Neutrino Flavor Oscillations in Core-collapse Supernovae

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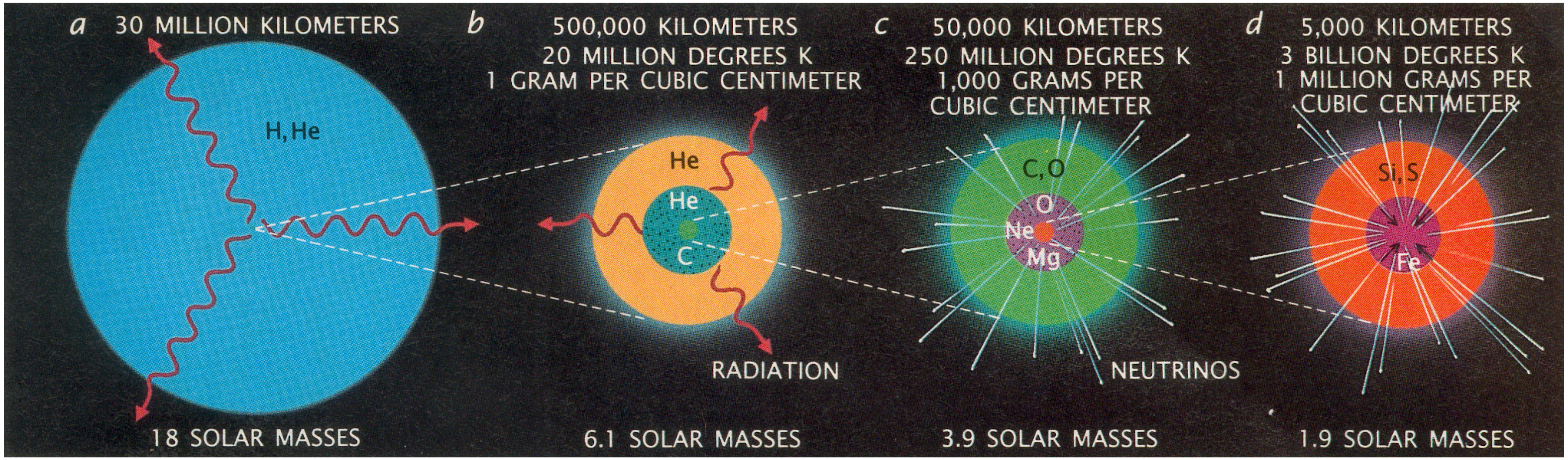
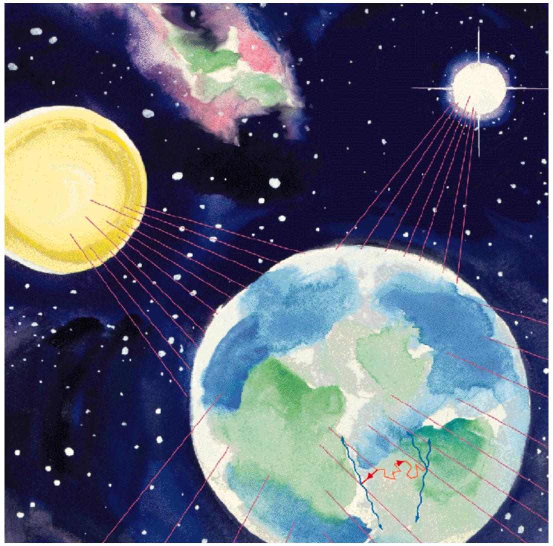
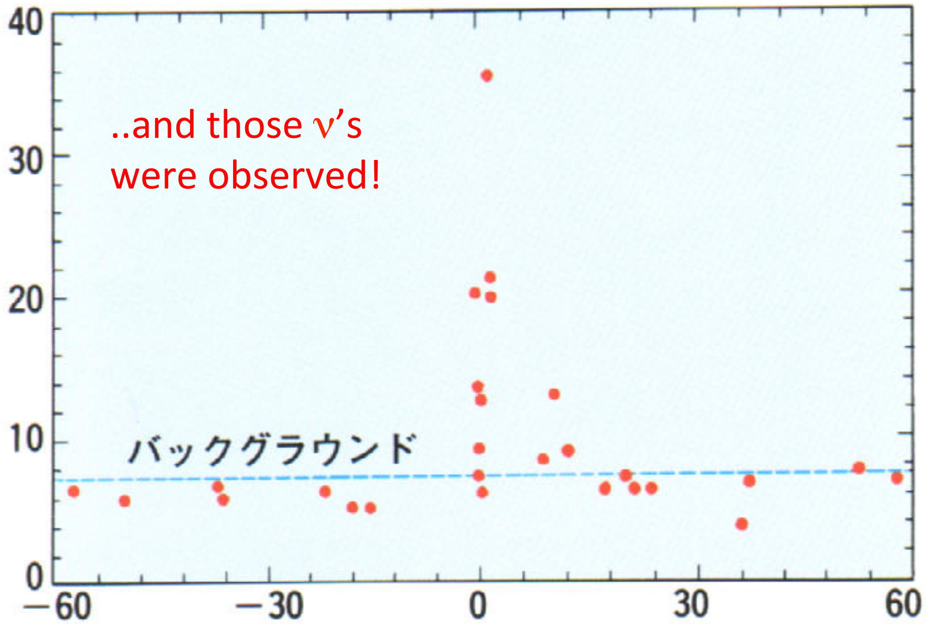
Neutrinos from core-collapse supernovae

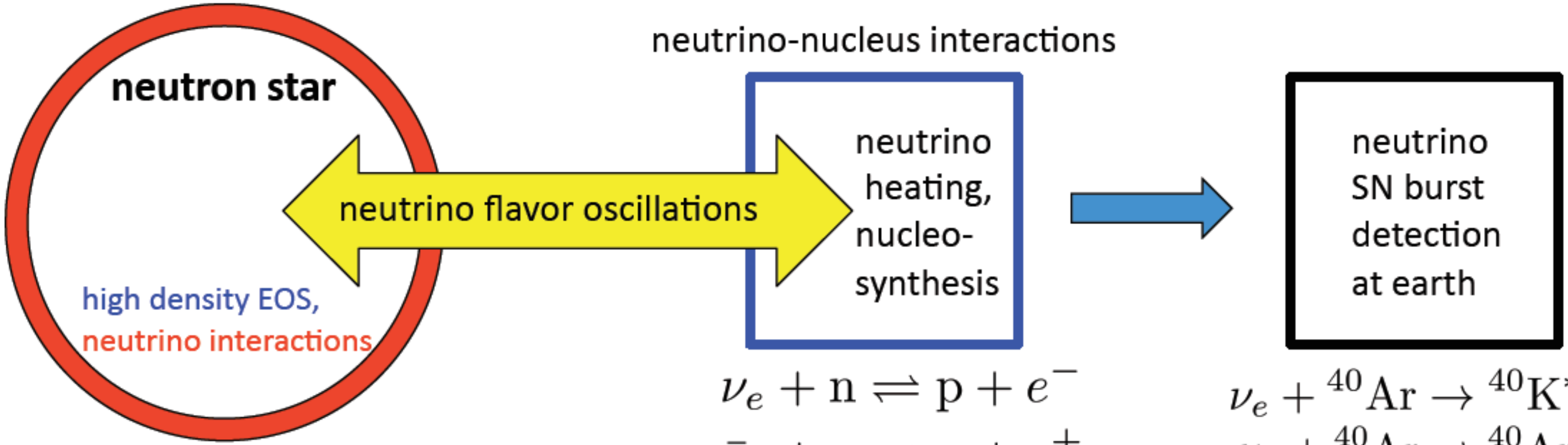
- $M_{prog} \geq 8\ M_{Sun}$
- $\Delta E \approx 10^{53}$ ergs

  $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with $10 \leq E_\nu \leq 30$ MeV
- 10^{58} Neutrinos!


a 30 MILLION KILOMETERS
H, He
18 SOLAR MASSES
b 500,000 KILOMETERS
20 MILLION DEGREES K
1 GRAM PER CUBIC CENTIMETER
He
He
C
RADIATION
6.1 SOLAR MASSES
c 50,000 KILOMETERS
250 MILLION DEGREES K
1,000 GRAMS PER
CUBIC CENTIMETER
C, O
O
Ne
Mg
NEUTRINOS
3.9 SOLAR MASSES
d 5,000 KILOMETERS
3 BILLION DEGREES K
1 MILLION GRAMS PER
CUBIC CENTIMETER
Si, S
Fe
1.9 SOLAR MASSES



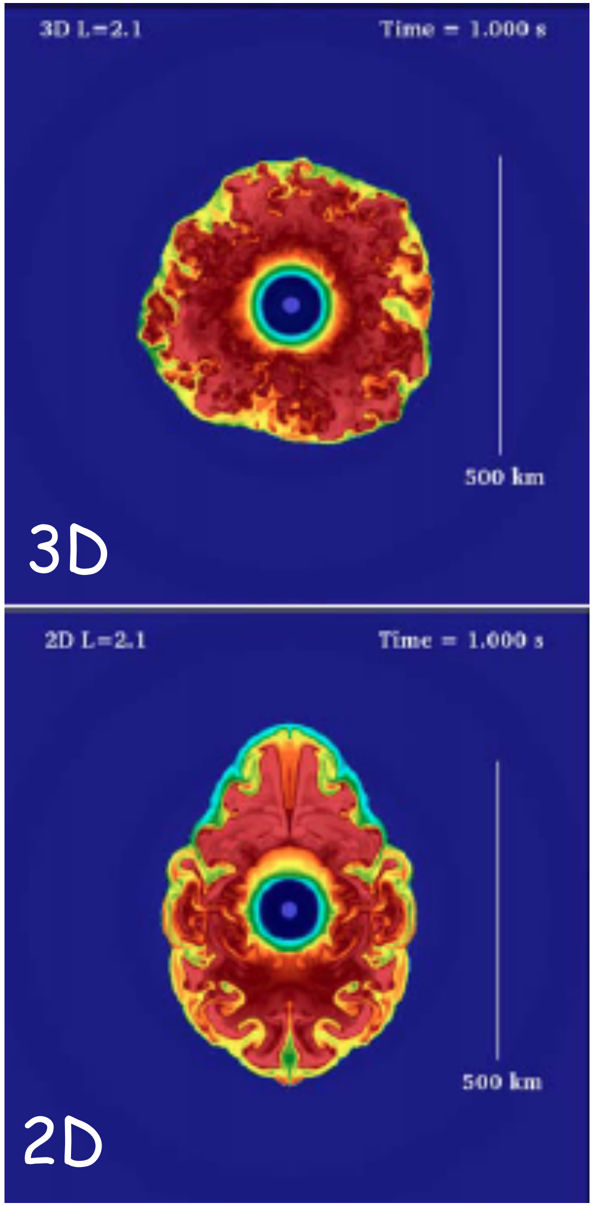

..and those ν's
were observed!
バックグラウンド
40
30
20
10
0
−60
−30
0
30
60



$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

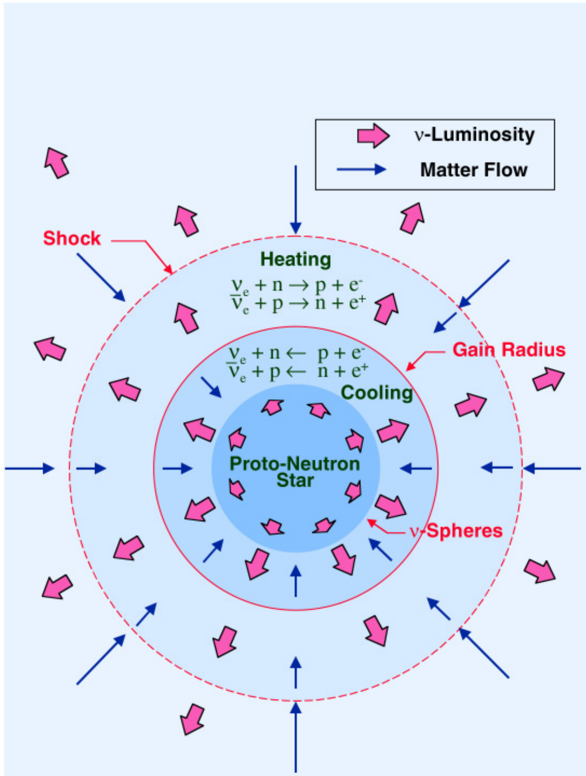
$$\nu_e + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \to {}^{40}\mathrm{Ar}^* + \nu_x$$

Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013).

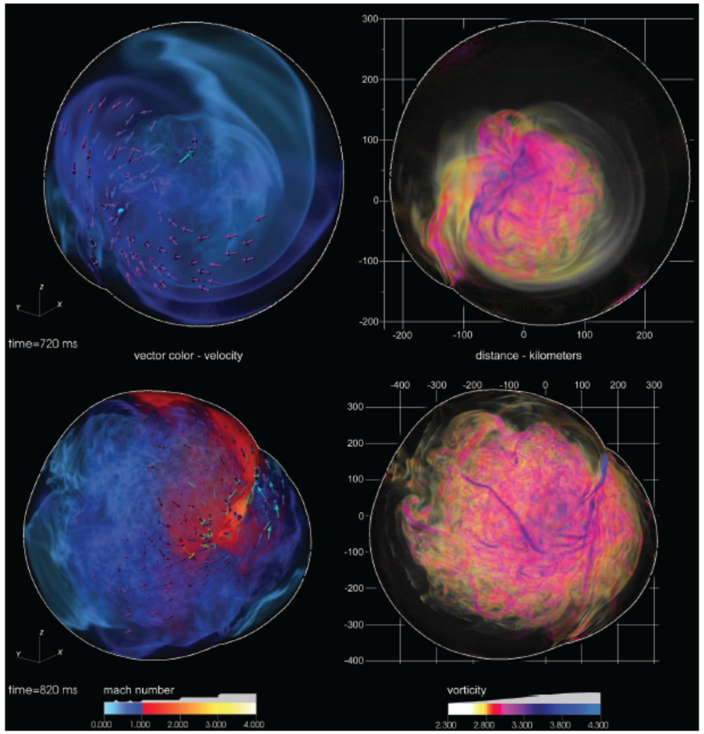


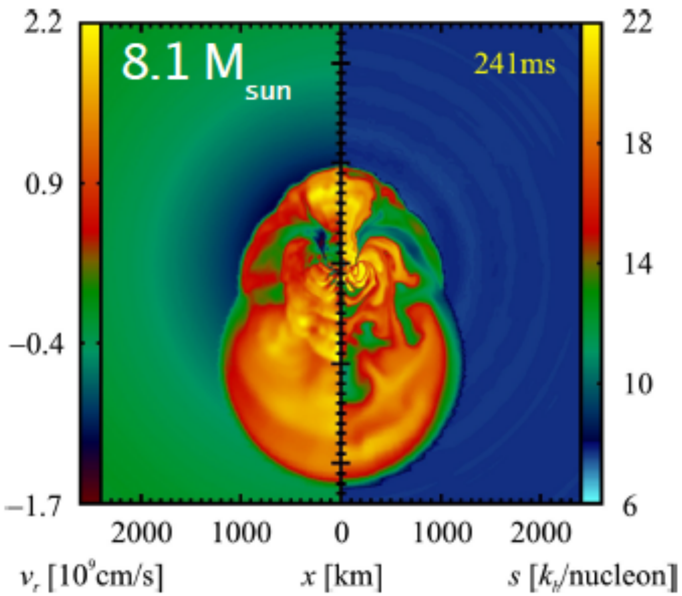
Princeton

Development of 2D and 3D models for core-collapse supernovae: Complex interplay between turbulence, neutrino physics and thermonuclear reactions.



ORNL/UT





Munich

Neutrinos dominate the energetics of core-collapse SN

Explosion only 1% of total energy

Total optical and kinetic energy = 10^{51} ergs

Total energy carried by neutrinos = 10^{53} ergs

10% of star's rest mass

$$E_{grav} \cong \frac{3}{5}\frac{GM_{ns}^2}{R_{ns}} \approx 3\times 10^{53} ergs \left(\frac{M_{ns}}{1.4M_{sun}}\right)^2 \left(\frac{10km}{R_{ns}}\right)$$

Neutrino diffusion time, τ_ν ~ 2-10 s

$$L_\nu \approx \frac{GM_{ns}^2}{6R_{ns}}\frac{1}{\tau_\nu} \approx 4\times 10^{51} ergs/s$$

Core-collapse supernovae are very sensitive to ν physics

Gravitational collapse yields very large values of the Fermi energy for electrons and ν_e's (~10^{57} units of electron lepton number). ν_μ's and ν_τ's are pair-produced, so they carry no μ or τ lepton number. Any process that changes neutrino flavor could increase electron capture and reduce electron lepton number.

Almost the entire gravitational binding energy of the progenitor star is emitted in neutrinos. Neutrinos transport entropy and the lepton number.

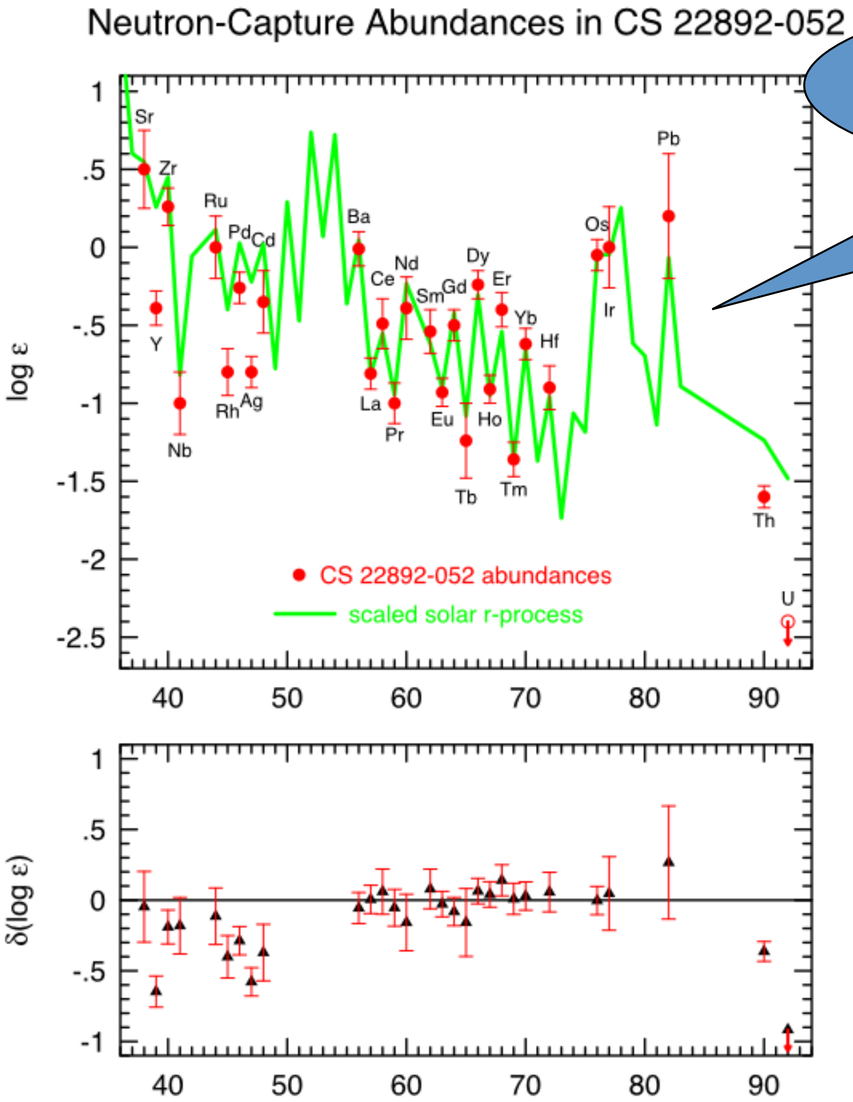
Electron fraction, or equivalently neutron-to-proton ratio (the controlling parameter for nucleosynthesis) is determined by the neutrino capture rates:

$$\nu_e + n \rightleftarrows p + e^-$$

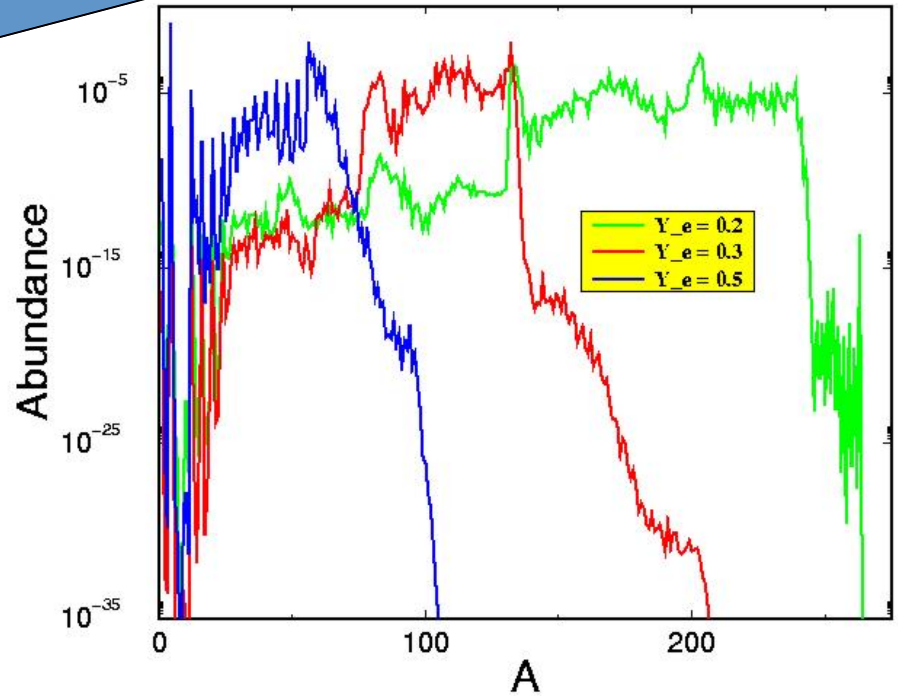
$$\overline{\nu_e} + p \rightleftarrows n + e^+$$

One way to produce most of the elements heavier than iron is via rapid neutron capture (r-process)

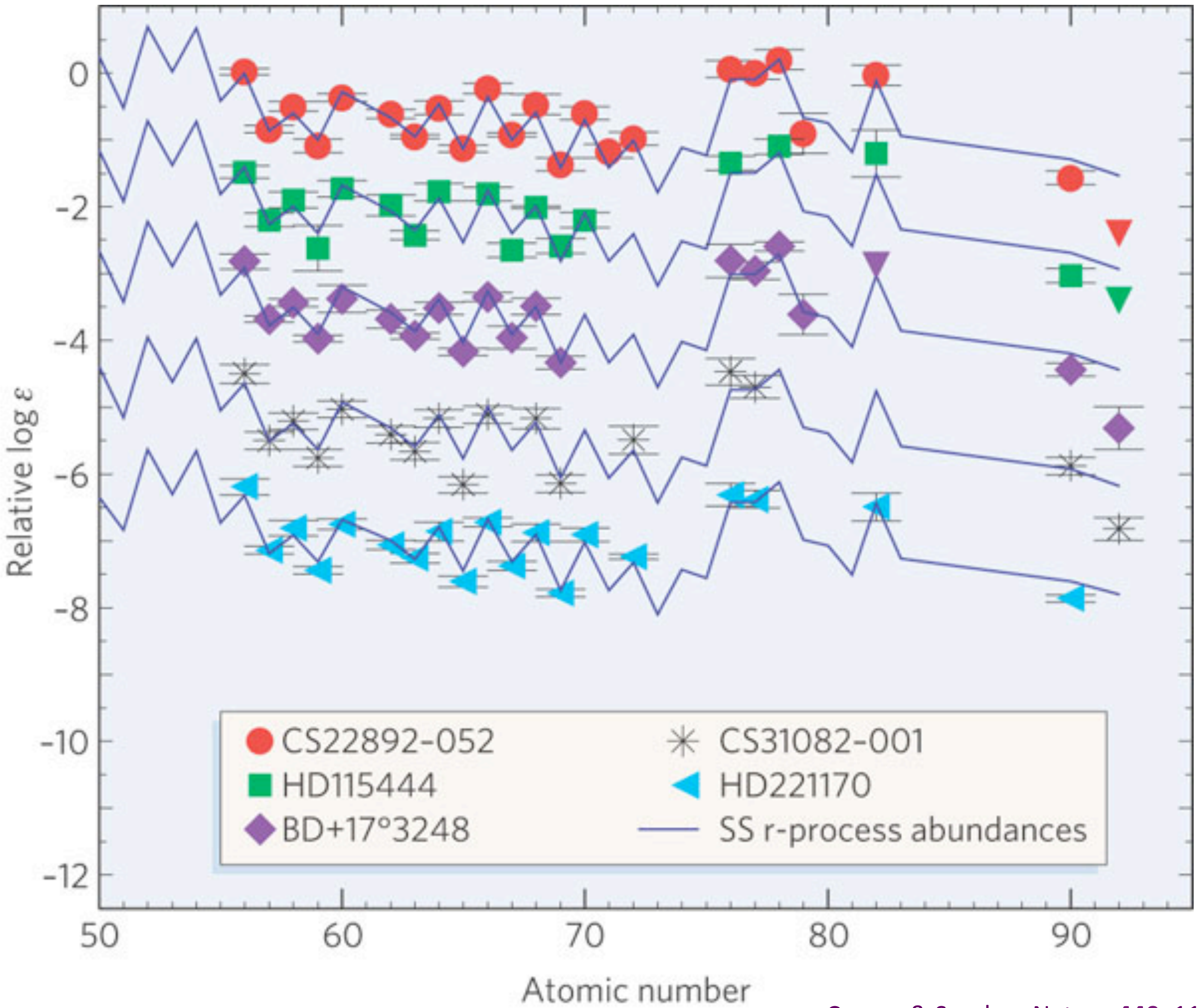
[Fe/H] ≈ -3.1



A > 100 abundance pattern fits the solar abundances well

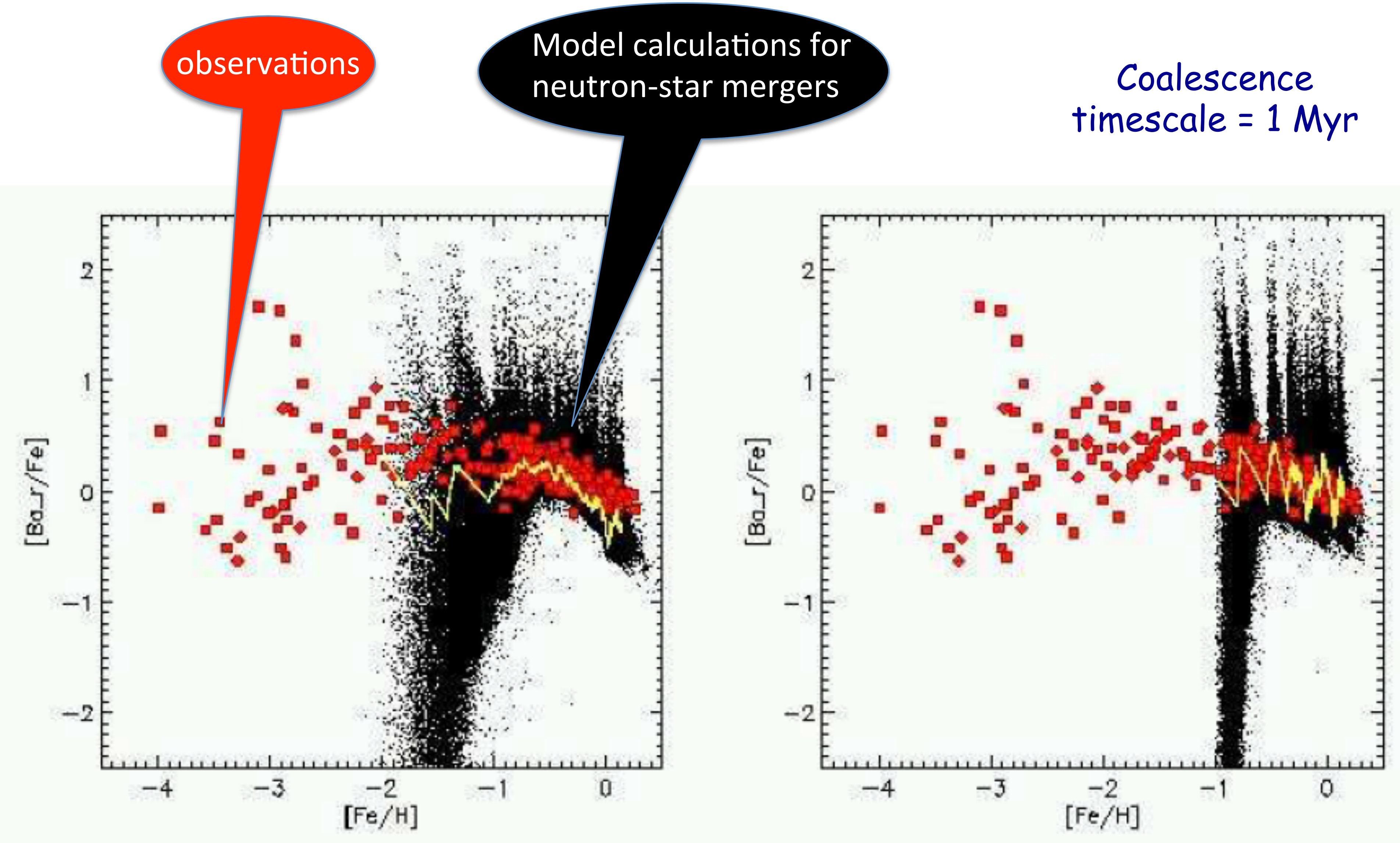


r-process abundances should depend very strongly on electron fraction Meyer



Cowan & Sneden, Nature **440**, 1151 (2006)

Coalescence timescale = 1 Myr

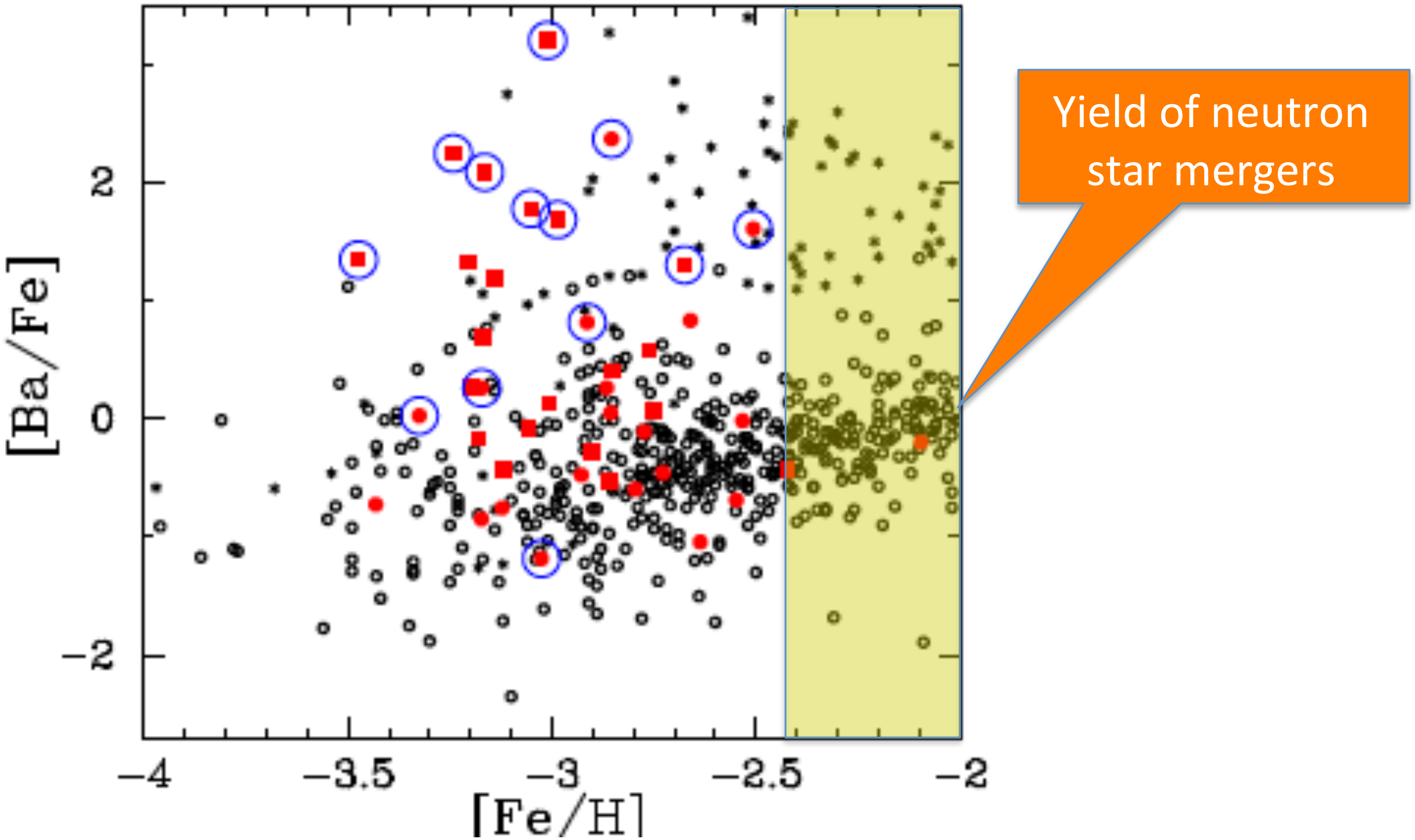


Average merger rate = 20/Myr

Average merger rate = 2/Myr

Star formation rate?

Argast *et al.*, A&A, **416**, 997 (2003)



SDSS Data from Aoki *et al.*, arXiv: 1210.1946 [astro-ph.SR]

- Yields of r-process nucleosynthesis are determined by the electron fraction, or equivalently by the neutron-to-proton ratio, n/p
- Interactions of the neutrinos and antineutrinos streaming out of the core both with nucleons and seed nuclei determine the n/p ratio. Hence it is crucial to understand neutrino properties and interactions.
- As these neutrinos reach the r-process region they undergo matter-enhanced neutrino oscillations as well as coherently scatter over other neutrinos. Many-body behavior of this neutrino gas is being explored, but may have significant impact on r-process nucleosynthesis.

The MSW Effect

In vacuum: $E^2 = \mathbf{p}^2 + m^2$

In matter:

$(E - V)^2 = (\mathbf{p} - \mathbf{A})^2 + m^2$

$\Rightarrow E^2 = \mathbf{p}^2 + m^2_{\mathrm{eff}}$

V $\propto$ background density

$\mathbf{A} \propto \mathbf{J}_{\mathrm{background}}$ (currents) or

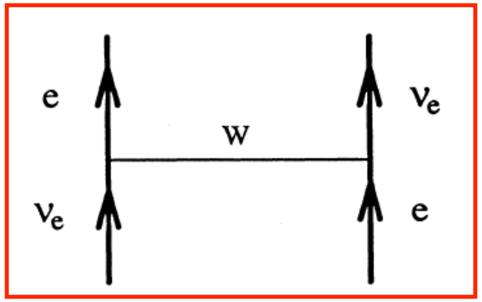
$\mathbf{A} \propto \mathbf{S}_{\mathrm{background}}$ (spin)

In the limit of static, charge-neutral, and unpolarized background

V$\propto N_e$ and $\mathbf{A} = 0$

$\Rightarrow m^2_{\mathrm{eff}} = m^2 + 2EV + \mathcal{O}(V^2)$

The potential is provided by the coherent forward scattering of ν_e's off the electrons in dense matter



There is a similar term with Z-exchange. But since it is the same for all neutrino flavors, it does not contribute to phase differences *unless* we invoke a sterile neutrino.

Note that matter effects induce an effective CP-violation since the matter in the Earth and the stars is not CP-symmetric!

$$\mathbf{T} = \mathbf{T}_{23}\mathbf{T}_{13}\mathbf{T}_{12}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix}}_{\mathbf{T}_{23}} \overbrace{\begin{pmatrix} C_{13} & 0 & S_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -S_{13}e^{i\delta_{CP}} & 0 & C_{13} \end{pmatrix}}^{\mathbf{T}_{13}} \underbrace{\begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{T}_{12}}$$

$C_{ij} = \cos\theta_{ij}$, $S_{ij} = \sin\theta_{ij}$

δ_{CP}: CP-violating phase

MSW Equations

$$i\frac{\partial}{\partial t}\begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix} = \mathbf{H}\begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \end{pmatrix}$$

$$\mathbf{H} = \mathbf{T}_{23}\mathbf{T}_{13}\mathbf{T}_{12}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}\mathbf{T}_{12}^{\dagger}\mathbf{T}_{13}^{\dagger}\mathbf{T}_{23}^{\dagger} + \begin{pmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{pmatrix}$$

$$\mathbf{T}_r = \mathbf{T}_{13}\mathbf{T}_{12}$$

One-Body Hamiltonian

$$\tilde{\mathbf{H}} = \mathbf{T}_r \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \mathbf{T}_r^\dagger + \begin{pmatrix} V_{e\mu} & 0 & 0 \\ 0 & S_{23}^2 V_{\tau\mu} & -C_{23}S_{23}V_{\tau\mu} \\ 0 & -C_{23}S_{23}V_{\tau\mu} & C_{23}^2 V_{\tau\mu} \end{pmatrix}$$

Dominant term, Wolfenstein

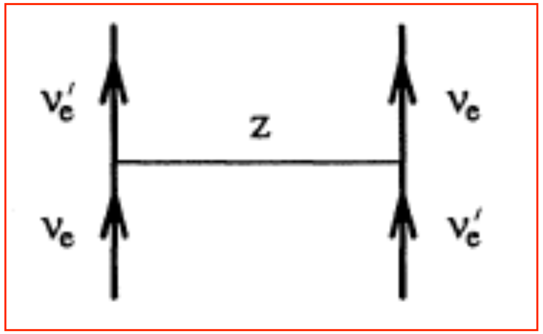
$$V_{e\mu}(x) = \sqrt{2}G_F N_e(x)$$

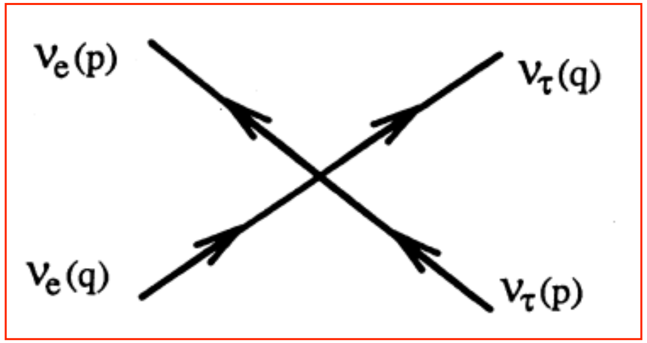
Sub-dominant term, Botella, Lim, Marciano, PRD **35**, 896 (1987)

$$V_{\tau\mu} = -\frac{3\sqrt{2}G_F\alpha}{\pi \sin^2\theta_W}\left(\frac{m_\tau}{m_W}\right)^2 \left\{(N_e + N_n)\log\frac{m_\tau}{m_W} + \left(\frac{N_e}{2} + \frac{N_n}{3}\right)\right\}$$

Coherent forward scattering of neutrinos off other neutrinos

If the neutrino density itself is also very high then one has to consider the effects of neutrinos scattering off other neutrinos. This is the case for a core-collapse supernova.





Not all neutrinos scatter in the forward direction, halo effect

Fuller, Qian, Raffelt, Smirnov, Duan, Balantekin, Pehlivan, Friedland, …

Many neutrino system

This is the only many-body system driven by the weak interactions:

Table: Many-body systems

Nuclei	Strong	at most ∼250 particles
Condensed matter	E&M	at most N_A particles
ν**'s in SN**	Weak	$\sim 10^{58}$particles

Astrophysical extremes allow us to test physics that cannot be tested elsewhere!

Algebraic description of the MSW effect

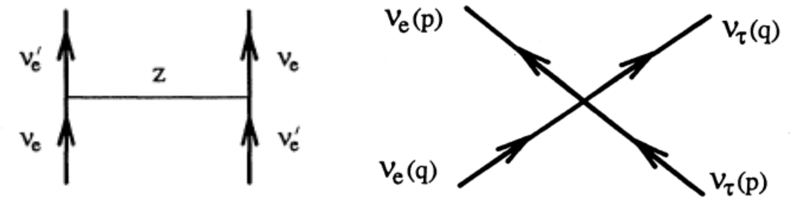
$$J_+(p) = a_x^\dagger(p) a_e(p), \quad J_-(p) = a_e^\dagger(p) a_x(p),$$
$$J_0(p) = \frac{1}{2}\left(a_x^\dagger(p) a_x(p) - a_e^\dagger(p) a_e(p)\right)$$

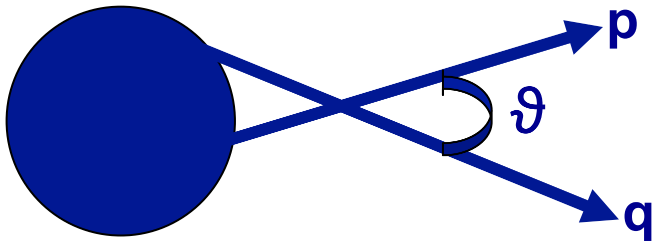
Neutrino flavor isospin algebra

$$H_\nu = \int dp \left(\frac{\delta m^2}{2p}\cos 2\theta - \sqrt{2} G_F N_e\right) J_0(p) + \frac{1}{2}\int dp \frac{\delta m^2}{2p} \sin 2\theta \,(J_+(p) + J_-(p))$$

Neutrino-Neutrino Interactions

Smirnov, Fuller and Qian, Pantaleone, McKellar, Friedland, Lunardini, Raffelt, Balantekin, Kajino, Pehlivan …





$$H_{\nu\nu} = \sqrt{2} G_F \int dp\, dq\, (1 - \cos\vartheta_{pq})\, \vec{J}(p) \cdot \vec{J}(q)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral “swaps” or “splits”).

The total neutrino Hamiltonian

$$\hat{H}_{\text{total}} = H_\nu + H_{\nu\nu} = \left(\sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p - \sqrt{2} G_F N_e J_p^0 \right) + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos \vartheta_{\mathbf{pq}}) \, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

Pantaleone, Dasgupta, Duan, Fogli, Fuller, Kostelecky, McKellar, Lisi, Mirizzi, Qian, Pastor, Raffelt, Samuel, Sawyer, Sigl, Smirnov, ...

$$\hat{B} = (\sin 2\theta, 0, -\cos 2\theta)$$

Including antineutrinos

$$H = H_\nu + H_{\bar{\nu}} + H_{\nu\nu} + H_{\bar{\nu}\bar{\nu}} + H_{\nu\bar{\nu}}$$

Requires introduction of a second set of SU(2) algebras!

Including three flavors

Requires introduction of SU(3) algebras.

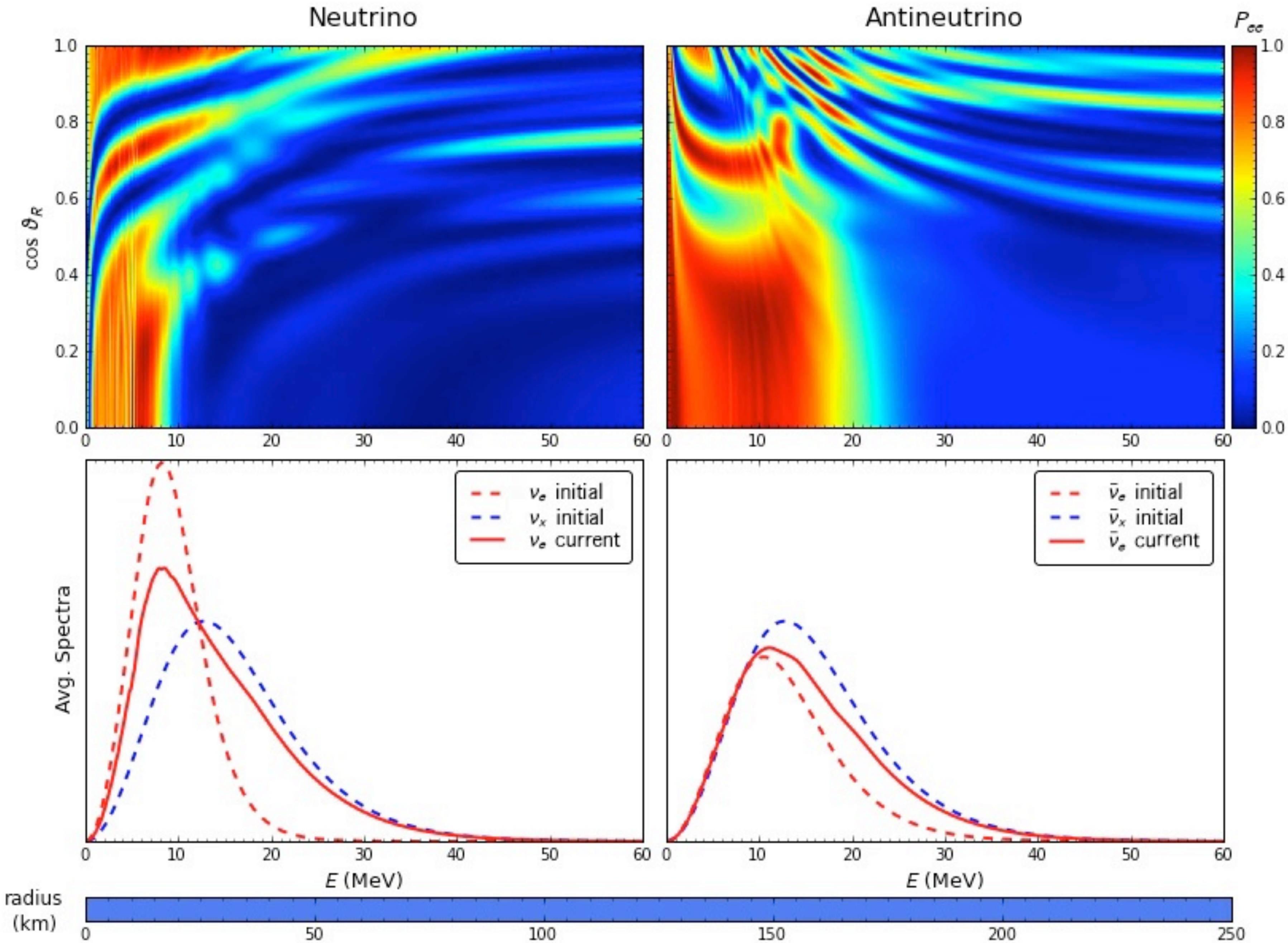
Both extensions are straightforward, but tedious!
Balantekin and Pehlivan, J. Phys. G **34**, 1783 (2007).

The total neutrino Hamiltonian

$$\hat{H}_{\text{total}} = H_\nu + H_{\nu\nu} = \left(\sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p - \sqrt{2} G_F N_e J_p^0 \right) + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos \vartheta_{\mathbf{pq}}) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

Pantaleone, Dasgupta, Duan, Fogli, Fuller, Kostelecky, McKellar, Lisi, Mirizzi, Qian, Pastor, Raffelt, Samuel, Sawyer, Sigl, Smirnov, ...

$$\hat{B} = (\sin 2\theta, 0, -\cos 2\theta)$$



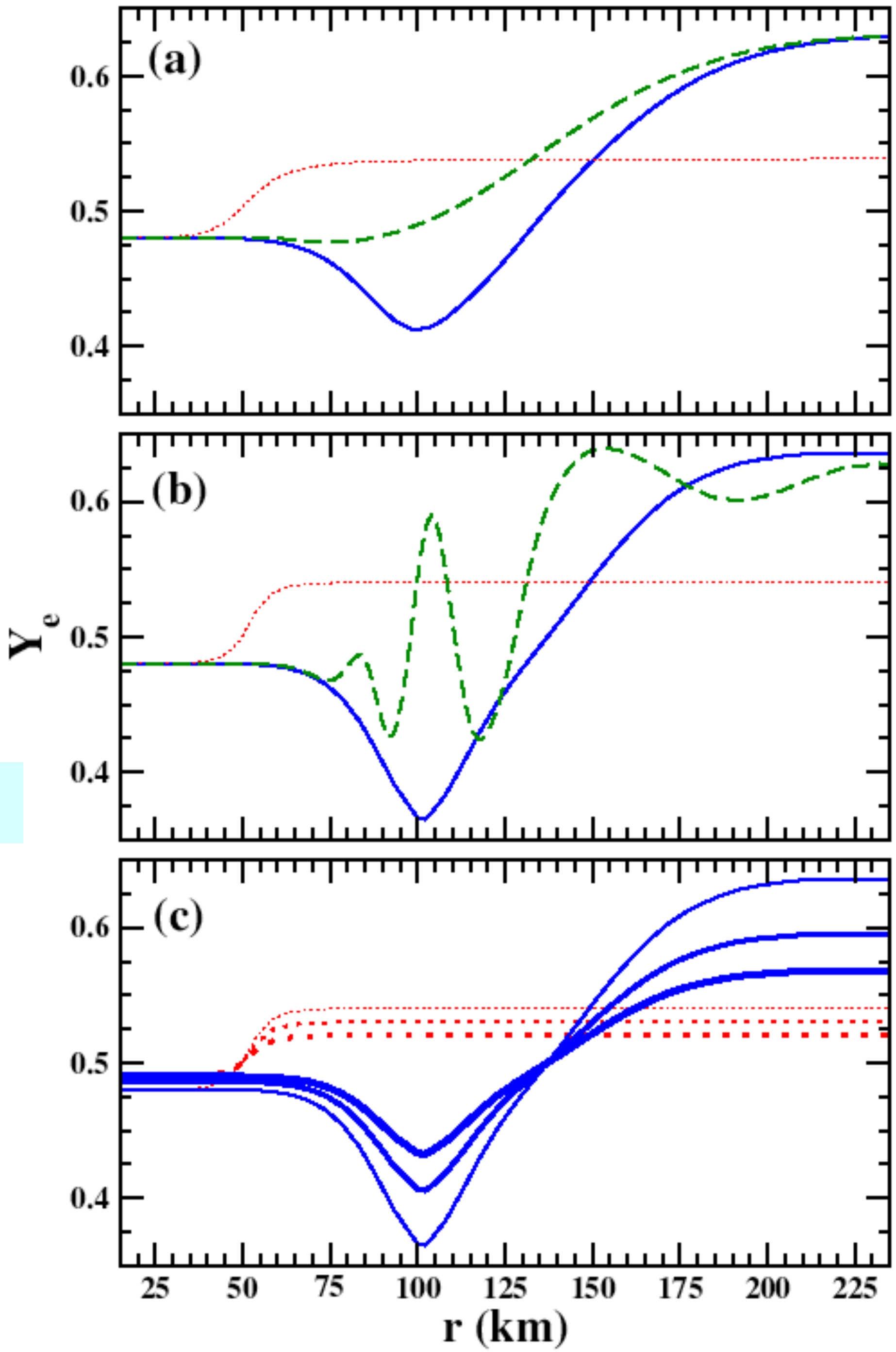
Fuller, Qian, Carlson, Duan

Equilibrium electron fraction with the inclusion of $\nu\nu$ interactions

L^{51} = 0.001, 0.1, 50

Balantekin and Yuksel

X_α= 0, 0.3, 0.5 (thin, medium, thick lines)



θ_{13}~π/10

θ_{13}~π/20

θ_{13}~π/20 with α effect

Neutrino Hamiltonian with $\nu - \nu$ interactions

$$\hat{H}_{\text{total}} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos \vartheta_{\mathbf{pq}}) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

Single-angle approximation $\Rightarrow$

$$\hat{H}_{\text{total}} = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}$$

Defining $\mu = \frac{\sqrt{2} G_F}{V}$, $\tau = \mu t$, and $\omega_p = \frac{1}{\mu} \frac{\delta m^2}{2p}$ one can write

$$\hat{H} = \sum_p \omega_p \hat{B} \cdot \vec{J}_p + \vec{J} \cdot \vec{J}$$

The duality between $H_{\nu\nu}$ and BCS Hamiltonians

The ν-ν Hamiltonian

$$\hat{H} = \sum_p \omega_p \hat{B} \cdot \vec{J}_p + \vec{J} \cdot \vec{J}$$

$\Longleftrightarrow$

The BCS Hamiltonian

$$\hat{H}_{\mathrm{BCS}} = \sum_k 2\epsilon_k \hat{t}^0_k - |G| \hat{T}^+ \hat{T}$$

Same symmetries leading to Analogous (dual) dynamics!

Some Invariants

$$\hat{H} = \sum_p \omega_p \hat{B} \cdot \vec{J}_p + \vec{J} \cdot \vec{J}$$

This Hamiltonian preserves the *length of each spin*

$$\hat{L}_p = \vec{J}_p \cdot \vec{J}_p \ , \qquad \left[\hat{H}, \hat{L}_p\right] = 0 \ ,$$

as well as the *total spin component* in the direction of the "external magnetic field", $\hat{B}$

$$\hat{C}_0 = \hat{B} \cdot \vec{J} \ , \qquad \left[\hat{H}, \hat{C}_0\right] = 0$$

Invariants

The collective neutrino Hamiltonian given has the following constants of motion:

$$\hat{h}_p = \hat{B} \cdot \vec{J}_p + 2 \sum_{q(\neq p)} \frac{\vec{J}_p \cdot \vec{J}_q}{\omega_p - \omega_q}.$$

The individual neutrino spin-length discussed before in an independent invariant. However $\hat{C}_0 = \sum_p \hat{h}_p$. The Hamiltonian itself is also a linear combination of these invariants.

$$\hat{H} = \sum_p w_p \hat{h}_p + \sum_p \hat{L}_p \ .$$

Pehlivan, Balantekin, Kajino and Yoshida, Phys. Rev. D**84**, 065008 (2011)

Spectral Splits

Lagrange multiplier to enforce neutrino number conservation:

$$\hat{H}^{\rm RPA} + \omega_c \hat{\mathcal{J}}^0 = \sum_p (\omega_c - \omega_p)\hat{\mathcal{J}}^0_p + \vec{\mathcal{P}} \cdot \vec{\mathcal{J}}$$

$$= \sum_{\mathbf{p},s} 2\lambda_p \hat{U}'^{\dagger} \hat{\mathcal{J}}^0_p \hat{U}'$$

$$\hat{U}' = e^{\sum_p z_p J^+_p} \; e^{\sum_p \ln(1+|z_p|^2) J^0_p} \; e^{-\sum_p z^*_p J^-_p}$$

$$z_p = e^{i\delta} \tan\theta_p$$

$$\cos\theta_p = \sqrt{\frac{1}{2}\left(1 + \frac{\omega_c - \omega_p + \mathcal{P}^0}{2\lambda_p}\right)}$$

Many people contributed to their explanation

Raffelt, Mirizzi, Dasgupta, Smirnov, Fuller, Qian, Duan, Carlson$\cdots$

$$\begin{aligned}\alpha_1(\mathbf{p},s) &= \hat{U}'^{\dagger} a_1(\mathbf{p},s)\hat{U}' = \cos\theta_p\, a_1(\mathbf{p},s) - e^{i\delta}\sin\theta_p\, a_2(\mathbf{p},s)\\ \alpha_2(\mathbf{p},s) &= \hat{U}'^{\dagger} a_2(\mathbf{p},s)\hat{U}' = e^{-i\delta}\sin\theta_p\, a_1(\mathbf{p},s) + \cos\theta_p\, a_2(\mathbf{p},s)\end{aligned}$$

$$\hat{H}^{\mathrm{RPA}} + \omega_c \hat{\mathcal{J}}^0 = \sum_{\mathbf{p},s} \lambda_p \left(\alpha_1^{\dagger}(\mathbf{p},s)\alpha_1(\mathbf{p},s) - \alpha_2^{\dagger}(\mathbf{p},s)\alpha_2(\mathbf{p},s)\right)$$

Assume that initially ($\mu \rightarrow \infty$) there are more ν_e's and all neutrinos are in flavor eigenstates:

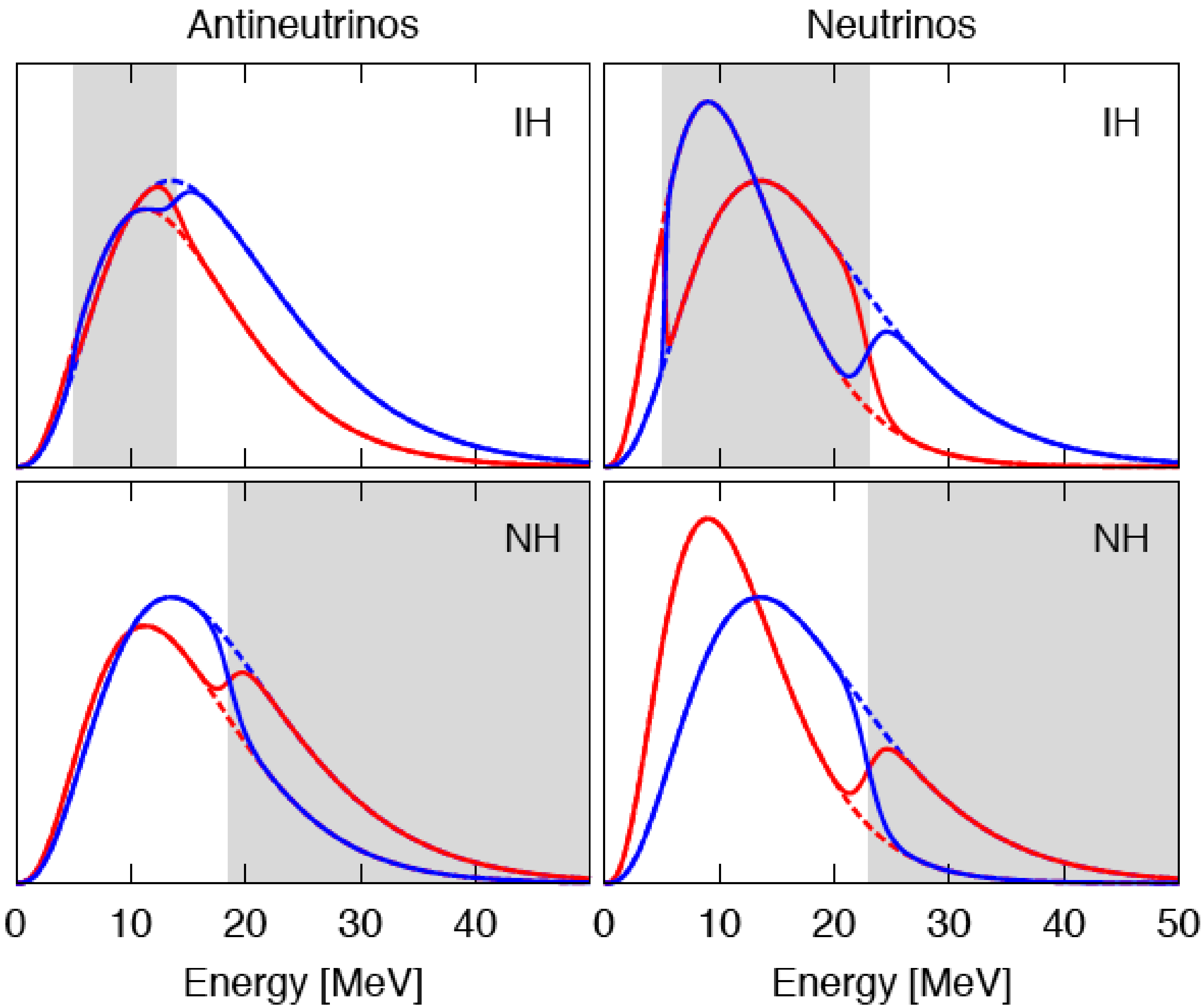
$$\cos\theta_p = \sqrt{\frac{1}{2}\left(1 + \frac{P^0}{|\vec{P}|}\cos 2\theta\right)} \rightarrow_{\lim \mu\rightarrow\infty} \cos\theta$$

$$\alpha_1(\mathbf{p}, s) = \hat{U}^\dagger a_1(\mathbf{p}, s)\hat{U} \Rightarrow a_e(\mathbf{p}, s)$$

At the end ($\mu \rightarrow 0$)

$$\cos\theta_p = \sqrt{\frac{1}{2}\left(1 + \frac{\omega_c - \omega_p}{|\omega_c - \omega_p|}\right)} \Rightarrow \begin{cases} 1 & \omega_p < \omega_c \\ 0 & \omega_p > \omega_c \end{cases}$$

$$\alpha_1(\mathbf{p}, s) = \hat{U}^\dagger a_1(\mathbf{p}, s)\hat{U} \Rightarrow a_1(\mathbf{p}, s)$$



Dasgupta *et al.*, 2009

CP (T) violation in Supernovae?

time

CP (T) violation in Supernovae?

time

"Hamiltonian" in the Rotated Basis

We define

$$\tilde{H} = \mathbf{T}_{13}\mathbf{T}_{12}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}\mathbf{T}_{12}^{\dagger}\mathbf{T}_{13}^{\dagger} + \begin{pmatrix} V_{e\mu} & 0 & 0 \\ 0 & S_{23}^2 V_{\tau\mu} & -C_{23}S_{23}V_{\tau\mu} \\ 0 & -C_{23}S_{23}V_{\tau\mu} & C_{23}^2 V_{\tau\mu} \end{pmatrix}$$

μ - τ Symmetry

If we can neglect the potential $V_{\tau\mu}$ we can write

$$\tilde{H} = \mathbf{T}_{13}\mathbf{T}_{12}\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}\mathbf{T}_{12}^{\dagger}\mathbf{T}_{13}^{\dagger} + \begin{pmatrix} V_{e\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

It is straightforward to show that

$$\tilde{H}(\delta) = \mathbf{S}\tilde{H}(\delta = 0)\mathbf{S}^{\dagger}$$

with

$$\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

Neutrino Luminosities

Electron Neutrino at a distance r from the neutrinosphere is

$$\mathcal{L}_e^{(r)} = \mathcal{L}_e^{(0)} P(\nu_e \to \nu_e, r) + \mathcal{L}_\mu^{(0)} P(\nu_\mu \to \nu_e, r) + \mathcal{L}_\tau^{(0)} P(\nu_\tau \to \nu_e, r)$$

If the ν_μ and ν_τ luminosities are the same at the neutrinosphere, i.e. $\mathcal{L}_\mu^{(0)} = \mathcal{L}_\tau^{(0)}$

$$\mathcal{L}_e^{(r)} = \mathcal{L}_e^{(0)} \{P(\nu_e \to \nu_e, r)\} + \mathcal{L}_\mu^{(0)} \{P(\nu_\mu \to \nu_e, r) + P(\nu_\tau \to \nu_e, r)\}$$

Since in the factorizable limit the quantities inside the curly brackets do not depend on the CP-violating phase, δ, we conclude that

$$\mathcal{L}_e^{(r)}(\delta \neq 0) = \mathcal{L}_e^{(r)}(\delta = 0)$$

These considerations give us interesting sum rules:

• Electron neutrino survival probability, $P(\nu_e \rightarrow \nu_e)$ is independent of the value of the CP-violating phase, δ; or equivalently

• The combination $P(\nu_\mu \rightarrow \nu_e) + P(\nu_\tau \rightarrow \nu_e)$ at a fixed energy is also independent of the value of the CP-violating phase. Balantekin, Gava, Volpe

• It is possible to derive similar sum rules for other amplitudes. Kneller, McLaughlin

This result holds even if the neutrino-neutrino interactions are included in the Hamiltonian,

Gava and Volpe

Under the stated assumptions, electron neutrino survival probability and, consequently, electron neutrino and antineutrino luminosities are independent of the CP-violating phase. To be able to observe the effects of δ, we need to relax the underlying assumptions:

- Permit the ν_μ and ν_τ luminosities to be different at the neutrinosphere. Standard Model (SM) loop corrections and also physics beyond the Standard Model may do this.

OR

- Explore when $V_{\tau\mu}$ is non-zero due to SM loop corrections and also physics beyond SM.


Thank you very much