“I’d rather be lucky than good”: electron-deuteron scattering in χEFT

Daniel Phillips
Ohio University

Research supported by the US department of energy

Outline

- χPT→χEFT: why we iterate
- Elastic electron-deuteron scattering: are χEFT calculations lucky or good?
- f_L in d(e,e'p)
- Implications of fine tuning in the $^1S_0$ for electromagnetic processes
- Summary and outlook

χPT for nuclear forces

χPT for nuclear forces

- χPT⇒pion interactions are weak at low energy.

  Weinberg (1990), apply χPT to V, i.e. expand it in P=(p/Λ_χ,m_π/Λ_χ)

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \ldots$$

Ordonez, Ray, van Kolck (1996); Epelbaum, Meissner, Gloeckle (1999); Entem, Machleidt (2001)

χPT for nuclear forces

- χPT⇒pion interactions are weak at low energy.

  Weinberg (1990), apply χPT to V, i.e. expand it in P=(p/Λ_χ,m_π/Λ_χ)

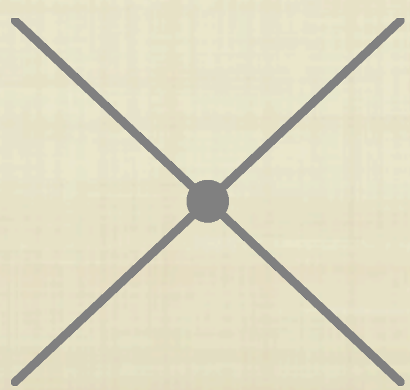
$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$V = V^{(0)} + V^{(2)} + V^{(3)} + \dots$$

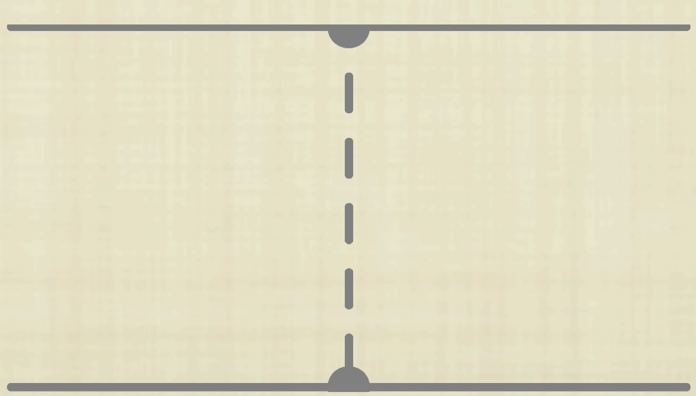
Ordonez, Ray, van Kolck (1996); Epelbaum, Meissner, Gloeckle (1999); Entem, Machleidt (2001)

- Leading-order V:

$V^{(0)} =$



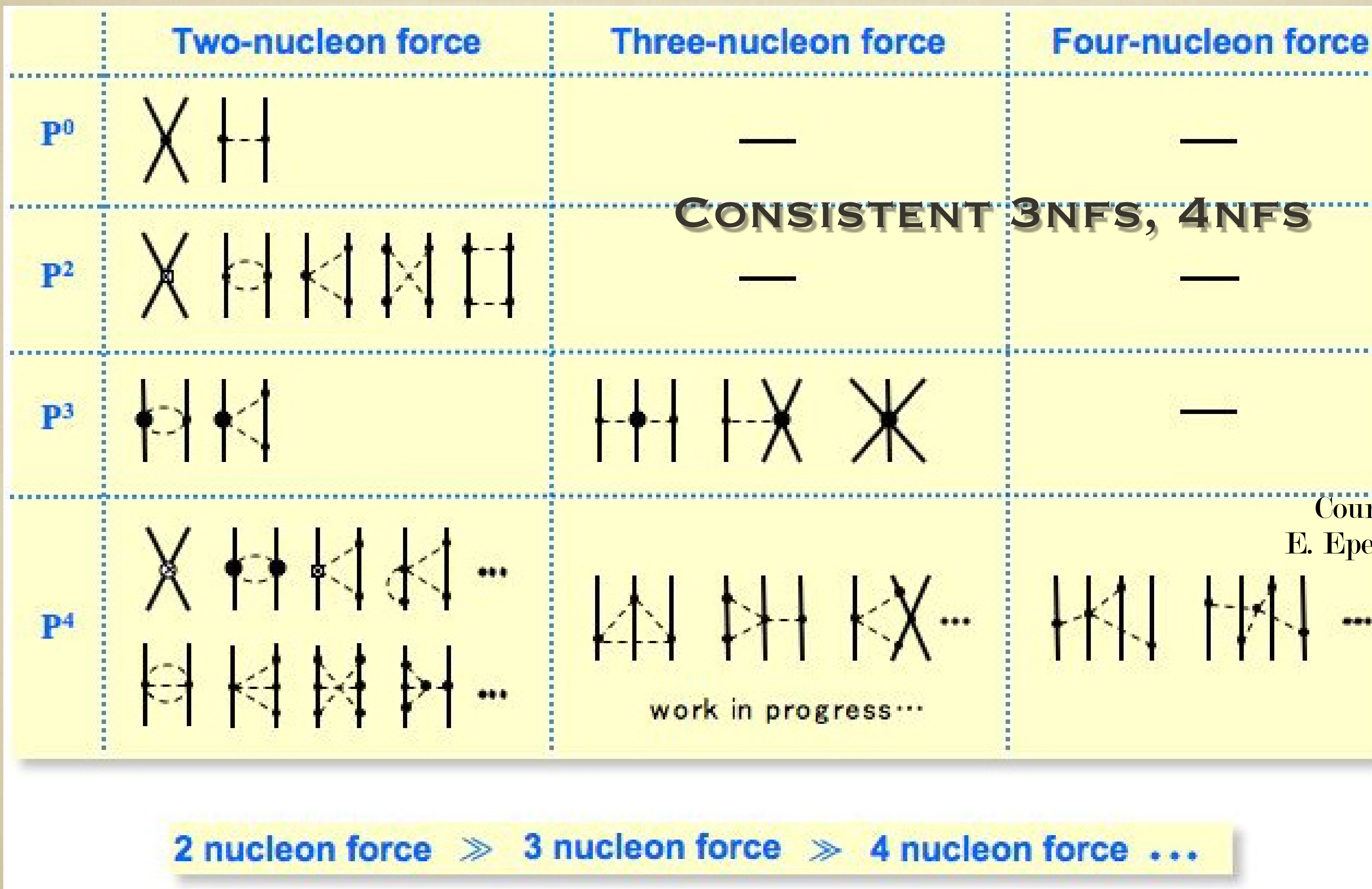
$+$



;

$$\langle \mathbf{p}'|V|\mathbf{p}\rangle = C^{3S1}P_{3S1} + C^{1S0}P_{1S0} + V_{1\pi}(\mathbf{p}' - \mathbf{p})$$

Higher orders in V

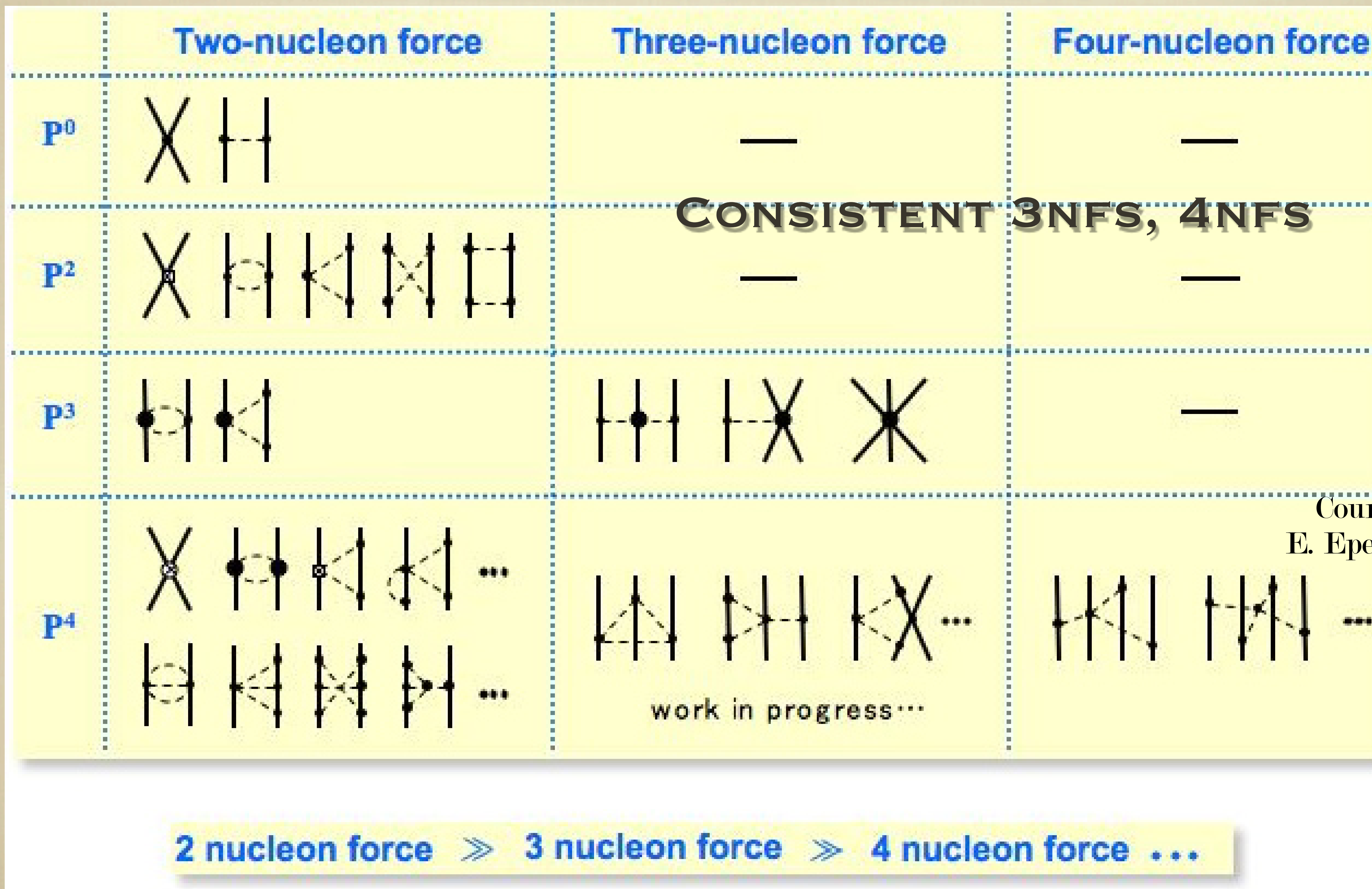


Consistent 3nfs, 4nfs

Courtesy
E. Epelbaum

Higher orders in V

(Ordonez, Ray, van Kolck; Kaiser, Brockmann, Weise; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)

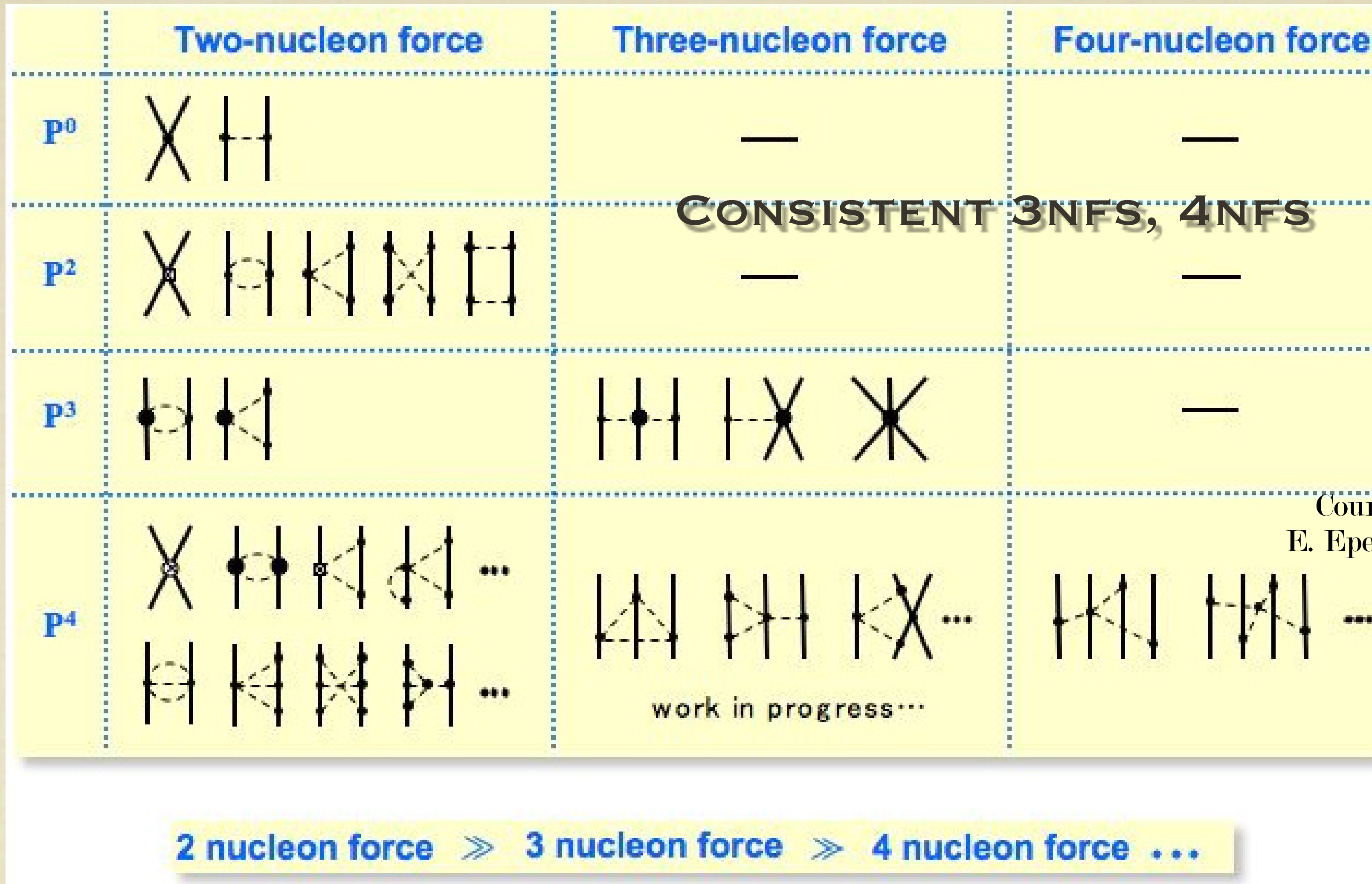


CONSISTENT 3NFS, 4NFS

Courtesy
E. Epelbaum

Higher orders in V

(Ordonez, Ray, van Kolck; Kaiser, Brockmann, Weise; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)



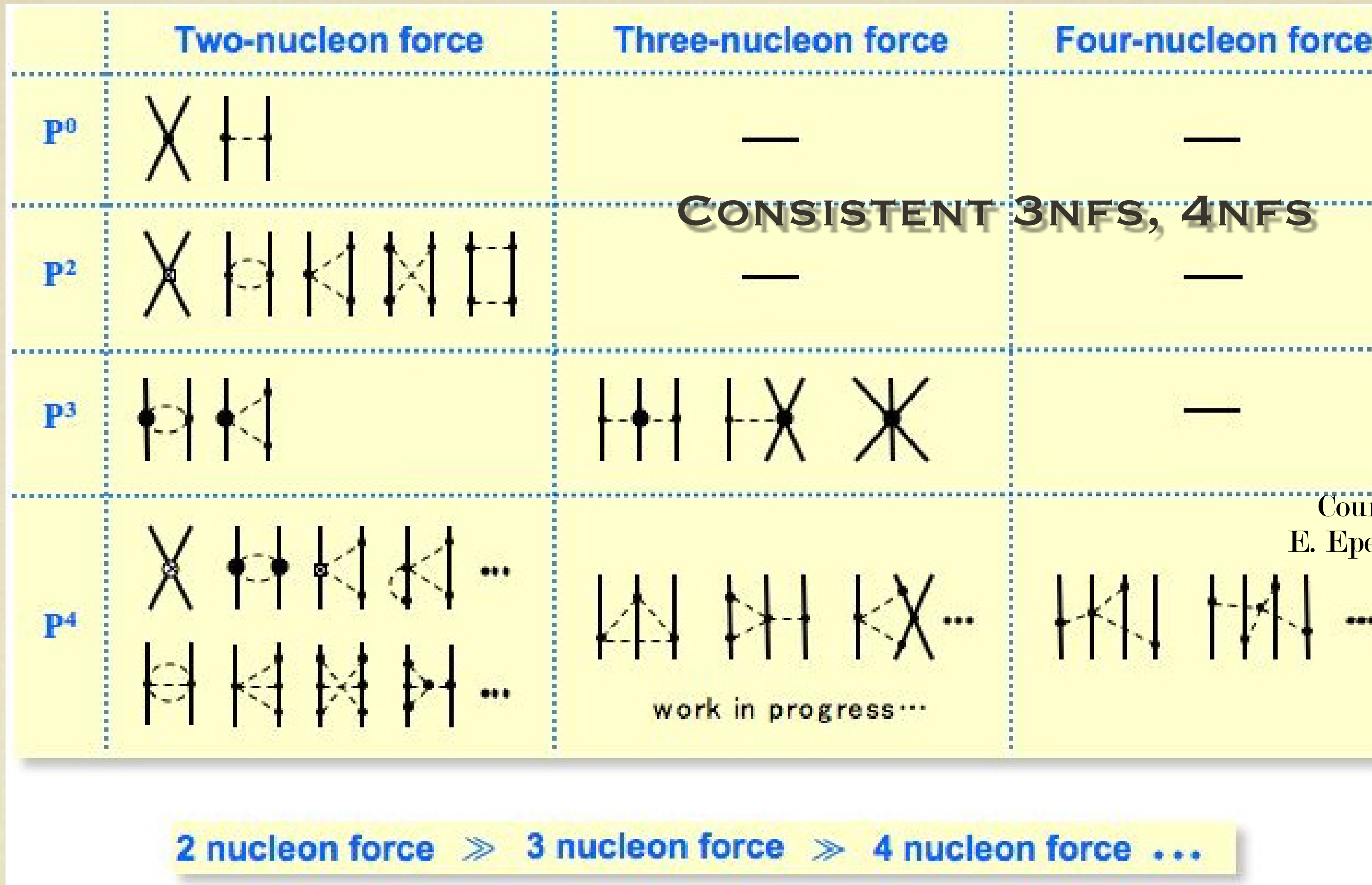
CONSISTENT 3NFS, 4NFS

Courtesy
E. Epelbaum

- No difficulties with counting for long-distance V

Higher orders in V

(Ordonez, Ray, van Kolck; Kaiser, Brockmann, Weise; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)



CONSISTENT 3NFS, 4NFS

Courtesy
E. Epelbaum

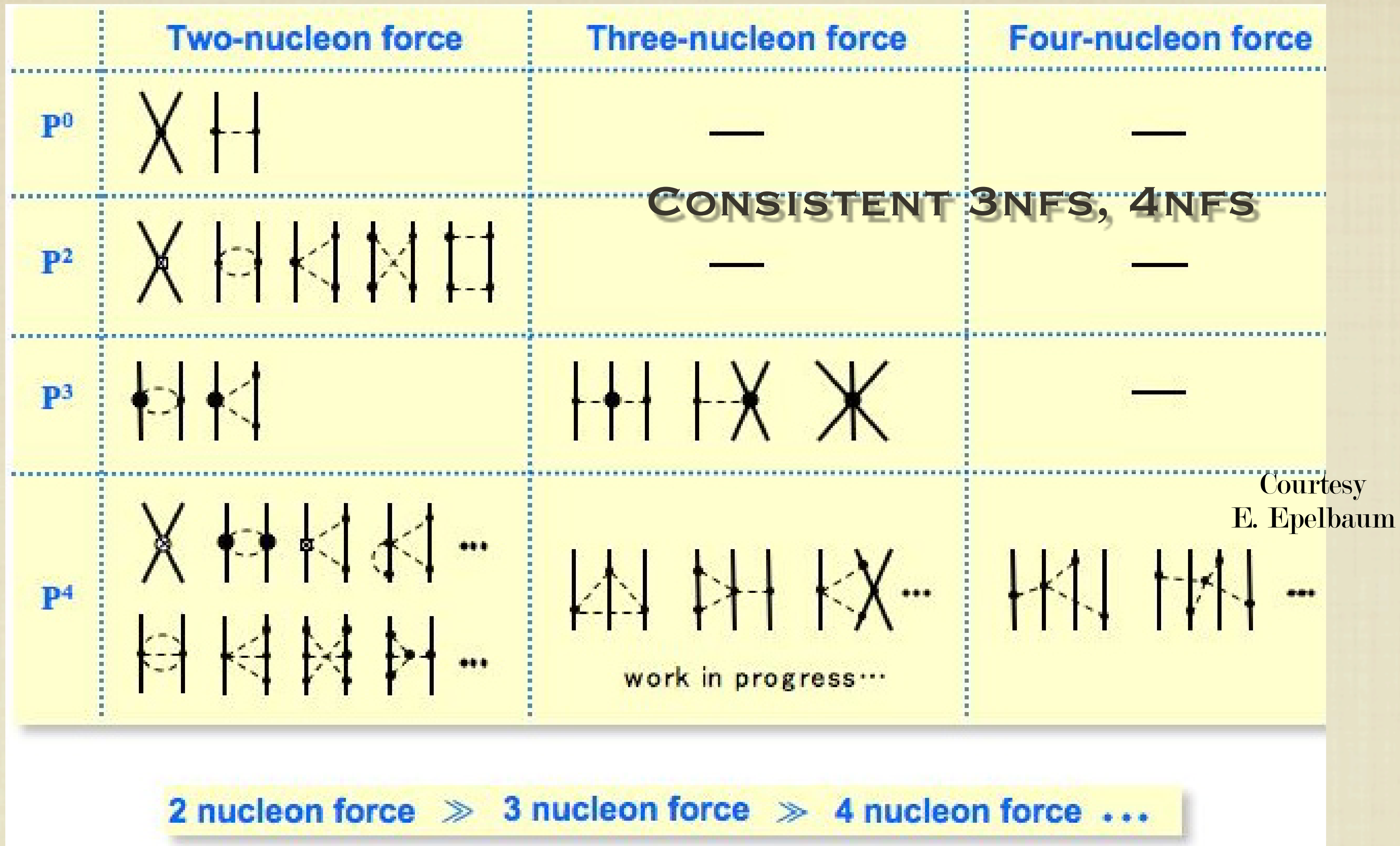
- No difficulties with counting for long-distance V

- Breakdown of expansion for that part of V: r≈0.9 fm

Baru, Epelbaum, Hanhart, Hoferichter, Kudratsyev, DP (2012)

Higher orders in V

(Ordonez, Ray, van Kolck; Kaiser, Brockmann, Weise; Epelbaum, Meissner, Gloeckle; Entem, Machleidt)



- No difficulties with counting for long-distance V
- Breakdown of expansion for that part of V: r≈0.9 fm

Baru, Epelbaum, Hanhart, Hoferichter, Kudratsyev, DP (2012)

- Here I present discussion of “Delta-less” EFT

But is it a (chiral) EFT?

But is it a (chiral) EFT?

- Existence of perturbative expansion?
- Renormalized?
- *A priori* error estimates?

But is it a (chiral) EFT?

- Existence of perturbative expansion?
- Renormalized?
- *A priori* error estimates?

Need to go back and re-examine why we iterate one-pion exchange, in order to obtain a well-defined, renormalized (i.e. cutoff-independent) leading order around which we can perturb

But is it a (chiral) EFT?

- Existence of perturbative expansion?
- Renormalized?
- *A priori* error estimates?

Need to go back and re-examine why we iterate one-pion exchange, in order to obtain a well-defined, renormalized (i.e. cutoff-independent) leading order around which we can perturb

- Note: don't need $\Lambda \to \infty$, just Λ varied by a factor~2 around $\Lambda_{\chi SB}$

But is it a (chiral) EFT?

- Existence of perturbative expansion?
- Renormalized?
- *A priori* error estimates?

Need to go back and re-examine why we iterate one-pion exchange, in order to obtain a well-defined, renormalized (i.e. cutoff-independent) leading order around which we can perturb

- Note: don't need $\Lambda \to \infty$, just Λ varied by a factor~2 around $\Lambda_{\chi SB}$

Goal: once we understand what terms are present in χEFT up to some order, we can include them in a potential, and use it with a low cutoff in order to do nuclear physics calculations

Fun facts about one-pion exchange

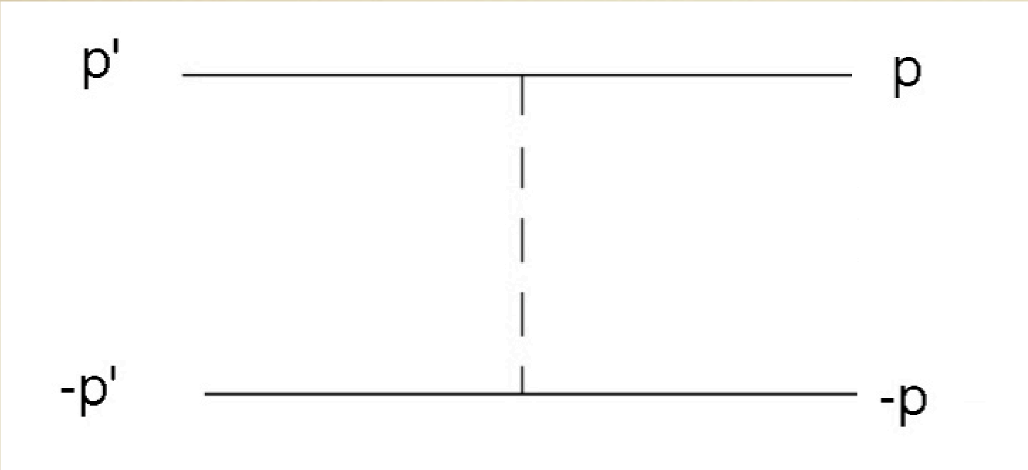
$$V(\mathbf{r}) = \tau_1^a \tau_2^a \left[\sigma_1 \cdot \sigma_2 Y(r) + S_{12}(\hat{r}) T(r)\right]$$

$$S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2;$$

$$Y(r) = \frac{g_A^2 m_\pi^2}{48\pi f_\pi^2} \frac{e^{-m_\pi r}}{r};$$

$$T(r) = \frac{g_A^2}{16\pi f_\pi^2} e^{-m_\pi r} \left[\frac{m_\pi^2}{3r} + \frac{m_\pi}{r^2} + \frac{1}{r^3}\right]$$

Fun facts about one-pion exchange



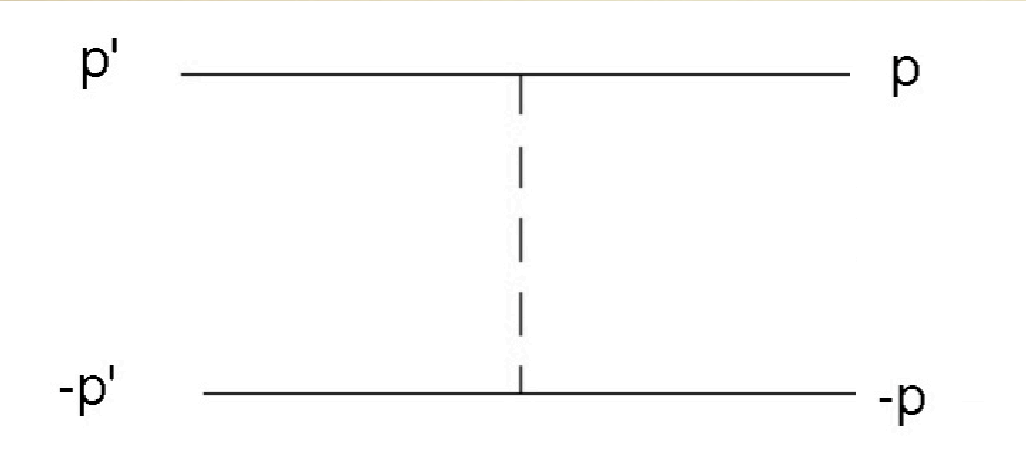
$$V(\mathbf{r}) = \tau_1^a \tau_2^a \left[\sigma_1 \cdot \sigma_2 Y(r) + S_{12}(\hat{r}) T(r)\right]$$

$$S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2;$$

$$Y(r) = \frac{g_A^2 m_\pi^2}{48\pi f_\pi^2} \frac{e^{-m_\pi r}}{r};$$

$$T(r) = \frac{g_A^2}{16\pi f_\pi^2} e^{-m_\pi r} \left[\frac{m_\pi^2}{3r} + \frac{m_\pi}{r^2} + \frac{1}{r^3}\right]$$

Fun facts about one-pion exchange



$$V(\mathbf{r}) = \tau_1^a \tau_2^a \left[\sigma_1 \cdot \sigma_2 Y(r) + S_{12}(\hat{r}) T(r)\right]$$

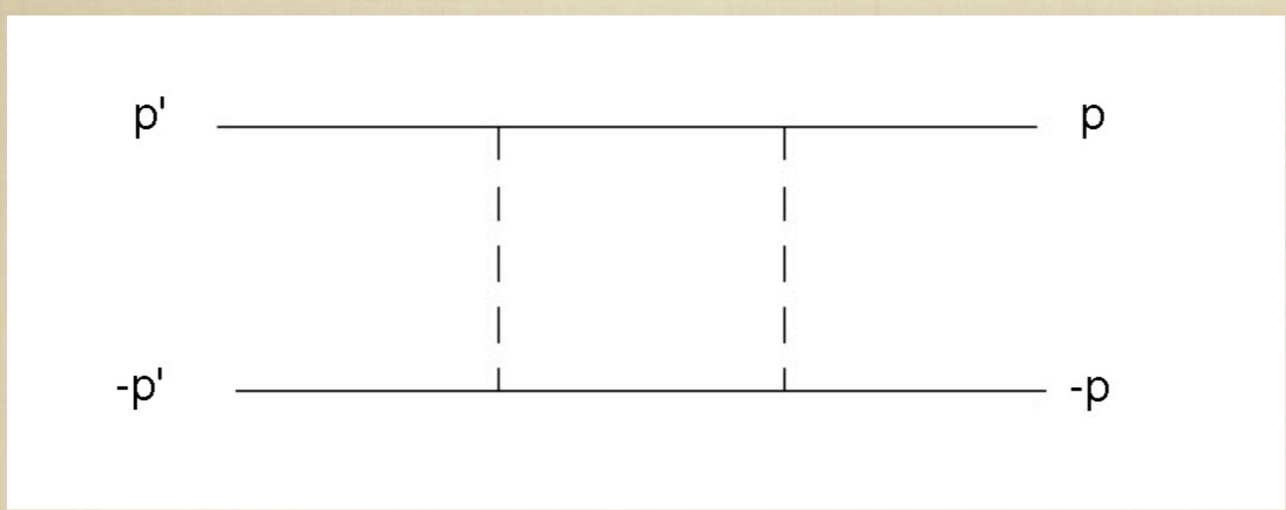
$$S_{12}(\hat{r}) = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2;$$

$$Y(r) = \frac{g_A^2 m_\pi^2}{48\pi f_\pi^2} \frac{e^{-m_\pi r}}{r};$$

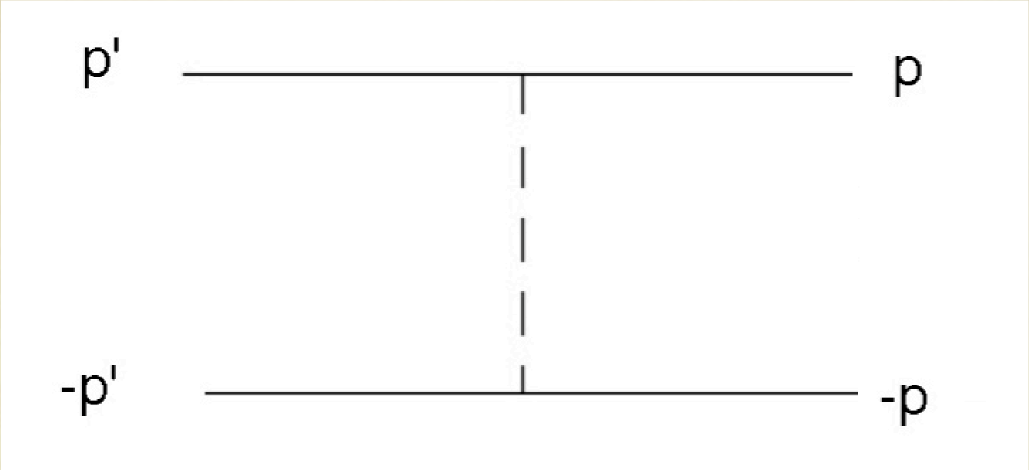
$$T(r) = \frac{g_A^2}{16\pi f_\pi^2} e^{-m_\pi r} \left[\frac{m_\pi^2}{3r} + \frac{m_\pi}{r^2} + \frac{1}{r^3}\right]$$

- Momentum scales present: m_π and $\Lambda_{NN} = \frac{16\pi f_\pi^2}{g_A^2 M} \approx 300$ MeV
- χSB predicts $1/r^3$ potential that couples waves with ΔL=2
- Tensor part of 1π exchange does not appear for S=0
- $1/r^3$ part of 1π exchange “screened” by centrifugal barrier for large L

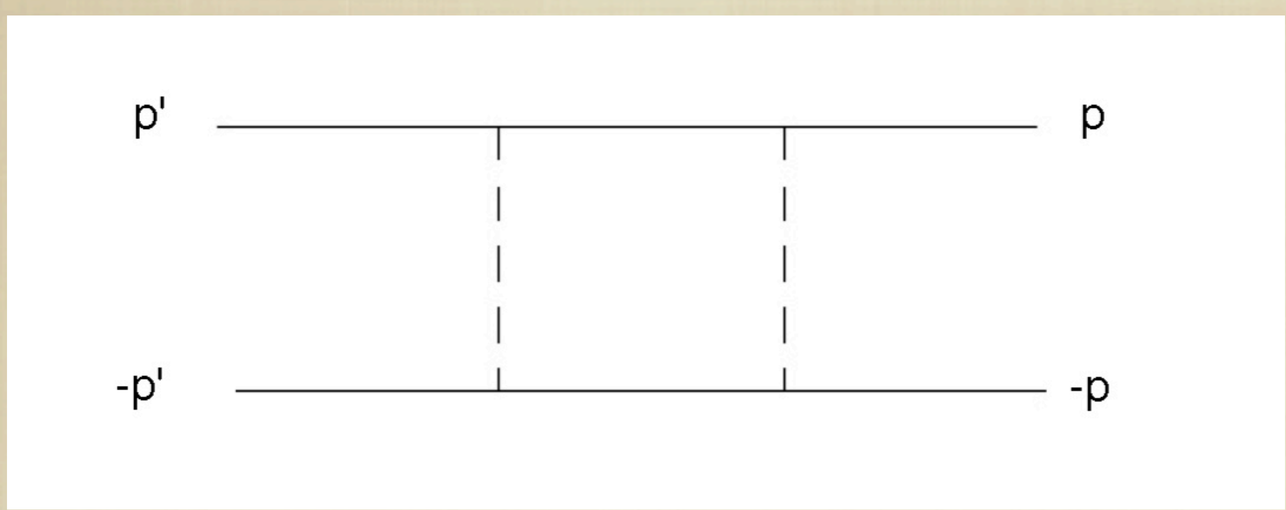
The quest for leading order



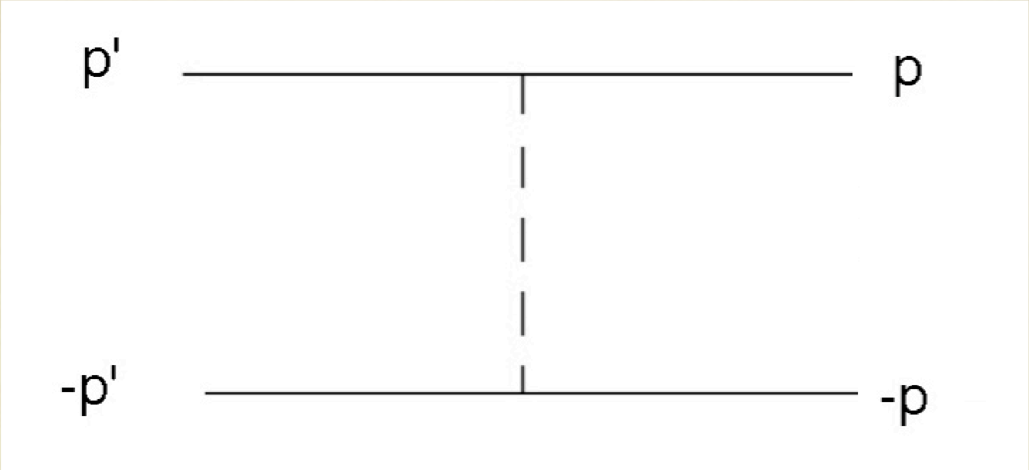
vs



The quest for leading order



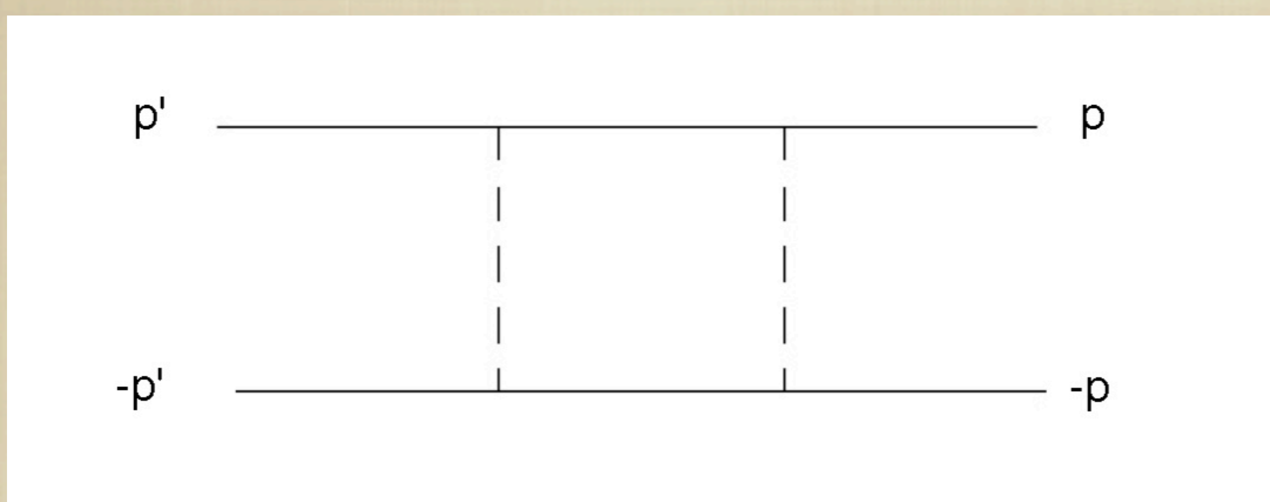
vs



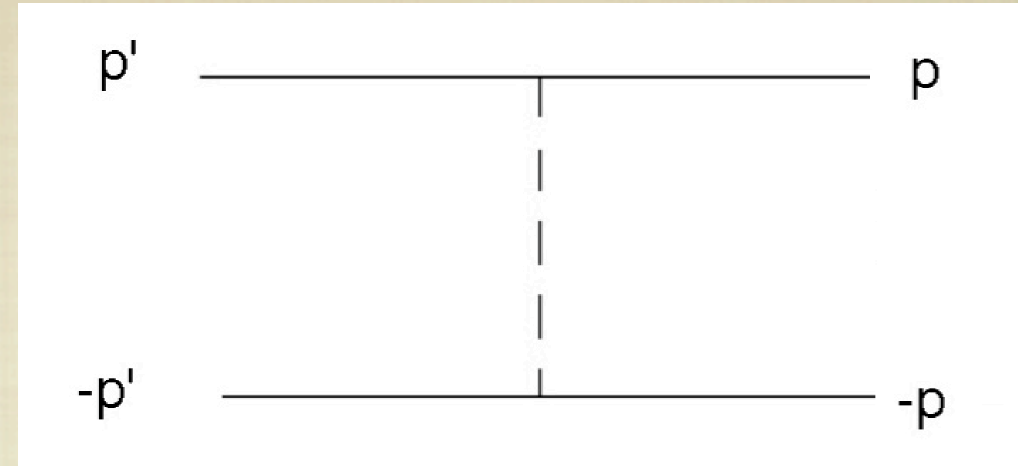
- Iterates of one-pion exchange become comparable with tree-level for momenta of order Λ_{NN}...in low partial waves

Fleming, Mehen, Stewart (2000); Beane, Bedaque, Savage, van Kolck (2002); Birse (2006)

The quest for leading order



vs

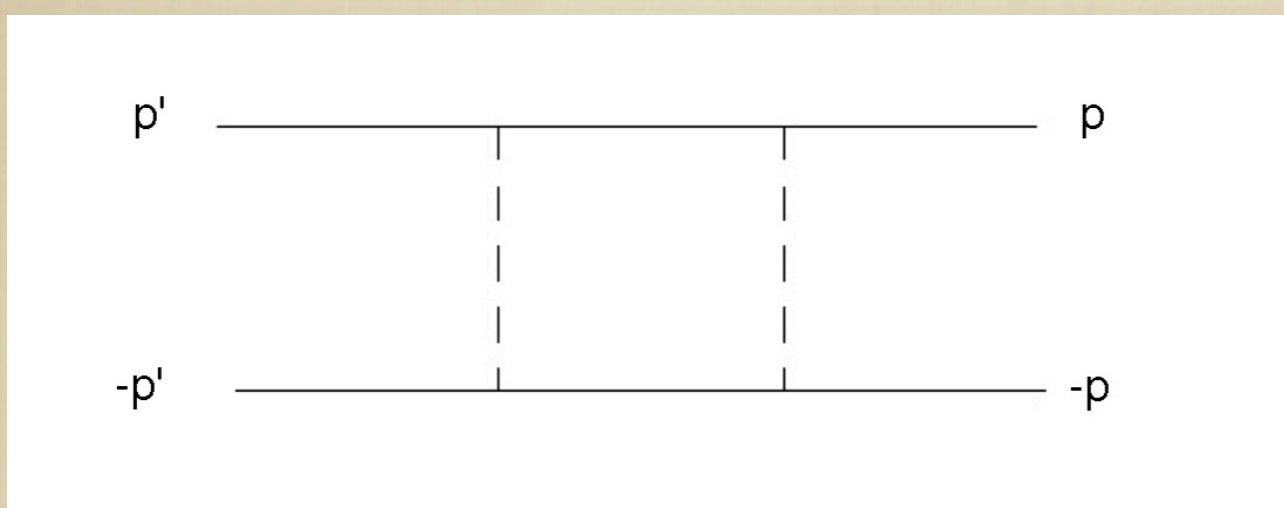


- Iterates of one-pion exchange become comparable with tree-level for momenta of order Λ_{NN}...in low partial waves

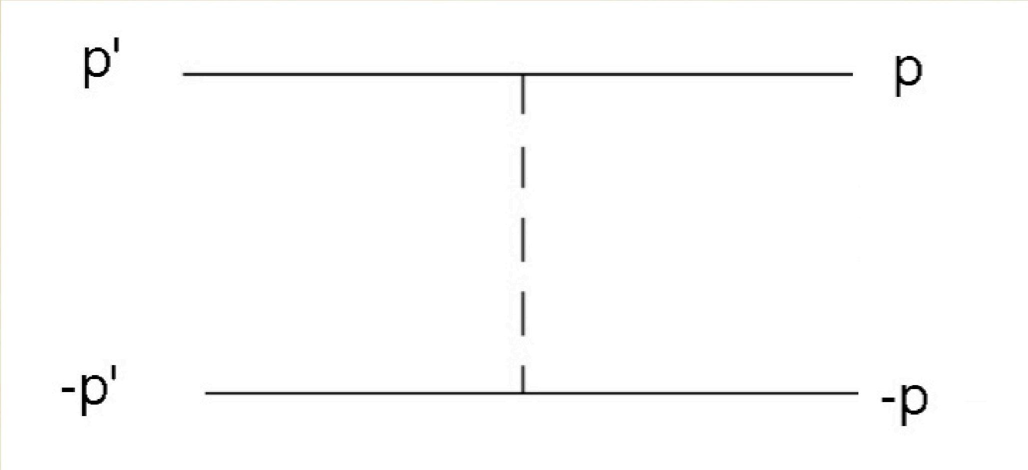
  Fleming, Mehen, Stewart (2000); Beane, Bedaque, Savage, van Kolck (2002); Birse (2006)

- To describe processes for $p \sim \Lambda_{NN}$ need to iterate (tensor part of) one-pion exchange to obtain the LO result

The quest for leading order



vs



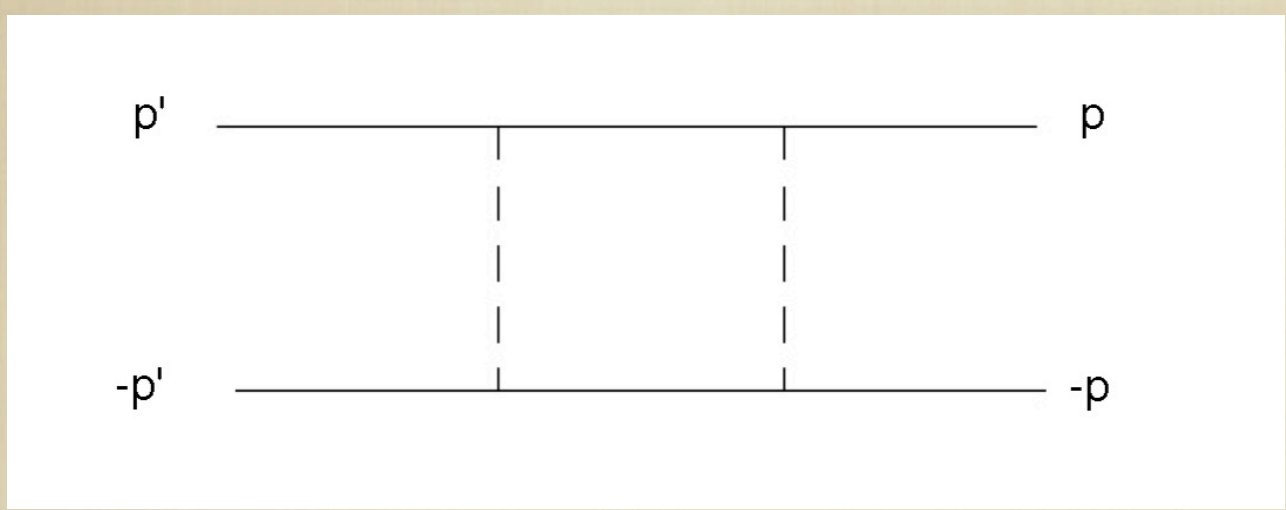
- Iterates of one-pion exchange become comparable with tree-level for momenta of order Λ_{NN}...in low partial waves

  Fleming, Mehen, Stewart (2000); Beane, Bedaque, Savage, van Kolck (2002); Birse (2006)

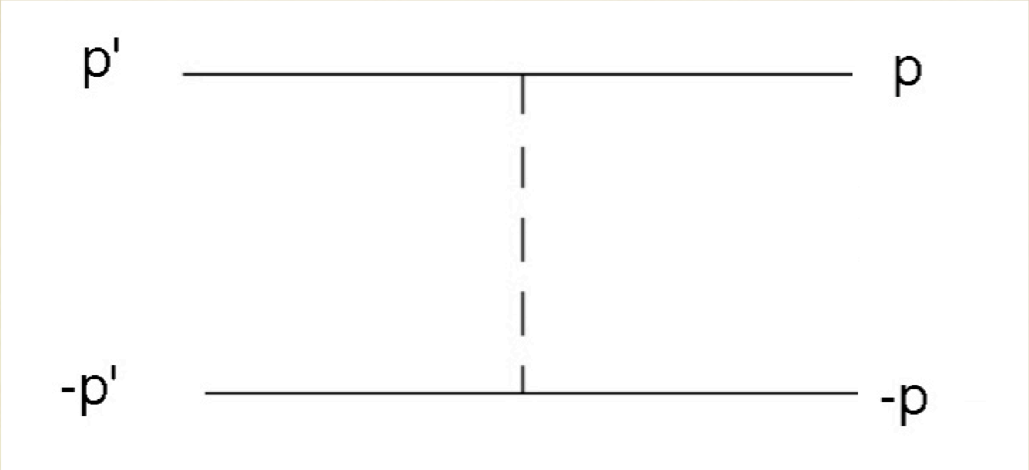
- To describe processes for $p \sim \Lambda_{NN}$ need to iterate (tensor part of) one-pion exchange to obtain the LO result

- Λ_{NN} is a new low-energy scale, thus this is not χPT. But, higher-order pieces of chiral potential suppressed by $\Lambda_{NN}/\Lambda_\chi$.

The quest for leading order



vs



- Iterates of one-pion exchange become comparable with tree-level for momenta of order Λ_{NN}...in low partial waves

  Fleming, Mehen, Stewart (2000); Beane, Bedaque, Savage, van Kolck (2002); Birse (2006)

- To describe processes for $p \sim \Lambda_{NN}$ need to iterate (tensor part of) one-pion exchange to obtain the LO result

- Λ_{NN} is a new low-energy scale, thus this is not χPT. But, higher-order pieces of chiral potential suppressed by $\Lambda_{NN}/\Lambda_{\chi}$.

- Perturbation theory should also be OK for: (a) higher partial waves; (b) 1π exchange in singlet waves; (c) $p \ll \Lambda_{NN}$

χEFT: a theory for light nuclei

χEFT: a theory for light nuclei

- χPT, low scales: m_π, p; high scales: m_ρ, M, Δ

χEFT: a theory for light nuclei

- χPT, low scales: m_π, p; high scales: m_ρ, M, Δ
- Integrate out pion production to get theory of potentials
- Also account for appearance of new light scales: γ, Λ_{NN}

χEFT: a theory for light nuclei

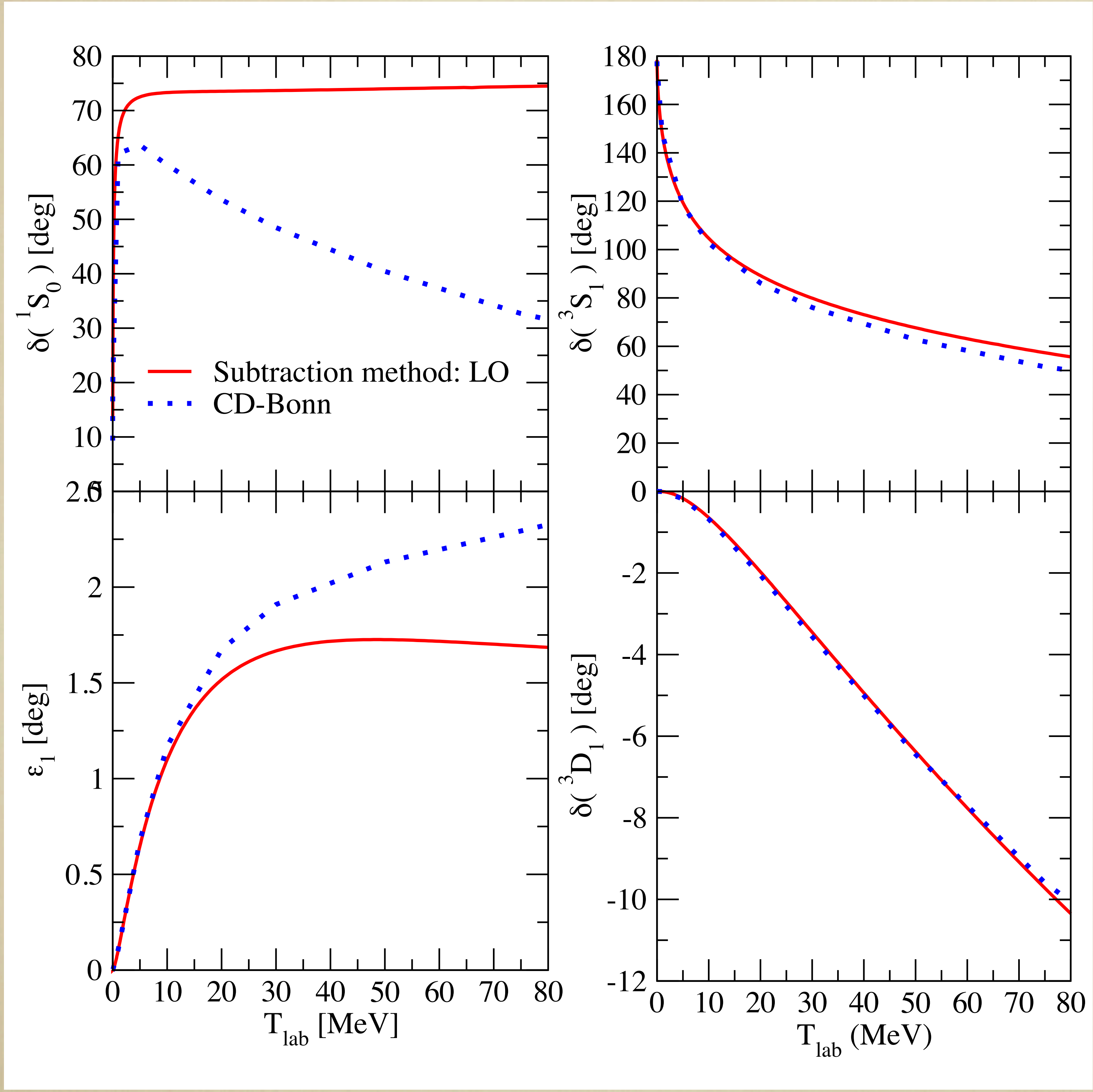
- χPT, low scales: m_π, p; high scales: m_ρ, M, Δ
- Integrate out pion production to get theory of potentials
- Also account for appearance of new light scales: γ, Λ_{NN}
- χEFT, low scales: γ, m_π, p, Λ_{NN}; high scales: m_ρ, M, Δ, $(Mm_\pi)^{1/2}$

χEFT: a theory for light nuclei

- χPT, low scales: m_π, p; high scales: m_ρ, M, Δ
- Integrate out pion production to get theory of potentials
- Also account for appearance of new light scales: γ, Λ_{NN}
- χEFT, low scales: γ, m_π, p, Λ_{NN}; high scales: m_ρ, M, Δ, $(Mm_\pi)^{1/2}$
- EFT($\not\pi$), low scales: γ, p; high scales: $(Mm_\pi)^{1/2}$, Δ, m_ρ, M, m_π, Λ_{NN}

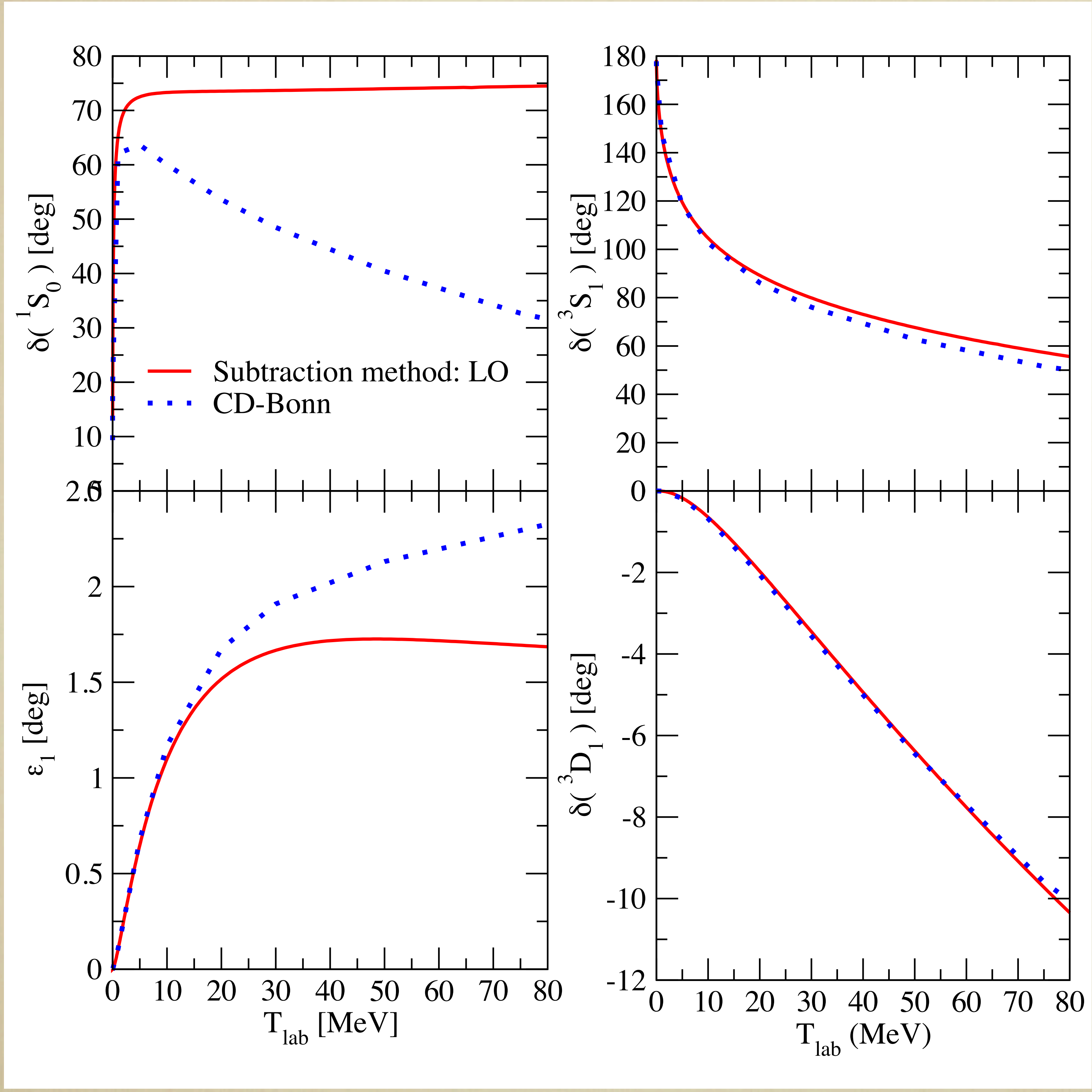
The quest continued: S waves

Beane, Bedaque, Savage, van Kolck (2002);
Pavon Valderrama, Ruiz Arriola (2005)



The quest continued: S waves

Beane, Bedaque, Savage, van Kolck (2002);
Pavon Valderrama, Ruiz Arriola (2005)

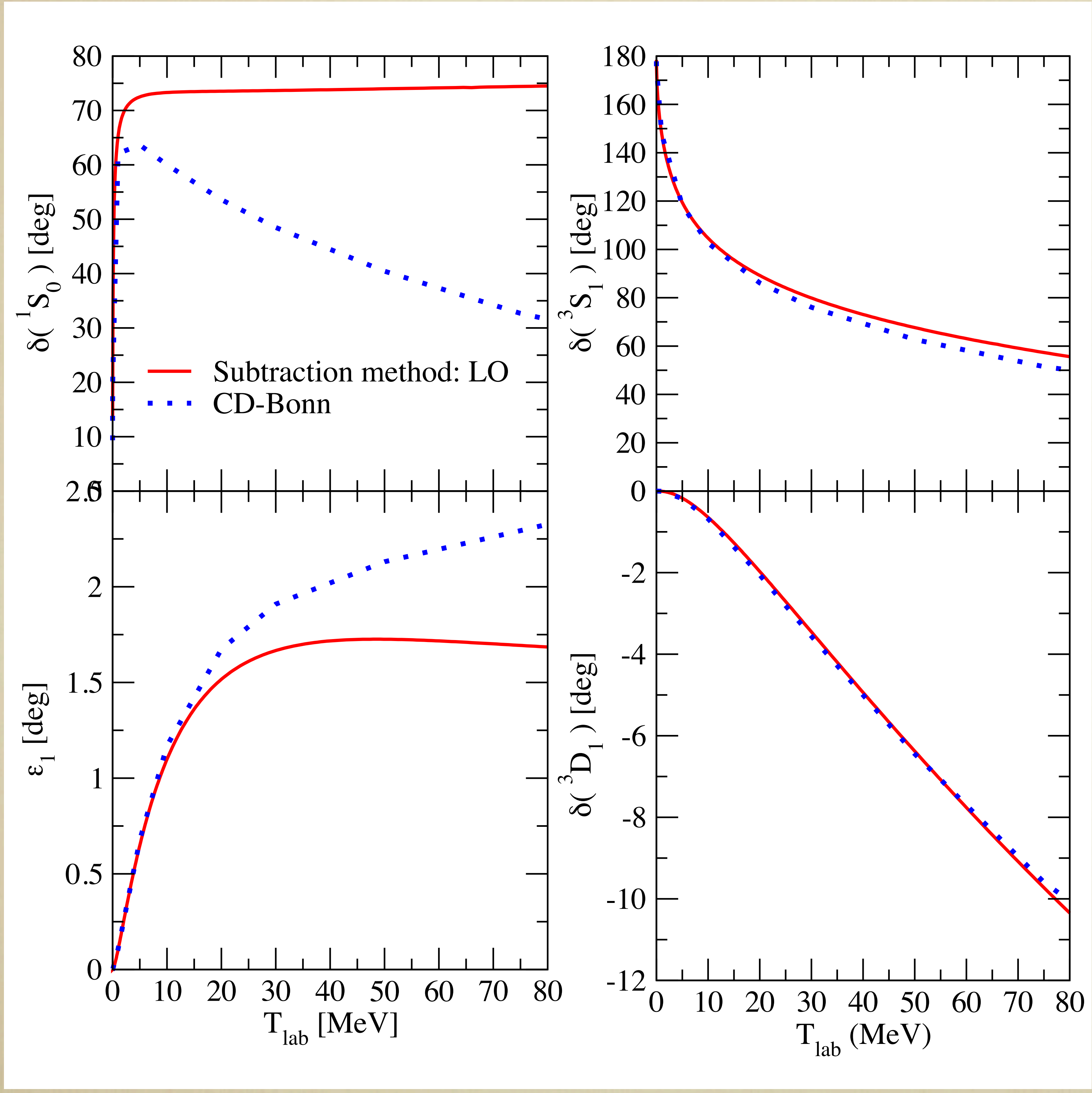


- Stable for wide range of cutoffs
- Subtractive renormalization numerically efficient

Yang, Elster, Phillips (2007)

The quest continued: S waves

Beane, Bedaque, Savage, van Kolck (2002);
Pavon Valderrama, Ruiz Arriola (2005)



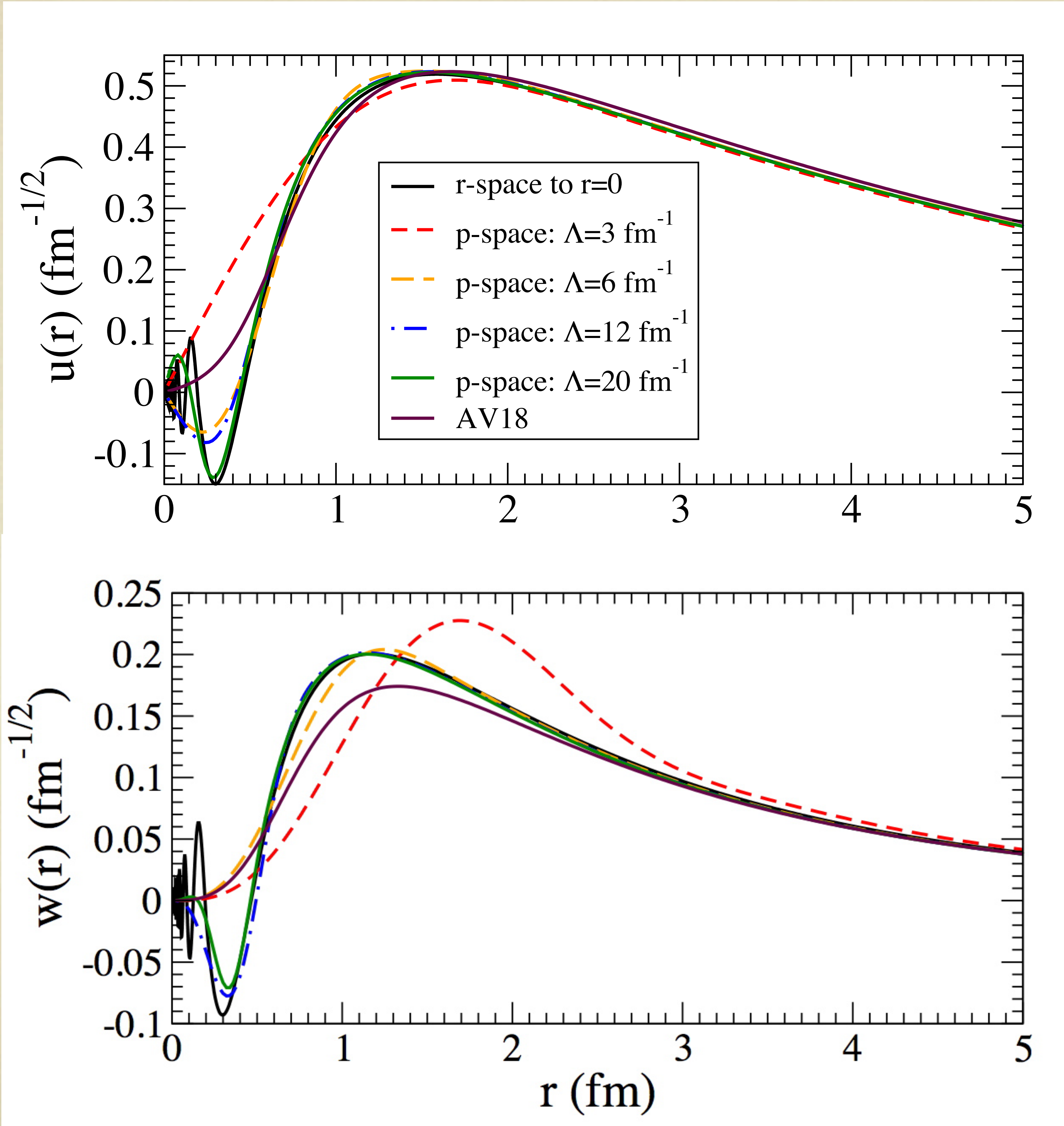
- Stable for wide range of cutoffs
- Subtractive renormalization numerically efficient

Yang, Elster, Phillips (2007)

- One-pion exchange weak in $^1S_0$

χEFT deuteron wave functions at leading order

Pavon Valderrama, Nogga, Ruiz Arriola, DP, EPJA 36, 315 (2008)



Those innocuous (?) wiggles

Case (1950), Sprung et al. (1994),
Beane et al. (2001),
Pavon Valderrama, Ruiz Arriola (2004-6)

- Attractive case, for r≪1/Λ_{NN}

$$u_1(r) = (\Lambda_{NN} r)^{3/4} \cos\left(4\sqrt{\frac{1}{\Lambda_{NN} r}}\right) ; u_2(r) = (\Lambda_{NN} r)^{3/4} \sin\left(4\sqrt{\frac{1}{\Lambda_{NN} r}}\right)$$

- Equally regular solutions, need boundary condition to fix phase

- c.f. $j_l(kr)$ and $n_l(kr)$ for plane waves as r→0

- Repulsive, for r≪1/Λ_{NN}

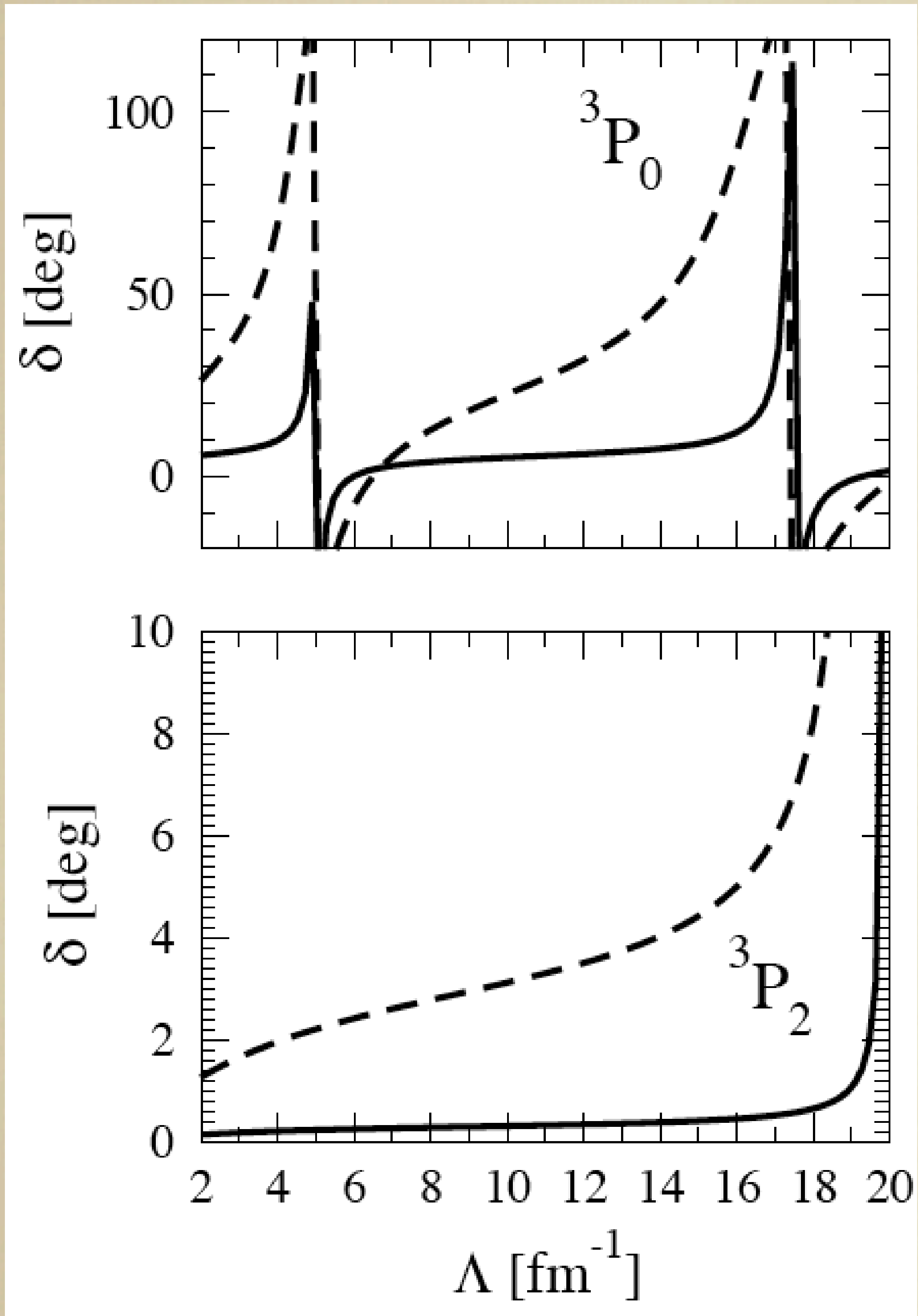
$$u_1(r) = (\Lambda_{NN} r)^{3/4} \exp\left(4\sqrt{\frac{1}{\Lambda_{NN} r}}\right) ; u_2(r) = (\Lambda_{NN} r)^{3/4} \exp\left(-4\sqrt{\frac{1}{\Lambda_{NN} r}}\right)$$

- Still need boundary condition to fix "phase", but results insensitive to choice

...are sometimes nocuous

- Need contact terms in certain P waves already at LO, in order to specify short-distance b.c.
- “New leading order”: 1π exchange plus contact interactions, iterated, in 3S1, 3P0 and 3P2
- Renormalization-group analysis
- Higher-order corrections to phase shifts calculated: promising results

...are sometimes nocuous



- Need contact terms in certain P waves already at LO, in order to specify short-distance b.c.

  Eiras, Soto (2002); Nogga, Timmermans, van Kolck (2005)

- "New leading order": 1π exchange plus contact interactions, iterated, in 3S1, 3P0 and 3P2

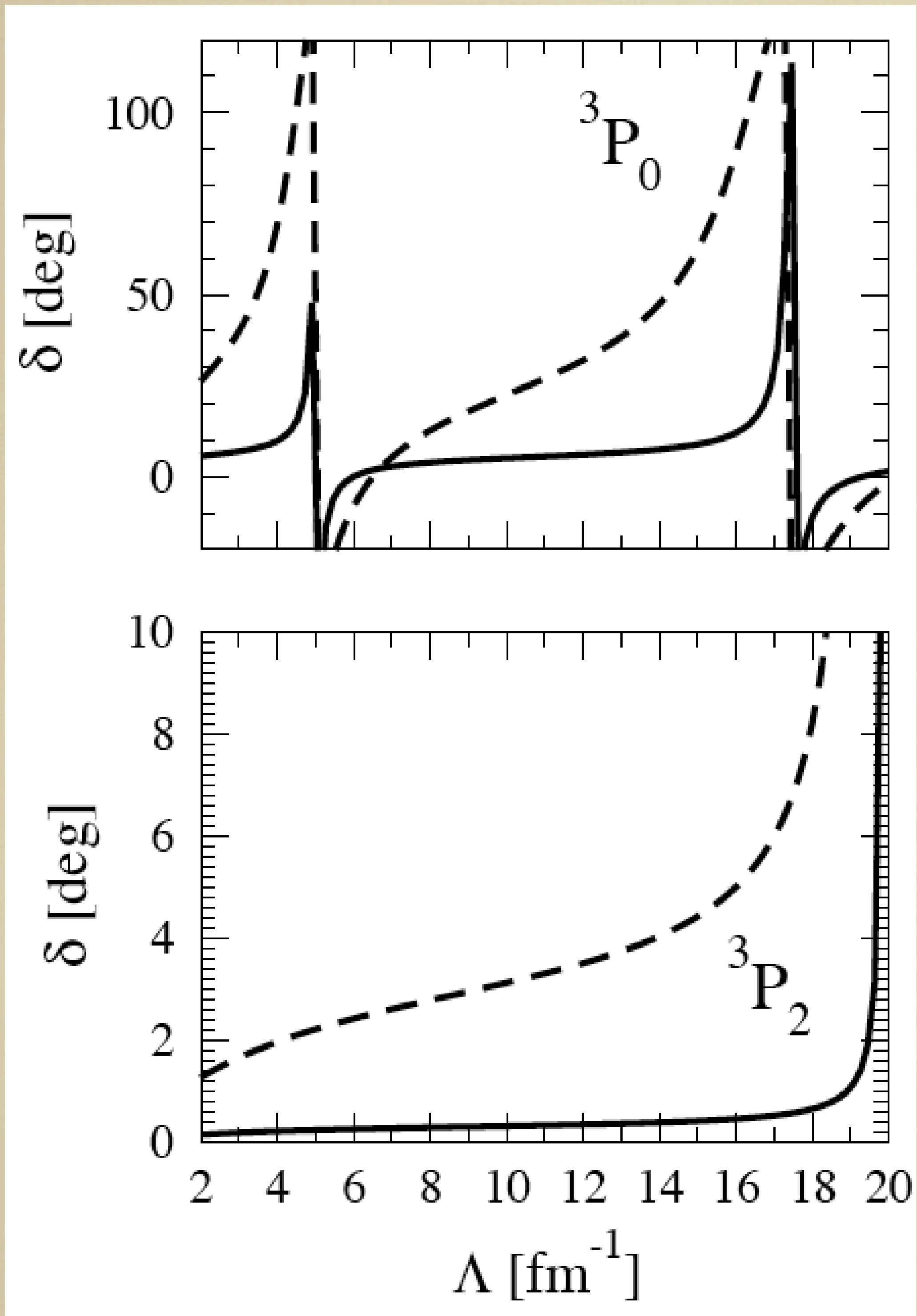
- Renormalization-group analysis

  Birse

- Higher-order corrections to phase shifts calculated: promising results

  Pavon Valderrama (2011, 2012); Long & Yang (2012)

...are sometimes nocuous



- Need contact terms in certain P waves already at LO, in order to specify short-distance b.c.

  Eiras, Soto (2002); Nogga, Timmermans, van Kolck (2005)

- "New leading order": 1π exchange plus contact interactions, iterated, in 3S1, 3P0 and 3P2

- Renormalization-group analysis

  Birse

- Higher-order corrections to phase shifts calculated: promising results

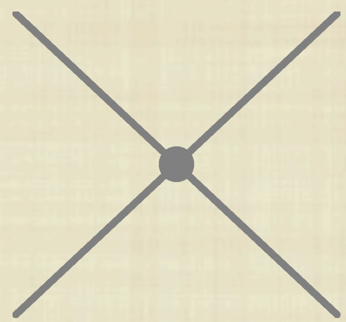
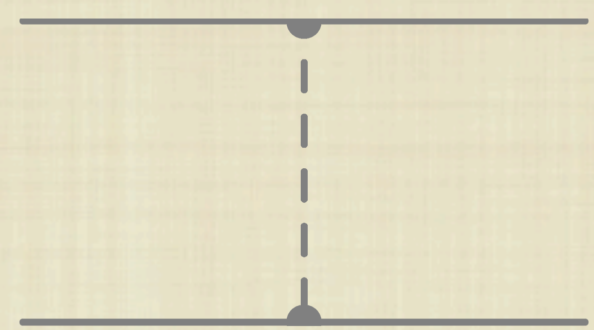
  Pavon Valderrama (2011, 2012); Long & Yang (2012)

Moral: NDA doesn't predict scaling of short-distance operators needed for renormalization if LO wave functions are not plane waves

Shallow poles: why the $^{1}S_{0}$ is special

Shallow poles: why the $^1S_0$ is special

- Let's talk about the $^1S_0$: almost a bound state, but one-pion exchange is weak (perturbative?) there.

$V^{(0)} =$  $+$  ;

- Existence of shallow pole results from tuning of contact interaction to be $O(P^{-1})$, stronger than indicated by NDA

Shallow poles: why the $^1S_0$ is special

- Let's talk about the $^1S_0$: almost a bound state, but one-pion exchange is weak (perturbative?) there.

$V^{(0)} =$  ;

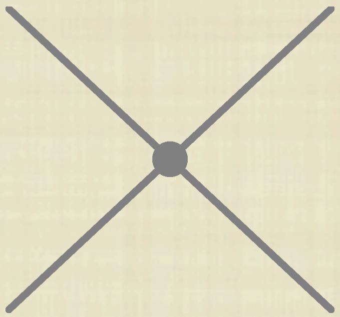
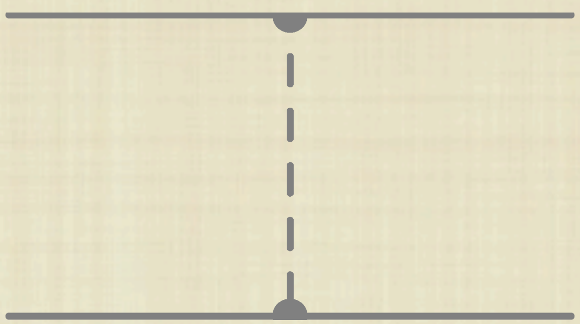
- Existence of shallow pole results from tuning of contact interaction to be $O(P^{-1})$, stronger than indicated by NDA

- $|\psi^{(0)}\rangle \sim 1/r$ at short distances$\Rightarrow$matrix elements very divergent

Birse (2009, 2010), Pavon Valderrama (2010), Long & Yang (2011)

Shallow poles: why the $^1S_0$ is special

- Let's talk about the $^1S_0$: almost a bound state, but one-pion exchange is weak (perturbative?) there.

$V^{(0)} =$  $+$  ;

- Existence of shallow pole results from tuning of contact interaction to be $O(P^{-1})$, stronger than indicated by NDA

- $|\psi^{(0)}\rangle \sim 1/r$ at short distances⇒matrix elements very divergent

Birse (2009, 2010), Pavon Valderrama (2010), Long & Yang (2011)

- C_2p^2, C_4p^4, etc. enhanced by two orders c.f. NDA

χEFT expansion for probes

χEFT expansion for probes

$$\mathcal{M}_\mu = \langle \psi | J_\mu | \psi \rangle$$

χEFT expansion for probes

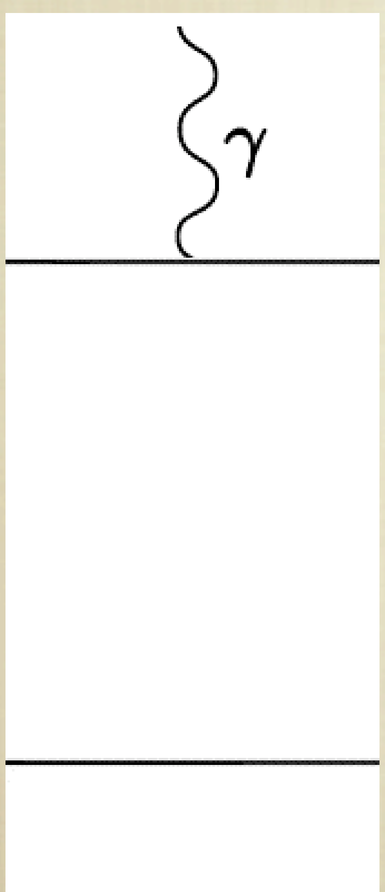
$$\mathcal{M}_\mu = \langle\psi|J_\mu|\psi\rangle$$

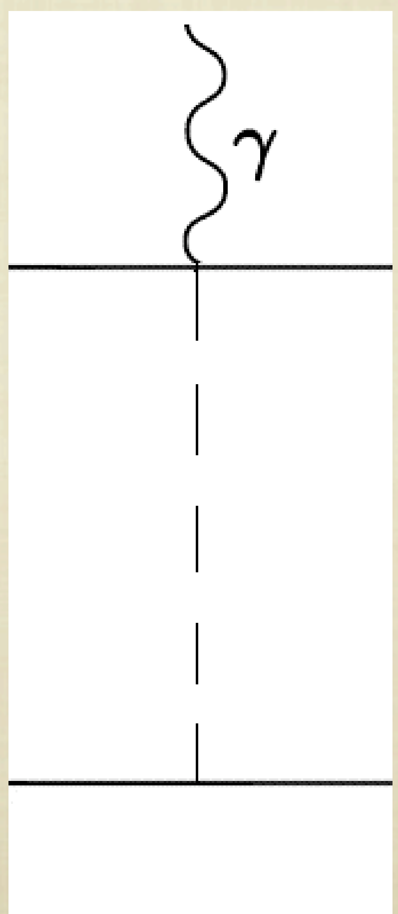
$$J_\mu = J_\mu^{(0)} + J_\mu^{(1)} + J_\mu^{(2)} + \ldots \qquad |\psi\rangle = |\psi\rangle^{(0)} + |\psi\rangle^{(2)} + \ldots$$

χEFT expansion for probes

$$\mathcal{M}_\mu = \langle \psi | J_\mu | \psi \rangle$$

$$J_\mu = J_\mu^{(0)} + J_\mu^{(1)} + J_\mu^{(2)} + \ldots \qquad |\psi\rangle = |\psi\rangle^{(0)} + |\psi\rangle^{(2)} + \ldots$$

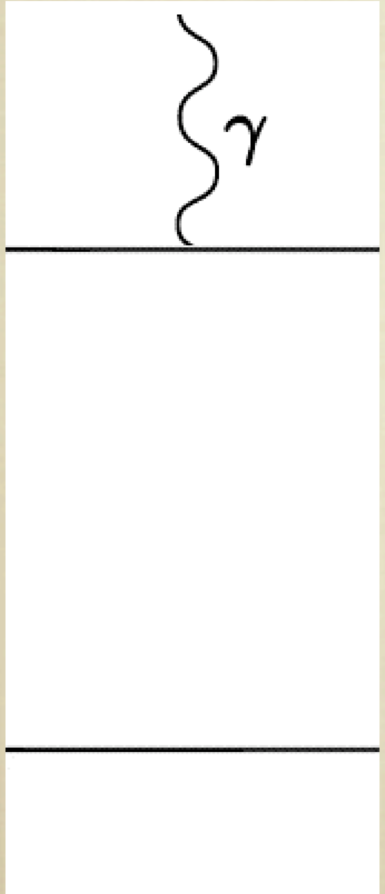


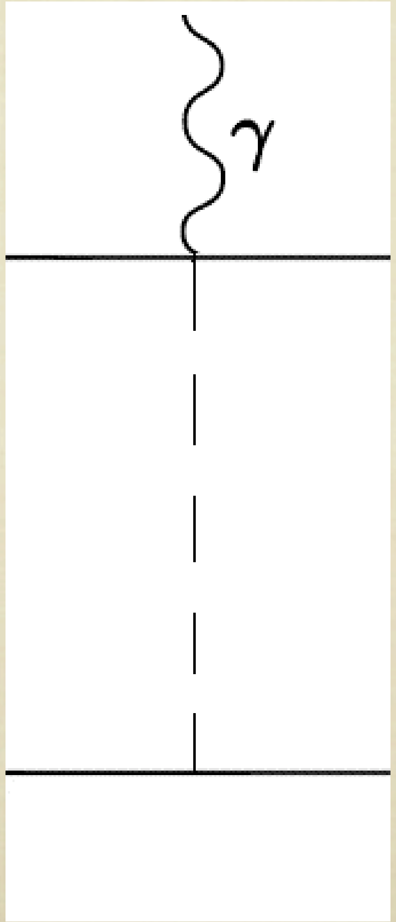


χEFT expansion for probes

$$\mathcal{M}_\mu = \langle\psi|J_\mu|\psi\rangle$$

$$J_\mu = J_\mu^{(0)} + J_\mu^{(1)} + J_\mu^{(2)} + \dots \qquad |\psi\rangle = |\psi\rangle^{(0)} + |\psi\rangle^{(2)} + \dots$$





NEED TO COMPUTE BOTH J_μ AND $|\psi>$ TO ORDER N TO GET $\mathcal{M}_\mu$ TO ORDER N

Computing $\mathcal{M}_\mu$

$$\mathcal{M}_\mu^{(0)} = \langle\psi|J_\mu^{(0)}|\psi\rangle$$

Computing $\mathcal{M}_\mu$

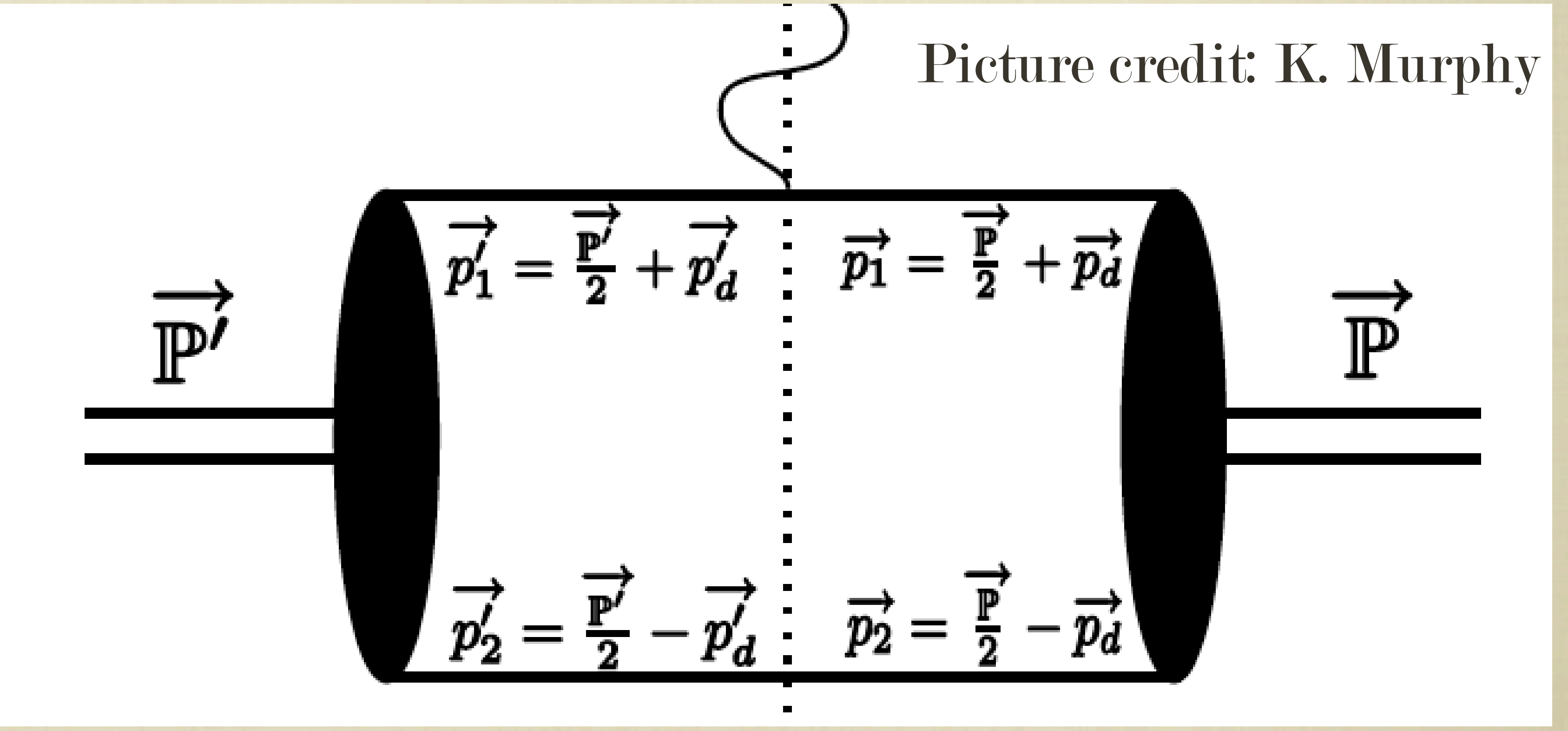
$$\mathcal{M}_\mu^{(0)} = \langle \psi | J_\mu^{(0)} | \psi \rangle \qquad \langle \mathbf{p}_1' | J_\mu^{(0)} | \mathbf{p}_1 \rangle = v_\mu |e| \delta^{(3)}(p_1' - p_1 - q)$$

Computing $\mathcal{M}_\mu$

$$\mathcal{M}_\mu^{(0)} = \langle\psi|J_\mu^{(0)}|\psi\rangle \qquad \langle\mathbf{p}_1'|J_\mu^{(0)}|\mathbf{p}_1\rangle = v_\mu|e|\delta^{(3)}(p_1' - p_1 - q)$$

$$\mathcal{M}_\mu^{(0)} = v_\mu|e|\int d^3p\,\psi^{(0)}(\mathbf{p}+\mathbf{q}/2)\psi^{(0)}(\mathbf{p})$$

Maps Out Nucleon Distribution Inside Deuterium



Results for G_C and G_Q at leading order

Pavon Valderrama, Ruiz Arriola, Nogga, DP, EPJA (2008)

LO χEFT: $J_0(\mathbf{r}) = |e|\delta^{(3)}(\mathbf{r}-\mathbf{r}_p) \Rightarrow$ $$G_C(|\mathbf{q}|) = \int dr\, j_0\left(\frac{|\mathbf{q}|r}{2}\right)\left[u^2(r) + w^2(r)\right]$$

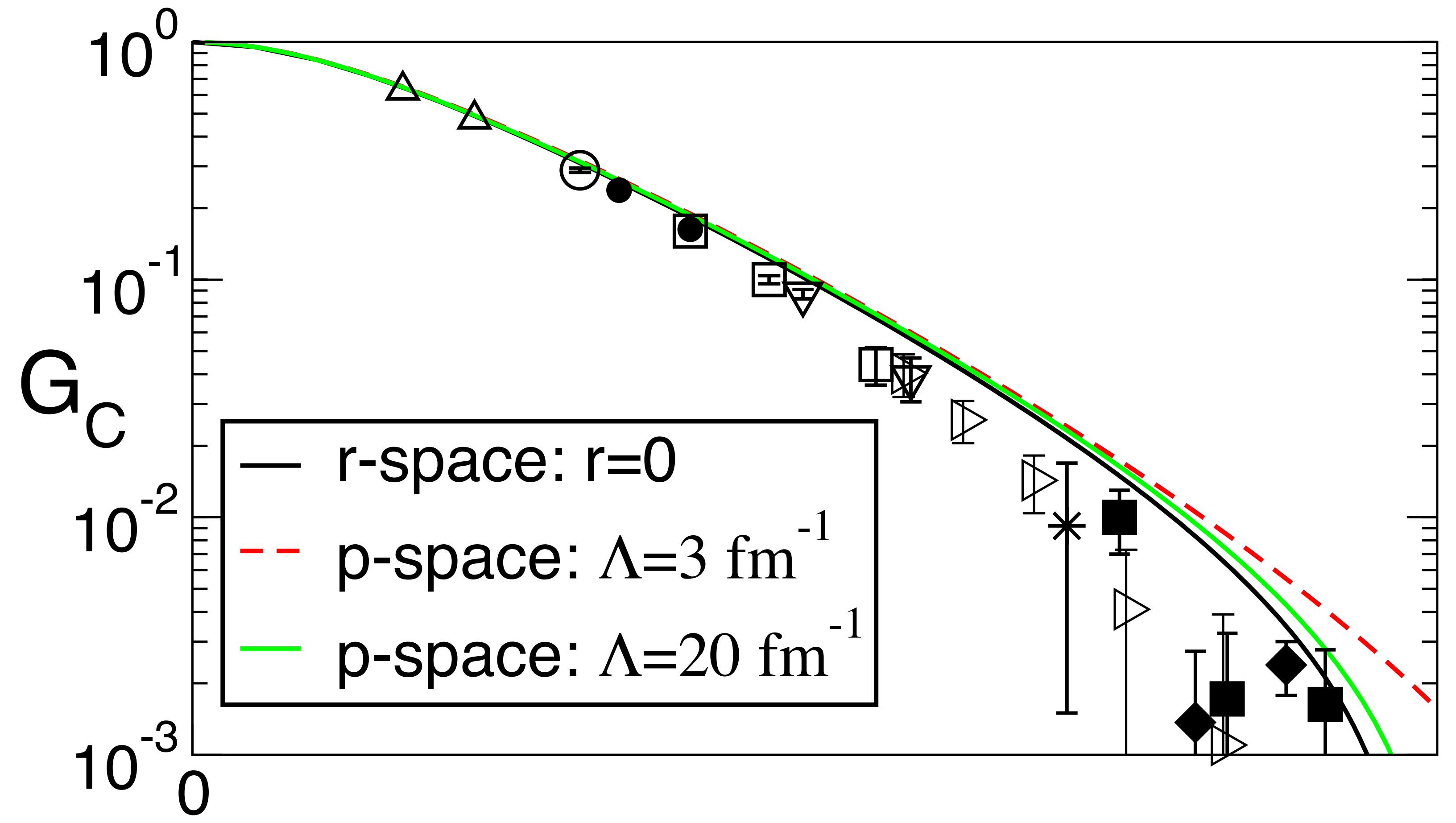
Results for G_C and G_Q at leading order

Pavon Valderrama, Ruiz Arriola, Nogga, DP, EPJA (2008)

LO χEFT: $J_0(\mathbf{r})=|e|\delta^{(3)}(\mathbf{r}-\mathbf{r}_p) \Rightarrow \; G_C(|\mathbf{q}|) = \int dr\, j_0\left(\frac{|\mathbf{q}|r}{2}\right)[u^2(r)+w^2(r)]$

Nucleon structure included via:

$$\frac{G_C}{G_E^{(s)}} = \langle\psi|e|\psi\rangle + O(eP^3)$$



Data: Abbott et al., EPJA 7, 421(2000)

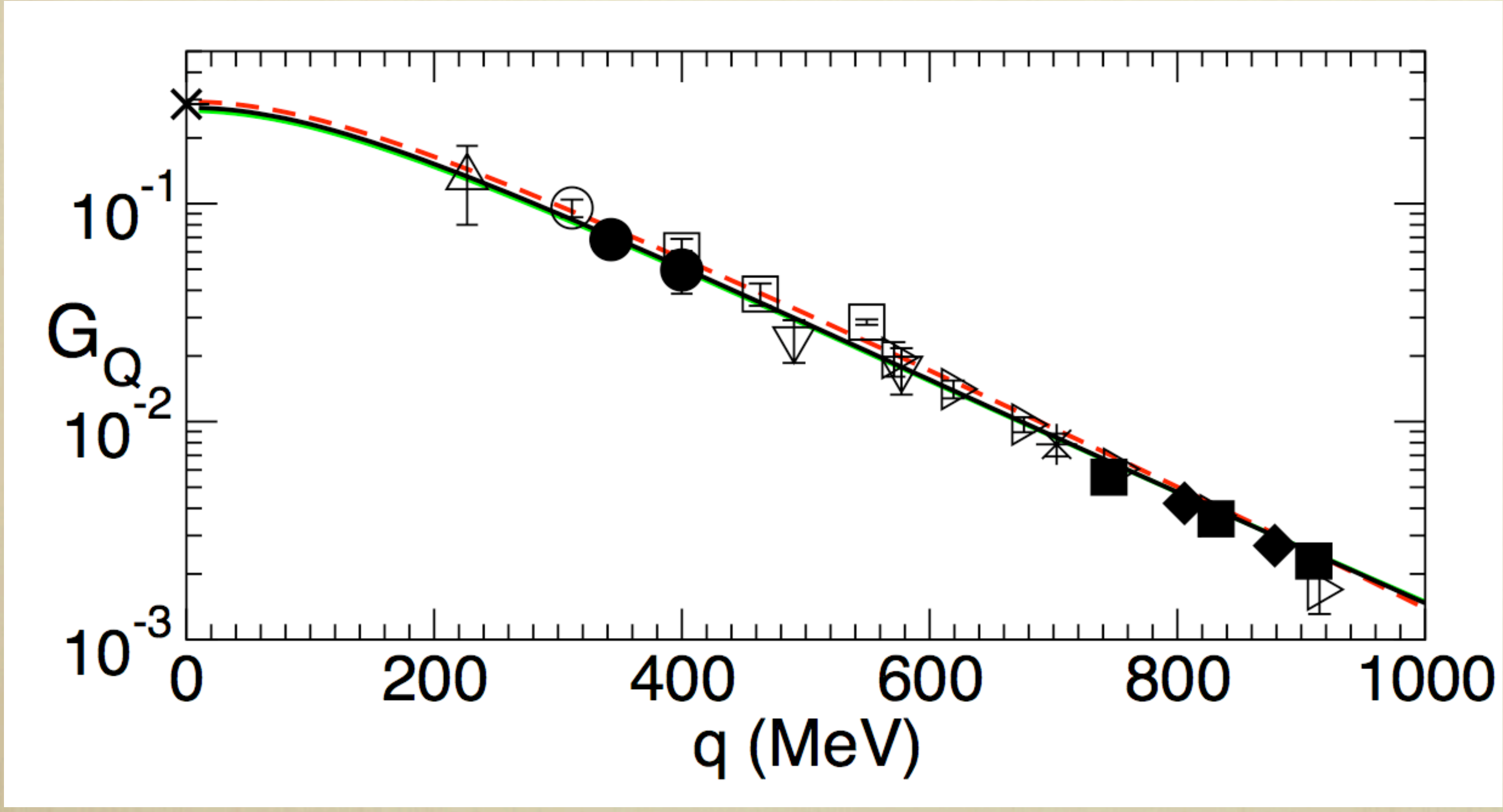
Results for G_C and G_Q at leading order

Pavon Valderrama, Ruiz Arriola, Nogga, DP, EPJA (2008)

LO χEFT: $J_0(\mathbf{r})=|e|\delta^{(3)}(\mathbf{r}-\mathbf{r}_p) \Rightarrow$ $G_C(|\mathbf{q}|) = \int dr\, j_0\left(\frac{|\mathbf{q}|r}{2}\right)\left[u^2(r)+w^2(r)\right]$

Nucleon structure included via:

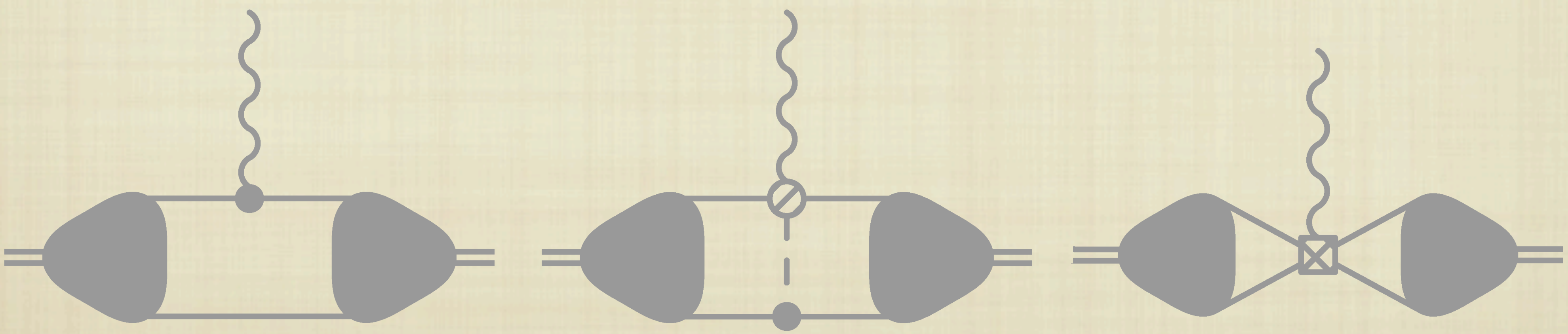
$$\frac{G_C}{G_E^{(s)}} = \langle\psi|e|\psi\rangle + O(eP^3)$$



Data: Abbott et al., EPJA 7, 421(2000)

Beyond LO I: two-body operators

DP and Cohen (1999); Park et al. (1999); DP (2003, 2007); Koelling et al. (2009)



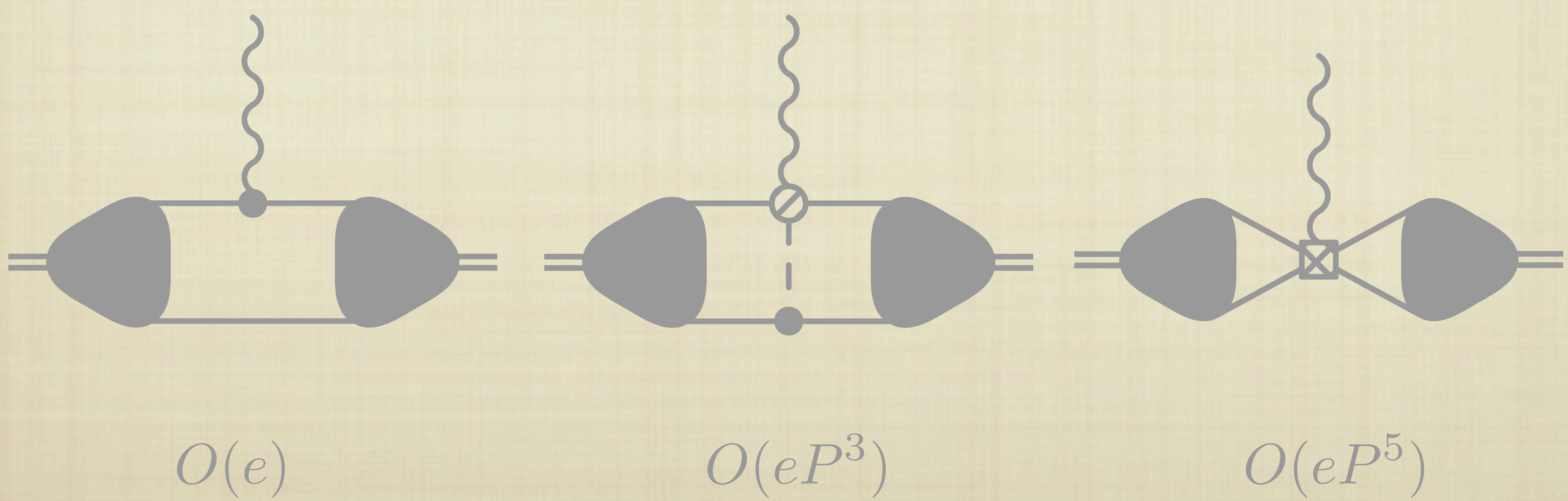
$O(e)$ $O(eP^3)$ $O(eP^5)$

Beyond LO I: two-body operators

DP and Cohen (1999); Park et al. (1999); DP (2003, 2007); Koelling et al. (2009)

- $O(eP^3/eP^4)$: 2B mechanism enters, but no free parameters

SUPPRESSION BY $1/M_N$



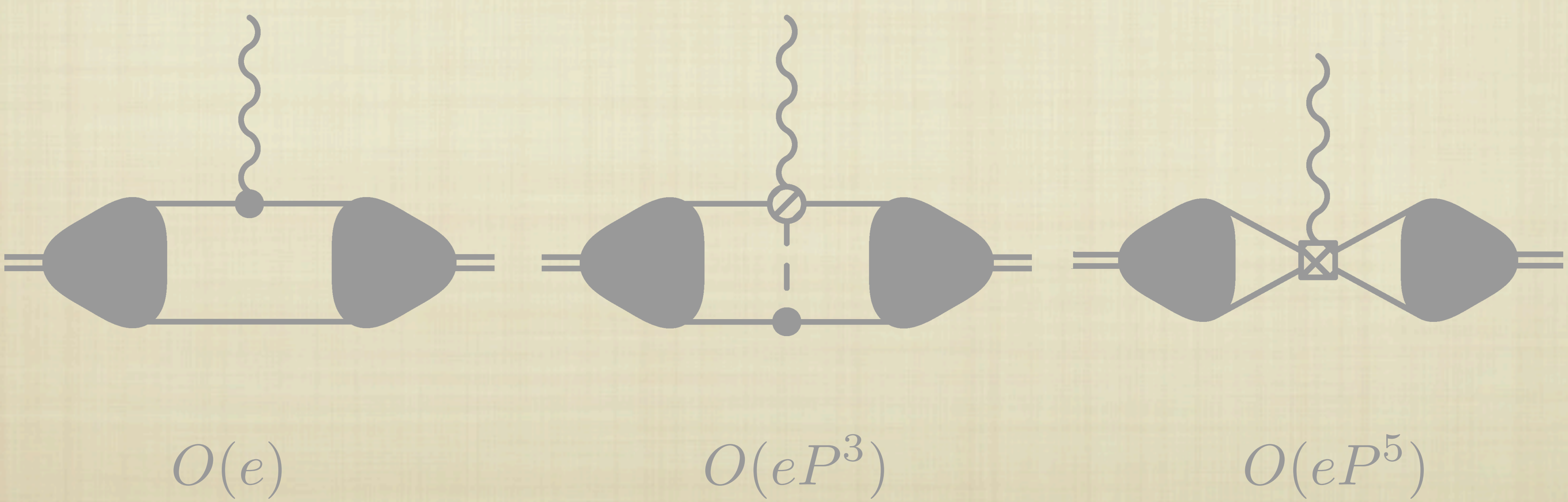
Beyond LO I: two-body operators

DP and Cohen (1999); Park et al. (1999); DP (2003, 2007); Koelling et al. (2009)

- $O(eP^3/eP^4)$: 2B mechanism enters, but no free parameters

SUPPRESSION BY $1/M_N$

- $O(eP^4)$: Two-pion exchange pieces of $J_0^{(s)}$. VANISH!



Beyond LO I: two-body operators

DP and Cohen (1999); Park et al. (1999); DP (2003, 2007); Koelling et al. (2009)

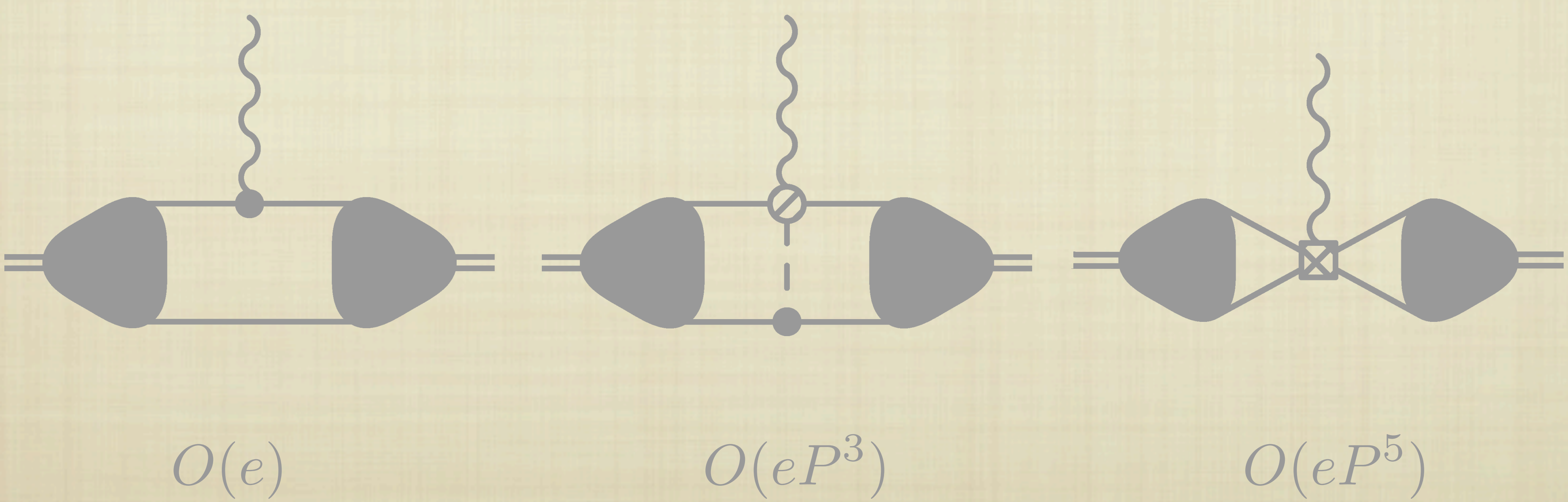
- $O(eP^3/eP^4)$: 2B mechanism enters, but no free parameters

SUPPRESSION BY $1/M_N$

- $O(eP^4)$: Two-pion exchange pieces of $J_0^{(s)}$. VANISH!

- $O(eP^5)$: Short-distance parts of operators

NDA COUNTING FOR SHORT-DISTANCE OPERATORS



Beyond LO II: contact operators

Beyond LO II: contact operators

Start by considering deuteron radius

$$\langle r_d^2 \rangle_{\rm IA} = \int_R^\infty dr\, r^2 \left[u^2(r) + w^2(r)\right]$$

Beyond LO II: contact operators

Start by considering deuteron radius

$$\langle r_d^2 \rangle_{\rm IA} = \int_R^\infty dr\, r^2 \left[u^2(r) + w^2(r)\right]$$

Renormalized by $\langle \phi | \hat{O}_{\Delta \langle r_d^2 \rangle} | \psi \rangle = D(R)\phi(R)\psi(R)$ $\qquad [D] = L^5$

Beyond LO II: contact operators

Start by considering deuteron radius

$$\langle r_d^2 \rangle_{\rm IA} = \int_R^\infty dr\, r^2 \left[u^2(r) + w^2(r)\right]$$

Renormalized by $\langle \phi | \hat{O}_{\Delta \langle r_d^2 \rangle} | \psi \rangle = D(R)\phi(R)\psi(R)$ $\qquad [D] = L^5$

RG equation (neglecting D-state contribution)

$$\frac{d}{dR}\left[D(R)\frac{u^2(R)}{R^2} + \int_R^\infty dr\, r^2 u^2(r)\right] = 0$$

Beyond LO II: contact operators

Start by considering deuteron radius

$$\langle r_d^2 \rangle_{\rm IA} = \int_R^\infty dr\, r^2 \left[u^2(r) + w^2(r)\right]$$

Renormalized by $\langle \phi | \hat{O}_{\Delta \langle r_d^2 \rangle} | \psi \rangle = D(R)\phi(R)\psi(R)$ $\qquad [D] = L^5$

RG equation (neglecting D-state contribution)

$$\frac{d}{dR}\left[D(R)\frac{u^2(R)}{R^2} + \int_R^\infty dr\, r^2 u^2(r)\right] = 0$$

Use this to "run" R between $1/\Lambda_0$ and $1/\Lambda_{NN}$ $\qquad \Lambda_{NN} \approx 300$ MeV

Beyond LO II: contact operators

Start by considering deuteron radius

$$\langle r_d^2 \rangle_{\rm IA} = \int_R^\infty dr\, r^2 \left[u^2(r) + w^2(r)\right]$$

Renormalized by $\langle \phi | \hat{O}_{\Delta \langle r_d^2 \rangle} | \psi \rangle = D(R)\phi(R)\psi(R)$ $\qquad [D] = L^5$

RG equation (neglecting D-state contribution)

$$\frac{d}{dR}\left[D(R)\frac{u^2(R)}{R^2} + \int_R^\infty dr\, r^2 u^2(r)\right] = 0$$

Use this to "run" R between $1/\Lambda_0$ and $1/\Lambda_{NN}$ $\qquad \Lambda_{NN} \approx 300$ MeV

Short-distance behaviour of LO u:

$$u(r) = C_{2A}\left(\Lambda_{NN} r\right)^{3/4} \cos\left(4\sqrt{\frac{1}{\Lambda_{NN} r}} + \phi\right) \text{ for } r \ll \frac{1}{\Lambda_{NN}}$$

Scaling of short-distance operators

Scaling of short-distance operators

RGE gives (looking only at power-law):

$$\frac{D(R)}{R^{1/2}} = \int_{1/\Lambda_0}^{R} R'^{7/2} dR' + D\left(\frac{1}{\Lambda_0}\right)\Lambda_0^{1/2}$$

Scaling of short-distance operators

RGE gives (looking only at power-law):

$$\frac{D(R)}{R^{1/2}} = \int_{1/\Lambda_0}^{R} R'^{7/2} dR' + D\left(\frac{1}{\Lambda_0}\right)\Lambda_0^{1/2}$$

Assuming naturalness at Λ_0: $D\left(\frac{1}{\Lambda_0}\right) = \frac{d_{\langle r^2 \rangle}}{\Lambda_0^5} \Longrightarrow$

$$\langle \hat{O}_{\Delta \langle r_d^2 \rangle} \rangle \sim \frac{2\gamma \Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}}$$

Scaling of short-distance operators

RGE gives (looking only at power-law):

$$\frac{D(R)}{R^{1/2}} = \int_{1/\Lambda_0}^{R} R'^{7/2} dR' + D\left(\frac{1}{\Lambda_0}\right)\Lambda_0^{1/2}$$

Assuming naturalness at Λ_0: $D\left(\frac{1}{\Lambda_0}\right) = \frac{d_{\langle r^2 \rangle}}{\Lambda_0^5} \implies$

$$\langle \hat{O}_{\Delta \langle r_d^2 \rangle} \rangle \sim \frac{2\gamma \Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}} \qquad \langle \hat{O}_{\Delta Q_d} \rangle \sim \frac{2\gamma \Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}}$$

Scaling of short-distance operators

RGE gives (looking only at power-law):

$$\frac{D(R)}{R^{1/2}} = \int_{1/\Lambda_0}^{R} R'^{7/2} dR' + D\left(\frac{1}{\Lambda_0}\right)\Lambda_0^{1/2}$$

Assuming naturalness at Λ_0: $D\left(\frac{1}{\Lambda_0}\right) = \frac{d_{\langle r^2\rangle}}{\Lambda_0^5} \implies$

$$\langle \hat{O}_{\Delta\langle r_d^2\rangle}\rangle \sim \frac{2\gamma\Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}} \qquad \langle \hat{O}_{\Delta Q_d}\rangle \sim \frac{2\gamma\Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}}$$

Both enhanced by $P^{1/2}$ c.f. Weinberg counting

Driven by different behaviour of short-distance wave function

It is not m_π that appears in numerator: dependence on IR scales!

Scaling of short-distance operators

RGE gives (looking only at power-law):

$$\frac{D(R)}{R^{1/2}} = \int_{1/\Lambda_0}^{R} R'^{7/2} dR' + D\left(\frac{1}{\Lambda_0}\right)\Lambda_0^{1/2}$$

Assuming naturalness at Λ_0: $D\left(\frac{1}{\Lambda_0}\right) = \frac{d_{\langle r^2\rangle}}{\Lambda_0^5} \Longrightarrow$

$$\langle \hat{O}_{\Delta\langle r_d^2\rangle}\rangle \sim \frac{2\gamma\Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}} \qquad \langle \hat{O}_{\Delta Q_d}\rangle \sim \frac{2\gamma\Lambda_{NN}^{3/2}}{\Lambda_0^{9/2}}$$

Both enhanced by $P^{1/2}$ c.f. Weinberg counting

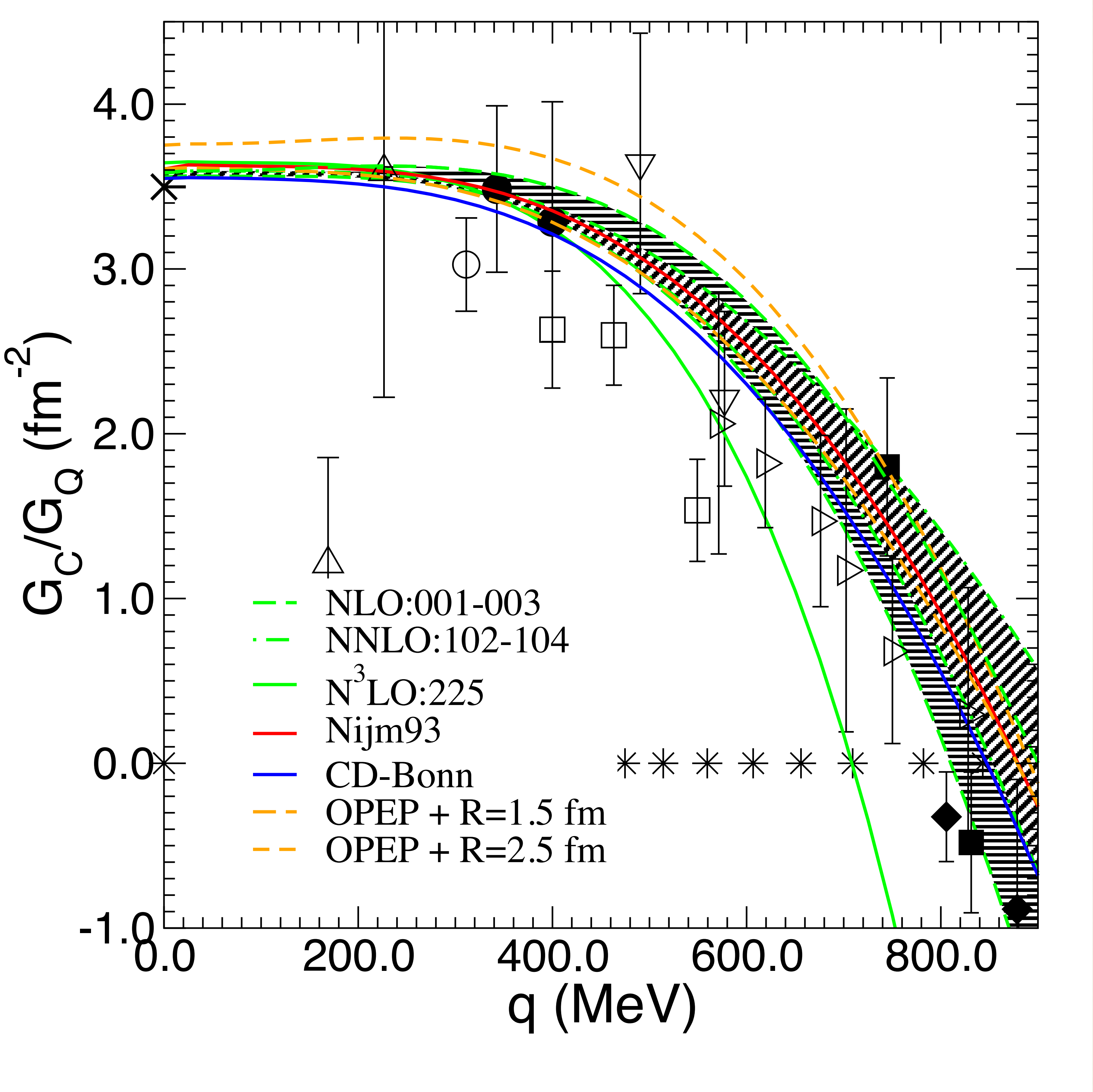
Driven by different behaviour of short-distance wave function

It is not m_π that appears in numerator: dependence on IR scales!

Similar size, but 0.002 fm^2 is a much larger percentage effect in Q_d

Renormalizing G_C/G_Q

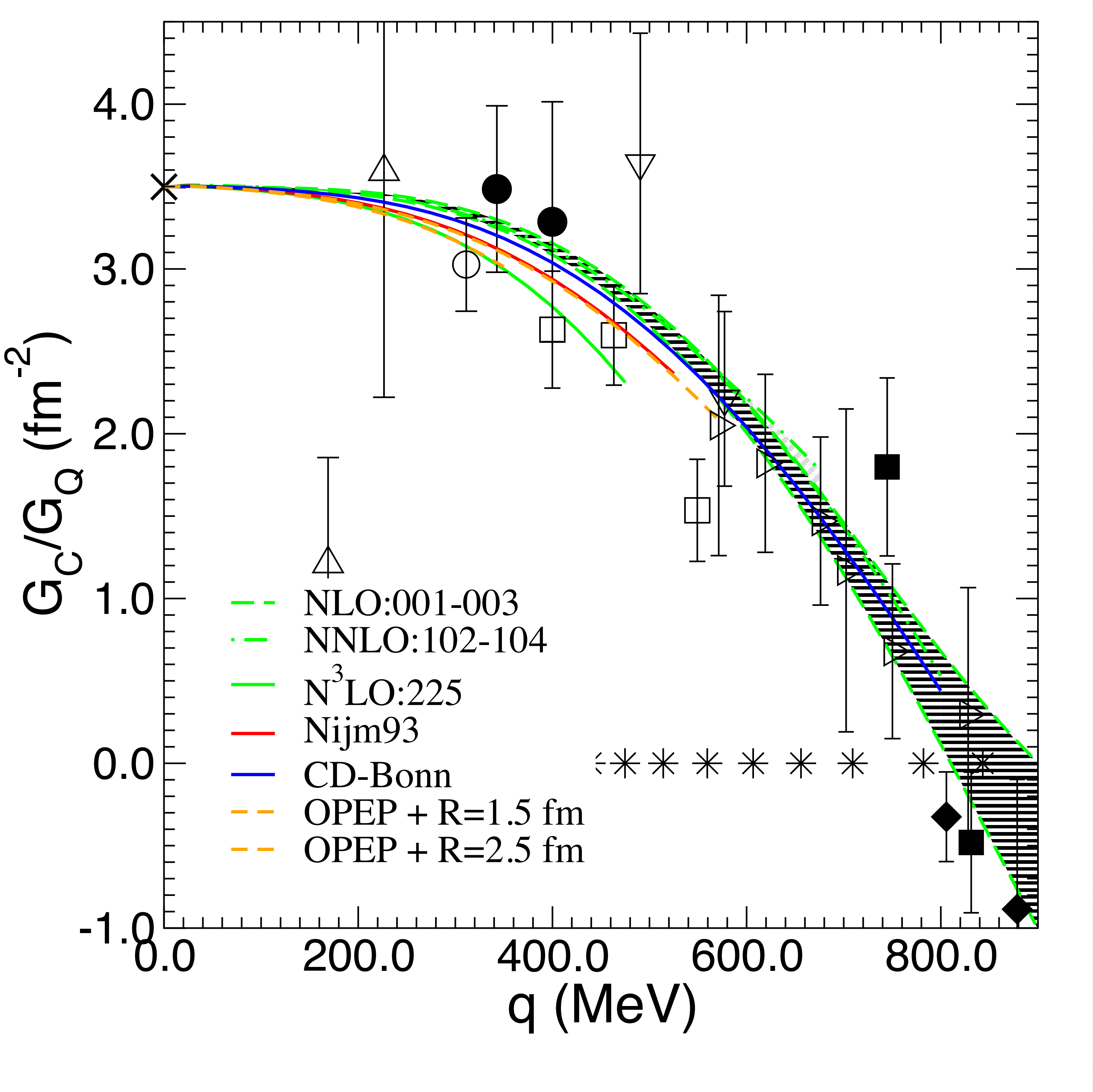
Phillips (2007)
c.f. Piarulli et al. (2013)



- Adjust $O(eP^{4.5})$ contact term to reproduce Q_d

Renormalizing G_C/G_Q

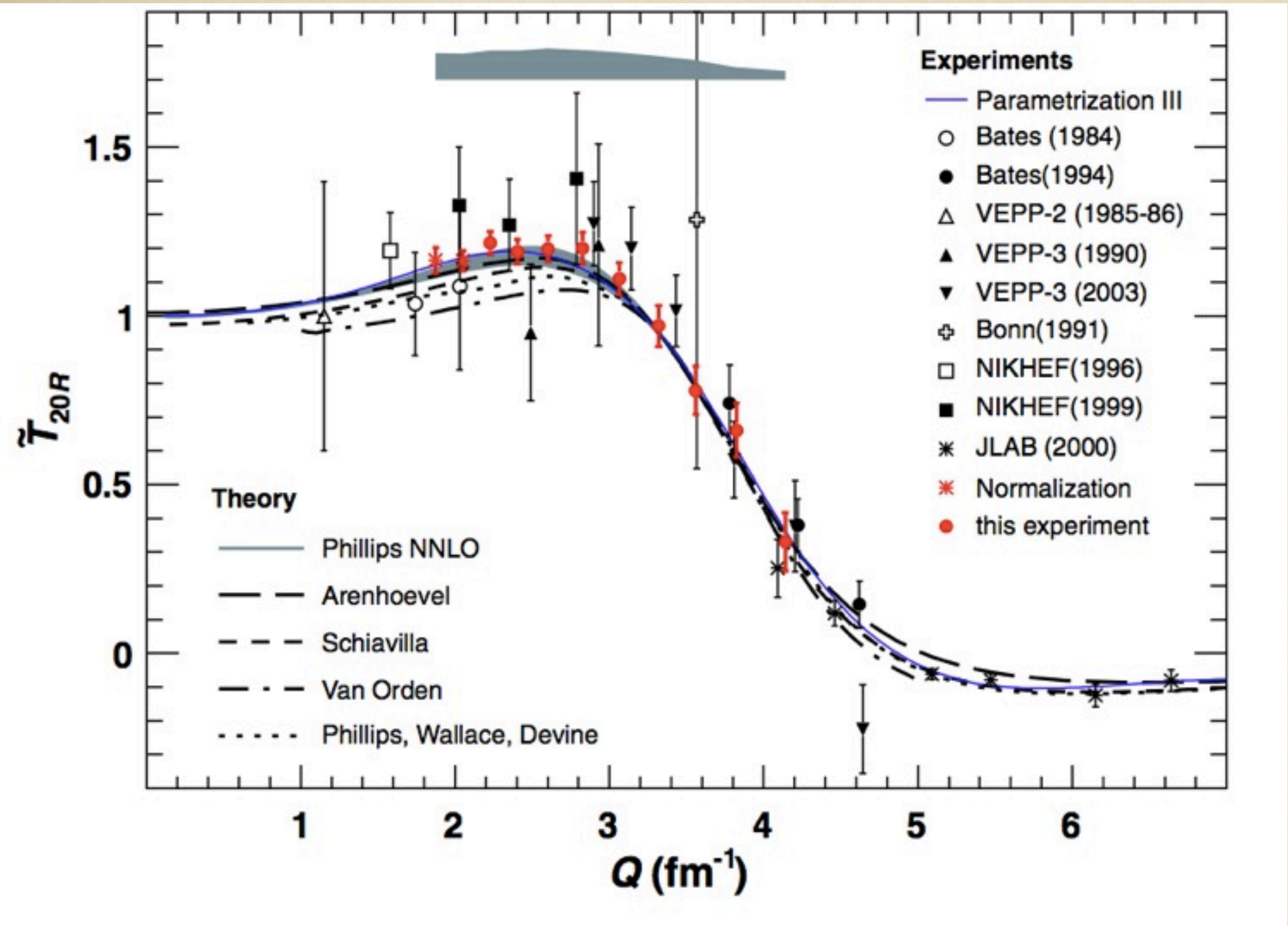
Phillips (2007)
c.f. Piarulli et al. (2013)



- Adjust $O(eP^{4.5})$ contact term to reproduce Q_d
- Ratio is largely independent of model for q<600 MeV
- G_C/G_Q to 3% at Q= 0.39 GeV

Confronting experiment

Zhang et al., PRL (2011)

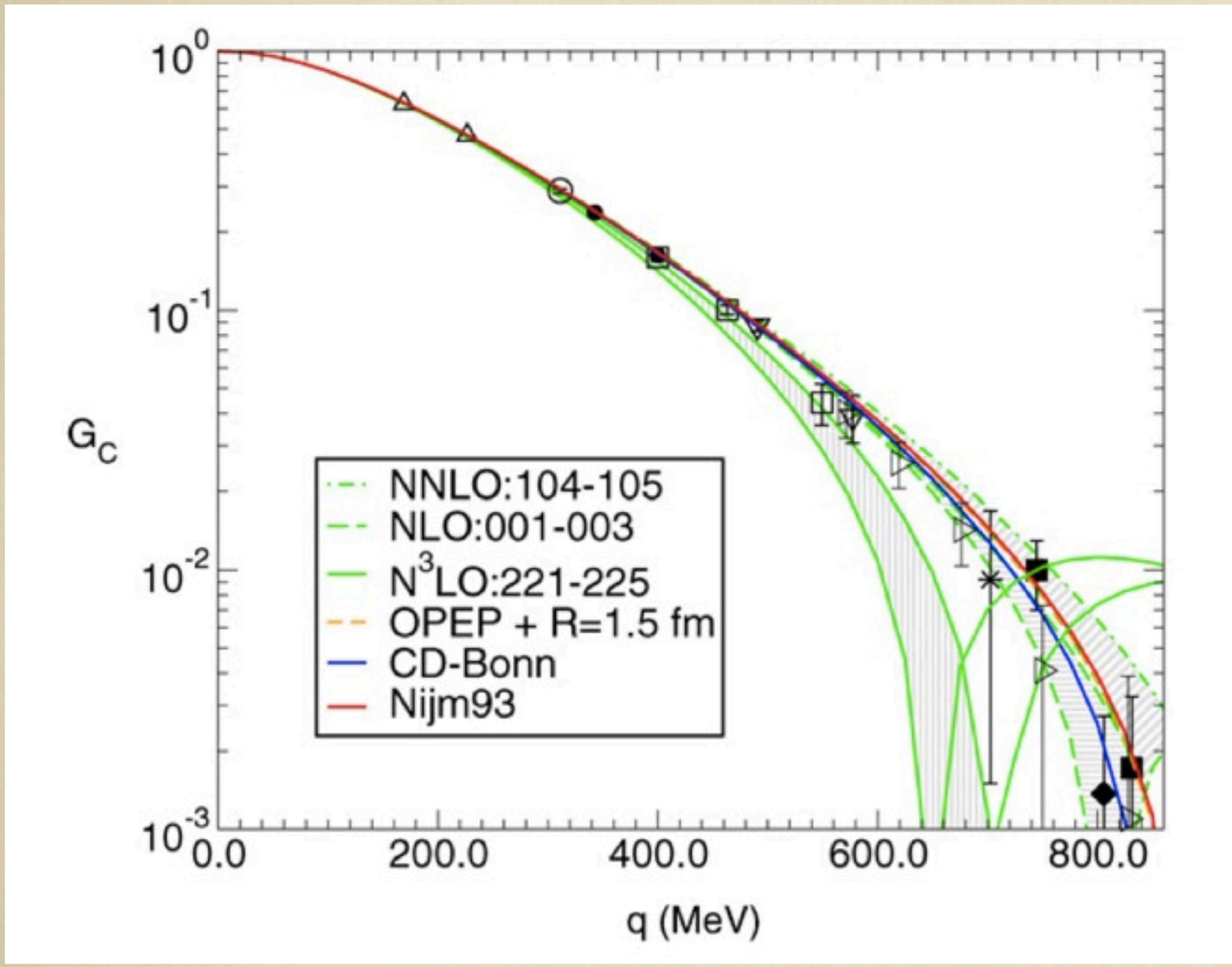


$$\tilde{T}_{20R} = -3\frac{\tilde{T}_{20}}{\sqrt{2}Q_d|Q|^2}$$

$$\leftrightarrow G_C/G_Q$$

Implications for ^{6}Li quadrupole moment?

χEFT for G_C up to $O(eP^4)$

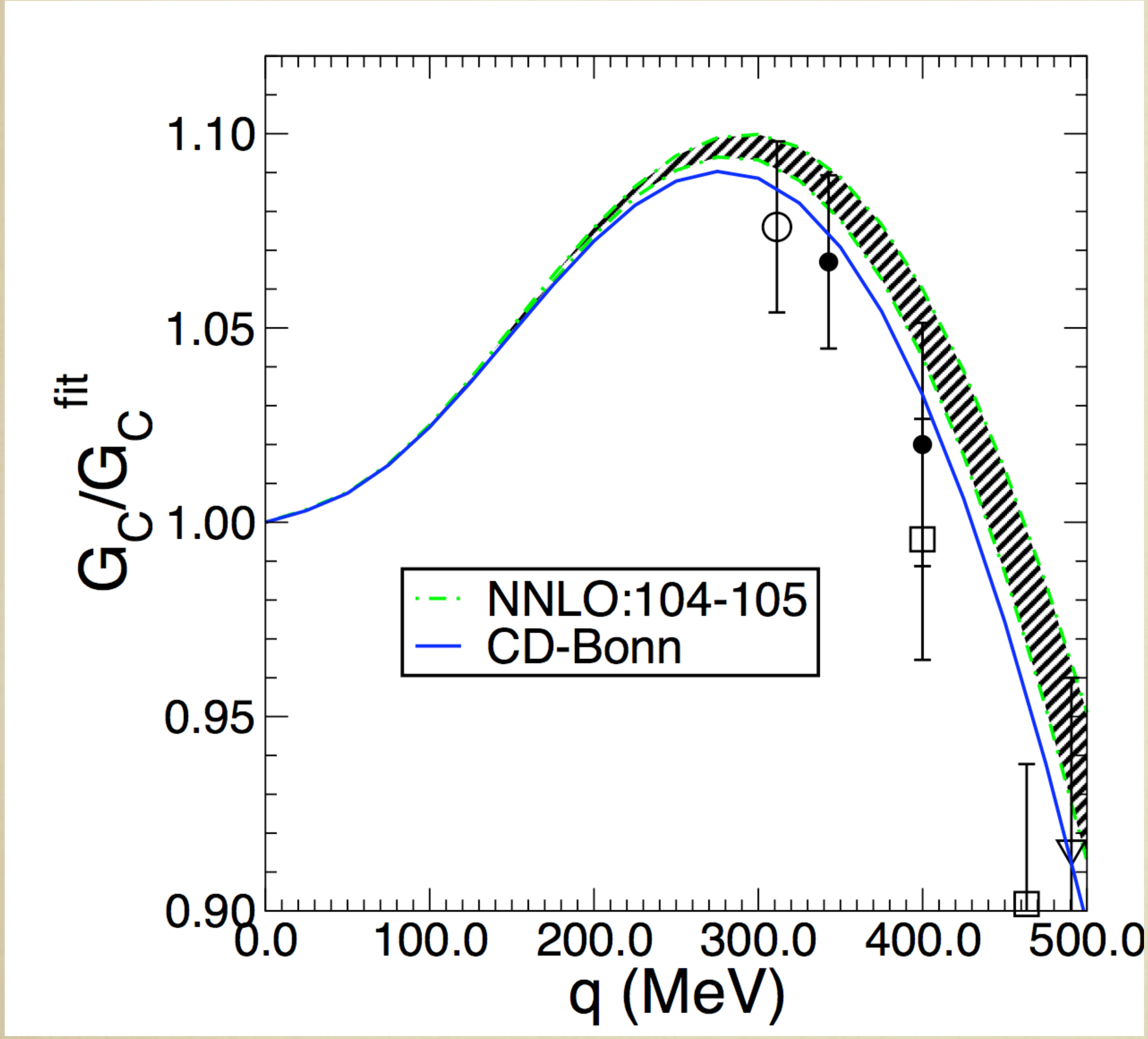


- Some sensitivity to deuteron wf
- Good J_0 convergence
- G_C dominated by $r \sim 1/m_\pi$ physics in this q range
- How to constrain interplay of contact pieces of J_0 and pion-range physics?

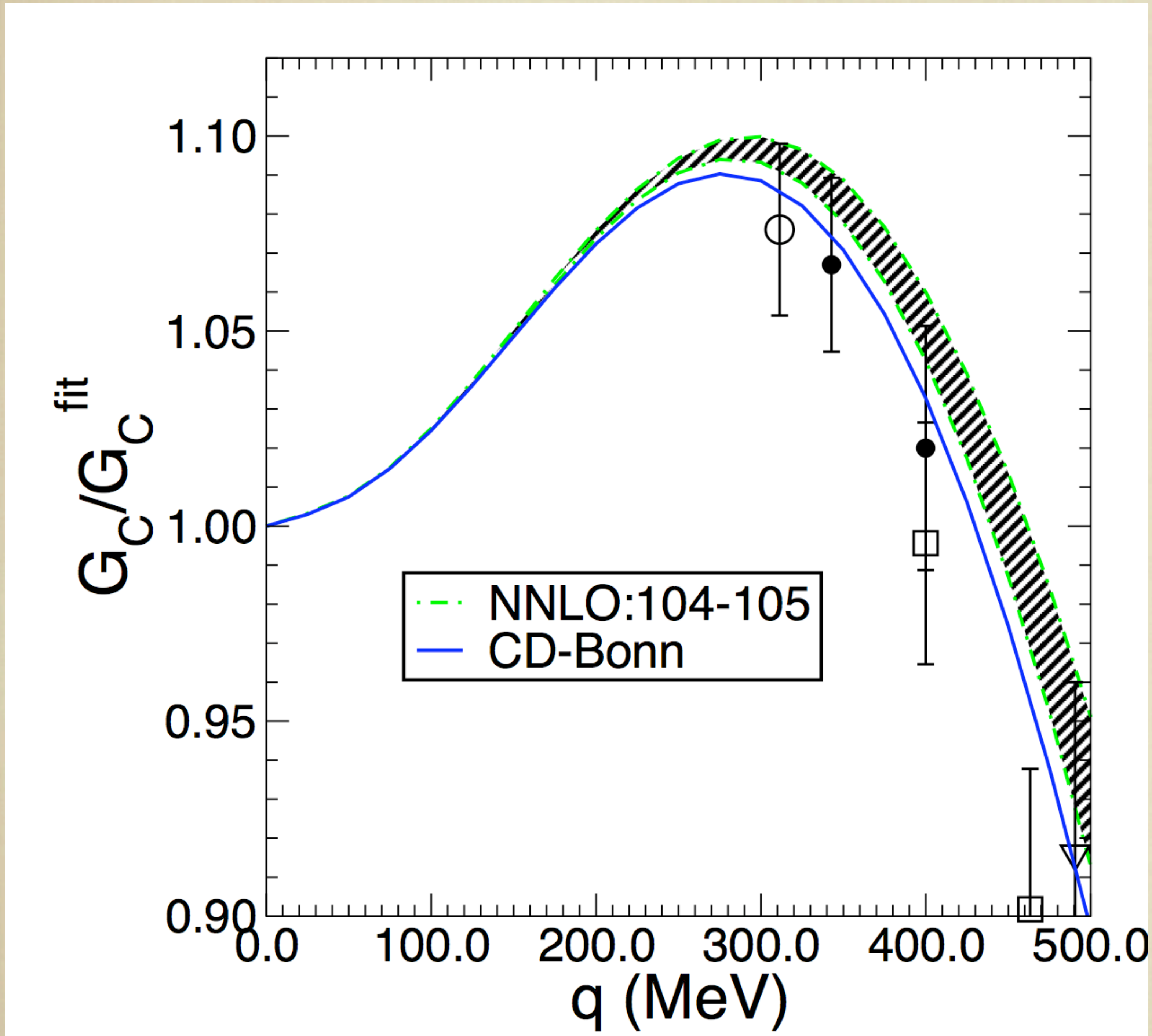
DP, J. Phys. G 34, 365 (2007)

c.f. Piarulli et al. (2013)

How much short-distance charge?

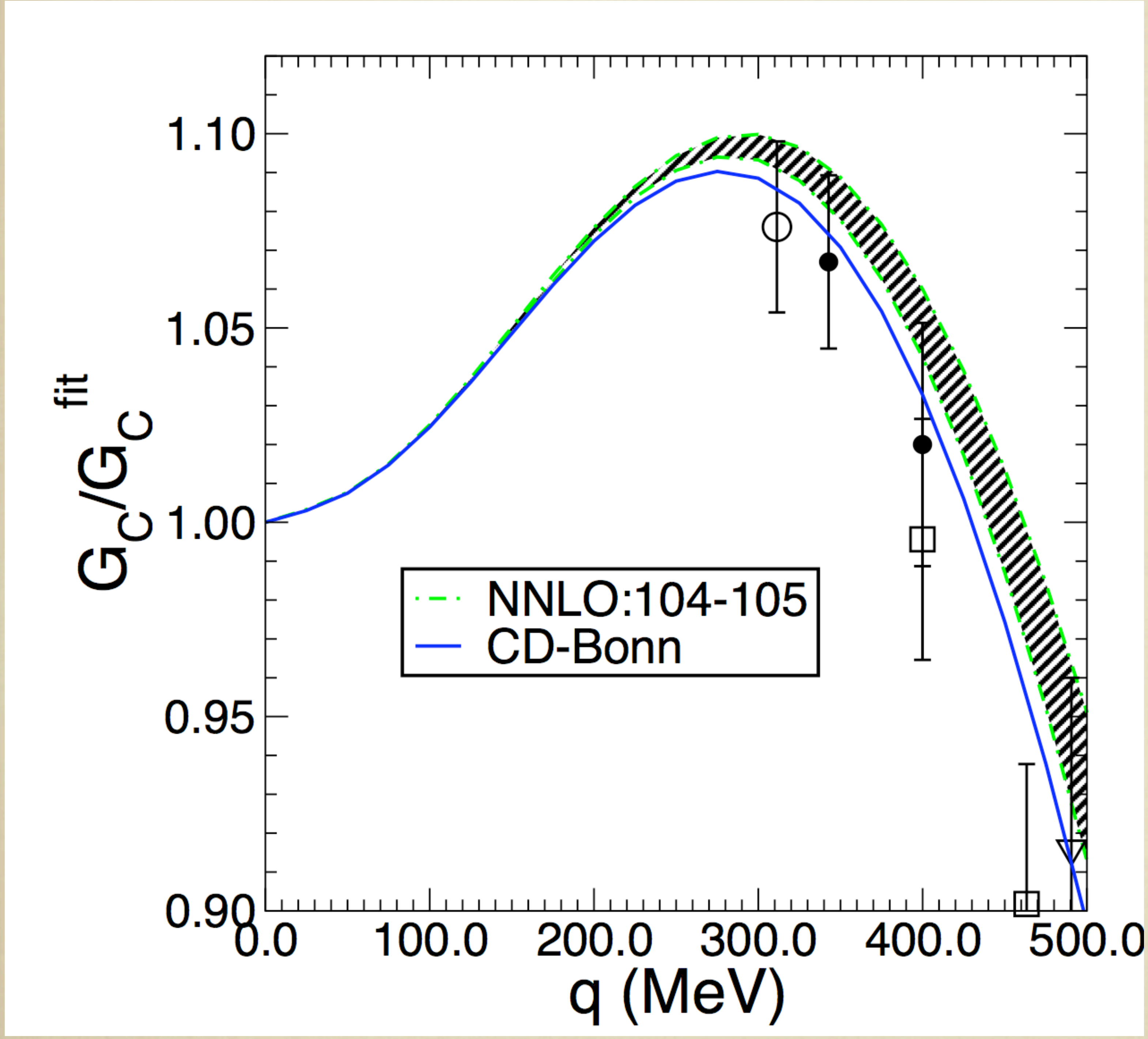


How much short-distance charge?



- $<r_{pt}^2>^{1/2}$=1.975(1) c.f. hydrogen level shift: $<r_{pt}^2>^{1/2}$=1.9753(10)
- At most 0.6% shift in ratio at Q=0.5 GeV/c

How much short-distance charge?



- $\langle r_{pt}^2 \rangle^{1/2}$=1.975(1) c.f. hydrogen level shift: $\langle r_{pt}^2 \rangle^{1/2}$=1.9753(10)
- At most 0.6% shift in ratio at Q=0.5 GeV/c
- Precision for A(Q) from JLab
- Caveat 1: more data
- Caveat 2: role of nucleon ffs

G_M beyond impulse approximation

	Expt.	NLO	NNLO	Nijm93
$\mu_d(\mu_N)$	0.857406(1)	0.856-0.862	0.853-0.860	0.848

G_M beyond impulse approximation

	Expt.	NLO	NNLO	Nijm93
$\mu_d(\mu_N)$	0.857406(1)	0.856-0.862	0.853-0.860	0.848

- 1% variation from short-distance effects

G_M beyond impulse approximation

	Expt.	NLO	NNLO	Nijm93
$\mu_d(\mu_N)$	0.857406(1)	0.856-0.862	0.853-0.860	0.848

- 1% variation from short-distance effects
- Up to (NDA) O(eP^4) there are two 2B contributions to $\mathbf{J}^{(s)}$: a pion-range current and a magnetic-moment contact interaction:

$$\mathcal{L}_{M1} = -eL_2(N^\dagger \sigma_i \epsilon^{ijk} F_{jk} N)(N^\dagger N)$$

$$\mathbf{J}^{(s)}_{d_9} = -2e \frac{g_A i}{f_\pi^2} d_9 \tau_1^a \tau_2^a \frac{\sigma_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + m_\pi^2} (\mathbf{q}_2 \times \mathbf{q}) + (1 \leftrightarrow 2)$$

G_M beyond impulse approximation

	Expt.	NLO	NNLO	Nijm93
$\mu_d(\mu_N)$	0.857406(1)	0.856-0.862	0.853-0.860	0.848

- 1% variation from short-distance effects

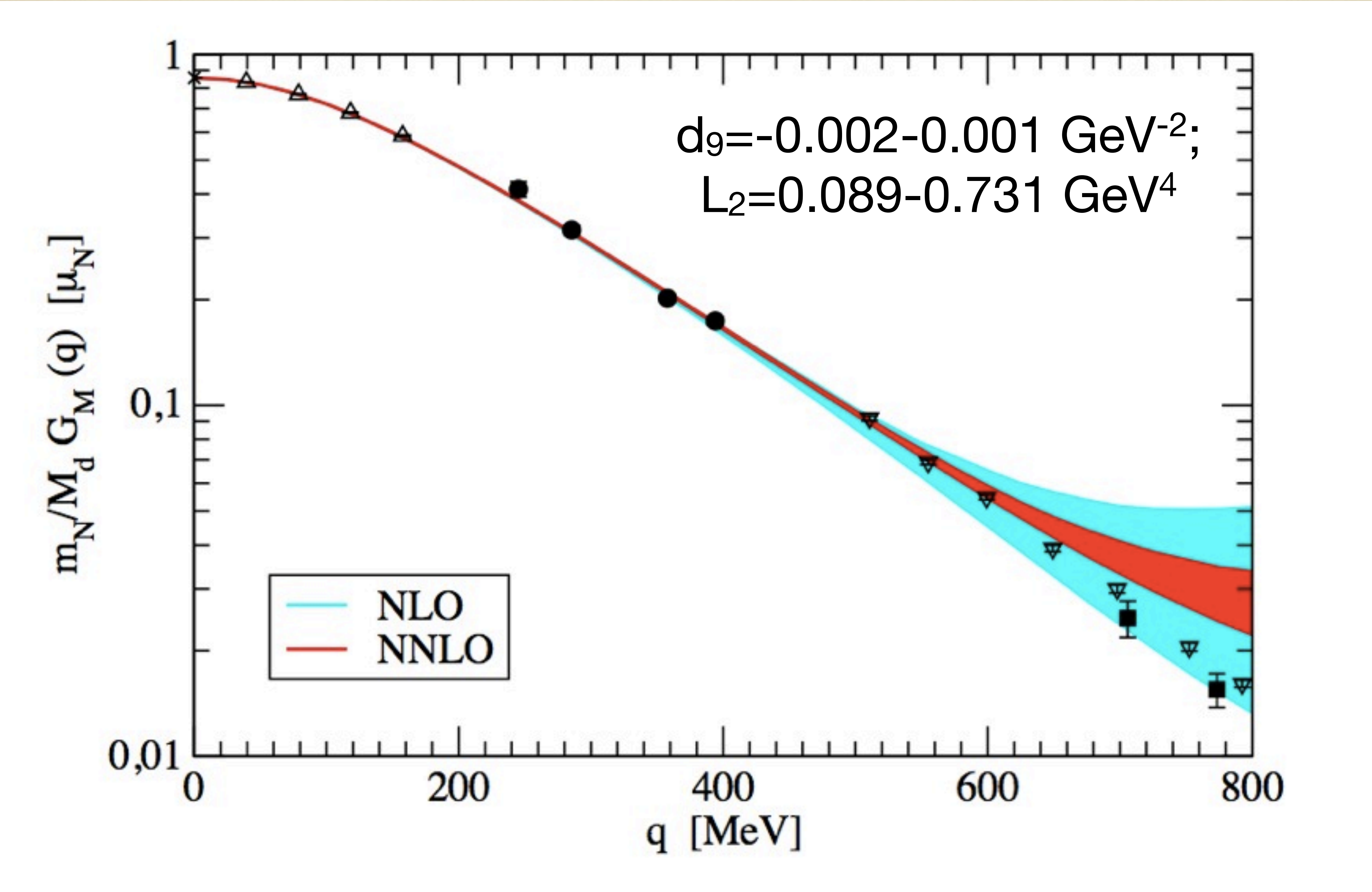
- Up to (NDA) $O(eP^4)$ there are two 2B contributions to $\mathbf{J}^{(s)}$: a pion-range current and a magnetic-moment contact interaction:

$$\mathcal{L}_{M1} = -eL_2(N^\dagger \sigma_i \epsilon^{ijk} F_{jk} N)(N^\dagger N)$$

$$\mathbf{J}^{(s)}_{d_9} = -2e\frac{g_A i}{f_\pi^2} d_9 \tau_1^a \tau_2^a \frac{\sigma_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + m_\pi^2}(\mathbf{q}_2 \times \mathbf{q}) + (1 \leftrightarrow 2)$$

- d_9 poorly constrained from single-nucleon sector

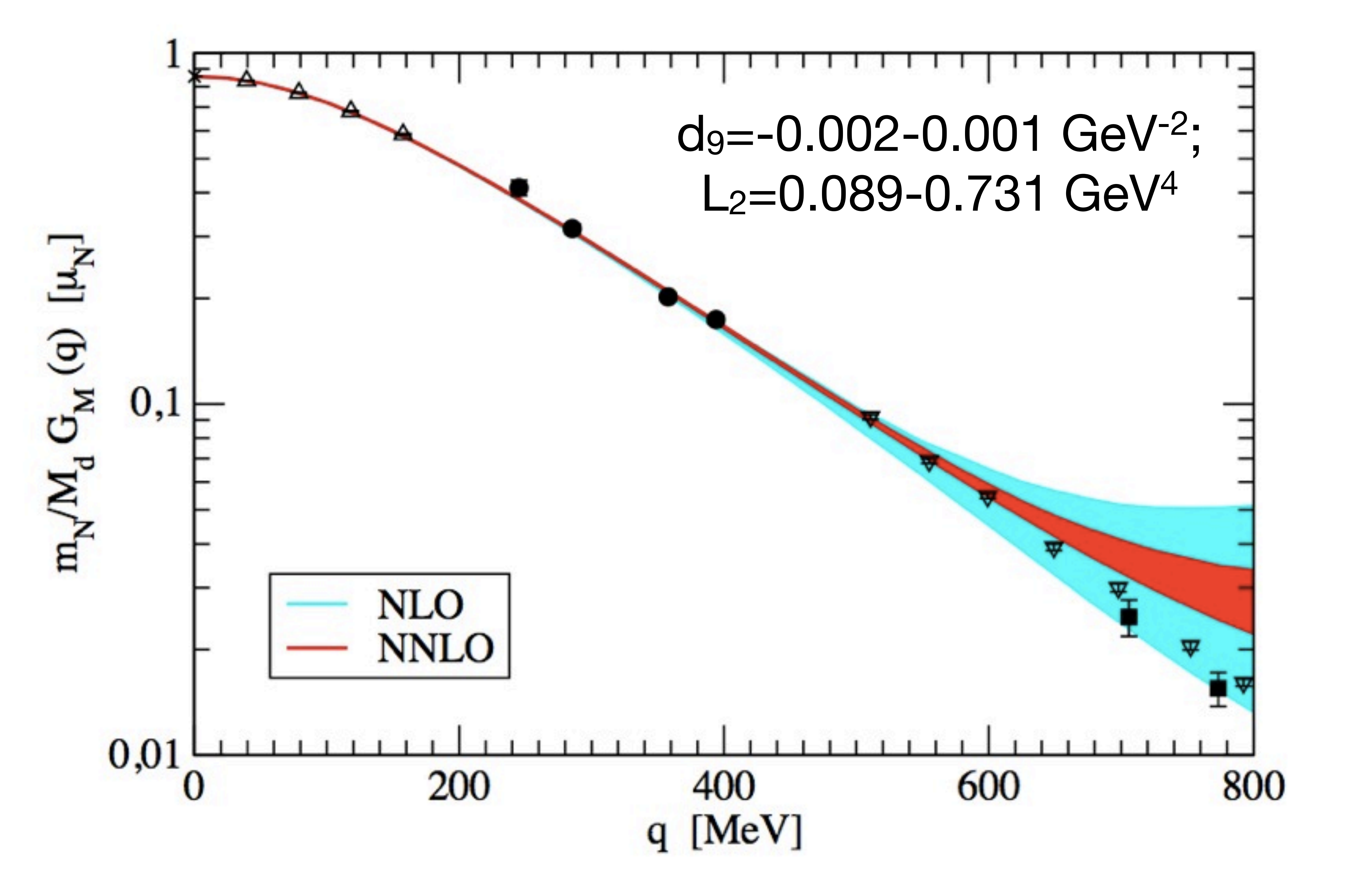
G_M to $O(eP^4)$



d9=-0.002-0.001 GeV-2;
L2=0.089-0.731 GeV4
NLO
NNLO
1
0,1
0,01
0
200
400
600
800
q [MeV]
mN/Md GM (q) [μN]

G_M to $O(eP^4)$

Koelling, Epelbaum, Phillips (2012)



c.f. Piarulli et al. (2013)

Lucky or good?

Lucky or good?

- χEFT, low scales: γ, m_π, p, Λ_{NN}; high scales: m_ρ, M, Δ,$(Mm_\pi)^{1/2}$

Lucky or good?

- χEFT, low scales: γ, m_π, p, Λ_{NN}; high scales: m_ρ, M, Δ,$(Mm_\pi)^{1/2}$

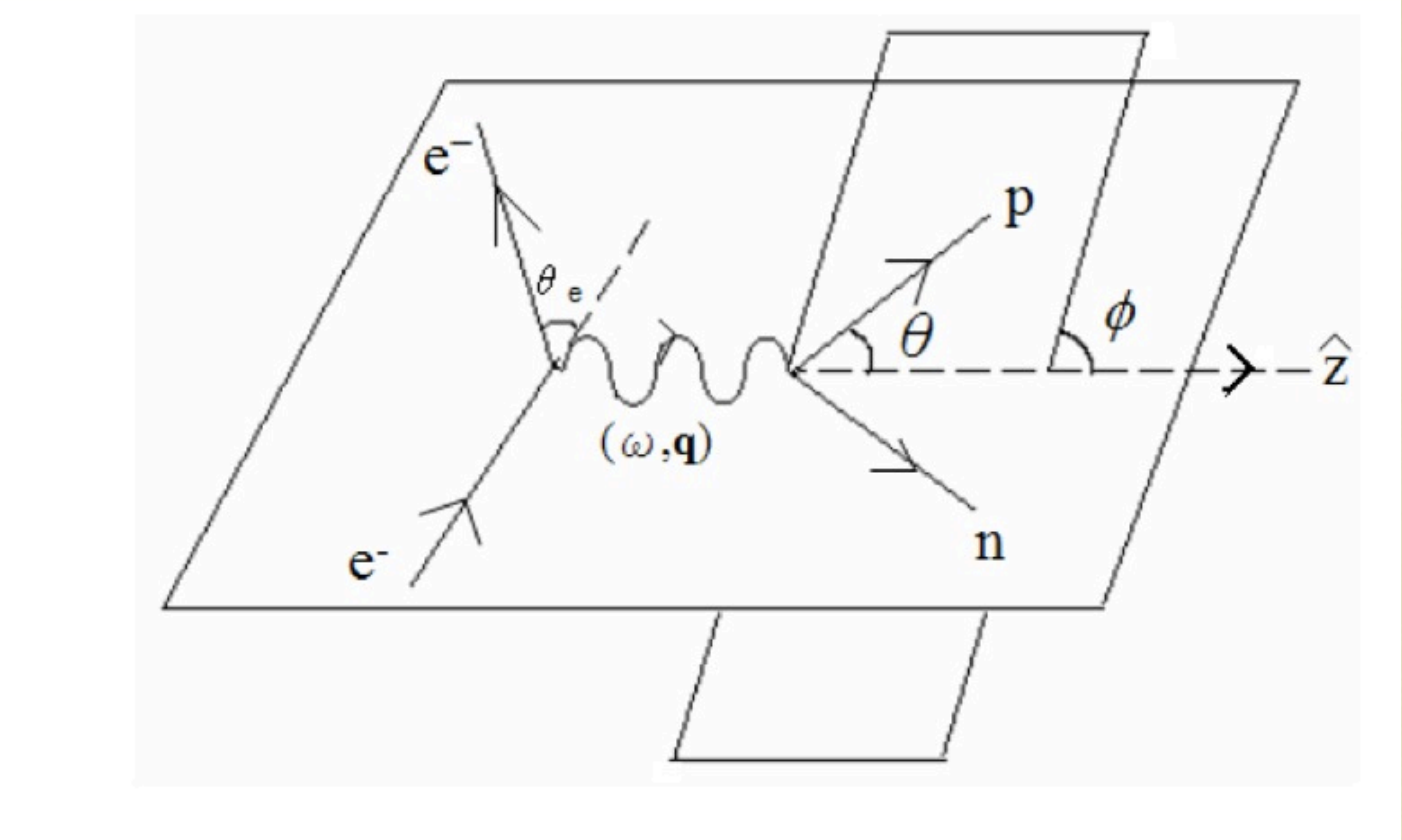
- $(Mm_\pi)^{1/2}$ doesn't play a role for space-like processes
- Factor of A/(A-1) is maximal for deuterium
- Since deuterium is isoscalar, no ΔN intermediate states

Lucky or good?

- χEFT, low scales: γ, m_π, p, Λ_{NN}; high scales: m_ρ, M, Δ,$(Mm_\pi)^{1/2}$

- $(Mm_\pi)^{1/2}$ doesn't play a role for space-like processes
- Factor of A/(A-1) is maximal for deuterium
- Since deuterium is isoscalar, no ΔN intermediate states
- Since deuterium is (mainly) $^3S_1$ only small deviations from NDA

Testing χEFT II: f_L in d(e,e'p)



$$\frac{d^3\sigma_L}{dk_2^{\rm lab} d\Omega_e^{\rm lab} d\Omega_p} = \frac{\alpha_{em}}{2\pi^2} \frac{k_2^{\rm lab}}{k_1^{\rm lab}(q^2)^2} \sum_{SM_S m_J} \rho_L |T_{SM_S 0 m_J}|^2$$

$$T_{SM_S \mu m_J} \sim \langle \Psi_{\mathbf{p}'} SM_S | J_\mu(\mathbf{q}) | m_J 0 \rangle$$

Our calculation

Yang, DP (2013)

$$\langle \Psi_{\mathbf{p}'} S M_S | J_0(\mathbf{q}) | m_J 0 \rangle = \langle \mathbf{p}' S M_S T | J_0(\mathbf{q}) | m_J 0 \rangle + \langle \mathbf{p}' S M_S T | t(E') G_0(E') J_0(\mathbf{q}) | m_J 0 \rangle$$

Our calculation

Yang, DP (2013)

$$\langle \Psi_{\mathbf{p}'} S M_S | J_0(\mathbf{q}) | m_J 0 \rangle = \langle \mathbf{p}' S M_S T | J_0(\mathbf{q}) | m_J 0 \rangle + \langle \mathbf{p}' S M_S T | t(E') G_0(E') J_0(\mathbf{q}) | m_J 0 \rangle$$

- Image deuteron wave function if final-state interaction is small
- FSI is not necessary if $\omega = \mathbf{q}^2/(2M_N) \Rightarrow \dfrac{E_{\mathrm{np}}}{1 \text{ MeV}} \approx 10 \dfrac{\mathbf{q}^2_{\mathrm{cm}}}{1 \text{ fm}^{-2}}$

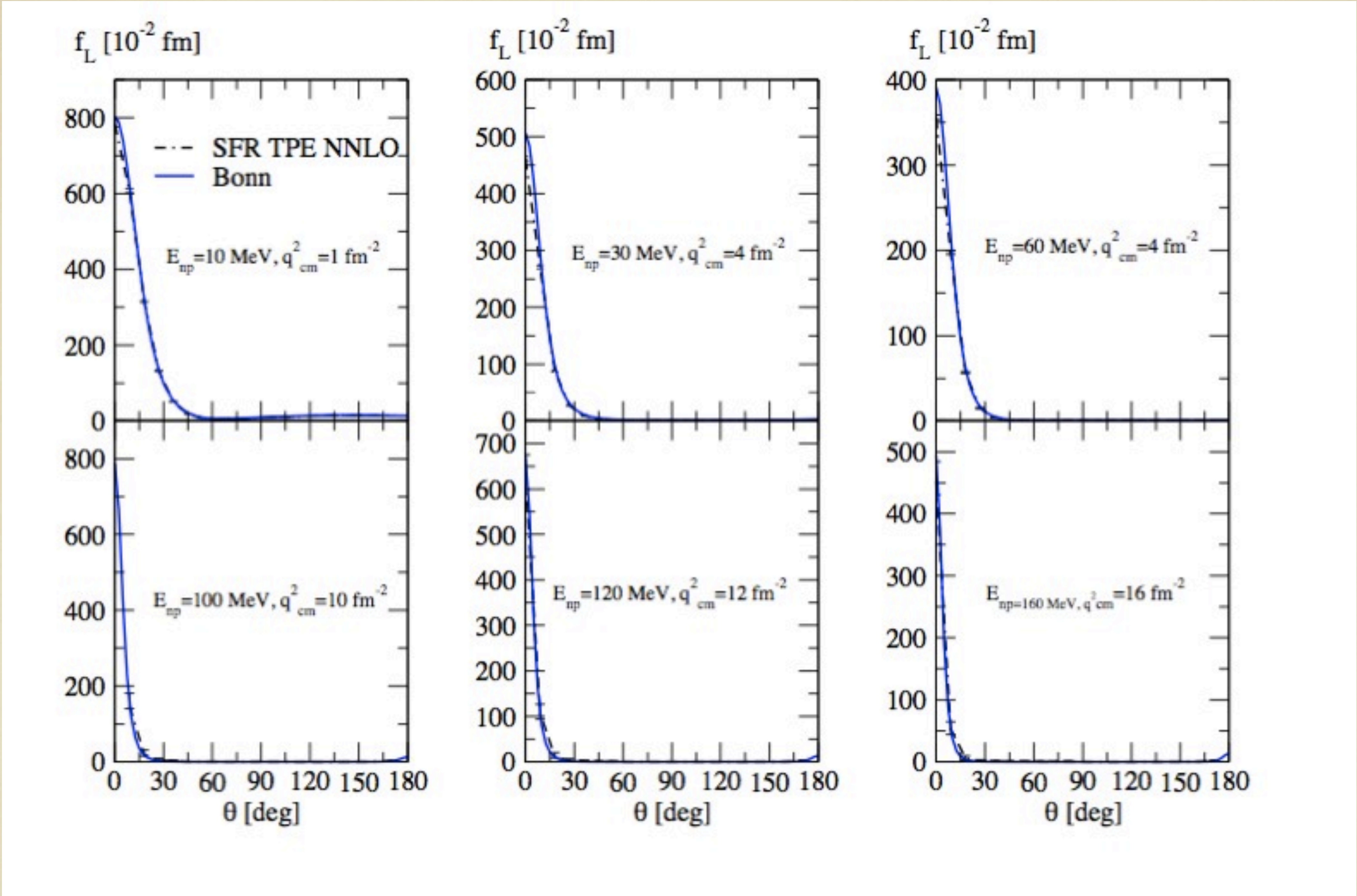
Our calculation

Yang, DP (2013)

$$\langle \Psi_{\mathbf{p}'} S M_S | J_0(\mathbf{q}) | m_J 0 \rangle = \langle \mathbf{p}' S M_S T | J_0(\mathbf{q}) | m_J 0 \rangle + \langle \mathbf{p}' S M_S T | t(E') G_0(E') J_0(\mathbf{q}) | m_J 0 \rangle$$

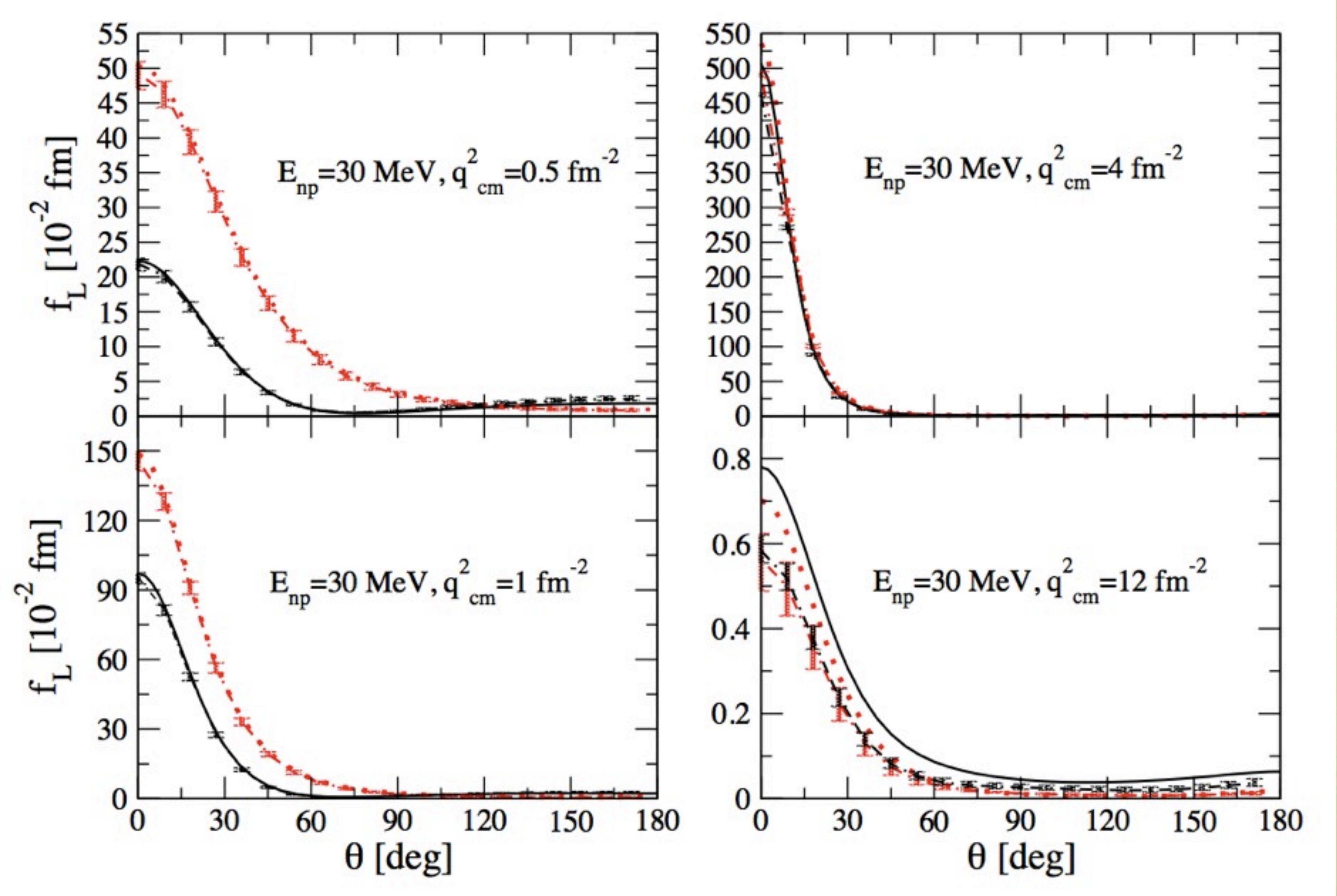
- Image deuteron wave function if final-state interaction is small
- FSI is not necessary if $\omega = \mathbf{q}^2/(2M_N) \Rightarrow \frac{E_{\rm np}}{1 \text{ MeV}} \approx 10 \frac{\mathbf{q}^2_{\rm cm}}{1 \text{ fm}^{-2}}$
- We compute J_0 to O(eP^3), use NNLO χEFT wave functions
- Computed using “subtraction method”
- Factorization + BHM form factors used for nucleon structure
- Comparison with Arenhoevel’s Bonn-potential calculation

Quasi-free ridge: impulse approx.

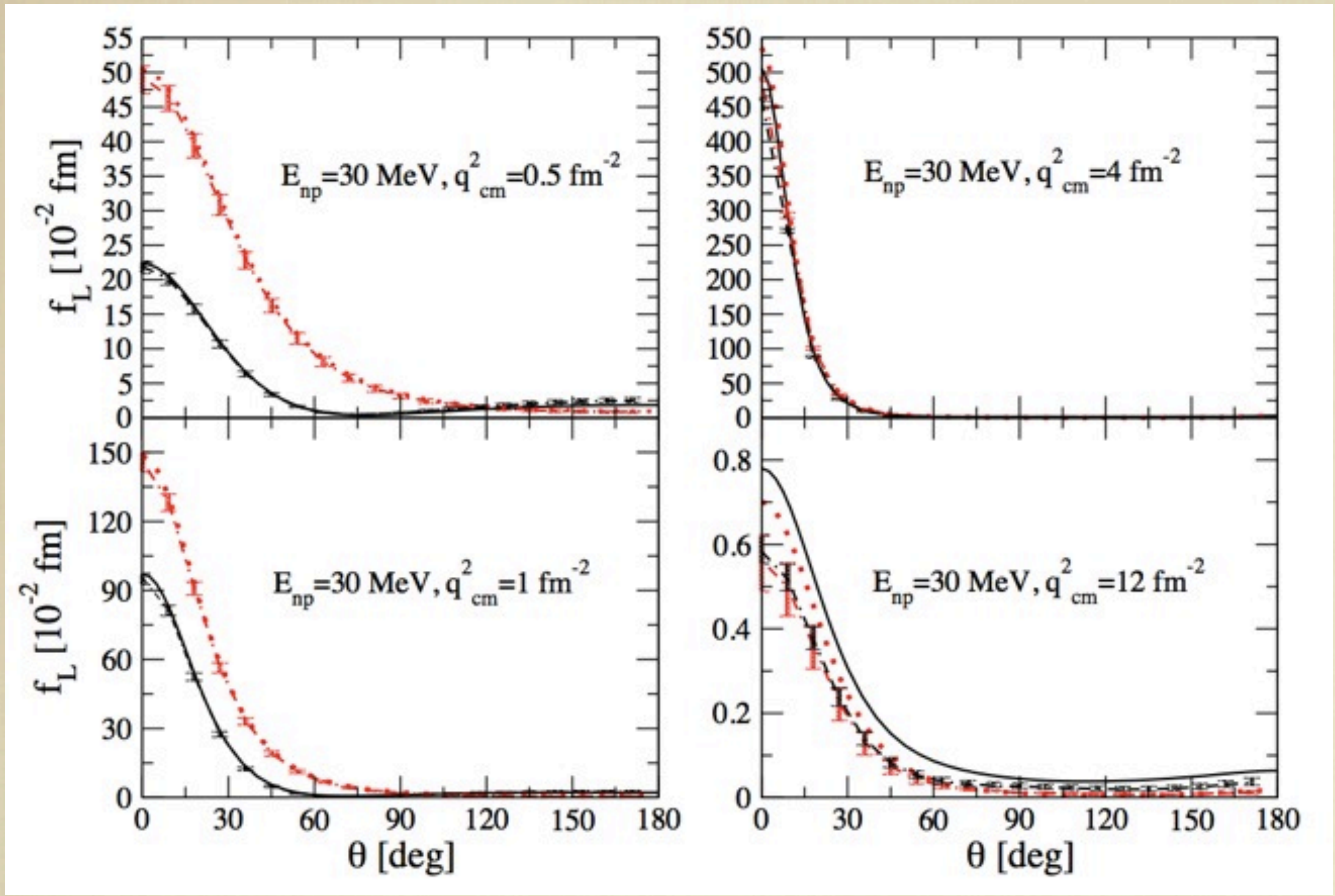


- Can be understood from scaling of wave function
- FSI corrections negligible from 30 MeV up

What about not quasi-free?

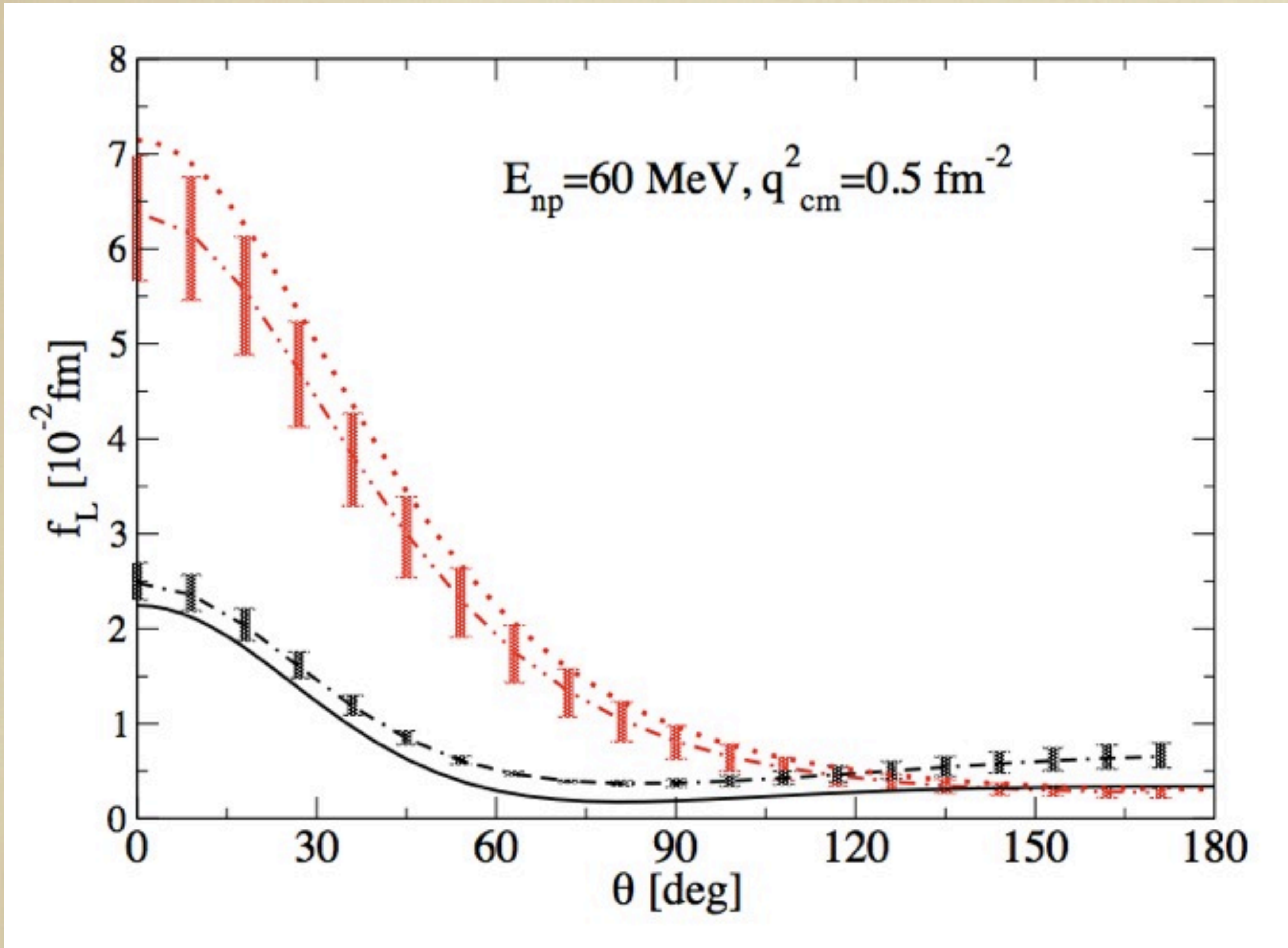


What about not quasi-free?



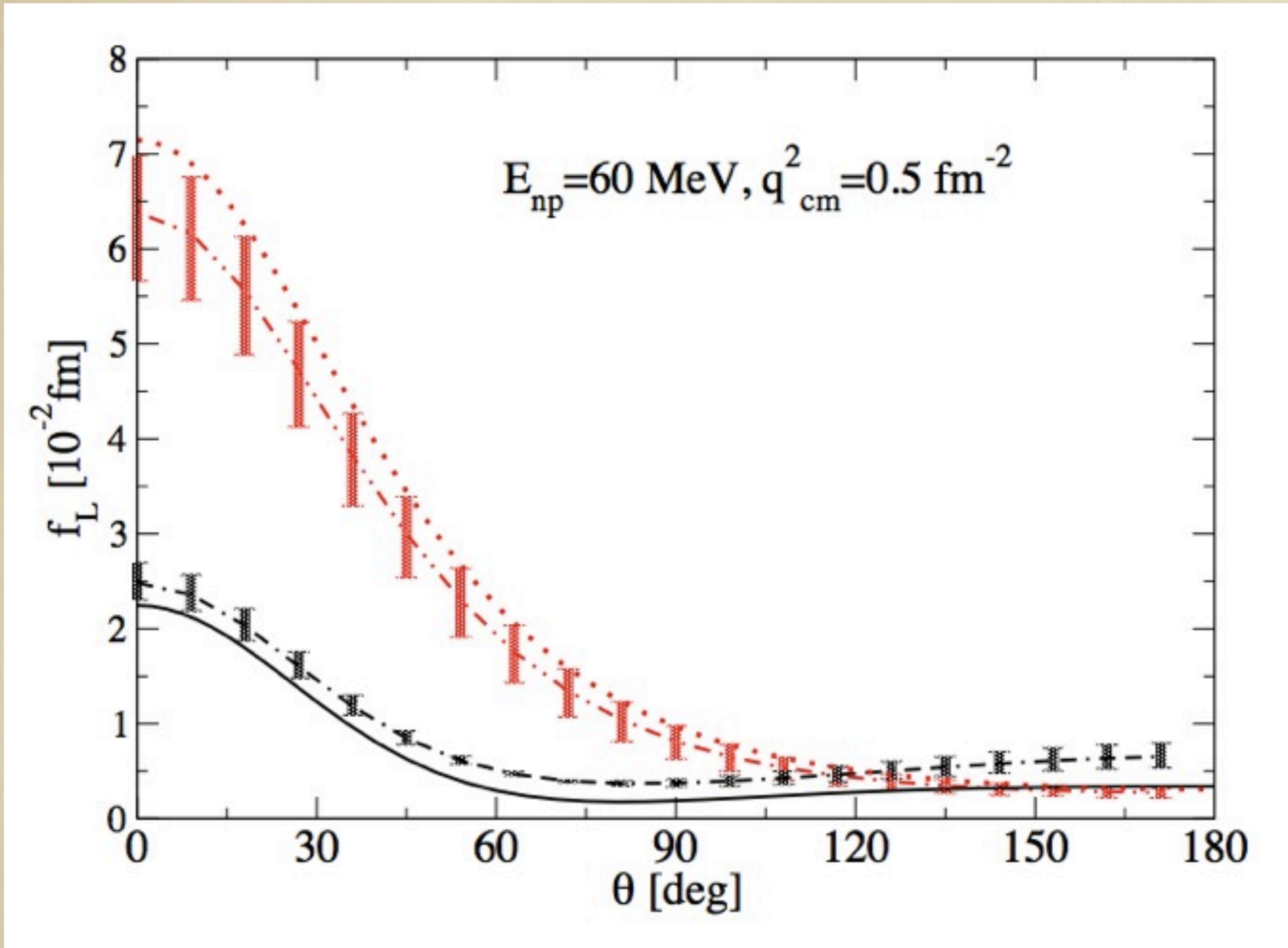
- Similar pattern at E_{np}=10 MeV, although FSI plays a bigger role in "QF" peak there
- Role of IA and FSI differences changes as $\mathbf{q}^2$ changes
- Big differences to Bonn⇒significant variation with cutoff

What is the maximum E_{np}?



- Big FSI
- Doesn't agree with Bonn
- Increasing issue above E_{np}=50 MeV
- Fit strategy?

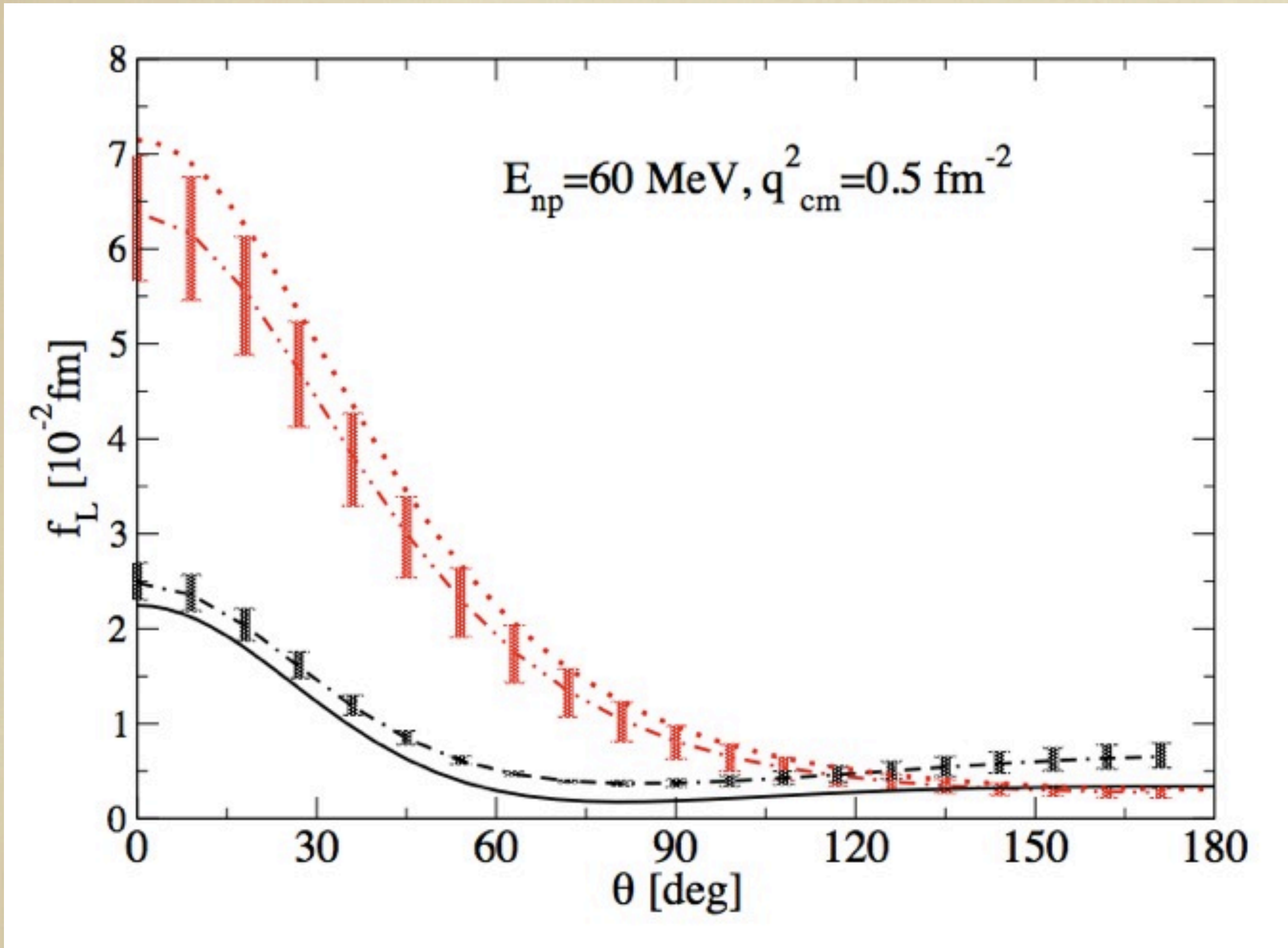
What is the maximum E_{np}?



- Big FSI
- Doesn't agree with Bonn
- Increasing issue above E_{np}=50 MeV
- Fit strategy?

E_{np}<60 MeV and $|\mathbf{q}^2-\mathbf{q}^2_{qf}|$<4 fm^{-2}, cutoff variation < 10%

What is the maximum E_{np}?



- Big FSI
- Doesn't agree with Bonn
- Increasing issue above E_{np}=50 MeV
- Fit strategy?

E_{np}<60 MeV and $|\mathbf{q}^2-\mathbf{q}^2_{qf}|$<4 fm^{-2}, cutoff variation < 10%

E_{np}<160 MeV and $|\mathbf{q}^2-\mathbf{q}^2_{qf}|$<2 fm^{-2}, |Bonn-χEFT| < 10%

The unnatural: $^3S_1 \rightarrow {}^1S_0$ transition

See computations of $np \rightarrow d\gamma$; $nd \rightarrow {}^3H\gamma$; $n^3He \rightarrow {}^4He\gamma$

Park et al. (1999); Song, Lazauskas, Park (2007-2009); Girlanda et al. (2010)

$$\langle \mathrm{M1V} \rangle = M\mu_V \int_R^\infty dr\, u(r)v(r) + L_{1V}(R)\frac{u(R)v(R)}{R^2}$$

The unnatural: $^3S_1 \rightarrow {}^1S_0$ transition

See computations of $np \rightarrow d\gamma$; $nd \rightarrow {}^3H\gamma$; $n^3He \rightarrow {}^4He\gamma$

Park et al. (1999); Song, Lazauskas, Park (2007-2009); Girlanda et al. (2010)

$$\langle \mathrm{M1V} \rangle = M\mu_V \int_R^\infty dr\, u(r)v(r) + L_{1V}(R)\frac{u(R)v(R)}{R^2}$$

But here, LO v(r)~1 at short distances, c.f. v(r)~r for regular potential

The unnatural: $^3S_1 \rightarrow {}^1S_0$ transition

See computations of $np \rightarrow d\gamma$; $nd \rightarrow {}^3H\gamma$; $n^3He \rightarrow {}^4He\gamma$

Park et al. (1999); Song, Lazauskas, Park (2007-2009); Girlanda et al. (2010)

$$\langle \mathrm{M1V} \rangle = M\mu_V \int_R^\infty dr\, u(r)v(r) + L_{1V}(R)\frac{u(R)v(R)}{R^2}$$

But here, LO v(r)~1 at short distances, c.f. v(r)~r for regular potential

RGE gives $L_{1V}(R) \sim \frac{1}{\Lambda_0^{7/4}} R^{5/4}$ $\quad \langle \hat{O}_{L_{1V}} \rangle \sim \frac{\sqrt{2\gamma}}{\Lambda_0}\left(\frac{\Lambda_{NN}}{\Lambda_0}\right)^{3/4}$

c.f. LO that scales as $\gamma^{-1/2}$

The unnatural: $^3S_1 \rightarrow {^1S_0}$ transition

See computations of np→dγ; nd→^{3}Hγ; n^{3}He→^{4}Heγ

Park et al. (1999); Song, Lazauskas, Park (2007-2009); Girlanda et al. (2010)

$$\langle \mathrm{M1V} \rangle = M\mu_V \int_R^\infty dr\, u(r)v(r) + L_{1V}(R)\frac{u(R)v(R)}{R^2}$$

But here, LO v(r)~1 at short distances, c.f. v(r)~r for regular potential

RGE gives $L_{1V}(R) \sim \frac{1}{\Lambda_0^{7/4}} R^{5/4}$ $\quad \langle \hat{O}_{L_{1V}} \rangle \sim \frac{\sqrt{2\gamma}}{\Lambda_0}\left(\frac{\Lambda_{NN}}{\Lambda_0}\right)^{3/4}$

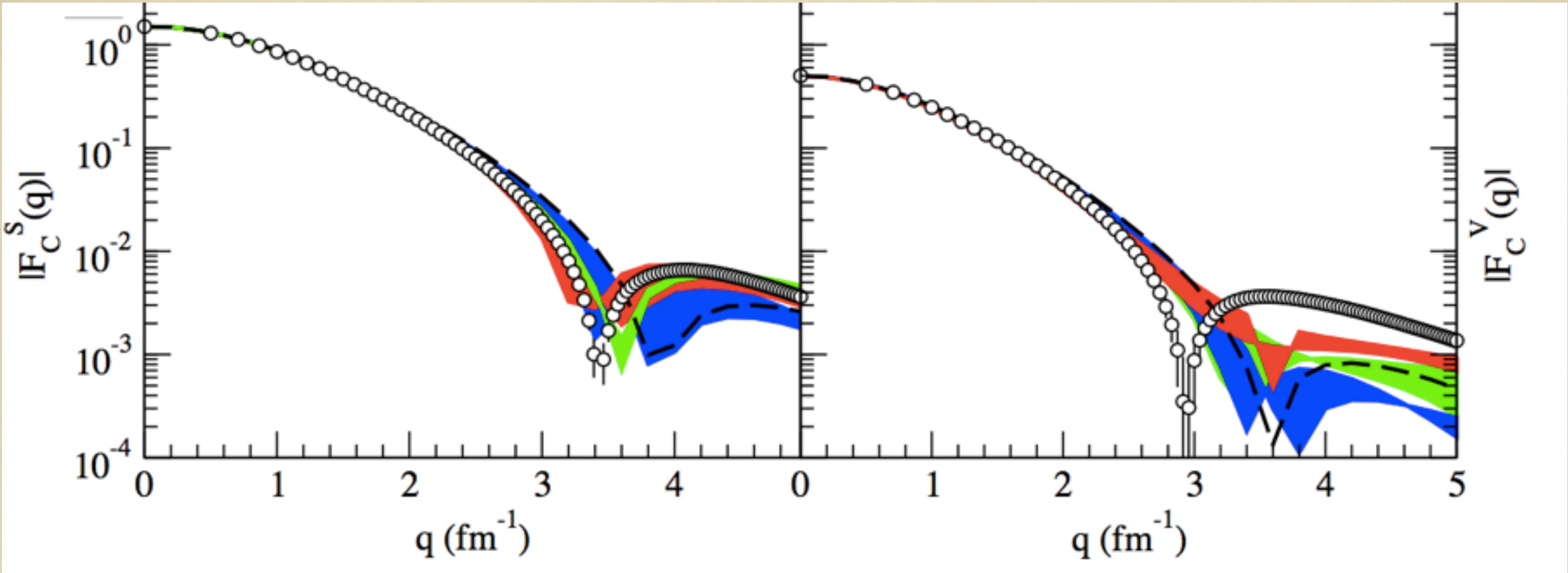
c.f. LO that scales as $\gamma^{-1/2}$

$P^{3/4}$ less important than in pionless EFT, but much more important than $O(eP^3)$, as indicated by NDA

Short-distance physics should be markedly more important than NDA indicates in isovector S-to-S transitions

Trinucleon form factors, results

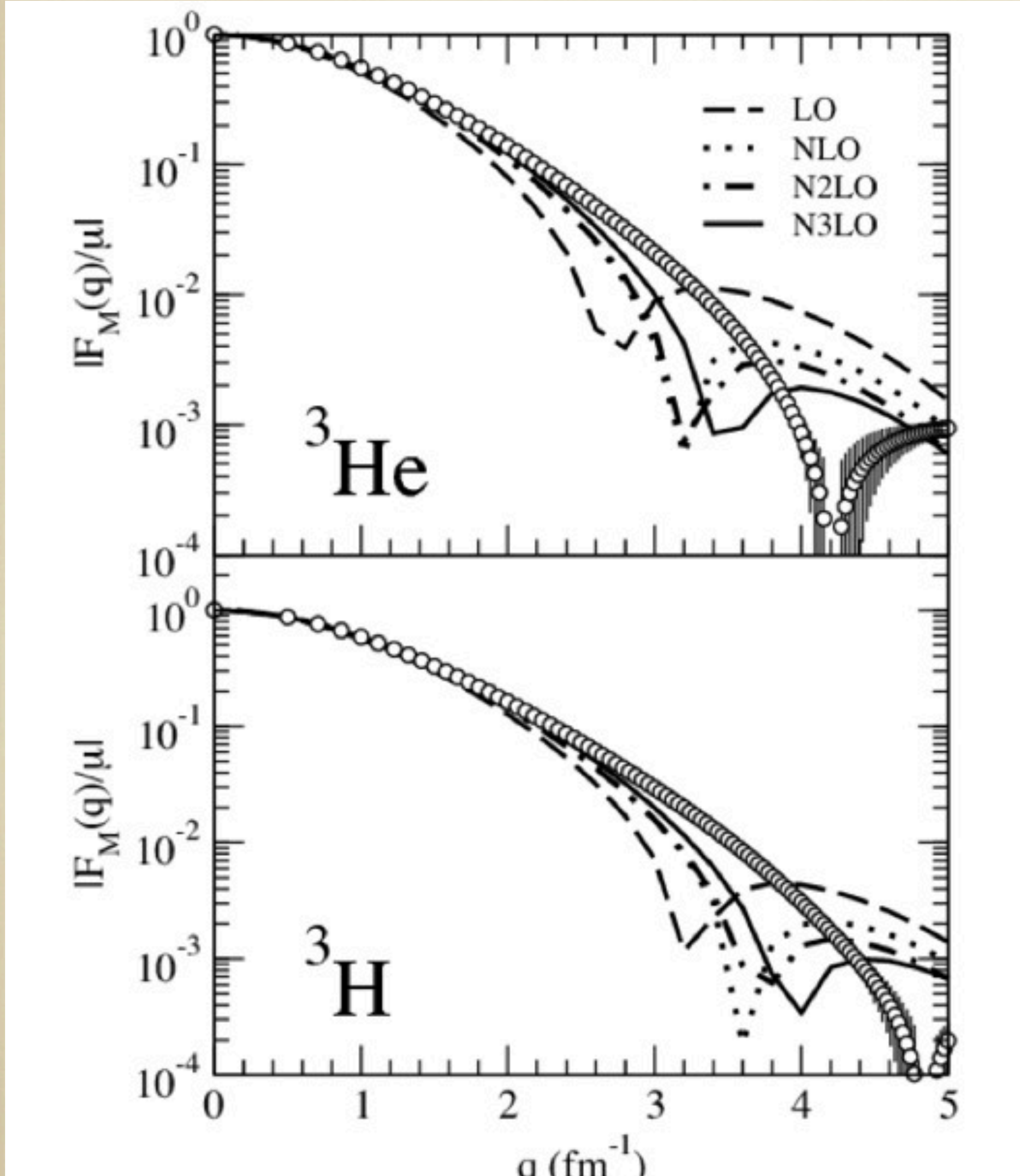
Piarulli et al. (2013)



and note radii

Trinucleon form factors, results

Piarulli et al. (2013)



O(eP^4) (nm) bigger than O(eP^3) at low q

Summary and outlook

- Electromagnetic reactions on light nuclei are a good place to test the efficacy of different χEFT variants
- Clear separation of “fast” evolution in $|\mathbf{q}|$ due to one-body operators, and “slow” pieces due to short-distance effects
- Elastic electron-deuteron developed up to at least $O(eP^4)$: only small enhancements of contact terms over NDA
- Trinucleon form factors: enhanced role for short-distance operators?
- d(e,e’p): f_L reasonable, f_T shows significant 2π exchange currents, but with sizable cutoff dependence
- Weak reactions: L_{1A} and modified counting?

Summary and outlook

- Electromagnetic reactions on light nuclei are a good place to test the efficacy of different χEFT variants
- Clear separation of "fast" evolution in $|\mathbf{q}|$ due to one-body operators, and "slow" pieces due to short-distance effects
- Elastic electron-deuteron developed up to at least $O(eP^4)$: only small enhancements of contact terms over NDA

  Perturbative calculation in progress: Pavon Valderrama and DRP
- Trinucleon form factors: enhanced role for short-distance operators?

  Piarulli et al. (2013)
- d(e,e'p): f_L reasonable, f_T shows significant 2π exchange currents, but with sizable cutoff dependence

  Rozpedzik et al. (2011)
- Weak reactions: L_{1A} and modified counting?

BACKUP SLIDES

Electron-deuteron observables

$$G_C = \frac{1}{3|e|}\left(\langle 1|\mathcal{M}^0|1\rangle + \langle 0|\mathcal{M}^0|0\rangle + \langle -1|\mathcal{M}^0|-1\rangle\right)$$

$$G_Q = \frac{1}{|e|Q^2}\left(\langle 0|\mathcal{M}^0|0\rangle - \langle 1|\mathcal{M}^0|1\rangle\right)$$

$$G_M = -\frac{1}{\sqrt{2\eta}|e|}\langle 1|\mathcal{M}^+|0\rangle; \qquad \eta = \frac{Q^2}{4M_d^2}$$

Evaluated in Breit Frame

Electron-deuteron observables

$$G_C = \frac{1}{3|e|}\left(\langle 1|\mathcal{M}^0|1\rangle + \langle 0|\mathcal{M}^0|0\rangle + \langle -1|\mathcal{M}^0|-1\rangle\right)$$

$$G_Q = \frac{1}{|e|Q^2}\left(\langle 0|\mathcal{M}^0|0\rangle - \langle 1|\mathcal{M}^0|1\rangle\right)$$

$$G_M = -\frac{1}{\sqrt{2\eta}|e|}\langle 1|\mathcal{M}^+|0\rangle; \qquad \eta = \frac{Q^2}{4M_d^2}$$

THEORY

Evaluated in Breit Frame

Electron-deuteron observables

$$G_C = \frac{1}{3|e|}\left(\langle 1|\mathcal{M}^0|1\rangle + \langle 0|\mathcal{M}^0|0\rangle + \langle -1|\mathcal{M}^0|-1\rangle\right)$$

$$G_Q = \frac{1}{|e|Q^2}\left(\langle 0|\mathcal{M}^0|0\rangle - \langle 1|\mathcal{M}^0|1\rangle\right)$$

THEORY

$$G_M = -\frac{1}{\sqrt{2\eta}|e|}\langle 1|\mathcal{M}^+|0\rangle; \qquad \eta = \frac{Q^2}{4M_d^2}$$

Evaluated in Breit Frame

EXPERIMENT

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\left[A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)\right]; \qquad T_{20}(Q^2;\theta_e)$$

Electron-deuteron observables

$$G_C = \frac{1}{3|e|}\left(\langle 1|\mathcal{M}^0|1\rangle + \langle 0|\mathcal{M}^0|0\rangle + \langle -1|\mathcal{M}^0|-1\rangle\right)$$

$$G_Q = \frac{1}{|e|Q^2}\left(\langle 0|\mathcal{M}^0|0\rangle - \langle 1|\mathcal{M}^0|1\rangle\right)$$

THEORY

$$G_M = -\frac{1}{\sqrt{2\eta}|e|}\langle 1|\mathcal{M}^+|0\rangle; \qquad \eta = \frac{Q^2}{4M_d^2}$$

$$\begin{aligned} A &= G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 M_d^4 G_Q^2, \\ B &= \frac{4}{3}\eta(1+\eta)G_M^2, \\ T_{20} &= -\frac{1}{\sqrt{2}}\frac{1}{A(Q^2)+B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)}\left[\frac{8}{3}\eta G_C(Q^2)G_Q(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2)\right. \\ &\quad \left. + \frac{1}{3}\eta\left\{1 + 2(1+\eta)\tan^2\left(\frac{\theta_e}{2}\right)\right\}G_M^2(Q^2)\right]. \end{aligned}$$

Evaluated in Breit Frame

EXPERIMENT

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}\left[A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta_e}{2}\right)\right]; \qquad T_{20}(Q^2;\theta_e)$$

Deuteron photodisintegration

Rozpedzik et al. (2011)