Towards Exploring Fundamental Symmetries with Lattice QCD

Brian Tiburzi

14 August 2013

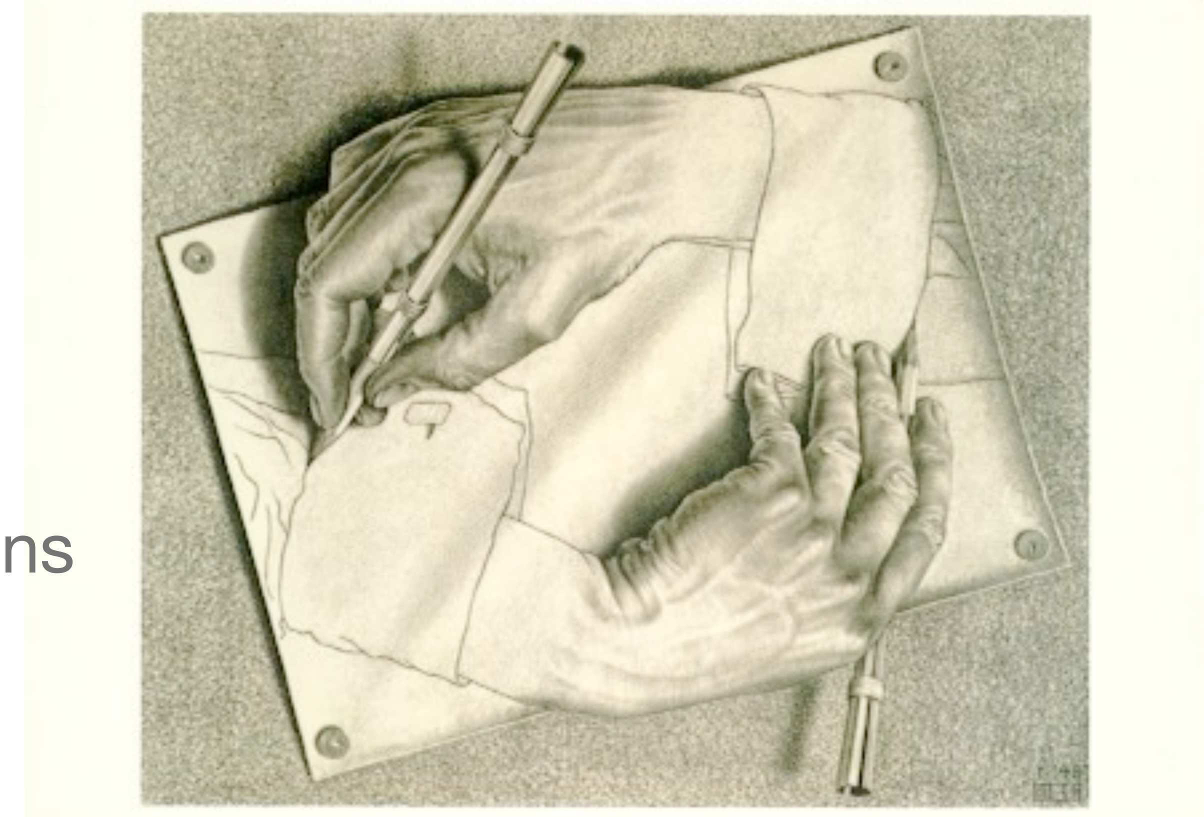
Towards Exploring Fundamental Symmetries with Lattice QCD

- **Lattice QCD**:
  compute single and few-body couplings

- **Hadronic Parity Violation**:
  isovector and isotensor

- **B Violation**: neutron-antineutron oscillations

- **T Violation**: nucleon EDMs



Goal: provide a sense of what challenges lattice QCD must confront

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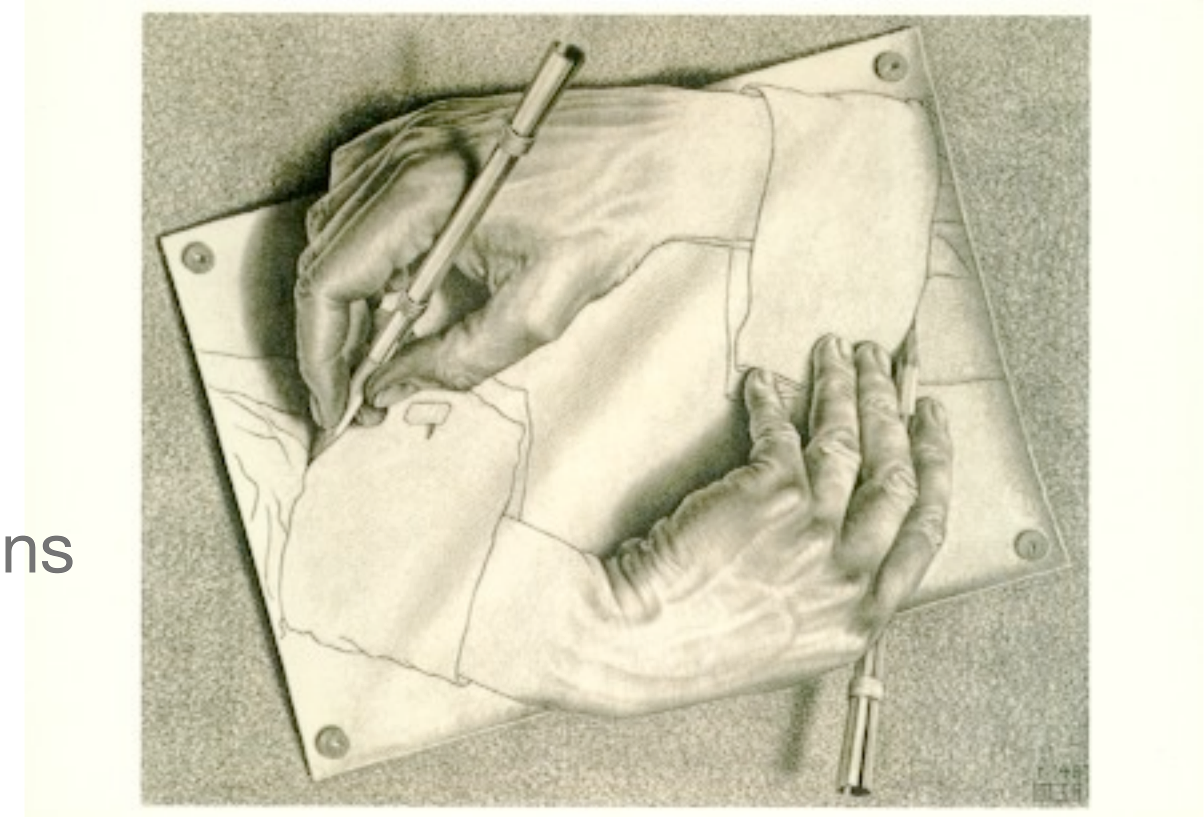
- ~~**T Violation**~~: nucleon EDMs



Goal: provide a sense of what challenges lattice QCD must confront

Quark Interactions to Hadronic Couplings

Ken Wilson
1936-2013

- **Textbook**: gauge theories defined in perturbation theory
- **QCD**: short distance perturbative, long distance non-perturbative

$$\overline{\psi}\left(\not{D}+m_q\right)\psi+\frac{1}{4}G_{\mu\nu}G_{\mu\nu}$$

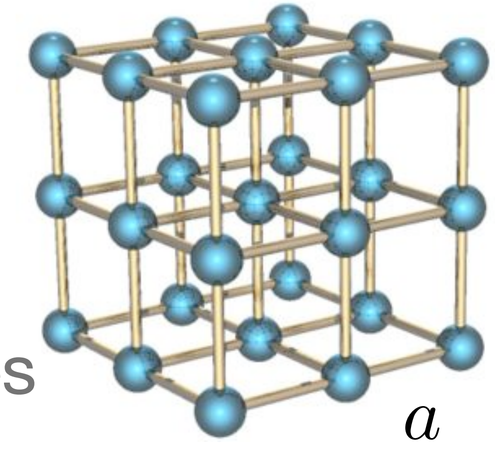
Many Technicalities

$$M_N \qquad \epsilon_b(D)$$

$$\delta_{NN}(k)$$

Wilson Lattice Action
Wilson Fermions

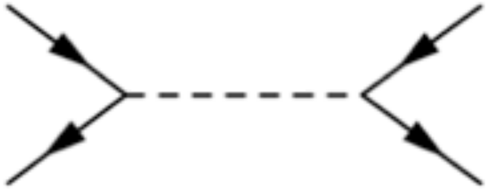
Non-perturbative definition of asymptotically free gauge theories



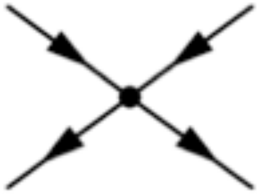
Spectrum
Interactions

Strong interaction observables

- **Quarks couple to other fundamental interactions**: e.g. weak interaction



$$J(x)D(x,0)J(0)=\sum_i C_i(\mu)\mathcal{O}_i(x,\mu)$$



Wilson Operator Product Expansion, Wilson Coefficients, Wilson Renormalization Group

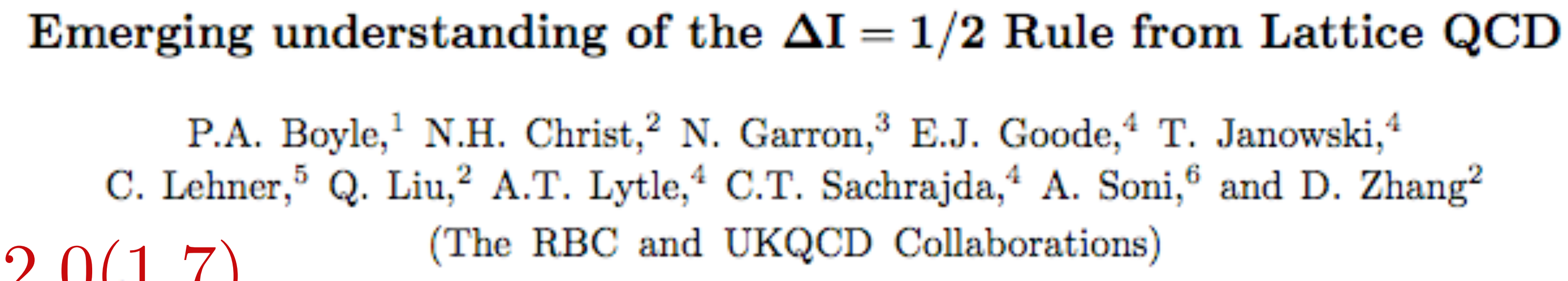
- **Hadronic weak (& *BSM*) interactions require all the Wilson brand names**

Example: K→ππ and ΔI = 1/2 Rule

- **Old Puzzle: I** = 0 weak decay channel experimentally observed ~500x over **I** = 2
- Amplitude level: A0 / A2 ~ 22.5
  pQCD contributes a factor of ~2
  Rest non-perturbative?

$$\mathcal{A} = \sum_i C_i(\mu)\langle\pi\pi|\mathcal{O}_i(\mu)|K\rangle_{\texttt{Lattice}}$$

PRL 110, 152001 (2013)



Emerging understanding of the ΔI = 1/2 Rule from Lattice QCD

P.A. Boyle,[1] N.H. Christ,[2] N. Garron,[3] E.J. Goode,[4] T. Janowski,[4]
C. Lehner,[5] Q. Liu,[2] A.T. Lytle,[4] C.T. Sachrajda,[4] A. Soni,[6] and D. Zhang[2]
(The RBC and UKQCD Collaborations)

- **Almost There?**

$$\mathcal{A}_0/\mathcal{A}_2(m_\pi = 330\,\texttt{MeV}) = 12.0(1.7)$$

- **Theoretical Challenges ΔS = 1 Processes**

Usual Suspects: pion mass, lattice spacing, lattice volume **underway**

Additional Challenges: Physical Kinematics **underway**

Multi-Hadron States and Normalization ✓

Operator Renormalization & Scale Invariance ✓

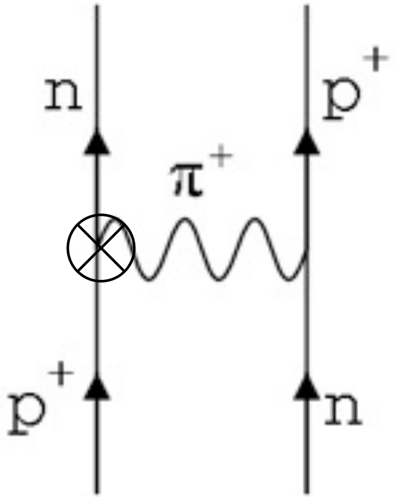
Statistically Noisy Operator Self-Contractions ✓

- **Can such success carry over to weak nuclear processes?**

Example: N→$(N\pi)_s$ and ΔI = 1 Parity Violation

- **Old Problem**: hadronic neutral weak interaction is the least constrained SM current

- **New experiments**: parity violation in few-body systems, map out NN weak interaction?



$$\mathcal{A} = \sum_i C_i(\mu)\langle(\pi N)_s|\mathcal{O}_i(\mu)|N\rangle_{\text{Lattice}}$$

Lattice QCD Calculation of Nuclear Parity Violation

Joseph Wasem **PRC 85, 022501 (2012)**

Signal Found $h^1_{\pi NN} = 1.1(5) \times 10^{-7}$

- **Theoretical Challenges ΔI = 1 Processes**

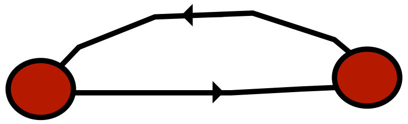
Usual Suspects: pion mass, lattice spacing, lattice volume		**to be done**
Additional Challenges:	Physical Kinematics	**partially solved**
	Multi-Hadron States and Normalization	**to be done**
	Operator Renormalization & Scale Invariance	**to be done**
	Statistically Noisy Operator Self-Contractions	**to be done**

- **How many lattice advances carry over to weak nuclear processes?**

Particle Physics (B=0) vs. Nuclear Physics (B>0)

Pion Correlation Function

Signal $$\sum_{\{A_\mu\}} \langle q\overline{q}(t) q\overline{q}(0)\rangle \sim e^{-m_\pi t}$$



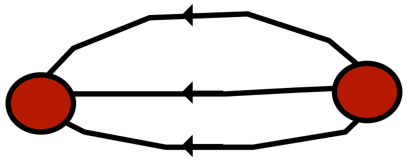
Noise^2 $$\sum_{\{A_\mu\}} \langle q\overline{q}(t) q\overline{q}(t) q\overline{q}(0) q\overline{q}(0)\rangle \sim e^{-2m_\pi t}$$

Signal/Noise

$$\sim \mathrm{const}$$

Nucleon Correlation Function

Signal $$\sum_{\{A_\mu\}} \langle qqq(t) \overline{qqq}(0)\rangle \sim e^{-Mt}$$



Noise^2 $$\sum_{\{A_\mu\}} \langle qqq(t) \overline{qqq}(t) qqq(0) \overline{qqq}(0)\rangle \sim e^{-3m_\pi t}$$

Signal/Noise

$$\sim e^{-(M-\frac{3}{2}m_\pi)t}$$

Baryons are statistically noisy.... scales exponentially with A

(Un)Physical Kinematics in N→(Nπ)$_s$

- Lattice states are created on-shell

$$G(\tau) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle N(\vec{x},\tau) N^\dagger(0,0)\rangle = Z e^{-\sqrt{\vec{p}^2+M_N^2}\,\tau} + \cdots$$

ground-state saturation

- Hadronic transition matrix elements have energy insertion

$$E_N = M_N$$

$$E_{(\pi N)_s} = M_N + m_\pi$$

$$\Rightarrow \quad \langle (\pi N)_s | \mathcal{O}_i(\mu) | N \rangle_{\mathrm{Lattice}} = h^1_{\pi NN}(\Delta E)$$

- Partial solution implemented (due to Beane, Bedaque, Parreno, Savage, **NUPHA:747,** 55 (2005))

$$p \to n\pi^+ \qquad h^1_{\pi NN}(m_\pi)$$

T-invariance

$$n\pi^+ \to p \qquad h^1_{\pi NN}(-m_\pi)$$

$$\Rightarrow \quad h^1_{\pi NN} = \frac{1}{2}\left[h^1_{\pi NN}(m_\pi) + h^1_{\pi NN}(-m_\pi)\right] + \mathcal{O}(m_\pi^2)$$

Consequence: remove via chiral extrapolation but then only can determine chiral limit coupling

Likely small ~10% at 400 MeV pion mass.
Precision demands in nuclear physics not as great as particle physics

- Full solution: determine form factors, extrapolate to zero, e.g. partially twisted BCs

Example: N→(Nπ)$_s$ and ΔI = 1 Parity Violation

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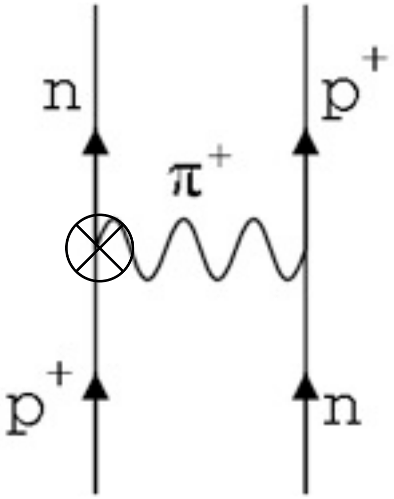

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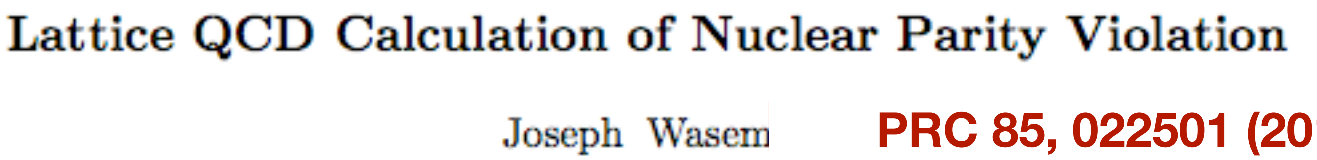

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Multi-Hadron States and Normalization **to be done**

Operator Renormalization & Scale Invariance **to be done**

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- **How many lattice advances carry over to weak nuclear processes?**

Multi-Hadron States and Normalization

- Multi-Hadron operator not used… Matrix element evaluated by a trick

$$G^*(\tau) = \langle 0|N^*(\tau)N^{*\dagger}(0)|0\rangle = Ze^{-E_{(N\pi)_s}\tau} + \cdots$$

$$M_{N^*} > M_N + m_\pi$$

$$(N\pi)_s$$

ground-state saturation

three-quark operator for odd-parity N

four-quarks + antiquark

Method requires this condition to hold for lattice parameters

$$= Ze^{-E_{(N\pi)_s}\tau} + Z'e^{-E^*\tau} + \cdots$$

Unfortunately likely Z << Z'

- Finite volume and infinite volume states have different normalizations

Lellouch, Lüscher, **Commun. Math. Phys. 291,** 31 (2001)

$$|1\rangle_\infty = N_1|1\rangle_V$$

$$|2\rangle_\infty = N_2|2\rangle_V$$

$$_\infty\langle n|n\rangle_\infty = N_n^2\,{}_V\langle n|n\rangle_V = N_n^2 e^{-E_n\tau} + \cdots$$

Not needed for spectrum

$$_\infty\langle 2|\mathcal{O}|1\rangle_\infty = N_2N_1\,{}_V\langle 2|\mathcal{O}|1\rangle_V = N_2N_1(h^1_{\pi NN})_V$$

Lellouch-Lüscher factor requires two-particle energy

Not Computed

Computed

Example: $N\rightarrow(N\pi)_s$ and ΔI = 1 Parity Violation

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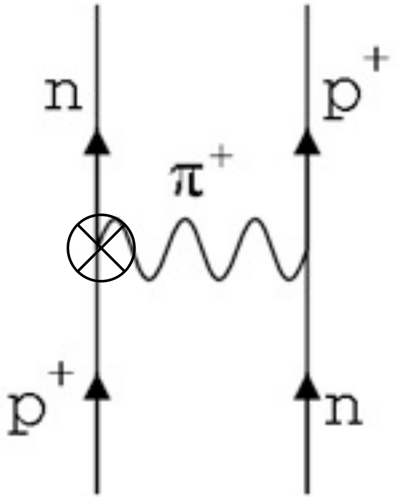

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Operator Renormalization and Scale Invariance

$$\mathcal{A} = \sum_i C_i(\mu)\langle(\pi N)_s|\mathcal{O}_i(\mu)|N\rangle$$

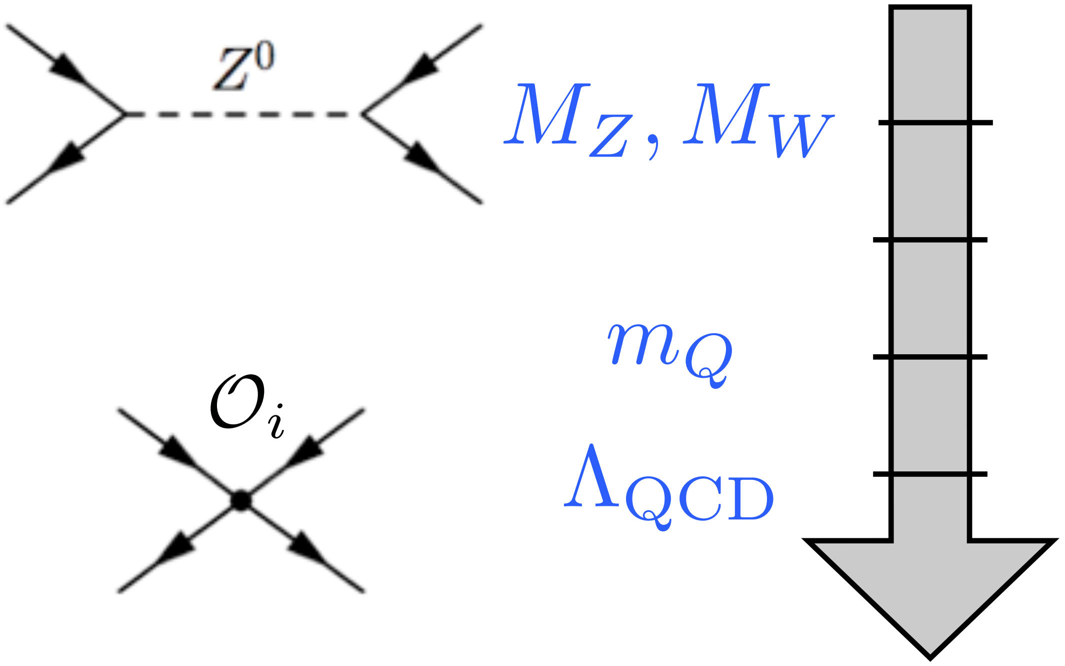
$\mu = 90\,\mathtt{GeV}$

computable in pQCD at high scale

$\mu = 1 - 2\,\mathtt{GeV}$

computable on lattice at low scale

Tree Level



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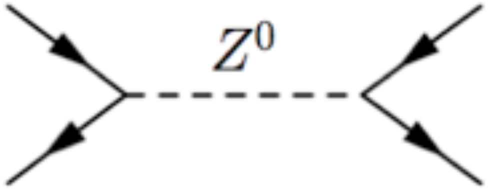
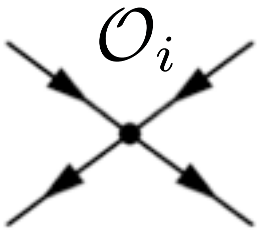
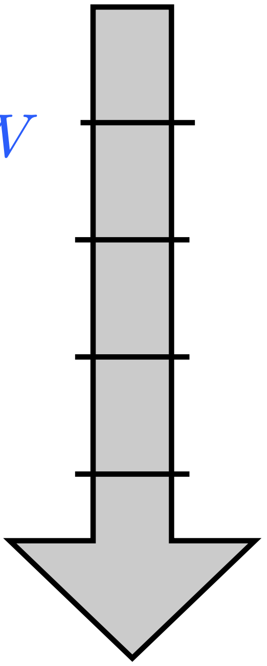


M_Z, M_W



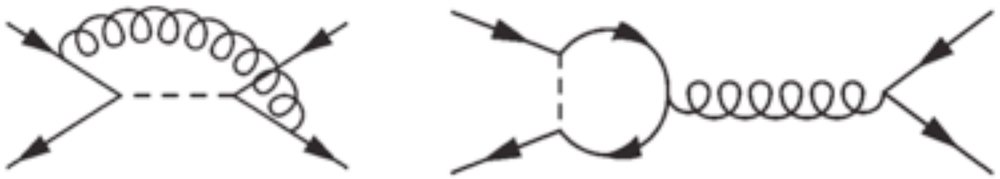
m_Q

Λ_{QCD}



One Loop

$\log \frac{M_Z^2}{p^2}$



$\log \frac{\mu^2}{p^2}$

$$\log \frac{M_Z^2}{p^2} = \log \frac{\mu^2}{p^2} - \log \frac{\mu^2}{M_Z^2}$$

$$\delta C(\mu) \sim -\alpha_s(\mu) \log \frac{\mu^2}{M_Z^2}$$

70's Donoghue, McKellar, . . . , 90's Dia Savage Liu Springer

Operator Renormalization and Scale Invariance

Tree Level

$$\mathcal{L}_{\rm PV}^{I=1} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

$\sin^2\theta_W$ Non-Strange

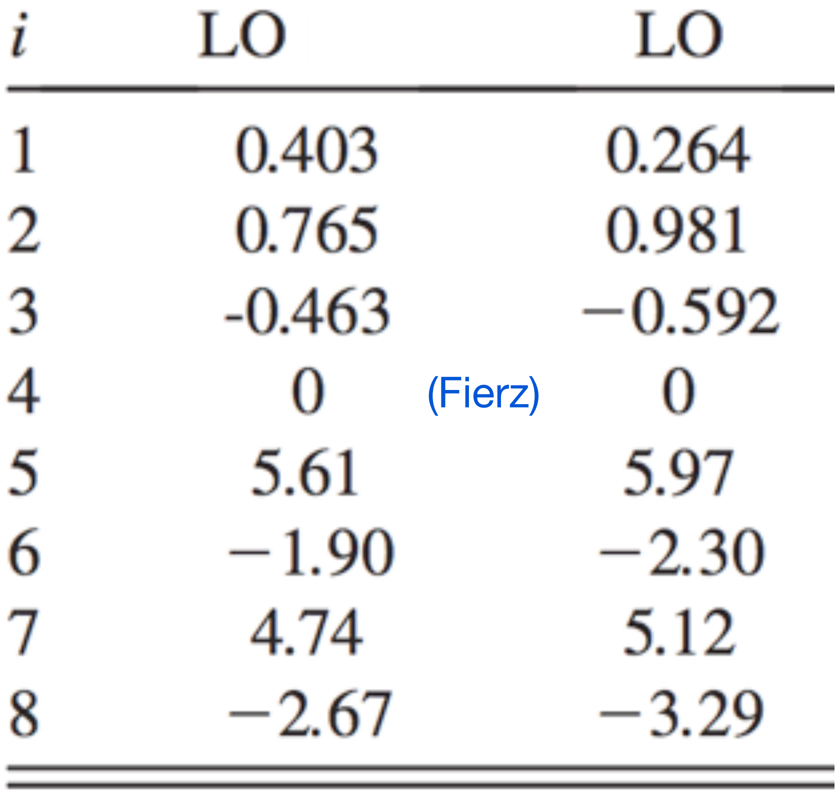
1 vs. Strange

One Loop Results

$$O_1 = (\bar{u}u - \bar{d}d)_A(\bar{u}u + \bar{d}d)_V,$$
$$O_2 = (\bar{u}u - \bar{d}d]_A[\bar{u}u + \bar{d}d)_V,$$
$$O_3 = (\bar{u}u - \bar{d}d)_V(\bar{u}u + \bar{d}d)_A,$$
$$O_4 = (\bar{u}u - \bar{d}d]_V[\bar{u}u + \bar{d}d)_A,$$

$$O_5 = (\overline{u}u - \overline{d}d)_A(\overline{s}s)_V$$
$$O_6 = (\overline{u}u - \overline{d}d]_A[\overline{s}s)_V$$
$$O_7 = (\overline{u}u - \overline{d}d)_V(\overline{s}s)_A$$
$$O_8 = (\overline{u}u - \overline{d}d]_V[\overline{s}s)_A$$

$C_i(\mu = 1\,\texttt{GeV})\,/\,C_1^{\rm Tree}$



i	LO	LO
1	0.403	0.264
2	0.765	0.981
3	-0.463	−0.592
4	0	0
5	5.61	5.97
6	−1.90	−2.30
7	4.74	5.12
8	−2.67	−3.29

(Fierz)

- Discrepancies

DSLS provide only ratios

$\alpha_s(m_c)/\alpha_s(m_b) = 1.44$

**Using their ratios,
I get their values**

No heavy quark masses
quoted in 1990 **PDG**

Dia, Savage, Liu, Springer PLB **271**, 403 (1991)

Tiburzi, PRD **85** 054020 (2012)

Operator Renormalization and Scale Invariance

Tree Level

$$\mathcal{L}^{I=1}_{\text{PV}} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

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$O_3 = (\bar{u}u - \bar{d}d)_V(\bar{u}u + \bar{d}d)_A,$

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$O_5 = (\overline{u}u - \overline{d}d)_A(\overline{s}s)_V$

$O_6 = (\overline{u}u - \overline{d}d]_A[\overline{s}s)_V$

$O_7 = (\overline{u}u - \overline{d}d)_V(\overline{s}s)_A$

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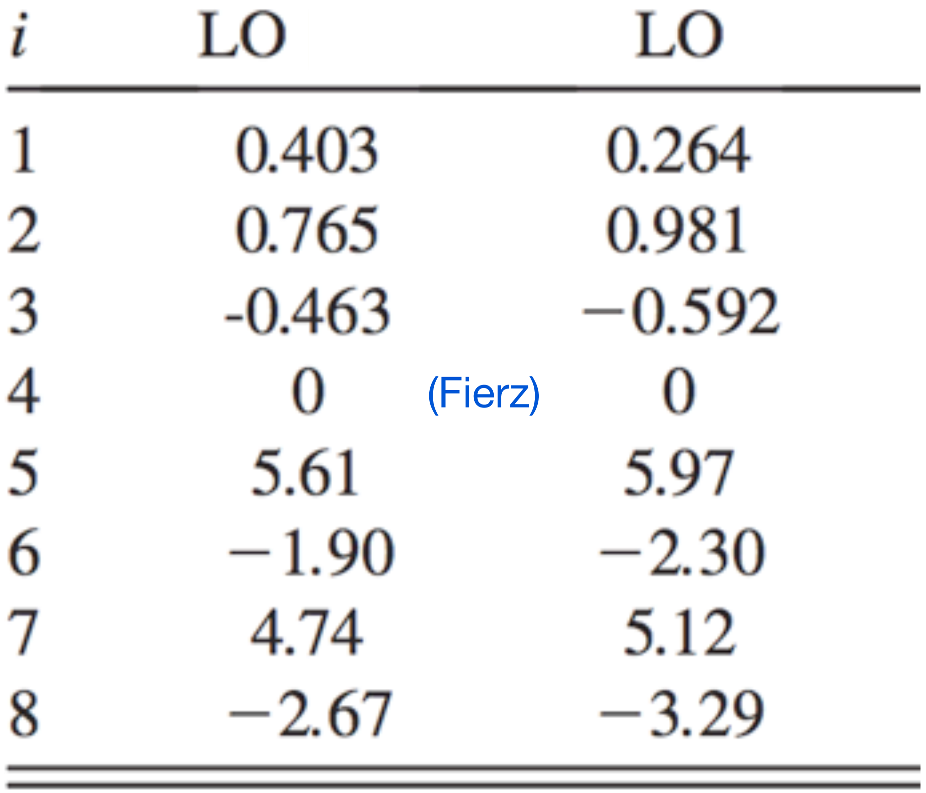
$C_i(\mu = 1\,\texttt{GeV})\,/\,C_1^{\text{Tree}}$



i	LO	LO	LO: 1992 **PDG**
1	0.403	0.264	$0.54(4)$
2	0.765	0.981	$0.55(6)$
3	-0.463	−0.592	$-0.35(3)$
4	0 (Fierz)	0	0
5	5.61	5.97	$5.35(7)$
6	−1.90	−2.30	$-1.57(10)$
7	4.74	5.12	$4.45(8)$
8	−2.67	−3.29	$-2.12(15)$

Dia, Savage, Liu, Springer PLB **271**, 403 (1991)

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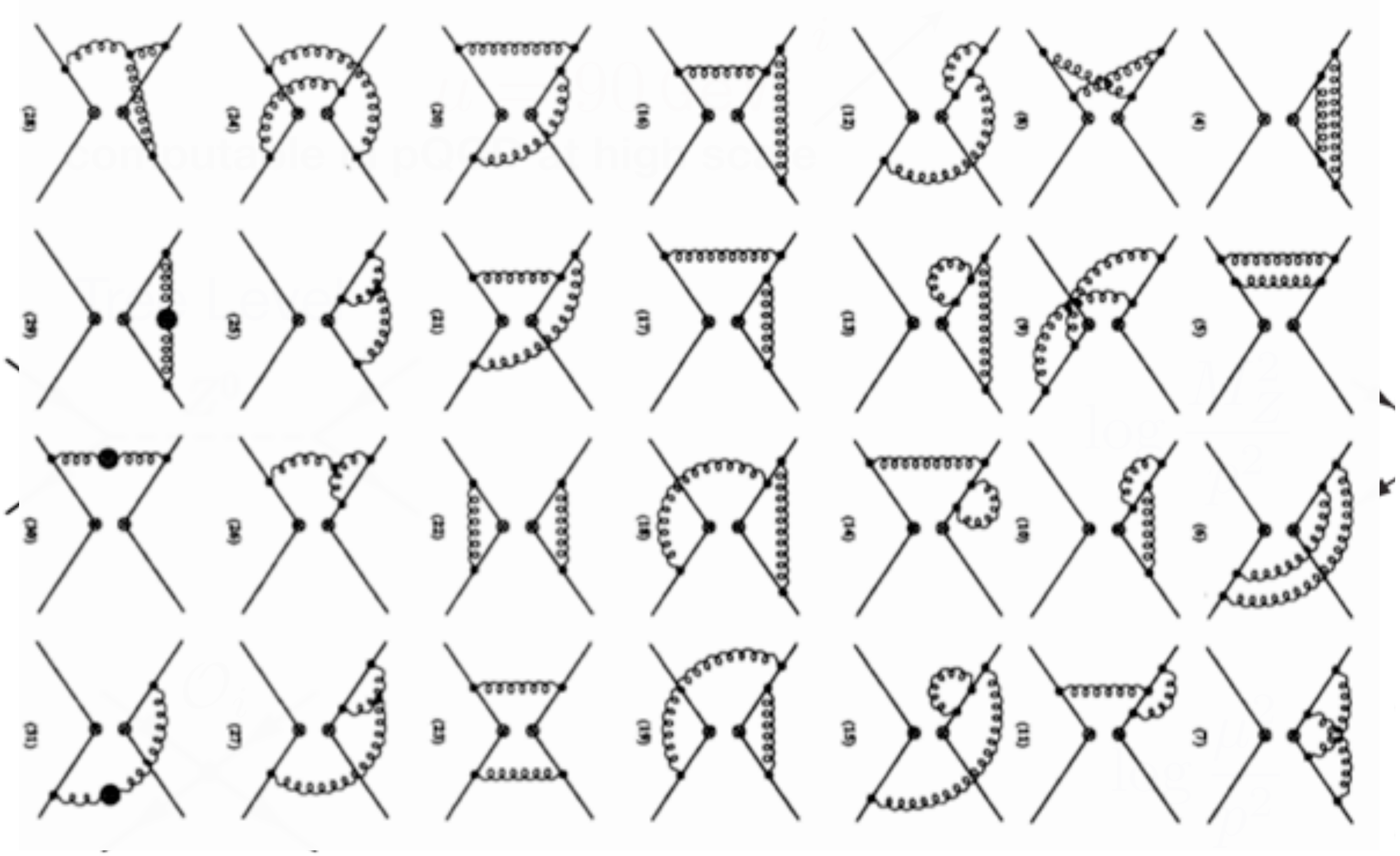
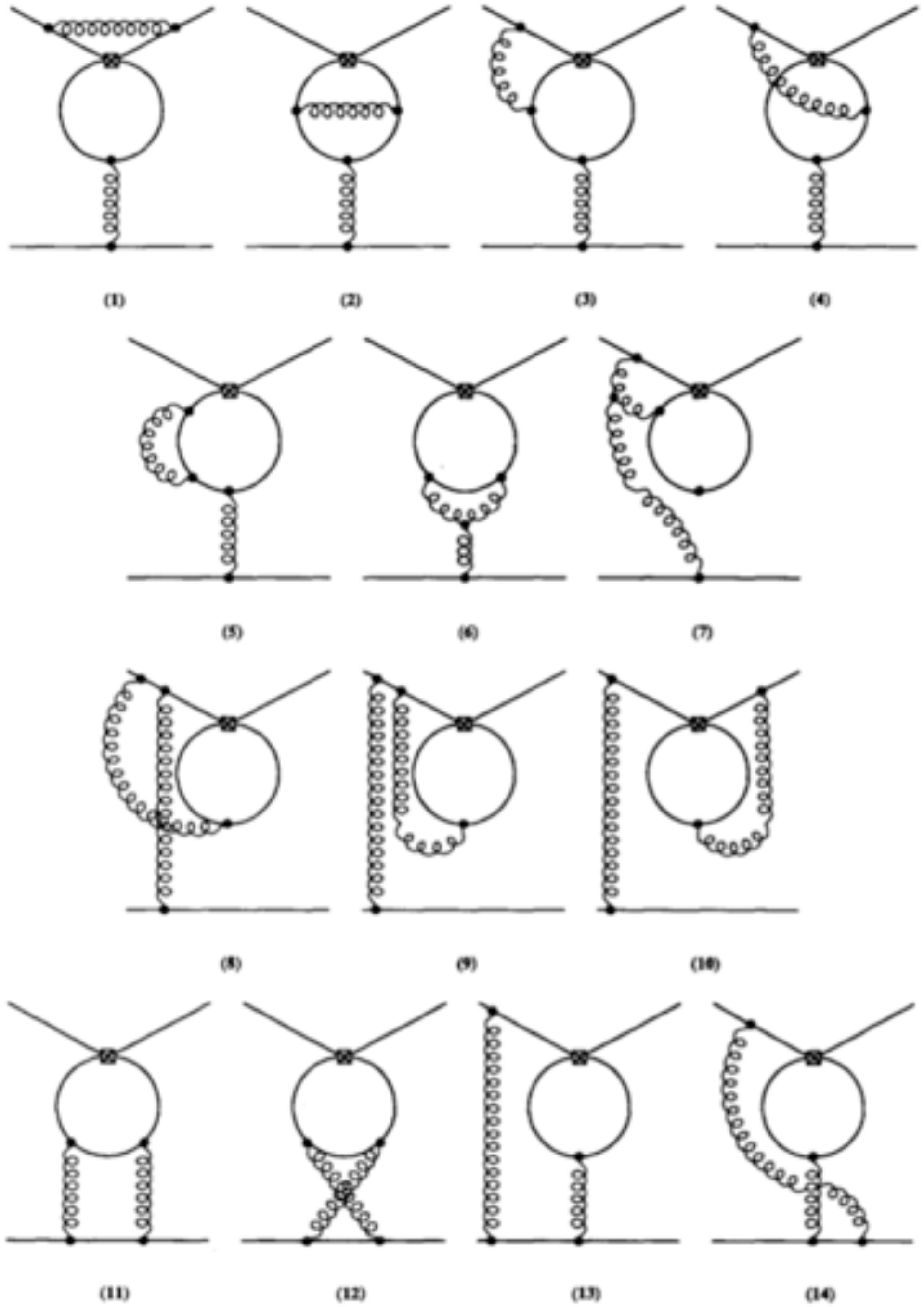
$$\mathcal{A} = \sum C_i(\mu)\langle(\pi N)_s|\mathcal{O}_i(\mu)|N\rangle$$





e

Two Loop

$\alpha_s(1\,\mathtt{GeV}) \sim 0.4$

QCD Renormalization of Isovector Parity Violation

Results ('t Hooft-Veltman scheme)

$$\mathcal{L}_{\text{PV}}^{I=1} = \sum_i C_i(\mu)\mathcal{O}_i(\mu)$$

Non-singlet chirality conservation:
only 5 independent operators

$$L \otimes L - R \otimes R$$

$$L \otimes R - R \otimes L$$

Alleged: 95% probe of hadronic neutral current

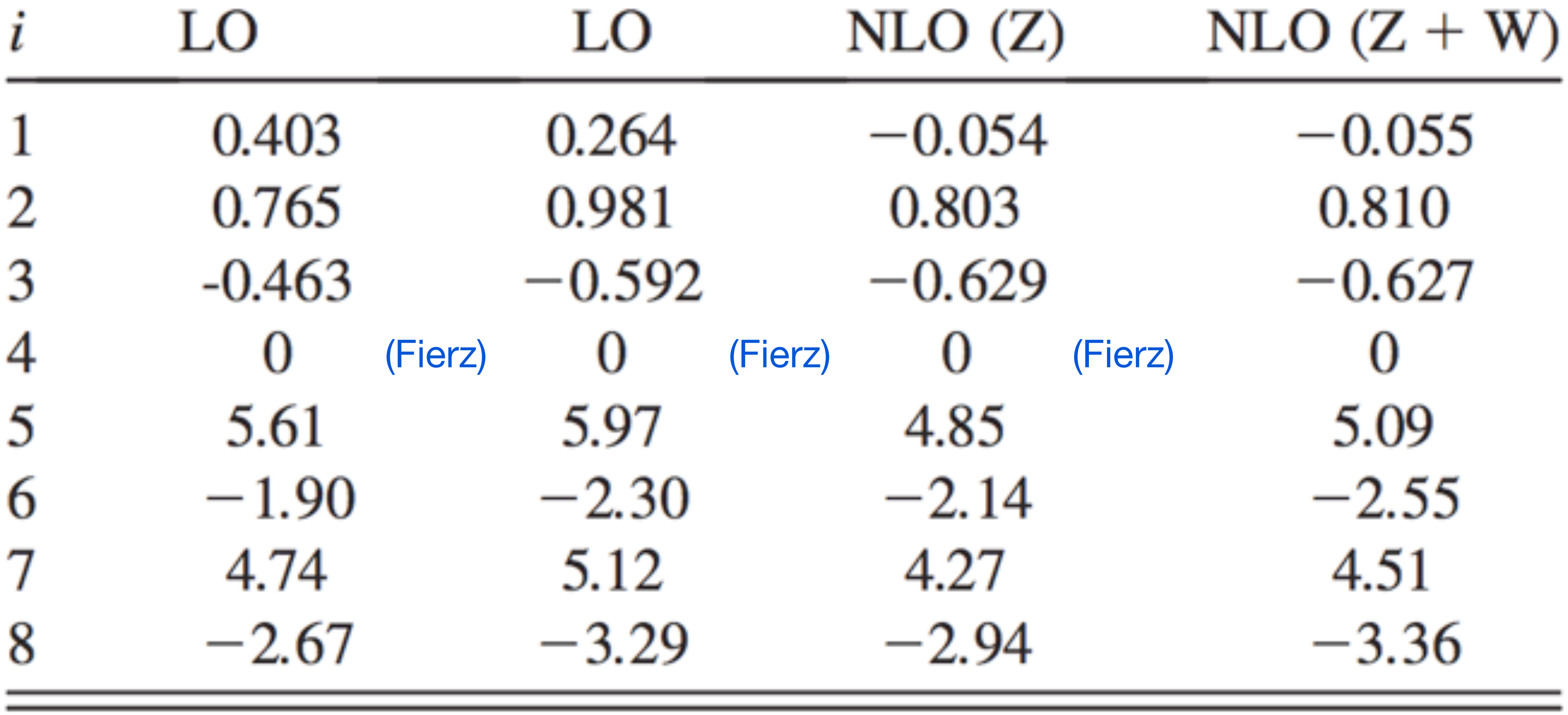
$\sin^2\theta_W$ Non-Strange

vs.

1 Strange

80 - 100%
Dynamical Question!

$C_i(\mu = 1\,\texttt{GeV}) \,/\, C_1^{\text{Tree}}$



i	LO	LO	NLO (Z)	NLO (Z + W)
1	0.403	0.264	−0.054	−0.055
2	0.765	0.981	0.803	0.810
3	-0.463	−0.592	−0.629	−0.627
4	0 (Fierz)	0 (Fierz)	0 (Fierz)	0
5	5.61	5.97	4.85	5.09
6	−1.90	−2.30	−2.14	−2.55
7	4.74	5.12	4.27	4.51
8	−2.67	−3.29	−2.94	−3.36

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$\mu = 90\,\mathtt{GeV}$

computable in pQCD at high scale

$\mu = 1 - 2\,\mathtt{GeV}$

computable on lattice at low scale

- Scale Invariance: requires same renormalization scheme

pQCD ‘t Hooft-Veltman scheme

5 independent PV operators in chiral basis

Anisotropic Lattice Regularization + Wilson Fermions

14 independent PV operators

Unphysical + unphysical chiral mixing

- Matching calculation required...

Example: N→(Nπ)$_s$ and ΔI = 1 Parity Violation

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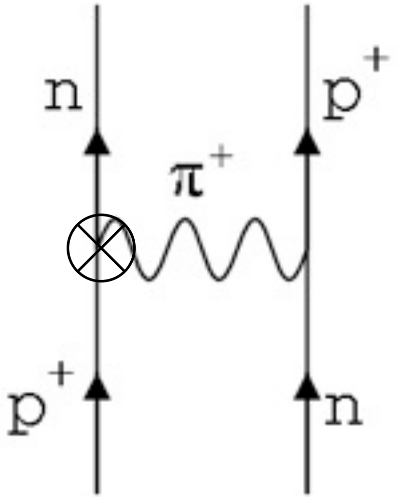

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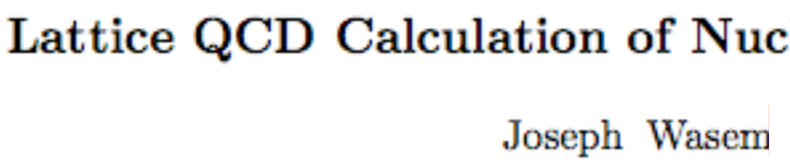
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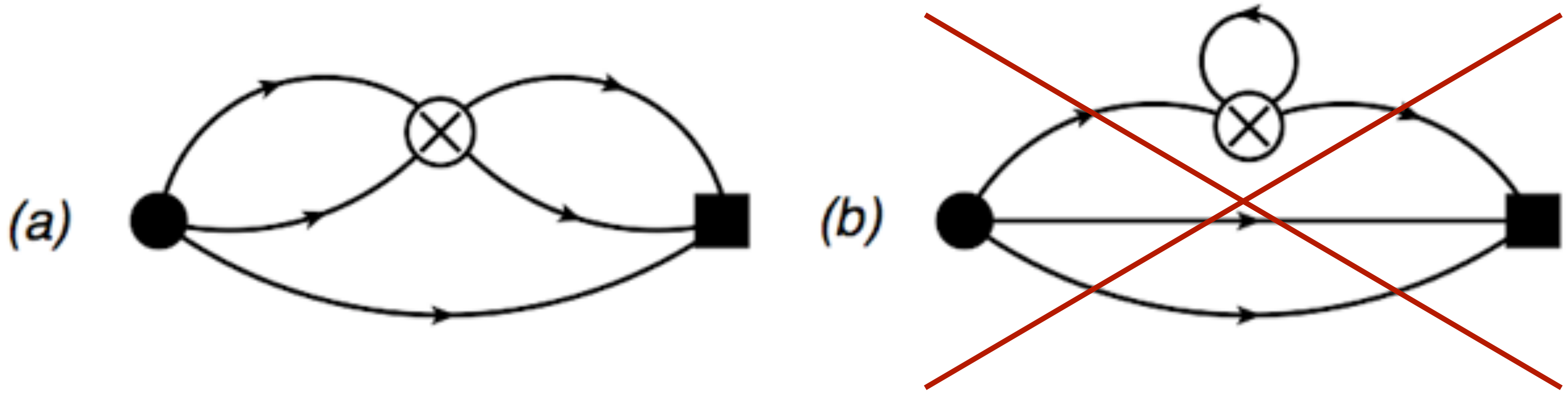
Statistically Noisy Operator Self-Contractions **to be done**

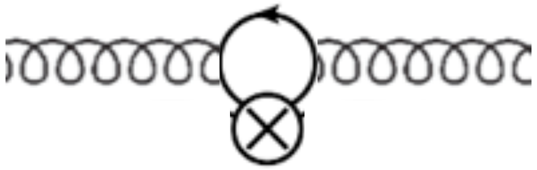
- **How many lattice advances carry over to weak nuclear processes?**

Statistically Noisy Operator Self-Contractions

$$G(\tau', \tau) = \langle 0 | N(\tau') \mathcal{O}_i(\tau) N^{*\dagger}(0) | 0 \rangle$$

Another notorious difficulty





quark disconnected diagrams

Vector and Axial-Vector self-contractions

$\sin^2 \theta_W$

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$$O_2 = (\bar{u}u - \bar{d}d]_A[\bar{u}u + \bar{d}d)_V,$$
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$$O_4 = (\bar{u}u - \bar{d}d]_V[\bar{u}u + \bar{d}d)_A,$$

$(a) + \cancel{(b)}$

1

$$O_5 = (\overline{u}u - \overline{d}d)_A(\overline{s}s)_V$$
$$O_6 = (\overline{u}u - \overline{d}d]_A[\overline{s}s)_V$$
$$O_7 = (\overline{u}u - \overline{d}d)_V(\overline{s}s)_A$$
$$O_8 = (\overline{u}u - \overline{d}d]_V[\overline{s}s)_A$$

$\cancel{(b)}$

Wilson coeffs.

Flavor dependence? ~m_q

Extend to SU(3) + chiral corrections?

Utilize Fierz redundancy?

$\overline{s}s$ $\overline{s}\gamma_\mu s$

small nucleon strangeness

$$\langle \overline{s}\gamma_\mu s \rangle \ll \langle \overline{q}\gamma_\mu q \rangle ?$$

0.16 from Adelaide

Isotensor Parity Violation

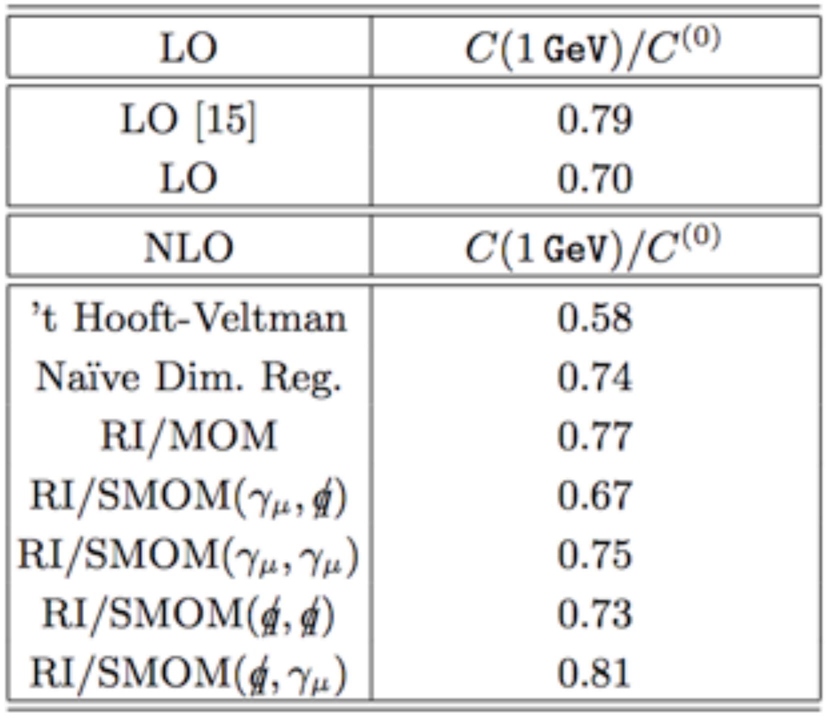
$$\mathcal{O} = (\bar{q}\tau^3 q)_A(\bar{q}\tau^3 q)_V - \frac{1}{3}(\bar{q}\vec{\tau}\, q)_A \cdot (\bar{q}\vec{\tau}\, q)_V$$

- Only **one** operator & **without** self-contractions

$$\mathcal{L}_{PV}^{\Delta I=2} = \frac{G_F}{\sqrt{2}} C(\mu)\mathcal{O}(\mu)$$

Operator Renormalization

Tiburzi, PRD**86:** 097501 (2012)



LO	$C(1\,\texttt{GeV})/C^{(0)}$
LO [15]	0.79
LO	0.70
NLO	$C(1\,\texttt{GeV})/C^{(0)}$
’t Hooft-Veltman	0.58
Naïve Dim. Reg.	0.74
RI/MOM	0.77
RI/SMOM$(\gamma_\mu, \not{q})$	0.67
RI/SMOM(γ_μ, γ_μ)	0.75
RI/SMOM$(\not{q}, \not{q})$	0.73
RI/SMOM$(\not{q}, \gamma_\mu)$	0.81

1992 PDG
0.78(1)

[15] Kaplan Savage, NuPhA **556** (1993)

Wilson fermions still to do...

Better proving ground for Lattice QCD?

$$\mathcal{L}_{NN} = [\vec{\nabla} p^\dagger \cdot \vec{\sigma}\, \sigma_2\, p^*] \cdot [n^T \sigma_2\, n] + \ldots$$

s- to p-wave **NN** interaction

Operator matrix element between
two hadrons (beyond current reach?)

πN interactions

$$\mathcal{L}_{\pi\pi N} + \mathcal{L}_{\pi\gamma N}$$

External fields could ``substitute” for pions

πPV

Isotensor pion interactions exist

Lattice compute parameters DDH potential?

... inevitably leads to chiral parity violating potential

Fundamental Symmetries and Lattice QCD

- **Lattice QCD**: Wilsonian machinery turns high-scale interactions (both SM & *Beyond*) into QCD-scale hadronic couplings
- After decades of dedicated work, trustworthy results emerging e.g. **K→ππ**

Theory Needs for Next-Decade Lattice QCD?

- **Hadronic Parity Violation**:
  πN-coupling more or less challenging than K→ππ?
  Methods for coupling to pions?
  NN-interactions?
  Isovector parity-violating lattices?

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Fundamental QCD Interaction Needed to Explore Fundamental Symmetries

Lellouch-Lüscher Factor

- **Single Particle Energy Quantization:** $E = \sqrt{\vec{p}^2 + M^2}$ $\qquad$ $\vec{p} = \frac{2\pi}{L}\vec{n}$
- **Two Particle Energy Quantization:** $E_{\text{total}} = \sqrt{k^2 + M^2} + \sqrt{k^2 + m^2}$ $\qquad$ $\vec{P} = 0$

$$n\pi - \delta_0(k) = \phi(k)$$

(known function for a torus)

- **One-to-Two Particle Amplitude:**

$$|\mathcal{M}_\infty|^2 = \frac{8\pi V^2 M E_{\text{total}}^2}{k^2}\left[\delta'(k) + \phi'(k)\right]|\mathcal{M}_V|^2$$

Generalization for energy insertion:

Lin, Martinelli, Pallante, Sachrajda, Villadoro **NuPhB:**650, 301 (2003)

Kim, Sachrajda, Sharpe **NuPhB:**727, 218 (2005)

Auxiliary Fields for Isovector Parity Violation

- Perhaps only a Gedankenexperiment until exascale computers materialize

E.g. $\mathcal{O} = \left(\bar{q}\gamma_\mu\gamma_5\tau^3 q\right)\left(\bar{q}\gamma_\mu q\right) \longrightarrow -a\left[\bar{q}\gamma_\mu\left(\gamma_5\tau^3 - b\cdot 1\right)q\right]^2$ $P\otimes\tau^1$

τ^3–chiral symmetry

Introduces PC and PV four-quark operators

Integrate in auxiliary field $\Delta\mathcal{L} = \sigma^2 + ia\,\sigma\left[\bar{q}\gamma_\mu\left(\gamma_5\tau^3 - b\cdot 1\right)q\right]$

No sign problem $\gamma_5\otimes\tau^1$–Hermiticity

- Can implement all isovector PV operators in sign-problem-free ways
  Continuum limit, parameter tuning (!?!?)

$$\langle p|\mathcal{L}_{\rm PV}^{I=1}|\pi n\rangle = h_\pi^1 \quad \to \langle p|\pi^+(x)|n\rangle_\sigma$$

Other PV observables: Nucleon anapole moment: just calculate anapole form factor

PV NN interactions from PV part of NN correlators

Bodies buried in gauge field generation