Nuclear Matrix Elements for Tensor Interactions that Violate Local Lorentz Invariance

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New Test of Local Lorentz Invariance Using a ^{21}Ne-Rb-K Comagnetometer

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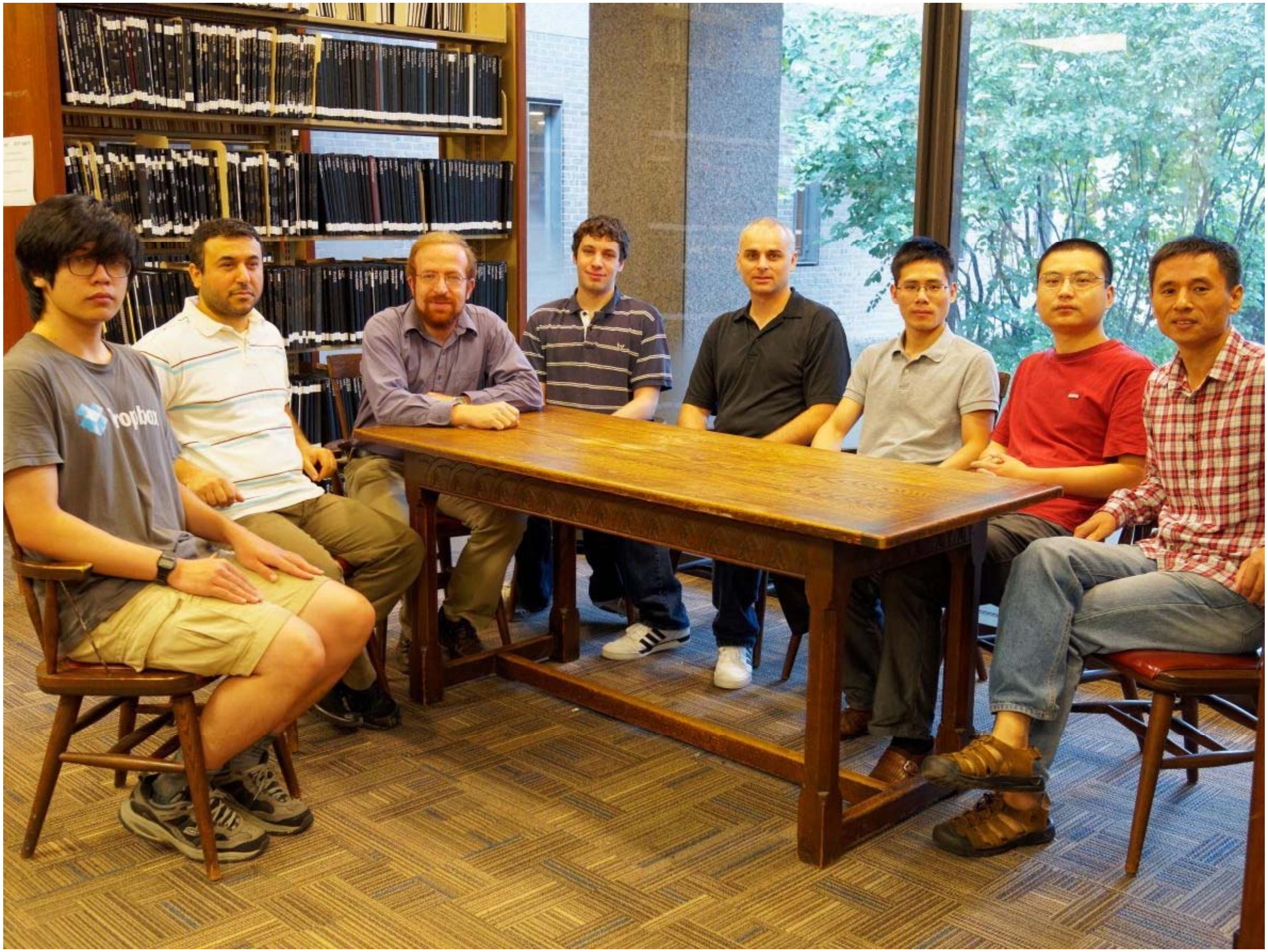
We develop a new comagnetometer using ^{21}Ne atoms with nuclear spin $I = 3/2$ and Rb atoms polarized by spin exchange with K atoms to search for tensor interactions that violate local Lorentz invariance. We frequently reverse the orientation of the experiment and search for signals at the first and second harmonics of the sidereal frequency. We constrain 4 of the 5 spatial Lorentz-violating coefficients c^n_{jk} that parametrize anisotropy of the maximum attainable velocity of a neutron at a level of 10^{-29}, improving previous limits by 2 to 4 orders of magnitude and placing the most stringent constraint on deviations from local Lorentz invariance.


ropbox

The Michelson-Morley experiment and its successors have established that the speed of light is isotropic to a part in 10^{17} [1,2]. Similarly, possible anisotropy in the maximum attainable velocity (MAV) for a massive particle [3] has been constrained by Hughes and Drever NMR experiments [4,5] and their successors to a part in 10^{27} [6]. These experiments form the basis for the principle of local Lorentz invariance (LLI). Together with the weak equivalence principle and the position invariance principle, they constitute the Einstein equivalence principle (EEP) that is the basis of general relativity [7].

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Here we describe a new comagnetometer that is sensitive to anisotropy in neutron MAV at 10^{-29} level. The idea of the experiment is based on the K-^{3}He comagnetometer, previously used to constrain Lorentz-violating vector spin interactions [18]. The ^{3}He ($I = 1/2$) is replaced by ^{21}Ne ($I = 3/2$) to allow measurements of tensor anisotropy. In addition, since the gyromagnetic ratio of ^{21}Ne is about an order of magnitude smaller than that of ^{3}He, the comagnetometer has an order of magnitude better energy resolution for the same level of magnetic field sensitivity. The electric quadrupole interactions of ^{21}Ne cause several difficulties.

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CPT-even Lorentz violation

$$\mathcal{L} = -\bar{\psi}(m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu)\psi + \frac{i}{2}\bar{\psi}(\gamma_\nu + c_{\mu\nu}\gamma^\mu + d_{\mu\nu}\gamma_5\gamma^\mu)\overleftrightarrow{\partial}^\nu \psi$$

a,*b* - *CPT-odd*
c,*d* - *CPT-even*

- Maximum attainable particle velocity

Coleman and Glashow
Jacobson

$$v_{MAX} = c(1 - c_{00} - c_{0j}\hat{v}_j - c_{jk}\hat{v}_j\hat{v}_k)$$

  - Implications for ultra-high energy cosmic rays, Cherenkov radiation, etc
  - Best limit $c_{00} \sim 10^{-23}$ from Auger ultra-high energy cosmic rays
  - Many laboratory limits (optical cavities, cold atoms, etc)
- Motivation for Lorentz violation (without breaking CPT)
  - Doubly-special relativity
  - Horava-Lifshitz gravity

Something special needs to happen when particle momentum reaches Plank scale!

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Search for CPT-even Lorentz violation with nuclear spin

- Need nuclei with orbital angular momentum and total spin >1/2
- Quadrupole energy shift proportional to the kinetic energy of the valence nucleon

$$E_Q \sim (c_{11} + c_{22} - 2c_{33})\left\langle p_x^2 + p_y^2 - 2p_z^2 \right\rangle$$

- Previosly has been searched for in two experiments using ^{201}Hg and ^{21}Ne with sensitivity of about 0.5 μHz
- Bounds on neutron $c_n \sim 10^{-27}$ – already most stringent bound on c coefficient!

Sidereal Variation | Semi-sidereal Variation

$$\Delta E(t) = E_0 + E_{1X}\cos\Omega t + E_{1Y}\sin\Omega t + E_{2X}\cos 2\Omega t + E_{2Y}\sin 2\Omega t$$

$$c_{\mu\nu} = \begin{pmatrix} c_{TT} & c_{TX} & c_{TY} & c_{TZ} \\ c_{XT} & c_{XX} & c_{XY} & c_{XZ} \\ c_{YT} & c_{YX} & c_{YY} & c_{YZ} \\ c_{ZT} & c_{ZX} & c_{ZY} & c_{ZZ} \end{pmatrix}$$

} 2nd Harmonic

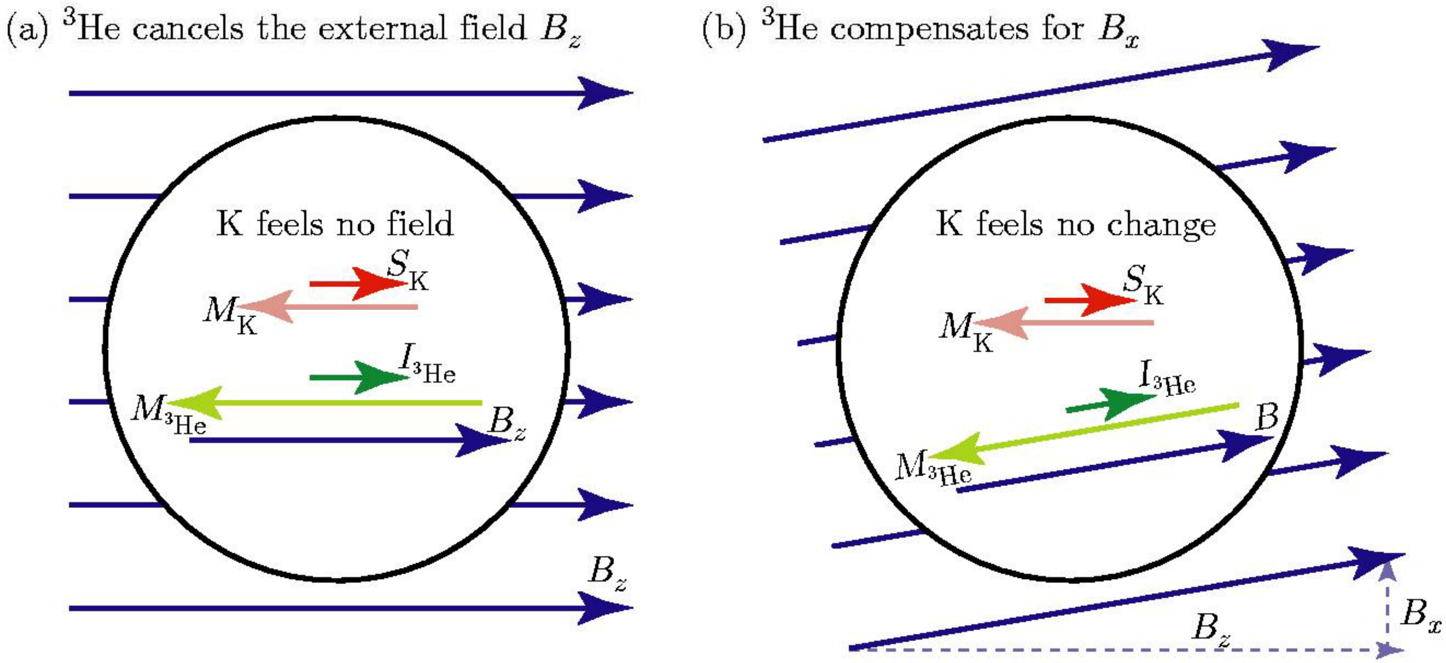
} 1st Harmonic

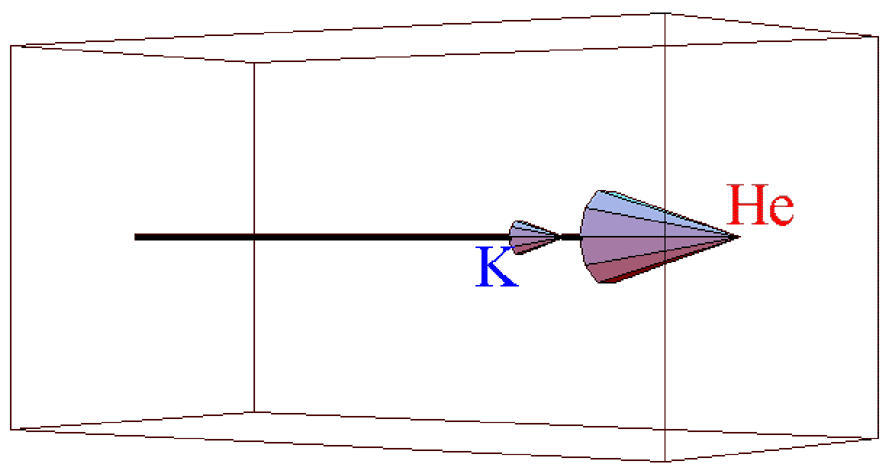
Suppressed by v_{Earth}

NSCL

Comagnetometer

Magnetic field self-compensation





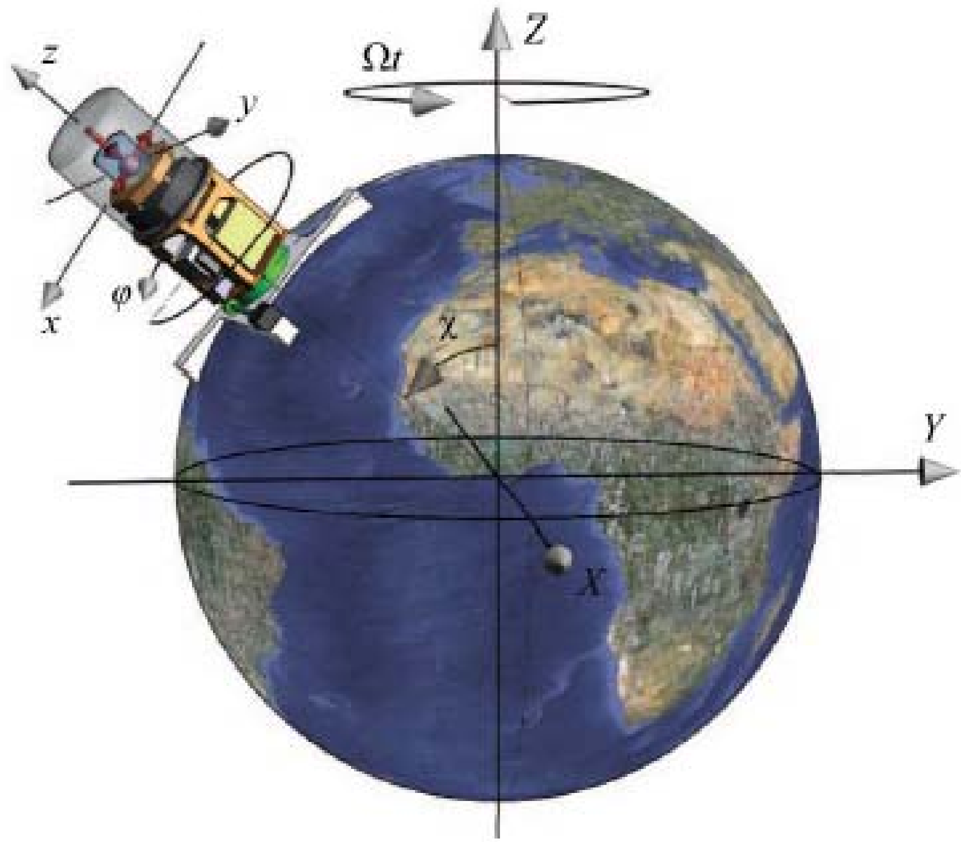


FIG. 1 (color online). The experimental apparatus is rotated around the local vertical. ^{21}Ne spins are polarized down along $-\hat{z}$ and the probe beam is directed horizontally along $-\hat{x}$.

Lots of data!

Signal:
Lock-In X
Lock-In Y
Background Signal
Background Times

Feedback monitors:
Probe Oscillator Intensity Monitor
Probe Wavelength Feedback
Probe Wavelength Fabry-Perot Signal
Probe Tapered Amplifier Current
Probe Intensity
Pump Intensity Feedback
Pump Intensity
Pump Wavelength Feedback
Pump Wavelength Signal
Pre-Oven Heater Power
Oven Heater Power
Shutter Duty Cycle

Motion:
Probe Horizontal Position
Probe Vertical Position
Probe Pre-Cell Horizontal Position
Probe Pre-Cell Vertical Position
Tube Position Z
Table Position X1
Table Position Y1
Table Position Z1
Table Position X2
Table Position Y2
Table Position Y3

Zeroing control:
Zero Pump Wavelength (Lz)
Zero Pockel Cell (Lx)
Pockel Cell Background
Zero Bz Orthogonal
Zero By Orthogonal
Zero Bx Orthogonal
Sensitivity Calibration
Zero Mirror Linear Stage
Polarization Pz

Environmental:
Shield #1 Temperature
Shield #3 Temperature
Shield #5 Temperature
Internal G10 Tube Temperature
Air Vent Temperature
Probe Laser Temperature
Cooling Shield Temperature
Inner Oven Temperature
Input Air Temperature
Outer Oven Temperature
Cell Temperature
Faraday Rotator Return Water Temperature
Barometric Pressure
Humidity

... and more to come!

Search for CPT-even Lorentz violation with nuclear spin

- Need nuclei with orbital angular momentum and total spin >1/2
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- Previosly has been searched for in two experiments using ^{201}Hg and ^{21}Ne with sensitivity of about 0.5 μHz
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Sidereal Variation — Semi-sidereal Variation

$$\Delta E(t) = E_0 + E_{1X}\cos\Omega t + E_{1Y}\sin\Omega t + E_{2X}\cos 2\Omega t + E_{2Y}\sin 2\Omega t$$

$$c_{\mu\nu} = \begin{pmatrix} c_{TT} & c_{TX} & c_{TY} & c_{TZ} \\ c_{XT} & c_{XX} & c_{XY} & c_{XZ} \\ c_{YT} & c_{YX} & c_{YY} & c_{YZ} \\ c_{ZT} & c_{ZX} & c_{ZY} & c_{ZZ} \end{pmatrix}$$

} 2nd Harmonic

} 1st Harmonic

Suppressed by v_{Earth}

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To convert our measurements from magnetic field units, we need an estimate of the nuclear operator $\langle I, I|\mathcal{P}_0^2|I, I\rangle = \langle I, I|2p_z^2 - p_x^2 - p_y^2|I, I\rangle/\sqrt{6}$. Within the Schmidt model, ^{21}Ne has a valence neutron in the $d_{3/2}$ state. This gives $\langle I, I|\mathcal{P}_0^2|I, I\rangle = -\sqrt{2/3}\langle p^2\rangle/5 = -\sqrt{8/3}mE_k/5$, and we take the kinetic energy of the valence nucleon $E_k \sim$ 5 MeV [6]. ^{21}Ne is better described by a collective wave function within the *sd* shell model, and it should be possible to calculate the nuclear operator more precisely [29].

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Matrix elements required for experiments involving nuclear spin coupling

μ_{th} = 1.148 ($d_{3/2}$ model)

μ_{th} = -0.750 (full sd model)

μ_{exp} = -0.662

(S_p, S_n) = (0.022, 0.292)

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Dimensionless harmonic-oscillator potential

$$H = (p^2 + r^2)/2$$

means that the form of the wf in momentum space are the same as the those in coordinate space. But there is a phase factor

$$\Psi(\vec{r}) = i^N \Psi(\vec{p})$$

where

$$N = n_x + n_y + n_z = 2n + \ell$$

The dimensions for $< r^2 >$ are $b^2 = \hbar/\mathrm{m}\omega$ and those for $< p^2 >$ are m $\hbar\omega$.

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Table 1: One-body transition density

a^+	a	simple		sd-shell	
		proton	neutron	proton	neutron
d5	d5	0	0	-0.253	-0.237
d5	d3	0	0	-0.038	-0.133
d5	s1	0	0	-0.270	-0.345
d3	d5	0	0	0.038	0.134
d3	d3	0	1	-0.035	-0.044
d3	s1	0	0	0.104	0.073
s1	d5	0	0	-0.270	-0.345
s1	d3	0	0	-0.104	-0.073

The sd-shell results for ^{21}Ne are

$$(< 2r_z^2 - r_y^2 - r_x^2 >_p, \ < 2r_z^2 - r_y^2 - r_x^2 >_n) = (2.37, 2.81) \ b^2$$

So the momentum results are

$$(< 2p_z^2 - p_y^2 - p_x^2 >_p, \ < 2p_z^2 - p_y^2 - p_x^2 >_n) = (2.37, 2.81) \text{ m } \hbar\omega$$

The simple neutron $d_{3/2}$ model gives

$$(< 2p_z^2 - p_y^2 - p_x^2 >_p, < 2p_z^2 - p_y^2 - p_x^2 >_n) = (0, -1.4) \text{ m } \hbar\omega$$

$(1.4 = 7/5)$.

With $b^2 = 3.40$ fm^2 (from ground-state rms charge radius) the Q moments are

$(Q_p, Q_n) = (5.7, 6.7)$ fm^2

The charge Q moment is 5.7 e^2 fm^2 compared to the experimental value of 10.3(8) e^2 fm^2.

E2 matrix elements for the sd-shell require effective changes of $e_p = 1.0 + 0.37$ and $e_n = 0 + 0.45$ that give

$Q_p = 5.7 + 5.7 \text{ x } 0.37 + 6.7 \text{ x } 0.45 = 10.8$ fm^2

$Q_n = 6.7 + 6.7 \text{ x } 0.45 + 5.7 \text{ x } 0.37 = 11.8$ fm^2

This is attributed to the core-polarization corrections coming from mixing of configurations with $\Delta N = 2$.

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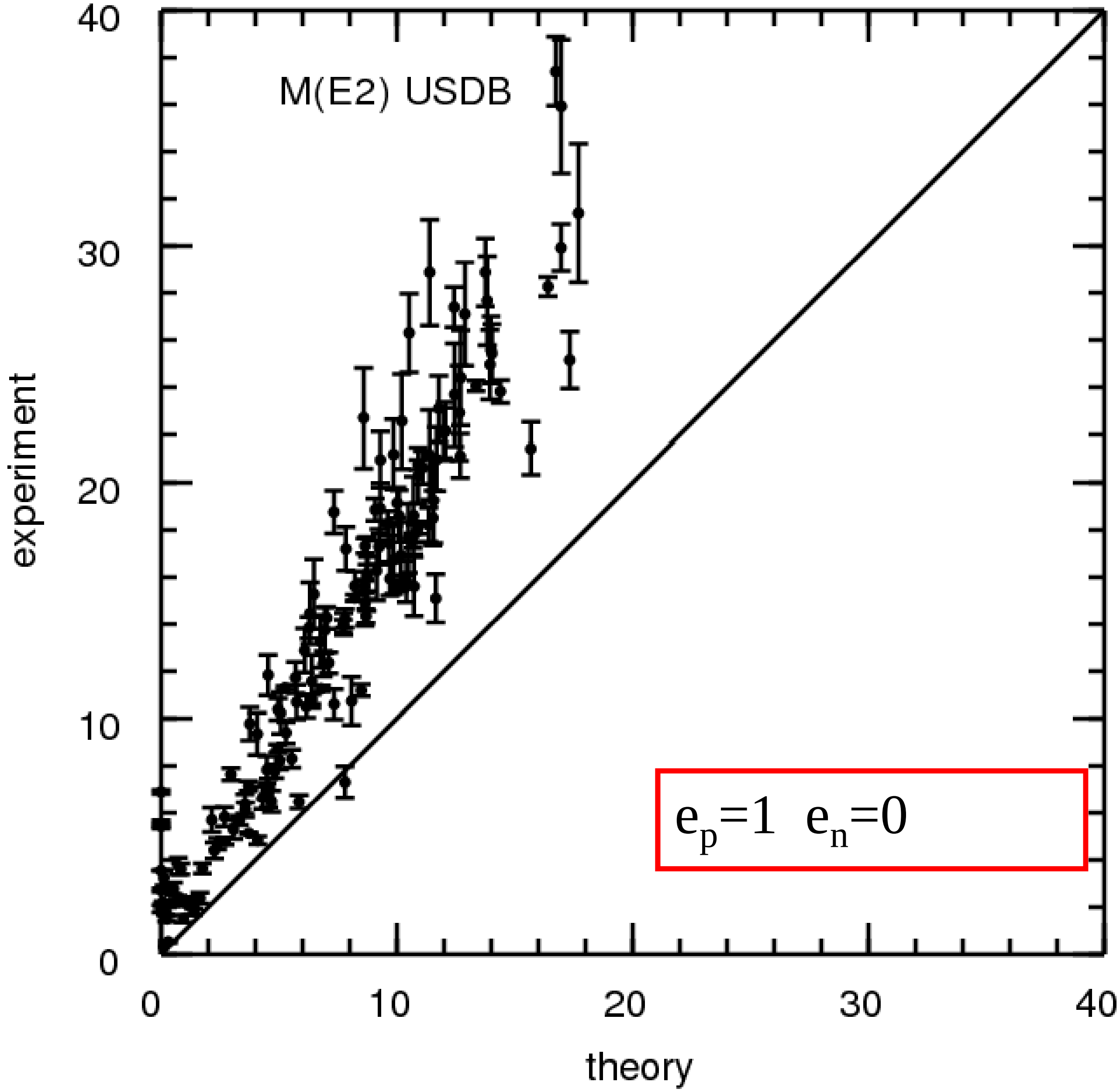
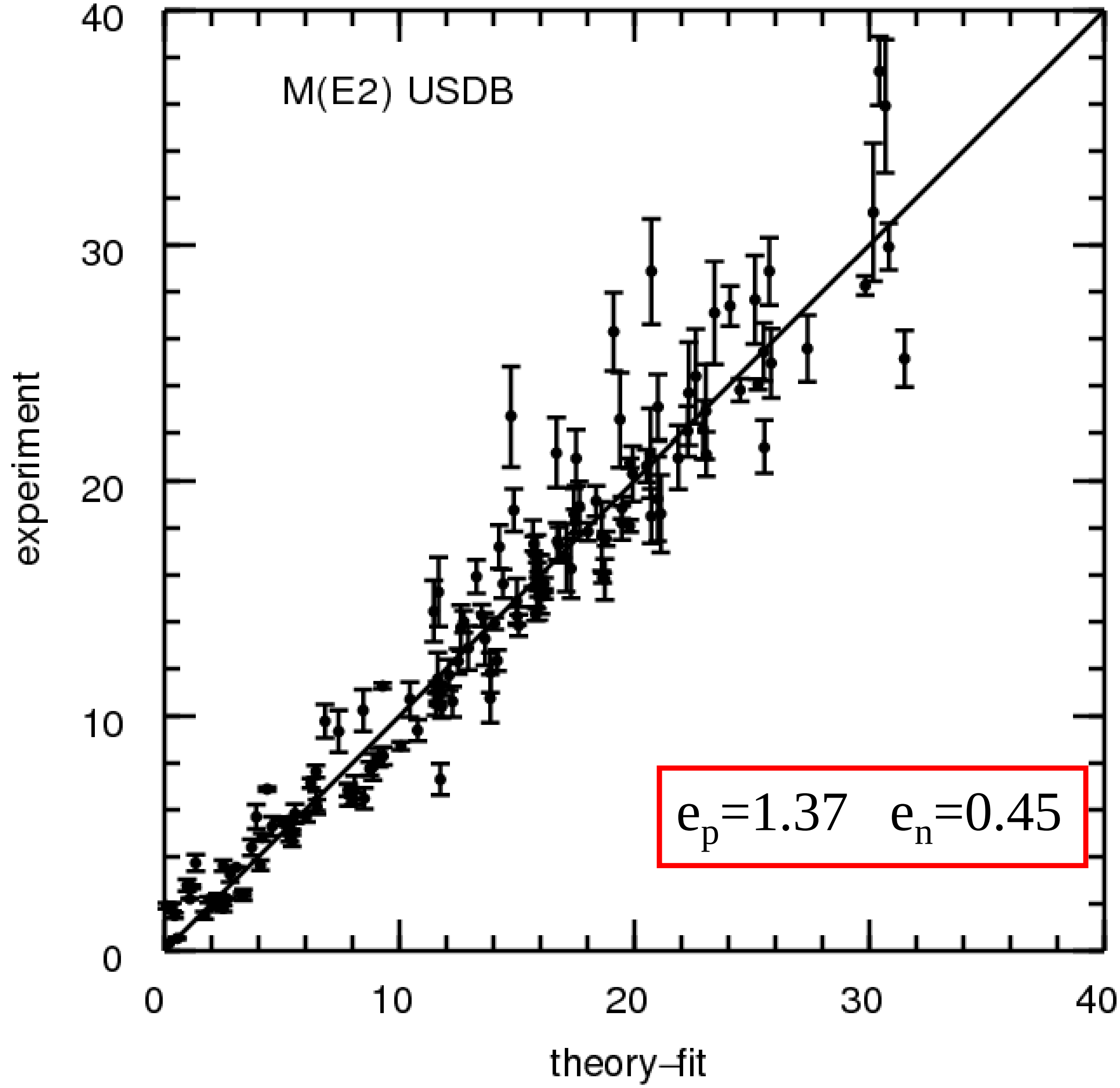
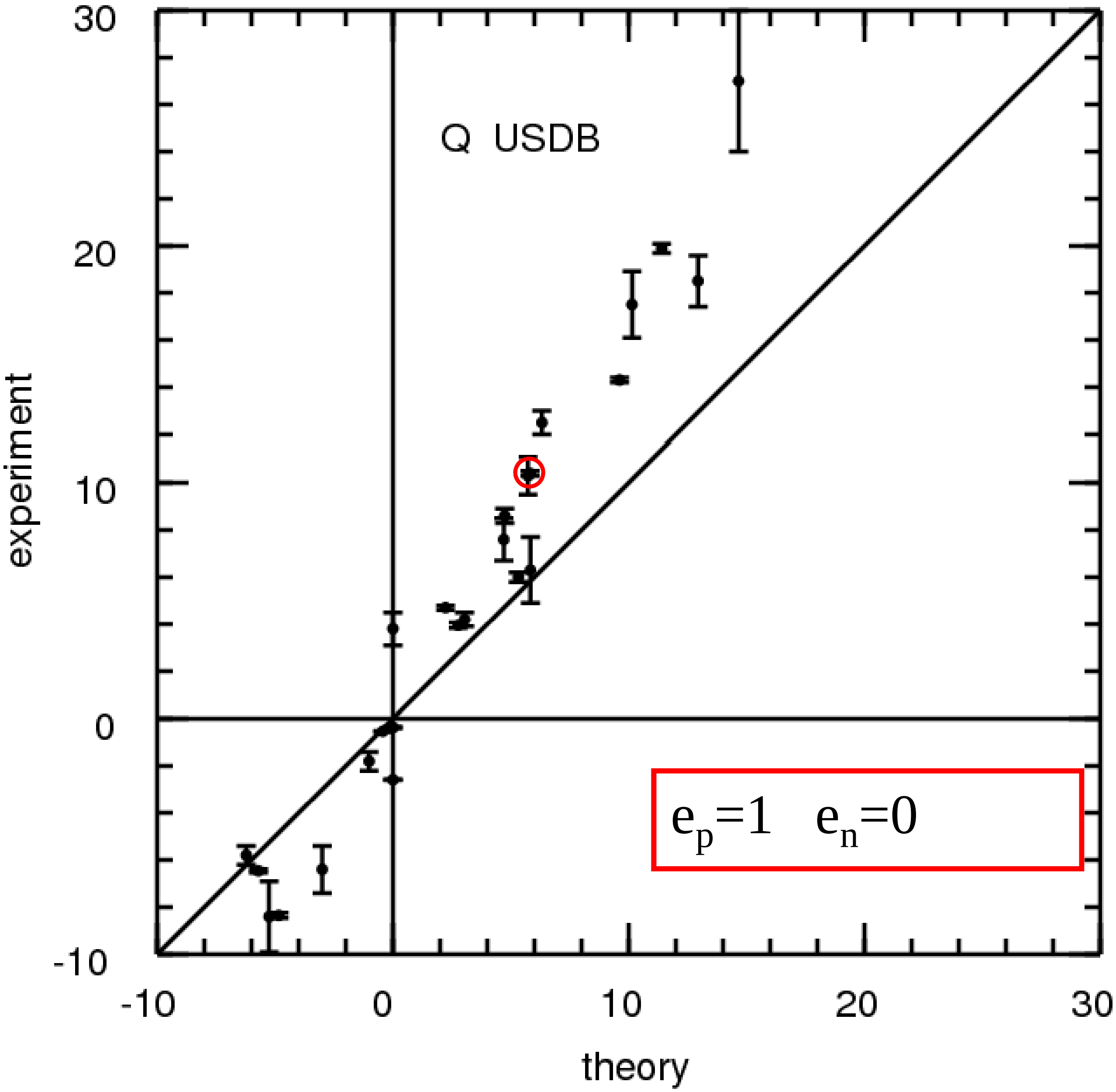
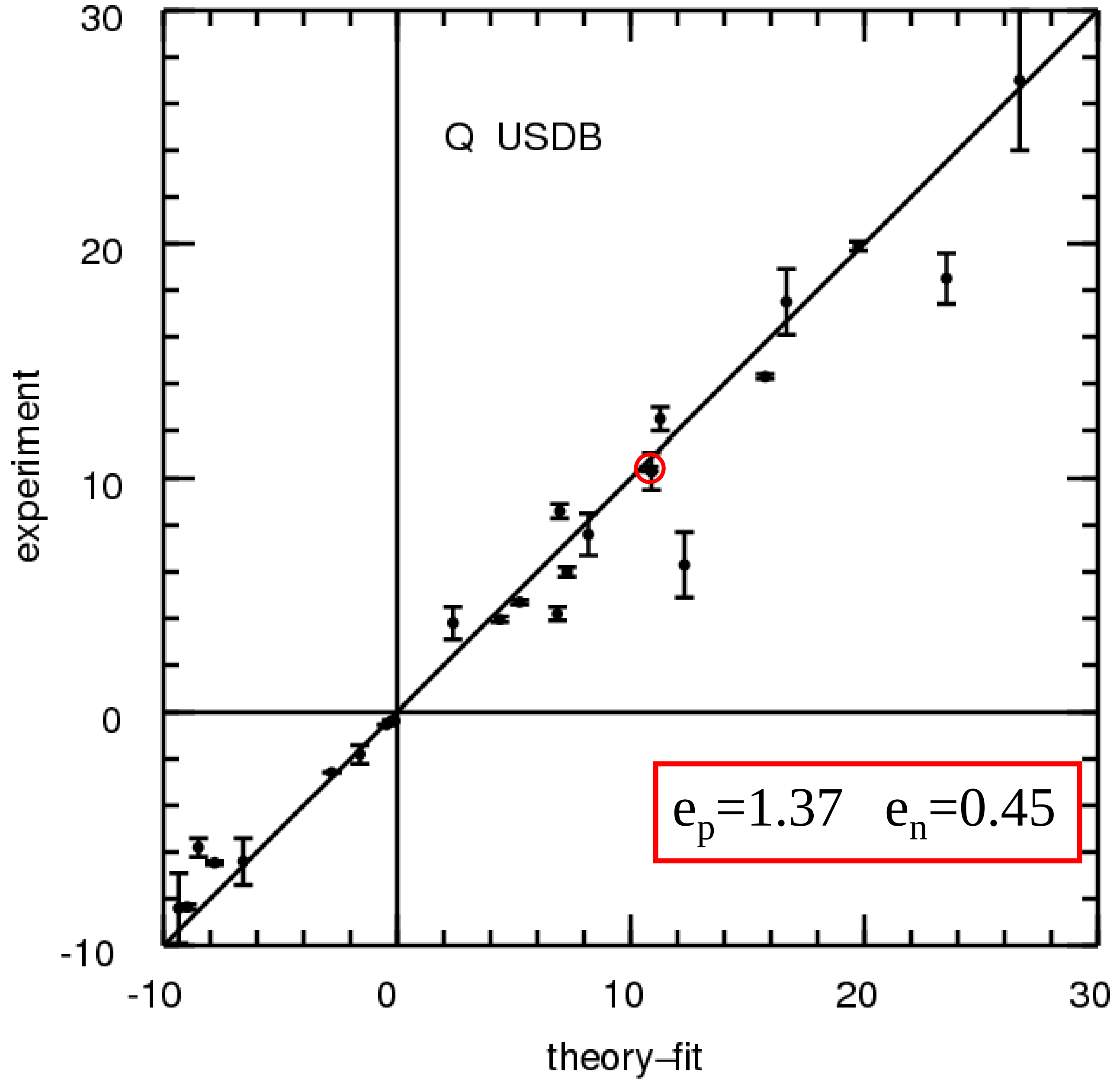

M(E2) USDB
experiment
theory
0
10
20
30
40
e_p=1 e_n=0



M(E2) USDB
experiment
theory–fit
0
10
20
30
40
e_p=1.37 e_n=0.45



Q USDB
experiment
theory
30
20
10
0
-10
-10
0
10
20
30
e_p=1 e_n=0



30
20
10
0
-10
-10
0
10
20
30
experiment
theory–fit
Q USDB
e_p=1.37 e_n=0.45

The i^N factor gives a reduction in the momentum matrix elements by factors of $e'_p = 1.0 - 0.37$ and $e'_n = -0.45$

$$(< 2p_z^2 - p_y^2 - p_x^2 >_p, \ < 2p_z^2 - p_y^2 - p_x^2 >_n) = (2.37, 2.81) \text{ m } \hbar\omega$$

to

$$(< 2p_z^2 - p_y^2 - p_x^2 >_p, \ < 2p_z^2 - p_y^2 - p_x^2 >_n) = (0.22, 0.67) \text{ m } \hbar\omega$$

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Bohr and Mottelson: if the deformation of the density distribution is equal to that of the self-consistent potential then for E2 we have $e_p + e_n = 1 + 1 = 2$ and for the momentum matrix elements $1 - 1 = 0$. That is $< p_z^2 > = < p_x^2 > = < p_y^2 >$.

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Other experiments of this type require these momentum matrix elements for ^{131}Xe (I=3/2) and ^{201}Hg (I=3/2).

We have a good Hamiltonian for the $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})$ (jj55) model space.

Results for ^{131}Xe can be obtained with NuShellX
M=3/2 dimension is 388,283,806
J=3/2 dimension is 14,021,322

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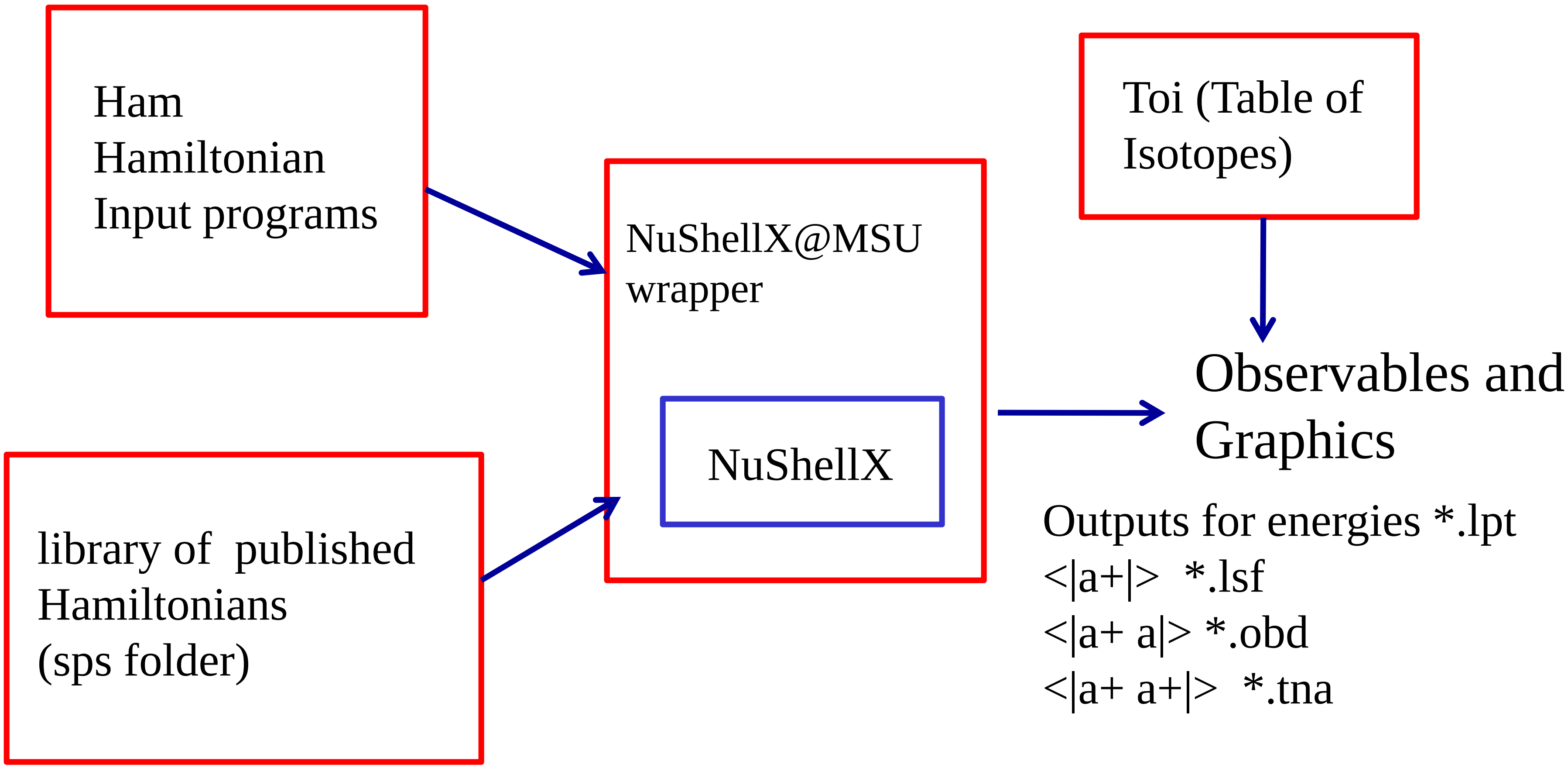


postscript (*.eps) (*.pdf)
figures

*.sp model space files
*.int Hamiltonian files

$$< B_f, J \mid H_{pn} \mid B_i, J >= \sum_{pp'nn'\lambda} F_\lambda(pp'nn')\ \Gamma_\lambda\ \mathrm{RDM}(p_f, p_i, p, p', \lambda)\ \mathrm{RDM}(n_f, n_i, n, n', \lambda)$$

where, for example, p_f, stands for labels (J_{p_f}, α_{p_f}),

$$\Gamma_\lambda = \left\{ \begin{array}{ccc} J_{p_f} & J_{p_i} & \lambda \\ J_{n_f} & J_{n_i} & \lambda \\ J & J & 0 \end{array} \right\}$$

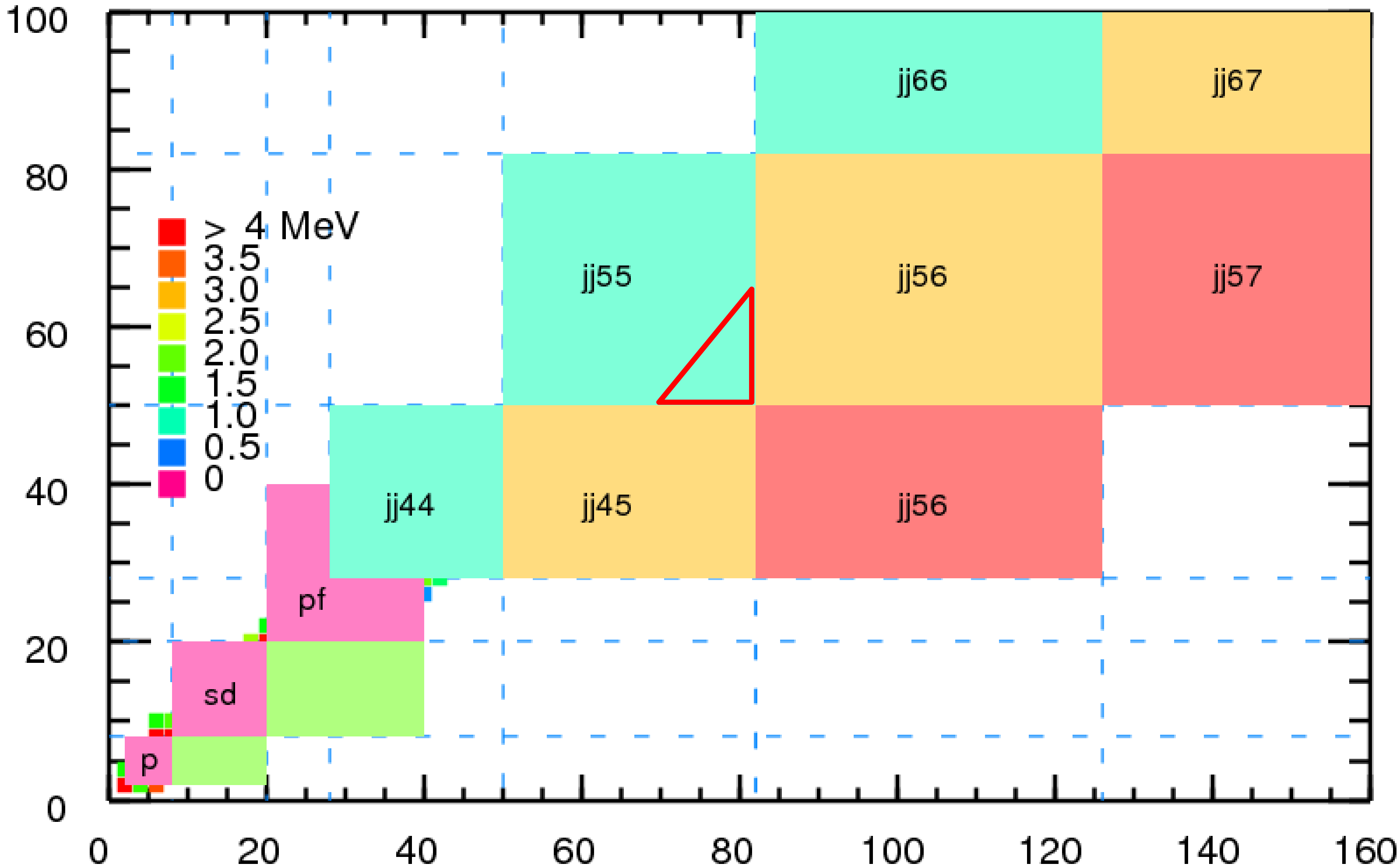
and RDM are the reduced density matricies:

$$\mathrm{RDM}(p_f, p_i, p, p', \lambda) = < [(J_{p_f}, \alpha_{p_f})||[a_p^+ \tilde{a}_{p'}]^\lambda||[(J_{p_i}, \alpha_{p_i}) >$$

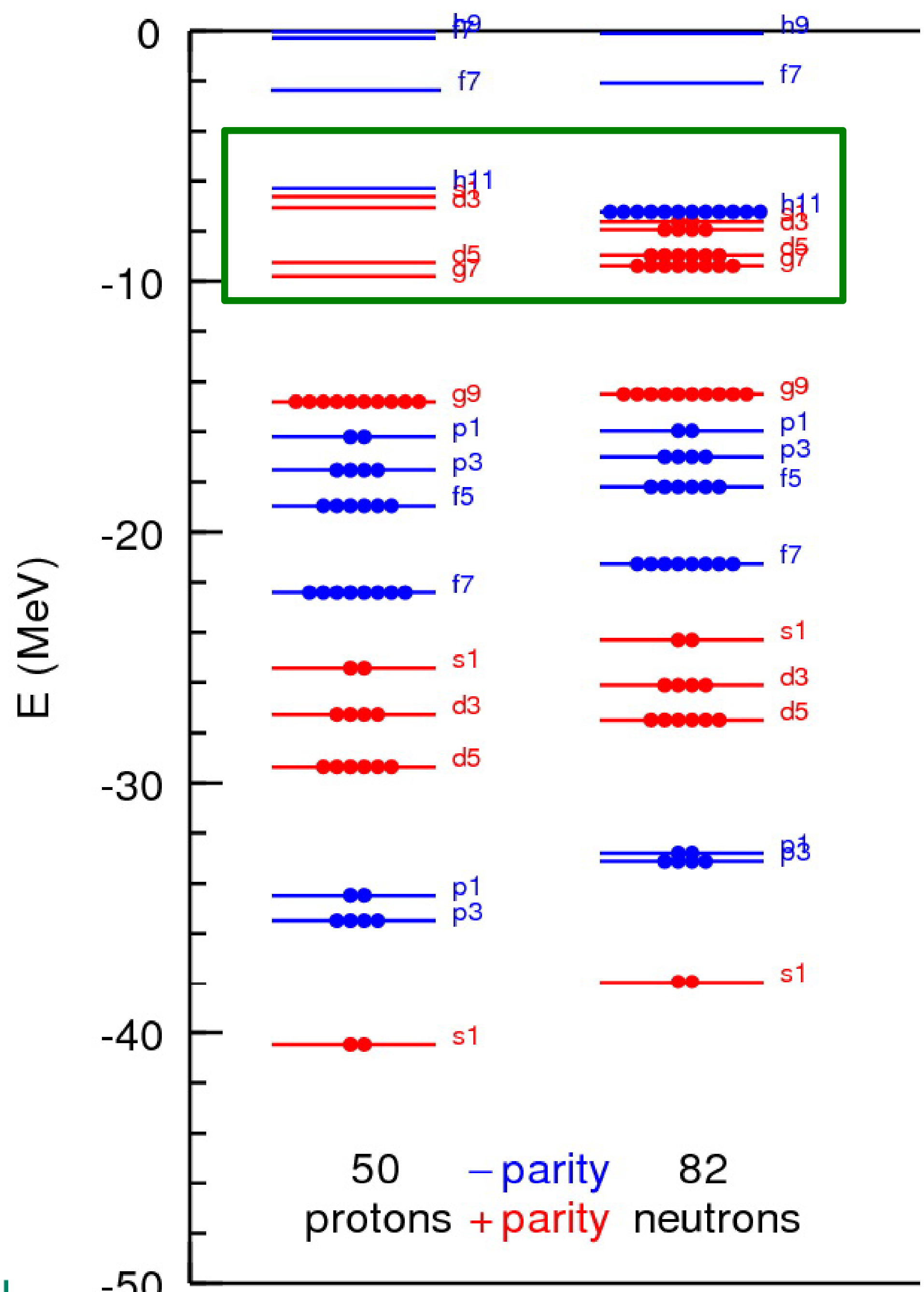
and

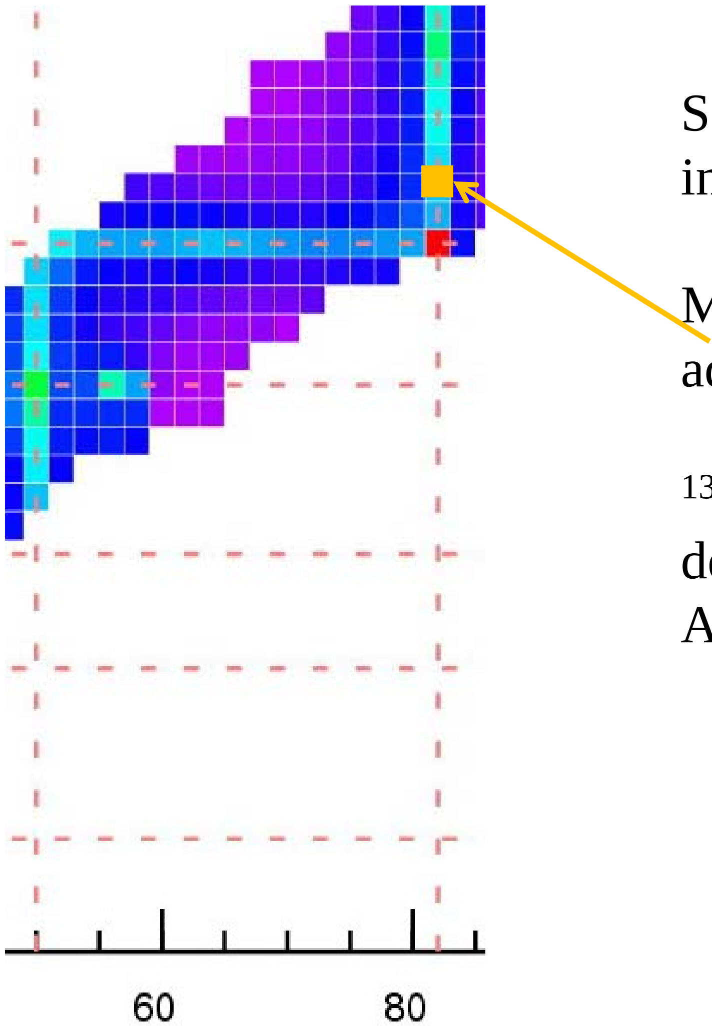
$$\mathrm{RDM}(n_f, n_i, n, n', \lambda) = < [(J_{n_f}, \alpha_{n_f})||[a_n^+ \tilde{a}_{n'}]^\lambda||[(J_{n_i}, \alpha_{n_i}) >$$

The key is to optimize the sums in this equation for OpenMP and/or MPI


100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
> 4 MeV
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
jj66
jj67
jj55
jj56
jj57
jj44
jj45
jj56
pf
sd
p

jj55 model space





Search for Neutrinoless Double-Beta Decay in ^{136}Xe with EXO-200

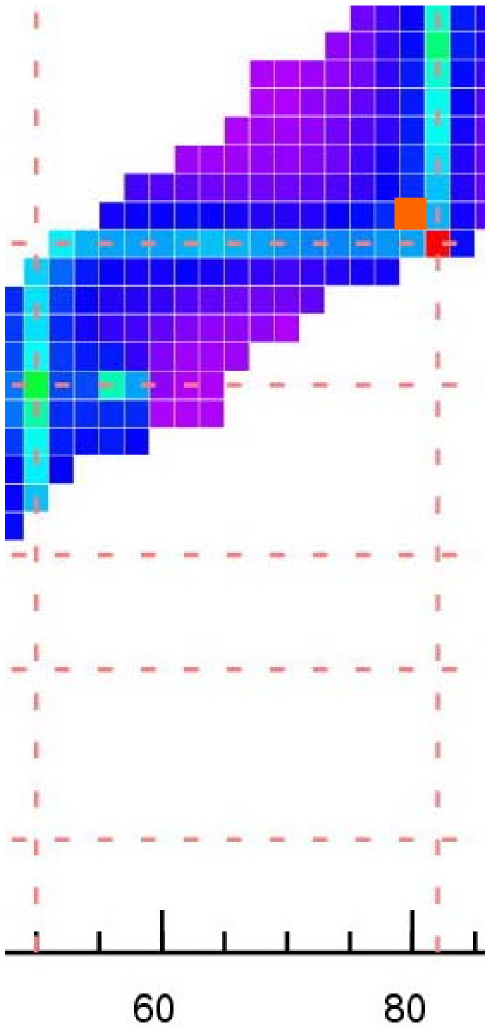
Mihai Horoi (next week) – also requires addition of spin-orbit partners

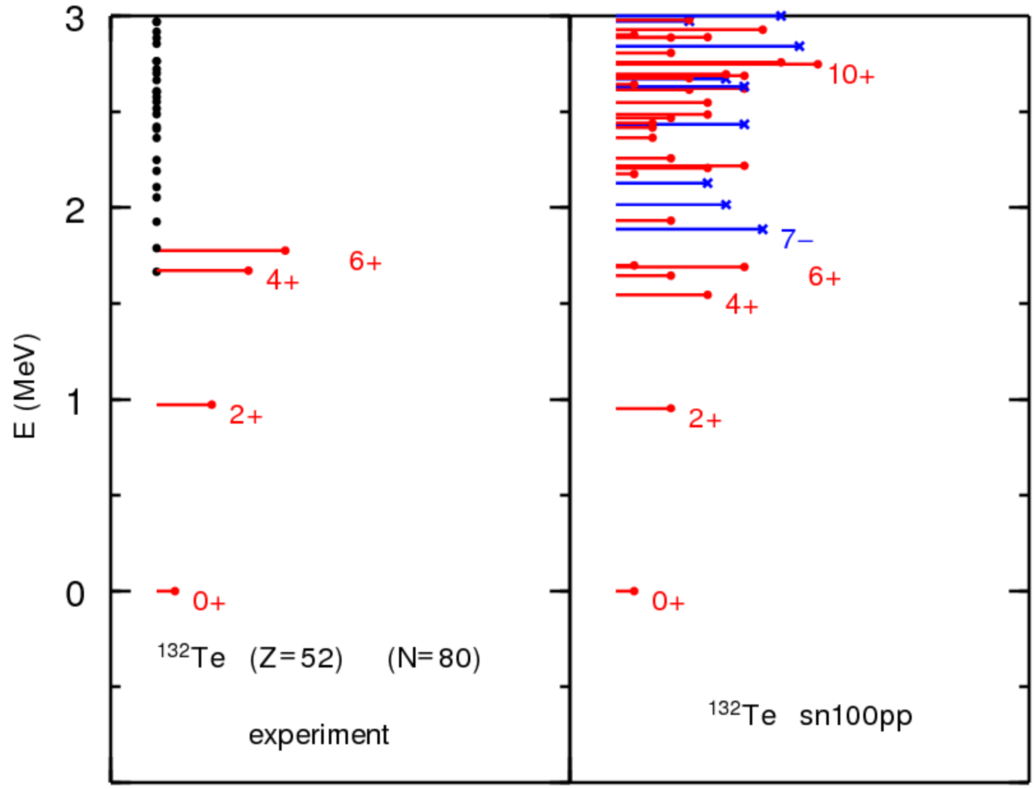
^{137}Ba – first observation of double-gamma decay (D.J. Millener, R.J. Sutter, D.E. Alburger)

(39) a

(10,052) b

^{132}Te

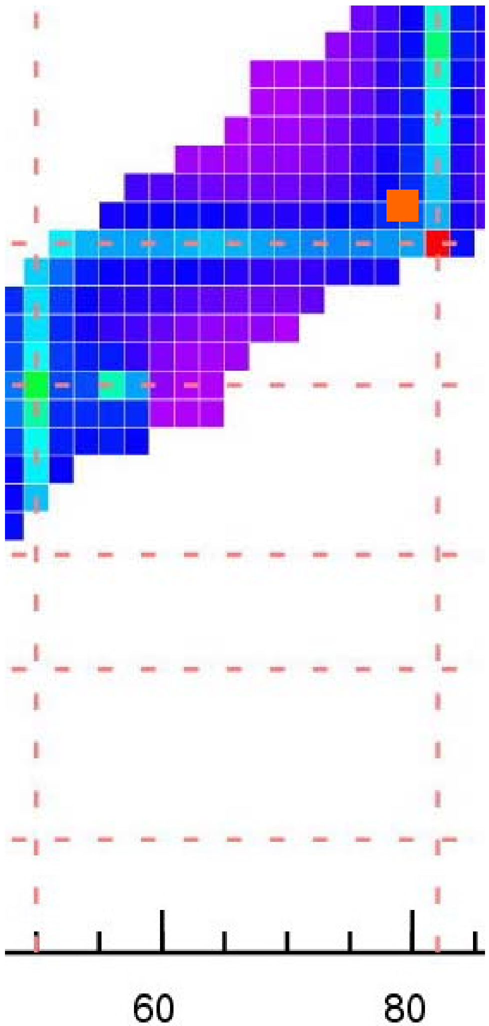


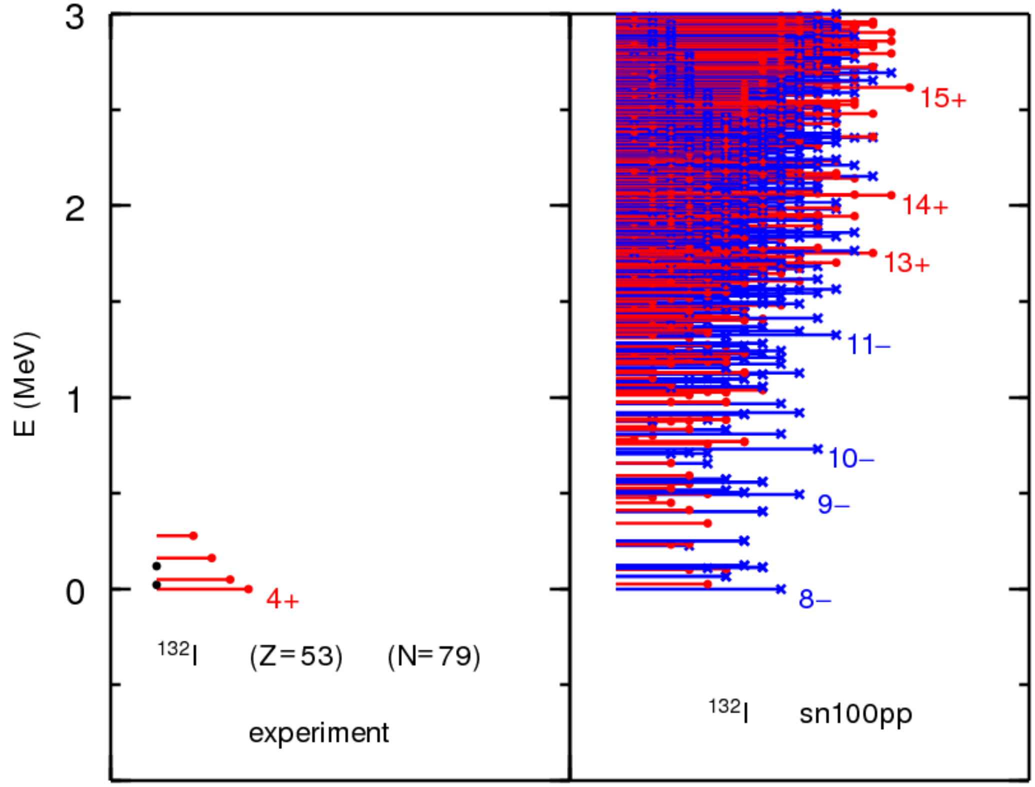


(1333) a

(359,934) b

^{132}I

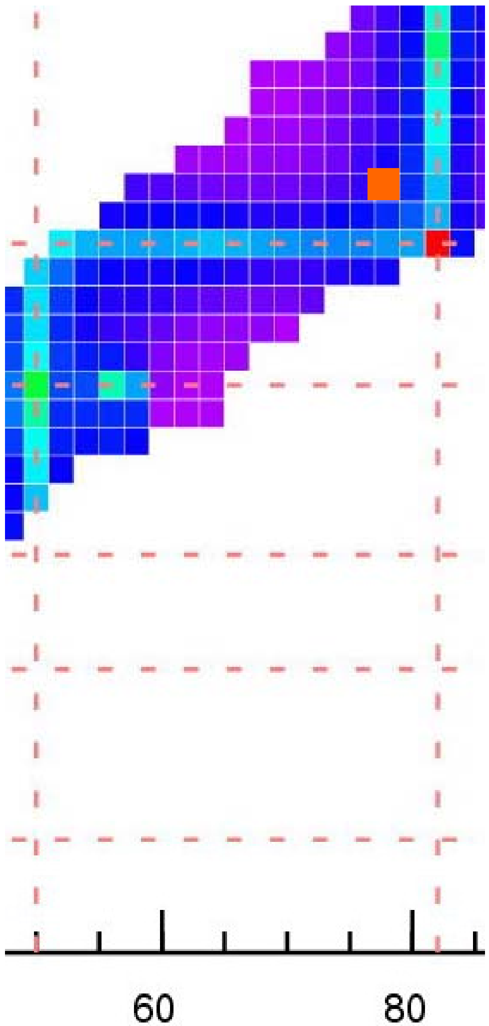


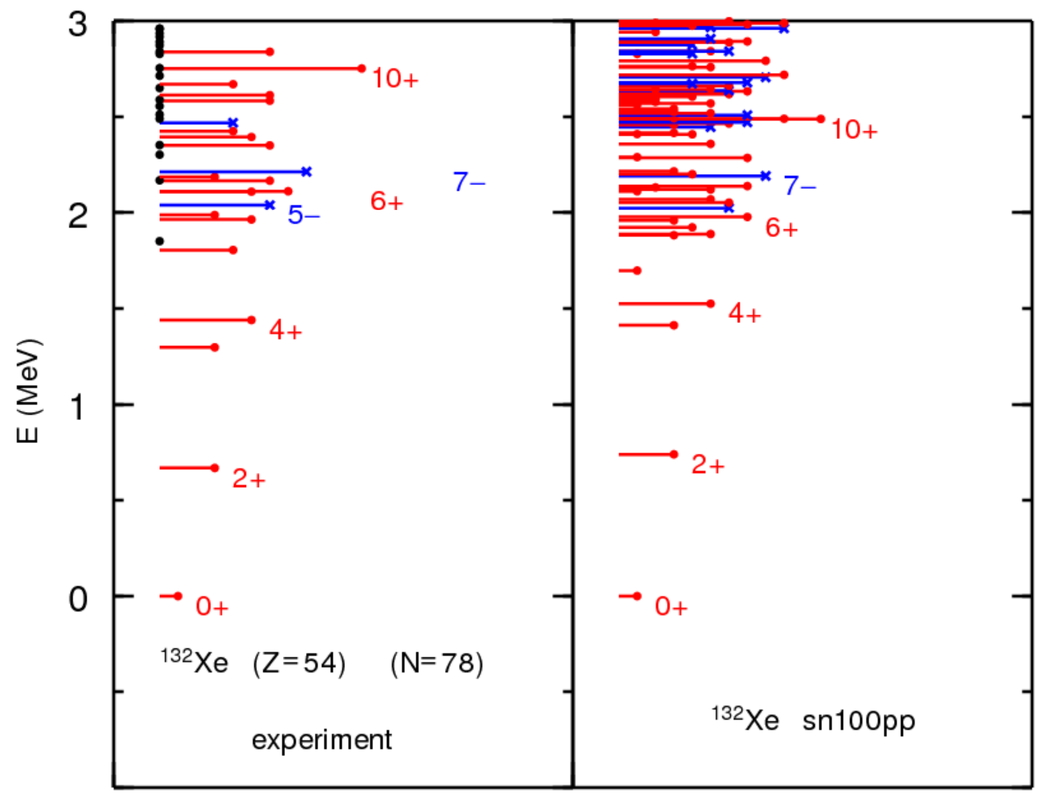


(72) (74,311,166)

a b

^{132}Xe





Result is (Q_p, Q_n) = (-1.91, -7.83) e^2 fm^2.
Compared to $Q_{\text{exp}} = -11.4(1)$ e^2 fm^2.

Now we need to add both $\Delta N = 0$ and $\Delta N = 2$ core-polarization corrections.

The results will be something like

Q_p = -1.91 (bare) - (1.91 + 7.83) (0.2) ($\Delta N = 0$)
- (1.91 + 7.83) (0.8) ($\Delta N = 2$) = -11.6

NSCL

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and the momentum matrix elements will be proportional to

P_p = -1.91 (bare) - (1.91 + 7.83) (0.2) ($\Delta N = 0$)
+ (1.91 + 7.83) (0.8) ($\Delta N = 2$) = +3.9

P_n = -7.83 (bare) - (1.91 + 7.83) (0.2) ($\Delta N = 0$)
+ (1.91 + 7.83) (0.8) ($\Delta N = 2$) = -2.0

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How small are these quadrupole momentum matrix elements in ab-initio and no-core approaches?

For example ^{9}Be 3/2- ground state