

The APS Council and the DNP have endorsed the establishment of the

Herman Feshbach Prize in Nuclear Physics

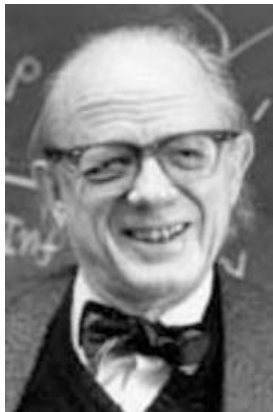
Purpose: To recognize and encourage outstanding research in theoretical nuclear physics. The prize will consist of \$10,000 and a certificate citing the contributions made by the recipient. The prize will be presented biannually or annually.

Herman Feshbach was a dominant force in Nuclear Physics for many years. The establishment of this prize depends entirely on the contributions of institutions, corporations and individuals associated with Nuclear Physics. So far, significant contributions have been made by MIT, the DNP, ORNL/U.Tenn, JSA/SURA, BSA, Elsevier Publishing, TUNL, TRIUMF, MSU, and a number of individuals. More than ~~\$150,000~~ **\$195,000** has been raised, ~~primarily through institutional contributions.~~ **It is very important that physicists make contributions to carry the endowment over the \$200,000 mark, so that the Prize will be eligible to be awarded annually.** Please help us reach that goal by making a contribution.

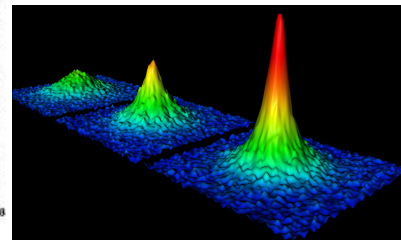
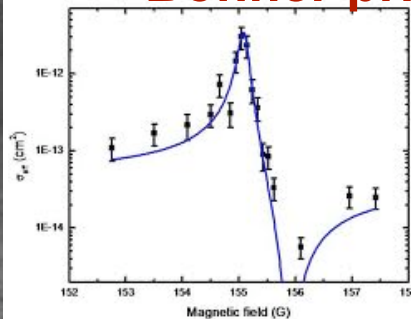
Go online at <http://www.aps.org/> Look for the support banner and click APS member (membership number needed) and look down the list of causes.



If you have any questions, please contact G. A. (Jerry) Miller UW, <mler@uw.edu>.



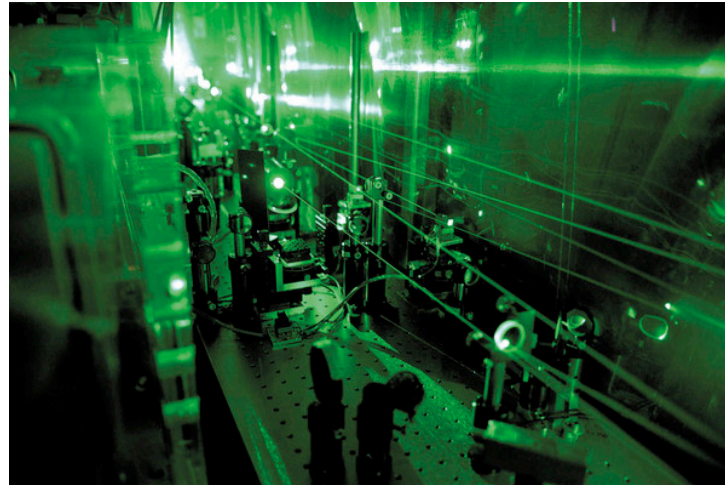
If annual- number of experimentalists winning Bonner prize goes up by >50%



The Proton Radius Puzzle: A challenge to all of us

Gerald A. Miller, University of Washington

Pohl et al Nature 466, 213 (8 July 2010)



muon H $r_p = 0.84184(67)$ fm
electron H $r_p = 0.8768(69)$ fm
electron-p scattering $r_p = 0.875(10)$ fm

arXiv:1301.0905

Pohl, Gilman, Miller, Pachucki
(ARNPS63, 2013)

$$r_p^2 \equiv -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

4 % in radius: why care?

- Can't be calculated to that accuracy
- 1/2 cm in radius of a basketball

4 % in radius: why care?

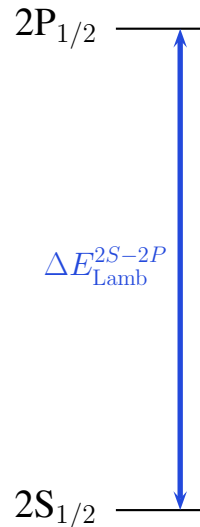
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Is the muon-proton interaction the same as the electron-proton interaction? - many possible ramifications

Experiment: Basic idea

The Experiment

Muonic Hydrogen



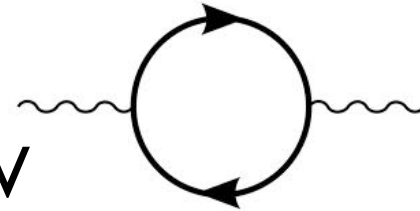
$2S_{1/2}, 2P_{1/2}$ states are degenerate—
Schroedinger, Dirac eqns.

The Lamb shift is the splitting of the degenerate $2S_{1/2}$ and $2P_{1/2}$ eigenstates, due to vacuum polarization

Dominant in μH

205 of 206 meV

Range is $1/m_e \sim a_B(\text{muon})$



Dominant in eH



Proton extent in hydrogen atom

$$\delta V(\mathbf{r}) \equiv V_C(\mathbf{r}) - V_C^{\text{pt}}(\mathbf{r}) = -4\pi\alpha \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{(G_E(\mathbf{q}^2) - 1)}{q^2}$$

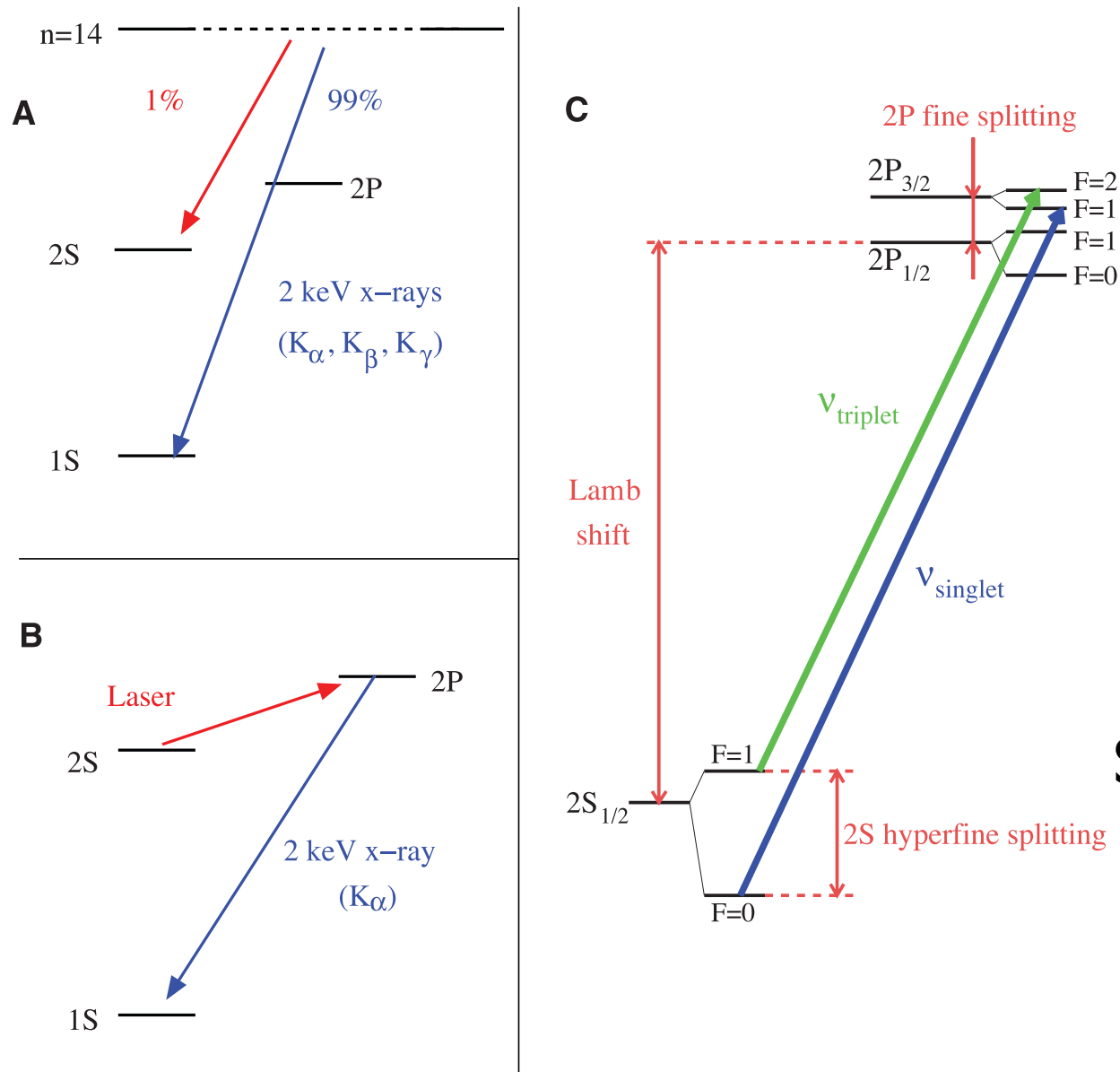
$$G_E(\mathbf{q}^2) - 1 \approx -\mathbf{q}^2 r_p^2 / 6$$

$$\Delta E = \langle \Psi_S | \delta V | \Psi_S \rangle = \frac{2}{3} \pi \alpha |\Psi_S(0)|^2 r_p^2$$



Square of wf at origin \sim lepton mass cubed

Muon/electron mass ratio 205! 8 million times larger for muon

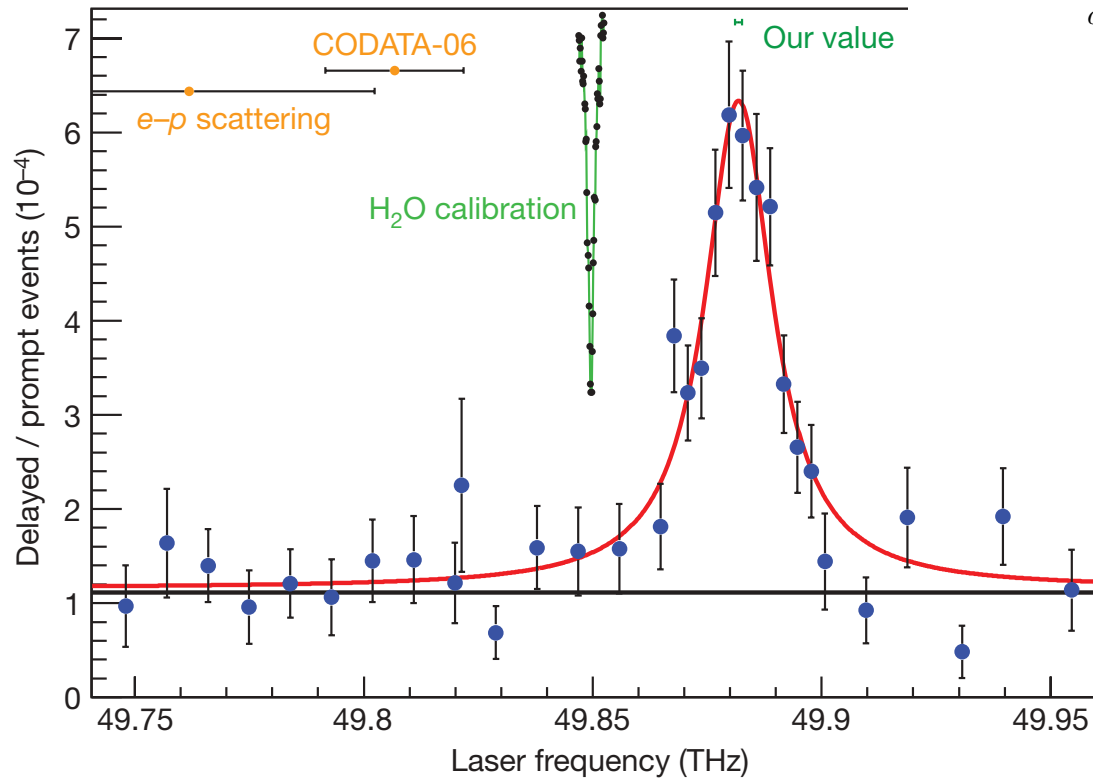


From 2013
Science paper

Fig. 1. (A) Formation of μp in highly excited states and subsequent cascade with emission of “prompt” $K_{\alpha, \beta, \gamma}$. (B) Laser excitation of the 2S-2P transition with subsequent decay to the ground state with K_α emission. (C) 2S and 2P energy levels. The measured transitions v_s and v_t are indicated together with the Lamb shift, 2S-HFS, and 2P-fine and hyperfine splitting.

The experiment: results disagree with previous measurements & world average

“The 1S-2S transition in H has been measured to 34 Hz, that is, 1.4×10^{-14} relative accuracy. Only an error of about 1,700 times the quoted experimental uncertainty could account for our observed discrepancy.”



2010 Rock Solid!

2010 Experimental summary

Pulsed laser spectroscopy

measure a muonic Lamb shift of 49,881.88(76) GHz. On the basis of “ present calculations¹¹⁻¹⁵ of fine and hyperfine splittings and QED terms, we find $r_p = 0.84184(67)$ fm, which differs by 5.0 standard deviations from the CODATA value³ of 0.8768(69) fm. Our result implies that either the Rydberg constant has to be shifted by -110 kHz/c (4.9 standard deviations), or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient. ”

Jan. 2013, 7 st. dev

Antogini -Sci. 339,417

- Rydberg is known to 12 figures

$$R_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} = 1.097\,373\,156\,852\,5\,(73) \times 10^7 \text{ m}^{-1},$$

- **Puzzle**- why muon H different than e H?

Pohl's Table of calculations

Lamb
shift:
vacuum
polarization
many, many
terms

Resolution I-
QED calcs not OK

α

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			Value	Unc.	Value	Unc.	Value	Unc.
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3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	-0.00103				-0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	-0.00500	0.0010	-0.006	0.001	-0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17-19	-0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21-23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
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19	Recoil correction to VP	5	-0.00410				-0.0041	
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21	Muon Lamb shift 4th order	5	-0.00169				-0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m_r}{M}$	2,5-7	-0.04497		-0.045		-0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m_r}{M}$	1,2,7	-0.00960		-0.0099		-0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	-0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-independent contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*⁷ Table 7.1. Item # 8 in Refs.^{2,5} is the sum of items #6 and #7, without the recent correction from Ref.¹². The error of #10 has been increased to 100% to account for a remark in Ref.⁷. Values are in meV and the uncertainties have been added in quadrature.

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Lamb shift:
vacuum polarization
many, many terms

Mostly irrelevant-theory replaced by experiment

Resolution I-QED calcs not OK

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QED calcs expand in α

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muon

electron

Possible resolutions

- ~~QED bound-state calculations not accurate-~~
very unlikely
- Electron experiments not so accurate
- Muon interacts differently than electron!
- Strong interaction effect in two photon exchange diagram

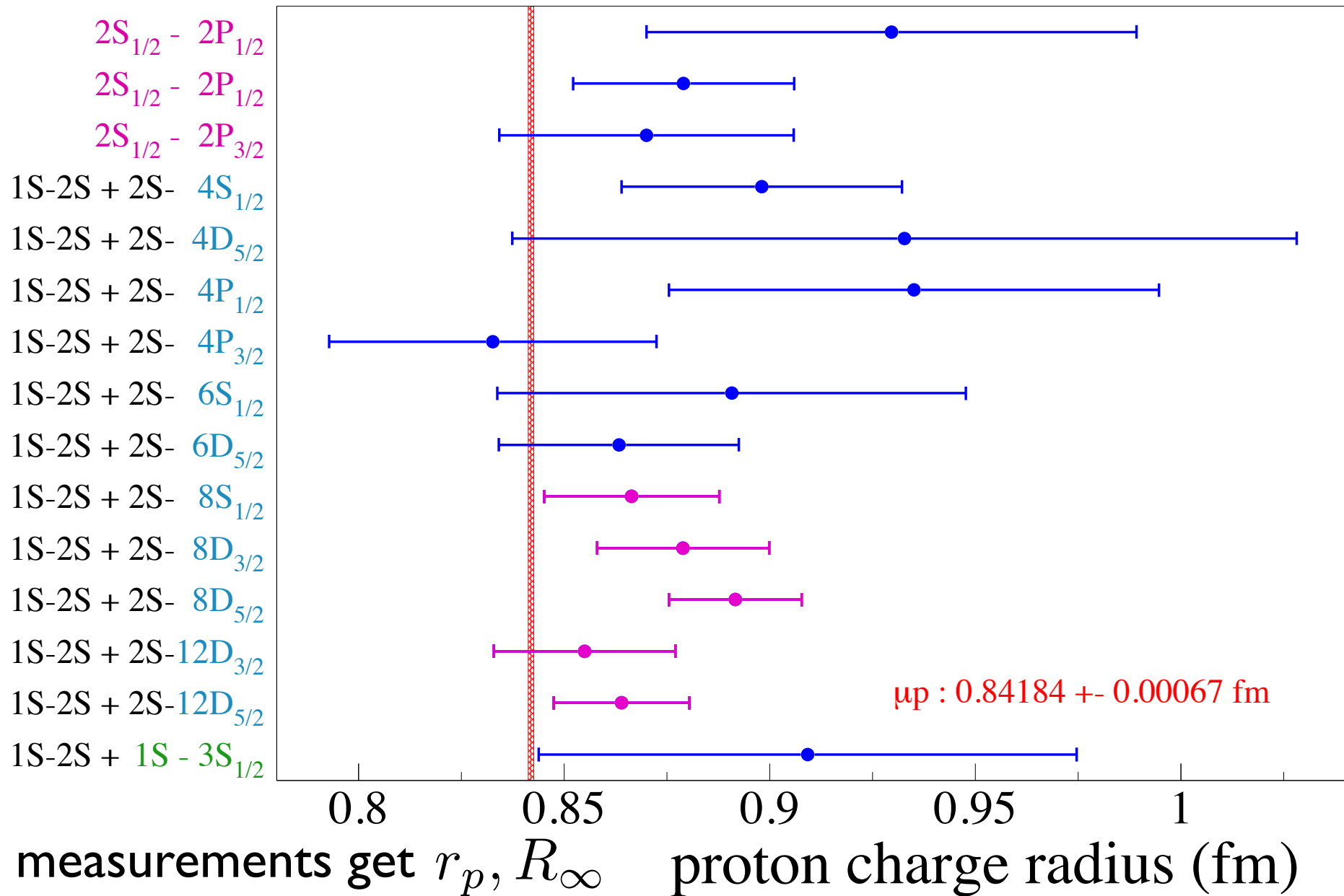
Experimental Electronic hydrogen energy levels

$$E(nS) \approx \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

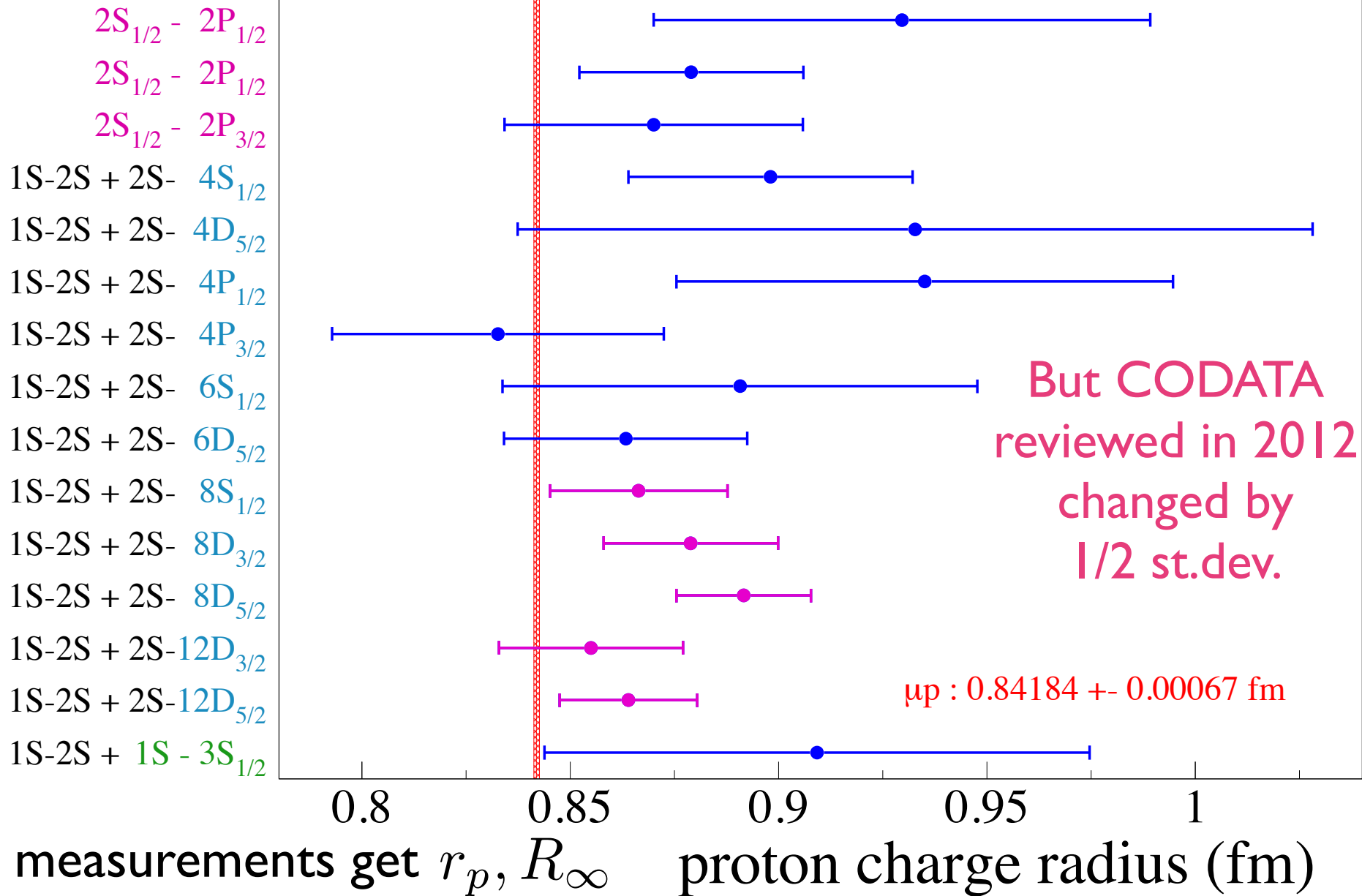
$$L_{1S} \approx (6172 + 1.56(r_p/fm)^2) MHz$$

- Need two levels to get Rydberg and Lamb shift-have ~ 20 available

Electronic Hydrogen -Pohl



Electronic Hydrogen -Pohl



Several new experiments planned

- Independent measurement of Rydberg constant
- This would change only extracted r_p nothing else
- 2S-6S UK, 2S-4P Germany, 1S-3S France
- 2S-2P classic, Canada
- Highly charged single electron ions NIST

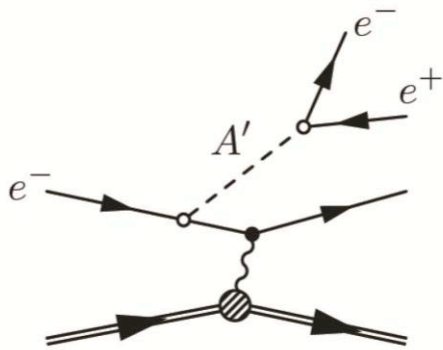
New forces, dark photons

- ordinary matter makes up 5 % of energy density of universe
- dark sector- energy density inferred through gravitational fields
- dark matter is 25 % (acts as matter gravitationally)
- dark energy 70 % of universe
- dark electromagnetism -dark photons-couple to dark matter not to standard model

Arkani-Hamed
Pospelov,
...

Arkani-Hamed: “The whole set-up is totally vanilla and conservative from a theorist’s point of view,”

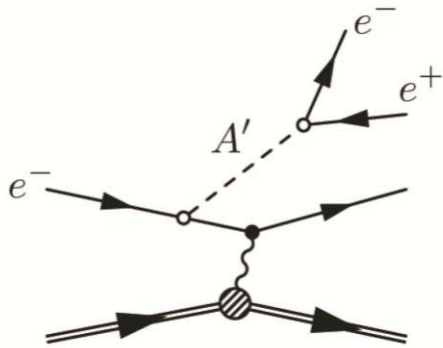
Searching for dark photons



JLab Aprime



Searching for dark photons

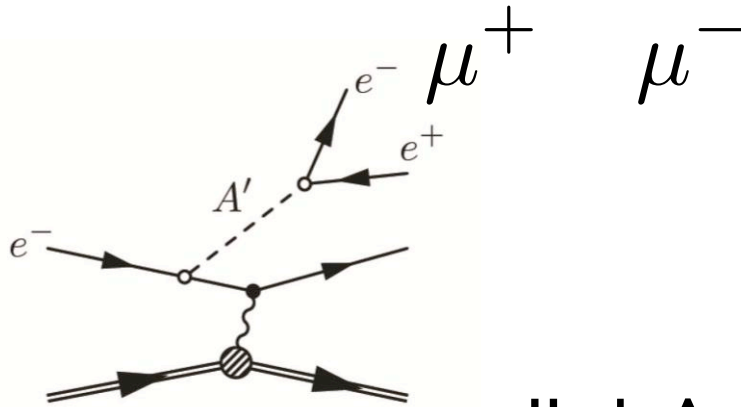


JLab Aprime

- But what about the muon?



Searching for dark photons



JLab Aprime

- But what about the muon?



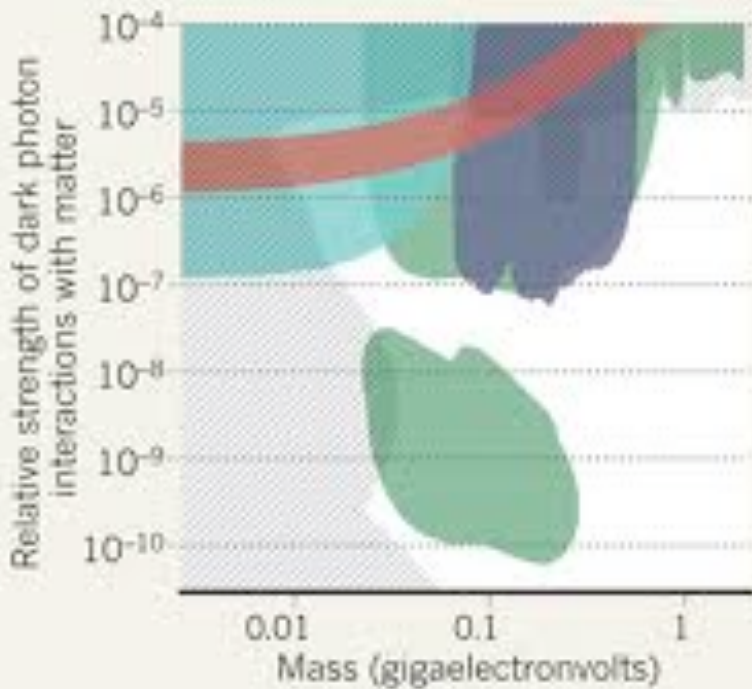
FEELING IN THE DARK

Three experiments will search unexplored mass regions for a dark photon, which could explain why muons flout the standard model.

Experiments: DarkLight APEX HPS

■ Where muon data hint dark photon may be

▨ Where dark photon is already ruled out



Three experiments at JLab



R. Essig

Muon data is $g-2$ - BNL exp't,
Hertzog- Kammel ...

muon anomalous moment

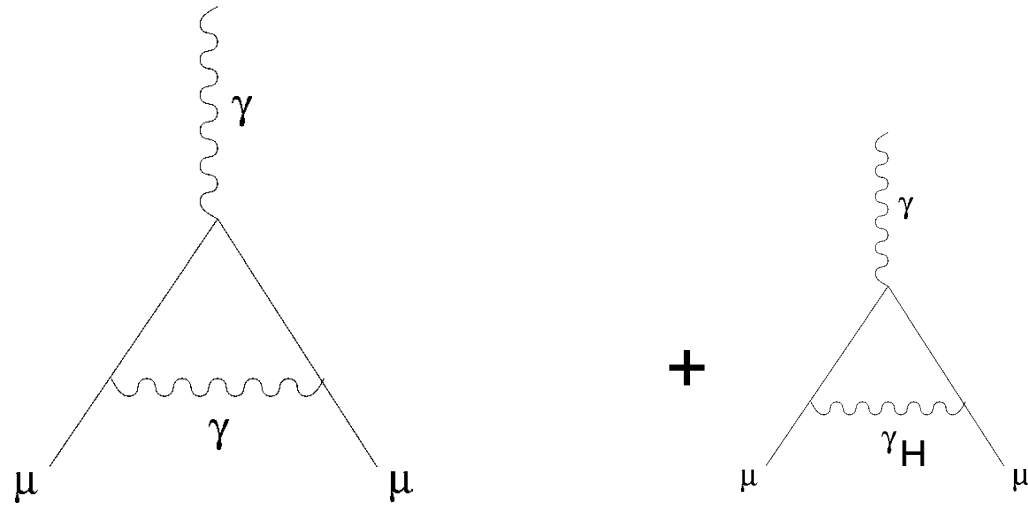


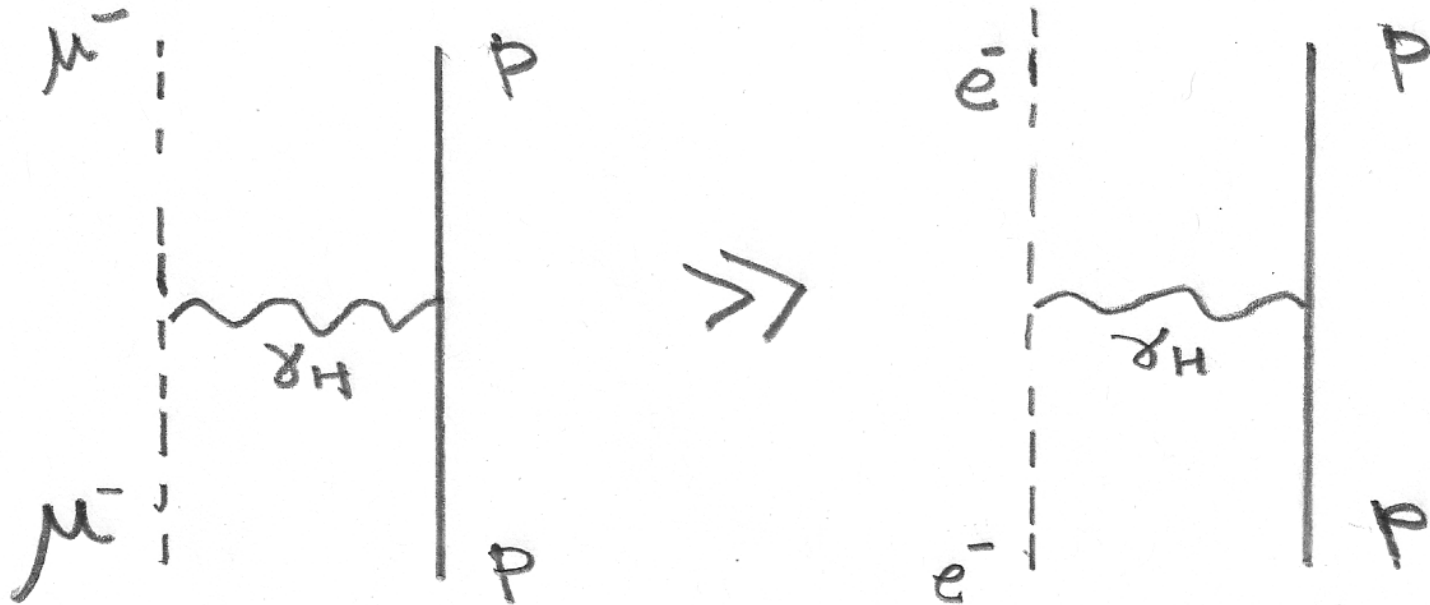
Figure 1 The first-order QED correction to g-2 of the muon.

Figure 1 The first-order QED correction to g-2 of the muon.

3.6 st. dev anomaly now - to fix add heavy photon that interacts preferentially with muon

$$\gamma \rightarrow \gamma + \gamma_H$$

Connection to Lamb shift



What theorists do

- make up new particles- compute shift
- study constraints -
- non-observation of new particles that couple mainly to muons

- Constraints are obtained from the decay of the Y resonances;
neutron interactions with nuclei;
the anomalous magnetic moment of the muon
x-ray transitions in ^{24}Mg and ^{28}Mg , Si atoms;
 J/ψ decay;
neutral pion decay
eta decay

Any time a photon appears can also have a diagram with heavy photon

$$\mu \neq e$$

- Marciano, INT Talk summer 2010-massive photon, violate mu-e universality, matter effects in neutrino oscillations too big by 10000
- Barger et al “We consider exotic particles that couple preferentially to muons, and mediate an attractive nucleon-muon interaction. Many constraints from low energy data disfavor new spin-0, spin-1 and spin-2 particles as an explanation. **PRL 106, 153001**
- Brax, Burrage “Combining these constraints with current particle physics bounds, the contribution of a scalar field to the recently claimed discrepancy in the proton radius is negligible.” *Phys.Rev.D*83:035020,2011
- Tucker-Smith & Yavin-Barger et al -many assumptions-scalars work
- [Batell](#), [McKeen](#), [Pospelov](#) **PRL 107,081802** New force differentiates between lepton species. Models with gauged right-handed muon number, contain new vector and scalar force carriers at the 100 MeV scale or lighter. Such forces would lead to an enhancement by several orders-of-magnitude of the parity-violating asymmetries in the scattering of low-energy muons on nuclei.
Related to muon g-2-- theory has anomaly
- Carlson, Rislw, **Phys.Rev. D86 (2012) 035013** Conclusions: New physics with fine tuned couplings may be entertained as a possible explanation for the Lamb shift discrepancy.
- Must consider HFS too!

Experimental analysis

Extract the proton radius from the transition energy,
compare measured ξ to the following sum of contributions:

$\xi = 206.2949(32)$ meV - One measured number

$$\xi = \boxed{206.0573(45)} - 5.2262r_p^2 + 0.0347r_p^3 \text{ meV}$$

three computed numbers

To explain puzzle:

increase 206.0573 meV by 0.31 meV = 3.1×10^{-10} MeV

Then radius is as in H atom

Pohl's Table of calculations

#	Contribution	Ref.	Our selection		Pachucki ¹⁻³		Borie ⁵	
			Value	Unc.	Value	Unc.	Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1-3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	-0.00103				-0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	-0.00500	0.0010	-0.006	0.001	-0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17-19	-0.00150					
13	Mixed electron and muon loops	20	0.00007				0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21-23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	-0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	-0.00410				-0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	-0.66770		-0.6677		-0.66788	
21	Muon Lamb shift 4th order	5	-0.00169				-0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m_r}{M}$	2,5-7	-0.04497		-0.045		-0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m_r}{M}$	1,2,7	-0.00960		-0.0099		-0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	-0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

Table 1: All known radius-**independent** contributions to the Lamb shift in μp from different authors, and the one we selected. We follow the nomenclature of Eides *et al.*⁷ Table 7.1. Item # 8 in Refs.^{2,5} is the sum of items #6 and #7, without the recent correction from Ref.¹². The error of #10 has been increased to 100% to account for a remark in Ref.⁷. Values are in meV and the uncertainties have been added in quadrature.

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Nuclear size correction of order $(Z\alpha)^6 \langle r_p^2 \rangle$	1,27-29	-0.001243 $\langle r_p^2 \rangle$		
Total $\langle r_p^2 \rangle$ contribution		-5.22619 $\langle r_p^2 \rangle$	-5.2256	-5.2244
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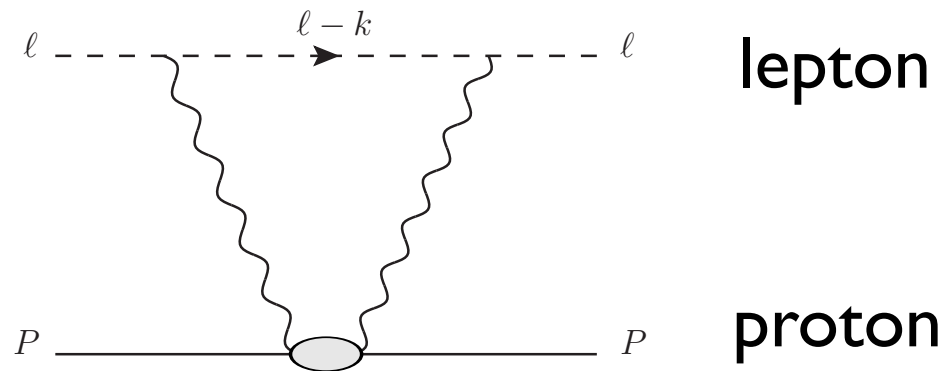
proportional to
lepton mass⁴

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Our idea



lepton propagator provides term so that energy shift is proportional to lepton mass⁴

Conventional approach \sim Pachucki

$$\Delta E \propto \alpha^5 m^3 \int \frac{d^4 q}{q^4} T^{\mu\nu} l_{\mu\nu}(m)$$

$T^{\mu\nu}$ is forward virtual-photon proton scattering amplitude,

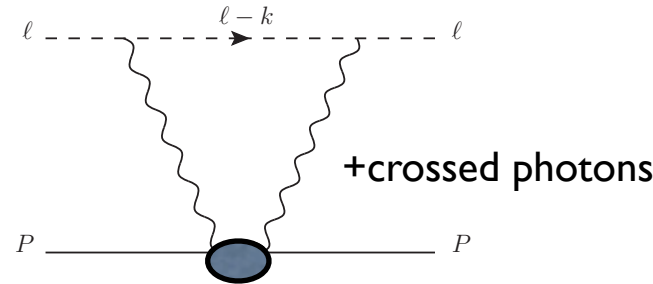
$l_{\mu\nu}(m)$ is lepton-tensor

$$T^{\mu\nu}(q, P) = -i \int d^4 x e^{iq \cdot x} \langle P | T(j^\mu(x) j^\nu(0)) | P \rangle$$

$$T^{\mu\nu}(q, P) = -(g^{\mu\nu} - \dots) T_1 + (P^\mu - \dots)(P^\nu - \dots) T_2$$

$Im(T_{1,2}) \propto W_{1,2}$ Measured structure functions

Cauchy plus data \rightarrow answers –rock solid (?)



$Im T_{1,2} \sim W_{1,2}(\nu, Q^2)$ measured

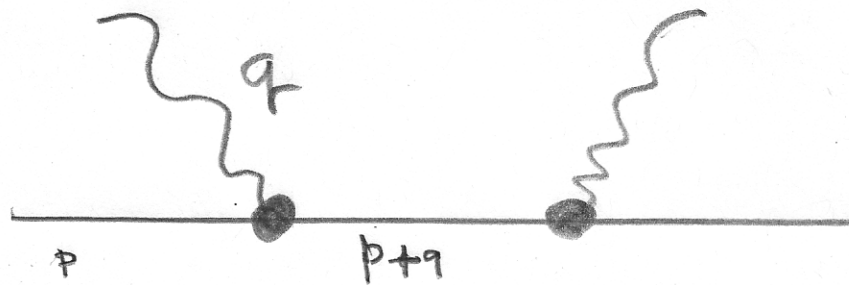
large ν $W_2 \sim 1/\nu$, $W_1 \sim \nu$

- Dispersion integral involving W_2 converges
- Dispersion integral involving W_1 diverges- uncertainty
- subtraction needed at **all** Q^2

Hill & Paz 2011 : dispersion approach
uncertainty order of mag larger than stated

Features

- need subtracted dispersion relation for T_1
- subtraction function ($q^0 = 0$, all q^2) mainly unknown $\overline{T}_1(0, Q^2)$ asymptotic $\sim 1/Q^2$
- Miller, Carroll, Thomas, Rafelski PRA 84,012506



$(p+q)^2 \neq M^2$ off-shell proton

- violates constraints on Compton- Carlson/VDH
Miller, Carroll, Thomas 1207.0549 better off-shell, but ruled out by $(e, e'p)$ nuclear reactions

Alternate: unknown $\bar{T}_1(0, Q^2)$ Miller PLB 2012

$$\Delta E^{\text{subt}} = \frac{\alpha^2}{m} \Psi_S^2(0) \int_0^\infty \frac{dQ^2}{Q^2} h(Q^2) \bar{T}_1(0, Q^2)$$

$$\lim_{Q^2 \rightarrow \infty} h(Q^2) \sim \frac{2m^2}{Q^2}, \text{ chiral PT : } \bar{T}_1(0, Q^2) = \frac{\beta_M}{\alpha} Q^2 + \dots$$

→ Logarithmic divergence

$$\bar{T}_1(0, Q^2) \rightarrow \frac{\beta_M}{\alpha} Q^2 F_{\text{loop}}(Q^2) \text{ Cuts off integral}$$

$$\text{Birse \& McGovern : } \bar{T}_1(0, Q^2) = \frac{\beta_M}{\alpha} Q^2 \left(1 - \frac{Q^2}{M_\beta^2} + \mathcal{O}(Q^4)\right)$$

$$\rightarrow \frac{\beta_M}{\alpha} Q^2 \frac{1}{\left(1 + \frac{Q^2}{2M_\beta^2}\right)^2}$$

$$M_\beta = 460 \pm 50 \text{ MeV}, \Delta E^{\text{subt}} = 4.1 \mu \text{ eV very small}$$

High Q^2 behavior is ASSUMED

Arbitrary functions

$$\bar{T}_1(0, Q^2) = \frac{\beta_M}{\alpha} Q^2 F_{\text{loop}}(Q^2).$$

$$F_{\text{loop}}(Q^2) = \left(\frac{Q^2}{M_0^2} \right)^n \frac{1}{(1 + aQ^2)^N}, \quad n \geq 2, \quad N \geq n + 3,$$

$$\bar{T}_1(0, Q^2) \sim \frac{1}{Q^4} \text{ or faster, } \beta_M \rightarrow \beta$$

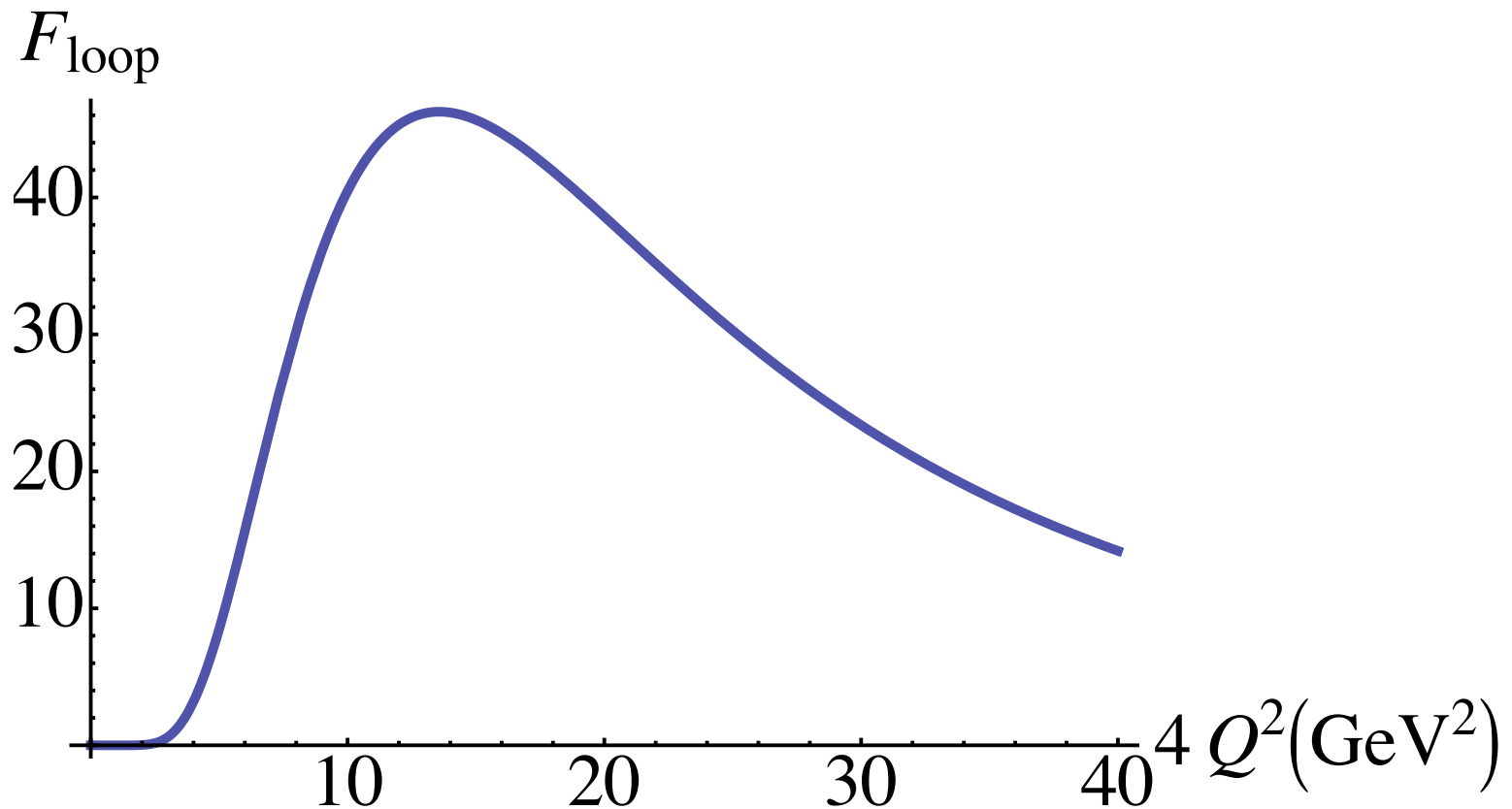
$$\Delta E^{\text{subt}} \approx 3\alpha^2 m \Psi_S^2(0) \frac{\beta}{\alpha} \gamma^n B(N, n), \quad \gamma \equiv \frac{1}{M_0^2 a}$$

If we take $N = 5, n = 2$ so that $B(5, 2) = 1/12$, and $\beta = 10^{-3} \text{ fm}^{-3}$, a value of $\gamma = 30.9$ reproduces $E = 0.31 \text{ meV}$. If we take $M_0 = 0.5 \text{ GeV}$ (as in [20]), then $a^{-1} = 15.4 \text{ GeV}^2$, and that the contribution to the integral comes from the region of very high values of Q^2 .

Can find functions that give big effect

Another example

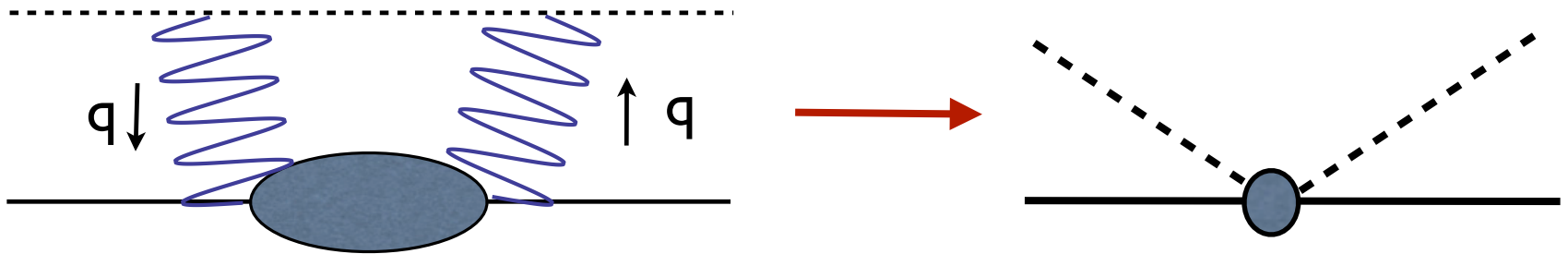
$n=23, N=26, 1/a=0.44 \text{ GeV}^2$



EFT of μp interaction

Caswell Lepage '86

- Compute Feynman diagram, remove log divergence using dimensional regularization
- include counter term in Lagrangian



$$\begin{aligned} \mathcal{M}_2^{DR} &= \frac{3}{2} i \alpha^2 m \frac{\beta_M}{\alpha} \left[\frac{2}{\epsilon} + \log \frac{\mu^2}{m^2} + \frac{5}{6} - \gamma_E + \log 4\pi \right] \bar{u}_f u_i \bar{U}_f U_i, \\ &= i \alpha^2 m \frac{\beta_M}{\alpha} (\lambda + 5/4) \bar{u}_f u_i \bar{U}_f U_i \end{aligned}$$

Choose λ to get 0.31 meV shift

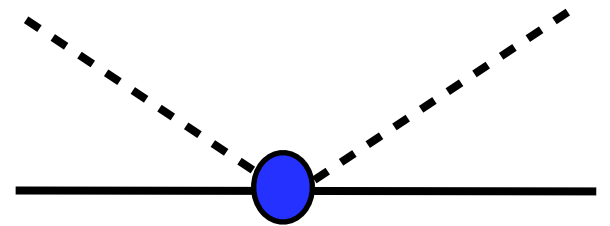
$$\Delta E^{\text{subt}}(DR) = \alpha^2 m \frac{\beta_M}{\alpha} \Psi_S^2(0) (\lambda + 5/4)$$

$$\Delta E^{\text{subt}}(DR) = 0.31 \text{ meV} \rightarrow \lambda = 769$$

β_M (magnetic polarizability) = $3.1 \times 10^{-4} \text{ fm}^3$ very small

Natural units $\beta_M/\alpha \sim 4\pi/(4\pi f_\pi)^3$ Butler & Savage '92

$$\mathcal{M}_2^{DR} = i 3.95 \alpha^2 m \frac{4\pi}{\Lambda_\chi^3} \bar{u}_f u_i \bar{U}_f U_i.$$



3.95 = natural

So what?

A Proposal for the Paul Scherrer Institute π M1 beam line

Studying the Proton “Radius” Puzzle with μp Elastic Scattering

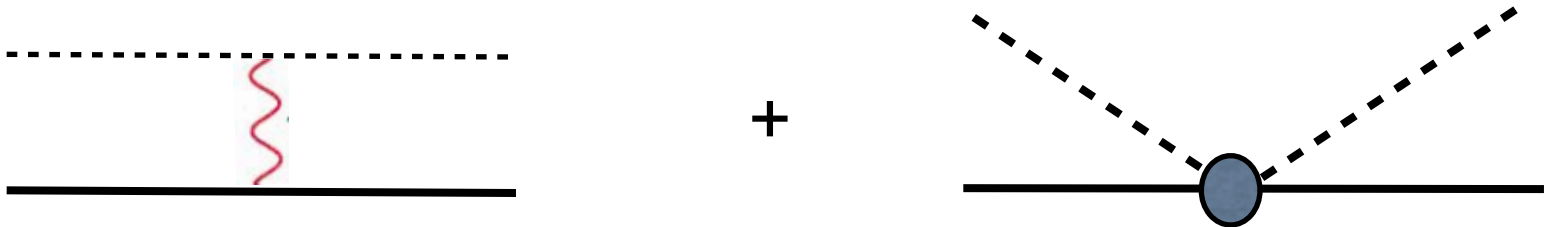
J. Arrington,¹ F. Benmokhtar,² E. Brash,² K. Deiters,³ C. Djalali,⁴ L. El Fassi,⁵ E. Fuchey,⁶ S. Gilad,⁷ R. Gilman (Contact person),⁵ R. Gothe,⁴ D. Higinbotham,⁸ Y. Ilieva,⁴ M. Kohl,⁹ G. Kumbartzki,⁵ J. Lichtenstadt,¹⁰ N. Liyanage,¹¹ M. Meziane,¹² Z.-E. Meziani,⁶ K. Myers,⁵ C. Perdrisat,¹³ E. Piassetzky (Spokesperson),¹⁰ V. Punjabi,¹⁴ R. Ransome,⁵ D. Reggiani,³ A. Richter,¹⁵ G. Ron,¹⁶ A. Sarty,¹⁷ E. Schulte,⁶ S. Strauch,⁴ V. Sulkosky,⁷ A.S. Tadapelli,⁵ and L. Weinstein¹⁸

PSI proposal R-12-01.1

2 photon exchange idea is testable

muon scattering

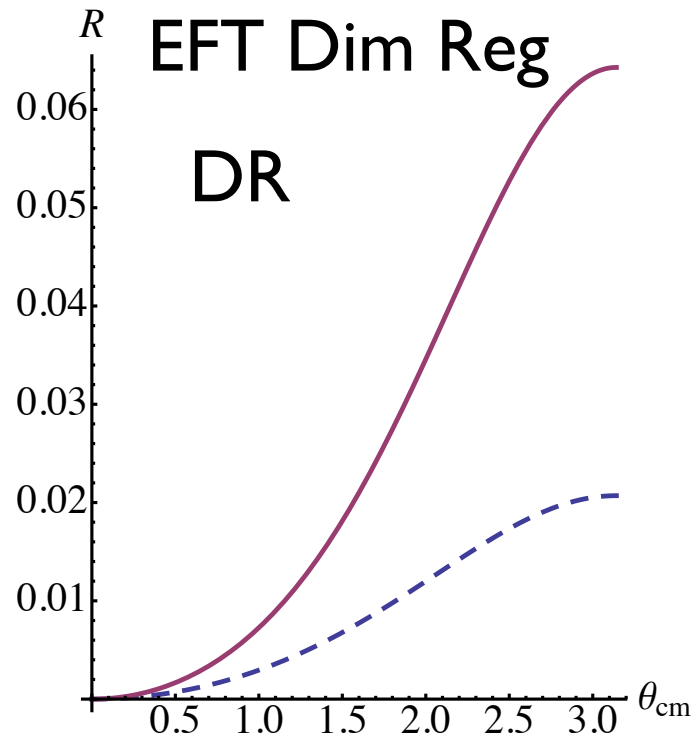
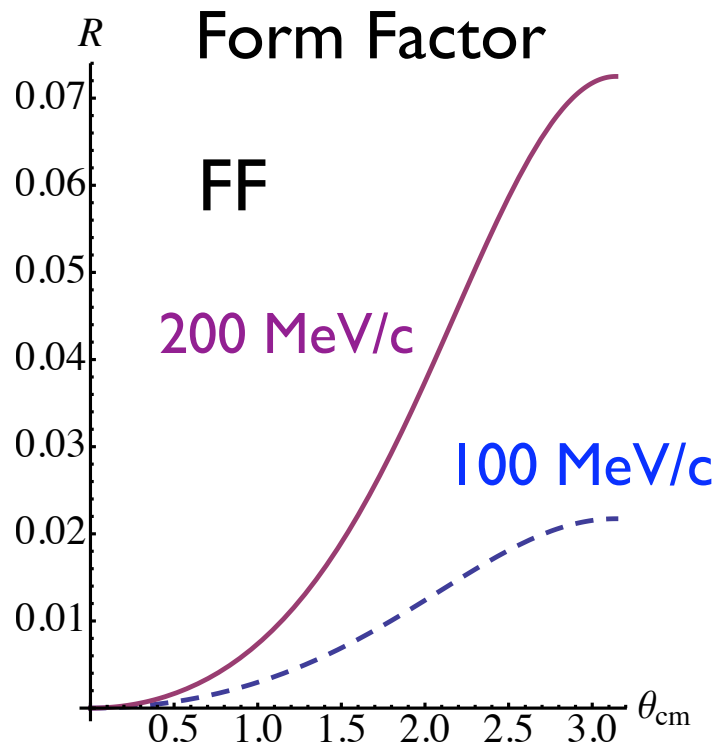
$$\mathcal{M} = \mathcal{M}^{(1)} + \mathcal{M}^{(2)}$$



- Is contact interaction too large??

Observable Effect in $\mu^- p$ Scattering

$$R = 2 \frac{\text{Re}[(\mathcal{M}^{(1)})^* \mathcal{M}^{(2)}]}{|\mathcal{M}^{(1)}|^2}$$



Deuteron as a test

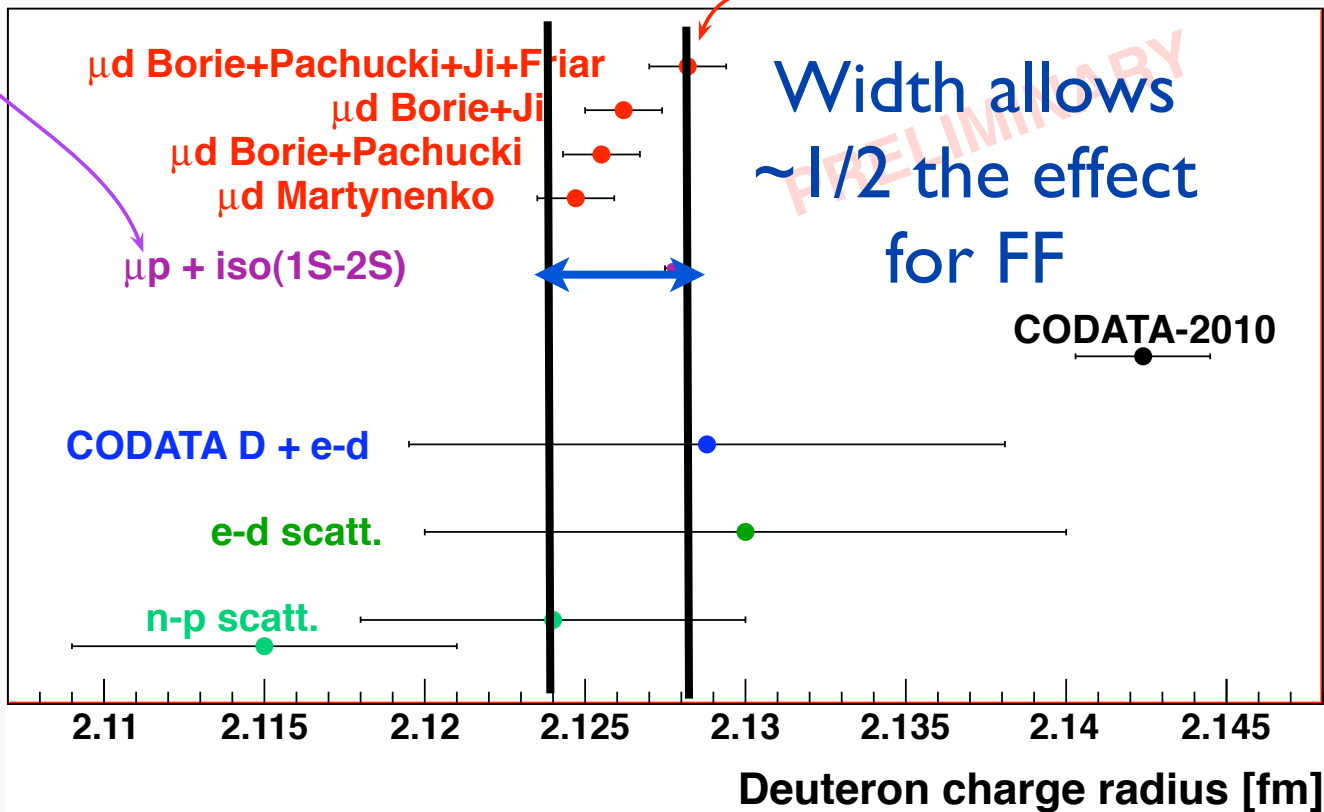
Need polarizability effect on **neutron**

- two versions of the hypothesis: form factor and EFT
- form factor- effect on neutron = effect on proton, otherwise n-p mass difference becomes gigantic, then in Deuteron **the TPE contribution to the Lamb shift effect is doubled -Aldo TPE contribution about the same**
- EFT- the unknown short distance μ -n interaction needs an unknown interaction constant, can't predict Deuteron

Deuteron radius from μd and μp (**preliminary**)

$$\left. \begin{array}{l} \text{H-D isot.-shift: } r_d^2 - r_p^2 = 3.820\,07(65) \text{ fm}^2 \\ \mu p : \quad r_p = 0.84087(39) \text{ fm} \end{array} \right\} \Rightarrow r_d = 2.12771(22) \text{ fm}$$

Directly from μd spectroscopy
using predictions of polarizability
with 0.0300 meV uncertainty



Summary

- Logarithmic divergence in the integrand that determines the value of ΔE^{subt} .
- The uncertainty in evaluation large enough to account for the proton radius puzzle.
- Logarithmic divergence controlled via form factor or dimensional regularization
- Either method account for the proton radius puzzle
- Either method predicts (same) observable few % effect- low energy $\mu - p$ scattering.

Explanations for the proton radius puzzle:

- Electronic-hydrogen experiments might not be as accurate as reported
- $\mu - e$ universality might be violated
- strong interaction effect important for muonic hydrogen, but not for electronic

Which correct ???

Strong-interaction effect discussed here is testable experimentally