Mass hierarchy determination in reactor antineutrino experiments at intermediate distances. Promises and challenges.

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- Note that for the NH $\Delta m_{31}^2 > \Delta m_{32}^2$ while for IH $|\Delta m_{31}^2| < |\Delta m_{32}^2|$

Why is the hierarchy determination difficult?

In a reasonable approximation (Oth order) the oscillation probabilities can be described in the two flavor picture with only one Δm^2 and only one mixing angle θ

$P(v_{\parallel} \rightarrow v_{\parallel}) = sin^2 2\theta sin^2 (1.27 \Delta m^2 L/E_v)$

In this case, obviously, there is no effect when $\Delta m^2 \rightarrow -\Delta m^2$. Thus, to separate the hierarchies we must to consider three flavor oscillations and thus effects that are small due to the smallness of θ_{13} and/or of $\Delta m^2_{31}/\Delta m^2_{21}$.

Recent oscillation parameter fits already separate the two hierarchies. But the fits do not allow to give significant preference to one of them.

TABLE I: Oscillation parameters: Fits of 2012

	Gonzales-Garcia et.al	Fogli et.al	Forero et.al	Parameter
	JHEP 12 , 2012(2012)	Phys. Rev.D86 013012(2012)	Phys. Rev.D86 073012(2012)	
$\sqrt{2}$)	$7.50_{-019}^{+0.18}$	$7.54^{+0.26}_{-0.22}$	$7.62^{+0.19}_{-0.19}$	$\Delta m^2_{21} \ (10^{-5} \ { m e}^{-5})$
eV^2) NH	$2.47^{+0.07}_{-0.07}$	$2.47^{+0.06}_{-0.10}$	$2.55^{+0.06}_{-0.09}$	$ \Delta m_{31}^2 (10^{-3})$
eV^2) IH	$2.35_{-0.07}^{+0.04}$	$2.38^{+0.07}_{-0.11}$	$2.43^{+0.07}_{-0.06}$	$ \Delta m_{31}^2 (10^{-3})$
+0.013 -0.012	$0.307^{+0.018}_{-0.016}$	$0.320^{+0.016}_{-0.017}$	$ \sin^2 \theta_{12} $	0.302
+0.037 -0.025	$0.386^{+0.024}_{-0.021}$	$0.427^{+0.034}_{-0.027}$	$\sin^2 \theta_{23}$ NH	0.41
+0.037 -0.025	$0.392^{+0.039}_{-0.022}$	$0.427^{+0.034}_{-0.027}$	$\sin^2 \theta_{23}$ IH	0.41
$27^{+0.0023}_{-0.0024}$	$0.0241^{+0.0025}_{-0.0025}$	$0.0246\substack{+0.0029\\-0.0028}$	$\sin^2 \theta_{13}$ NH	0.02
$27^{+0.0023}_{-0.0024}$	$0.0244_{-0.0025}^{+0.0023}$	$0.0250\substack{+0.0026\\-0.0027}$	$\sin^2 \theta_{13}$ IH	0.02

Survival probability for 3-neutrino mixing for the ν_{e} and its antineutrinos in vacuum is

$$P_{ee} = 1 - \{ \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta_{21}) \\ + \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{31}) \\ + \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{32}) \}$$

Where $\Delta_{ij} = 1.27 |\delta m_{ji}^2 (eV^2)| L(m)/E_v(MeV)$. Since $\Delta_{31} \sim \Delta_{32} = \Delta_{atm}$ the P_{ee} exhibits low frequency oscillations governed by Δ_{21} (dominant) and high frequency oscillations governed by Δ_{atm} (subdominant) with amplitude proportional to $\sin^2(2\theta_{13})$. With the relatively large $\sin^2(2\theta_{13}) = 0.092 + 0.017$ these subdominant oscillations are more easily visible.

Moreover, since for normal mass hierarchy (NH) $\Delta_{31} = \Delta_{32} + \Delta_{21}$ while for inverted mass hierarchy (IH) $\Delta_{31} = \Delta_{32} - \Delta_{21}$, there is a phase shift between the two hierarchies proportional to L/E_v.

(For proposals to use reactor neutrinos at intermediate distances see e.g. Choubey, Petcov, Piai (2003) or Schoenert, Lasserre, Oberauer (2003).)



There are two oscillation lengths in the problem. The `solar' Here for $\Delta m^2 + \Delta m^2 = 2x2.4$ And the `atmospheric' $(L/E)_{osc}^{atm} \sim 1000 \text{ m/MeV}.$

> Depending on the hierarchy the atmospheric oscillation lengths are slightly different.

After several oscillations the phase difference between these two possibilities increases; this allows determination of the correct hierarchy.

For realistic input, and the typical reactor neutrino energy ~4 MeV the optimum distance is $L \sim 60 \, \text{km}$.



Observable reactor spectrum with and without oscillations at 60 km



As an aside, when this is measured, we will have another even better proof that the `oscillation' idea is valid. Until now most experiments see just disappearance (in few recent cases also appearance), but no down and up changes of the flux. The example here, from KamLAND (Araki et al., 2005) is a notable exception.



First difficulty: Δm_{31}^2 and Δm_{32}^2 are not known accurately enough. ($\Delta m_{atm}^2 = (\Delta m_{31}^2 + \Delta m_{32}^2)/2 \approx 2.49(0.06)\times 10^{-3} \text{ eV}^2$).



Spectra as functions of E_{vis} at 60 km for 100 kT year exposure



Ideal spectra, no statistical fluctuations. The same Δm^2_{32} for both hierarchies.

Ideal spectra, no statistical fluctuations. Different (degenerate) Δm_{32}^2 for the two hierarchies.

Realistic spectrum with statistical fluctuations for NH and ideal spectrum for IH

Spectra are plotted assuming the energy resolution $\delta E/E = (a^2/E + 1)^{1/2}$ %, a = 2.6 for E in MeV. This value corresponds to the estimated performance of an ideal 100% photon coverage. Note that $E_{vis} \approx E_v$ -0.8 MeV.

(figure from Qian, Dwyer, McKeown, Vogel, Wang and Zhang, (2013).)

Same as in the previous slide, but now the NH/IH ratio is plotted.





Ideal spectra, no statistical fluctuations. The same Δm_{32}^2 for both hierarchies.

Ideal spectra, no statistical fluctuations. Different (degenerate) ∆m²₃₂ for the two hierarchies

Realistic spectrum with statistical fluctuations for NH and ideal spectrum for IH. Note the change in the range of Y axis.

Since, at the present time, the uncertainty in Δm^2_{atm} is comparable (actually a bit larger) to Δm^2_{21} the degeneracy problem means that for a fixed E_v one cannot separate the NH and IH. However, the degeneracy could be, in principle, overcome, by considering a range of L/E_v or, realistically, a range of E_v .

However, with a finite energy resolution, the high frequency (atm. L_{osc}) oscillations of the spectrum, whose phase contains the MH information, will be, at least partially, smeared out.

Lets call the phase difference of the NH and IH oscillatory behavior 2ϕ . The corresponding mass square difference is $\Delta m_{\phi}^2 = (\phi/1.27) (E_v/L)$. When this Δm_{ϕ}^2 remains small and essentially unchanged with E_{vis} , it is impossible to determine the MH. Our simulation suggests that the dividing line is $\Delta m_{\phi}^2 = 0.128 \times 10^{-3} \text{ eV}^2$. For smaller Δm_{ϕ}^2 the degeneracy cannot be overcome.

(see Qian et al. , Phys. Rev. D87, 033005 (2013))



⁽figure from Qian, Dwyer, McKeown, Vogel, Wang and Zhang, (2013).)

Since the goal is determine the both frequencies Δ_{31} and Δ_{32} , the logical approach is to use the Fourier transform. Note that since $\theta_{12} \sim 34^{\circ}$ the amplitude of the Δ_{31} oscillations will be larger than of the Δ_{32} oscillations.

 $\begin{aligned} \mathsf{P}_{ee} &= 1 - \{ \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta_{21}) \\ &+ \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{31}) \\ &+ \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{32}) \} \end{aligned}$

Determination of the mass hierarchy using the Fourier transform:

By performing the integration (or discrete summation) over T = L/E from $t_{min} = L/E_{max}$ till $t_{max} = L/E_{min}$ one can find the peak corresponding to δm_{atm}^2 .

That peak, in fact, consists of two close frequencies one for $|\delta m^2_{31}|$ and the other one for $|\delta m^2_{32}|$.

Since the part of P_{ee} with δm_{31}^2 is proportional to $\cos^2\theta_{12}$ and the part with δm_{32}^2 is proportional to $\sin^2\theta_{12}$, and because $\theta_{12} \sim 34^0$, the peak at δm_{31}^2 is stronger.

(see Learned, Dye, Pakvasa, Svoboda (2008) and Zhan, Wang, Cao, Wen (2008, 2009))



The main peak of the Fourier transform of the data in reactor experiment at 50 km. Full line (NH) and dashed line (IH). The MH can be distinguished by the left or right shoulder, reduced in power by $\sim \cot^4(\theta_{12})$.

(figure from Learned, Dye, Pakvasa, and Svoboda (2008).)

 $FCT(\omega) = \int F(t) \cos(\omega t) dt$, $FST(\omega) = \int F(t) \sin(\omega t) dt$

and ω = 2.54 δ m², t = L/E_v



Zhan, Wang, Cao and Wen, 2008

But the problem is not so simple. In our paper the quantity PV+RL was evaluated with two reactor fluxes (older and new) that differ only by ~ 3-4%. While in both of them the two hierarchies are separated, with the older flux the separation is noticeably less. The problem is made even worse due to the inexact knowledge of Δm^2_{32} .



This figure is for 100 kT year exposure.

The average probability for determining the correct MH evaluated using Monte-Carlo simulation, PV+RL and the pre-2011 flux is only 93%. If instead the Fourier Transform is used, that probability is further reduced, since only part of the information is utilized.



See Ciuffoli et al. , 1302.0624 have shown, in reaction to our finding that the oscillations can be suppressed by using weighted Fourier transforms with weight $exp(-cE^2/MeV^2)$ as indicated in the figure (c = 0.02,0.04,0.08). The x-axis is Δm^2_{32} running from 0.002 to 0.0028 eV². Note, however, that using that trick reduces on average the ability to separate the hierarchies.

In general, one should consider two problems:

- 1) Once the data are collected, what are the constrains on the matter hierarchy they represent.
- 2) How to evaluate the ability of a future experiment to determine the matter hierarchy. This means, loosely, how to judge the sensitivity of a future experiment.

Unlike the usual issue of determining the correct value (and the confidence level) of a parameter that has continuous possible values (say a mixing angle or Δm^2), we are dealing with a quantity that has only two discrete values (sign of Δm^2_{31} and Δm^2_{32} or sign of ($|\Delta m^2_{31}| - |\Delta m^2_{32}|$)).

The way to do it is indicated here: Use Monte-Carlo to simulate a spectrum assuming either NH or IH. Minimize $\chi^2_{\rm NH}$

$$\chi^{2}_{\rm NH} = \sum_{i} \frac{(S^{i}_{m} - S^{i}_{e \rm NH}(\Delta m^{2}))^{2}}{(\delta S^{i}_{m})^{2}} + \chi^{2}_{p}(\Delta m^{2})$$

Where S_{m}^{i} (S_{NH}^{i}) is the measured (expected for NH) spectrum, and the last term is the penalty term from the error in $|\Delta m_{atm}^{2}|$. Find the $\Delta m_{min \ NH}^{2}$. Repeat for IH.

Once this is done, evaluate the difference

$$\Delta \chi^2 \equiv \chi^2_{\rm NH} (\Delta m^2_{\rm min\,NH}) - \chi^2_{\rm IH} (\Delta m^2_{\rm min\,IH}).$$

Neglecting the uncertainties in Δm_{21}^2 , θ_{12} and θ_{13} we obtained the plots of probability densities (areas normalized to unity)



Once the measurement is done, the $\Delta \chi^2$ can be determined, and the $P_{NH}/(P_{NH} + P_{IH})$ can be determined.

Also, the average probability can be calculated. With 100 kT year exposure, resolution a = 2.6, the average probability is 98.9%. This is idealized situation (perfect knowledge of the reactor spectrum and energy scale (see next), this represents the best estimate for the separation of mass hierarchy.

(Note that the proponents of the Daya-Bay II experiment do not agree with our conclusions. They believe that the MH can be determined with > 5σ confidence)

The relation between the $\Delta \chi^2$ and Confidence level (or probability) need to be modified in this case. The probability is substantially less than it would be for the Gaussian case.



Additional challenge: Energy scale nonlinearity.

A small nonlinearity of the energy scale can lead to a substantial reduction of the hierarchy discovery potential (in particular in association with the Δm_{32}^2 uncertainty).

As an illustration, lets assume that the ratio $E_{reconstructed}/E_{real}$ is like in the



figure, for the case when the true hierarchy is IH (blue) or NH (red). In that case the spectrum analysis would lead to wrong MH.

Thus, the nonlinearity of E_{rec}/E_{real} need to be controlled to a fraction of 1% over a wide range of $E_{vis.}$. Current state-of-the-art is ~1.9%. Substantial improvement is required.

Conclusions:

- 1) Determination of the MH in a reactor experiment at intermediate distance is obviously very challenging, but not really unrealistic.
- 2) Besides the necessity of sufficient count rate (hence very large detector), it is necessary to have very good energy resolution, better than existing large detectors.
- 3) Improvement in the accuracy of the known oscillation parameters, in particular Δm_{atm}^2 would help.
- 4) The energy scale nonlinearity need to be improved as well.
- 5) One needs to be careful in determining the degree of confidence with which the MH was determined; the usual relation between the number of σ and CL cannot be used.

Nevertheless, the method is clean in the sense that the outcome is independent of other things, like matter effects, CP phase etc. It appears to be probably the best way to determine the MH.

Daya Bay II: A multi-purpose LS-based experiment

(Now called JUNO for Jiangmen Underground Neutrino Observatory)

Yifang Wang Institute of High Energy Physics NeuTel'2013, March 13, 2013

Idea of the Daya Bay-II Experiment



- 20 kton LS detector
- 3% energy resolution
- **Rich physics possibilities**
 - ⇒ Mass hierarchy
 - Precision measurement of 4 mixing parameters
 - ⇒ Supernovae neutrinos
 - ⇒ Geoneutrinos
 - ⇒ Sterile neutrinos
 - ⇒ Atmospheric neutrinos
 - ⇒ Exotic searches

Estimated IBD rate: ~40/day

Talk by Y.F. Wang at ICFA seminar 2008, Neutel 2011; by J. Cao at Nutel 2009, NuTurn 2012; Paper by L. Zhan, Y.F. Wang, J. Cao, L.J. Wen, PRD78:111103,2008; PRD79:073007,2009

New site: Kaiping county, Jiangmen city



Brief schedule

- Civil preparation: 2013-2014
- Civil construction: 2014-2017
- Detector R&D: 2013-2016
- Detector component production: 2016-2017
- PMT production: 2016-2019
- Detector assembly & installation: 2018-2019
- Filling & data taking: 2020

After a number of reviews, we are approved by the CAS(~CD1)