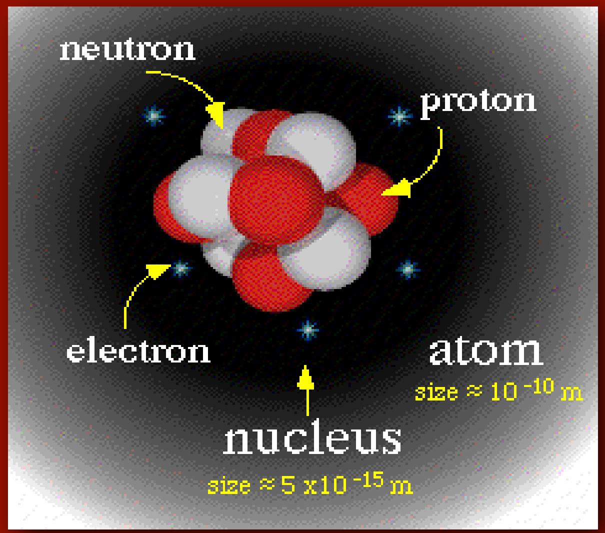
Nuclear Forces and Few-Nucleons Dynamics in Break-up Reactions

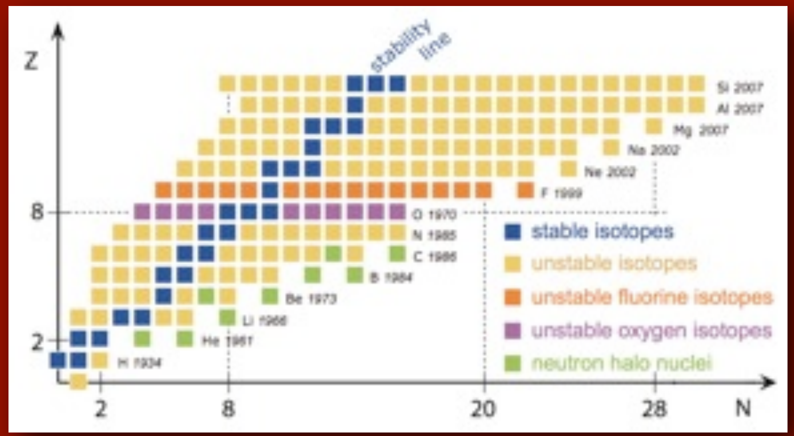
Sonia Bacca

TRIUMF Theory Group

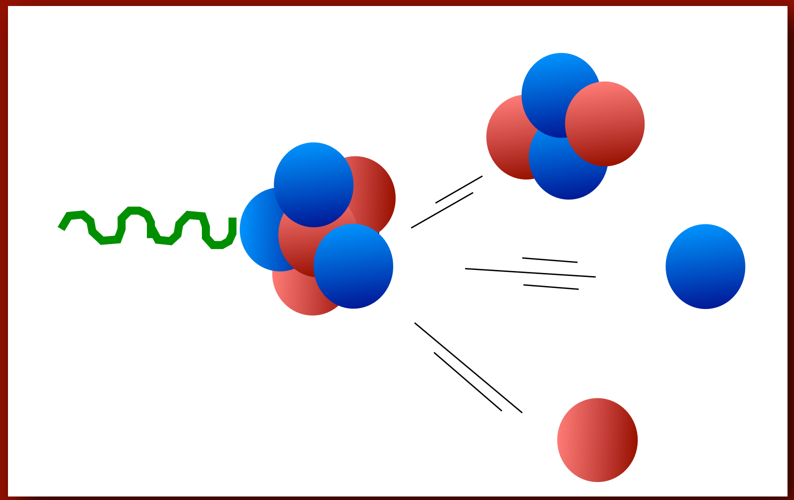
INT program on "Nuclei and Fundamental Symmetries: Theory Needs of the Next-Decade Experiments"
August 6th, 2013



The Atomic Nucleus

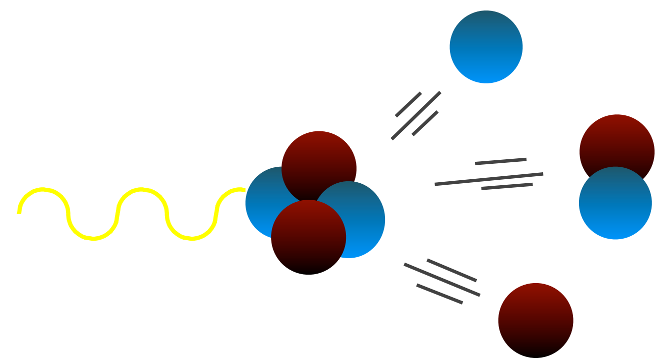


Nuclear Chart


Nuclear Reactions

Motivations

- For few-nucleons one can perform exact calculations both for bound and scattering observables ➡ test the nuclear theory on light nuclei and extend it to heavier mass number

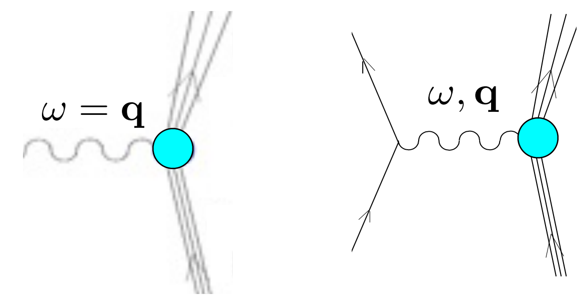


- Electroweak probes (coupling constant <<1)

"*With the electro-magnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself*"

[De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$



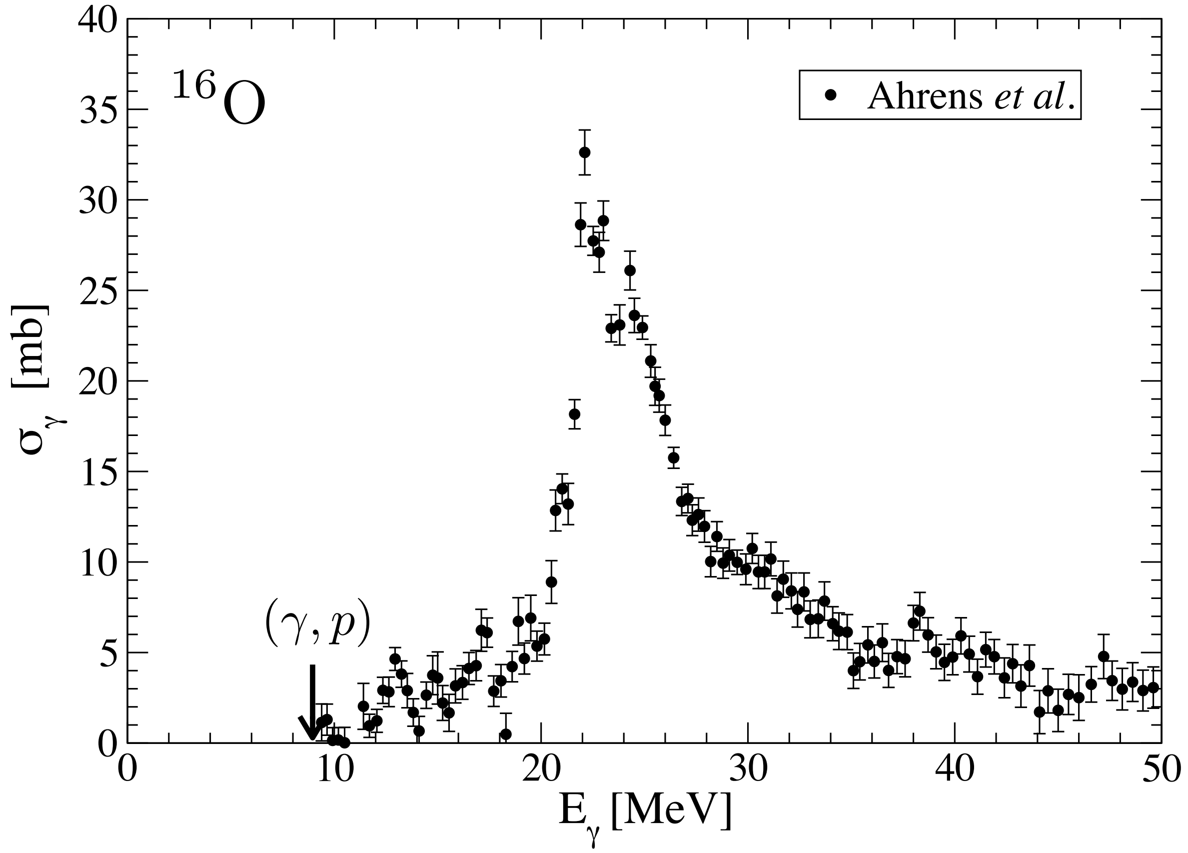
- Provide important informations in other fields of physics, where nuclear physics plays a crucial role:
  - Astrophysics: γ interactions with nucleonic matter, radiative capture reactions, ν interactions with nucleonic matter (vector current as em)
  - Atomic physics (nuclear corrections to atomic levels, etc.)
  - Particle physics (neutrino experiments,...)

Electromagnetic Reactions

Photo-nuclear Reactions

Reactions resulting from the interaction of a photon with the nucleus

For photon energy 15-25 MeV stable nuclei across the periodic table show wide and large peak



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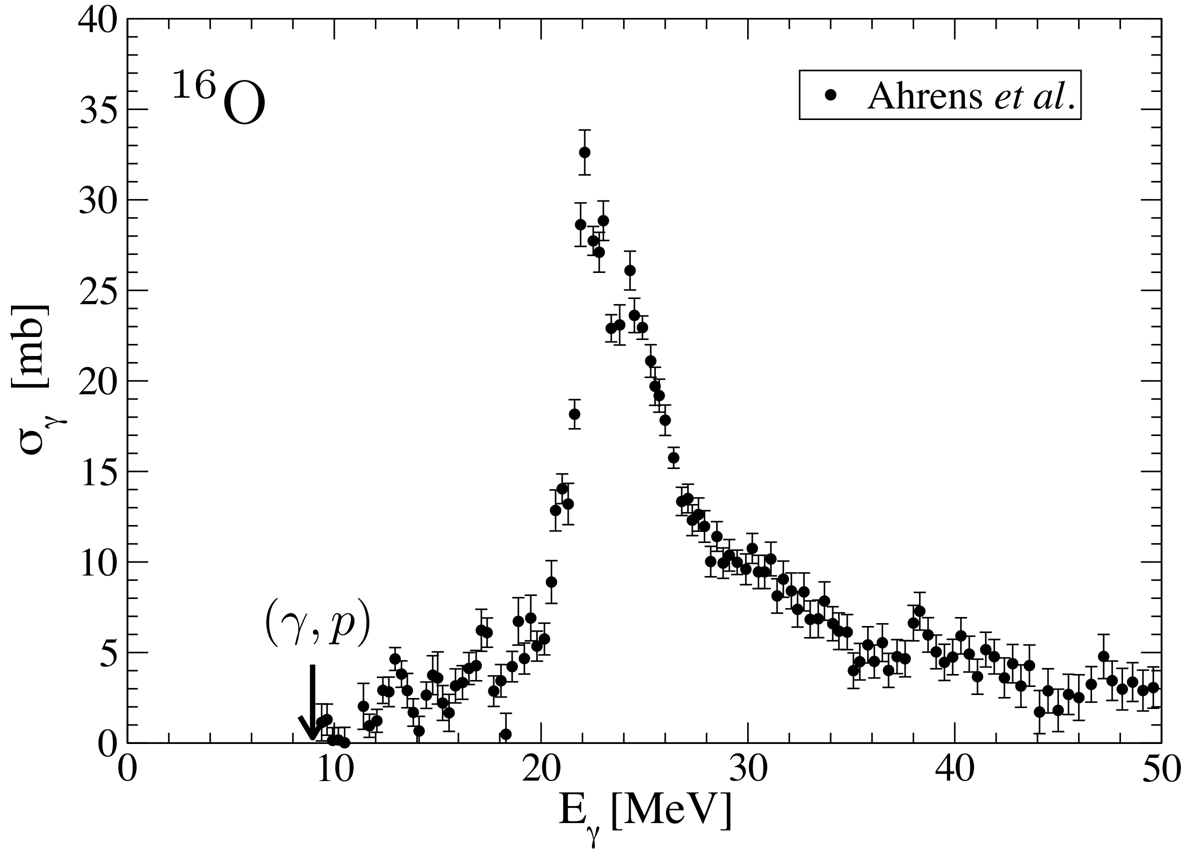
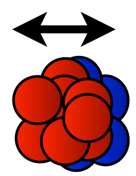


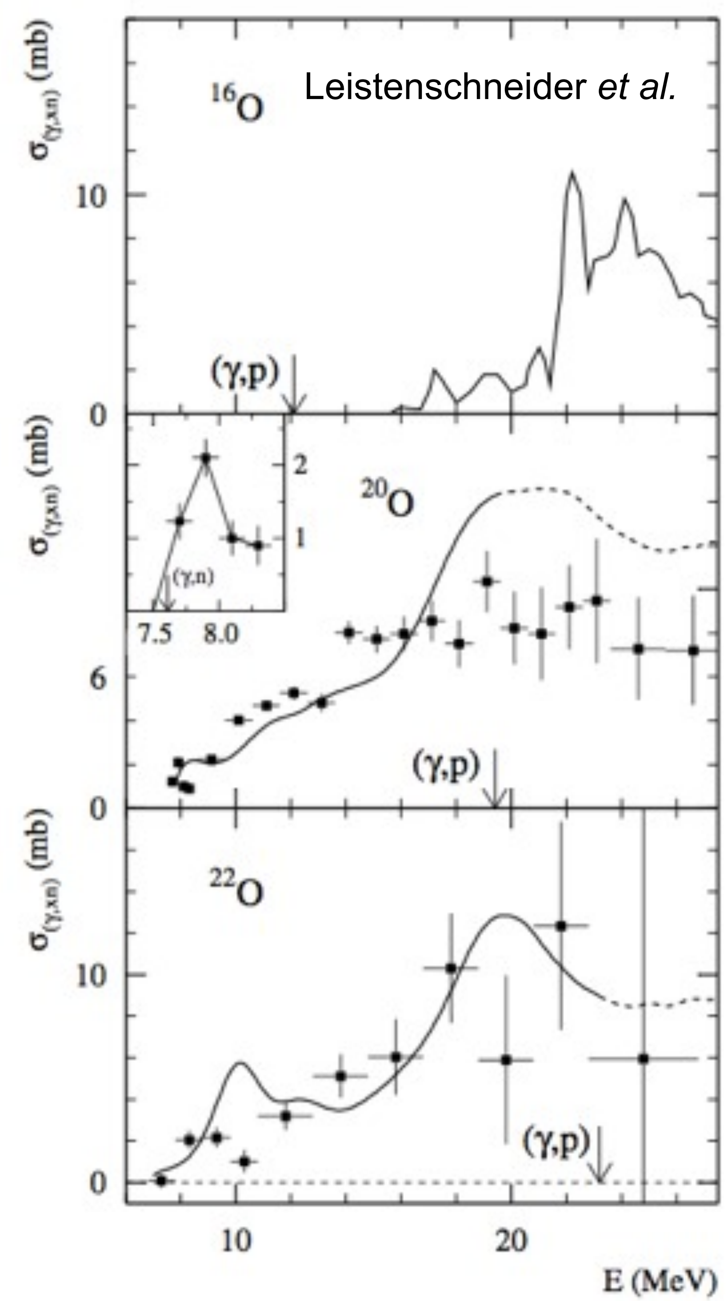
Giant Dipole Resonance



Coulomb excitations

Inelastic scattering between two charged particles. Can use unstable nuclei as projectiles.

Neutron-rich nuclei show fragmented low-lying strength



- Can we give a microscopic explanation of these observations?

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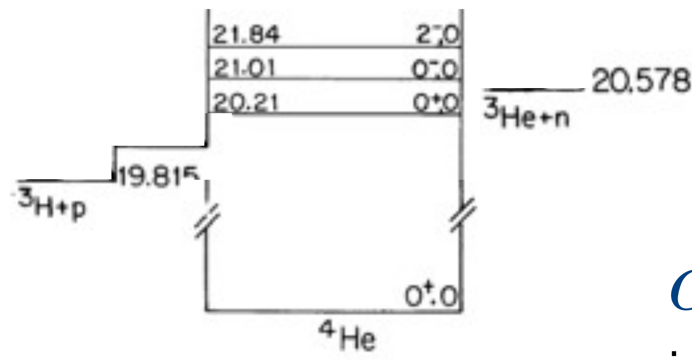
Electromagnetic Reactions

Monopole Resonance in ^{4}He



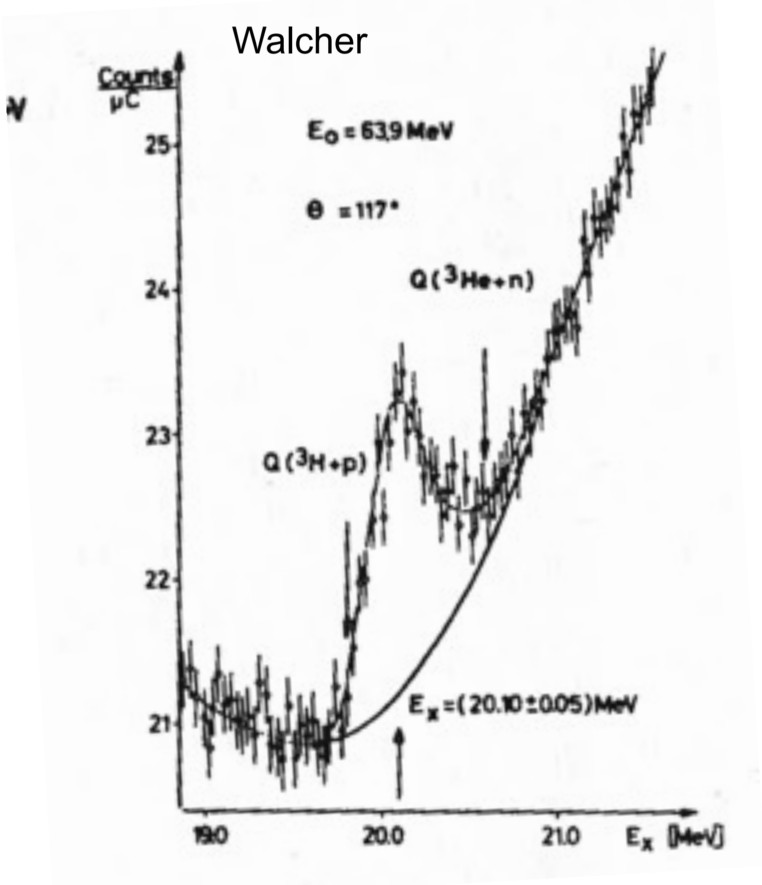
Electron scattering

interaction of a virtual photon with the nucleus

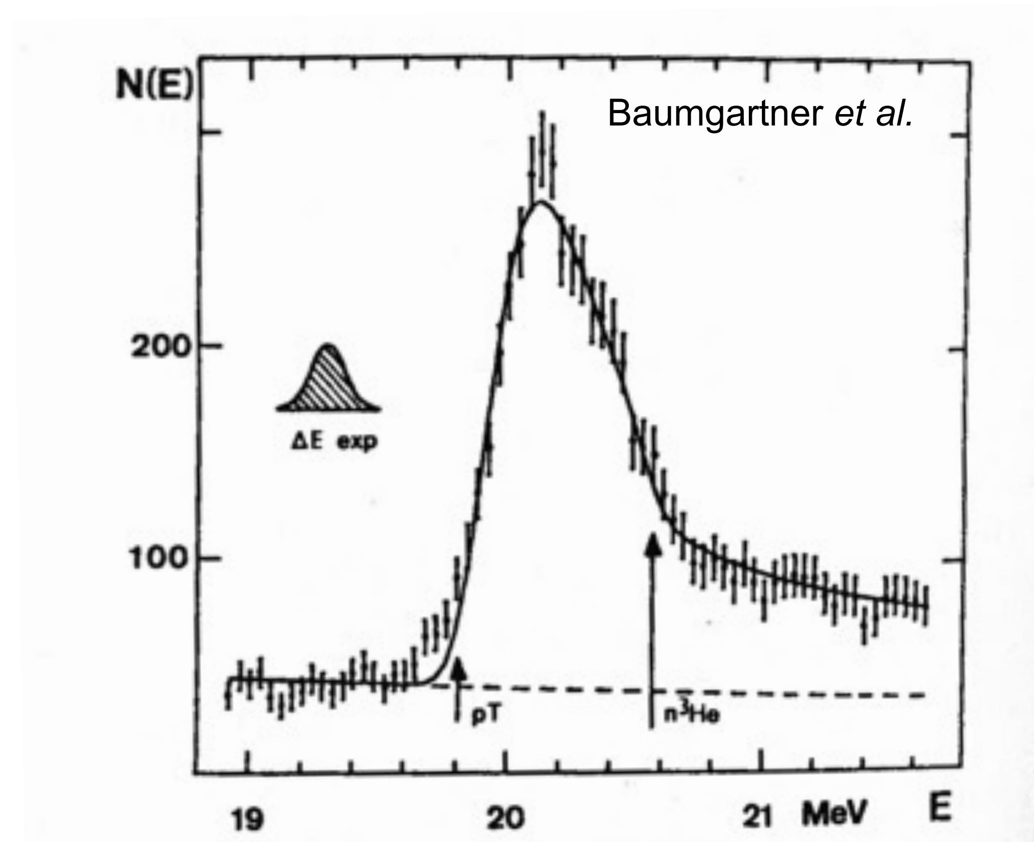
α - scattering

interaction of a virtual photon with the nucleus + strong interaction

Walcher



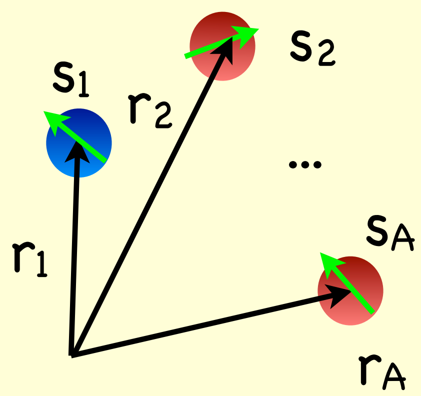
$$E_x = 20.10(5)\ \mathrm{MeV}$$
$$\Gamma = 0.27(5)\ \mathrm{MeV}$$



$$E_x = 20.29(2)\ \mathrm{MeV}$$
$$\Gamma = 0.89(4)\ \mathrm{MeV}$$

- Can we understanding this difference? Can microscopic theories help?

Ab-initio Theory Tools



$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} + \ldots$$

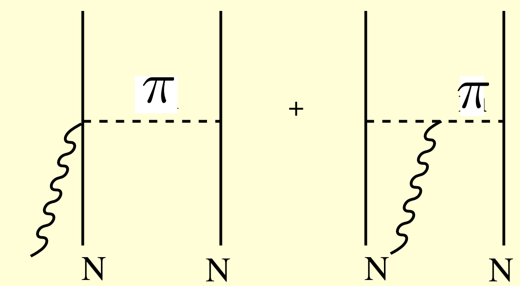
High precision two-nucleon potentials:
well constraint on NN phase shifts

Three nucleon forces:
less known, constraint on A>2 observables

Traditional Nuclear Physics
AV18+UIX, ..., J_2

Effective Field Theory
N^2LO, N^3LO ...

$$J^\mu = J^\mu_N + J^\mu_{NN} + \ldots$$



two-body currents (or MEC)
subnuclear d.o.f.

J^μ consistent with V

$$\nabla \cdot J = -i[V, \rho]$$

$$\sigma \propto \left|\langle \Psi_f | J^\mu | \Psi_0 \rangle\right|^2$$

Exact Initial state &
Final state in the continuum at different energies and for different A

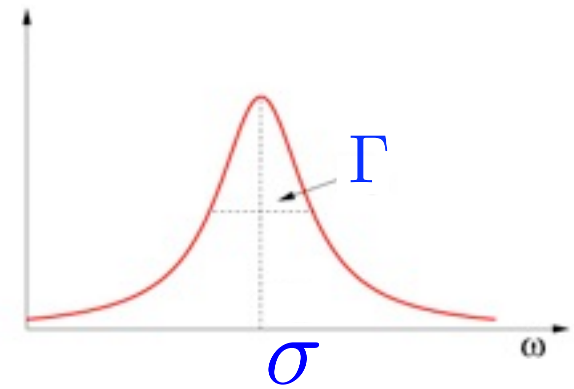
Final State Interaction

Exact evaluation of the final state in the continuum is limited in energy and A

Solution: The Lorentz Integral Transform Method   Efros *et al.*, Nucl.Part.Phys. **34** (2007) R459

Response in the continuum

$$R(\omega) = \sum_f \left| \left\langle \psi_f \middle| J^\mu \middle| \psi_0 \right\rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

$$(H - E_0 - \sigma + i\Gamma) \, | \, \tilde{\psi} \rangle = J^\mu \, | \, \psi_0 \rangle$$

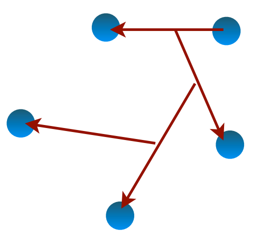
- Due to imaginary part Γ the solution $|\tilde{\psi}\rangle$ is unique
- Since the r.h.s. is finite, then $|\tilde{\psi}\rangle$ has bound state asymptotic behavior

$L(\sigma, \Gamma)$ $\xrightarrow{\text{inversion}}$ $R(\omega)$ with the exact final state interaction

You can use any good bound state method!   e.g. Hyperspherical Harmonics, No Core Shell Model, Coupled Cluster Theory

Hypersperical Harmonics

Starts from relative coordinates $|\psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})\rangle$



$\vec{\eta}_0 = \sqrt{A}\vec{R}_{CM} \quad \vec{\eta}_1, \ldots, \vec{\eta}_{A-1}$

Recursive definition of hyper-spherical coordinates

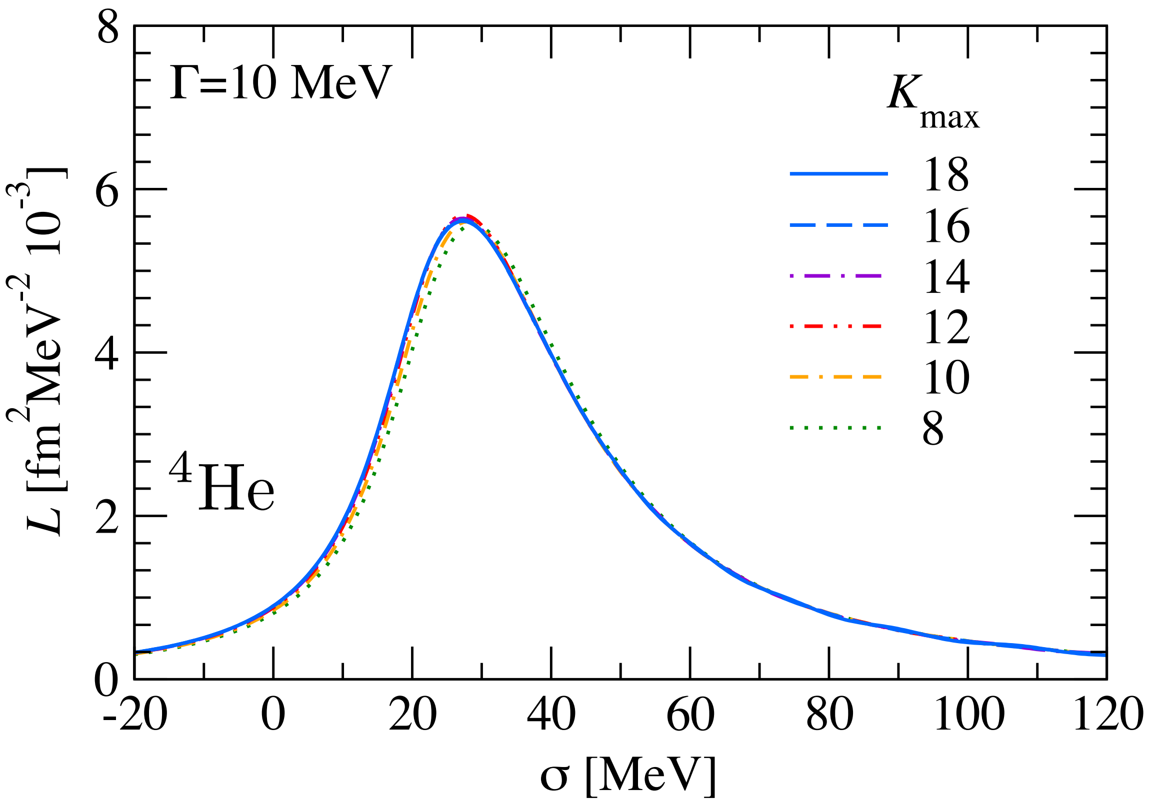
$$\rho, \Omega \qquad \rho^2 = \sum_{i=1}^{A} r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

Kinetic Energy $H_0(\rho, \Omega) = T_\rho - \frac{K^2(\Omega)}{\rho^2}$ → HH eigenstates of K^2

- Use HH as a basis to expand the wf $$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2b} \rho^{n/2} L_\nu^n\left(\frac{\rho}{b}\right) [\mathcal{Y}_{[K]}^{\mu}(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^{a}$$
- Model space truncation $K \leq K_{max}$ → Matrix diagonalization
- Anstisymmetrization algorithm Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

The LIT with Hyperspherical Harmonics

Numerical example: Dipole Response Function of ^{4}He $J^\mu \longrightarrow \hat{D}_z = \sum_i^Z (z_i - Z_{\rm cm})$ with NN(N^{3}LO)



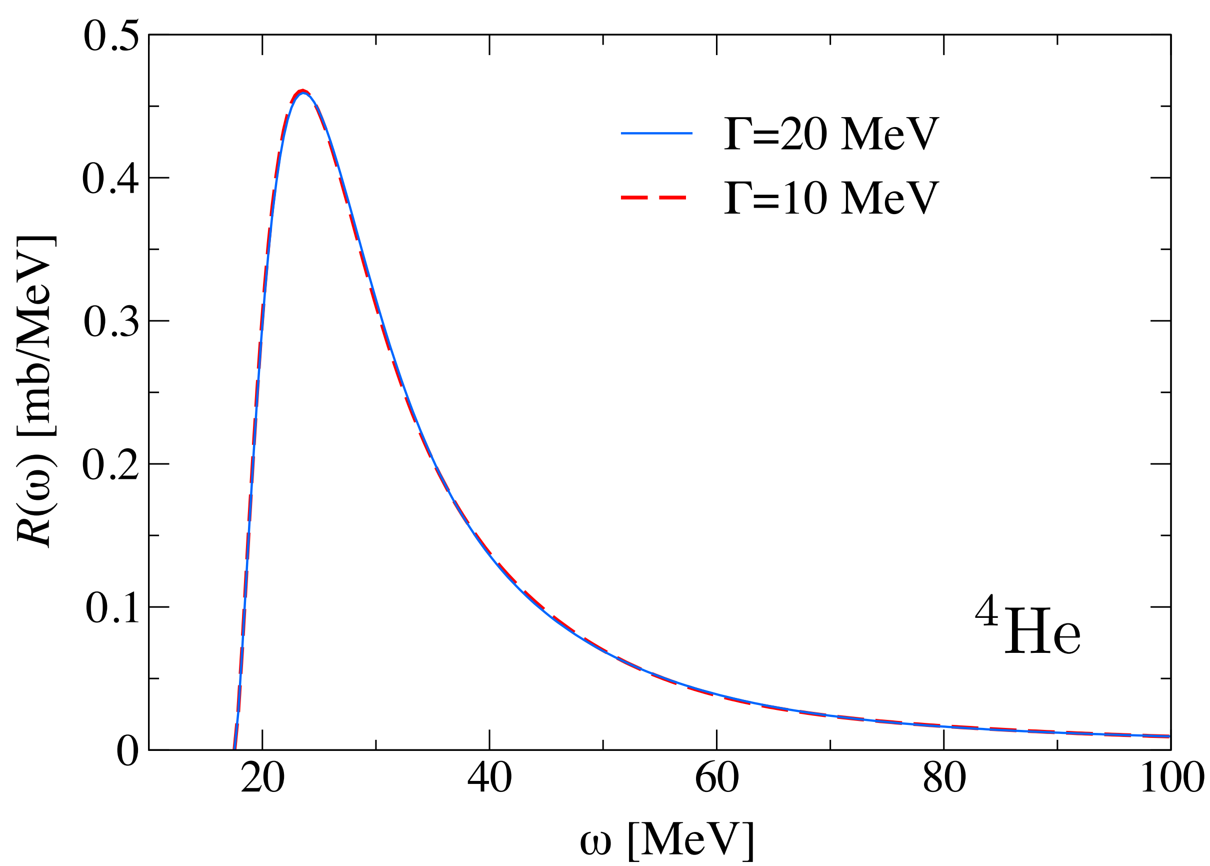
Inversion of the LIT

Ansatz

$$R(\omega) = \sum_i^{I_{\rm max}} c_i \chi_i(\omega, \alpha)$$

$$L(\sigma, \Gamma) = \sum_i^{I_{\rm max}} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

Least square fit of the coefficients c_i to reconstruct the response function



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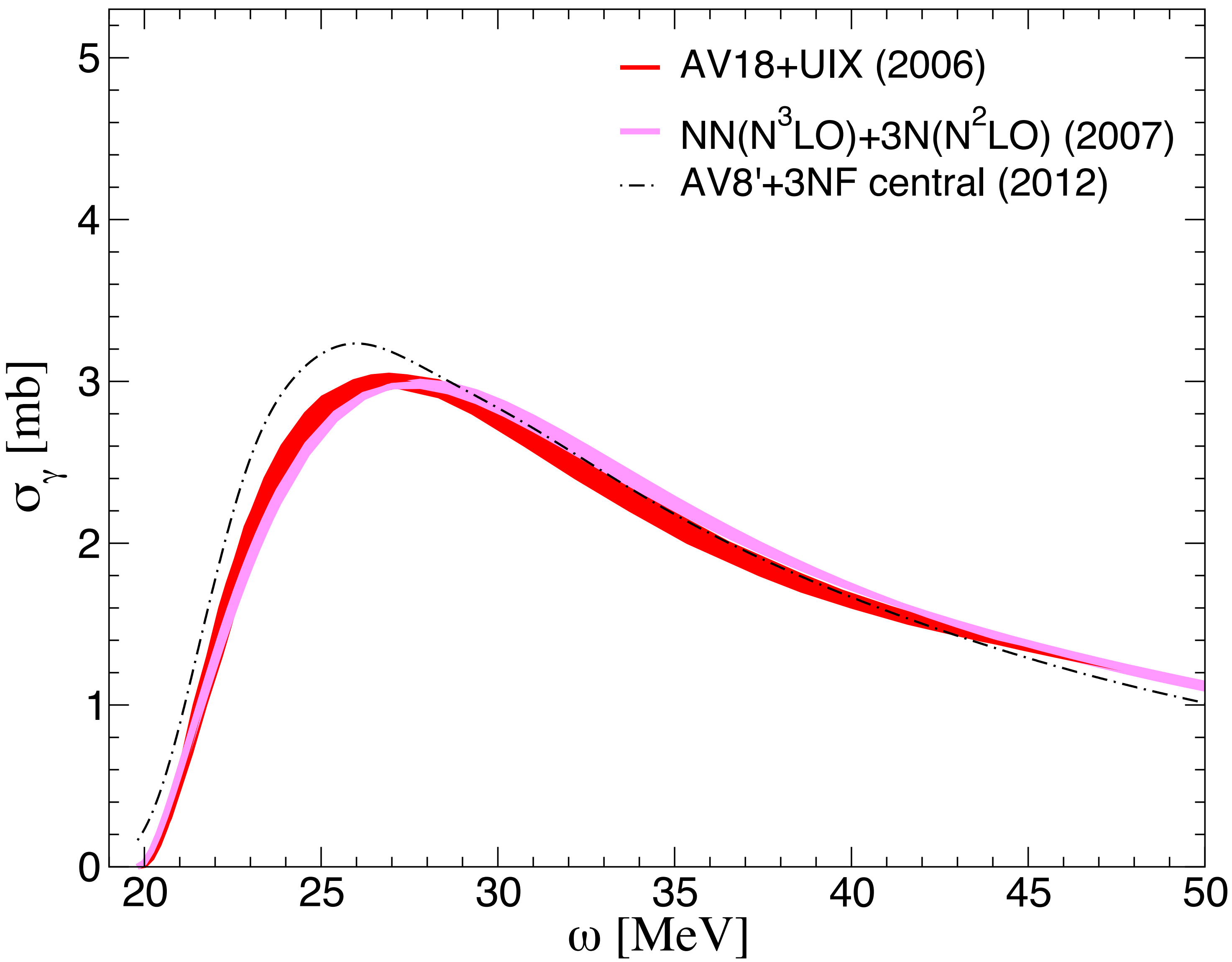
Applications

$\gamma + {}^4\mathrm{He} \longrightarrow X$



$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

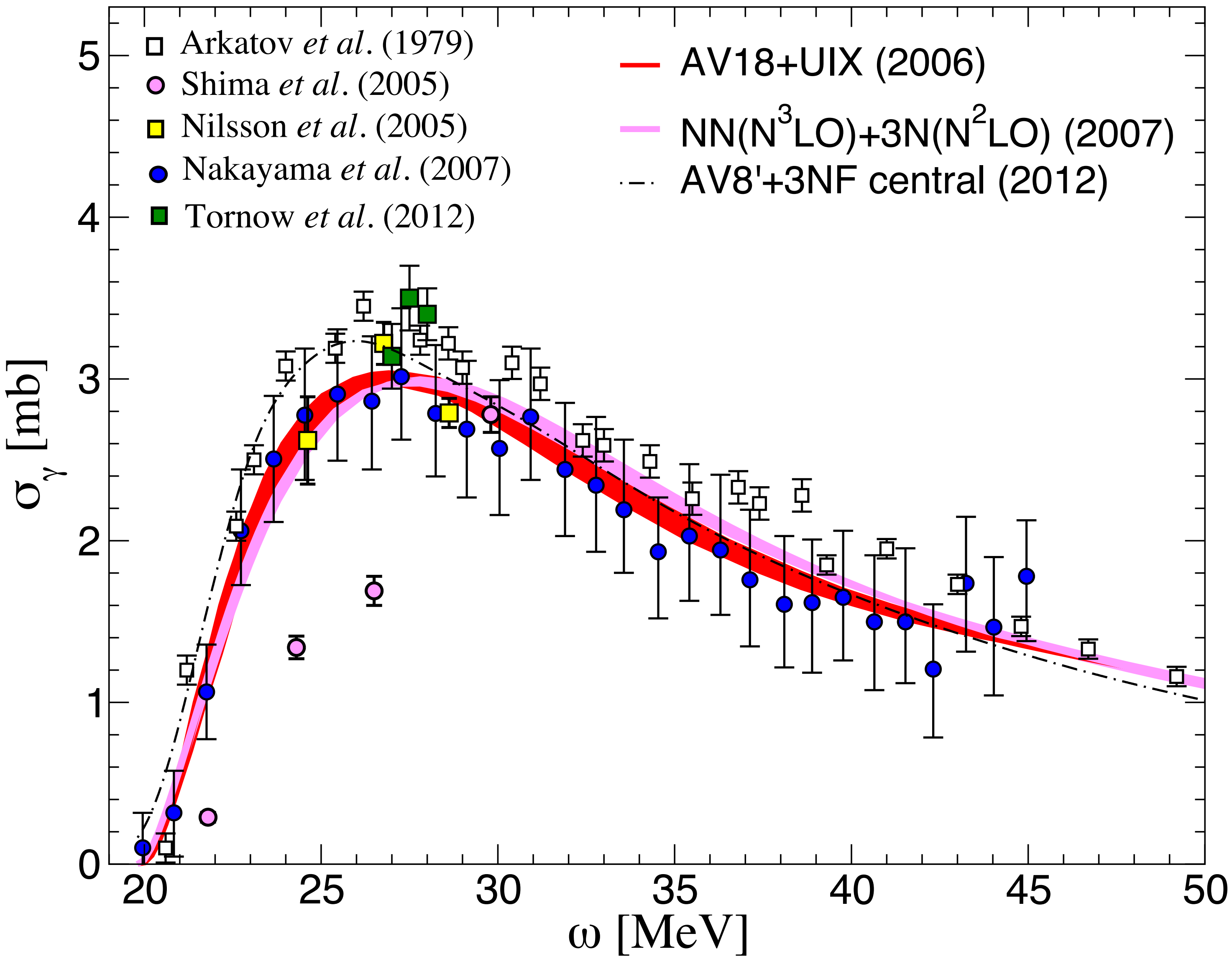
Traditional Hamiltonian
PRL 96 112301 (2006)

Hamiltonian from χEFT
S.Quaglioni and P.Navratil PLB 652 (2007)

Realistic NN + phenomenological central 3NF
W.Horiuchi et al. PRC 85 054002 (2012)

➡ Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak

$\gamma + {}^4\text{He} \longrightarrow X$



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Realistic NN + phenomenological central 3NF
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- Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak
- More recent experimental activity seems to confirm higher data with peak around 27 MeV
- Theoretical precision is better than experimental error

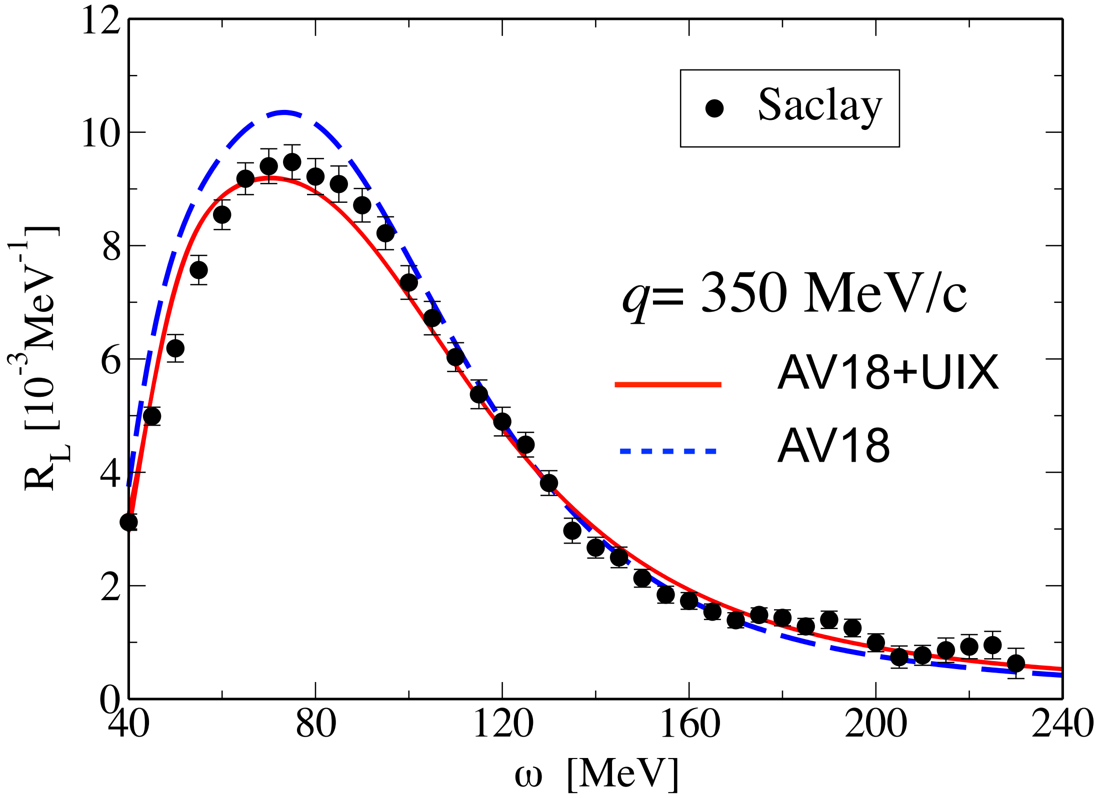
Useful in Muonic Atoms arXiv:1307.6577
See talk by C.Ji

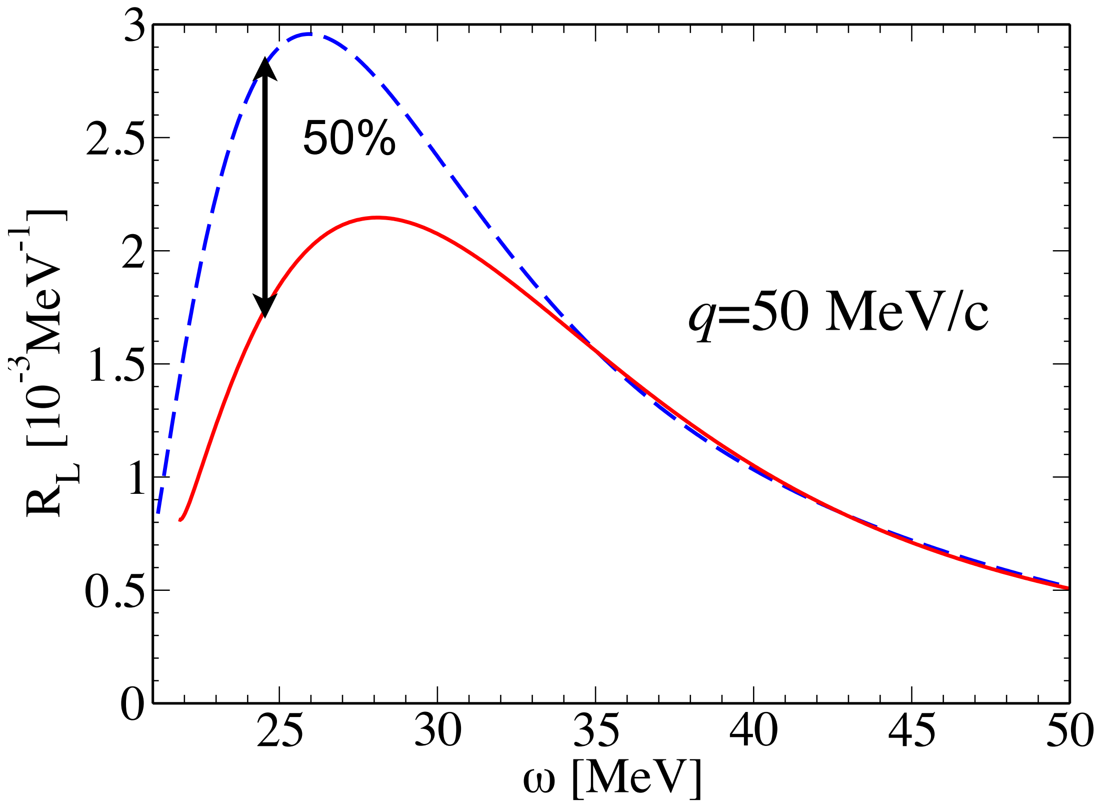
$^4He(e,e')X$

PRL **102**, 162501 (2009)
PRC **80**, 064001 (2009)

$$R_L(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle \Psi_f | \rho(\mathbf{q}) | \Psi_0 \rangle|^2 \, \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

two-body currents are not important





Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%

^{4}He(e,e')X

PRL **102**, 162501 (2009)
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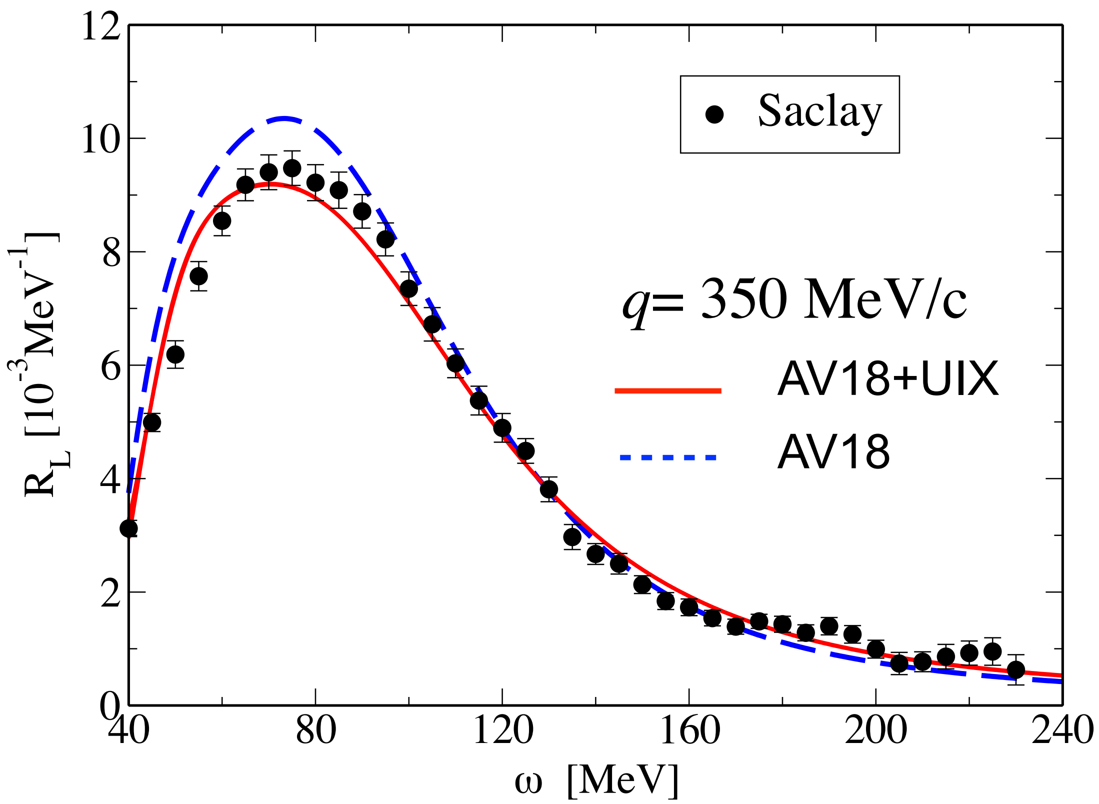
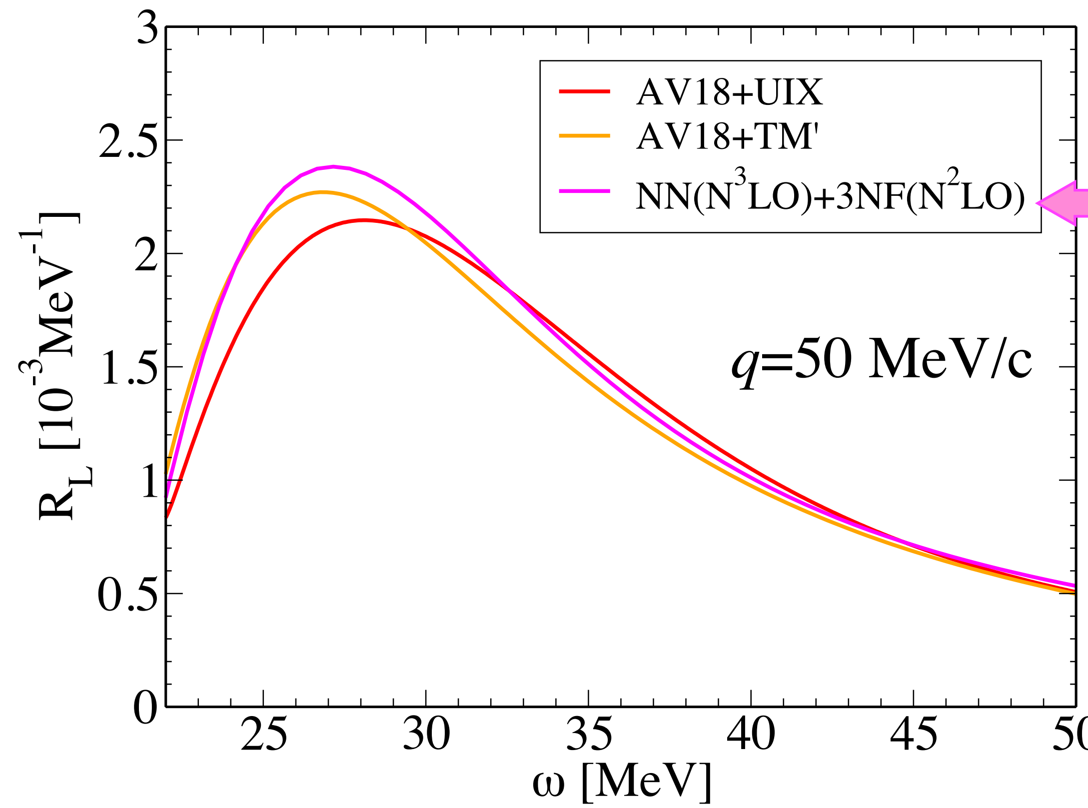
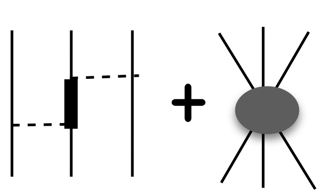
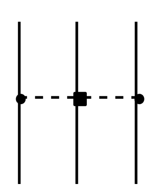


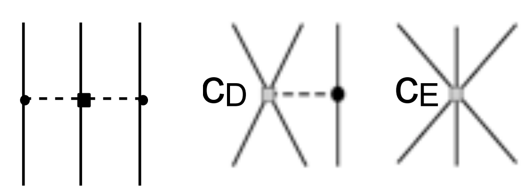


- Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%
- It is not a simple binding effect!

	AV18+UIX	AV18+TM'	NN(N^{3}LO)+3NF(N^{2}LO)
B.E./MeV	28.40	28.46	28.36(2)







- Stimulating new experiments:
  - MAMI taken data q>150
  - S-DALINAC can possibly take data at lower q

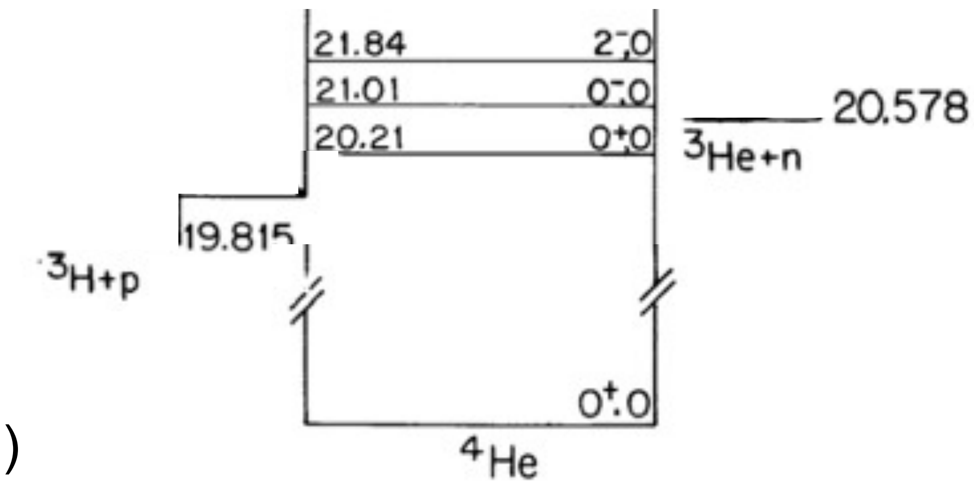
^{4}He(e,e')0$^+$

PRL **110**, 042503 (2013)

Monopole Transition Form Factor

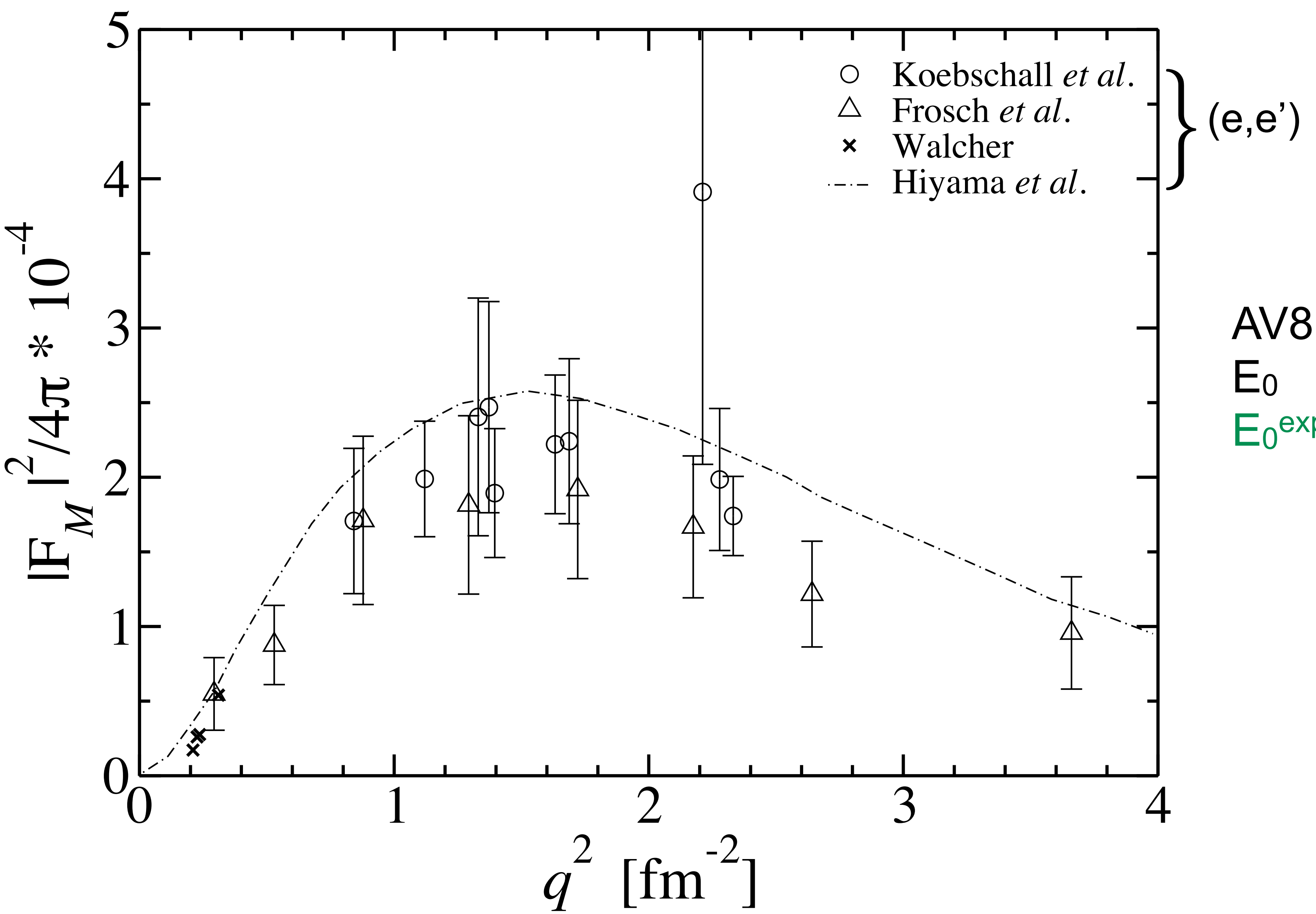
$$0_1^+ \longrightarrow 0_2^+$$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2}\int d\omega R_{\mathcal{M}}^{\rm res}(q,\omega)$$



First *ab-initio* calculation: Hiyama *et al.*, PRC **70** 031001 (2004)

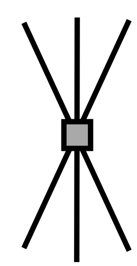
obtained good description of data with phenomenological central 3N



AV8' + central 3NF

E_0 = -28.44 MeV

E_0^{exp}= -28.30 MeV



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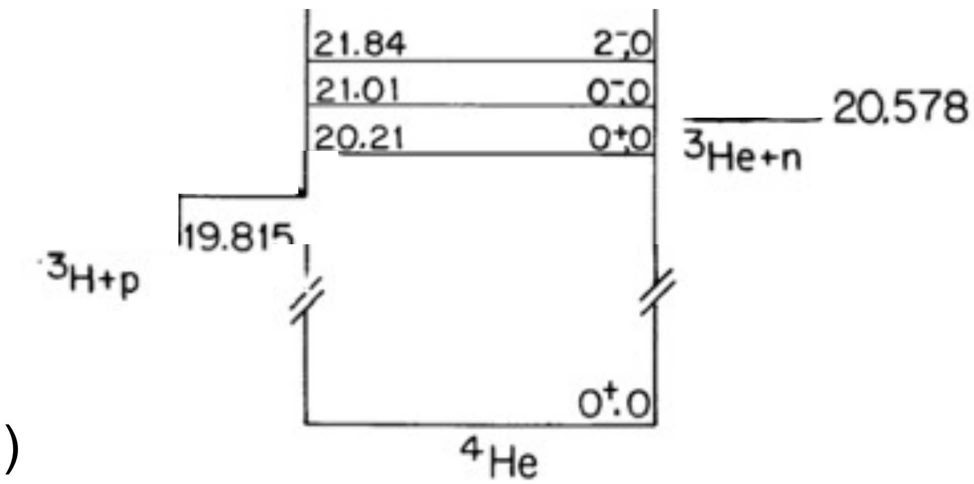
^{4}He(e,e')0$^+$

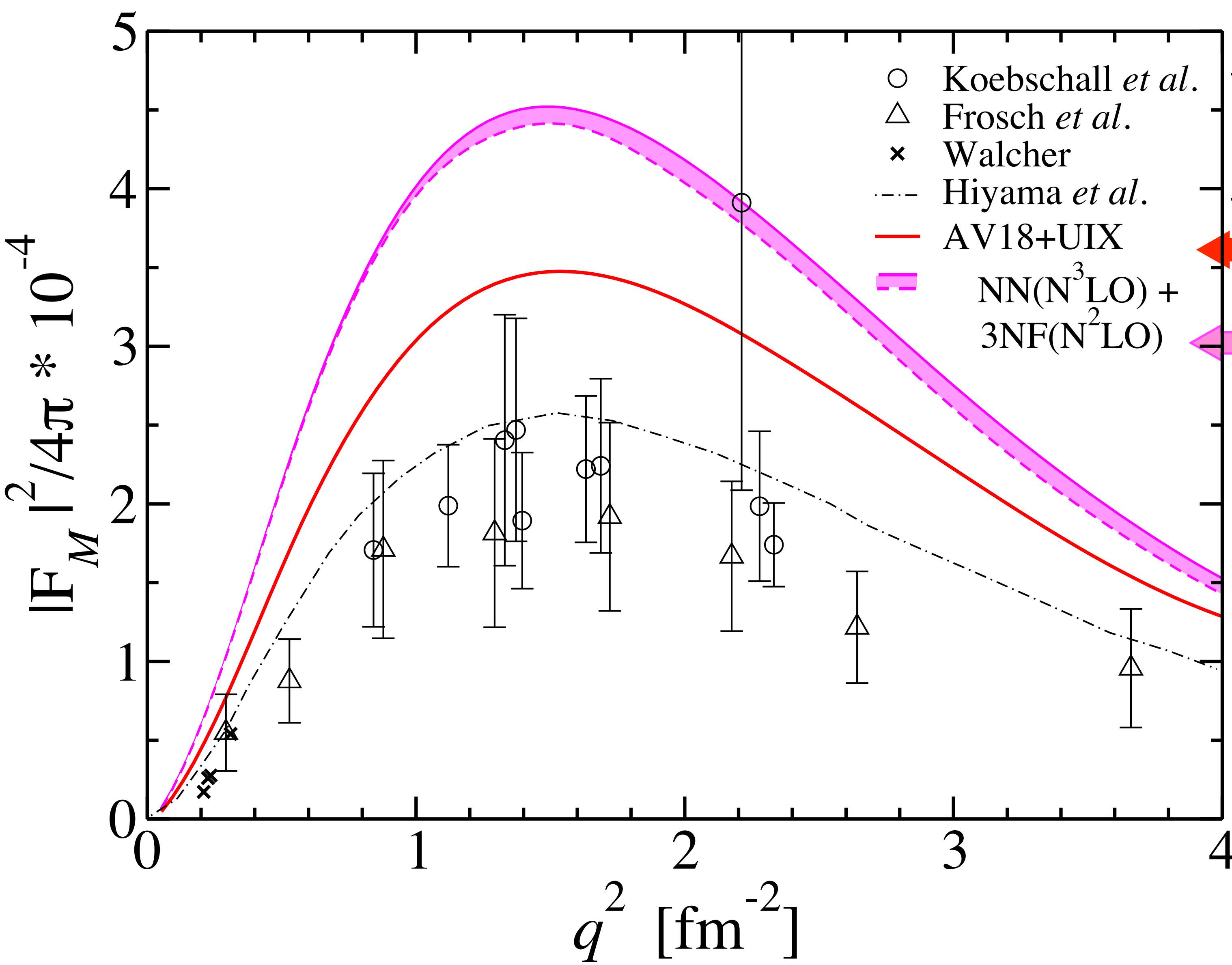
PRL **110**, 042503 (2013)

Monopole Transition Form Factor $0_1^+ \longrightarrow 0_2^+$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2}\int d\omega R_{\mathcal{M}}^{\text{res}}(q,\omega)$$



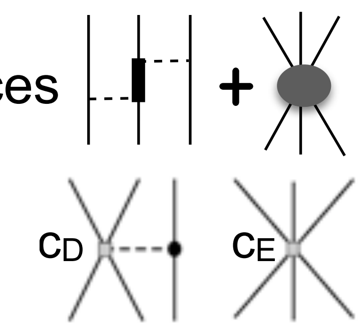
First *ab-initio* calculation with realistic three-nucleon forces and with the Lorentz Integral Transform method



(e,e')

traditional forces

χEFT forces



F.Cappuzzello: plan to measure $F_{\mathcal{M}}$ with α-scattering in Catania

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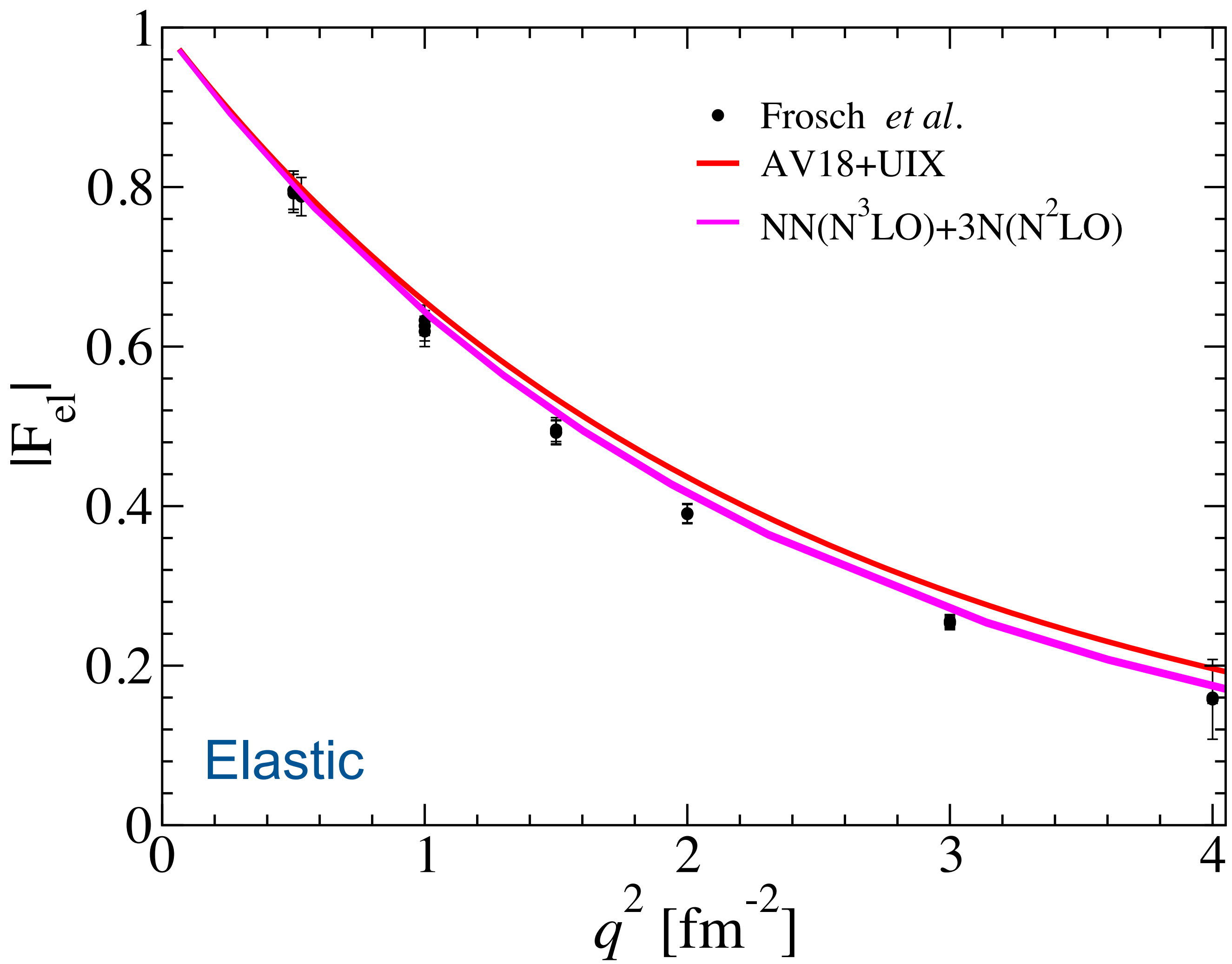
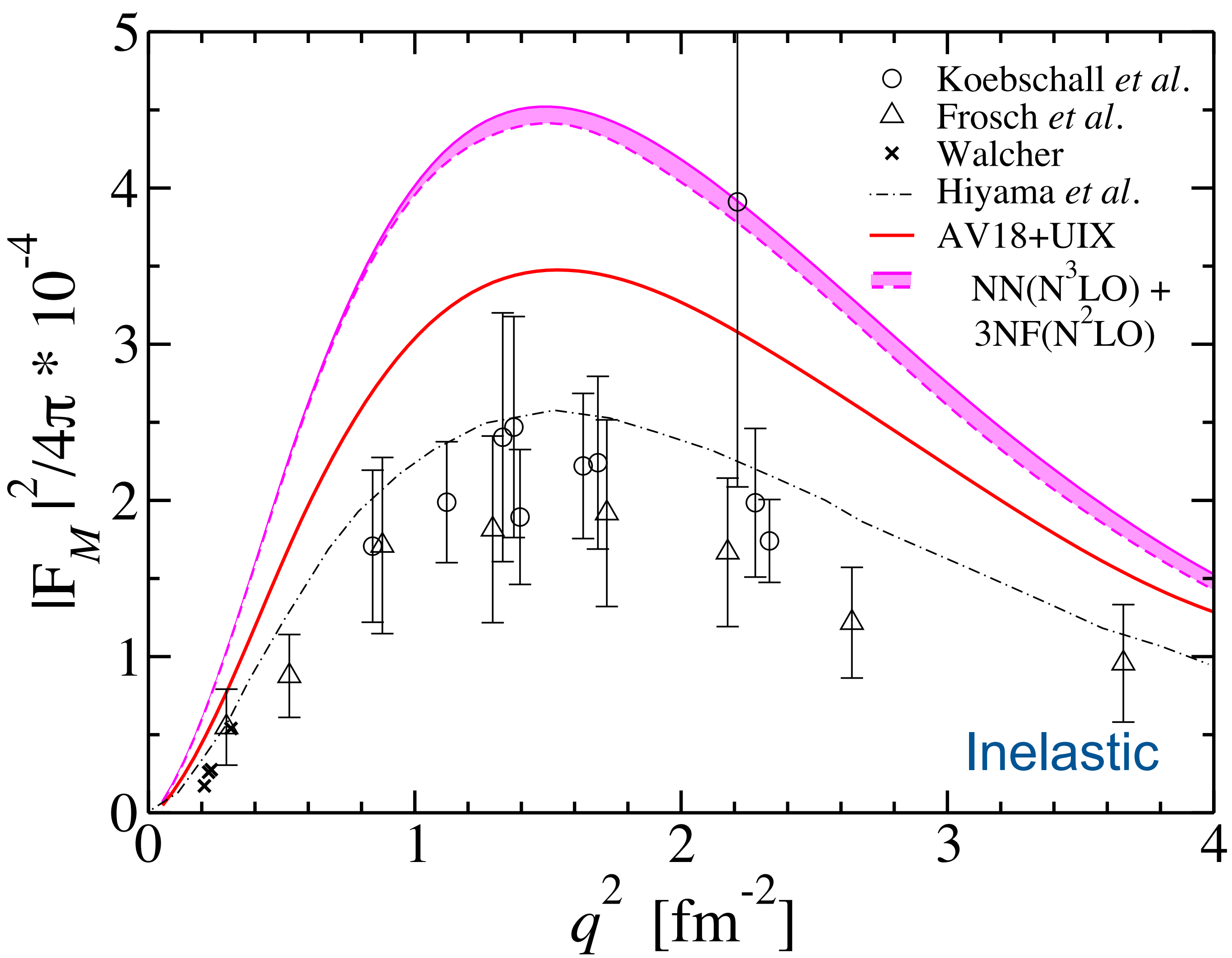
Sensitivity to Nuclear Hamiltonians

PRL **110**, 042503 (2013)

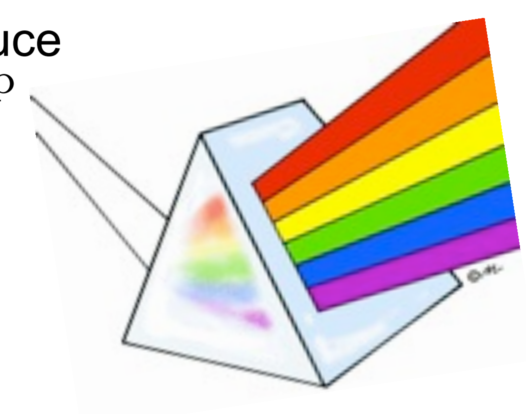




➡ The inelastic monopole resonance acts as a prism to nuclear Hamiltonians.

AV8' + central 3NF	E_0 = -28.44 MeV
AV18+UIX	E_0 = -28.40 MeV
NN(N^{3}LO)+3NF(N^{2}LO)	E_0 = -28.36 MeV
	E_0^{exp}= -28.30 MeV

H reproduce $E_0^{\rm exp}$



spectrum in $|F_{\mathcal{M}}|^2$

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Analysis of this result

Realistic three-nucleon forces do not reproduce the data for $|F_{\mathcal{M}}|^2$. Particularly large difference are found with chiral EFT potentials. This is unexpected! What can be the source of this behaviour?

- **Numerics?** Our calculations are well converged (few % level) in the HH basis

$K_{\max}$	12	14	16	18
$10^4\|F_{\mathcal{M}}\|^2$	4.59	4.75	4.85	4.87

- **Many-body charge operators?**

  Conventional Nuclear Physics

  Impulse approximation valid for elastic form factor below 2 fm^{-1}
  Viviani *et al.*, PRL **99** (2007) 112002

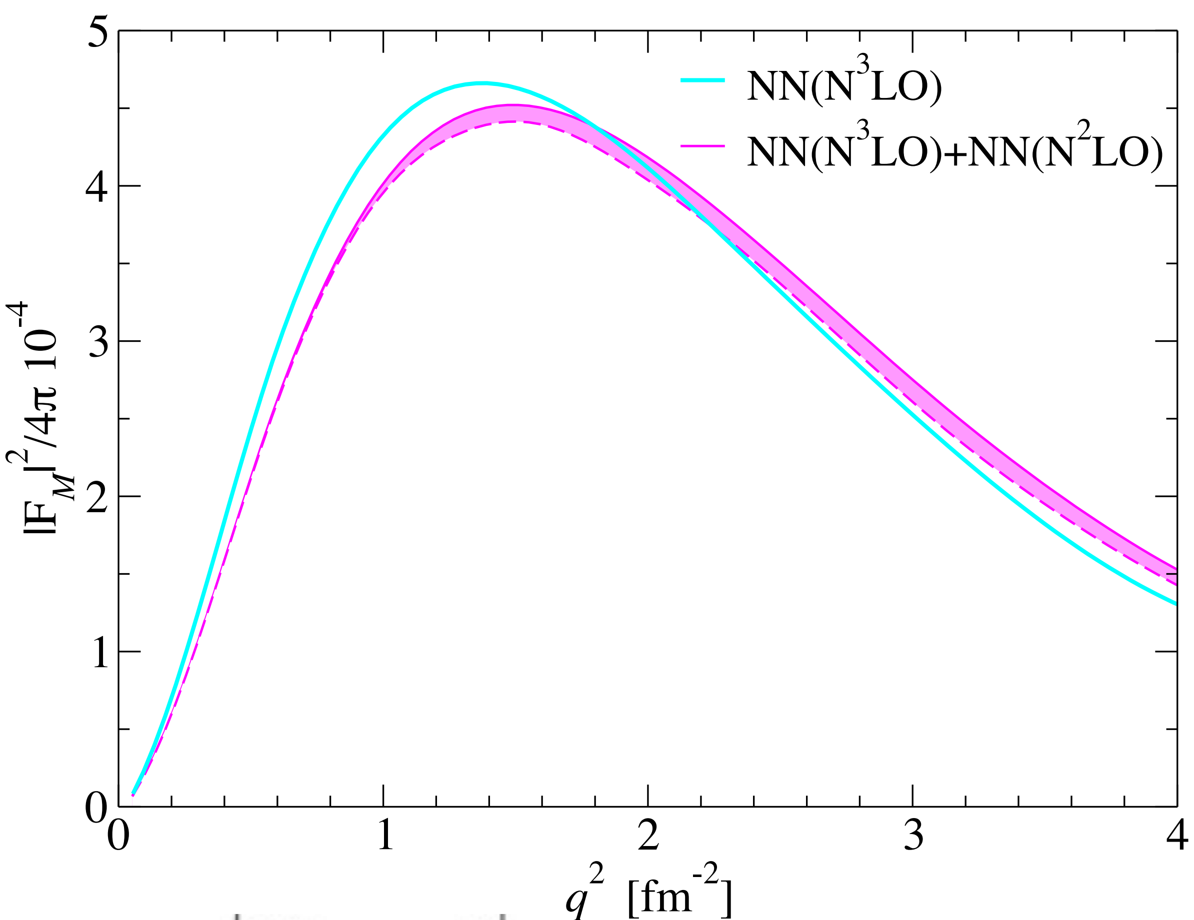
  EFT approach

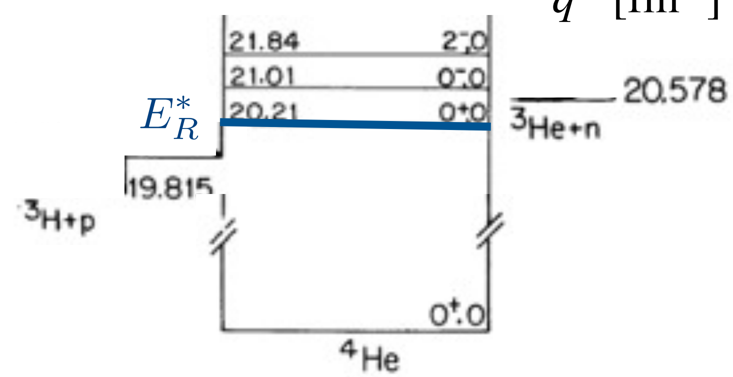
  Park *et al.,* Epelbaum, Koelling *et al.,* Pastore *et al.:* many-body operators appear at high oder in EFT

- **Higher order 3NF (N^3LO)?** Unlikely...

- **Location of the resonance?**

AV8' + central 3NF	E_R^* = 20.25 MeV
AV18+UIX	E_R^* = 21.00(20) MeV
NN(N^3LO)+3NF(N^2LO)	E_R^* = 21.01(30) MeV
	E_R^* = 20.21 MeV





Tuesday, 6 August, 13

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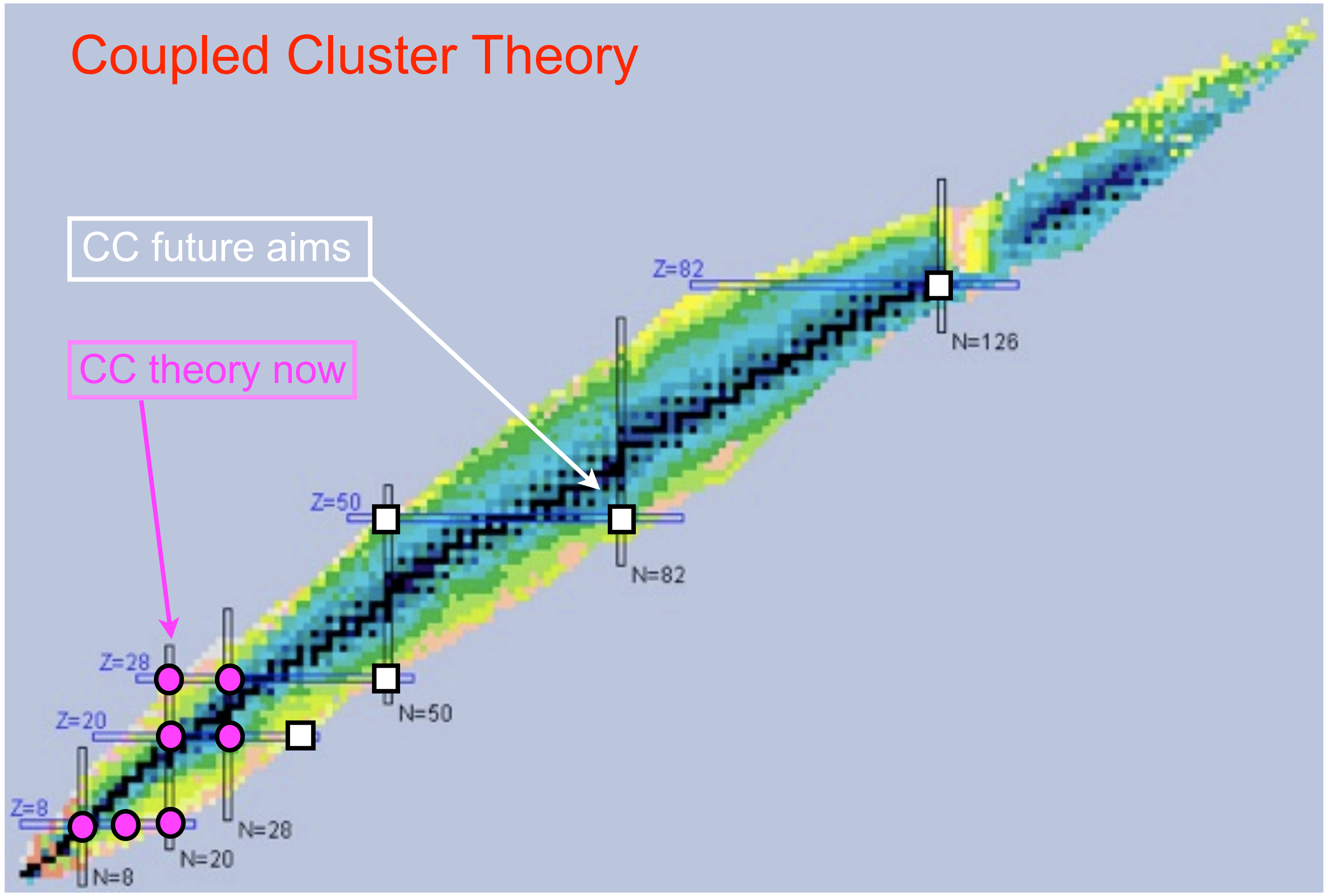
Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei



- CC is optimal for closed shell nuclei (±1, ±2)

Uses particle coordinates

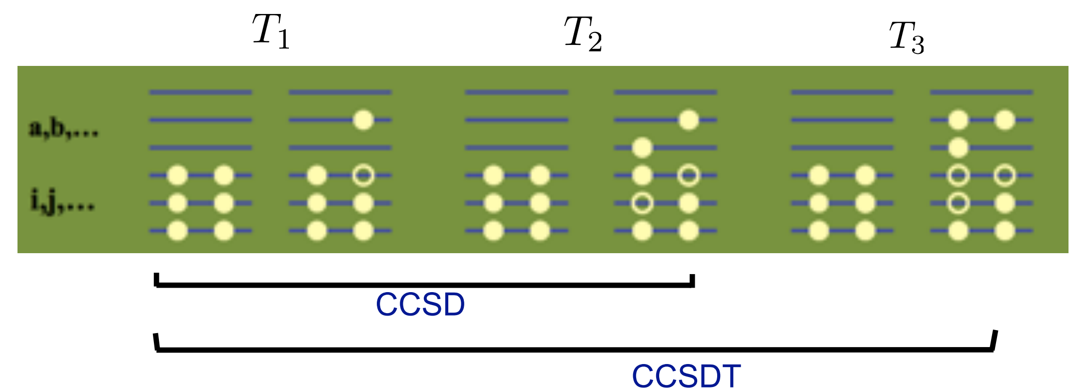
$$|\psi_0(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)\rangle = e^T|\phi_0(\vec{r}_1,\vec{r}_2,...,\vec{r}_A)\rangle$$

reference SD with any sp states

$T = \sum T_{(A)}$ cluster expansion

$$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$

$$T_2 = \frac{1}{4}\sum_{ij,ab} t_{ij}^{ab}\ a_a^\dagger a_b^\dagger a_j a_i \qquad \cdots$$



For the ground state energy

$$E_0 = \langle\phi_0|e^{-T}He^{T}|\phi_0\rangle \qquad \bar{H} = e^{-T}He^{T}$$

similarity transformed Hamiltonian

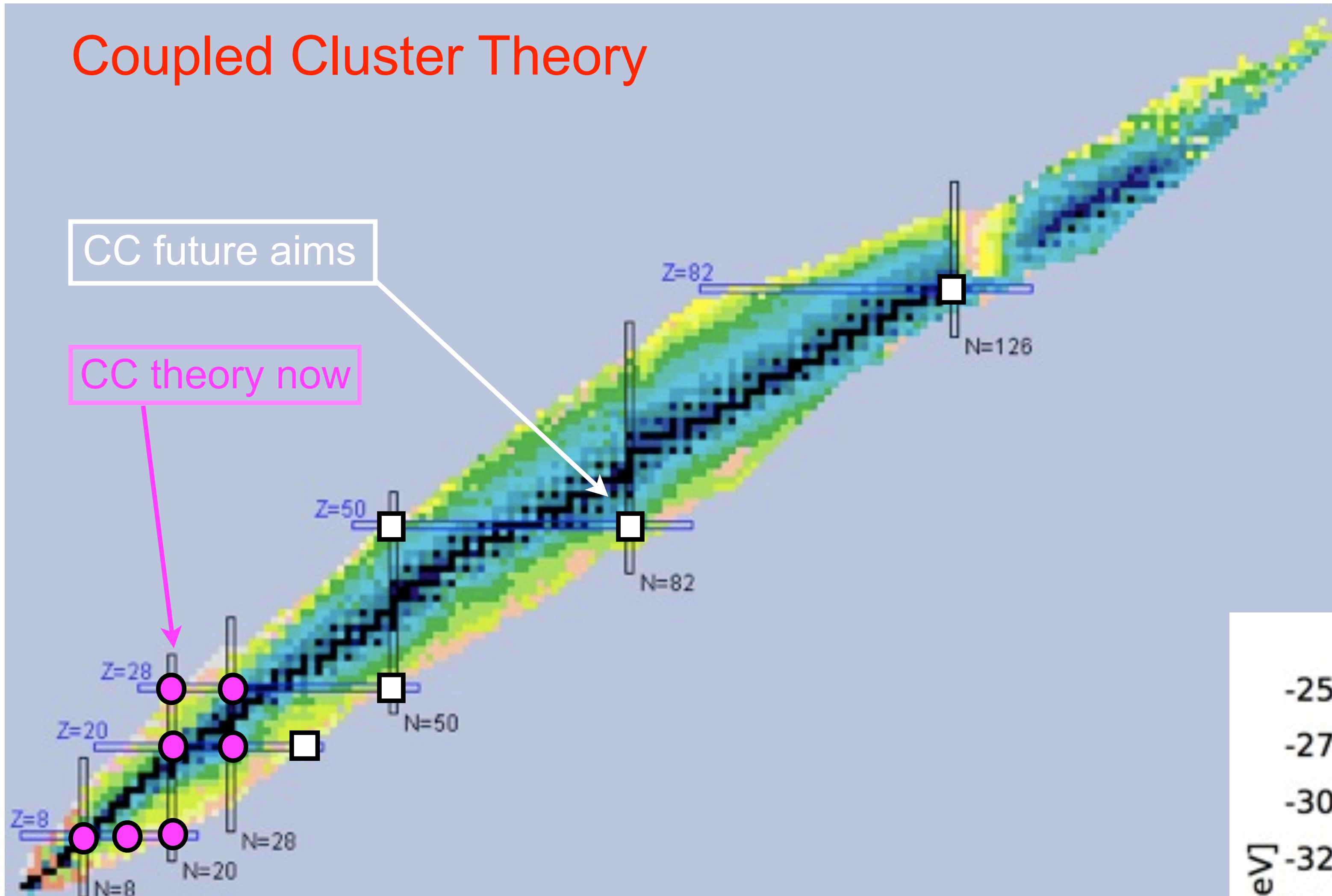
$$0 = \langle\phi_i^a|e^{-T}He^{T}|\phi_0\rangle$$

$$0 = \langle\phi_{ij}^{ab}|e^{-T}He^{T}|\phi_0\rangle$$

Leads to CCSD equations for the t-amplitudes

Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei



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↳ reference SD with any sp states

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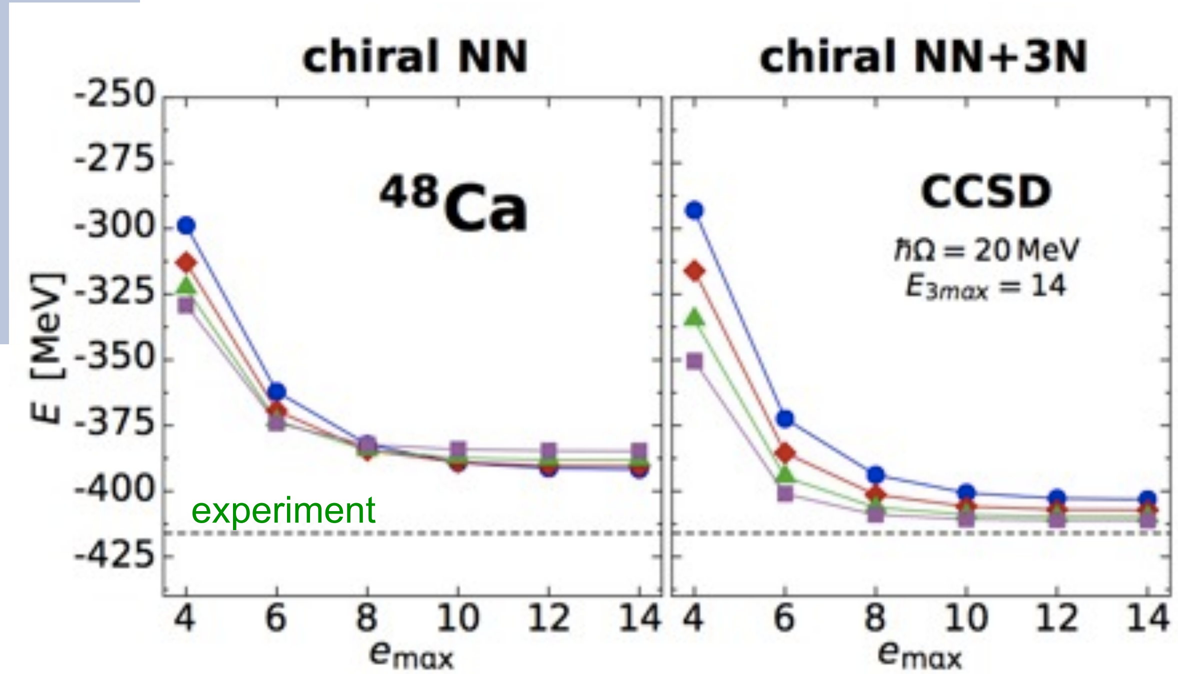


R. Roth *et al.*, Phys. Rev. Lett. **109**, 052501 (2012)

CC is a very mature theory for g.s., see e.g.

Hagen *et al.* PRL **101**, 092502 (2008),PRC **82**, 03433 (2010)
PRL **108**, 242501 (2012), PRL **109,** 032502 (2012)

What about electromagnetic reactions?

LIT with Coupled Cluster Theory

arXiv:1303.7446

New theoretical method aimed at extending *ab-initio* calculations towards medium mass

$$L(\sigma,\Gamma) = \int d\omega \frac{R(\omega)}{(\omega-\sigma)^2+\Gamma^2} \rightarrow \left\langle \tilde{\Psi}_L | \tilde{\Psi}_R \right\rangle$$

$$[\bar{H}, \hat{R}(z^*)]|\Phi_0\rangle = (z^* - E_0)\hat{R}(z^*)|\Phi_0\rangle + \bar{\Theta}|\Phi_0\rangle$$

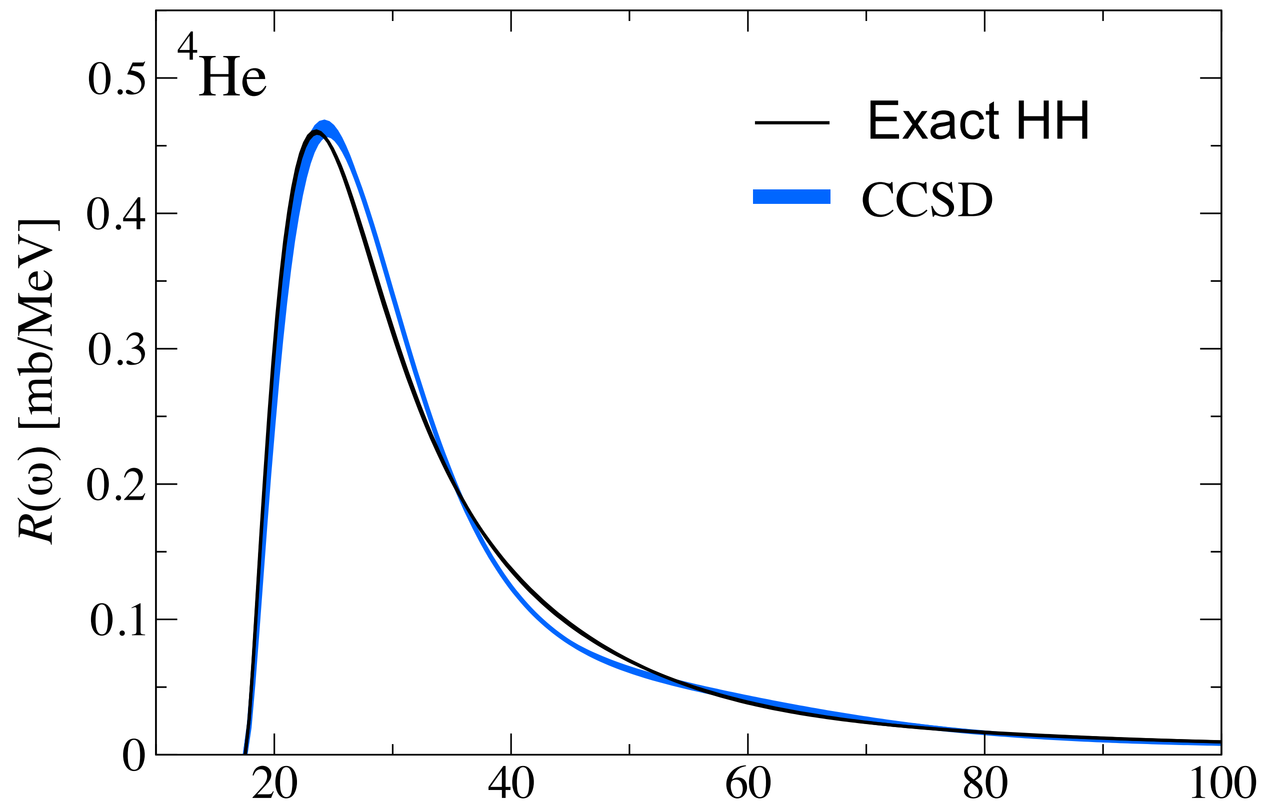
The LIT equation becomes EoM with $z = E_0 + \sigma + i\Gamma$

CCSD scheme $\bar{\Theta} = e^{-T}\Theta e^{T}$

$$\hat{R} = \hat{R}_0 + \sum_{ia} \hat{R}_i^a \hat{c}_a^\dagger \hat{c}_i + \frac{1}{4}\sum_{ijab} \hat{R}_{ij}^{ab} \hat{c}_a^\dagger \hat{c}_b^\dagger \hat{c}_j \hat{c}_i + \dots$$

Validation for ^{4}He

Dipole Response Functions
with NN forces from χEFT (N^{3}LO)



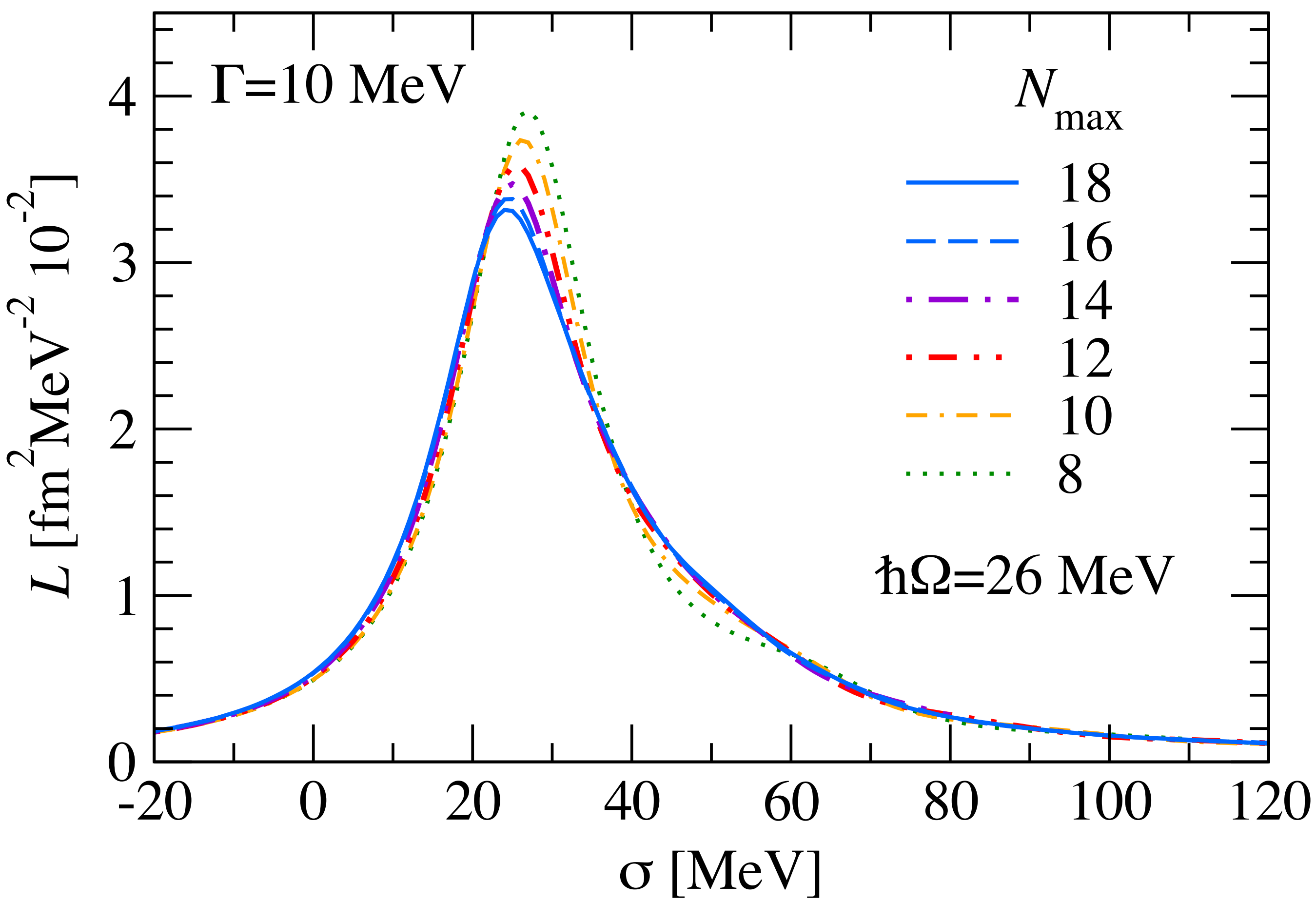
LIT with Coupled Cluster Theory

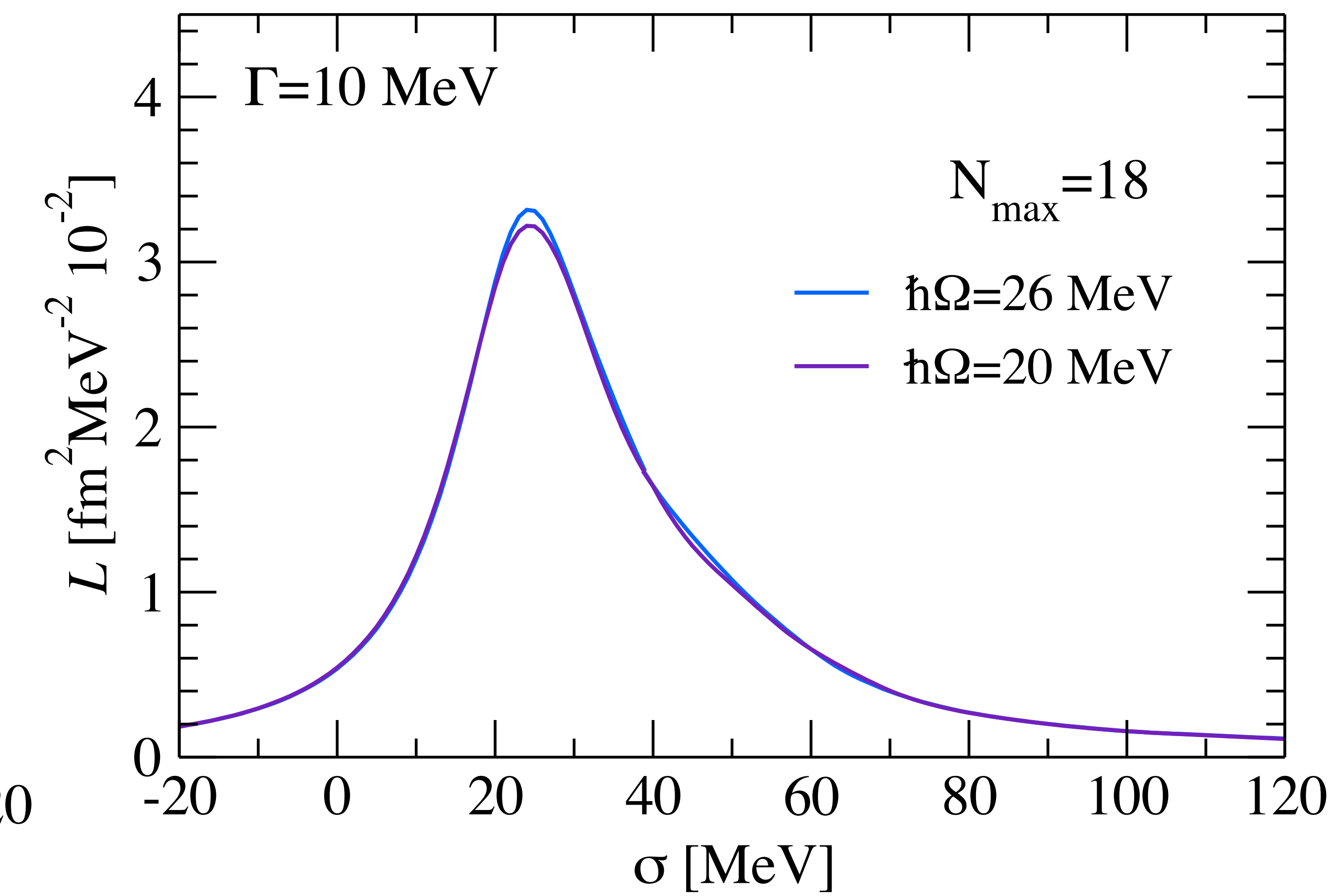
arXiv:1303.7446

New theoretical method aimed at extending *ab-initio* calculations towards medium mass

Extension to Dipole Response Function in ^{16}O with NN forces derived from χEFT (N^3LO)

Convergence in the model space expansion





Good convergence!

Small HO dependence: use it as error bar

Sonia Bacca

LIT with Coupled Cluster Theory

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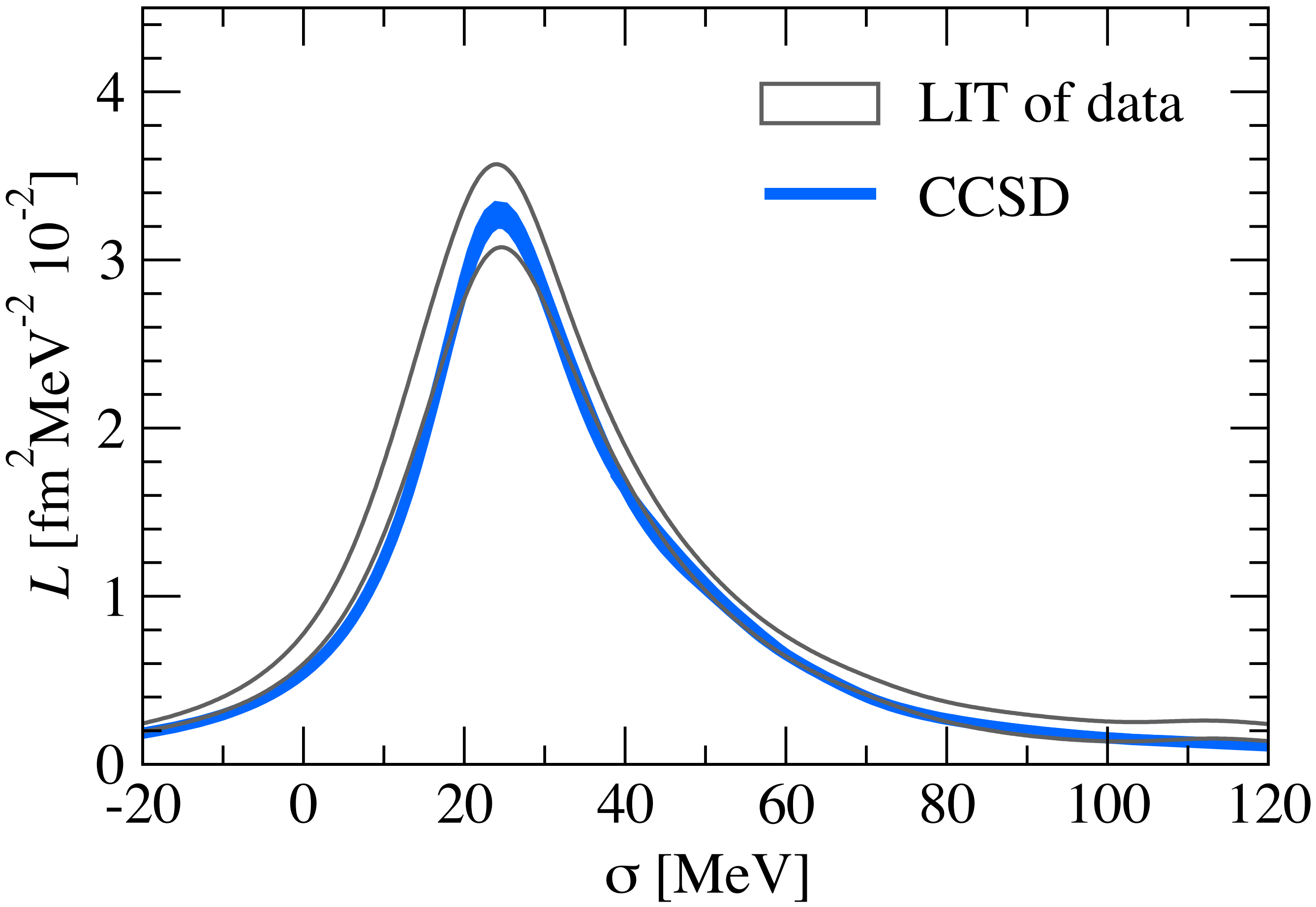
Extension to Dipole Response Function in ^{16}O with NN forces derived from χEFT (N^3LO)

Comparison to the experiment



LIT with Coupled Cluster Theory

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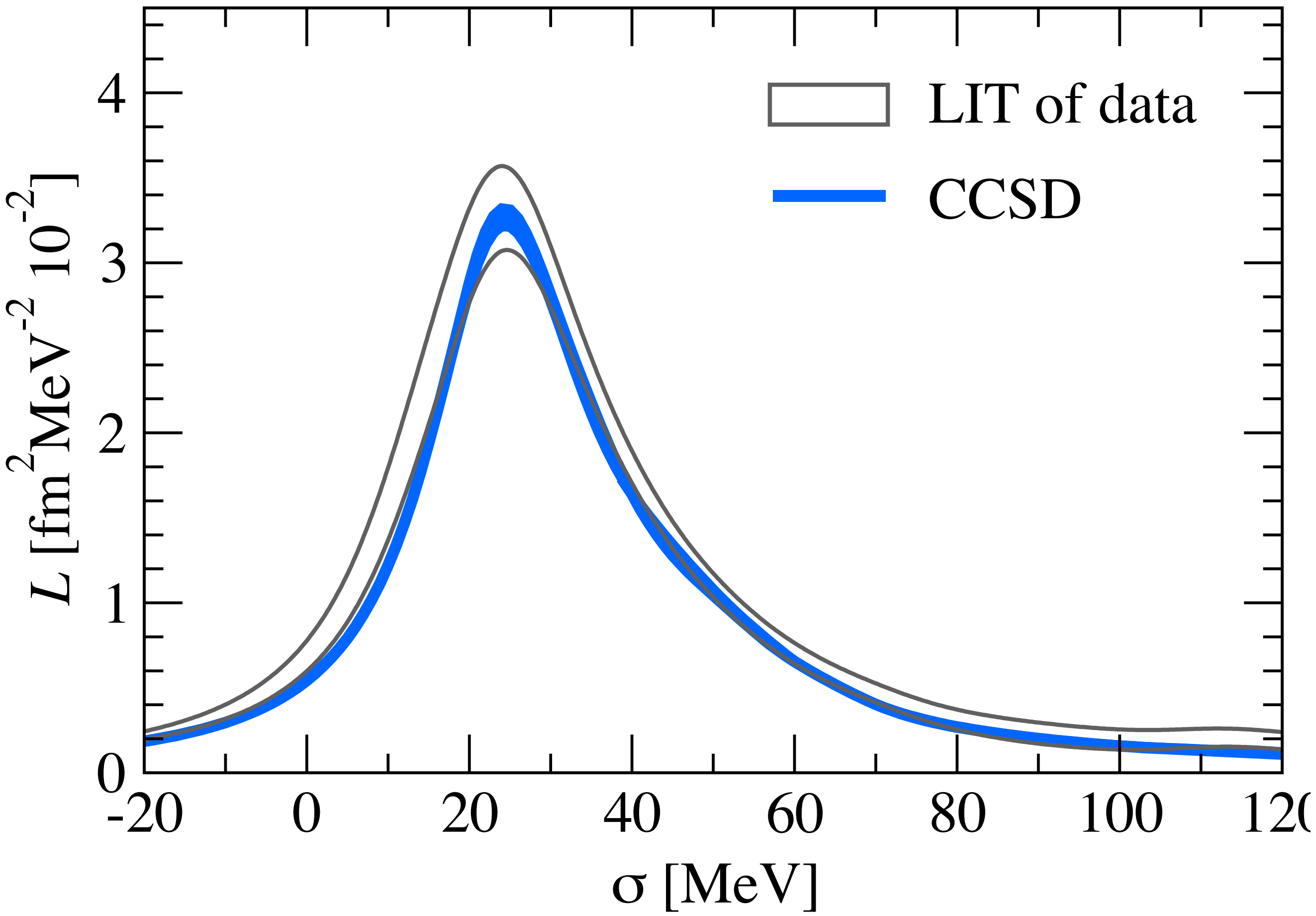
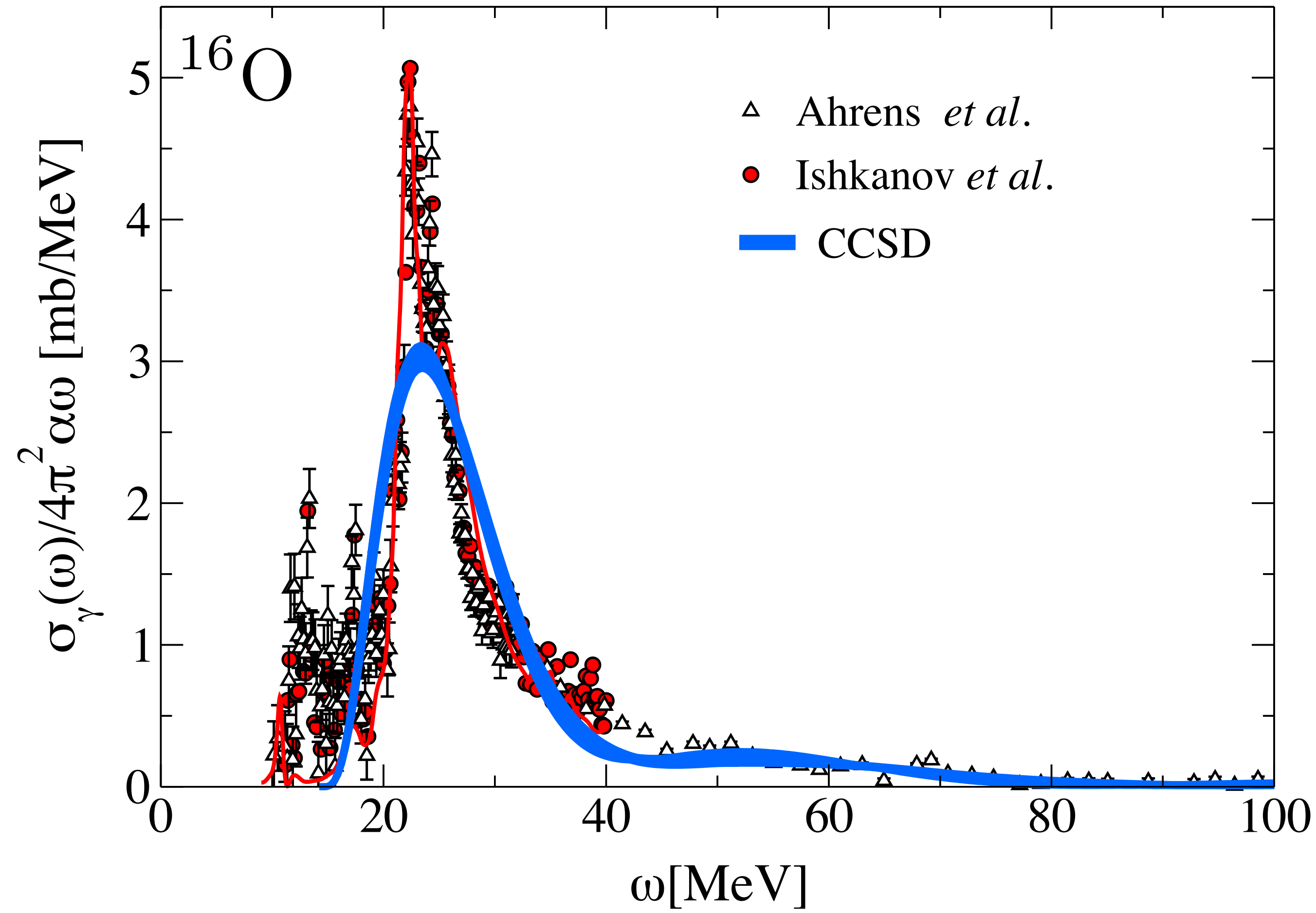
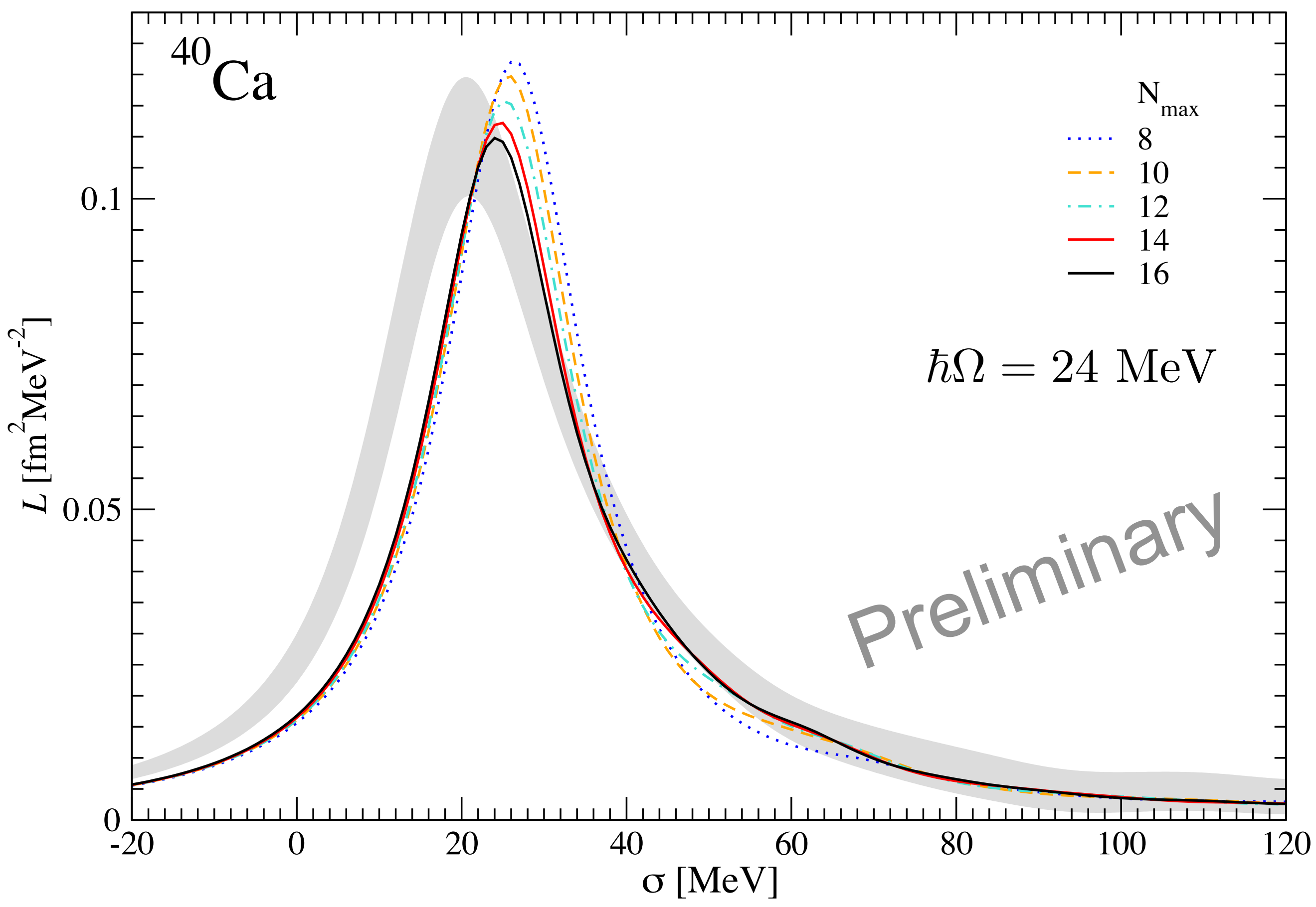


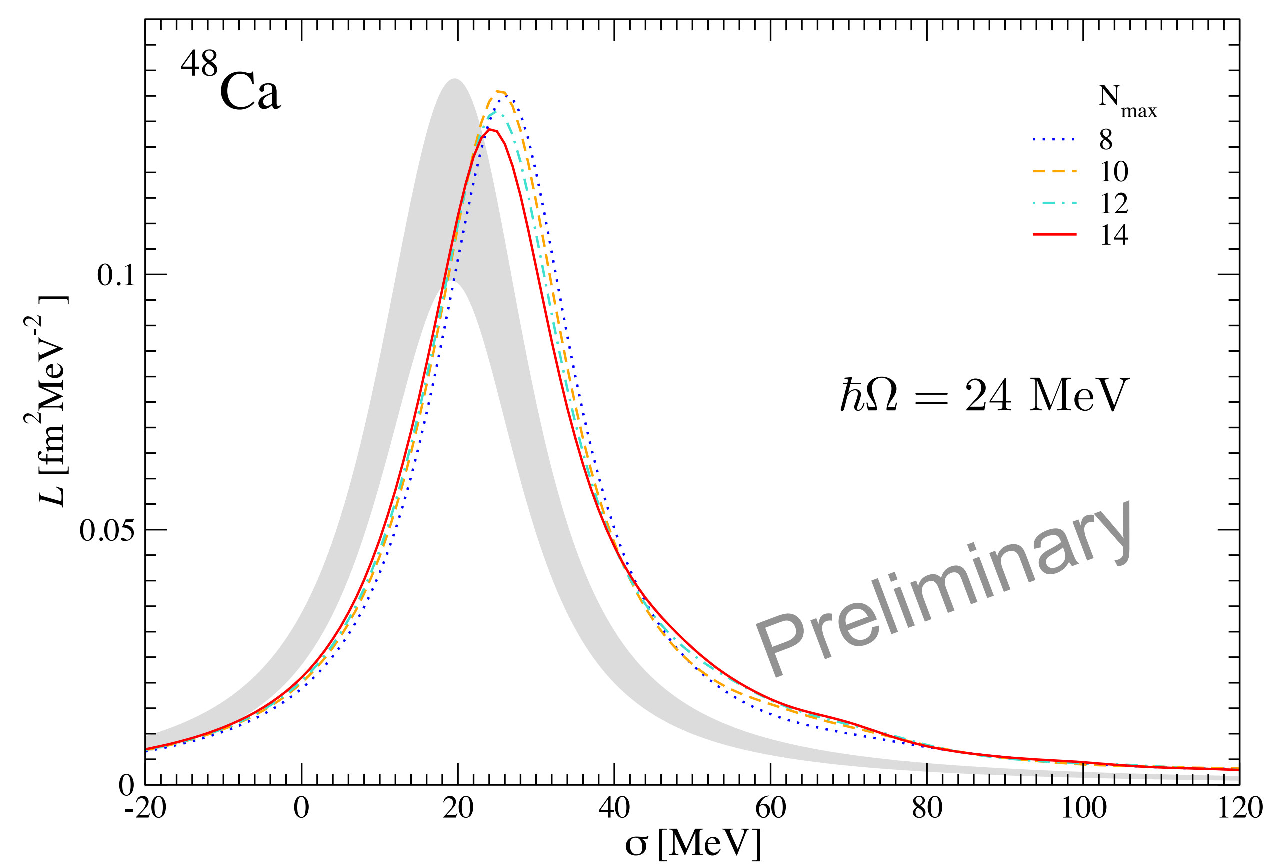


The GDR of ^{16}O is described from first principles for the first time!

LIT with Coupled Cluster Theory
with M.Miorelli

Calcium isotopes with NN(N^{3}LO)





LIT with Coupled Cluster Theory

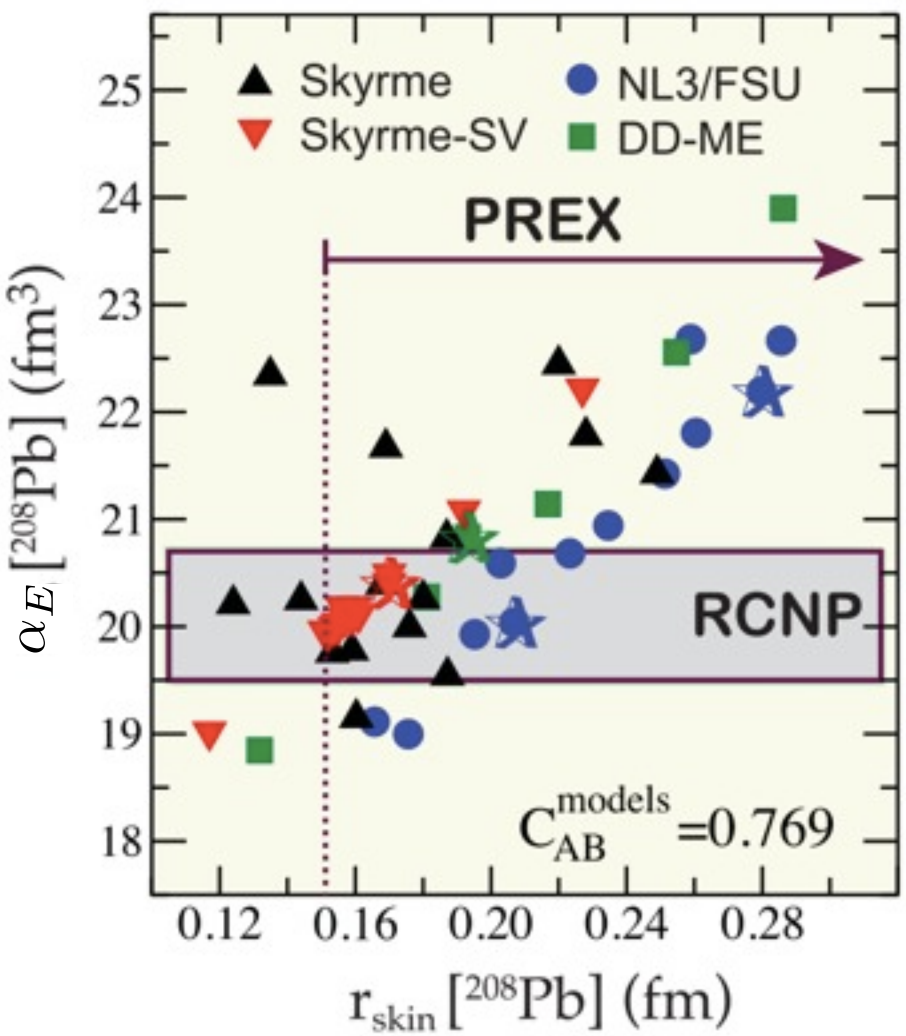
with M.Miorelli

Electric Dipole Polarizability

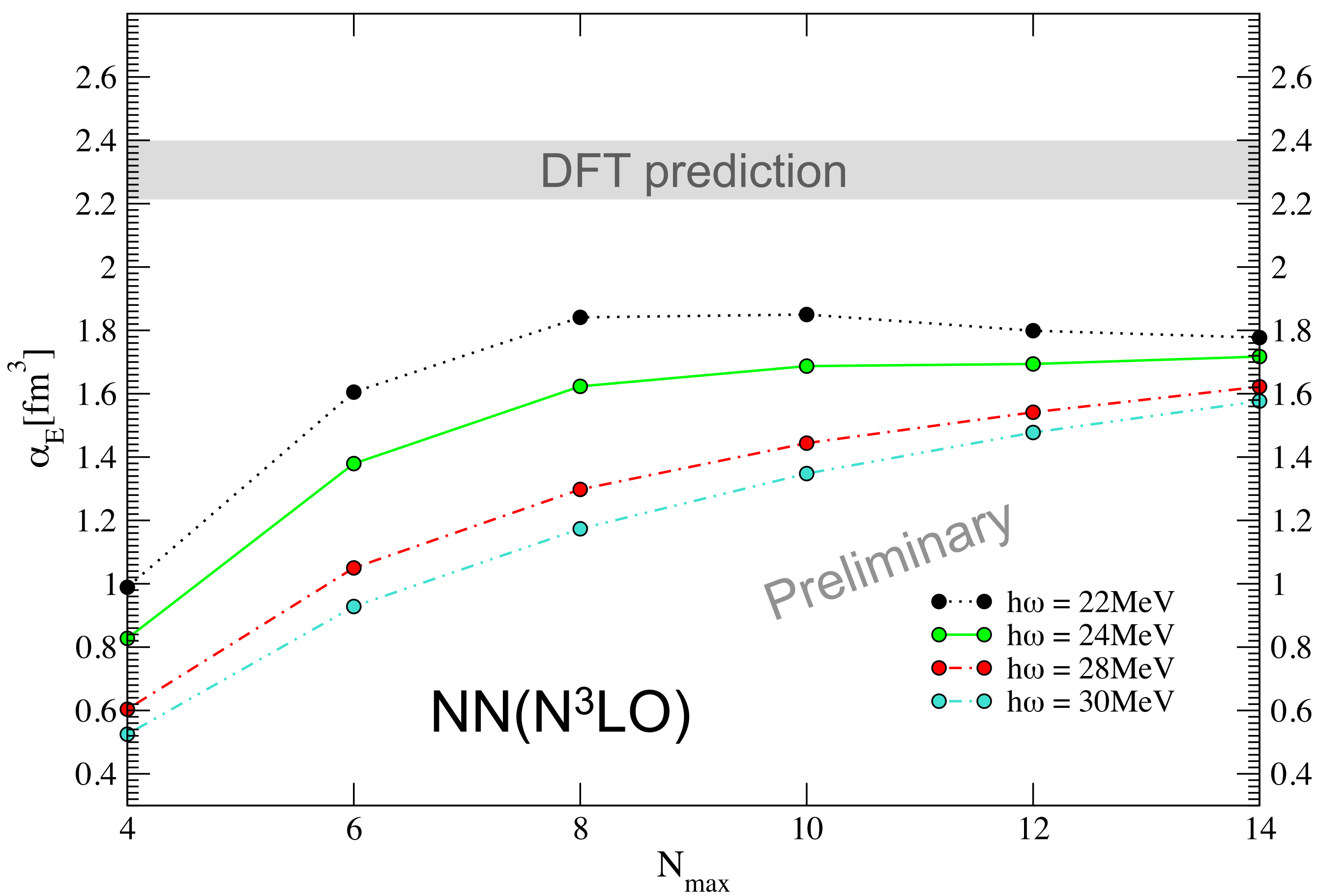
$$\alpha_E = \frac{1}{2\pi^2}\int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_\gamma(\omega)}{\omega^2}$$

Phys. Rev. C 85, 041302 (2012) very correlated to the neutron-skin radius

Energy Density Functional Theory



Towards an ab-initio theory for ^{48}Ca



^{48}Ca α_E being measured at RCNP

^{48}Ca parity violating electron scattering CREX

Future: study correlation $\alpha_E - r_{\rm skin}$ with ab-initio methods

TRIUMF

Conclusions and Outlook

- Electromagnetic break up reactions are very rich observables to test our understanding on nuclear forces
- Interesting applications to other fields of physics ➡ muonic atoms
- Extending these calculations to medium mass nuclei is possible and very exciting, with hopefully more applications/impact on future experiments on fundamental symmetries.

Perspectives

- Dipole response function of neutron-rich Oxygen isotope
- Other multipole excitation (quadrupole or monopole) of medium mass nuclei
  ➡ need extension of LIT/CCSD to two-body operator
- Add triples and three-nucleon forces

Thanks to my collaborators

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Thank you!