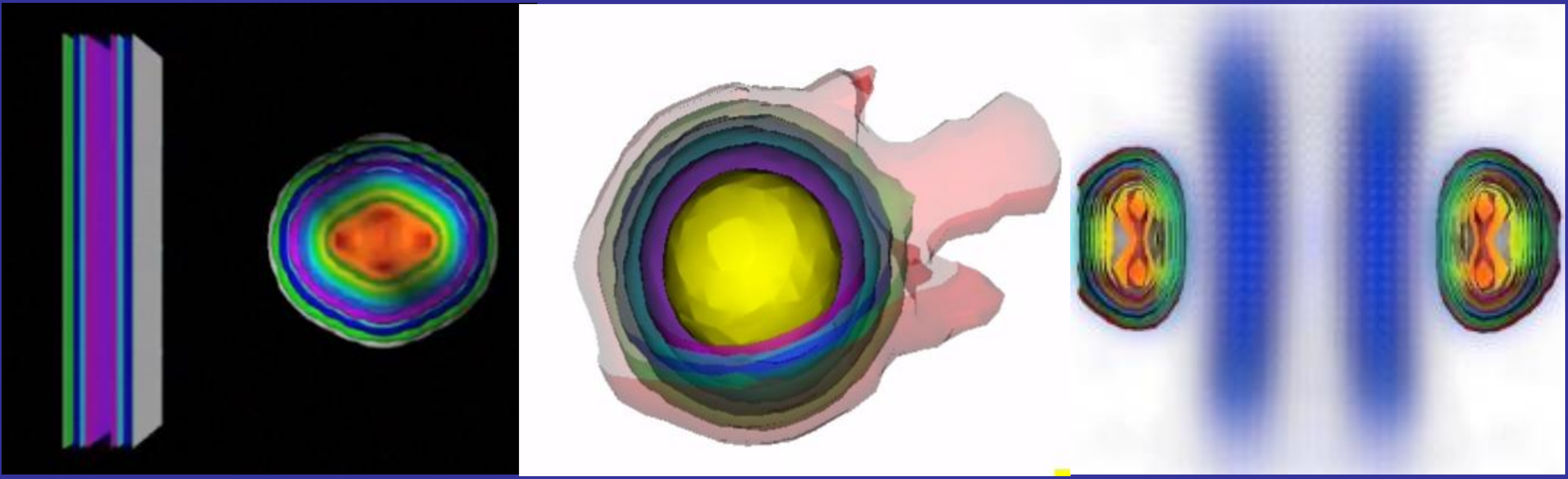
Nuclear Dynamics within Time Dependent Superfluid Local Density Approximation (TDSLDA)



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GOAL:

Description of nuclear dynamics far from equilibrium within the framework of TDDFT.

Nuclear Skyrme functional

$$E = \int d^3 r \mathcal{H}(\mathbf{r})$$

where

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &= C^{\rho}\rho^2 + C^{s}\vec{s}\cdot\vec{s} + C^{\Delta\rho}\rho\nabla^2\rho + C^{\Delta s}\vec{s}\cdot\nabla^2\vec{s} + C^{\tau}(\rho\tau - \vec{j}\cdot\vec{j}) + \\ &+ C^{sT}(\vec{s}\cdot\vec{T} - \mathbf{J}^2) + C^{\nabla J}(\rho\vec{\nabla}\cdot\vec{J} + \vec{s}\cdot(\vec{\nabla}\times\vec{j})) + C^{\nabla s}(\vec{\nabla}\cdot\vec{s})^2 + C^{\gamma}\rho^{\gamma} - \Delta\chi^*\end{aligned}$$

where

$$\begin{aligned} J_i &= \sum_{k,l} \epsilon_{ikl}\mathbf{J}_{kl} \\ \mathbf{J}^2 &= \sum_{k,l} \mathbf{J}_{kl}^2 \end{aligned}$$

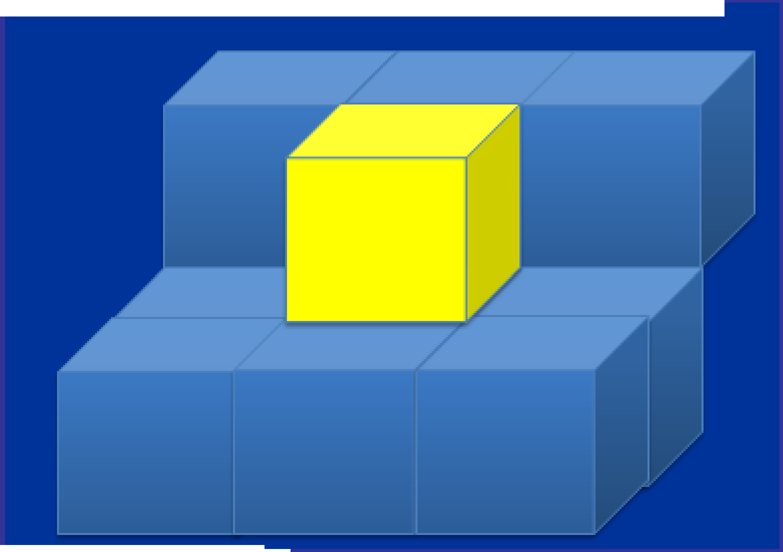
- density: $\rho(\mathbf{r}) = \rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin density: $\vec{s}(\mathbf{r}) = \vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- current: $\vec{j}(\mathbf{r}) = \frac{1}{2i}(\vec{\nabla} - \vec{\nabla}')\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin current (2nd rank tensor): $\mathbf{J}(\mathbf{r}) = \frac{1}{2i}(\vec{\nabla} - \vec{\nabla}')\otimes\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- kinetic energy density: $\tau(\mathbf{r}) = \vec{\nabla}\cdot\vec{\nabla}'\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin kinetic energy density: $\vec{T}(\mathbf{r}) = \vec{\nabla}\cdot\vec{\nabla}'\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- anomalous (pairing) density: $\chi(\mathbf{r}) = \chi(\mathbf{r},\mathbf{r}')|_{r=r'}$

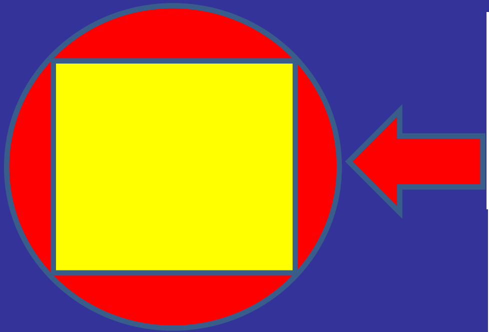
Treatment of the Coulomb potential

$$\nabla^2\Phi(\mathbf{r}) = 4\pi e^2\rho(\mathbf{r})$$
$$\Phi(\mathbf{r}) = \int d^3r' \frac{e^2\rho(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}$$

$$\Phi(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\frac{e^2\rho(\vec{k})}{k^2}\exp(i\vec{k}\cdot\mathbf{r}) = \frac{1}{27N_xN_yN_z}\sum_{\vec{k}\in L_xL_yL_z} e^2\rho(\vec{k})f(k)\exp(i\vec{k}\cdot\mathbf{r})$$

Defining an auxiliary potential *f(r)* on can get rid of spurious interaction with neighboring cells at the cost of performing FFT in 3 times larger box.





$$f(r) = 1/r \text{ for } r < \sqrt{L_x^2+L_y^2+L_z^2}$$
$$f(r) = 0 \text{ otherwise}$$

$$f(k) = 4\pi\frac{1-\cos(k\sqrt{L_x^2+L_y^2+L_z^2})}{k^2}$$

Castro, Rubio, Stott, arXive:0012024v1

However taking into account that FFT in a larger box means simply denser momentum space one can replace one FFT in 3 times larger box with 27 FFT's in the original box.

$$\Phi(\mathbf{r}) =$$
$$= \frac{1}{27N_xN_yN_z}\sum_{k,l,m=0}^{2}\left[\sum_{\vec{k}\in L^3} e^2\rho_{klm}(\vec{k}) f\left(\vec{k}+\left(k\frac{2\pi}{3L_x}, l\frac{2\pi}{3L_y}, m\frac{2\pi}{3L_z}\right)\right)\exp(i\vec{k}\cdot\mathbf{r})\right]$$
$$\times \exp\left(i\left(k\frac{2\pi}{3L_x}x + l\frac{2\pi}{3L_y}y + m\frac{2\pi}{3L_z}z\right)\right)$$

where

$$\rho_{klm}(\vec{k}) = \sum_{\mathbf{r}\in L^3}\rho(x,y,z)\exp\left(-i\left(k\frac{2\pi}{3L_x}x + l\frac{2\pi}{3L_y}y + m\frac{2\pi}{3L_z}z\right)\right)\exp(-i\vec{k}\cdot\mathbf{r})$$

Gain in computational cost:

$$27\cdot N^3 Log N^3 < (3N)^3 Log(3N)^3$$

Formalism for Time Dependent Phenomena: TDSLDA

Local density approximation (no memory terms – adiabatic TDDFT)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k\uparrow}(\mathbf{r},t) \\ u_{k\downarrow}(\mathbf{r},t) \\ v_{k\uparrow}(\mathbf{r},t) \\ v_{k\downarrow}(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow,\uparrow}(\mathbf{r},t) & h_{\uparrow,\downarrow}(\mathbf{r},t) & 0 & \Delta(\mathbf{r},t) \\ h_{\downarrow,\uparrow}(\mathbf{r},t) & h_{\downarrow,\downarrow}(\mathbf{r},t) & -\Delta(\mathbf{r},t) & 0 \\ 0 & -\Delta^*(\mathbf{r},t) & -h^*_{\uparrow,\uparrow}(\mathbf{r},t) & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) \\ \Delta^*(\mathbf{r},t) & 0 & -h^*_{\uparrow,\downarrow}(\mathbf{r},t) & -h^*_{\downarrow,\downarrow}(\mathbf{r},t) \end{pmatrix}\begin{pmatrix} u_{k\uparrow}(\mathbf{r},t) \\ u_{k\downarrow}(\mathbf{r},t) \\ v_{k\uparrow}(\mathbf{r},t) \\ v_{k\downarrow}(\mathbf{r},t) \end{pmatrix}$$

Density functional contains normal densities, anomalous density (pairing) and currents:

$$E(t) = \int d^3r \left[\varepsilon(n(\vec{r},t), \tau(\vec{r},t), \nu(\vec{r},t), \underline{\vec{j}(\vec{r},t)}) + V_{ext}(\vec{r},t)n(\vec{r},t) + \ldots \right]$$

- **The system is placed on a large 3D spatial lattice.**
- **Derivatives are computed with FFTW**
- **Fully self-consistent treatment with fundamental symmetries respected (isospin, gauge, Galilean, rotation, translation)**
- **for TD high-accuracy and numerically stable Adams–Bashforth–Milne 5th order predictor-corrector-modifier algorithm with only 2 evaluations of the rhs per time step and with no matrix operations**
- **No symmetry restrictions**
- **Number of PDEs is of the order of the number of spatial lattice points**
- **Initial conditions for TDSLDA are generated from static SLDA code.**

In future: ground state may be generated through adiabatic switching and quantum friction (Bulgac et al. arXiv:1305.6891)

Single particle potential (Skyrme):

$$h(\mathbf{r}) = -\vec{\nabla}\cdot\left(B(\mathbf{r}) + \vec{\sigma}\cdot\vec{C}(\mathbf{r})\right)\vec{\nabla} + U(\mathbf{r}) + \frac{1}{2i}\left[\vec{W}(\mathbf{r})\cdot(\vec{\nabla}\times\vec{\sigma}) + \vec{\nabla}\cdot(\vec{\sigma}\times\vec{W}(\mathbf{r}))\right]$$
$$+\vec{U}_\sigma(\mathbf{r})\cdot\vec{\sigma} + \frac{1}{i}\left(\vec{\nabla}\cdot\vec{U}_\Delta(\mathbf{r}) + \vec{U}_\Delta(\mathbf{r})\cdot\vec{\nabla}\right)$$

where

$$\begin{aligned}
B(\mathbf{r}) &= \frac{\hbar^2}{2m} + C^\tau\rho \\
\vec{C}(\mathbf{r}) &= C^{sT}\vec{s} \\
U(\mathbf{r}) &= 2C^\rho\rho + 2C^{\Delta\rho}\nabla^2\rho + C^\tau\tau + C^{\nabla J}\vec{\nabla}\cdot\vec{J} + C^\gamma(\gamma+2)\rho^{\gamma+1} \\
\vec{W}(\mathbf{r}) &= -C^{\nabla J}\vec{\nabla}\rho \\
\vec{U}_\sigma(\mathbf{r}) &= 2C^s\vec{s} + 2C^{\Delta s}\nabla^2\vec{s} + C^{sT}\vec{T} + C^{\nabla J}\vec{\nabla}\times\vec{j} \\
\vec{U}_\Delta(\mathbf{r}) &= C^j\vec{j} + \frac{1}{2}C^{\nabla j}\vec{\nabla}\times\vec{s}
\end{aligned}$$

and pairing potential:

$$\Delta(\mathbf{r},t) = -g_{eff}(\mathbf{r})\chi(\mathbf{r},t)$$

Linear response regime:
GDR of deformed nuclei

Box size: 32.5fm (mesh size: 1.25fm)
Energy deposited into a nucleus: 45-50MeV
Adiabatic switching of external perturbation: C*exp[-(t-10)^2/2]
Time window for Fourier transform: 1600 fm/c
Time step: 0.12fm/c -> relative accuracy: 10^(-7)

Photoabsorption cross section for heavy, deformed nuclei.

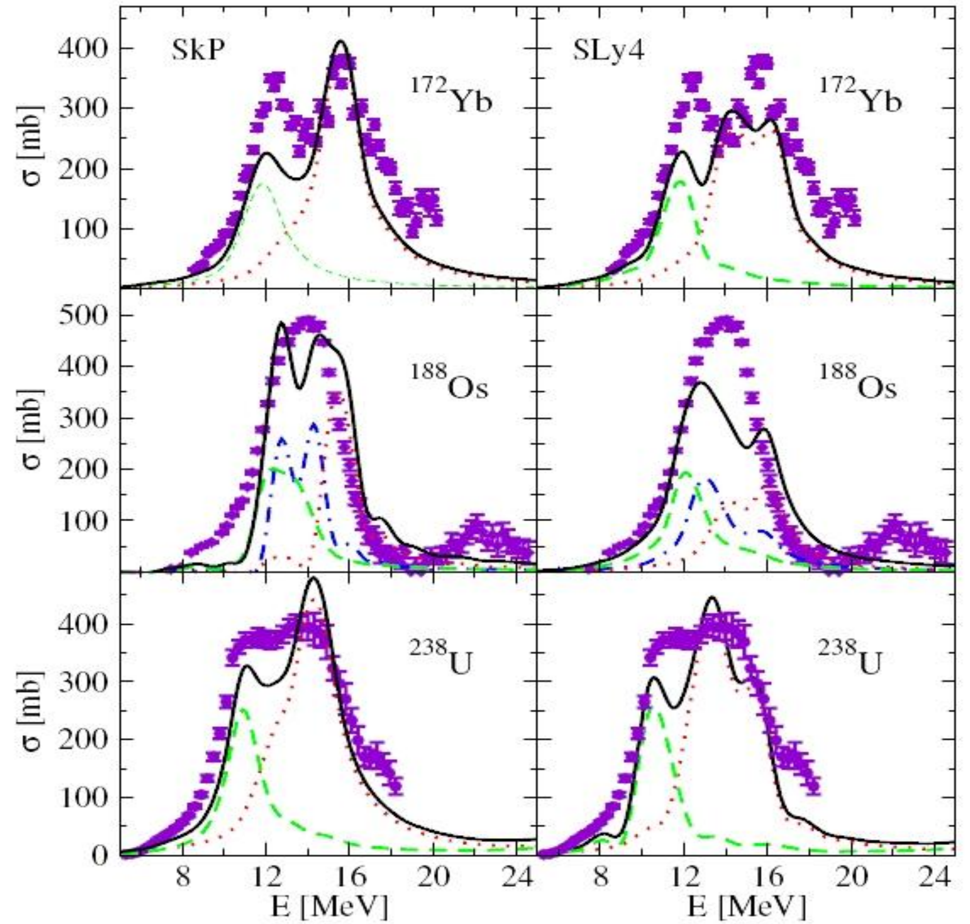
$$h_{\tau,\sigma\sigma}(\mathbf{r},t) \Rightarrow h_{\tau,\sigma\sigma}(\mathbf{r},t) + F_\tau(\mathbf{r}) f(t) \qquad F_\tau(\mathbf{r}) = N_\tau \sin(\mathbf{k}\cdot\mathbf{r}_\tau)/|\mathbf{k}|$$

$$S(E) = \sum_\nu |\langle \nu|\hat{F}|0\rangle|^2 \delta(E - E_\nu)$$

$$S(\omega) = \mathrm{Im}\{\delta F(\omega)/[\pi f(\omega)]\}$$

$$\delta F(t) = \langle \hat{F}\rangle_t - \langle \hat{F}\rangle_0 = \int d^3r\, \delta\rho(\mathbf{r},t) F(\mathbf{r}) \quad f(t) = C \exp[-(t-10)^2/2]$$

(gamma,n) reaction
through the excitation of GDR



I.Stetcu, A.Bulgac, P. Magierski, K.J. Roche, Phys. Rev. C84 051309 (2011)

Evolution of occupation probabilities

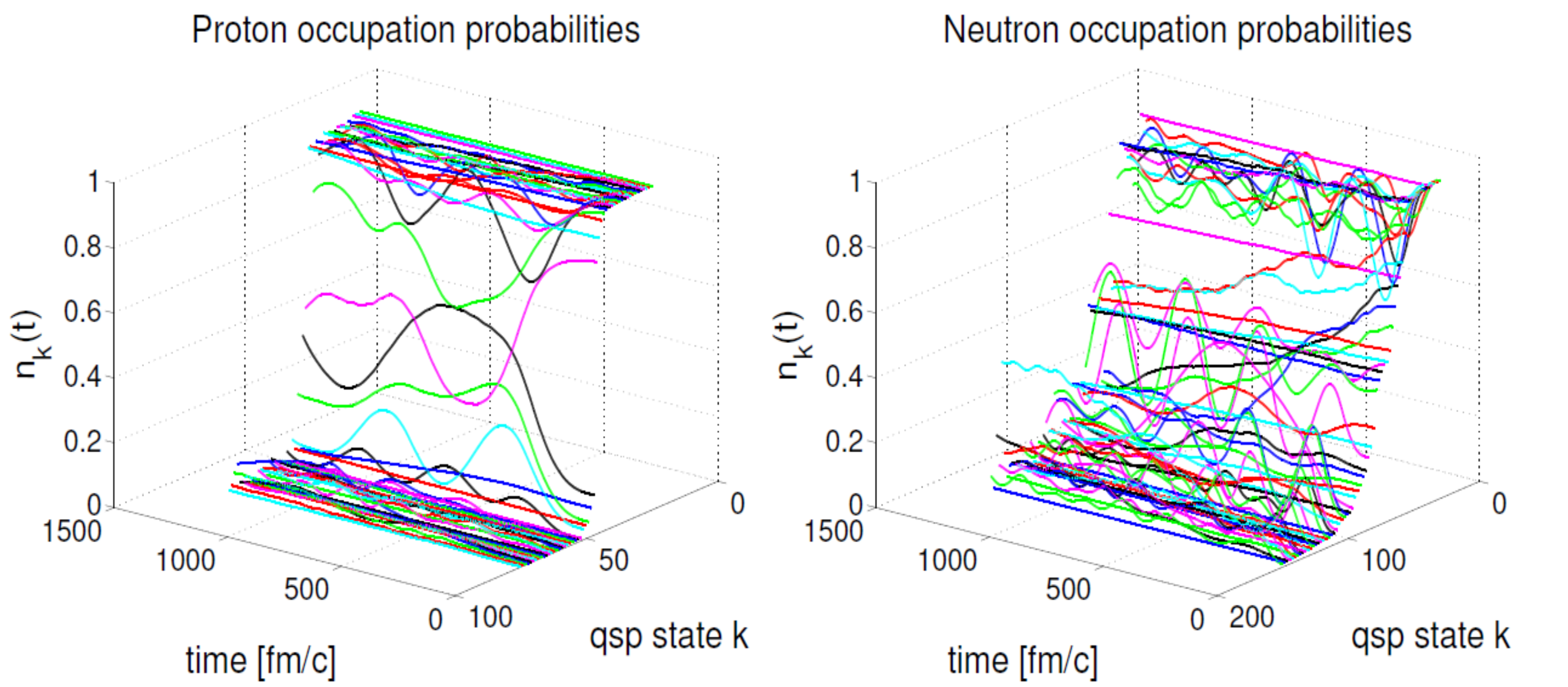


FIG. 1. (Color online) The time-dependent proton and neutron occupation probabilities of a a number of quasiparticle states around the Fermi level for ^{238}U calculated as described in the main text with SLy4.

Occupation probabilities vary significantly in time.

Pairing has to be treated fully selfconsistently!

Beyond linear regime:
Relativistic Coulomb excitation

Coupling to e.m. field:

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{\nabla}\psi \to \vec{\nabla}_A\psi = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right)\psi$$
$$\vec{\nabla}\psi^* \to \vec{\nabla}_{-A}\psi^* = \left(\vec{\nabla} + i\frac{e}{\hbar c}\vec{A}\right)\psi^*$$
$$i\hbar\frac{\partial}{\partial t}\psi \to \left(i\hbar\frac{\partial}{\partial t} - e\phi\right)\psi$$

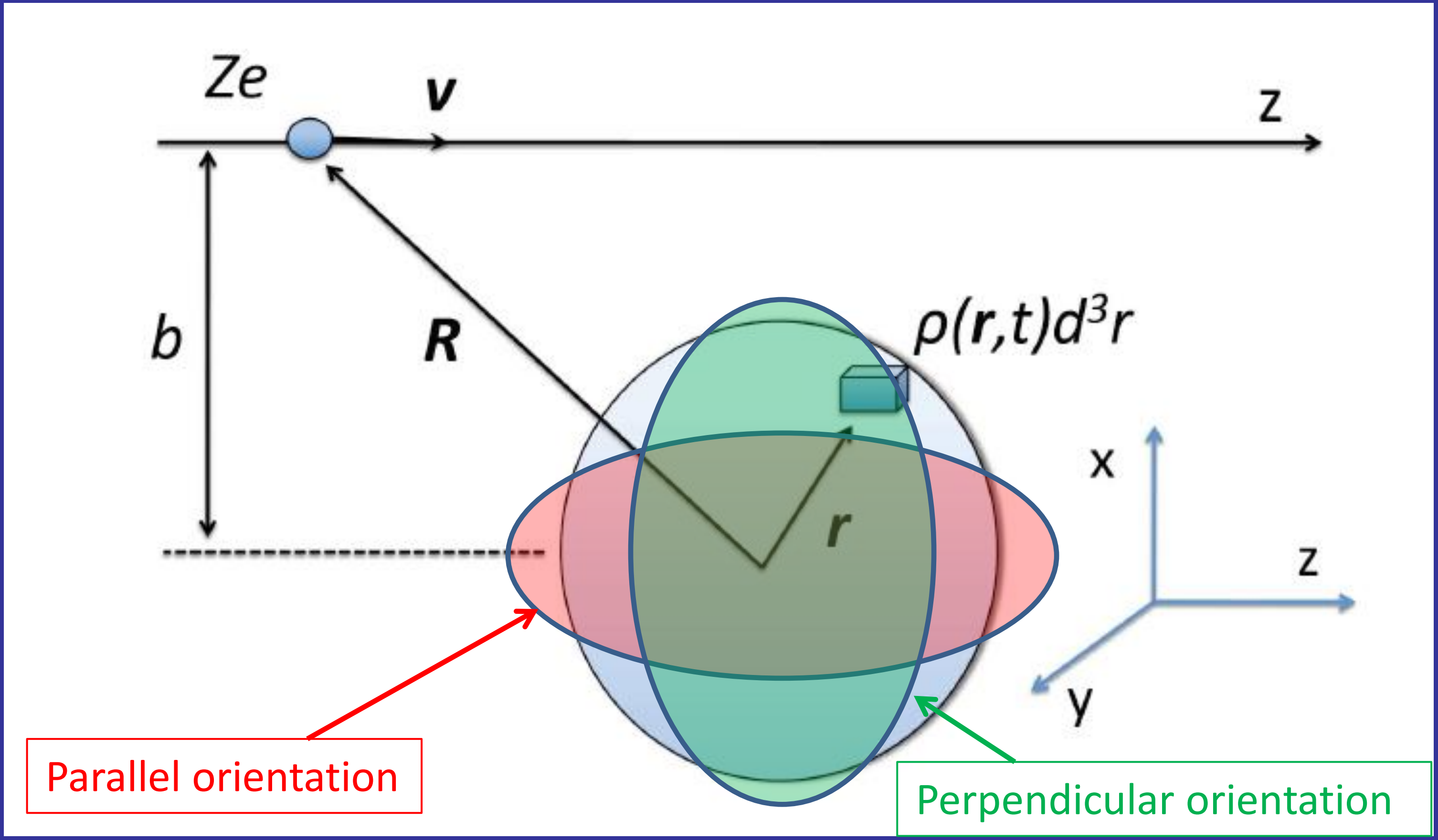
which implies that $\vec{\nabla}\psi\psi^* \to \vec{\nabla}\psi\psi^*$.

Consequently the densities change according to:

- density: $\rho_A(\mathbf{r}) = \rho_A(\mathbf{r})$
- spin density: $\vec{s}_A(\mathbf{r}) = \vec{s}(\mathbf{r})$
- current: $\vec{j}_A(\mathbf{r}) = \vec{j}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A}\rho(\mathbf{r})$
- spin current (2nd rank tensor): $\mathbf{J}_A(\mathbf{r}) = \mathbf{J}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A}\otimes\vec{s}(\mathbf{r})$
- spin current (vector): $\vec{J}_A(\mathbf{r}) = \vec{J}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A}\times\vec{s}(\mathbf{r})$
- kinetic energy density: $\tau_A(\mathbf{r}) = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right)\cdot\left(\vec{\nabla}' + i\frac{e}{\hbar c}\vec{A}\right)\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
  $= \tau(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}\cdot\vec{j}(\mathbf{r}) + \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\rho(\mathbf{r}) = \tau(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}\cdot\vec{j}_A(\mathbf{r}) - \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\rho(\mathbf{r})$
- spin kinetic energy density: $\vec{T}_A(\mathbf{r}) = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right)\cdot\left(\vec{\nabla}' + i\frac{e}{\hbar c}\vec{A}\right)\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
  $= \vec{T}(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}^T\cdot\mathbf{J}(\mathbf{r}) + \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\vec{s}(\mathbf{r}) = \vec{T}(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}^T\cdot\mathbf{J}_A(\mathbf{r}) - \frac{e^2}{\hbar^2c^2}|\vec{A}|^2\vec{s}(\mathbf{r})$

Relativistic Coulomb excitation

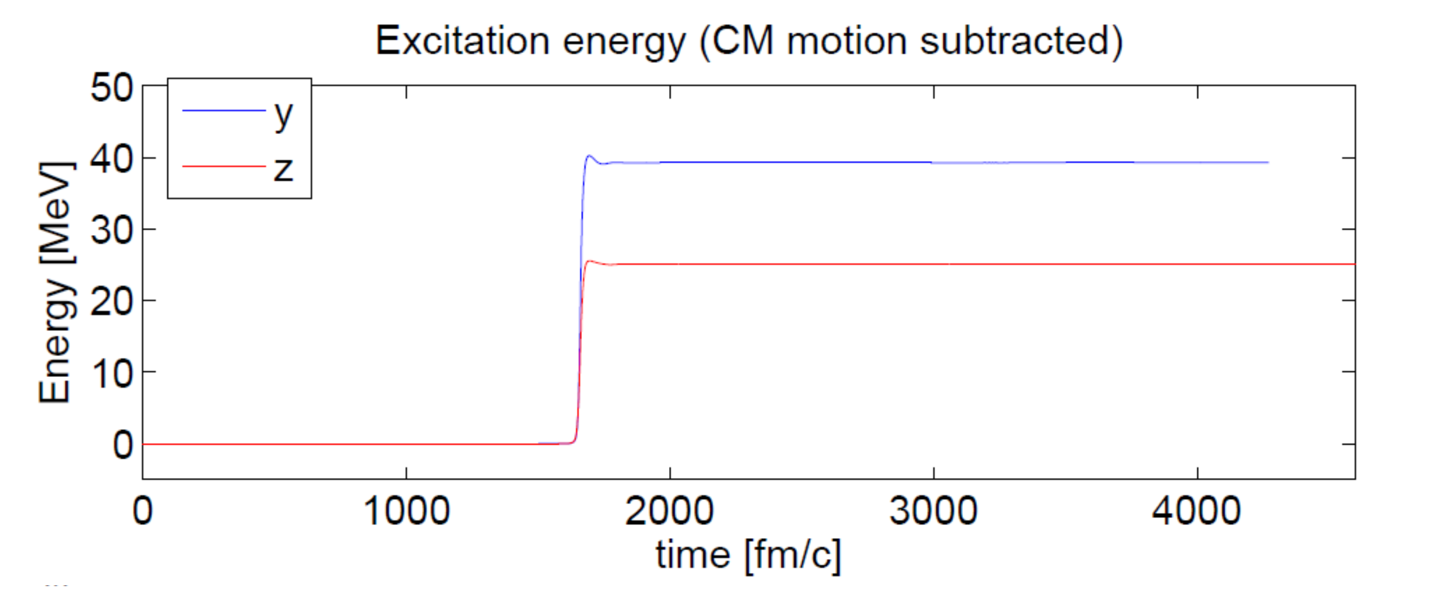
$$^{238}U + ^{238}U \rightarrow ^{238}U^* + \ldots \text{ at about 700 MeV/n}$$



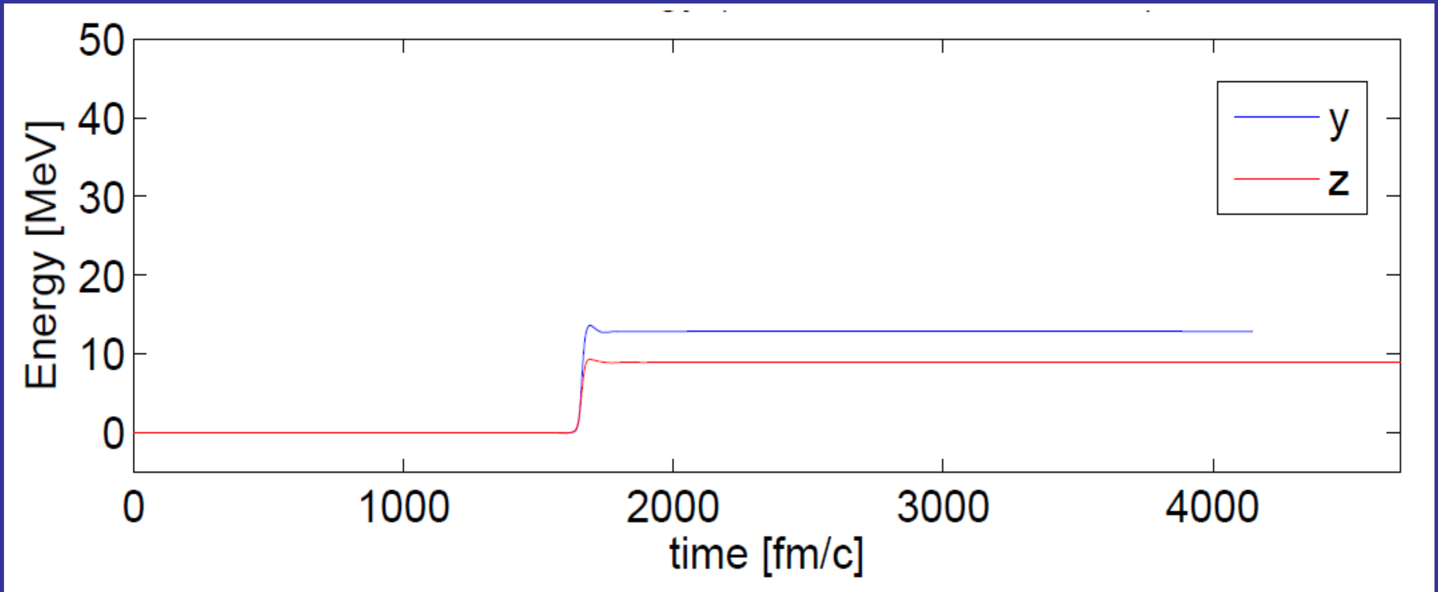
The coordinate transformation has been applied to keep CM in the center of the box at all times.

Energy deposited for two nuclear orientations (y – perpendicular, z – parallel)

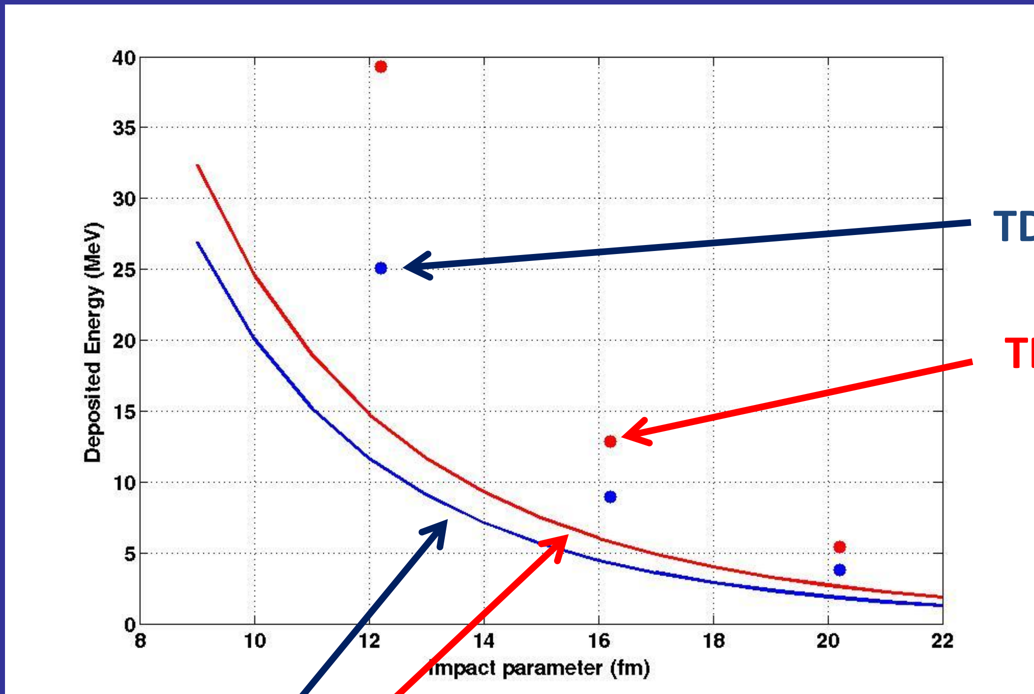
Impact parameter b=12.2fm



Impact parameter b=16.2fm



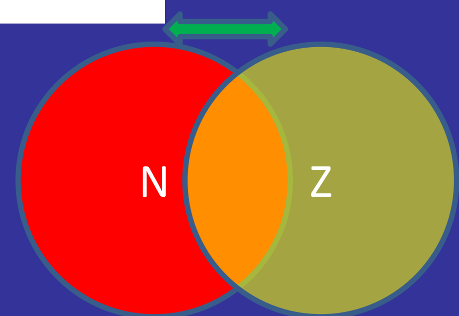
Energy transferred to the target nucleus in the form of internal excitations



TDSLDA – parallel orientation

TDSLDA – perpendicular orientation

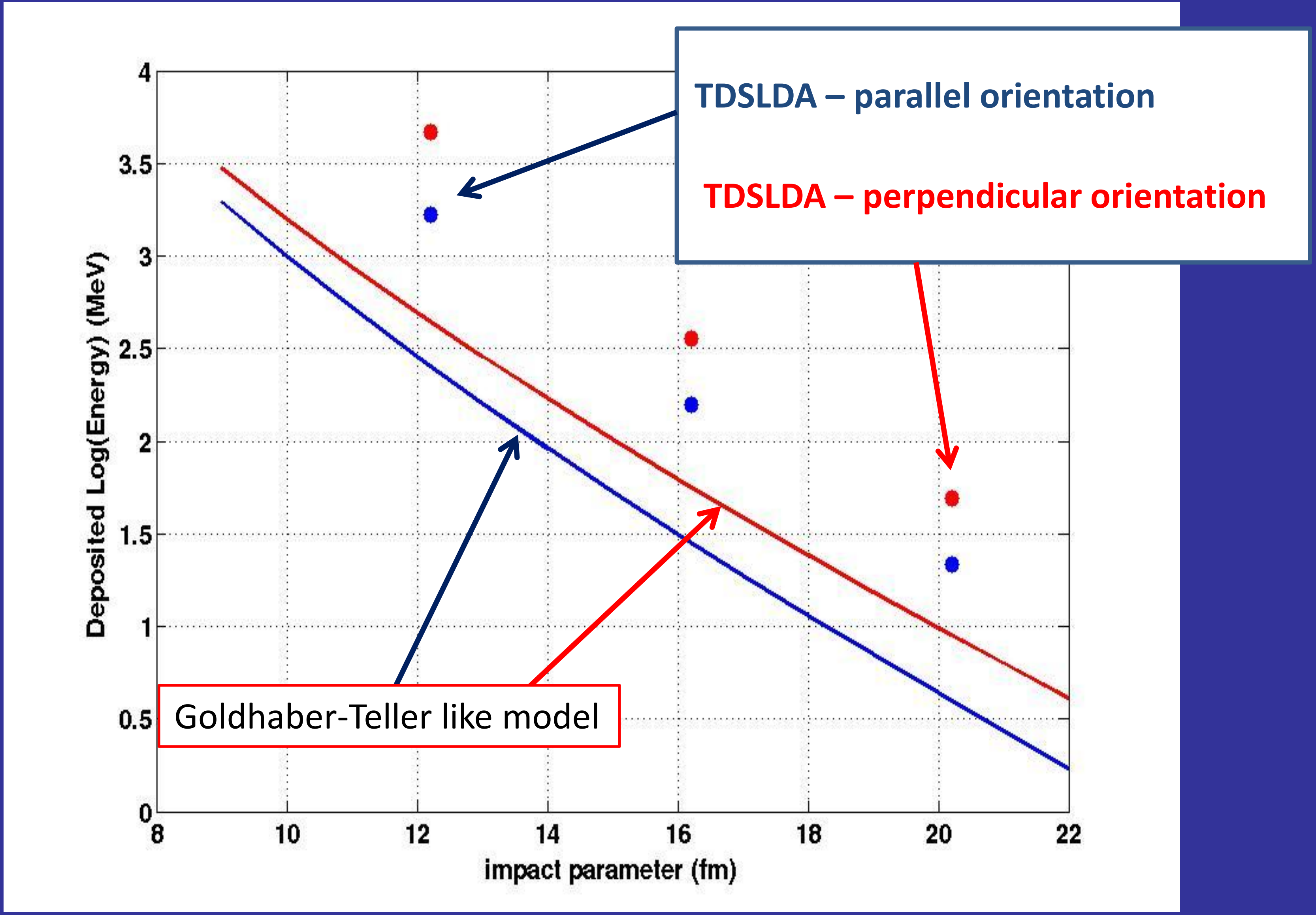
Goldhaber-Teller like model:
proton and neutron density distributions
oscillating against each other



Two characteristic frequencies

$$\hbar\omega_1 = 12 MeV$$
$$\hbar\omega_2 = 16 MeV$$

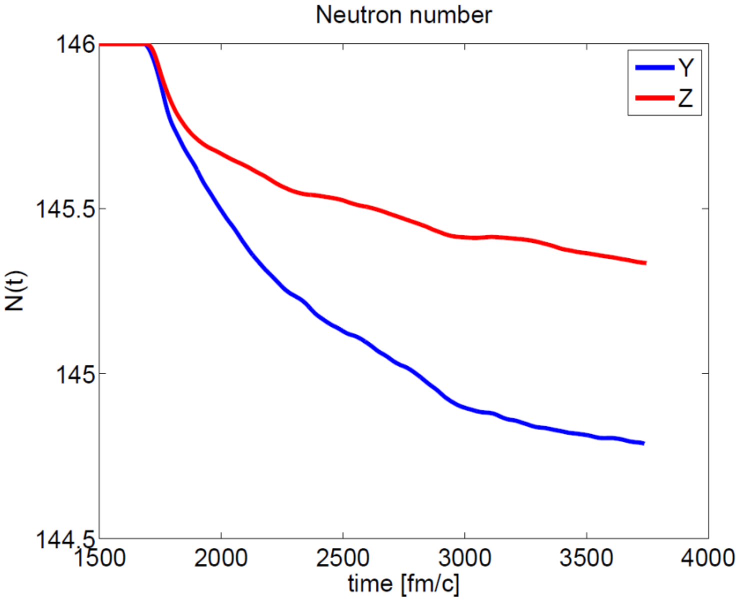
**Part of the energy is transferred
to other degrees of freedom
than pure dipole moment oscillations.**



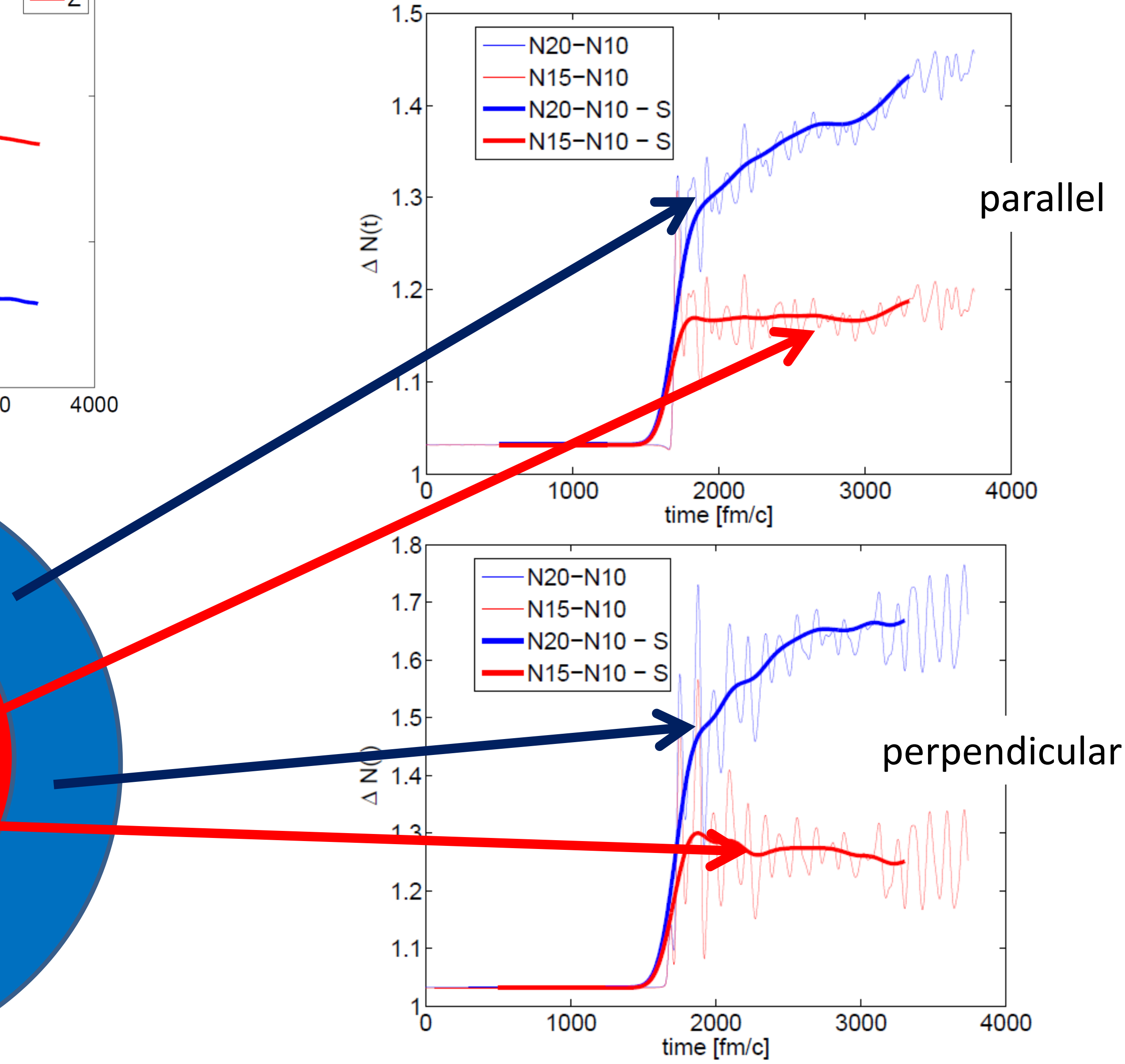
To get the same slope in GT model as in the corresponding TDSLDA results the frequencies of GDR should lie in the interval (10,18) MeV

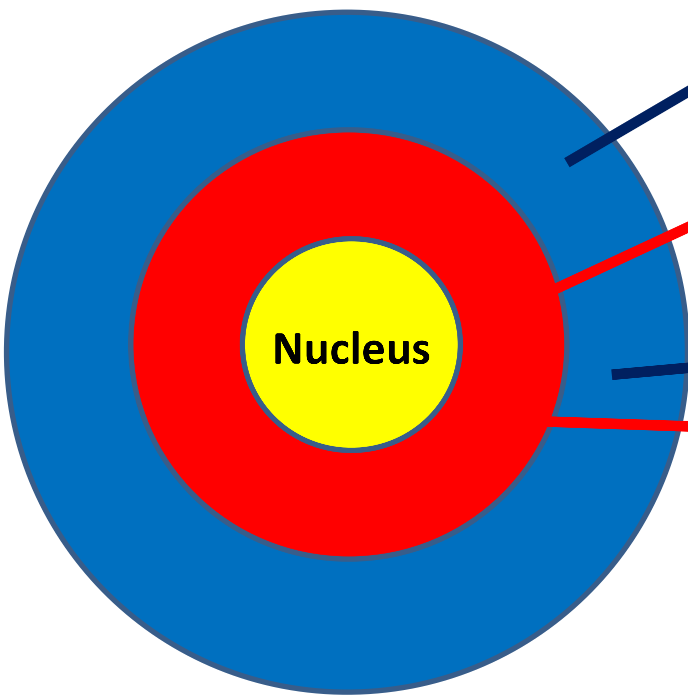
Neutron emission

Impact parameter b=12.2fm



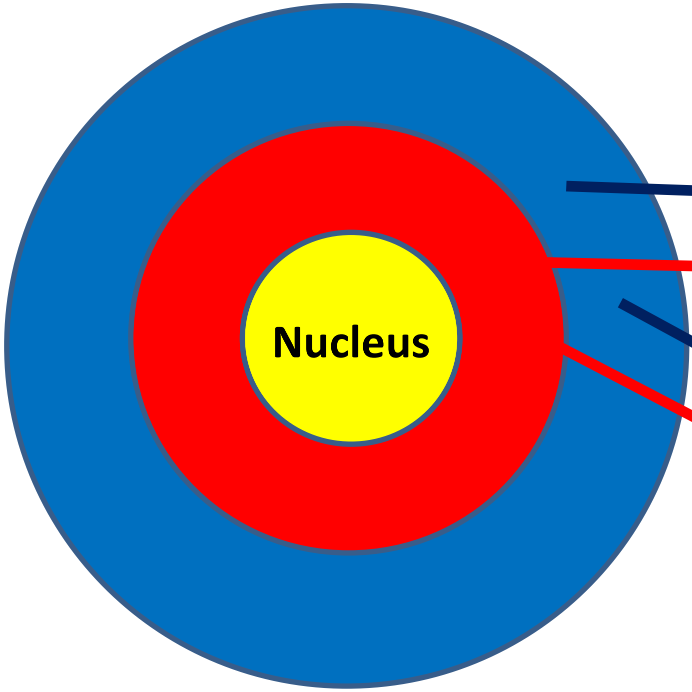
Number of neutrons in two shells surrounding nucleus for two nuclear orientations with respect to incoming projectile:



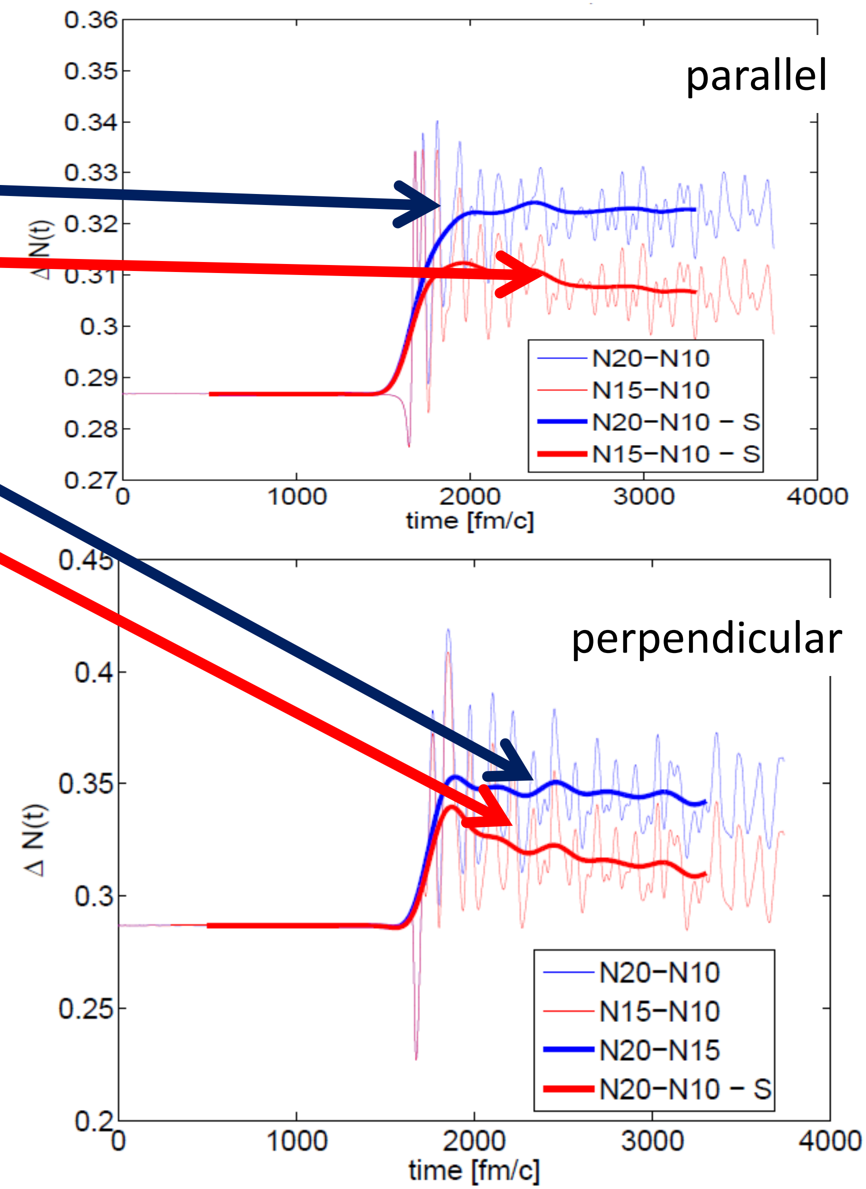


Protons

Impact parameter b=12.2fm



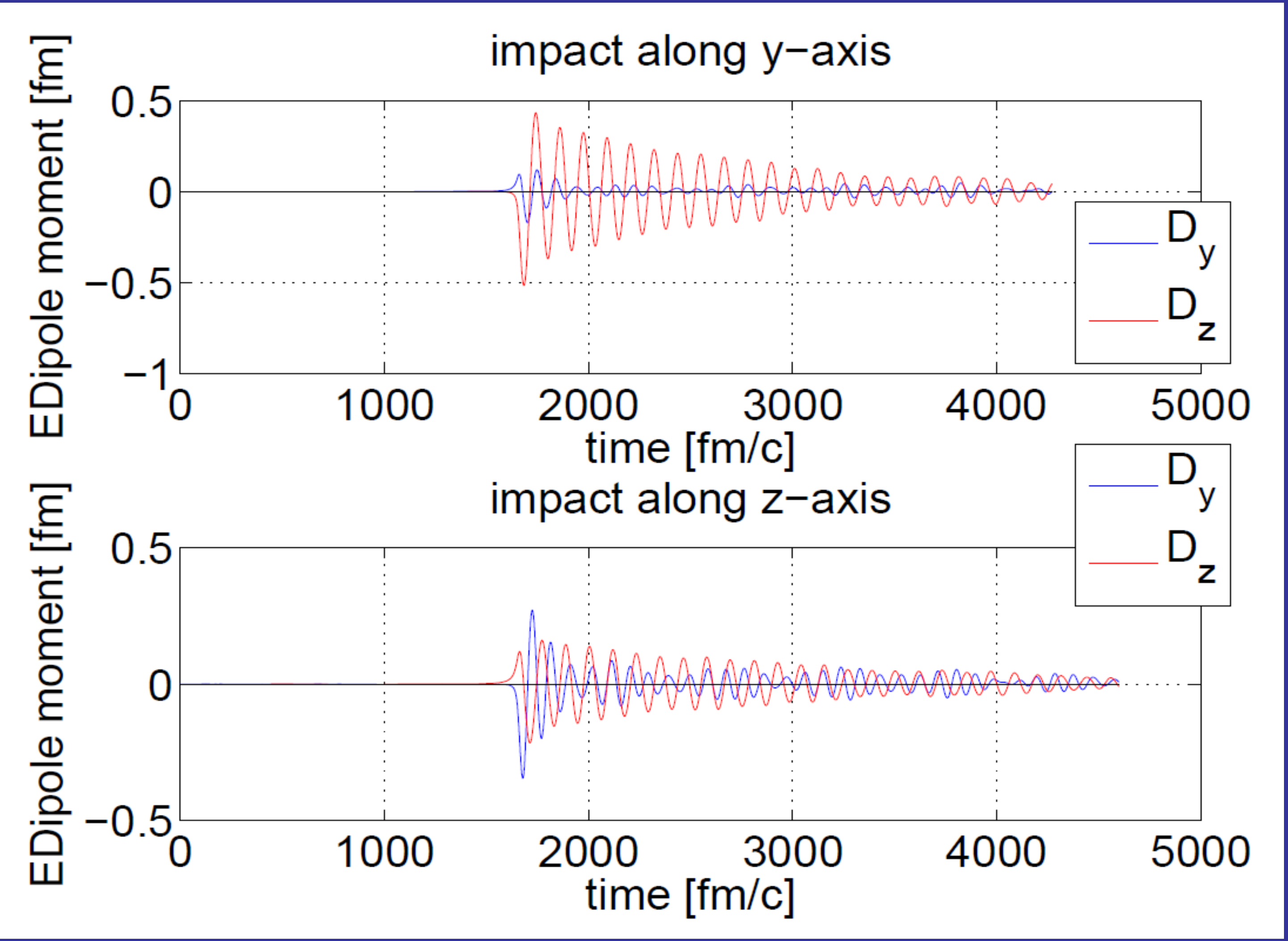
Number of protons in two shells surrounding nucleus:



Contrary to protons, neutrons exhibit approximately steady flow out of nucleus

Internal nuclear excitations

Electric dipole moment (along two axes: y, z) as a function of time

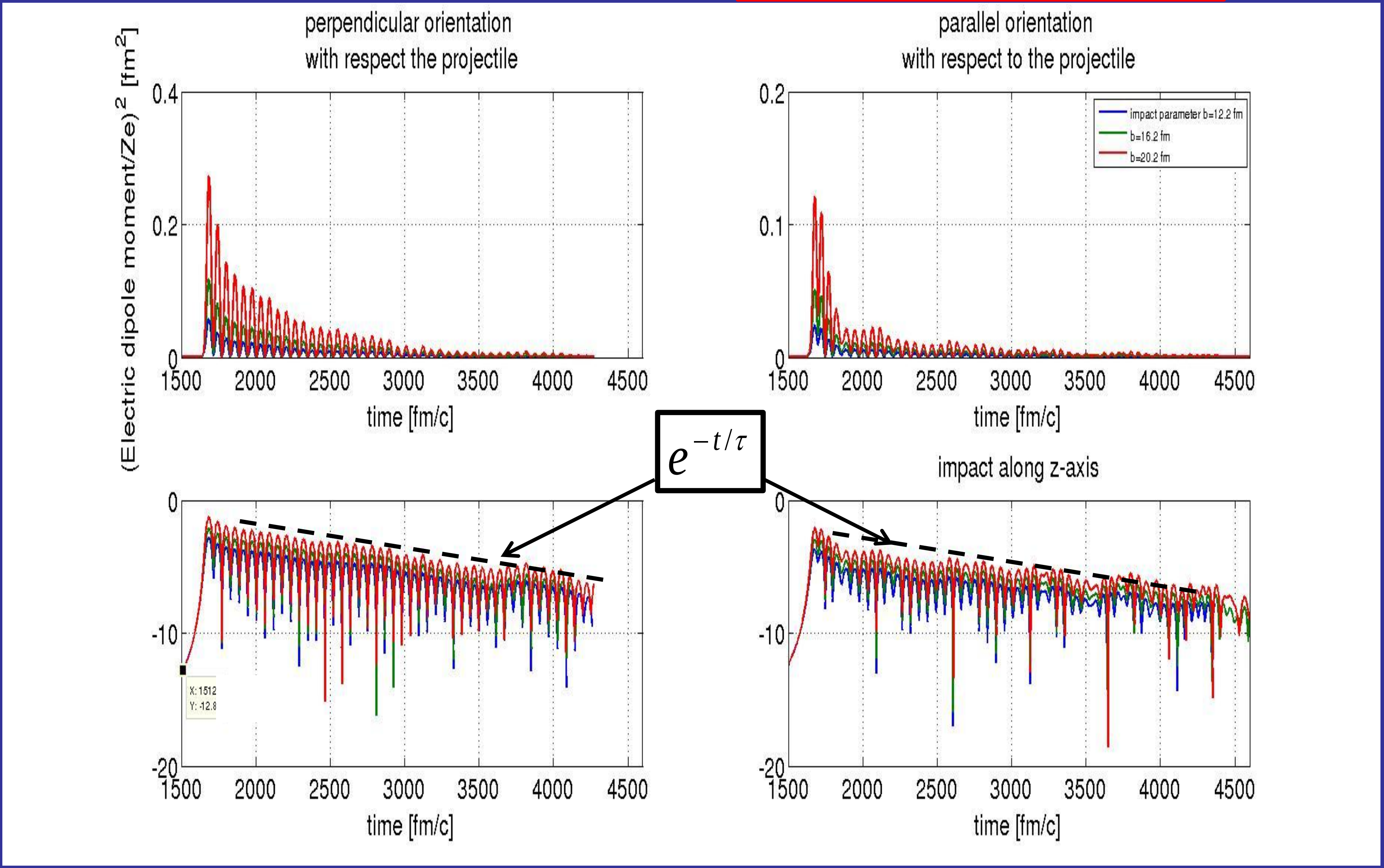


Oscillations are damped due to the one-body dissipation mechanism

One body dissipation

Let us assume that the collective energy of dipole oscillation is proportional the square of the amplitude of electric dipole moment:

$$E_{coll}(t) \propto \left[D_{\max}(t)\right]^2$$



The rate of the dissipation weakly depends on the amplitude and approximately behaves like:

$$E_{coll}(t) \propto e^{-t/\tau}; \quad \tau \approx 500\, fm/c$$

From the wall formula (assuming classically chaotic single particle motion) :

$$\frac{d\langle v^n \rangle}{dt} = \frac{n(n+2)}{4}\langle v^{n-1}\rangle \frac{1}{V}\int \dot{n}^2 d\sigma$$

- Relation between velocity moments from F-P eq.

$$\langle v^n \rangle_0 = \frac{3}{n+3} v_F^n$$

$$\frac{\Delta E}{E_0} = \frac{\langle E \rangle - \langle E \rangle_0}{\langle E \rangle_0} = \frac{3}{4}\tilde{\beta}\eta\left[\omega t - \frac{1}{2}\sin(2\omega t)\right] + O(\eta^2)$$

- Wall formula

$$\eta = \frac{\tilde{\beta}\omega R_0}{v_F}$$

- Adiabaticity parameter   for GDR: $\eta \approx 0.3$

$\tilde{\beta}$ - Deformation parameter

Note that wall formula does not predict exponential damping and it scales with the square of the oscillation amplitude.

Estimation of τ gives $250\, fm/c < \tau_{wall} < 500\, fm/c$

Electromagnetic radiation from excited nucleus

$$\rho(\mathrm{r},t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\rho(\mathrm{r},\omega)\exp(-i\omega t)$$

$$\vec{j}(\mathrm{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \vec{j}(\mathrm{r},\omega)\exp(-i\omega t)$$

From TDSLDA

$$\vec{B}(\mathrm{r},\omega) = \frac{ie}{c}\frac{\exp(ikr)}{r}\int d^3r'\vec{k}\times\vec{j}(\mathrm{r}',\omega)\exp(-i\vec{k}\cdot\mathrm{r}') = \frac{ie}{c}\frac{\exp(ikr)}{r}\vec{k}\times\vec{j}(\vec{k},\omega)$$

$$\vec{E}(\mathrm{r},\omega) = \frac{ie}{c}\frac{\exp(ikr)}{r}\frac{\mathrm{r}}{r}\times\int d^3r'(\vec{j}(\mathrm{r}',\omega)\times\vec{k})\exp(-i\vec{k}\cdot\mathrm{r}') = \frac{ie}{c}\frac{\exp(ikr)}{r}\frac{\mathrm{r}}{r}\times\left(\vec{j}(\vec{k},\omega)\times\vec{k}\right)$$

$$\frac{dP}{d\Omega}(t) = \frac{e^2}{4\pi c}\left|\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}(\vec{k}\times\vec{j}(\vec{k},\omega))\exp(-i\omega(t-r/c))\right|^2$$

Angular distribution of radiated power

$$\frac{dE}{d\Omega d\omega}(\omega) = \frac{e^2}{4\pi^2 c}\left|\vec{k}\times\vec{j}(\vec{k},\omega)\right|^2 = \frac{e^2}{4\pi^2 c}\left|\int d^3r\left(\nabla\times\vec{j}(\mathrm{r},\omega)\right)\exp(-i\vec{k}\cdot\mathrm{r})\right|^2$$

Angular distribution and ferquency distribution of emitted radiation

In practice it is better to perform multipole expansion:

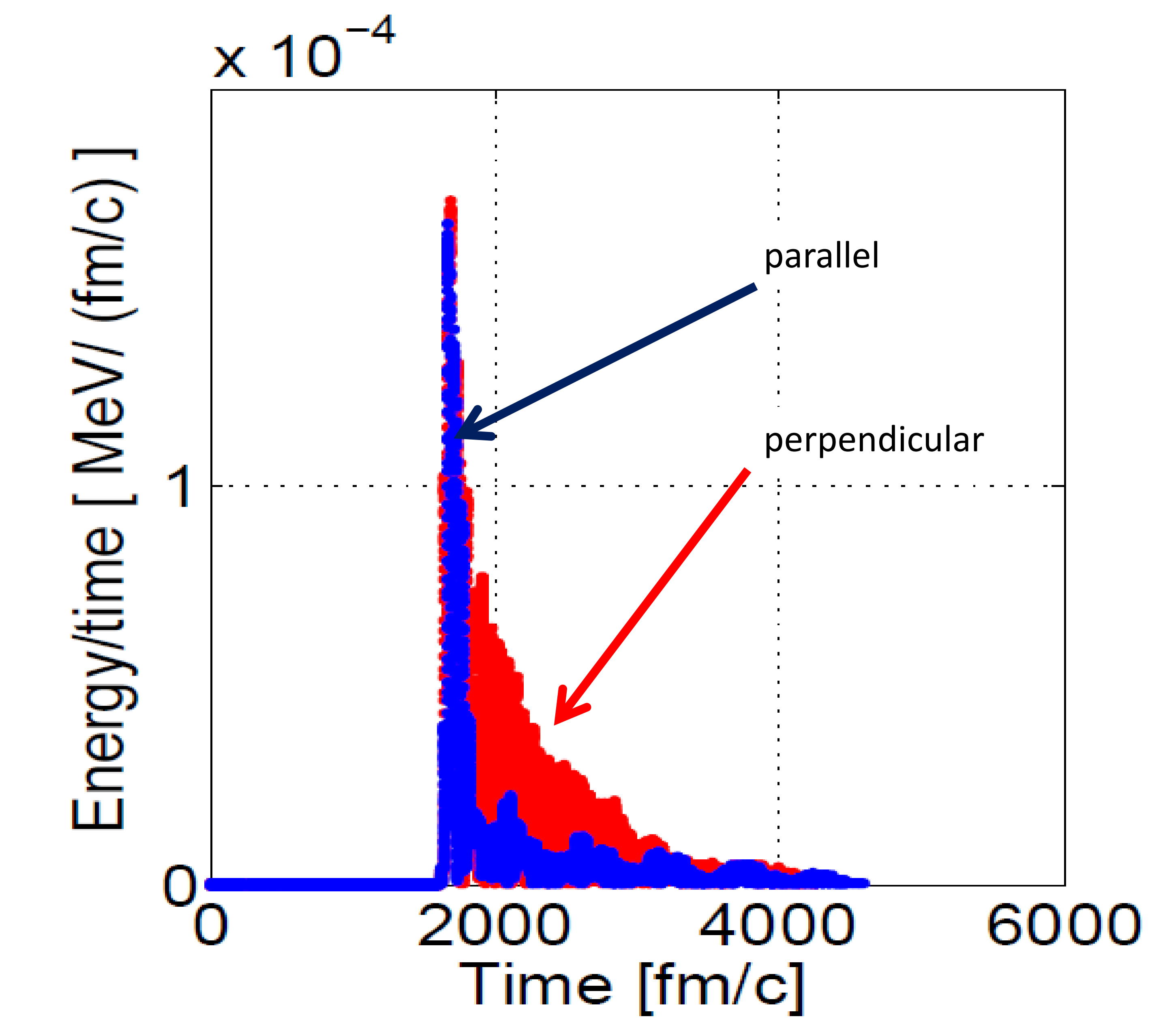
$$\frac{dE}{d\omega} = \frac{4e^2}{c}\sum_{l,m}|\vec{b}_{lm}(k,\omega)|^2$$

$$P(t+r/c) = \int\frac{dP}{d\Omega}(t+r/c)d\Omega = \frac{e^2}{\pi c}\sum_{l,m}\left|\int_{-\infty}^{\infty}\vec{b}_{lm}(k,\omega)\exp(-i\omega t)d\omega\right|^2$$

$$\vec{b}_{lm}(k,t) = \int d^3r\vec{b}(\mathrm{r},t)j_l(kr)Y^*_{lm}(\hat{r})$$

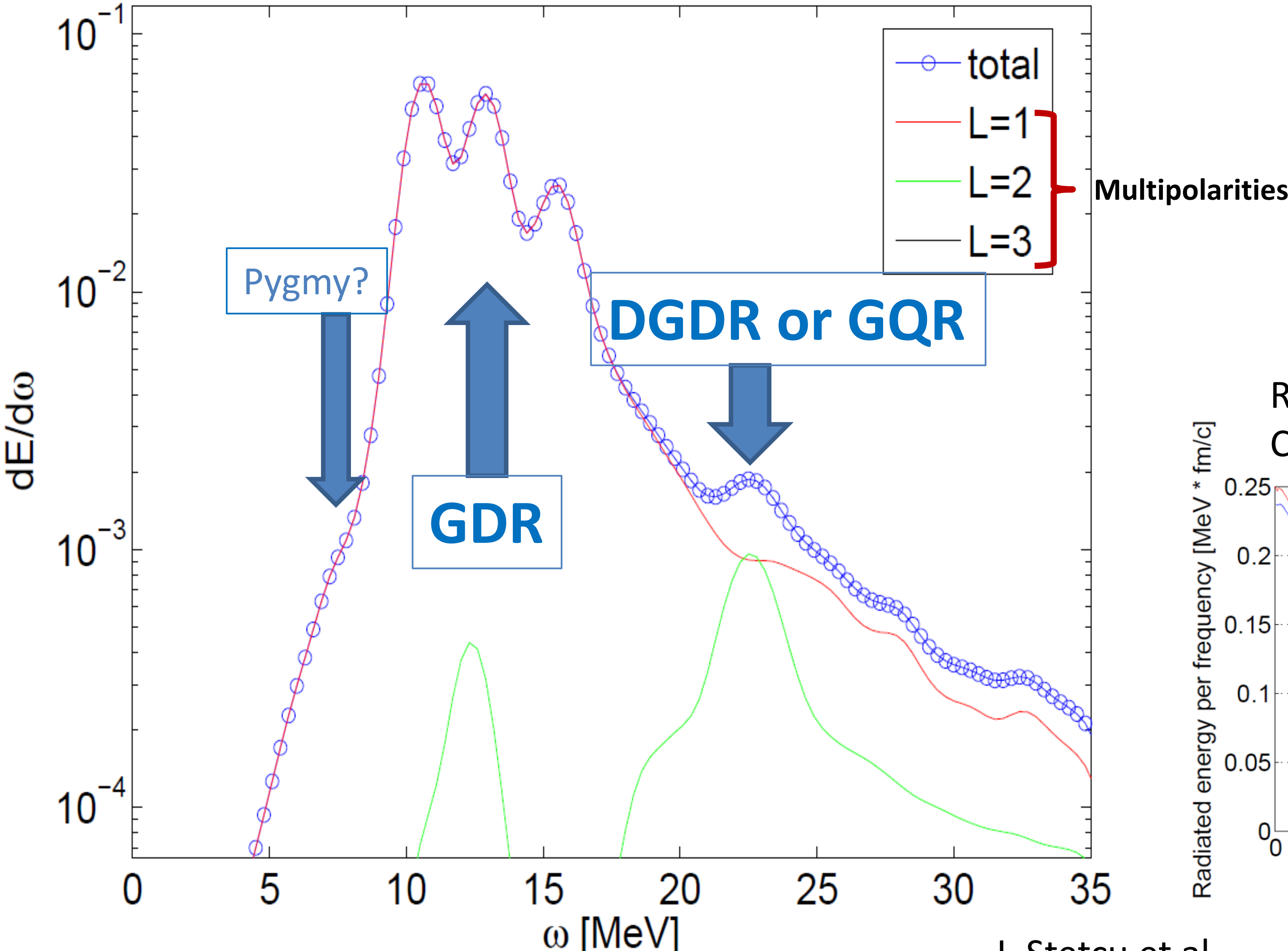
$$\vec{b}_{lm}(k,\omega) = \int_{-\infty}^{\infty}\vec{b}_{lm}(k,t)\exp(i\omega t)dt$$

Electromagnetic radiation rate due to the internal motion

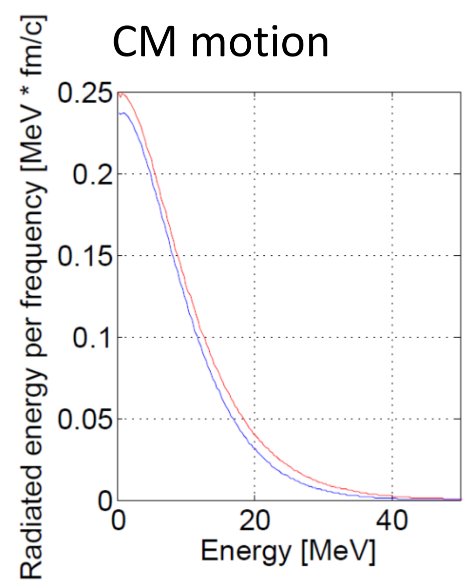


Electromagnetic radiation due to the internal nuclear motion

b (en, z)= 12.2 fm - impact parameter



Radiation due to CM motion



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Summary

- ***TDSLDA is a flexible tool to study nuclear dynamics.***
- ***Pairing field is treated on the same footing like single particle potentais (no frozen occupation number approximation).***
- ***Nuclear excitation modes (beyond linear response!) can be identified from e.m. radiation.***
- ***Various nonequilibrium nuclear processes can be studied:***
  - ***Nuclear large amplitude collective motion (LACM)***
  - ***(induced) nuclear fission***
  - ***Excitation of nuclei with gamma rays and neutrons***
  - ***Coulomb excitation of nuclei with relativistic heavy-ions***
  - ***Nuclear reactions, fusion between colliding heavy-ions***
  - ***Neutron star crust and dynamics of vortices and their pinning mechanism***

Current capabilities of the code:

- **volumes of the order of ($L = 80^3$) capable of simulating time evolution of 42000 neutrons at saturation density (possible application: neutron stars)**
- **capable of simulating up to times of the order of 10^{-19} s (a few million time steps)**
- **CPU vs GPU on Titan ≈ 15 speed-up** **(likely an additional factor of 4 possible)**

  Eg. for 137062 two component wave functions:

  CPU version (4096 nodes x 16 PEs) - 27.90 sec for 10 time steps

  GPU version (4096 PEs + 4096GPU) - 1.84 sec for 10 time steps