

# Towards a microscopic theory for low-energy heavy-ion reactions

*Role of internal degrees of freedom  
in low-energy nuclear reactions*

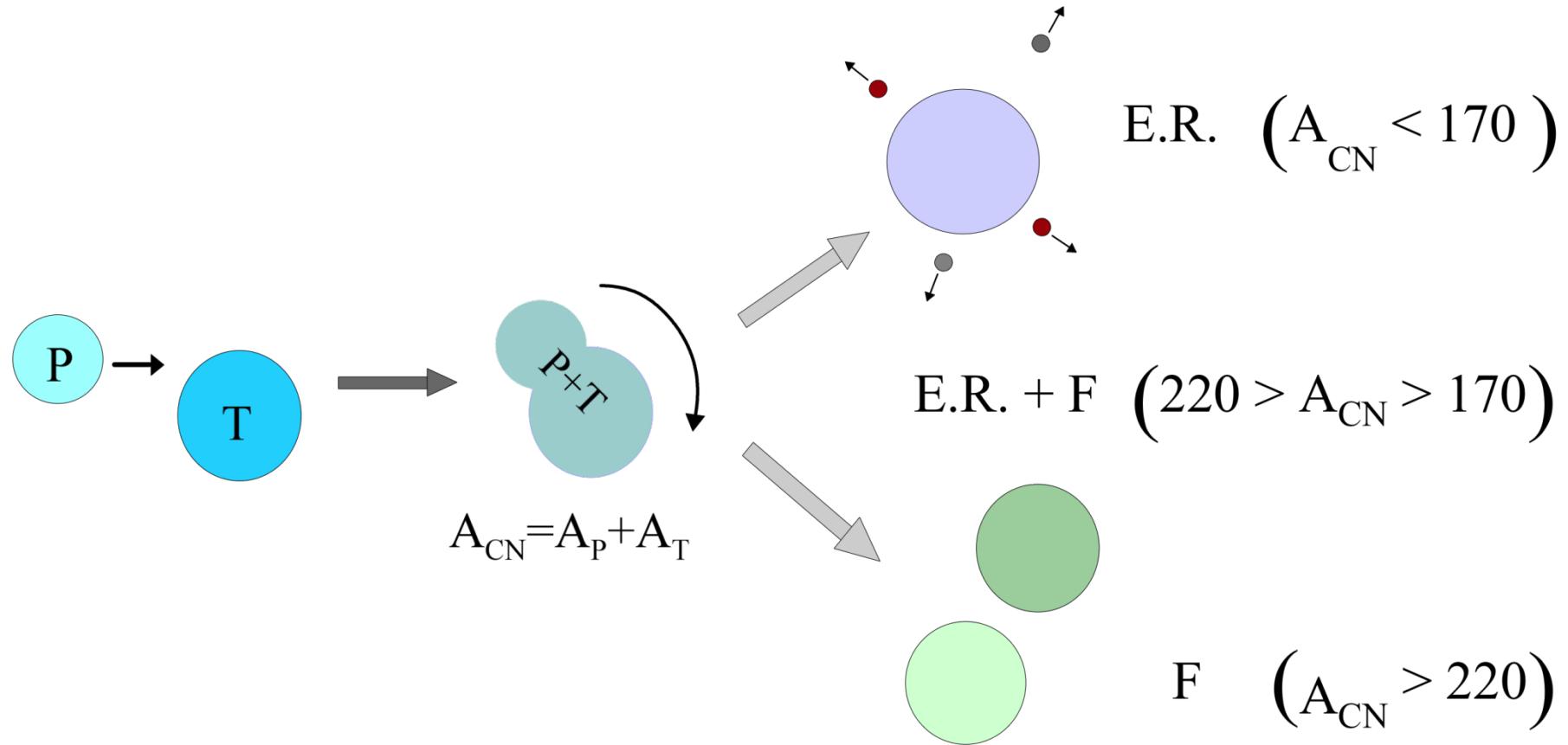


Kouichi Hagino (Tohoku University)

1. *Introduction: Environmental Degrees of Freedom*
2. *Application of RMT to subbarrier fusion*
3. *Discussions: Towards a microscopic theory for low-energy heavy-ion reactions*
4. *Summary*

# Introduction

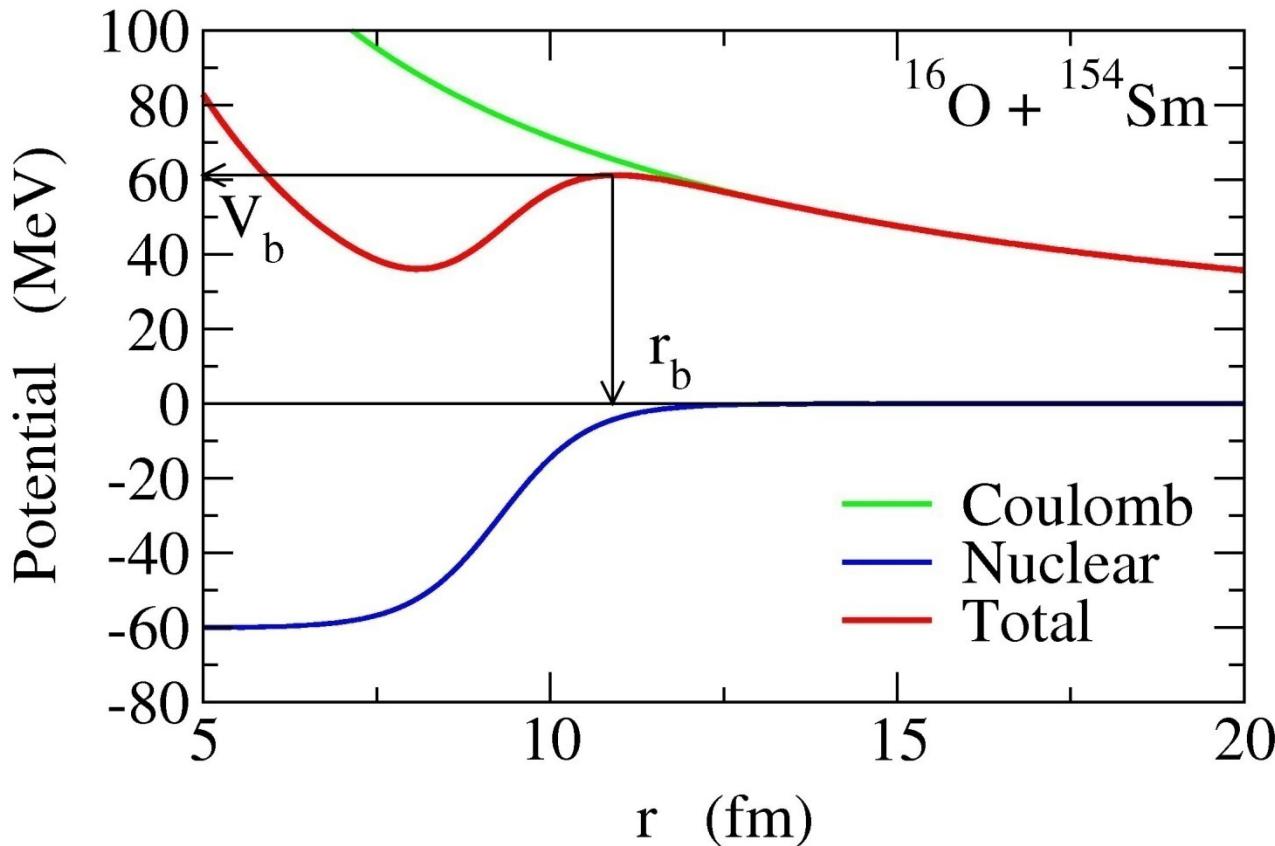
## Fusion: compound nucleus formation



Recent review:

K. Hagino and N. Takigawa,  
Prog. Theo. Phys. 128 (2012) 1061

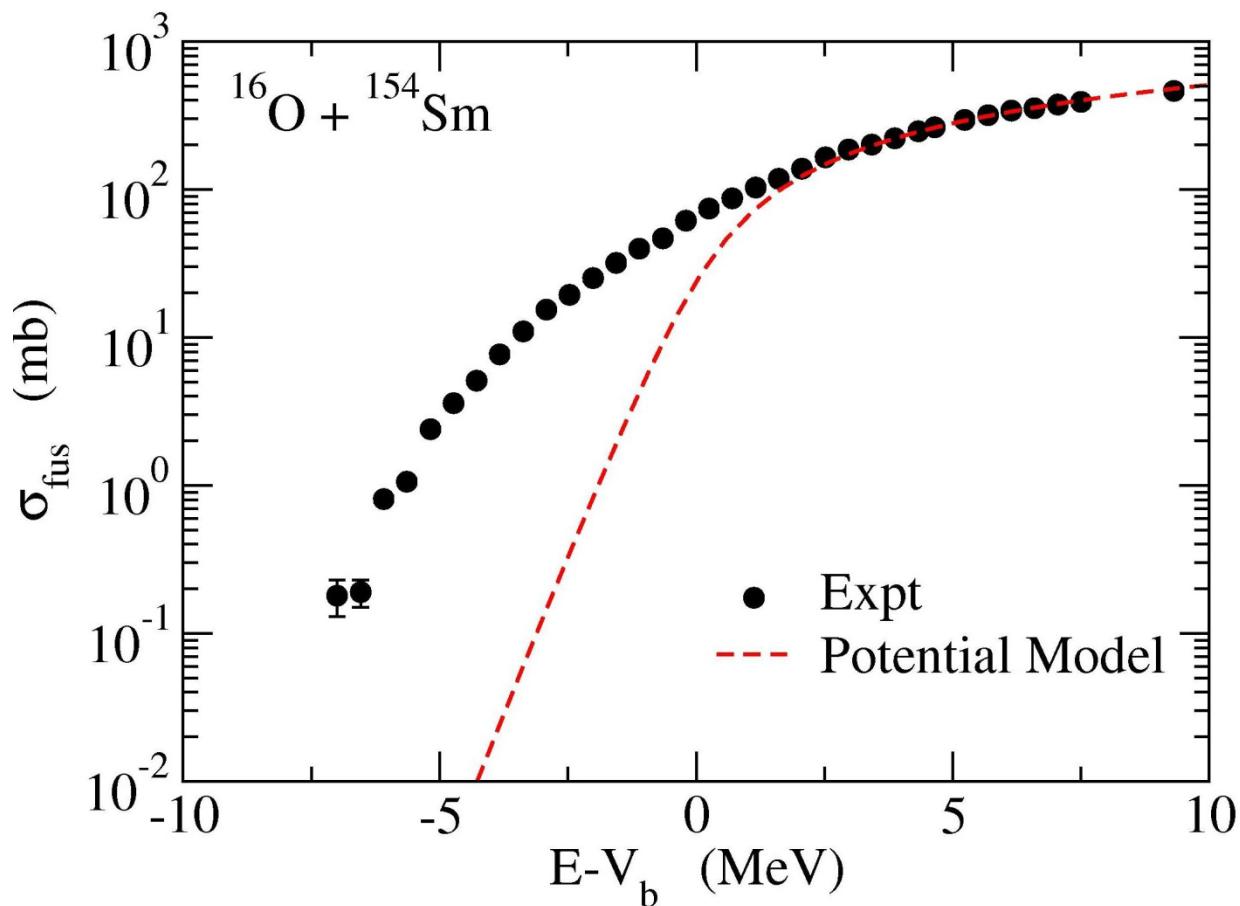
courtesy: Felipe Canto



the simplest approach to fusion cross sections: [potential model](#)

$$\sigma_{\text{fus}}(E) = \frac{\pi}{k^2} \sum_l (2l + 1) P_l(E)$$

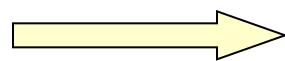
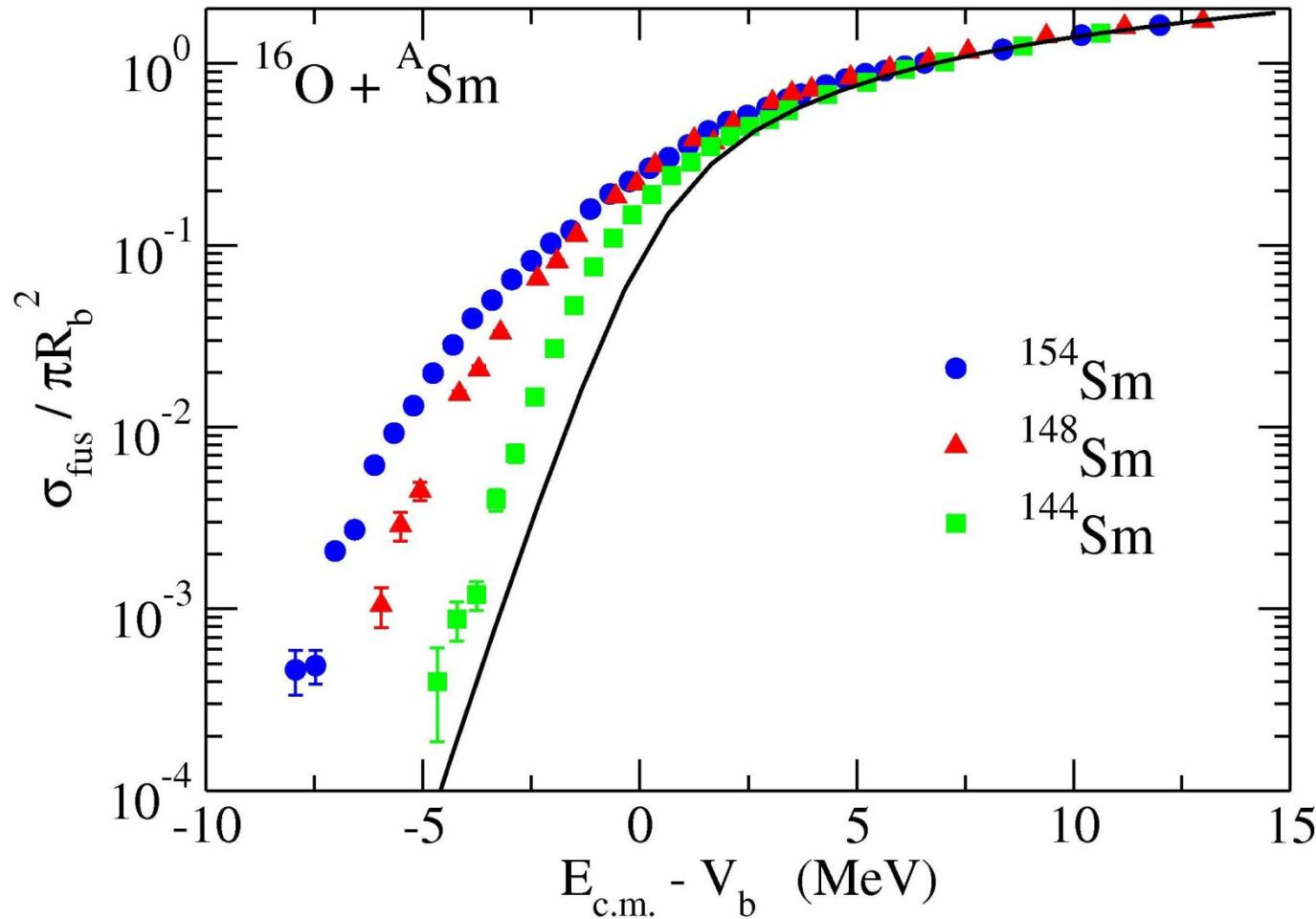
## Subbarrier fusion reactions



Potential model:  
Reproduces the data  
reasonably well for  
 $E > V_b$   
Underpredicts  $\sigma_{\text{fus}}$  for  
 $E < V_b$

cf. seminal work:

R.G. Stokstad et al., PRL41('78)465  
PRC21('80)2427



Strong target dependence at  $E < V_b$

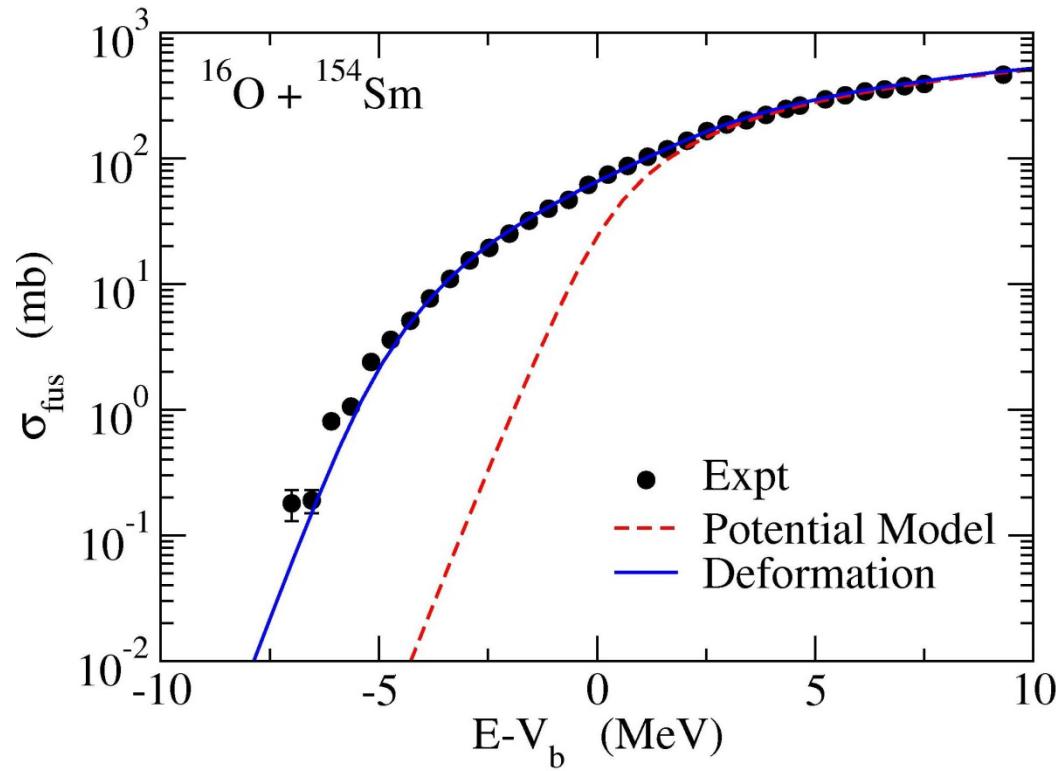
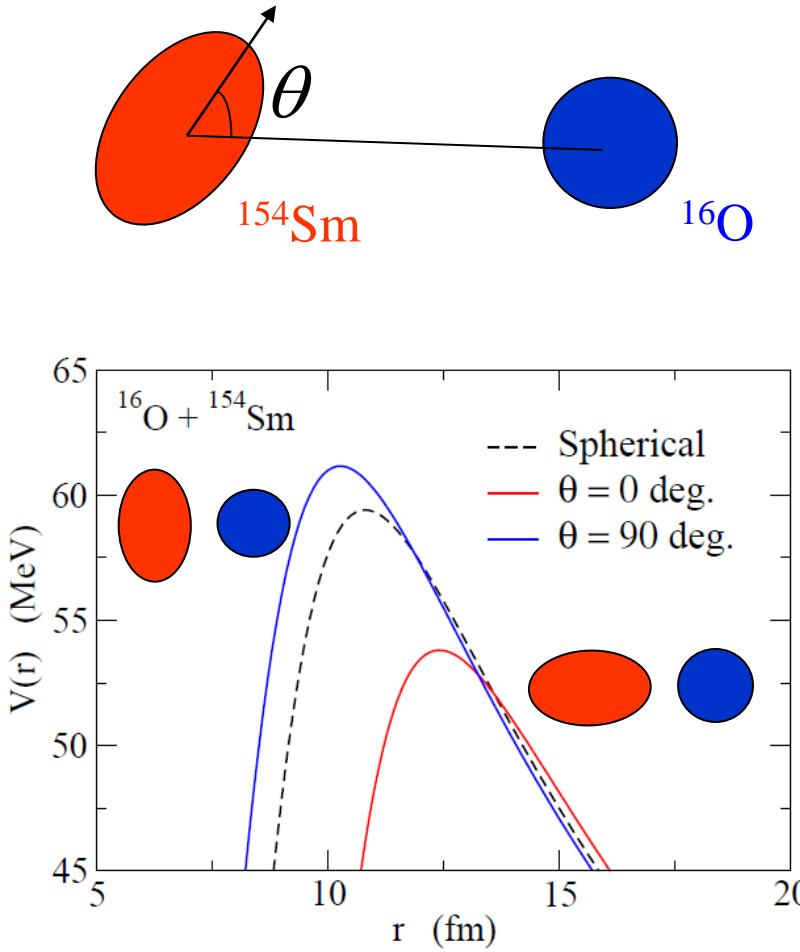


low-lying collective excitations

**Subbarrier fusion:**  
strong interplay between  
reaction and structure

coupled-channels equations

$$\rightarrow \sigma_{\text{fus}}(E) = \int_0^1 d(\cos \theta) \sigma_{\text{fus}}(E; \theta)$$

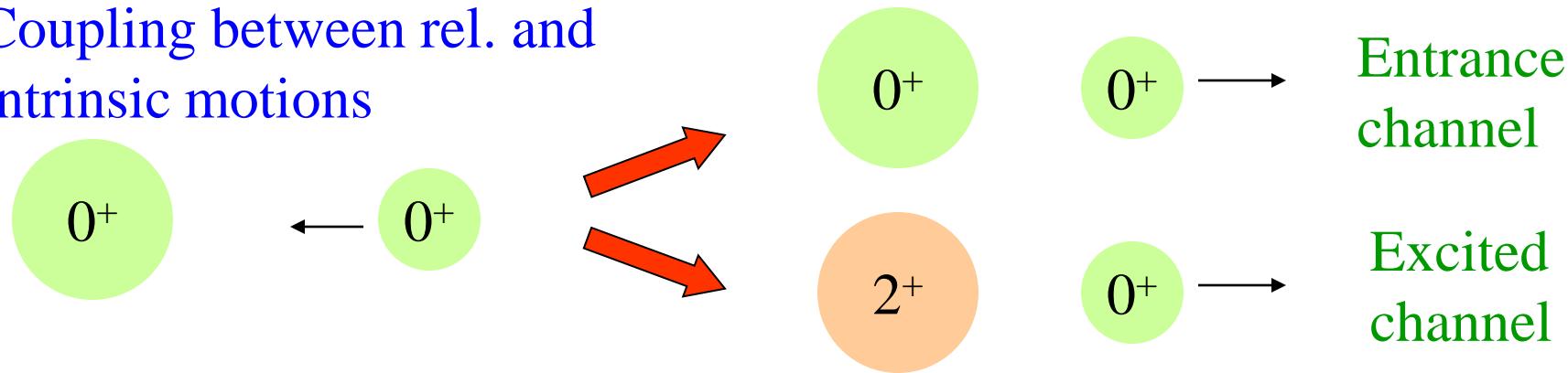


**Def. Effect:** enhances  $\sigma_{\text{fus}}$  by a factor  
of  $10 \sim 100$

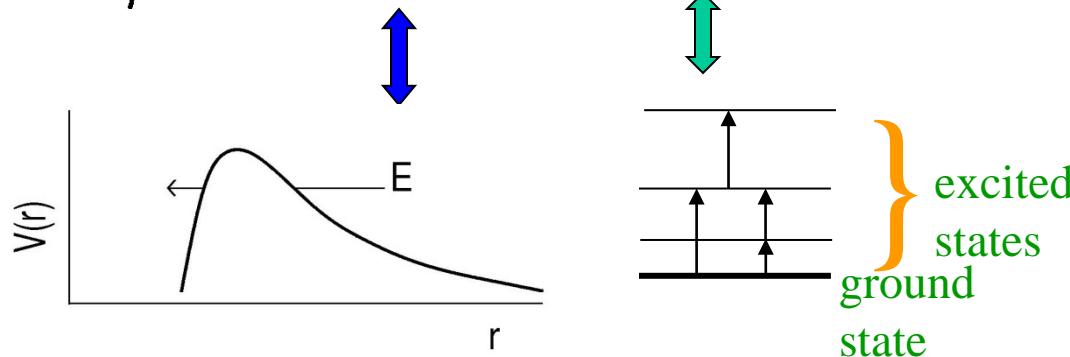
$\rightarrow$  **Fusion:** interesting probe for  
nuclear structure

# Coupled-Channels method

Coupling between rel. and intrinsic motions



$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + H_0(\xi) + V_{\text{coup}}(r, \xi)$$



$$H_0(\xi)\phi_k(\xi) = \epsilon_k \phi_k(\xi)$$

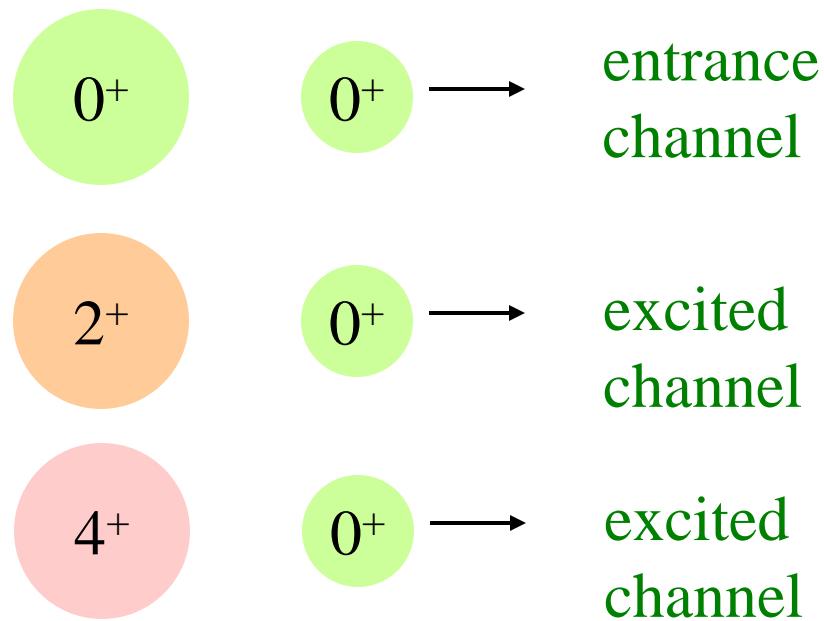
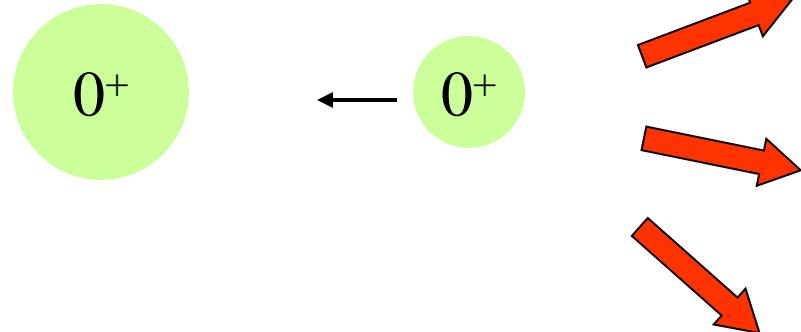
$$\Psi(r, \xi) = \sum_k \psi_k(r) \phi_k(\xi)$$



coupled Schroedinger  
equations for  $\psi_k(r)$

## Coupled-channels framework

Coupling between rel.  
and intrinsic motions



- Quantum theory which incorporates excitations in the colliding nuclei
- a few collective states (vibration and rotation) which couple strongly to the ground state + transfer channel
- several codes in the market: ECIS, FRESCO, CCFULL.....

→ has been successful in describing heavy-ion reactions

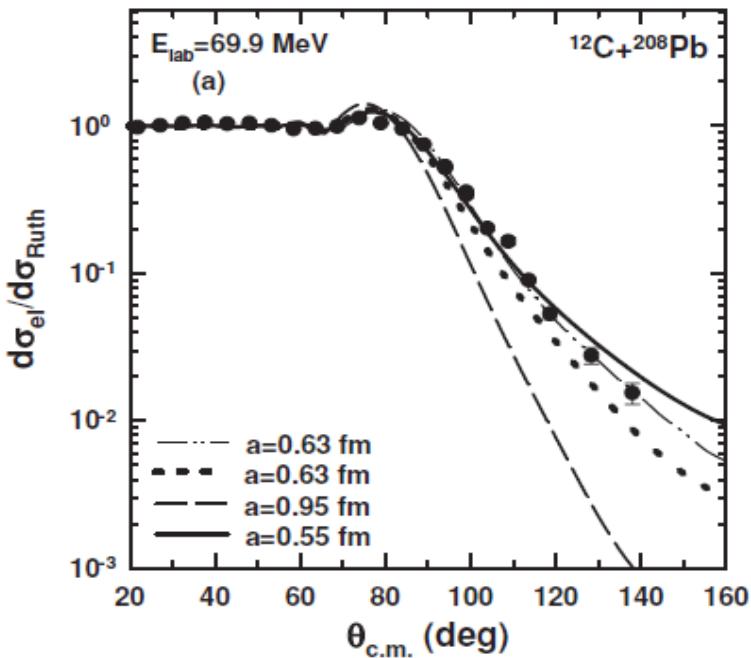
However, many recent challenges in C.C. calculations

# surface diffuseness anomaly

*Scattering processes:*

Double folding potential  
Woods-Saxon ( $a \sim 0.63$  fm)

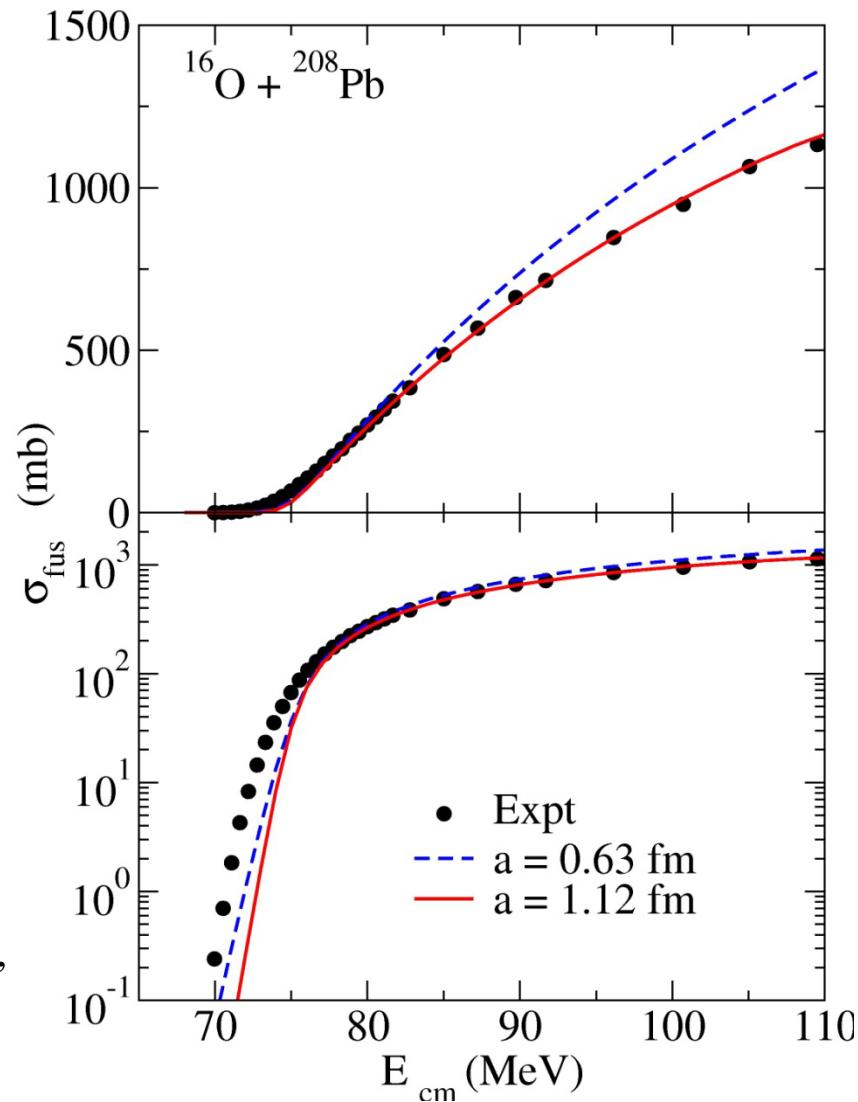
→ successful



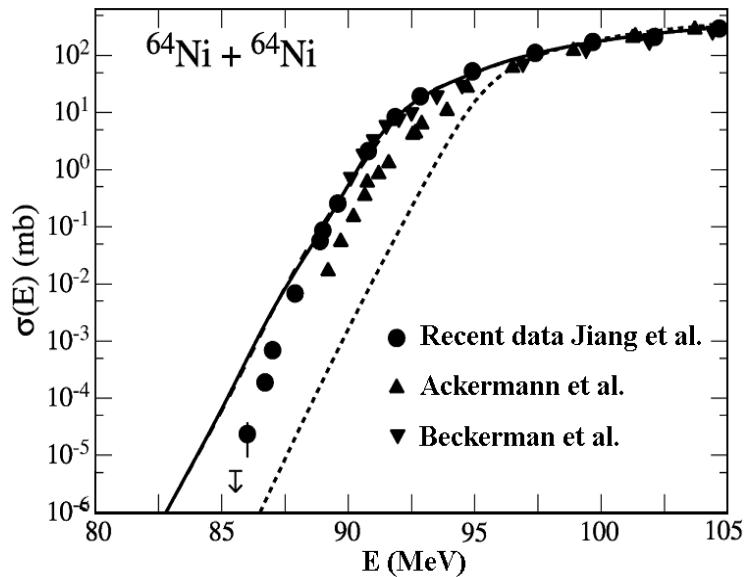
A. Mukherjee, D.J. Hinde, M. Dasgupta, K.H., et al.,  
PRC75('07)044608

*Fusion process: not successful*

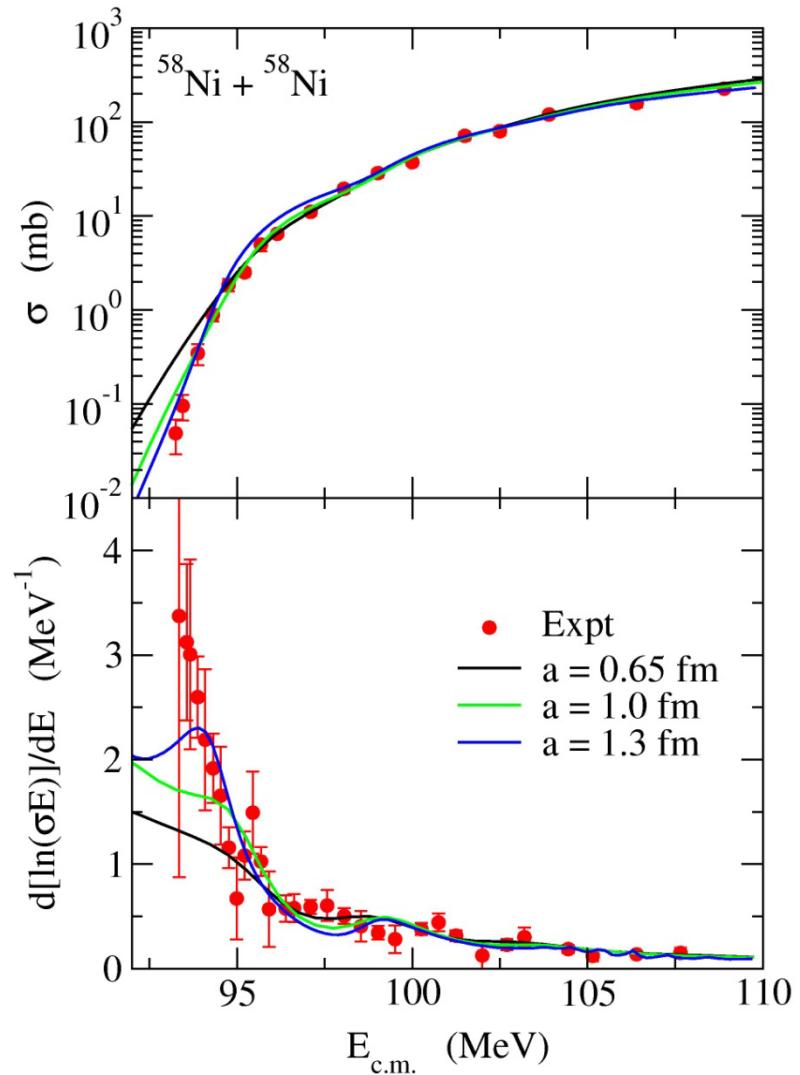
→  $a \sim 1.0$  fm required (if WS)



## Deep subbarrier fusion data

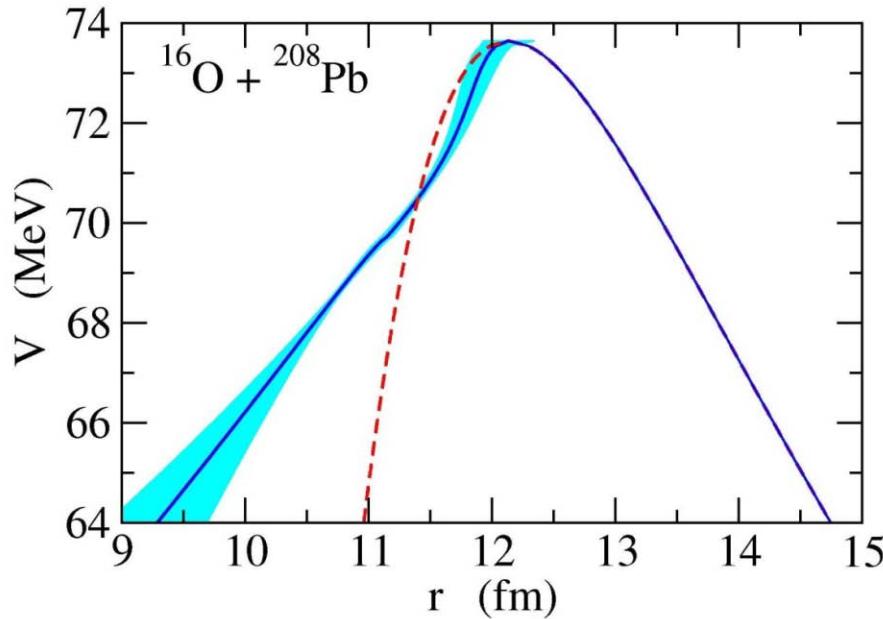


C.L. Jiang et al., PRL93('04)012701  
“steep fall-off of fusion cross section”



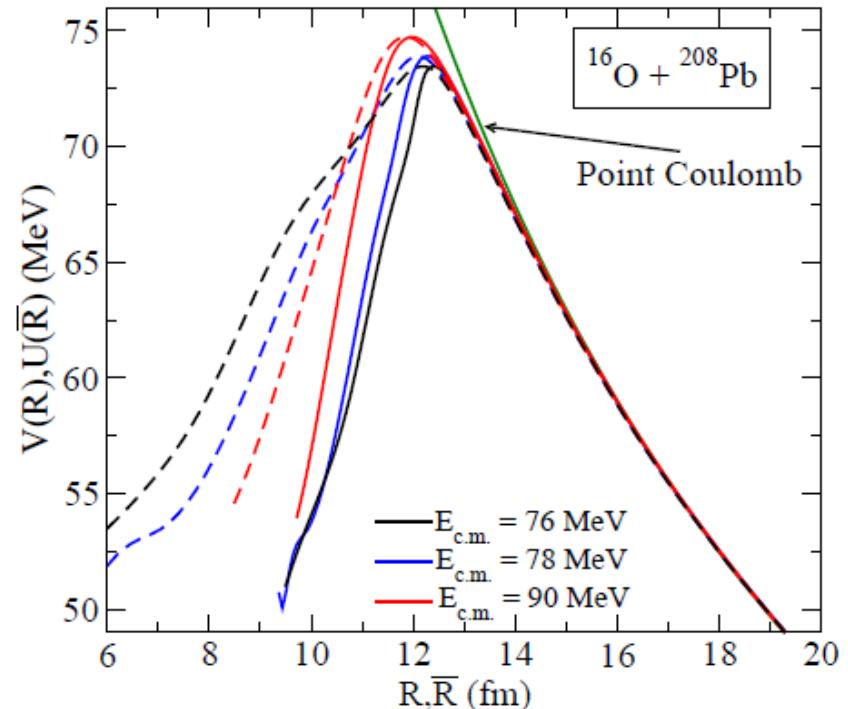
K. H., N. Rowley, and M. Dasgupta,  
PRC67('03)054603

# potential inversion with deep subbarrier data



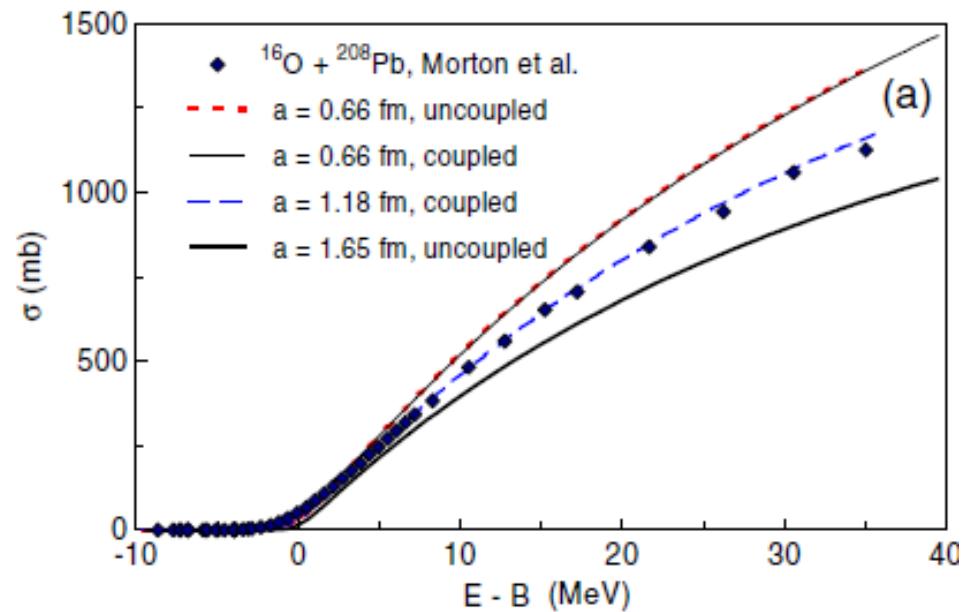
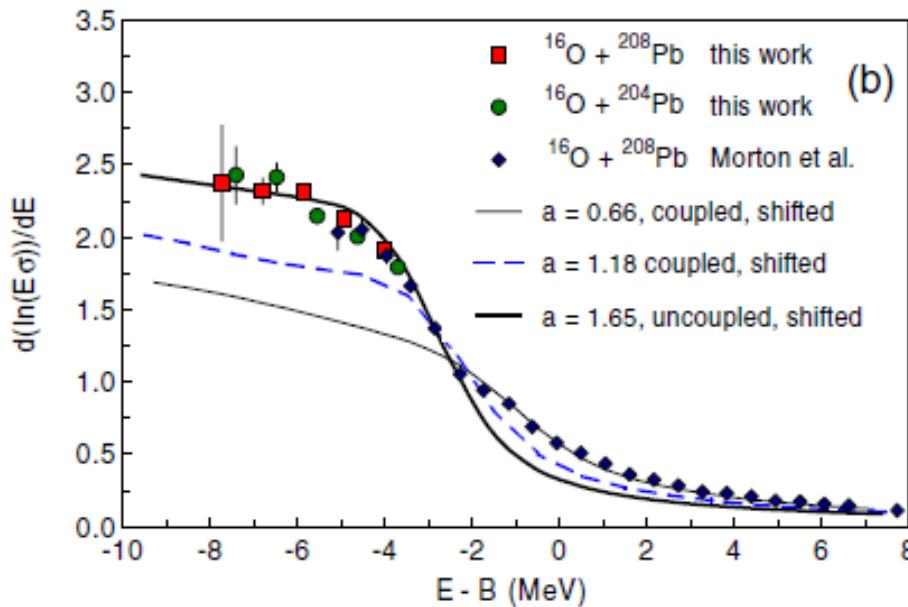
K.H. and Y. Watanabe,  
PRC76 ('07) 021601(R)

cf. Earlier work on potential inversion:  
A.B. Balantekin, S.E. Koonin, and  
J.W. Negele, PRC28('83)1565



cf. Density-Constrained TDHF:  
A.S. Umar and V.E. Oberacker,  
Euro. Phys. J. A39 ('09)243

## energy dependence of surface diffuseness parameter



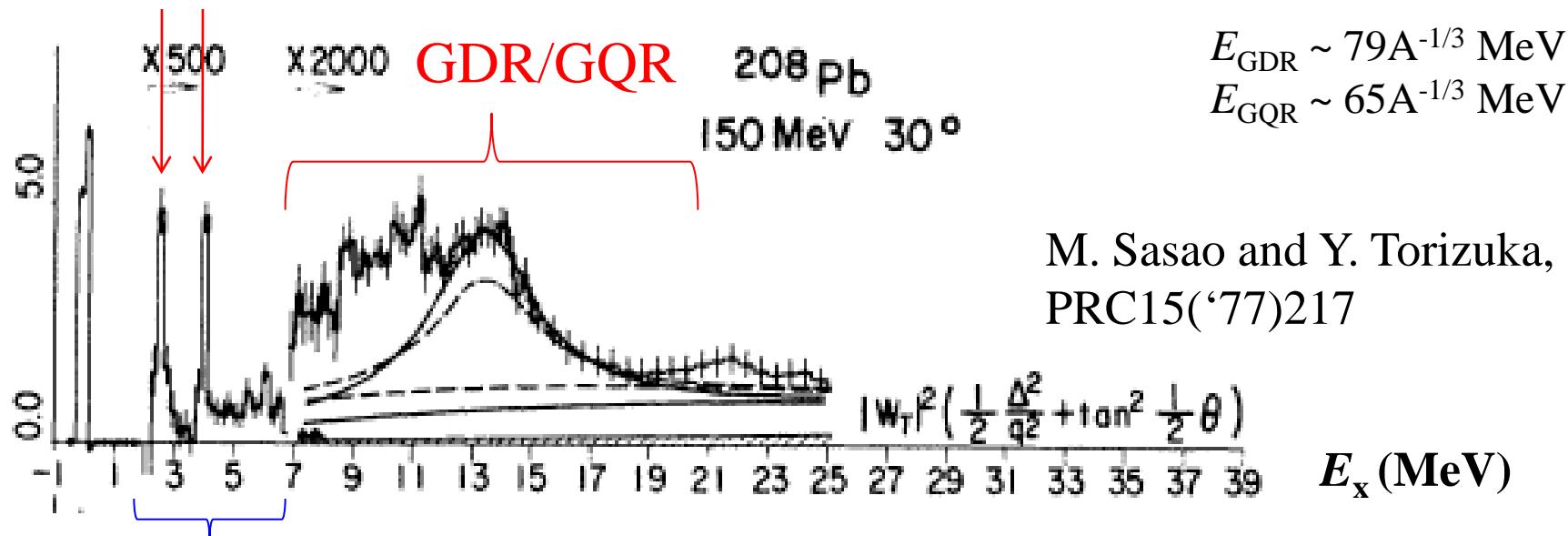
M. Dasgupta et al., PRL99('07)192701



- dynamical effects not included in C.C. calculation?
- energy and angular momentum dissipation?
- weak channels? ← this talk

## typical excitation spectrum: electron scattering data

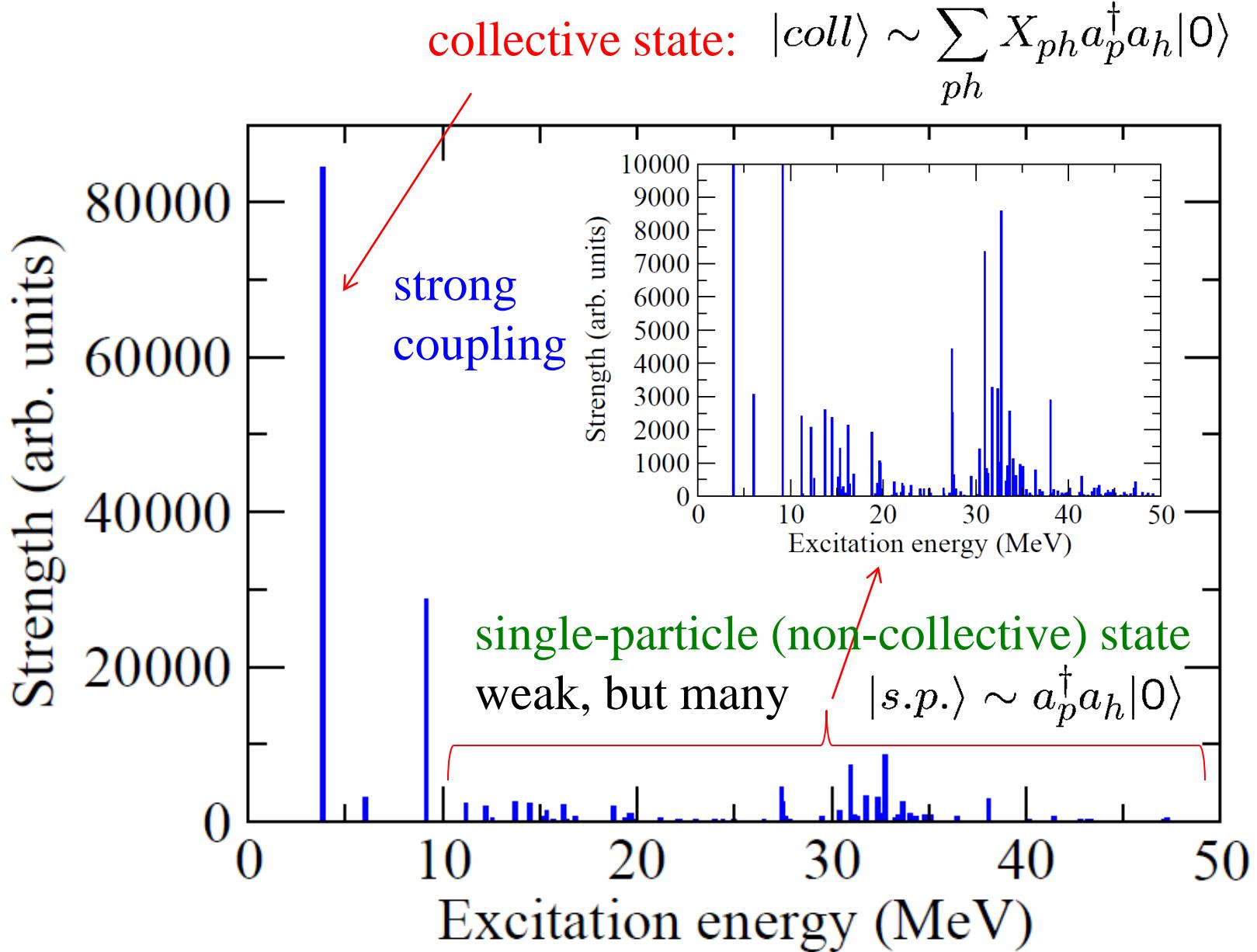
### low-lying collective excitations



### low-lying non-collective excitations

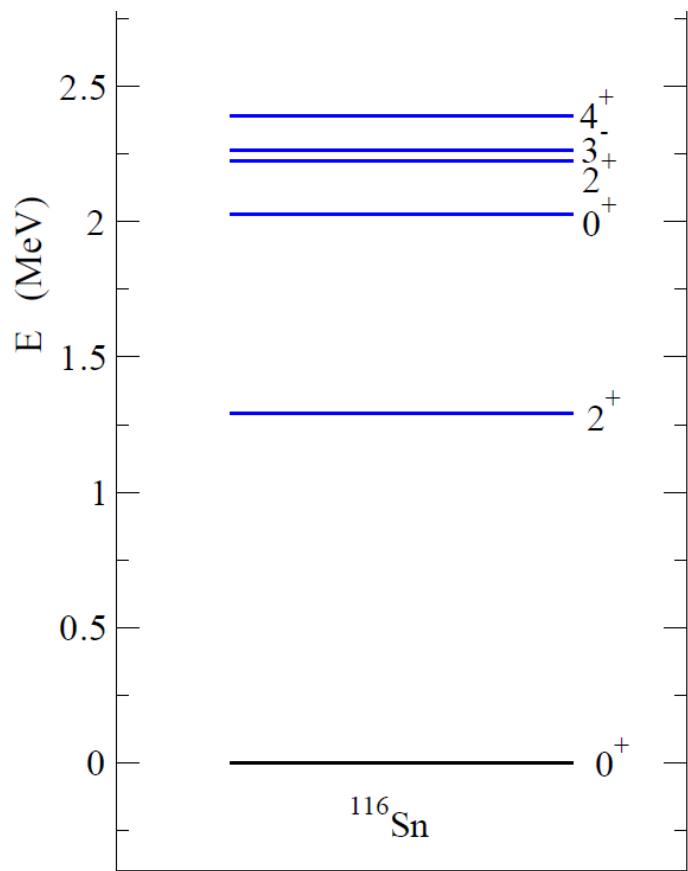
- Giant Resonances: high  $E_x$ , smooth mass number dependence  
→ adiabatic potential renormalization
- Low-lying collective excitations: barrier distributions,  
strong isotope dependence
- Non-collective excitations: either neglected completely or  
implicitly treated through an absorptive potential

# IS Octupole response of $^{48}\text{Ca}$ (Skyrme HF + RPA calculation: SLy4)



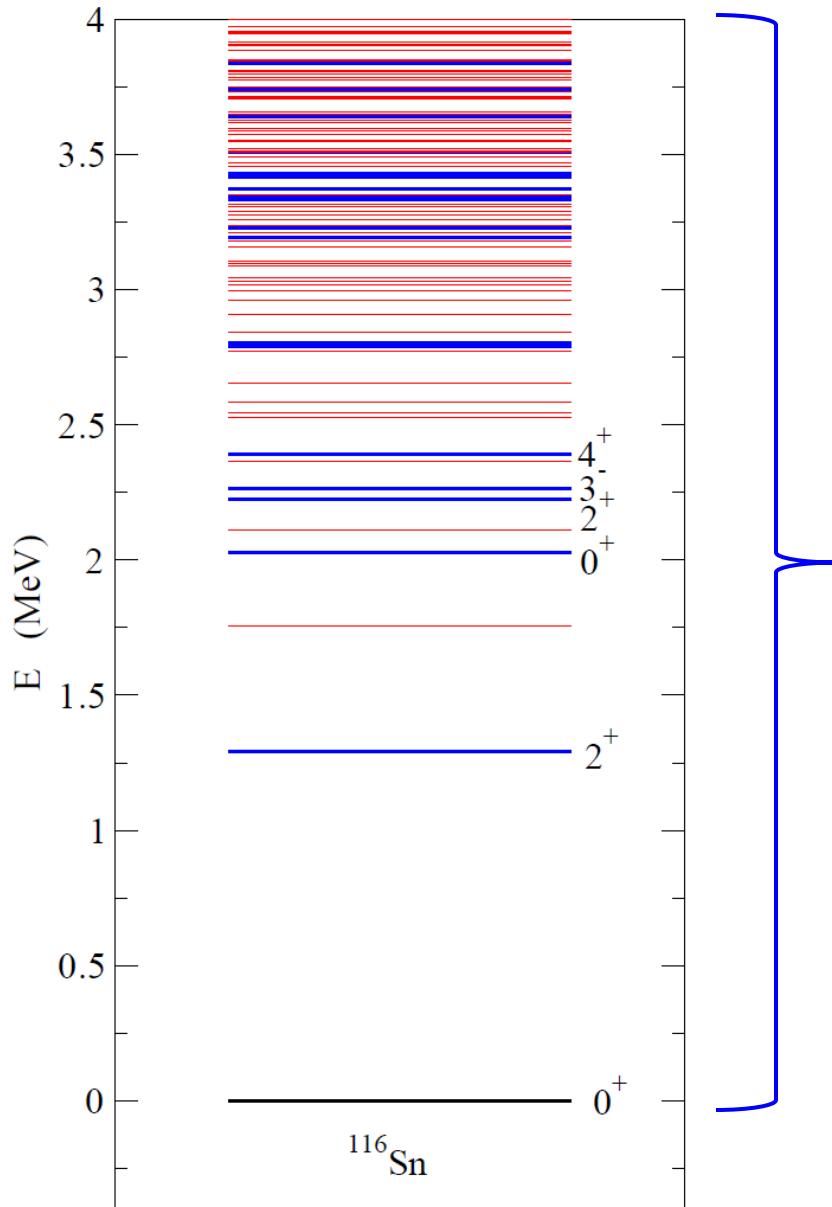
Our interest: couplings to (relatively) low-lying single-particle levels

e.g., collective levels in  $^{116}\text{Sn}$



model space in a typical  
C.C. calculation

# Our interest: couplings to (relatively) low-lying single-particle levels



112 levels up to 4.1 MeV  
(93 single-particle levels)  
nearly “complete” level scheme

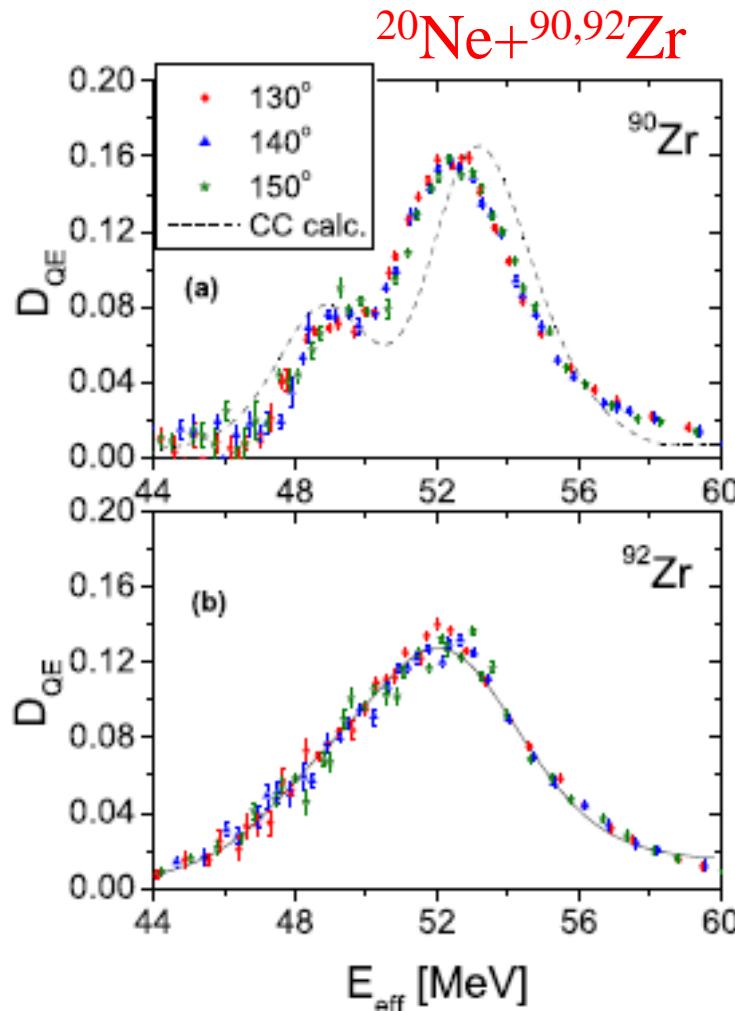
S. Raman et al.,  
PRC43('91)521



role of these s.p. levels in  
reaction dynamics?

## Indications of non-collective excitations

: a comparison between  $^{20}\text{Ne}+^{90}\text{Zr}$  and  $^{20}\text{Ne}+^{92}\text{Zr}$



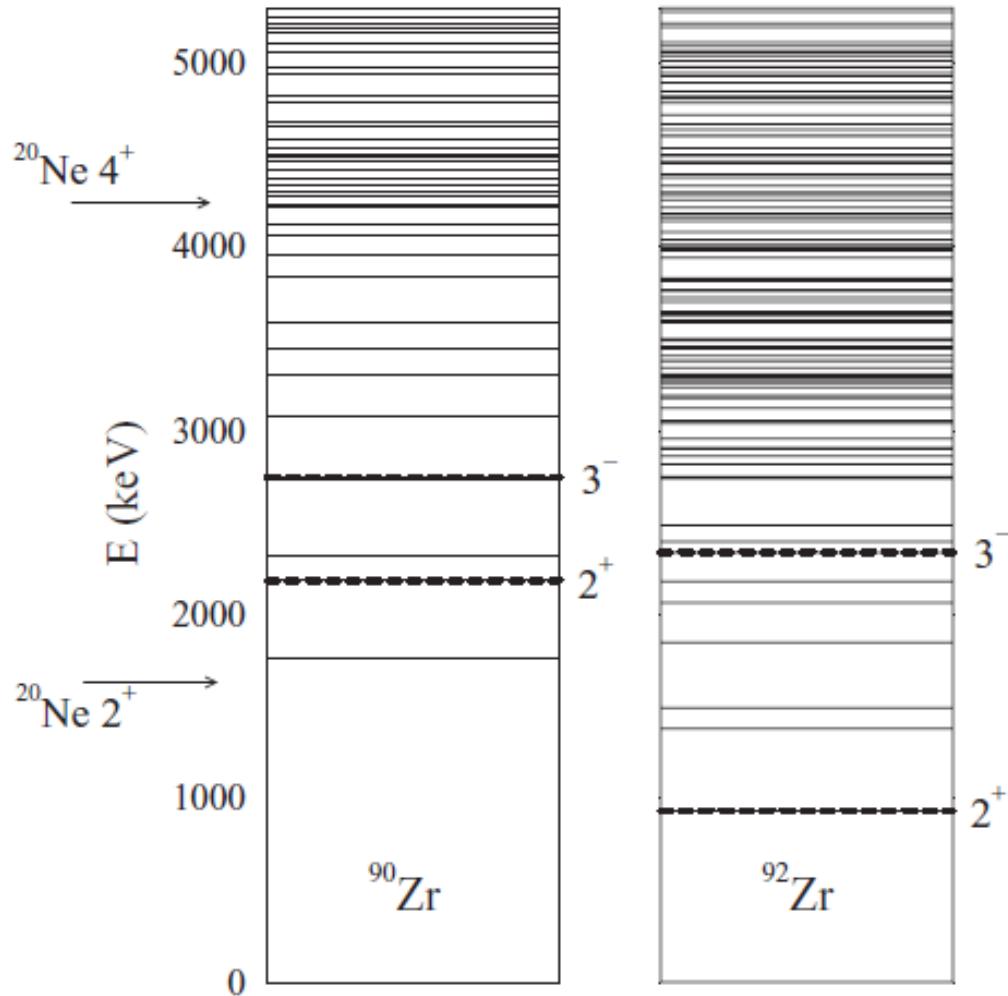
$$D_{\text{QEL}}(E) = -\frac{d}{dE} \left( \frac{\sigma_{\text{QEL}}(E, \pi)}{\sigma_R(E, \pi)} \right)$$

QEL = elastic + inelastic + transfer

- C.C. results are almost the same between the two systems
- Yet, quite different barrier distribution and Q-value distribution



non-collective excitations?



$^{90}\text{Zr}$  ( $Z=40$  sub-shell closure,  
 $N=50$  shell closure)  
 $^{92}\text{Zr} = ^{90}\text{Zr} + 2\text{n}$

a problem: the nature of non-collective states is  
 poorly known (the energy, spin, parity only)  
 i.e., no information on the coupling strengths

# Random Matrix Model

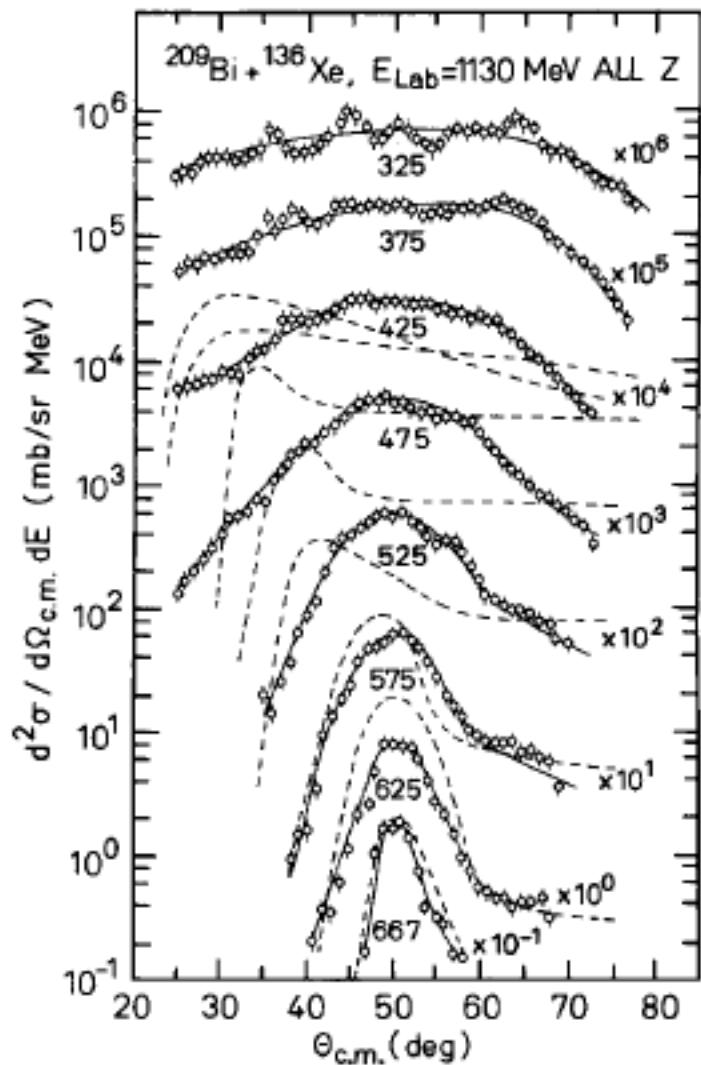
Coupled-channels equations:

$$\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(r) + \epsilon_k - E \right] \psi_k(\mathbf{r}) + \sum_{k'} \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle \psi_{k'}(\mathbf{r}) = 0$$

$|\phi_k\rangle$  : complicated single-particle states

coupling matrix elements  $V_{kk'} = \langle \phi_k | V_{\text{coup}} | \phi_{k'} \rangle$  are **random numbers** generated from a Gaussian distribution:

$$\begin{aligned}\overline{V_{ij}(r)} &= 0, \\ \overline{V_{ij}(r)V_{kl}(r')} &= (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \frac{w_0}{\sqrt{\rho(\epsilon_i)\rho(\epsilon_j)}} \\ &\quad \times e^{-\frac{(\epsilon_i-\epsilon_j)^2}{2\Delta^2}} \cdot e^{-\frac{(r-r')^2}{2\sigma^2}} \cdot h(r)h(r')\end{aligned}$$



## RMT model for H.I. reactions:

- ✓ originally developed by Weidenmuller et al. to analyze DIC
- ✓ similar models have been applied to discuss *quantum dissipation*
  - M. Wilkinson, PRA41('90)4645
  - A. Bulgac, G.D. Dang, and D. Kusnezov, PRE54('96)3468
  - S. Mizutori and S. Aberg, PRE56('97)6311

D. Agassi, H.A. Weidenmuller, and  
C.M. Ko, PL 73B('78)284

## Application to $^{20}\text{Ne} + {}^{90,92}\text{Zr}$ reactions

### ➤ Internuclear potential

Woods-Saxon potential

$$V_0 = 55 \text{ MeV } ({}^{90}\text{Zr}), 62.3 \text{ MeV } ({}^{92}\text{Zr}), \\ r_0 = 1.2 \text{ fm}, a=0.65 \text{ fm}$$

### ➤ Coupling form factor $h(r)$

derivative of Woods-Saxon

### ➤ Non-collective couplings

up to 5.7 MeV, only from the ground state

→ 38 channels for  ${}^{90}\text{Zr}$ , 75 channels for  ${}^{92}\text{Zr}$

energy and radial coherence lengths: Weidenmuller et al.

$$\Delta = 7 \text{ MeV}, \sigma = 4 \text{ fm}$$

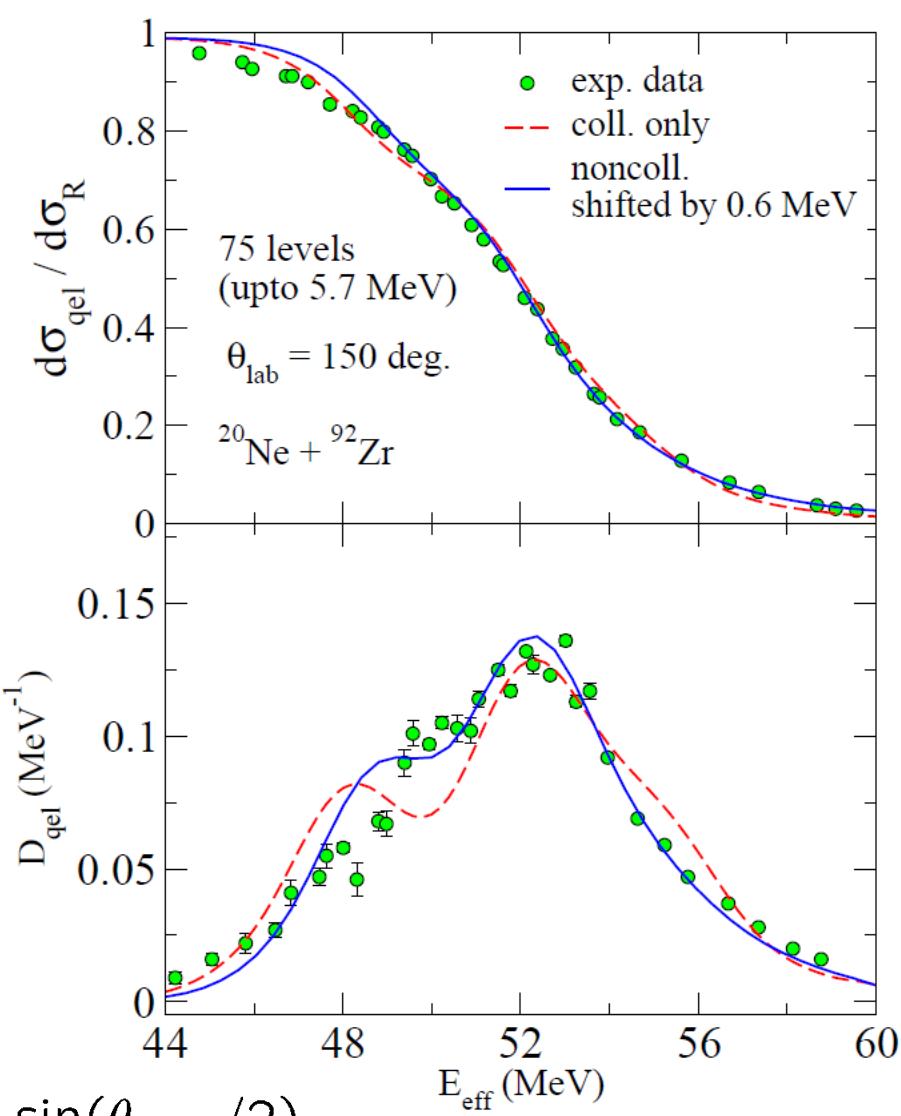
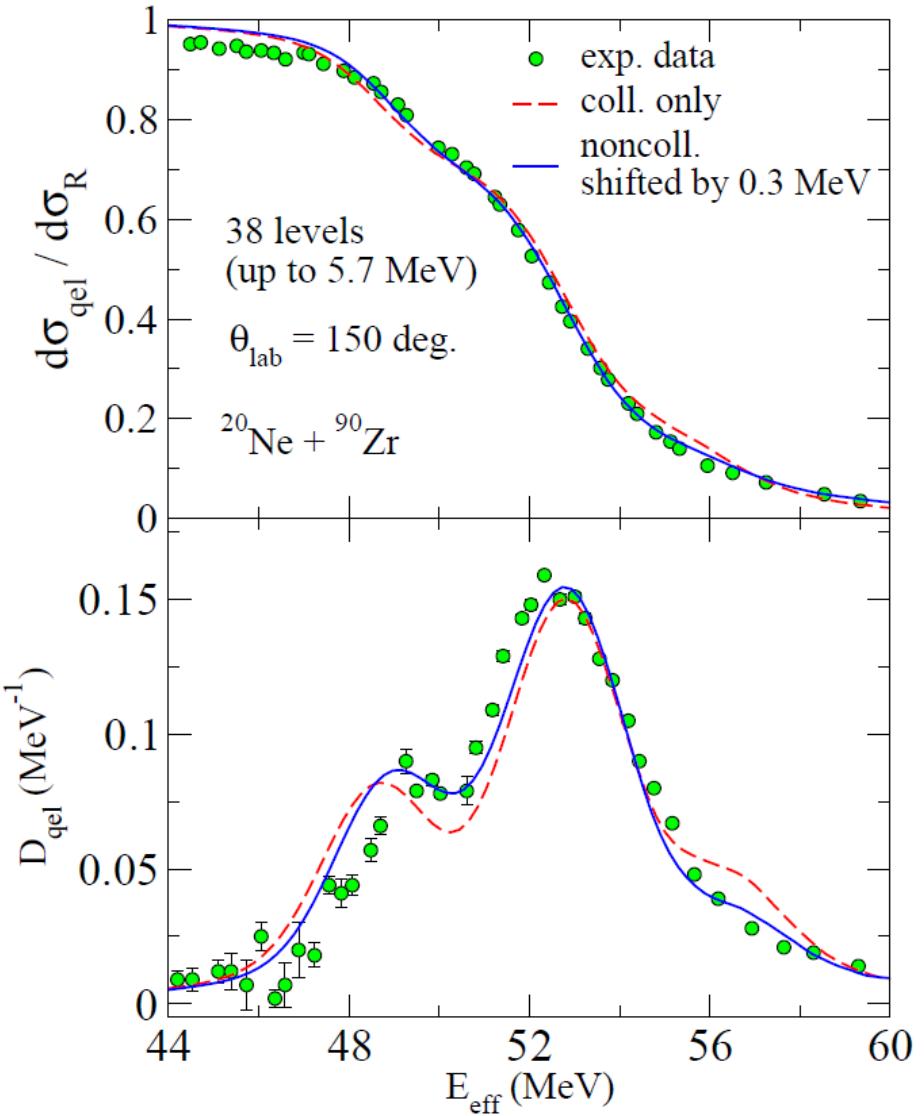
the overall coupling strength: adjustable parameter

(the same value between  ${}^{90}\text{Zr}$  and  ${}^{92}\text{Zr}$ )

### ➤ Collective couplings

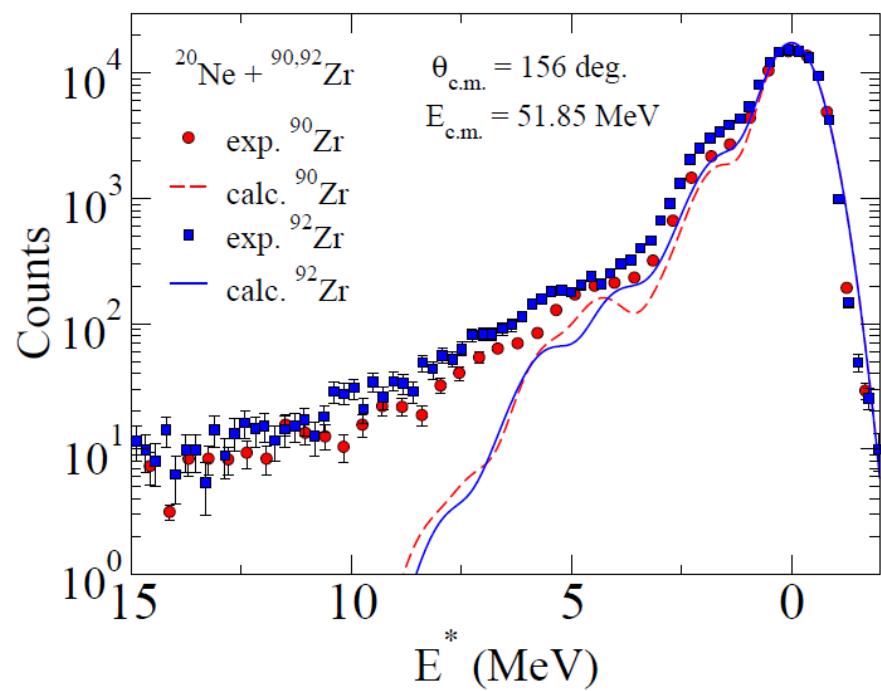
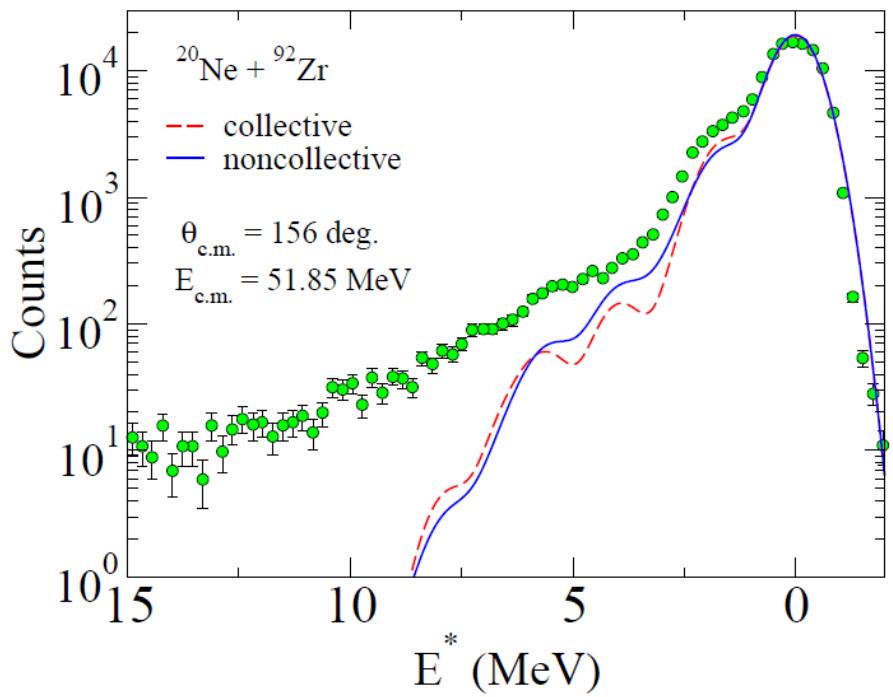
rot. states of  $^{20}\text{Ne}$  up to  $6^+ + 2^+$  and  $3^-$  two-phonons in Zr

# Quasi-elastic cross sections

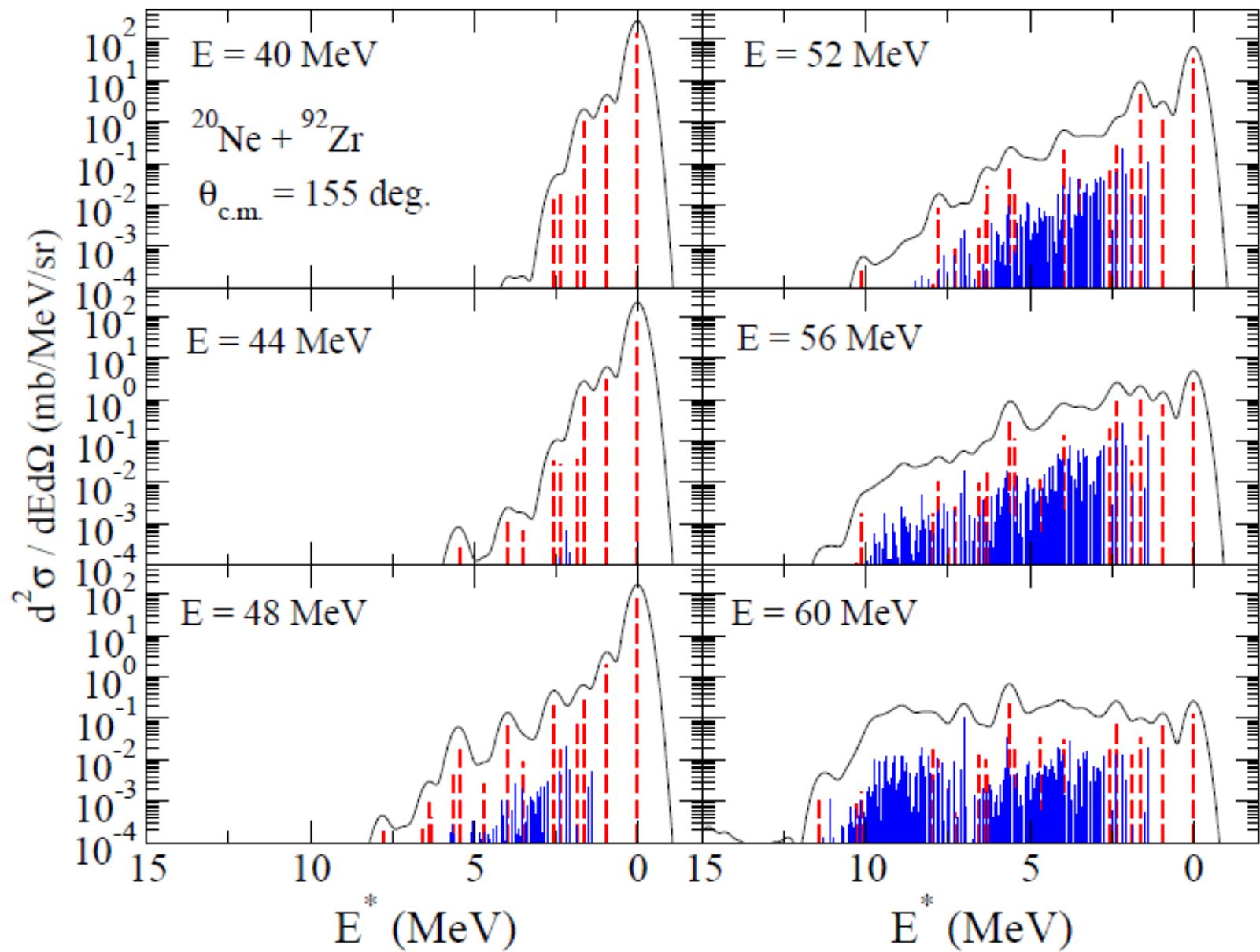


$$E_{\text{eff}} = 2E \frac{\sin(\theta_{\text{c.m.}}/2)}{1 + \sin(\theta_{\text{c.m.}}/2)}$$

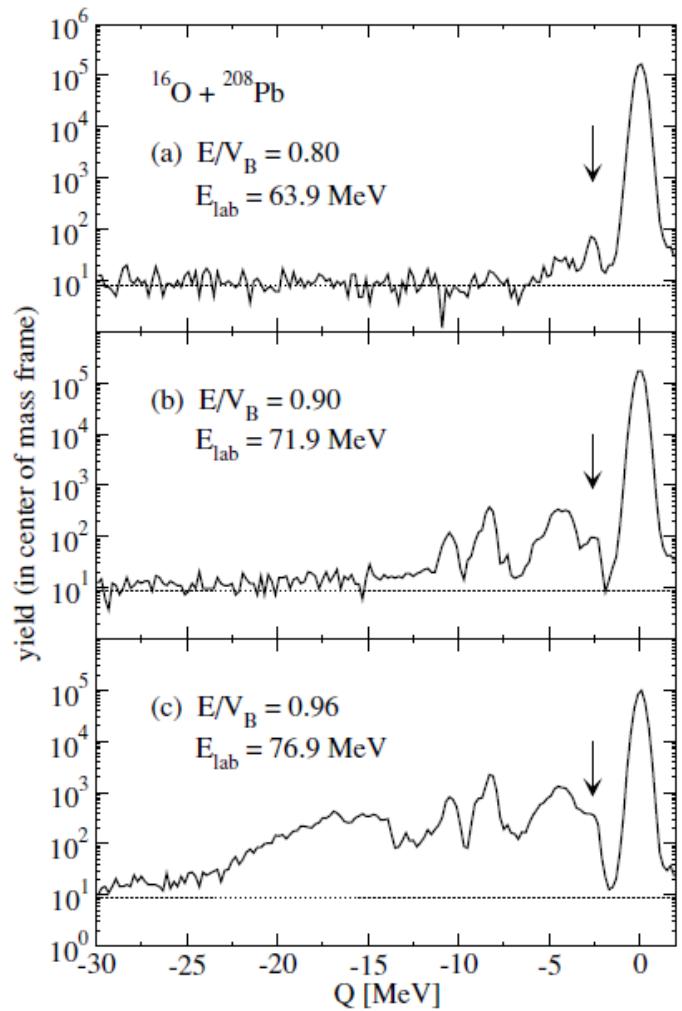
# Q-value distributions



S. Yusa, K.H., and N. Rowley, arXiv:1309.4674

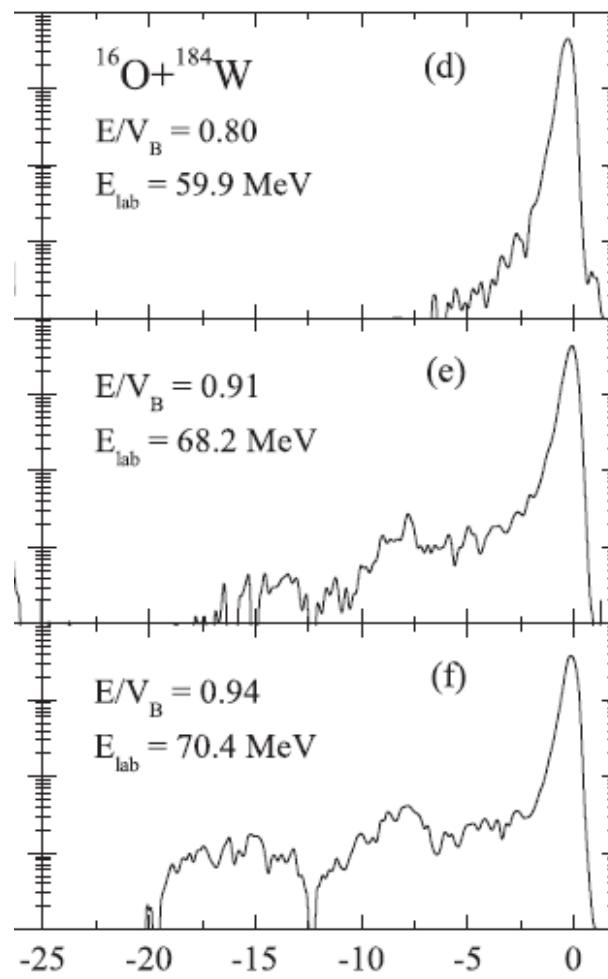


# cf. Q-value distribution from backward scattering:



M. Evers et al.,  
PRC78('08)034614

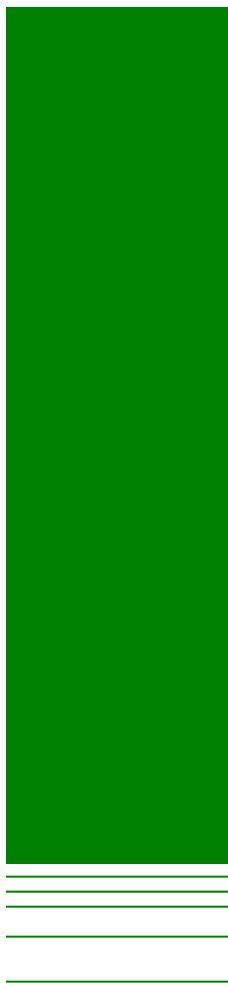
(elastic + collective) peaks + non-collective bumps



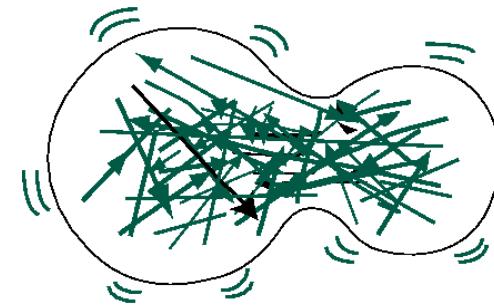
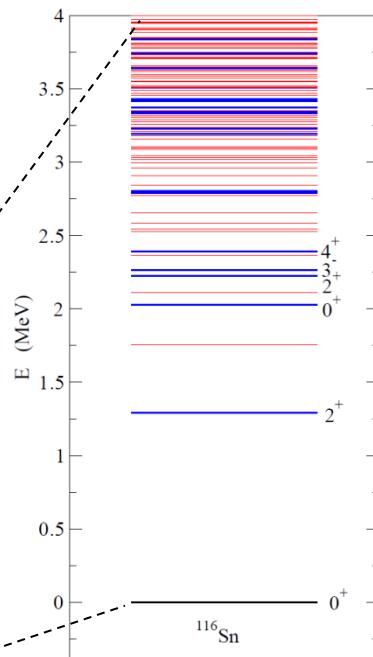
C.J. Lin et al.,  
PRC79('09)064603

# Discussions: towards a microscopic reaction theory

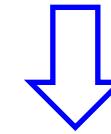
$E^*$



$$\rho(E) \sim e^{2\sqrt{aE^*}}$$



These states are excited during nuclear reactions in a complicated way.



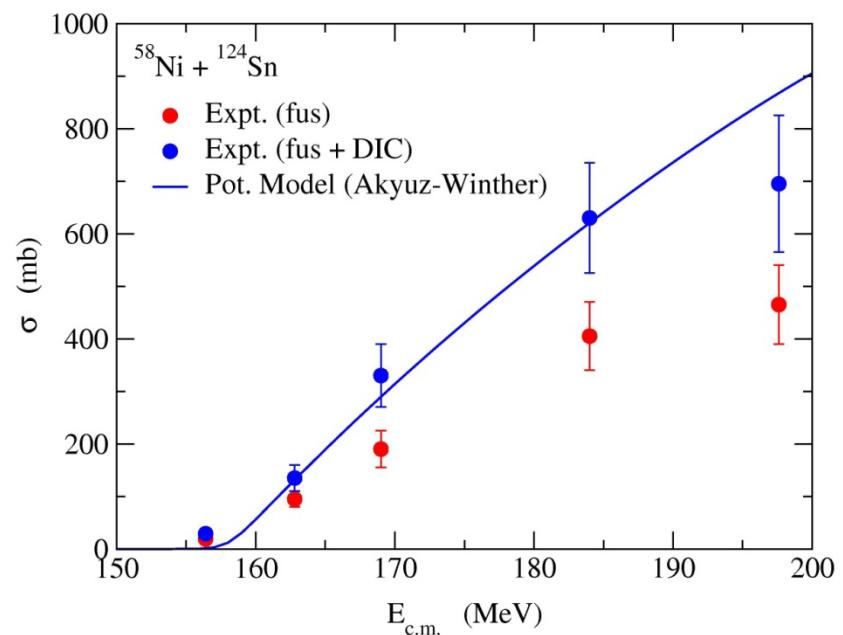
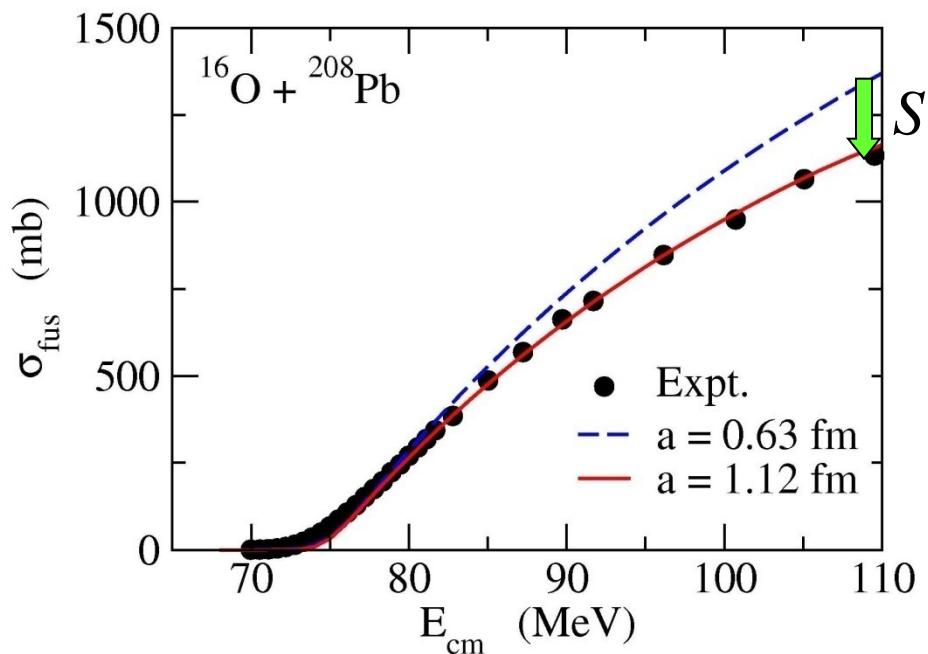
nuclear intrinsic d.o.f.  
act as environment for  
nuclear reaction processes

*“intrinsic environment”*

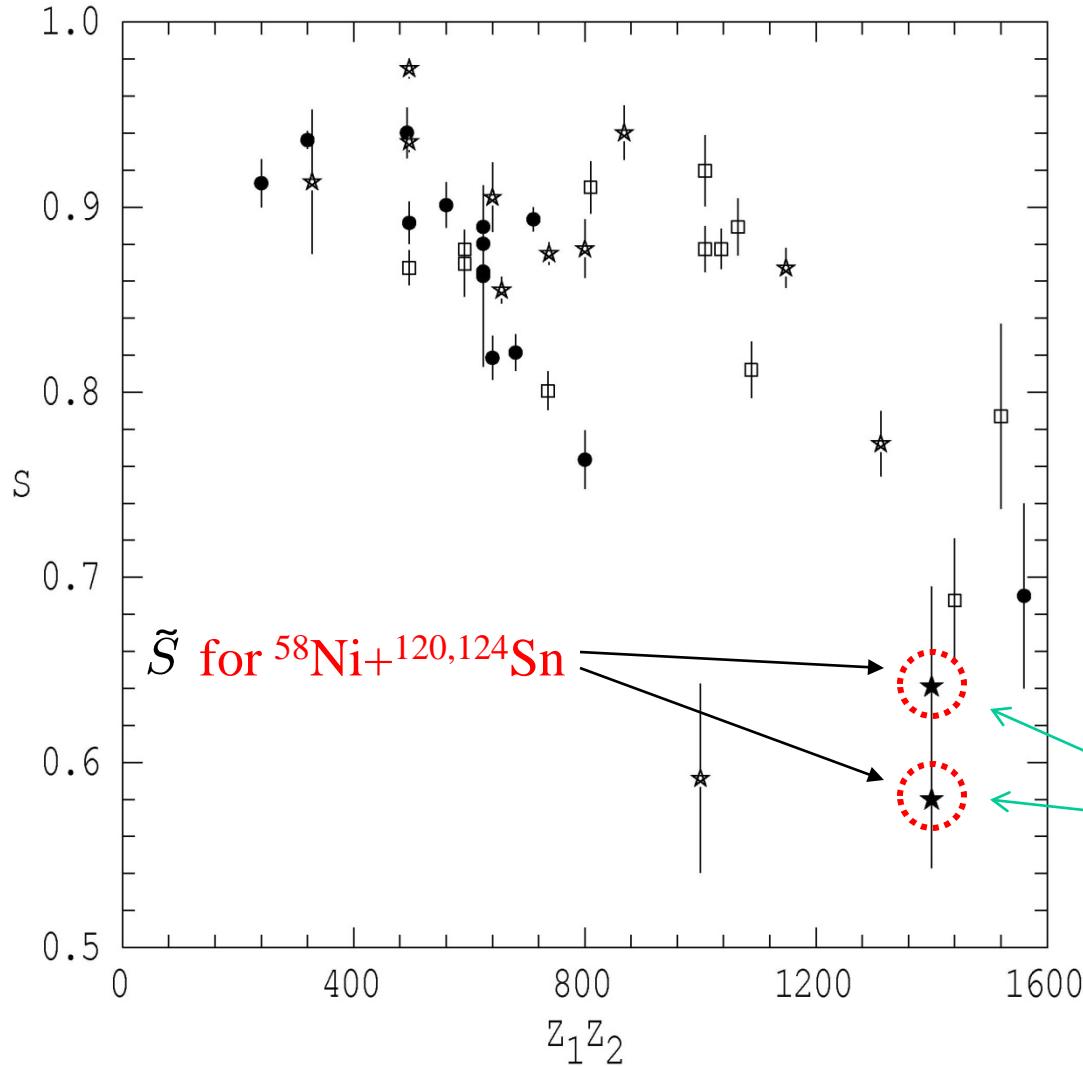
coupling to environment  $\longleftrightarrow$  dissipation & friction

*How much do we know about “friction”?*

Fusion model  $\longrightarrow$  friction free: strong absorption inside the barrier



$$\sigma_{\text{fus}}^{(\text{exp})}(E) = S \cdot \sigma_{\text{capt}}^{(\text{th})}(E; a = 0.63)$$



DIC also in light systems?

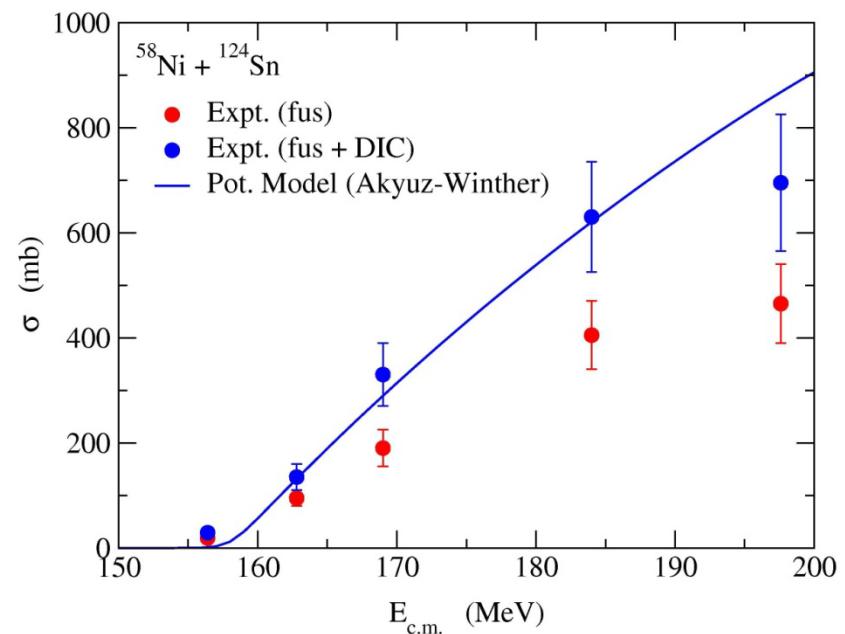
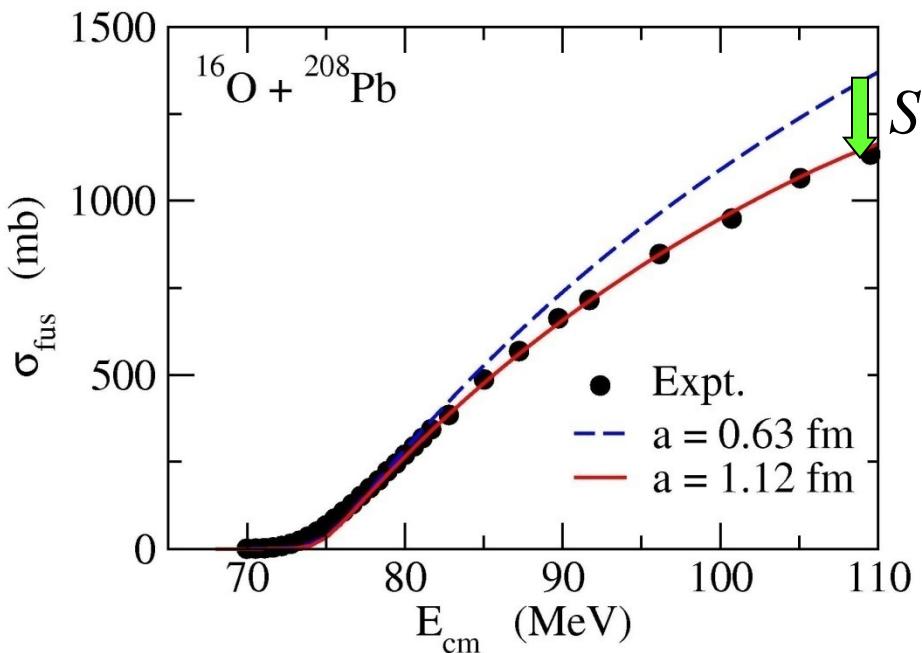
Transfer processes with large negative Q-value?

$$\tilde{S} \equiv \frac{\sigma_{\text{fus}}^{(\text{exp})}(E)}{\sigma_{\text{fus}}^{(\text{exp})}(E) + \sigma_{\text{DIC}}^{(\text{exp})}(E)}$$

coupling to environment  $\longleftrightarrow$  dissipation & friction

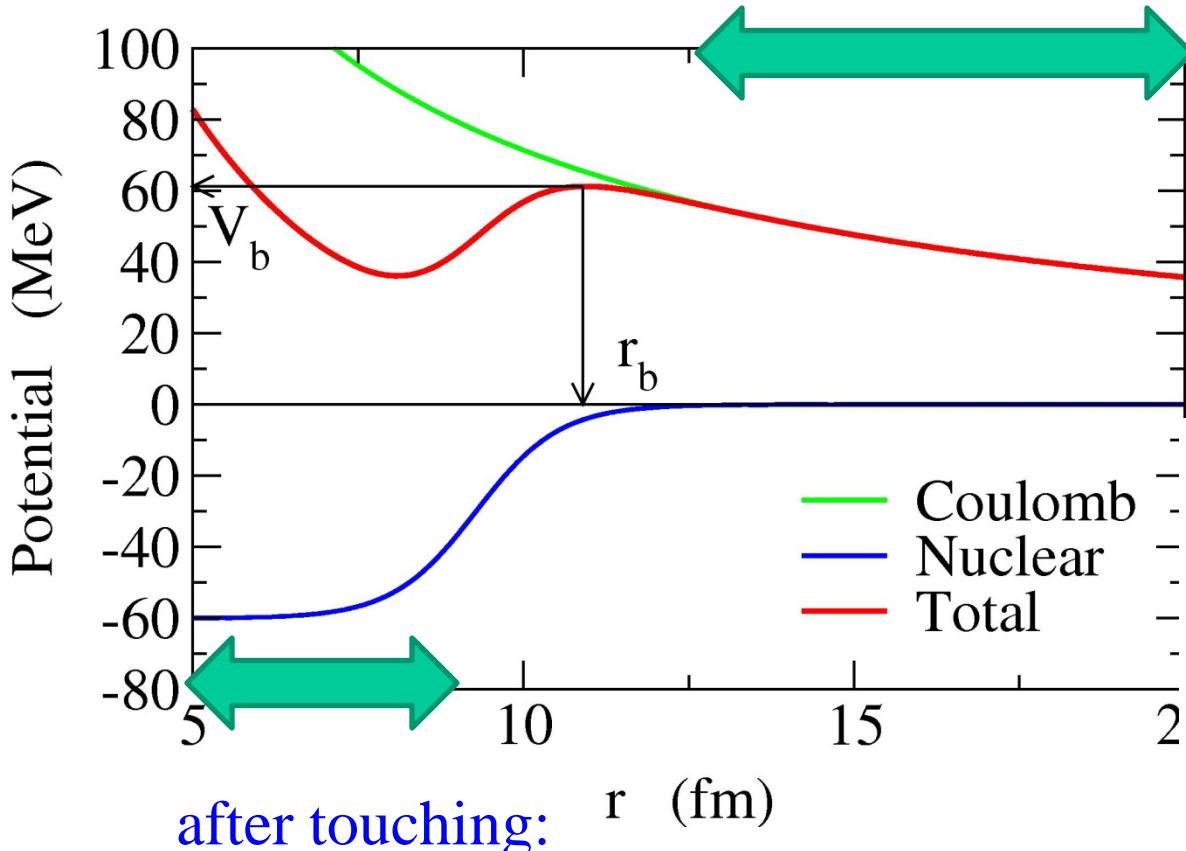
*How much do we know about “friction”?*

Fusion model  $\longrightarrow$  friction free: strong absorption inside the barrier



 The topic of energy dissipation in fusion should be re-visited

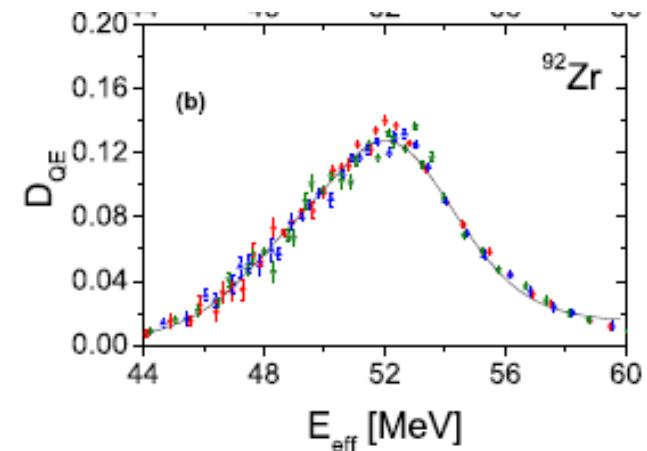
- re-analyses of DIC data: maybe helpful
- Consistent theoretical model (dissipative quantum tunneling)



molecular excitations     

- Deep subbarrier fusion

Non-collective  
excitations in isolated  
nuclei



- Random matrix  
model?

Unified quantum theory for fusion (subbarrier, deep subbarrier) & DIC?

↔ Single-particle (non-collective) excitations in H.I. reactions  
quantum mechanical model for Wall-Window friction?

## (Big) open question:

➤ Construction of a microscopic nuclear reaction model applicable at low energies?

→ many-particle tunneling

cf. nuclear structure calculations

- 2-body nn interaction → mean-field → RPA  
                                ↓  
                                residual interaction → TDHF

advantage: non-empirical

disadvantage: difficult to control a mean-field



- mean-field pot. → residual interaction → RPA  
                                ↓  
                                TDHF

- 2-body nn interaction  $\rightarrow$  mean-field  
residual interaction  $\rightarrow$  RPA  
 $\nearrow$  TDHF

many reaction theories correspond to this type



- mean-field pot.  $\rightarrow$  residual interaction  $\rightarrow$  RPA  
 $\nearrow$  TDHF

## Microscopic nuclear reaction theories

TDHF, QMD, AMD  not applicable to low-energy fusion  
(classical nature)

Cluster approach (RGM)

 only for light systems  
H.O. wave function (separation of  
cm motion)

Double Folding approach

 surface region: OK, but inside?  
role of antisymmetrization?  
validity of frozen density approximation?

Full microscopic theory: ATDHF, TD-GCM, ASCC ?  
imaginary-time TDHF?

how to understand quantum tunneling from many-particle point of view?

## Another issue

Is reaction fast or slow?

Many-body (N-particle system) Hamiltonian

$$H = \sum_i t_i + \sum_{i < j} v_{ij}$$

Large Amplitude Collective Motion

$$H = H_{rel} + H_{s.p.} + H_{coup}$$

✧ Sudden approach (fast collision)

Double Folding Model

Optical Model

Coupled-channels model

Resonating Group Method (RGM)

} const. reduced mass  $\mu$

✧ Adiabatic approach (slow collision)

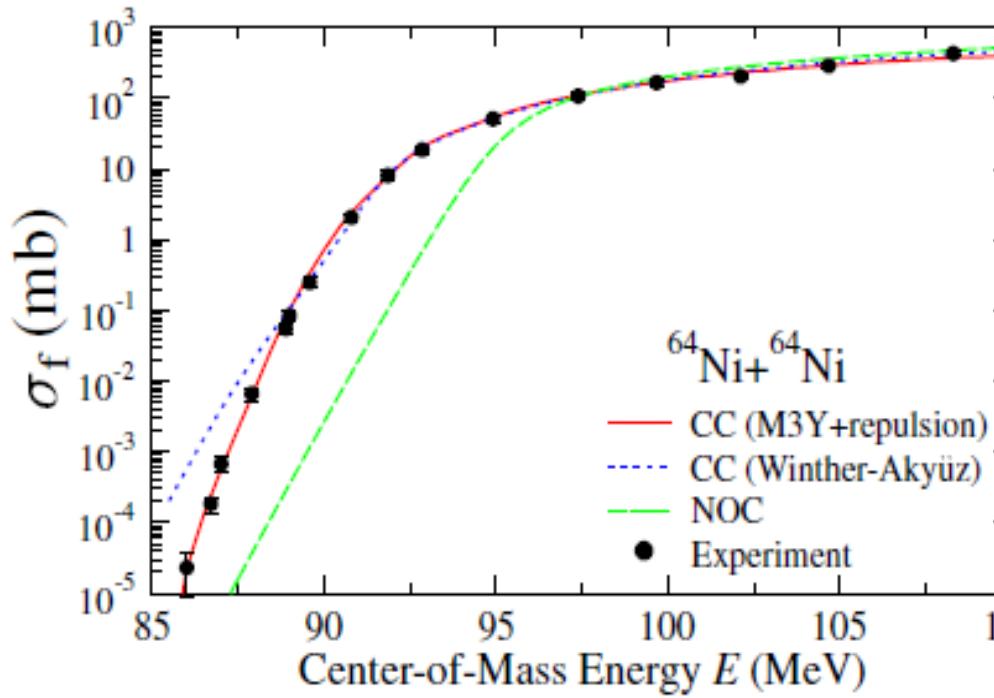
Liquid-drop model (+ shell correction)

Adiabatic TDHF

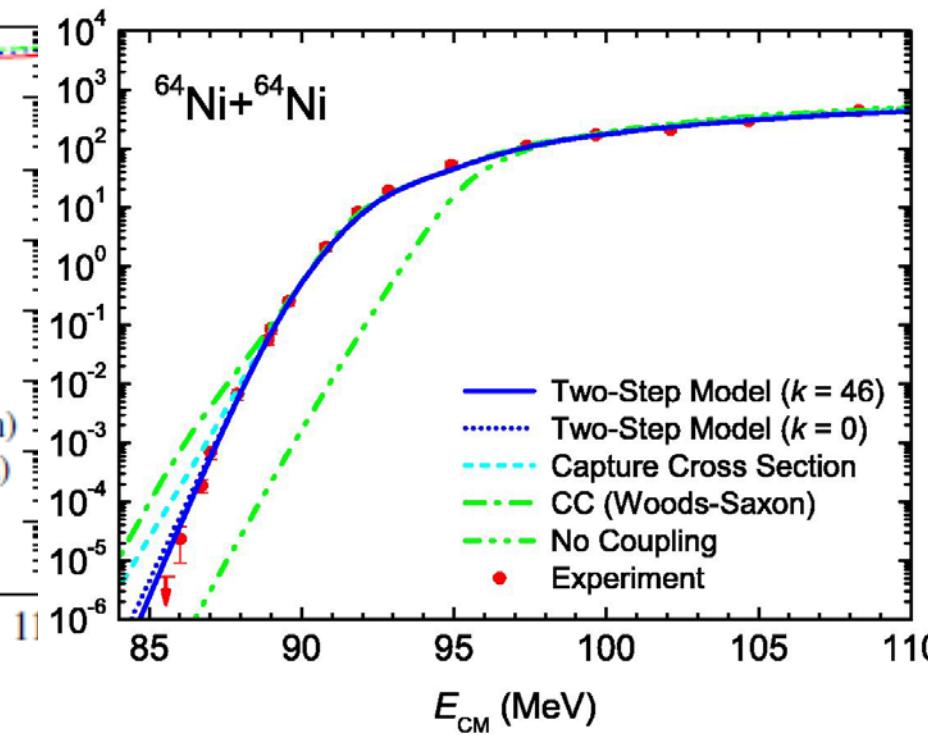
← Coordinate dependent mass  $\mu(r)$

cannot disreminate one of them at present

sudden approach (frozen density)



adiabatic approach



S. Misic and H. Esbensen,  
PRL96('06)112701

T. Ichikawa, K.H., A. Iwamoto,  
PRC75('07)057603



- ✓ need further studies from several perspectives
- ✓ construction of dynamical model without any assumption on adiabaticity

# Summary

## Heavy-ion subbarrier fusion reactions

- ✓ strong interplay between reaction and structure
- ✓ quantum tunneling with several kinds of environment

## Open questions

- ✓ how do we understand many-particle tunneling?
  - related topics: fission, alpha decays, two-proton radioactivities  
Large amplitude collective motions
- ✓ role of noncollective excitations?
  - dissipation, friction
- ✓ microscopic understanding of subbarrier fusion?
- ✓ unified theory of fusion and DIC?