LATTICE QCD STUDY OF BARYON PROPERTIES IN A MESON MEDIUM

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in collaboration with W. Detmold (MIT)

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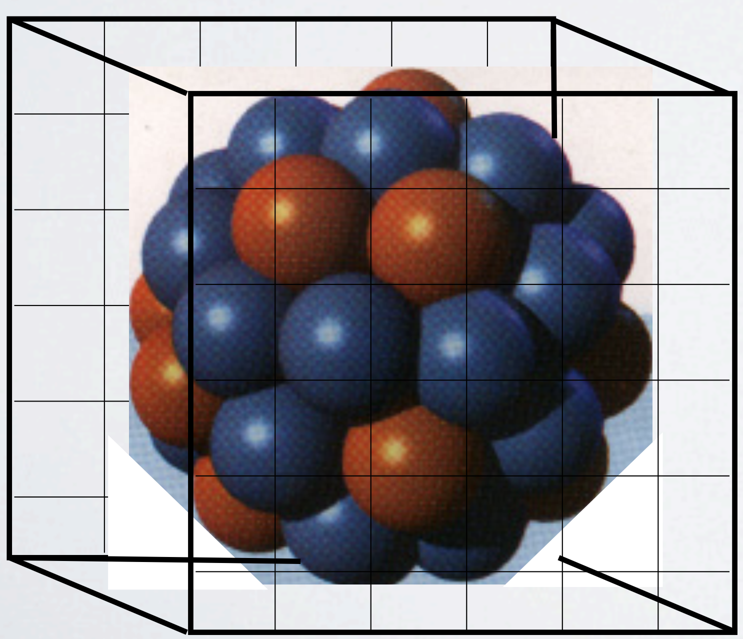
OVERVIEW

- Multi-hadron states in lattice QCD
  - Nucleons
  - Mesons



- Many mesons + a single baryon
  - Lattice calculation details
  - Results
    - Ground-state energies
    - 2- and 3-body interactions
    - χPT low-energy constants

LATTICE METHOD

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle \underset{t\to\infty}{\longrightarrow} \langle 0|\mathcal{O}|E_0\rangle\langle E_0|\mathcal{O}|0\rangle e^{-E_0 t}$$

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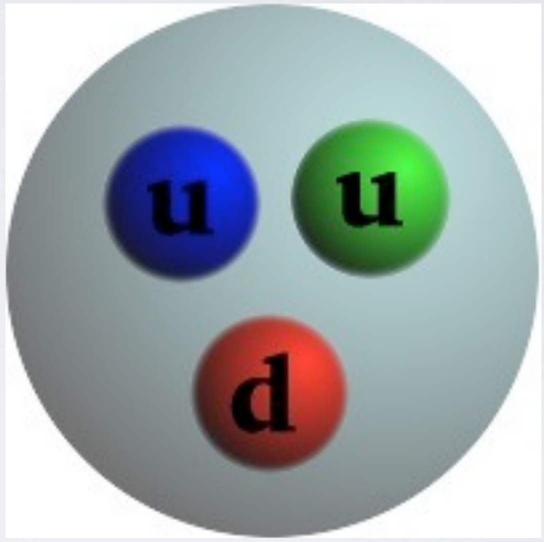
Example: Single baryon



$$\mathcal{O}(x,t) = \psi_{x_1,t}\psi_{x_2,t}\psi_{x_3,t}$$

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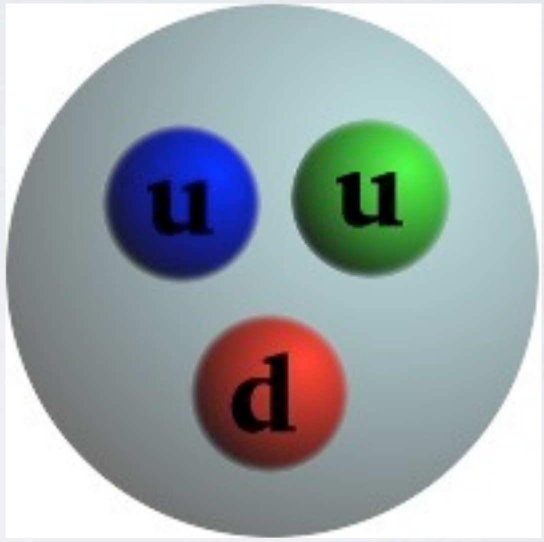


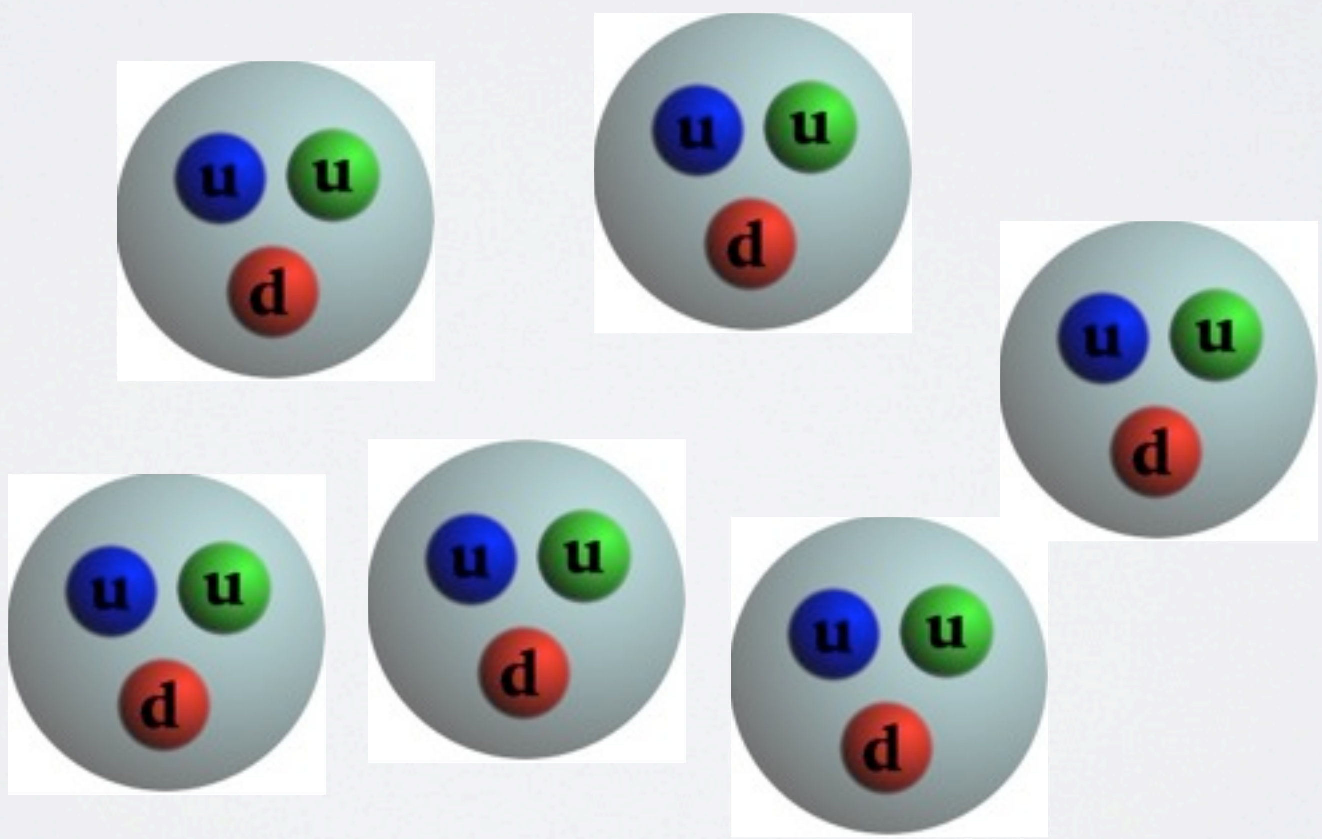
$$\mathcal{O}(x,t) = \psi_{x_1,t}\psi_{x_2,t}\psi_{x_3,t}$$

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle \sim \langle [S(0,t)]^3\rangle_A \underset{t\to\infty}{\longrightarrow} |\langle 0|\mathcal{O}|B\rangle|^2 e^{-M_B t}$$

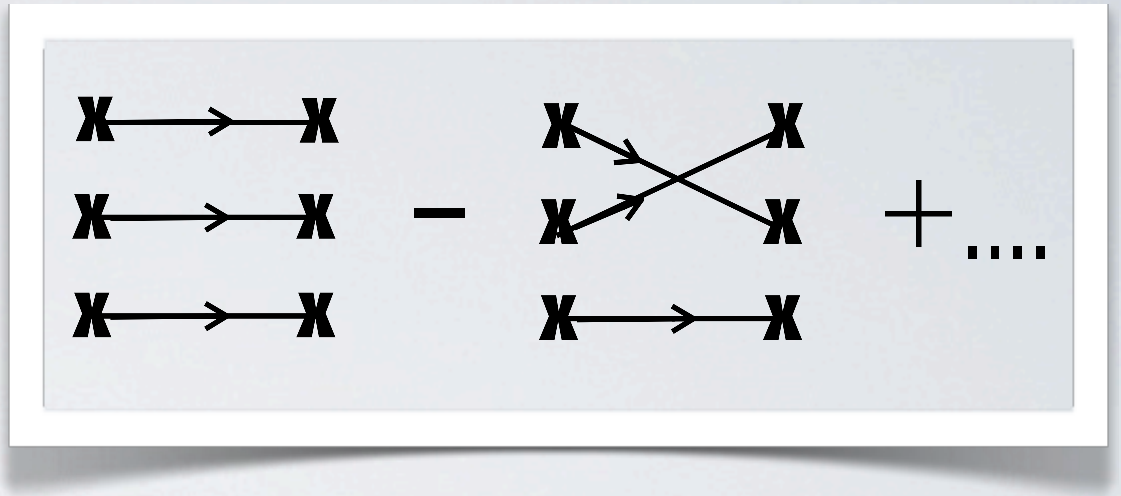
$$Z_A = \int \mathcal{D}A \det M_F(A) e^{-S_{YM}}$$

MULTI-HADRON STATES

- In principle, can choose operator which produces multi-hadron states
- Until recently, mostly one and two particle properties calculated with lattice QCD



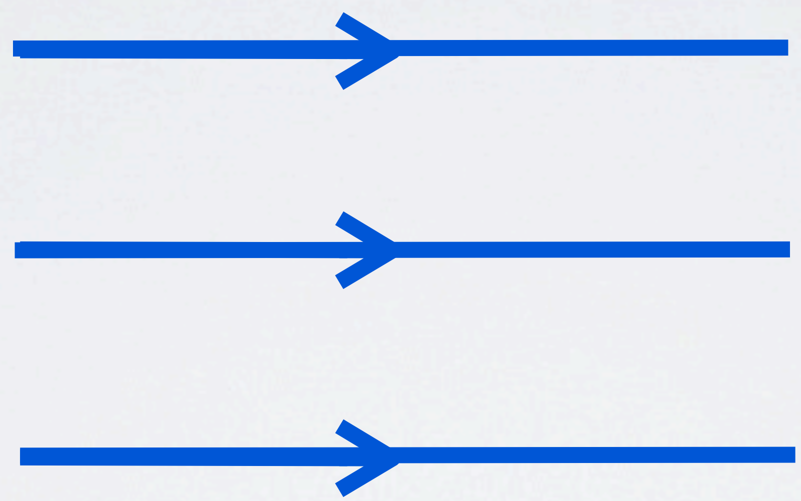
MULTI-HADRON STATES



- Why?
  - Small energy splittings
  - Numerical precision
  - Propagator contractions: (A+Z)!(2A-Z)!
  - Statistics
    - baryon noise/sign problem
    - overlap problem?

BARYON SNR

$$C_N(\tau) = \Bigg\langle$$

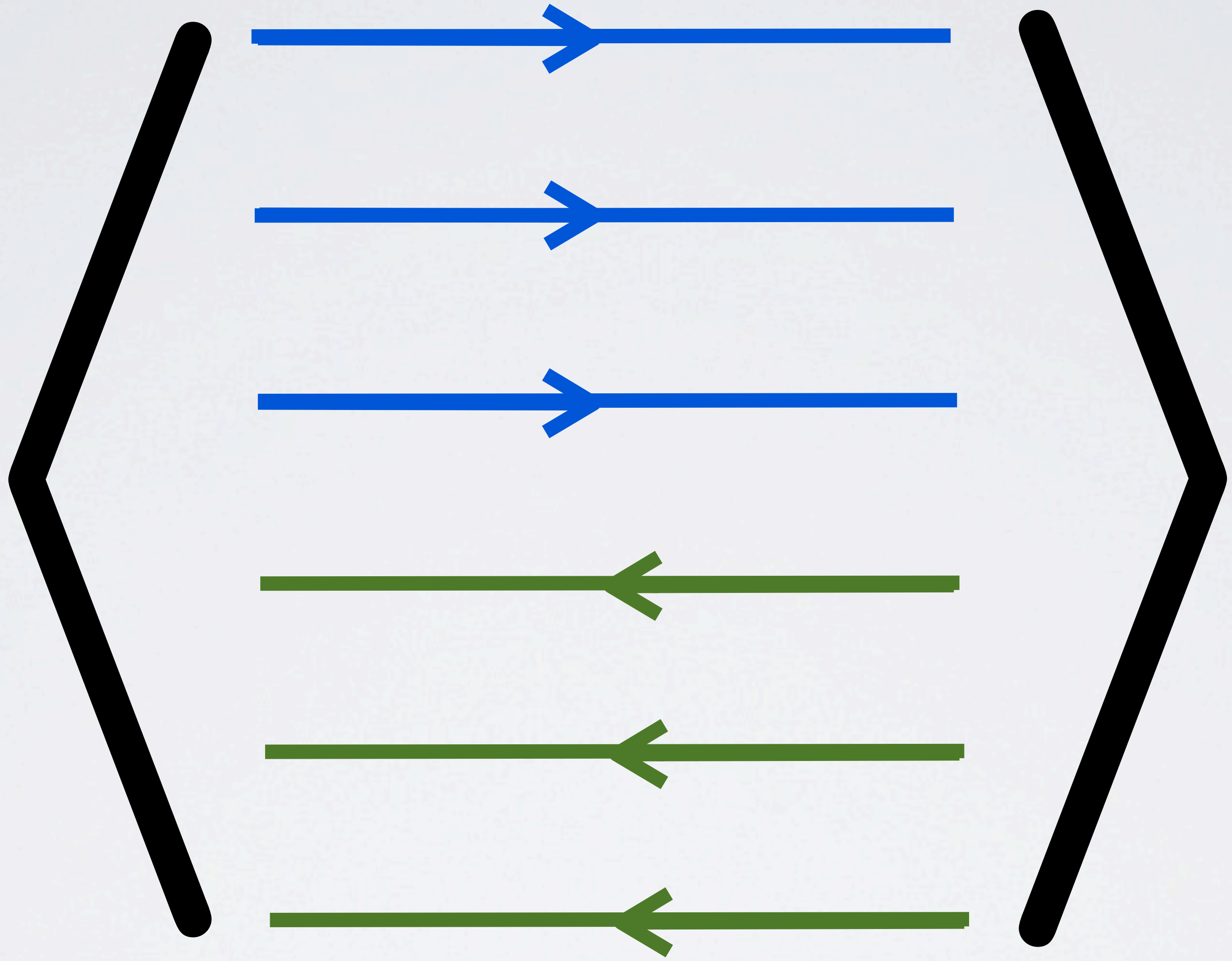


$$\Bigg\rangle \underset{\tau \to \infty}{\longrightarrow} e^{-M_N \tau}$$

Lepage (1989)

BARYON SNR

$$C_{N^\dagger N}(\tau) =$$

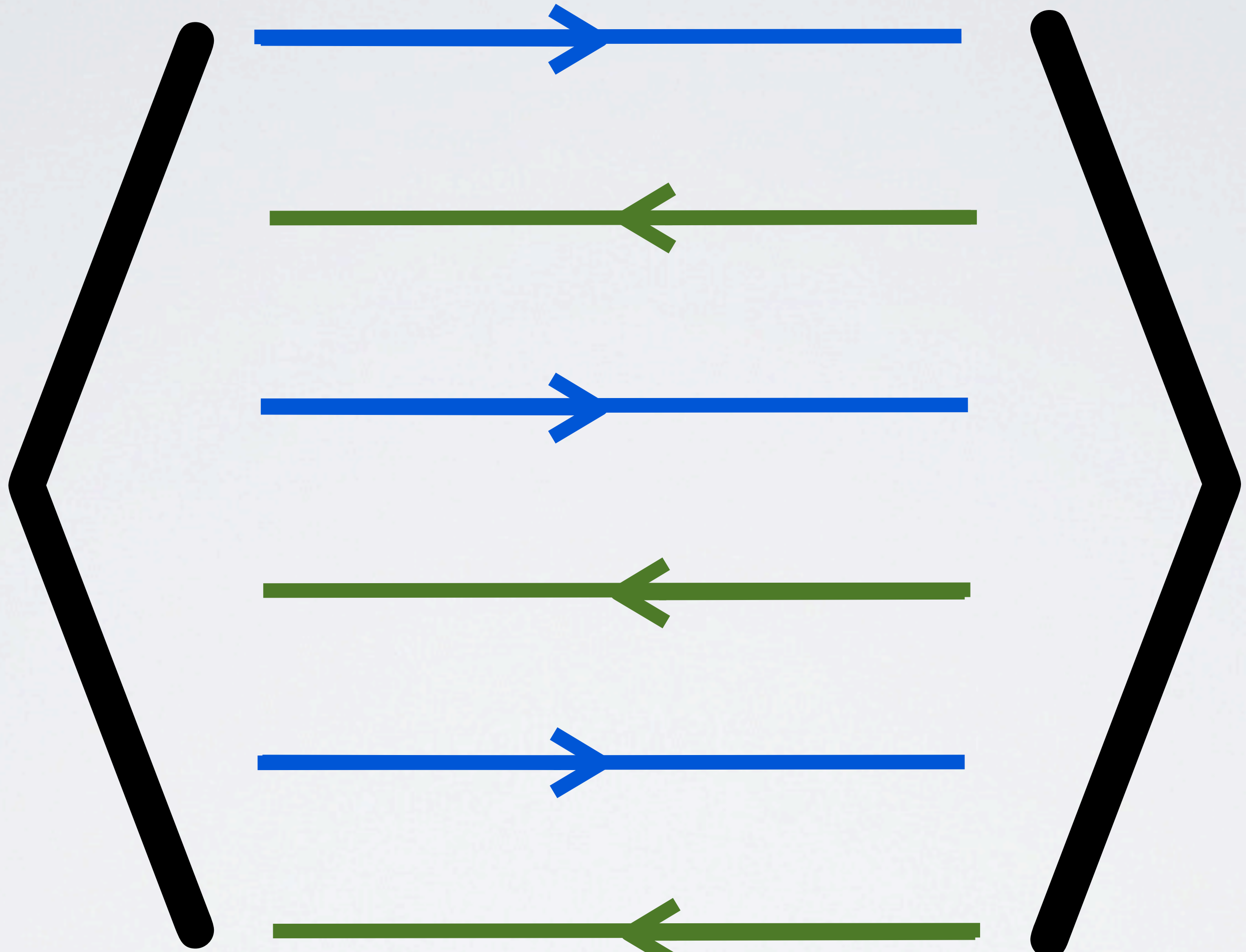


$$\xrightarrow[\tau\to\infty]{} e^{-2M_N\tau}$$

Lepage (1989)

BARYON SNR

$$C_{N^\dagger N}(\tau) =$$



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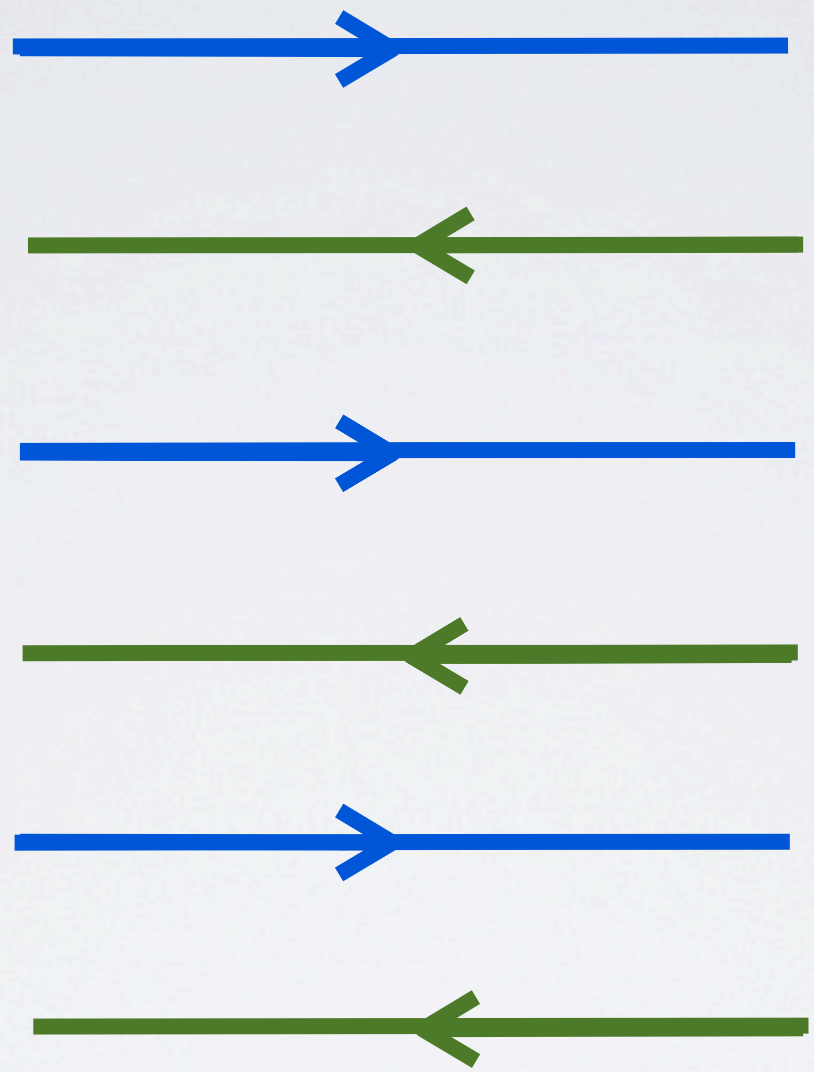


$$\Big\rangle \xrightarrow[\tau \to \infty]{} e^{-3 m_\pi \tau}$$

Lepage (1989)

BARYON SNR

Signal-to-noise ratio:

$$\frac{C_N(\tau)}{\sigma(\tau)} \underset{\tau\to\infty}{\longrightarrow} \sqrt{N_{cfg}}e^{-(M_N-3/2m_\pi)\tau}$$

Exponentially poor signal-to-noise!

BARYON SNR

Signal-to-noise ratio:

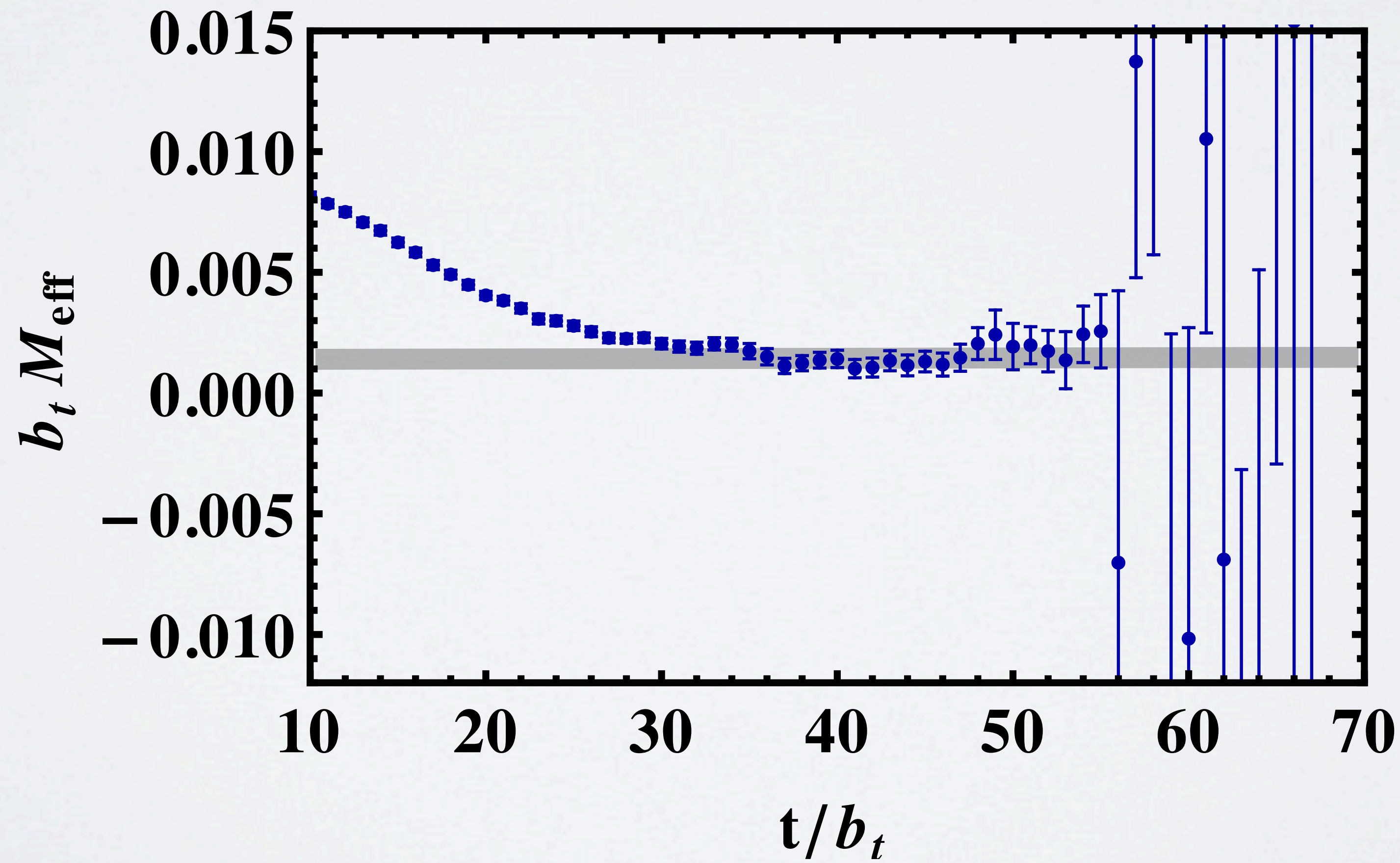
$$\frac{C_{N^A}(\tau)}{\sigma(\tau)} \underset{\tau\to\infty}{\longrightarrow} \sqrt{N_{cfg}}e^{-A(M_N-3/2m_\pi)\tau}$$

Exponentially poor signal-to-noise!

BARYON SNR

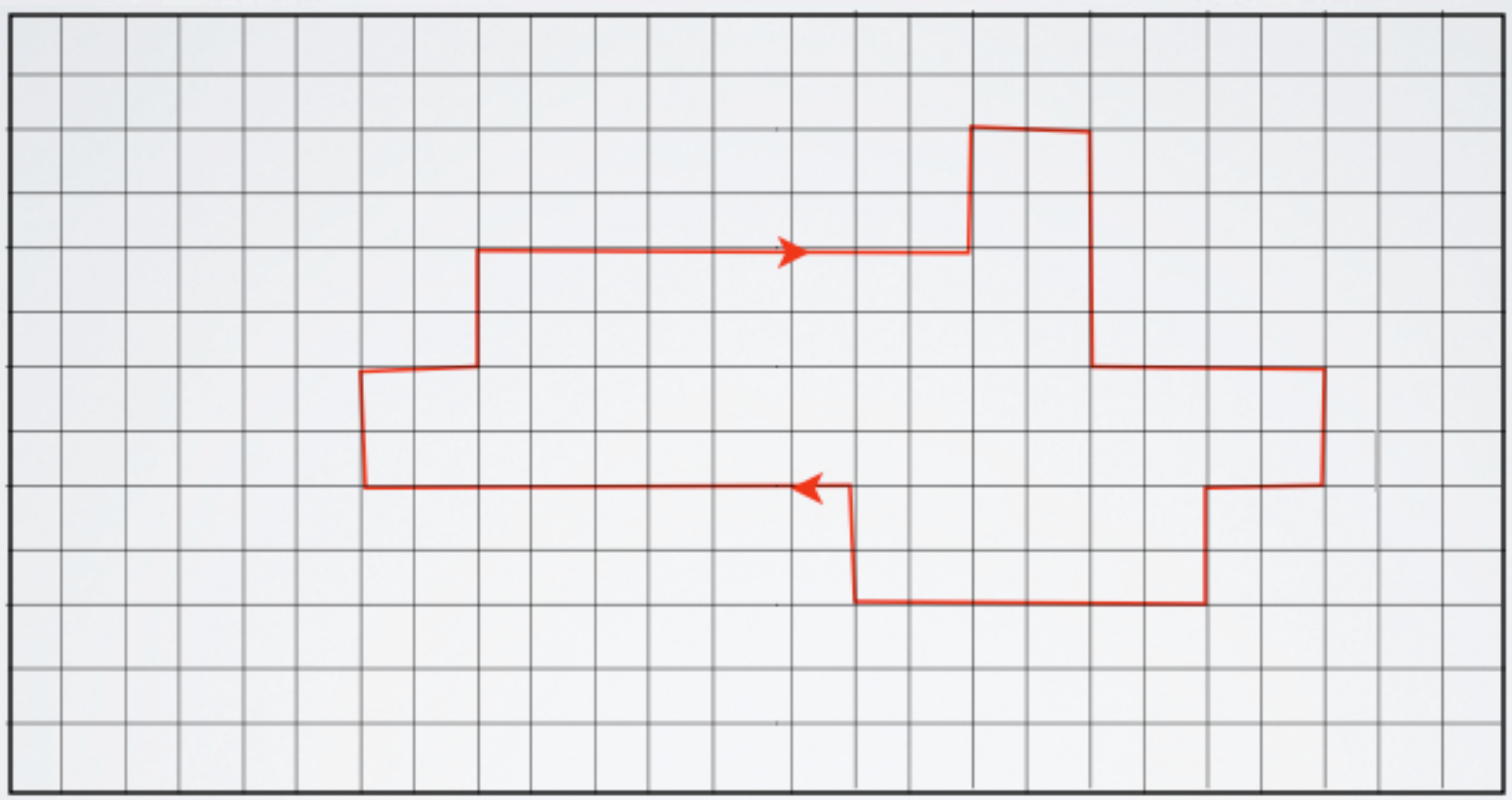
$$M_{\text{eff}} \equiv \ln \frac{C(t)}{C(t+1)}$$

$$C(t) \underset{t\to\infty}{\longrightarrow} Ae^{-E_0 t}$$



OVERLAP PROBLEM

$$Z_A = \int \mathcal{D}A \det M_F(A) e^{-S_{YM}}$$



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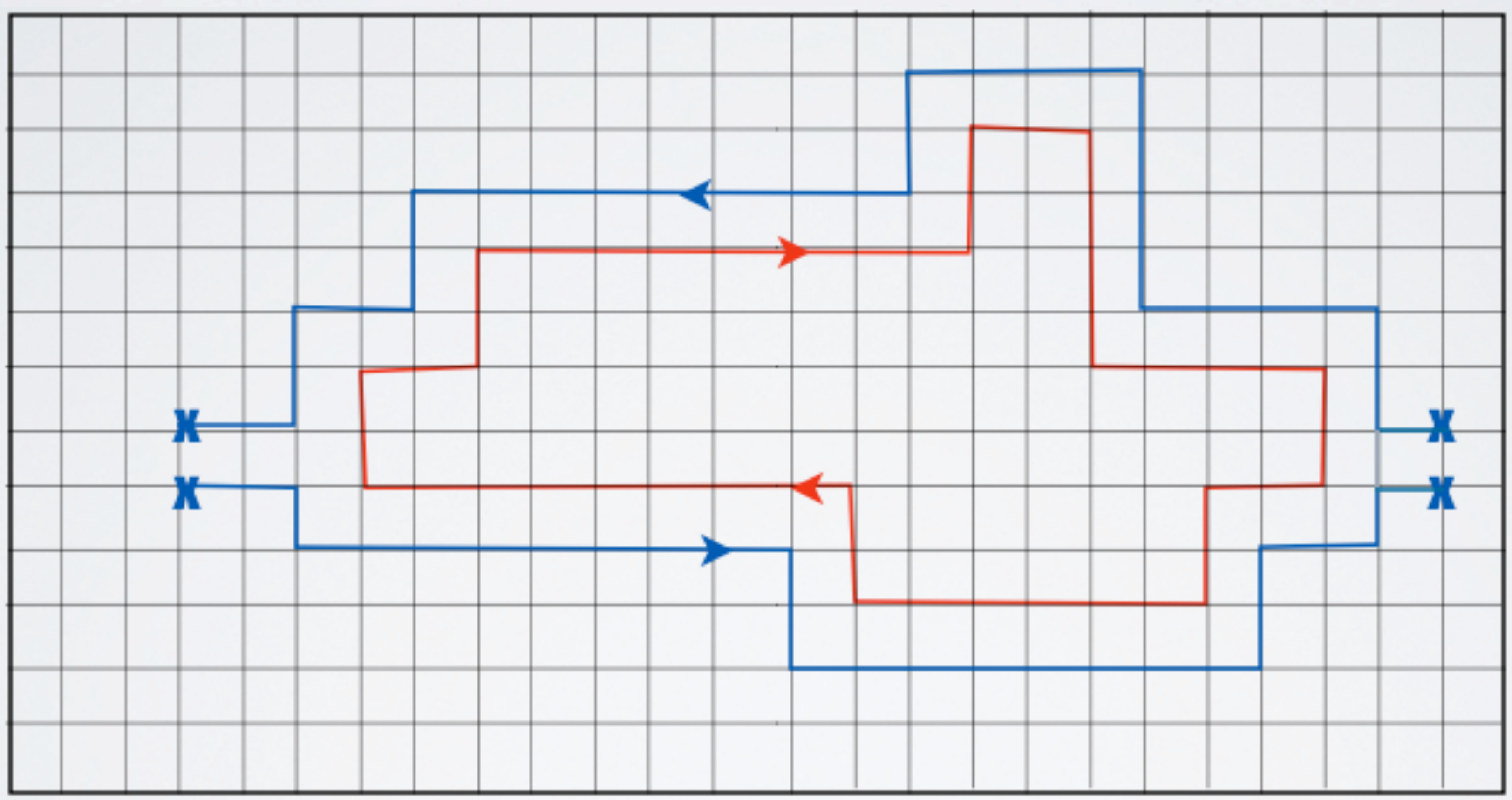
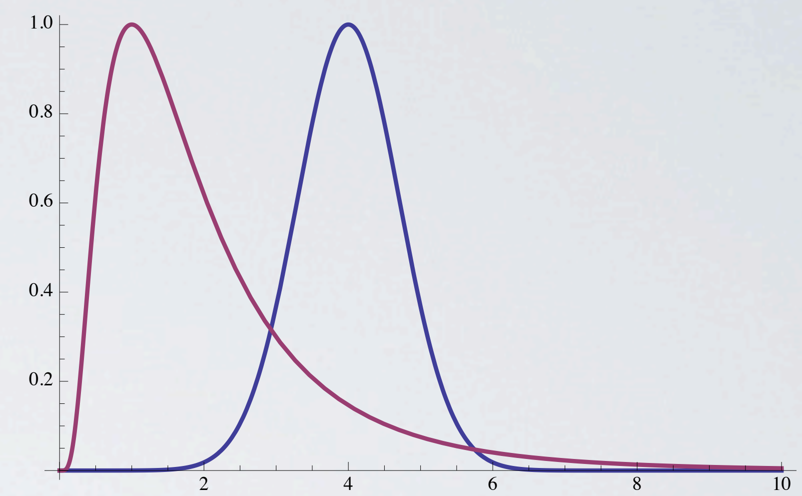
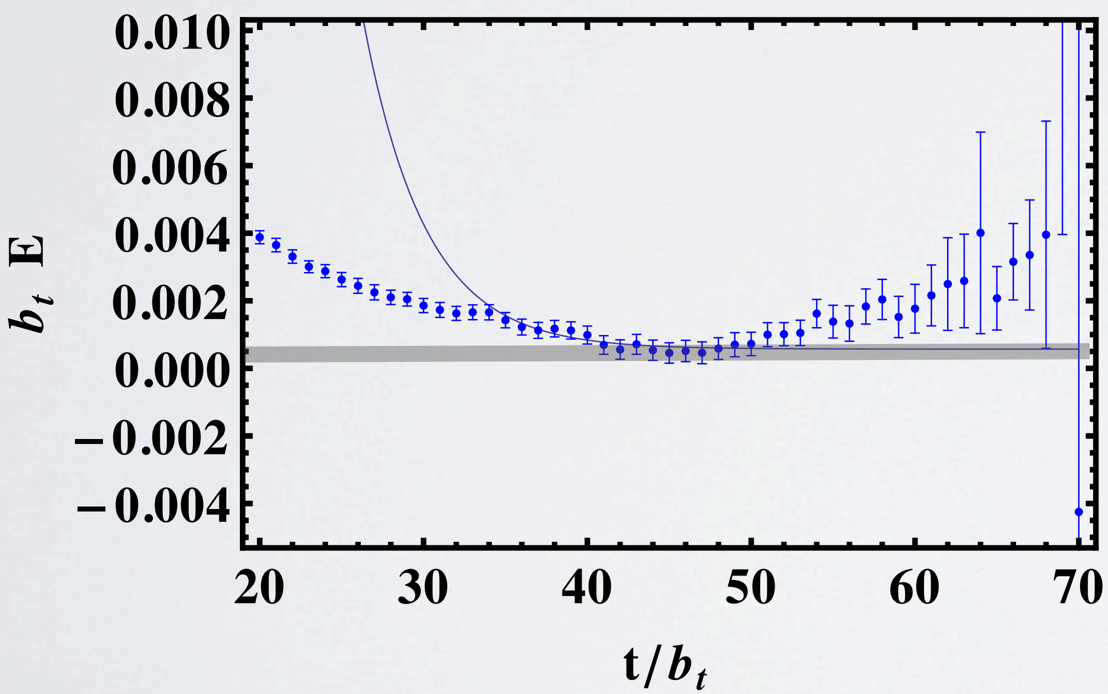


OVERLAP PROBLEM

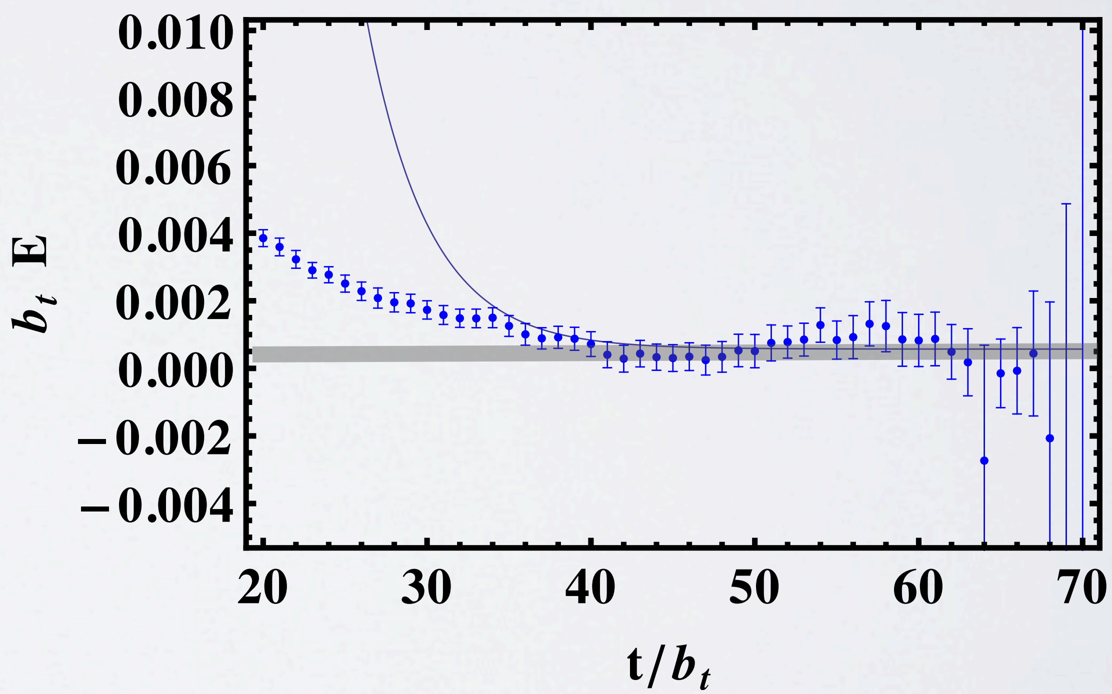
Kaplan, Endres, Lee, A.N. (2011)

Correlator distributions





Standard



Cumulant Expansion

OVERLAP PROBLEM

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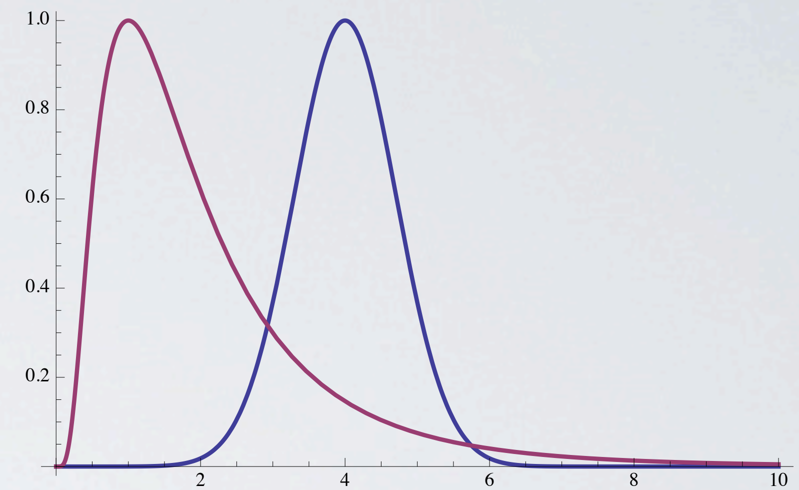
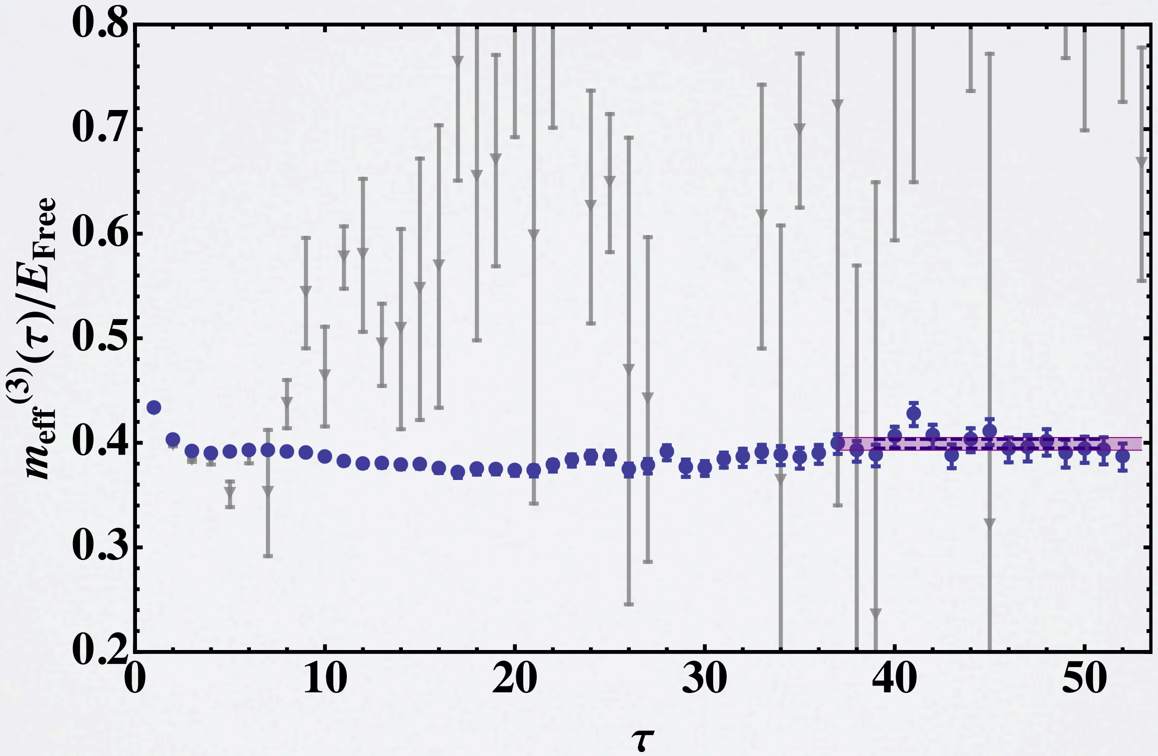
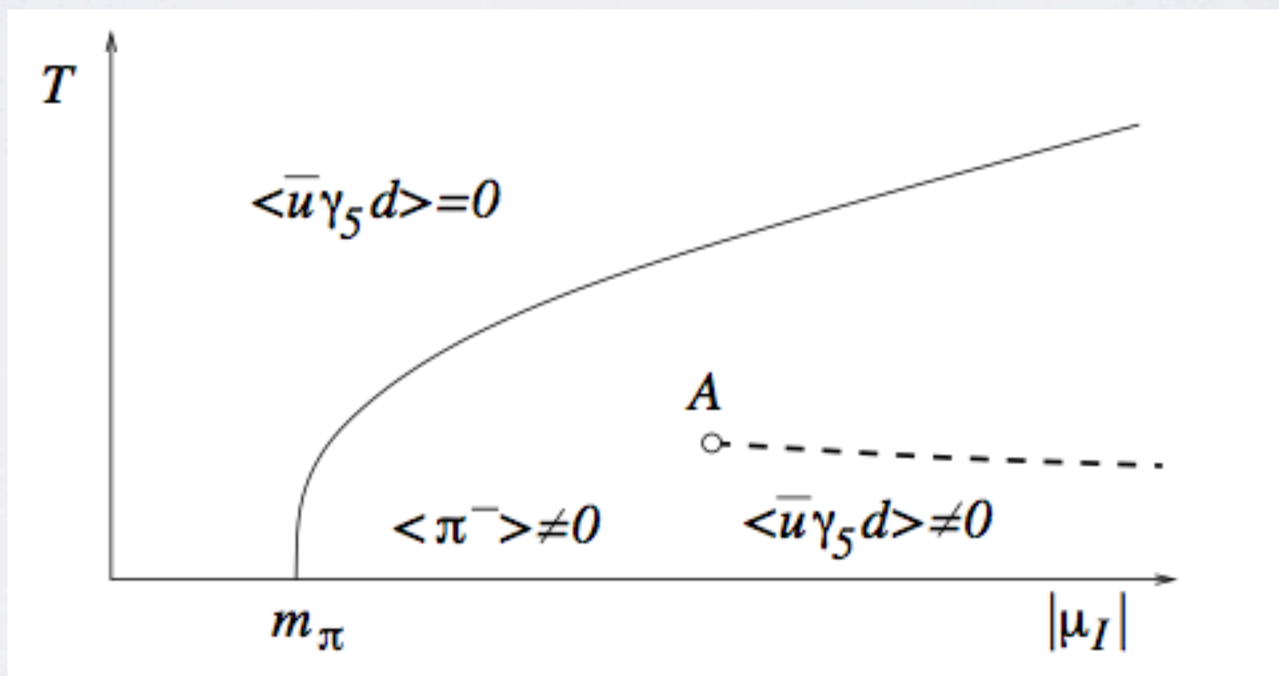




WHAT ABOUT MESONS?

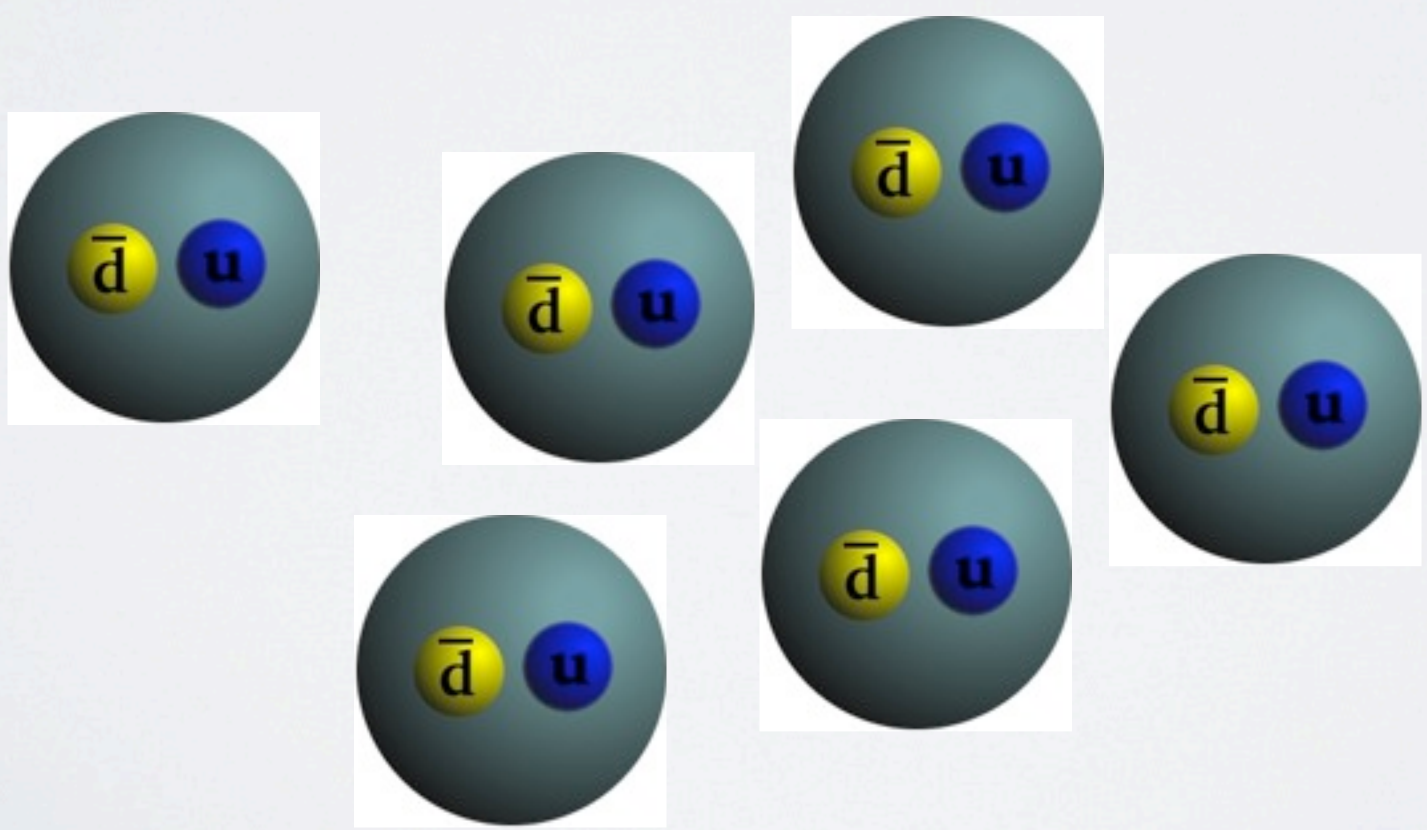
- SNR ~ $\sqrt{N_{\mathrm{cfg}}}$
- Explore lattice methods for complex hadronic systems
- Interesting phase diagram (BEC)
- Possibly relevant for neutron stars



Son & Stephanov (2001)

WHAT ABOUT MESONS?

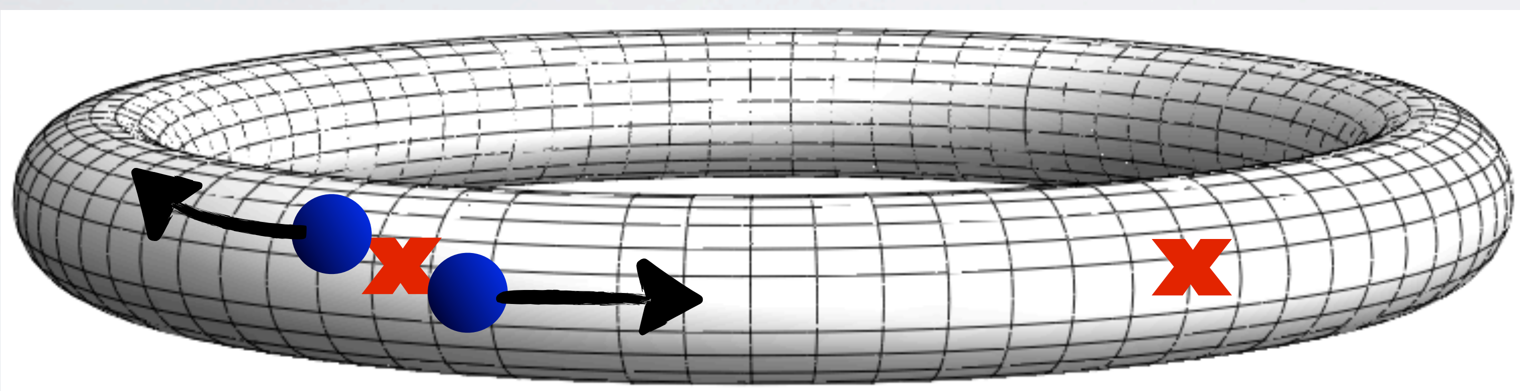
- Multi-meson systems studied extensively by NPLQCD
- Would like to add baryons
- First step: investigate properties of single baryon in meson medium

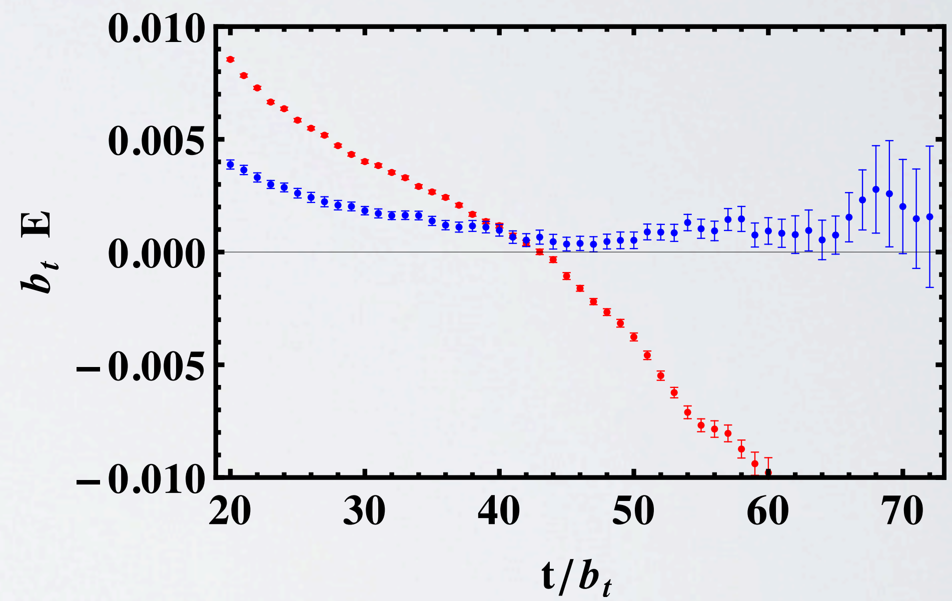


WHAT ABOUT MESONS?

- Still have to deal with contractions

- Thermal effects can be large





- Possibility of annihilation diagrams

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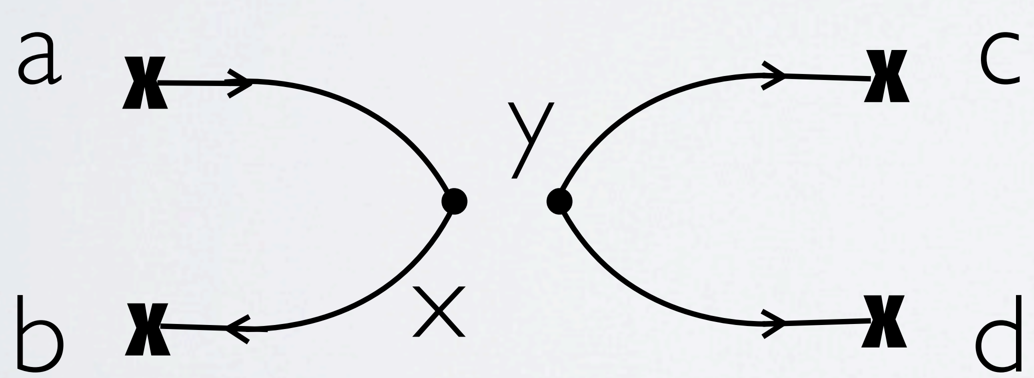
- Thermal effects can be large
- Possibility of annihilation diagrams



$$\sim \sum_{x,y} \langle S_{a,x} S^{\dagger}_{b,x} S_{y,c} S^{\dagger}_{y,d} \rangle$$

THIS WORK:

System	Quark content
$\Sigma^+(\pi^+)^n$	$uus(u\bar{d})^n$
$\Xi^0(\pi^+)^n$	$uss(u\bar{d})^n$
$p(K^+)^n$	$uud(u\bar{s})^n$
$n(K^+)^n$	$udd(u\bar{s})^n$

Will calculate:

- Ground-state energies
- 2- and 3-body interaction parameters
- LECs - tree-level ChiPT

CONTRACTIONS

NPLQCD (2007)

First let's look at the simpler case for n mesons*

$$\Pi_{a,\alpha}^{b,\beta} \equiv \sum_{c,\gamma} \sum_{\mathbf{x}} \left[S_d(\mathbf{x},t;\mathbf{0},0)\gamma_5\right]^{b,\beta,c,\gamma} \left[S_u^{\dagger}(\mathbf{x},t;\mathbf{0},0)\gamma_5\right]_{a,\alpha,c,\gamma}$$

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$$\longrightarrow \quad \Pi_a^b$$

12x12 matrix for 12 dof

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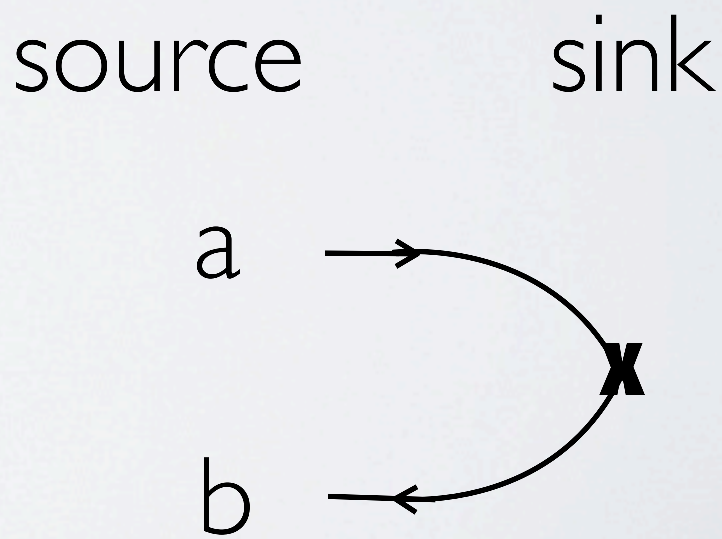
$$\longrightarrow \quad \Pi^{b}_{a}$$

Graphically:



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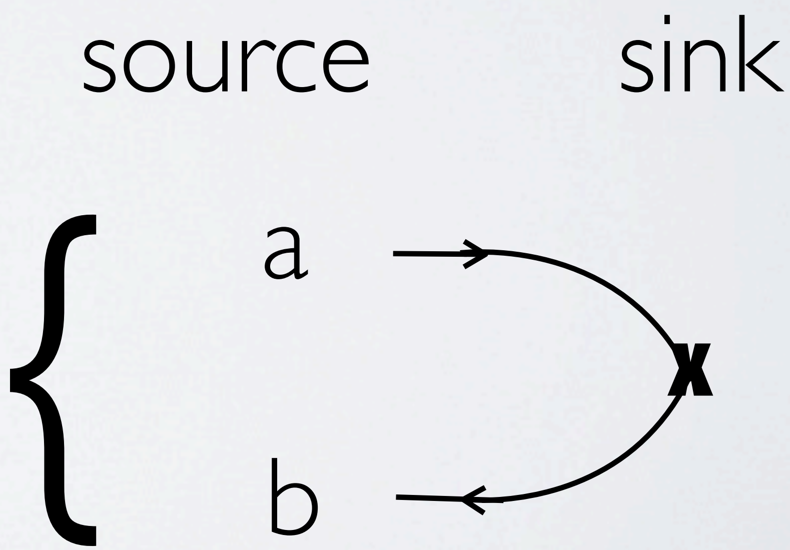
$$\longrightarrow \quad \Pi_a^b$$

Graphically: need to tie up source indices



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$$\det(1 + \lambda\Pi) = \frac{1}{12!}\sum_{m=1}^{12} \lambda^m C_m(t)$$

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$$C_n(t)$$

We use:

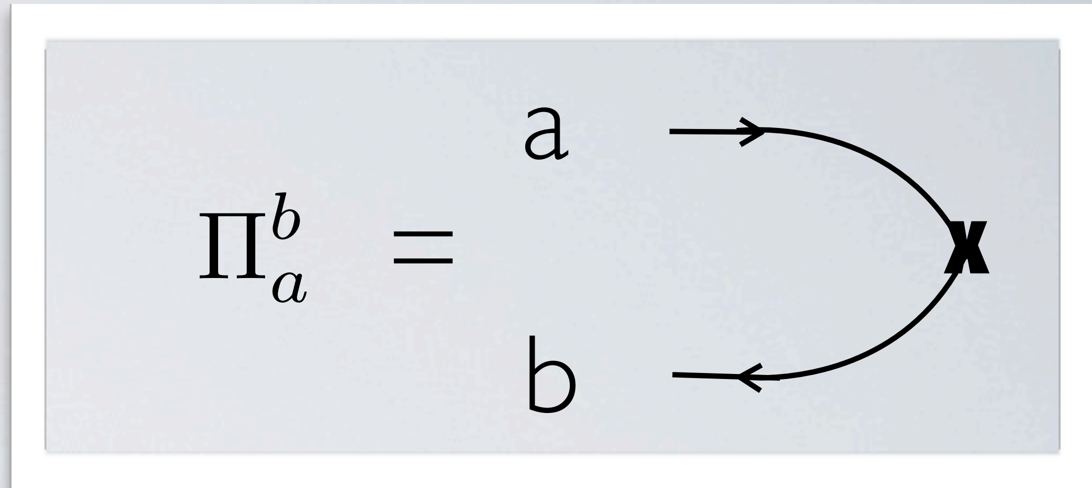
$$\det(1+\lambda\Pi) = \frac{1}{12!}\sum_{m=1}^{12} \lambda^m C_m(t)$$

Then expand: $\det(1+\lambda\Pi) = e^{\mathrm{Tr}\ln(1+\lambda\Pi)}$

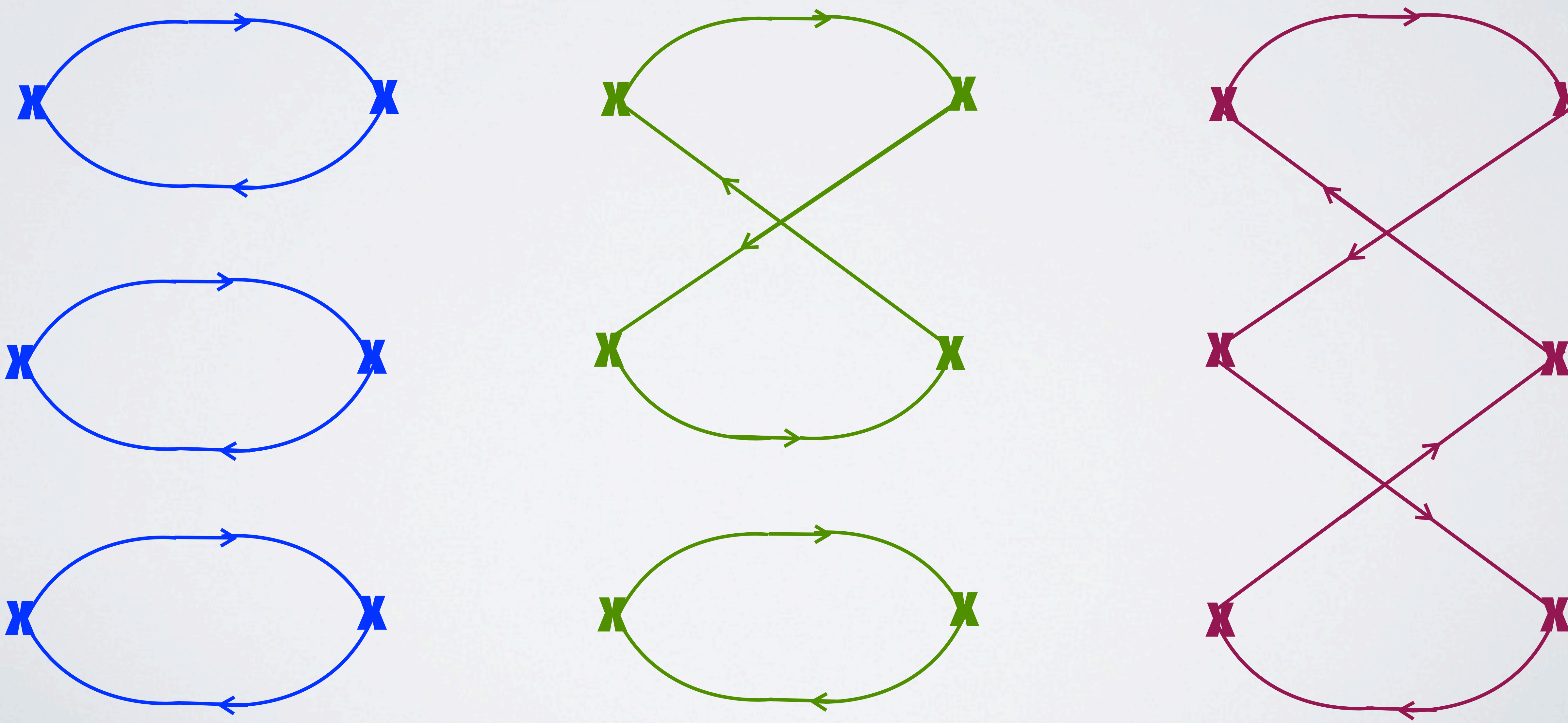
and pick off the terms with n powers of λ

AN EXAMPLE

3 mesons



$$\mathcal{O}(\lambda^3) : C_3(t) = (\mathrm{tr}[\Pi])^3 - 3\,\mathrm{tr}[\Pi^2]\mathrm{tr}[\Pi] + 2\,\mathrm{tr}[\Pi^3]$$



CONTRACTIONS

Detmold & Smigielski (2011)

- Easily extended for multiple species of mesons, e.g. pions and kaons

$$\det(1+\lambda\Pi) \longrightarrow \det(1+\lambda\Pi+\kappa K)$$

2 pions, 1 kaon:

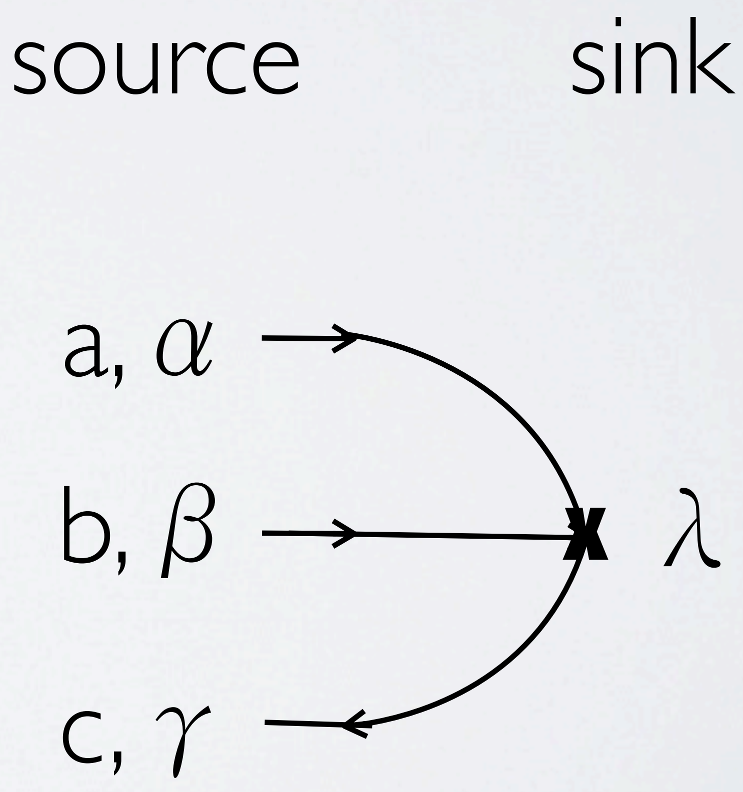
$$C_{1,2} = 2\mathrm{tr}[K\Pi\Pi] - 2\mathrm{tr}[K\Pi]\mathrm{tr}[\Pi] + (\mathrm{tr}[\Pi])^2\,\mathrm{tr}[K] - \mathrm{tr}[K]\mathrm{tr}[\Pi\Pi]$$

ADDING A BARYON

Baryon “block”

$$B_{a,\alpha,b,\beta,c,\gamma,\lambda} \equiv \sum_{\sigma,h,i,j} [S_{q_1} C\gamma_5]_{a,\alpha,h,\sigma} [S_{q_2}]_{b,\beta,i,\sigma} [S_{q_3}]_{c,\gamma,j,\lambda} \, \epsilon_{h,i,j}$$

Graphically:



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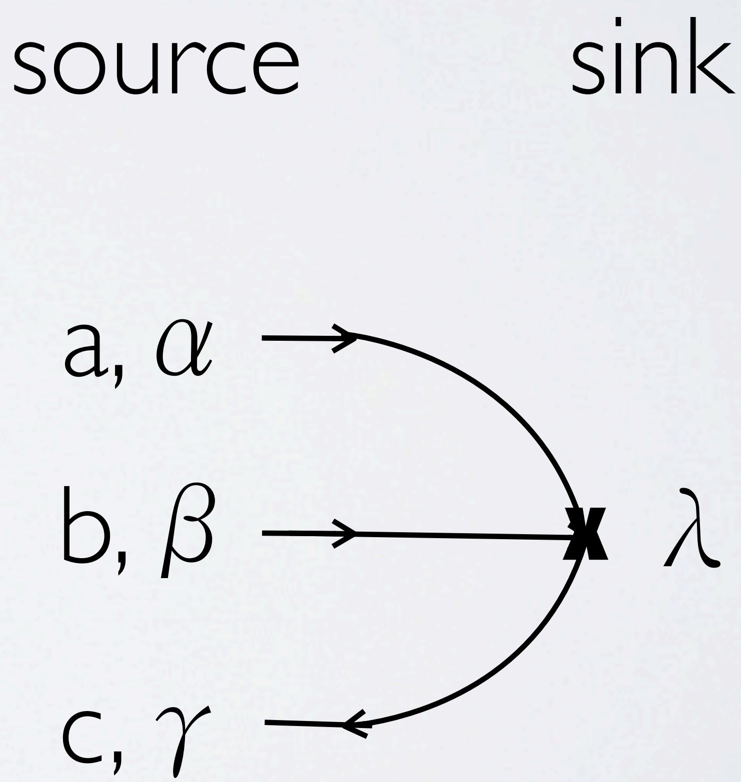

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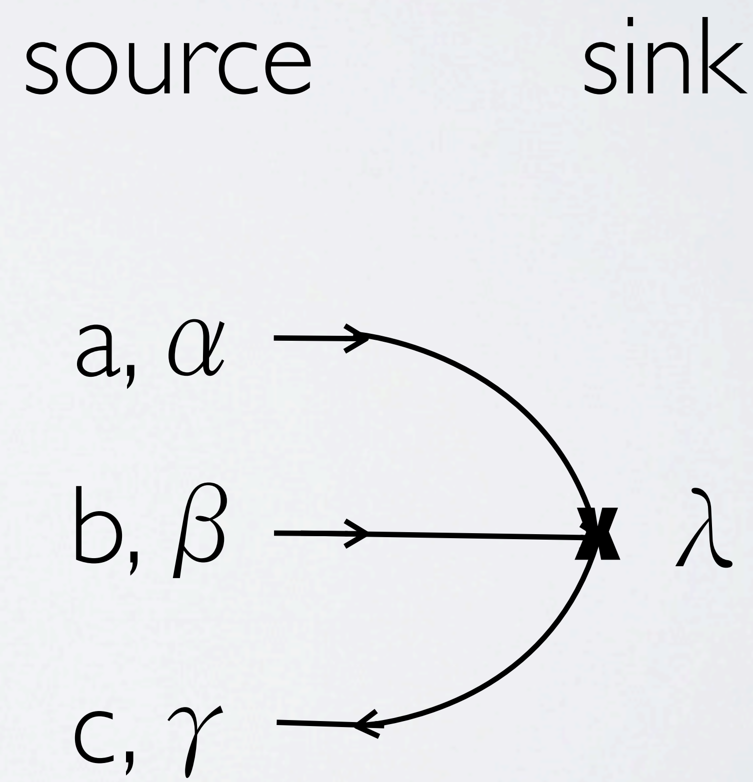

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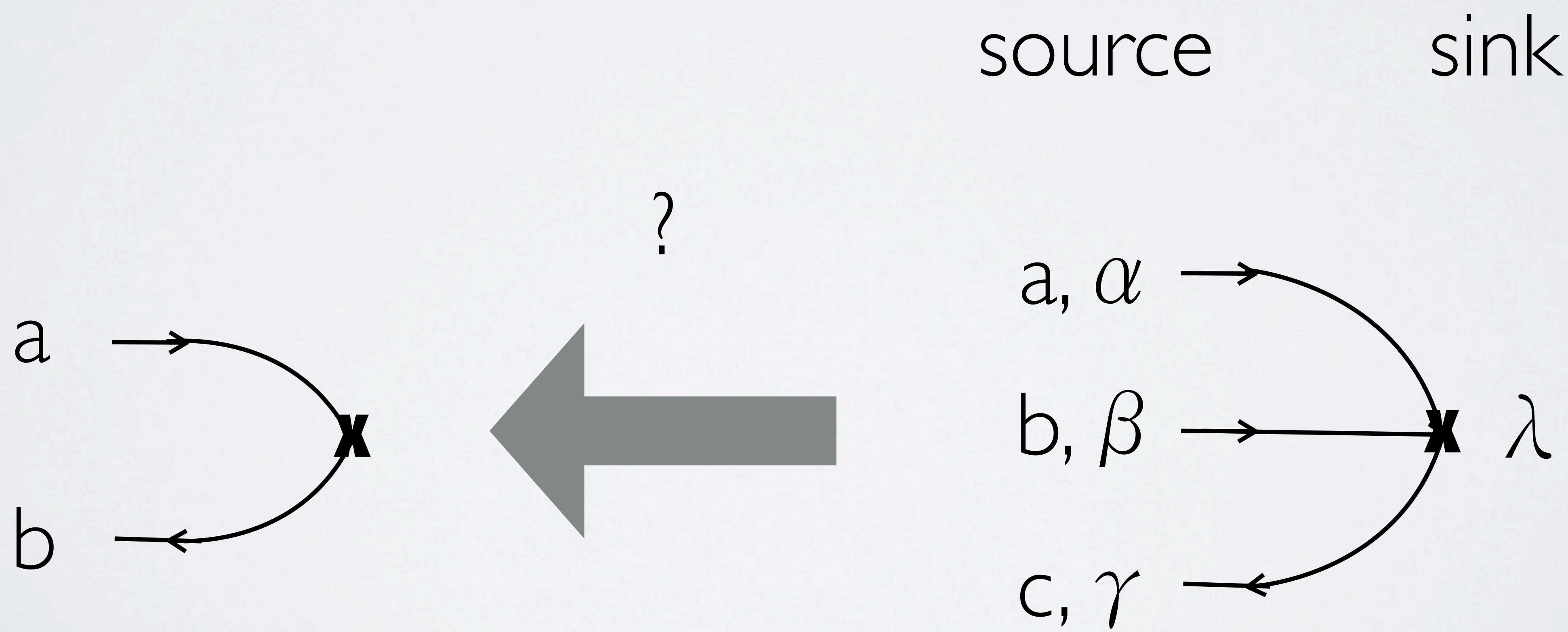
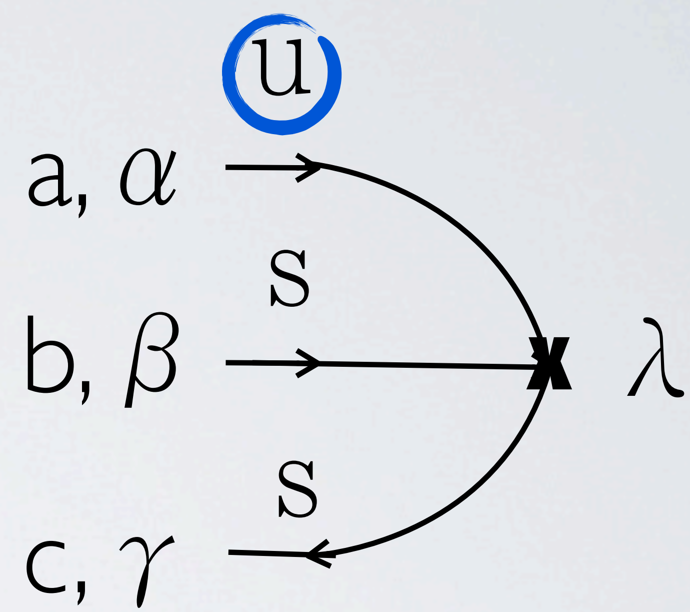


Ξ +PIONS, N+KAONS

Ξ^0 +pions

only u quarks need to be contracted with pions



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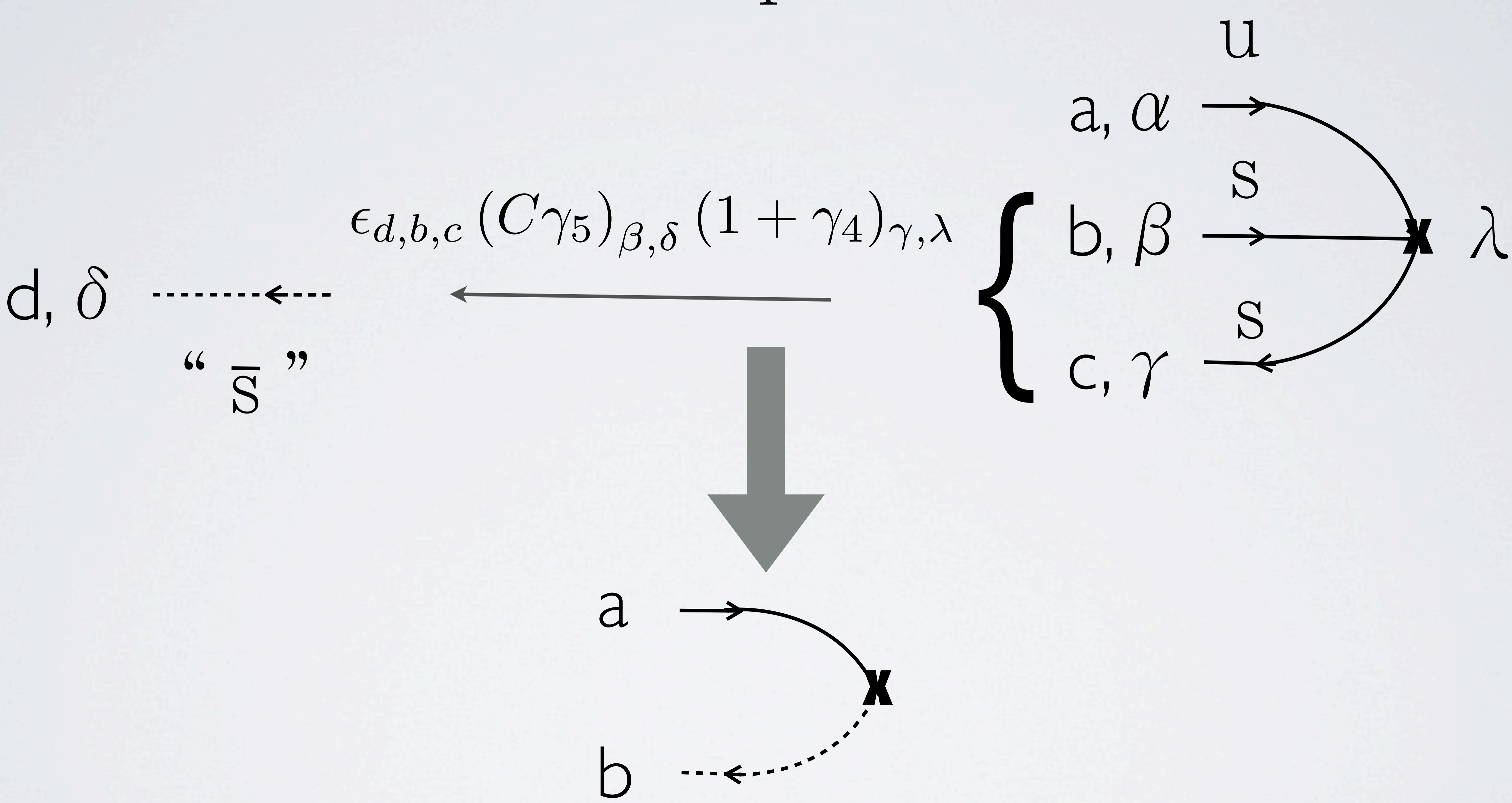

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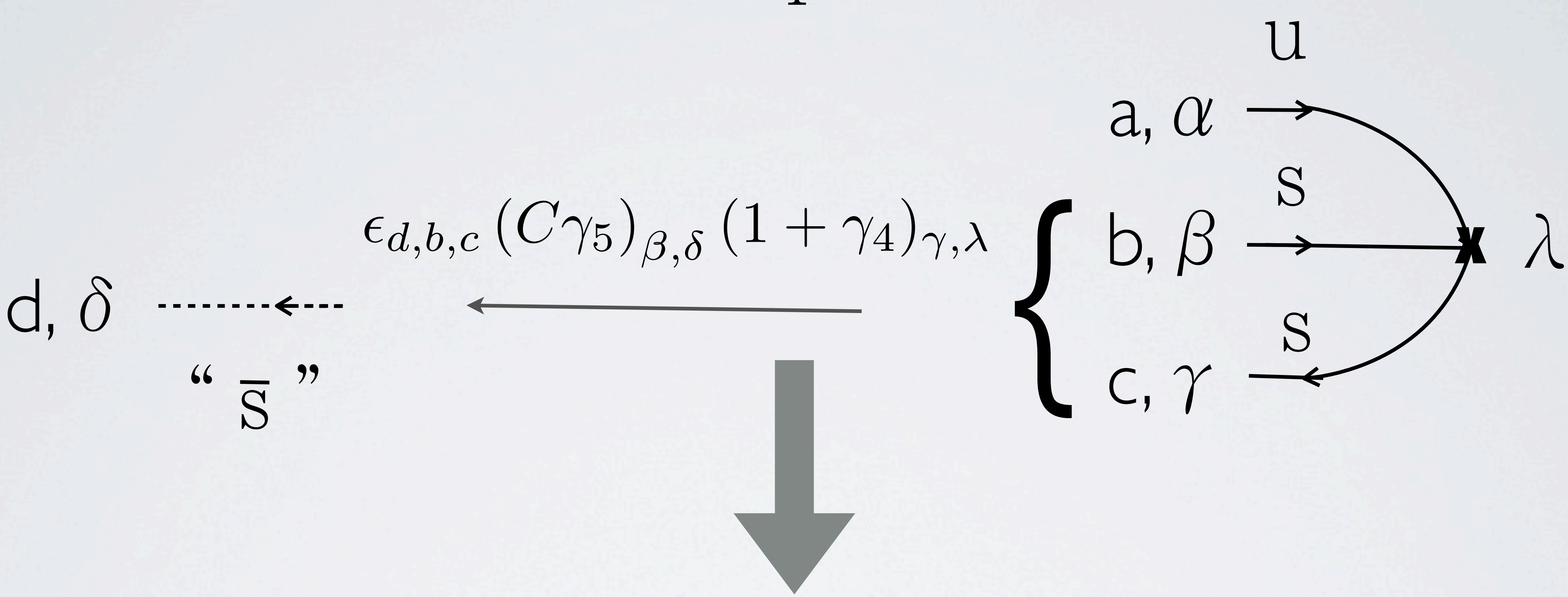
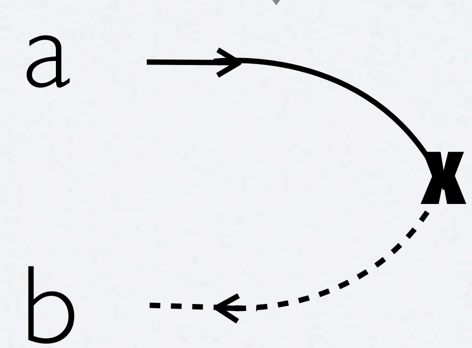
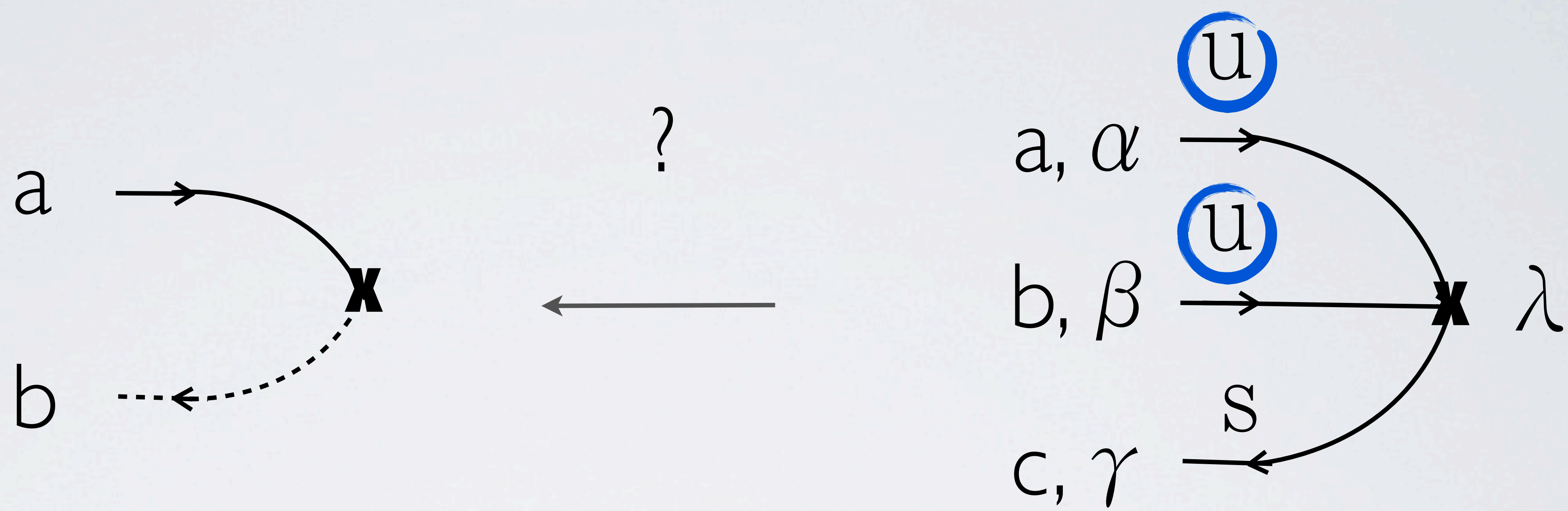




Plug in to formula for mixed species

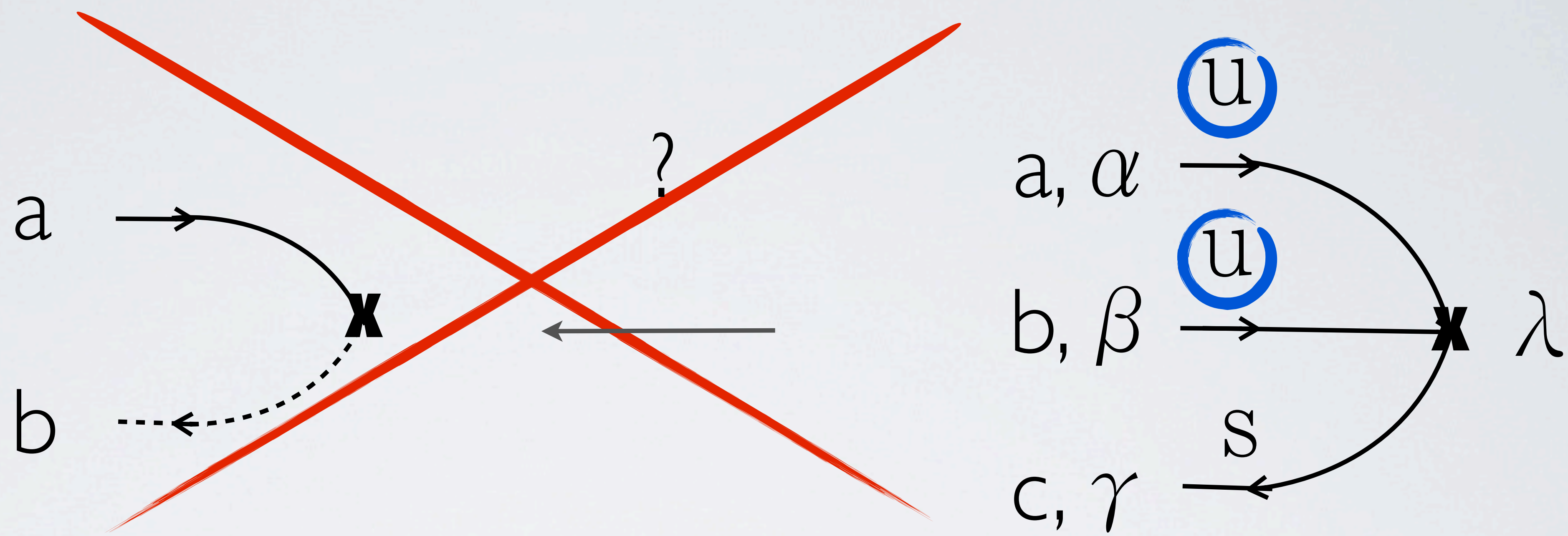
Σ +PIONS, P+KAONS

Σ^+ +pions



Σ +PIONS, P+KAONS

Σ^+ +pions



Σ +PIONS, P+KAONS

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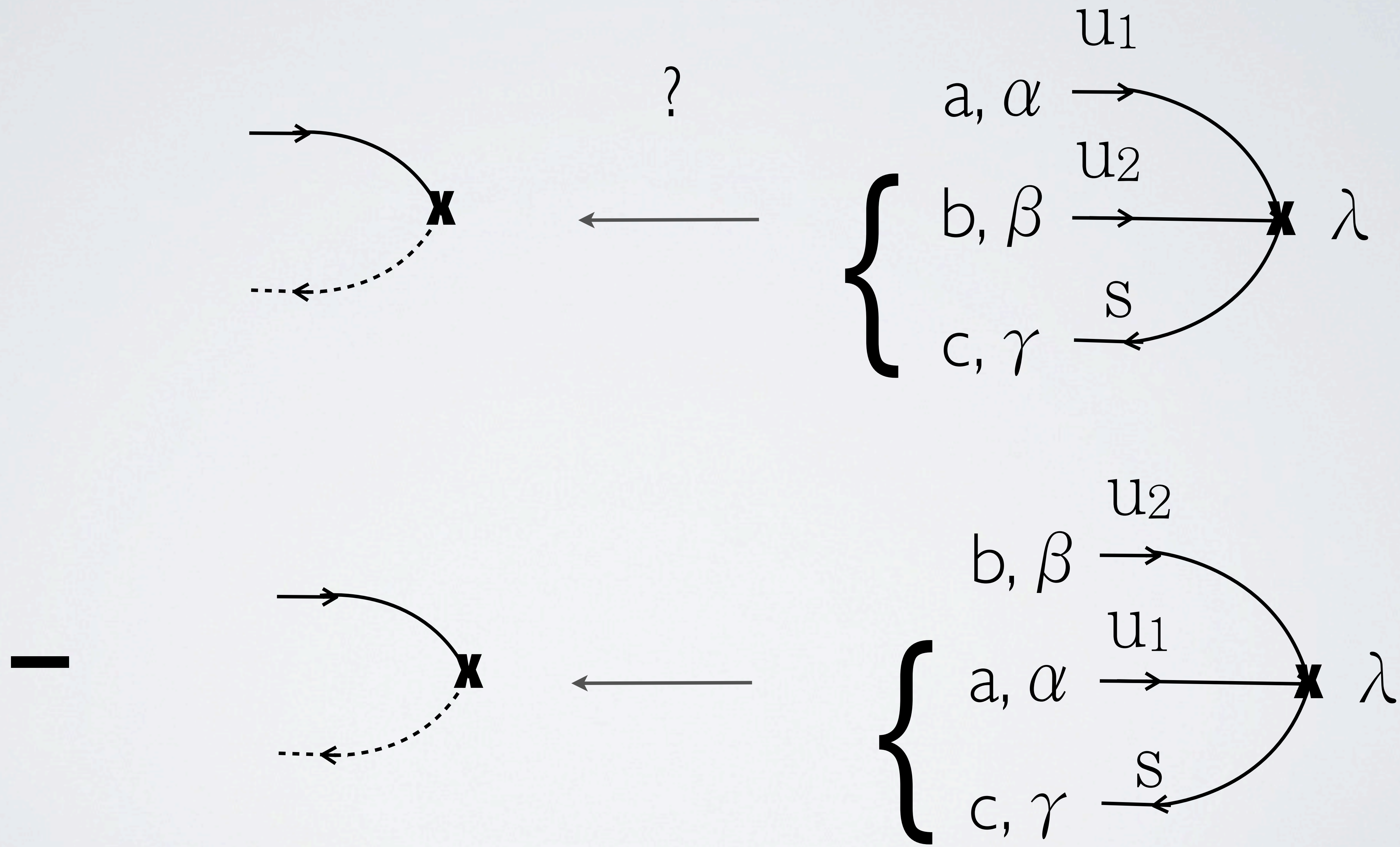

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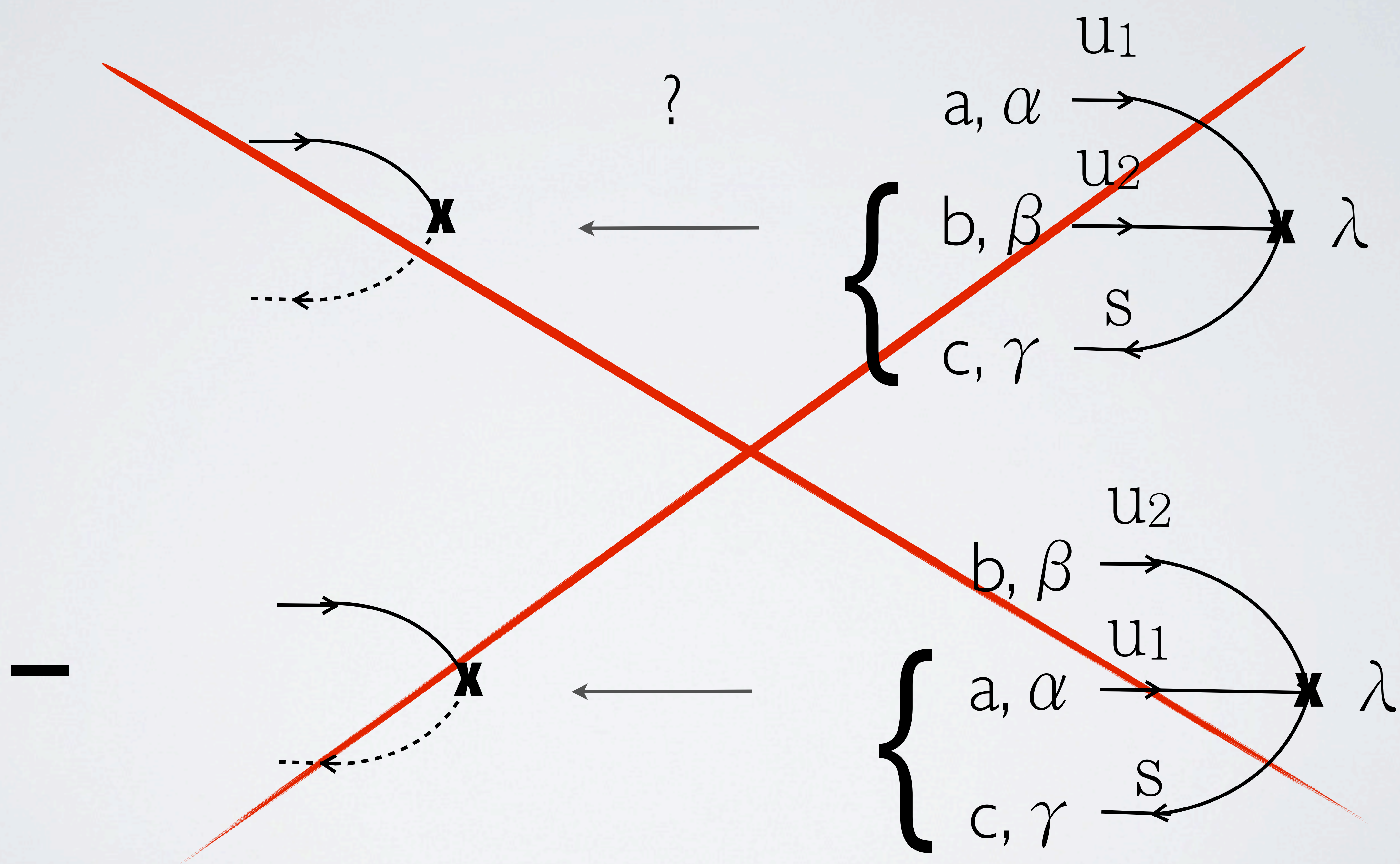
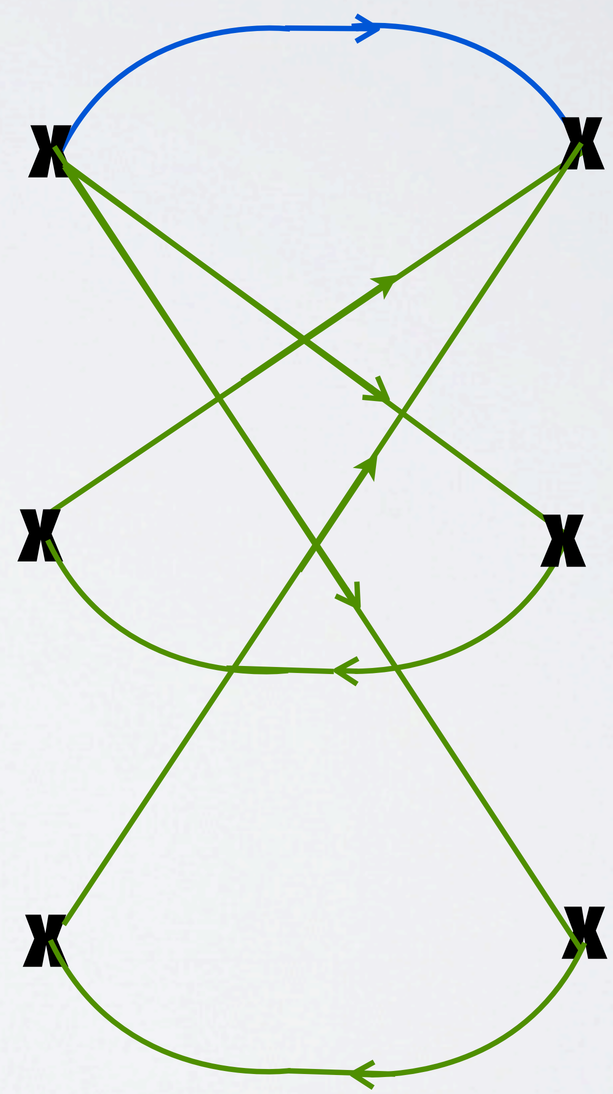


Σ +PIONS, P+KAONS

Σ^+ +pions

Missing diagrams where baryon exchanges both quarks



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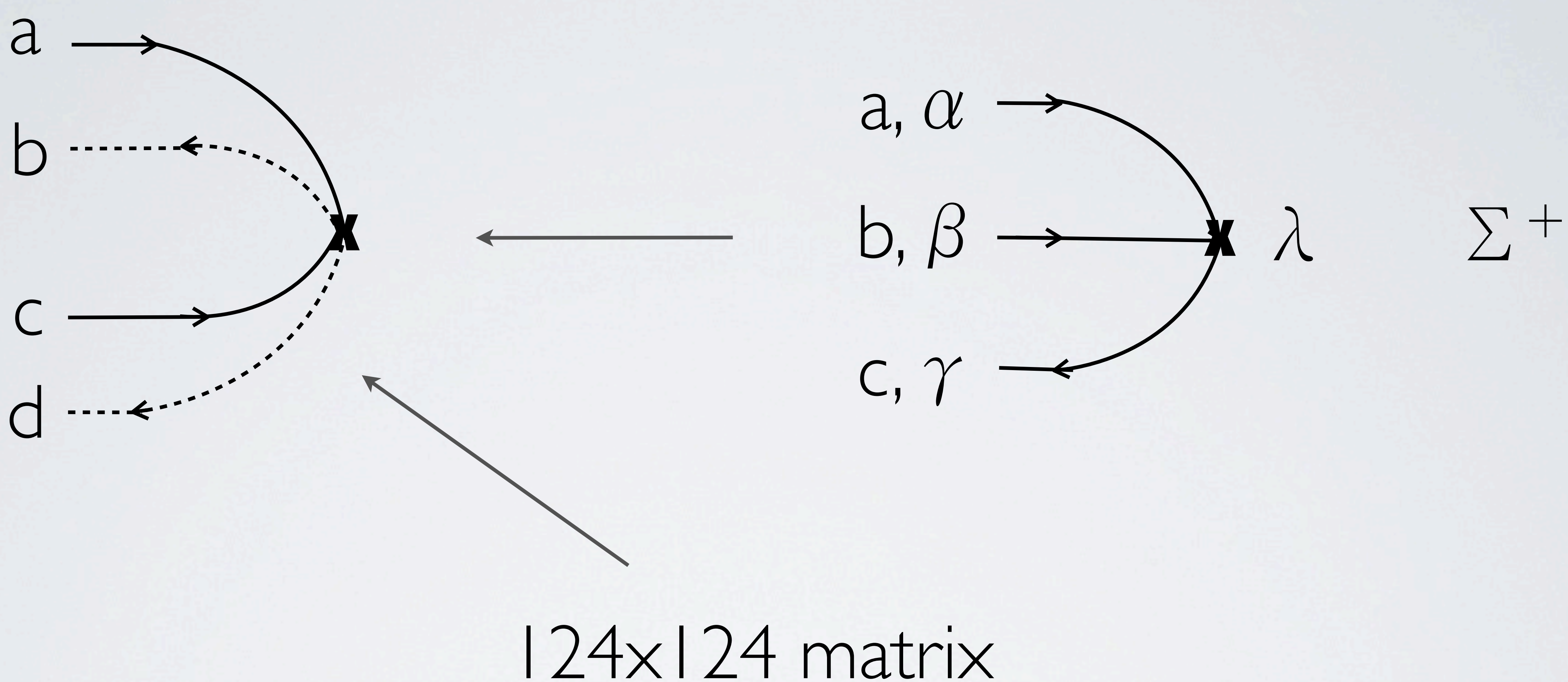

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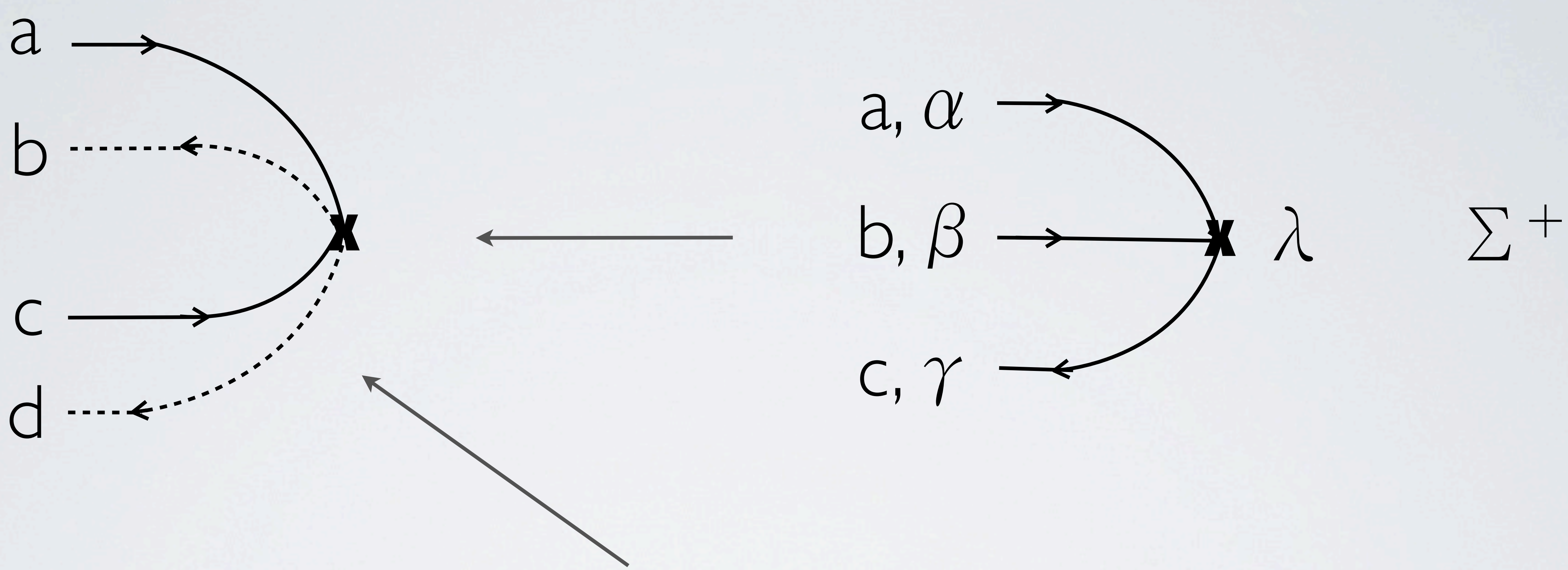


124x124 matrix

$$\Pi \otimes \Pi, \qquad 1 \otimes \Pi, \qquad \Pi \otimes 1$$

LATTICE DETAILS

- HSC lattices
  - clover, tadpole improved
  - a_s=0.125 fm, a_t=a_s/3.5,

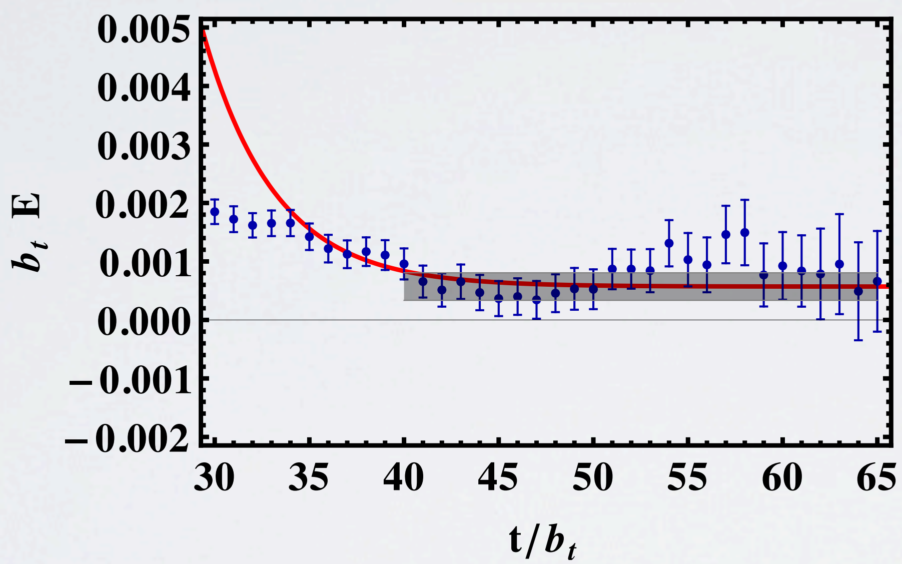
    m_π=390 MeV, 32^3x256
- NPLQCD propagators
  - same discretization as gauge fields
  - ~ 200 per configuration

ENERGY SPLITTINGS

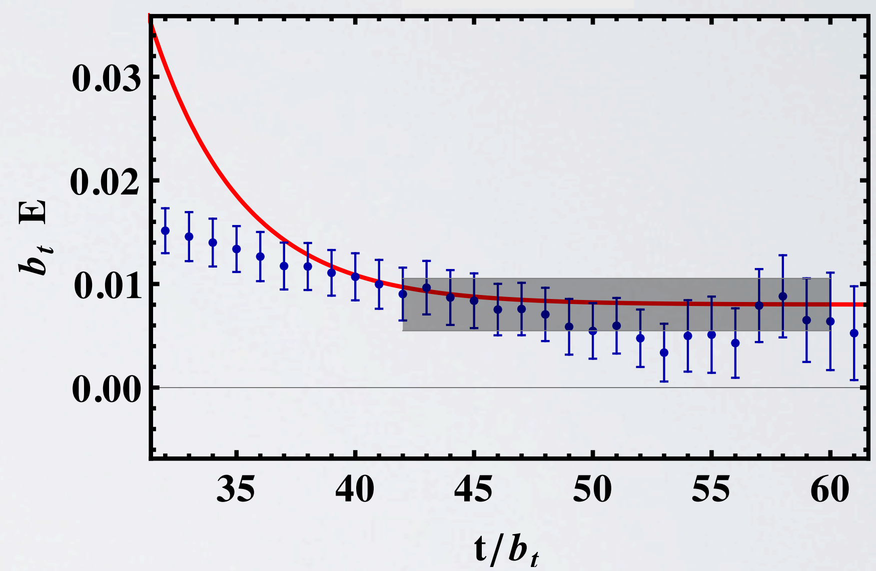
$$\Delta M_{\text{eff}}^{(n)}(t) = \ln\left(\frac{C_{B,n}(t)/C_{B,n}(t+1)}{[C_B(t)/C_B(t+1)][C_n(t)/C_n(t+1)]}\right)$$

Ξ^0

$N_\pi = 1$

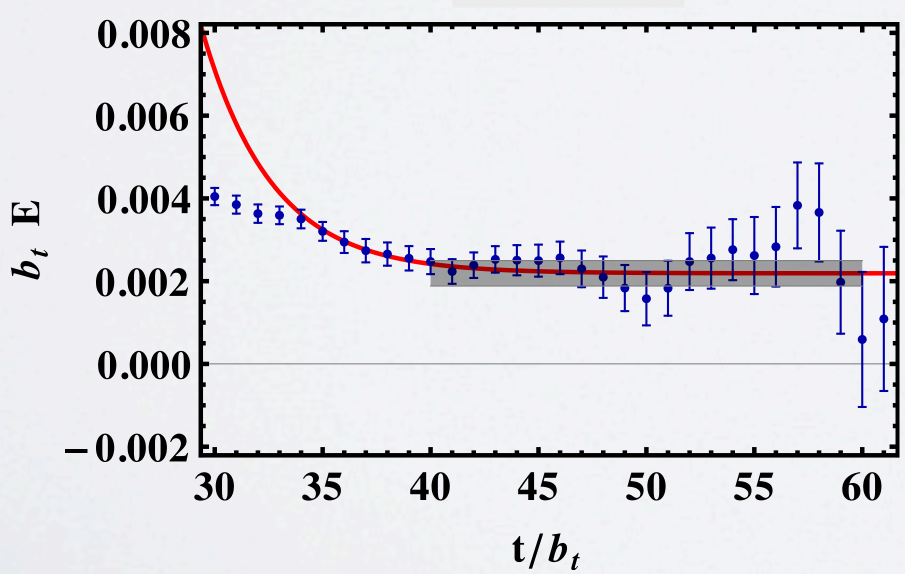


$N_\pi = 7$

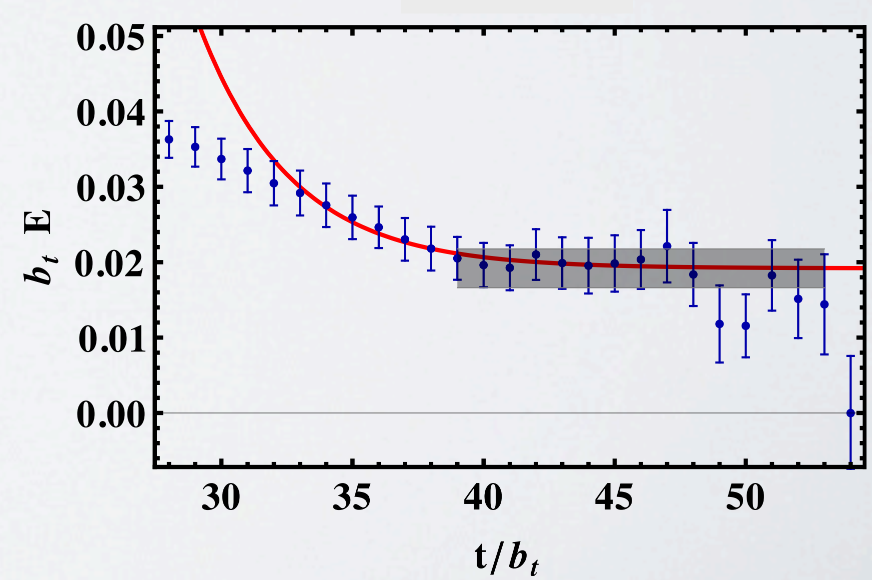


Σ^+

$N_\pi = 1$



$N_\pi = 8$

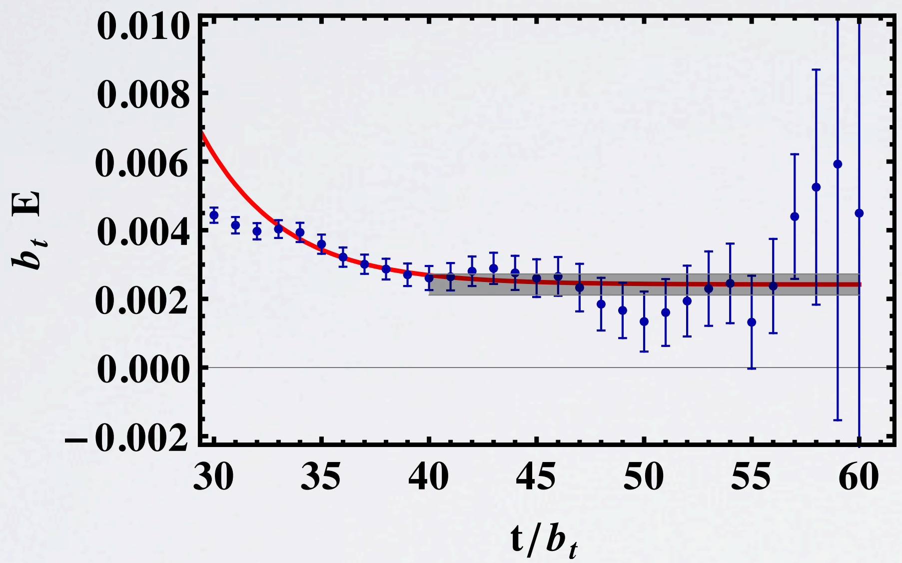


ENERGY SPLITTINGS

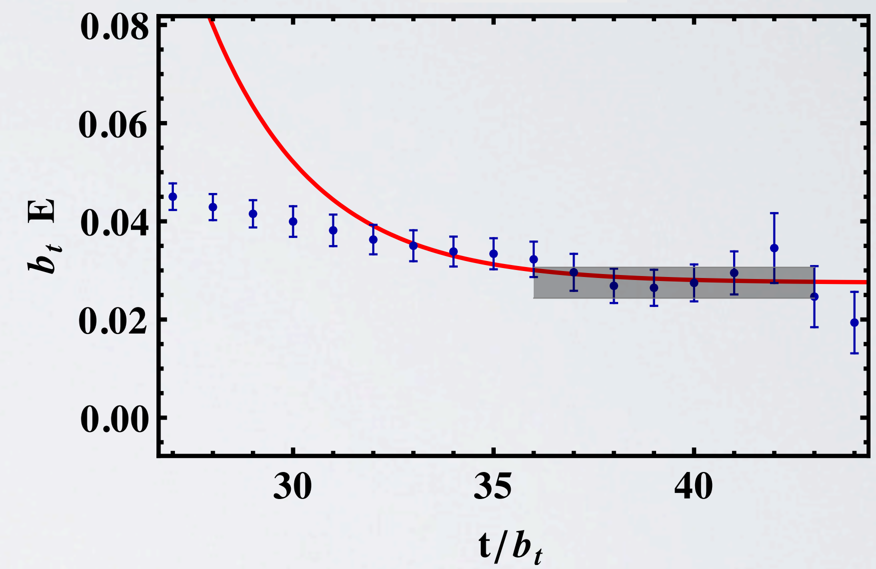
$$\Delta M_{\rm eff}^{(n)}(t) = \ln\left(\frac{C_{B,n}(t)/C_{B,n}(t+1)}{[C_B(t)/C_B(t+1)][C_n(t)/C_n(t+1)]}\right)$$

proton

$N_K=1$

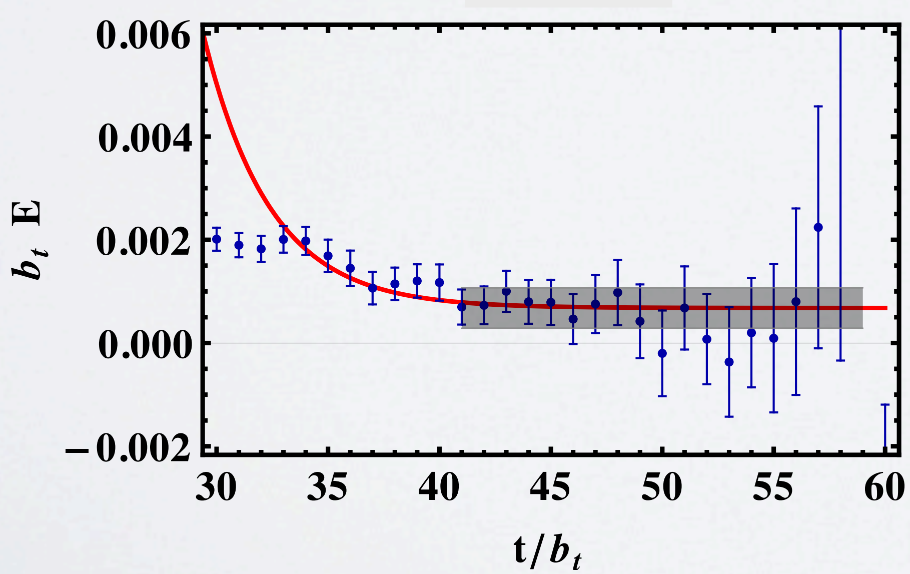


$N_K=9$

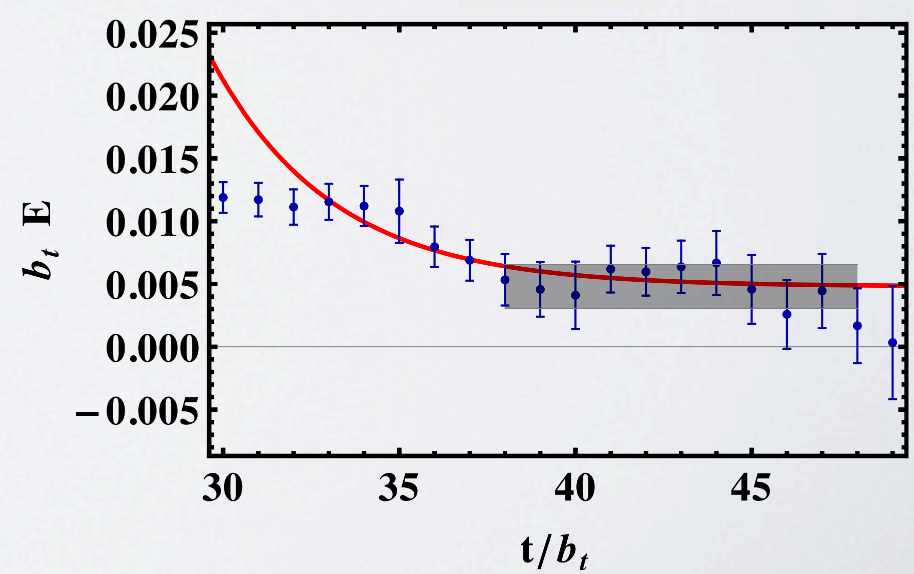


neutron

$N_K=1$



$N_K=5$



ENERGIES IN A BOX

Beane, Detmold & Savage (2007)
Smigielski & Wasem (2008)

- Large volume expansion of g.s. energy for two species of bosons in a box to $O(L^{-6})$
  - extension of Lüscher's relation for 2 particles in a box
  - includes 2- and 3-body parameters
- Since single baryon carries the spin for the entire system, can treat like different species of boson

ENERGIES IN A BOX

Beane, Detmold & Savage (2007)
Smigielski & Wasem (2008)

$$\Delta E_{MB}(n,L) = \frac{2\pi \bar{a}_{MB} n}{\mu_{MB} L^3} \Big[1 - \Big(\frac{\bar{a}_{MB}}{\pi L}\Big) \mathcal{I} + \Big(\frac{\bar{a}_{MB}}{\pi L}\Big)^2 \left(\mathcal{I}^2 + \mathcal{J} \left[-1 + 2\frac{\bar{a}_{MM}}{\bar{a}_{MB}}(n-1)\left(1 + \frac{\mu_{MB}}{m_M}\right) \right.\right.$$

$$\left.\left. + \Big(\frac{\bar{a}_{MB}}{\pi L}\Big)^3 \left(-\mathcal{I}^3 + \sum_{i=0}^{2} \left(f_i^{\mathcal{I}\mathcal{J}} \mathcal{I}\mathcal{J} + f_i^{\mathcal{K}} \mathcal{K}\right) \left(\frac{\bar{a}_{MM}}{\bar{a}_{MB}}\right)^i \right) \right]$$

$$+ \frac{n(n-1)\bar{\eta}_{3,MMB}(L)}{2L^6} + \mathcal{O}(L^{-7})$$

ENERGIES IN A BOX

Beane, Detmold & Savage (2007)
Smigielski & Wasem (2008)

2-body parameters

$$\Delta E_{MB}(n,L) = \frac{2\pi \bar{a}_{MB} n}{\mu_{MB} L^3}\left[1 - \left(\frac{\bar{a}_{MB}}{\pi L}\right)\mathcal{I} + \left(\frac{\bar{a}_{MB}}{\pi L}\right)^2 \left(\mathcal{I}^2 + \mathcal{J}\left[-1 + 2\frac{\bar{a}_{MM}}{\bar{a}_{MB}}(n-1)\left(1 + \frac{\mu_{MB}}{m_M}\right)\right.\right.\right.$$

$$\left.\left. + \left(\frac{\bar{a}_{MB}}{\pi L}\right)^3 \left(-\mathcal{I}^3 + \sum_{i=0}^{2}\left(f_i^{\mathcal{I}\mathcal{J}}\mathcal{I}\mathcal{J} + f_i^{\mathcal{K}}\mathcal{K}\right)\left(\frac{\bar{a}_{MM}}{\bar{a}_{MB}}\right)^i\right)\right] + \frac{n(n-1)\bar{\eta}_{3,MMB}(L)}{2L^6} + \mathcal{O}(L^{-7})$$

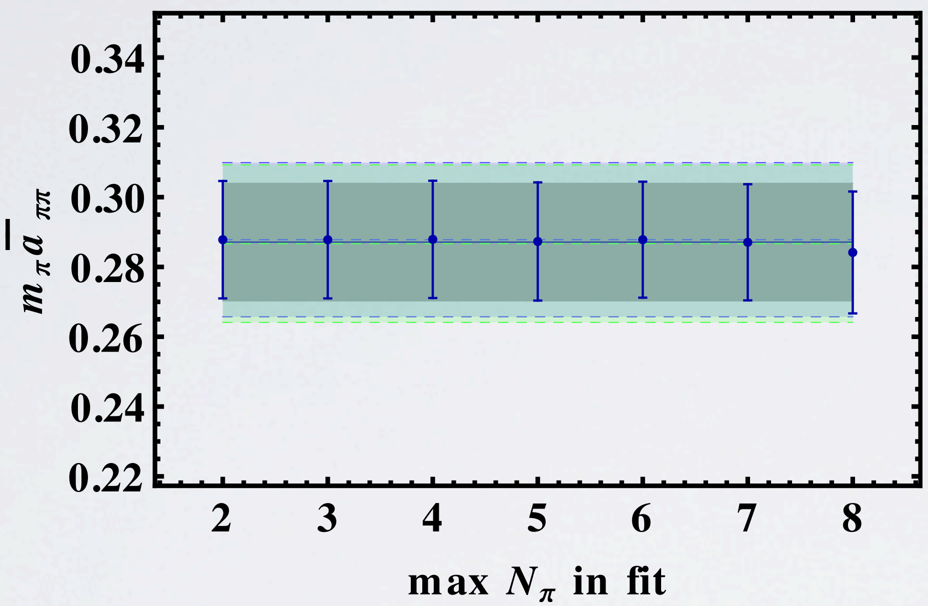
3-body parameter

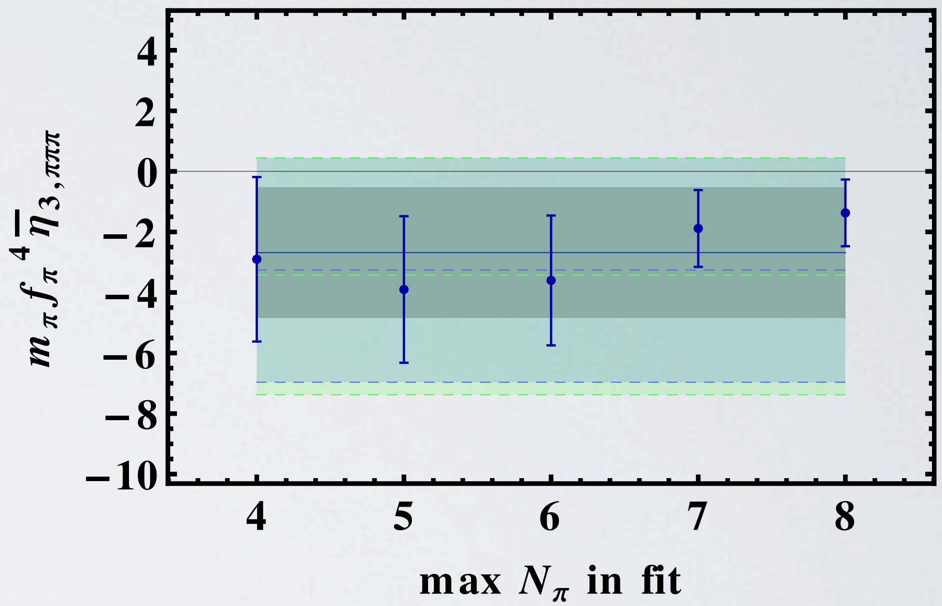
PURE MESON SYSTEMS

2-body

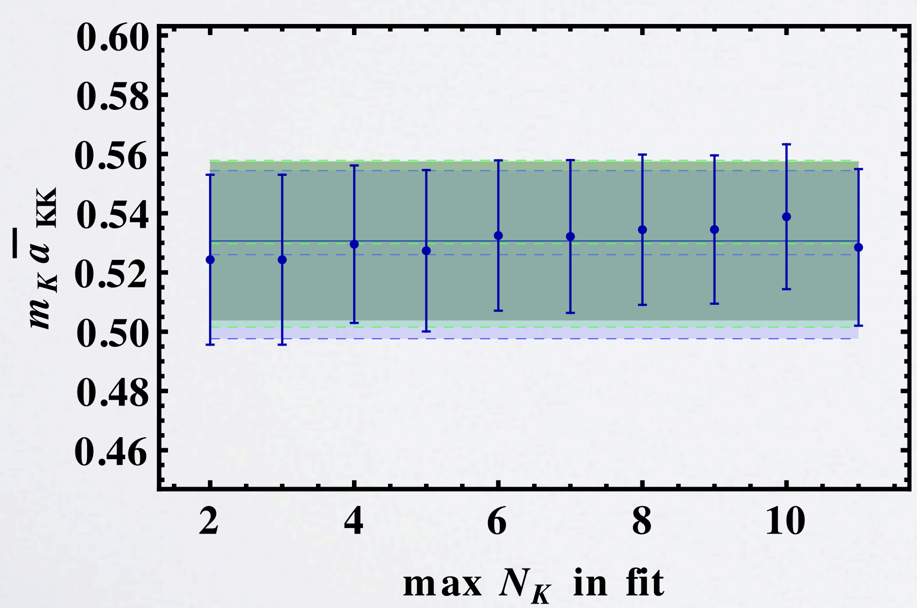
3-body

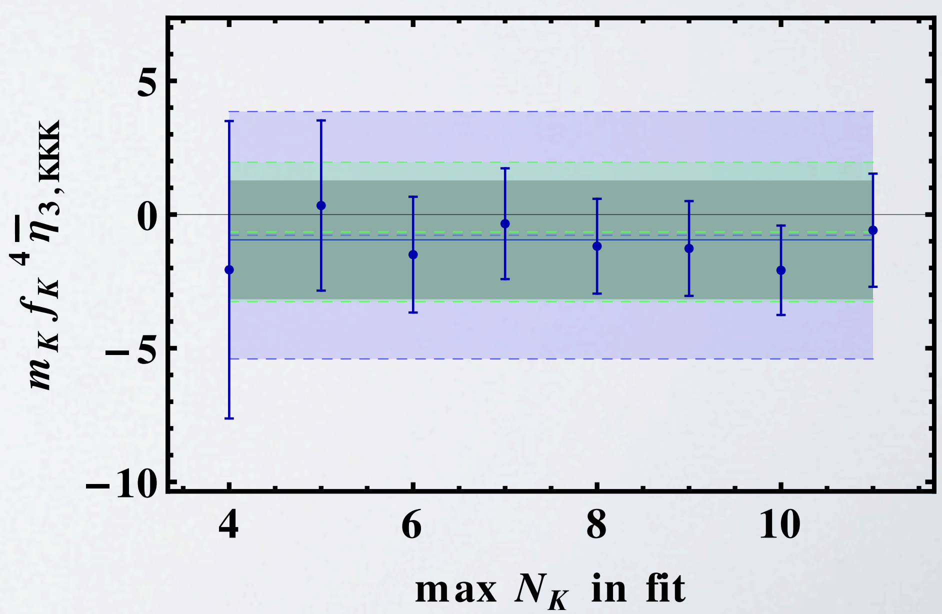
pions



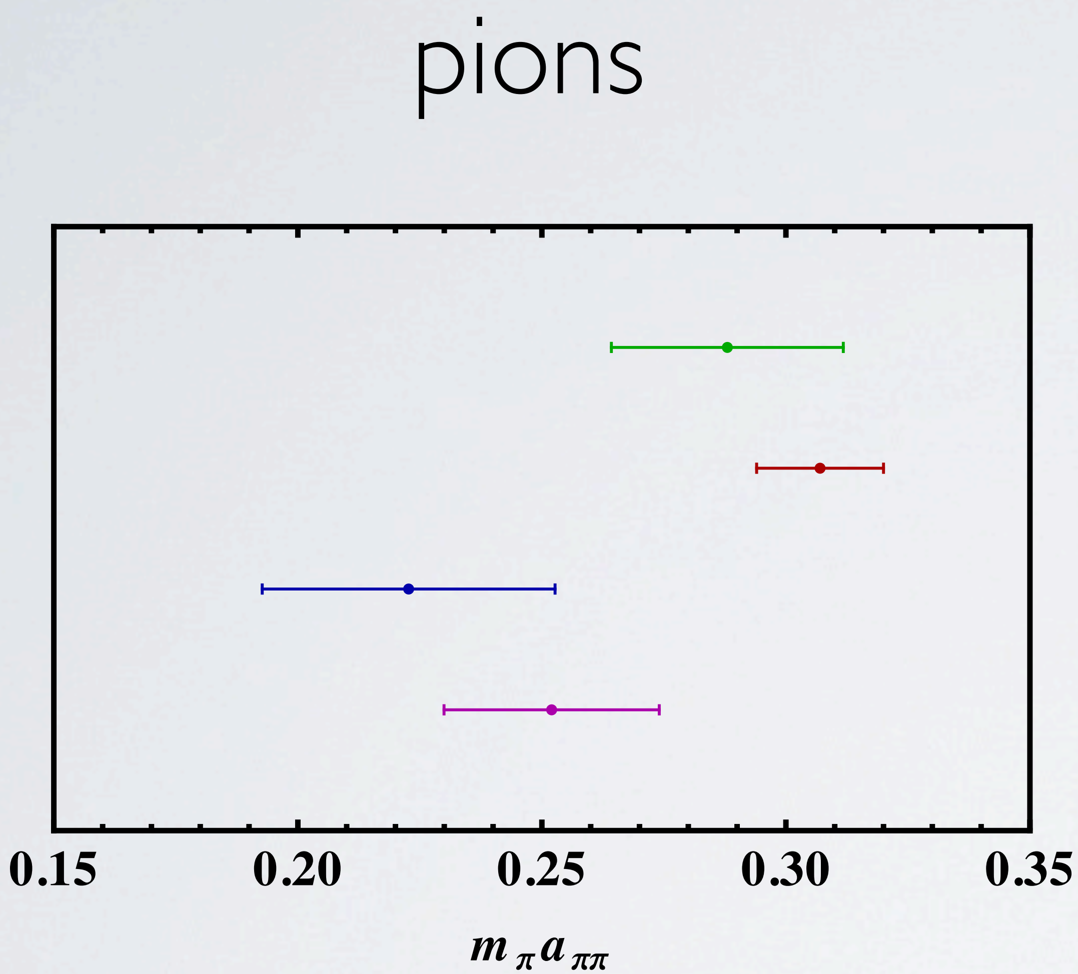


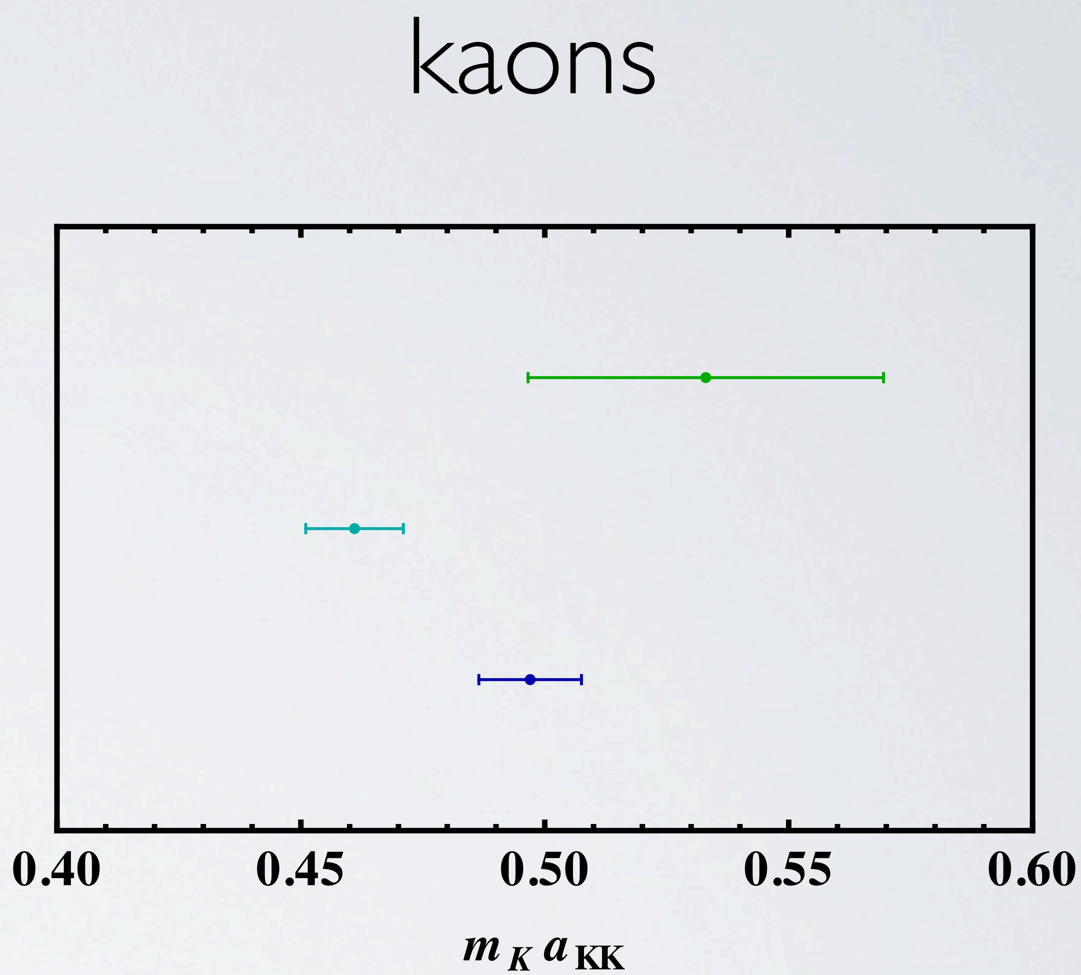
kaons

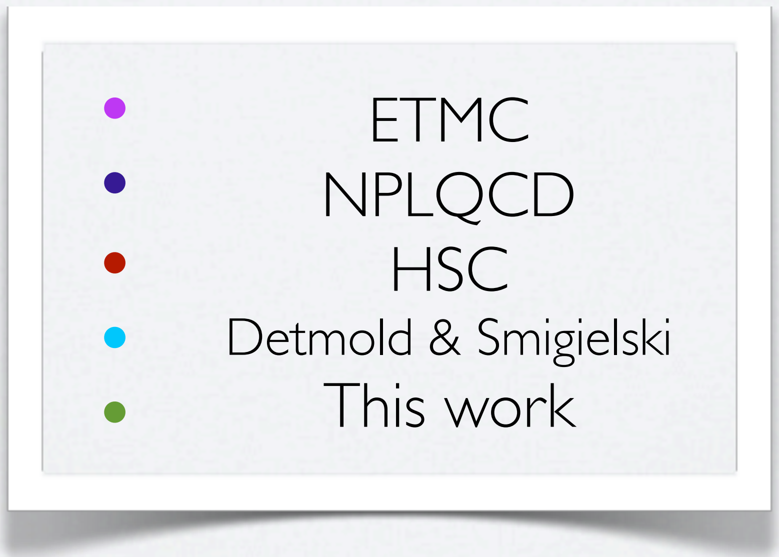




PURE MESON SYSTEMS

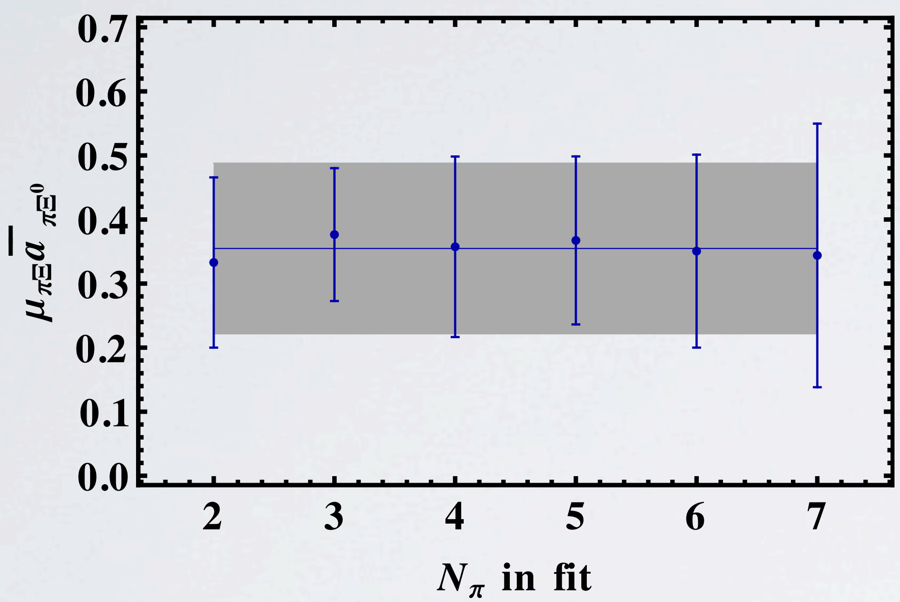




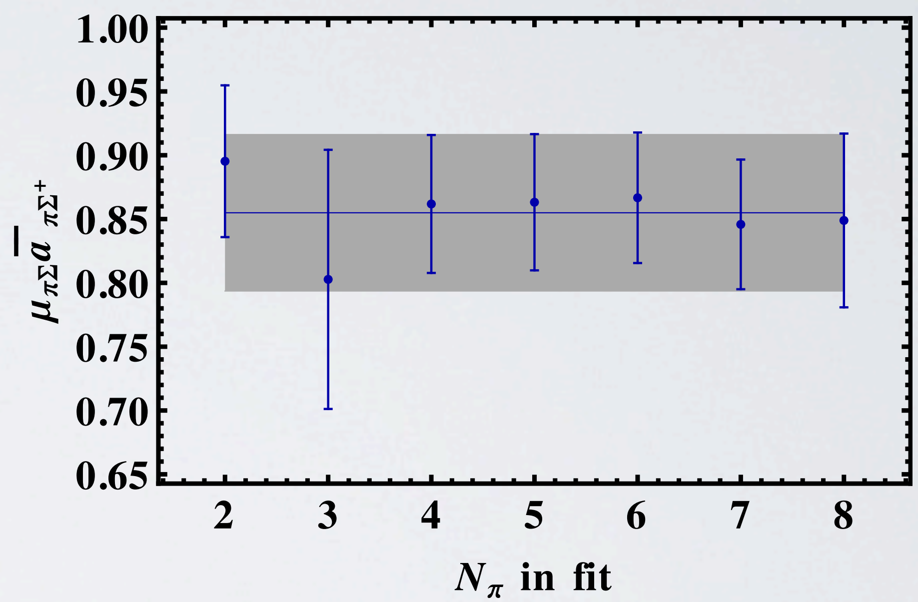


2-BODY PARAMETERS

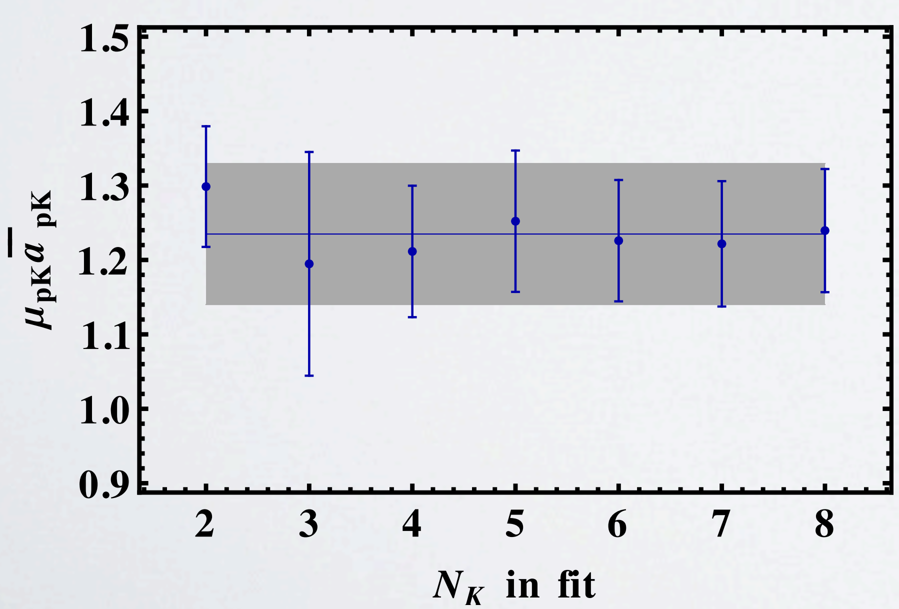
Ξ^0, π^+



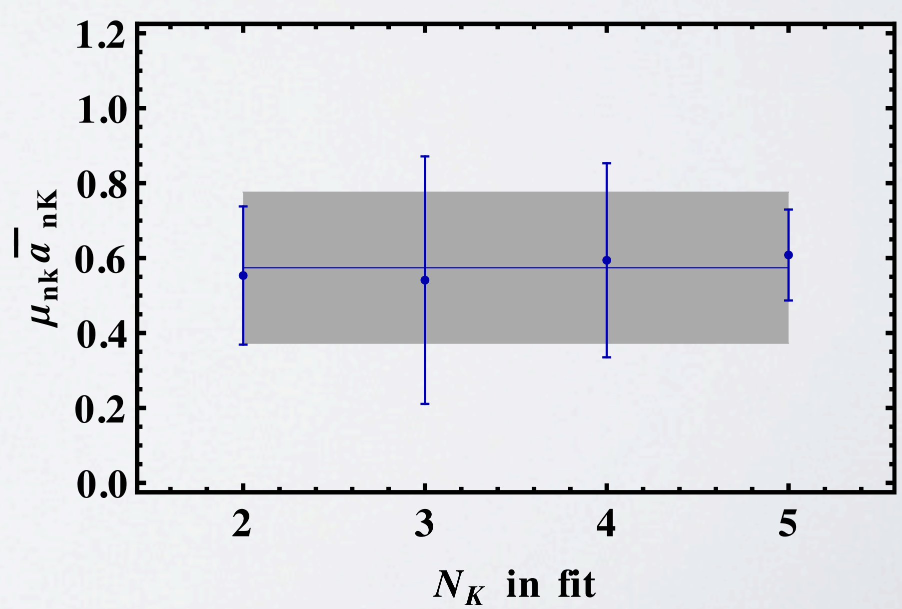
Σ^+, π^+



p,K^+



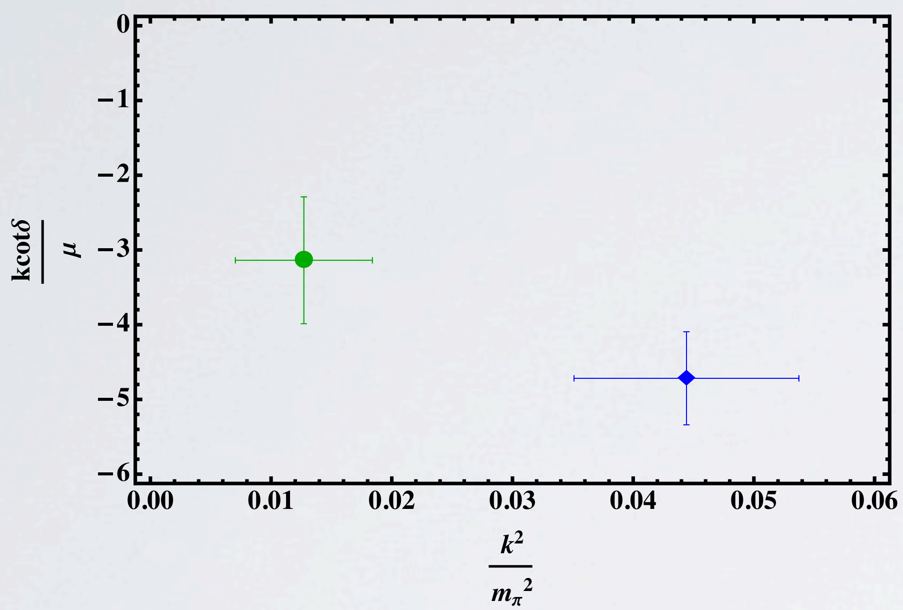
n,K^+



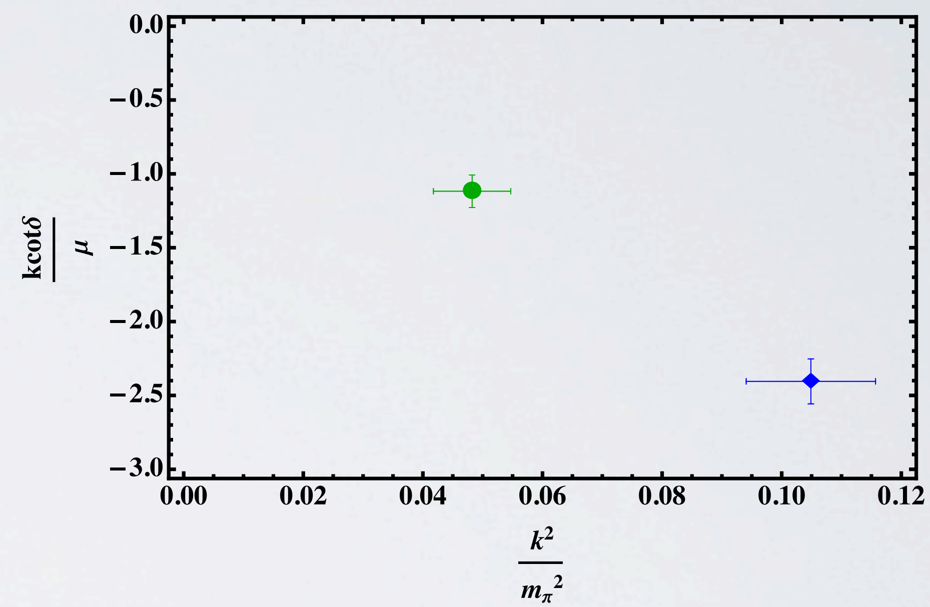
2-BODY PARAMETERS

NPLQCD (2009)
L=2.5 fm

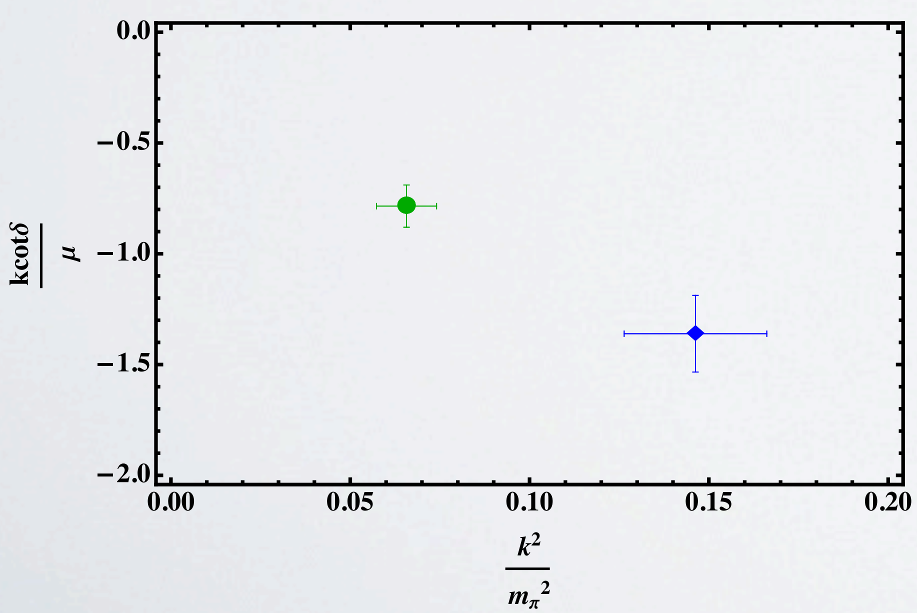
Ξ^0, π^+



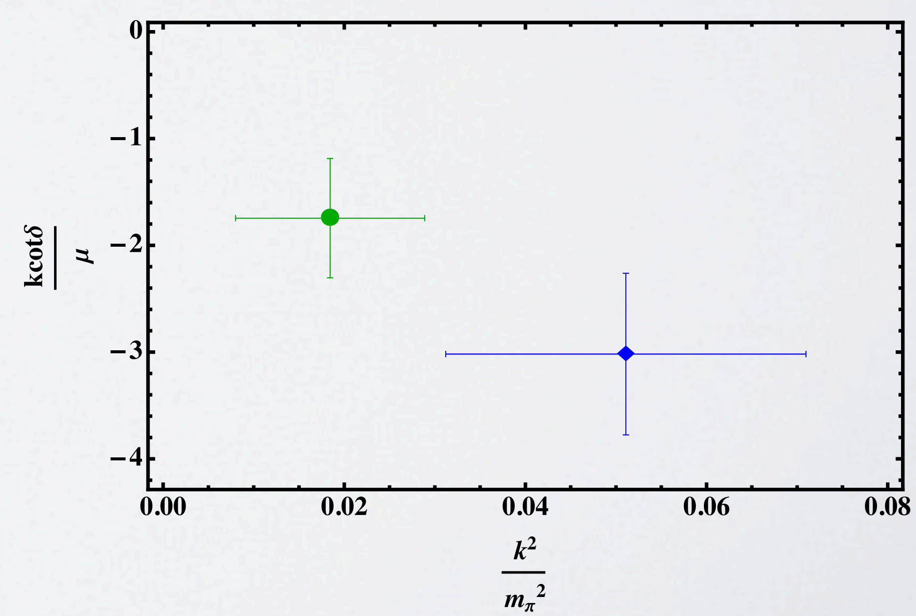
Σ^+, π^+



p, K^+

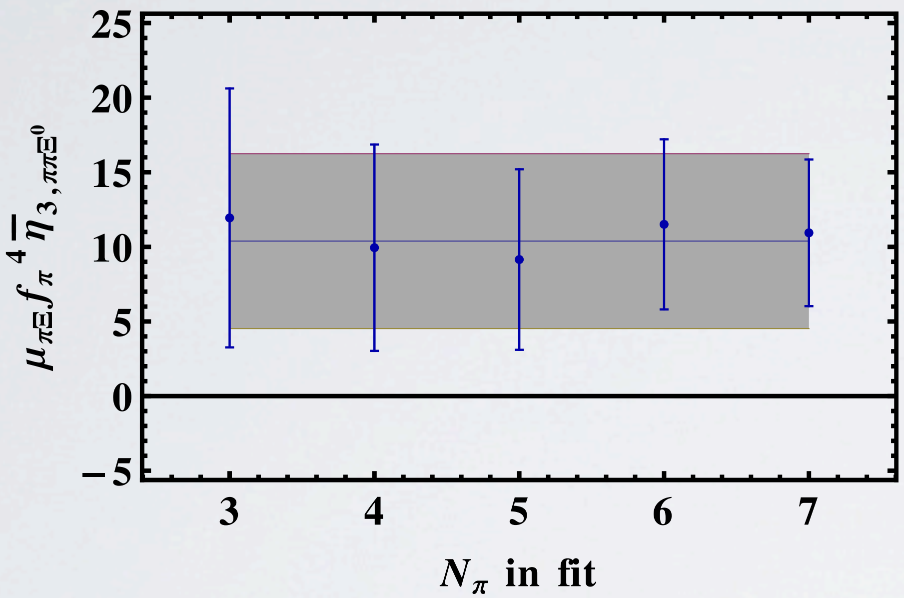


n, K^+

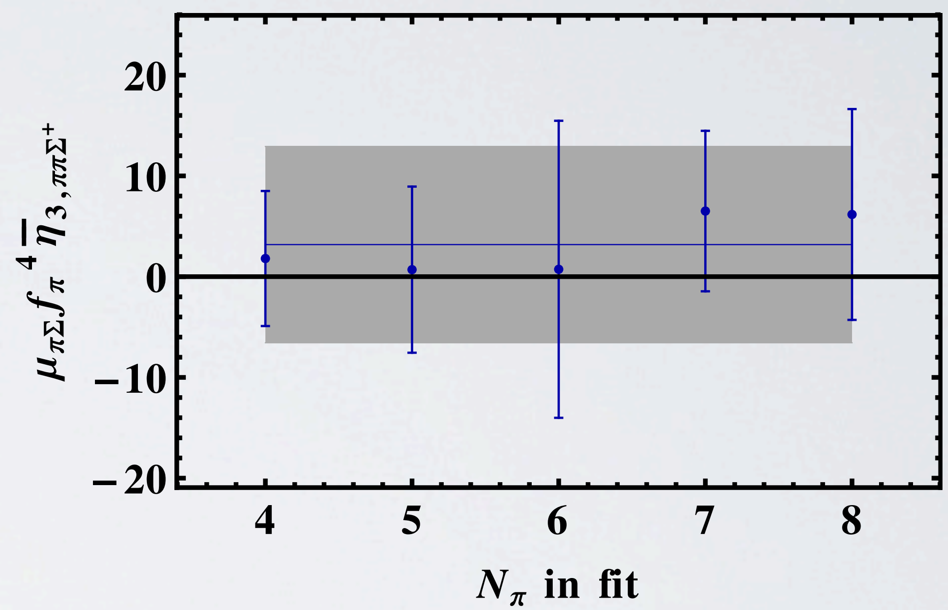


3-BODY PARAMETERS

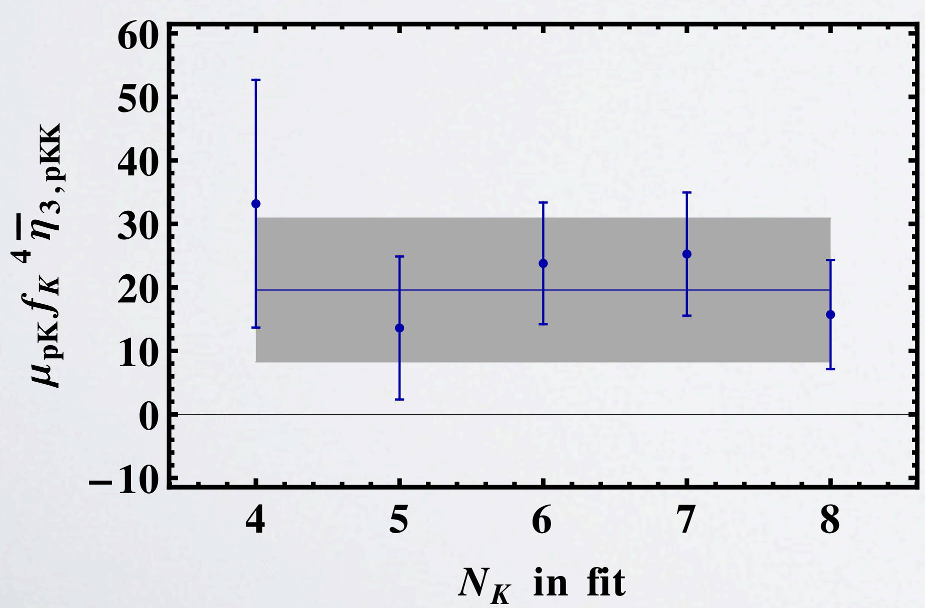
Ξ^0, π^+



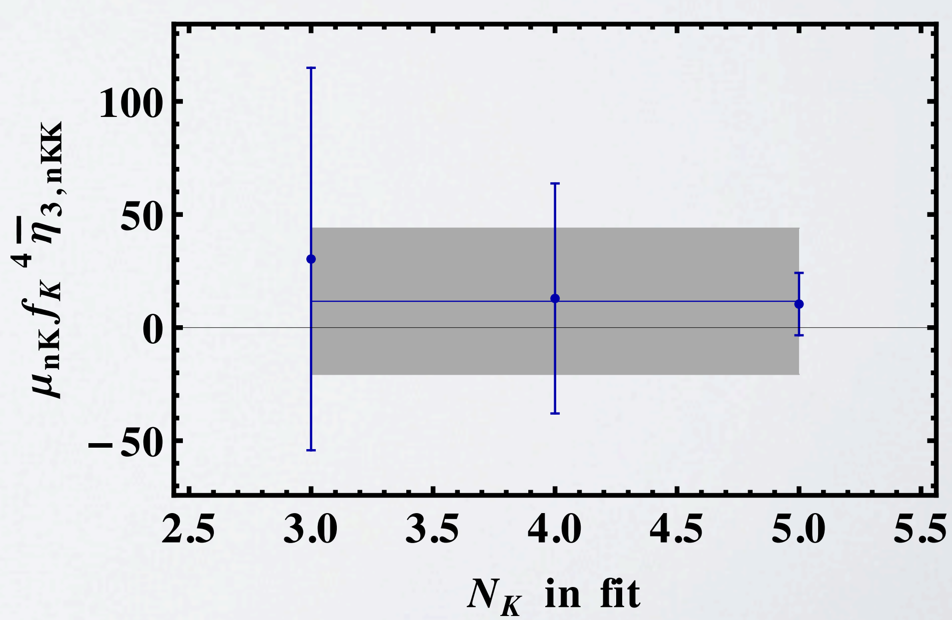
Σ^+, π^+



p,K$^+$



n,K$^+$



TREE-LEVEL χPT

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases}$$

$$M_{\Xi^0}(\mu_I) = M_{\Xi^{(0)}} - \frac{\mu_I}{2}\cos\alpha + 4c_1^\Xi m_\pi^2 \cos\alpha + \left(c_2^\Xi - \frac{g_{\Xi\Xi}^2}{8M_\Xi^{(0)}} + c_3^\Xi\right)\mu_I^2 \sin^2\alpha$$

Son & Stephanov (2001)
Bedaque, Buchoff, Tiburzi (2009)

$$M_{\Sigma^+}(\mu_I) = M_{\Sigma^{(0)}} + 4c_1^\Sigma m_\pi^2 \cos\alpha + (c_2^\Sigma + c_3^\Sigma + c_6^\Sigma + c_7^\Sigma)\mu_I^2 \sin^2\alpha - \mu_I\sqrt{\cos^2\alpha + (c_6^\Sigma + c_7^\Sigma)^2 \mu_I^2 \sin^4\alpha}$$

TREE-LEVEL χPT

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases}$$

$$M_{\Xi^0}(\mu_I) = M_{\Xi^{(0)}} - \frac{\mu_I}{2}\cos\alpha + 4c_1^{\Xi} m_\pi^2 \cos\alpha + \left(c_2^{\Xi} - \frac{g_{\Xi\Xi}^2}{8M_\Xi^{(0)}} + c_3^{\Xi}\right)\mu_I^2 \sin^2\alpha$$

$$M_{\Sigma^+}(\mu_I) = M_{\Sigma^{(0)}} + 4c_1^{\Sigma} m_\pi^2 \cos\alpha + (c_2^{\Sigma} + c_3^{\Sigma} + c_6^{\Sigma} + c_7^{\Sigma})\mu_I^2 \sin^2\alpha - \mu_I\sqrt{\cos^2\alpha + (c_6^{\Sigma} + c_7^{\Sigma})^2 \mu_I^2 \sin^4\alpha}$$

Tree level mass corrections in vacuum ($\cos\alpha = 1$)

TREE-LEVEL χPT

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases}$$

$$M_{\Xi^0}(\mu_I) = M_{\Xi^{(0)}} - \frac{\mu_I}{2}\cos\alpha + 4c_1^{\Xi} m_\pi^2 \cos\alpha + \left(c_2^{\Xi} - \frac{g_{\Xi\Xi}^2}{8M_{\Xi}^{(0)}} + c_3^{\Xi}\right)\mu_I^2 \sin^2\alpha$$

$$M_{\Sigma^+}(\mu_I) = M_{\Sigma^{(0)}} + 4c_1^{\Sigma} m_\pi^2 \cos\alpha + (c_2^{\Sigma} + c_3^{\Sigma} + c_6^{\Sigma} + c_7^{\Sigma})\mu_I^2 \sin^2\alpha - \mu_I\sqrt{\cos^2\alpha + (c_6^{\Sigma} + c_7^{\Sigma})^2 \mu_I^2 \sin^4\alpha}$$

Direct coupling to chemical potential in vacuum ($\cos\alpha$ =1)

TREE-LEVEL χPT

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases}$$

$$M_{\Xi^0}(\mu_I) = M_{\Xi^{(0)}} - \frac{\mu_I}{2}\cos\alpha + 4c_1^{\Xi} m_\pi^2 \cos\alpha + \left(c_2^{\Xi} - \frac{g_{\Xi\Xi}^2}{8M_{\Xi}^{(0)}} + c_3^{\Xi}\right)\mu_I^2 \sin^2\alpha$$

$$-M_{\Xi^{(0)}} - 4c_1^{\Xi} m_\pi^2 + \frac{\mu_I}{2}$$

$$M_{\Sigma^+}(\mu_I) = M_{\Sigma^{(0)}} + 4c_1^{\Sigma} m_\pi^2 \cos\alpha + (c_2^{\Sigma} + c_3^{\Sigma} + c_6^{\Sigma} + c_7^{\Sigma})\mu_I^2 \sin^2\alpha - \mu_I\sqrt{\cos^2\alpha + (c_6^{\Sigma} + c_7^{\Sigma})^2 \mu_I^2 \sin^4\alpha}$$

$$-M_{\Sigma^{(0)}} - 4c_1^{\Sigma} m_\pi^2 + \mu_I$$

TREE-LEVEL χPT

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \dfrac{m_\pi^2}{\mu_I^2} & (\langle\pi\rangle \neq 0) \end{cases}$$

$$M_{\Xi^0}(\mu_I) = M_{\Xi^{(0)}} - \frac{\mu_I}{2}\cos\alpha + 4c_1^\Xi m_\pi^2 \cos\alpha + \left(c_2^\Xi - \frac{g_{\Xi\Xi}^2}{8M_\Xi^{(0)}} + c_3^\Xi\right)\mu_I^2 \sin^2\alpha$$

$$-M_{\Xi^{(0)}} - 4c_1^\Xi m_\pi^2 + \frac{\mu_I}{2}$$

$$M_{\Sigma^+}(\mu_I) = M_{\Sigma^{(0)}} + 4c_1^\Sigma m_\pi^2 \cos\alpha + \left(c_2^\Sigma + c_3^\Sigma + c_6^\Sigma + c_7^\Sigma\right)\mu_I^2 \sin^2\alpha - \mu_I\sqrt{\cos^2\alpha + \left(c_6^\Sigma + c_7^\Sigma\right)^2 \mu_I^2 \sin^4\alpha}$$

$$-M_{\Sigma^{(0)}} - 4c_1^\Sigma m_\pi^2 + \mu_I$$

TREE-LEVEL χPT

$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_K^2}{\mu_K^2} & (\langle K\rangle \neq 0) \end{cases}$$

$$M_n(\mu_K) = M_{n^{(0)}} - \frac{\mu_K}{2}\cos\alpha + (2b_0 + b_D - b_F)m_K^2\cos\alpha + \frac{1}{4}(b_1 - b_2 + b_3 + b_4 - b_5 + b_6 + 2b_7 + 2b_8)\mu_K^2\sin^2\alpha$$

$$M_p(\mu_K) = M_{p^{(0)}} + 2(b_0 + b_D)m_K^2\cos\alpha + \frac{1}{2}(b_1 + b_3 + b_4 + b_6 + b_7 + b_8)\mu_K^2\sin^2\alpha - \sqrt{(2b_F(m_K^2 - m_\pi^2) + \mu_K\cos\alpha)^2 + \frac{1}{4}(b_1 - b_3 + b_4 - b_6)^2\mu_K^4\sin^4\alpha}$$

+ μ -independent quark mass terms

TREE-LEVEL χPT

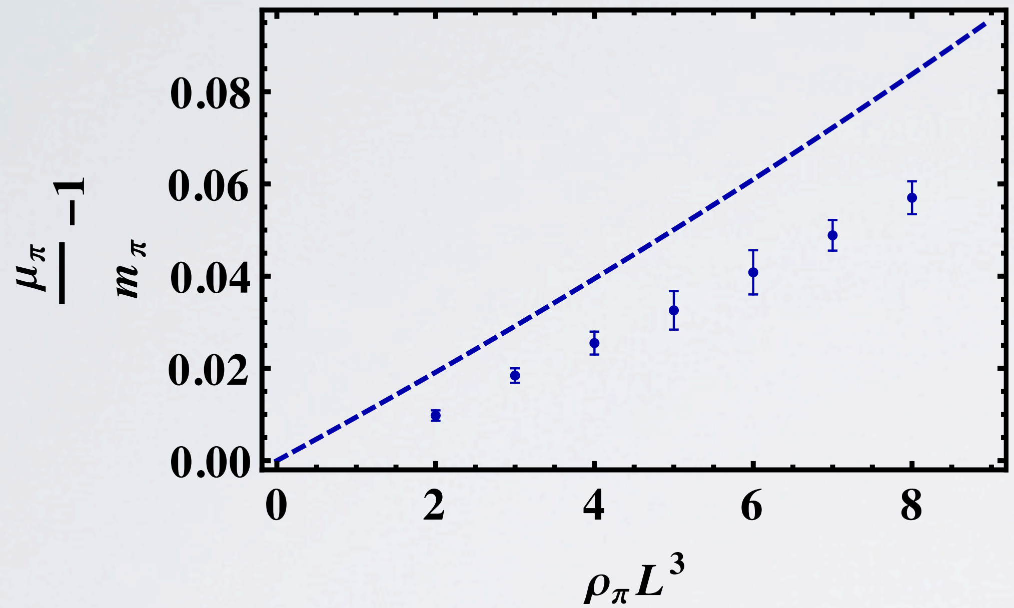
$$\cos\alpha = \begin{cases} 1 & \text{(vacuum)} \\ \frac{m_K^2}{\mu_K^2} & (\langle \mathrm{K} \rangle \neq 0) \end{cases}$$

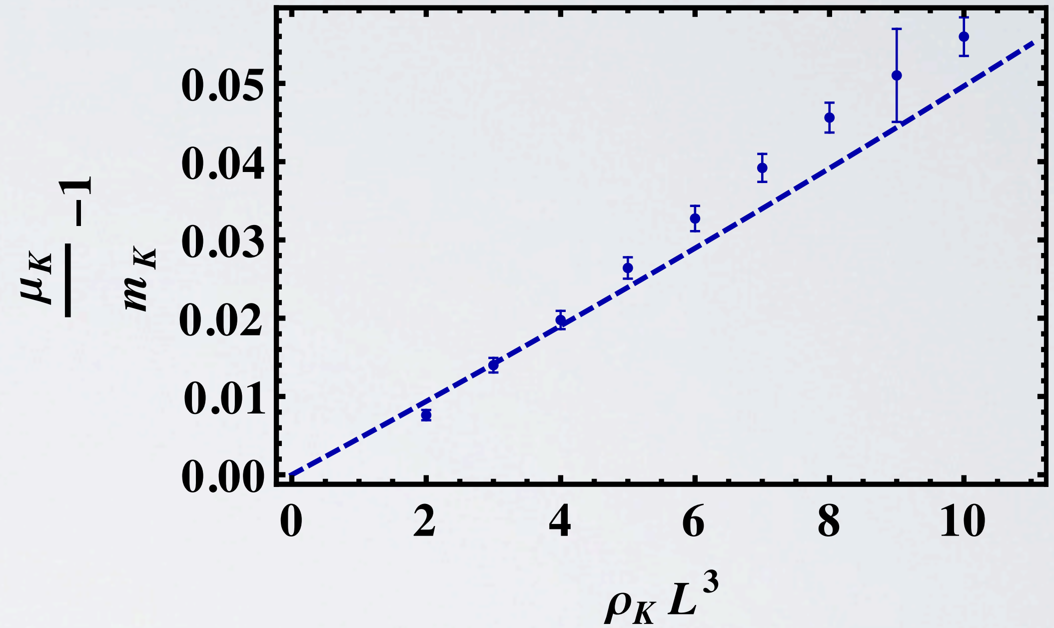
$$M_n(\mu_K) = M_{n^{(0)}} - \frac{\mu_K}{2}\cos\alpha + (2b_0 + b_D - b_F)m_K^2\cos\alpha + \frac{1}{4}(b_1 - b_2 + b_3 + b_4 - b_5 + b_6 + 2b_7 + 2b_8)\mu_K^2\sin^2\alpha$$

$$M_p(\mu_K) = M_{p^{(0)}} + 2(b_0 + b_D)m_K^2\cos\alpha + \frac{1}{2}(b_1 + b_3 + b_4 + b_6 + b_7 + b_8)\mu_K^2\sin^2\alpha - \sqrt{\left(2b_F(m_K^2 - m_\pi^2) + \mu_K\cos\alpha\right)^2 + \frac{1}{4}(b_1 - b_3 + b_4 - b_6)^2\mu_K^4\sin^4\alpha}$$

CHEMICAL POTENTIAL

$$\rho_{\pi,K} = -\frac{\partial \mathcal{L}_{stat}}{\partial \mu_{\pi,K}} = f_{\pi,K}^2 \mu_{\pi,K} \left(1 - \frac{m_{\pi,K}^4}{\mu_{\pi,K}^4}\right)^*$$



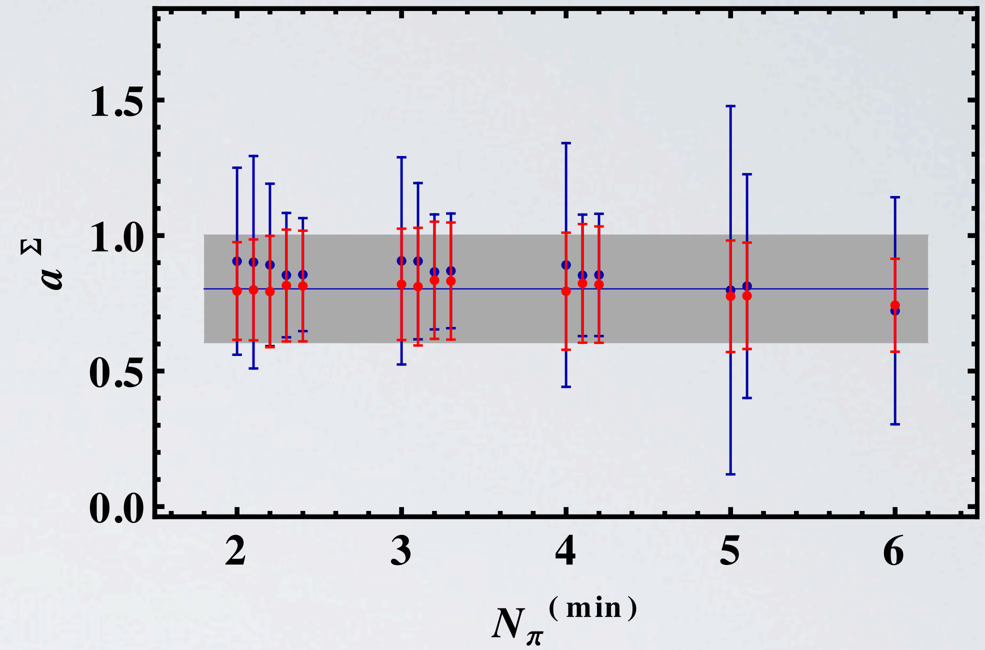


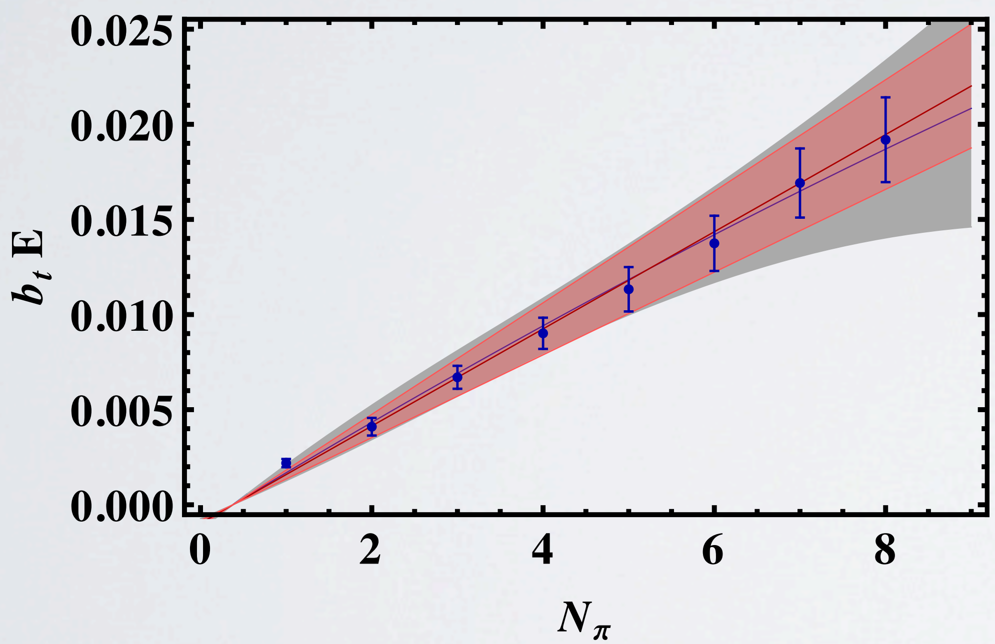
- $\mu_{\pi,K}/m_{\pi,K}-1$ very small
- Expanding mass relations around $\mu_{\pi,K}=m_{\pi,K}$ gives different linear combinations of LECs
  - fits much more stable

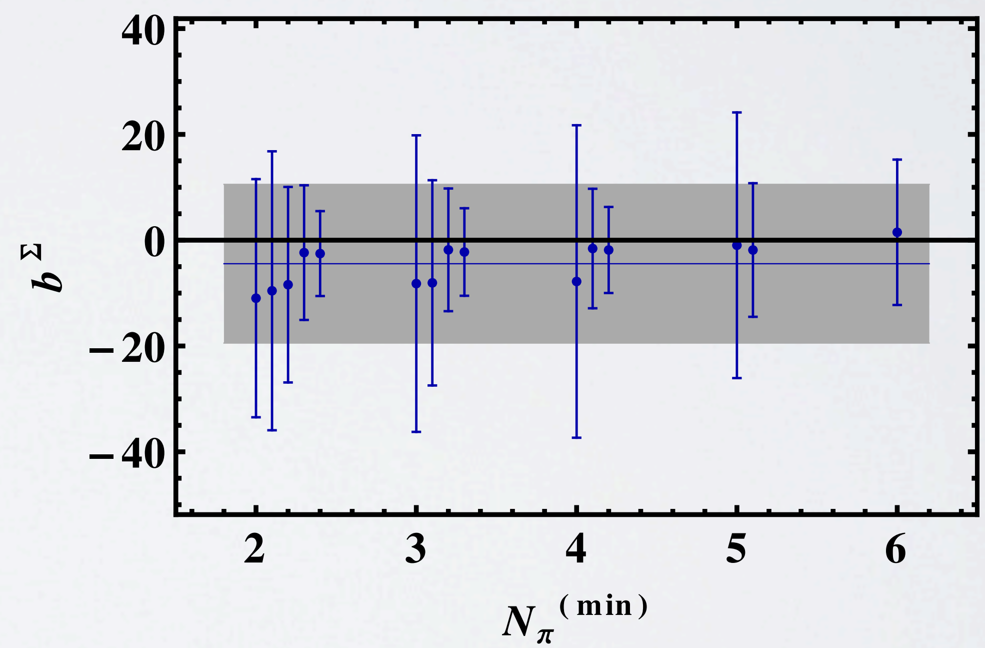
Son & Stephanov (2001)

LOW-ENERGY CONSTANTS

$\Sigma^+ +$ pions



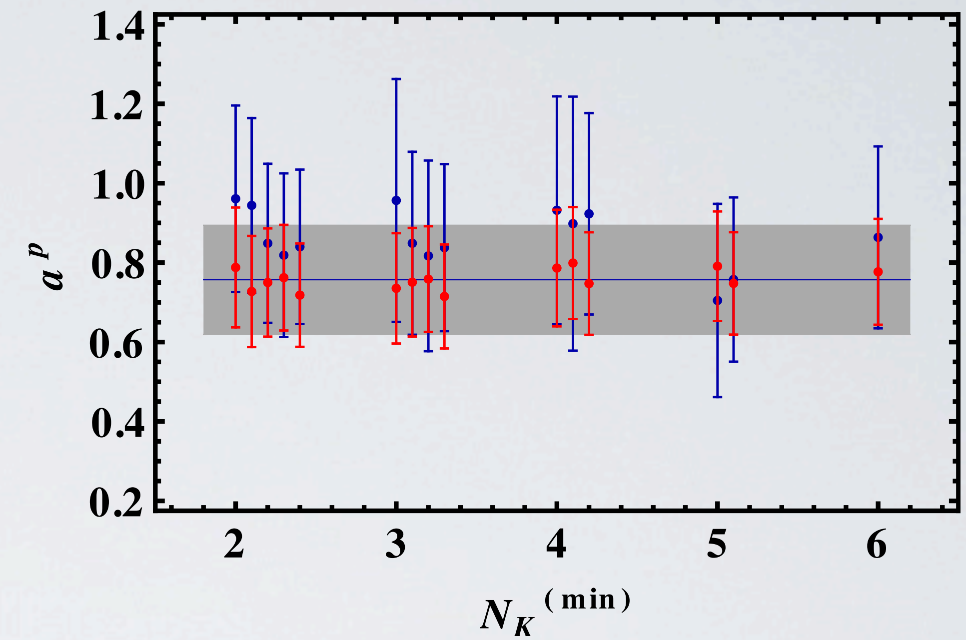


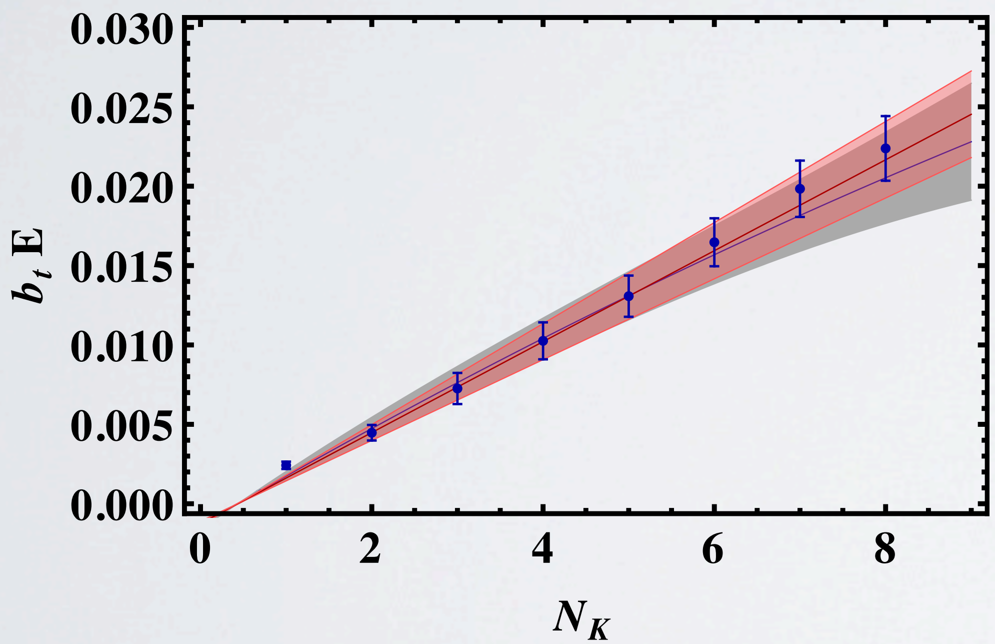


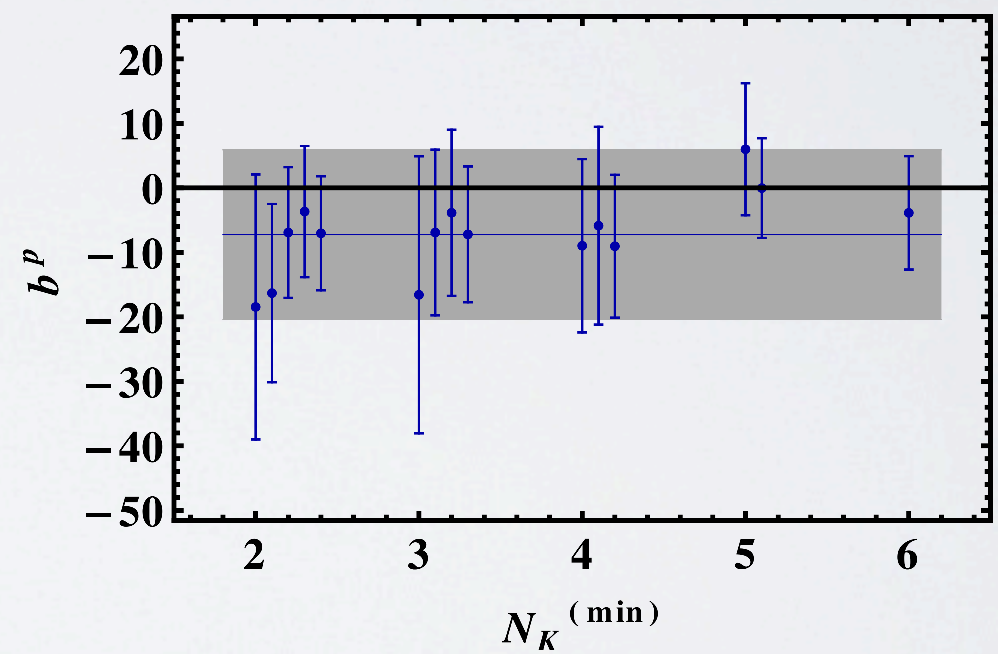
$$M(\mu_{I,K}) \approx a\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right) + b\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right)^2 + \cdots$$

LOW-ENERGY CONSTANTS

Proton + kaons



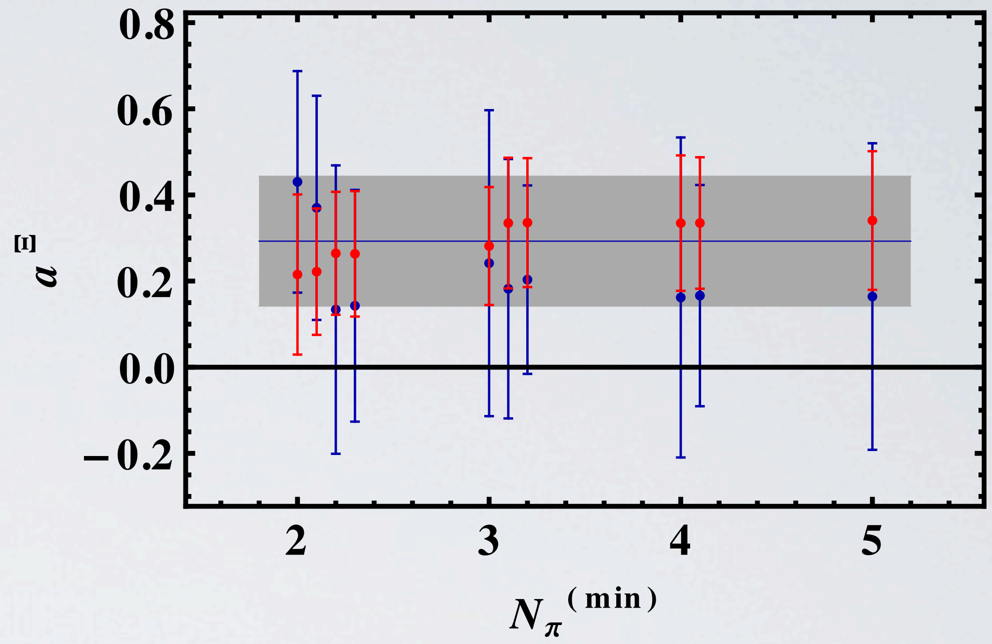


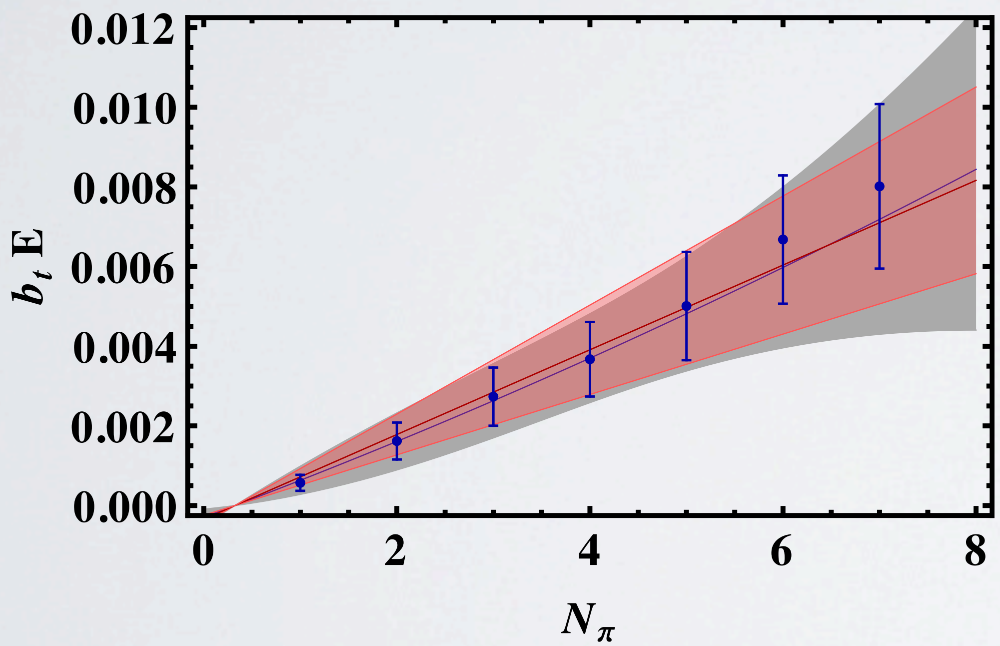


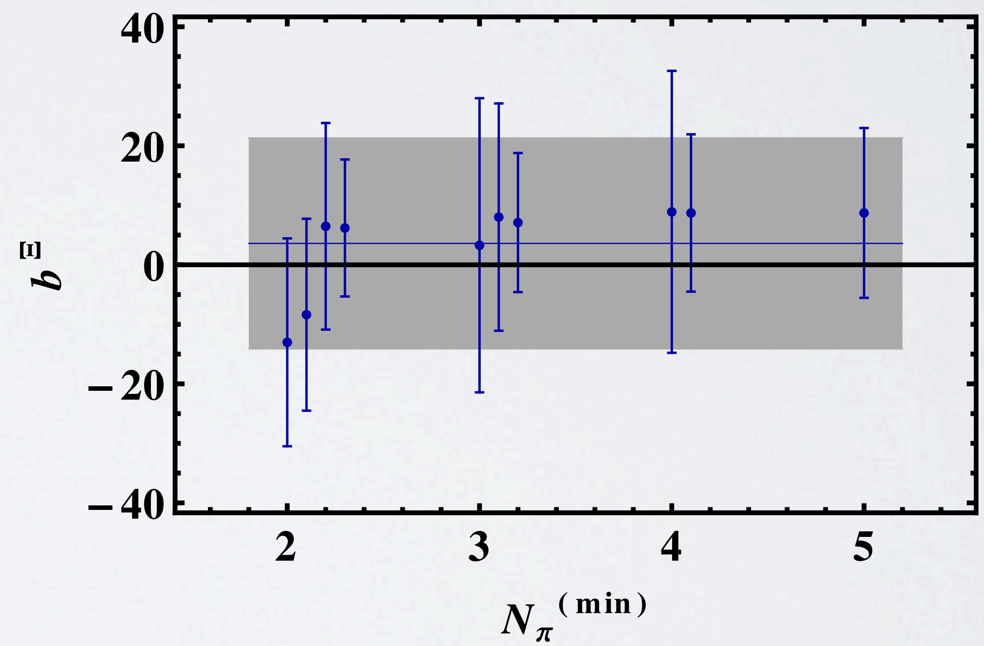
$$M(\mu_{I,K}) \approx a\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right) + b\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right)^2 + \cdots$$

LOW-ENERGY CONSTANTS

Ξ^0 + pions

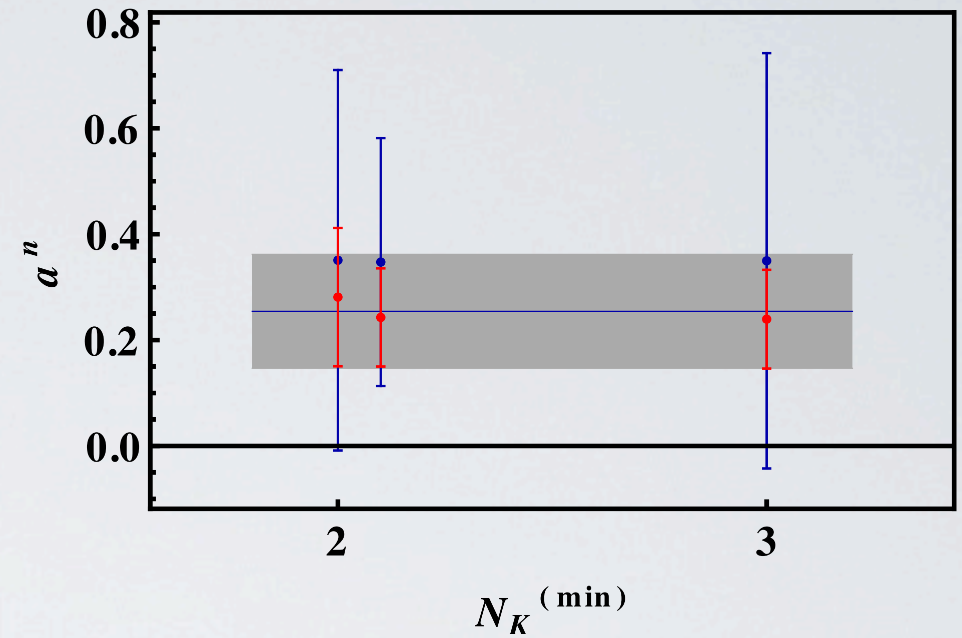




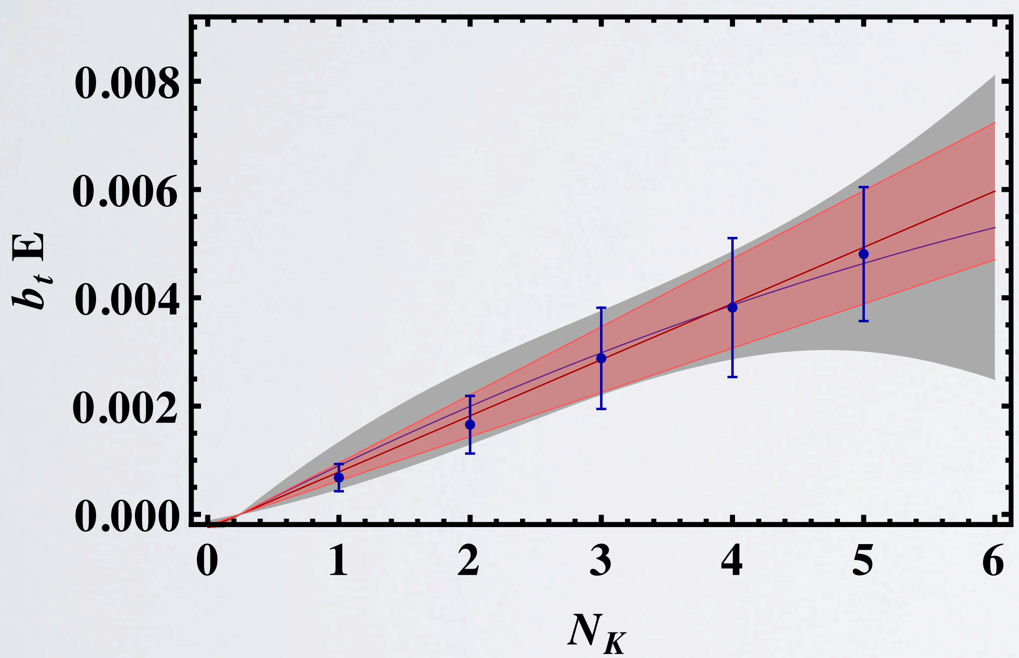


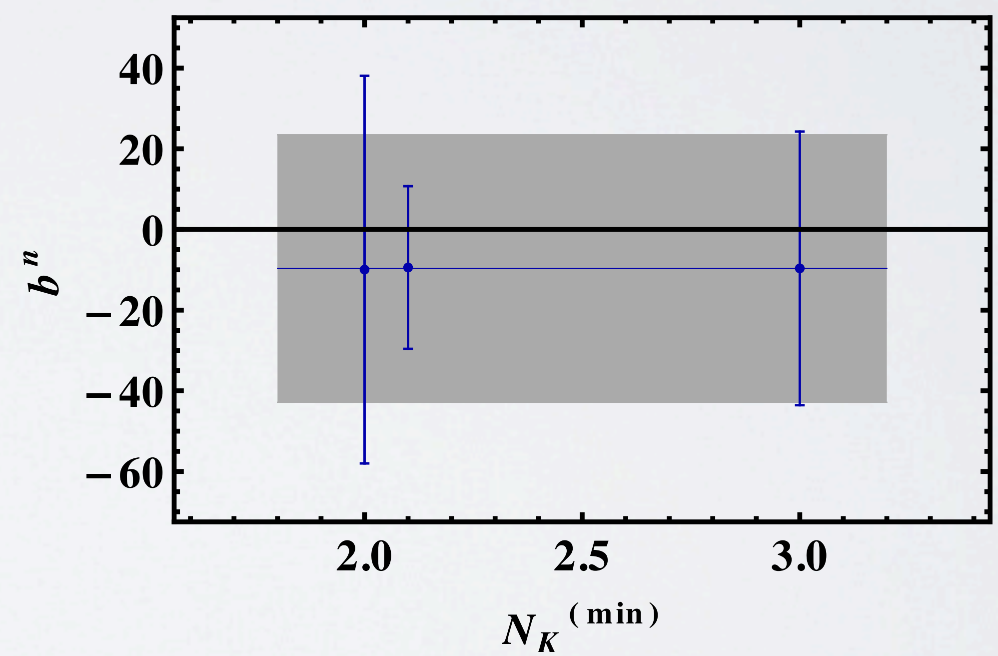
$$M(\mu_{I,K}) \approx a\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right) + b\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right)^2 + \cdots$$

LOW-ENERGY CONSTANTS



Neutron + kaons

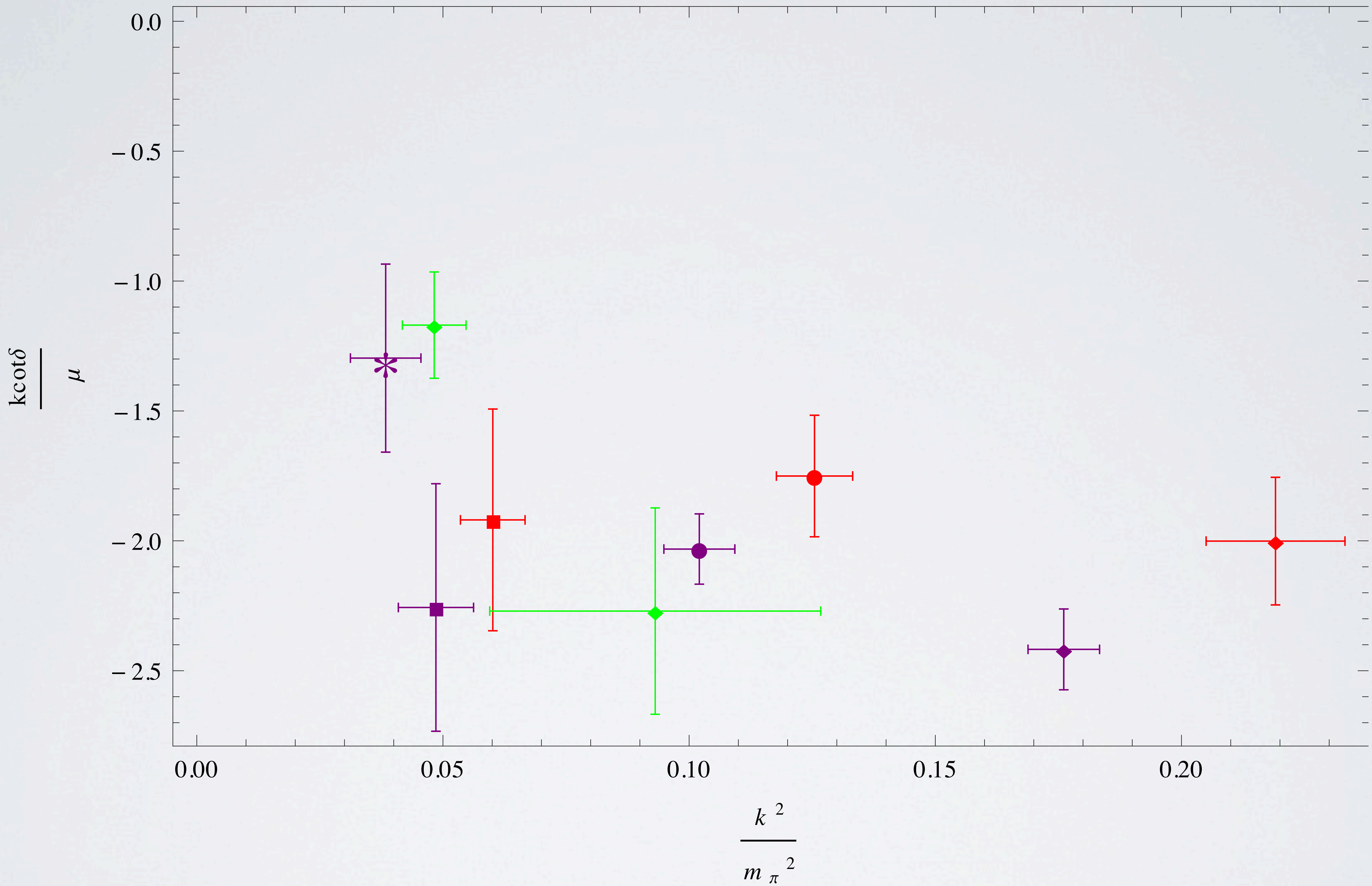




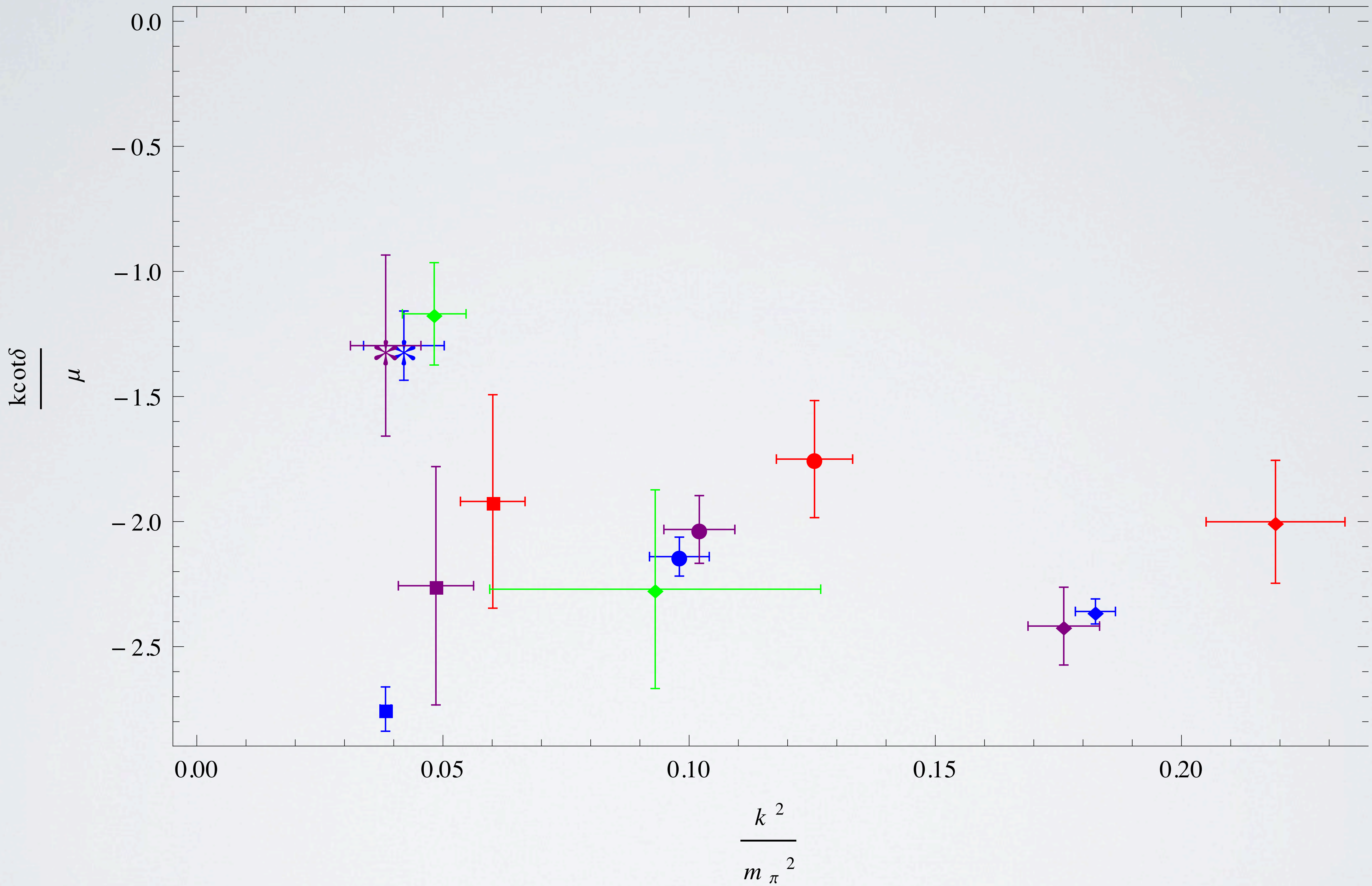
$$M(\mu_{I,K}) \approx a\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right) + b\left(\frac{\mu_{I,K}}{m_{\pi,K}} - 1\right)^2 + \cdots$$

SUMMARY

- Investigated systems of up to 9 mesons + 1 baryon
  - 2-body parameters
    - significant volume-dependence found for meson-baryon scattering phase shifts
    - may indicate large effective range contribution and/or inelasticities
  - First calculation of meson-meson-baryon 3-body interaction
  - Some combinations of LECs accessible
- Thermal effects & noise current limitation to system size
- Would like to add more baryons (solve noise problem!)



0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0.00
0.05
0.10
0.15
0.20
kcotδ / μ
k 2 / m π 2

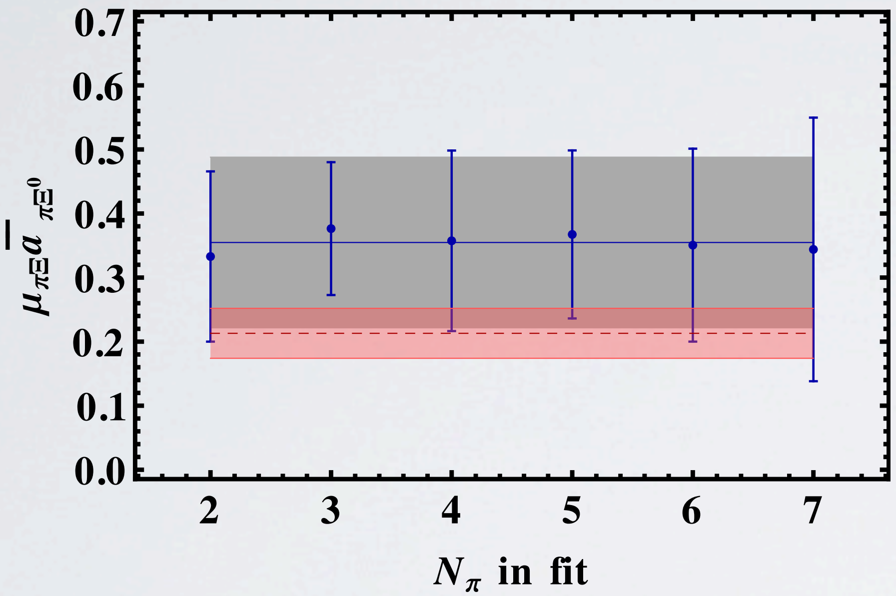

0.0
−0.5
−1.0
−1.5
−2.0
−2.5
kcotδ / μ
0.00
0.05
0.10
0.15
0.20
k 2 / m π 2

$$
\begin{aligned}
\det(1+\lambda A) &= \frac{1}{12!}\,\varepsilon^{\alpha_1\alpha_2..\alpha_{12}}\,\varepsilon_{\beta_1\beta_2..\beta_{12}}\,(1+\lambda A)^{\beta_1}_{\alpha_1}(1+\lambda A)^{\beta_2}_{\alpha_2}\dots(1+\lambda A)^{\beta_{12}}_{\alpha_{12}} \\
&= \frac{1}{12!}\Big[\,\varepsilon^{\alpha_1\alpha_2..\alpha_{12}}\,\varepsilon_{\alpha_1\alpha_2..\alpha_{12}} \;+\; \lambda\;{}^{12}C_1\,\varepsilon^{\alpha_1\alpha_2..\alpha_{12}}\,\varepsilon_{\beta_1\alpha_2..\alpha_{12}}\;(\,A\,)^{\beta_1}_{\alpha_1}+\dots \\
&\qquad +\lambda^n\;{}^{12}C_n\,\varepsilon^{\alpha_1\alpha_2..\alpha_n\xi_1..\xi_{12-n}}\,\varepsilon_{\beta_1\beta_2..\beta_n\xi_1..\xi_{12-n}}\;(\,A\,)^{\beta_1}_{\alpha_1}(\,A\,)^{\beta_2}_{\alpha_2}\dots(\,A\,)^{\beta_n}_{\alpha_n} \\
&\qquad \dots\;+\;\lambda^{12}\varepsilon^{\alpha_1\alpha_2..\alpha_{12}}\,\varepsilon_{\beta_1\beta_2..\beta_{12}}\;(\,A\,)^{\beta_1}_{\alpha_1}\dots(\,A\,)^{\beta_{12}}_{\alpha_{12}}\Big] \\
&= \frac{1}{12!}\sum_{j=1}^{12}{}^{n}C_j\;\lambda^j\;C_j(t)\quad, \qquad (18)
\end{aligned}
$$

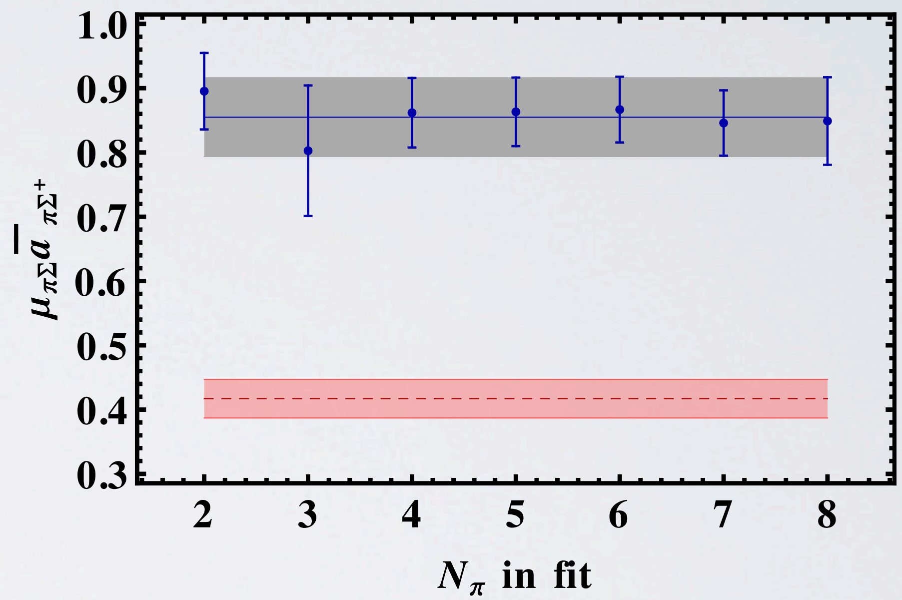
2-BODY PARAMETERS

NPLQCD (2009)
L=2.5 fm

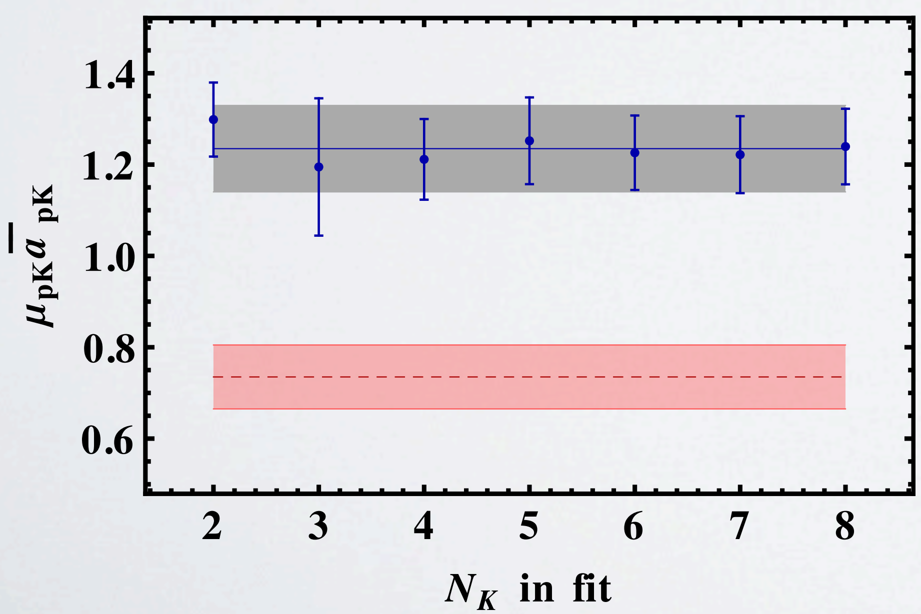
Ξ^0, π^+



Σ^+, π^+



p,K$^+$



n,K$^+$