

Role of the Nucleus-Nucleus Potential in Sub-barrier Fusion

Șerban Mișicu
NIPNE-HH, Bucharest

INT Program INT-13-3

Quantitative Large Amplitude Shape Dynamics:

Fission and Heavy Ion Fusion

Nuclear Physics – Presence and Future

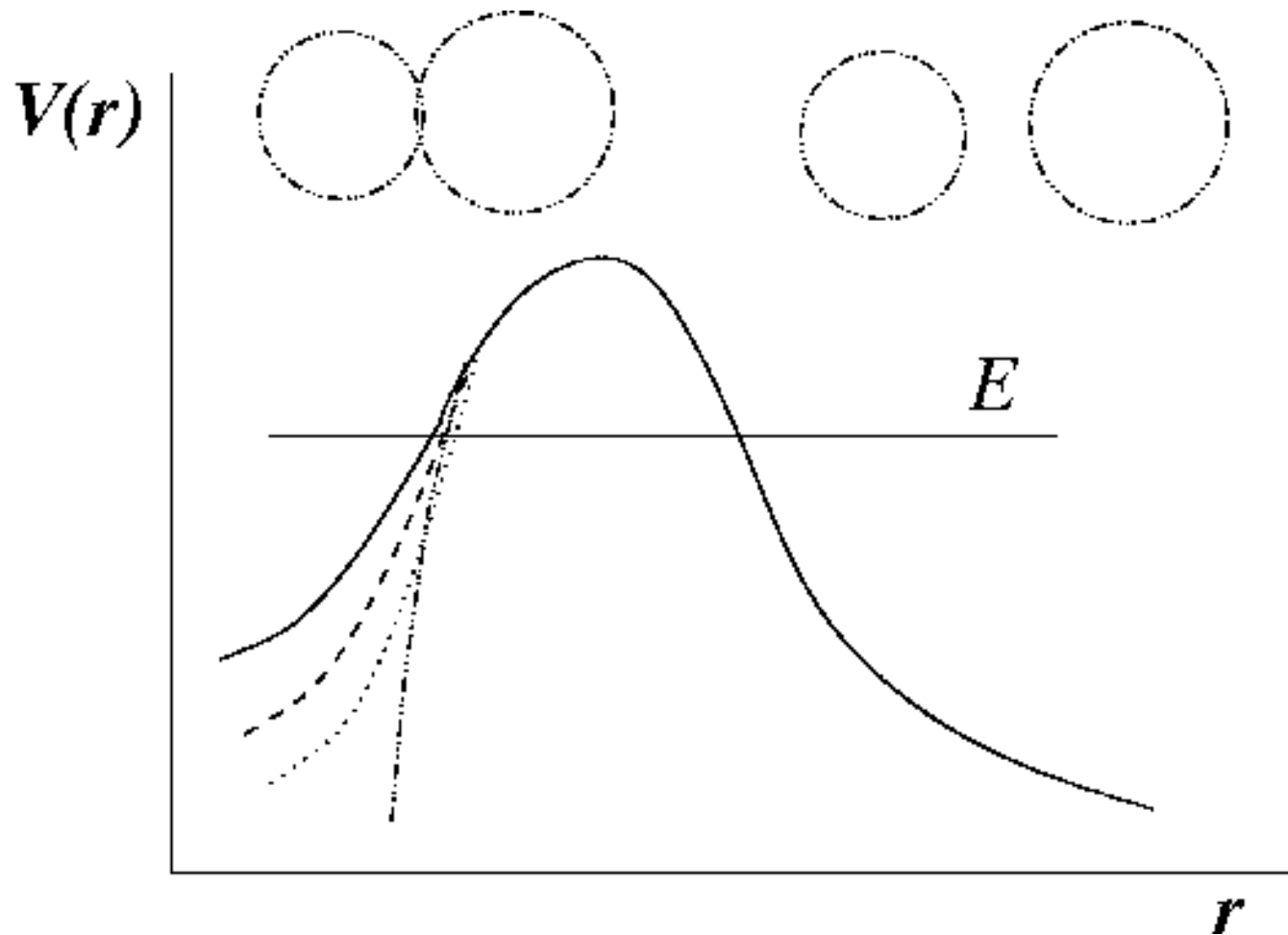
Seattle, 18 October 2013

Role of the Nucleus-Nucleus Potential in Sub-barrier Fusion

- * Introductory remarks
- * Nucleus-Nucleus Potential :
 - *Nucleon-Nucleon interaction*
 - *Orientation*
- * Sub-barrier Fusion
 - *Capture Reactions with ^{48}Ca projectile*
 - *Hindrance to fusion deep under the barrier*

SUB-BARRIER FUSION

- Formation of the compound nucleus through the quantum tunneling of the Coulomb barrier of two nuclei ($E < B_c$)

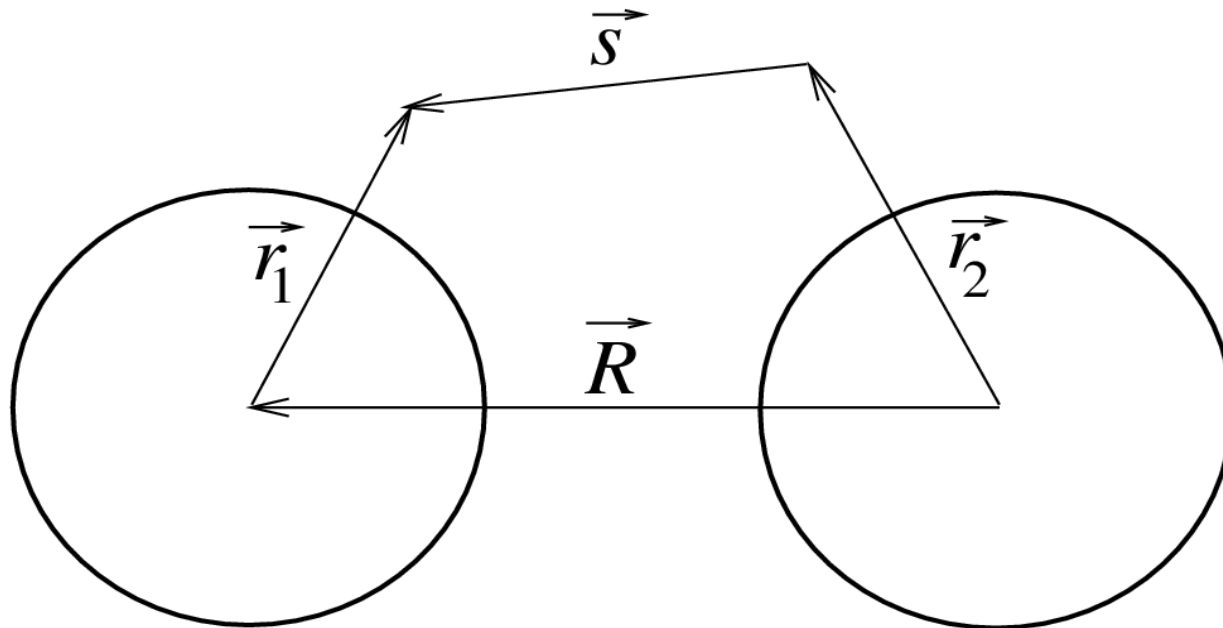


SUB-BARRIER FUSION : Introductory remarks

- Test the nuclear potential inside the barrier
- Enhancement of cross-sections due to nuclear structure :
*vibrational nuclei(sensitivity to multi-phonon excitations) ,
rotational excitations (effect of higher multipole deformations)*
- Gate to the synthesis of heavy and superheavy nuclei
- Burning Cycles (C & O) in massive stars – extrapolation of near-barrier data on σ_F of C and O reactions down to astrophysical energies

Double folding Heavy Ion Potential

$$V_{DF}(\mathbf{R}) = \int d\mathbf{r}_1 d\mathbf{r}_2 \rho(\mathbf{r}_1) \rho(\mathbf{r}_2) v(\mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2) = (\rho_1 * (\rho_2 * v))$$



Double folding Heavy Ion Potential

$$V_{DF}(\mathbf{R}) = \int d\mathbf{r}_1 d\mathbf{r}_2 \rho(\mathbf{r}_1) \rho(\mathbf{r}_2) v(\mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2) = (\rho_1 * (\rho_2 * v))$$

Ground state one-body densities(spherical):

M3Y : Fermi-Dirac with FRLDM parameters

Skyrme : HFBRAD

Gogny : HFB

Ground state one-body densities(deformed):

$$\rho(\mathbf{r}) = \rho_0 \left[1 + \exp \frac{1}{a} \left(r - R_0 \left(1 + \sum_{\lambda=2,3,4} \beta_\lambda Y_{\lambda 0}(\theta, 0) \right) \right) \right]^{-1}$$

Double folding Heavy Ion Potential

$$V_{\text{DF}}(\mathbf{R}) = \int d\mathbf{r}_1 d\mathbf{r}_2 \rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2) v((\mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2)) \equiv \rho_1 * \rho_2 * v$$

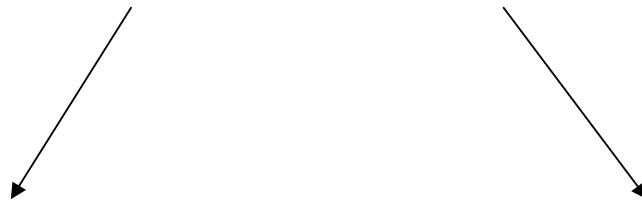
central part of the effective $N - N$ interactions may be written as

$$v = v_{00}(\mathbf{r}) + v_{01}(\mathbf{r})(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + v_{10}(\mathbf{r})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + v_{11}(\mathbf{r})(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

(σ, τ) representation

From the exceedingly large number of exchange terms retain only the knock-on (KOE) term :
 two nucleons are interacting and in the same time are exchanged

$$v = v^{\text{d}}(\mathbf{r}, \rho) + v^{\text{ex}}(\mathbf{r}, \rho) P_{12}^x$$



$$v^{\text{d}} = \sum_{S,T} v_{S,T} P_s^\sigma P_s^\tau, \quad v^{\text{ex}} = \sum_{S,T} (-)^{S+T+1} v_{S,T} P_s^\sigma P_s^\tau,$$

G-matrix interactions

G-matrix elements of a bare NN interaction in an oscillator basis were obtained solving Bethe-Goldstone equation and then fitted with a sum of Yukawa f.f.

$$v(r) = \sum_{i=1}^{n_{max}} V_i \frac{\exp(-\mu_i r)}{\mu_i r}$$

- (I) M3Y-Reid (standard) [Bertsch et al., NPA 284, 399 (1977)]
- (ii) M3Y-Paris [Antaraman, Toki and Bertsch, NPA 398, 269 (1978)]
- (iii) M4Y-Paris [Hosaka, Kubo, Toki, NPA 444, 76 (1985)]

G-matrix interactions are local and purely real. Density dependence is implicit through the oscillator parameter and it is characteristic to $\approx \rho_{NM}/3$

Density-dependent effective interactions

Gogny interaction : a sum of central, finite-range term and a zero-range d.d. Term
neglect spin-orbit term for spin-saturated nuclei

$$v(\mathbf{r}_{12}) = \sum_{i=1}^2 (W_i + B_i P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau) e^{-\frac{r_{12}^2}{\mu_i^2}} + t_3 (1 + P^\sigma) \rho^\alpha \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \delta(\mathbf{r}_{12})$$

(σ, τ) representation

$$v_{00}^d = \frac{1}{4} \sum_{i=1}^2 (4W_i + 2B_i - 2H_i - M_i) e^{-\frac{r_{12}^2}{\mu_i^2}} + \frac{3}{2} t_3 \rho^\alpha \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \delta(\mathbf{r}_{12})$$

$$v_{00}^{\text{ex}} = -\frac{1}{4} \sum_{i=1}^2 (W_i + 2B_i - 2H_i - 4M_i) e^{-\frac{r_{12}^2}{\mu_i^2}} - \frac{3}{4} t_3 \rho^\alpha \delta(\mathbf{r}_{12})$$

Density-dependent effective interactions

Skyrme interaction : a sum of a zero-range local, non-local and d.d. terms

$$\begin{aligned}
 v(\mathbf{r}_{12}) &= t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_{12}) \\
 &+ \frac{1}{2} t_1 (1 + x_1 P_\sigma) \left(\mathbf{p}^\dagger{}^2 \delta(\mathbf{r}_{12}) + \delta(\mathbf{r}_{12}) \mathbf{p}^2 \right) + t_2 (1 + x_2 P_\sigma) \mathbf{p}^\dagger \cdot \delta(\mathbf{r}_{12}) \mathbf{p} \\
 &+ \frac{1}{6} t_3 (1 + x_3 P_\sigma) \left[\rho \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \right]^\alpha \delta(\mathbf{r}_{12}) + iW_0 \boldsymbol{\sigma} \cdot [\mathbf{p}^\dagger \times \delta(\mathbf{r}_{12}) \mathbf{p}]
 \end{aligned}$$

$(\mathcal{O}, \mathcal{T})$ representation

$$\begin{aligned}
 v_{00}^d &= t_0 \left(1 + \frac{1}{2} x_0 \right) \delta(\mathbf{r}) + \frac{1}{6} t_3 \left(1 + \frac{1}{2} x_3 \right) [\rho(\mathbf{R})]^\alpha \delta(\mathbf{r}) \\
 &+ \frac{1}{2} t_1 \left(1 + \frac{1}{2} x_1 \right) \left(\mathbf{p}^\dagger{}^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{p}^2 \right) + t_2 \left(1 + \frac{1}{2} x_2 \right) \mathbf{p}^\dagger \cdot \delta(\mathbf{r}) \mathbf{p} \\
 v_{00}^{\text{ex}} &= - \frac{1}{4} t_0 (1 + 2x_0) \delta(\mathbf{r}) \\
 &- \frac{1}{8} t_1 (1 + 2x_1) \left(\mathbf{p}^\dagger{}^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{p}^2 \right) - \frac{1}{4} t_2 (1 + 2x_2) \mathbf{p}^\dagger \cdot \delta(\mathbf{r}) \mathbf{p} \\
 &- \frac{1}{24} t_3 (1 + 2x_3) [\rho(\mathbf{R})]^\alpha \delta(\mathbf{r})
 \end{aligned}$$

Exchange Kernels (finding a local equivalent)

Pseudopotential approximation : $v_{ex} P_{12}^x \rightarrow f(\rho_0, K) \delta(\vec{s})$

$$U_L(R) = f(\rho_0, K_0/\mu) \int \rho_1(r_1) \rho_2(r_2) \delta(\mathbf{R} + \mathbf{r}_1 - \mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2$$

$$f(\rho_0, K) = 4\pi \int v_{ex}(s) \hat{j}_1^2(\hat{k}(\rho_0)) j_0(Ks) s^2 ds$$

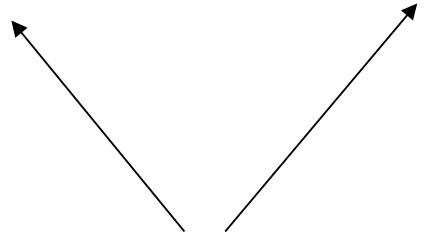
Energy dependence



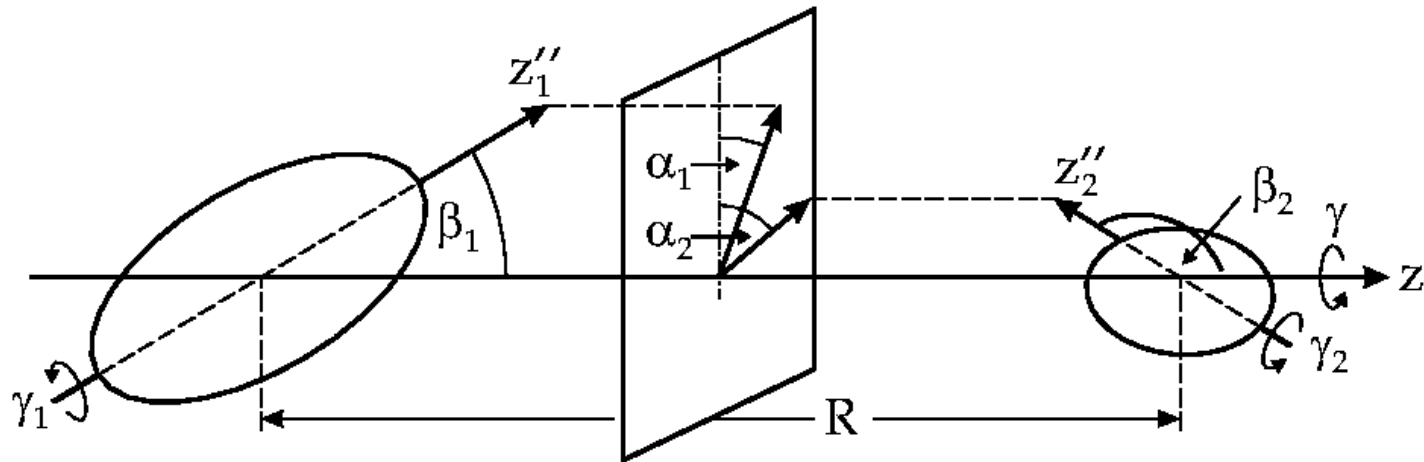
Perey-Saxon localization procedure:

$$U_L(R) = 4\pi \int d\vec{x} \rho_1(x) \rho_2(|\vec{R}-\vec{x}|) \int s^2 ds v_{ex}(s) \hat{j}_1(k_{f1}(x) \frac{A_1-1}{A_1} s) \hat{j}_1(k_{f2}(|\vec{R}-\vec{x}|) \frac{A_2-1}{A_2} s) j_0\left(\frac{K(R)s}{\mu}\right)$$

Full recoil included



Orientation



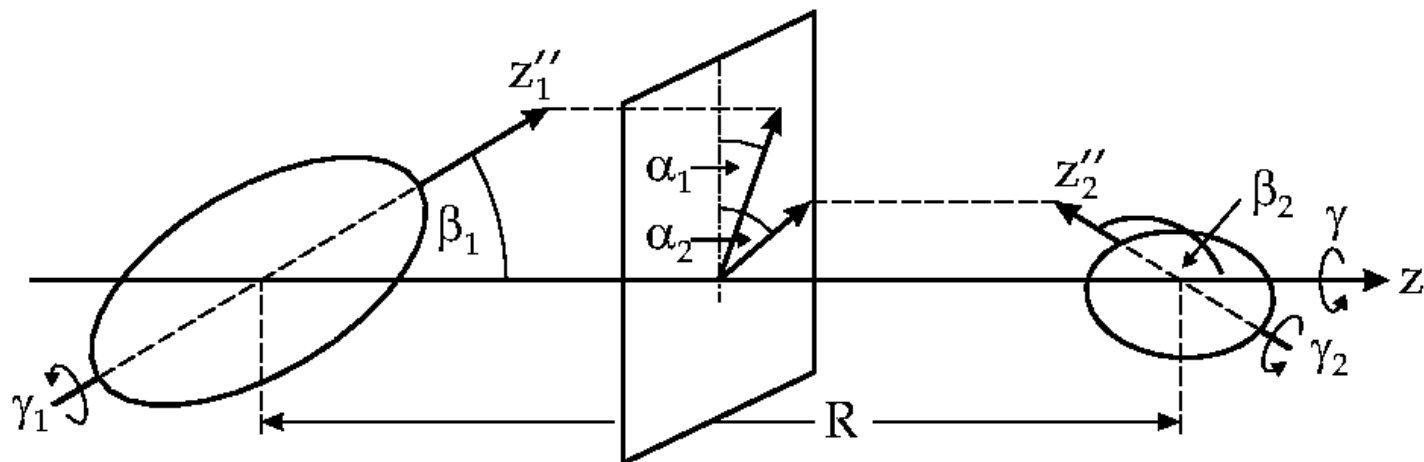
$$\omega_i = (\alpha_i, \beta_i, \gamma_i)$$

$$V_{DF}(\mathbf{R}) = \frac{1}{(2\pi)^3} \int d\mathbf{q} \tilde{V}(\mathbf{q}) \exp(-i\mathbf{q} \cdot \mathbf{R})$$

$$\tilde{V}(\mathbf{q}) = \tilde{\rho}_1(\mathbf{q}) \tilde{\rho}_2(-\mathbf{q}) \tilde{v}(\mathbf{q})$$

$$\rho_i(\mathbf{r}') = \mathcal{R}(\alpha_i, \beta_i, \gamma_i) \rho_i(\mathbf{r}'') \quad \rho_i(\mathbf{r}'') = \sum_{\lambda} \rho_{\lambda}(\mathbf{r}'') Y_{\lambda 0}(\hat{r}'')$$

Orientation



$$\omega_i = (\alpha_i, \beta_i, \gamma_i)$$

$$V(\mathbf{R}) = \sum V_{\lambda_1 \lambda_2 \lambda_3}^{\mu_1 \mu_2 \mu_3}(R) D_{\mu_1 0}^{\lambda_1}(\omega_1) D_{\mu_2 0}^{\lambda_2}(\omega_2) D_{\mu_3 0}^{\lambda_3}(\Phi, \Theta, 0)$$

$$V_{\lambda_1 \lambda_2 \lambda_3}^{\mu_1 \mu_2 \mu_3}(R) = \frac{1}{(2\pi)^3} i^{\lambda_1 - \lambda_2 - \lambda_3} \hat{\lambda}_1 \hat{\lambda}_2 \hat{\lambda}_3^2 \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \mu_1 & \mu_2 & \mu_3 \end{pmatrix} F_{\lambda_1 \lambda_2 \lambda_3}(R)$$

Fixed Molecular Axis

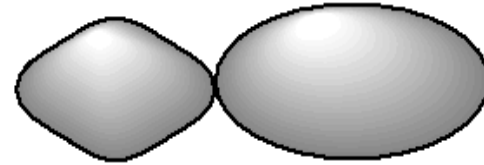
$$V(\mathbf{R}) = \sum_{\lambda_i \mu} V_{\lambda_1 \lambda_2 \lambda_3}^{\mu - \mu \ 0}(R) \cos \mu(\alpha_2 - \alpha_1) d_{\mu 0}^{\lambda_1}(\beta_1) d_{-\mu 0}^{\lambda_2}(\beta_2)$$

Orientation

Touching Configurations

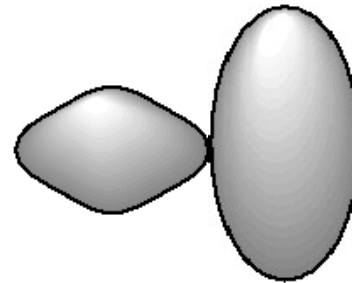
(i) Pole-Pole(nose-to-nose)

$$\alpha_1 = \alpha_2 = 0, \beta_1 = \beta_2 = 0$$



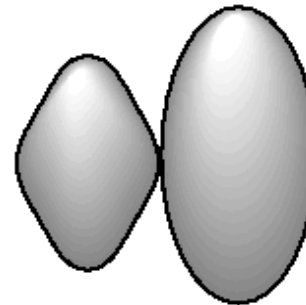
(ii) Pole-Equator(nose-to-belly)

$$\alpha_1 - \alpha_2 = 0, \beta_1 = 0 \text{ and } \beta_2 = \frac{\pi}{2}$$



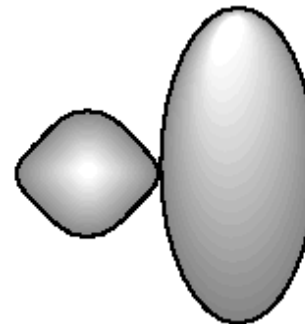
(iii) Equator-Equator(belly-to-belly)

$$\alpha_1 - \alpha_2 = 0, \beta_1 = \beta_2 = \frac{\pi}{2}$$



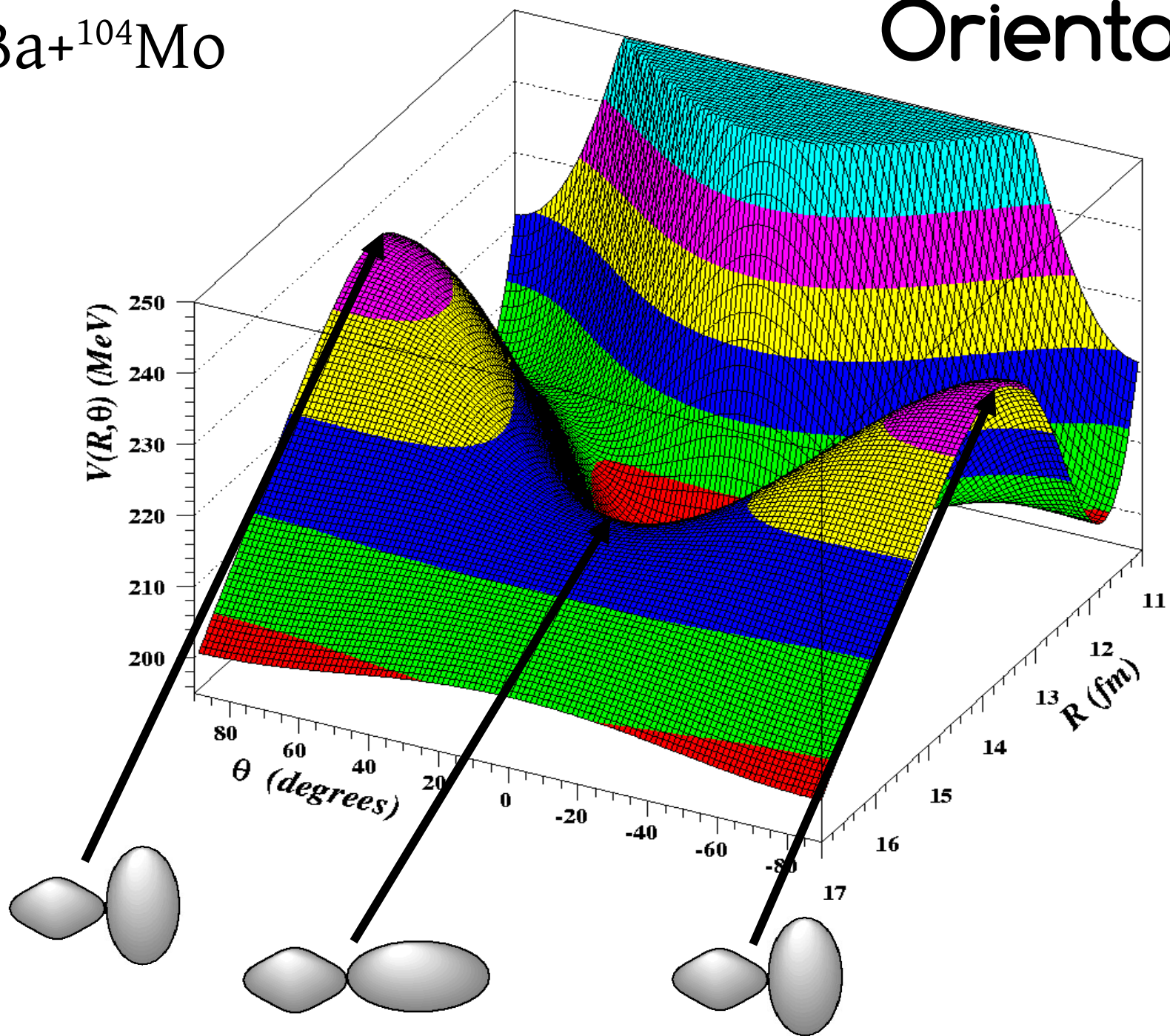
(iv) Equator-Equator Twisted(crossed bellies)

$$\alpha_1 - \alpha_2 = \frac{\pi}{2}, \beta_1 = \beta_2 = \frac{\pi}{2}$$

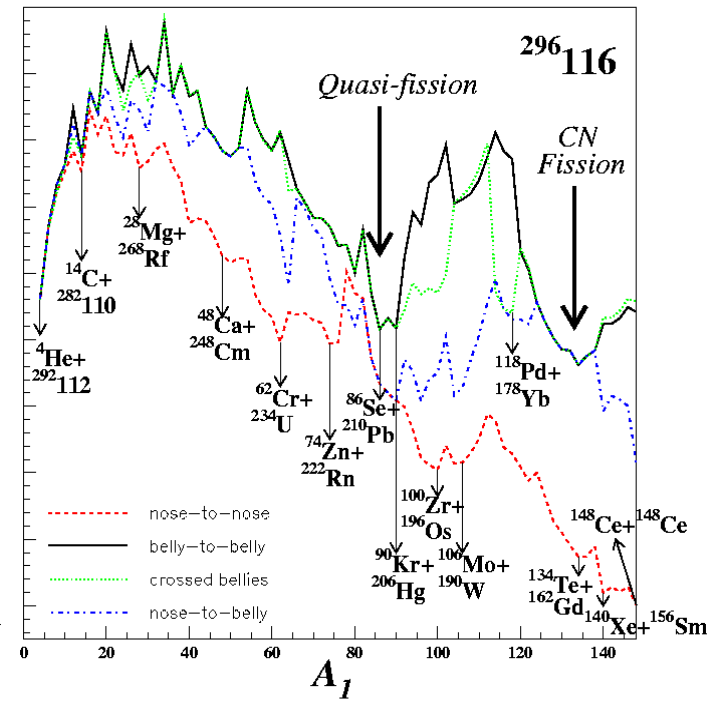
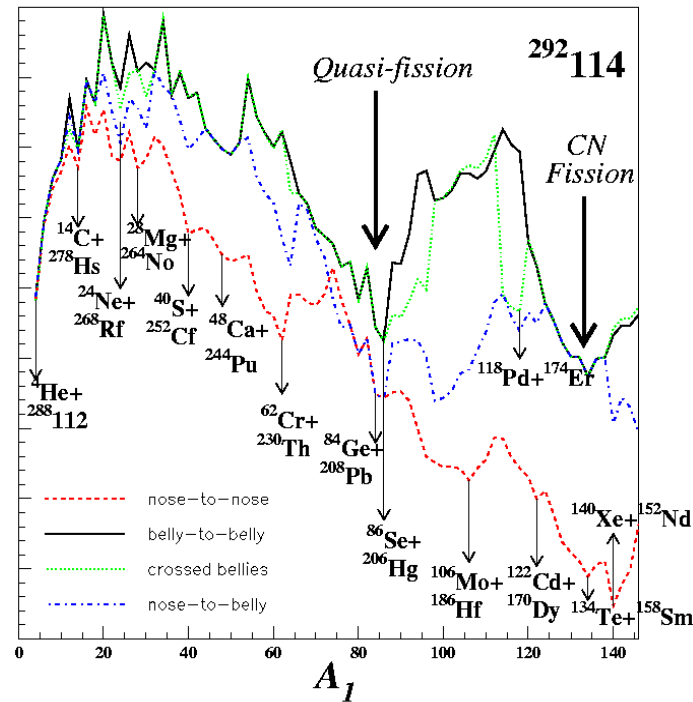
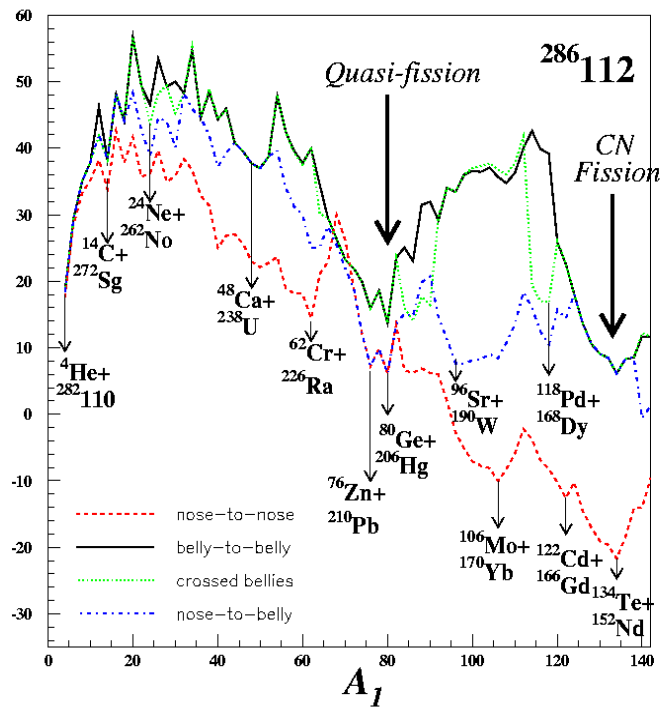


$^{148}\text{Ba} + ^{104}\text{Mo}$

Orientation



Capture Reactions with ^{48}Ca projectile



Coupling with Rotational Band States

$$H = T + H_{\text{rot}}(\omega) + V(R, \omega)$$

$$\Psi(\mathbf{R}, \omega) = \sum_{JM} a_{JM} \psi_{JM}(\mathbf{R}, \omega)$$

$$\psi_{JM}(\mathbf{R}, \omega) = \mathcal{R}_{J\Omega}(R) \Phi_{JM\Omega}(R, \omega) + \sum_{I' \nu'} \mathcal{R}_{JI' \nu'}(R) \Phi_{JM I' \nu'}(R, \omega)$$

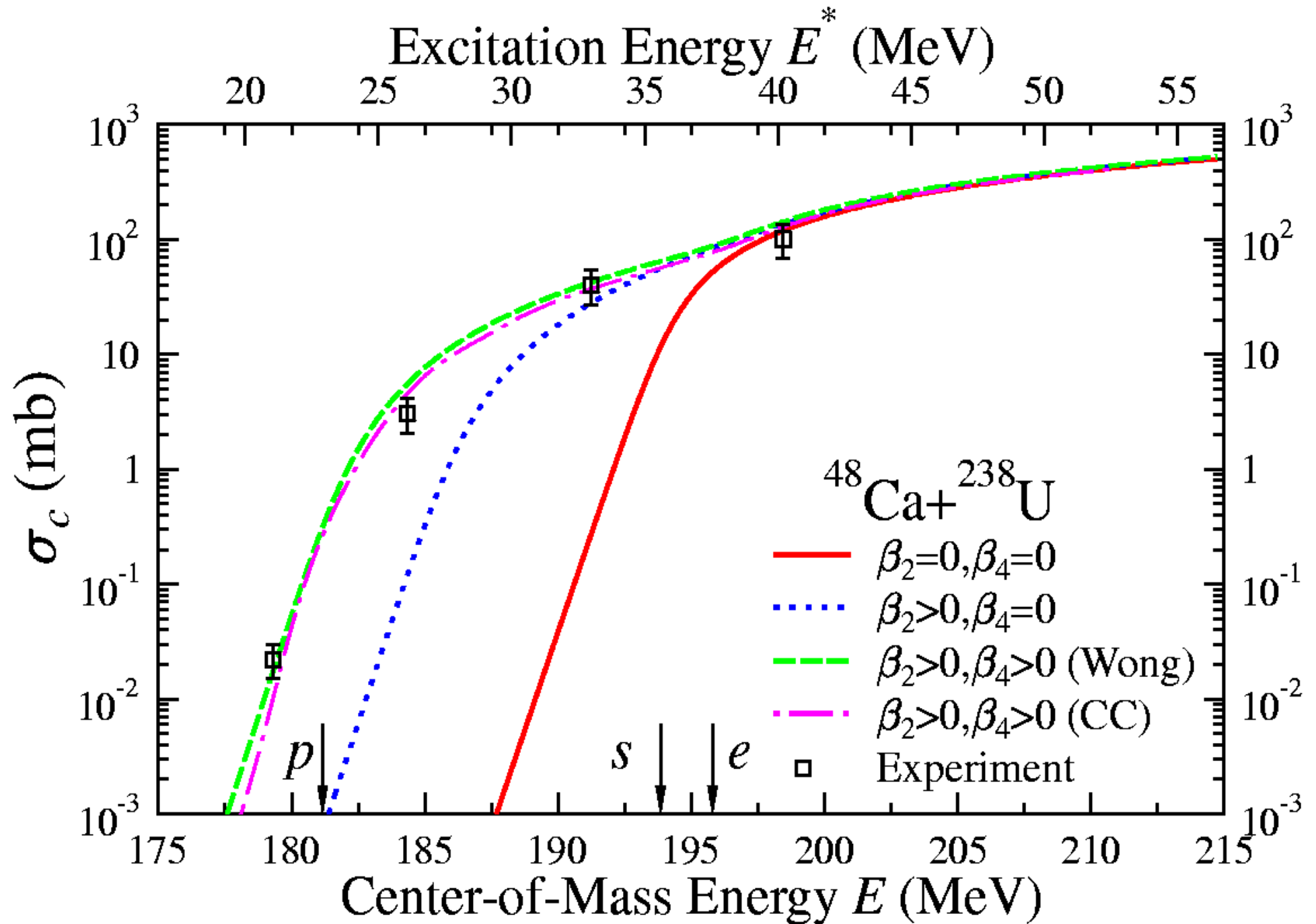
$$\left\{ -\frac{\hbar^2}{2\mu} \left(\frac{\partial^2}{\partial R^2} - \frac{l(l+1)}{R^2} \right) + \varepsilon_I - E \right\} \mathcal{R}_{J\Omega}(R) + \sum_{I' \nu'} \mathcal{R}_{JI' \nu'}(R) \langle \Phi_{JM\Omega} | V(R, \omega) | \Phi_{JM I' \nu'} \rangle = 0$$

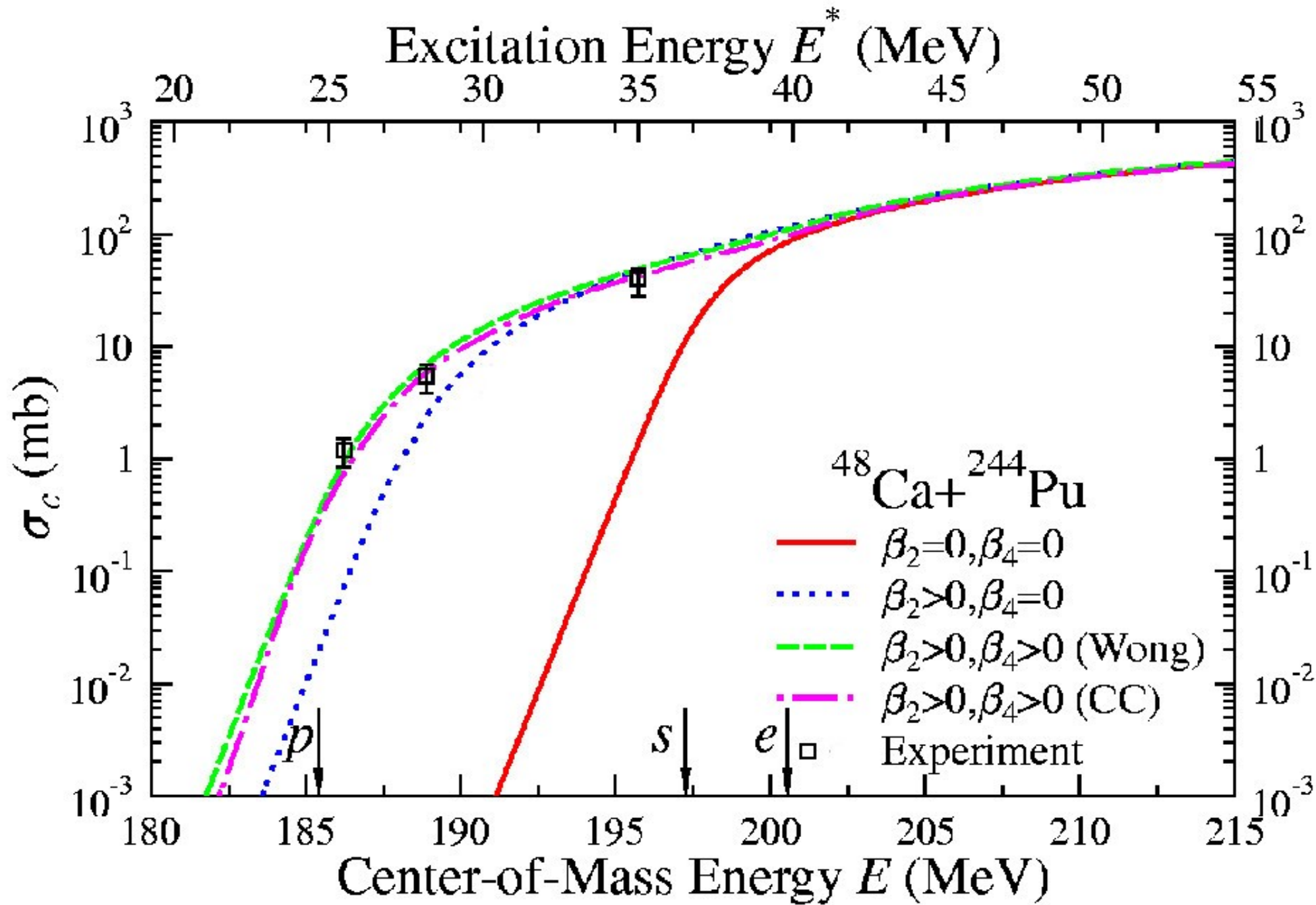
$$\langle \Phi_{JM\Omega} | V(R, \omega) | \Phi_{JM I' \nu'} \rangle = (-)^{l+I} \hat{I} \hat{I}' \hat{l} \hat{l}' \sum_{\lambda} \begin{pmatrix} I & I' & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} l & I & J \\ I' & l' & \lambda \end{Bmatrix} V_{\lambda 0 0 \lambda}^0(R)$$

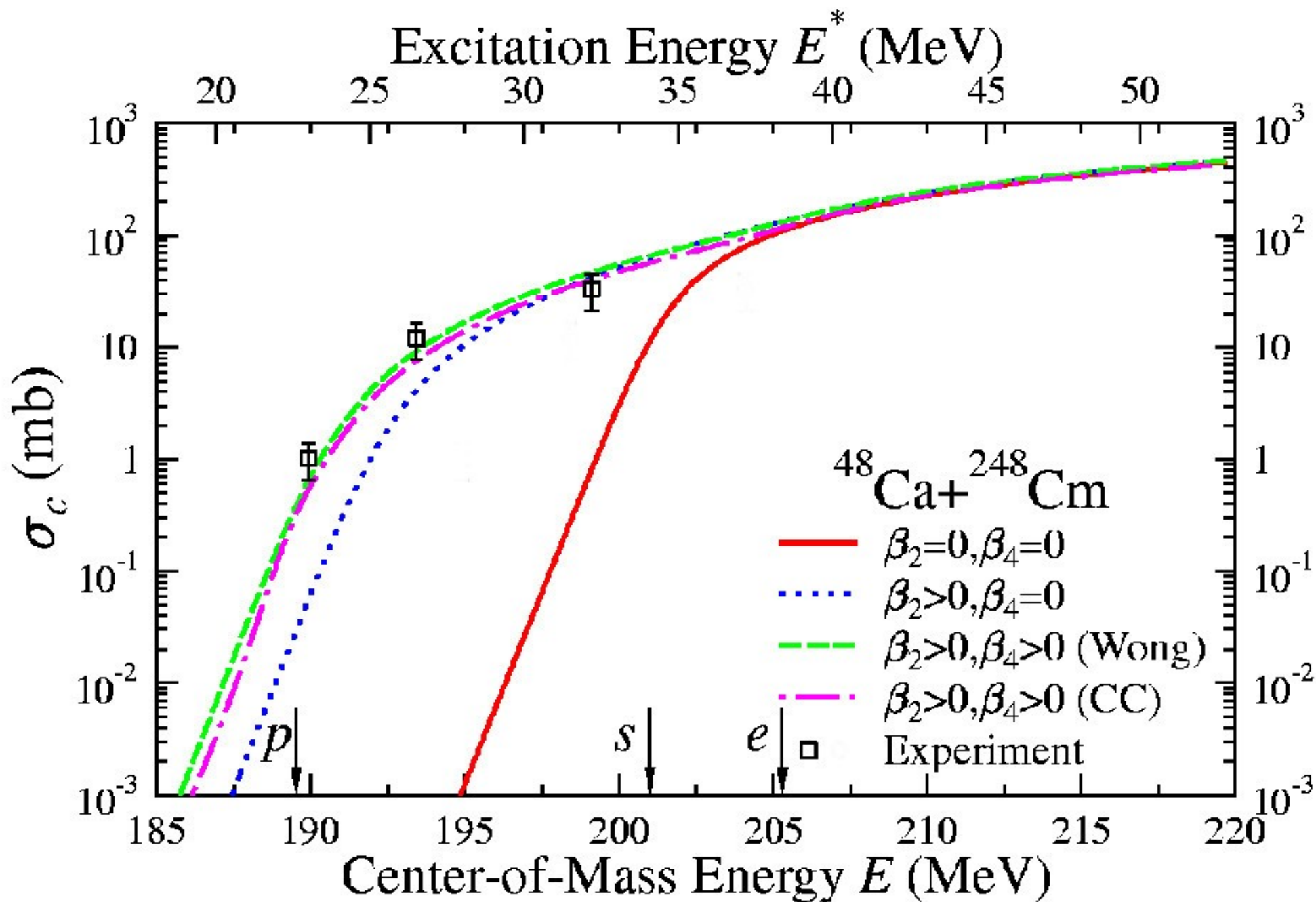
$$\psi_{IJ} = \sum_I \sqrt{2l+1} \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \mathcal{R}_{J\Omega}(R)$$

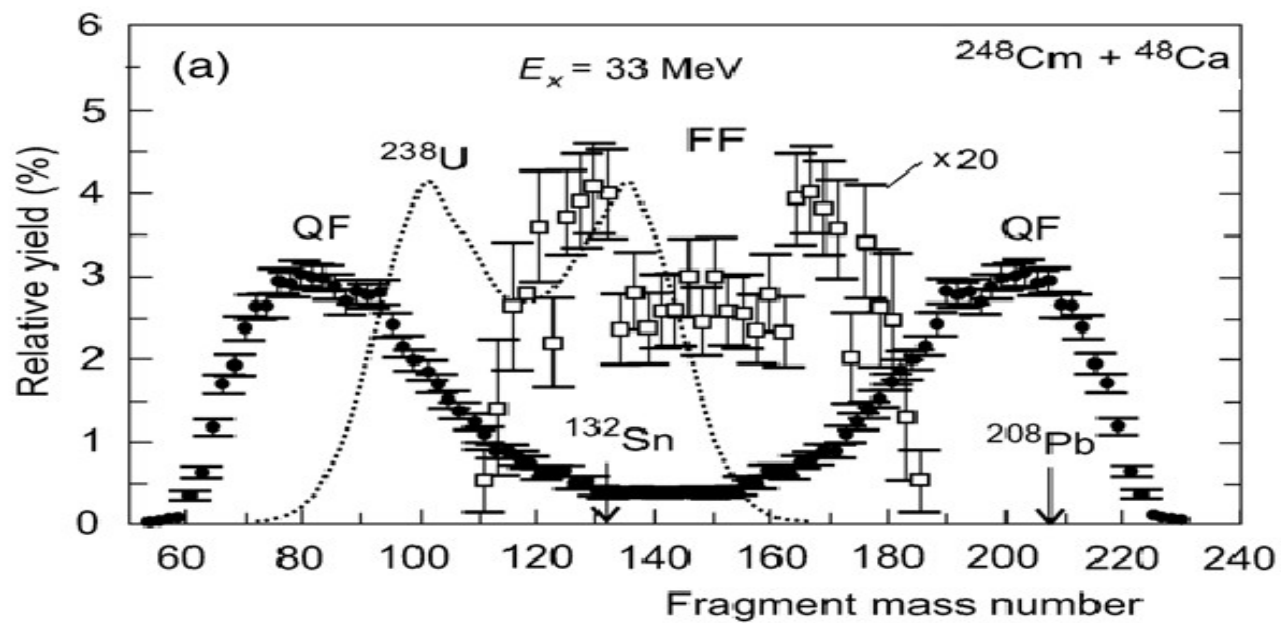
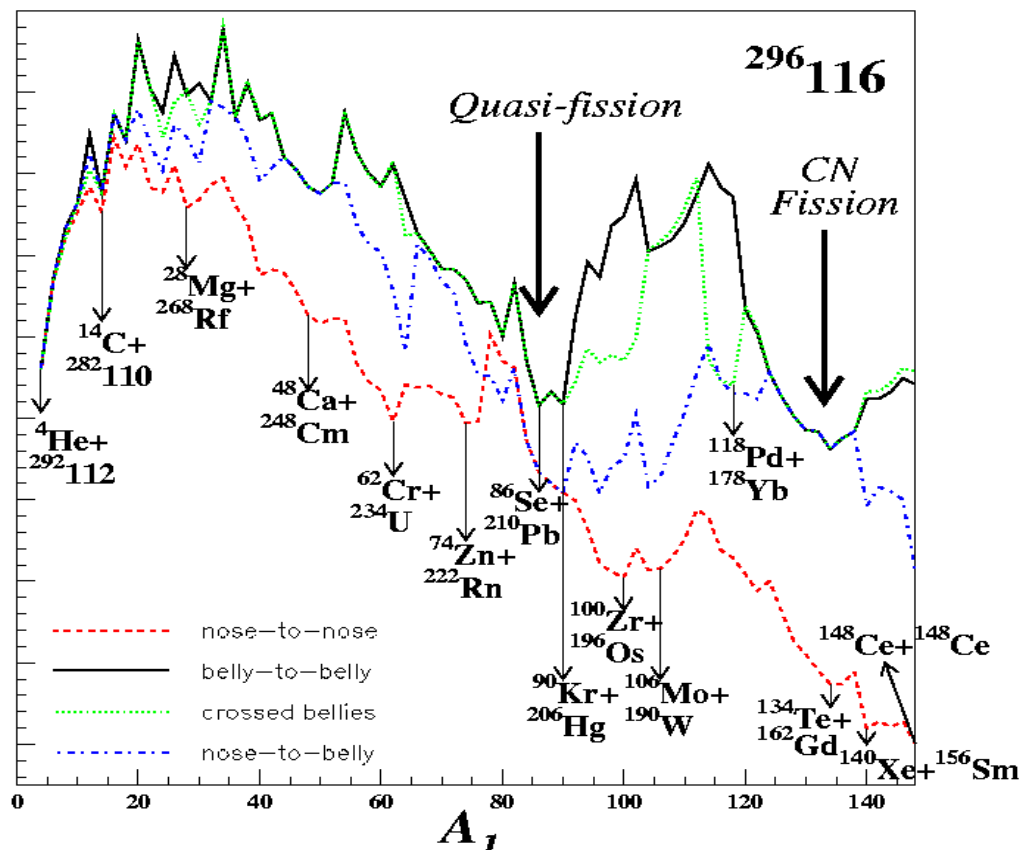
$$\left\{ -\frac{\hbar^2}{2\mu} \left(\frac{\partial^2}{\partial R^2} - \frac{J(J+1)}{R^2} \right) + \varepsilon_I - E \right\} \psi_{IJ} + \sum_{I'} \hat{I} \hat{I}' \sum_{\lambda} V_{\lambda 0 0 \lambda}^0(R) \begin{pmatrix} I & \lambda & I' \\ 0 & 0 & 0 \end{pmatrix}^2 \psi_{I'J} = 0$$

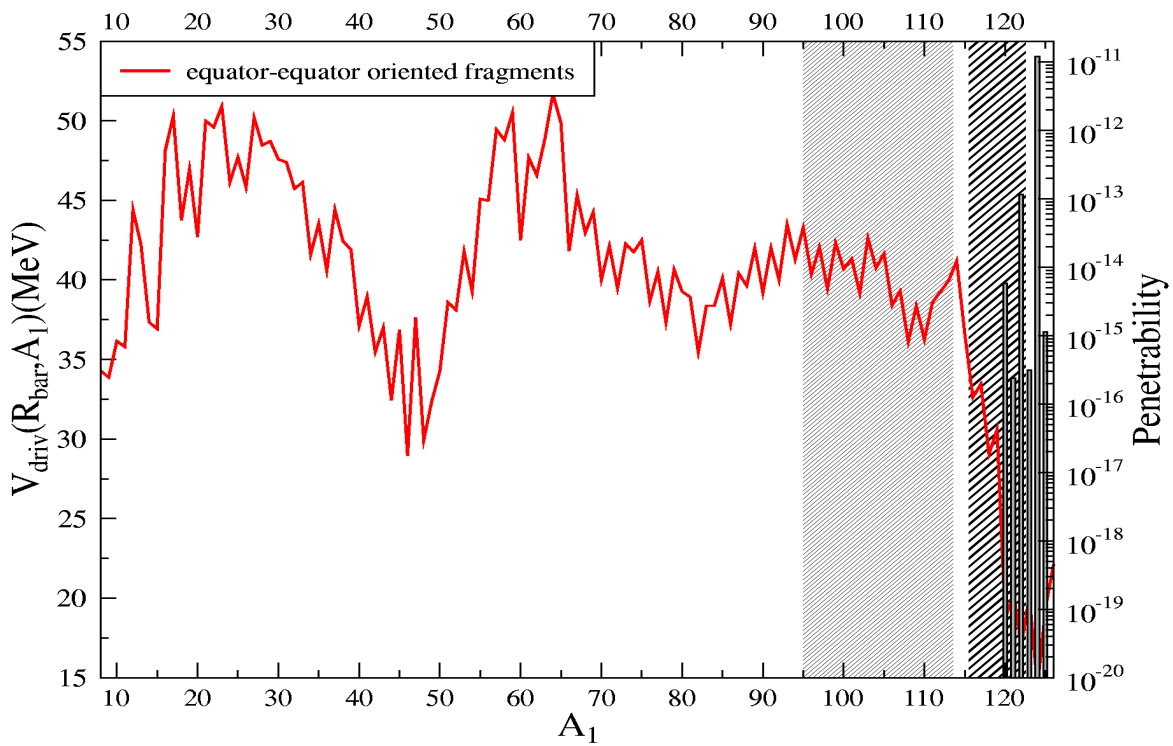
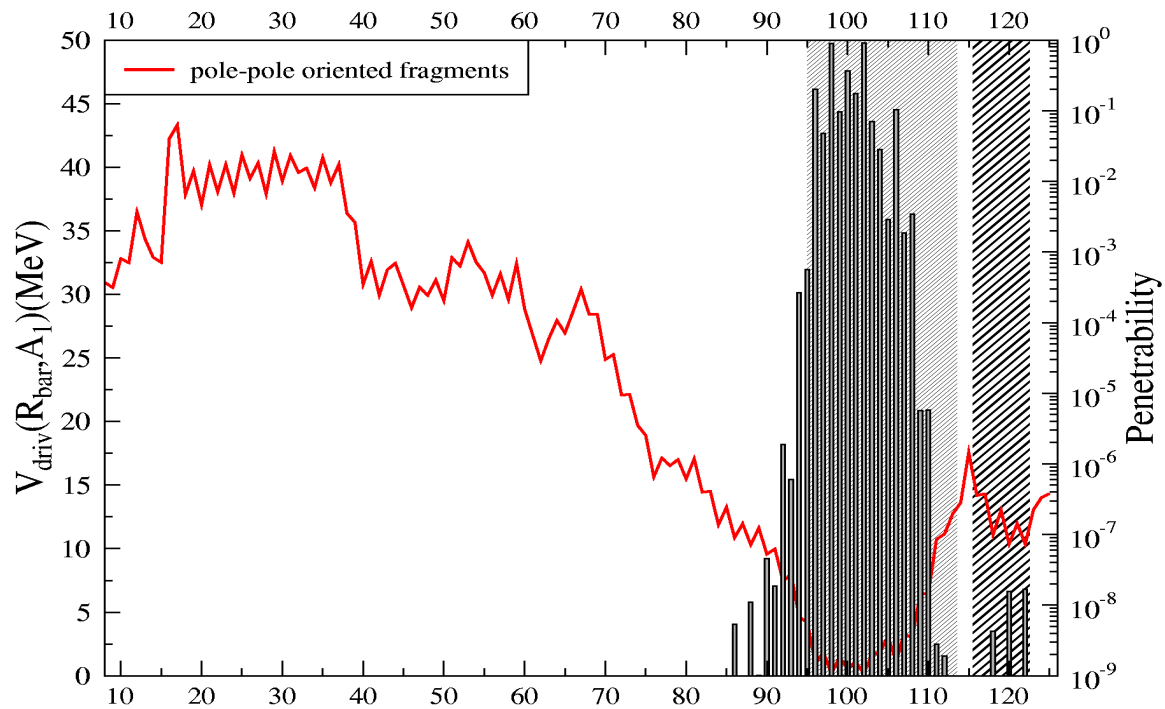
$$\sigma_c(E) = \frac{\pi}{k^2} \sum_l (2J+1) P_J(\theta)$$





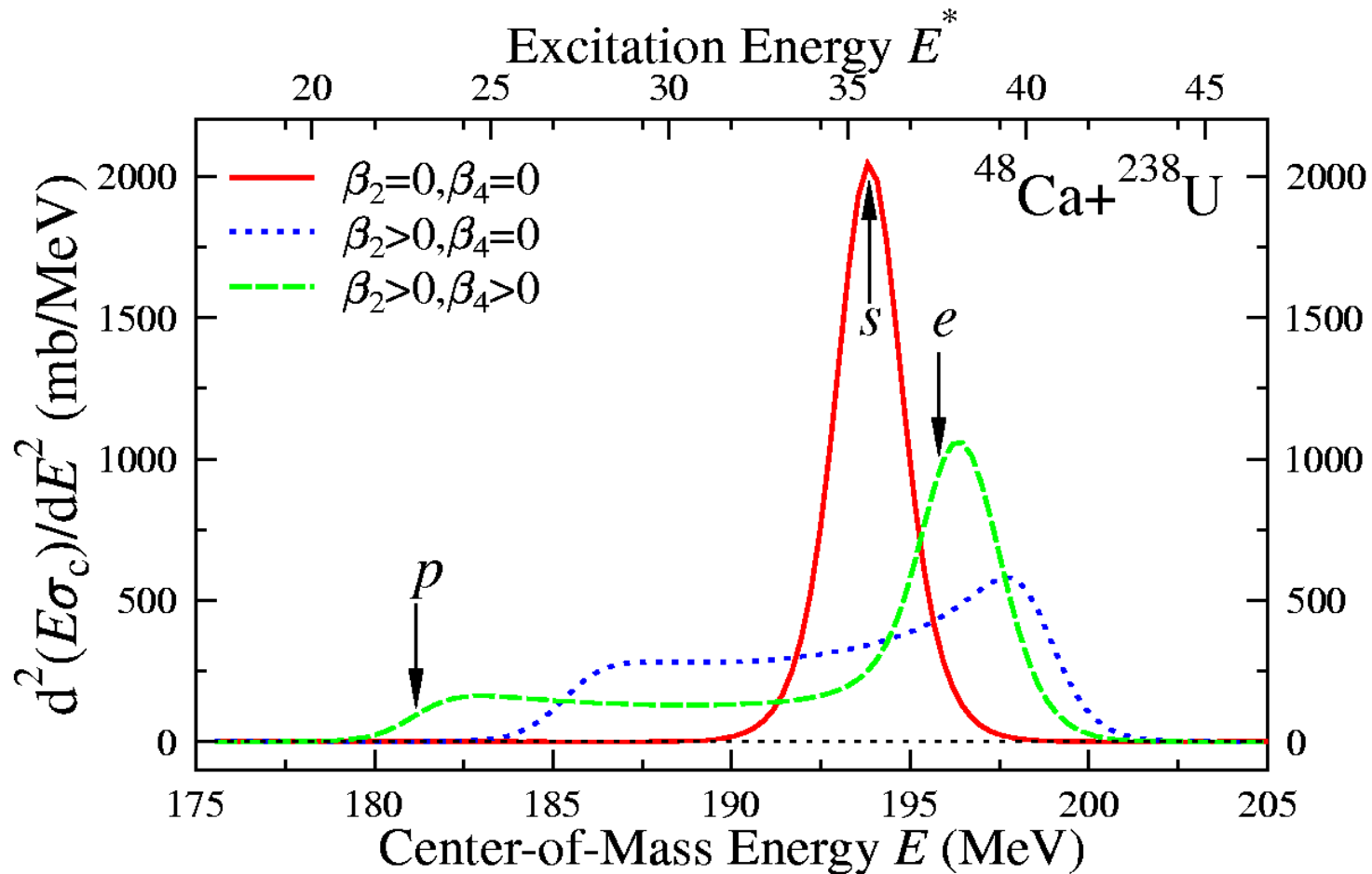




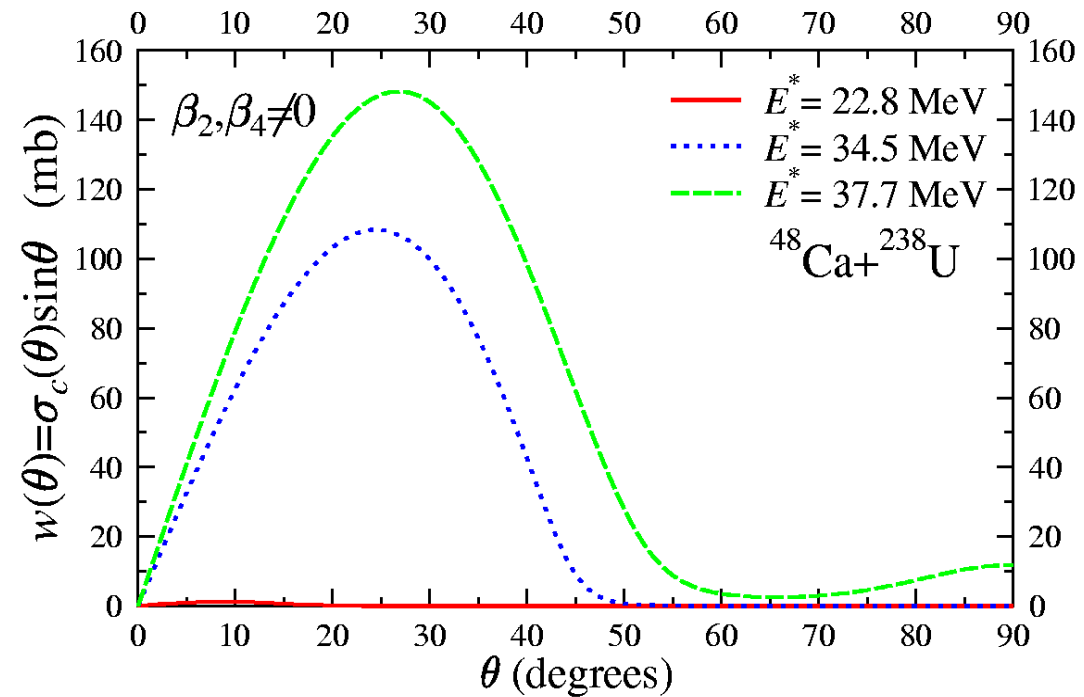
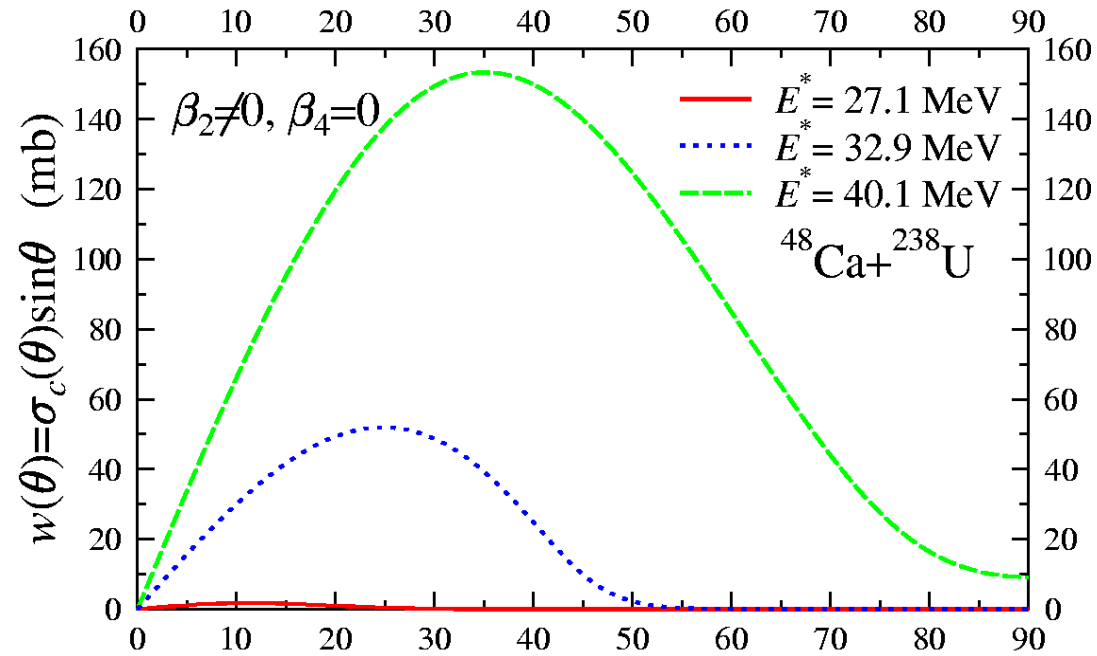


Barrier Distribution

$$D(E) = \frac{d^2(E\sigma_c)}{dE^2} \quad \text{Probability to encounter a fusion barrier of height } E$$



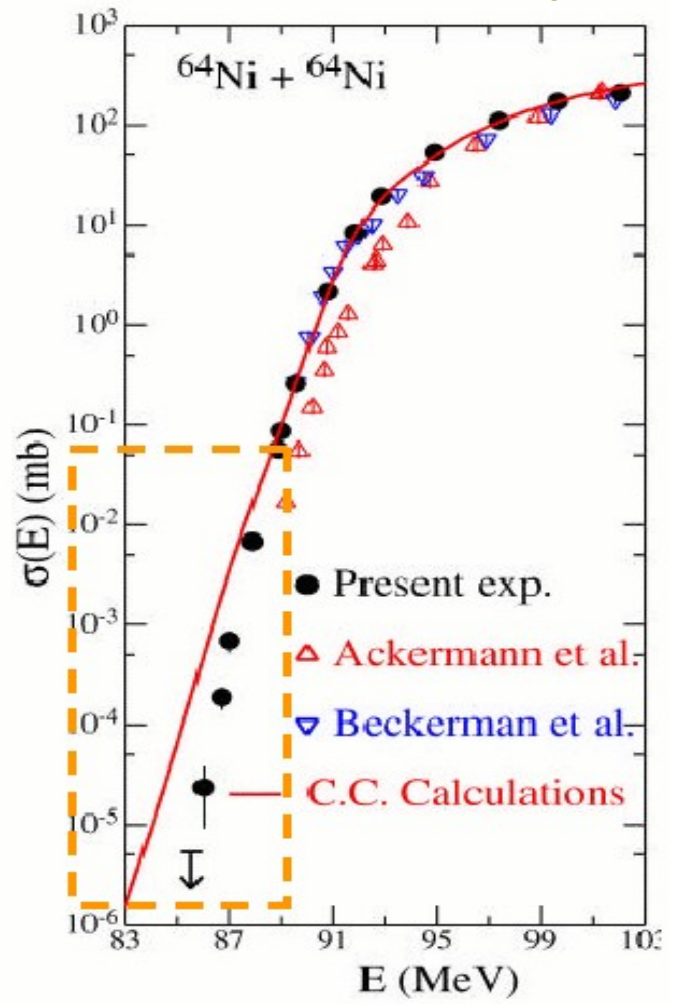
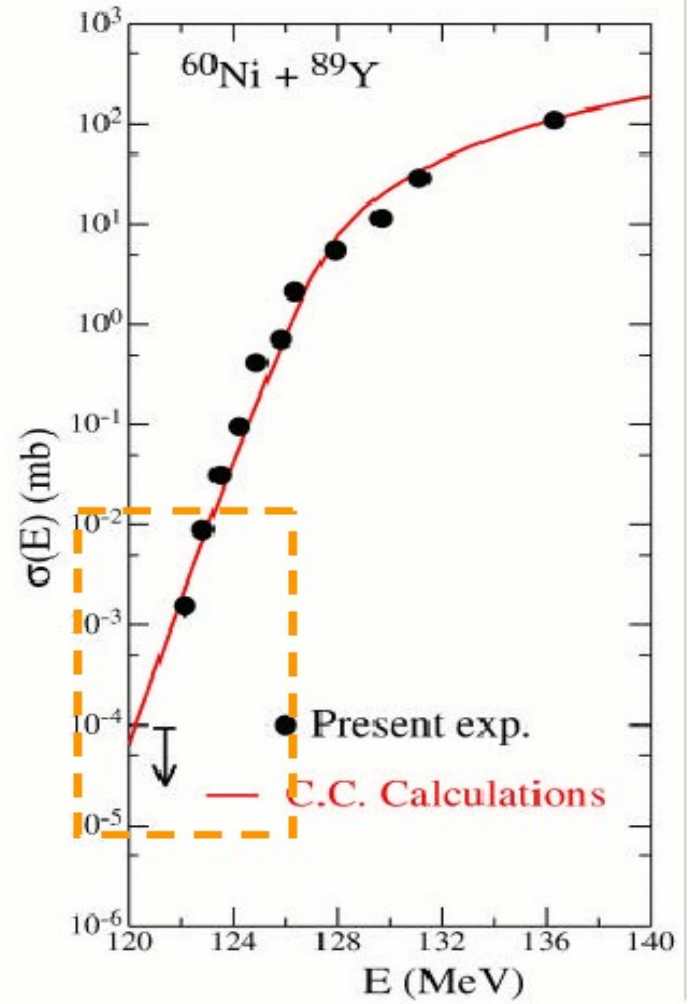
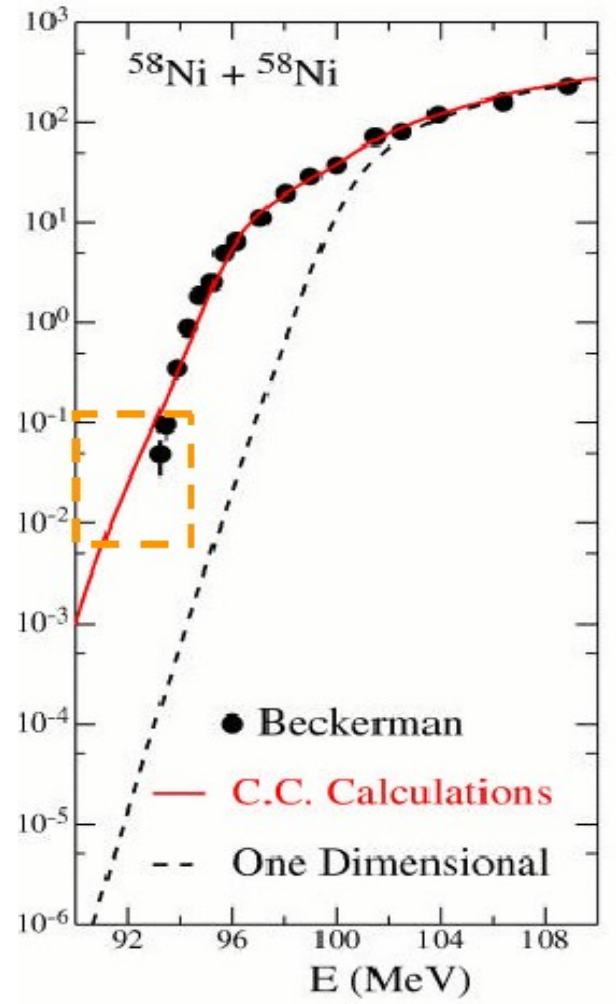
Effect of various Orientations



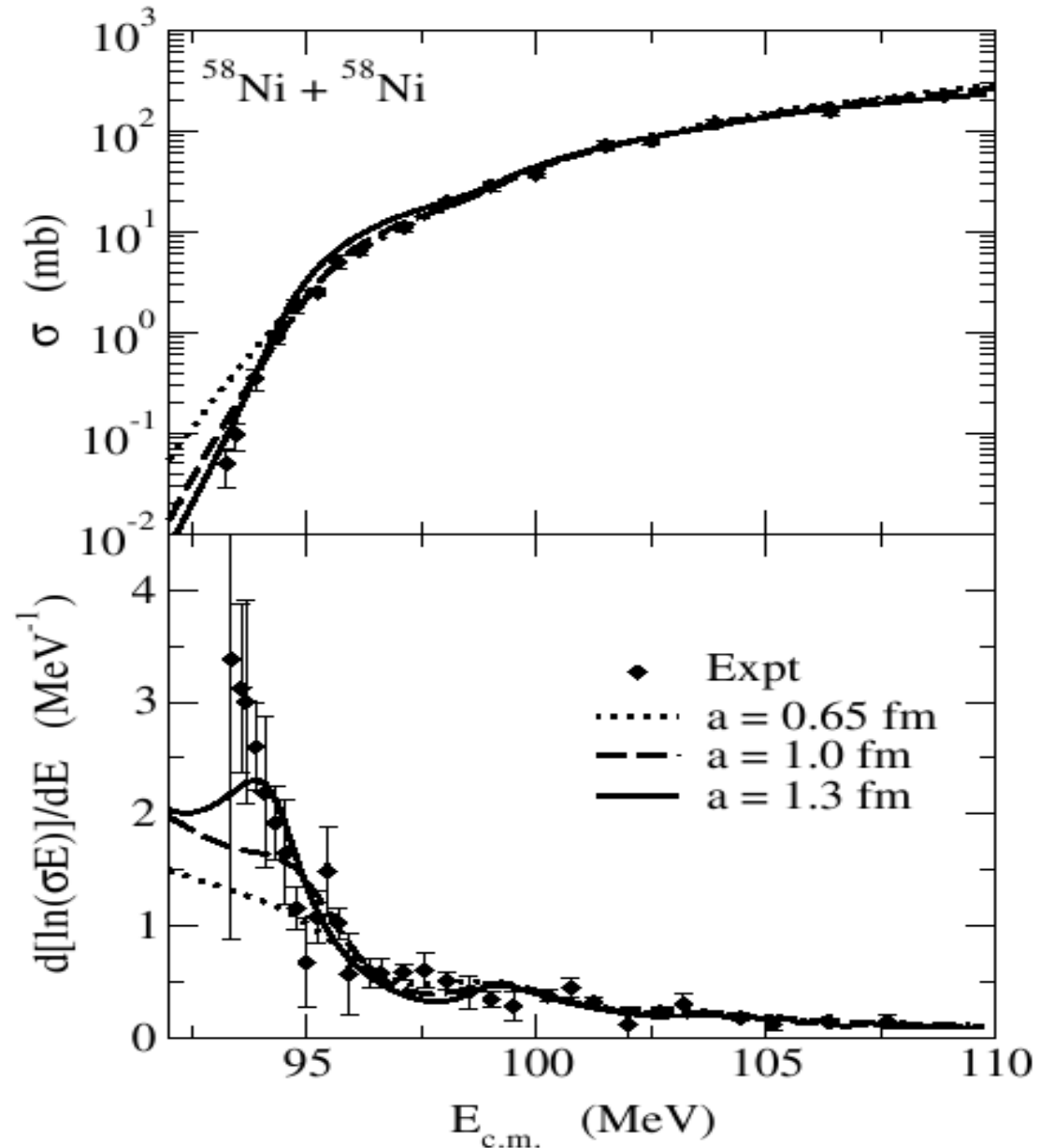
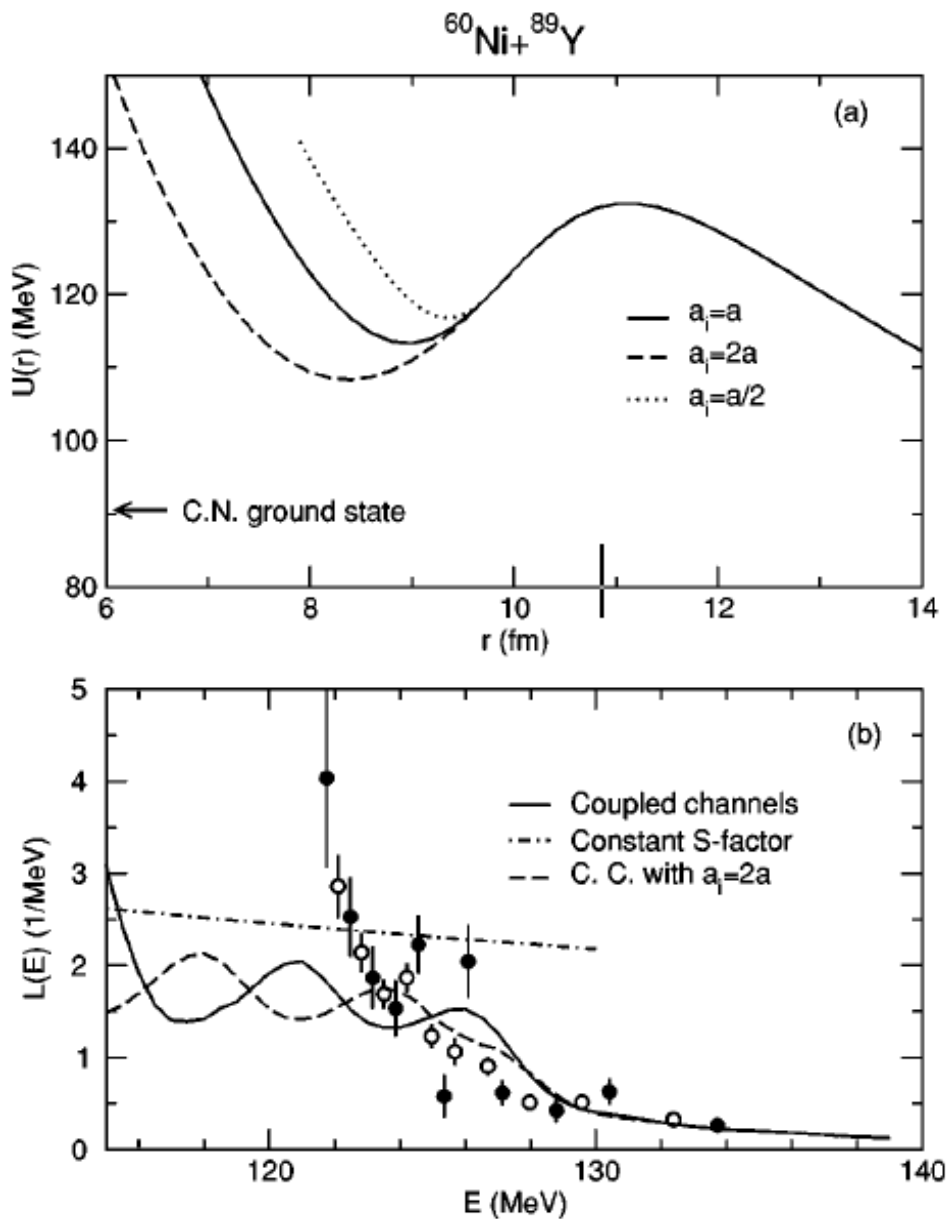
SUB-BARRIER FUSION : Hindrance for $E < E_s$

2001-Jiang conjectures that the hindrance of fusion far below the Coulomb barrier is a general phenomenon for many heavy-ion systems.

$E < E_s$



SUB-BARRIER FUSION : early attempts to solve the puzzle



Outgoing boundary conditions

$$u_n^{LM}(r) \longrightarrow \delta_{n0} F_L(k_n r) + T_n H_L^{(+)}(k_n r)$$

Transmission coefficient

$$T_n = \frac{1}{2\pi} (\delta_{n0} - S_{n0})$$

$$T = 1 - \sum_n |S_{n0}|^2 \quad (T = e^{-\frac{2}{\hbar} \int_{R_{t1}}^{R_{t2}} \sqrt{2\mu(E-V(r'))} dr'} \quad -WKB)$$

Cross section

$$\sigma_F(E) = \frac{\pi}{k_0^2} \sum_L (2L+1) T_L$$

Repulsive Core

Calculate the cost of overlapping completely the ions from the EOS
Equate the cost to the (increase in HI potential)/particle

$$\Delta V \approx 2A_p [\varepsilon(2\rho_0, \delta) - \varepsilon(\rho_0, \delta)]$$

EOS – Thomas-Fermi Model (Myers&Swiatecki)

$$\varepsilon(\rho, \delta) = \varepsilon_F \left[A(\delta) \left(\frac{\rho}{\rho_0} \right)^{2/3} + B(\delta) \left(\frac{\rho}{\rho_0} \right) + C(\delta) \left(\frac{\rho}{\rho_0} \right)^{5/3} \right]$$

Incompressibility of Cold Nuclear Matter at saturation

$$K = 9 \left(\rho^2 \frac{\partial^2 \varepsilon}{\partial \rho^2} \right)_{\rho=\rho_0}$$

Repulsive Core

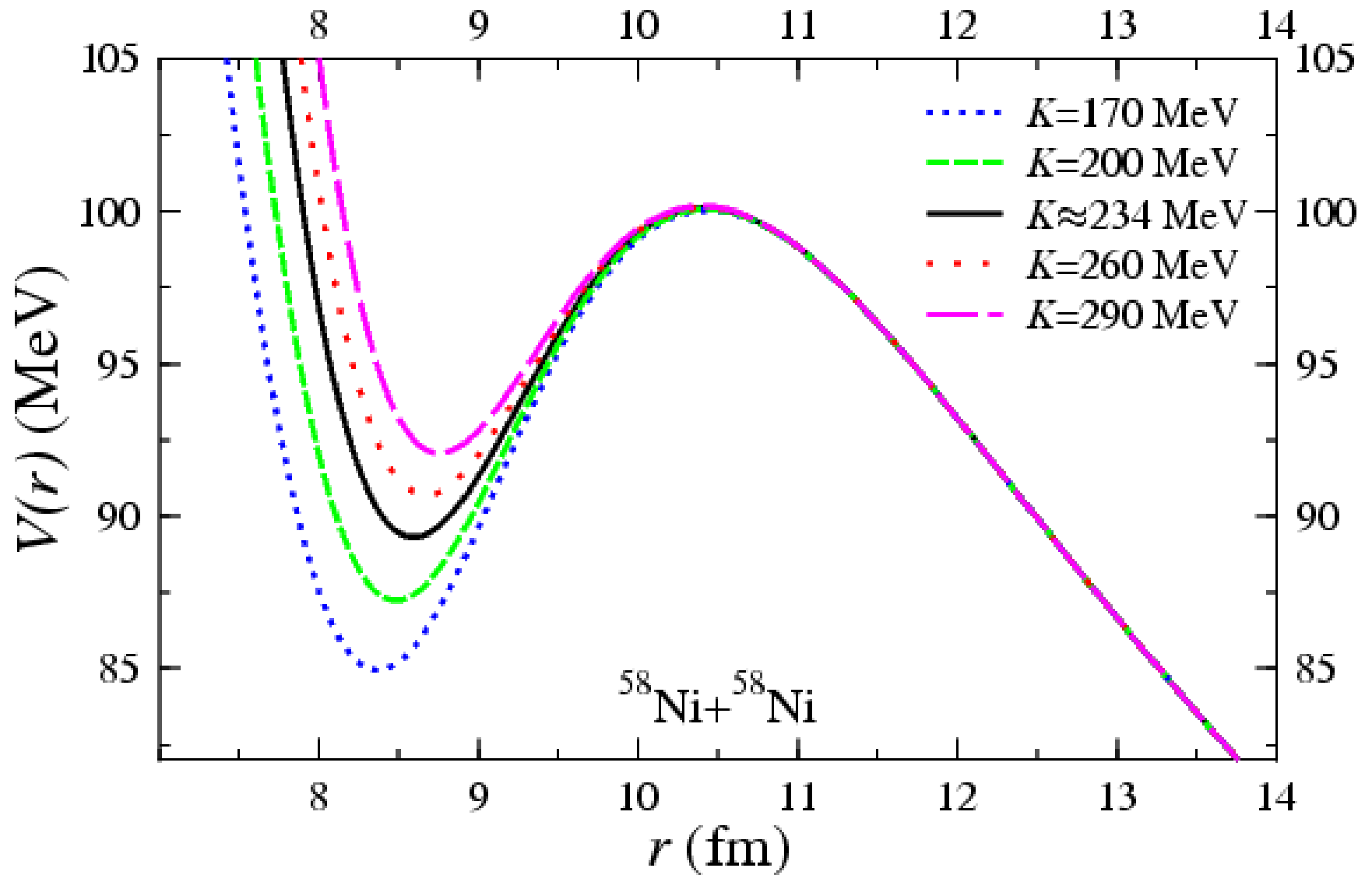
$$V_{\text{rep}}(\mathbf{R}) = V_p \int d\mathbf{r}_1 \int d\mathbf{r}_2 \tilde{\rho}_1(\mathbf{r}_1) \tilde{\rho}_2(\mathbf{r}_2) \delta(\mathbf{r}_{12})$$

Approximations to calibrate the strength of the repulsion

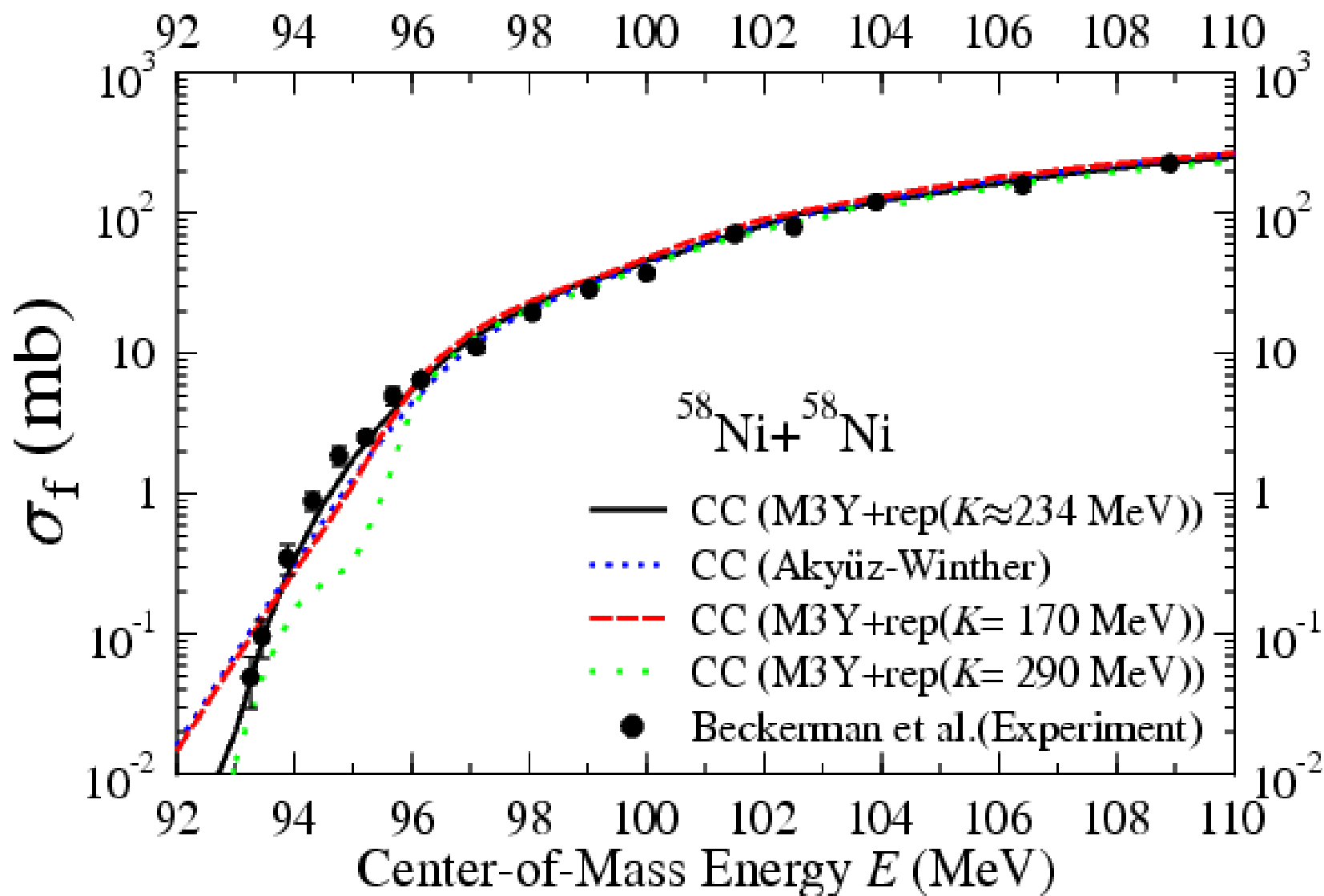
$$\varepsilon(\rho, \delta) = \varepsilon(\rho_0, \delta) + \frac{K}{18\rho_0^2} (\rho - \rho_0)^2$$

$$\Delta V = V_N(0)$$

Shallow potential



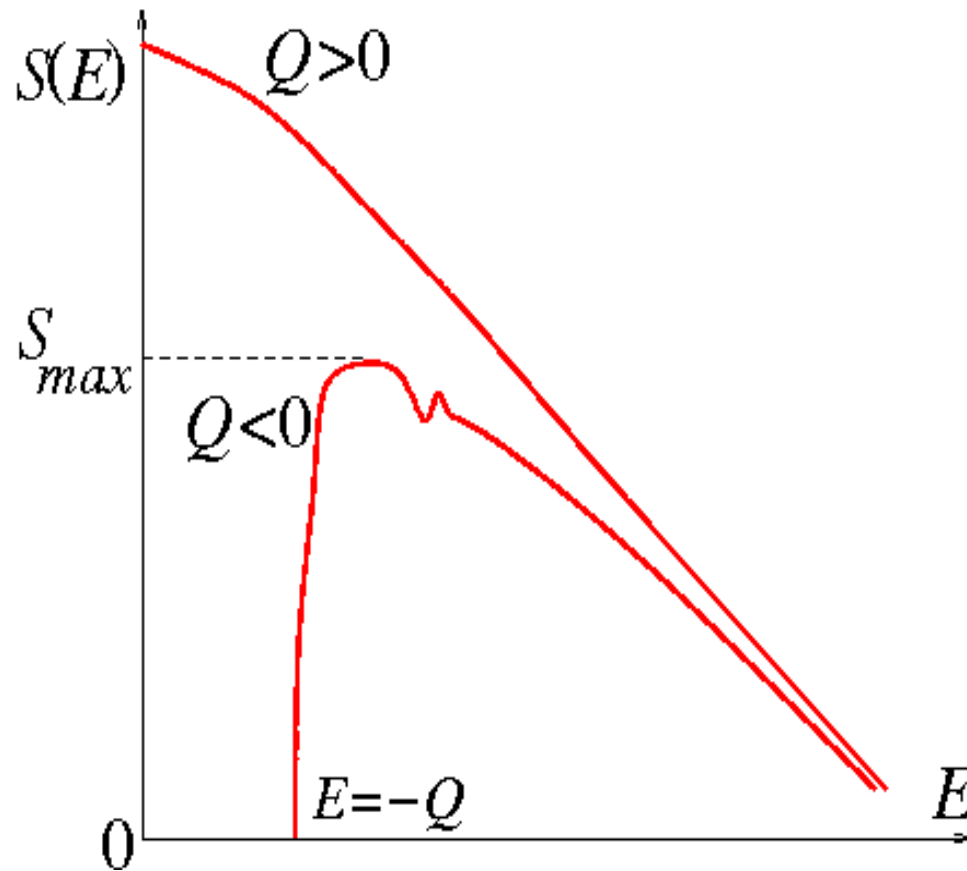
Fusion Cross-Sections vs. EOS



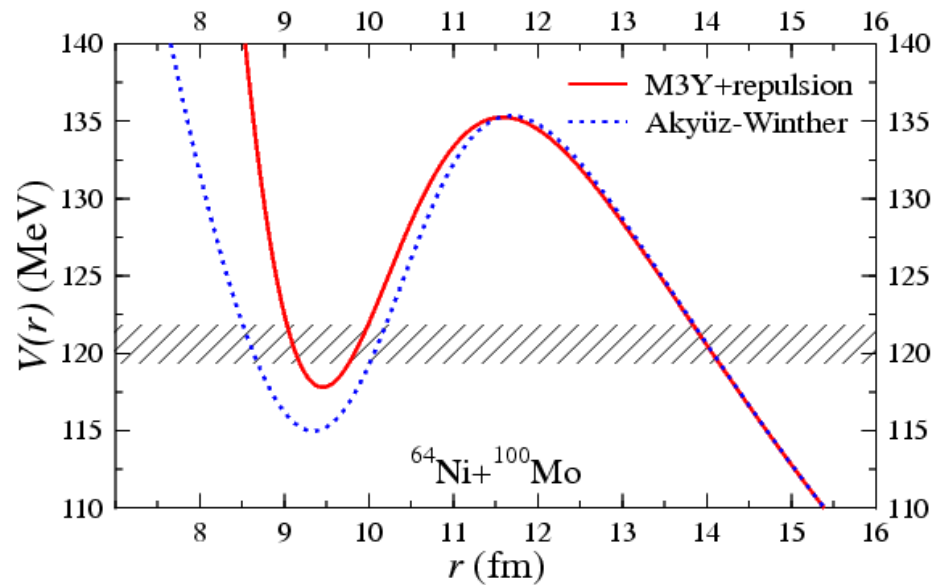
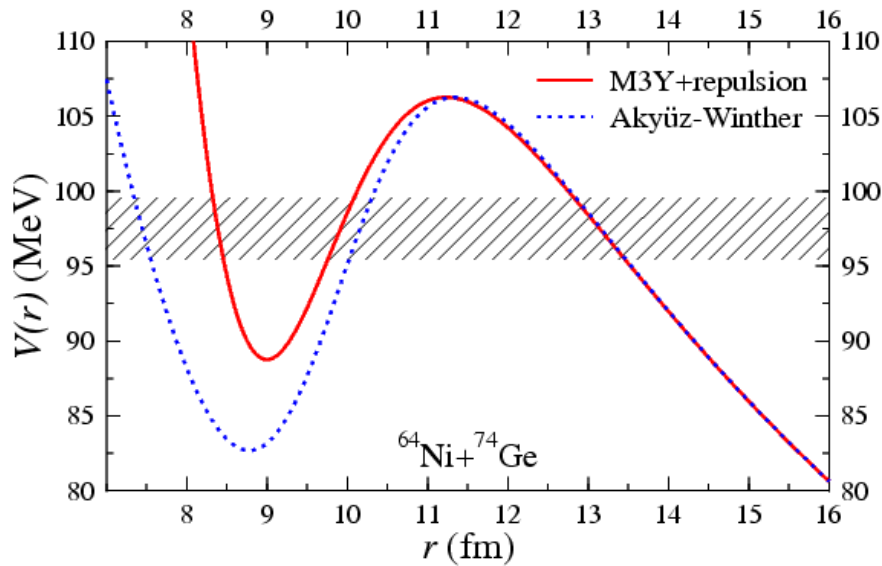
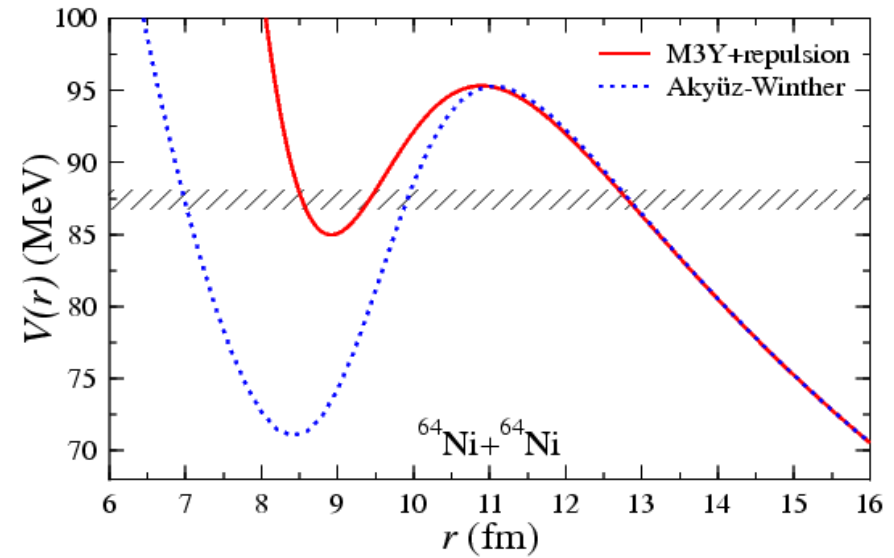
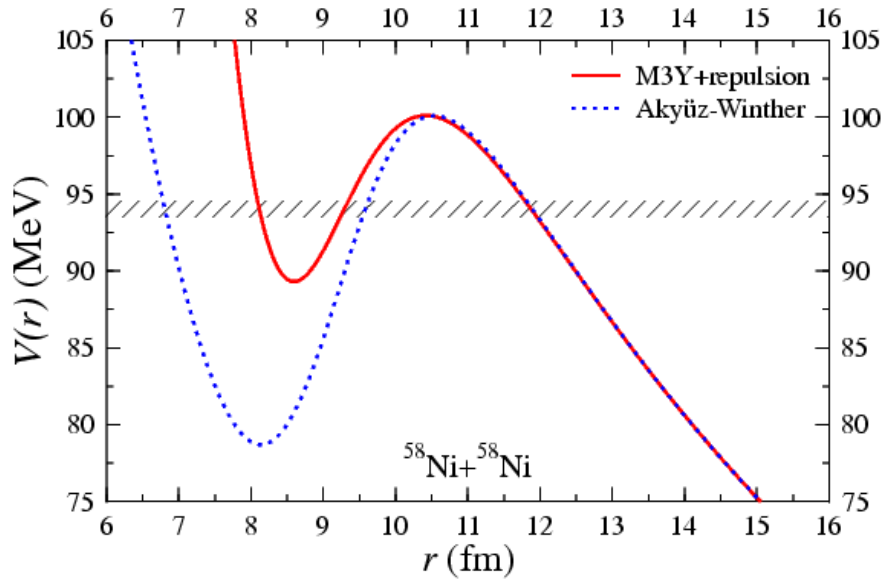
Diagnostic Tools: Astrophysical S-factor

$$S = E\sigma_F(E) \exp(2\pi\eta), \quad \eta = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar v}$$

- Magnify the low-energy behavior of cross-sections



M3Y+repulsion vs. Woods-Saxon

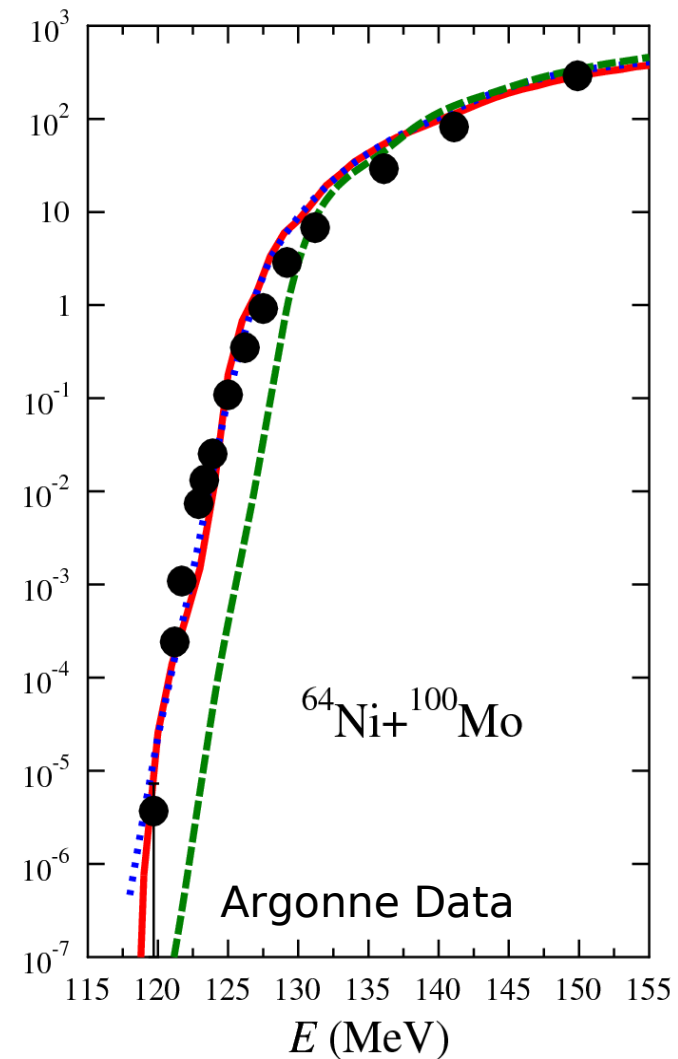
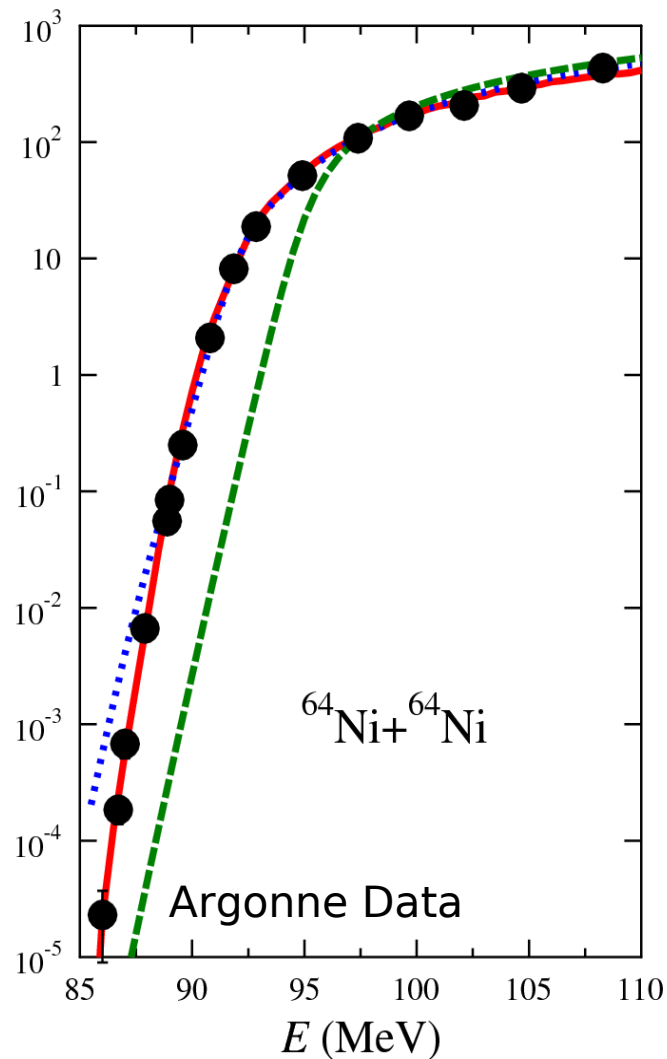
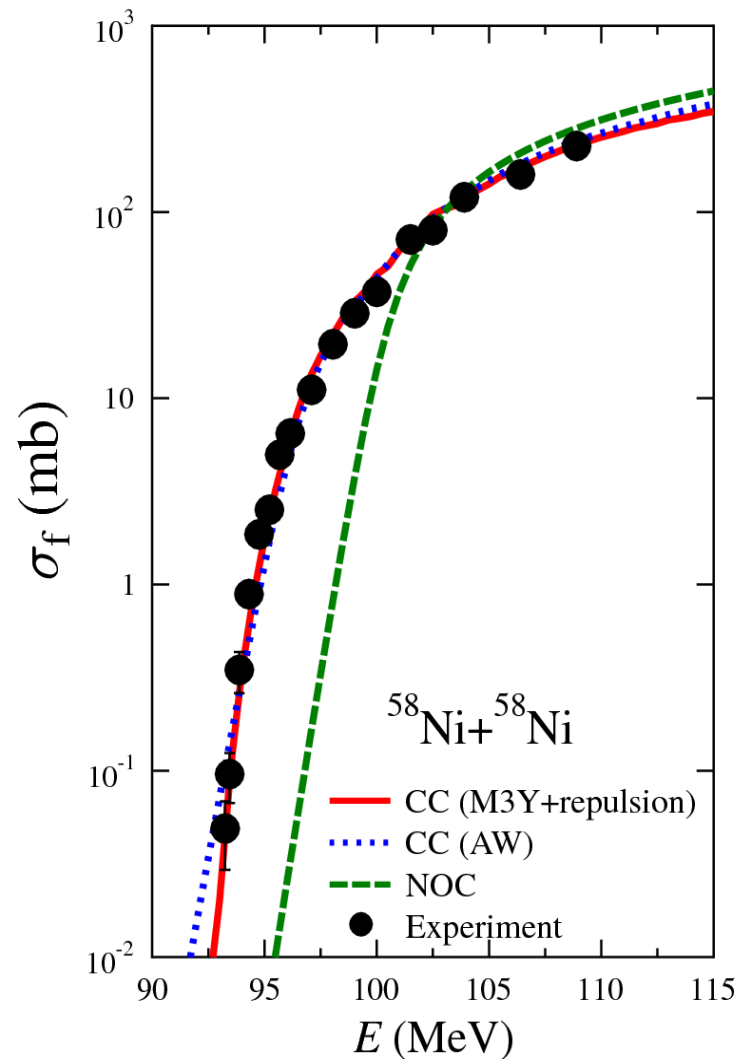


SUB-BARRIER FUSION : Maximum of S

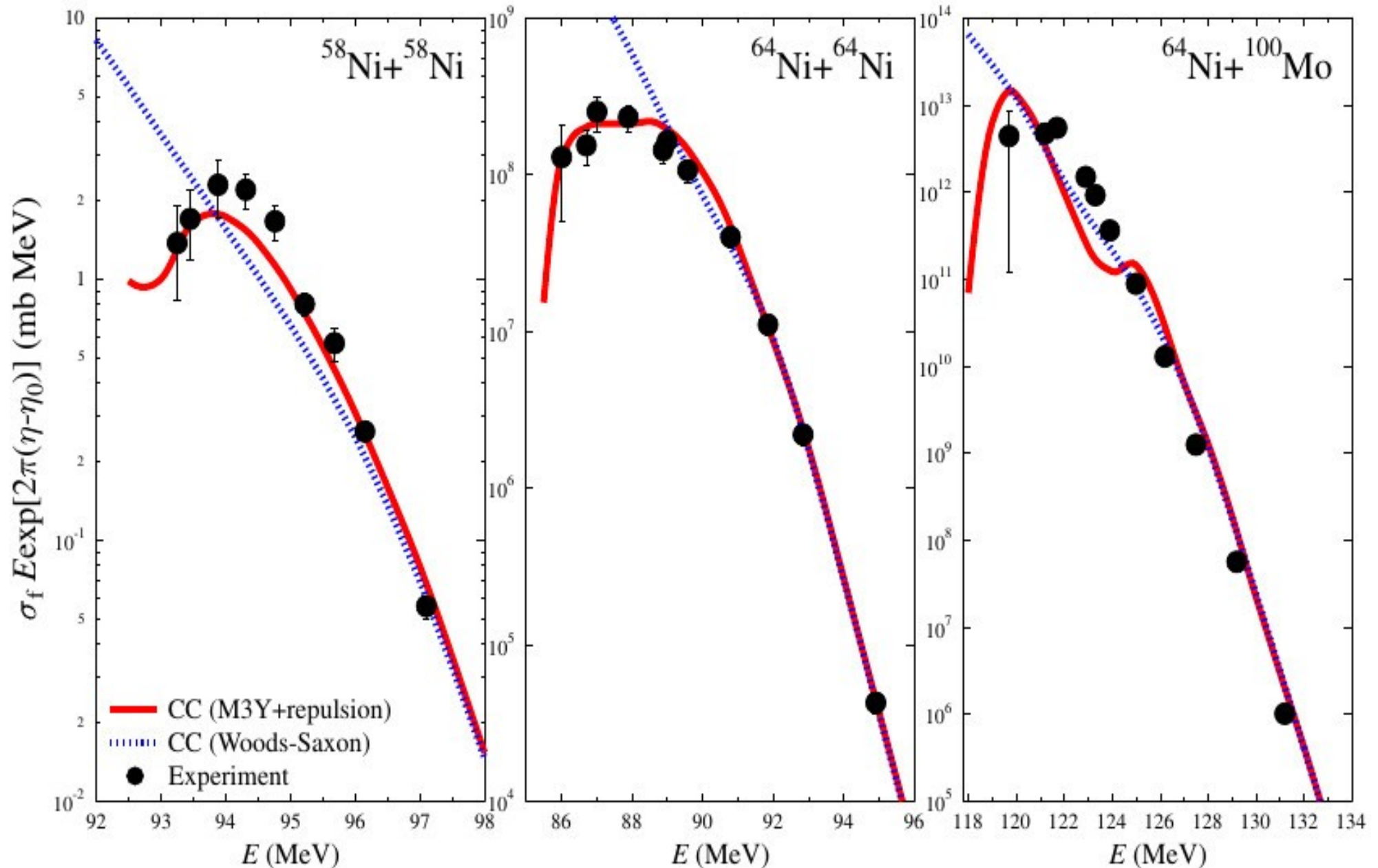
$^{58}\text{Ni}+^{58}\text{Ni}$, $Q=-66.12$ MeV

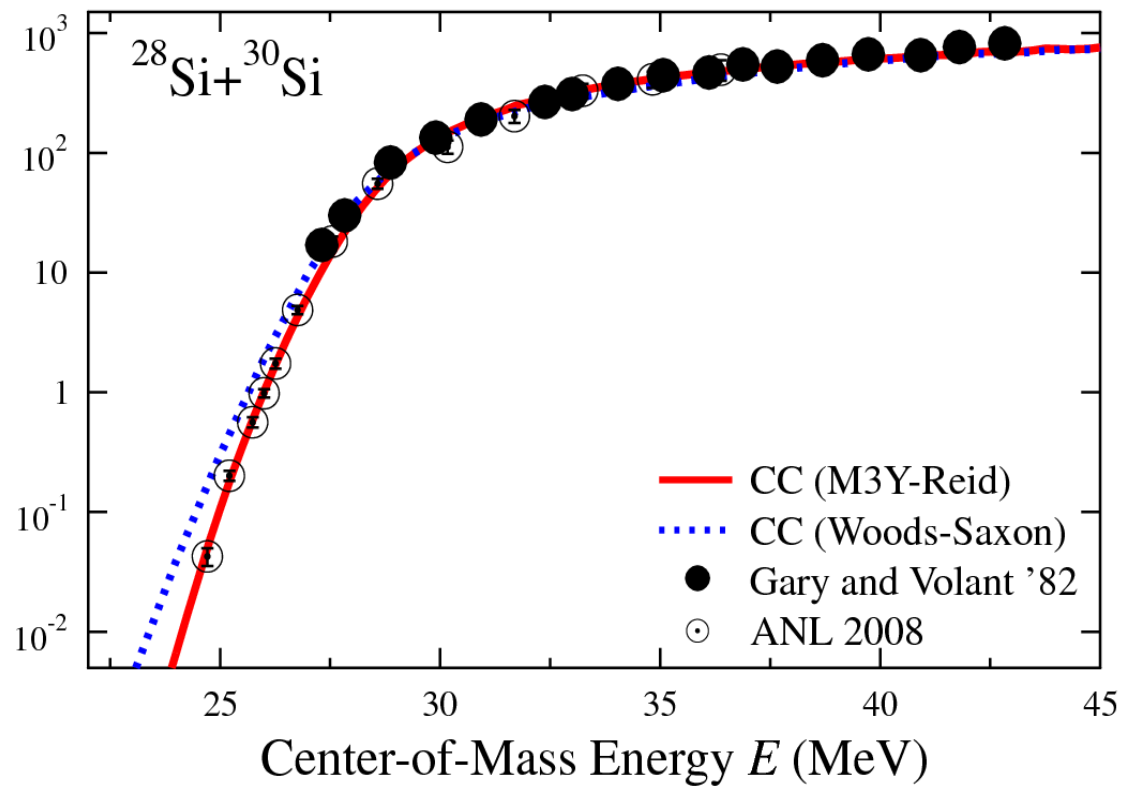
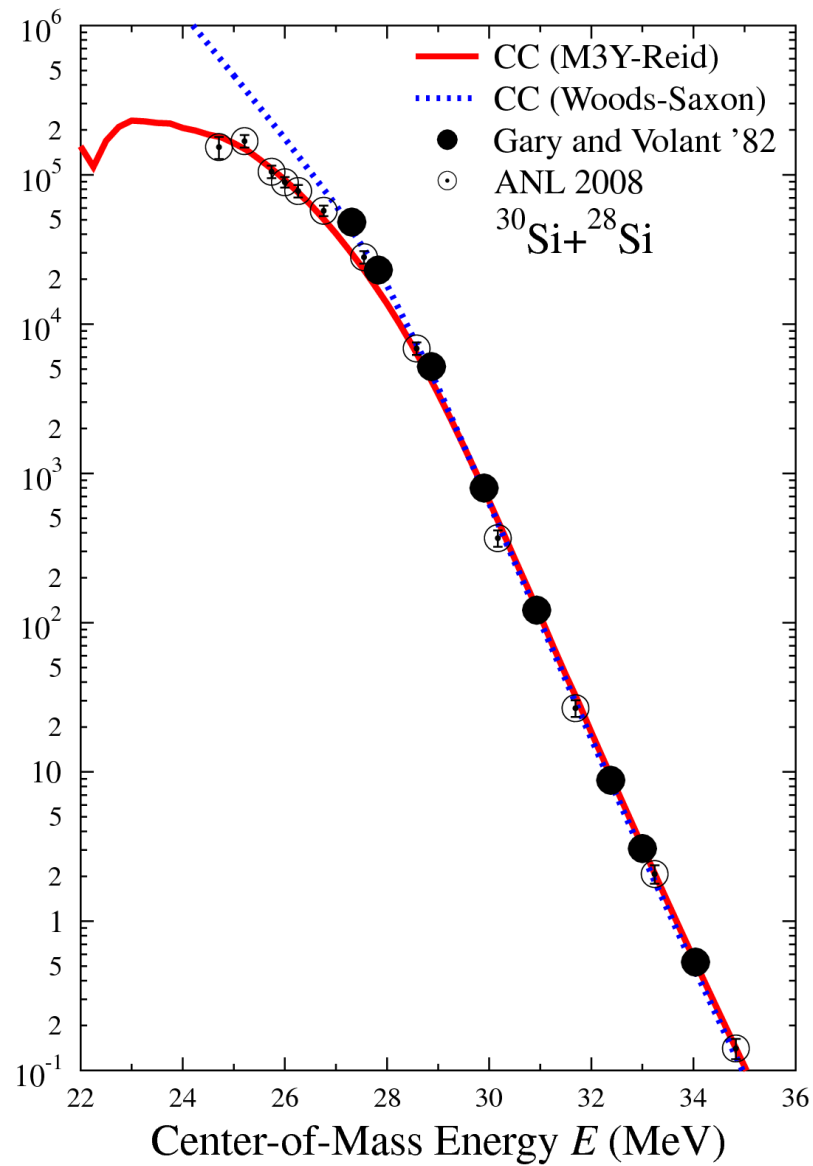
$^{64}\text{Ni}+^{64}\text{Ni}$, $Q=-48.78$ MeV

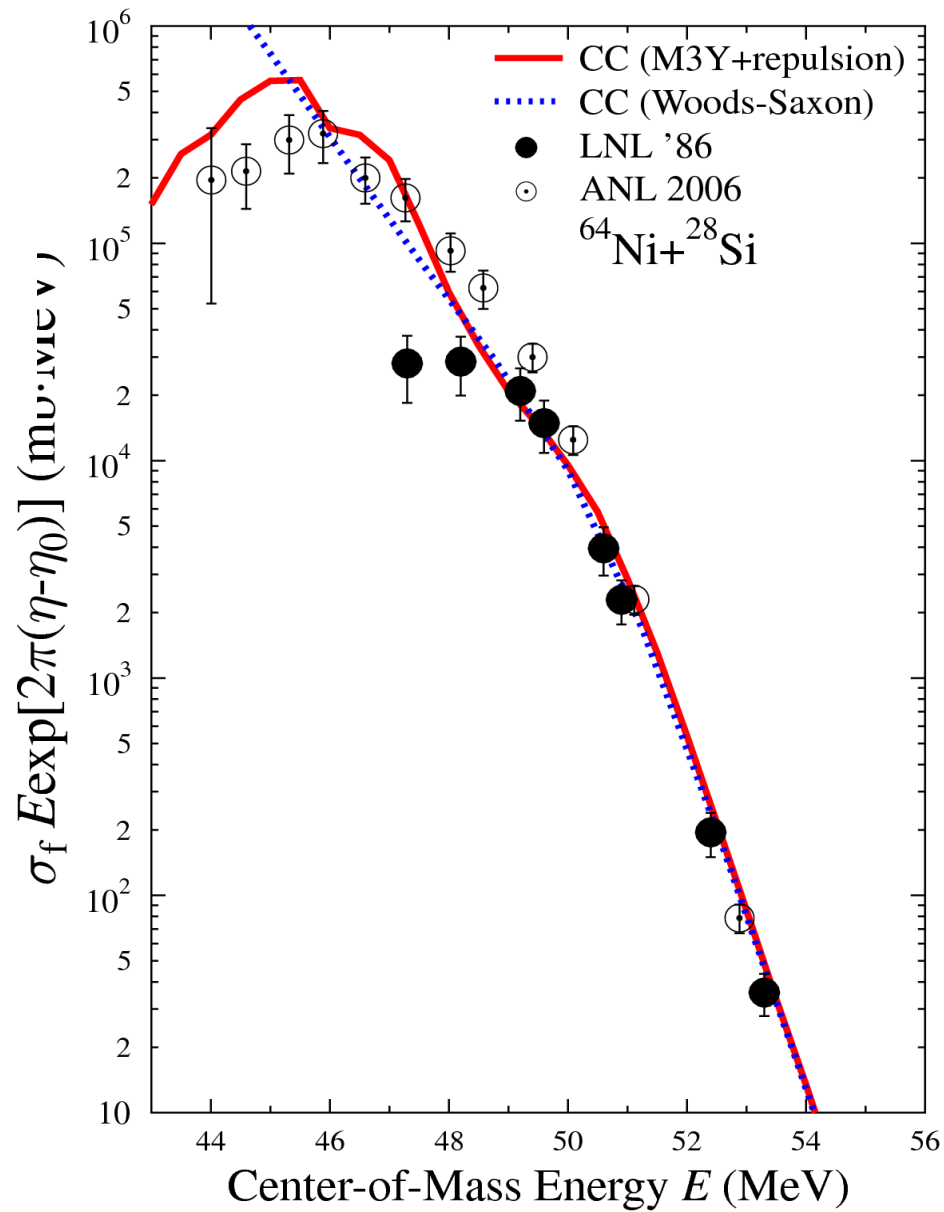
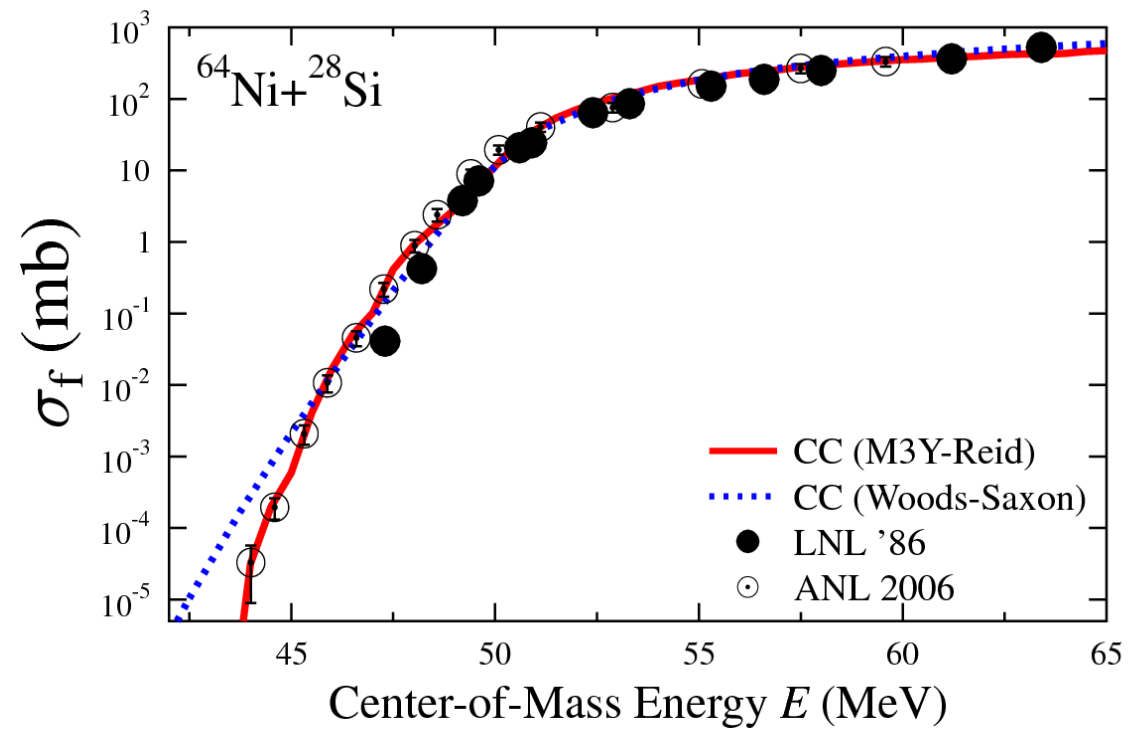
$^{64}\text{Ni}+^{100}\text{Mo}$, $Q=92.29$ MeV



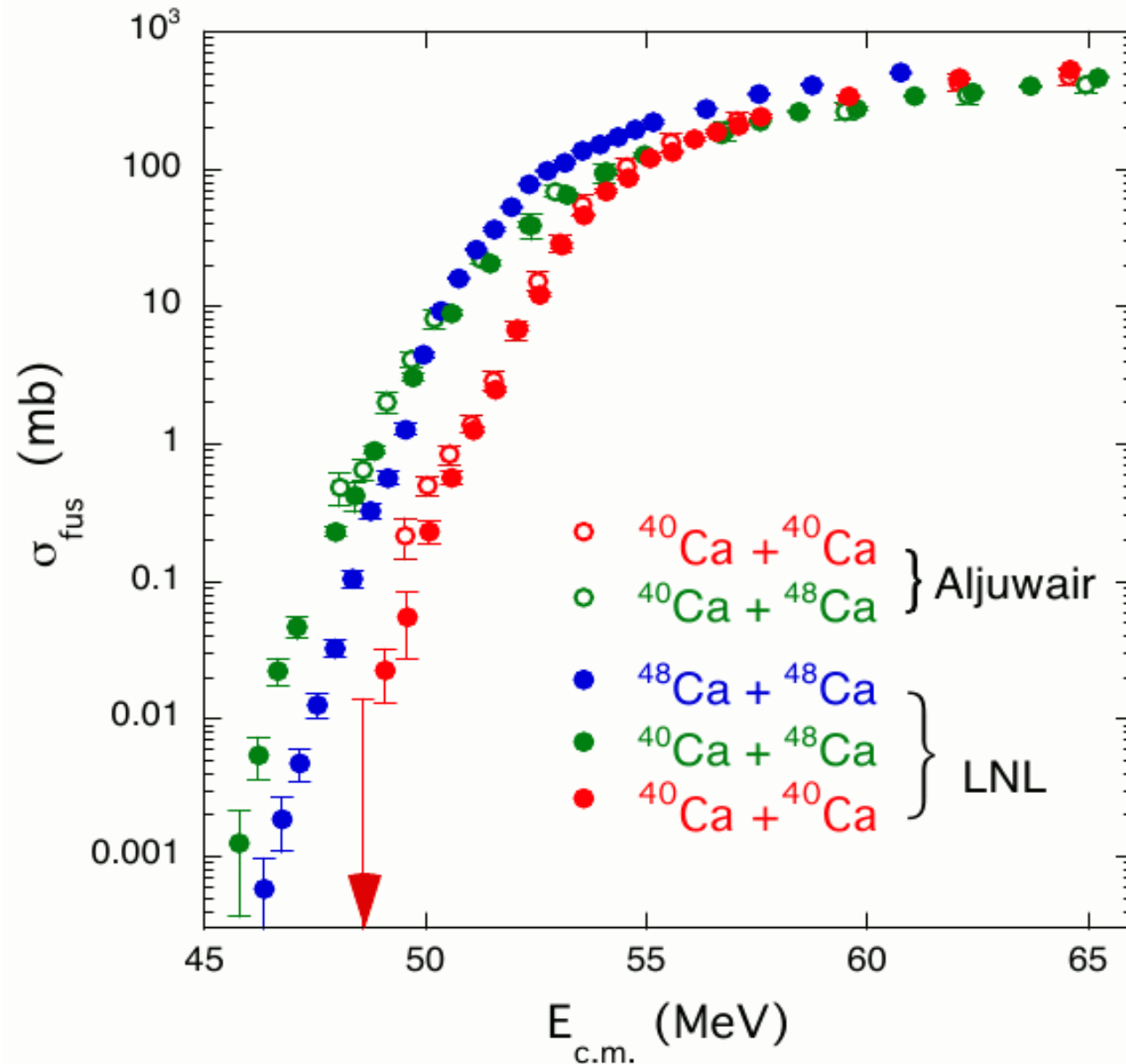
SUB-BARRIER FUSION : Maximum of S



σ_f (mb) $\sigma_f E \exp[2\pi(\eta - \eta_0)]$ (mb·MeV)

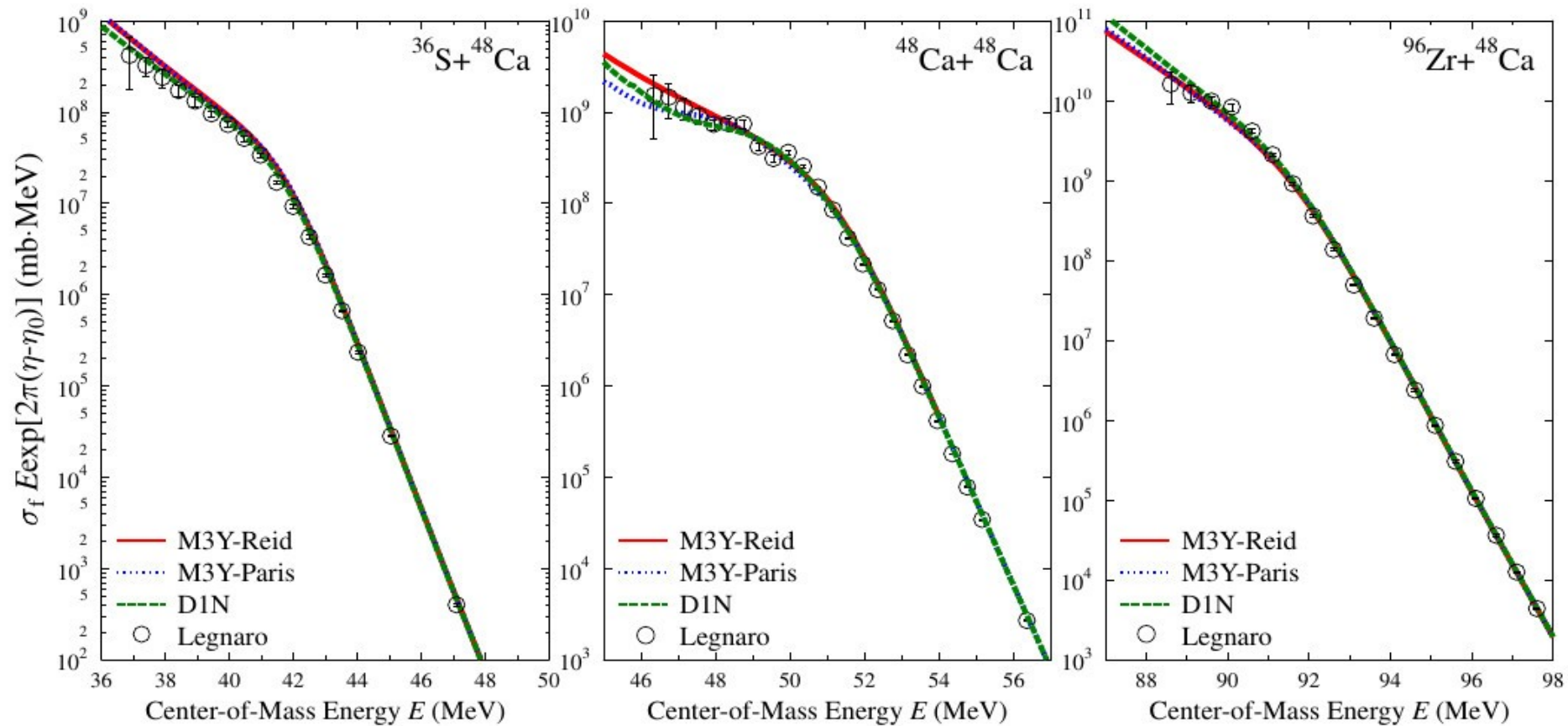
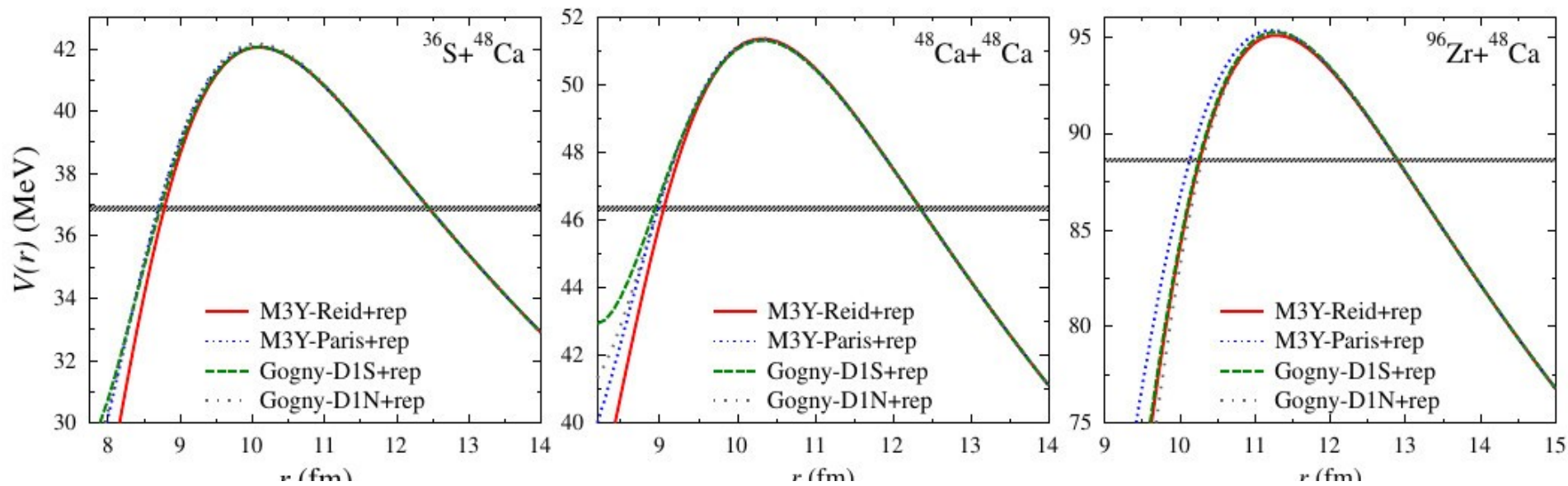


SUB-BARRIER FUSION : Hindrance but no maximum of S

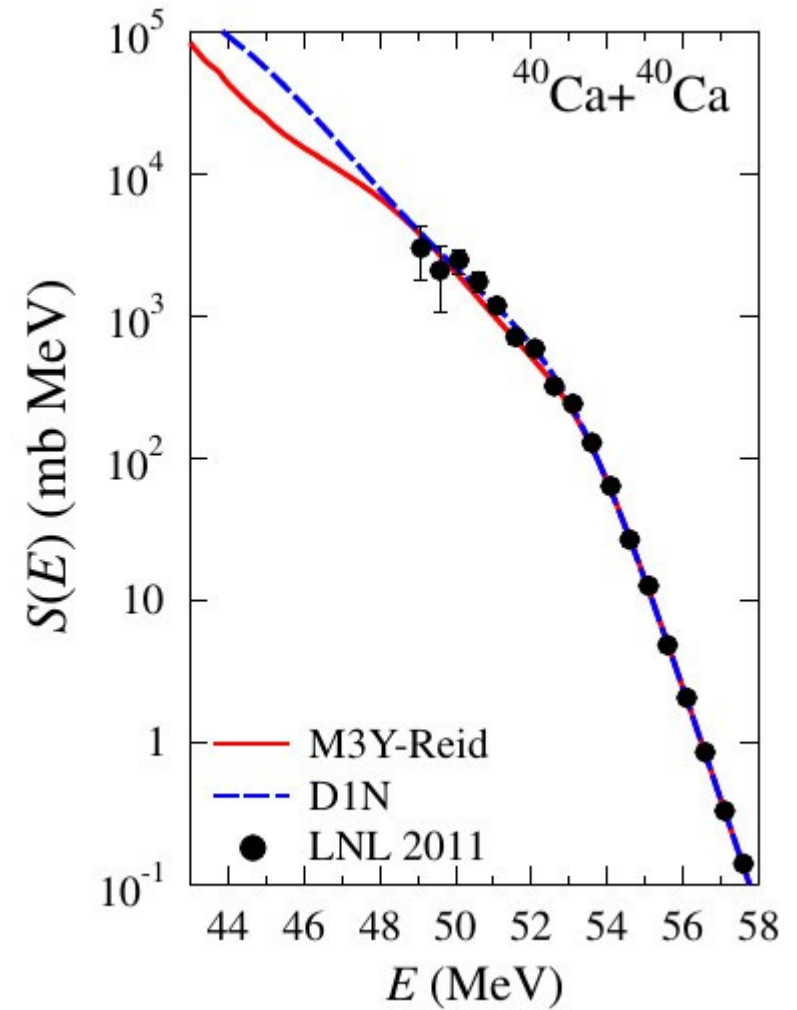
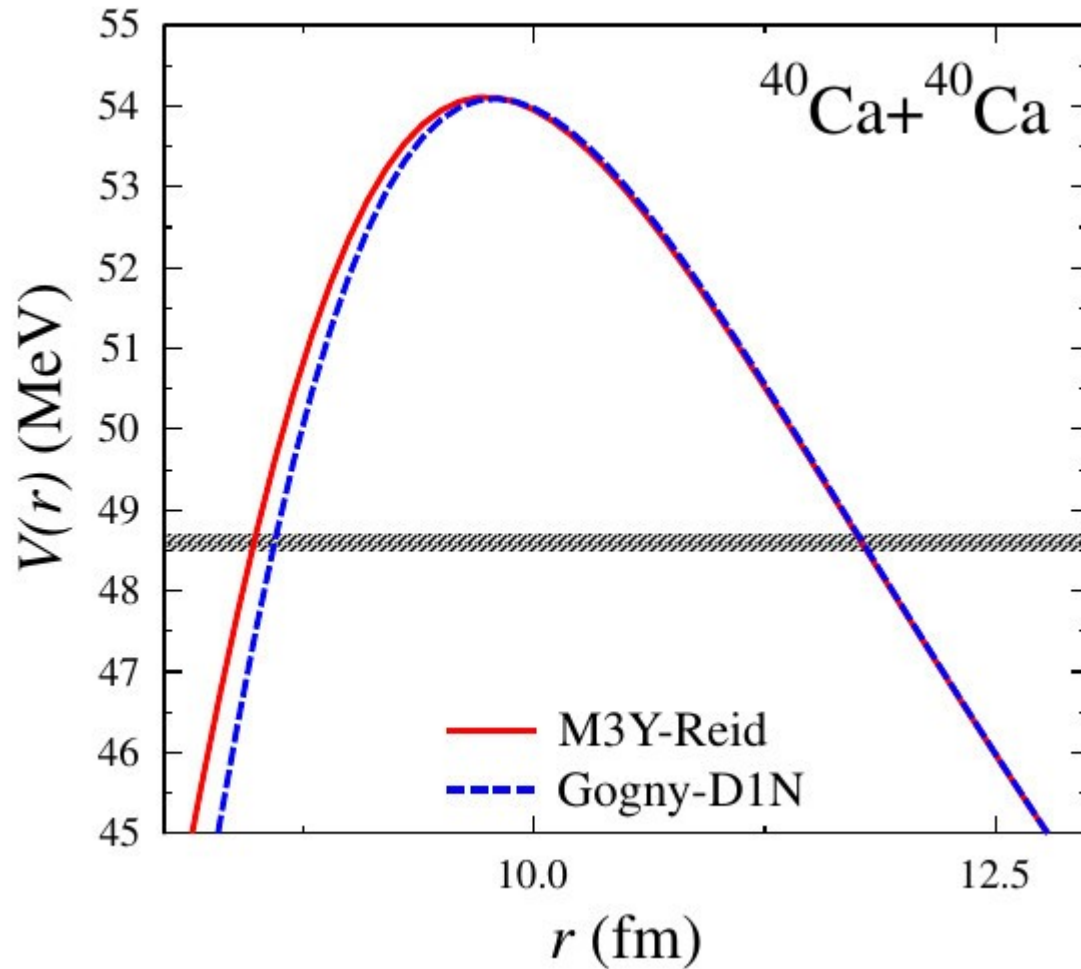


Legnaro (2009)

SUB-BARRIER FUSION : Hindrance but no maximum of S



SUB-BARRIER FUSION : Hindrance but no maximum of S



SUB-BARRIER FUSION : Lighter systems

- **WILL THE HINDRANCE PERSIST**, and how will it affect the extrapolation to astrophysical reaction rates ?

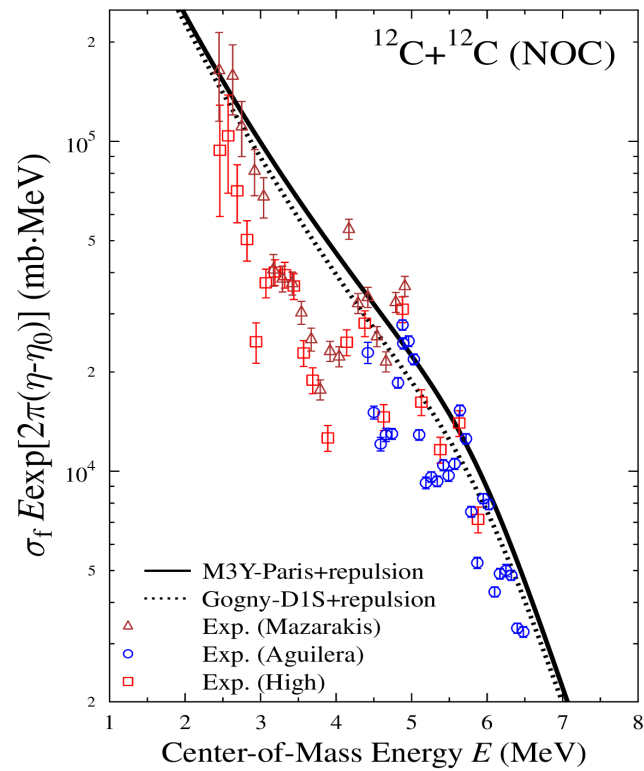
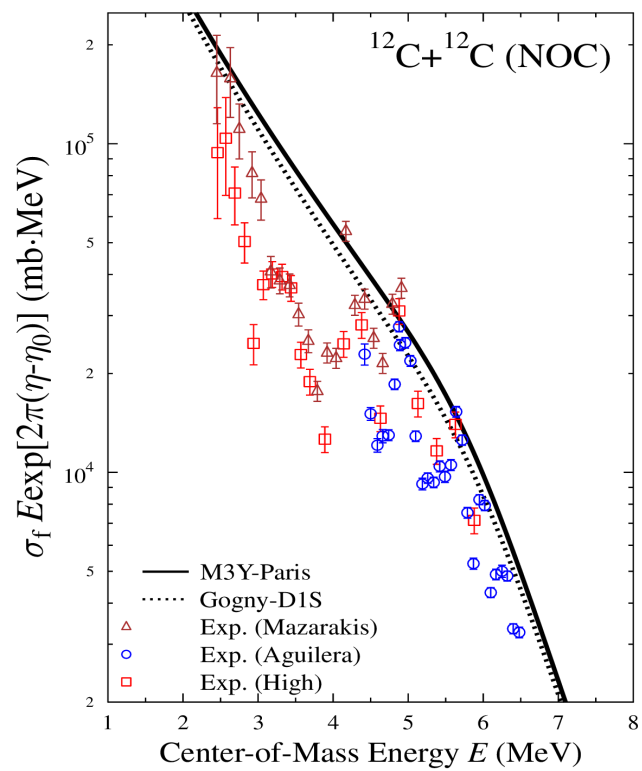
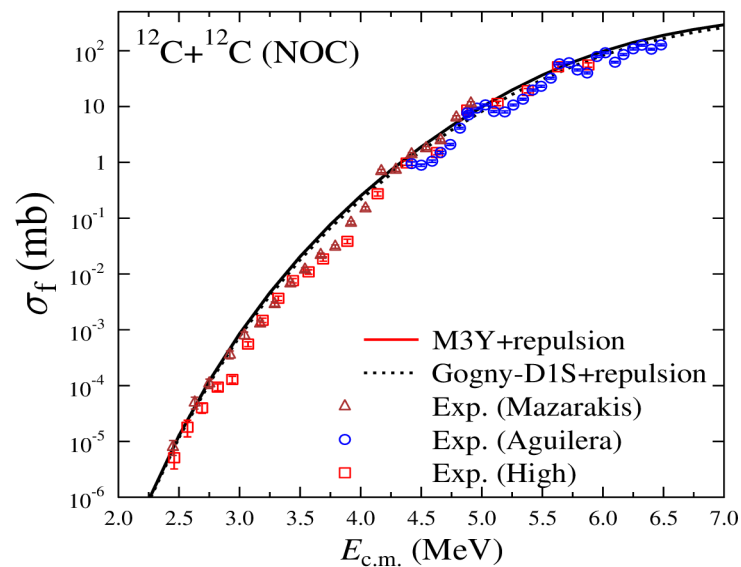
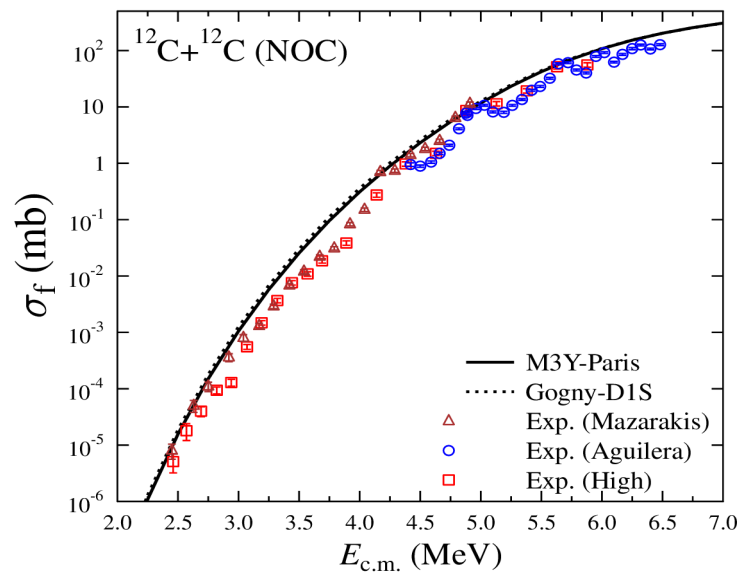
YES!

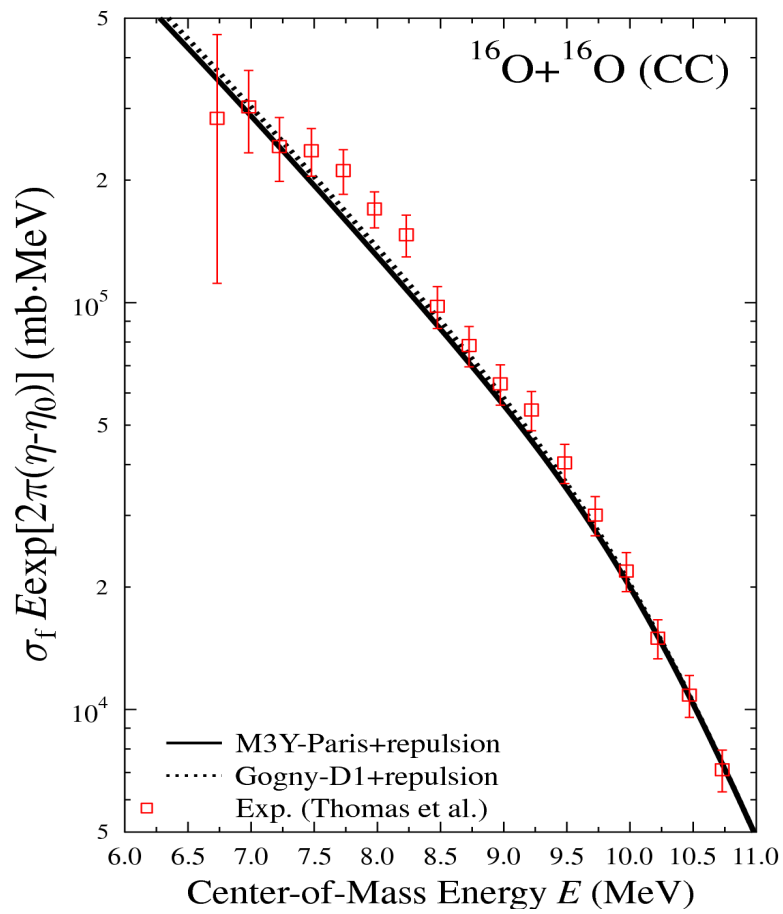
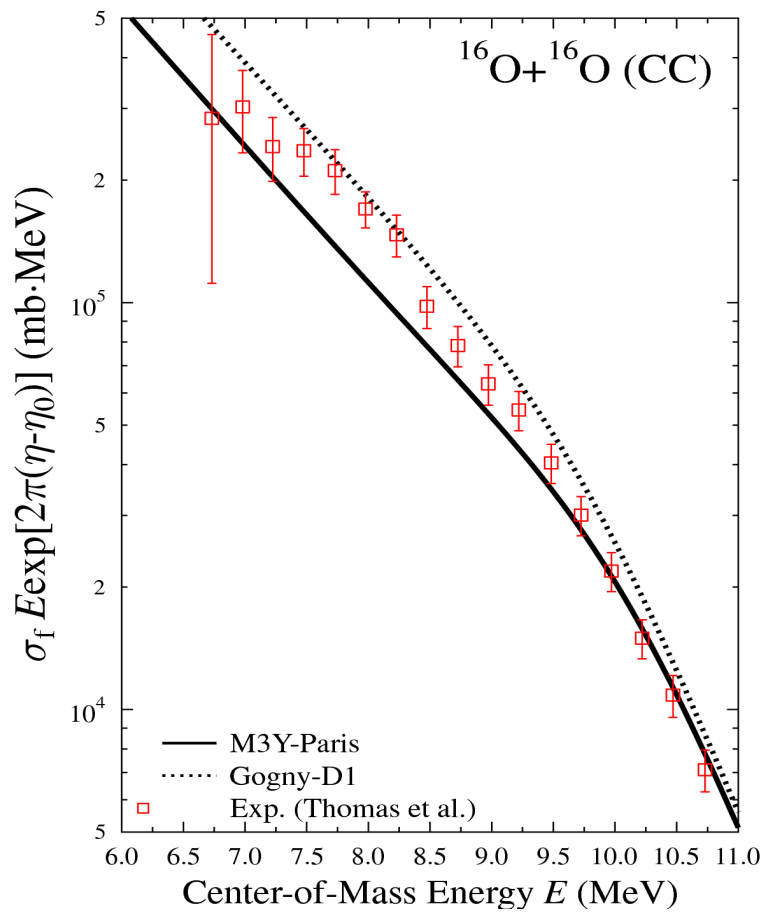
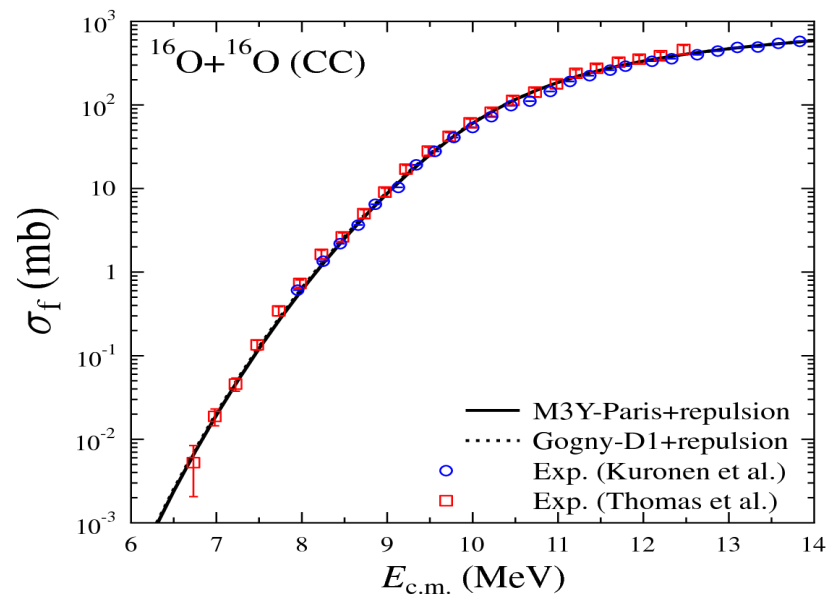
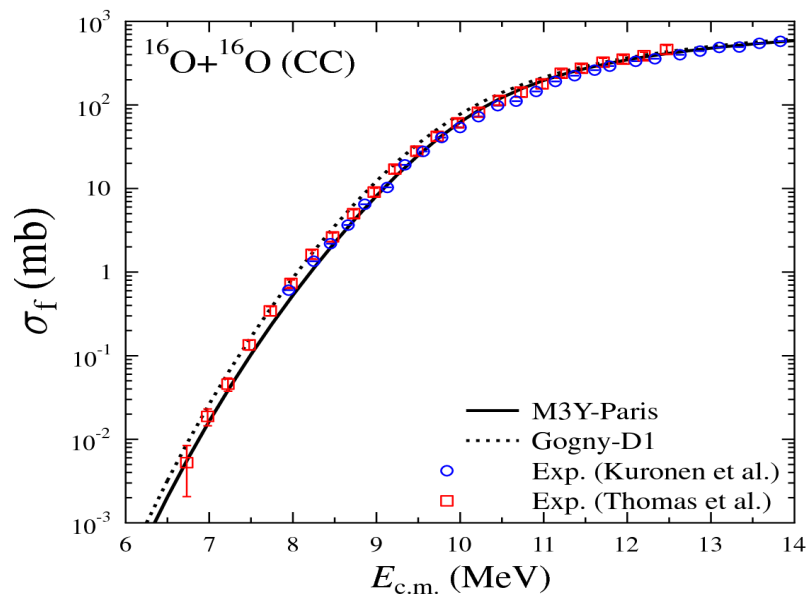
Jiang et al. PRC 75, 015803, 2007

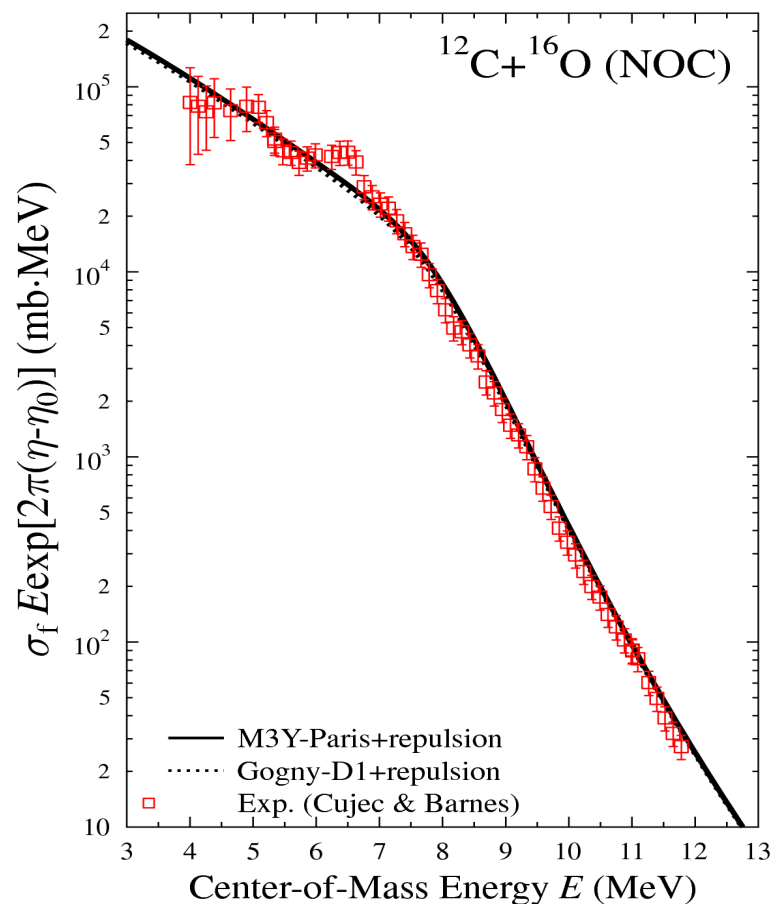
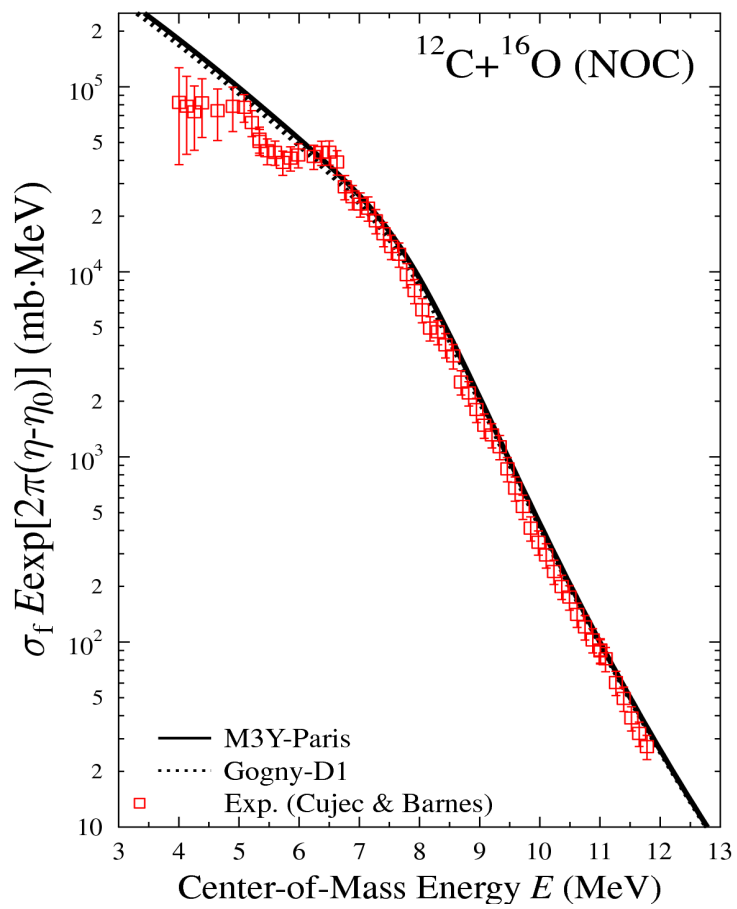
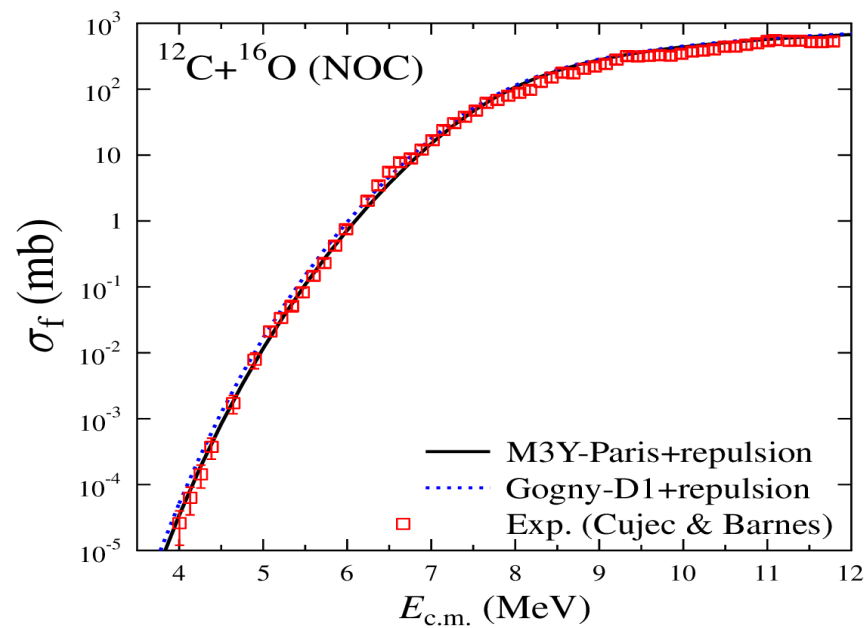
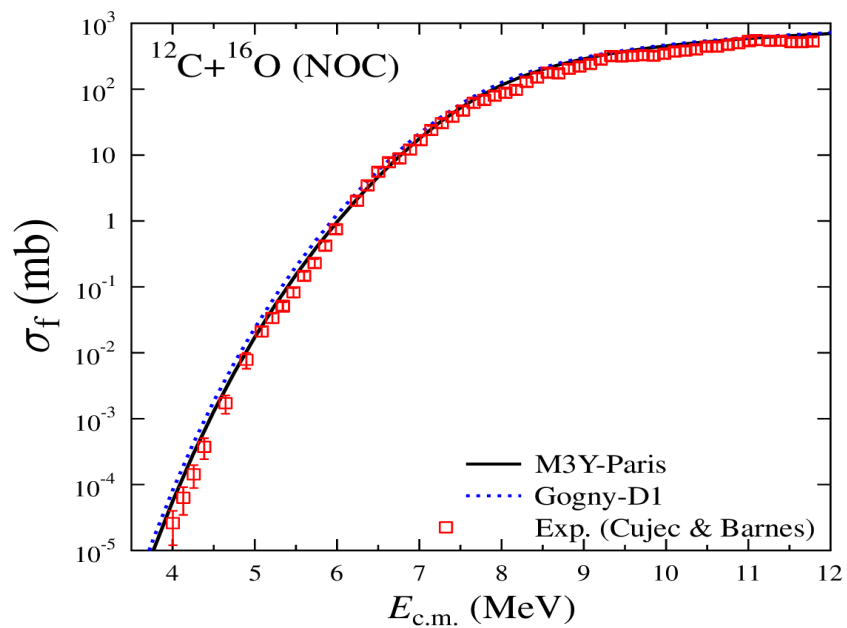
Gasques et al., PRC 76, 03582, 2007

NO!

Misicu&Carstoiu, NPA 834, 180c(2010)







Acknowledgements

Work supported by

MSR, CNCSIS (Romania)

