Matrix elements for processes that could compete in double beta decay

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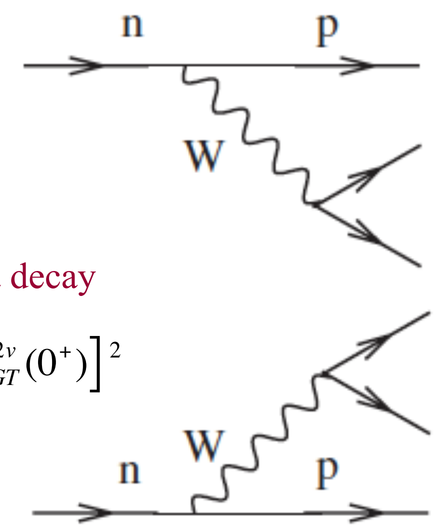
Plan of the talk

- Short overview of “standard” (light neutrino exchange) DBD mechanism
- Other mechanisms: right-handed currents, heavy neutrinos, R-parity SUSY, etc, …
- ^{48}Ca: 2ν and 0ν nuclear matrix elements
  - The right-handed currents contribution
  - The effect of larger model spaces
  - Beyond closure approximation
- ^{136}Xe results

Classical Double Beta Decay Problem

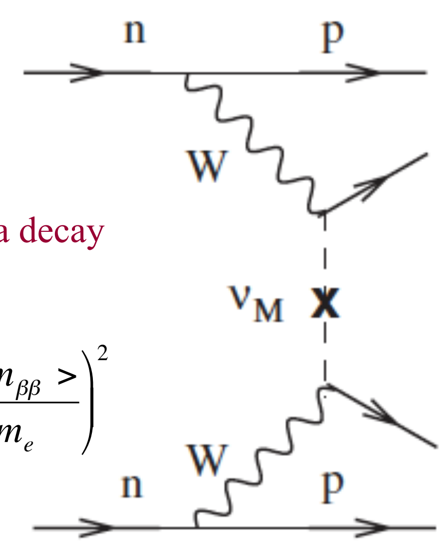
Isotope	$T_{1/2}(2\nu)$ (years)	$M^{2\nu}$
^{48}Ca	$4.4^{+0.6}_{-0.5} \times 10^{19}$	$0.0238^{+0.0015}_{-0.0017}$
^{76}Ge	$(1.5 \pm 0.1) \times 10^{21}$	$0.0716^{+0.0025}_{-0.0023}$
^{82}Se	$(0.92 \pm 0.07) \times 10^{20}$	$0.0503^{+0.0020}_{-0.0018}$
^{96}Zr	$(2.3 \pm 0.2) \times 10^{19}$	$0.0491^{+0.0023}_{-0.0020}$
^{100}Mo	$(7.1 \pm 0.4) \times 10^{18}$	$0.1258^{+0.0037}_{-0.0034}$
^{100}Mo-^{100}Ru(0_1^+)	$5.9^{+0.8}_{-0.6} \times 10^{20}$	$0.1017^{+0.0056}_{-0.0063}$
^{116}Cd	$(2.8 \pm 0.2) \times 10^{19}$	$0.0695^{+0.0025}_{-0.0024}$
^{128}Te	$(1.9 \pm 0.4) \times 10^{24}$	$0.0249^{+0.0031}_{-0.0023}$
^{130}Te	$(6.8^{+1.2}_{-1.1}) \times 10^{20}$	$0.0175^{+0.0016}_{-0.0014}$
^{150}Nd	$(8.2 \pm 0.9) \times 10^{18}$	$0.0320^{+0.0018}_{-0.0017}$
^{150}Nd-^{150}Sm(0_1^+)	$1.33^{+0.45}_{-0.26} \times 10^{20}$	$0.0250^{+0.0029}_{-0.0034}$
^{238}U	$(2.0 \pm 0.6) \times 10^{21}$	$0.0271^{+0.0053}_{-0.0033}$
^{136}Xe	2.23×10^{21}	0.020

A.S. Barabash, PRC 81 (2010)



2-neutrino double beta decay

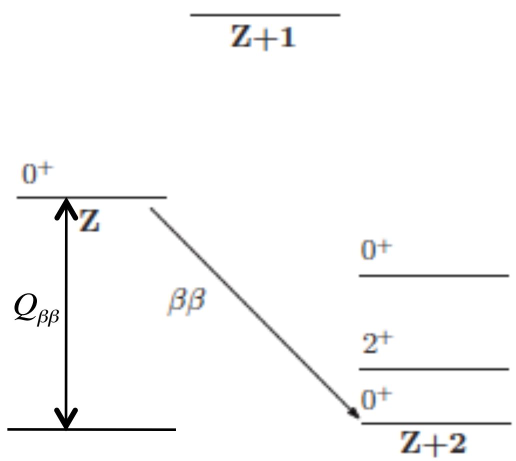
$$T_{1/2}^{-1}(2\nu) = G^{2\nu}(Q_{\beta\beta})\left[M_{GT}^{2\nu}(0^+)\right]^2$$



neutrinoless double beta decay

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

$$\langle m_{\beta\beta}\rangle = \left|\sum_k m_k U_{ek}^2\right|$$



Adapted from Avignone, Elliot, Engel, Rev. Mod. Phys. 80, 481 (2008) -> RMP08

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$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle$$
$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}\,|\nu_\alpha\rangle$$

Neutrino Masses

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

PMNS – matrix $\qquad c_{12} \equiv \cos\theta_{12},\; s_{12} = \sin\theta_{12},\; etc$

$$\tan^2\theta_{12} \cong 0.452,\;\; \sin^2 2\theta_{23} > 0.92,\;\; \sin^2 2\theta_{13} \cong 0.1$$

$$\Delta m^2_{12} \approx 8\times 10^{-5}\; eV^2\,(solar) \qquad \left|\Delta m^2_{23}\right| \approx 2.4\times 10^{-3}\; eV^2\;\;(atmospheric)$$

- Tritium decay:

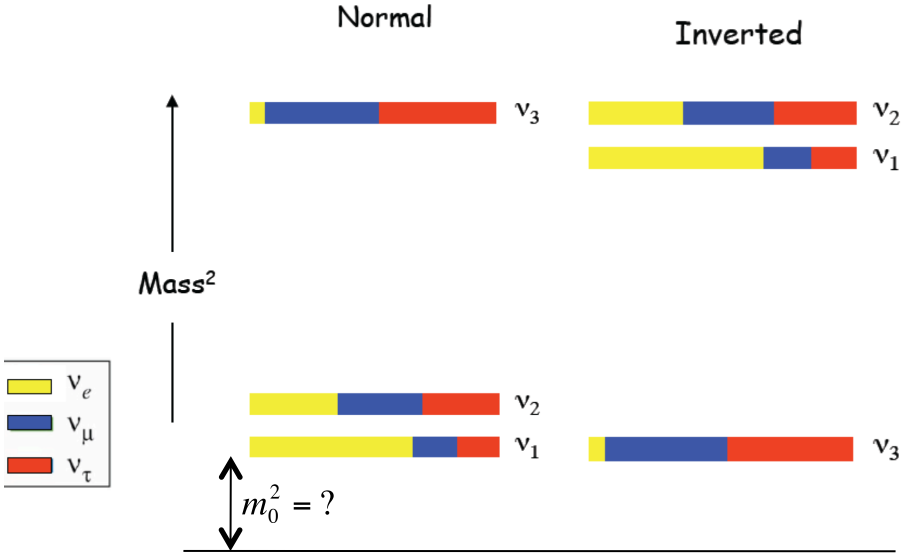
$$^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$$

$$m_{\nu_e} = \sqrt{\sum_i \left|U_{ei}\right|^2 m_i^2} < 2.2eV\,(Mainz\;\exp.)$$

$$Katrin\;\exp.(in\;progress):\,goal\;\; m_{\nu_e} < 0.3eV$$

- Cosmic Microwave Background (CMB) power spectrum:

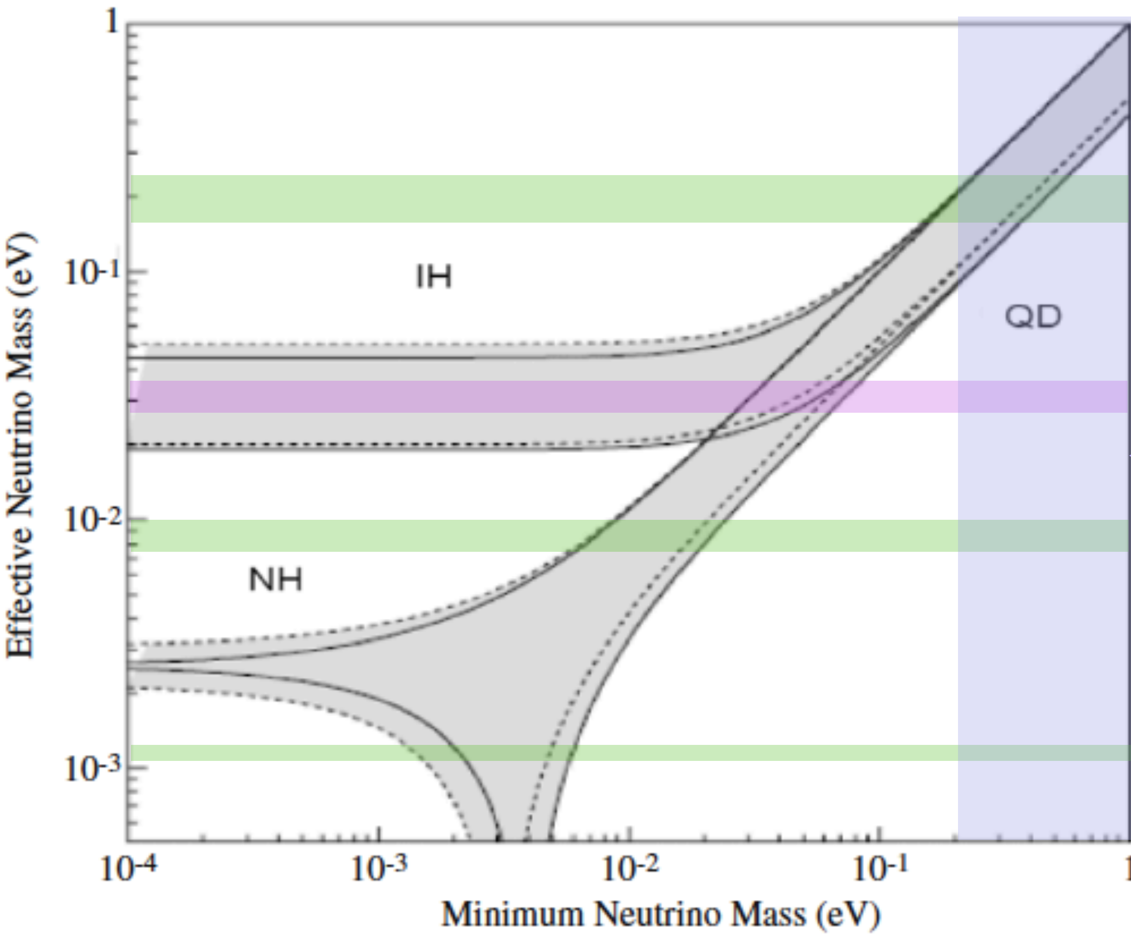
$$m_1 + m_2 + m_3 < 0.6eV$$



Two neutrino mass hierarchies

Neutrino ββ effective mass

H. Ejiri / Progress in Particle and Nuclear Physics 64 (2010) 249–257



$$\langle m_{\beta\beta}\rangle = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right|$$

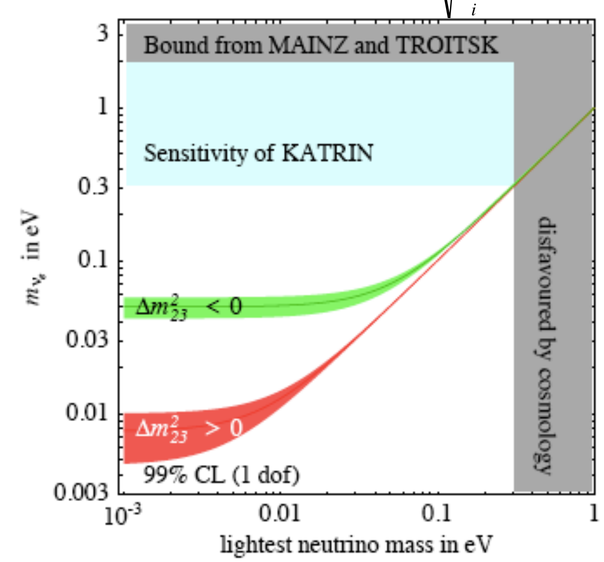
$$= \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

$$\phi_2 = \alpha_2 - \alpha_1 \qquad \phi_3 = -\alpha_1 - 2\delta$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2 \left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

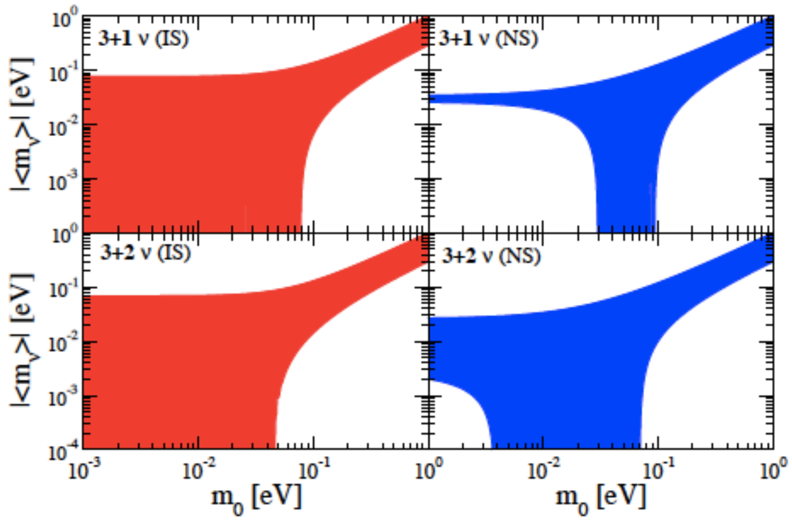
← CMB constraint

$$m_{\nu_e} = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$



Neutrino effective mass: the sterile effects

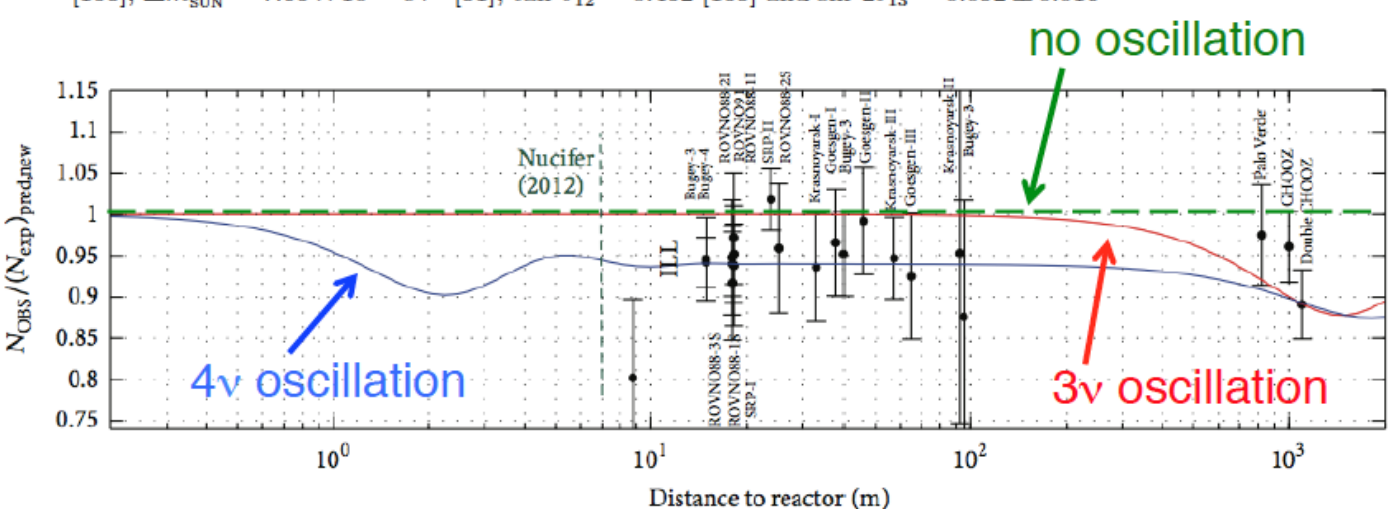
Vergados, Ejiri, Simkovic, Rep. Prog. Phys. **75**, 106301 (2012)

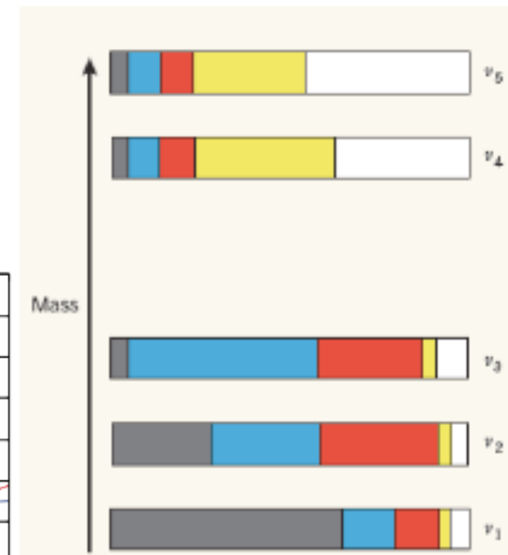


$$\langle m_{\beta\beta}\rangle = \left|\sum_{k=1}^{4,5} m_k U_{ek}^2\right|$$

$$T_{1/2}^{-1}(0\nu) = G_{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

Figure 7: (Color online) The same as in Fig. 6, if one considers one (version 3+1) or two (version 3+2) sterile neutrinos, which are heavier than the standard neutrinos. Best fit points for the 3+1 ($\Delta m^2_{41} = 1.78\ eV^2$, $U_{e4} = 0.151$) and 3+2 ($\Delta m^2_{41} = 0.46\ eV^2$, $U_{e4} = 0.108$ and $\Delta m^2_{51} = 0.89\ eV^2$, $U_{e5} = 0.124$) scenarios from reactor antineutrino data are taken into account [186]. In addition, best fit values $\Delta m^2_{\rm ATM} = 2.43 \times 10^{-3}$ eV^2 [158], $\Delta m^2_{\rm SUN} = 7.65 \times 10^{-5}$ eV^2 [81], $\tan^2\theta_{12} = 0.452$ [159] and $\sin^2 2\theta_{13} = 0.092 \pm 0.016$





The origin of (Majorana) neutrino masses

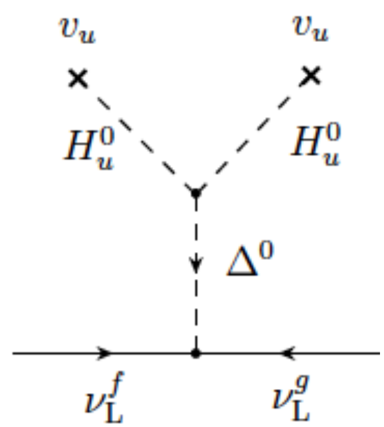
$\mathcal{L} \supset$

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^C} \end{pmatrix} \begin{pmatrix} 0 & m_{LR}^{\nu} \\ m_{LR}^{\nu T} & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} + h.c.$$

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^C} \end{pmatrix} \begin{pmatrix} m_{LL}^{II} & m_{LR}^{\nu} \\ m_{LR}^{\nu T} & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} + h.c.$$

$$m_{LL}^{\nu} \approx m_{LL}^{II} + m_{LL}^{I}$$

$$m_{LL}^{\nu} = -m_{LR}^{\nu} M_{RR}^{-1} m_{LR}^{\nu T}$$



See-saw mechanisms

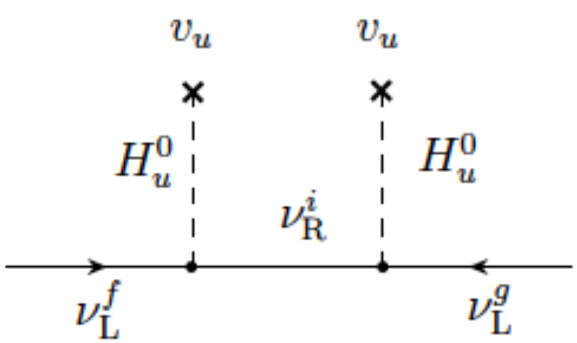
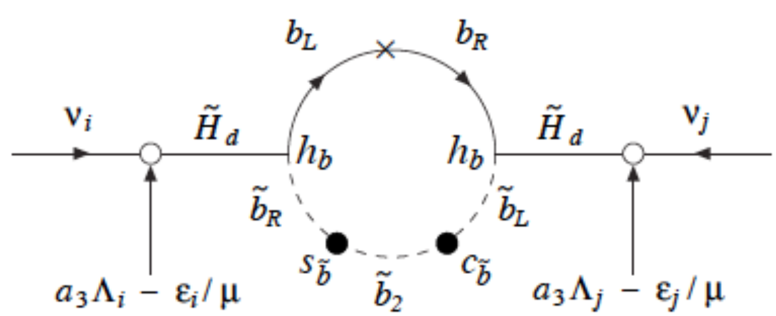
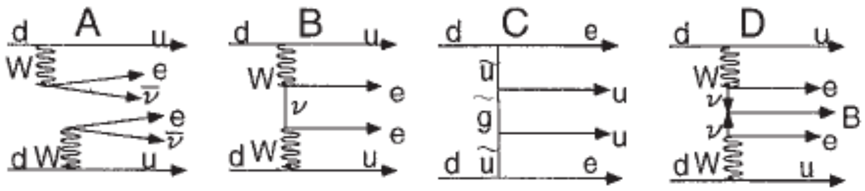


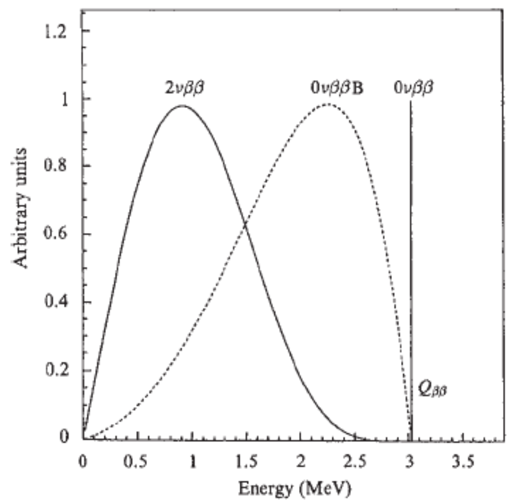
Diagram illustrating the type I see-saw mechanism

GUT/SUSY R-parity v. mechanism



Possible contributions of other mechanisms





$$C_{mm} = G^{0\nu}\left|M^{0\nu}\right|, \ldots$$

$$[T_{1/2}^{0\nu}]^{-1} = \left[C_{mm}\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 + C_{\lambda\lambda}\langle\lambda\rangle^2 + C_{\eta\eta}\langle\eta\rangle^2 + C_{m\lambda}\frac{\langle m_{\beta\beta}\rangle}{m_e}\langle\lambda\rangle\cos\phi_1 + C_{m\eta}\frac{\langle m_{\beta\beta}\rangle}{m_e}\langle\eta\rangle\cos\phi_2 + C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle\cos(\phi_1-\phi_2)\right]$$

$$\langle\eta\rangle = \zeta\sum_k^{light} U_{ek}V_{ek}$$

$$\langle\lambda\rangle = \left(\frac{M_{WL}}{M_{WR}}\right)^2 \sum_k^{light} U_{ek}V_{ek} \qquad W_R \approx \zeta W_1 + W_2$$

Hiroyasu EJIRI J. Phys. Soc. Jpn., Vol. 74, No. 8, August, 2005

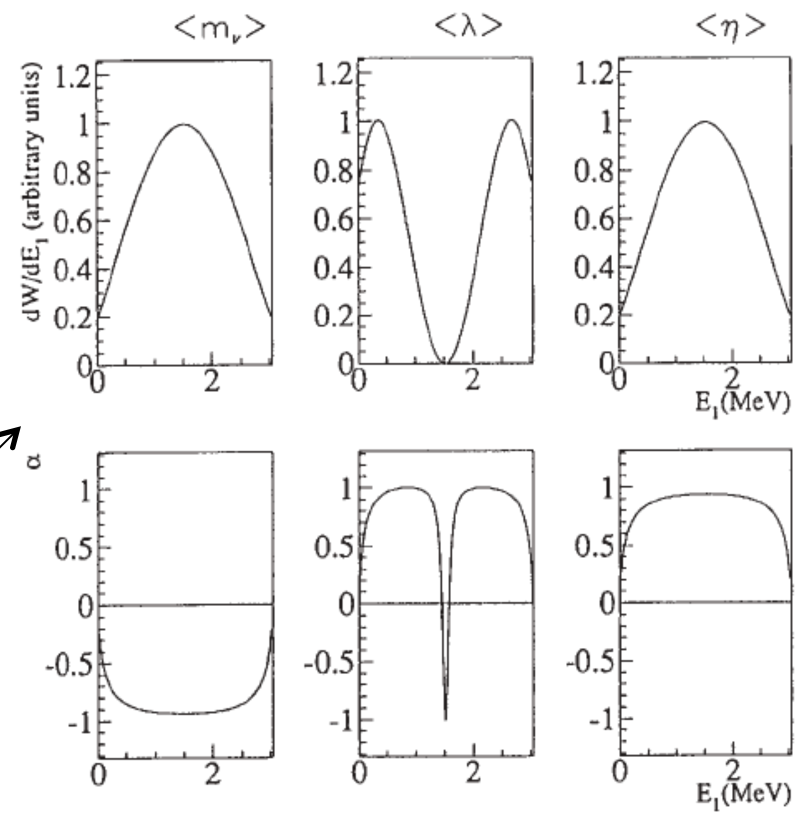
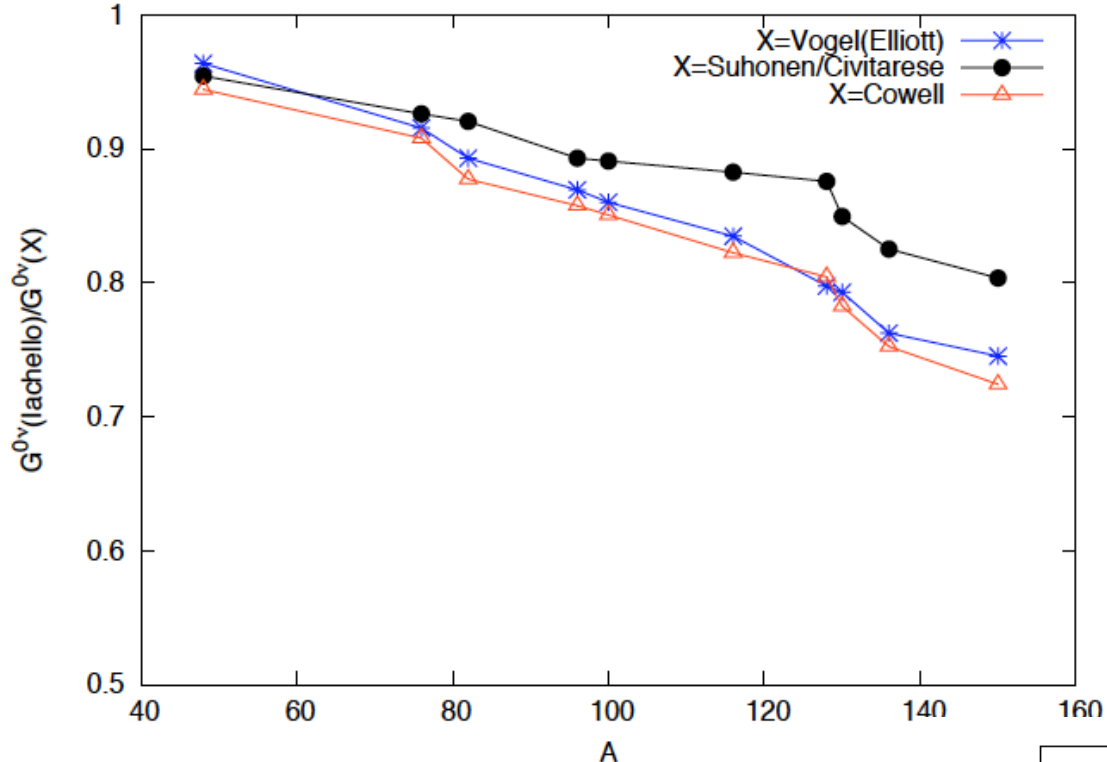


Fig. 4. Energy and angular correlations for the ^{100}Mo $0\nu\beta\beta$ process caused by the mass and right-handed current terms of $\langle m\rangle$, $\langle\lambda\rangle$ and $\langle\eta\rangle$. Top: Calculated single-β spectra. Bottom: $\beta_1 - \beta_2$ angular correlation coefficients α defined by $W(\theta_{12}) = 1 + \alpha\cos\theta_{12}$.[4]

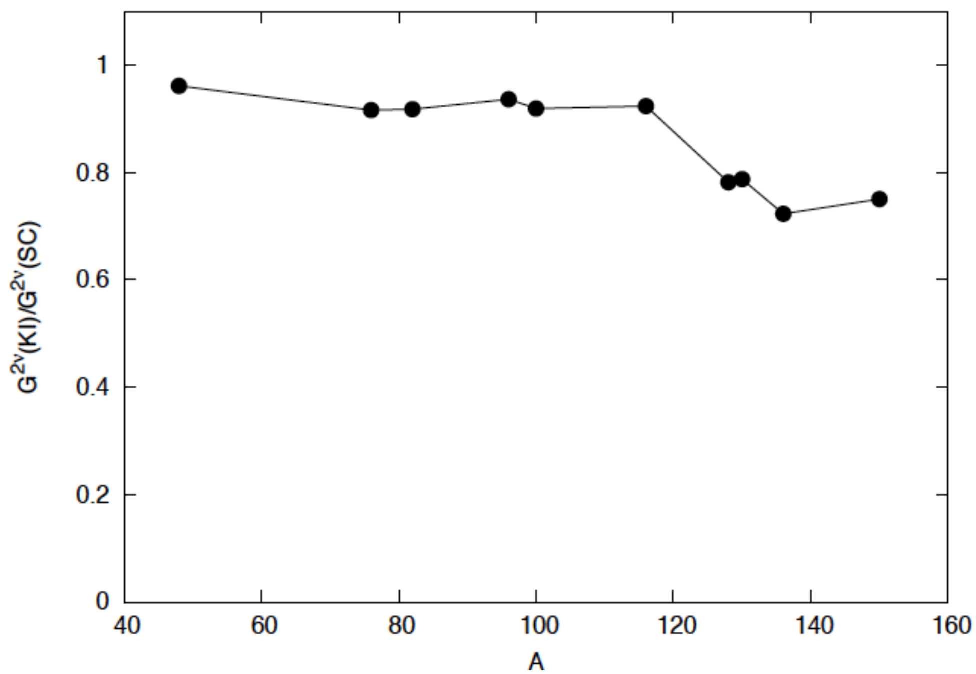
Phase Space Factors



All calculated with:
g_A = 1.254

Iachello (KI): PRC 85, 0342316 (2012)
(see also: Stoica & Mirea arXiv:1307.0290)

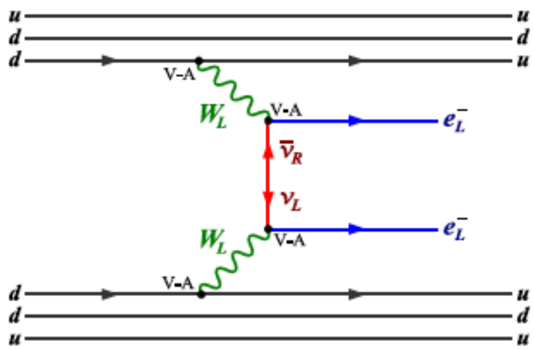
Suhonen (SC): Phys. Rep 300, 123 (1998)
Vogel (Elliott): J. Phys. G 34, 667 (2007)
Cowell: PRC 73, 028501 (2006)



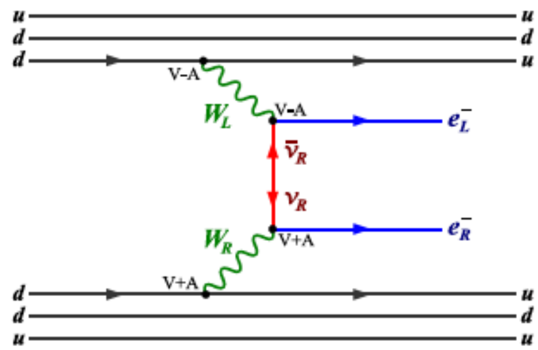
Possible contributions of other mechanisms

R. Arnold et al.: Probing New Physics Models of Neutrinoless Double Beta Decay with SuperNEMO

arXiv:1005.1241



(a)



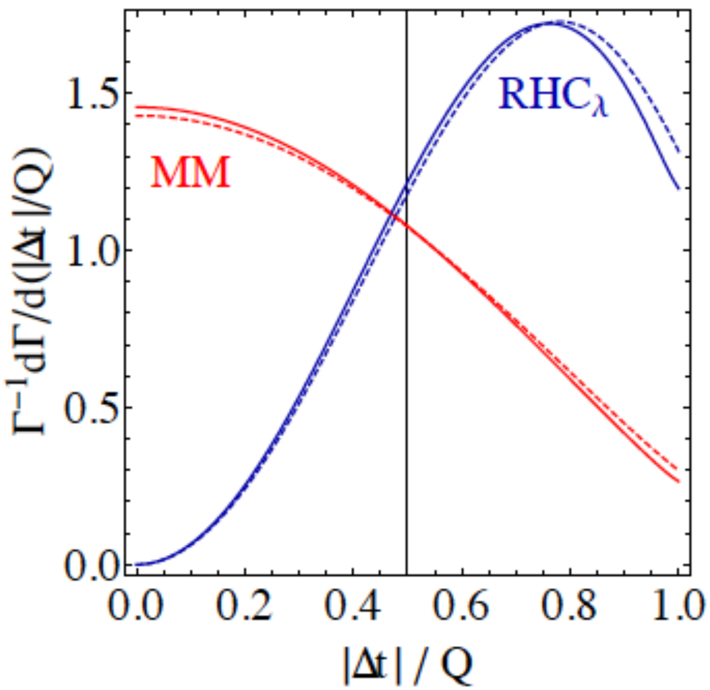
(b)

$$\mu \approx \frac{m_\nu}{m_e}, \tag{10}$$

$$\eta \approx \tan\zeta \sqrt{\frac{m_\nu}{M_R}}, \tag{11}$$

$$\lambda \approx \left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \sqrt{\frac{m_\nu}{M_R}}. \tag{12}$$

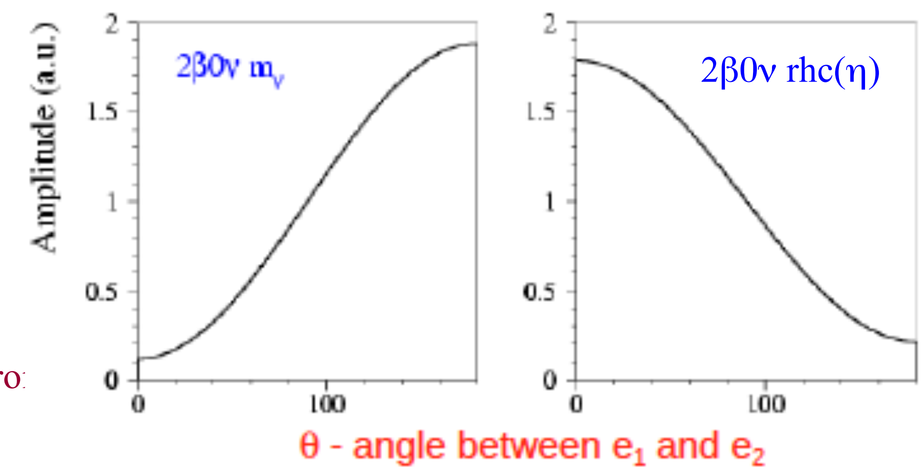
$$[T_{1/2}]^{-1} = C_{mm}\mu^2 + C_{\lambda\lambda}\lambda^2 + C_{m\lambda}\mu\lambda. \tag{13}$$



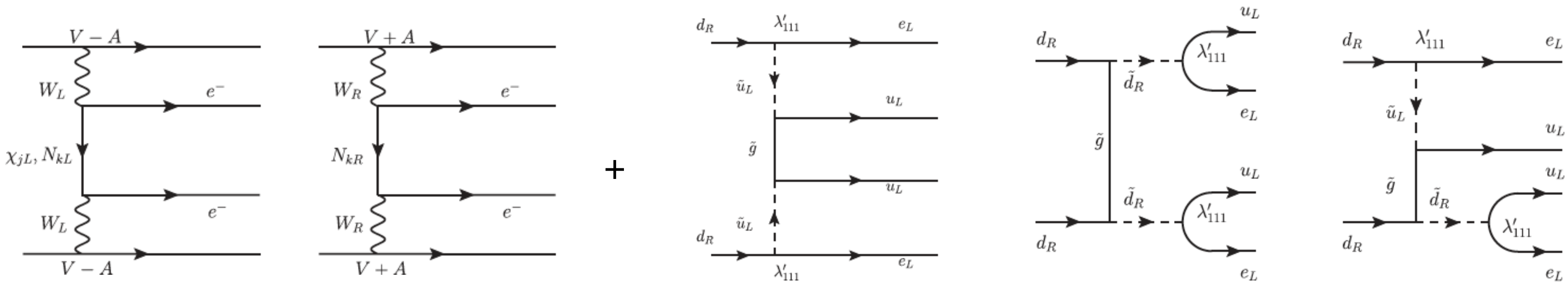
←

Isotope	C_{mm} [y^{-1}]	$C_{\lambda\lambda}$ [y^{-1}]	$C_{m\lambda}$ [y^{-1}]
^{76}Ge	1.12×10^{-13}	1.36×10^{-13}	-4.11×10^{-14}
^{82}Se	4.33×10^{-13}	1.01×10^{-12}	-1.60×10^{-13}
^{150}Nd	7.74×10^{-12}	2.68×10^{-11}	-3.57×10^{-12}

Table 1: Coefficients used in calculating the $0\nu\beta\beta$ decay rate [30].



Possible contributions from other mechanisms



PRD 83, 113003 (2011)

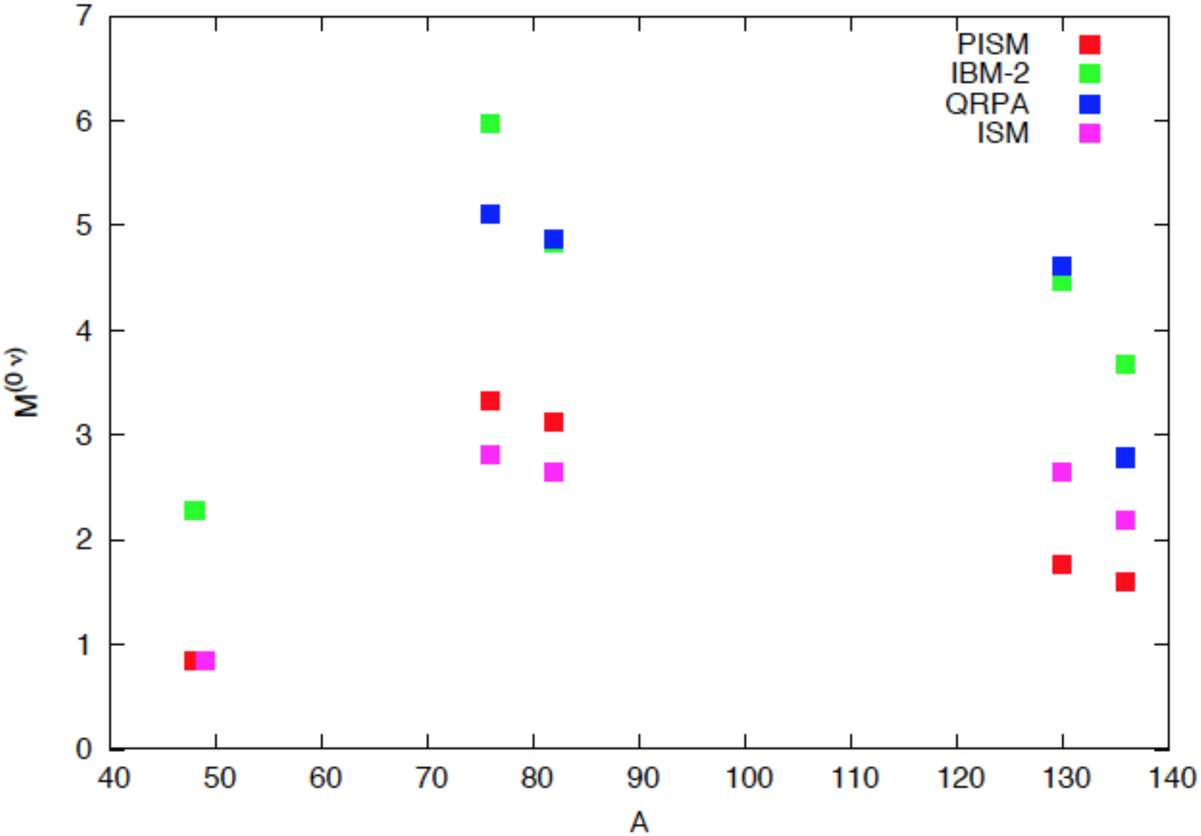
$$\begin{cases} \nu_{eL} = P_L\left[\sum_k^{light} U_{ek}\nu_k + \sum_k^{heavy} U_{ek}N_k\right] \\ \nu'_{eR} = P_R\left[\sum_k^{light} V_{ek}\nu_k + \sum_k^{heavy} V_{ek}N_k\right] \end{cases} \qquad \eta_{\nu L} = \frac{\langle m_{\beta\beta}\rangle}{m_e} \qquad \eta_{NL} = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k} \qquad \eta_{NR} = \left(\frac{M_{WL}}{M_{WR}}\right)^4 \sum_k^{heavy} V_{ek}^2 \frac{m_p}{M_k}$$

$$<\lambda>, <\eta> - \ mass-independent: \ |<\lambda>|, \ |<\eta>| \ << \ |\eta_{\nu L}|, \ |\eta_N|$$

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|\sum_j M_j \eta_j\right|^2 = G^{0\nu}\left|M^{(0\nu)}\eta_{\nu L} + M^{(0N)}(\eta_{NL}+\eta_{NR}) + \tilde{X}_\lambda <\lambda> + \tilde{X}_\eta <\eta> + M^{(0\lambda')}\eta_{\lambda'} + M^{(0\tilde{q})}\eta_{\tilde{q}} + \cdots\right|^2$$

$$\left[T_{1/2}^{0\nu}\right]^{-1} \cong G^{0\nu}\left|M^{(0\nu)}\eta_{\nu L} + M^{(0N)}\eta_{NR}\right|^2 \approx G^{0\nu}\left[\left|M^{(0\nu)}\right|^2\left|\eta_{\nu L}\right|^2 + \left|M^{(0N)}\right|^2\left|\eta_{NR}\right|^2\right]$$

Matrix Elements: Light Neutrinos



PRL 109, 042501 (2012)

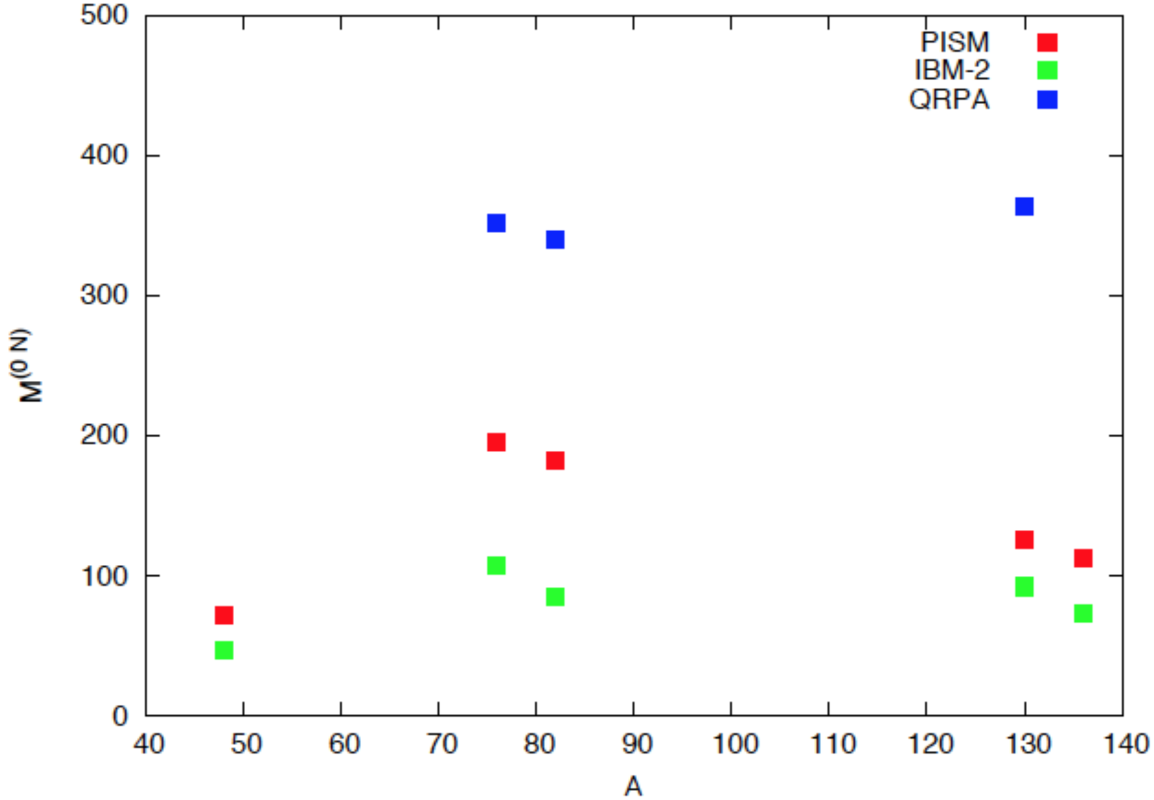
PRD 83, 113003 (2011)

NPA 818, 139 (2009)

$$\eta_{\nu L} = \frac{\langle m_{\beta\beta} \rangle}{m_e}$$

$$\left[T_{1/2}^{0\nu}\right]^{-1} \cong G^{0\nu}\left|M^{(0\nu)}\eta_{\nu L} + M^{(0N)}\eta_{NR}\right|^2 \approx G^{0\nu}\left[\left|M^{(0\nu)}\right|^2\left|\eta_{\nu L}\right|^2 + \left|M^{(0N)}\right|^2\left|\eta_{NR}\right|^2\right]$$

Matrix Elements: Heavy Neutrinos



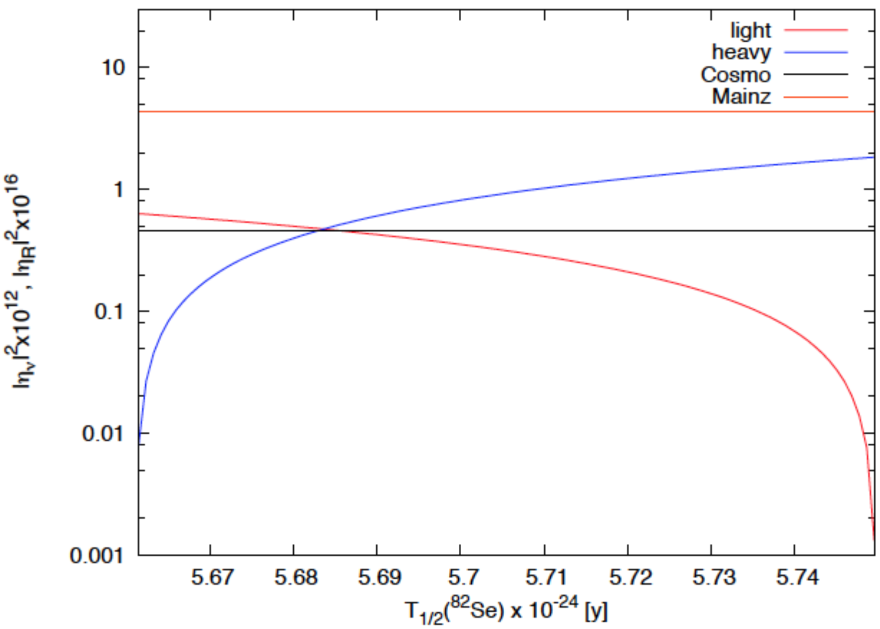
PRL 109, 042501 (2012)

PRD 83, 113003 (2011)

$$\eta_{NR} = \left(\frac{M_{WL}}{M_{WR}}\right)^4 \sum_k^{heavy} V_{ek}^2 \frac{m_p}{M_k}$$

$$\left[T_{1/2}^{0\nu}\right]^{-1} \cong G^{0\nu}\left|M^{(0\nu)}\eta_{\nu L} + M^{(0N)}\eta_{NR}\right|^2 \approx G^{0\nu}\left[\left|M^{(0\nu)}\right|^2\left|\eta_{\nu L}\right|^2 + \left|M^{(0N)}\right|^2\left|\eta_{NR}\right|^2\right]$$

Two Non-Interfering Mechanisms

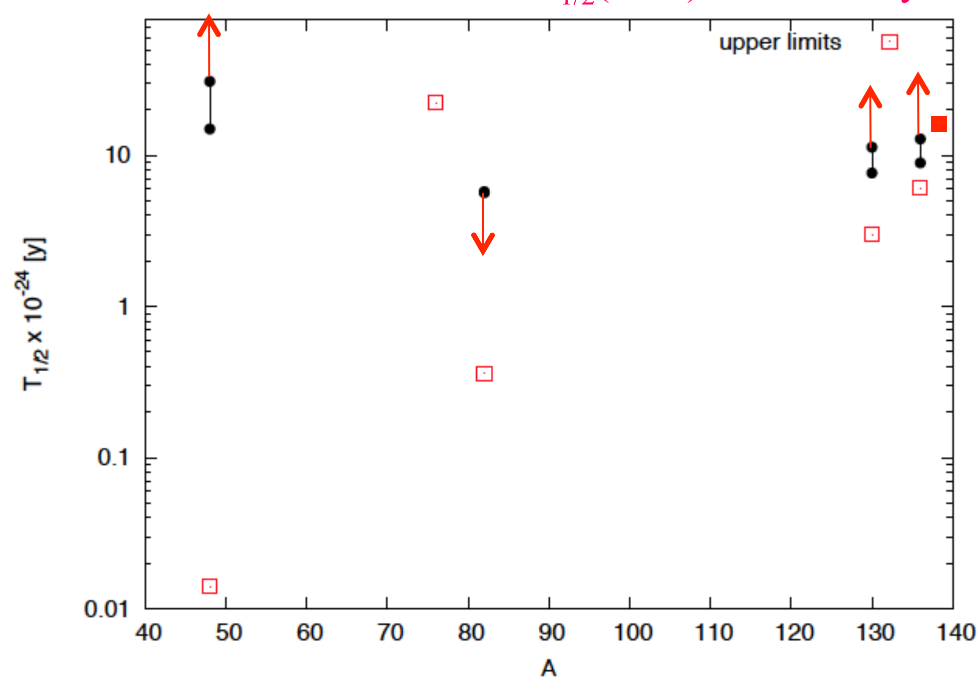


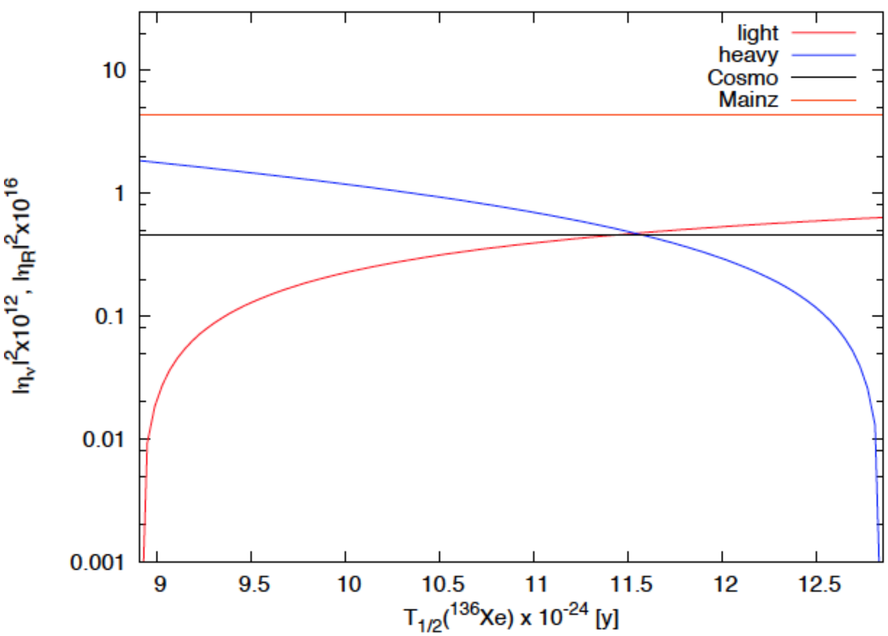
$$\eta_\nu = \frac{\langle m_{\beta\beta} \rangle}{m_e} \approx 10^{-6} \qquad \eta_{NR} = \left(\frac{M_{WL}}{M_{WR}}\right)^4 \sum_k^{heavy} V_{ek}^2 \frac{m_p}{M_k} \approx 10^{-8}$$

$$\left[G_i^{0\nu} T_{1/2\,i}^{0\nu}\right]^{-1} = \left|M_i^{(0\nu)}\right|^2 \left|\eta_\nu\right|^2 + \left|M_i^{(0N)}\right|^2 \left|\eta_{NR}\right|^2 \quad i = 1,2$$

$$\left|\eta_\nu\right|^2 > 0 \ and \ \left|\eta_{NR}\right|^2 > 0$$

$T_{1/2}(^{76}Ge) = 22.3 \times 10^{24}$ y





Horoi CMU

Neutrinoless Double Beta Decay Requires a Massive Majorana Neutrino

J. Schechter and J.W.F Valle, PRD 25, 2951 (1982)

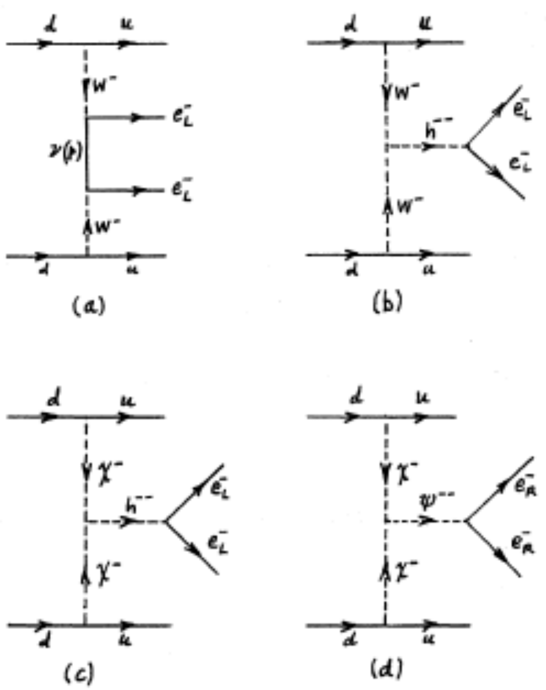


FIG. 1. Diagrams for neutrinoless double-β decay in an SU(2)$\times$U(1) gauge theory. The standard diagram is Fig. 1(a). It is the only one which contains a virtual neutrino (of four-momentum p). d and u are the down and up quarks.

$0\nu\beta\beta$ obs consequences:

-Neutrinos are Majorana fermions (with m > 0).

- Lepton numbers conservation is violated by 2 units

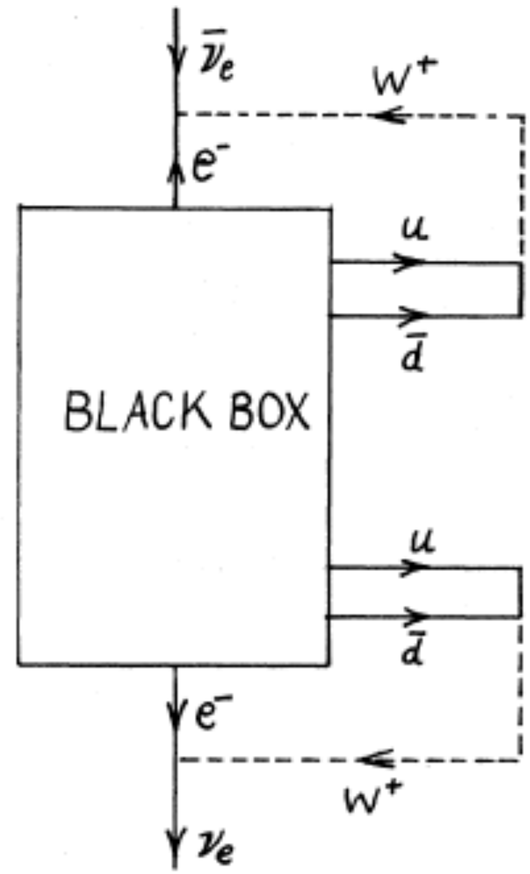


FIG. 2. Diagram showing how any neutrinoless double-β decay process induces a $\bar{\nu}_e$-to-ν_e transition, that is, an effective Majorana mass term.

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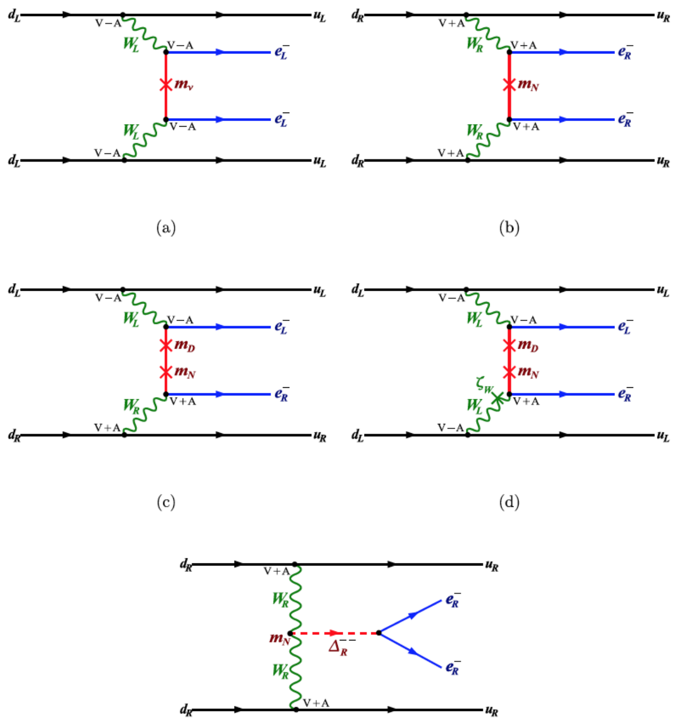
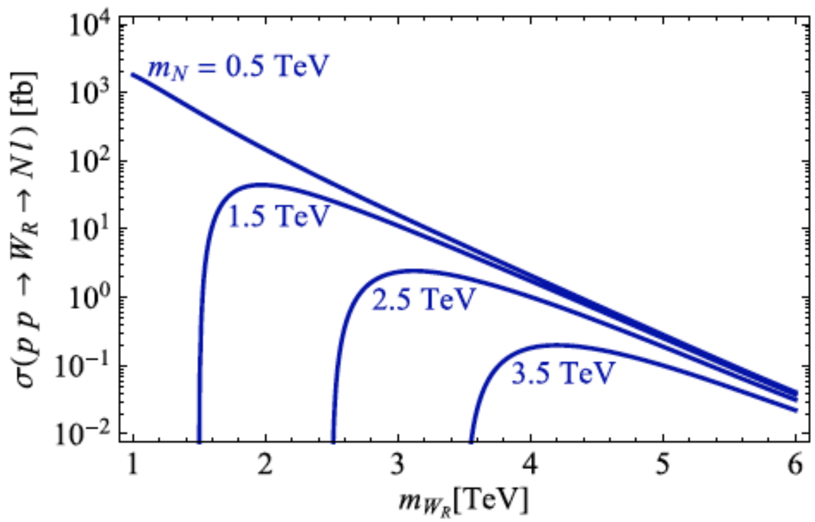
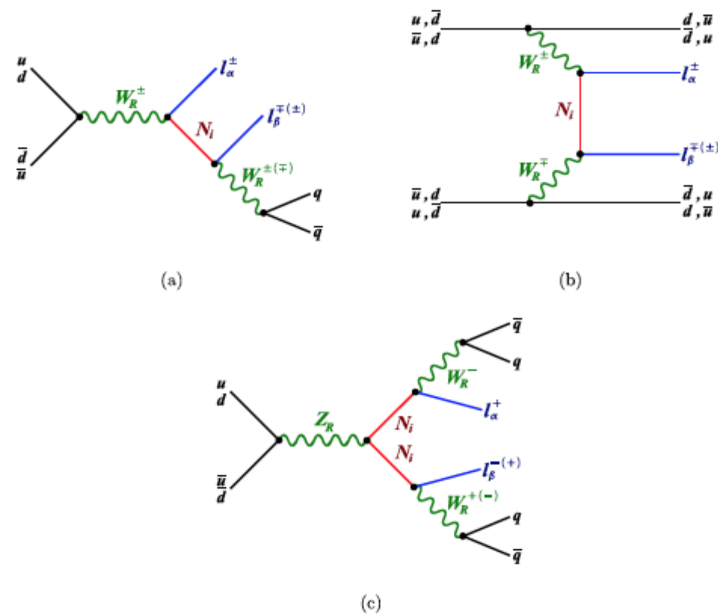
Some mechanisms tested at LHC

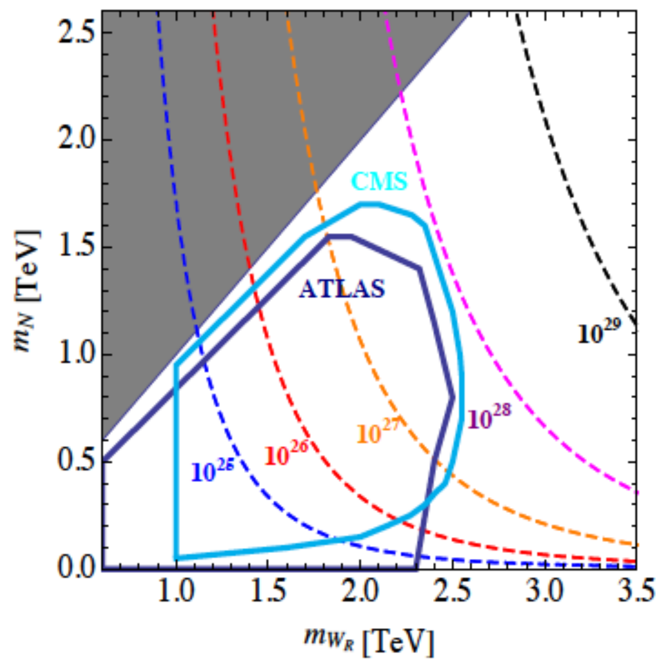
PRD 86, 055006 (2012)





CMU



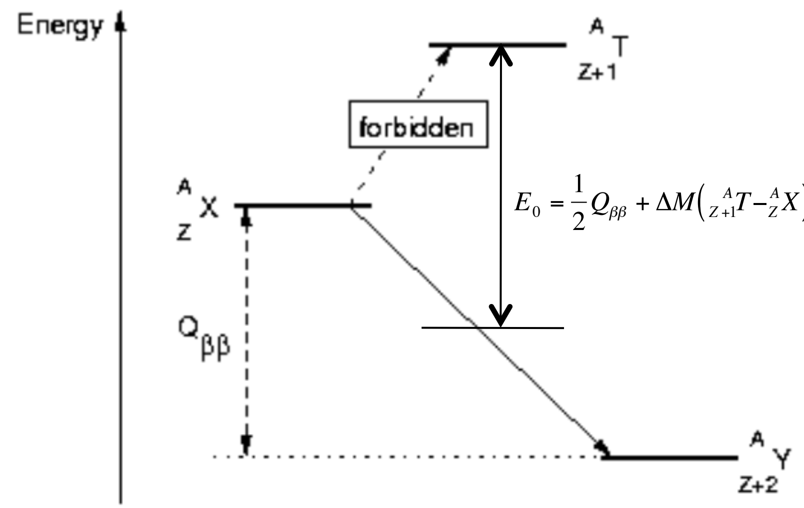


2ν Double Beta Decay (DBD) of ^{48}Ca

$$T_{1/2}^{-1} = G_{2\nu}(Q_{\beta\beta})\left[M_{GT}^{2\nu}(0^+)\right]^2$$

$$M_{\mathrm{GT}}^{2\nu}(0^+) = \sum_k \frac{\langle 0_f \| \sigma\tau^- \| 1_k^+ \rangle \langle 1_k^+ \| \sigma\tau^- \| 0_i \rangle}{E_k + E_0}$$

$$^{48}Ca \xrightarrow{2\nu\,\beta\beta} {}^{48}Ti$$



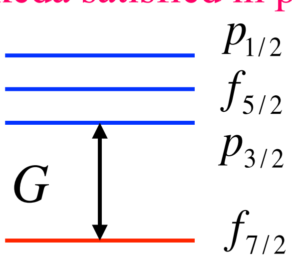
The choice of valence space is important!

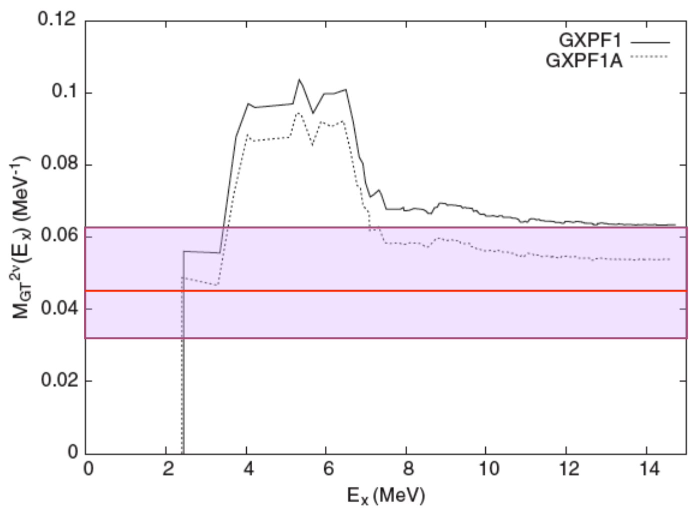
$$B(GT) = \frac{\left|\langle f \| \sigma\cdot\tau \| i \rangle\right|^2}{(2J_i + 1)}$$

ISR	48Ca	48Ti
pf	24.0	12.0
f7 p3	10.3	5.2

$$Ikeda\ sum\ rule(ISR) = \sum B(GT; Z \to Z+1) - \sum B(GT; Z \to Z-1) = 3(N-Z)$$

Ikeda satisfied in pf !





$$\sigma\tau \xrightarrow{quenched} 0.77\sigma\tau$$

Horoi, Stoica, Brown,

PRC **75**, 034303 (2007)

INT
August 23, 2013

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Double Beta Decay NME for ^{48}Ca

TABLE I. Matrix elements and half-lives for 2ν decay calculated using GXPF1A interaction and two quenching factors. Matrix elements are in MeV^{-1} for transitions to 0^+ states and in MeV^{-3} for transitions to 2^+ states.

	$qf = 0.77$		$qf = 0.74$	
J_n^π	$M^{2\nu}$	$T_{1/2}^{2\nu}$ (yr)	$M^{2\nu}$	$T_{1/2}^{2\nu}$ (yr)
0_1^+	0.054	3.3×10^{19}	0.050	3.9×10^{19}
2_1^+	0.012	8.5×10^{23}	0.010	1.0×10^{24}
0_2^+	0.050	1.6×10^{24}	0.043	1.9×10^{24}

M. Horoi, PRC **87**, 014320 (2013)

$$M_{GT}^{2\nu}(0^+) = \sum_k \frac{\langle 0_f \| \sigma\tau^- \| 1_k^+ \rangle \langle 1_k^+ \| \sigma\tau^- \| 0_i \rangle}{E_k + E_0}$$

$$\longleftarrow \left(T_{1/2}^{2\nu}\right)_{exp} = \left[4.4^{+0.6}_{-0.5}(stat) \pm 0.4(syst)\right] \times 10^{19}\ yr$$

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu} \left| \tilde{\eta}_{\nu L} M_\nu^{0\nu} + \tilde{\eta}_N M_N^{0\nu} + \eta_{\lambda'} M_{\lambda'}^{0\nu} + \eta_{\tilde{q}} M_{\tilde{q}}^{0\nu} \right|^2,$$

TABLE II. Matrix elements for 0ν decay using the GXPF1A interaction and two SRC models [61], CD-Bonn (SRC1) and Argonne (SRC2). For comparison, the values labeled (a) are taken from Ref. [27], and the value labeled (b) is taken from Ref. [62] for $g_{pp} = 1$ and no SRC.

	Model	$M_\nu^{0\nu}$	$M_N^{0\nu}$	$M_{\lambda'}^{0\nu}$	$M_{\tilde{q}}^{0\nu}$
0_1^+	SRC1	0.90	75.5	618	86.7
	SRC2	0.82	52.9	453	81.8
	others	2.3(a)	46.3(a)	392(b)	
0_2^+	SRC1	0.80	57.2	486	84.2
	SRC2	0.75	40.6	357	80.6

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Possible contributions from right-handed currents

$$dW^{0\nu}_{0^+\to 0^+} = \frac{a_{0\nu}}{(m_e R)^2}\left[A(\varepsilon_1) + B(\varepsilon_1)\cos\theta_{12}\right] w_{0\nu}(\varepsilon_1)\, d\varepsilon_1\, d(\cos\theta_{12})$$

$$\left[T^{0\nu}_{1/2}\right]^{-1} = \frac{1}{\ln 2}\int dW^{0\nu}_{0^+\to 0^+} = \frac{a_{0\nu}}{\ln 2 (m_e R)^2}\int A(\varepsilon_1) d\varepsilon_1\, d(\cos\theta_{12})$$

$$\eta_{\nu L} \equiv \frac{|< m_{\beta\beta} >|}{m_e} = \frac{1}{m_e}\sum_k^{light} U^2_{ek} m_k \approx 10^{-6}$$

$$\langle\eta\rangle = \zeta \sum_k^{light} U_{ek} V_{ek} \approx \zeta \sqrt{\frac{m_\nu}{M_{N_R}}} \approx 10^{-3}\sqrt{\frac{10^{-1}}{10^{11}}} \approx 10^{-9}$$

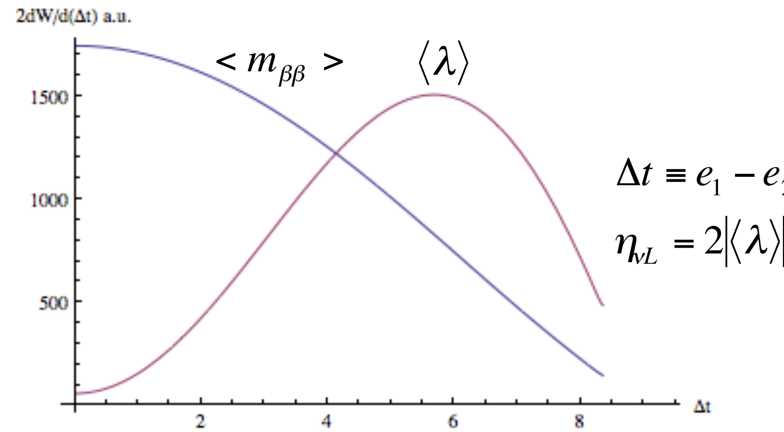
$$\langle\lambda\rangle = \left(\frac{M_{WL}}{M_{WR}}\right)^2 \sum_k^{light} U_{ek} V_{ek} \approx \left(\frac{M_{WL}}{M_{WR}}\right)^2 \sqrt{\frac{m_\nu}{M_{N_R}}} \approx 10^{-9}$$

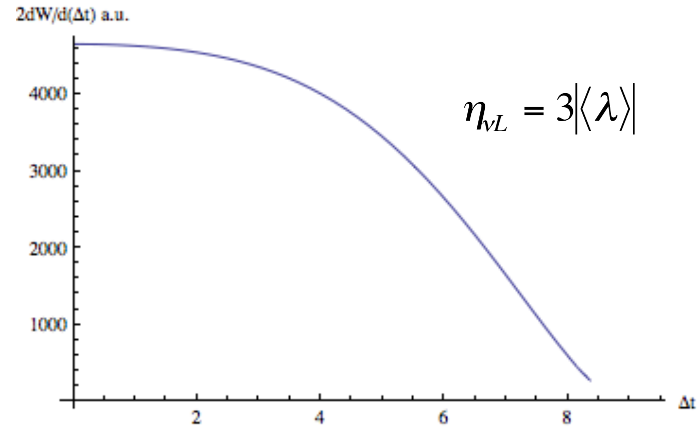
$$W_R \approx \zeta W_1 + W_2$$

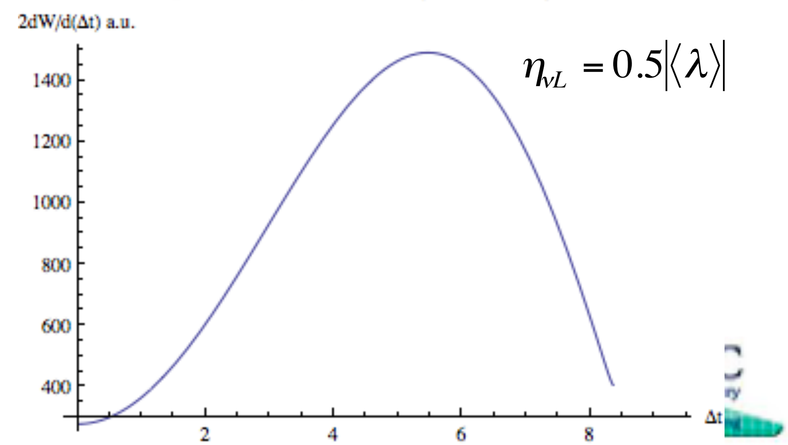
$$[T^{0\nu}_{1/2}]^{-1} = \Bigg[C_{mm}\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 + C_{\lambda\lambda}\langle\lambda\rangle^2 + C_{\eta\eta}\langle\eta\rangle^2 +$$

$$C_{m\lambda}\frac{\langle m_{\beta\beta}\rangle}{m_e}\langle\lambda\rangle\cos\phi_1 + C_{m\eta}\frac{\langle m_{\beta\beta}\rangle}{m_e}\langle\eta\rangle\cos\phi_2$$

$$+ C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle\cos(\phi_1 - \phi_2)\Bigg]$$







The effect of larger model spaces for ^{48}Ca

M(0ν)	SDPFU	SDPFMUP
$0\ \hbar\omega$	0.941	0.623
$0{+}2\,\hbar\omega$	1.182 (26%)	1.004 (61%)

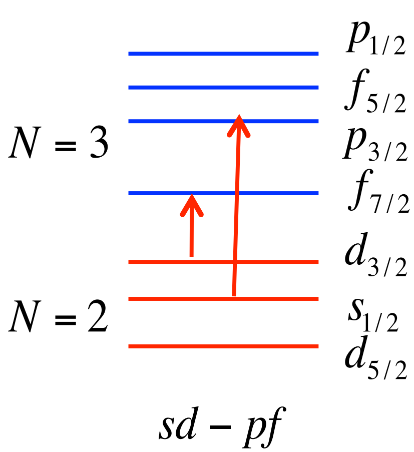
	M(0ν)
$0\ \hbar\omega$ / GXPF1A	0.733
$0\ \hbar\omega + 2^{nd}$ ord./GXPF1A	1.301 (77%)

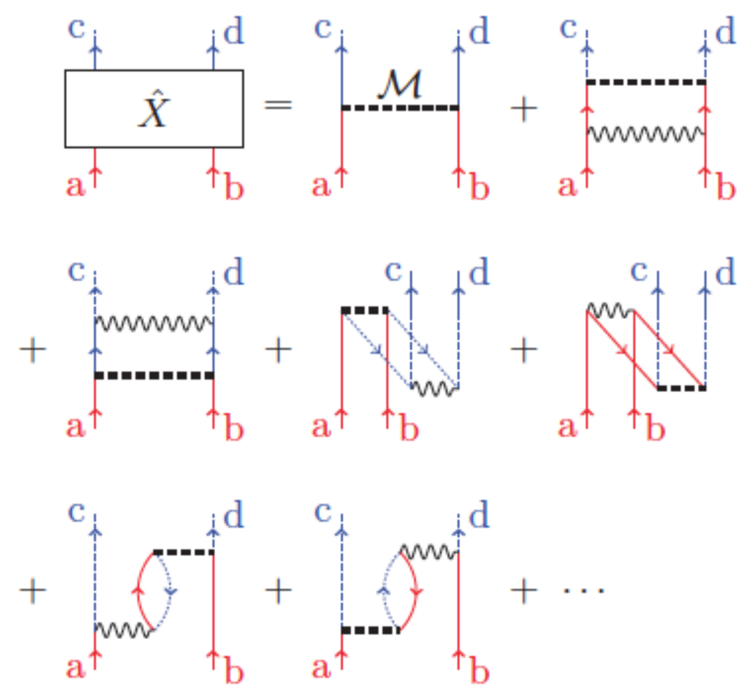
SDPFU: PRC 79, 014310 (2009)

SDPFMUP: PRC 86, 051301(R) (2012)

arXiv:1308.3815

PRC 87, 064315 (2013)





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Closure approximation for the $M^{0\nu}$

PRC **81**, 024321 (2010)

$$T_{1/2}^{-1}(0\nu) = G_{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{<m_{\beta\beta}>}{m_e}\right)^2$$

$$M^{0\nu} = \left(M_{GT}^{0\nu}\right) - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

- Closure approximation
- Includes higher order corrections in the nucleon currents

$$M_S^{0\nu} = \sum_{\substack{p<p' \\ n<n' \\ p<n}} (\Gamma)\left\langle 0_f^+ \left\| \left[\left(a_p^+ a_{p'}^+\right)^J \left(\tilde{a}_{n'}\tilde{a}_n\right)^J\right]^0 \right\| 0_i^+ \right\rangle \left\langle p\,p';J \left| \int q^2 dq \left[\hat{S}\,\frac{h(q)\,j_\kappa(qr)\,G_{FS}^2 f_{SRC}^2}{q(q+<E>)}\,\tau_{1-}\tau_{2-}\right] \right| n\,n';J \right\rangle_{as} - closure$$

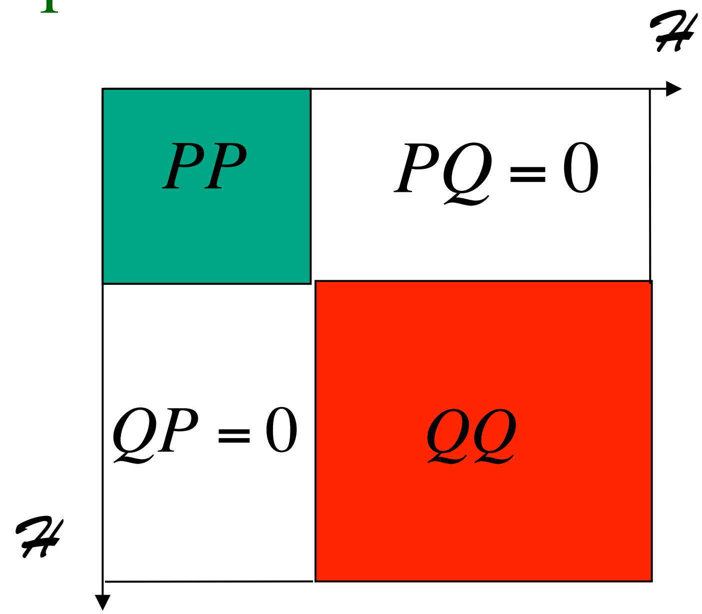
- Old and new short range correlations included
- **No quenching**
- New technique to calculate the many body part of M

$0^+ T = 4$ (^{48}Ca) $\longrightarrow$ $0^+ T = 4$

$$\left\langle 0_f^+ T = 2 \left\| \left[\left(a_i^+ a_j^+\right)^J \left(\tilde{a}_k \tilde{a}_m\right)^J\right]^0 \right\| 0_i^+ T = 4 \right\rangle_{Ti}$$

$0^+ T = 2$ (^{48}Ti)

Effective Hamiltonians for Large N $\hbar\omega$ Excitation Model Spaces



Renormalization methods:

- G-matrix: Physics Reports 261, 125 (1995)
- Lee-Suzuki (NCSM): PRC 61, 044001 (2000)
- $V_{\text{low } k}$: PRC 65, 051301(R) (2002)
- Unitary Correlation Operator: PRC 72, 034002 (2004)
- Similarity Renormalization Group (SRG): PRL 103, 082501 (2009)

$$H = T + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots$$

$$\Psi_{\mathcal{H}} \rightarrow \Psi_P = P\Psi_{\mathcal{H}}$$

"Bare" Nucleon-Nucleon Potentials:

- Argonne V18: PRC 56, 1720 (1997)
- CD-Bonn 2000: PRC 63, 024001 (2000)
- N^3LO: PRC 68, 041001 (2003)
- INOY: PRC 69, 054001 (2004)

$$\mathcal{H} = U\,H U^{+} = \mathcal{H}_2 + \mathcal{H}_3 + \mathcal{H}_4 + \ldots$$

$$O \rightarrow U\,O\,U^{+}$$

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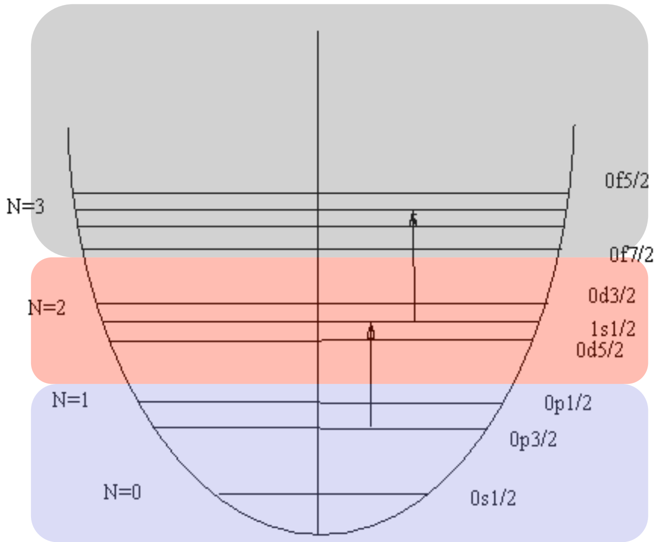
Shell Model Effective Hamiltonians

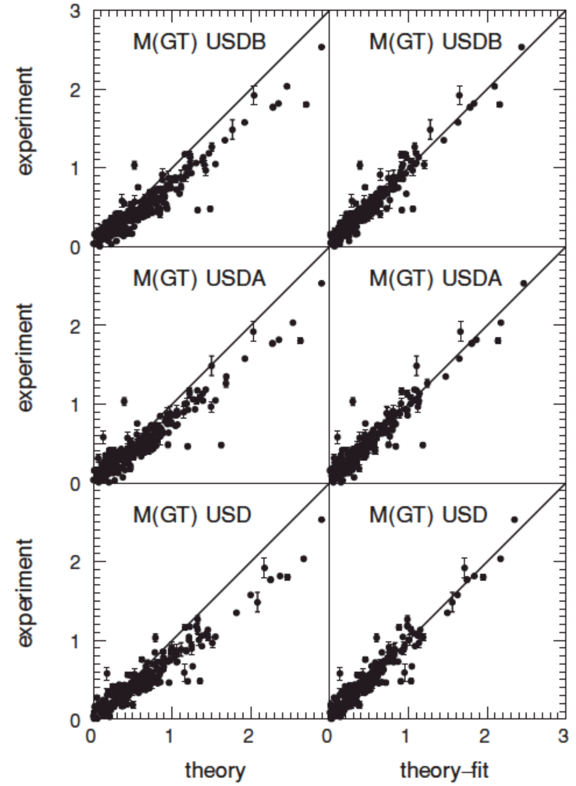
$$\sigma\tau \xrightarrow{quenched} 0.77\sigma\tau$$



$$H_{valence}\Psi = E_n\Psi$$

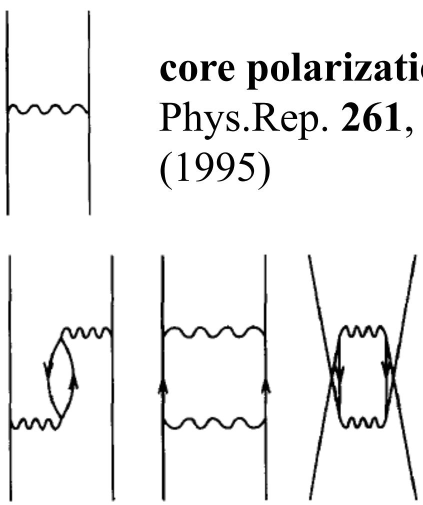
$$H_{valence} = H_{2-body}$$

can describe most correlations around the Fermi surface!

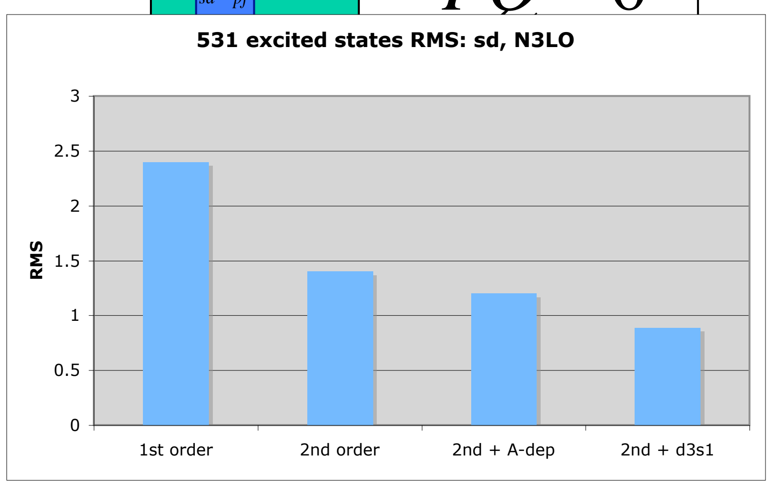


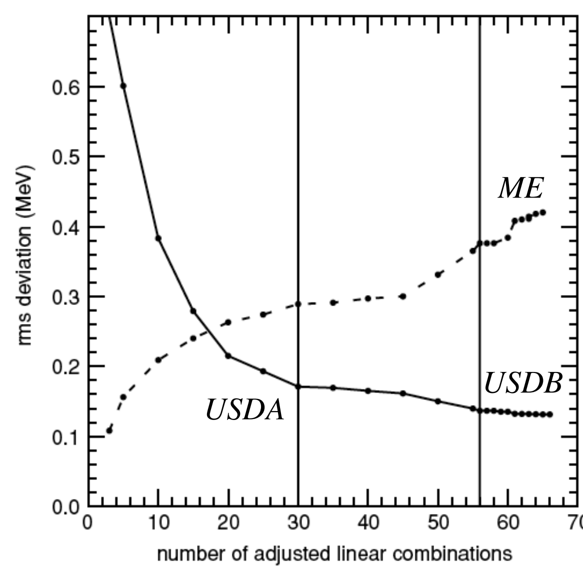
$\mathcal{H}$

PRC 74, 34315 (2006), 78, 064302 (2008)



core polarization:
Phys.Rep. **261**, 125 (1995)





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Beyond Closure in Shell Model

$$M_S^{0\nu} = \sum_{\substack{p<p' \\ n<n' \\ p<n}} (\Gamma) \left\langle 0_f^+ \middle\| \left[\left(a_p^+ a_{p'}^+\right)^J \left(\tilde{a}_{n'} \tilde{a}_n\right)^J \right]^0 \middle\| 0_i^+ \right\rangle \left\langle p\, p'; J \middle| \int q^2 dq \left[\hat{S} \frac{h(q)\, j_\kappa(qr) G_{FS}^2 f_{SRC}^2}{q\left(q + \langle E \rangle\right)} \tau_{1-} \tau_{2-} \right] \middle| n\, n'; J \right\rangle_{as} - closure$$

$$M_S^{0\nu} = \sum_{\substack{pp'nn' \\ J\, k\, \mathcal{J}}} (\tilde{\Gamma}) \left\langle 0_f^+ \middle\| \left(a_p^+ \tilde{a}_n\right)^J \middle\| J_k \right\rangle \left\langle J_k \middle\| \left(a_{p'}^+ \tilde{a}_{n'}\right)^J \middle\| 0_i^+ \right\rangle \left\langle p\, p'; \mathcal{J} \middle| \int q^2 dq \left[\hat{S} \frac{h(q)\, j_\kappa(qr) G_{FS}^2 f_{SRC}^2}{q\left(q + E_k^J\right)} \tau_{1-} \tau_{2-} \right] \middle| n\, n'; \mathcal{J} \right\rangle - exact$$

Challenge: there are about 100,000 J_k states in the sum for 48Ca

Much more intermediate states for heavier nuclei, such as ^{76}Ge!!!

No-closure may need states out of the model space (not considered).

Minimal model spaces

^{82}Se : 6,146,681

^{130}Te : 22,437,983

^{76}Ge : 89,472,767

Volume 242, number 1 PHYSICS LETTERS B

NEUTRINOLESS DOUBLE BETA DECAY MATRIX ELEMENTS BEYOND CLOSURE APPROXIMATION

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K. Muto, NPA (1994) QRPA

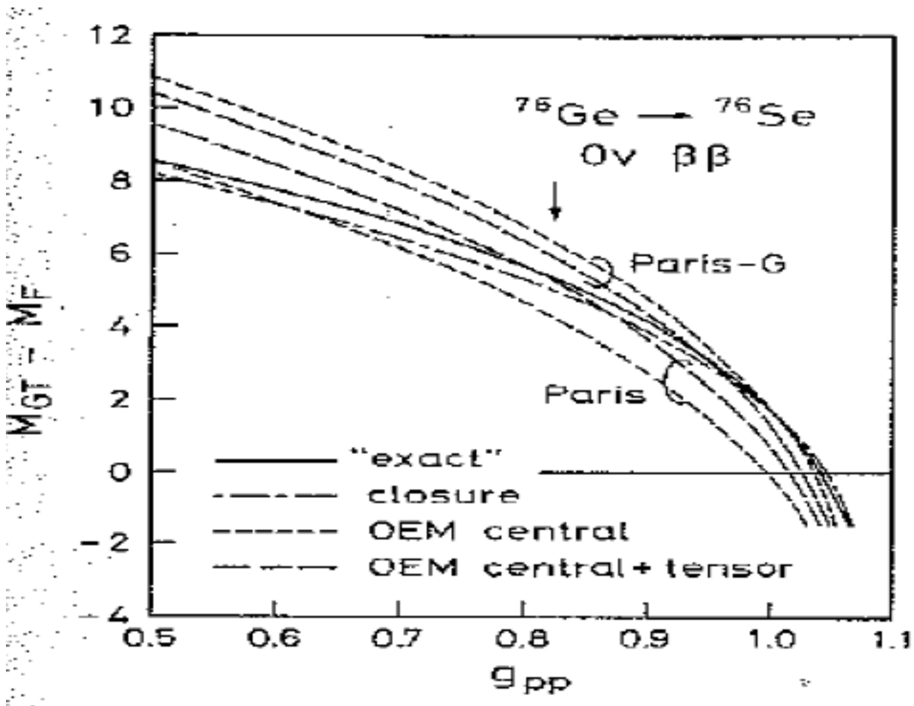


Table 1
The matrix elements of all the operators discussed in the text which appear in the $0\nu\,\beta\beta$-decay of $^{48}Ca\rightarrow$
an average excitation energy of 2.5 MeV and ED corresponds to the exact energy dependent treatment
nucleon form factor, (B) correlation and nucleon form factor, (C) correlation without nucleon form fa
The decay to the 0_1^+ is experimentally interesting. 0_2^+ and 0_3^+ were included for comparison only.

Shell model: only $f_{7/2}$

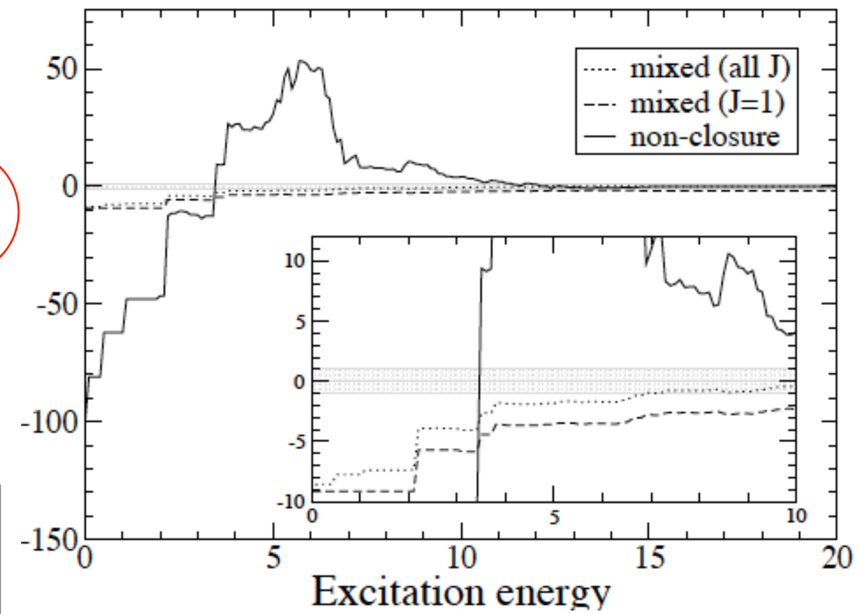
Final state	Case		M_F	M_{GT}	M'_F	M'_{GT}	M'_T	$M_{F\omega}$	$M_{GT\omega}$
	A	CL	0.2906	−1.216	0.2906	−1.216	0.3441	0.2906	−1.2160
		CL(E)	0.2864	−1.110	0.2905	−1.1860	0.3283	0.2757	−1.0441
		ED	0.2873	−1.125	0.2908	−1.1880	0.3348	0.2837	−1.0712
	B	CL	0.1841	−0.7308	0.0871	−0.5324	0.3299	0.1541	−0.7309
		CL(E)	0.1508	−0.6550	0.0896	−0.5170	0.3156	0.1473	−0.6101
0^+		ED	0.1447	−0.6983	0.0808	−0.5624	0.3162	0.1433	−0.6516
	C	CL	0.1703	−0.8588	0.1654	−0.8412	0.3458	0.1703	−0.8558
		ED	0.1688	−0.7700	0.1660	−0.8151	0.3364	0.1671	−0.7218
	D	CL	0.2100	−0.9861	0.0989	−0.6349	0.3224	0.2137	−0.9861
0_1^+	B	ED	0.2971	−1.3323	0.2457	−1.4110	0.2800	0.2906	−1.1847
0_2^+		ED	0.2826	−4.1478	0.2169	−4.7930	0.3233	0.2772	−3.5370
0_3^+		ED	7.3790	−1.7124	8.9921	−1.7470	0.3935	6.3653	−1.5186

Beyond Closure in Shell Model

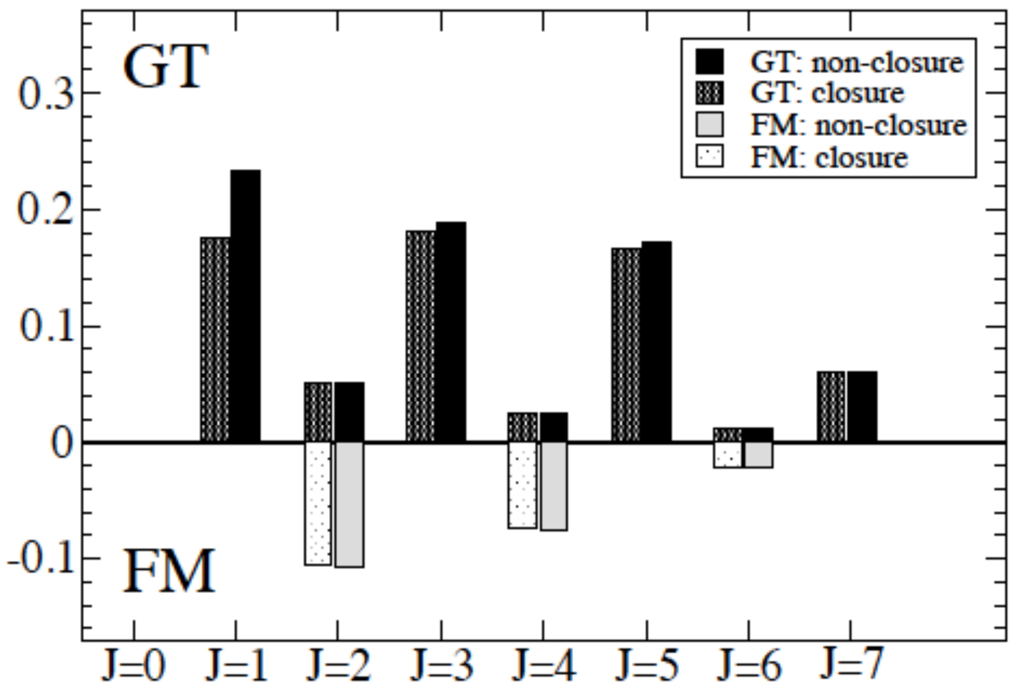
$$M_S^{0\nu} = \sum_{\substack{pp'nn' \\ Jk\mathcal{J}}} (\Gamma) \left\langle 0_f^+ \left\| \left(a_p^+ \tilde{a}_n\right)^J \right\| J_k \right\rangle \left\langle J_k \left\| \left(a_p^+ \tilde{a}_n\right)^J \right\| 0_i^+ \right\rangle \times$$

$$\left\langle p\, p'; \mathcal{J} \left| \int q^2 dq \left[\hat{S} \frac{h(q) j_\kappa(qr) G_{FS}^2 f_{SRC}^2}{q\left(q + E_k^J\right)} \tau_{1-} \tau_{2-} \right] \right| n\, n'; \mathcal{J} \right\rangle$$

^{48}Ca, GXPF1A, 100%×δM/M



Challenge: there are about 100,000 J_k states in the sum for 48Ca !!!

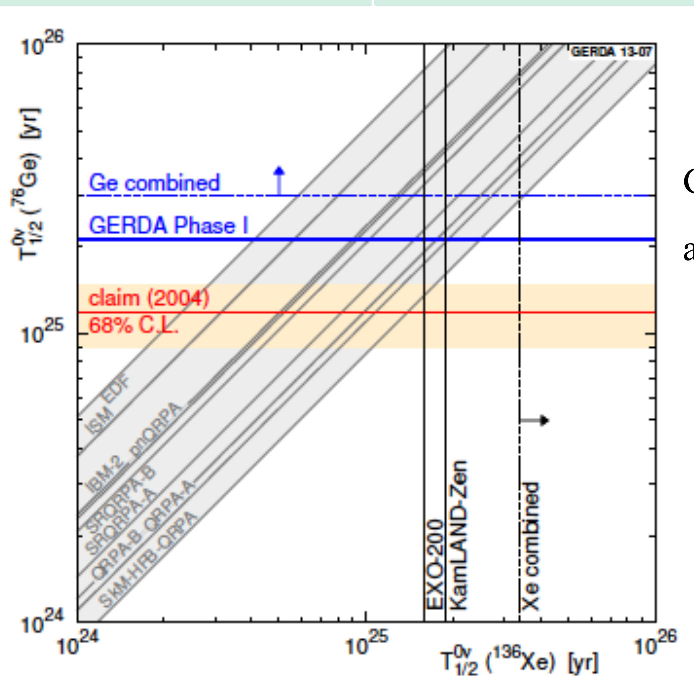


- About 300 intermediate states for each spin are (more than) enough
- GT dominates and experience the largest change
- A 8-12% increase from closure was found

^{136}Xe ββ Experimental Results

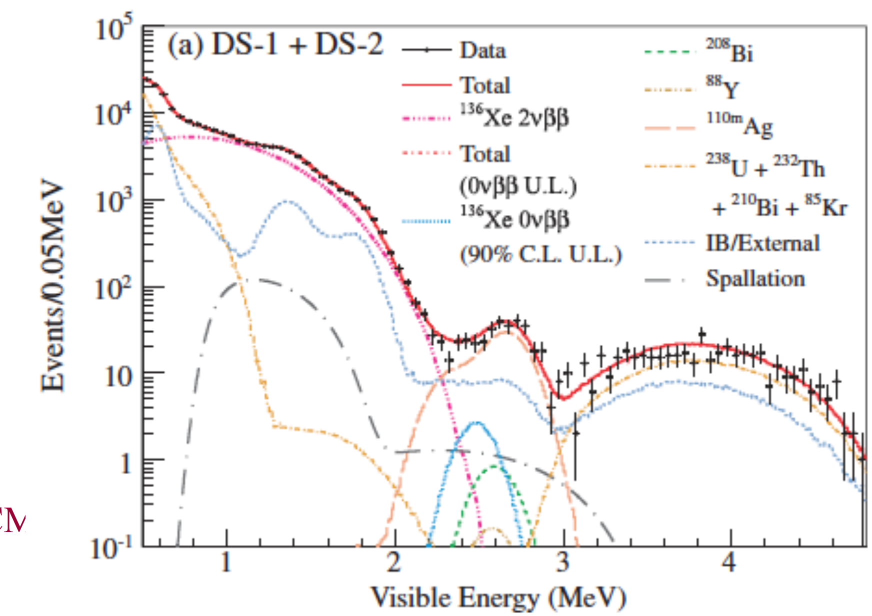
$M^{2\nu}_{exp} = 0.019\ MeV^{-1}$

Publication	Experiment	$T^{2\nu}_{1/2}$	$T^{0\nu}_{1/2}$	$T^{0\nu}_{1/2}$(Maj)
PRL 110, 062502	KamLAND-Zen		> 1.9×10^{25} y	
PRL 107, 212501	EXO-200	$(2.11 \pm 0.04 \pm 0.21) \times 10^{21}$ y		
PRL 109, 032505	EXO-200	$(2.23 \pm 0.017 \pm 0.22) \times 10^{21}$ y	> 1.6×10^{25} y	
PRC 85, 045504	KamLAND-Zen	$(2.38 \pm 0.02 \pm 0.14) \times 10^{21}$ y	> 5.7×10^{24} y	
PRC 86, 021601	KamLAND-Zen		> 6.2×10^{24} y	> 2.6×10^{24} y



GERDA

arXiv:1307.4720



Other Shell Model Results

Shell Model description of the $\beta\beta$ decay of ^{136}Xe

E. Caurier[a], F. Nowacki[a], A. Poves[b,*]

Physics Letters B 711 (2012) 62–64

$$\sigma\tau \xrightarrow{quenched} q\sigma\tau$$

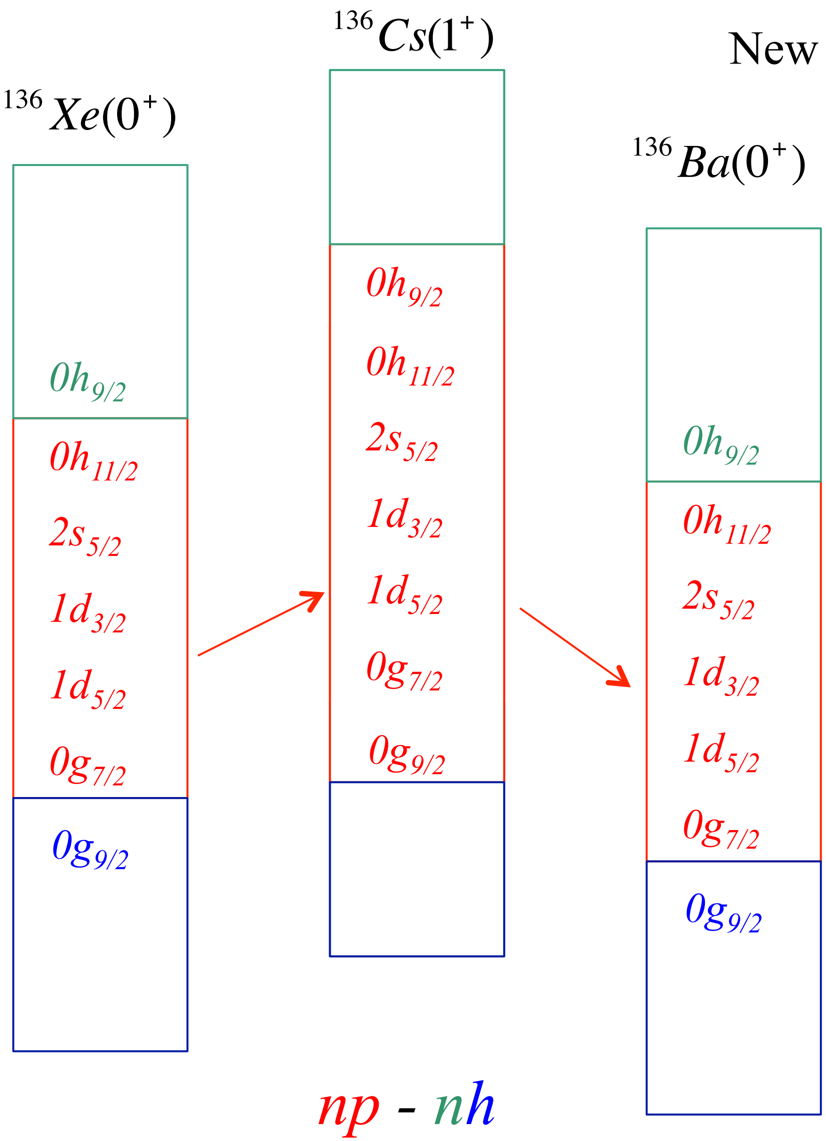
Table 2
The ISM predictions for the matrix element of several 2ν double beta decays (in MeV^{-1}). See text for the definitions of the valence spaces and interactions.

	$M^{2\nu}$(exp)	q	$M^{2\nu}$(th)	INT
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.047	kb3
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.048	kb3g
$^{48}Ca \rightarrow {}^{48}Ti$	0.047 ± 0.003	0.74	0.065	gxpf1
$^{76}Ge \rightarrow {}^{76}Se$	0.140 ± 0.005	0.60	0.116	gcn28:50
$^{76}Ge \rightarrow {}^{76}Se$	0.140 ± 0.005	0.60	0.120	jun45
$^{82}Se \rightarrow {}^{82}Kr$	0.098 ± 0.004	0.60	0.126	gcn28:50
$^{82}Se \rightarrow {}^{82}Kr$	0.098 ± 0.004	0.60	0.124	jun45
$^{128}Te \rightarrow {}^{128}Xe$	0.049 ± 0.006	0.57	0.059	gcn50:82
$^{130}Te \rightarrow {}^{130}Xe$	0.034 ± 0.003	0.57	0.043	gcn50:82
$^{136}Xe \rightarrow {}^{136}Ba$	0.019 ± 0.002	0.45	0.025	gcn50:82

$$M^{2\nu}_{GT}(0^+) = \sum_k \frac{\langle 0_f \| \sigma\tau^- \| 1_k^+ \rangle \langle 1_k^+ \| \sigma\tau^- \| 0_i \rangle}{E_k + E_0}$$

$0g_{7/2}\ 1d_{5/2}\ 1d_{3/2}\ 2s_{5/2}\ 0h_{11/2}$ valence space

^{136}Xe 2νββ Results $M^{2\nu}_{exp} = 0.019\ MeV^{-1}$



New effective interaction, $\sigma\tau \rightarrow 0.74\,\sigma\tau$ quenching

$0g_{7/2}\ 1d_{5/2}\ 1d_{3/2}\ 2s_{5/2}\ 0h_{11/2}$ *model space*

$$\sum B(GT;Z \rightarrow Z+1) - \sum B(GT;Z \rightarrow Z-1) = 52$$

$$Ikeda: \quad 3(N-Z) = 84$$

$$M^{2\nu} = 0.064\ MeV^{-1}$$

$0g_{9/2}\ 0g_{7/2} 1d_{5/2}\ 1d_{3/2}\ 2s_{5/2}\ 0h_{11/2}\ 0h_{9/2}$

$$\sum B(GT;Z \rightarrow Z+1) - \sum B(GT;Z \rightarrow Z-1) = 84$$

$$Ikeda: \quad 3(N-Z) = 84$$

n (0+)	n (1+)	M(2v)
0	0	0.062
0	1	0.091
1	1	0.037
1	2	0.020

^{136}Xe 0νββ Results

M. Horoi and B.A. Brown, Phys. Rev. Lett. **110**, 222502 (2013)

TABLE II. Matrix elements for 0ν decay using two SRC models [13], CD-Bonn (SRC1), and Argonne (SRC2). The upper values of the neutrino physics parameters η_j^{up} in units of 10^{-7} are calculated using the $G^{0\nu}$ from Refs. [9,35].

		$M_\nu^{0\nu}$	$M_N^{0\nu}$	$M_{\lambda'}^{0\nu}$	$M_{\tilde{q}}^{0\nu}$
$n = 0$	SRC1	2.21	143.0	1106	206.8
	SRC2	2.06	98.79	849.0	197.2
$n = 1$	SRC1	1.46	128.0	1007	157.8
	$\lvert\eta_j^{up}\rvert$ [9]	8.19	0.093	0.012	0.075
	$\lvert\eta_j^{up}\rvert$ [35]	9.02	0.103	0.013	0.083

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}|\eta_{\nu L} M_\nu^{0\nu} + \eta_N M_N^{0\nu} + \eta_{\lambda'} M_{\lambda'}^{0\nu} + \eta_{\tilde{q}} M_{\tilde{q}}^{0\nu}|^2,$$

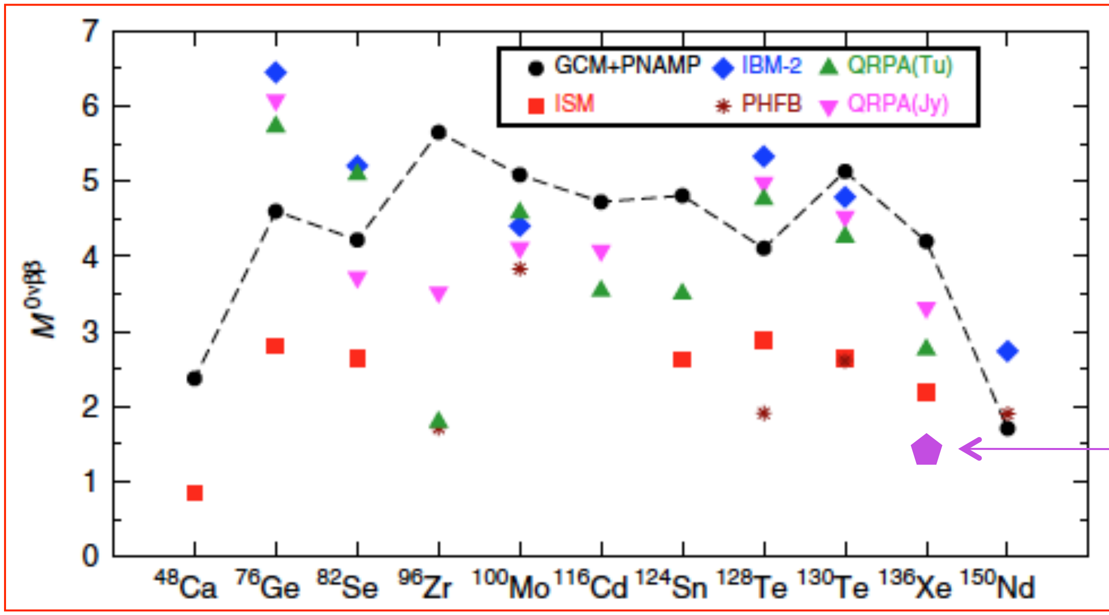
$$^{134}Te \rightarrow {}^{134}Xe$$

n(0+)	n(1+)	M(2v)	M(0v)
0	0	0.045	1.8
0	1	0.071	
1	1	0.20	1.2
1	2	0.012	
2	2	0.016	1.5

$$\eta_{NR} = \left(\frac{M_{WL}}{M_{WR}}\right)^4 \sum_k^{heavy} V_{ek}^2 \frac{m_p}{M_k}$$

$$\approx \left(\frac{80}{2400}\right)^4 \frac{1}{100} = 10^{-8}$$

Comparisons of $M^{0\nu}$ 0νββ Results



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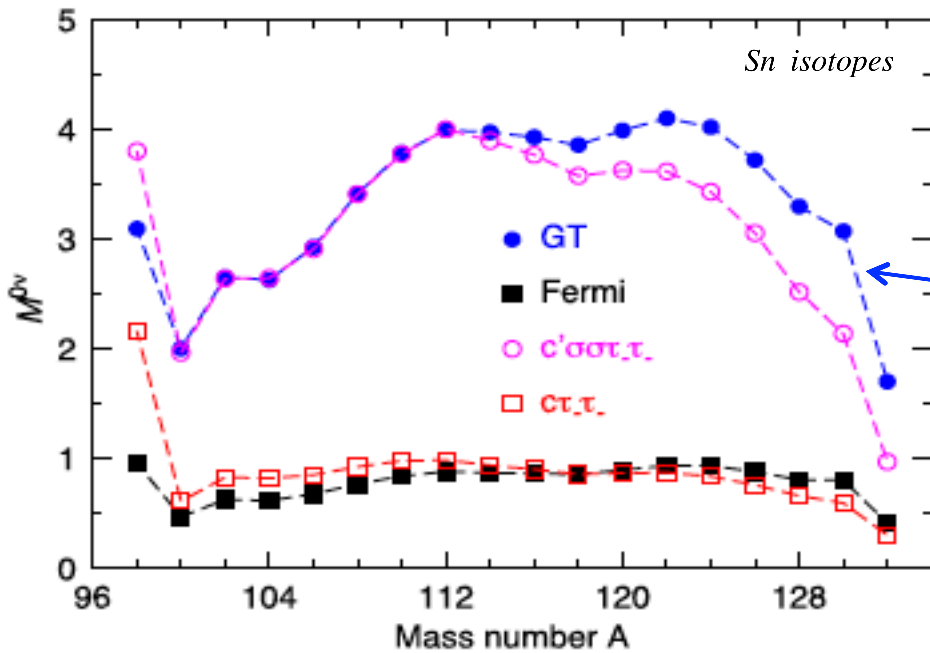

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Large jump down for magic no of neutrons !!!

MU

Summary and Outlook

- Observation of neutrinoless double beta decay would signal physics beyond the Standard Model: **massive Majorana neutrinos, right-handed currents, SUSY LNV, etc**
- ^{48}Ca case suggests that 2ν double-beta decay can be described reasonably within the shell model with standard quenching, provided that all spin-orbit partners are included.
- Higher order effects for 0ν NME included: range 0.6 − 1.4
- Reliable 0νββ nuclear matrix elements could be used to identify the dominant mechanism if energy/angular correlations and data for several isotopes become available.
- The effects of the quenching and the missing spin-orbit partners are important (see the ^{136}Xe case), and they need to be further investigated for ^{76}Ge, ^{82}Se and ^{130}Te.