

# Some thoughts on Fission Yield Data in Estimating Reactor Core Radionuclide Activities (for anti-neutrino estimation)

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- Introduction
- Fission Yield Evaluation
  - Evaluation
  - Measurements and their analysis
  - Models
  - Adjustment
  - Cumulative and independent yield uncertainty estimation
- Error propagation in nuclear codes
  - “Total Monte Carlo”
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  - Summation example using TENDL decay heat
- Some FISPIN calculations of anti-neutrinos in a Pressurised Water Reactor

- Anti-Neutrino production in reactors is a decay process and thus is dependent upon the summation over all radio-nuclides of the decay rate present multiplied by the probability of (anti-) neutrino production.

$$N_{\bar{\nu}_e} = \sum_i \lambda_i N_i * N_{\bar{\nu}_e} \text{ per decay of } i$$

- This probability is equal to the  $\beta^+$  (or  $\beta^-$ ) probability.
- This requires the number densities or activities of the radio-nuclides present in the core

- The production and destruction of all nuclides are governed by the standard equations:

$$\begin{aligned}\frac{dN_i}{dt} = & -\lambda_i N_i + \sum_j \lambda_j N_j B_{j,i} \\ & - \sum_l N_l \sigma_{l,d} \phi + \sum_m N_m \sigma_{m,i} \phi \\ & + \sum_k N_k \sigma_{k,f} \phi Y_{k,i}^i\end{aligned}$$

- Depends on flux, cross-sections, fission yields, decay constants and branching ratios decay paths.

## What is nuclear data “evaluation”

- Can be described as the processing of giving numeric value to a quantity

.... but

- Has to be consistent with measurements and constraints of physical laws, at least within currently accepted knowledge.
- Has to be consistent with semi-empirical or theoretical models.
- Has to be reported/distributed in a form that all can use easily (e.g. ENDF format)

## History of UK evaluations

1960-1981 Crouch

Atomic and Nuclear Data Tables (1977)

1981-1987

James and Banai UKFY1 / JEF1 (1986)

1988-1995

Mills, James and Weaver UKFY2 / JEF-2.2 (1993)

1995-present

Mills UKFY3.x series - JEFF-3.1.1 (UKFY3.6A)

(UKFY4 photon, neutron and charged particle induced energy dependent or spontaneous fission using Wahl code).

## Definitions

The independent yield  $y(A,Z,I)$  is the number of atoms of  $(A,Z,I)$  produced directly from one fission, but after the emission of prompt neutrons (but before any radioactive decay and hence the emission of delayed neutrons). It can be written as the product of 3 factors:

$$y(A, Z, I) = Y(A) \times f(A, Z) \times R(A, Z, I)$$

where the **sum yield** or **mass yield**  $Y(A)$  is the total of the independent yields (before delayed neutron emission) of all fission products of mass number  $A$ ;  $f(A,Z)$  is the **fractional independent yield** of all isomers of  $(A,Z)$ ; and  $R(A,Z,I)$ , the **isomeric yield ratio**, is the fraction of  $(A,Z)$  produced directly as isomer  $I$ .

## Independent yields

- Direct probability of forming a nuclide after prompt neutron emission but before decay
- Needed for general inventory calculations
- Difficult to measure, especially for short half-lives or where corrections for major contributions from other nuclides' decay need to be considered.
- In JEFF fits to Wahl  $Z_p$  model are used to fill extrapolate from small number of measurements to complete set and then adjusted for physical constraints.
- Large uncertainties on values.

## Definitions

The **cumulative yield**  $c(A,Z,I)$  of nuclide  $(A,Z,I)$  is the total number of atoms of that nuclide produced over all time after one fission. If the nuclide is stable the cumulative yield is the total number of atoms of that nuclide remaining per fission after all precursor decays (ignoring the effects of other nuclear reactions e.g. neutron capture). However, for a radioactive nuclide for which this is not the case, some atoms will have decayed before all have been produced.

An equivalent definition that is more useful is the following: immediately at the end of an “infinite” irradiation at the rate of 1 fission per second,  $c(A,Z,I)$  is the rate of decay of  $(A,Z,I)$  if that nuclide is radioactive, or its rate of production if it is stable.

The **chain yield**  $Ch(A)$  is equal to the sum of all stable or long-lived cumulative yields for a given mass chain. It should be noted that the chain yield,  $Ch(A)$ , and the sum or mass yield,  $Y(A)$ , for a mass chain  $A$  may differ by a few per cent because the former applies after, and the latter before, delayed neutron emission.

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## Cumulative yields

- Probability of forming a nuclide directly after prompt neutron emission or by decay, or after a **“long” irradiation at 1 fission per second = decay rate (production=destruction)**
- Not used in inventory calculations
- Easier to measure and many more measurements
- In JEFF independent yields and decay data used to calculate and then merged with experimental data and their uncertainties using an adjustment process and a “backward-forward” error calculation.
- Smaller uncertainties on values with expt. data

# Fission Yield Evaluation

| Maximum fraction of fission rate  |   |   | Spontaneous fission  |
|---|---|---|--|
| >10%  | 1-10%   | 0.1-1%  |  |
| Nuclides: <b>5</b>  | <b>2</b>                                      | <b>12</b>   | <b>3</b>   |
| $^{233}\text{U}^*$ TFH<br>$^{235}\text{U}^*$ TFH<br>$^{238}\text{U}^*$ FH<br>$^{239}\text{Pu}^*$ TF<br>$^{241}\text{Pu}^*$ TF | $^{240}\text{Pu}^*$ F<br>$^{245}\text{Cm}$ TF | $^{232}\text{Th}^*$ FH<br>$^{234}\text{U}$ F<br>$^{236}\text{U}$ F<br>$^{237}\text{Np}$ TF<br>$^{238}\text{Np}$ TF<br>$^{238}\text{Pu}$ TF<br>$^{242}\text{Pu}$ F<br>$^{241}\text{Am}$ TF<br>$^{242\text{m}}\text{Am}$ TF<br>$^{243}\text{Am}$ TF<br>$^{243}\text{Cm}$ TF<br>$^{244}\text{Cm}$ TF | $^{252}\text{Cf}$ Sp<br>$^{242}\text{Cm}$ Sp<br>$^{244}\text{Cm}$ Sp |

\* Nuclides in UKFY1 and previous UK libraries [5,7].  
 T Thermal fission.  
 F Fast fission.  
 H 14 MeV fission.  
 Sp Spontaneous fission.

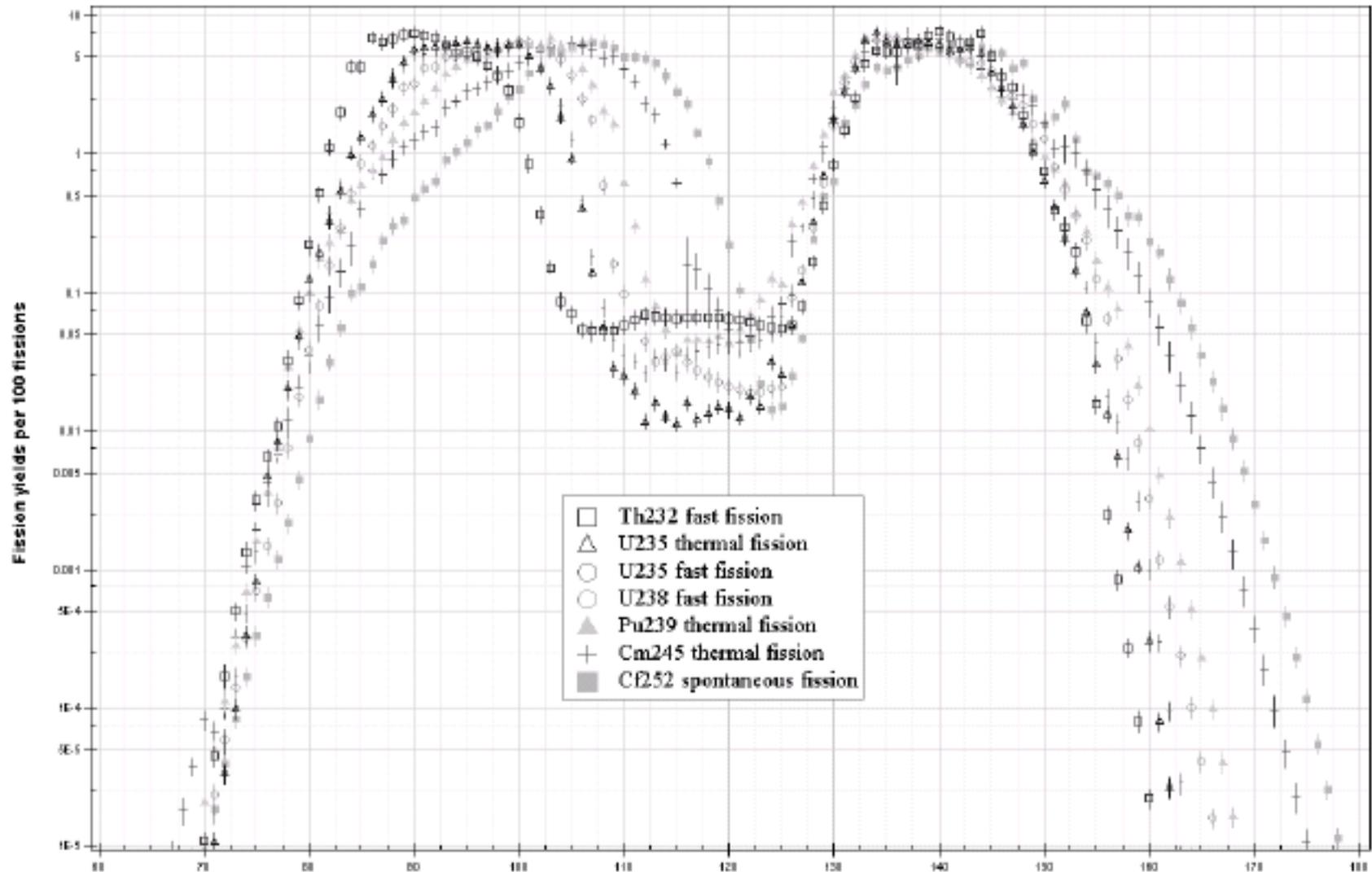
## Data Analysis

- UKFY3 based upon experimental data from >2000 papers, reports etc.
- Measurements analyzed to given “best estimates”  
of each measured yield

**Table 1: Number of data items in the UKFY3 experimental database**

| Absolute measurements | ratio measurements | ratio of ratio measurements | Total |
|-----------------------|--------------------|-----------------------------|-------|
| 11887                 | 1352               | 1471                        | 14710 |

# Mass distribution



Historically empirical fitting of Gaussian distributions have been used to model chain yields. Work in the 1980s showed that the best results were obtained using five Gaussians. Due to physical constraints there were only 7 free parameters to fit:

$$Y(A) = \frac{N_1}{\sigma_1 \sqrt{2\pi}} \left[ e^{-\left(\frac{(A-\bar{A}-D_1)^2}{2\sigma_1^2}\right)} + e^{-\left(\frac{(A-\bar{A}+D_1)^2}{2\sigma_1^2}\right)} \right] \\ + \frac{N_2}{\sigma_2 \sqrt{2\pi}} \left[ e^{-\left(\frac{(A-\bar{A}-D_2)^2}{2\sigma_2^2}\right)} + e^{-\left(\frac{(A-\bar{A}+D_2)^2}{2\sigma_2^2}\right)} \right] \\ + \frac{N_3}{\sigma_3 \sqrt{2\pi}} e^{-\left(\frac{(A-\bar{A})^2}{2\sigma_3^2}\right)}$$

with  $N_3=2(1-N_1-N_2)$ .

- The independent yield is calculated as the integral of a normal distribution

$$FI(A, Z) = \frac{1}{2} F(A, Z) N(A) (\text{erf}(V) - \text{erf}(W))$$

where

$$V = \frac{Z(A) - Z_p(A) + 0.5}{\sigma_z(A) \sqrt{2}} \quad W = \frac{Z(A) - Z_p(A) - 0.5}{\sigma_z(A) \sqrt{2}}$$

$F(A, Z)$  describes the odd-even effect and  $N(A)$  is a normalization constant

- $Z_p(A)$  is the most probable charge for mass  $A$

This is calculated as the "unchanged charge distribution" corrected for prompt neutron emission with a term describing the variation of the charge offset  $\Delta Z$ . In the heavy mass peak this is:

$$Z_p(A_H) = A'_H \frac{Z_f}{A_f} + \Delta Z(A'_H)$$

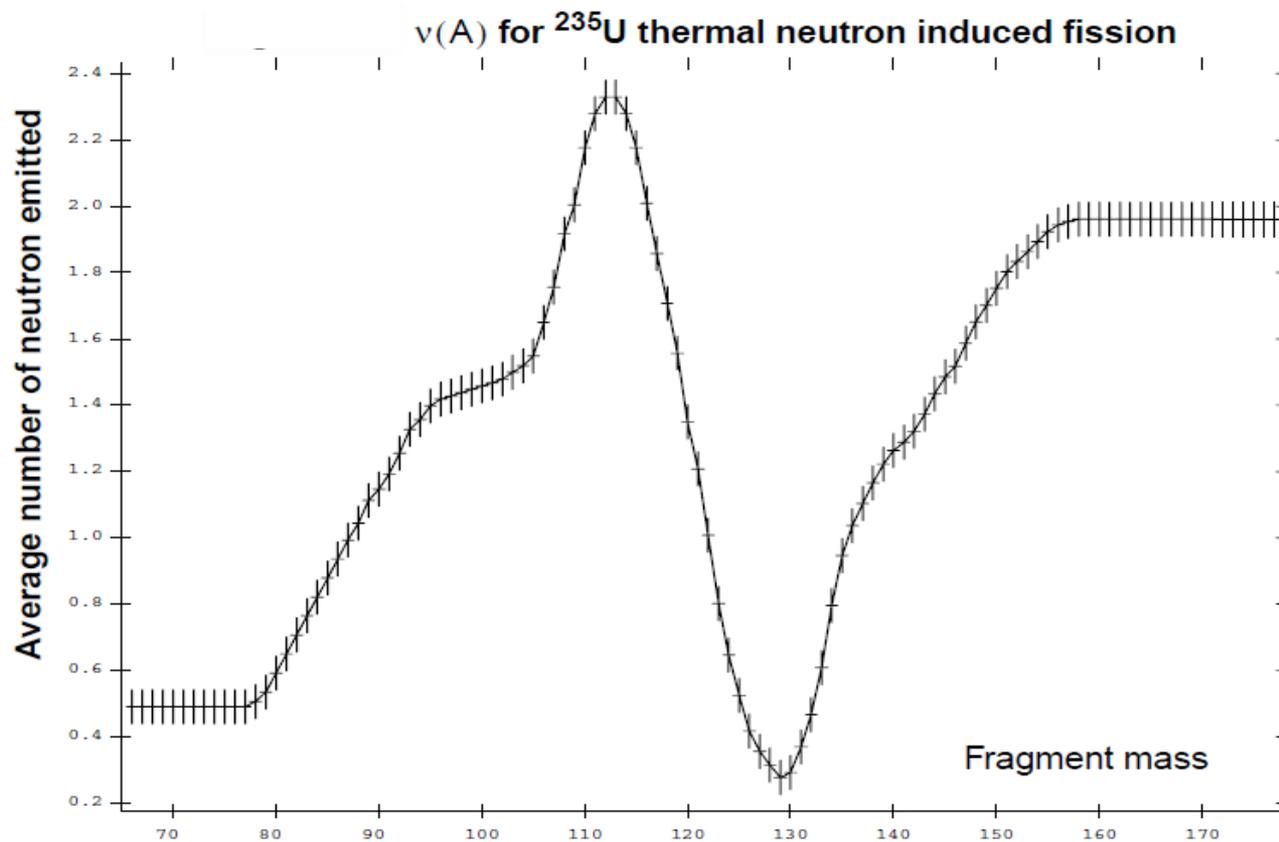
In the light mass peak, by conservation of mass and charge

$$Z_p(A_L) = A'_L \frac{Z_f}{A_f} + \Delta Z(A'_{Hc}) \quad A'_{Hc} = A_f - A'_L$$

The mass after prompt neutron emission is  $A'$  and the mass before is  $A$ . Thus

$$A' = A - \nu(A)$$

- $\nu(A)$  can be estimated using chain yields by the method of Terrell.



# Charge distribution (Wahl Zp)

- The odd-even effect,  $F(A,Z)$  is defined by two parameters  $\bar{F}_Z$  and  $\bar{F}_n$ .

Where  $F(A,Z)$  is given by:

| $F(A,Z)$                        | proton number<br>$Z$ | neutron number<br>$N$ |
|---------------------------------|----------------------|-----------------------|
| $\bar{F}_Z \bar{F}_n$           | even                 | even                  |
| $\frac{\bar{F}_Z}{\bar{F}_n}$   | even                 | odd                   |
| $\frac{\bar{F}_n}{\bar{F}_Z}$   | odd                  | even                  |
| $\frac{1}{\bar{F}_Z \bar{F}_n}$ | odd                  | odd                   |

- There exists 388 nuclides with two long-lived isomeric states and 21 with three or more.
- Only small number of measurements.
- The main predictive model available is that of Madland and England that assumed the fragments with a spin near a long-lived isomer would feed that isomer. Model used in UKFY2.
- Rudstam proposed a modification that included energetic feasibility. Results used in UKFY3.

# Ternary fission (light charge particle emission)

- The emission of protons, deuterons, tritons, alpha particles and other light fragments up to 30 amu have been observed from fission.
- The most common emission is an alpha particle followed by tritons.
- The light charged particles are produced between the two heavy fragments very close in time to the scission of the nucleus.
- In UKFY2 empirical relationships were used to fill gaps, but in UKFY3 improved model results published by Serot et al at ND2004 were used.

- $\sum_A Y(A) = 2.0$

- $\sum_{A > \frac{A_f}{2}} Y(A) = 1$

- From conservation of mass

$$\sum_A AY(A) = A_f - \bar{\nu}_p - A_{LCP}$$

- From conservation of charge

$$\sum_{ZA} Z f(A, Z) Y(A) = Z_f - Z_{LCP}$$

and as  $Z_1 + Z_2 = Z_f$

then

$$\sum_A f(A, Z) Y(A) = \sum_A f(A, Z_f - Z) Y(A)$$

for all  $Z < \frac{Z_f}{2}$

- Spent Fuel Assay (typically 1 – 10 years)
- Spent Fuel Decay heat (typically 1 – 30 years)
- Pulse irradiation of pure fissile species – beta/gamma decay heat (typically 1-100000 seconds)
- Delayed neutron emission (typically 1-60 seconds)

Summary of PWR spent fuel decay heatnew validation results.

| Reactor      | C/E values<br>(JEF-2.2) | C/E values<br>(JEFF-3.1.1) |
|--------------|-------------------------|----------------------------|
| Point Beach  | $0.993 \pm 0.016$       | $0.983 \pm 0.016$          |
| Turkey Point | $1.040 \pm 0.025$       | $1.026 \pm 0.030$          |
| Ringhals 2   | $1.039 \pm 0.013$       | $1.025 \pm 0.014$          |
| Ringhals 3   | $1.034 \pm 0.011$       | $1.021 \pm 0.012$          |

- Given the individual decay branches for all nuclides in the decay paths from one nuclide to a distant daughter it is possible to calculate the fraction of  $j$  that decays to  $i$

$$Q_{j,i} = \sum_{\text{all paths}} \left( \prod_{\text{each } j \rightarrow i} B_{j,j+1} B_{j+1,j+2} \dots B_{i-1,i} \right)$$

- If  $Q_{i,i}$  is defined as 1 and  $Q_{k,i} = 0$  (where  $k$  does not decay to  $i$ ), Thus any cumulative yield can be calculated from the independent yield.

$$Y_i^c = \sum_j Y_j^i Q_{j,i}$$

- As this is a Q weighted sum of  $Y^i$  then the variance of the result is given by

$$\begin{aligned} \text{var}(Y_i^c) &= \text{var}\left(\sum_j Y_j^i Q_{j,i}\right) \\ &= \sum_j \text{var}(Y_j^i) Q_{j,i}^2 \\ &\quad + 2 \sum_j \sum_k Q_{j,i} Q_{k,i} \text{cov}(Y_j^i, Y_k^i) \end{aligned}$$

- If all terms except the covariance is known, these can be calculated.

“Forward-Backward” technique

- C is weighted sum of independent yields

$$C(Z) = I(Z) + I(Z-1)*Q(z-1,z) + I(z-2)*Q(z-2,z) + \dots$$

$$C(Z+1) = I(Z+1) + Q(z,z+1) * ( I(Z) + I(Z-1)*Q(z-1,z) + I(z-2)*Q(z-2,z) + \dots )$$

$$C(Z+1) = I(Z+1) + C(Z) * Q(z,z+1)$$

“Forward-Backward” technique

$$C(Z) = (C(Z+1) - I(Z+1)) / Q(z,z+1)$$

- Equation is now a function of only one cumulative yield, one independent yield and decay data.
- If you know  $C(Z+1)$  and its error from measurement and have an estimate of its independent yield and uncertainty can determine uncertainty of  $C(Z)$ .
- In JEFF-3.1.1, if  $C(Z)$  has been adjusted by  $x$  to fit independent yield constraints then uncertainty is increased by  $x$ .

# Mass 85 example

- What does the Q matrix look like

| $Q_{i,j}$         | $^{85}\text{Ge}$ | $^{85}\text{As}$ | $^{86}\text{Ga}$ | $^{86}\text{Ge}$ | $^{86}\text{As}$ | $^{85}\text{Se}$ | $^{85}\text{Br}$ | $^{85m}\text{Kr}$ | $^{85}\text{Kr}$ | $^{85}\text{Rb}$ |
|-------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|-------------------|------------------|------------------|
| $^{85}\text{Ge}$  | 1.00             | 0.85             |                  |                  |                  | 0.67             | 0.67             | 0.67              | 0.14             | 0.67             |
| $^{85}\text{As}$  |                  | 1.00             |                  |                  |                  | 0.78             | 0.78             | 0.78              | 0.17             | 0.78             |
| $^{86}\text{Ga}$  |                  |                  | 1.00             | 1.00             | 1.00             | 0.33             | 0.33             | 0.33              | 0.07             | 0.33             |
| $^{86}\text{Ge}$  |                  |                  |                  | 1.00             | 1.00             | 0.33             | 0.33             | 0.33              | 0.07             | 0.33             |
| $^{86}\text{As}$  |                  |                  |                  |                  | 1.00             | 0.33             | 0.33             | 0.33              | 0.07             | 0.33             |
| $^{85}\text{Se}$  |                  |                  |                  |                  |                  | 1.00             | 1.00             | 1.00              | 0.22             | 1.00             |
| $^{85}\text{Br}$  |                  |                  |                  |                  |                  |                  | 1.00             | 1.00              | 0.22             | 1.00             |
| $^{85m}\text{Kr}$ |                  |                  |                  |                  |                  |                  |                  | 1.00              | 0.21             | 1.00             |
| $^{85}\text{Kr}$  |                  |                  |                  |                  |                  |                  |                  |                   | 1.00             | 1.00             |
| $^{85}\text{Rb}$  |                  |                  |                  |                  |                  |                  |                  |                   |                  | 1.00             |

# Effect on cumulative yield uncertainties

- The following shows the JEFF-3.1.1 file data and a calculation of  $Y^c$  and its uncertainty without the covariance terms.

| Quantities            | $^{85}\text{Ge}$ | $^{85}\text{As}$ | $^{86}\text{Ga}$ | $^{86}\text{Ge}$ | $^{86}\text{As}$ | $^{85}\text{Se}$ | $^{85}\text{Br}$ | $^{85m}\text{Kr}$ | $^{85}\text{Kr}$ | $^{85}\text{Rb}$ |
|-----------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|-------------------|------------------|------------------|
| JEFF-3.1.1 $Y^f$      | 2.44e-5          | 1.41e-3          | 1.82e-6          | 2.85e-6          | 4.42e-4          | 9.58e-3          | 2.19e-3          | 1.12e-5           | 4.85e-5          | 3.30e-8          |
| JEFF-3.1.1 $Y^f$ 1FPY | 9.08e-6          | 4.69e-4          | 6.65e-11         | 1.04e-6          | 1.59e-4          | 9.47e-4          | 7.14e-4          | 4.18e-6           | 1.81e-5          | 1.23e-8          |
| JEFF-3.1.1 $Y^c$      | 2.44e-5          | 1.43e-3          | 1.82e-10         | 2.85e-6          | 4.45e-4          | 1.08e-2          | 1.30e-2          | 1.30e-2           | 2.86e-3          | 1.31e-2          |
| JEFF-3.1.1 $Y^c$ 1sd  | 9.06e-6          | 4.19e-4          | 6.66e-11         | 1.04e-6          | 1.55e-4          | 2.10e-4          | 1.19e-4          | 1.19e-4           | 2.10e-4          | 1.19e-4          |
| Calculated $Y^c$      | 2.44e-5          | 1.43e-3          | 1.82e-10         | 2.85e-6          | 4.45e-4          | 1.08e-2          | 1.30e-2          | 1.30e-2           | 2.86e-3          | 1.31e-2          |
| Calculated $Y^c$ 1sd  | 9.08e-6          | 4.69e-4          | 6.65e-11         | 1.04e-6          | 1.59e-4          | 1.02e-3          | 1.24e-3          | 1.24e-3           | 2.68e-4          | 1.24e-3          |
| $Y^c$ ratio file/calc | 1.000            | 1.000            | 1.000            | 1.000            | 1.000            | 1.000            | 1.000            | 1.000             | 1.000            | 1.000            |
| $Y^c$ 1sd file/calc   | 0.998            | 0.893            | 1.000            | 1.000            | 0.978            | 0.206            | 0.096            | 0.096             | 0.785            | 0.095            |

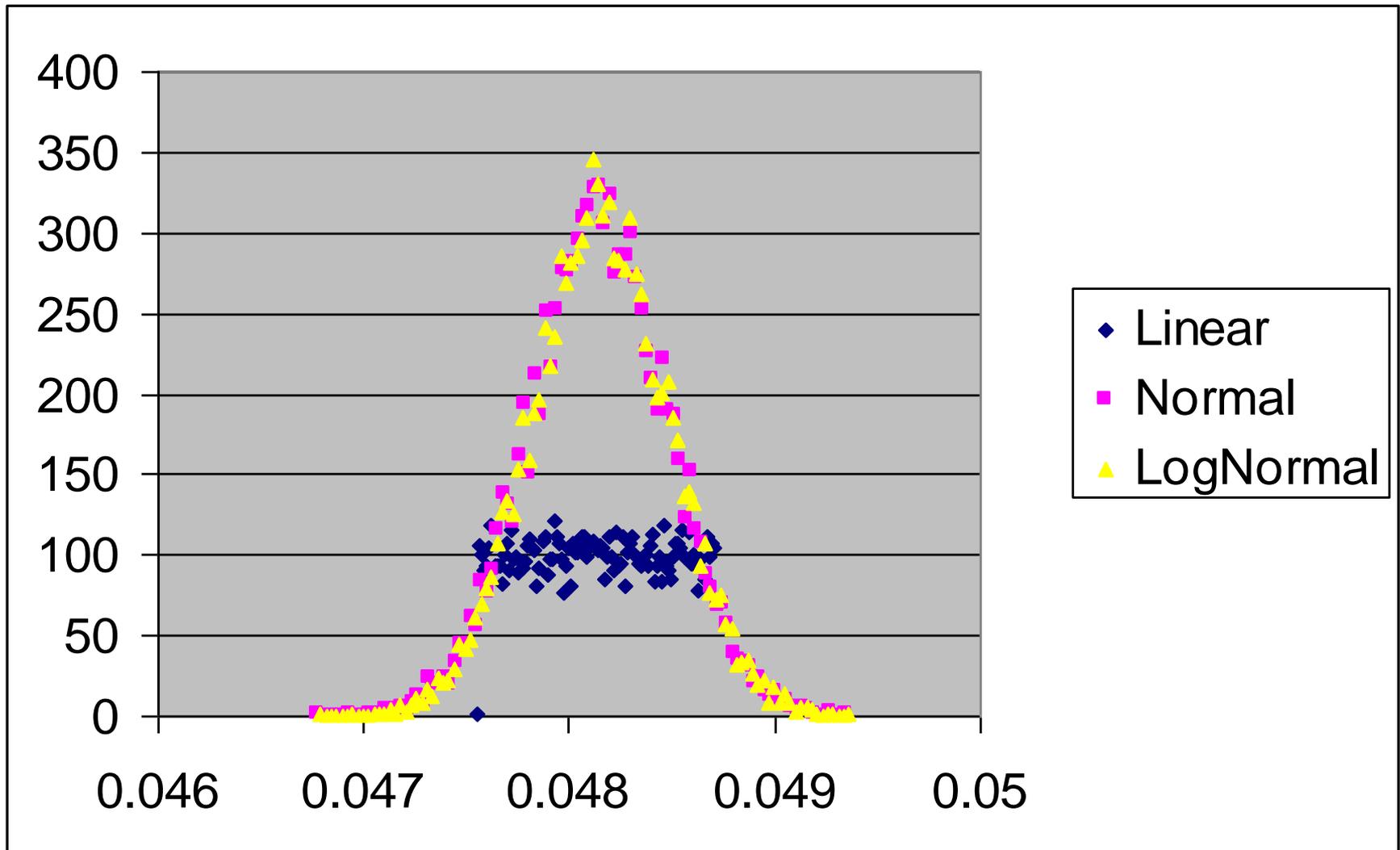
- The results show that without the covariance terms the yields are over-predicted.

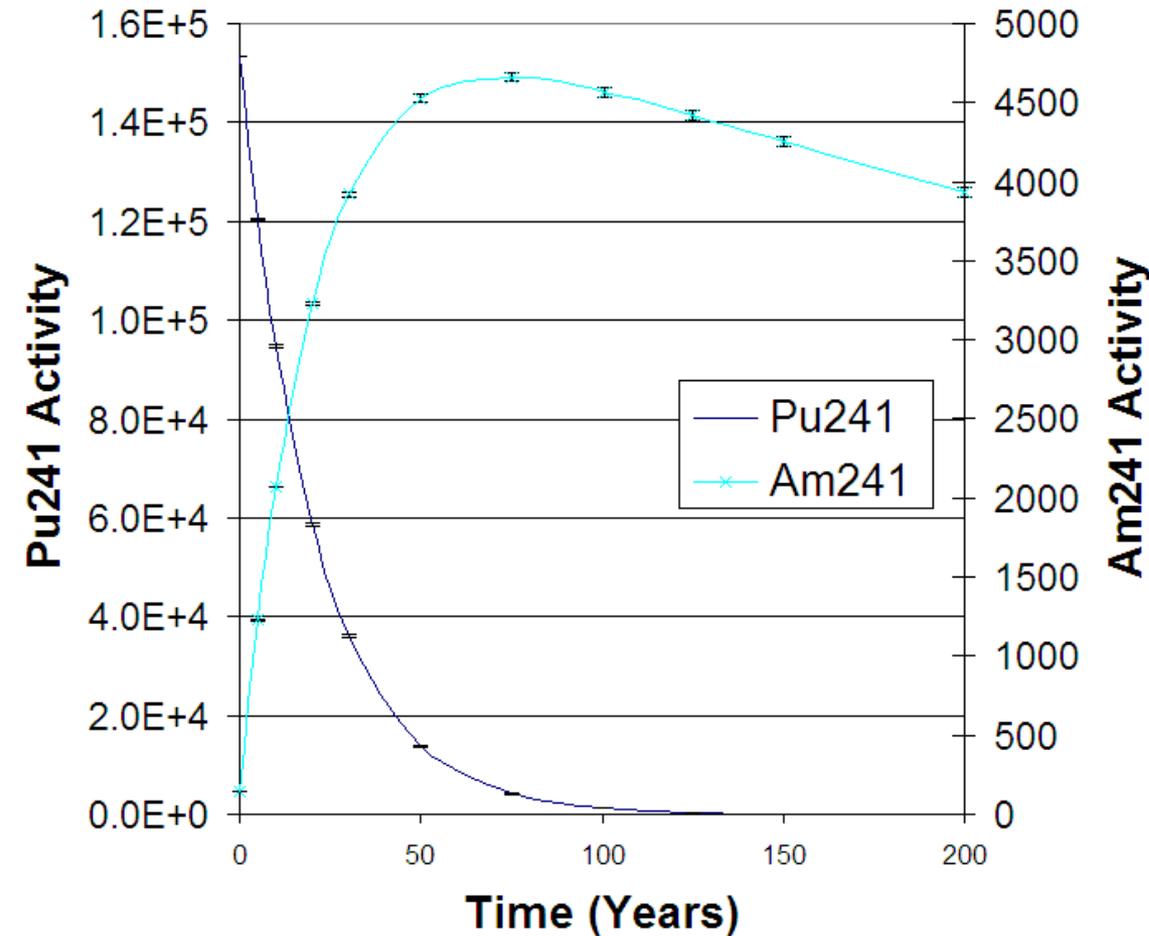
- Monte Carlo method is a well understood technique where random sampling of variables is used to solve a mathematical or physical problem.
- “Total Monte Carlo” is where you randomly adjust the parameters going into the evaluation to produce randomised data sets each of which can then be run through a problem to give an output probability distribution on a parameter based upon the entire set of randomised nuclear data that because the evaluation ensure consistency will agree with physical constrains.
  - e.g. TENDL

# Simple “Total Monte Carlo” for practical decay data example

- Consider a mass of plutonium containing an initial activity of  $^{241}\text{Pu}$  and  $^{241}\text{Am}$ . How do these vary with time and what are their uncertainties?
- Given the decay constants are  $4.8134 \times 10^{-2} \pm 3.342 \times 10^{-4}$  per year and  $1.6019 \times 10^{-3} \pm 1.852 \times 10^{-6}$  per year respectively.
- The “best estimate” result can easily be calculated.
- If we sample the decay constants from different probability distributions with the required mean and standard deviation, run each for 10000 times and then analyse the results we get ....

# Probability Distribution functions

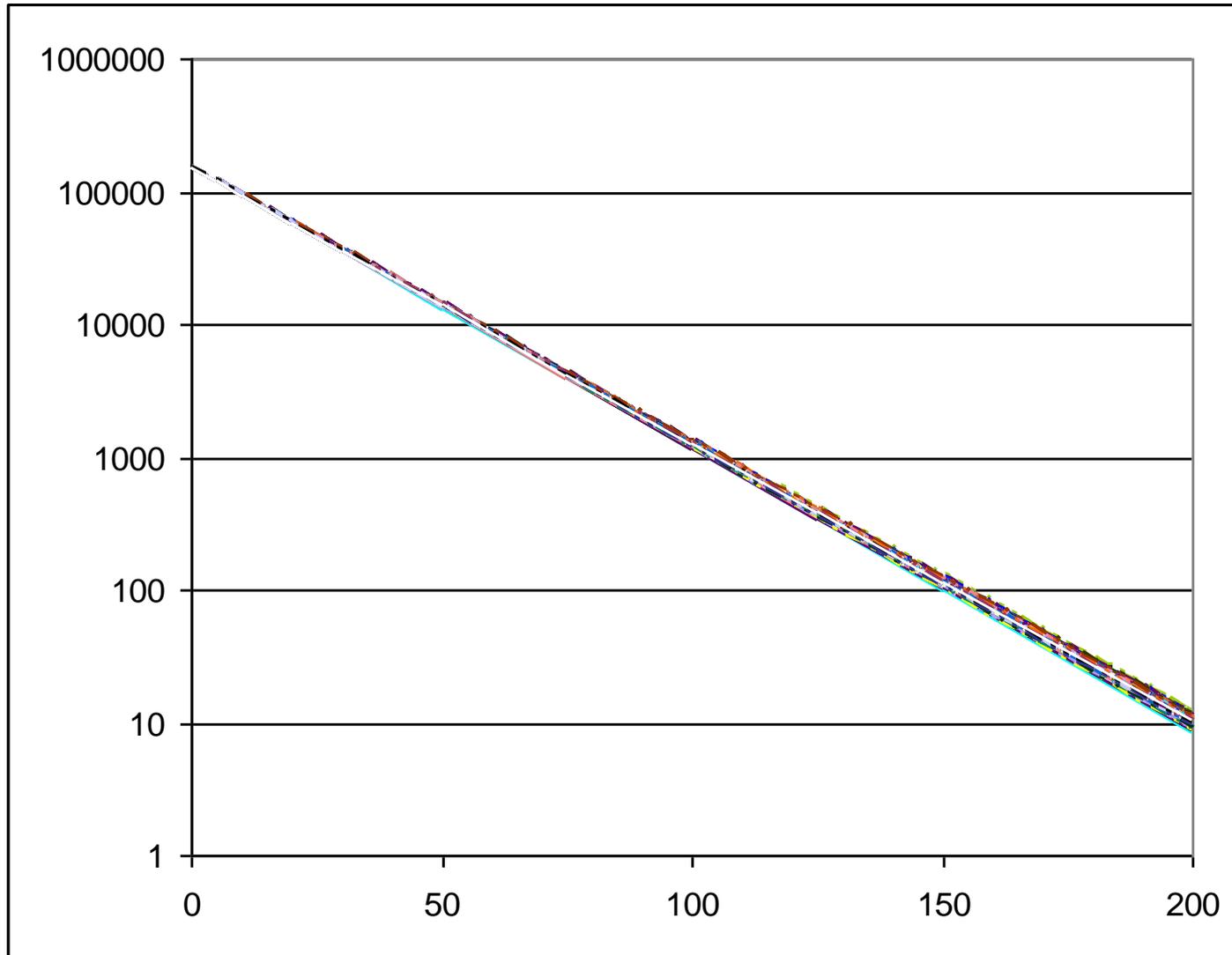




## Activities at 200 years

| PDF        | <sup>241</sup> Pu  | <sup>241</sup> Am |
|------------|--------------------|-------------------|
| Normal     | 10.13<br>±<br>0.68 | 3933<br>±<br>28   |
| Linear     | 10.12<br>±<br>0.67 | 3933<br>±<br>28   |
| Log Normal | 10.13<br>±<br>0.68 | 3933<br>±<br>27   |

# Plot of 255 $^{241}\text{Pu}$ decay curves from normal distribution calcs



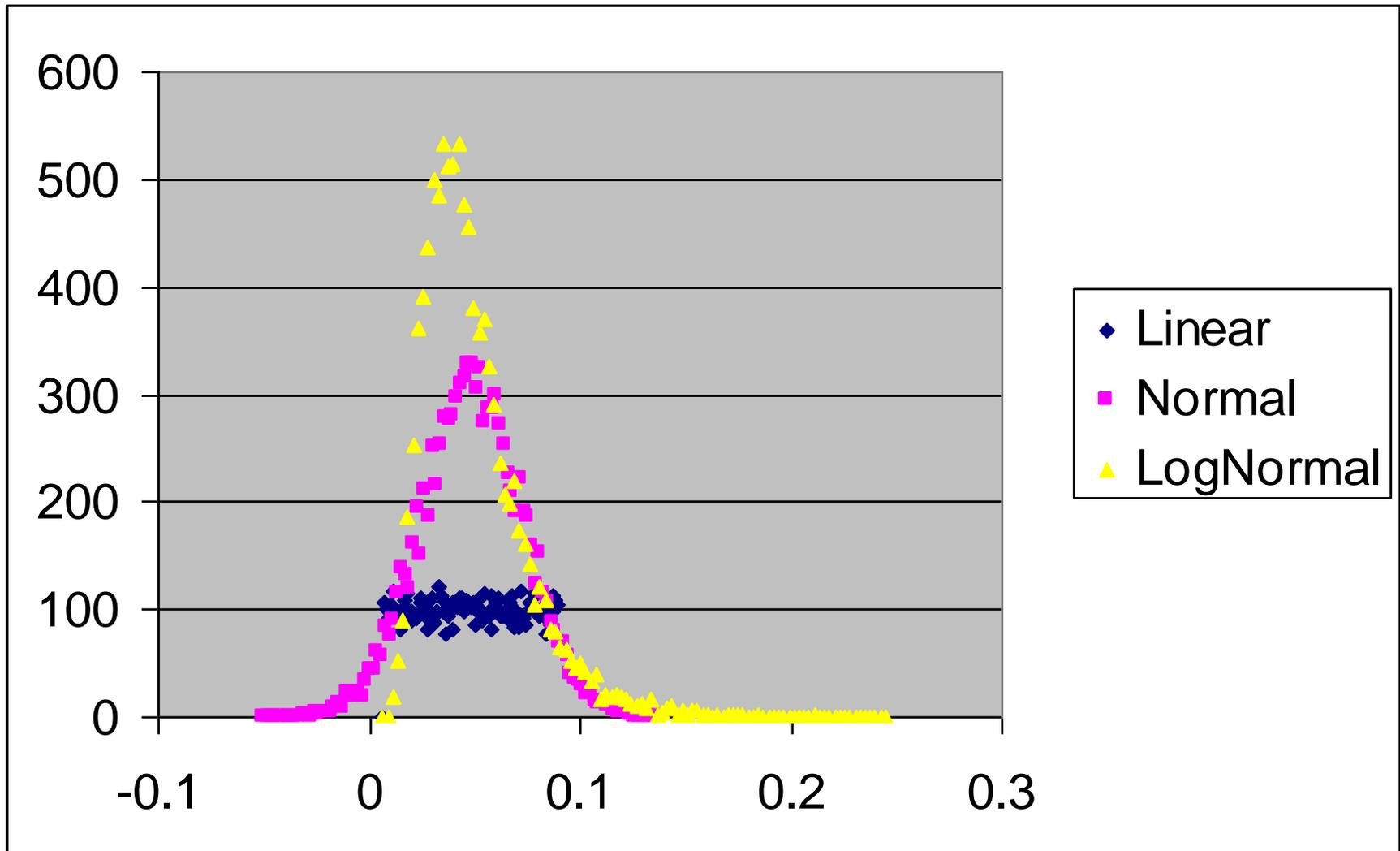
- In nuclear data work it is often necessary to consider larger uncertainties, even orders of magnitude.
- Repeating the calculations using 50 percent uncertainties on the decay constants, we get activities at 200 years for

$^{241}\text{Pu}$  activities of  
 $4.62 \times 10^5 \pm 3.42 \times 10^7$ ,  
 $560 \pm 1762$   
and  
 $2595 \pm 7094$   
(should be 10.1)

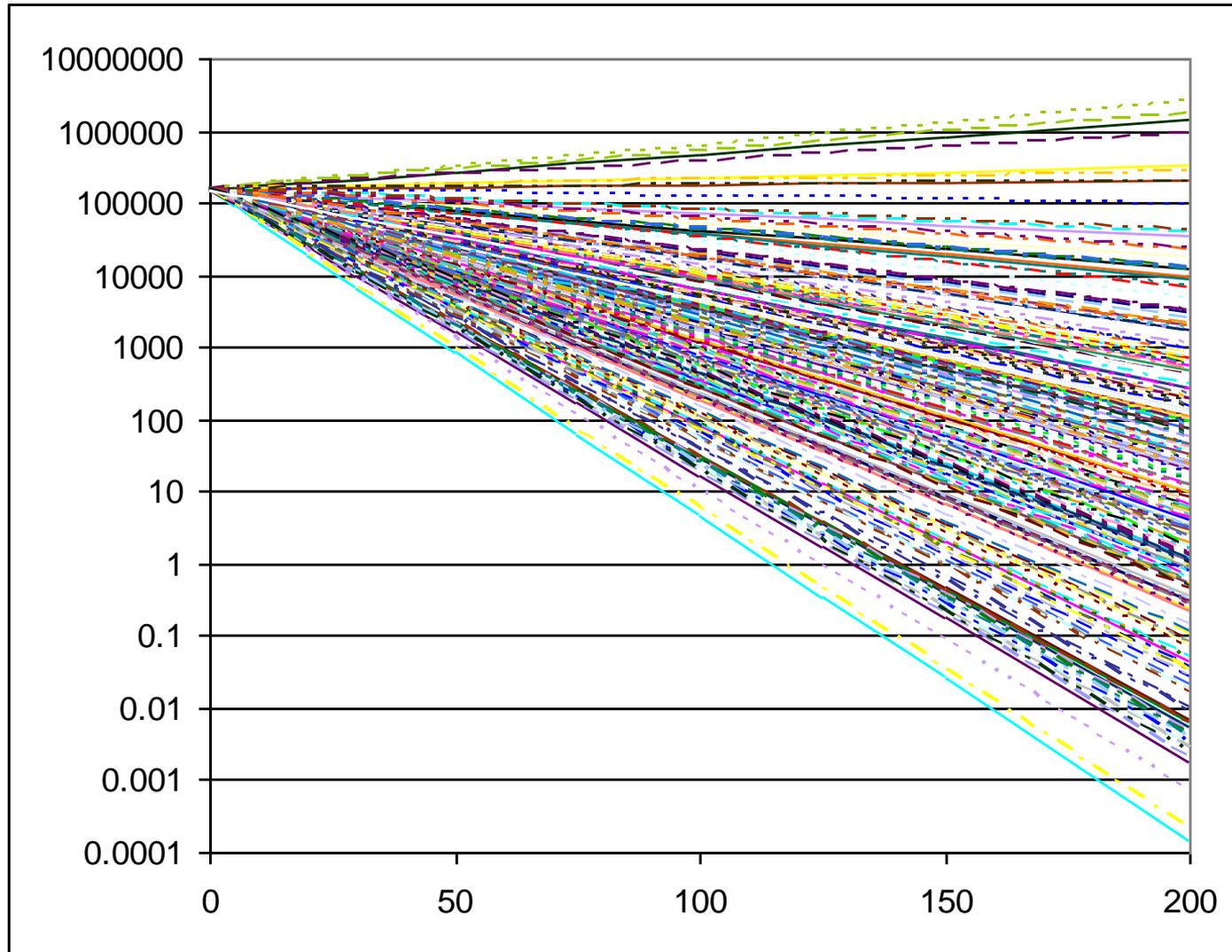
and  $^{241}\text{Am}$  activities of  
 $18946 \pm 727627$ ,  
 $4683 \pm 2888$   
and  
 $5522 \pm 5221$   
(should be 3933)

- Why?
- Mean and standard deviations of  $^{241}\text{Pu}$   $\lambda$  are
  - Normal  $0.048224 \pm 0.02405$
  - Linear  $0.048175 \pm 0.02411$
  - LogNormal  $0.048210 \pm 0.02403$
- Compared to input 0.048134 looks okay!
- Examining PDFs and results ...

# Probability Distribution functions



# Plot of 255 $^{241}\text{Pu}$ decay curves from normal distribution calcs

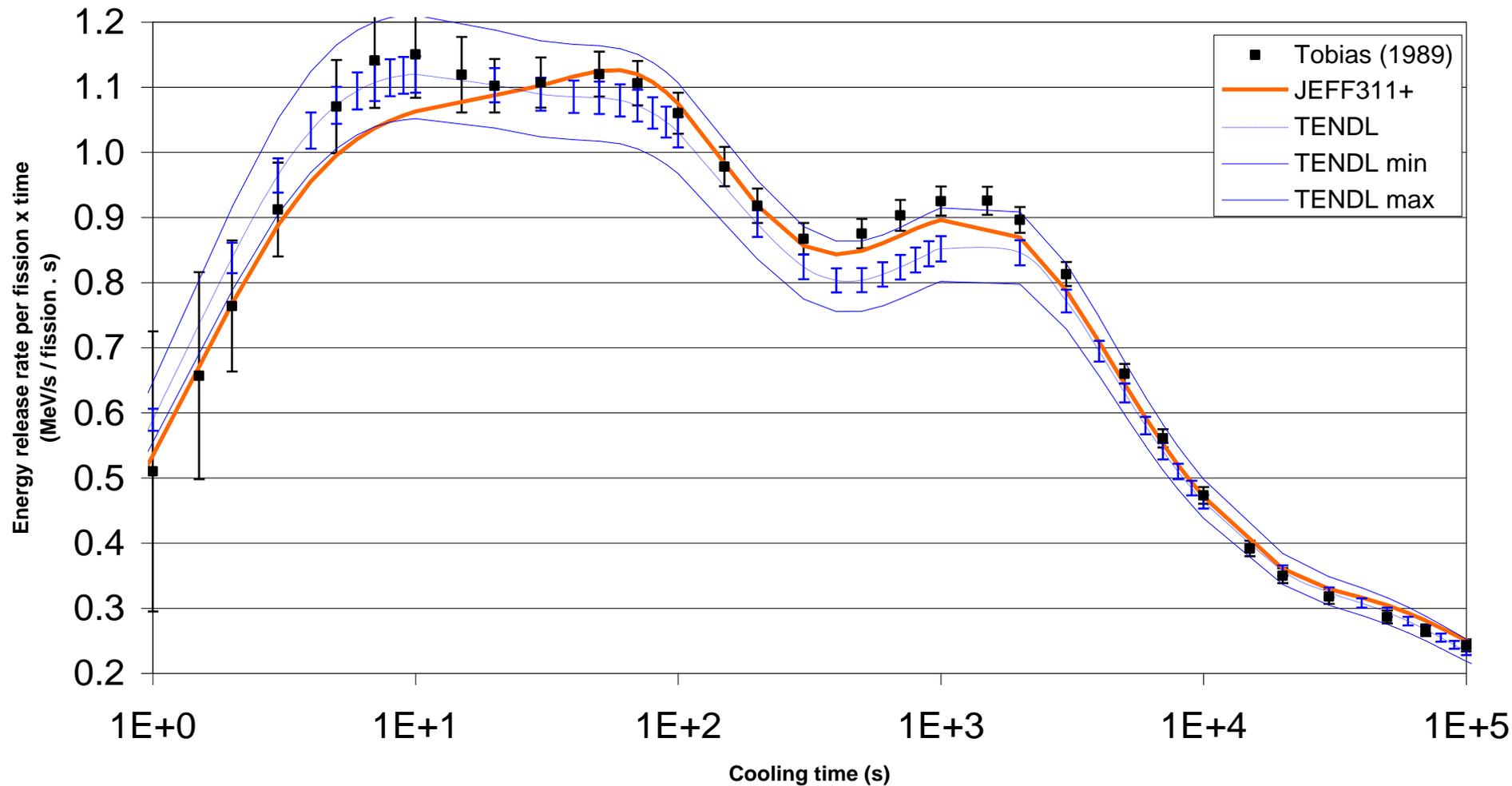


- This shows the method fails with high uncertainties, resulting from their physical interpretation.
- What do these large uncertainties mean?
- An uncertainty of, say,  $> 35$  percent has little meaning without specifying its distribution and any constraints on the value from the expt. analysis, theoretical model or evaluation process.
  - For example  $\lambda$  must be greater than zero.
- Without these it may only be possible to quote a "best estimate" result.

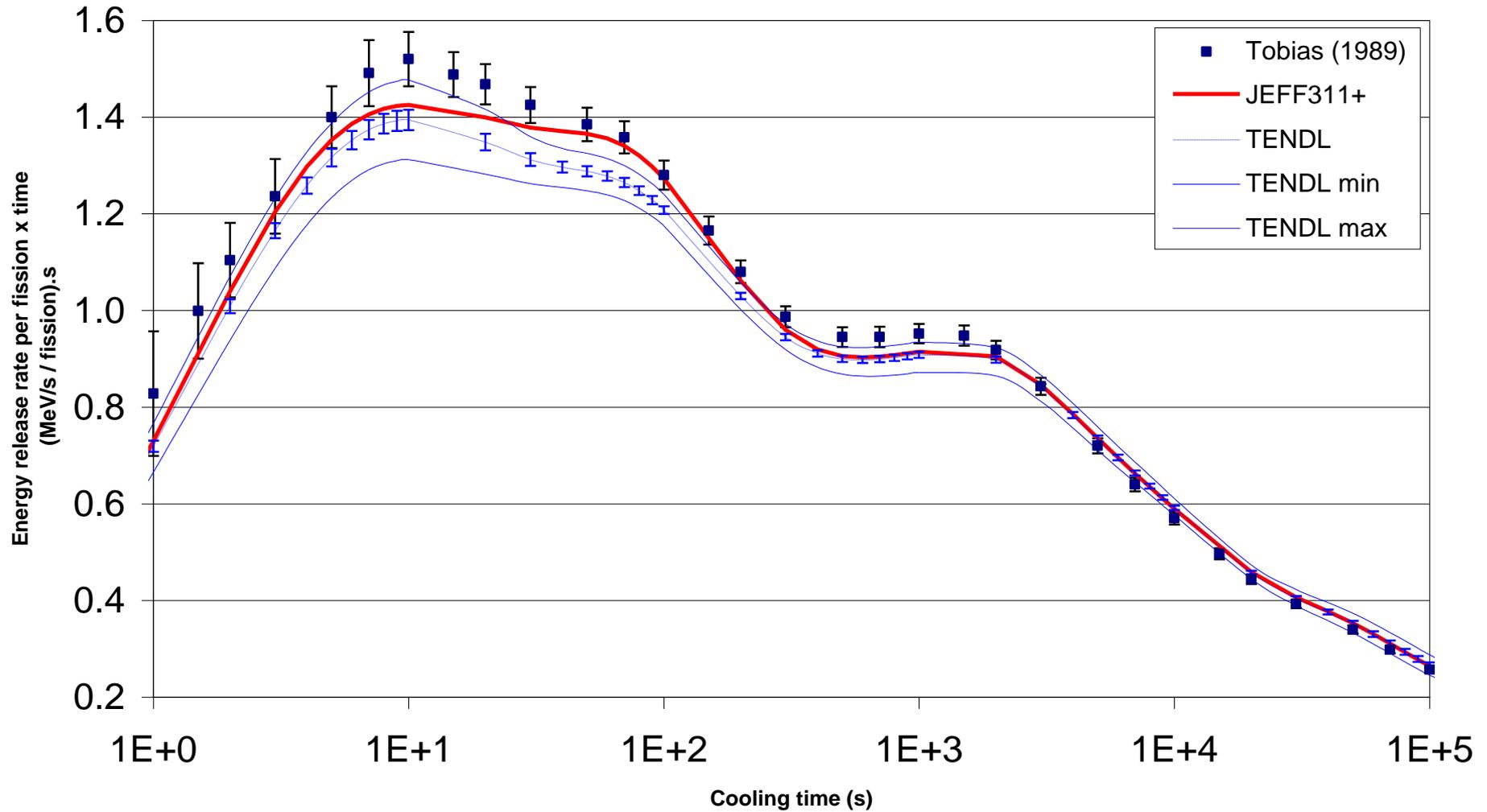
# Fission Product decay heat following a fission pulse

- TENDL-2010 included FPY and decay data files with up to 1000 randomly perturbed data libraries for used in uncertainty propagation.
- Using these FPY libraries, the UK spent fuel inventory code FISPIN was used to calculate decay heat from a fission pulse with each library.
- The unperturbed JEFF-3.1.1 decay data library was used and thus the uncertainties from energy release per decay and half-lives are not considered. Although some published Algora/Tain TAGS data was included extending JEFF-3.1.1.
- The results were compared Tobias (1989).

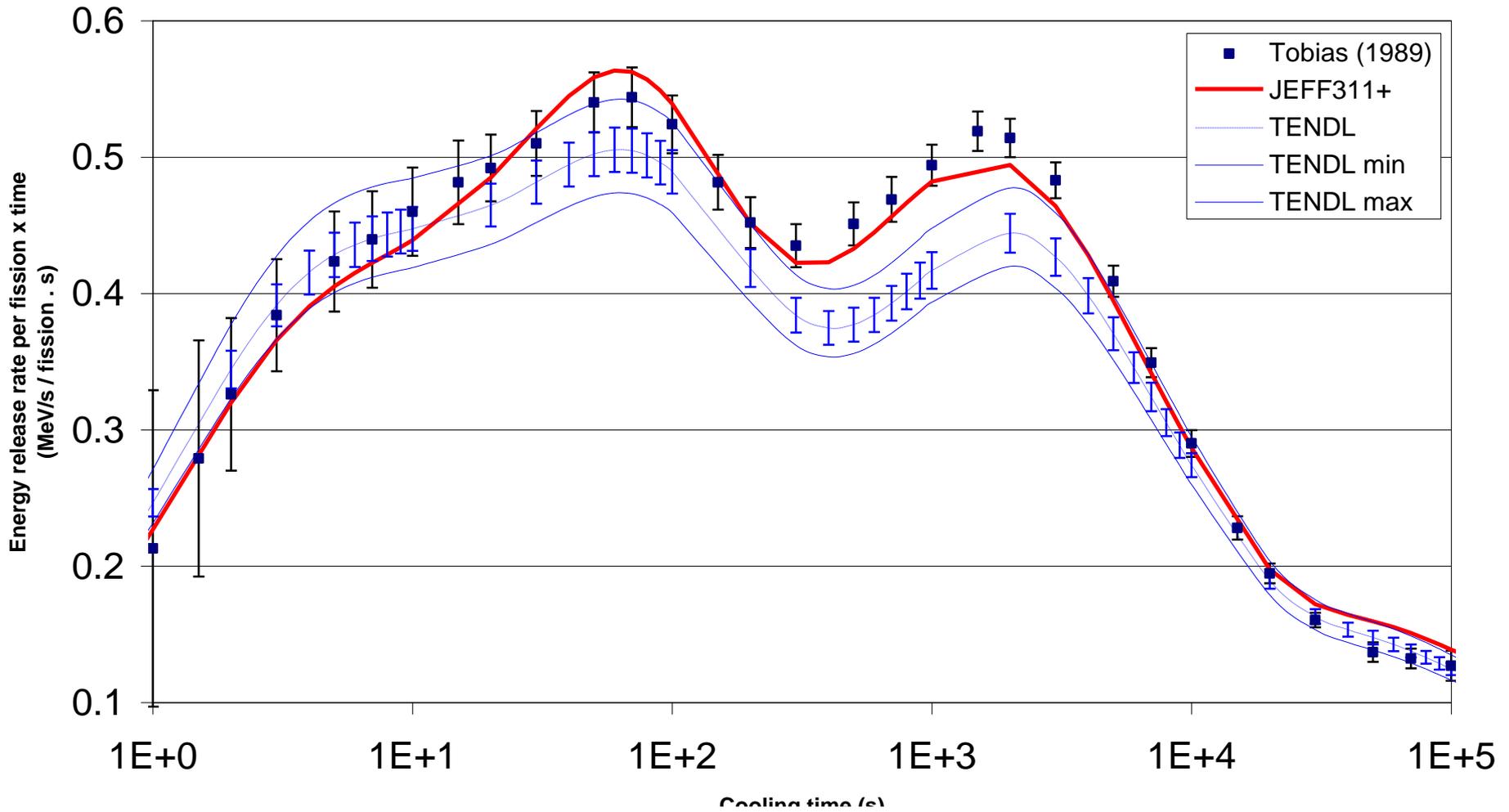
# $^{239}\text{Pu}$ total energy release



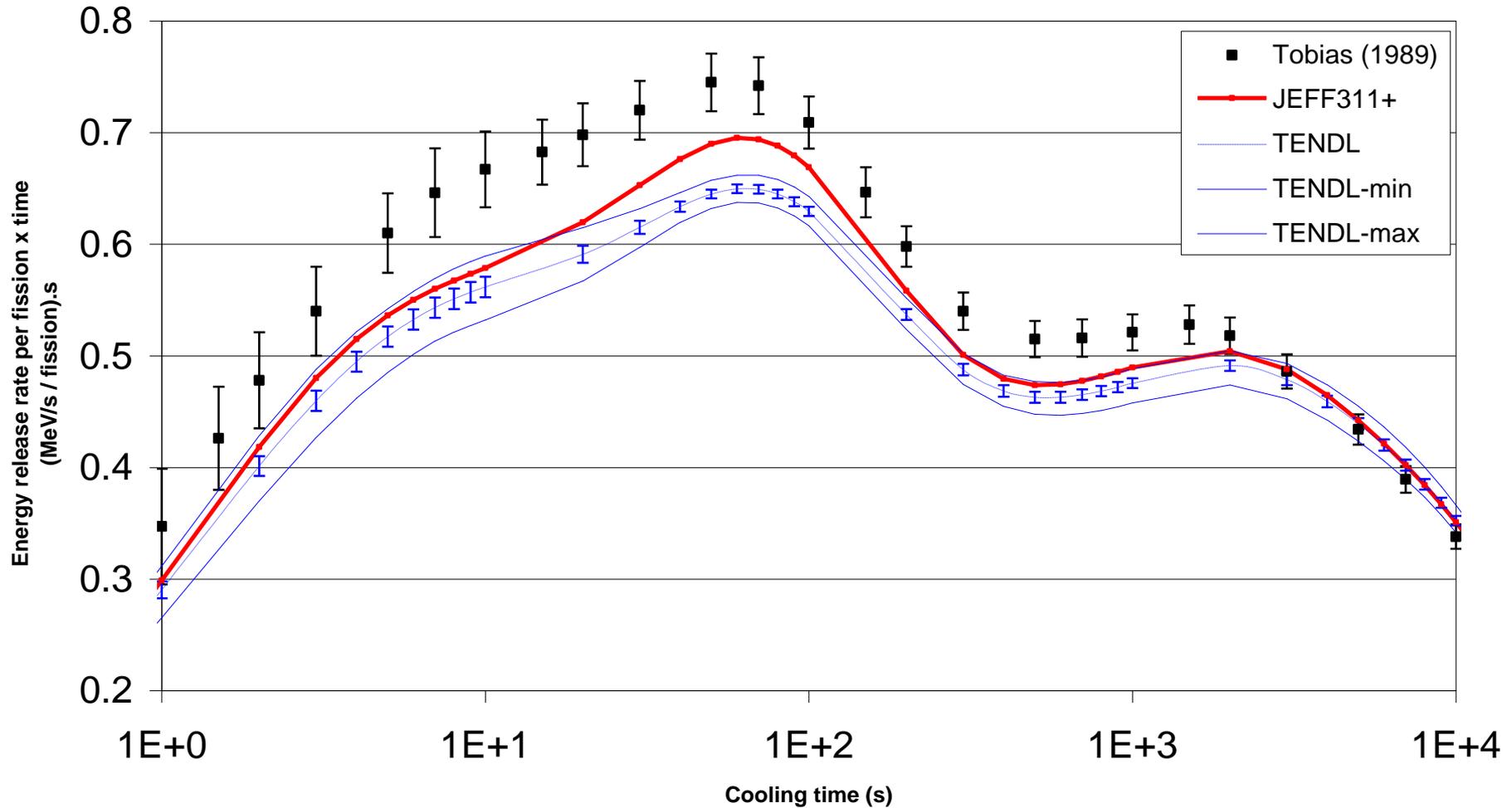
# $^{235}\text{U}$ total energy release



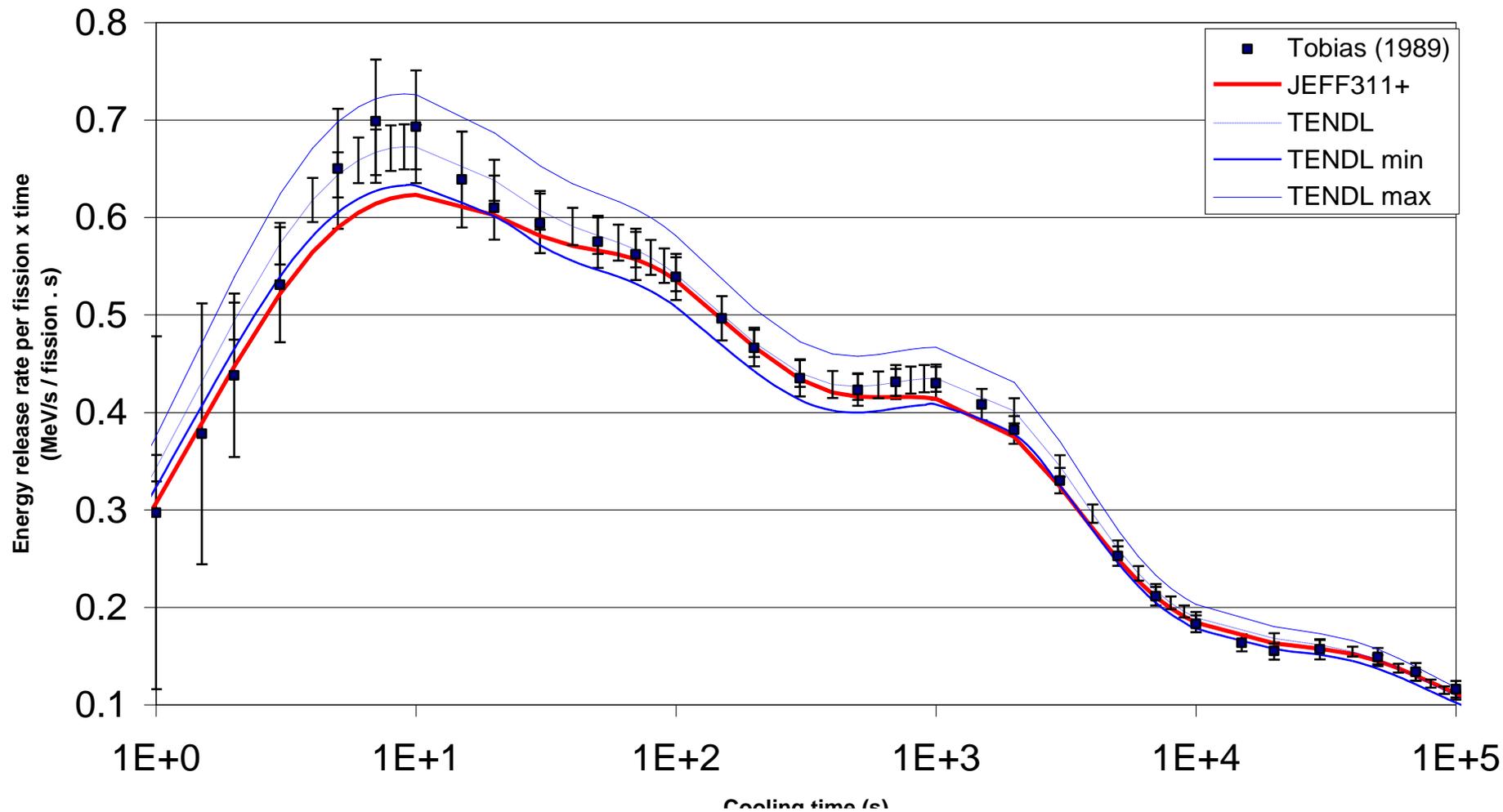
# $^{239}\text{Pu}$ $\gamma$ -ray energy release



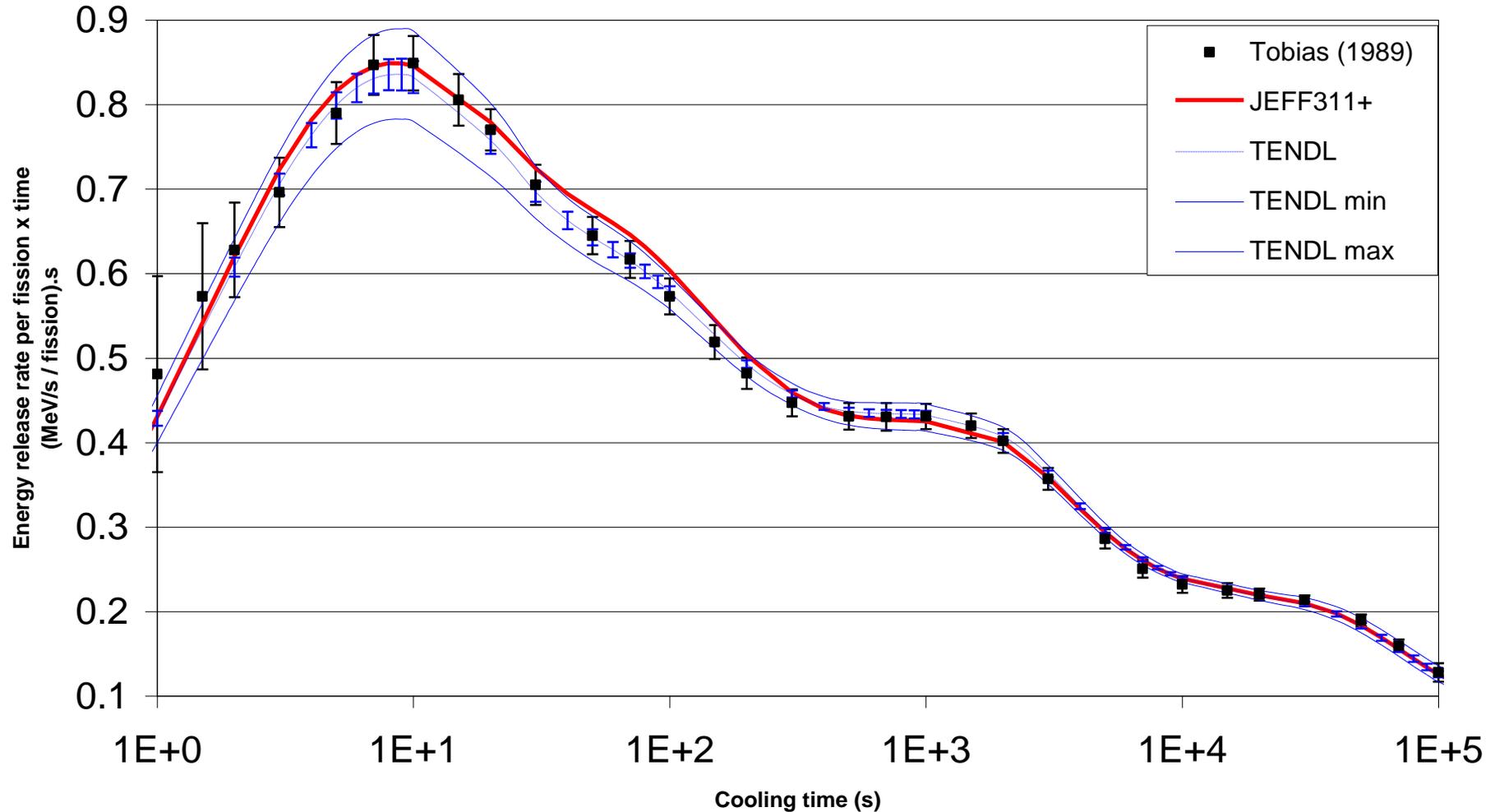
# $^{235}\text{U}$ $\gamma$ -ray energy release



# $^{239}\text{Pu}$ $\beta$ -particle energy release



# $^{235}\text{U}$ $\beta$ -particle energy release



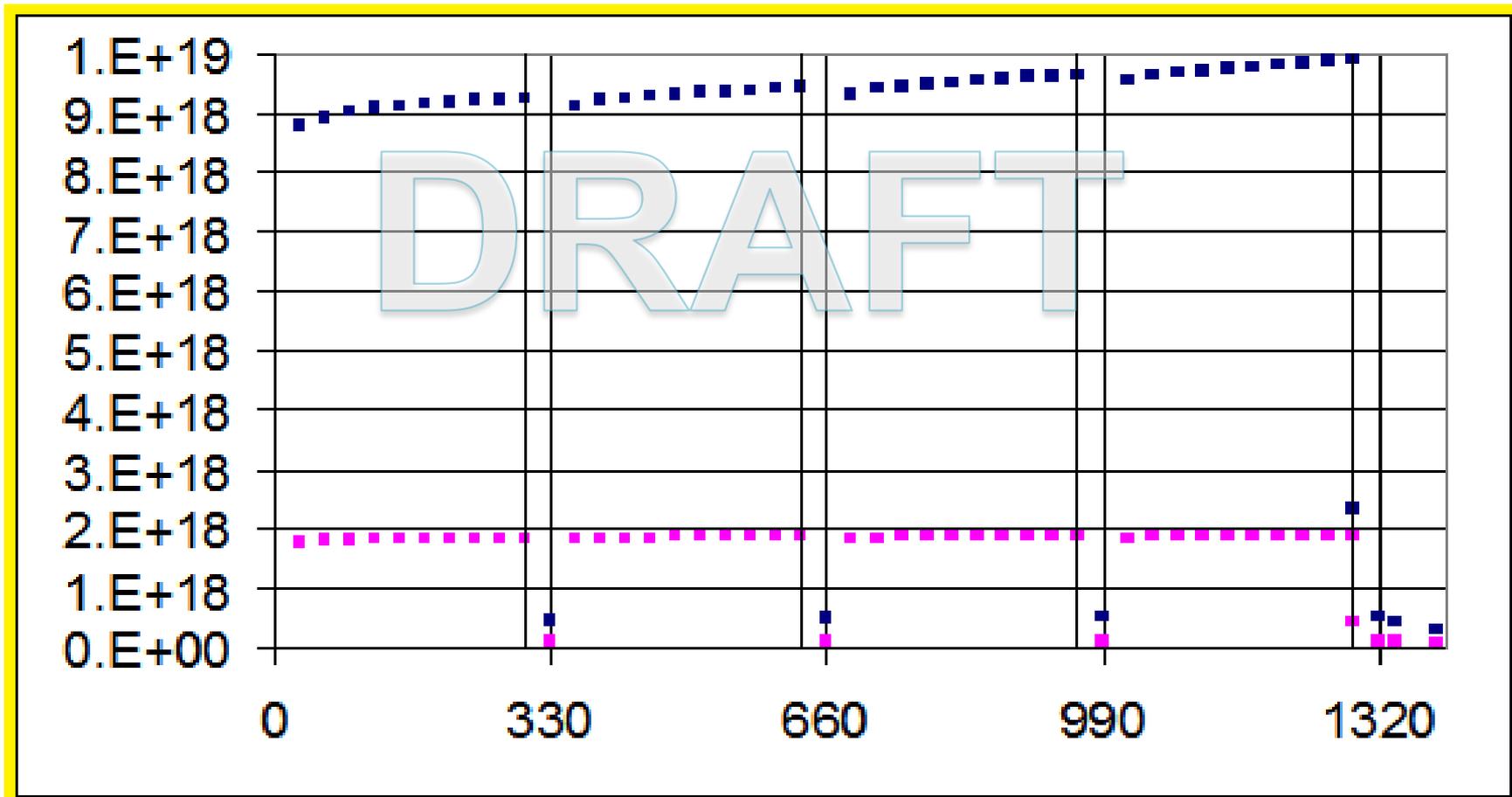
# Fission Product decay heat following a fission pulse

- As expected the TENDL results show larger uncertainties at shorter times when more short-lived fission products contribute. Note scaling by multiplying by time, reduces value and uncertainties below 1 second.
- The results show the expected under-prediction for  $^{235}\text{U}$  gamma-ray energy release reported by other workers. As the corresponding beta particle energy release appears centred about the Tobias data it appears that this is more about fission yields rather than only a decay data effect.

- Consider a typical modern PWR reactor containing 75.14 metric tonnes of uranium in a  $\text{UO}_2$  fuel with a 4.5% enrichment.
- Consider a mean rating of 45.833 MW/t (i.e. a total power of 3444 MW thermal) with fuel residing in the core for four 300 day cycles with 30 day gaps for refuelling and maintenance what neutrino source term would we expect.
- In a real core there will be variation both axially and radially in the core e.g. at the end of the assemblies where the power could considerably less.
- The following considers the mean power and to show effects 20% of this but with the same residency time.

- Using the FISGUI (FISPIN Graphical User Interface) calculations of were made every 30 days during irradiation and shutdown, after the irradiation the fuel was considered as removed and cooled to 1, 30, 50, 100 and 365.25 days.
- The activities were then multiplied by the probability of  $\beta^-$  emission and summed to get the neutrino emission (i.e. no weighting by detection probability).

Anti-neutrino emission rate (per second) per tonne of fuel  
against time in days.



## Fractional anti-neutrino contribution 30 days into cycle

| First cycle                   |         |               |          |            | Fourth cycle                  |         |               |          |            |
|-------------------------------|---------|---------------|----------|------------|-------------------------------|---------|---------------|----------|------------|
| #                             | Nuclide | half-life (d) | fraction | cumulative | #                             | Nuclide | half-life (d) | fraction | cumulative |
| 1                             | U239    | 0.0163        | 0.07534  | 0.07534    | 1                             | U239    | 0.0163        | 0.0984   | 0.0984     |
| 2                             | NP239   | 2.3550        | 0.07510  | 0.15044    | 2                             | NP239   | 2.3550        | 0.0980   | 0.1964     |
| 3                             | I134    | 0.0365        | 0.01236  | 0.16280    | 3                             | I134    | 0.0365        | 0.0109   | 0.2072     |
| 4                             | CS138   | 0.0224        | 0.01074  | 0.17354    | 4                             | I133    | 0.8667        | 0.0098   | 0.2170     |
| 5                             | TE134   | 0.0291        | 0.01072  | 0.18426    | 5                             | XE133   | 5.2450        | 0.0095   | 0.2265     |
| 6                             | I133    | 0.8667        | 0.01068  | 0.19494    | 6                             | I135    | 0.2754        | 0.0092   | 0.2358     |
| 7                             | XE133   | 5.2450        | 0.01040  | 0.20534    | 7                             | CS138   | 0.0224        | 0.0092   | 0.2449     |
| 8                             | BA139   | 0.0577        | 0.01034  | 0.21568    | 8                             | MO99    | 2.7500        | 0.0090   | 0.2539     |
| 9                             | Y95     | 0.0072        | 0.01031  | 0.22599    | 9                             | XE137   | 0.0027        | 0.0089   | 0.2627     |
| 10                            | CS139   | 0.0064        | 0.01026  | 0.23625    | 10                            | NB100   | 0.0000        | 0.0085   | 0.2713     |
| 11                            | Y94     | 0.0133        | 0.01024  | 0.24649    | 11                            | BA139   | 0.0577        | 0.0085   | 0.2798     |
| 12                            | XE138   | 0.0098        | 0.01024  | 0.25673    | 12                            | TE134   | 0.0291        | 0.0085   | 0.2883     |
| 13                            | I135    | 0.2754        | 0.01020  | 0.26692    | 13                            | TC101   | 0.0099        | 0.0085   | 0.2968     |
| 14                            | Y93     | 0.4208        | 0.00998  | 0.27690    | 14                            | MO101   | 0.0101        | 0.0085   | 0.3053     |
| 15                            | MO99    | 2.7500        | 0.00996  | 0.28686    | 15                            | NB98    | 0.0000        | 0.0083   | 0.3136     |
| 241 nuclides contribute 99.5% |         |               |          |            | 292 nuclides contribute 99.5% |         |               |          |            |

## Fractional anti-neutrino contribution after irradiation

| 1 Day |         |               |          |            |
|-------|---------|---------------|----------|------------|
| #     | Nuclide | half-life (d) | fraction | cumulative |
| 1     | NP239   | 2.355         | 0.33     | 0.33       |
| 2     | XE133   | 5.245         | 0.04     | 0.37       |
| 3     | LA140   | 1.678         | 0.03     | 0.40       |
| 4     | RU103   | 39.350        | 0.03     | 0.44       |
| 5     | BA140   | 12.740        | 0.03     | 0.47       |
| 6     | CE141   | 32.500        | 0.03     | 0.50       |
| 7     | NB95    | 35.150        | 0.03     | 0.53       |
| 8     | ZR95    | 63.980        | 0.03     | 0.56       |
| 9     | PR143   | 13.580        | 0.03     | 0.59       |
| 10    | MO99    | 2.750         | 0.03     | 0.62       |
| 11    | CE144   | 284.896       | 0.02     | 0.64       |
| 12    | PR144   | 0.012         | 0.02     | 0.66       |
| 13    | I132    | 0.096         | 0.02     | 0.69       |
| 14    | TE132   | 3.258         | 0.02     | 0.71       |
| 15    | Y91     | 58.510        | 0.02     | 0.73       |
| 16    | I131    | 8.040         | 0.02     | 0.75       |
| 17    | I133    | 0.867         | 0.02     | 0.77       |
| 18    | CE143   | 1.375         | 0.02     | 0.79       |
| 19    | RH105   | 1.473         | 0.02     | 0.80       |

| 1 year cooling |         |               |          |            |
|----------------|---------|---------------|----------|------------|
| #              | Nuclide | half-life (d) | fraction | cumulative |
| 1              | PR144   | 0.012         | 0.19     | 0.19       |
| 2              | CE144   | 284.896       | 0.19     | 0.38       |
| 3              | RU106   | 368.200       | 0.13     | 0.51       |
| 4              | RH106   | 0.000         | 0.13     | 0.64       |
| 5              | CS134   | 753.146       | 0.07     | 0.71       |
| 6              | PU241   | 5259.594      | 0.06     | 0.76       |
| 7              | CS137   | 10957.498     | 0.05     | 0.82       |
| 8              | PM147   | 958.051       | 0.05     | 0.87       |
| 9              | Y90     | 2.667         | 0.03     | 0.90       |
| 10             | SR90    | 10636.074     | 0.03     | 0.94       |
| 11             | NB95    | 35.150        | 0.02     | 0.96       |
| 12             | ZR95    | 63.980        | 0.01     | 0.97       |
| 13             | EU154   | 3141.146      | 0.01     | 0.98       |
| 14             | SB125   | 996.447       | 0.01     | 0.98       |
| 15             | Y91     | 58.510        | 0.01     | 0.99       |
| 16             | KR85    | 3915.474      | 0.00     | 0.99       |
| 17             | EU155   | 1810.417      | 0.00     | 0.99       |
| 18             | SR89    | 50.500        | 0.00     | 1.00       |

- Anti-neutrino emission is approximately proportional to reactor power.
- Varies between  $1.96E20$  and  $2.18E20$  neutrinos per GW for a single assembly
- Small variation of value during irradiation  $\sim 10\%$ .
- Assuming all fuel reaches the same final irradiation in 4 cycles an equilibrium core will have a starting value of 20.65 GWd/t and finishing value of 34.75 GWd/t (mid-point 27.5 GWd/t)
- Then expect neutrino emission to be nearer to  $2.07E20$  per GW with  $\sim 3\%$  variation during cycle.
- In a real core expect variation to be larger.