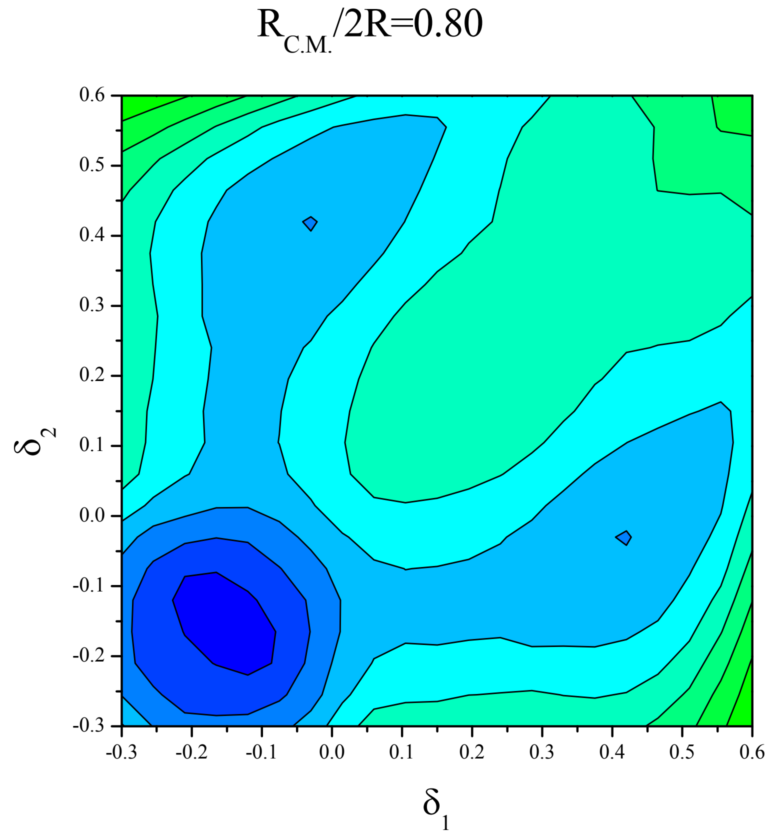


Independent deformation of fragments

Four-dimensional energy surface

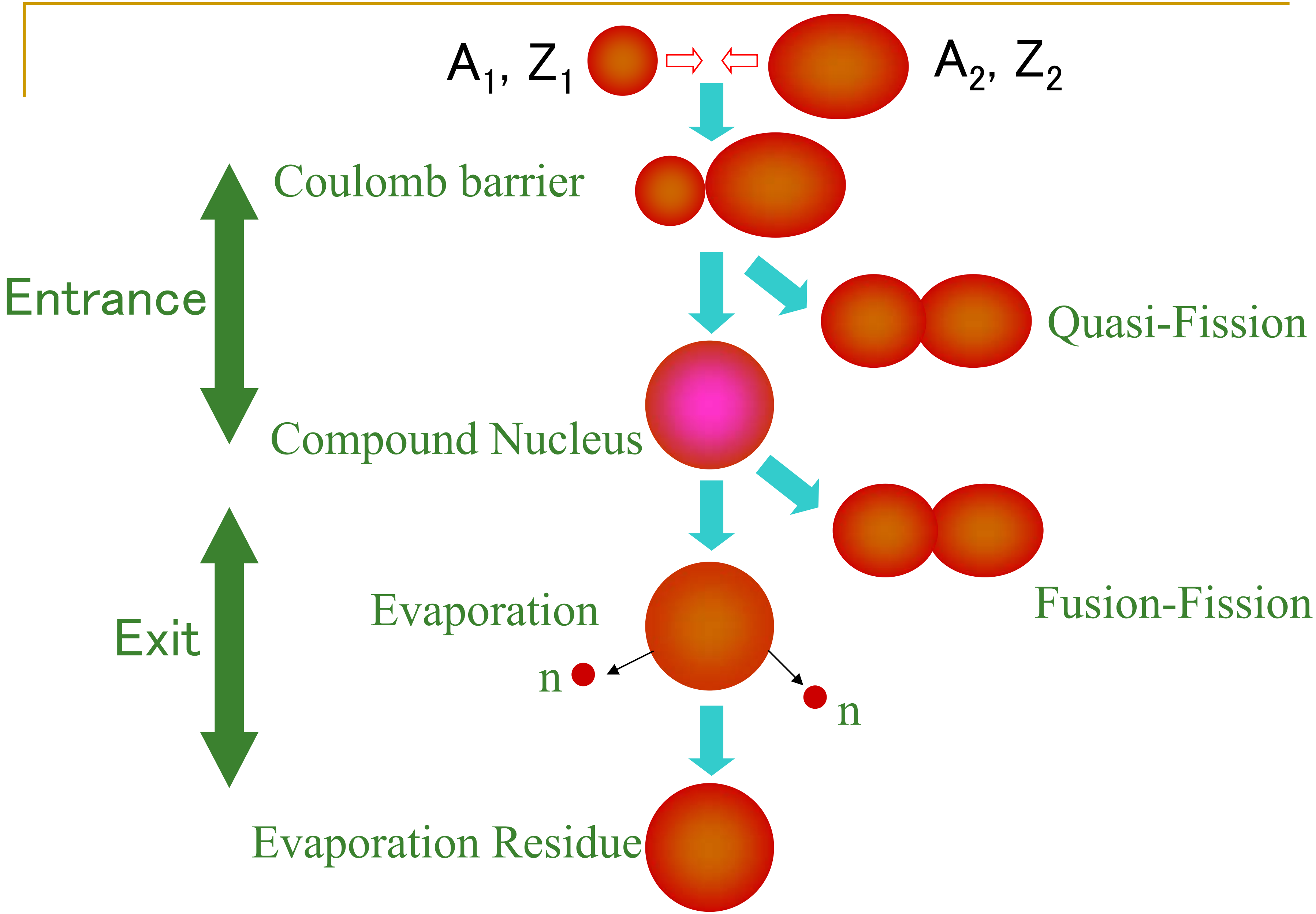
elongation, 2 fragment deformations, mass-asymmetry





Superheavy Elements

- How many elements can exist in nature?
  - Stability against fission
- Superheavy elements
  - Shell-stabilized : No macroscopic fission barrier
- Heavy-ion fusion reaction
  - Hot fusion reaction
    - Actinide target + ^{48}Ca projectile
    - E_{ex} = 30-40 MeV
  - Cold fusion reaction
    - Pb, Bi target
    - E_{ex} = 10-15 MeV



A1, Z1
A2, Z2
Coulomb barrier
Entrance
Quasi-Fission
Compound Nucleus
Exit
Evaporation
Fusion-Fission
n
n
Evaporation Residue

Evaporation residue cross section

$$\sigma_{ER} = \pi \lambdabar^2 \sum_l (2l+1) T_l(E_{cm}) P_l^{for}(E_{cm}) P_l^{sur}(E^*)$$

- Evaporation residue cross section is extremely small in the synthesis of SHE

Picobarn = 10^{-12} barn

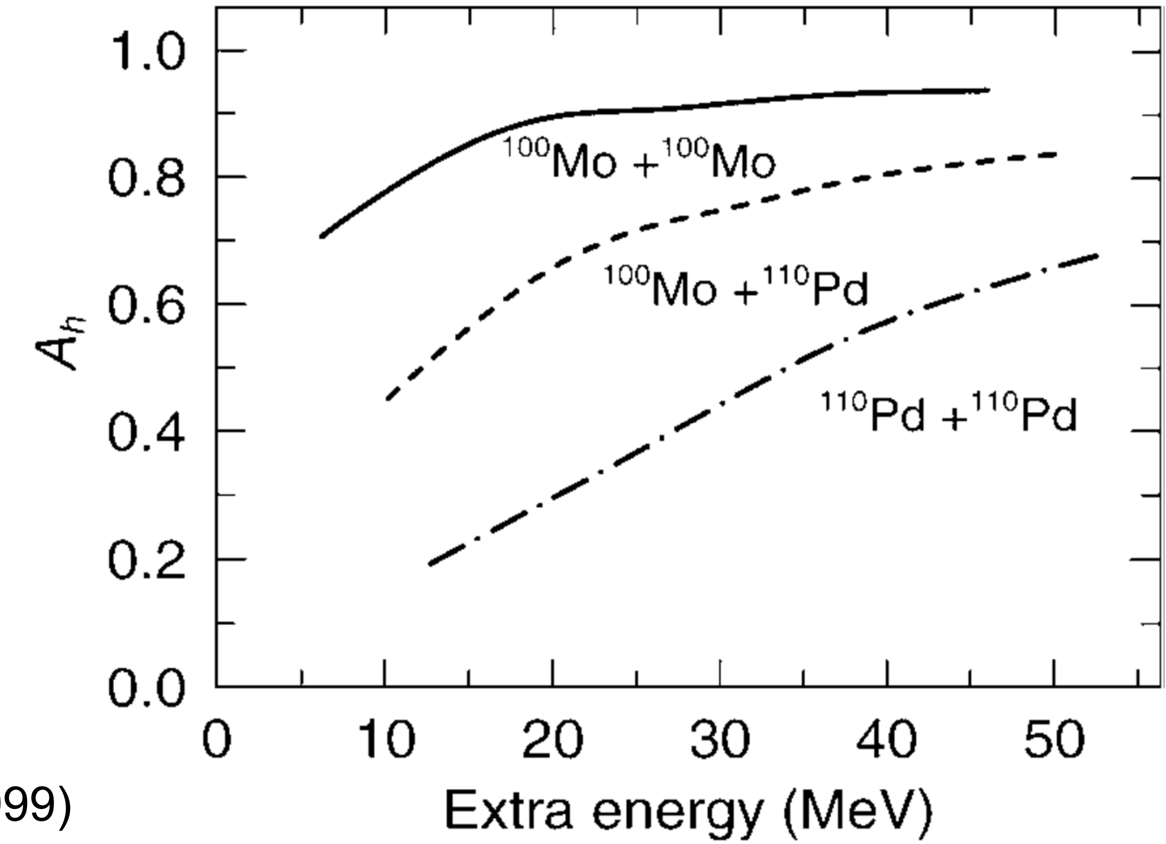
- T_l : sticking probability
  - Optical potential, dissipation
- P_l^{for} : formation probability small
  - Fluctuation-dissipation dynamics
- P_l^{sur} : survival probability small
  - Statistical approach

Fusion reaction of two heavy nuclei

- For lighter systems, fusion occurs when two nuclei touch each other.
- For heavier systems, fusion does not always occur even when two nuclei touch.
  - Fusion hindrance
  - Extra fusion barrier inside the Coulomb barrier of the entrance channel.
- For the synthesis of SHE, the fusion hindrance plays an essential roll.
  - Fusion probability by Langevin approach

Fusion hindrance

- Extra energy is necessary for fusion
  - Two-dimensional Langevin calculation
  - $Z_1 X Z_2 > 1600$
  - Dissipation of relative motion



Tokuda, Wada (1999)

Fusion hindrance (one-dimensional model)

$$U = U_B - \frac{1}{2} m \omega_B^2 q^2$$ Parabolic barrier

- Formation probability

$$P^{for} = \frac{1}{2}\mathrm{erfc}\left[\left(\frac{aB^2}{K-B}\right)^{1/4}\sqrt{\frac{\beta+\beta'}{2\beta}}\left(1-\frac{2\omega_B}{\beta+\beta'}\sqrt{\frac{K}{B}}\right)\right]$$

B: extra barrier height

K: extra kinetic energy (at contact)

a: level density parameter

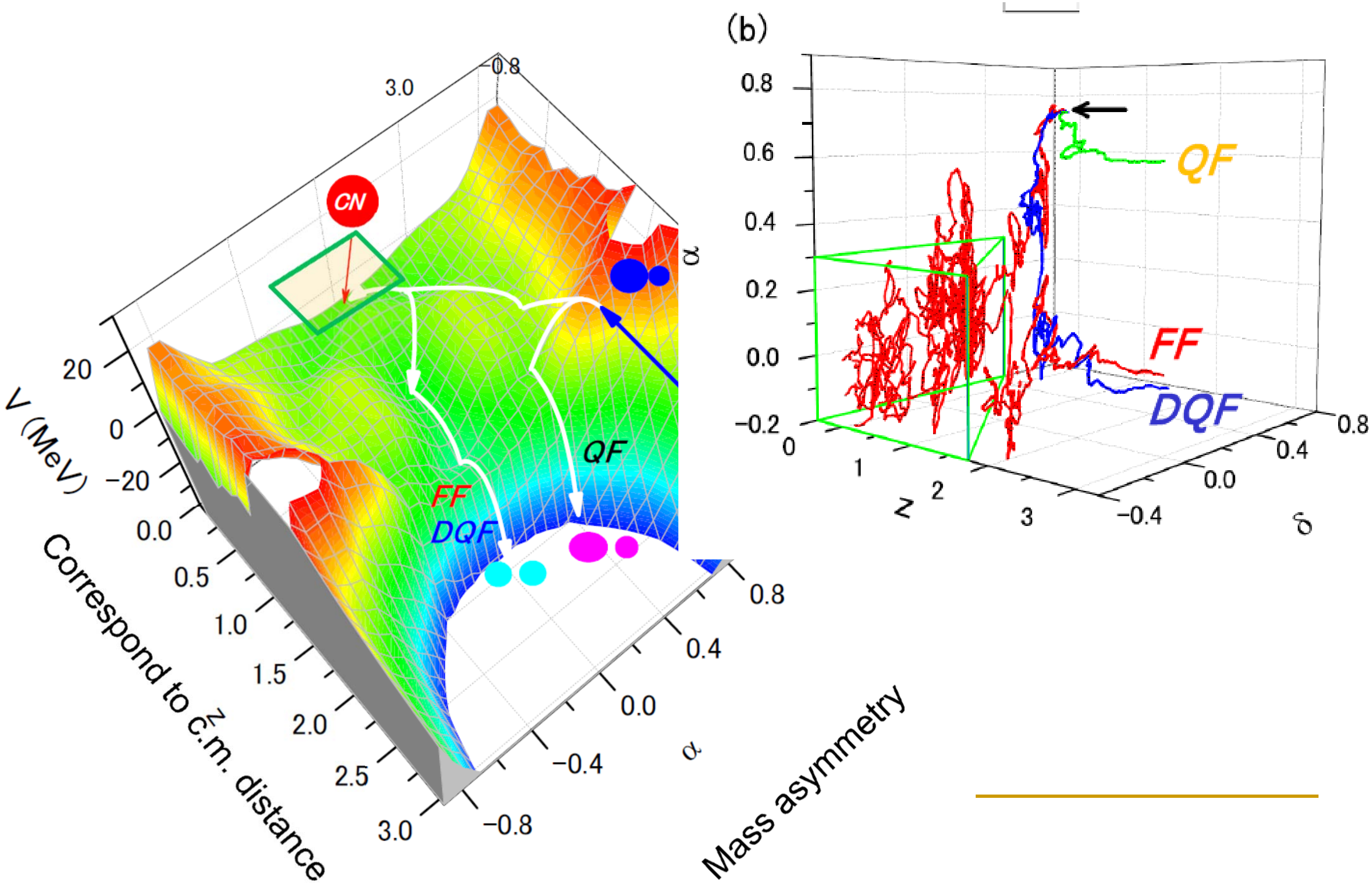
β: reduced friction parameter

- Strong friction case

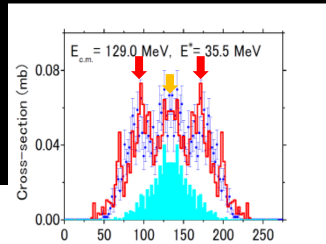
$$P^{for} = \frac{1}{2}\mathrm{erfc}\left[\left(\frac{aB^2}{K-B}\right)^{1/4}\left(1-\frac{\omega_B}{\beta}\sqrt{\frac{K}{B}}\right)\right]$$

$$\beta' = \sqrt{\beta^2 + 4\omega_B^2}$$

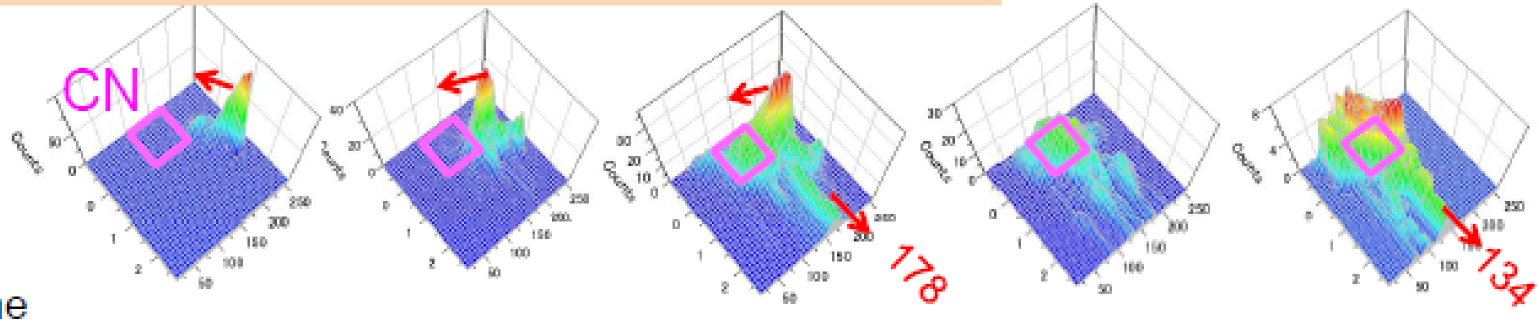
Overview of Dynamical Process in reaction $^{36}S+^{238}U$



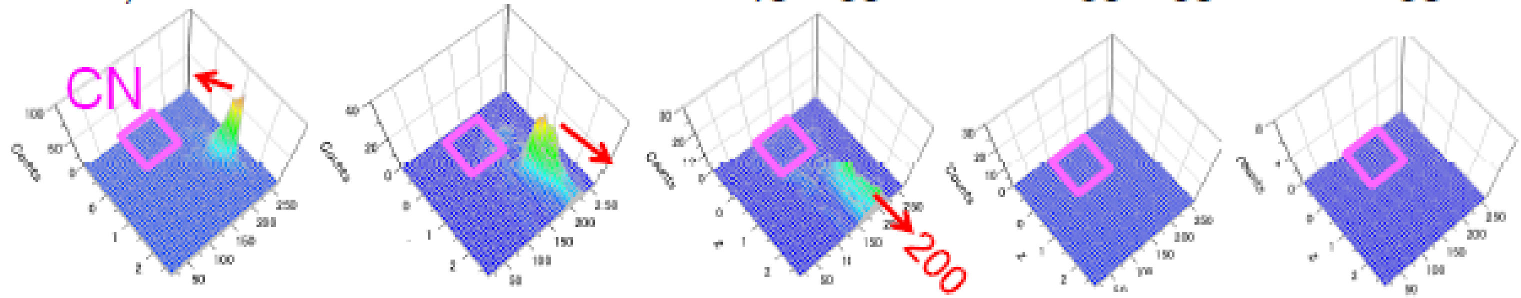
Time evolution of probability distribution



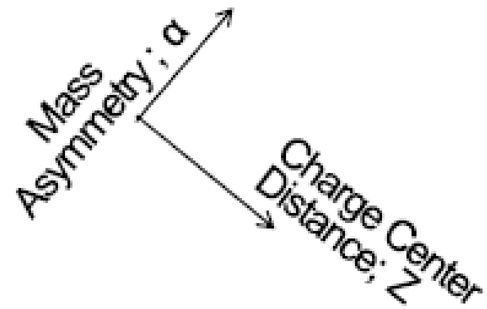
$^{30}Si + ^{238}U \rightarrow ^{268}Sg$ (E*=35.5 MeV)

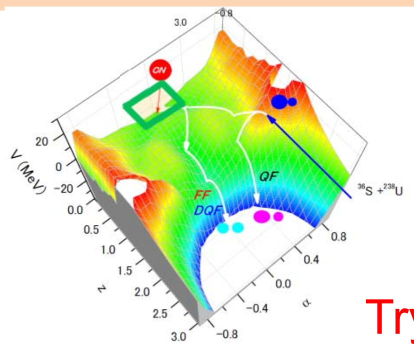


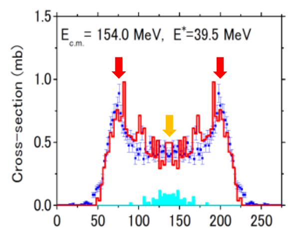
Time
(× 10^{-21} s) 0 - 5 | 5 – 10 | 10 – 30 | 30 – 50 | 50 -



$^{36}S + ^{238}U \rightarrow ^{274}Hs$ (E*=40.5 MeV)

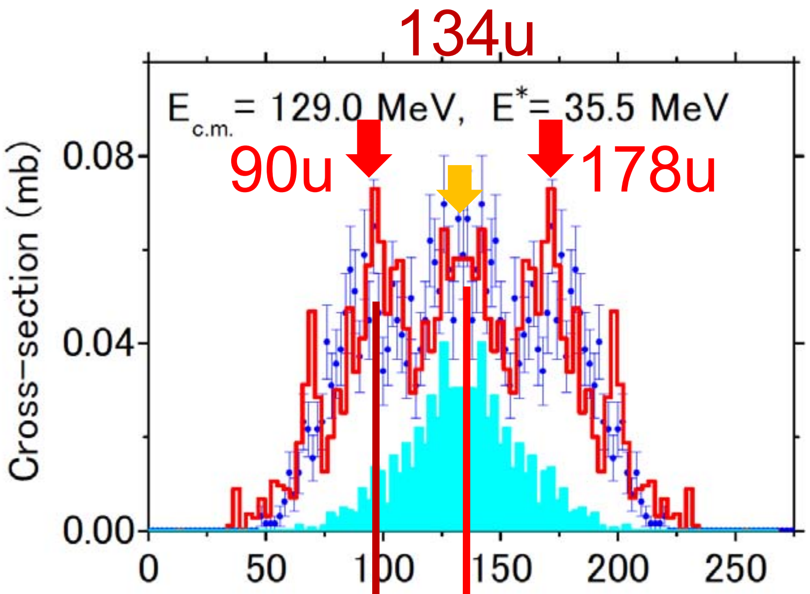




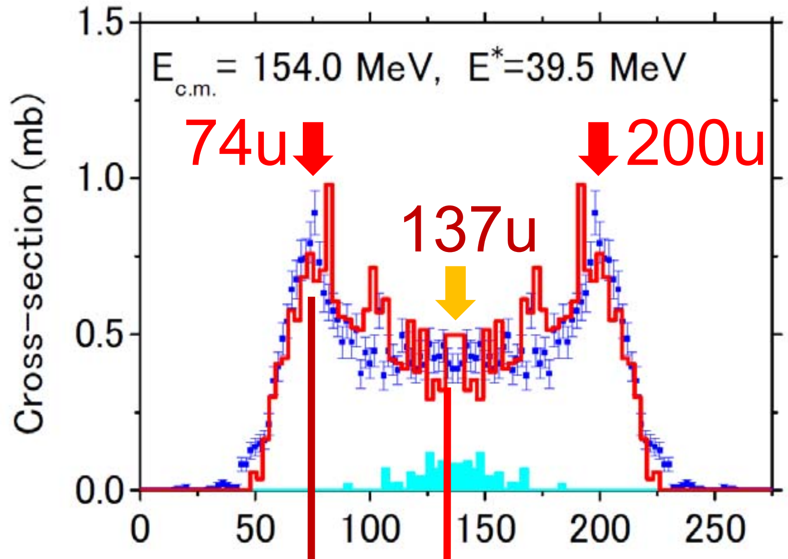


Try to clarify the origin of difference between the both cases →

^{30}Si + ^{238}U



^{36}S + ^{238}U



FF and DQF
$t > 50 \times 10^{-21}$ sec
$-0.2 < \delta < 0.2$ (peak 0)

QF via mono-nucleus
$t < 30 \times 10^{-21}$ sec
$0.2 < \delta < 0.5$ (peak 0.4)

FF and DQF
$t < 30 \times 10^{-21}$ sec
$0 < \delta < 0.4$ (peak 0.2)

QF
$t < 10 \times 10^{-21}$ sec
$0 < \delta < 0.2$ (peak 0)

(1) Origin of reaction process
(2) Building times
(3) Deformation of fragments

Summary

- The fluctuation-dissipation dynamics is a general framework to describe the dynamics of a few slow (collective) variables interacting with many fast variables that can be treated as a heat bath.
- Powerful tool to study fission and heavy-ion fusion
- Fission
  - Time scale of fission
  - Fragment mass and TKE distributions
- Heavy-ion fusion reaction
  - Fusion hindrance
  - Quasi-fission and fusion-fission and more