

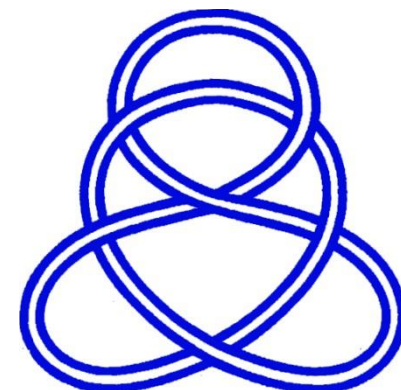
Gluing Coupled Cluster to Geminal Powers with Quantum Monte Carlo

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July 10, 2013



UC Berkeley Dept. of Chemistry
&
The Miller Institute for Basic
Research in Science



outline

motivation

geminals

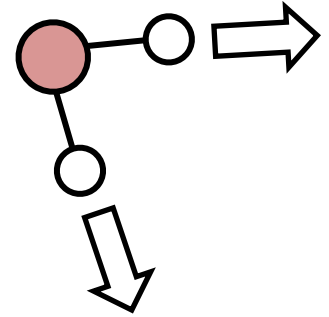
coupled cluster

results

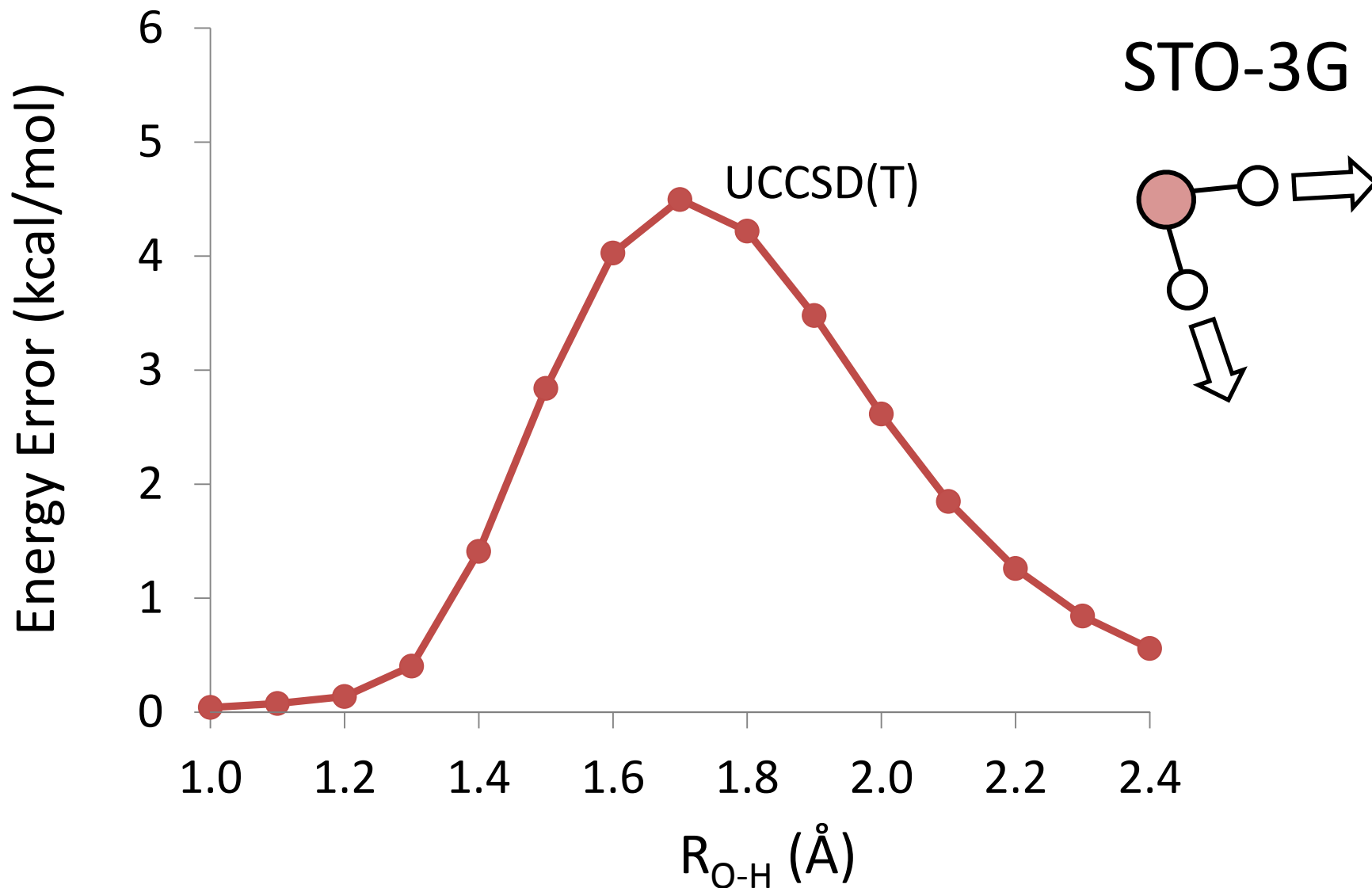
conclusions

wetting one's appetite

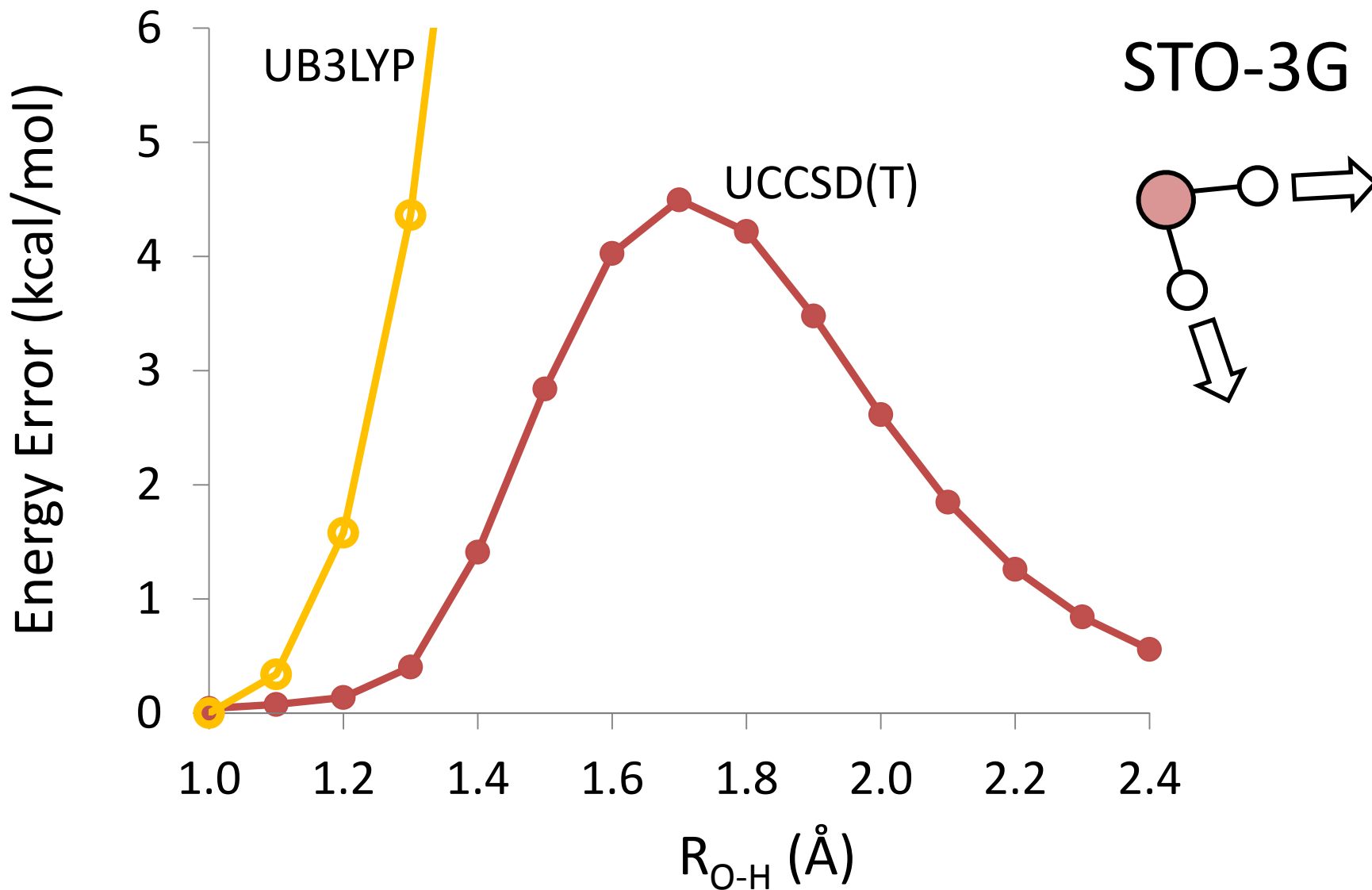
STO-3G



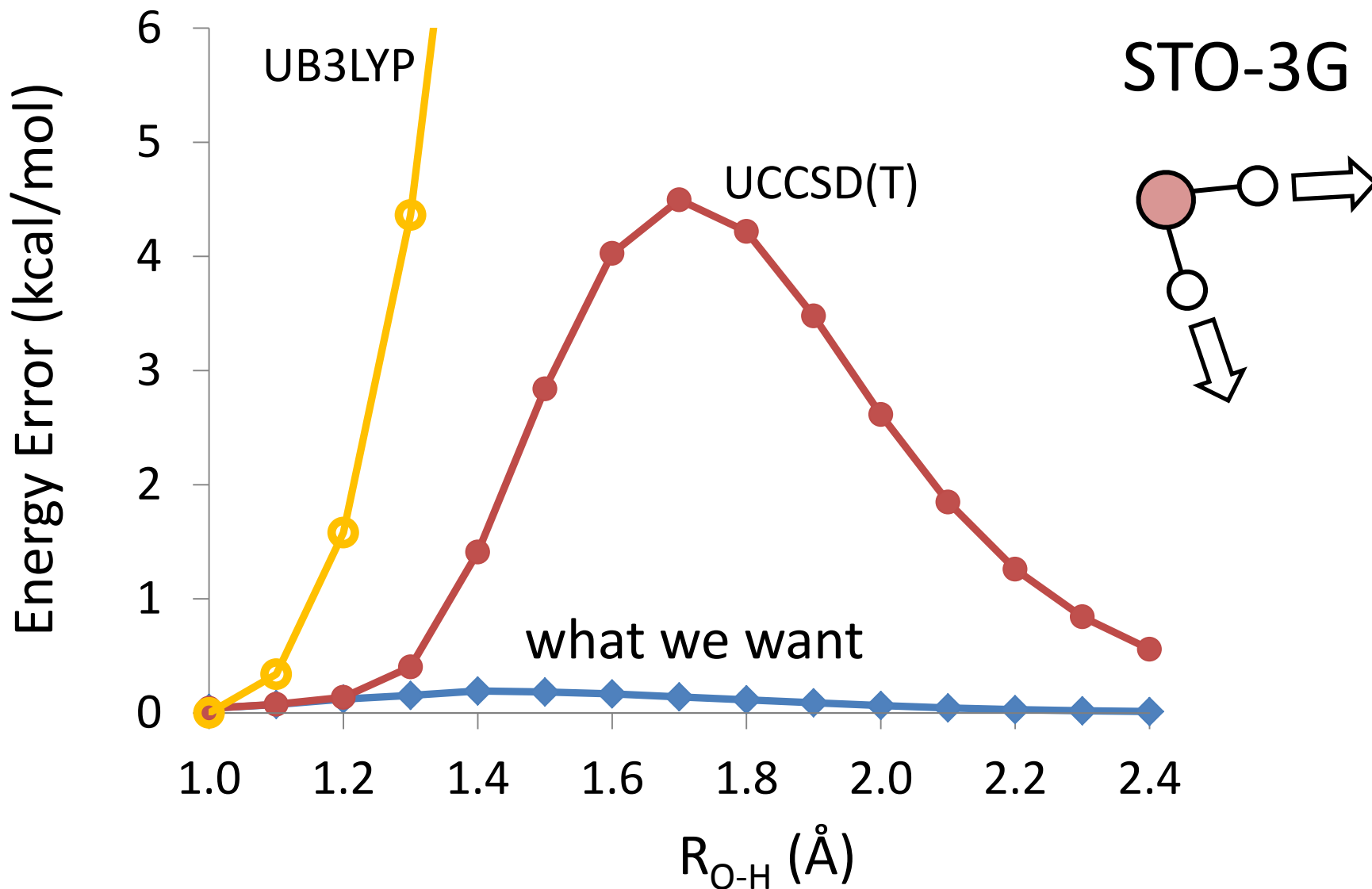
wetting one's appetite



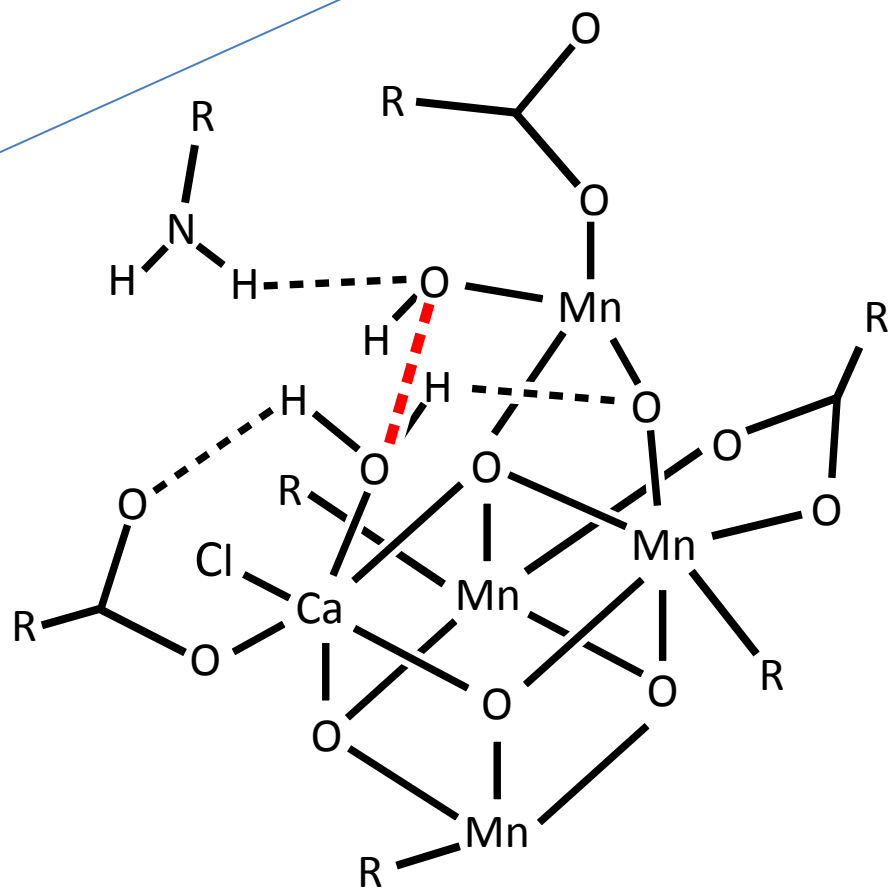
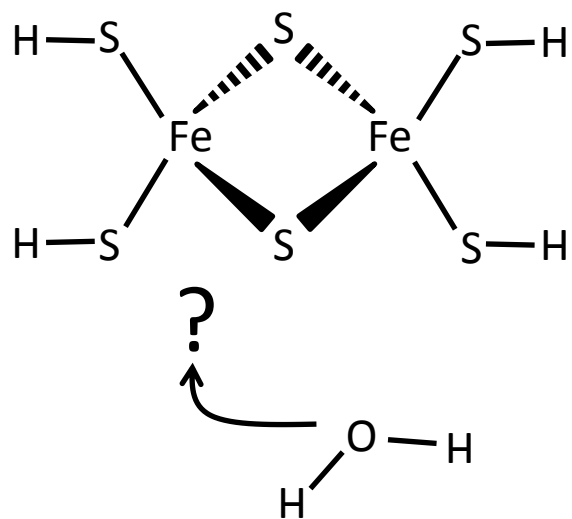
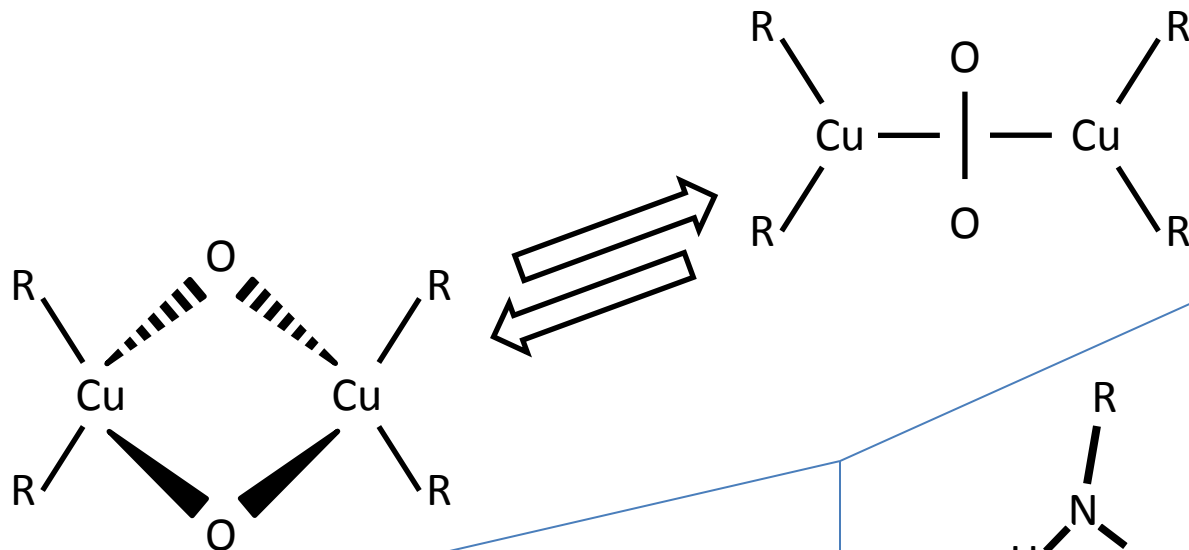
wetting one's appetite



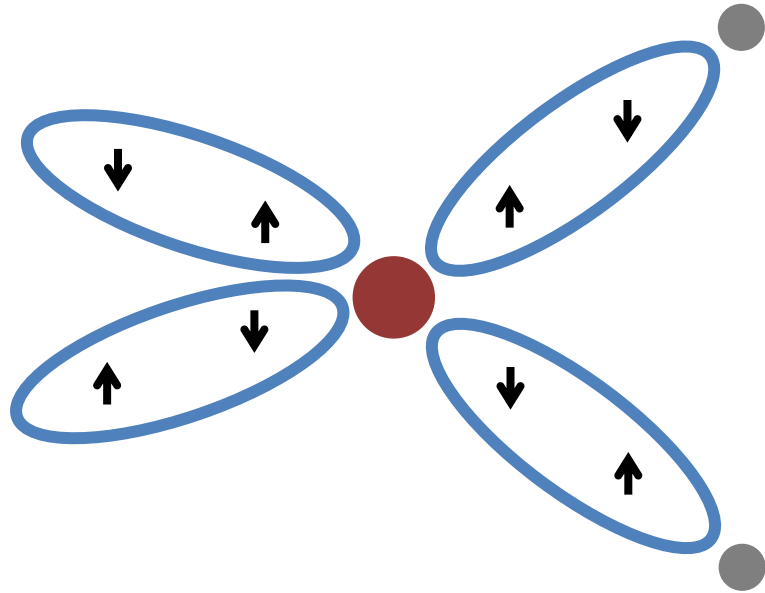
wetting one's appetite



stuff real chemists care about

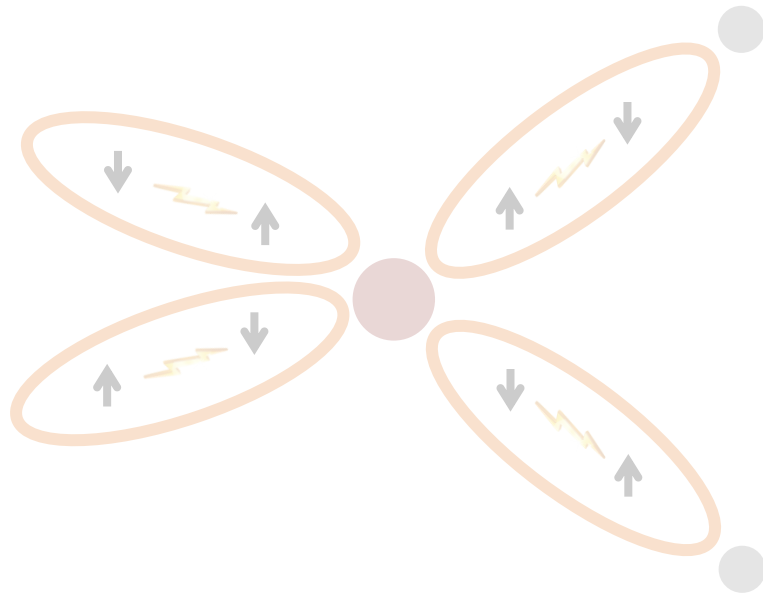


geminals: two electron building blocks



$$\hat{A} [\phi_1 \phi_2 \phi_3 \phi_4 \dots]$$

geminals: two electron building blocks



$$\hat{A} [\phi_1 \phi_2 \phi_3 \phi_4 \dots]$$

~~exponential scaling!~~

PP/Q/H Head-Gordon

PMF Scuseria

QMC Sorella, Bajdich,
Schmidt, others...

size consistent/extensive

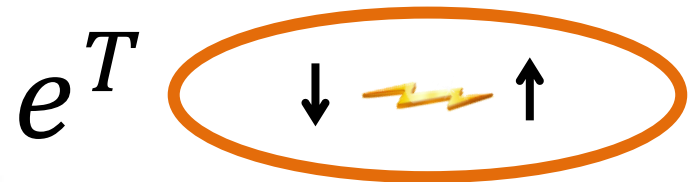
$$1 + 1 + 1 \dots \propto N$$

variational

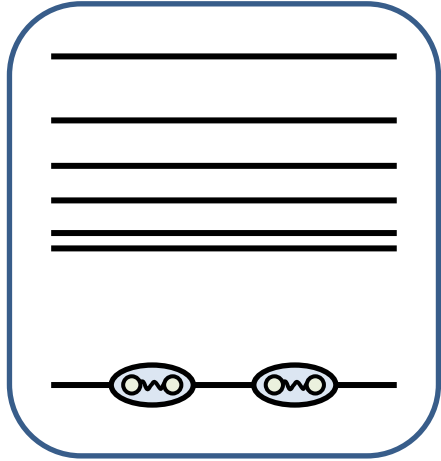
$$1 + 1 > 0$$

accurate

$$1 + 1 = 2$$



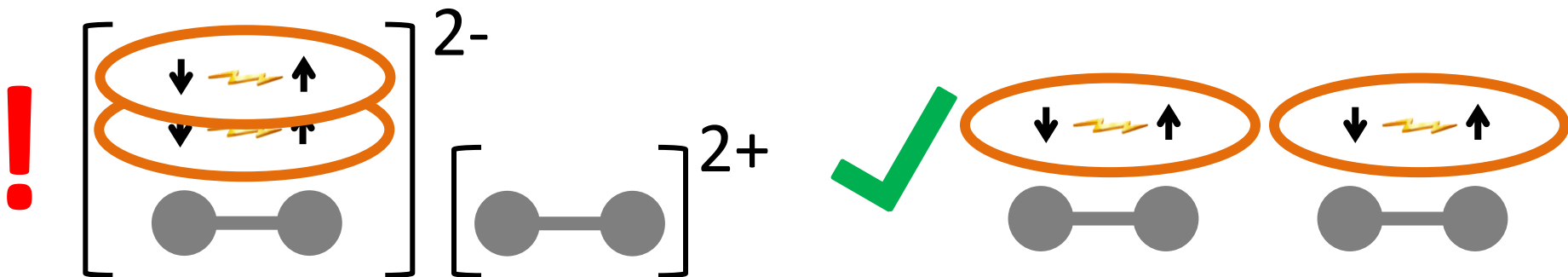
the antisymmetric geminal power



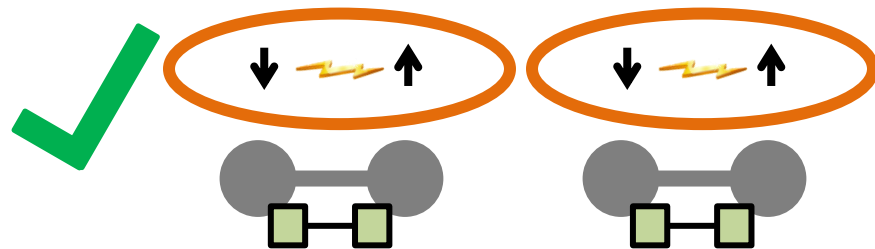
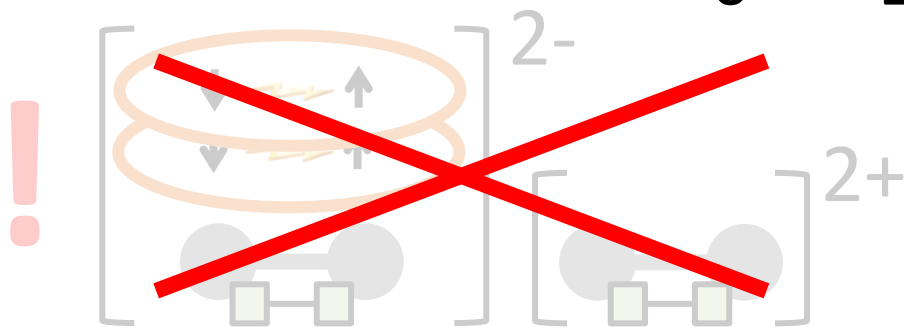
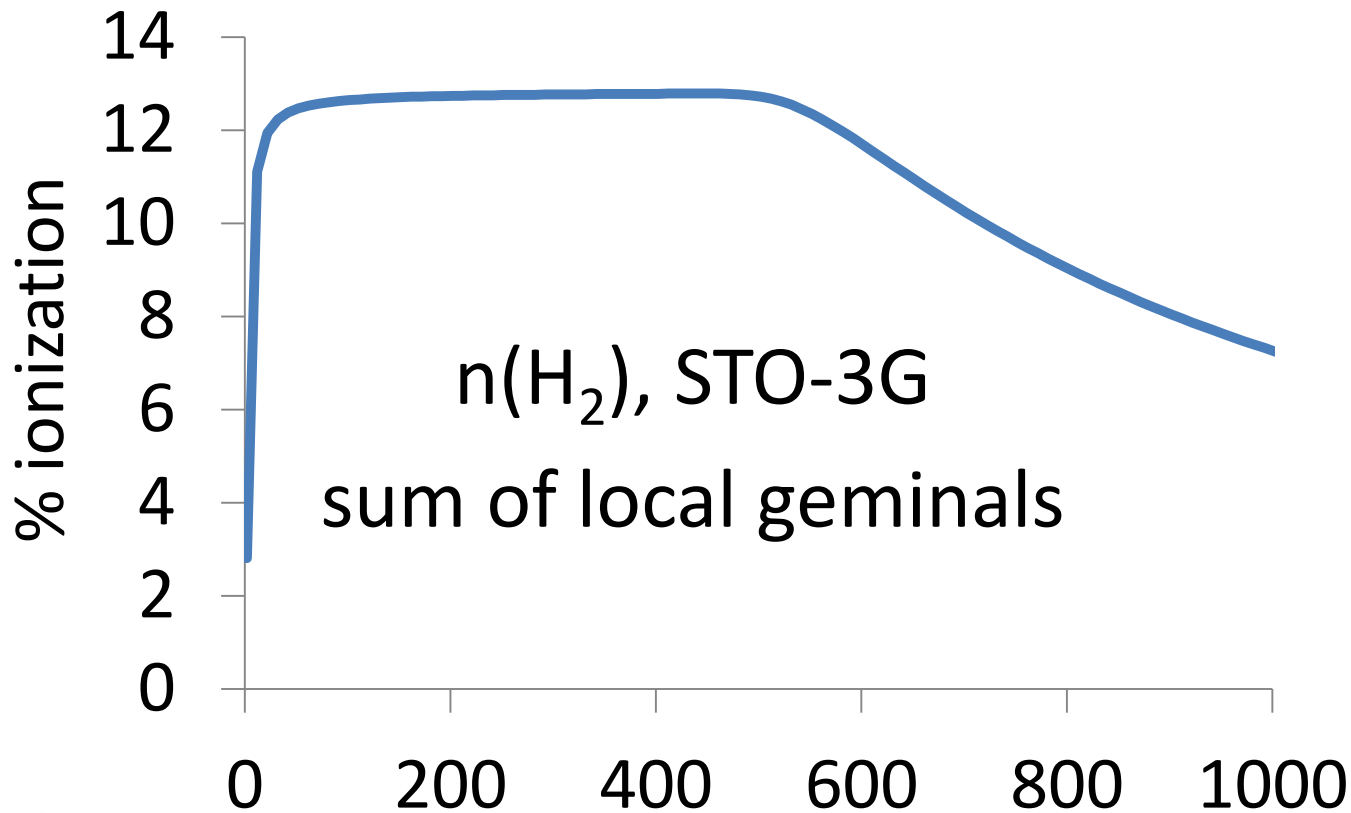
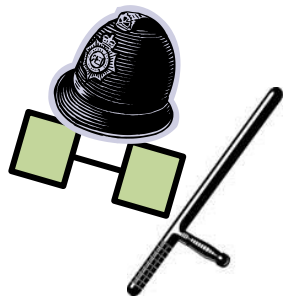
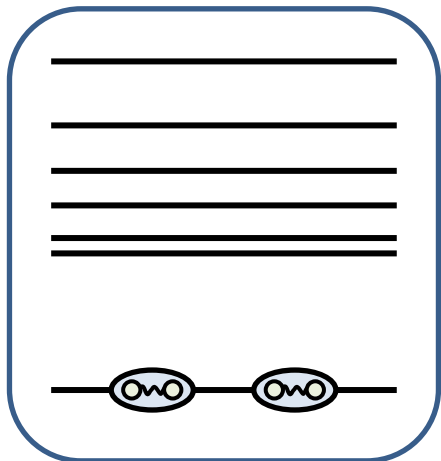
$$\Psi = \phi_A(\vec{r}_1, \vec{r}_2) \times \phi_B(\vec{r}_3, \vec{r}_4)$$

$$\left(\begin{array}{c} \downarrow \text{---} \uparrow \\ \downarrow \text{---} \uparrow \end{array} \right)^2$$

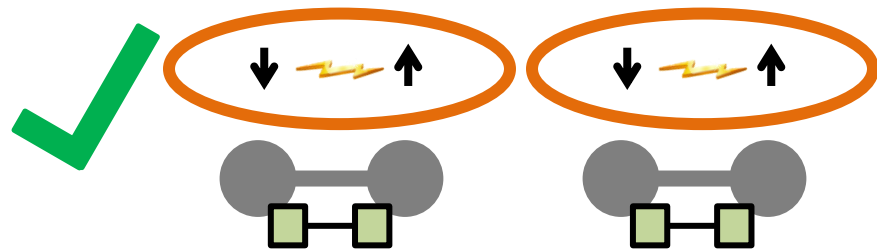
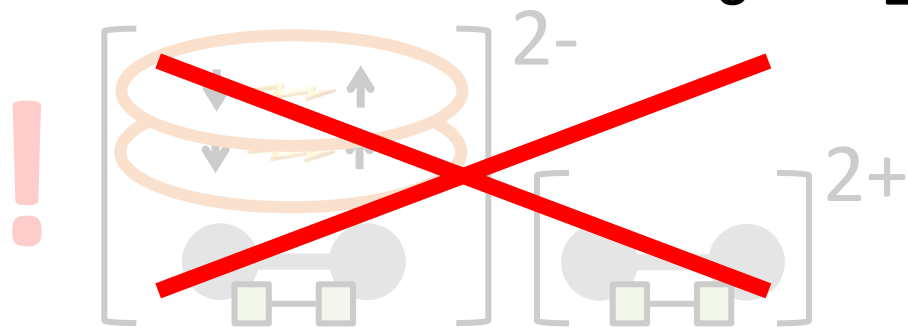
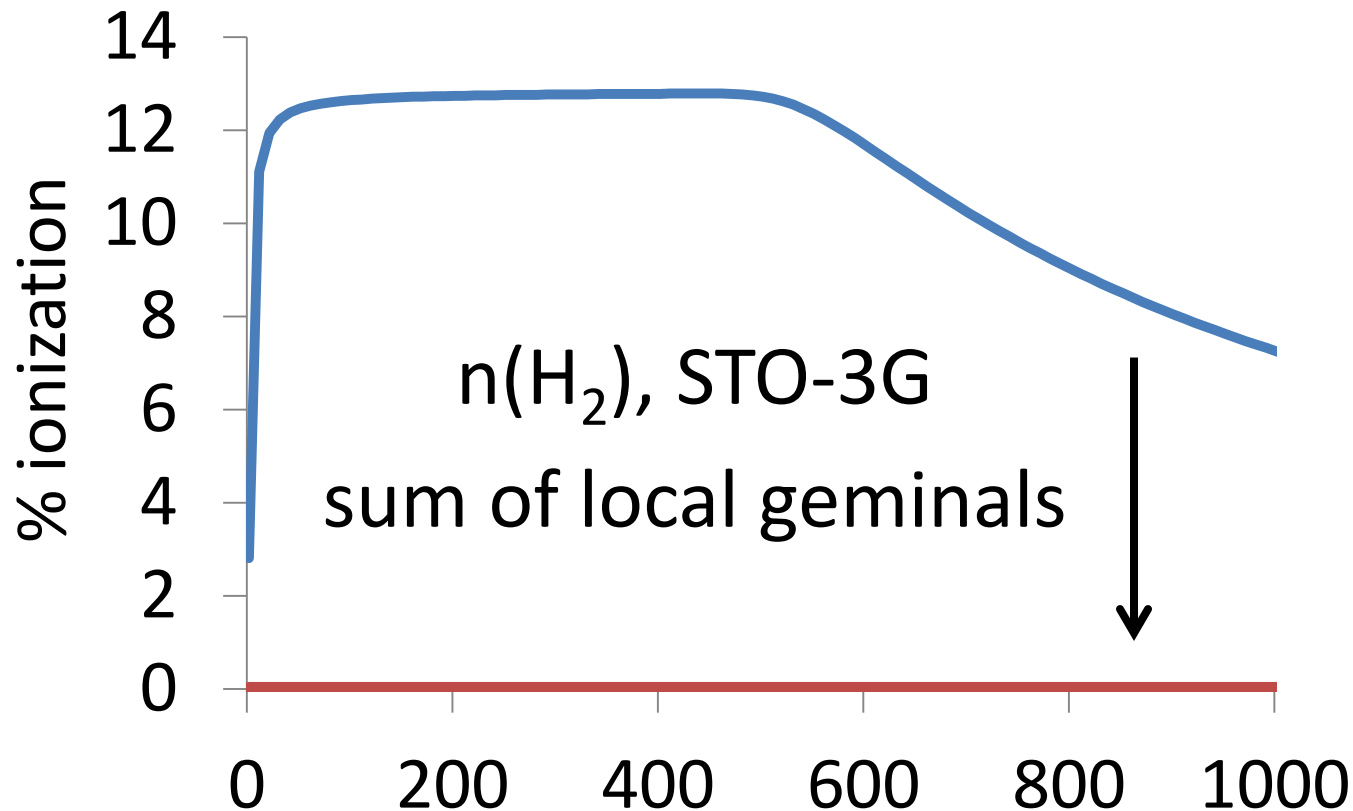
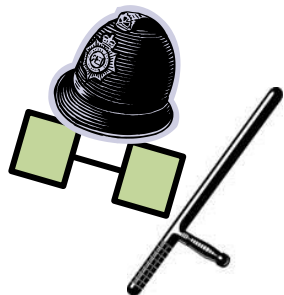
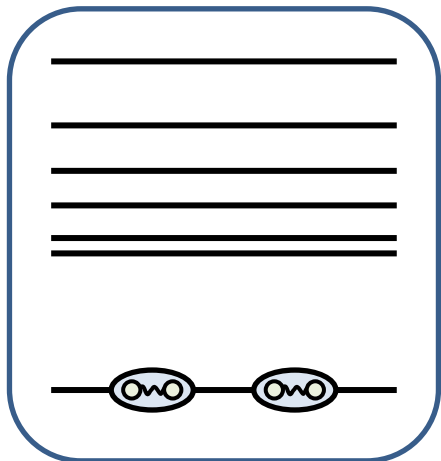
$$\Psi = \Phi(\vec{r}_1, \vec{r}_2) \times \Phi(\vec{r}_3, \vec{r}_4)$$



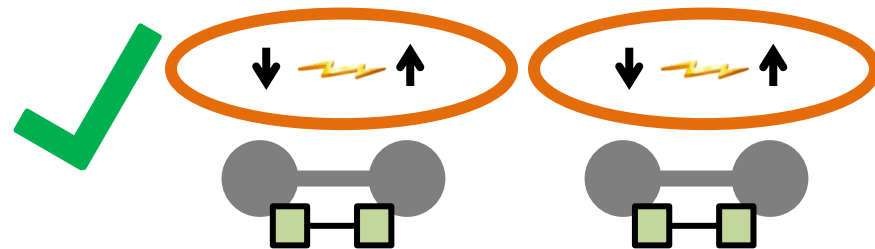
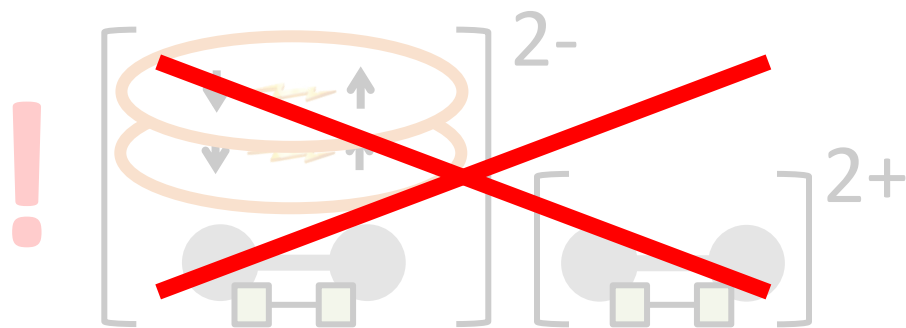
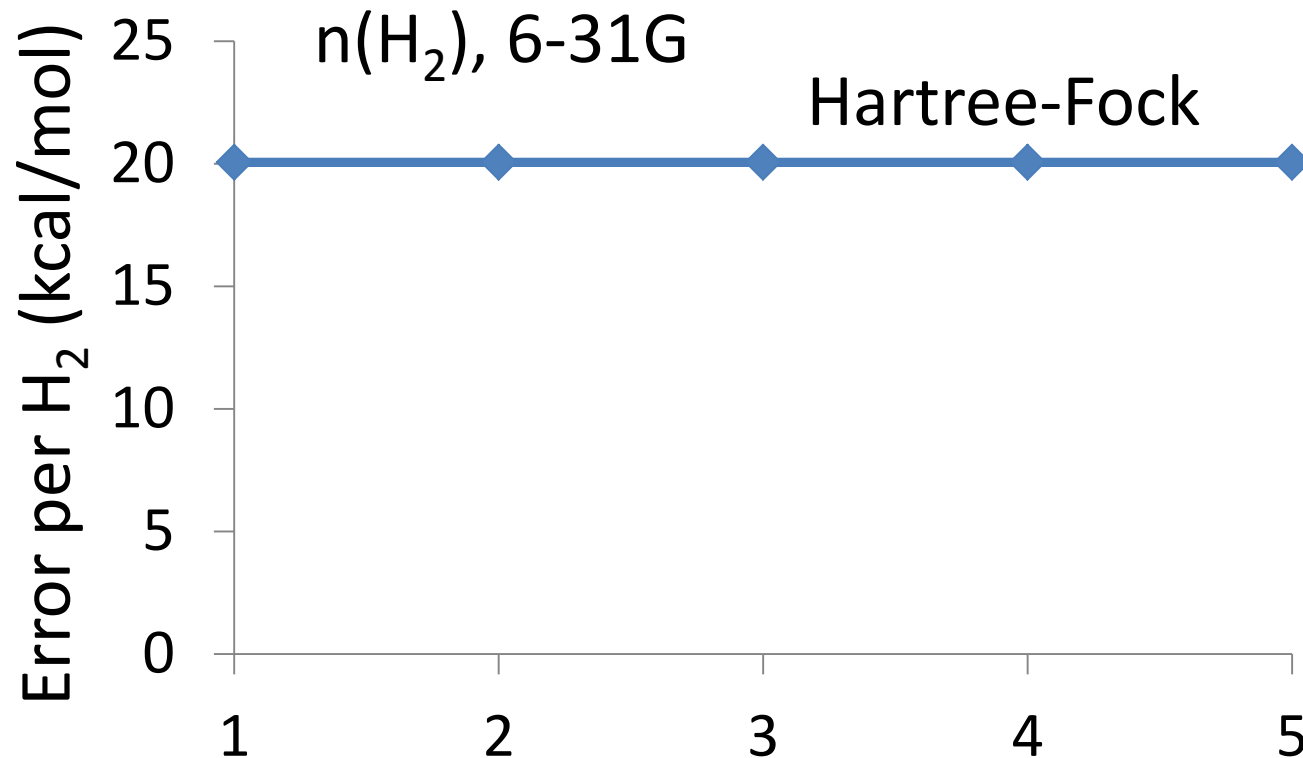
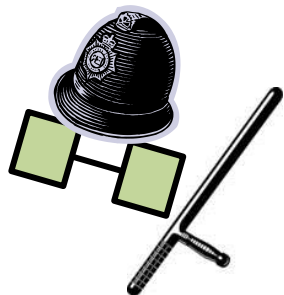
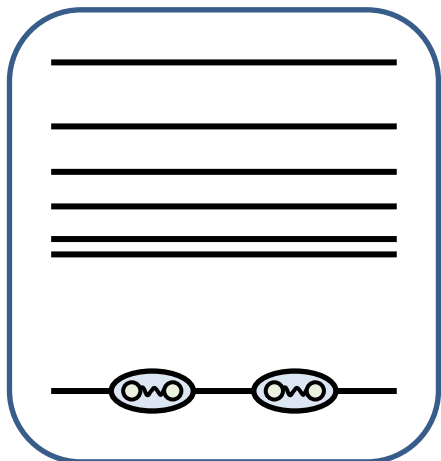
the antisymmetric geminal power



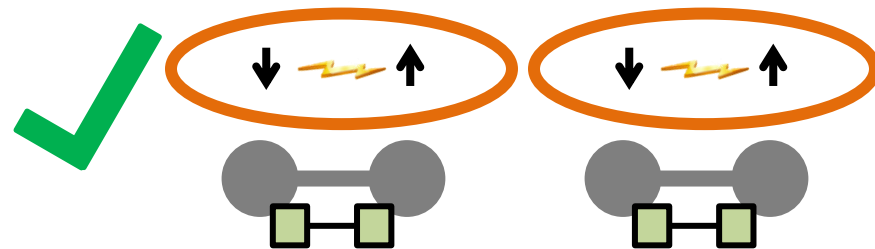
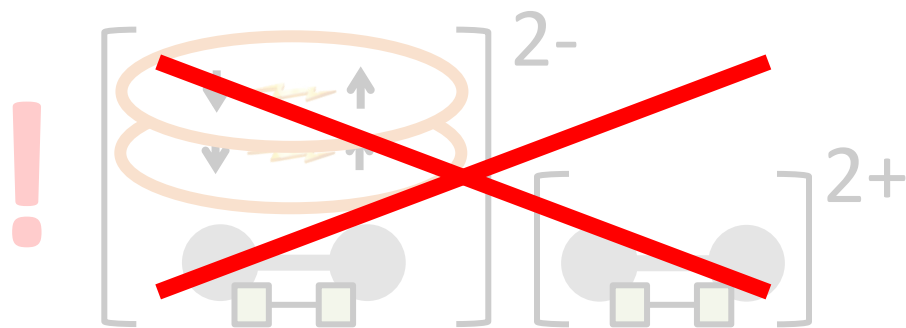
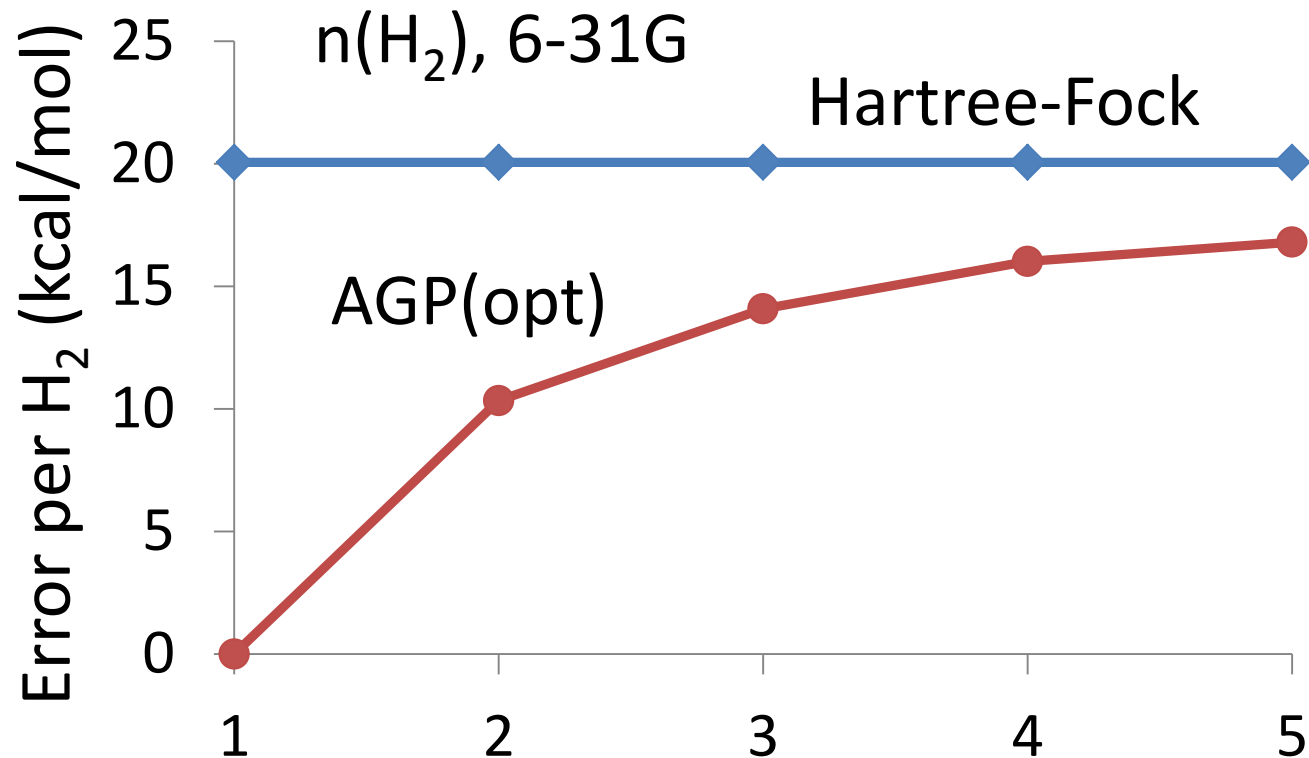
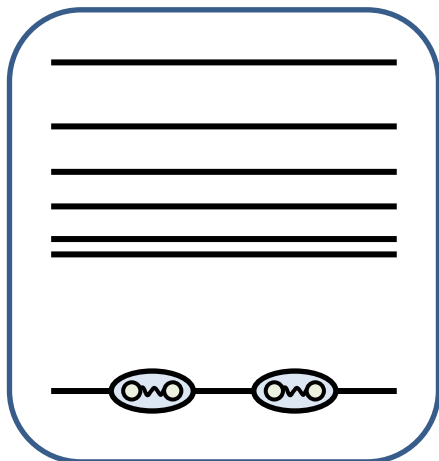
the antisymmetric geminal power



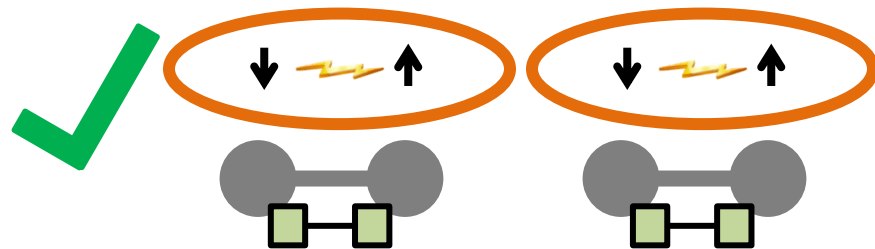
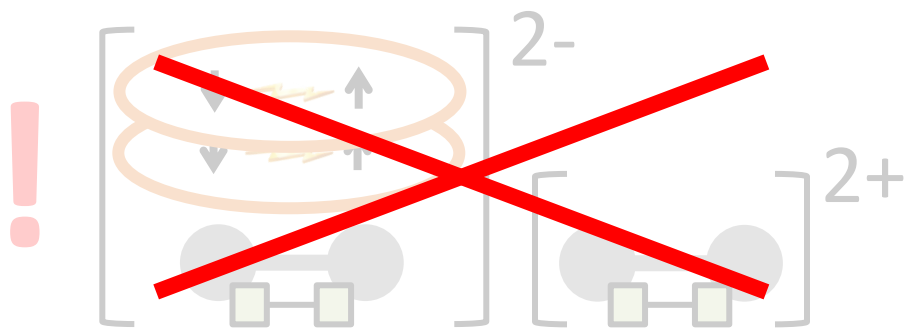
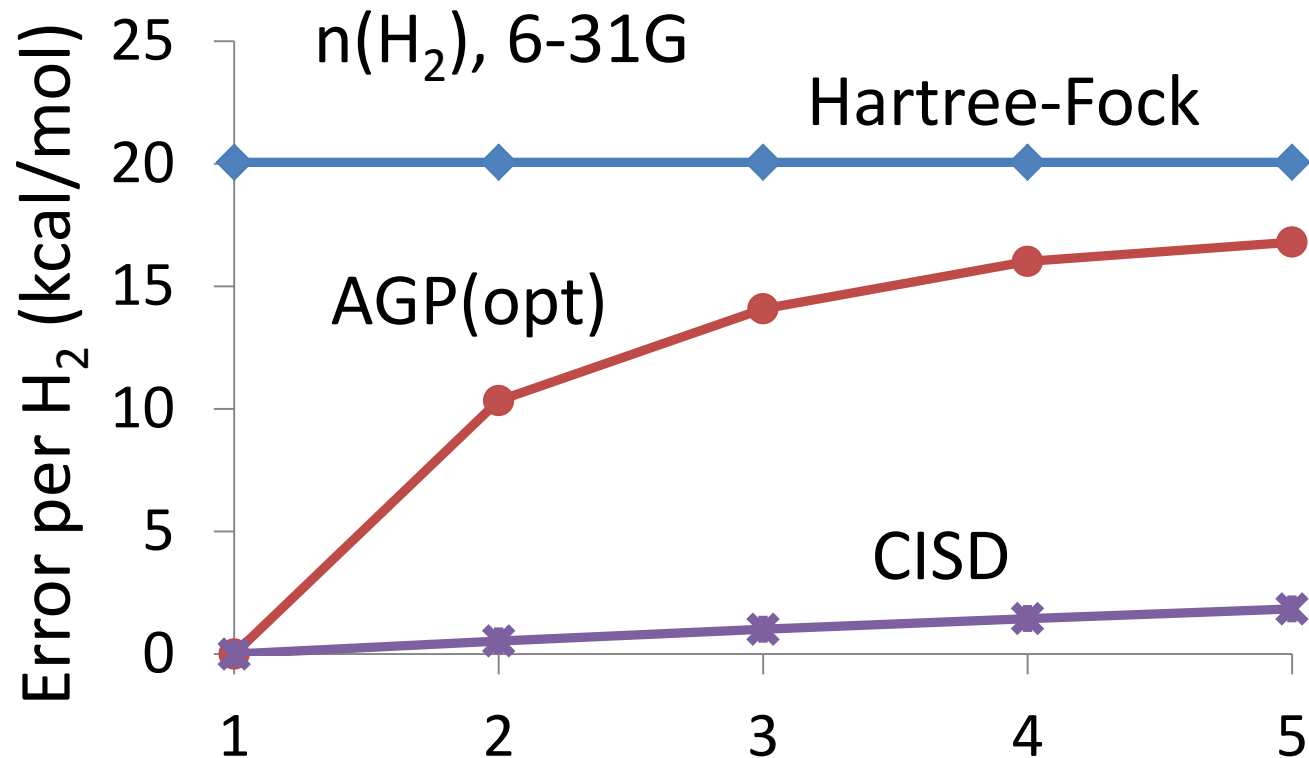
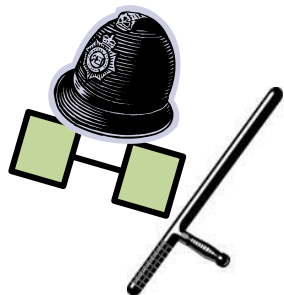
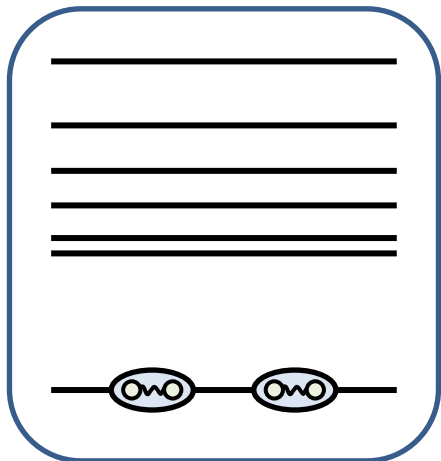
the antisymmetric geminal power



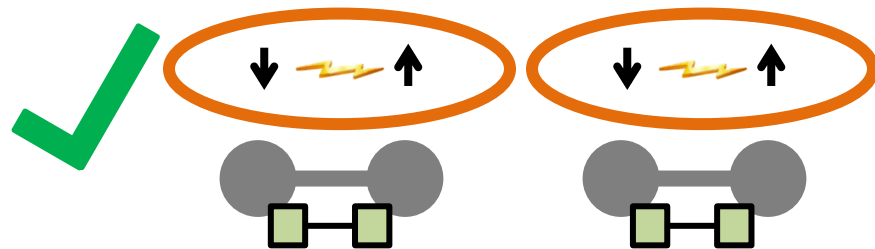
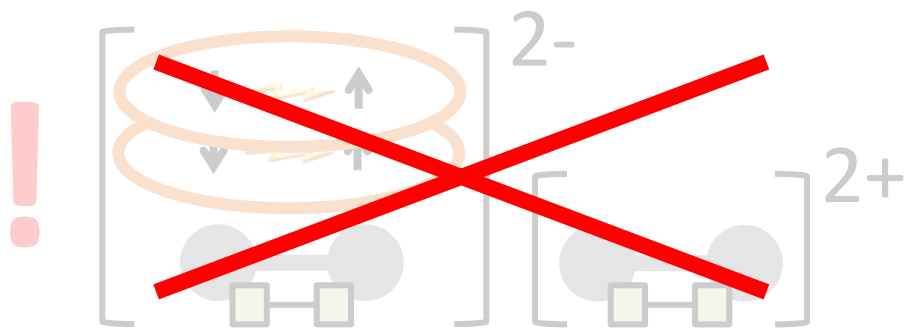
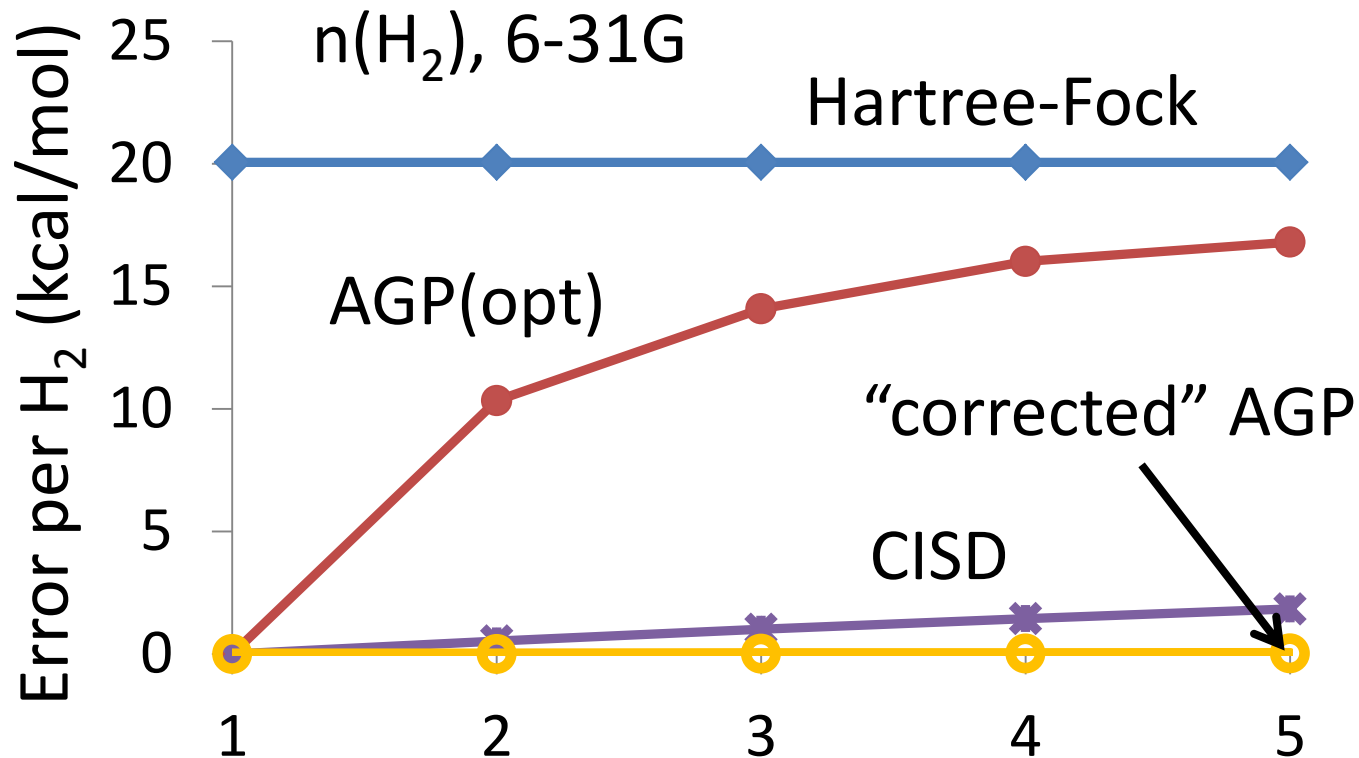
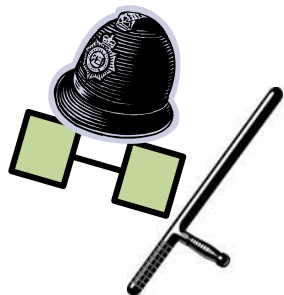
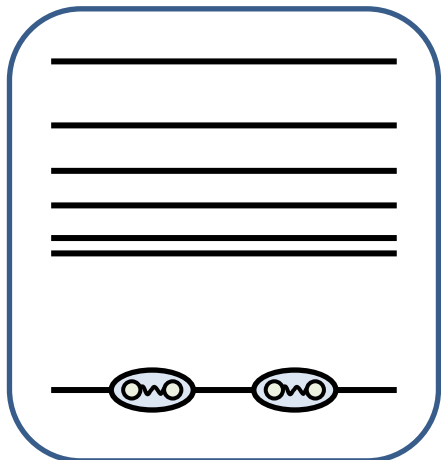
the antisymmetric geminal power



the antisymmetric geminal power



the antisymmetric geminal power

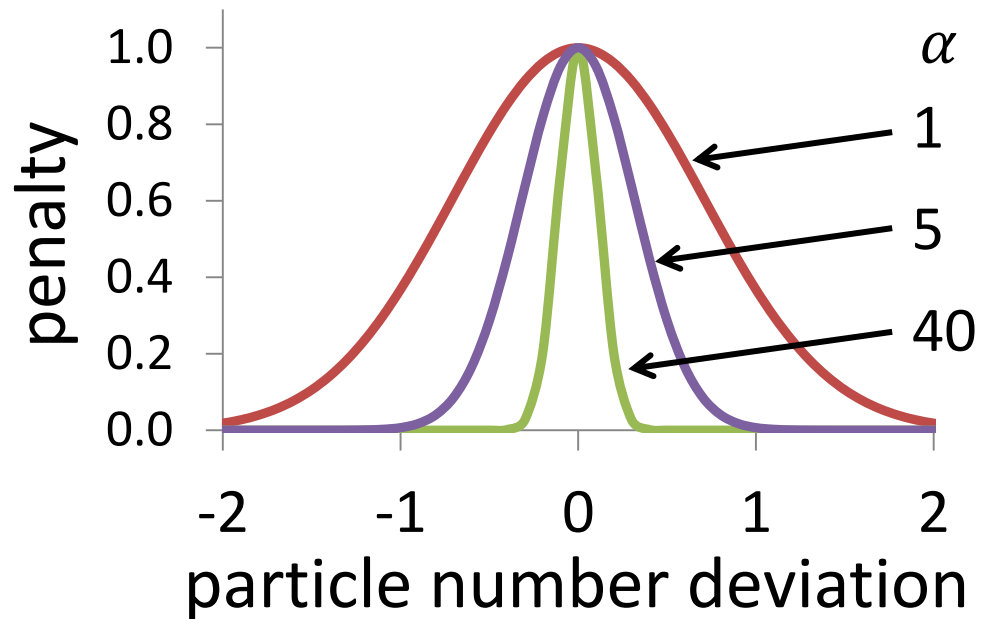


Hilbert space Jastrow factors

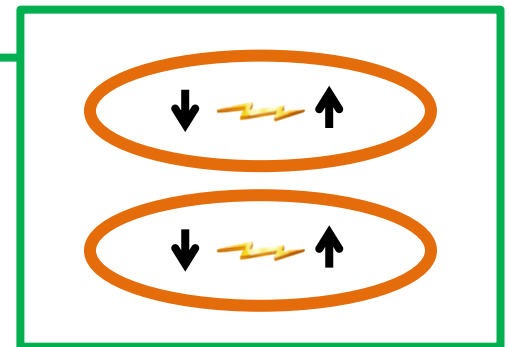
$$\exp \left[-\alpha \left(M - \sum_i \hat{n}_i \right)^2 \right] |\Psi\rangle$$



**size
consistent**

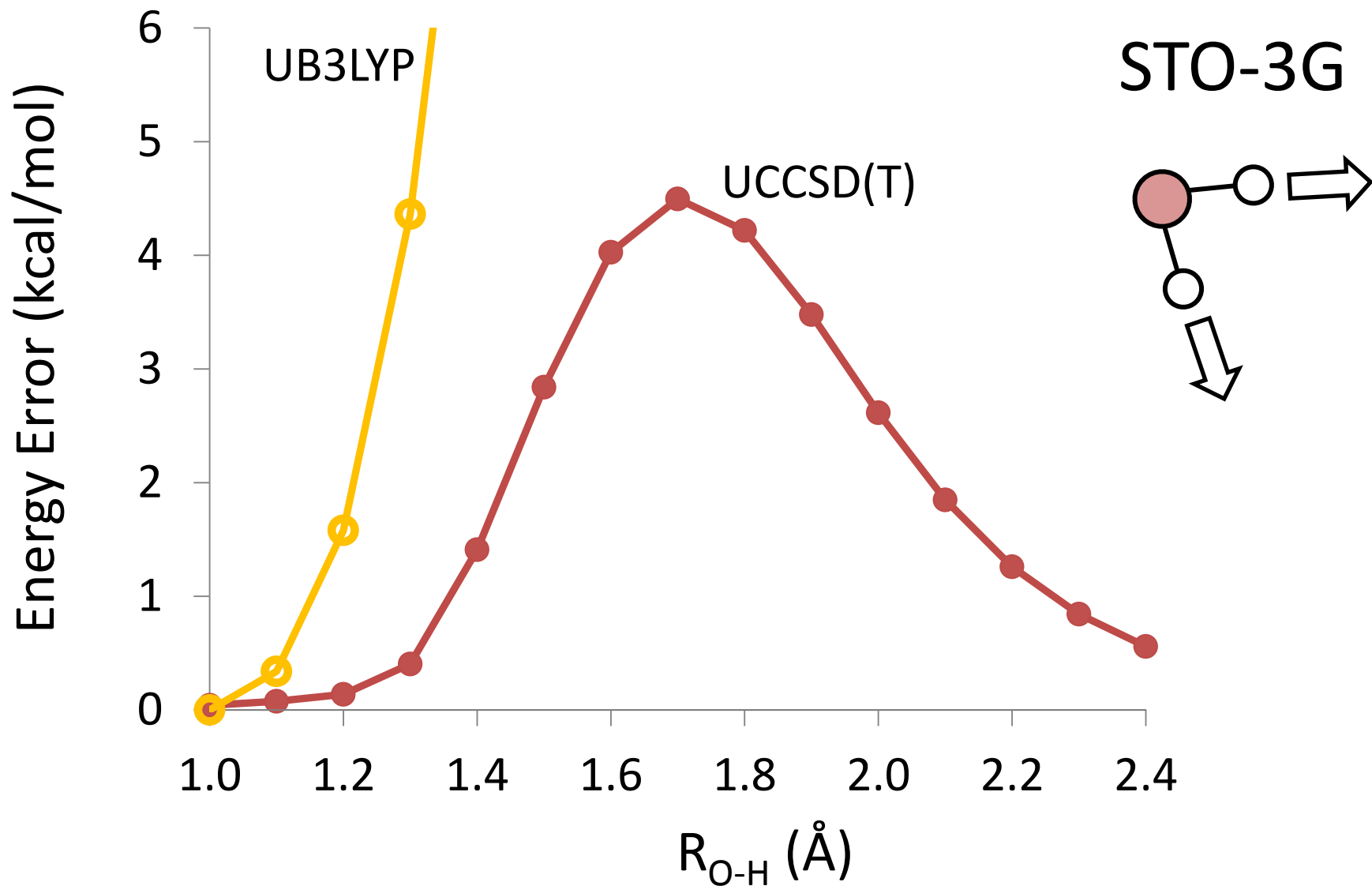


$$|\Psi_{\text{JAGP}}\rangle = \exp \left[\sum_{ij} J_{ij} \hat{n}_i \hat{n}_j \right] \left(\sum_{pq} \phi_{pq} a_p^+ a_q^+ \right)^{\frac{N}{2}} |0\rangle$$

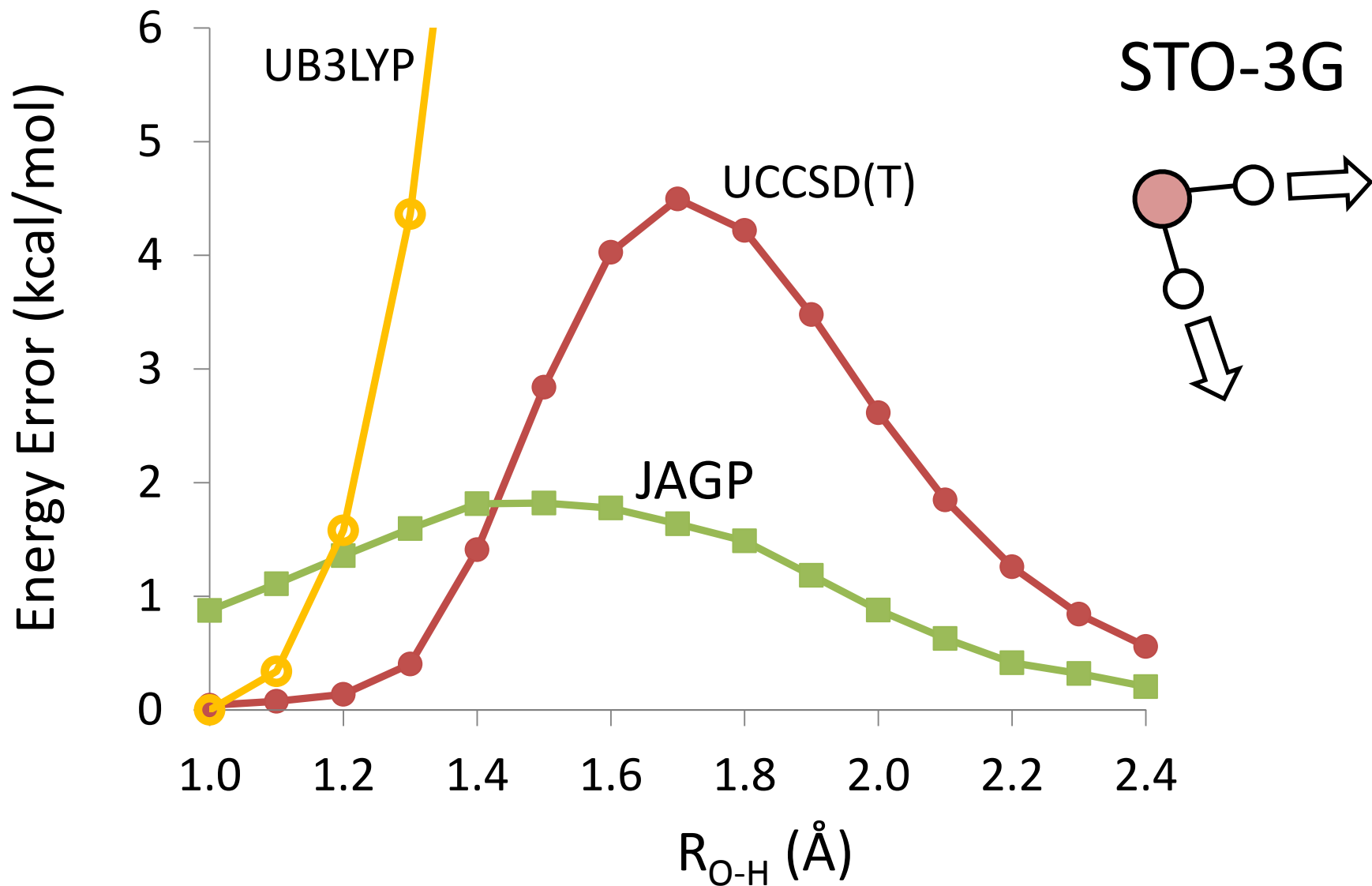


The (Hilbert space) JAGP Ansatz

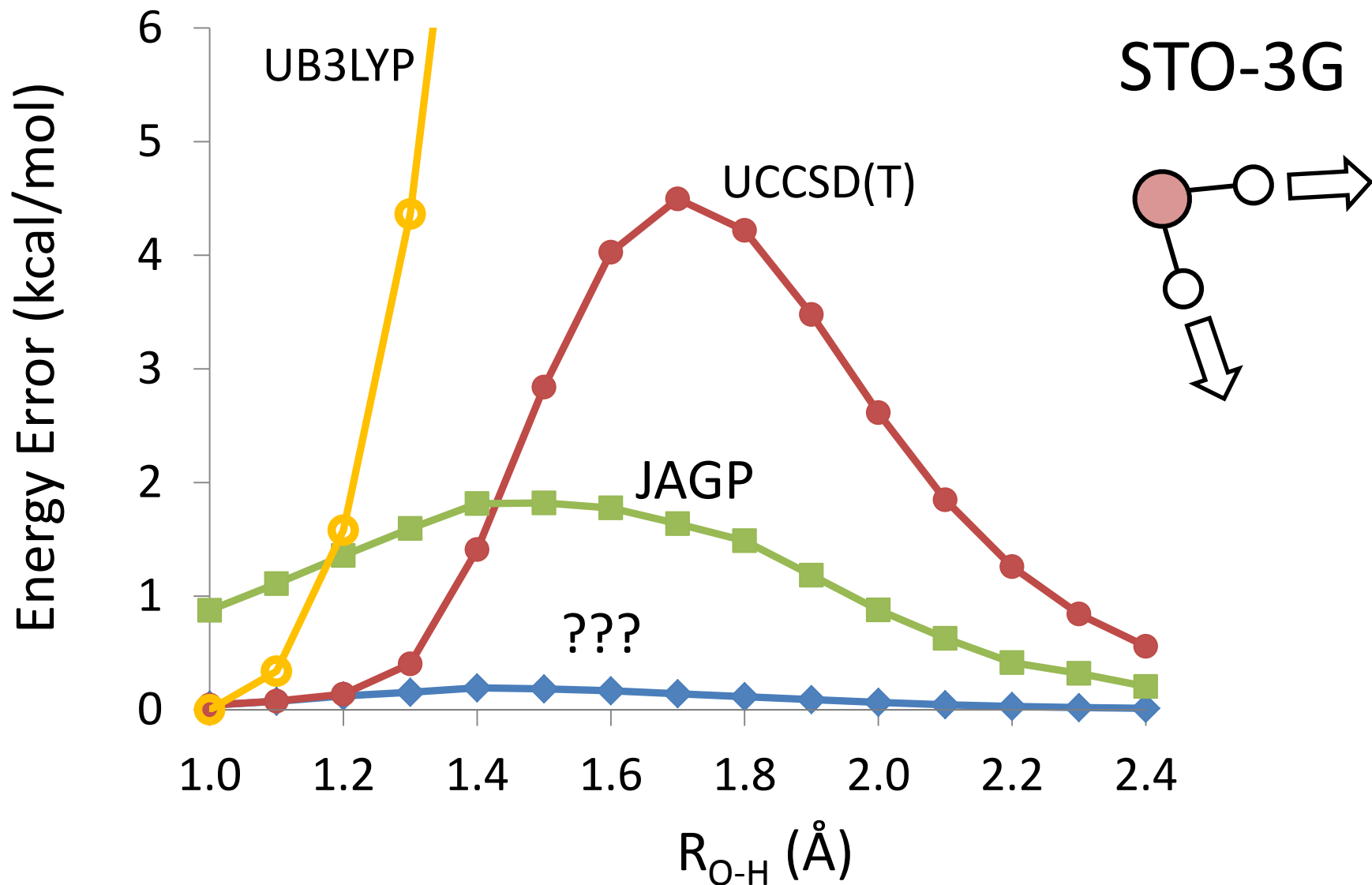
getting closer



getting closer



getting closer



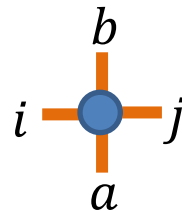
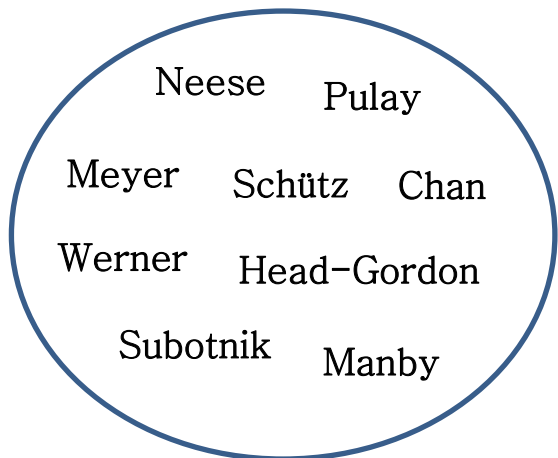
coupled cluster theory

$$\exp \left[\sum_{iajb} T_{ij}^{ab} a_a^+ a_i a_b^+ a_j \right]$$

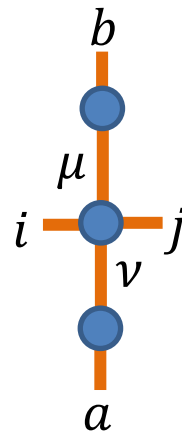
$$\left(\langle \vec{n} | = \langle 010011010 | \right)$$

$$\langle \vec{n} | e^T | \Phi \rangle$$

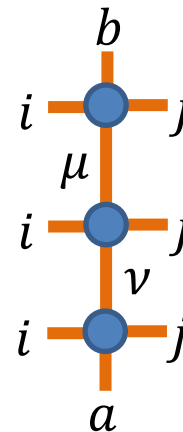
exponential cost



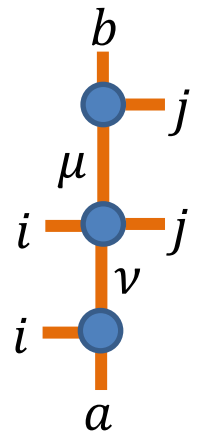
full T



PAO



PNO



OSV

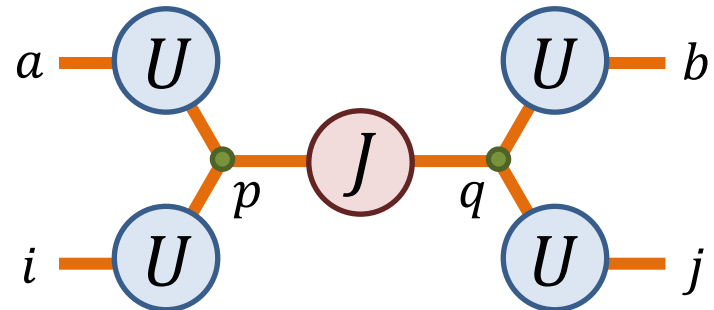
the Jastrow cluster operator

$$\sum_{pq} a_p^\dagger a_p J_{pq} a_q^\dagger a_q$$

orbital rotation
with unitary U

$$\sum_{pq} \sum_{iajb} (U_{pa}^* a_a^\dagger) (U_{pi} a_i) J_{pq} (U_{qb}^* a_b^\dagger) (U_{qj} a_j)$$

$$T_{ij}^{ab} \approx$$



$$\sum_{iajb} T_{ij}^{ab} a_a^\dagger a_i a_b^\dagger a_j$$

optimization

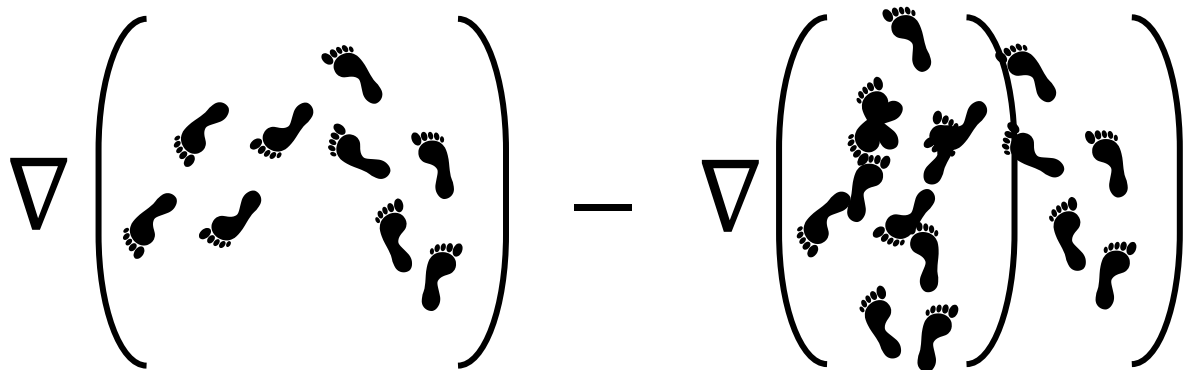
$$\langle \Phi | e^{T^\dagger} H e^T | \Phi \rangle \xrightarrow{T \approx U^\dagger J U} \left[\langle \tilde{\Phi} | \right] e^{J^\dagger} \left[\tilde{H} \right] e^J \left[| \tilde{\Phi} \rangle \right]$$

$$U = e^{K - K^\dagger}$$

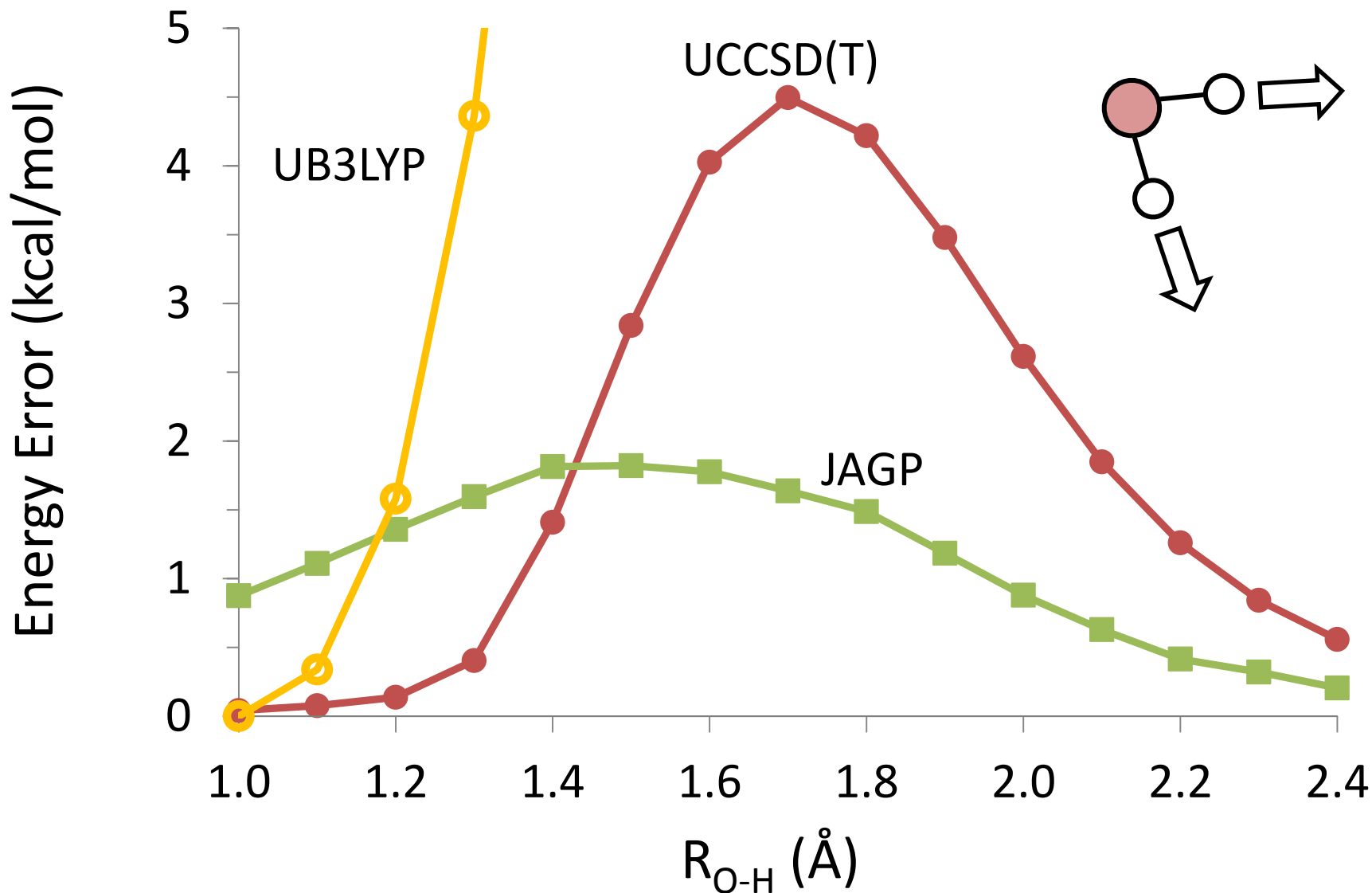
$$\frac{\partial E}{\partial K_{pq}} = \sum_{ij} \frac{\partial E}{\partial \tilde{t}_{ij}} \frac{\partial \tilde{t}_{ij}}{\partial K_{pq}} + \sum_{ijkl} \frac{\partial E}{\partial \tilde{V}_{ijkl}} \frac{\partial \tilde{V}_{ijkl}}{\partial K_{pq}}$$

correlated sampling

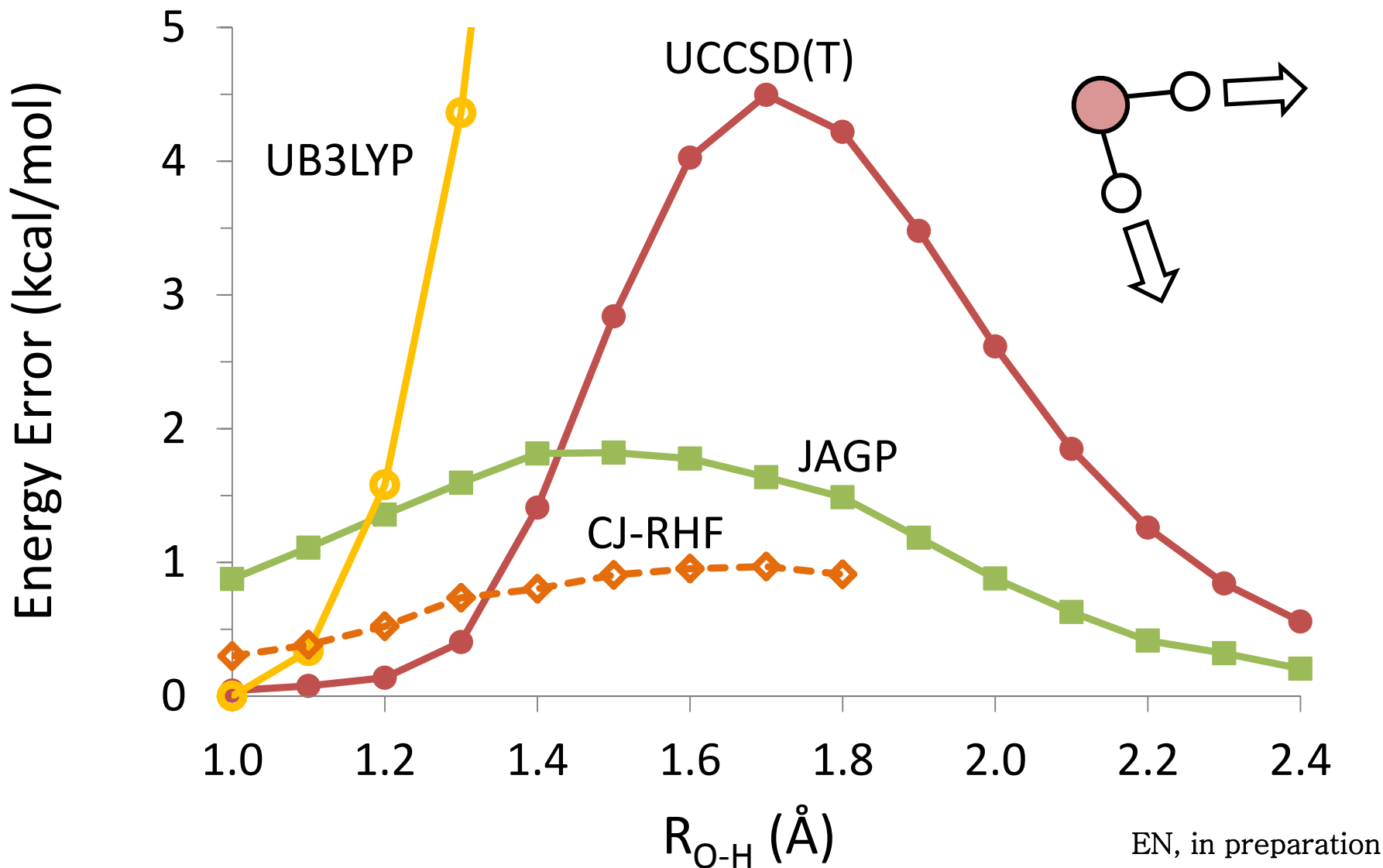
derivatives ✓
quasi-Newton ✓



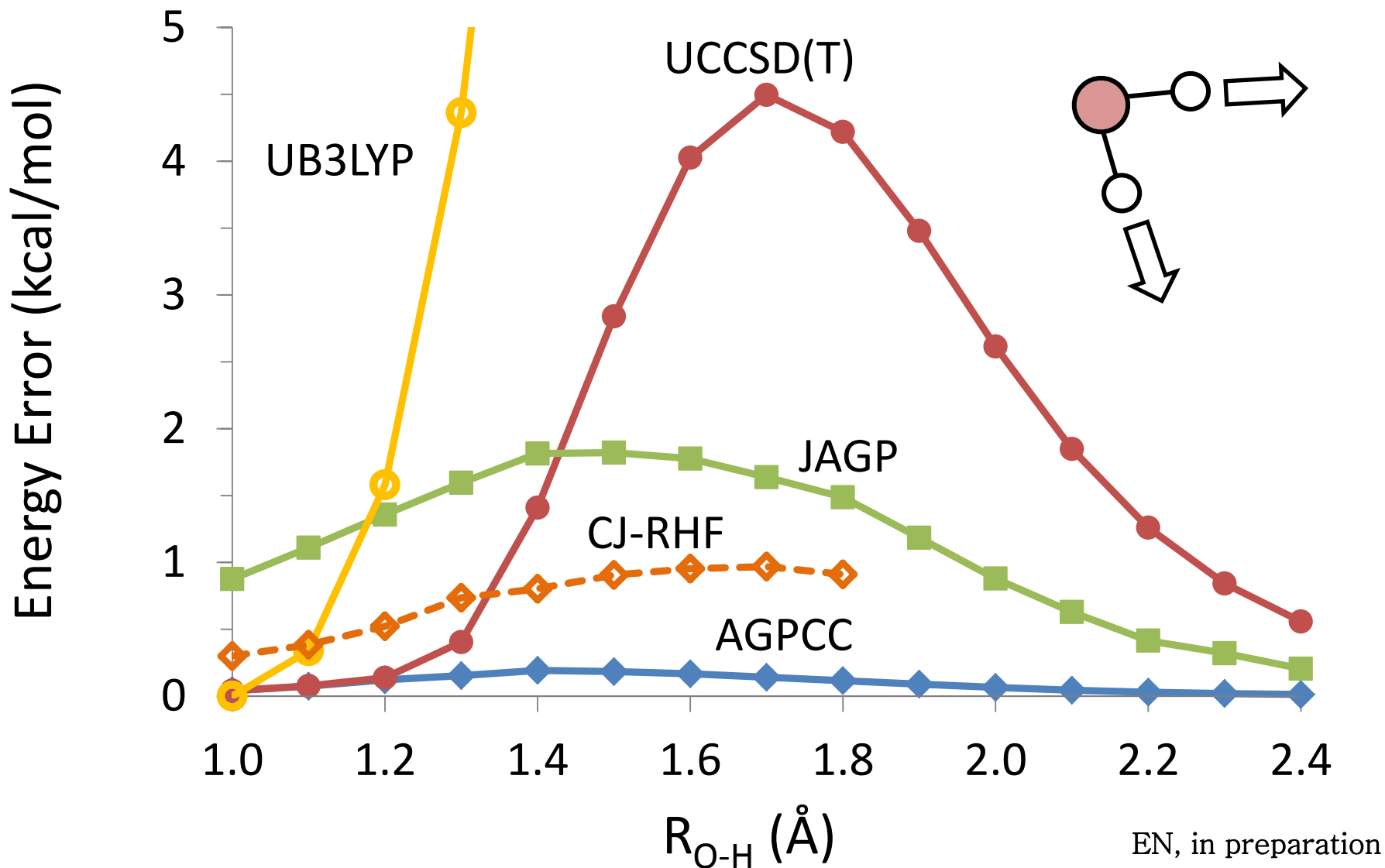
strong correlation (H₂O, STO-3G)



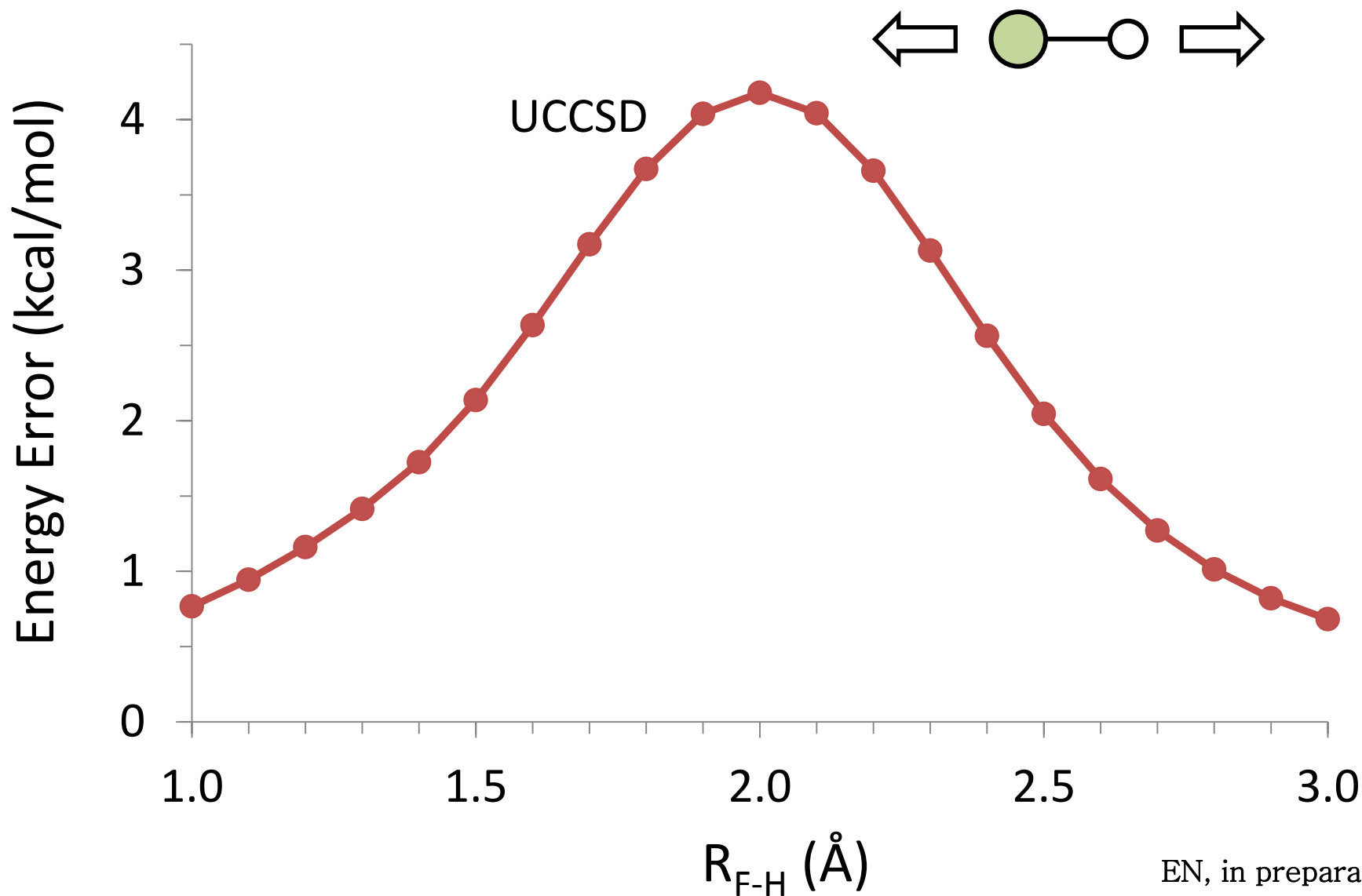
strong correlation (H_2O , STO-3G)



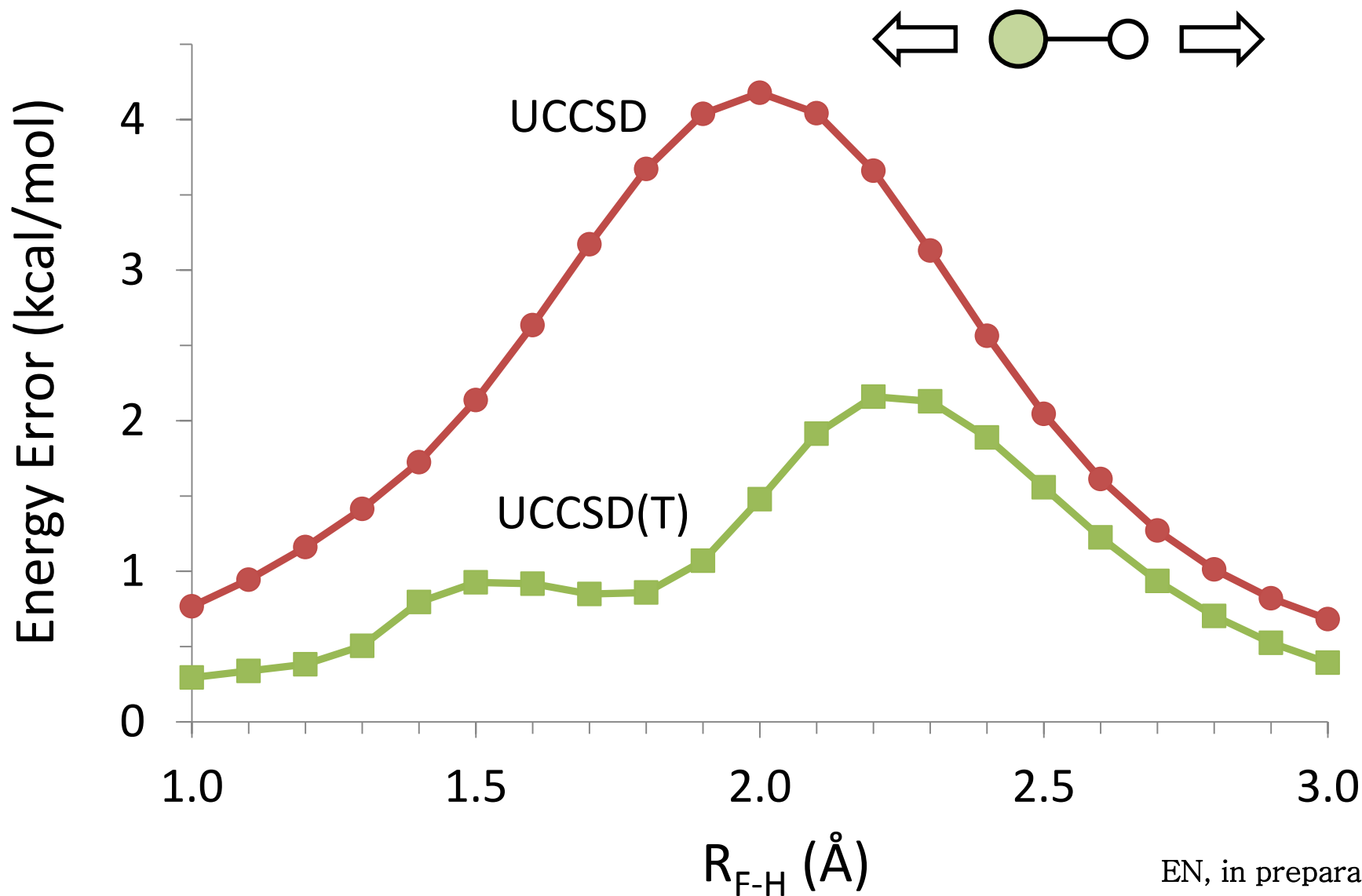
strong correlation (H_2O , STO-3G)



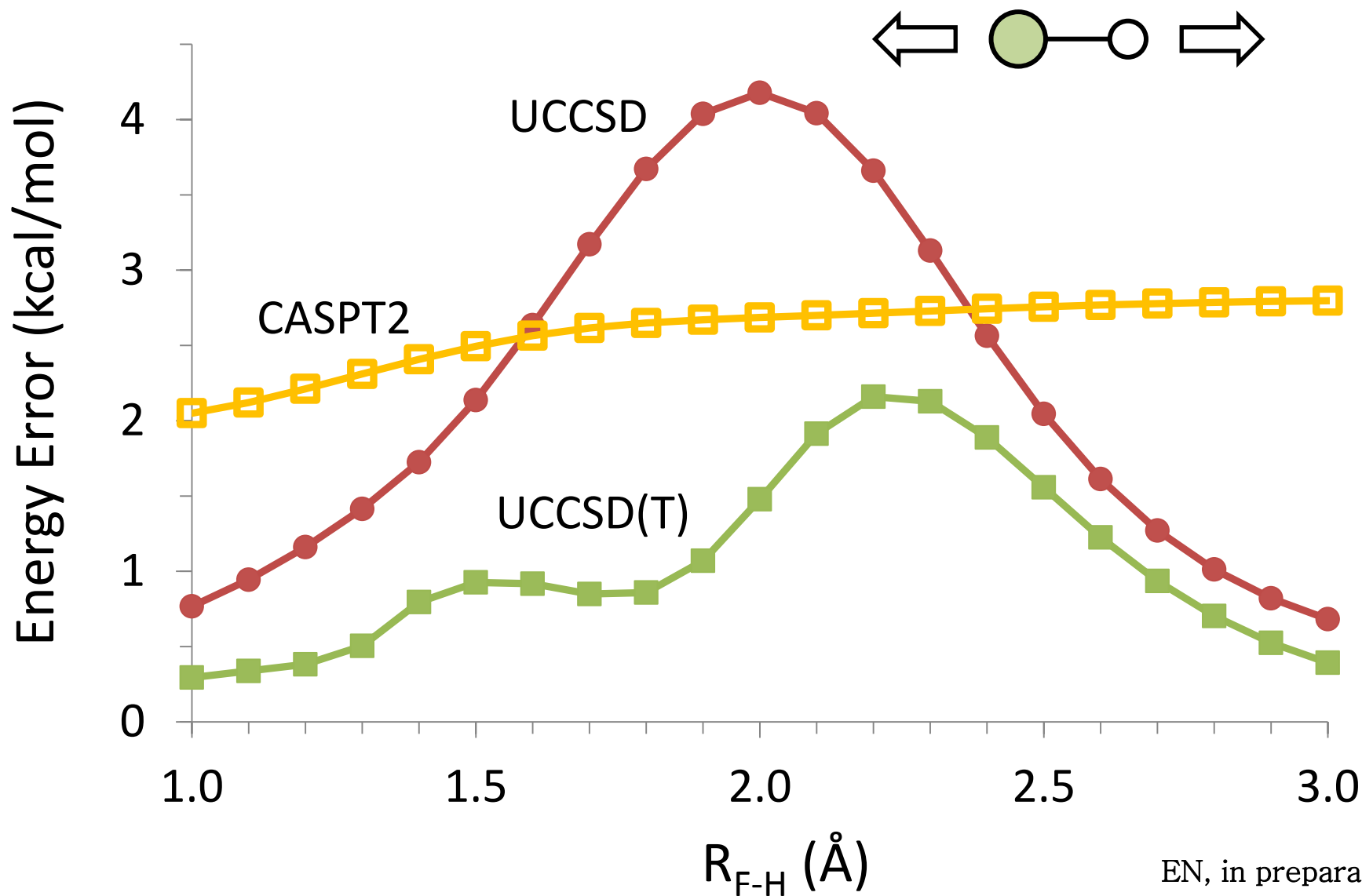
strong and weak (HF, 6-31G)



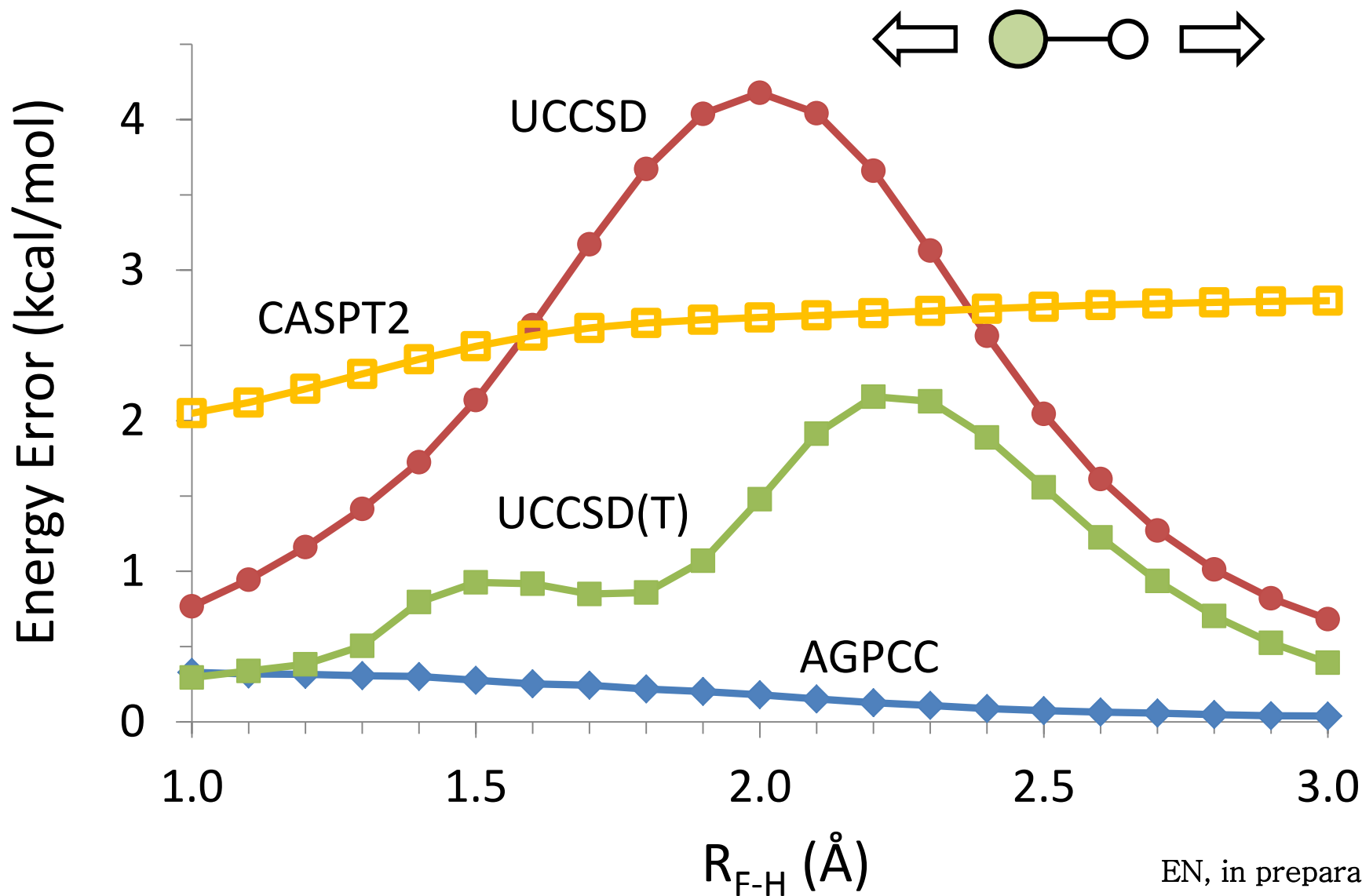
strong and weak (HF, 6-31G)



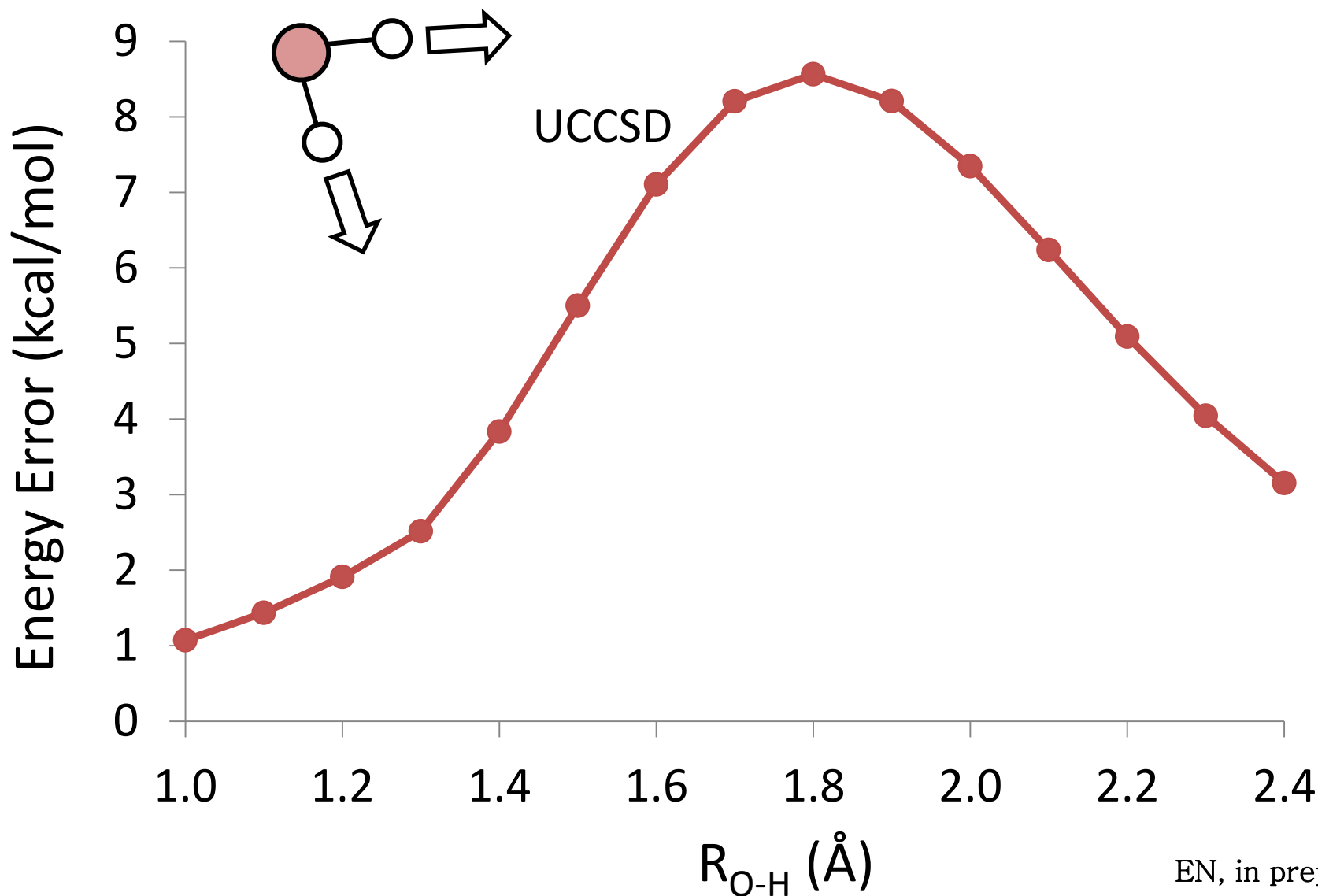
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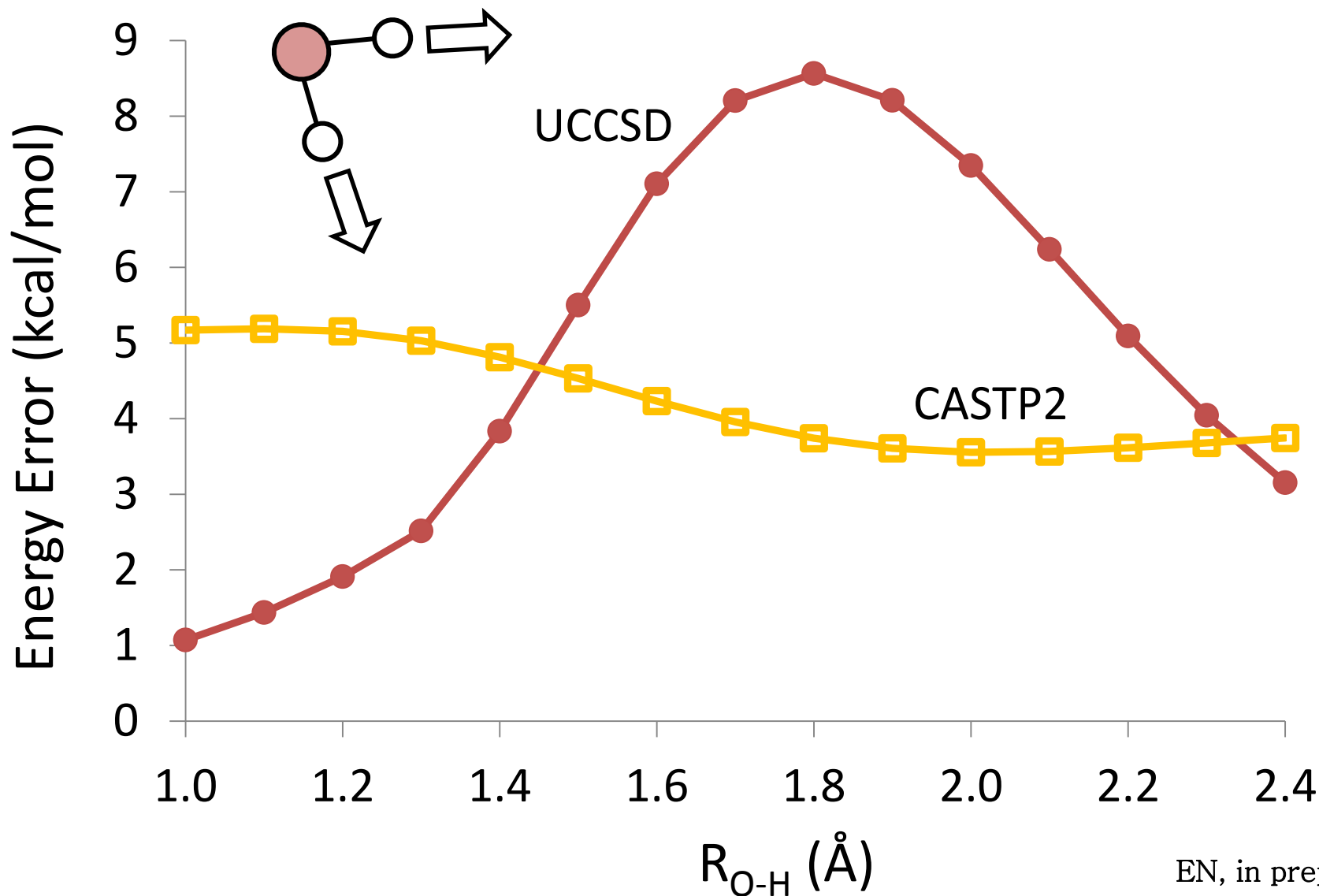
strong and weak (HF, 6-31G)



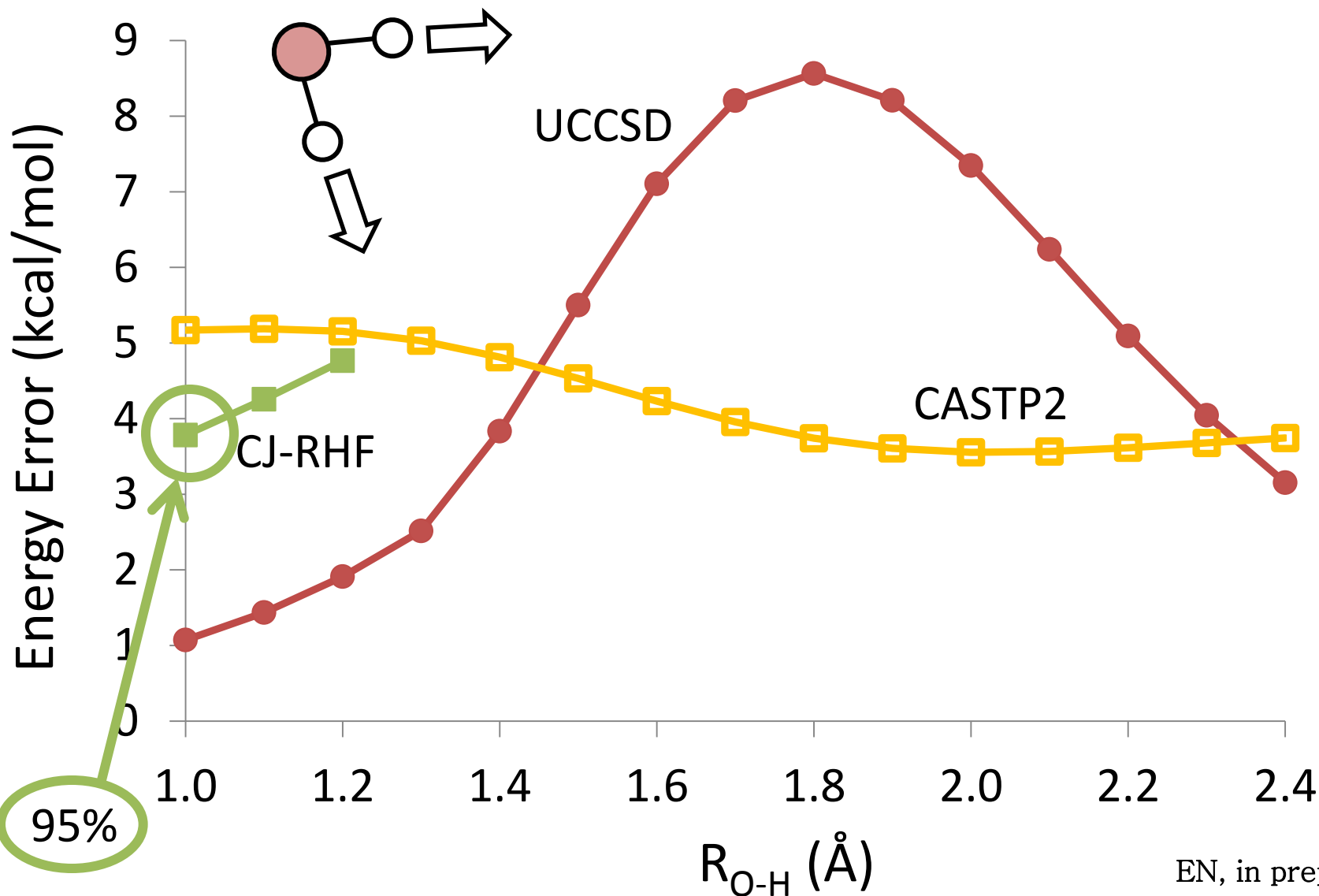
strong and weak (H₂O, 6-31G)



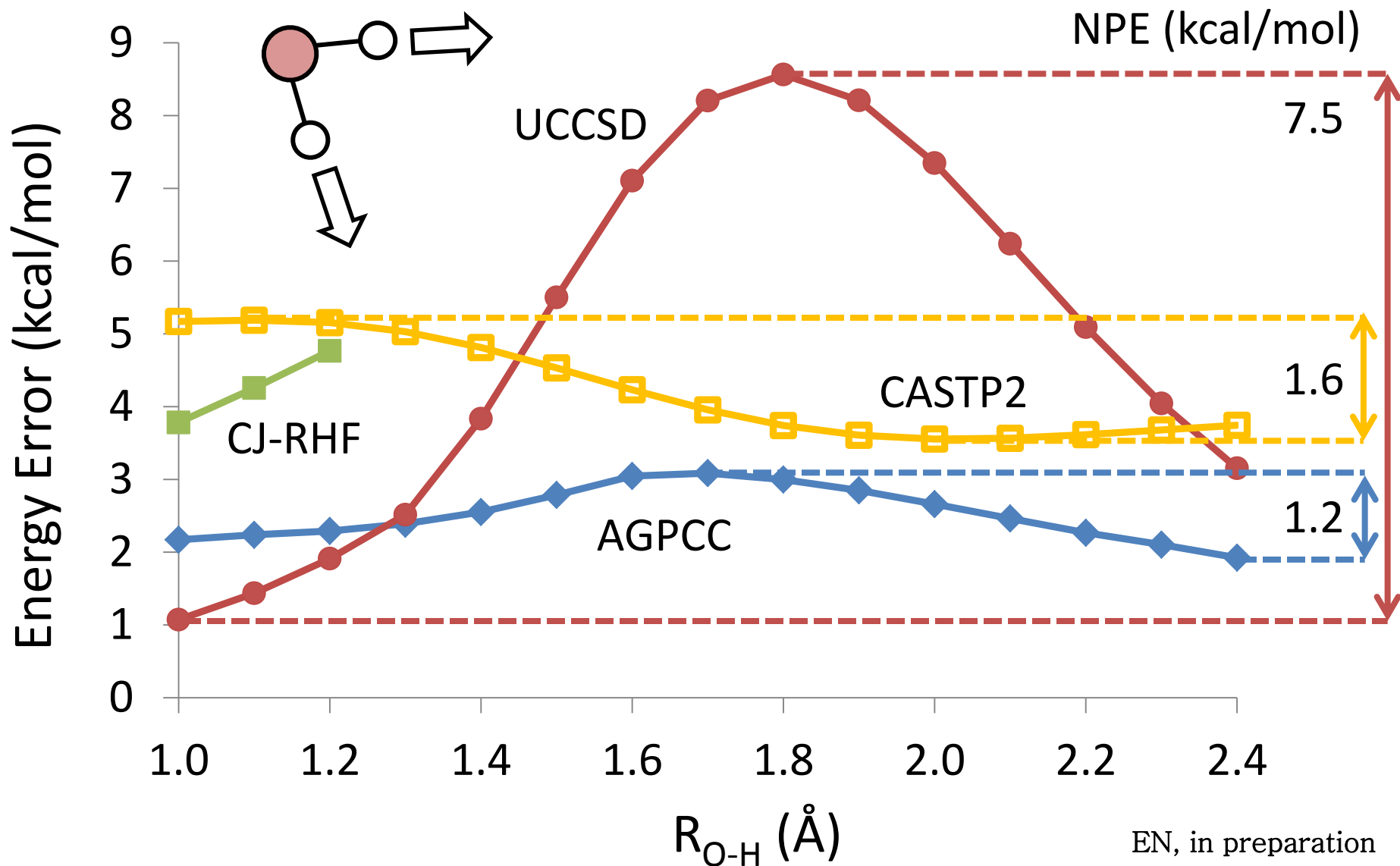
strong and weak (H₂O, 6-31G)



strong and weak (H₂O, 6-31G)



strong and weak (H₂O, 6-31G)



a word on cost

accurate (strong & weak)

$$1 + 1 = 2$$

size consistent/extensive

$$1 + 1 + 1 \dots \propto N$$

variational

$$1 + 1 > 0$$

$$e^T \quad \text{⬇} \text{⚡} \text{⬆}$$

$$\text{cost} = C \times N^5$$

very large (10^8 🦶)

~~quasi-Newton $\nabla \left(\begin{matrix} \text{🦶} \\ \text{🦶} \\ \text{🦶} \end{matrix} \right)$~~

analytic Hessian

acknowledgements

Martin Head-Gordon

Eric Sundstrum

Evgeny Epifanovsky



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Miller Institute for Basic
Research in Science**
University of California, Berkeley