Computation of reactor fluxes – state of the art

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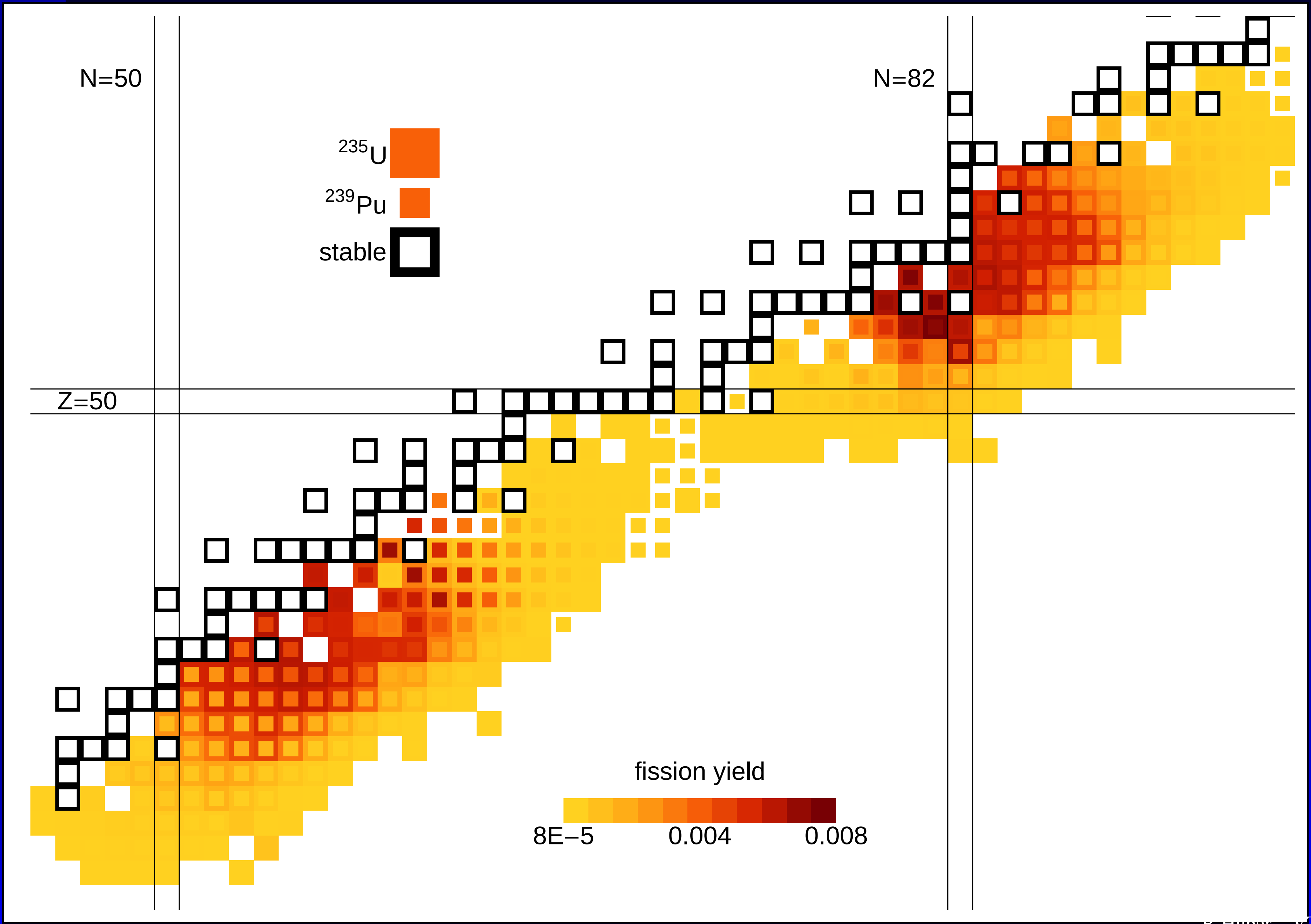
Fission yields of β emitters



Neutrinos from fission

$$^{235}U + n \to X_1 + X_2 + 2n$$

with average masses of X_1 of about A=94 and X_2 of about A=140. X_1 and X_2 have together 142 neutrons.

The stable nuclei with A=94 and A=140 are $^{94}_{40}Zr$ and $^{140}_{58}Ce$, which together have only 136 neutrons.

Thus 6 β-decays will occur, yielding 6 $\bar{\nu}_e$. About 2 will be above inverse β-decay threshold.

How does one compute the number and spectrum of neutrinos above inverse β-decay threshold?

Beta decay theory

In Fermi theory, the spectrum of massless neutrinos is obtained from

$$E_\nu = E_0 - E_e$$

In reality there are many corrections: finite nuclear size, radiative corrections, screening effects, induced currents, … which in principle can be computed for allowed decays but not for forbidden ones.

There is a sizable fraction of around 40% of all neutrinos coming from forbidden decays, essentially for reasons of combinatorics.

β-decay – Fermi theory

$$N_\beta(W) = K \underbrace{p^2(W - W_0)^2}_{\text{phase space}} F(Z,W)\,,$$

where $W = E/(m_e c^2) + 1$ and W_0 is the value of W at the endpoint. K is a normalization constant. $F(Z,W)$ is the so called Fermi function and given by

$$F(Z,W) = 2(\gamma+1)(2pR)^{2(\gamma-1)} e^{\pi\alpha Z W/p} \frac{|\Gamma(\gamma + i\alpha Z W/p)|^2}{\Gamma(2\gamma+1)^2}$$

$\gamma = \sqrt{1-(\alpha Z)^2}$

The Fermi function is the modulus square of the electron wave function at the origin.

Corrections to Fermi theory

$$N_\beta(W) = K\,p^2(W-W_0)^2\,F(Z,W)\,L_0(Z,W)\,C(Z,W)\,S(Z,W) \times G_\beta(Z,W)\,(1+\delta_{\mathrm{WM}}W)\,.$$

The neutrino spectrum is obtained by the replacements $W \to W_0 - W$ and $G_\beta \to G_\nu$.

All these correction have been studied 15-30 years ago.

Finite size corrections – I

Finite size of charge distribution affects outgoing electron wave function

$$L_0(Z,W) = 1 + 13\frac{(\alpha Z)^2}{60} - W R\alpha Z\frac{41-26\gamma}{15(2\gamma-1)}$$
$$-\alpha Z R\gamma\frac{17-2\gamma}{30W(2\gamma-1)}\dots$$

Parameterization of numerical solutions, only small associated error. Specifically, this is a parameterization by Wilkinson, 1990 based on numerical results by Behrens, Bühring, 1982.

Finite size corrections – II

Convolution of electron wave function with nucleon wave function over the volume of the nucleus, again following Wilkinson, 1990

$$\begin{aligned} C(Z,W) &= 1 + C_0 + C_1 W + C_2 W^2 \quad \text{with} \\ C_0 &= -\frac{233}{630}(\alpha Z)^2 - \frac{(W_0 R)^2}{5} + \frac{2}{35} W_0 R \alpha Z \,, \\ C_1 &= -\frac{21}{35} R \alpha Z + \frac{4}{9} W_0 R^2 \,, \\ C_2 &= -\frac{4}{9} R^2 \,. \end{aligned}$$

Small associated theory error (?). Assuming the n/p ratio is constant within the nucleus this should have the same uncertainty as L_0.

Screening correction

All of the atomic bound state electrons screen the charge of the nucleus – correction to Fermi function using the formalism of Behrens, Bühring, 1982

$$\bar{W} = W - V_0\,, \quad \bar{p} = \sqrt{\bar{W}^2 - 1}\,, \quad y = \frac{\alpha Z W}{p} \quad \bar{y} = \frac{\alpha Z \bar{W}}{\bar{p}} \quad \tilde{Z} = Z - 1\,.$$

V_0 is the so called screening potential

$$V_0 = \alpha^2 \tilde{Z}^{4/3} N(\tilde{Z})\,,$$

and $N(\tilde{Z})$ is taken from numerics.

$$S(Z,W) = \frac{\bar{W}}{W} \left(\frac{\bar{p}}{p}\right)^{(2\gamma-1)} e^{\pi(\bar{y}-y)} \frac{|\Gamma(\gamma + i\bar{y})|^2}{\Gamma(2\gamma+1)^2} \quad \text{for} \quad W > V_0\,,$$

Small associated theory error (overall small effect)

Radiative correction - I

Order α QED correction to electron spectrum, by Sirlin, 1967

$$g_\beta = 3\log M_N - \frac{3}{4} + 4\left(\frac{\tanh^{-1}\beta}{\beta}\right)\left(\frac{W_0 - W}{3W} - \frac{3}{2} + \log\left[2(W_0 - W)\right]\right) + \frac{4}{\beta} L\left(\frac{2\beta}{1+\beta}\right) + \frac{1}{\beta}\tanh^{-1}\beta\left(2(1+\beta^2) + \frac{(W_0 - W)^2}{6W^2} - 4\tanh^{-1}\beta\right)$$

where $L(x)$ is the Spence function, The complete correction is then given by

$$G_\beta(Z, W) = 1 + \frac{\alpha}{2\pi} g_\beta \,.$$

Small associated theory error.

Radiative correction - II

Order α QED correction to neutrino spectrum, recent calculation by Sirlin, Phys. Rev. **D84**, 014021 (2011).

$$h_\nu = 3\ln M_N + \frac{23}{4} - \frac{8}{\hat\beta} L\left(\frac{2\hat\beta}{1+\hat\beta}\right) + 8\left(\frac{\tanh^{-1}\hat\beta}{\hat\beta} - 1\right)\ln(2\hat W\hat\beta)$$
$$+4\frac{\tanh^{-1}\hat\beta}{\hat\beta}\left(\frac{7+3\hat\beta^2}{8} - 2\tanh^{-1}\hat\beta\right)$$

$$G_\nu(Z,W) = 1 + \frac{\alpha}{2\pi}h_\nu\,.$$

Very small correction.

Weak currents

In the following we assume $q^2 \ll M_{\mathrm{W}}$ and hence charged current weak interactions can be described by a current-current interaction.

$$-\frac{G_F}{\sqrt{2}} V_{ud} J^h_\mu J^l_\mu$$

where

$$J^h_\mu = \bar{\psi}_u \gamma_\mu (1+\gamma_5) \psi_d = V^h_\mu + A^h_\mu$$

However, we are not dealing with free quarks …

Induced currents

Describe protons and neutrons as spinors which are solutions to the free Dirac equation, but which are not point-like, we obtain for the hadronic current

$$V^h_\mu = i\bar{\psi}_p \left[g_V(q^2)\gamma_\mu + \frac{g_M(q^2)}{8M}\sigma_{\mu\nu}q_\nu + i g_S(q^2) q_\mu \right] \psi_n$$

$$A^h_\mu = i\bar{\psi}_p \left[g_A(q^2)\gamma_\mu\gamma_5 + \frac{g_T(q^2)}{8M}\sigma_{\mu\nu}q_\nu\gamma_5 + i g_P(q^2) q_\mu\gamma_5 \right] \psi_n$$

In the limit $q^2 \to 0$ the form factors $g_X(q^2) \to g_X$, *i.e.* new induced couplings, which are not present in the SM Lagrangian, but are induced by the bound state QCD dynamics. Note, that some form factors are absent in the SM.

Weak magnetism & β-spectra

g_M is call weak magnetism and the question is how it manifests itself in nuclear β-decay. Nuclear structure effects can be summarized by the use of appropriate form factors F_X^N.

The weak magnetic nuclear, F_M^N form factor by virtue of CVC is given in terms of the analog EM form factor as

$$F_M^N(0) = \sqrt{2}\mu(0)$$

The effect on the β decay spectrum is given by

$$1 + \delta_{\mathrm{WM}} W \simeq 1 + \frac{4}{3M} \frac{F_M^N(0)}{F_A^N(0)} W$$

Impulse approximation

In the impulse approximation nuclear β-decay is described as the decay of a free nucleon inside the nucleus. The sole effect of the nucleus is to modify the initial and final state densities.

In impulse approximation

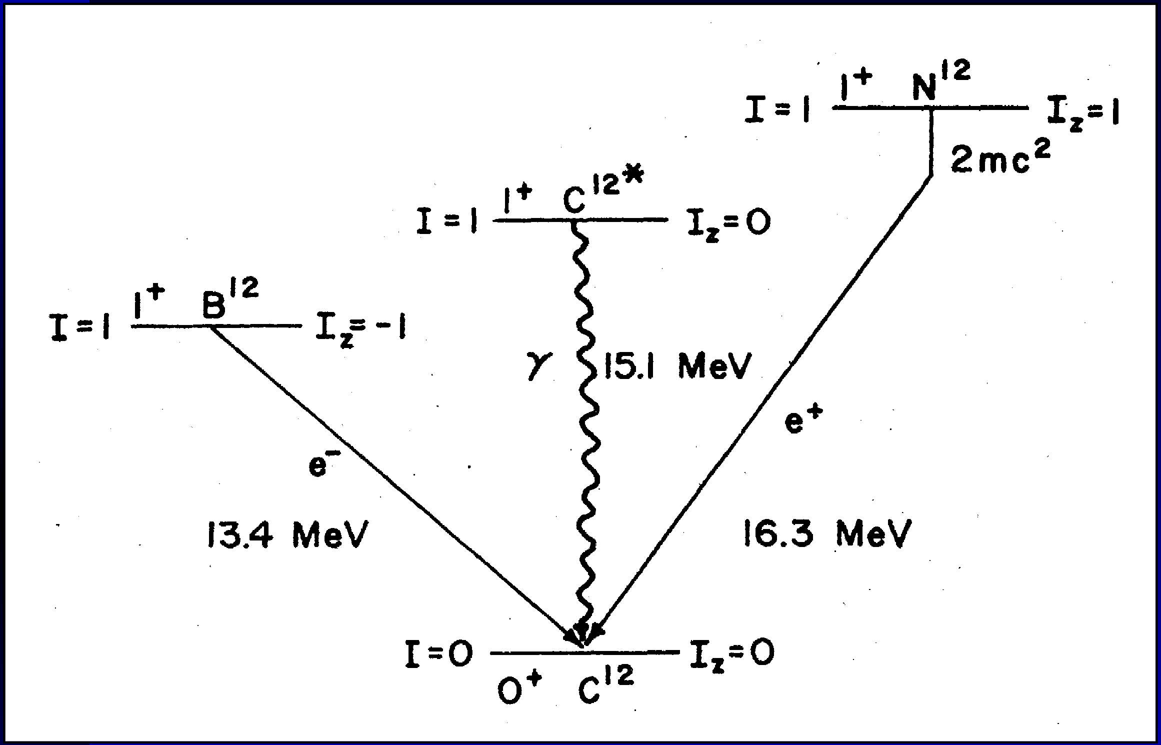
$$F_M^N(0) = \mu_p - \mu_n \simeq 4.7 \quad \text{and} \quad F_A^N(0) = C_A \simeq 1.27\,,$$

and thus

$$\delta_{WM} \simeq 0.5\%\,\mathrm{MeV}^{-1}$$

This value, in impulse approximation, is universal for all β-decays since it relies only on free nucleon parameters.

Isospin analog γ-decays



B. Holstein, Rev. Mod. Phys. **46**, 789, 1974.

$$\Gamma(C^{12*} - C^{12})_{M1} = \frac{\alpha E_\gamma^3}{3M^2}\left|\sqrt{2}\mu(0)\right|^2$$

$$b := \sqrt{2}\mu(0) = F_M^N(0)$$

Gamow-Teller matrix element c

$$c = F_A^N(0) = \sqrt{\frac{2ft_{\text{Fermi}}}{ft}}$$

and thanks to CVC $ft_{\text{Fermi}} \simeq 3080\,\text{s}$ is universal.

What is the value of δ_{WM}?

Three ways to determine δ_{WM}

- impulse approximation – universal value $0.5\%\,\mathrm{MeV}^{-1}$
- using CVC – F_M from analog M1 γ-decay width, F_A from ft value
- direct measurement in β-spectrum – only very few, light nuclei have been studied. In those cases the CVC predictions are confirmed within (sizable) errors.

In the following, we will compare the results from CVC with the ones from the impulse approximation.

CVC at work

Collect all nuclei for which we

- can identify the isospin analog energy level
- and know Γ_{M1}

then, compute the resulting δ_{WM}. This exercise has been done in Calaprice, Holstein, Nucl. Phys. **A273** (1976) 301. and they find for nuclei with $ft < 10^6$

$$\delta_{WM} = 0.82 \pm 0.4\% \, \text{MeV}^{-1}$$

which is in reasonable agreement with the impulse approximated value of $\delta_{WM} = 0.5\%\text{MeV}^{-1}$. Our result for $ft < 10^6$ is $\delta_{WM} = (0.67 \pm 0.26)\,\%\,\text{MeV}^{-1}$.

CVC at work

Decay	$J_i \rightarrow J_f$	E_γ (keV)	Γ_{M1} (eV)	b_γ	ft (s)	c	b_γ/Ac	$\lvert dN/dE \rvert$ (% MeV^{-1})
^{6}He $\rightarrow ^6$ Li	$0^+ \rightarrow 1^+$	3563	8.2	71.8	805.2	2.76	4.33	0.646
^{12}B $\rightarrow ^{12}$ C	$1^+ \rightarrow 0^+$	15110	43.6	37.9	11640.	0.726	4.35	0.62
^{12}N $\rightarrow ^{12}$ C	$1^+ \rightarrow 0^+$	15110	43.6	37.9	13120.	0.684	4.62	0.6
^{18}Ne $\rightarrow ^{18}$ F	$0^+ \rightarrow 1^+$	1042	0.258	242.	1233.	2.23	6.02	0.8
^{20}F $\rightarrow ^{20}$ Ne	$2^+ \rightarrow 2^+$	8640	4.26	45.7	93260.	0.257	8.9	1.23
^{22}Mg $\rightarrow ^{22}$ Na	$0^+ \rightarrow 1^+$	74	0.0000233	148.	4365.	1.19	5.67	0.757
^{24}Al $\rightarrow ^{24}$ Mg	$4^+ \rightarrow 4^+$	1077	0.046	129.	8511.	0.85	6.35	0.85
^{26}Si $\rightarrow ^{26}$ Al	$0^+ \rightarrow 1^+$	829	0.018	130.	3548.	1.32	3.79	0.503
^{28}Al $\rightarrow ^{28}$ Si	$3^+ \rightarrow 2^+$	7537	0.3	20.8	73280.	0.29	2.57	0.362
^{28}P $\rightarrow ^{28}$ Si	$3^+ \rightarrow 2^+$	7537	0.3	20.8	70790.	0.295	2.53	0.331
^{14}C $\rightarrow ^{14}$ N	$0^+ \rightarrow 1^+$	2313	0.0067	9.16	1.096×10^9	0.00237	276.	37.6
^{14}O $\rightarrow ^{14}$ N	$0^+ \rightarrow 1^+$	2313	0.0067	9.16	1.901×10^7	0.018	36.4	4.92
^{32}P $\rightarrow ^{32}$ S	$1^+ \rightarrow 0^+$	7002	0.3	26.6	7.943×10^7	0.00879	94.4	12.9

None of this is anywhere close to A=90…

What happens for large ft?

Decay	$J_i \to J_f$	E_γ (keV)	Γ_{M1} (eV)	b_γ	ft (s)	c	b_γ/Ac	$\lvert dN/dE \rvert$ (% MeV^{-1})
$^{14}C \to ^{14}N$	$0^+ \to 1^+$	2313	0.0067	9.16	1.096×10^9	0.00237	276.	37.6
$^{14}O \to ^{14}N$	$0^+ \to 1^+$	2313	0.0067	9.16	1.901×10^7	0.018	36.4	4.92
$^{32}P \to ^{32}S$	$1^+ \to 0^+$	7002	0.3	26.6	7.943×10^7	0.00879	94.4	12.9

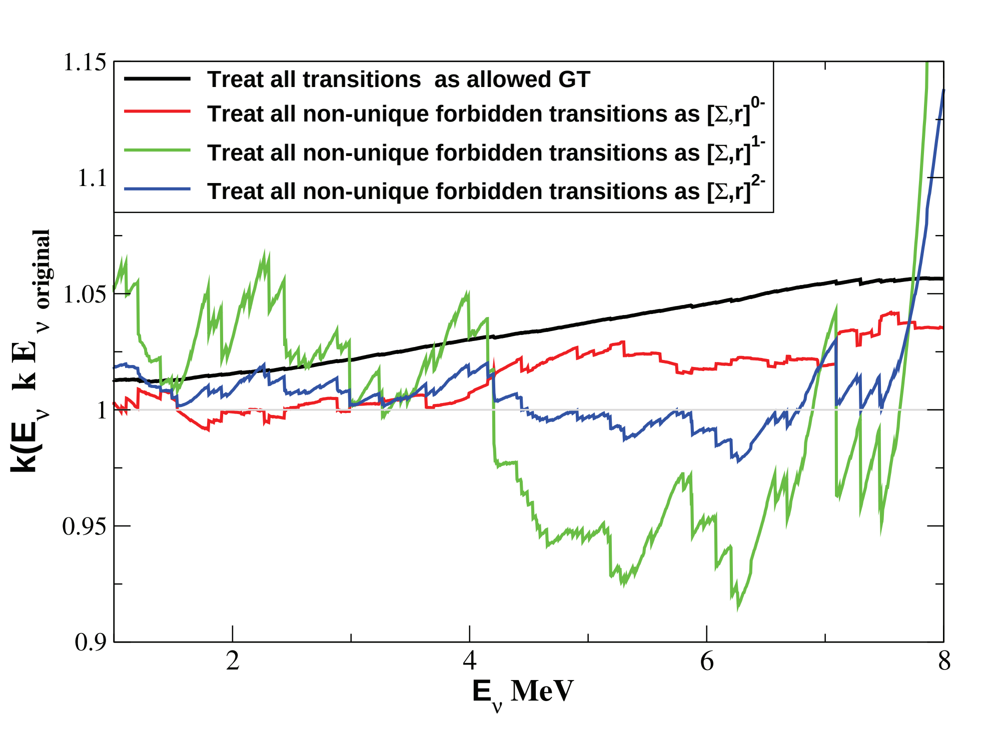
Including these large ft nuclei, we have

$$\delta_{WM} = (4.78 \pm 10.5)\,\%\,\mathrm{MeV}^{-1}$$

which is about 10 times the impulse approximated value and this are about 3 nuclei out of 10-20...

NB, a shift of δ_{WM} by $1\%\mathrm{MeV}^{-1}$ shifts the total neutrino flux above inverse β-decay threshold by $\sim 2\%$.

WM in forbidden decays



Approximate upper bound for the flux error due to forbidden decays.

Hayes *et. al*, arXiv:1309.4146 point out that in forbidden decays a mixture of different operators are involved, and that while for many of the individual operators the corrections can be computed, the relative contribution of each operator is generally unknown.

My interpretation: it is again the WM which is the leading cause for the large combined uncertainty they find. see talk by A. Hayes

Impact on fluxes

Following the nomenclature of Hayes *et al.*, the forbidden correction, κ, reads

$$\kappa(E_e) = C(E_e)\,[1 + \delta_{\mathrm{WM}}(E_e)]$$

and the neutrino correction $\Lambda(E_\nu)$ is obtained by

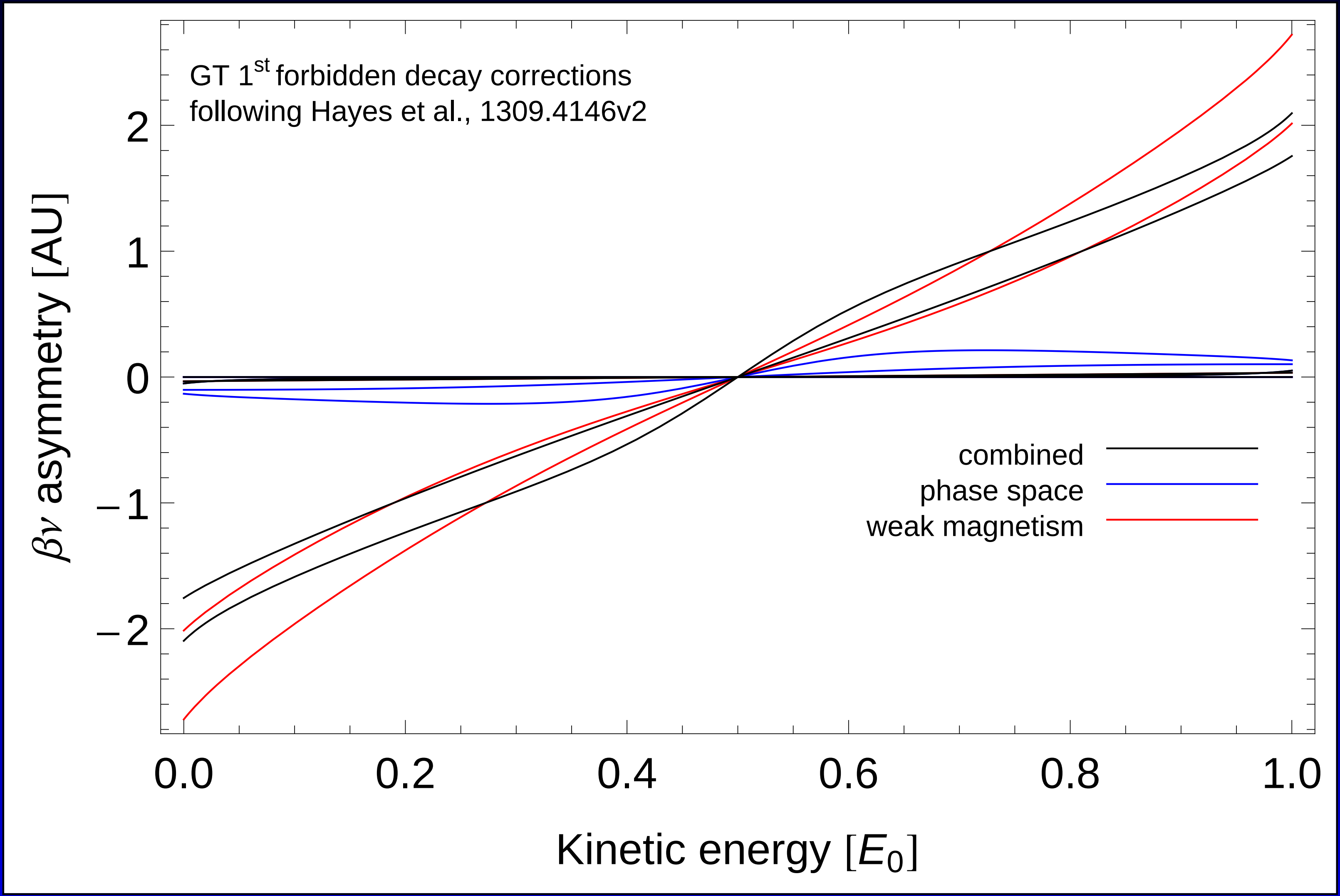
$$\Lambda(E_\nu) = \kappa(E_0 - E_\nu)$$

Given that the total β-spectrum is fixed by the ILL measurements, what matters are effects which change the neutrino and β-spectrum in different ways, and we define

$$\beta\nu(T) := \frac{\kappa(T) - \Lambda(T)}{\kappa(T) + \Lambda(T)}\,,$$

with T being the lepton kinetic energy.

Impact on fluxes



The vast majority of the effect is due to weak magnetism

Neutrinos from fission

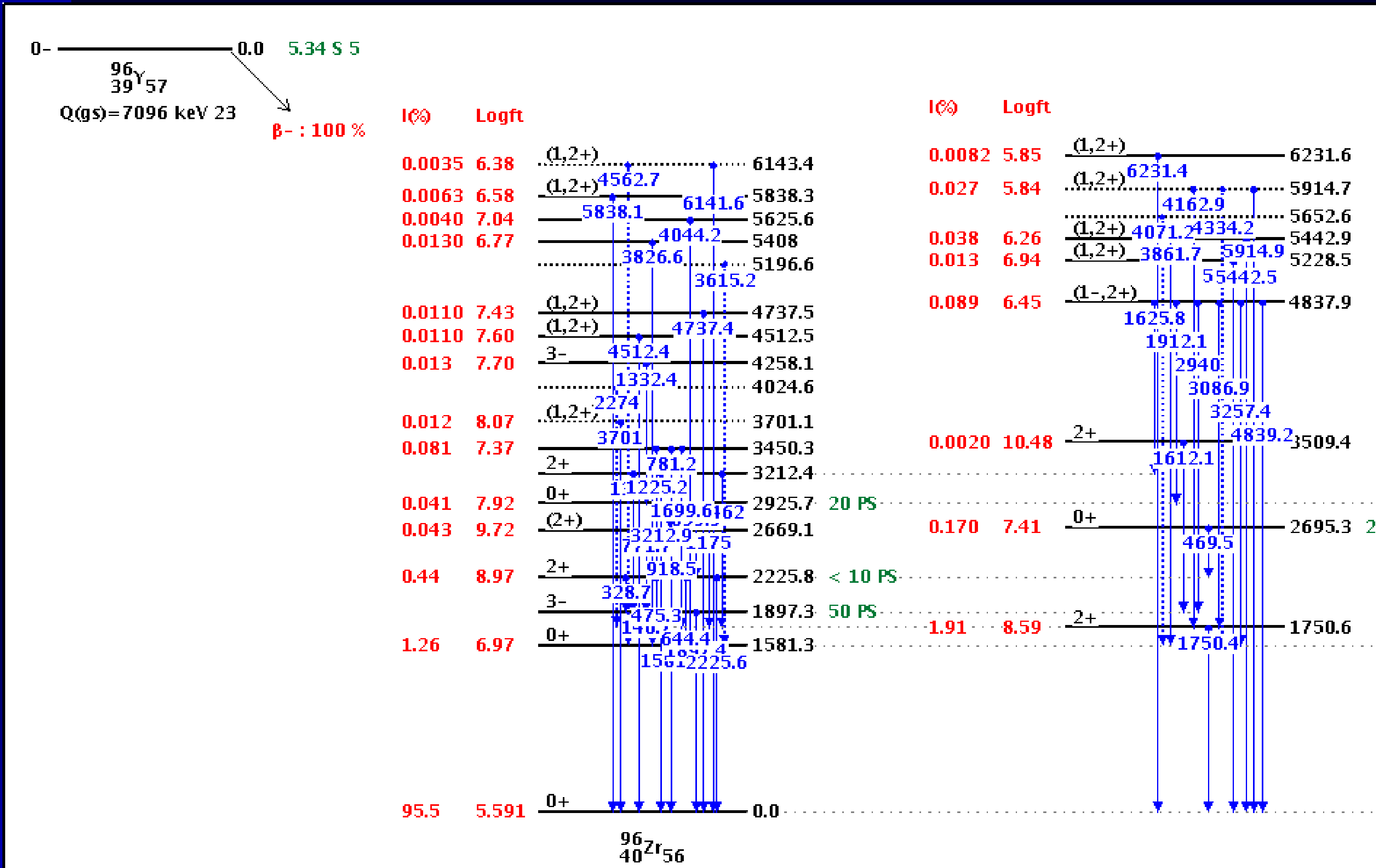
For a single branch energy conservation implies a one-to-one correspondence between β and $\bar{\nu}$ spectrum.

However, here there are about 500 nuclei and 10 000 individual β-branches involved; many are far away from stability.

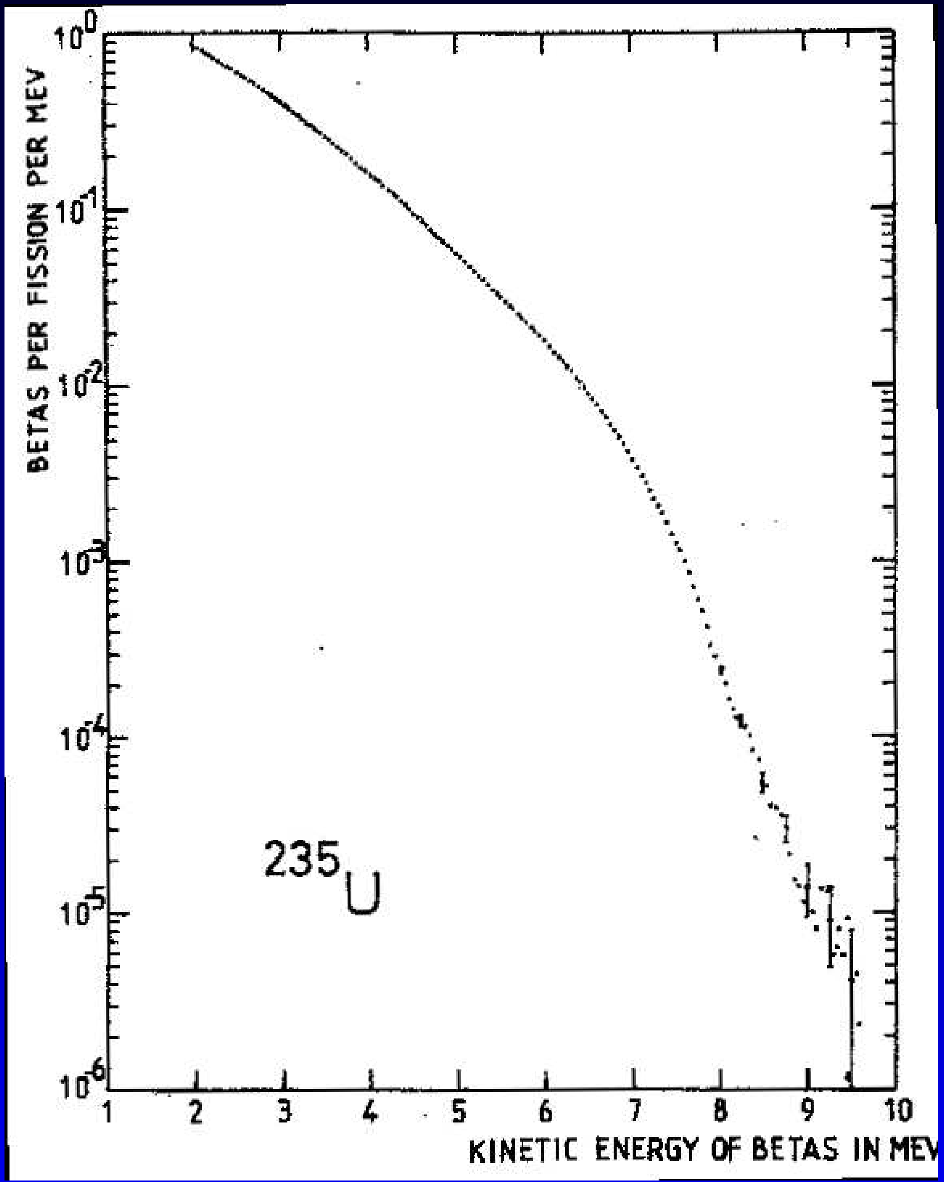
Direct β spectroscopy of single nuclei never will be complete, and even then one has to untangle the various branches

γ spectroscopy yields energy levels and branching fractions, but with limitations, *cf.* pandemonium effect

β branches



β-spectrum from fission



Schreckenbach, *et al.* 1985.

^{235}U foil inside the High Flux Reactor at ILL

Electron spectroscopy with a magnetic spectrometer

Same method used for ^{239}Pu and ^{241}Pu

For ^{238}U reliance on the theory – small contribution to overall neutrino spectrum

Extraction of ν-spectrum

The total β-spectrum is a sum of all decay branches

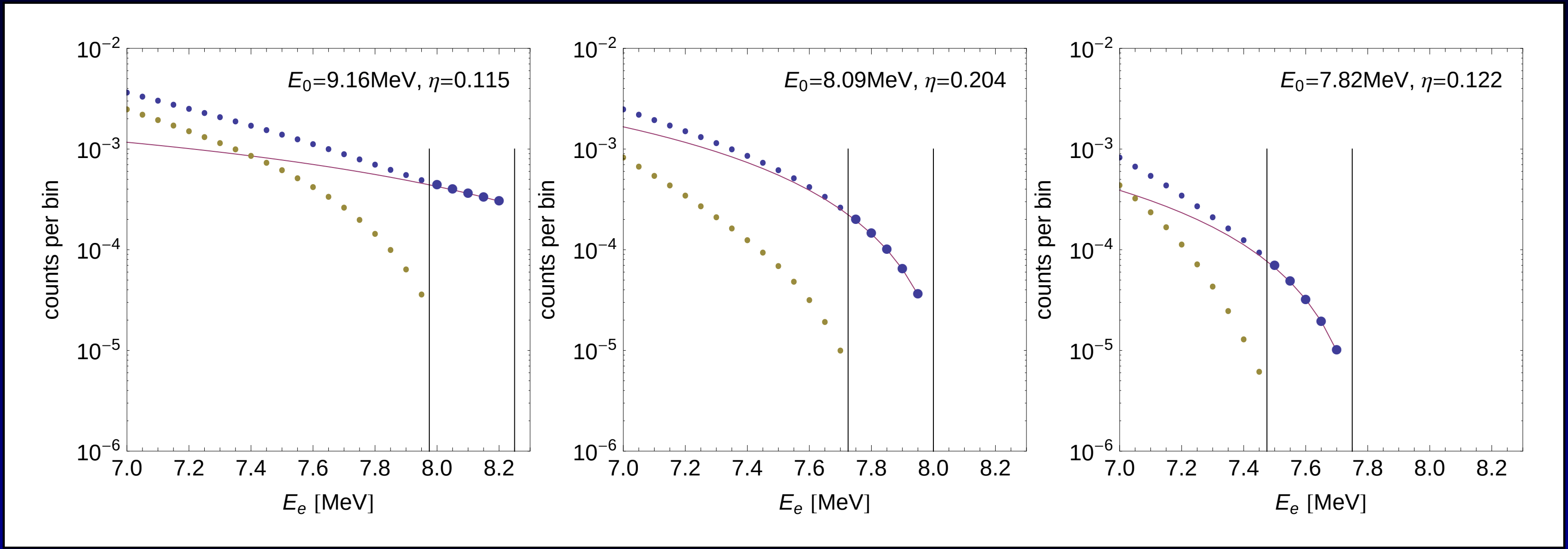
$$\mathcal{N}_\beta(E_e) = \int dE_0 N_\beta(E_e, E_0; \bar{Z})\, \eta(E_0)\,.$$

with $\bar{Z}$ effective nuclear charge and $\eta(E_0)$, the underlying distribution of endpoints

This is a so called Fredholm integral equation of the first kind – mathematically ill-posed, *i.e.* solutions tend to oscillate, needs regulator.

This approach is the basis for "virtual branches" Schreckenbach *et al.*, 1982, 1985, 1989 and is used in the modern calculations as well Mueller *et al.* 2011, Huber 2011

Virtual branches



1 – fit an allowed β-spectrum with free normalization η and endpoint energy E_0 the last s data points

2 – delete the last s data points

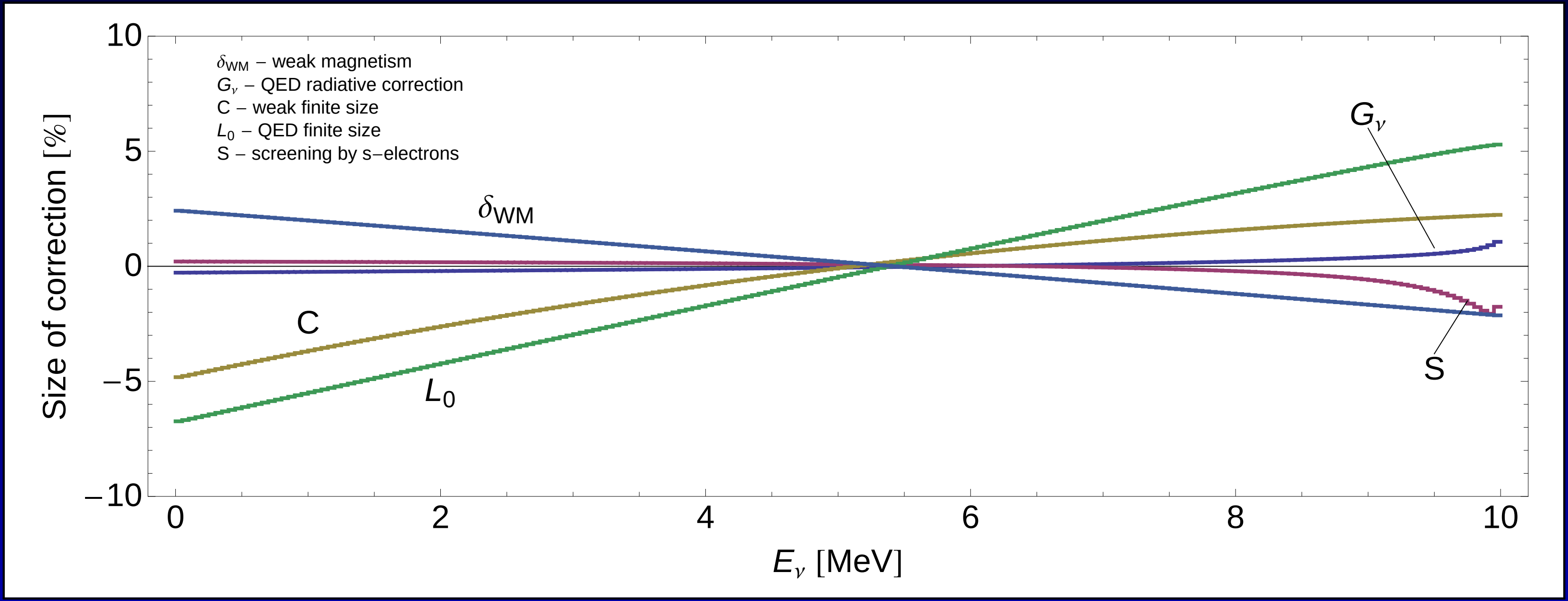
3 – subtract the fitted spectrum from the data

4 – goto 1

Invert each virtual branch using energy conservation into a neutrino spectrum and add them all. *e.g.* Vogel, 2007

Corrections to β-shape

There are numerous correction to the β-spectrum



Many of these correction depend on the nuclear charge Z, but Z is not determined by the β-spectrum measurement $\Rightarrow$ nuclear databases.

Effective nuclear charge

In order to compute all the QED corrections we need to know the nuclear charge Z of the decaying nucleus.

Using virtual branches, the fit itself cannot determine Z since many choices for Z will produce an excellent fit of the β-spectrum

$\Rightarrow$ use nuclear database to find how the average nuclear charge changes as a function of E_0, this is what is called effective nuclear charge $\bar{Z}(E_0)$.

Weigh each nucleus by its fission yield and bin the resulting distribution in E_0 and fit a second order polynomial to it.

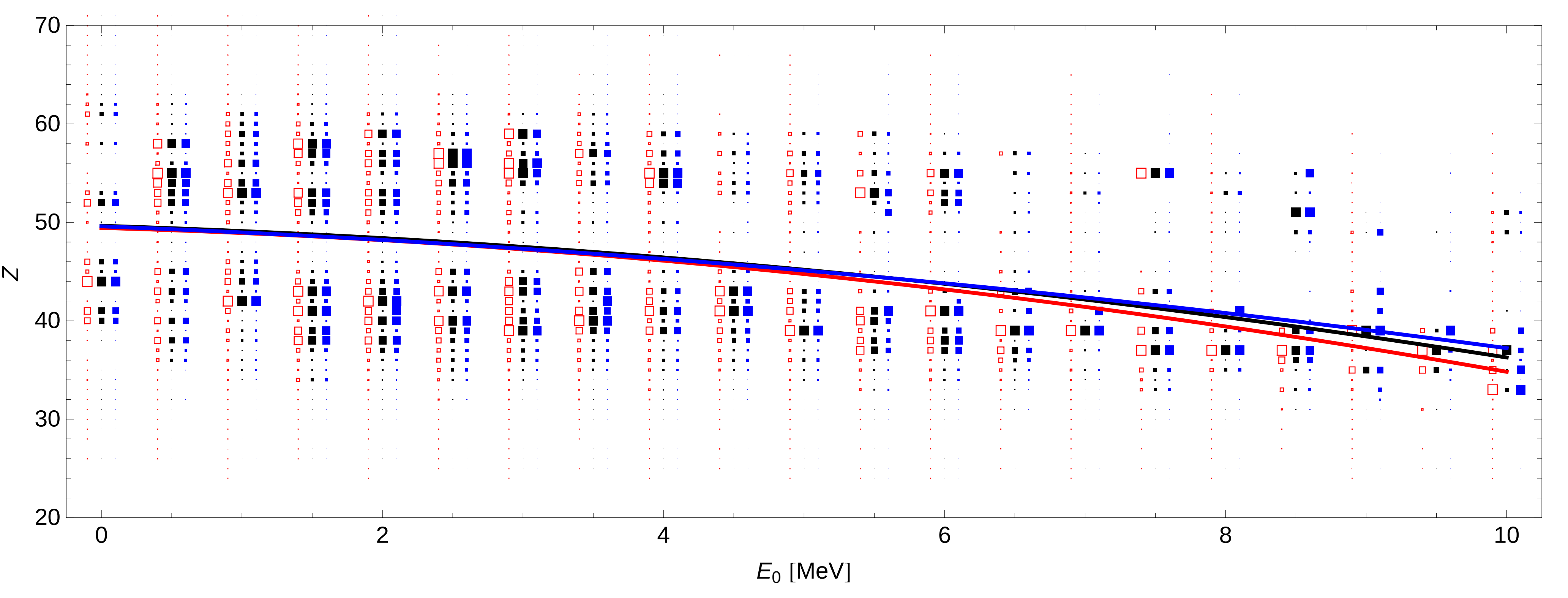
Effective nuclear charge

The nuclear databases have two fundamental shortcomings

- they are incomplete – for the most neutron-rich nuclei we only know the $Q_{gs\to gs}$, *i.e.* the mass differences
- they are incorrect – for many of the neutron-rich nuclei, γ-spectroscopy tends to overlook faint lines and thus too much weight is given to branches with large values of E_0, aka pandemonium effect

Simulation using our synthetic data set: by removing a fraction of the most neutron-rich nuclei and/or by randomly distributing the decays of a given branch onto several branches with $0 < E_0 < Q_{gs\to gs}$.

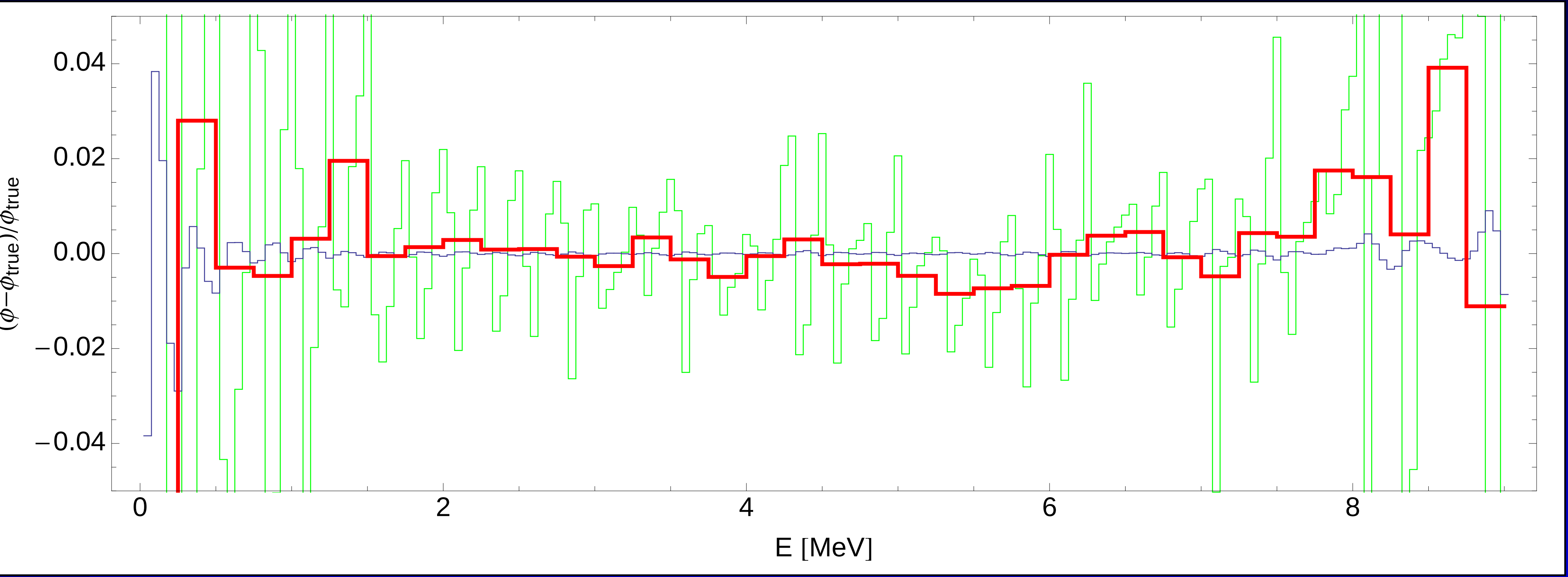
Effective nuclear charge



Spread between lines – effect of incompleteness and incorrectness of nuclear database (ENSDF). Only place in this analysis, where database enters directly.

Bias

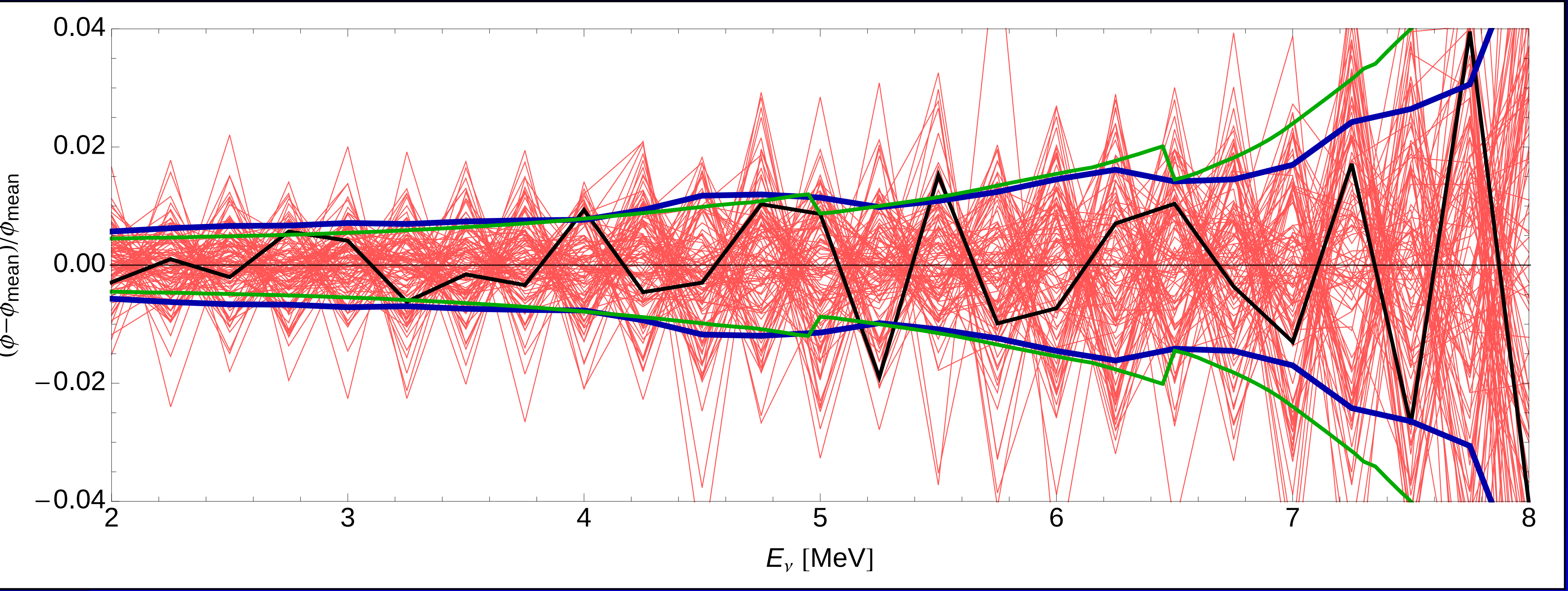
Use synthetic data sets derived from cumulative fission yields and ENSDF, which represent the real data within 10-20% and compute bias



Approximately 500 nuclei and 8000 β-branches.

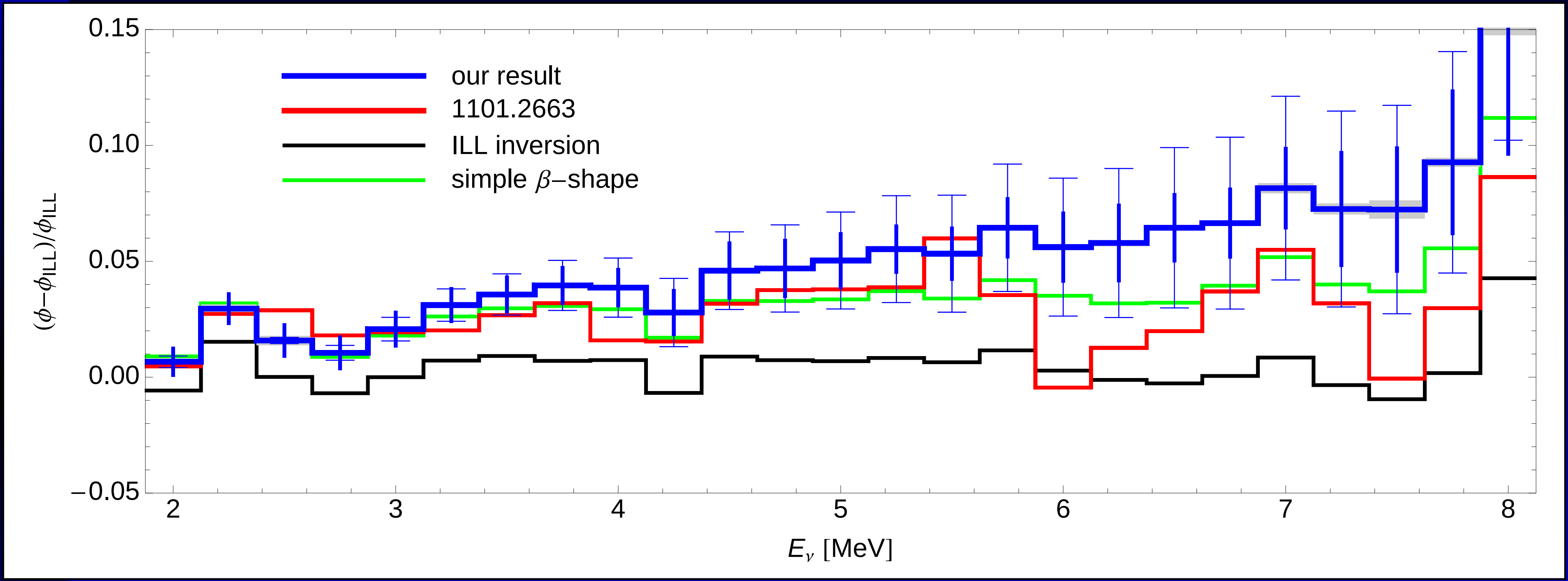
Statistical Error

Use synthetic data sets and fluctuate β-spectrum within the variance of the actual data.



Amplification of stat. errors of input data by factor 7.

Reactor antineutrino fluxes

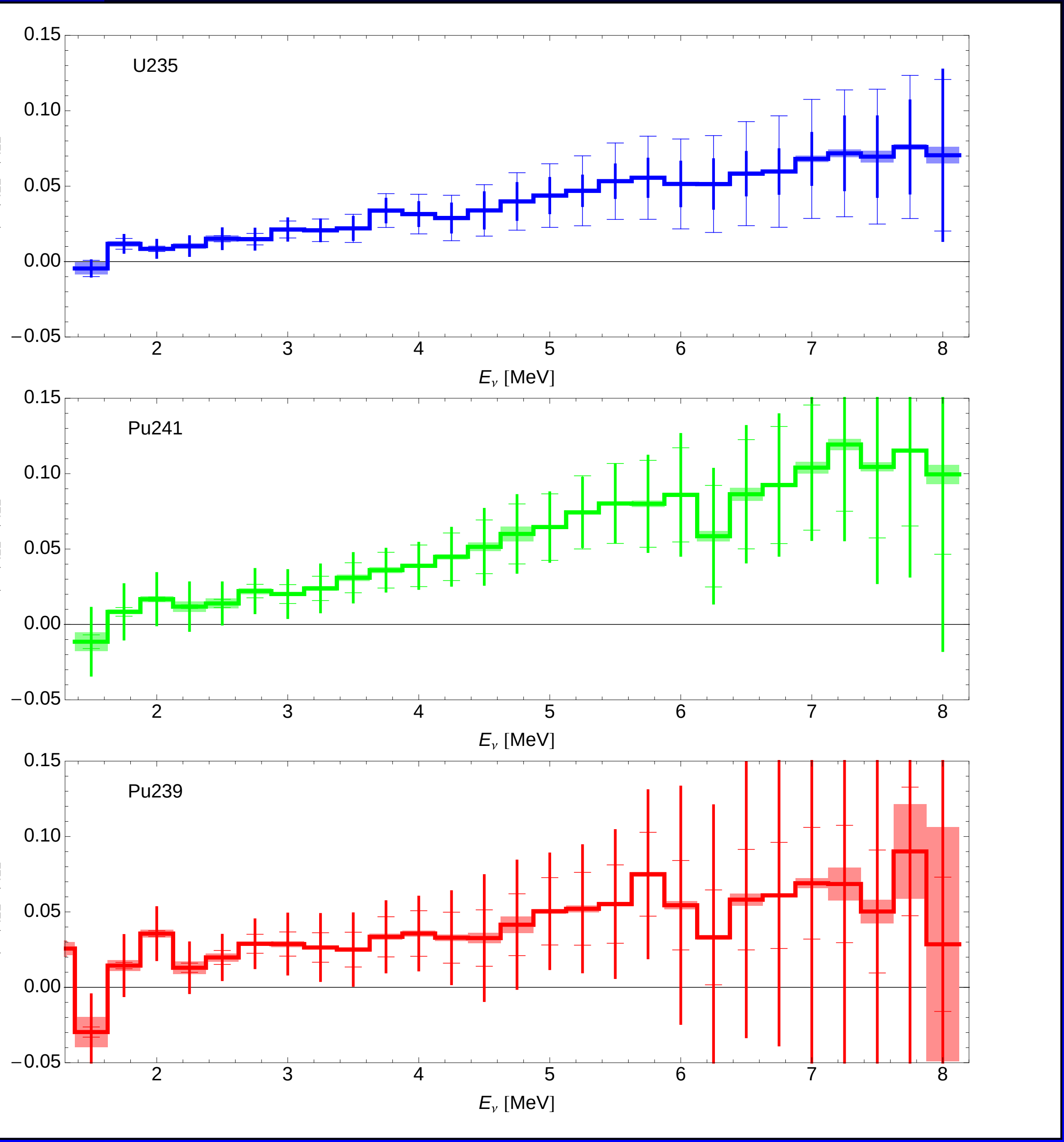


Shift with respect to ILL results, due to

a) different effective nuclear charge distribution

b) branch-by-branch application of shape corrections

Comparison of isotopes

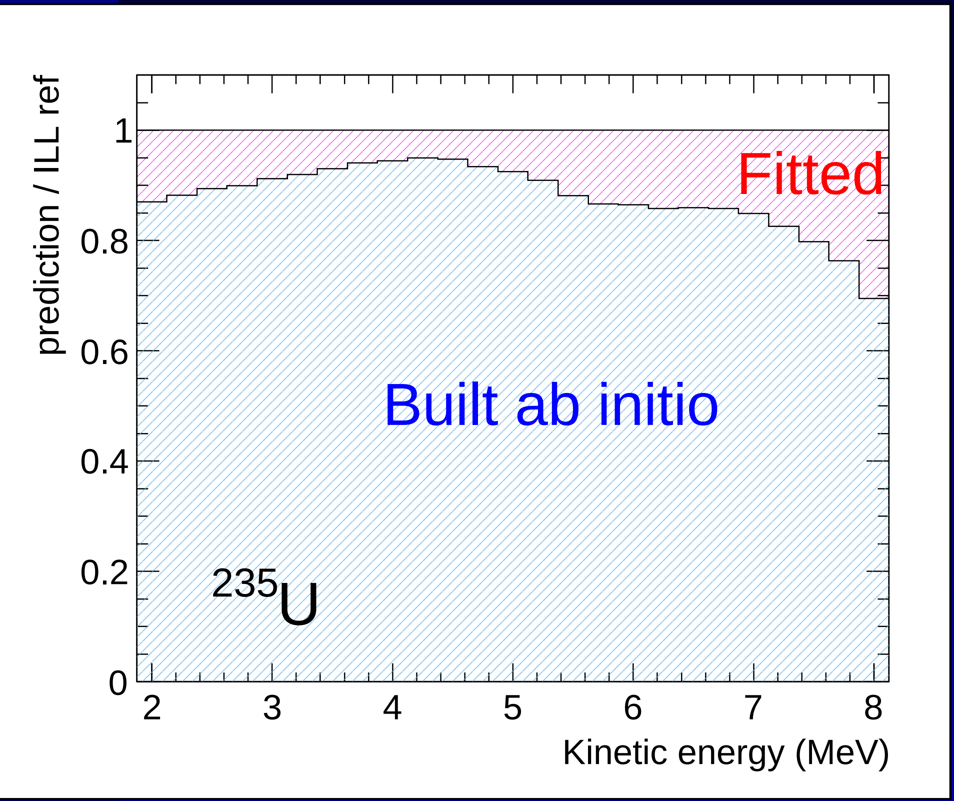


Same shift in all isotopes

Statistical errors of different size, direct consequence of different ILL data quality

^{239}Pu most problematic due to large fission fraction

From first principles?



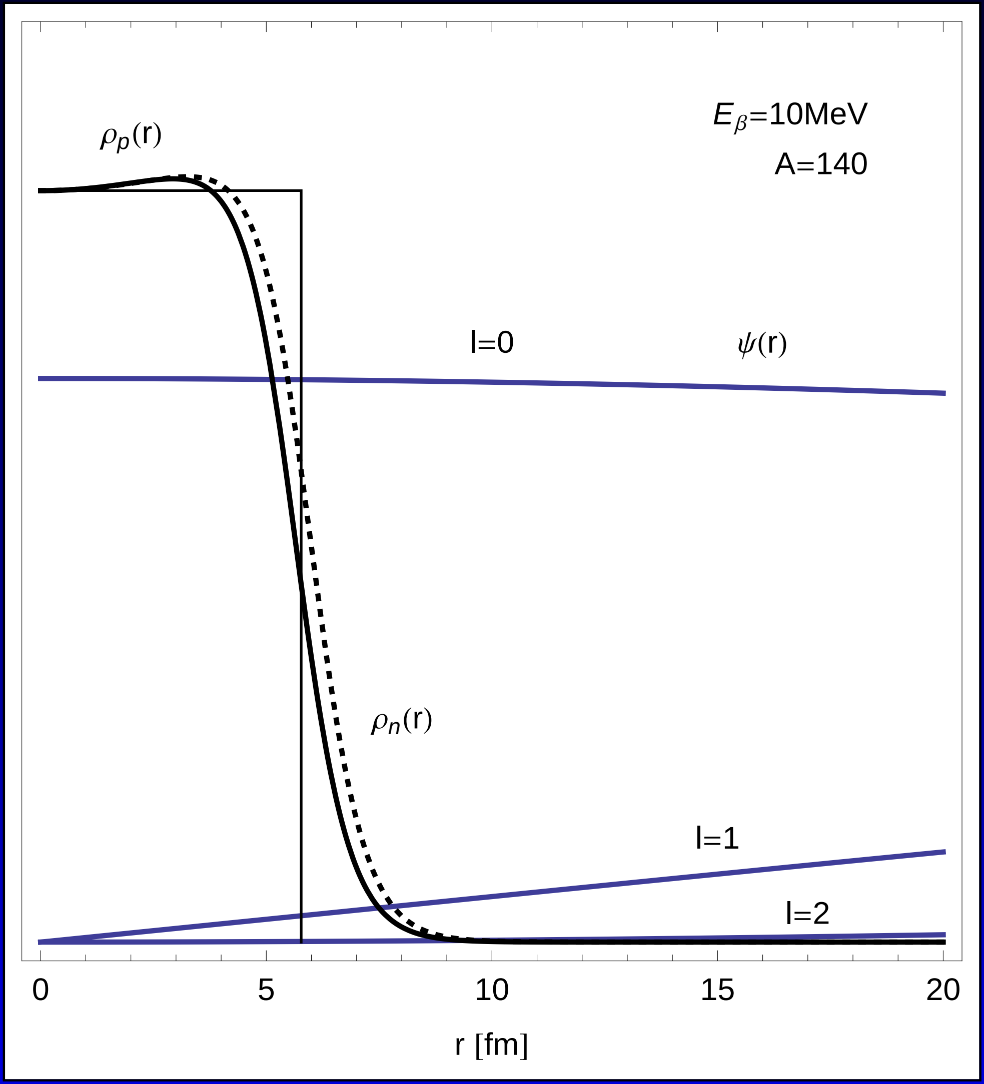
In Mueller *et al.*, Phys.Rev. **C83** (2011) 054615 an attempt was made to compute the neutrino spectrum from fission yields and information on individual β decay branches from databases.

The resulting cumulative β spectrum should match the ILL measurement.

About 10-15% of electrons are missing, Mueller *et al.* use virtual branches for that small remainder.

see talk by M. Fallot

Improving finite size effects



Shape effect for allowed decays presumably small

Not so small for forbidden decays

$\rho_p(r) \neq \rho_n(r)$ effects?

All this can be done with numerics

Industrial structure calculations

If we knew the nuclear wavefunction of parent and daughter we could compute everything we need to know.

On the other hand we do not need to compute the whole β-spectrum from scratch, we just want to know the size of certain corrections like WM. Therefore, an approximate wave function may be all that is needed.

Question: Is there a technology to perform approximate (!) calculations of nuclear wave functions which can be automatized?

Future neutrino measurements

The Daya Bay near detectors collect about 1M events per year

They do this at a distance where all sterile oscillations are averaged away – no confusion between nuclear physics and new physics

Daya Bay detectors are nearly completely active volume – nonetheless the events the acrylic vessel have an impact on the energy response

Daya Bay surface to volume ratio much smaller than for any of the new experiments (and Daya Bay has a γ-catcher)

see talk by K. Heeger

Future β-measurements

To my knowledge we can expect a ^{238}U spectrum soon from a group working at the FRM II in Germany – important to reduce reliance on a priori spectra

There is a proposal to trap Cf spontaneous fission products (CARIBU) in an ion trap and perform detailed β-spectroscopy for a group of isotopes (LLNL).

There is a proposal to use spallation neutrons to essentially redo the ILL measurements at FNAL Asner *et al.*, 1304.4205.

All these will provide useful information – quantitative impact depends very strongly on experimental accuracies and systematics!

Open issues

- Reactors are complex neutrino sources – our current understanding is at the 2-5% level
- New data will have to have systematics around 1% or better to make a real difference
- The Daya Bay data set will remain a benchmark which we need to exploit to its fullest
- Low energy, total rate neutrino measurements may offer a robust tool PH, AAP2012
- Pushing into the 1-2% region (or below) will require better theory…