Nuclear Polarization Contribution to the Lamb Shift in Muonic Atoms

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INT Program INT-13-2b

How small is the proton?

- **electron-proton**
  1. e-p scattering: $r_p = 0.875(10)$ fm
  2. eH atomic spectroscopy: $r_p = 0.8768(69)$ fm
  3. CODATA-2010: $r_p = 0.8775(51)$ fm

     Mohr *et al.*, Rev. Mod. Phys. (2012)

Proton radius puzzle

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- **muonic hydrogen Lamb shift (2S-2P)**
  1. μH $2S_{1/2}^{F=1}$-$2P_{3/2}^{F=2}$: $r_p = 0.84184(67)$ fm (5σ)

     Pohl *et al.*, Nature (2010)
  2. Combine μH $2S_{1/2}^{F=0}$-$2P_{3/2}^{F=1}$: $r_p = 0.84087(39)$ fm (7σ)

     Antognini *et al.*, Science (2013)

μH Lamb shift experiment

Lamb Shift:
2S-2P splitting in atomic spectrum

$a.$ prompt X-ray ($t \sim 0$)

- μ^- stopped in H_2 gases
- 99% $\rightarrow$ 1S
- 1% $\rightarrow$ 2S ($\tau_{2S} \approx 1\mu s$)

$b.$ delayed X-ray ($t \sim 1\mu s$)

- laser induced 2S$\rightarrow$2P
- measure $K_\alpha^{\mathrm{delayed}} / K_\alpha^{\mathrm{prompt}}$
- $f_{res} = \Delta E_{LS}$

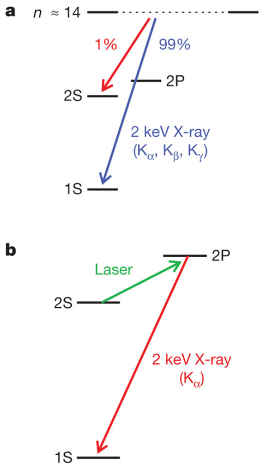


Figure from Pohl *et al.* Nature (2010)

μH Lamb shift experiment

r_p **from** μ**H experiment disagrees with** eH **(**ep**) by** 7σ**!**

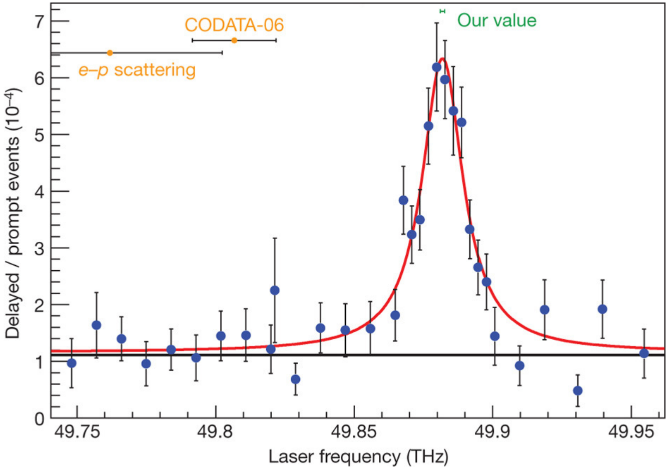
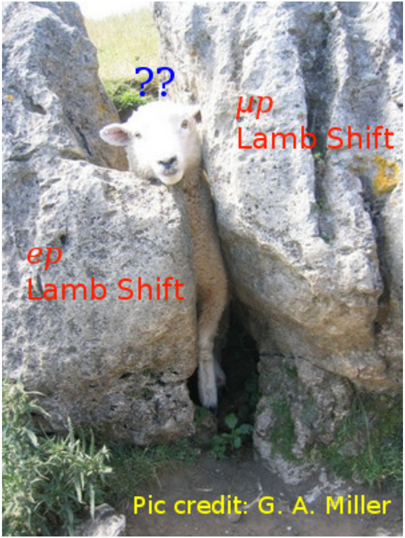


Figure from Pohl *et al.* Nature (2010)

Origins of the discrepancy?

Discrepancy of r_p between ep and μp measurements



Origins of the discrepancy?

- **study r_p's discrepancy between μp and ep experiments**
  - systematic errors in ep scattering
  - new physics that distinguishes μp and ep interactions
  - high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs

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- **study r_p's discrepancy between μp and ep experiments**
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  - high-precision measurements $\Longleftrightarrow$ accurate theoretical inputs
- **new experiments at PSI**
  - Lamb shift in μD
    CREMA collaboration, finishing
  - Lamb shift in muonic helium
    CREMA collaboration, planned in 2013
  - μp scattering experiment
    MUSE collaboration, in development

Extract nuclear charge radius

- $\langle r^2 \rangle$ from Lamb shift

$$\Delta E_{LS} = \delta_{QED} + \delta_{pol} + \frac{m_r^3}{12}(Z\alpha)^4 \langle r^2 \rangle - \frac{m_r^4}{24}(Z\alpha)^5 \langle r^3 \rangle_{(2)}$$

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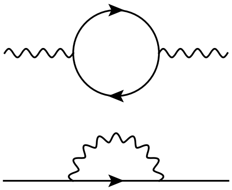
- QED corrections:
  - vacuum polarization
  - lepton self energy
  - relativistic recoil effects



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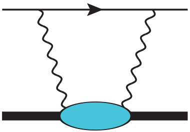
- Nuclear polarization corrections (inelastic):
  - exchange of two virtual photons
  - dominant contribution $\sim (Z\alpha)^5$



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- Nuclear finite-size corrections (elastic):
  - leading term $\sim (Z\alpha)^4$: $\frac{m_r^3}{12}(Z\alpha)^4 \langle r^2 \rangle$
  - Zemach moment $\sim (Z\alpha)^5$: $-\frac{m_r^4}{24}(Z\alpha)^5 \langle r^3 \rangle_{(2)} \quad \propto \langle r^2 \rangle^{3/2}$

Uncertainty in nuclear polarization

$$\Delta E_{LS} = \delta_{QED} + \boldsymbol{\delta_{pol}} + \frac{m_r^3}{12}(Z\alpha)^4\langle r^2\rangle - \frac{m_r^4}{24}(Z\alpha)^5\langle r^3\rangle_{(2)}$$

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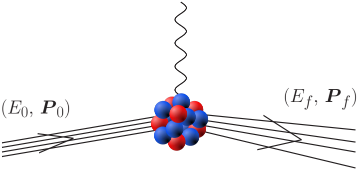
- Nuclear polarization $\Longrightarrow$ inputs from nuclear response function

$$S_O(\omega) = \sum\!\!\!\!\!\!\int_f |\langle\psi_f|\hat{O}|\psi_0\rangle|^2\delta(E_f - E_0 - \omega)$$



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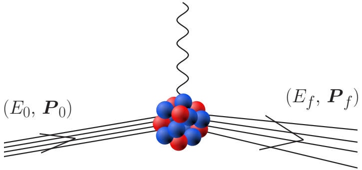

- Early calculations of δ_{pol} in muonic atoms:

  $\Rightarrow S_O(\omega)$ inputs were not accurate enough

Calculations of $S_O(\omega)$ & δ_{pol}

- **Toy models**
  - $\mu\,^{12}$C (square-well) Rosenfelder '83
  - μD (Yamaguchi) Lu & Rosenfelder '93

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- **$S_O(\omega)$ from photoabsorption cross sections**
  - $\mu\,^4$He$^+$: Bernabeu & Jarlskog '74; Rinker '76; Friar '77
  - $\delta_{pol} = -3.1$ meV $\pm 20\%$
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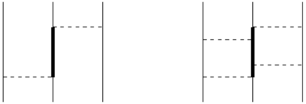
- **state of the art potentials**
  - μD: AV14 (Leidemann & Rosenfelder, '95); AV18 (Pachucki, '11)
- **δ_{pol} in other light muonic atoms (*e.g.*, $\mu\,^{3}$He^{+}, $\mu\,^{4}$He^{+}, $\cdots$)**
  - need to calculate S_O using **modern potentials *ab-initio* methods**

- **Argonne** v_{18} fitted to
  - 1787 pp & 2514 np observables for $E_{lab} \leq 350$ MeV with $\chi^2/\text{datum} = 1.1$
  - nn scattering length & ^{2}H binding energy

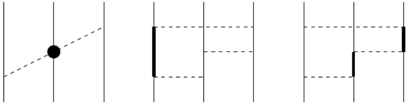
- **Urbana IX**

  $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$



- **Illinois**

  $+V_{ijk}^{2\pi S} + V_{ijk}^{3\pi \Delta R}$



Phenomenological potential

- **Argonne v_{18}** fitted to
  - 1787 pp & 2514 np observables for $E_{lab} \leq 350$ MeV with $\chi^2/\mathrm{datum} = 1.1$
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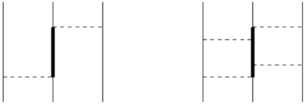
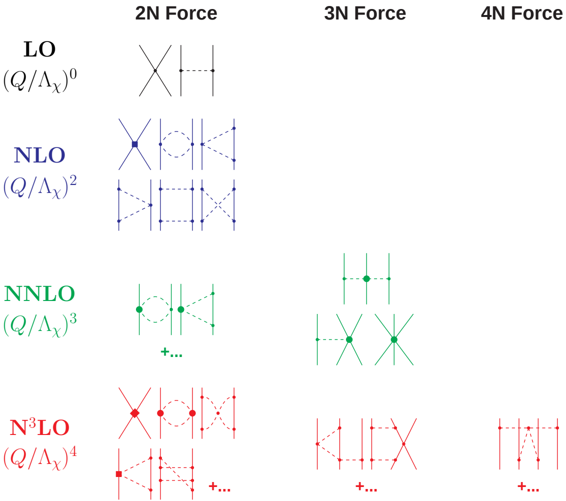


Chiral EFT potential

- **based on**
  spontaneous breaking of chiral symmetry
- **separation of scales**
  $Q \ll \Lambda \sim 500$ MeV
- **nuclear forces**
  are built in systematic expansions of Q/Λ
- **coupling constants**
  fitted to nuclear data



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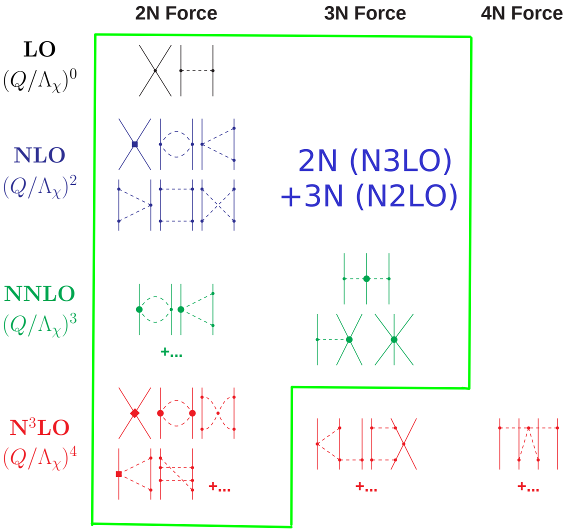

Calculations of δ_{pol} in $\mu\,^4$He$^+$

We perform the first *ab-initio* calculation of nuclear polarization in $\mu\,^4$He$^+$ with state-of-the-art potentials

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- Hyperspherical Harmonics + AV18/UIX and EFT $\Longrightarrow$ response functions
- response functions $\Longleftrightarrow$ nuclear polarization in $\mu\,^4$He$^+$

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- **Final Goal:**

  provide δ_{pol} with an accuracy comparable to the $\pm 5\%$ experimental needs

Ab-initio response functions

- Response in continuum

$$S_O(\omega) = \sum\!\!\!\!\!\!\!\int_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$

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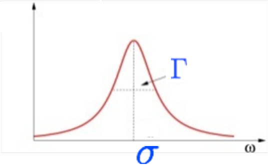
- Lorentz integral transform (LIT) method

$$\mathcal{L}(\sigma,\Gamma) = \int d\omega \frac{S_O(\omega)}{(\sigma-\omega)^2+\Gamma^2} = \langle\tilde{\psi}|\tilde{\psi}\rangle$$

$$(H - E_0 - \sigma + i\Gamma)|\tilde{\psi}\rangle = \hat{O}|\psi_0\rangle$$

  - Since r.h.s. is finite, $|\tilde{\psi}\rangle$ has bound-state asymptotic behavior

Efros et al., '07



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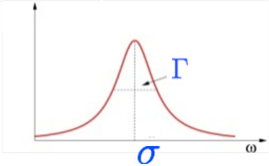


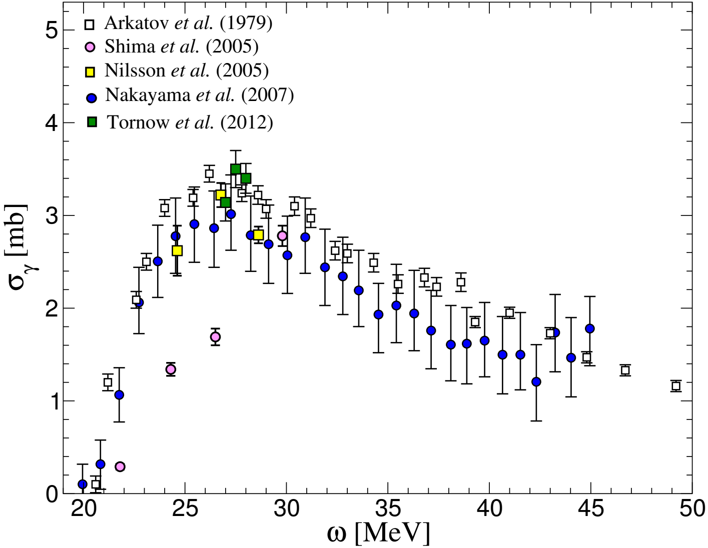
- Few-body methods for bound-state problems → **Hyperspherical Harmonics**
  - applicable for $3 \leqslant A \leqslant 8$
  - can accommodate local and non-local two/three body forces

    AV18 + UIX & NN(N^{3}LO) + NNN(N^{2}LO)

Details in tomorrow's talk by Sonia Bacca

^{4}He photoabsorption cross sections

electric-dipole photoabsorption cross section $\sigma_\gamma(\omega) = 4\pi^2\alpha\omega S_{D1}(\omega)$



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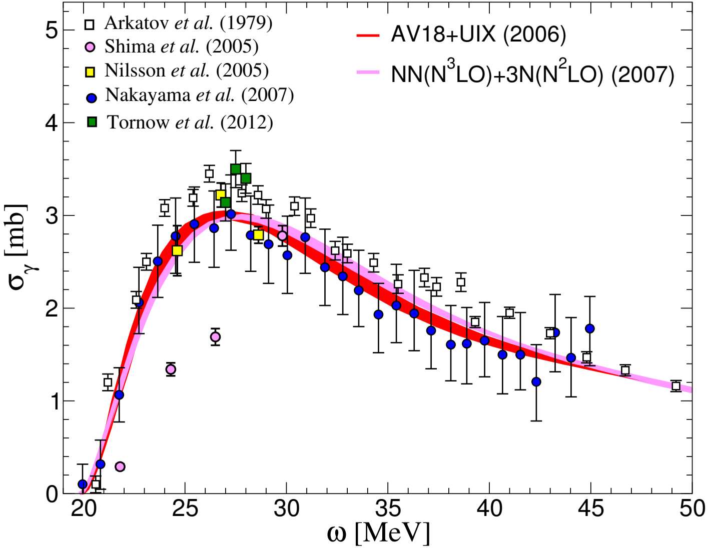
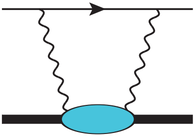


Nuclear polarization: basic idea

- Hamiltonian for muonic atoms

$$H = H_{nucl} + H_\mu + \Delta H$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



- Corrections to the point Coulomb

$$\Delta H = \alpha \sum_i^Z \Delta V(\boldsymbol{r}, \boldsymbol{R}_i) \equiv \alpha \sum_i^Z \left(\frac{1}{r} - \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} \right)$$

- Evaluate ΔH's inelastic effects to the muonic atom spectrum in 2nd-order perturbation theory

$$\delta_{pol} = \sum_{N \neq N_0} \langle N_0 \phi_x | \Delta H | N \rangle \frac{1}{E_{N_0} - E_N + \epsilon_{\mu_0} - H_\mu} \langle N | \Delta H | \phi_x N_0 \rangle$$

ϕ_x: muon wave function for $2S/2P$ state

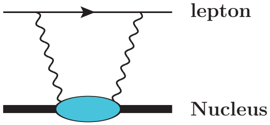
Nuclear polarization: contributions

Systematic contributions to nuclear polarization

- non-relativistic limit (multipole expansion)
- relativistic dipole polarization
- Coulomb distortion in dipole polarization
- corrections from finite nucleon sizes

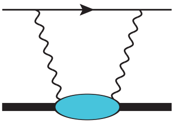
Non-relativistic limit

- **Neglect Coulomb interactions in the intermediate state**



Non-relativistic limit

- Neglect Coulomb interactions in the intermediate state
- Expand muon matrix element in powers $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

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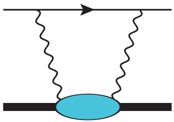

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- $|\boldsymbol{R}-\boldsymbol{R}'| \Longrightarrow$ "virtual" distance a proton travels in 2γ exchange
- uncertainty principal $|\boldsymbol{R}-\boldsymbol{R}'| \sim 1/\sqrt{2m_N\omega}$
- $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} \approx 0.17$ for $\mu\,{}^4\mathrm{He}^+$

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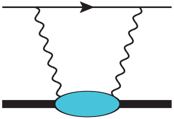

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- $\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)} \Longrightarrow$ LO + NLO + N^{2}LO

NR limit at LO: $\delta^{(0)}_{NR}$

$\delta_{NR} = \delta^{(0)}_{NR} + \delta^{(1)}_{NR} + \delta^{(2)}_{NR}$

- $\delta^{(0)}_{NR} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

$$\delta^{(0)}_{D1} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

- $S_{D_1}(\omega) \Longrightarrow$ electric dipole response function
- $\delta^{(0)}_{D1}$ is the dominant contribution to δ_{pol}

NR limit at NLO: $\delta_{NR}^{(1)}$

$$\delta_{NR} = \delta_{NR}^{(0)} + \delta_{NR}^{(1)} + \delta_{NR}^{(2)}$$

- $\delta_{NR}^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$

$$\boxed{\delta_{NR}^{(1)} = \delta_{R3pp}^{(1)} + \delta_{Z3}^{(1)}}$$

$$\delta_{R3pp}^{(1)} = -\frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \langle N_0 | \hat{\rho}^\dagger(\boldsymbol{R}) \hat{\rho}(\boldsymbol{R}') | N_0 \rangle$$

$$\delta_{Z3}^{(1)} = \frac{m_r^4}{24}(Z\alpha)^5 \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'|^3 \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}')$$

- $\delta_{R3pp}^{(1)} \Longrightarrow$ 3rd-order proton charge correlation
- $\delta_{Z3}^{(1)} \Longrightarrow$ 3rd-order Zemach moment
  cancels Zemach moment in finite-size corrections
  c.f. Pachucki '11 & Friar '13 (μ D)

NR limit at N^{2}LO: $\delta^{(2)}_{NR}$

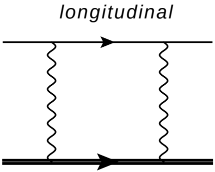
$\delta_{NR} = \delta^{(0)}_{NR} + \delta^{(1)}_{NR} + \delta^{(2)}_{NR}$

- $\delta^{(2)}_{NR} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

$$\delta^{(2)}_{NR} = \frac{m_r^5}{18}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} \left[S_{R^2}(\omega) + \frac{16\pi}{25} S_Q(\omega) + \frac{16\pi}{5} \mathcal{S}_{D_1 D_3}(\omega) \right]$$

- $S_{R^2}(\omega) \Longrightarrow$ monopole response function
- $S_Q(\omega) \Longrightarrow$ quadrupole response function
- $S_{D_1 D_3}(\omega) \Longrightarrow$ interference between D_1 and D_3 $[\ \hat{D}_3 = R^3 Y_1(\hat{R})\ ]$

- **Longitudinal contributions**
  - exchange Coulomb photon



$$\delta_L^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_L(\frac{\omega}{m_r})\, S_{D_1}(\omega)$$

$$K_L \approx \frac{\pi}{2}\sqrt{\frac{\omega}{2m_r}} - \frac{2\omega}{3m_r} + \cdots \qquad \left(\textbf{c.f. } \boldsymbol{\delta}^{(0)}_{\mathbf{D1}} : \quad \sqrt{\frac{\mathbf{2m_r}}{\boldsymbol{\omega}}} \right)$$

Relativistic dipole polarization

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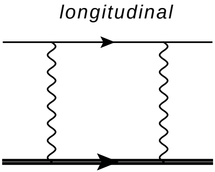

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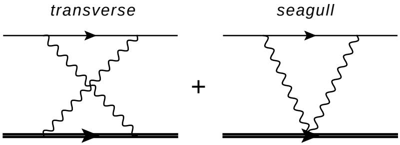
- **Transverse contributions**
  - convection current & spin current
  - seagull term: cancels infrared divergence
    restore gauge invariance

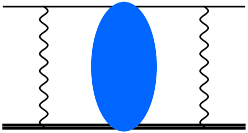


$$\delta_T^{(0)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_T(\frac{\omega}{m_r})\, S_{D_1}(\omega)$$

$$K_T \approx \frac{\omega}{m_r}\left(1 + \ln\frac{2\omega}{m_r}\right) + \cdots$$

Coulomb distortion

- **Non-perturbative Coulomb interaction in intermediate state**
  - contributes to both $2S$ and $2P$ atomic states



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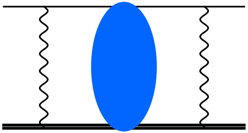
  - naive estimation: $\delta_C^{(0)} \sim (Z\alpha)^6$
  - full analysis: correct $\delta_{D1}^{(0)}$ in order of $Z\alpha\sqrt{\frac{m_r}{\omega}}$

    μD: $Z\alpha\sqrt{\frac{m_r}{\omega}} \sim 0.05$ & μ^4He: $Z\alpha\sqrt{\frac{m_r}{\omega}} \sim 0.03$



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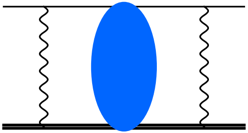

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μD: $Z\alpha\sqrt{\frac{m_r}{\omega}} \sim 0.05$ & μ^4He: $Z\alpha\sqrt{\frac{m_r}{\omega}} \sim 0.03$

$$\delta_C^{(0)} = -\frac{2\pi m_r^3}{9}(Z\alpha)^6 \int_{\omega_{\rm th}}^{\infty} d\omega\, \frac{m_r}{\omega} \left(\frac{1}{6} + \ln \frac{2Z^2\alpha^2 m_r}{\omega}\right) S_{D_1}(\omega)$$

Friar '77 & Pachucki '11

Finite nucleon size corrections

- In point-nucleon limit

$$\Delta H = -\alpha \sum_{i}^{Z} \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|} + \frac{Z\alpha}{r}$$

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- a low-momentum approximation of nucleon charge form factor

$$\begin{aligned} \tilde{n}_p(q) &\simeq 1 - 2q^2/\beta^2 \quad \Longrightarrow \quad \beta = \sqrt{12/\langle r_p^2 \rangle} = 4.12 \text{ fm}^{-1} \\ \tilde{n}_n(q) &\simeq \lambda q^2 \quad \Longrightarrow \quad \lambda = -\langle r_n^2 \rangle / 6 = 0.0191 \text{ fm}^2 \end{aligned}$$

Finite nucleon size corrections

- at LO $\delta^{(0)}$: zero contribution

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- **at NLO $\delta^{(1)}$:**

$$\delta^{(1)}_{NS} = \delta^{(1)}_{R1pp} + \delta^{(1)}_{Z1}$$

$$= -m_r^4 (Z\alpha)^5 \left[\frac{2}{\beta^2} - \lambda\right] \iint d\boldsymbol{R} d\boldsymbol{R}' |\boldsymbol{R} - \boldsymbol{R}'| \left[\langle N_0 | \hat{\rho}^\dagger(\boldsymbol{R}) \hat{\rho}(\boldsymbol{R}') | N_0 \rangle - \rho_0(\boldsymbol{R}) \rho_0(\boldsymbol{R}')\right]$$

  - $\delta^{(1)}_{R1pp} \Longrightarrow$ 1st-order proton charge correlation
  - $\delta^{(1)}_{Z1} \Longrightarrow$ 1st-order Zemach moment

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  - $\delta^{(1)}_{R1pp} \Longrightarrow$ 1st-order proton charge correlation
  - $\delta^{(1)}_{Z1} \Longrightarrow$ 1st-order Zemach moment

- at N²LO $\delta^{(2)}$:

$$\boxed{\delta^{(2)}_{NS} = -\frac{16\pi}{9} m_r^5 (Z\alpha)^5 \left[\frac{2}{\beta^2} - \lambda\right] \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{\omega}{2m_r}} S_{D_1}(\omega)}$$

- **Test Run: electric-dipole polarization effects in μ D**
  - μ-D's nuclear polarization (AV18): Pachucki, '11
  - we calculate contributions from dipole excitation (AV18) [$S_{D_1}(\omega)$ from Bampa, Leidemann & Arenhövel '11]

$\delta^{(0)}$ [meV]		Pachucki, '11	Our work
non-rel dipole	$\delta^{(0)}_{D1}$	-1.910	-1.907
relativistic	$\delta^{(0)}_{L}$	0.035	0.029
	$\delta^{(0)}_{T}$	–	-0.012
Coulomb	$\delta^{(0)}_{C}$	0.261	0.259

In relativistic corrections

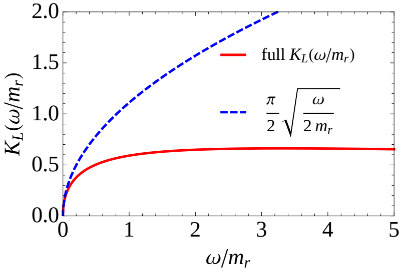
$$\delta^{(0)}_{L(T)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right) S_{D_1}(\omega)$$

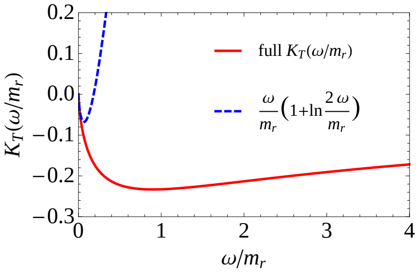
- Pachucki '11:

  low-energy approximation of $K_{L(T)}$

  $K_L = \frac{\pi}{2}\sqrt{\frac{\omega}{m_r}}$ & $K_T = 0$

- this approximation is not accurate enough





[meV]		AV18/UIX	χEFT★
$\delta^{(0)}$	$\delta^{(0)}_{D1}$	-4.418	-4.701
	$\delta^{(0)}_{L}$	0.289	0.308
	$\delta^{(0)}_{T}$	-0.126	-0.134
	$\delta^{(0)}_{C}$	0.512	0.546

★ NN(N^{3}LO)/$3N$(N^{2}LO)
c_D=1, c_E=-0.029

nuclear polarization in $\mu\,^4$He$^+$

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	$\delta^{(1)}_{Z3}$	4.183	4.526
$\delta^{(2)}$	$\delta^{(2)}_{R2}$	0.259	0.324
	$\delta^{(2)}_{Q}$	0.484	0.561
	$\delta^{(2)}_{D1D3}$	-0.666	-0.784

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nuclear polarization in $\mu\,{}^4\mathrm{He}^+$

[meV]	AV18/UIX	χEFT★
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{pol}	-2.408	-2.542

- Convergence from $\delta^{(0)}$ to $\delta^{(2)}$
  in a systematic expansion of
  $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{m_r/M_N} \approx 0.17$

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- δ_{pol} with AV18/UIX & χEFT differ: $\sim 5.5\%$ (0.134 meV)

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Error estimate

The work is not completed yet ...

Nuclear physics uncertainty

^{4}He		AV18/UIX	χEFT	Difference
$\mu\,^4$He$^+$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

Nuclear physics uncertainty

^{4}He		AV18/UIX	χEFT	Difference
binding energy	B_0 [MeV]	28.422	28.343	0.28%
point-proton radius	R_{pp} [fm]	1.432	1.475	3.0%
electric-dipole polarizability	α_E [fm^3]	0.0651	0.0694	6.4%
$\mu\,^4$He$^+$ nuclear polarization	δ_{pol} [meV]	-2.408	-2.542	5.5%

- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09

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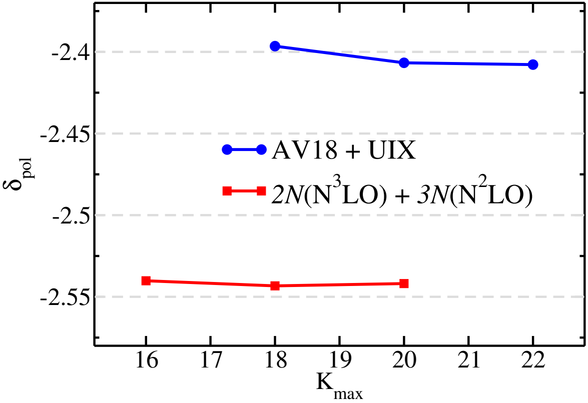
- B_0, R_{pp} & α_E in good agreement with previous calculations
  Kievsky *et al.* '08, Gazit *et al.* '06 & Stetcu *et al.* '09
- systematic uncertainty in δ_{pol} from nuclear physics:
  $\Longrightarrow \frac{5.5\%}{\sqrt{2}} \rightarrow \pm 4\%$

Numerical accuracy

- Convergence with the largest model space K_{max}
- Difference btw K_{max} & $K_{max}-4$
  - AV18/UIX $\sim 0.4\%$
  - EFT $\sim 0.2\%$



Atomic physics uncertainty

- $(Z\alpha)^6$ effects (beyond 2nd-order perturbation theory)
- relativistic & Coulomb corrections to other multipoles (other than dipole)
- higher-order nucleon-size corrections

- combine these corrections $\Longrightarrow$ an additional few percent error

δ_{pol} in $\mu\,^4$He$^+$

- combine all errors in a quadratic sum
- our prediction: $\delta_{pol} = -2.47$ meV $\pm 6\%$
- more accurate than early calculations: $\delta_{pol} = -3.1$ meV $\pm 20\%$
  Bernabeu & Jarlskog '74; Rinker '76; Friar '77
- our accuracy is comparable to the 5% requirement for the future $\mu\,^4$He$^+$ Lamb shift measurement
  Antognini *et al.* '11

Summary

- Lamb shifts in muonic atoms
  - raise interesting questions about lepton symmetry
  - connect nuclear and atomic physics
- We perform the first *ab-initio* calculation for $\mu\,^4\mathrm{He}^+$ polarization corrections
  - combine Hyperspherical Harmonics methods with modern phenomenological & chiral potentials
- We obtain $\delta_{pol} = -2.47$ meV $\pm 6\%$
  - more accurate than early calculations
  - will significantly improve the precision of $\langle r^2 \rangle$ extracted from future $\mu\,^4\mathrm{He}^+$ Lamb shift measurement (2013)

Outlook

- Study higher-order atomic-physics corrections
- Narrow uncertainty in nuclear physics
  - understand the discrepancy btw AV18/UIX & EFT results
  - explore other choices for potential parameterizations
  - include higher-order χEFT forces
- Investigate nuclear polarization in *e.g.* $\mu\,^3\text{He}^+$, $\cdots$