Real-time Methods for Cooling and Simulating Macroscopic Fermi Systems

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Goal: Simulate Large Fermi Systems

- •Neutron stars
	- •Glitching (thousands of vortices pinning on nuclei)
	- •Macroscopic dynamic properties
- 10⁶ cold atoms in traps
	- •Preparation
	- •Imaging
- •Quantum turbulence, vortex tangles

Problem: Fermions are Expensive

• Even Fermionic DFTS too costly:

- How to find ground state?
- Limited to few thousand particles

• How to scale up to study macroscopic systems?

Outline

•Resolving a Mystery: MIT Heavy Solitons = Vortex Rings

Fermionic DFT for small systems validates bosonic model for realistic systems

• Fermionic DFTs

- •Real-time State Preparation Adiabatic Switching + Quantum Friction
- Real-time extraction of forces
- Modelling Fermions with Bosons **in a fermionic superfluiders** · ETF model (like GPE) a, Superfluid pairing gap D(z) for a stationary soliton, normalized by the bulk \mathcal{N} pairing gap D0, and density near the localized bosonic (fermionic) state versus $\mathcal{C}(\mathcal{C})$ state versus $\mathcal{C}(\mathcal{C})$

imprinting laser beam twists the phase of one-half of the trapped superfluid by

Fermionic Superfluids Universality

Fermionic Superfluids

Unitary

Nuclei neutrons

 $k_F \sim fm^{-1}$ $a_{nn} = -19$ fm $r_{nn} = 2 fm$

Neutron Matter

Fermi Gas $a = \infty$

Cold Atoms $k_F \sim \mu m^{-1}$

 $r_e=0$

and protons

Other Superfluids

- Superconductors (charged + phonons)
- Quarks (gluon interactions, Dark Matter?)
- \bullet $\overline{^3He}$ (p-wave)

Many systems

Tuneable a

 $r_{nn} \sim 0.1$ nm

- different species
- dipole interactions
- optical lattices
- quantum simulators

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Unitary Fermi Gas (UFG) H ⇤ = \int $\widehat{a}^{\dagger} \widehat{a} E_{\alpha} + \widehat{b}^{\dagger}$ b bE_b ⇥ - $\sqrt{ }$ Va ⇤*†* b $\mathbf{\hat{b}}^{\dagger}$ b ⇤ $\widehat{\mathfrak{a}}$ ⇤ $E_{a,b} =$ \mathfrak{p}^2 $\frac{P}{2m} - \mu_{a,b}, \quad \mu_{\pm} =$ $\mu_a \pm \mu_b$ 2

•Take regulator $\lambda \rightarrow \infty$ and coupling $g \rightarrow 0$ to fix s-wave scattering length $a^{-1} \propto (\lambda - g^{-1}) = 0$ (unitary limit)

Universality

• Short distance irrelevant:

- At long distance $(r>R)$ potentials equivalent $V_1 \equiv V_2$
- •Characterized by scattering length a

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- Unitary limit $a=\infty$: No interaction length scale!
- •Universal physics: $\cdot \mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}, \xi_{exp} = 0.370(5)(8)$

• Simple, but hard to calculate! Bertsch Many Body X-challenge

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- •Universal physics:
	- $\cdot \mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}, \xi = 0.376(5)$

• Lithium 6 (⁶Li)

•Dilute neutron matter in neutron stars \cdot a_{nn} $=$ -19 fm

MIT Experiment

- •⁶Li atoms (N≈10⁶) cooled in harmonic trap
- Step potential used to imprint a soliton
- Let system evolve
- Image after ramping magnetic field B and expanding
- •Observe an oscillating soliton with long period $T\approx 12T_z$
	- •Bosonic solitons (BECs) oscillate with $T\approx\sqrt{2T_z}\approx1.4T_z$
	- Fermionic solitons ($BddG$) oscillate with T≈1.7T_z
	- •Interpret as "Heavy Solitons"

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

MIT Experiment

 $\hbar \partial_t(\delta \varphi) = \delta V$

Imprint soliton

Step potential and with soliton with solid waves, the solid waves, and the solid waves, the solid waves, the s phases evolve to π phase shift $\qquad \qquad$ ϵ it is expected to decay into phonons or \mathbb{R}

Flat domain wall (dark/grey soliton) potential m of the condensate to be not much larger than the transverse

μ

against quantum fluctuations10,21–25,33.

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

MIT Experiment $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}$ leading to its acceleration. When the soliton reaches a critical velocity, Time

each image is a (each image is a different run)

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Trapped Domain Walls

- •Bosonic solitons (GPE) have T≈√2Tz ≈1.4Tz Busch and Anglin (2000)
- Fermionic solitons (BdG) have $T\approx\sqrt{3}T_{z}\approx1.7T_{z}$ Liao, Brand (2011); Scott, Dalfovo, Pitaevskii, Stringari (2011)
- Experiment sees $T \approx 10T_z 20T_z$ •Order of magnitude larger than theory!

MIT Experiment . Our correct fifting that scales as na3 instead of the correct fifting instead of the correct fifting instead
Na3 p scales as na3 p scales for the correct fifting instead of the correct field of the correct field of the experiment thus direction N direction in beyond mean-field N effects for the dynamics of strongly interacting fermionic superfluids. S ignificant soliton filling was found theoretically inter- \mathcal{S} inter-section filling inter-section filling \mathcal{S} smaller in magnitude than M* when the soliton is filled. For weakly interacting BECs, where solitons are devoid of particles, the effective mass is still of the same order of the bare mass, (M*/M)BEC 5 2. This leads to an oscillation period that is only ffiffi ² ^p times longer than Tz \mathbf{r} , as has been observed in experiments. where only a minute fraction $\mathcal{L}_\mathcal{F}$ on $\mathcal{L}_\mathcal{F}$ 'snaking'10,13,15 (for examples, see Supplementary Information). $W' = -4$ attribute mass \mathcal{A} interaction and filling of the soliton with α mion pairs resulting from strong quantum fluctuations. Similar filling with uncondensed particles has been predicted for solid and the solid structure in structure in structure in s interacting Bose condensates10,22–25,33. A substantial filling of the soliton will reduce the number jNsj of atoms missing inside the soliton,

Supplementary Information). Mean-field theory for the BEC–BCS

Period depends on: •Aspect ratio $\lambda \in \{3.3, 6.2, 12\}$ •Interaction

Much longer than predicted for domain walls

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

MIT Experiment sions with the thermally induced phonons \mathbf{N} \blacksquare in the anti-damping time constant as the \blacksquare is raised (Fig. 5b). At even higher temperatures, the soliton's position effects, which are likely to be due to be due to unconducted fermion pairs filling \mathbf{r} the soliton, in addition to purely fermionic Andreev bound states. Our \blacksquare ituditative benchmark for the important for the important for the important for the intervals of the intervals of

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736] an El al. Ivaluit 499 (420) 2013 fai λ iv.1302.4/30

Finite temperature: · Anti-decay • (Negative mass) Tz 10 **a**

and c, the soliton lifetime, are found to be soliton lifetime, are found to be strongly dependent on the thermal

MIT Experiment

Figure S 1: Imaging solitons. a Optical density, b integrated 1D profiles and c corresponding residuals of a fermionic

Subtle imaging: •Need expansion (turn off trap) •Must ramp to B<700G •~10% depletion

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736] refsan et al. Nature 499 (420) 2013 JarXIV:1302.4730J

MIT Experiment

Domain walls should have snake instability

•They observe something for small aspect ratios

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

MIT Experiment Interpretation

- •"Heavy solitons"
	- •Effective mass larger by orders of magnitude
	- •Extremely stable (thick) filled domain walls
	- •Interpreted as a new quantum phenomenon not described by current theories
- What do fully 3D simulations see?

SLDA Simulations

Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]: 32x32x128

Friday, October 18, 13

SLDA Simulations

RACK AND CO

ALCOHOL: A MARKET A MARKET

Bulgac, Forbes, Forbes, Kelley, Roche, R

Friday, October 18, 13

34/111

DOTE

10015

Vortex Ring Oscillation ⁸

fully full set on itself, re-formal as a dark-soliton near the turning points. This behavior mirrors that see

Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266] (right). We start with a cylindrical cloud (not shown, see Ref. [30, 38]) with central density *n^F* = *k*³ *^F /*3⇡² where the Fermi *zy*, inoche, whaziowsni (2013) [al λ iv.1300.4200]

Vortex Rings

$$
E \sim \frac{mn\kappa^2}{2} R \ln \frac{R}{l_{coh}}, \qquad \nu = \frac{dE}{dp} \sim \frac{\kappa}{4\pi} \frac{1}{R} \ln \frac{R}{l_{coh}}
$$

•Thin vortex approximation in infinite matter (follows essentially from Biot-Savart law)

•Approximately valid for rings near core (but not too near)

Near-Harmonic Motion

Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]

Vortex Rings in a Trap

$$
M_{I} = \frac{F}{\dot{v}} \sim 8\pi^2 m n R^3 \left(\ln \frac{R}{l_{\text{coh}}} \right)^{-1}
$$

$$
M_{VR} = m N_{VR} \sim m n \ 2\pi R \ \pi l_{\text{coh}}^2
$$

- M_I: Inertial (kinetic mass) differs significantly from
- \cdot M_{VR} : Mass depletion • Long periods T T_z \sim $\big/ \,{\sf M}_{\rm I}$ M_{VR} \sim $\sqrt{ }$ $2R/l_{\mathsf{coh}}$ $ln(R/l_{coh})$

Vortex Rings in a Trap

- \bullet Behaviour depends on $\mathsf{T} \sim \mathsf{R} / \mathsf{l_{coh}} \sim \mathsf{k_{F}} \mathsf{R}$
- Large traps have long periods (k FR \sim 20 for experiment)
- Small (narrow) approach domain wall $T\simeq \sqrt{2T_z}$ Formula does not apply
- Depends on l_{coh}

Characterizes dependence on scattering length

Vortex Rings in a Trap

$$
M_{I} = \frac{F}{\dot{v}} \sim 8\pi^2 m n R^3 \left(\ln \frac{R}{l_{con}} \right)^{-1}
$$

$$
M_{VR} = m N_{VR} \sim m n 2\pi R \pi l_{con}^2
$$

- M_I: Inertial (kinetic mass) differs significantly from
- M_{VR}: Mass depletion
- Long periods

$$
\boxed{\frac{T}{T_z}} \sim \sqrt{\frac{M_I}{M_{VR}}} \sim \frac{2R/l_{coh}}{\sqrt{ln(R/l_{coh})}}
$$

Does MIT measure vortex rings?

•Reproduces all qualitative dependences: ✓Long periods ✓Anti-decay at "finite temperature" ✓Dependence on aspect ratio and interaction strength

But MIT sees domain walls, not rings LOIIIdIII WdII

200

Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

Imaging Vortex Rings (small ring) **Supplementary information:** monthon *Vortey* Tarik Yefsah, Ariel T. Sommer, Mark J.H. Ku, Lawrence W. Lawrence W. Cheuk, Waseem S. Bakr, and Mark J.H. Ku, A B C DISCOVERED A DISCOVERED AT A B C DISCOVERED AT A B C DISCOVERED AT A B C DISCOVERED AT A DISCOVERED AT A

to the BEC-side before expansion at 760 reduces interactions but still only reveals a very faint trace of the soliton.

Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266] \mathbf{F} 1. Imaging solitons. A Optical density, b integrated 1D profiles and c corresponding residuals of a fermionic re superfluid, prepared at 832 G, after expansion and rapid ramp to various final magnetic fields Bmin. Without any ramp,

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Ramp

Image

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Image after expansion (integrated average)

Image after expansion (simulate noise)

Imaging Vortex Rings (large ring) **Supplementary information:** monthon *Vortey* Tarik Yefsah, Ariel T. Sommer, Mark J.H. Ku, Lawrence W. Lawrence W. Cheuk, Waseem S. Bakr, and Mark J.H. Ku, Field / / TOF A B C DISCOVERED A B C DI

to the BEC-side before expansion at 760 reduces interactions but still only reveals a very faint trace of the soliton.

800

Ramp

Image

750

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Image after expansion (integrated average)

Image after expansion (simulate noise)

Image after expansion borderline Bmin=702G

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Explains Dependence \sum_{min}

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Solitions? Finds \sim soliton \sim to decay rapidly into the such dissiparation of \sim 1 $\frac{1}{\sqrt{2}}\int_{\mathcal{C}}\sqrt{2\pi}\sqrt{2\pi}$ and $\frac{1}{\sqrt{2}}\int_{\mathcal{C}}\sqrt{2\pi}$ and with soliton with soliton with sound waves, $\frac{1}{\sqrt{2}}\int_{\mathcal{C}}\sqrt{2\pi}$ \mathcal{L} to the soliton reaches and the soliton reaches a contract of \mathcal{L} it is expected to decay into phonons or $\mathcal{U}(\mathcal{U})$ in the case of fermionic supervortices via the so-called snake instability13,15,31,32. In the case of weakly interacting BECs in elongated traps, stability requires the chemical potential m of the condensation than the transverse to be not much larger than the transverse \bigcirc <u> Andreas Andreas and Andreas Andreas Andreas and Andreas Andr</u> 4IMEMS "MIN

Here we can also and observe long-lived solitons in a strongerve long-lived solitons in a strongly inter- $\overline{}$ MIT experiment · Long periods

- •Dependence on
- $\frac{1}{\text{costion vs time } (T=9.9 T_{z})}$. Imaging limitations
	- $\frac{1}{2}$, where af is isomorphism. and time of flight, for the \bullet V alied the validation of $\mathbf v$

Time (ms)

SLDA: Superfluid Local Density Approximation

$$
\mathcal{E}(\mathfrak{n},\tau,\nu)=\alpha\frac{\tau}{\mathfrak{m}}+\beta\frac{(3\pi^2\mathfrak{n})^{5/3}}{10\mathfrak{m}\pi^2}+9\mathfrak{e}\mathfrak{n}\nu^\dagger\nu
$$

- •Three densities: $n \approx \langle a^{\dagger}a \rangle$, $\tau \approx \langle \nabla a^{\dagger} \nabla a \rangle$, $\nu \approx \langle ab \rangle$
- •Three parameters:
	- Effective mass (m/α)
	- Hartree (β), Pairing (g)

Forbes, Gandolfi, Gezerlis (2012)

BdG: contained in SLDA

$$
\langle \nabla \hat{a}^{\dagger} \nabla \hat{a} \rangle + \langle \nabla \hat{b}^{\dagger} \nabla \hat{b} \rangle \qquad \langle \hat{a}^{\dagger} \hat{b}^{\dagger} \rangle \langle \hat{b} \hat{a} \rangle
$$
\n
$$
\mathcal{E}(\mathbf{n}, \tau, \mathbf{v}) = \alpha \frac{\tau}{\mathbf{m}} + \beta \frac{(3\pi^2 \mathbf{n})^{5/3}}{10 \mathbf{m} \pi^2} + \mathbf{g}_{\text{eff}} \mathbf{v}^{\dagger} \mathbf{v}
$$

- •Variational: ℰ=〈H〉 (minimize over Gaussian states)
- Bogoliubov-de Gennes (BdG) contained in SLDA
- Unit mass $(\alpha=1)$
- No Hartree term $(\beta=0)$
	- •(No polaron properties)

SLDA: Superfluid Local Density Approximation

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Forbes, Gandolfi, Gezerlis (2012)

SLDA: Superfluid Local

Forbes, Gandolfi, Gezerlis (2012)

TDDFT (TDSLDA)

$$
\iota \partial_t \Psi_n = H[\Psi] \Psi_n = \begin{pmatrix} \frac{-\alpha \nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha \nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}
$$

•No diagonalization needed for evolution Just apply Hamiltonian Use FFT for kinetic term

•Efficient real-time evolution the scales well

Distribute wavefunctions over nodes Utilize GPUS

TDDFT (TDSLDA)

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$$

•Computational challenge: Finding initial (ground) state?

Root-finders requires repeated diagonalization of s.p. Hamiltonian

- Slow and does not scale well
- Only suitable for small problems or if symmetries can be used

State Preparation?

- How to find initial (ground) state?
- •Root-finders repeatedly diagonalize s.p. Hamiltonian Slow and does not scale well
- •Imaginary time evolution? Non-unitary: spoils orthogonality of wavefunctions
	- Re-orthogonalization unfeasible (communication)

Quantum Friction
\n
$$
V_t \propto -\frac{\hbar \vec{\nabla} \cdot \vec{j}_t}{\rho_t} = \frac{\hbar \dot{\rho}_t}{\rho_t} \propto \frac{-\mathfrak{I}(\psi_t^\dagger \nabla^2 \psi_t)}{\rho_t}
$$

- •Unitary evolution (preserves orthonormality)
- •Easy to compute: local time-dependent potential Acts to remove local currents
- •Couple with quasi-adiabatic state preparation Bulgac, Forbes, Roche, and Wlazłowski (2013) [arXiv:1305.6891]

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\n
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$$

• Consider evolution with potential $H+V_t$:

 $\partial_t E = -i Tr$ ([H, ρ] $\cdot V_t$)

•Therefore $V_t = i[H,\rho]$ † guarantees $\partial_t E \leq 0$

Non-local potential equivalent to "complex time" evolution Not suitable for fermionic problem

• Diagonal version is a local potential: $V_t = diag(i[H, \rho]^\dagger)$

State Preparation

Quantum Friction

Potential counteracts currents

Use with dynamics to minimize energy

Harmonic oscillator with an excited state

Quantum Friction

Harmonic oscillator with an excited state

Potential counteracts currents

Use with dynamics to minimize energy

Quantum Friction

Harmonic oscillator with an excited state

Potential counteracts currents

Use with dynamics to minimize energy

Quantum Friction $V_t \propto -\frac{\hbar \vec{\nabla}}{\rho}$ *·* $\overline{\mathbf{i}}$ $j_{\rm t}$ ρ_t = $\hbar \dot{\rho}_t$ ρ_t $\propto \frac{-\Im(\psi_t^{\dagger}\nabla^2\psi_t)}{\Omega_t}$ ρ_t

- •General method: (works for many problems) Needs a good initial state to ensure reasonable occupation numbers
- •Easy to compute: local time-dependent potential Acts to remove local currents
- •Couple with quasi-adiabatic state preparation Bulgac, Forbes, Roche, and Wlazłowski (2013) [arXiv:1305.6891]

TDDFT (TDSLDA)

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$$

- Still Computationally expensive: Need to evolve each hundreds of thousands of wavefunctions
- •Possible for moderate systems (nuclei) using supercomputers, resonances, induced fission etc. Maybe cold atoms (if axially symmetric etc.) Probably not for neutron stars (glitching dynamics)

A Tale of Two Simulations

• ETF: (Effective Thomas Fermi model)

• "Bosonic" DFT simulation of dimers

Gross-Pitaevskii equation (GPE) tuned to model the unitary Fermi gas (UFG) Quantum hydrodynamics

- •Easy to compute
- SLDA: (Superfluid Local Density Approximation)
	- Fermionic Kohn-Sham

Like HFB or BdG mean-field theory with tuned parameters

•Hard to compute, but more accurate

Comparison

With Rishi Sharma [arXiv:1308.4387]

0.8

 0.6

 0.4

0.2

5

 \circ

 -5

 -10

 -15

95

90

85

 $-15 - 10 - 5$

 \circ

Fermions SLDA TDDFT

Gross Pitaevskii model

t=894.477/eF, frame=1657

Bulgac et al. (Science 2011)

15 7.2 10 6.4 5.6 5 4.8 $\mathbf 0$ 4.0 3.2 -5 2.4 1.6 -10 0.8 -15 0.0 Delta (eF) 15 0.28 10

Fermi Potential (eF)

 2.4

1.6

 0.8

0.0

 -0.8

 -1.6

 -2.4

•Fermions:

- Simulation hard!
- Evolve 10⁴-10⁶ wavefunctions
- Requires supercomputers

·GPE:

- <u>Comparison Comparison</u>
Comparison Comparison
Comparison Comparison • Simulation much easier!
	- efunctions Evolve 1 wavefunction
		- · Use supercomputers to study large volumes

GPE model for UFG

$$
E[\Psi]=\int d^3\vec{x}\,\left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F}+V_F(\vec{x})\rho_F+\xi\mathcal{E}(\rho_F)\right)
$$

$$
i\partial_t \Psi = \left(-\frac{\nabla^2}{4m_F} + 2[V_F + \xi \varepsilon(\rho_F)]\right)\Psi
$$

- •Think:
	- •Boson = Fermion pair (dimer)
- •Galilean Covariant (fixes mass)
- •Match Unitary Equation of State

 $\rho_F = 2|\Psi|^2$ ${\cal E}_{\rm FG} \propto \rho_{\rm F}^{5/2}$ $\epsilon_{\rm F} = \mathcal{E}_{\rm F}^{\,\prime}$ $_{\rm FG}^{\prime}(\rho_{\rm F}) \propto \rho_{\rm F}^{3/2}$

GPE model = Extended Thomas Fermi (ETF)

$$
E[\Psi]=\int d^3\vec{x}\; \left(\frac{|\nabla\sqrt{\rho_F}|^2}{8m_F}+V_F(\vec{x})\rho_F+\xi\mathcal{E}_{FG}(\rho_F)\right)
$$

• Vortices etc. appear as kinks in $\sqrt{\rho_F}$

GPE model for UFG

$$
E[\Psi]=\int d^3\vec{x}\,\left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F}+V_F(\vec{x})\rho_F+\xi\mathcal{E}(\rho_F)\right)
$$

$$
i\partial_t \Psi = \left(-\frac{\nabla^2}{4m_F} + 2[V_F + \xi \varepsilon(\rho_F)]\right) \Psi
$$

- Dynamics are much easier than SLDA •Only one wavefunction to evolve
- •Contains superfluid hydrodynamic equations
- Match to low-energy physics
Matching Theories: The Good

- •Galilean Covariance (fixes mass/density relationship)
- •Equation of State
- Hydrodynamics
	- •speed of sound (exact)
	- phonon dispersion (to order q^3)
	- static response (to order q^2)

Linear Response

With Rishi Sharma [arXiv:1308.4387]

Data from Joseph, Thomas, Kulkarni, and Abanov PRL (2011 \frown

D GPE simulation

2D GPE simulation

GPE vs. Experiment

Fig. 4 10 Ancilotto, L. Salasnich, and F. Toigo (2012) clouds. *Left part*: our calculations [25]. *Right part*: experimental data from Ref. [40]. The normalized

Matching Theories: The Bad

- \cdot ETF has $\rho=2|\Psi|^2$ •Density vanishes in core of vortex •Implies ∫|Ψ|2 conserved (Conservation of coarse-grained ∫|Ψ|2 provides a measure of validity) **Figure 7**. (color online) Comparison of the power spectrum
- No "normal state" in the slow drop down at a faster rate than the group \mathcal{C}
	- •Two fluid model needed? l model needed' short-wavelength modes.
	- Coarse graining (transfer to "normal" component) raining (transfer to norn breaking excitations above \mathcal{L} pairing gap (squared smoothed) for the simulations \mathcal{L} $v^{\prime\prime}$ component) variations v $\mathbf u \cdot \mathbf v$

tdslda that allows the vortex lattice to crystallize. In

With Rishi Sharma [arXiv:1308.4387] \mathbf{V} break is a superfluid pairs, transferred to the normal pairs, the normal pairs, the normal pairs of \sim

than).

Vortex Structure

With Rishi Sharma [arXiv:1308.4387] **Figure 2**. Structure of a single static vortex in the slda [53]

(solid blue curve), and in the matching ethnic blue curve), and in the matching ethnic e

further *ab initio* confirmation). If this correction turns

Defect motion

- Like GPE, the ETF has $\overline{T}{\approx}\sqrt{2T_z}{\approx}1.4T_z$ for domain walls
	- Fermionic theories (SLDA, BdG) have $T\approx 1.7T_z$
	- •Consistent with occupation of fermionic cores (fermionic walls are heavy)
- ETF vortex rings have period 1.8 shorter than experiment. Consistent when compared with SLDA. threshold set by the gap ~! *>* 2 ⇡ *E^F* . The theory, however, has the same symmetries, and is tuned to have the same equation of ρ of this approach over traditional fermionic time-dependent distant diese leave period 10 sleepter theo PULLEX HITSS HAVE PEHUU T.O SHULLET LHA

TABLE II. Benchmark of the ETF periods to the SLDA periods for sizes $24 \times 24 \times 96$, $32 \times 32 \times 128$, and $48 \times 48 \times 128$.

on the initial radius of the imprinted vortex ring *R*0. This

Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266] The observations are consistently larger than the ETF predictions bulgac, r $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n}$

Disagreement with MIT experiment?

•Periods slightly underestimated

- Will probably be resolved with full SLDA simulation
- Fringe pattern does not exactly match
	- Again, likely resolved by full SLDA

Fringe Pattern

Figure S 1: Imaging solitons. A Optical density, b integrations. Notice Γ integrated residuals residuals Γ fermionic residuals Γ fermionic residuals of a fermionic residuals of a fermionic residuals of a fermioni superfluid, prepared at 832 G, after expansion and rapid ramp to various final magnetic fields Bmin. Without a
The state big any ramp, with any ra

Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013)

Conclusion

•Virtually all aspects of the MIT experiment are explained by vortex rings:

Long periods, dependence on aspect ratio and interaction strength, anti-decay at finite temperature, imaging after expansion and dependence on B_{min}

Solitions? Finds \sim soliton \sim to decay rapidly into the such dissiparation of \sim 1 $\frac{1}{\sqrt{2}}\int_{\mathcal{C}}\sqrt{2\pi}\sqrt{2\pi}$ and $\frac{1}{\sqrt{2}}\int_{\mathcal{C}}\sqrt{2\pi}$ and with soliton with soliton with sound waves, $\frac{1}{\sqrt{2}}\int_{\mathcal{C}}\sqrt{2\pi}$ \mathcal{L} to the soliton reaches and the soliton reaches a contract of \mathcal{L} it is expected to decay into phonons or $\mathcal{U}(\mathcal{U})$ in the case of fermionic supervortices via the so-called snake instability13,15,31,32. In the case of weakly interacting BECs in elongated traps, stability requires the chemical potential m of the condensation than the transverse to be not much larger than the transverse \bigcirc <u> Andreas Andreas and Andreas Andreas Andreas and Andreas Andr</u> 4IMEMS "MIN

Here we can also and observe long-lived solitons in a strongerve long-lived solitons in a strongly inter- $\overline{}$ MIT experiment · Long periods

- •Dependence on aspect ratio and
	- $\frac{1}{\text{costion vs time } (T=9.9 T_{z})}$. Imaging limitations
		- $\frac{1}{2}$, where af is isomorphism. and time of flight, for the \bullet V alied the validation of $\mathbf v$

see Supplementary Information).

Time (ms)

Conclusion

•Virtually all aspects of the MIT experiment are explained by vortex rings:

Long periods, dependence on aspect ratio and interaction strength, anti-decay at finite temperature, imaging after expansion and dependence on B_{min}

- •Combined approach:
	- ETF for large systems validated with SLDA
	- •Efficient realtime methods for cooling, analyzing
		- •Quantum Friction

Pinning Force

dE dt $=-\bar{\mathbf{v}}$ $\vec{\mathbf{v}}$ · Ē F

Bulgac, Forbes, Sharma PRL 110 (2013) 241102 [arXiv: 1302.2172]

Thermodynamics

•Well defined: (unlike vortex mass)

- •Accessible from dynamic simulations
- •Extract from stirring simulations

Conclusion

•Virtually all aspects of the MIT experiment are explained by vortex rings:

Long periods, dependence on aspect ratio and interaction strength, anti-decay at finite temperature, imaging after expansion and dependence on B_{min}

- •Combined approach:
	- ETF for large systems validated with SLDA
	- •Efficient realtime methods for cooling, analyzing •Quantum Friction, Pinning force
- Details validate reliability of DFTS for dynamical simulations of defects etc. in neutron stars.