The strange way from nuclei to neutron stars: a Quantum Monte Carlo approach

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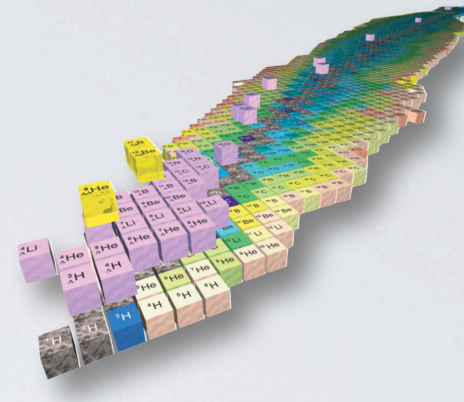
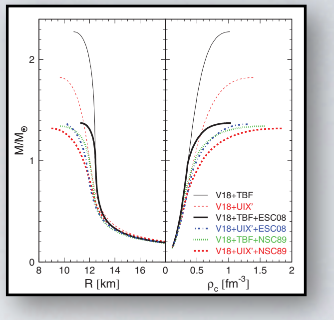
- S. Gandolfi (LANL, US-NM)

INT, Seattle - July 29, 2013

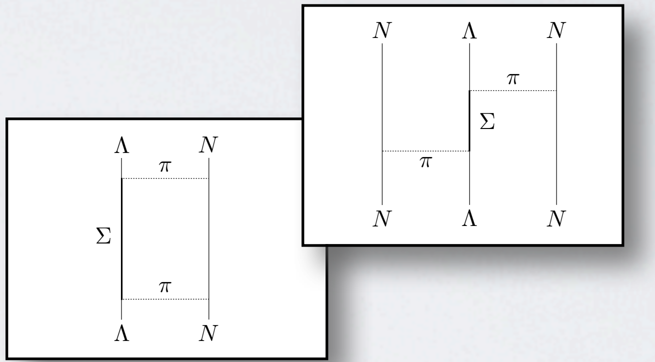
Outline

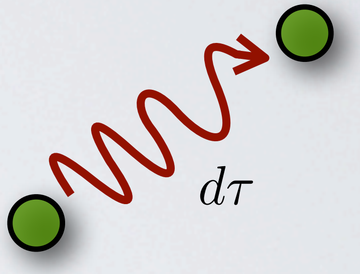
- Strangeness in nuclear physics: why?
  - terrestrial experiments: hypernuclei
  - neutron stars: theory vs. observations



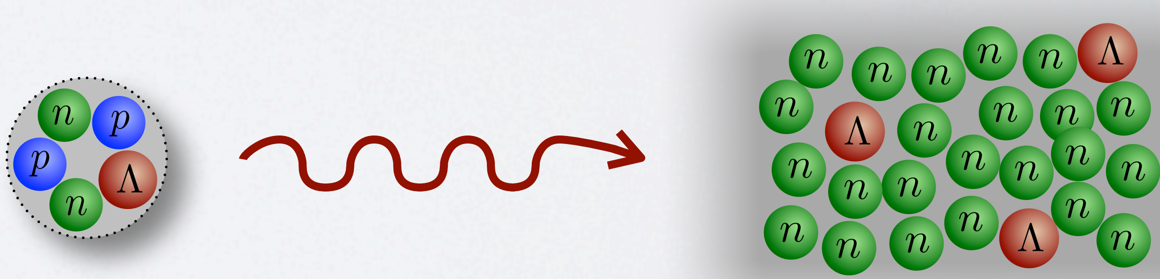


- The strange AFDMC project
  - the idea
  - the hyperon-nucleon interaction
  - the AFDMC code



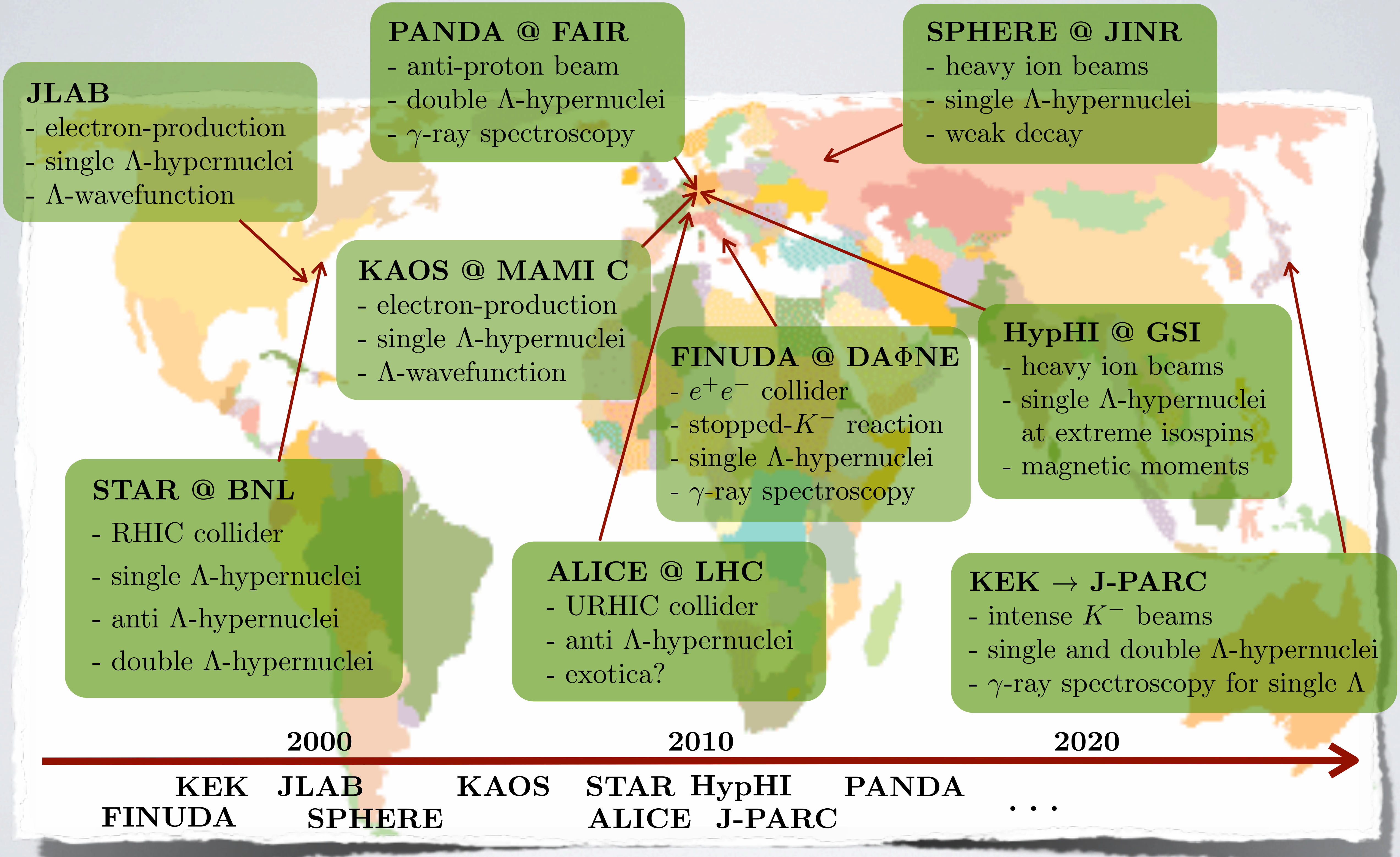


- Results
  - Λ-hypernuclei
  - Λ-neutron matter



- Conclusions

Strangeness in nuclear physics: hypernuclei experiments



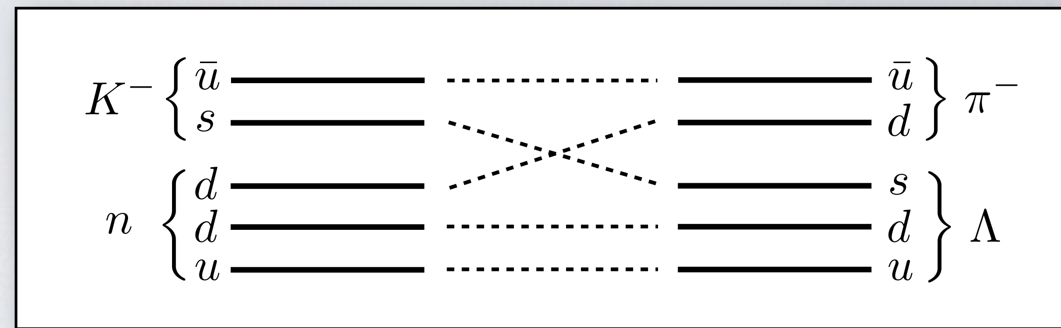
Adapted from: J. Pochodzalla, arXiv:1101.2790v1 [nucl-ex] (2011)

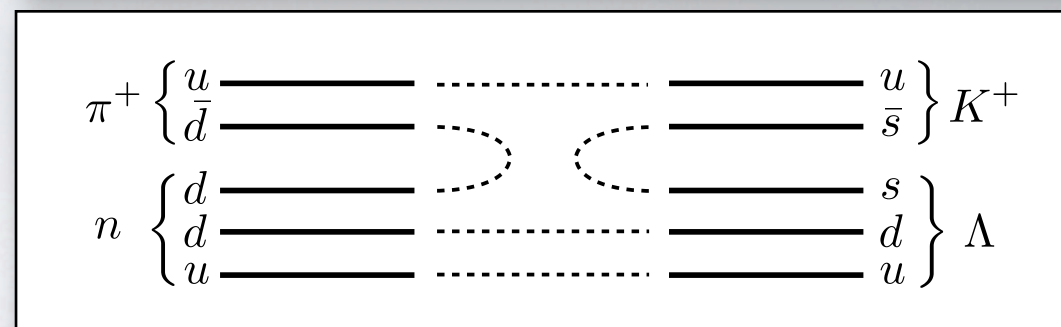
Strangeness in nuclear physics: hypernuclei experiments

- ✓ Charge conserving reactions

$$^{A}Z\,(K^-,\pi^-)\,^{A}_{\Lambda}Z$$

$$^{A}Z\,(\pi^+,K^+)\,^{A}_{\Lambda}Z$$





- ✓ Single charge exchange reactions (SCX)

$$^{A}Z\,(K^-,\pi^0)\,^{A}_{\Lambda}[Z-1]$$

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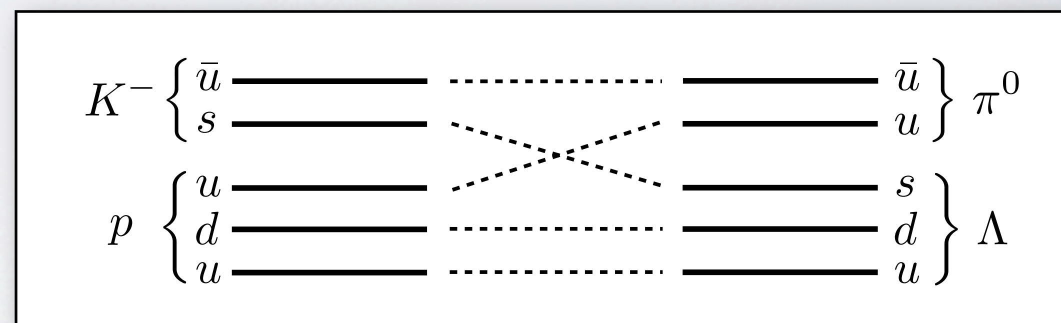
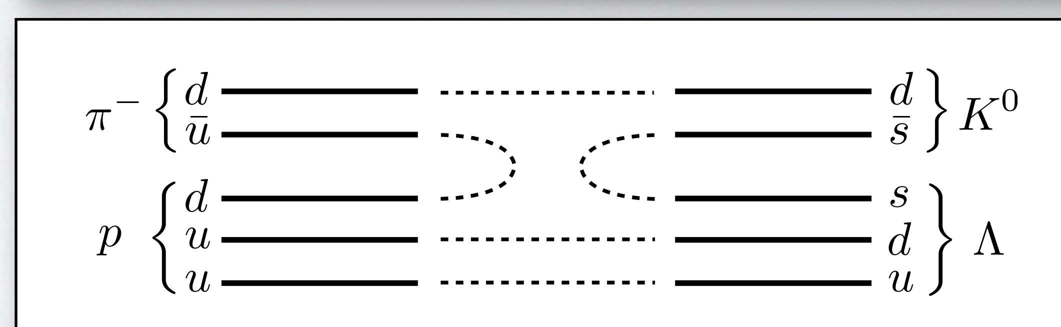
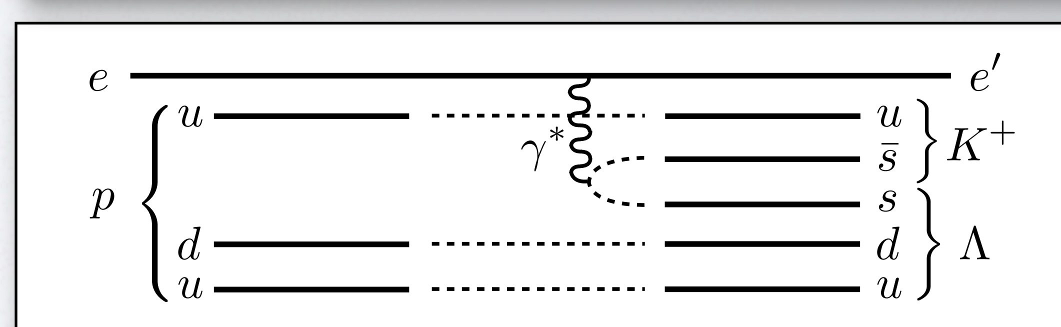
$$^{A}Z\,(e,e'K^+)\,^{A}_{\Lambda}[Z-1]$$





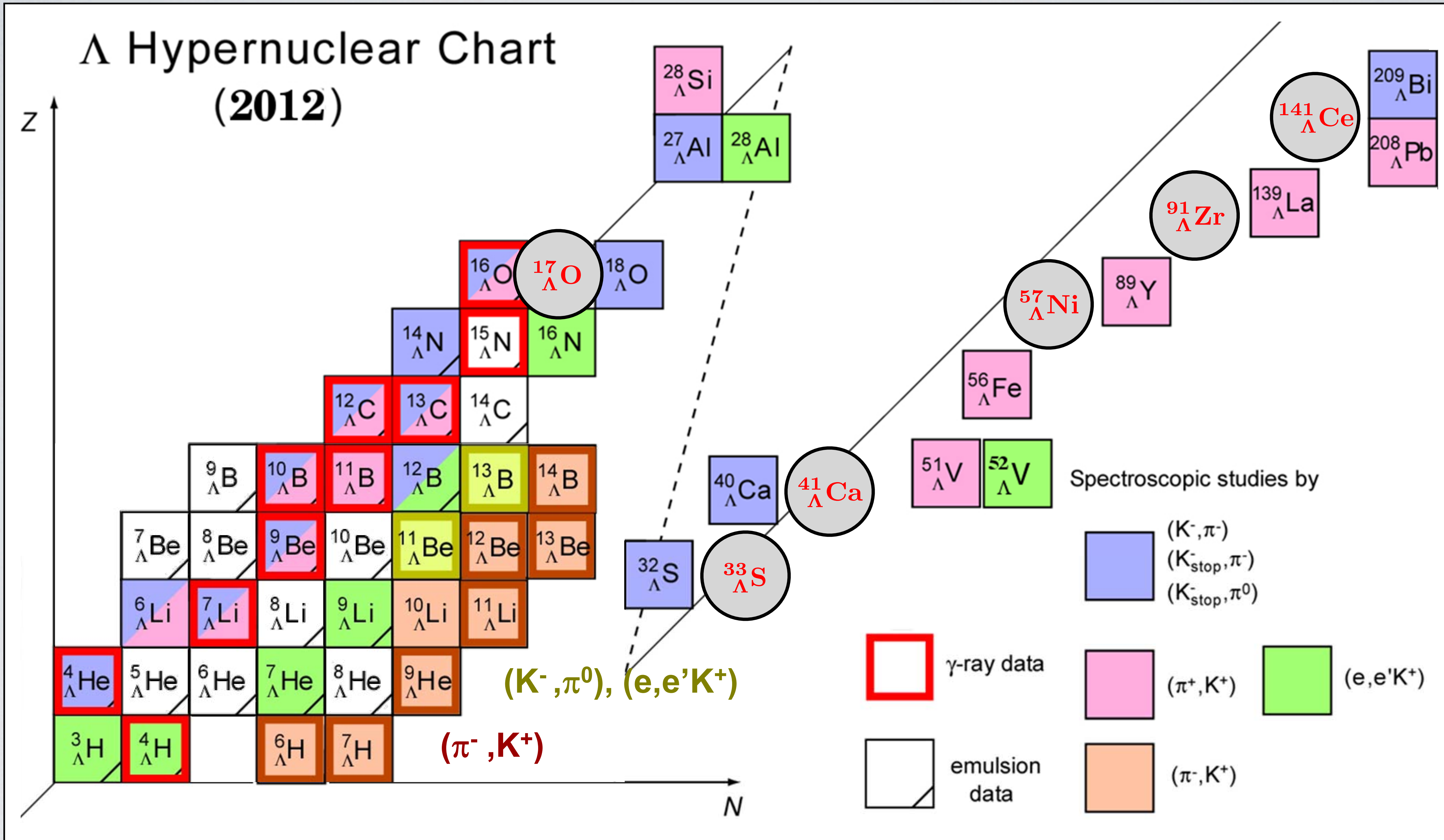


- ✓ Double charge exchange reactions (DCX)

$$^{A}Z\,(\pi^-,K^+)\,^{A+1}_{\quad\Lambda}[Z-2]$$

$$^{A}Z\,(K^-,\pi^+)\,^{A+1}_{\quad\Lambda}[Z-2]$$

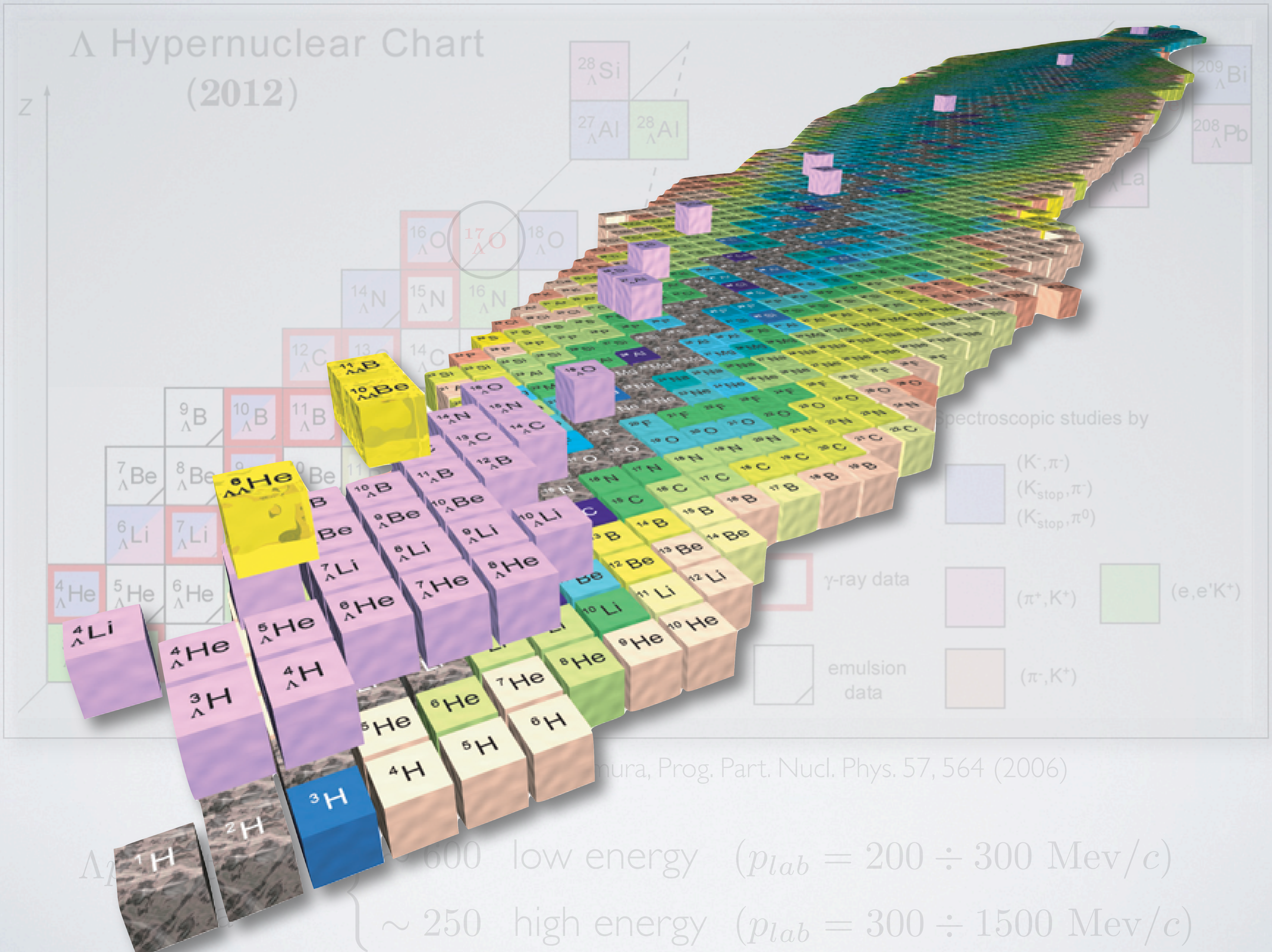
Strangeness in nuclear physics: hypernuclei experiments



Updated from: O. Hashimoto, H. Tamura, Prog. Part. Nucl. Phys. 57, 564 (2006)

Λp scattering data $\begin{cases} \sim 600 & \text{low energy} \quad (p_{lab} = 200 \div 300\ \mathrm{Mev}/c) \\ \sim 250 & \text{high energy} \quad (p_{lab} = 300 \div 1500\ \mathrm{Mev}/c) \end{cases}$

Strangeness in nuclear physics: hypernuclei experiments



mura, Prog. Part. Nucl. Phys. 57, 564 (2006)

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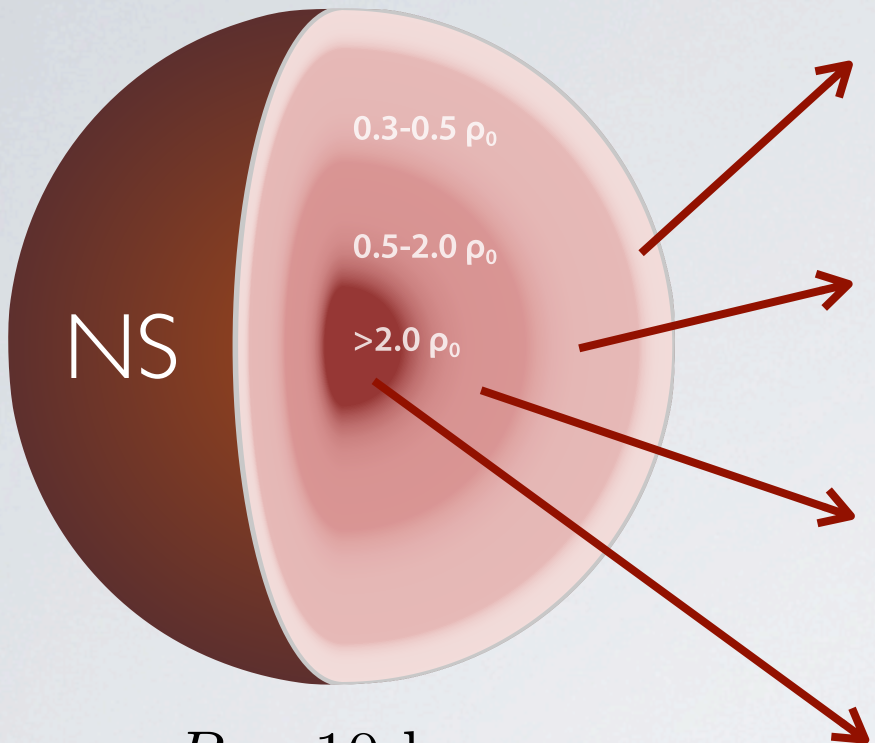
Strangeness in nuclear physics: neutron stars



$R \sim 10$ km

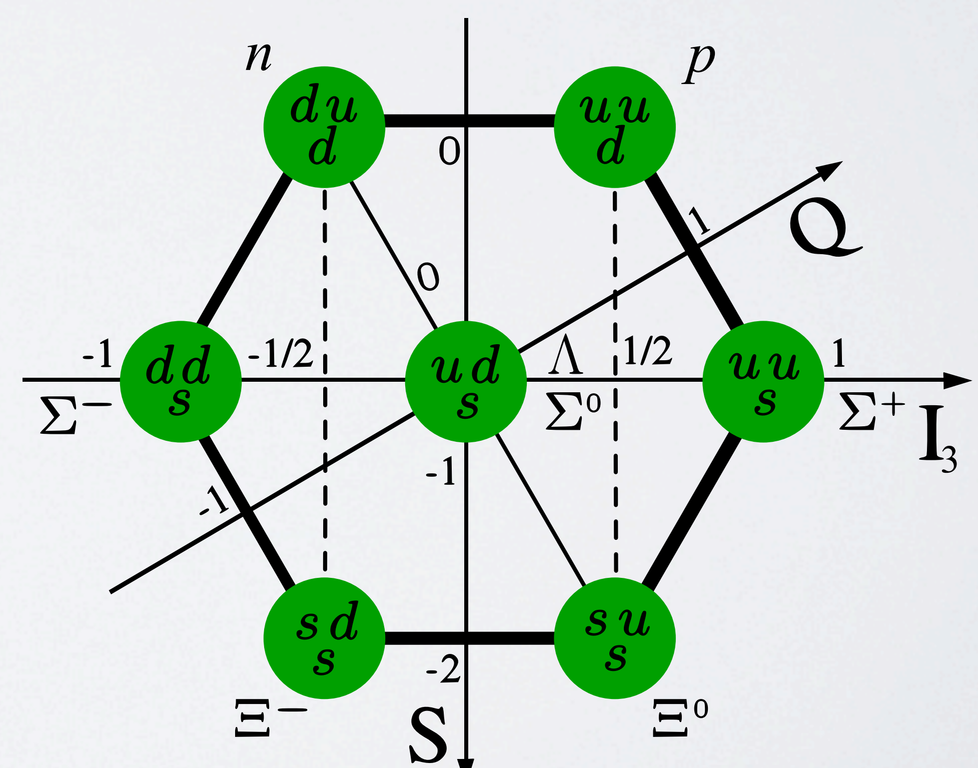
$M \sim 1.4\ M_\odot$

outer crust: $Z\ e$
$(0.3 \div 0.5$ km$)$

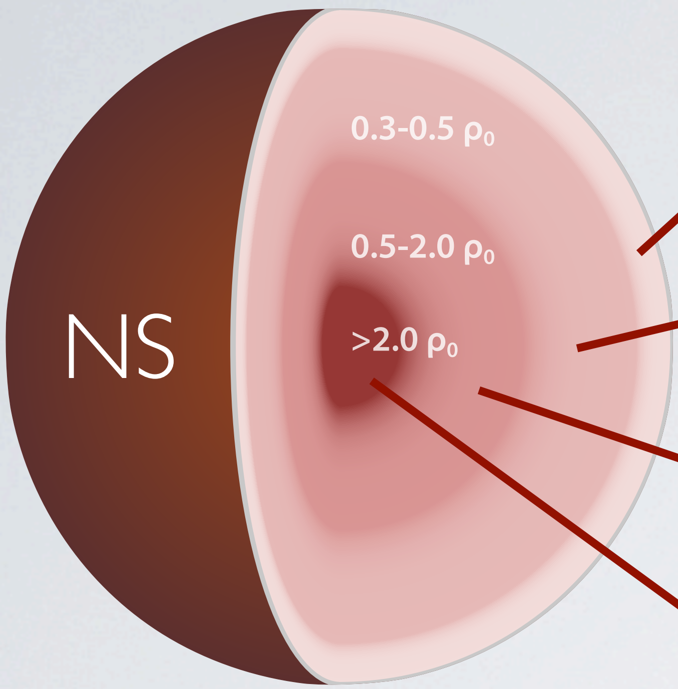
inner crust: $Z\ n\ e$
$(1 \div 2$ km$)$

outer core: $n\ p\ e\ \mu$
$(\sim 9$ km$)$

inner core: $n\ p\ e\ \mu\ \Lambda\ \Sigma\ \Xi\ \pi_c\ K_c\ q_p$?
$(0 \div 3$ km$)$



Strangeness in nuclear physics: neutron stars



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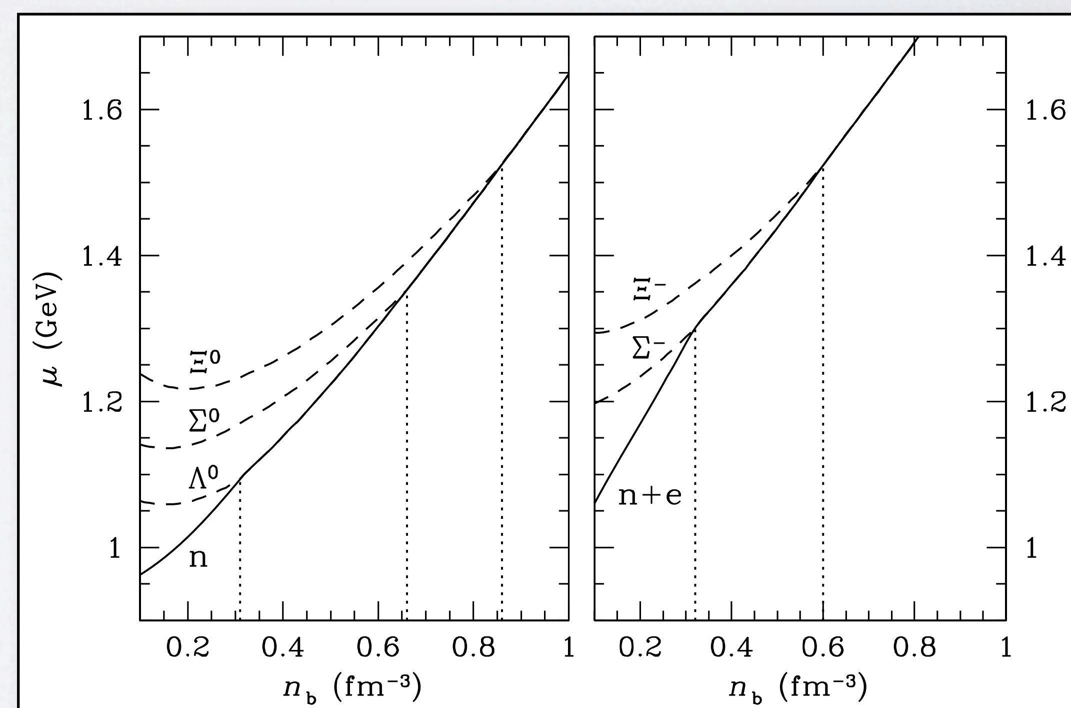
$(0 \div 3$ km$)$

$$Q = -1\ :\ \mu_{b^-} = \mu_n + \mu_e$$
$$Q = \ \ 0\ :\ \mu_{b^0} = \mu_n$$
$$Q = +1\ :\ \mu_{b^+} = \mu_n - \mu_e$$



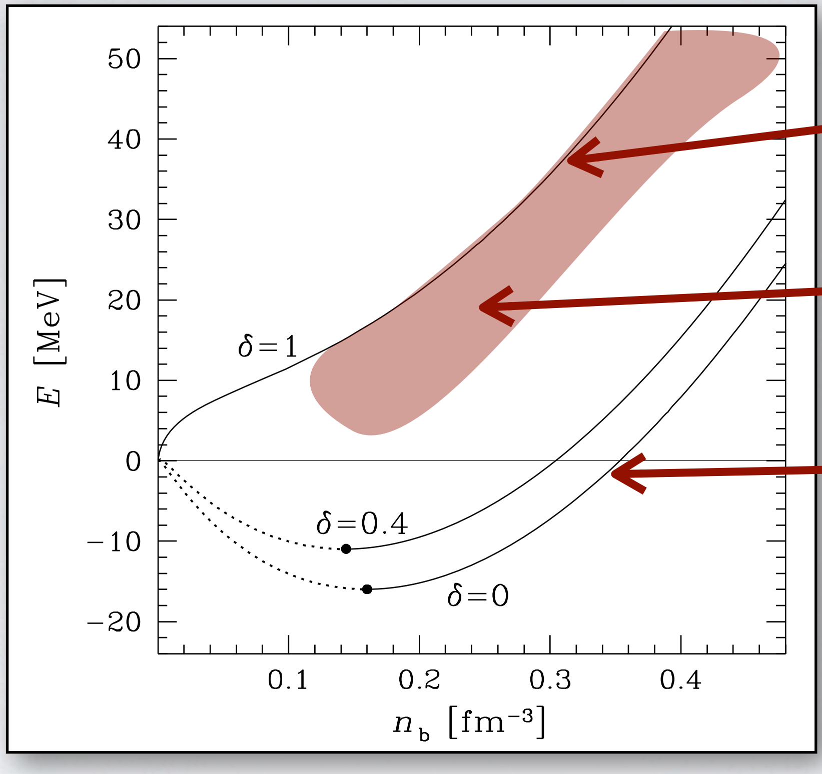
P. Haensel, A.Y. Potekhin, D.G. Yakovlev, Neutron Stars 1, Springer 2007

Strangeness in nuclear physics: neutron stars

$$\begin{cases} E \equiv E(n_b, \delta) \\ P = n_b^2 \dfrac{\partial E(n_b, \delta)}{\partial n_b} \end{cases}$$

$$n_b = n_p + n_n = A/V$$

$$\delta = \frac{n_n - n_p}{n_b}$$



P. Haensel, A.Y. Potekhin, D.G. Yakovlev, Neutron Stars 1, Springer 2007

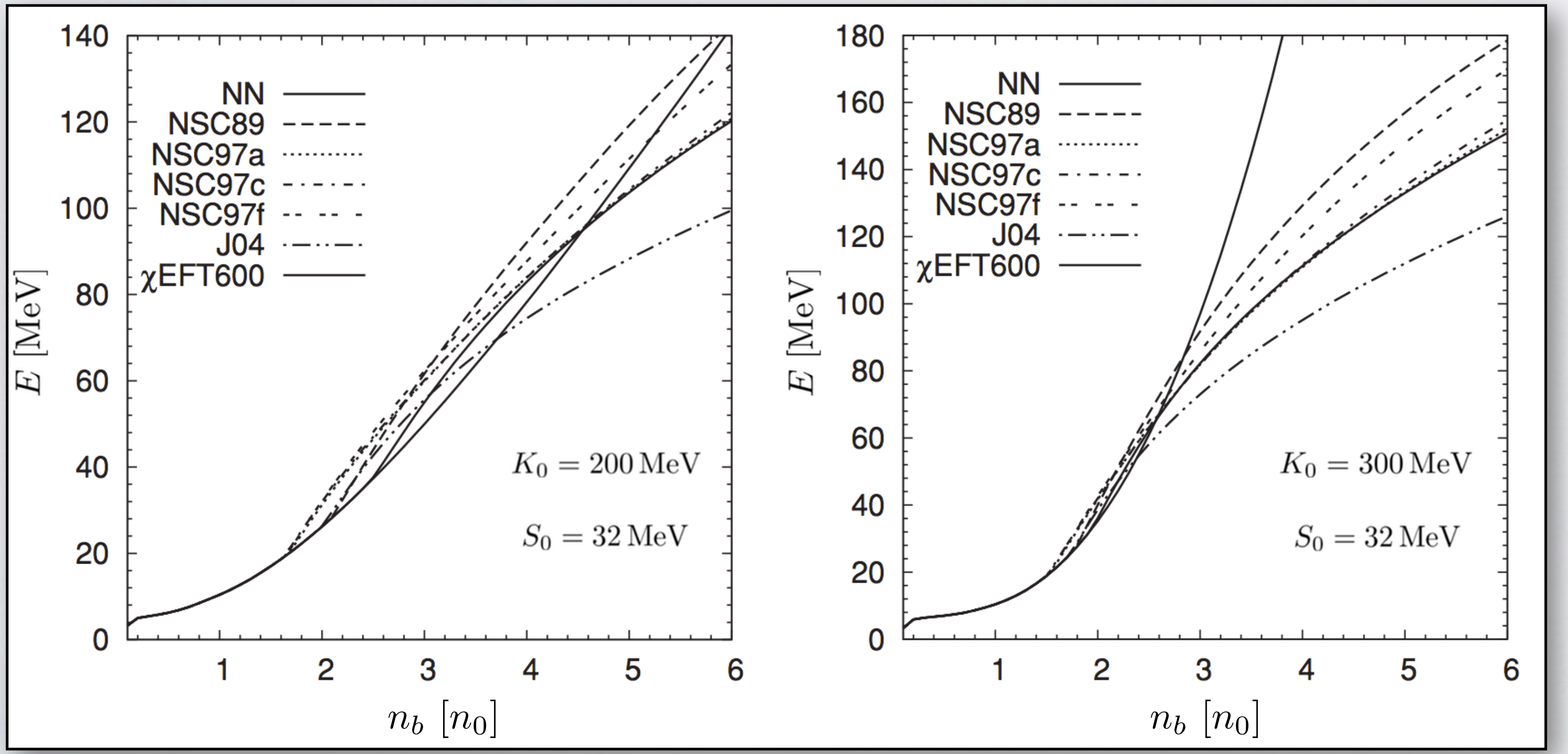
pure n matter

NS core

symmetric matter

$$n_0 = 0.16\ \mathrm{fm}^{-3}$$

$$E_0 = -16\ \mathrm{MeV}$$



H. Đapo, B.-J. Schaefer, and J. Wambach, Phys. Rev. C 81, 035803 (2010)

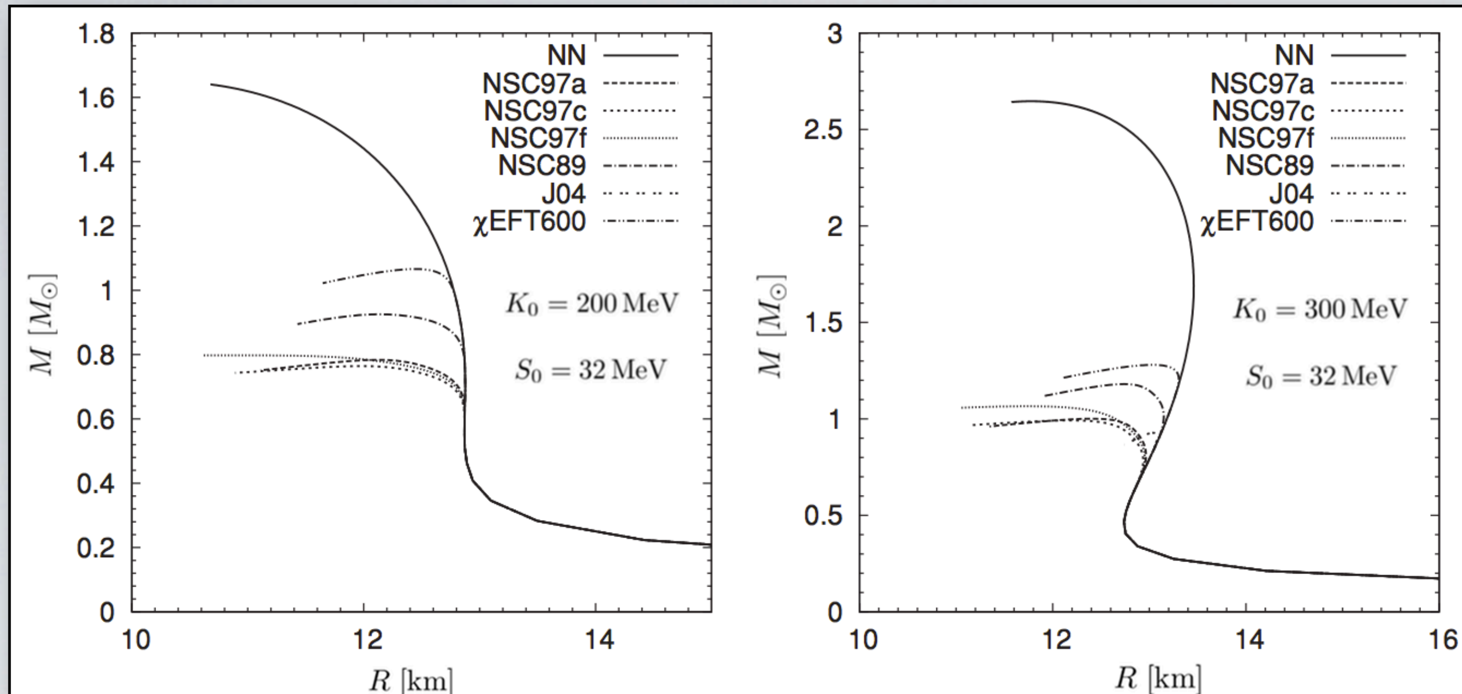
hyperons: softening of the EOS

↓

M(R) & Mmax (TOV)

model dependent

Strangeness in nuclear physics: neutron stars



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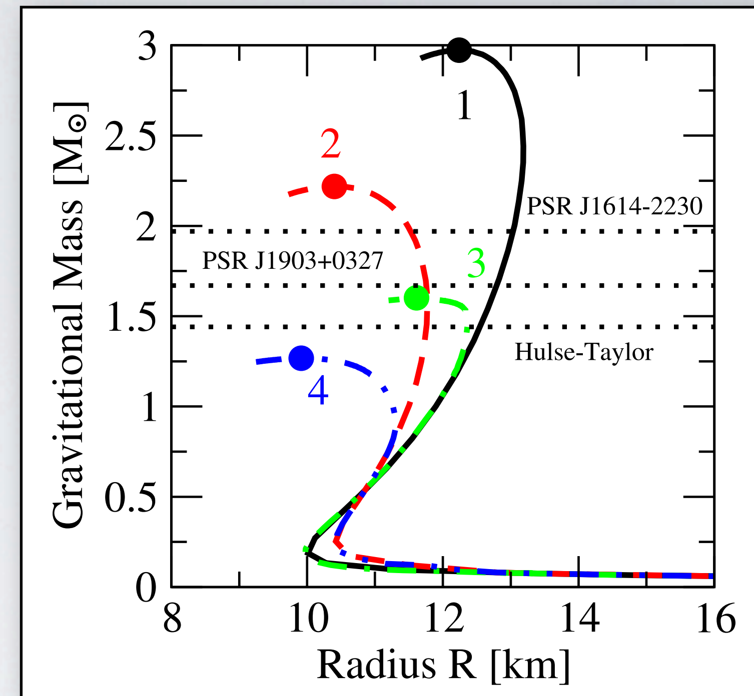
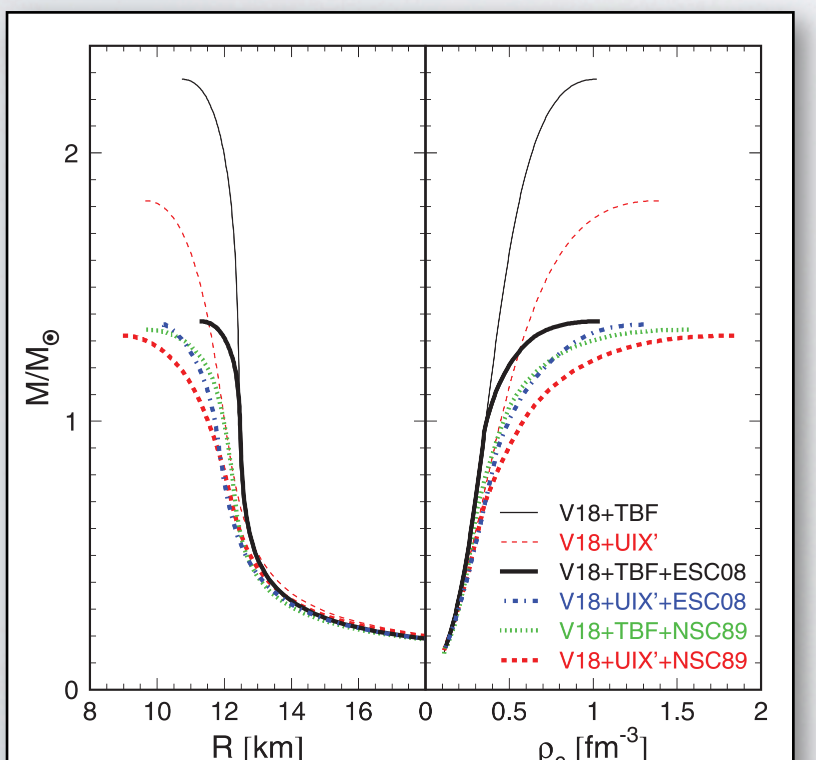
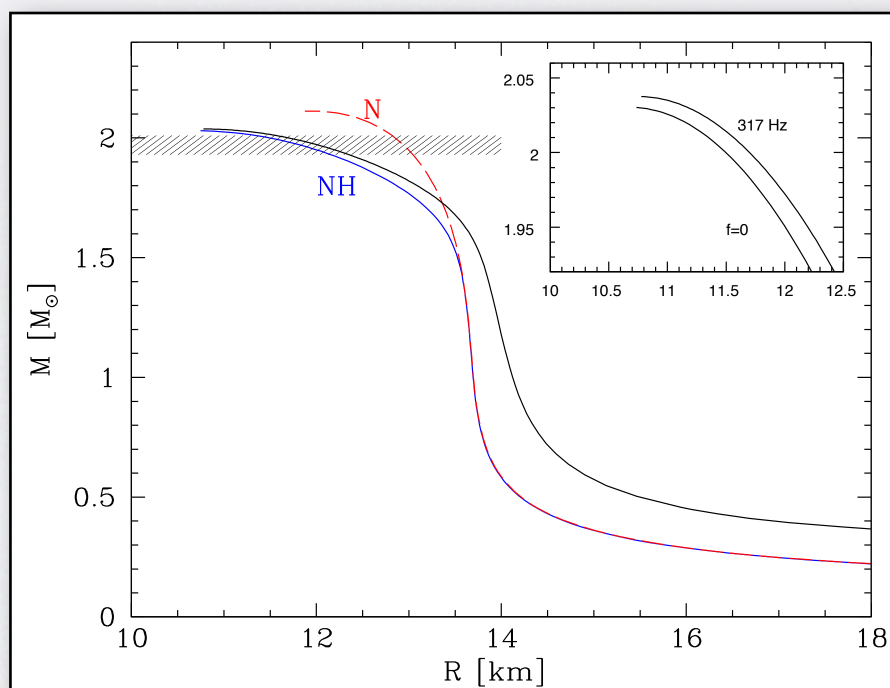


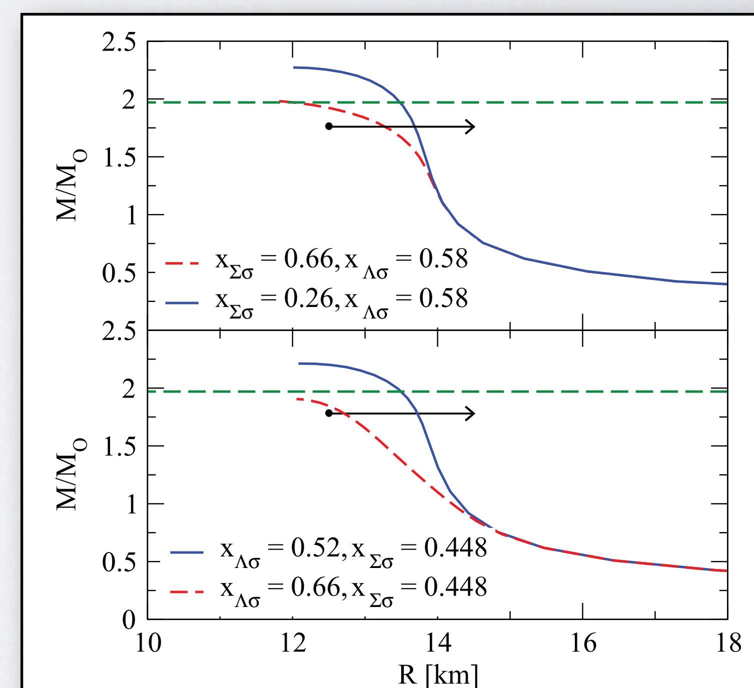
I. Vidaña, D. Logoteta, C. Providência,
A. Polls, I. Bombaci, EPL 94, 11002 (2011)



H.-J. Schulze, T. Rijken,
Phys. Rev. C 84, 035801 (2011)

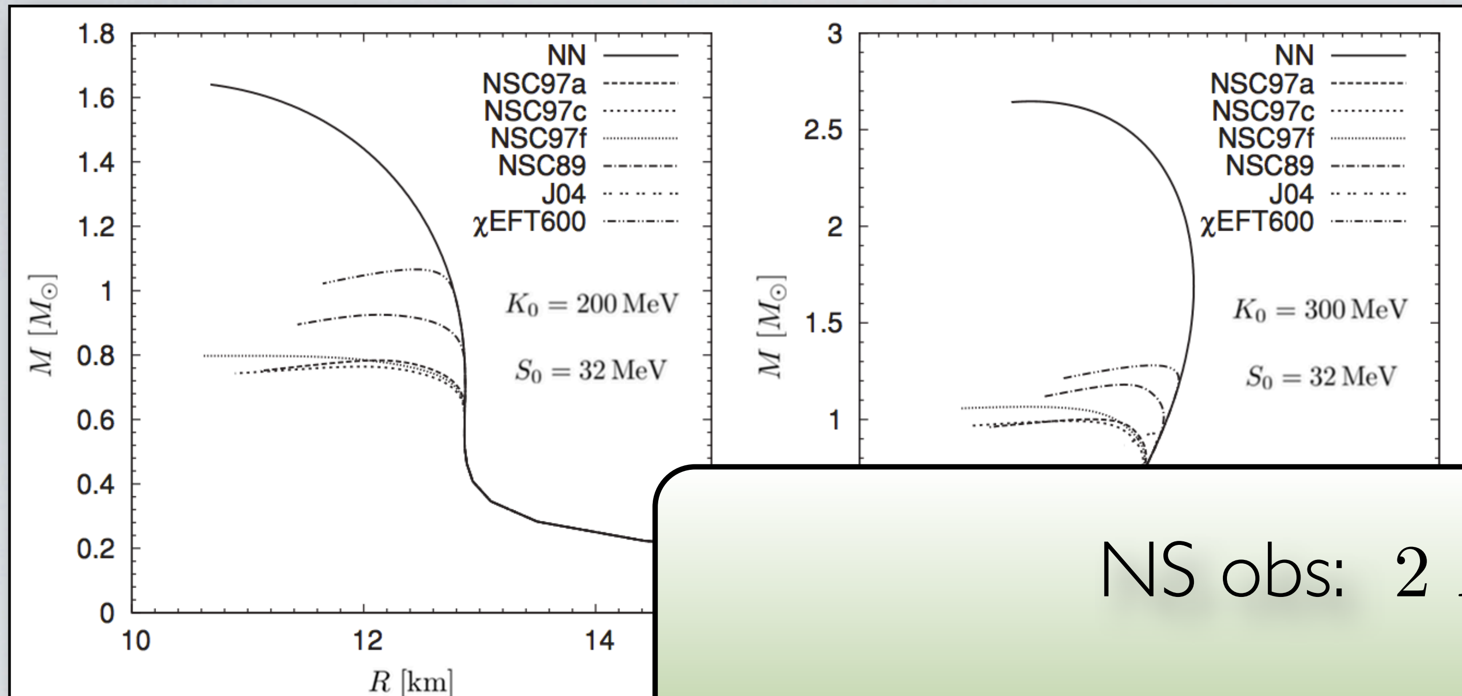


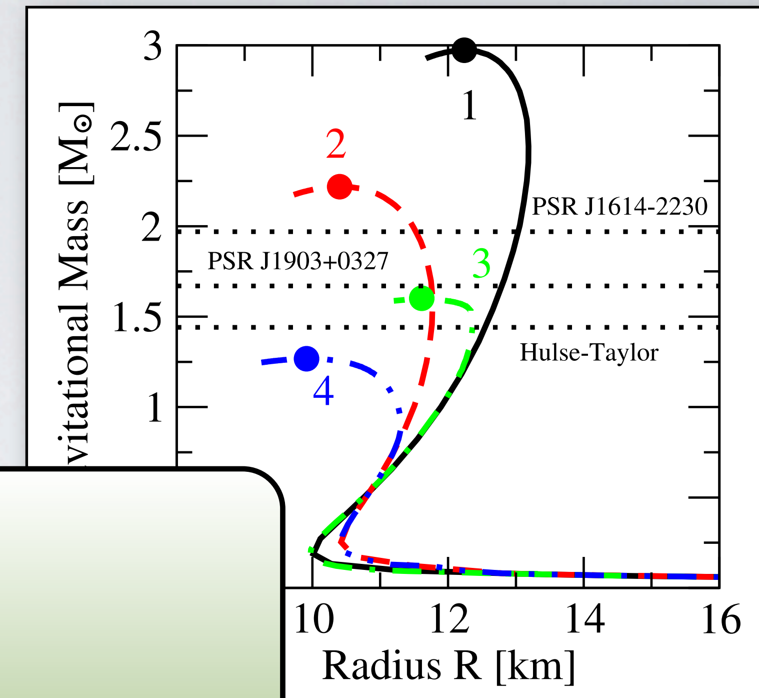
I. Bednarek, P. Haensel, J.~L. Zdunik, M. Bejger,
R. Mańka, Astron. Astrophys. 543, A157 (2012)



G. Colucci, A. Sedrakian,
Phys. Rev. C 87, 055806 (2013)

Strangeness in nuclear physics: neutron stars





H. Đapo, B.-J. Schaefer, J. Wam

ogoteta, C. Providência,

aci, EPL 94, 11002 (2011)

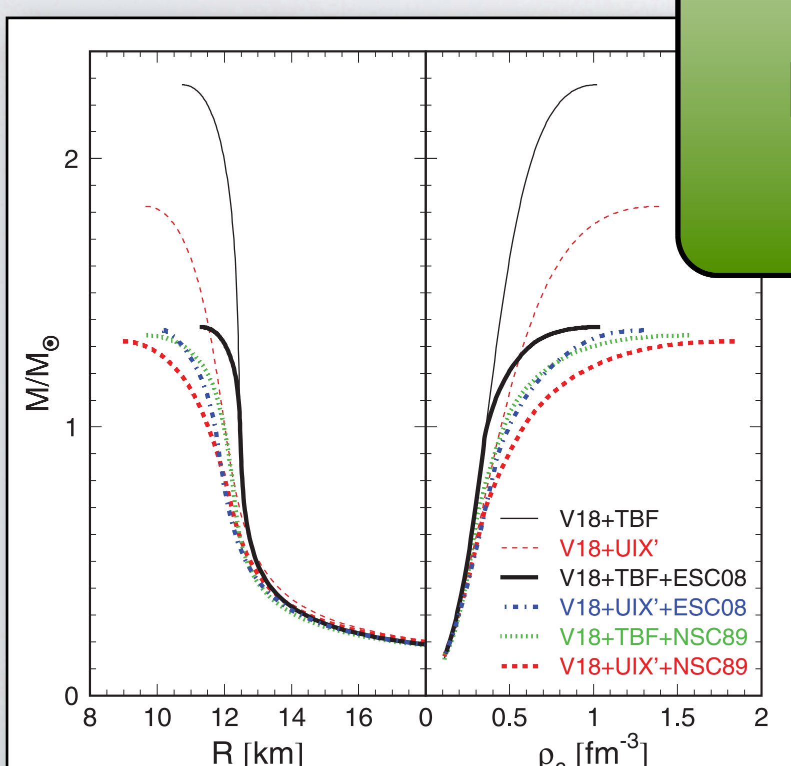
NS obs: $2\ M_\odot$

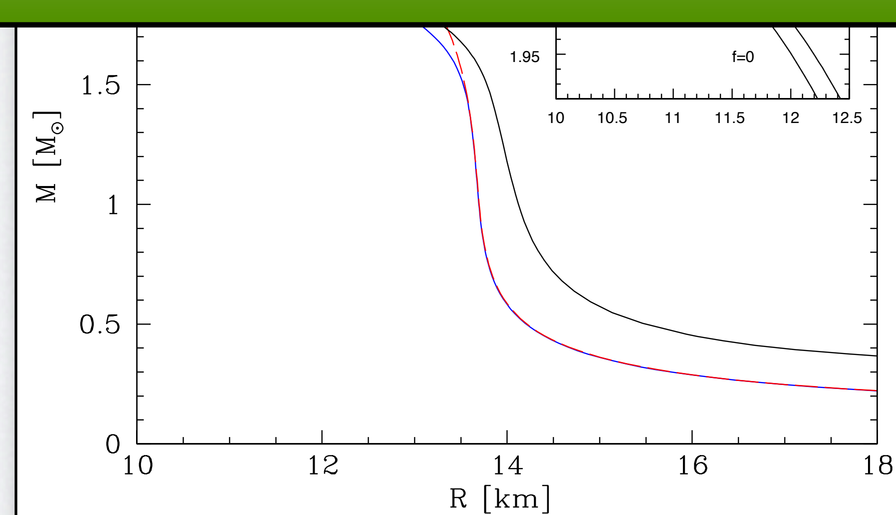
PSR J1614-2230: $M = 1.97(4)\ M_\odot$

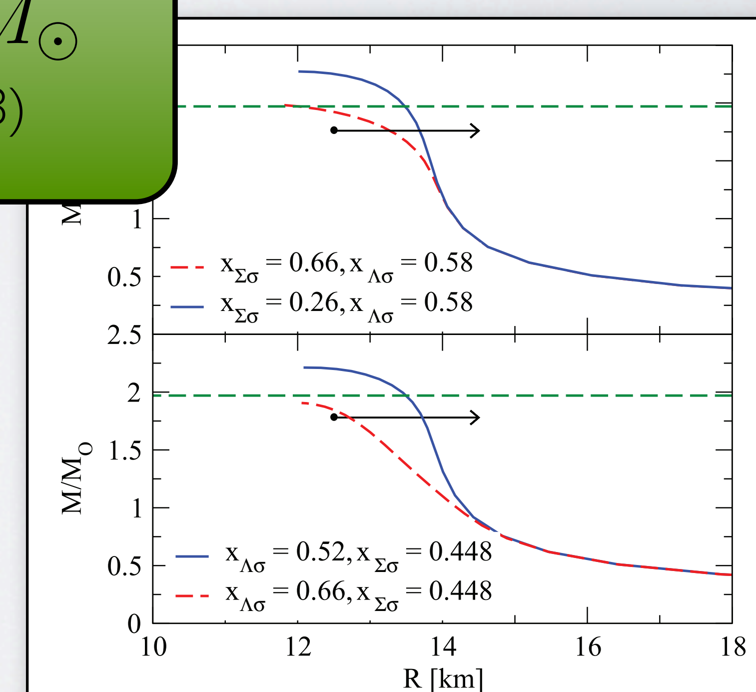
P. B. Demorest et al. , Nature 467, 1081 (2010)

PSR J0348+0432: $M = 2.01(4)\ M_\odot$

J. Antoniadis et al. , Science 340, 1233232 (2013)







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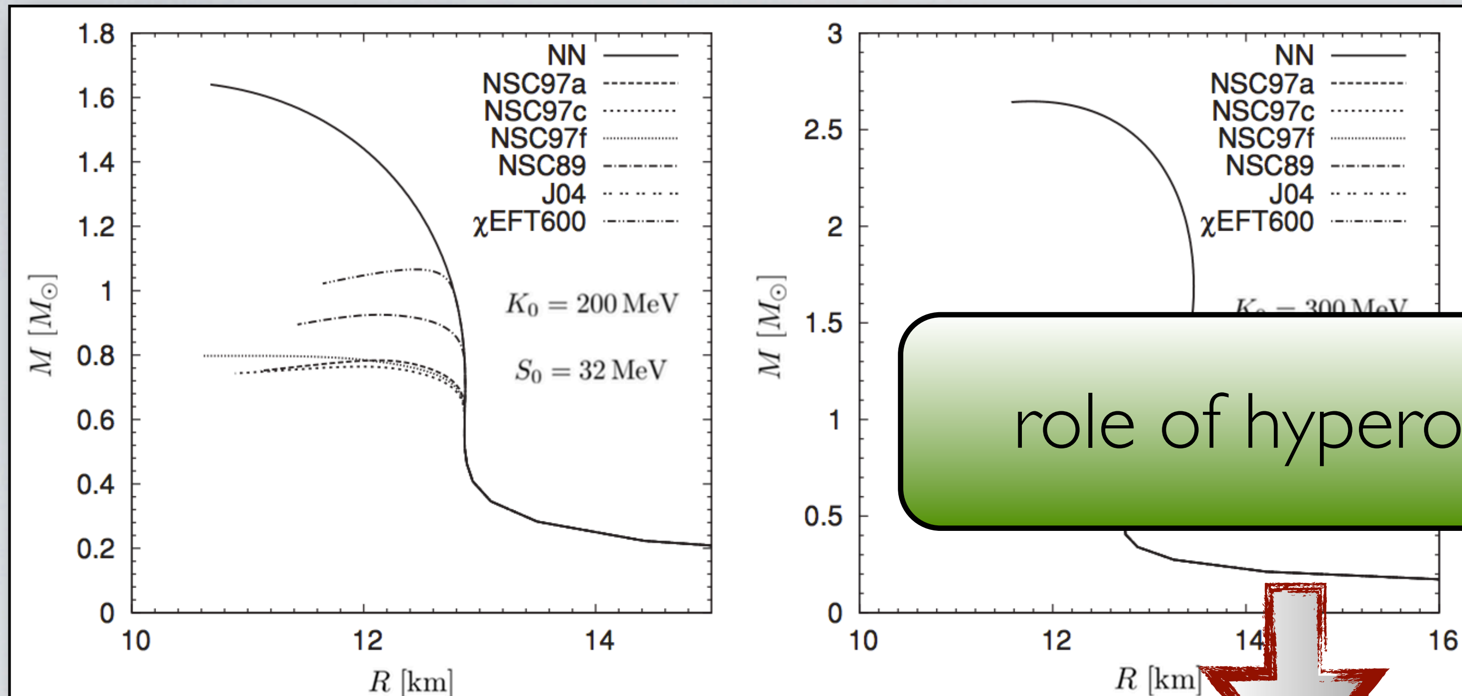
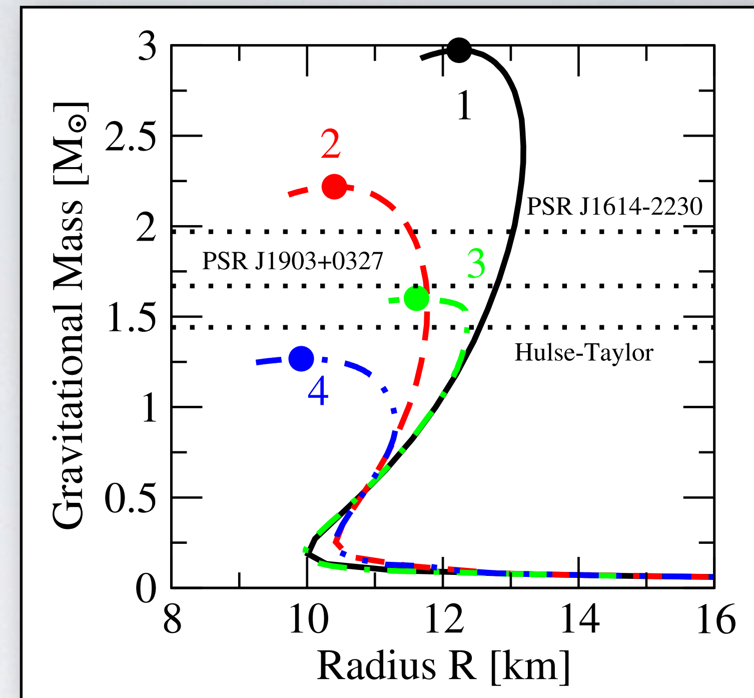
Strangeness in nuclear physics: neutron stars



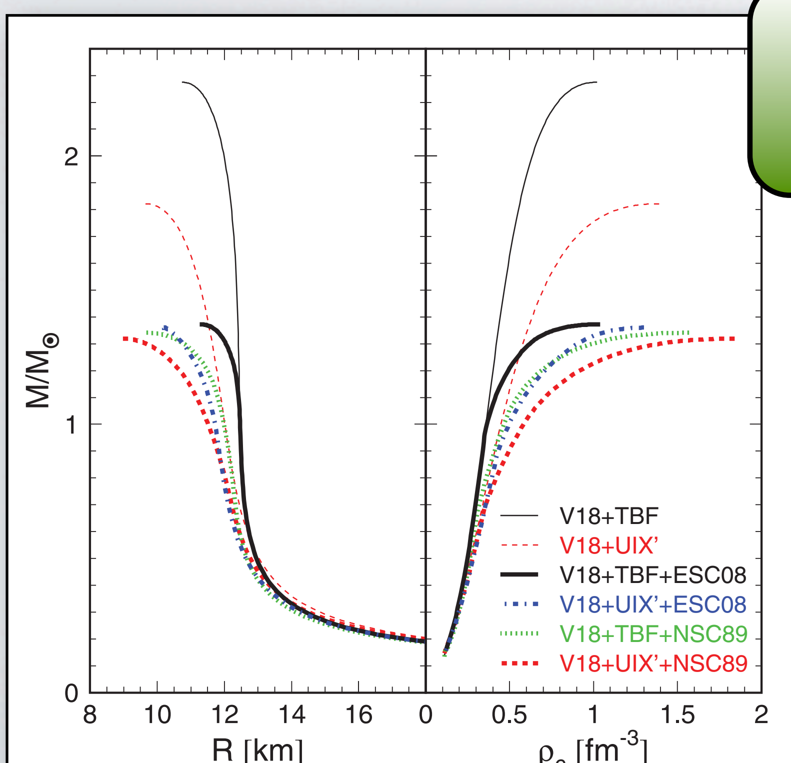
role of hyperons ?

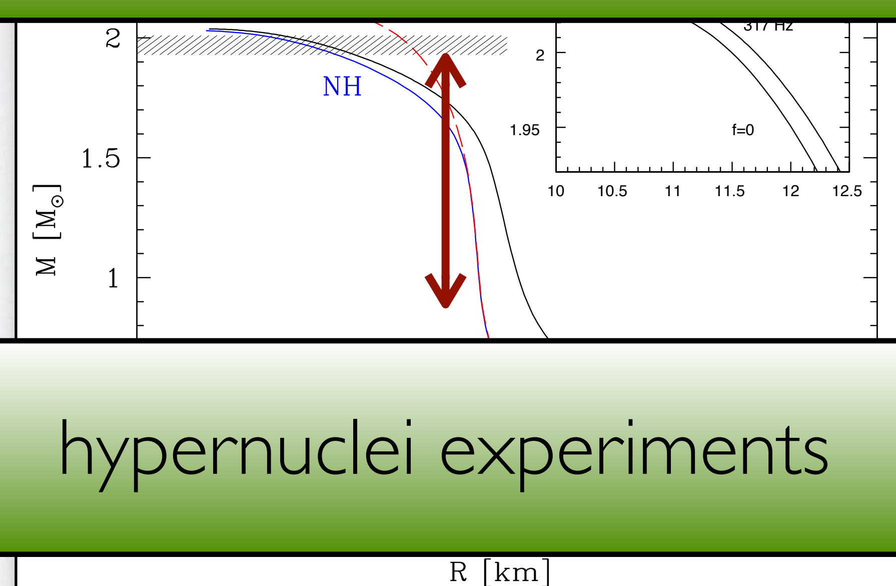
H. Đapo, B.-J. Schaefer, J. Wambach, Phys. Rev. C 81, 035803 (2010)

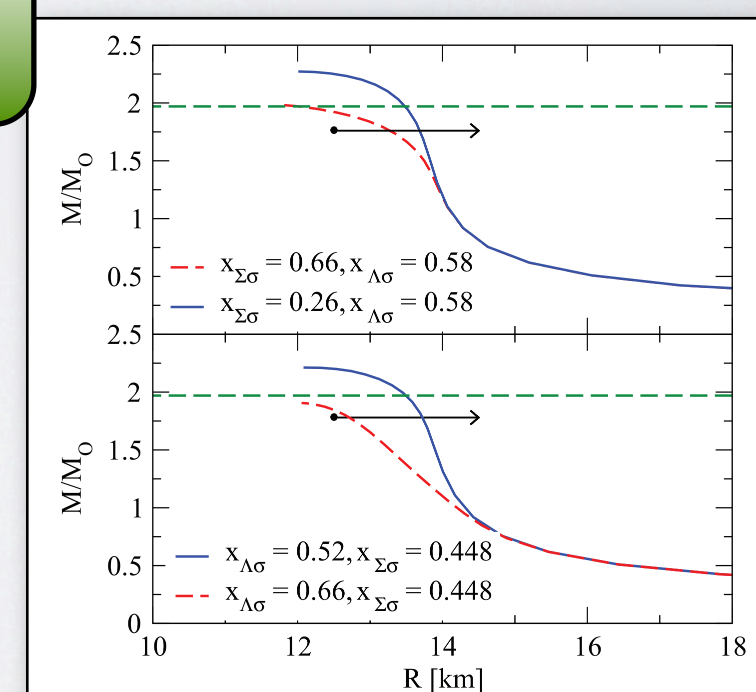


I. Vidaña, D. Logoteta, C. Providência, A. Polls, I. Bombaci, EPL 94, 11002 (2011)

hyperon-nucleon interaction







hypernuclei experiments

H.-J. Schulze, T. Rijken, Phys. Rev. C 84, 035801 (2011)

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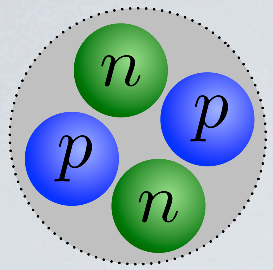
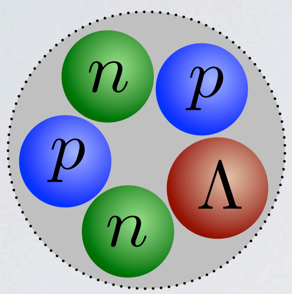
The strange AFDMC project: the idea



^{A-1}Z

nucleus $\longrightarrow$ $BE_{nuc} = \dfrac{\langle\psi_{nuc}|\mathcal{H}_{nuc}|\psi_{nuc}\rangle}{\langle\psi_{nuc}|\psi_{nuc}\rangle}$ $\rightarrow$ $\mathcal{H}_{NN}$

ab-initio method : AFDMC



$^{A}_{\Lambda}Z$

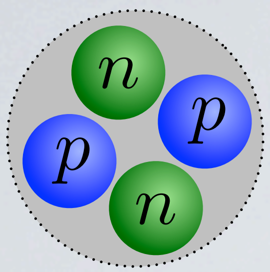
Λ-hypernucleus $\longrightarrow$ $BE_{hyp} = \dfrac{\langle\psi_{hyp}|\mathcal{H}_{hyp}|\psi_{hyp}\rangle}{\langle\psi_{hyp}|\psi_{hyp}\rangle}$ $\rightarrow$ $\mathcal{H}_{NN}+\mathcal{H}_{\Lambda N(N)}$

$$B_\Lambda = BE_{nuc} - BE_{hyp}$$

Hyp. : nuclear effects cancel at most

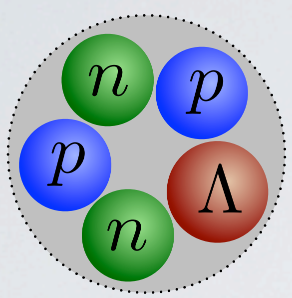
information about the hyperon-nucleon interaction

The strange AFDMC project: the idea



^{A-1}Z

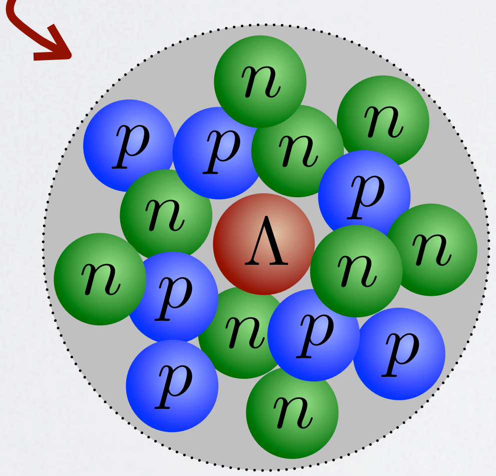
nucleus



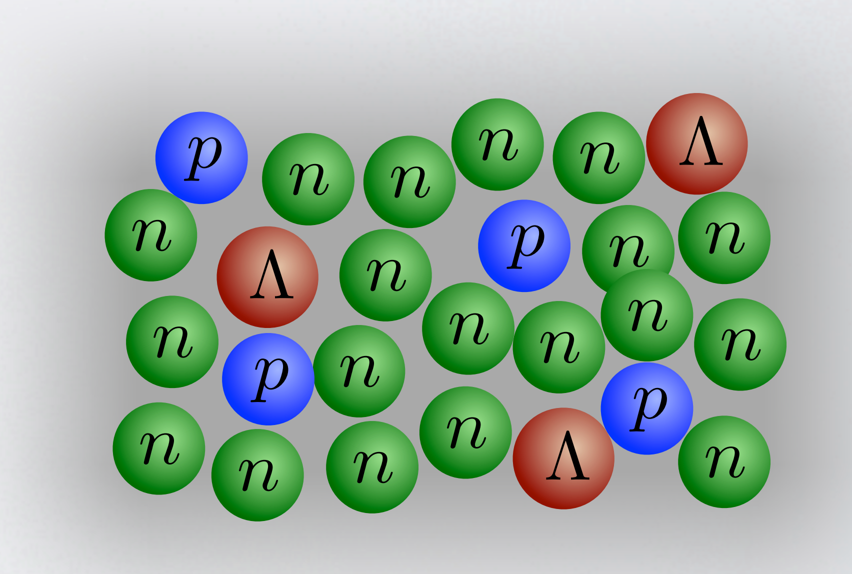
$^{A}_{\Lambda}Z$

Λ-hypernucleus

comparison with experimental data in a wide mass range



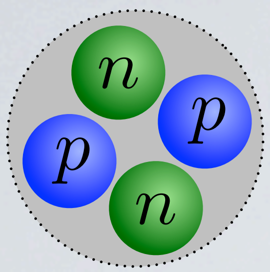
heavy Λ-hypernuclei



Λ-nuclear matter

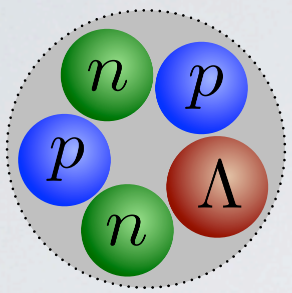
extrapolation

The strange AFDMC project: the idea



^{A-1}Z

nucleus



$^{A}_{\Lambda}Z$

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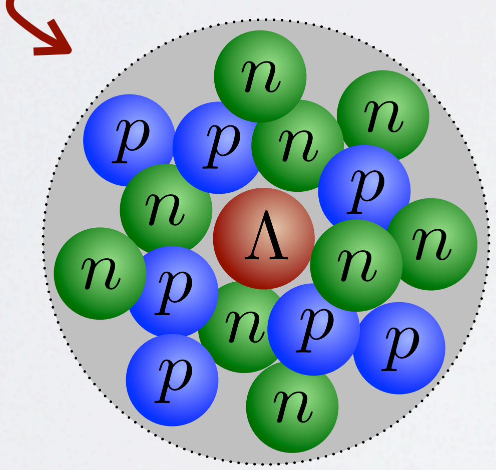

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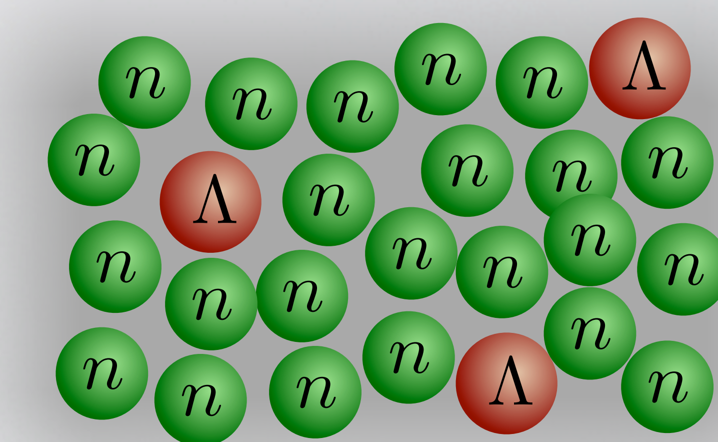


Λ-~~nuclear~~ matter

Λ-neutron matter

The strange AFDMC project: the hyperon-nucleon interaction

✓ Phenomenologic

- phenomenological nature
- 2-body terms
- shell & cluster model calculations

D.J. Millener, Nucl. Phys. A 1 (2013), in press

E. Hiyama, Nucl. Phys. A 1 (2013), in press

✓ Nijmegen & Jülich

- one meson exchange model
- 2-body terms, several parametrizations
- G-matrix calculations

I. Vidaña, A. Polls, A. Ramos, M. Hjorth-Jensen, Nucl. Phys. A 644, 201 (1998)

✓ χ-EFT

- derived from chiral EFT (NLO)
- 2-body terms (@ NLO)
- Faddeev-Yakubovsky calculations

A. Nogga, Nucl. Phys. A 1 (2013), in press

✓ Usmani & co.

- diagrammatic contributions, due to pion exchange
- 2-body and 3-body terms
- variational calculations

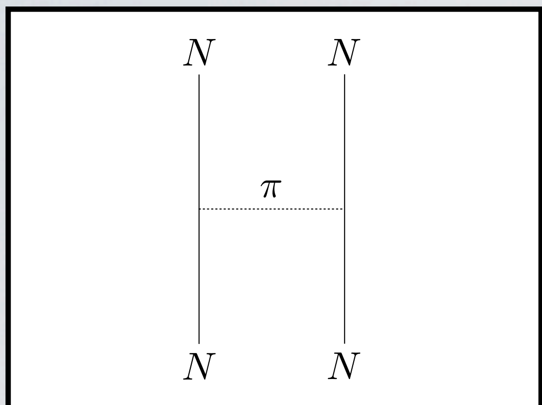
A. A. Usmani, F. C. Khanna, J. Phys. G: Nucl. Part. Phys. 35, 025105 (2008)

AV18+UIX like

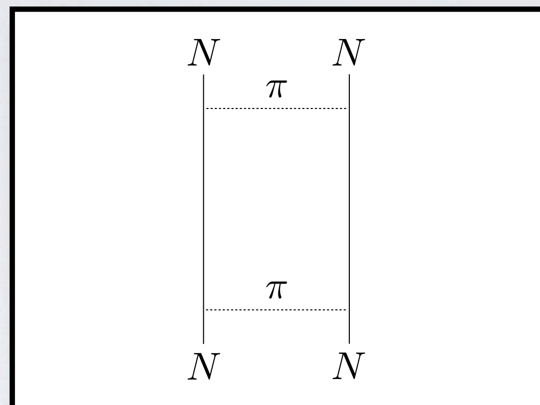
The strange AFDMC project: the hyperon-nucleon interaction

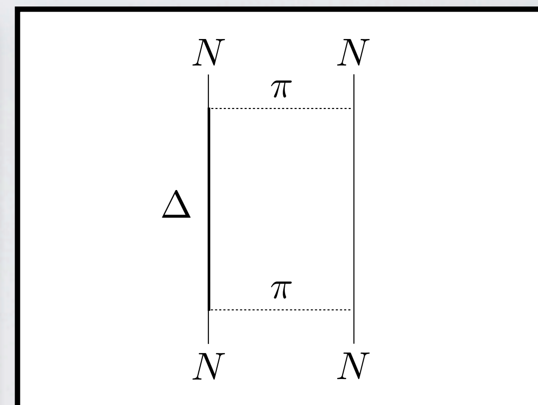
2-body

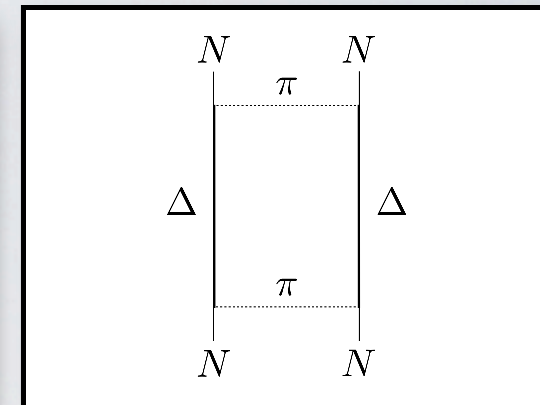
1π



2π



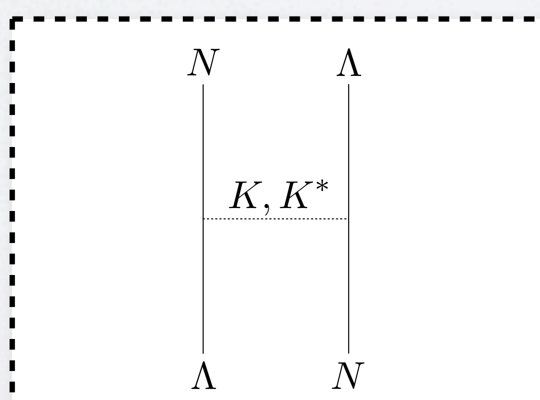


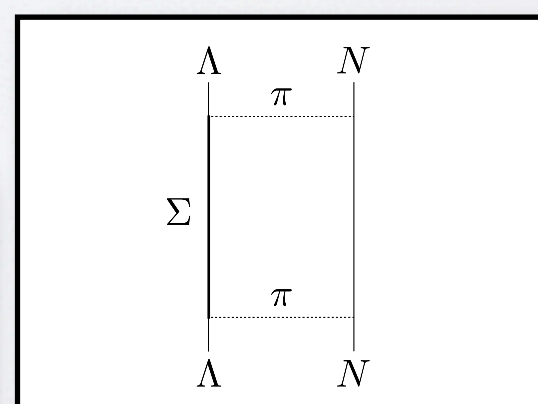


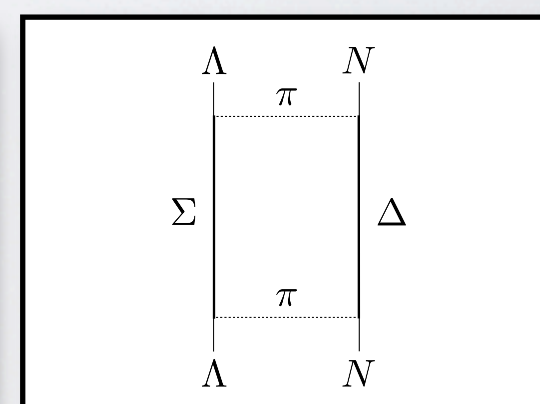
$\Lambda\pi\Sigma$ vertex

forbidden

1 meson





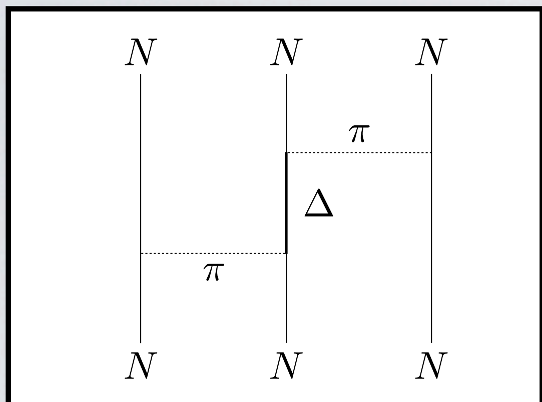


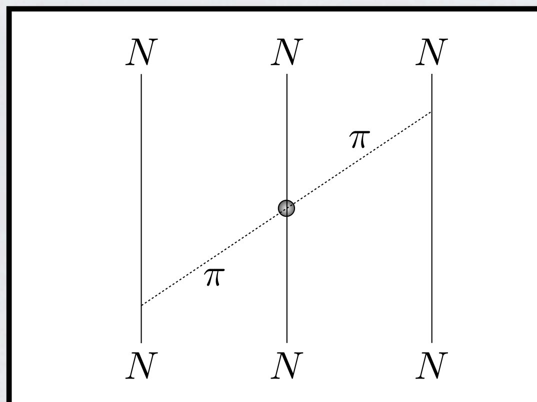
CSB $(A = 4)$

The strange AFDMC project: the hyperon-nucleon interaction

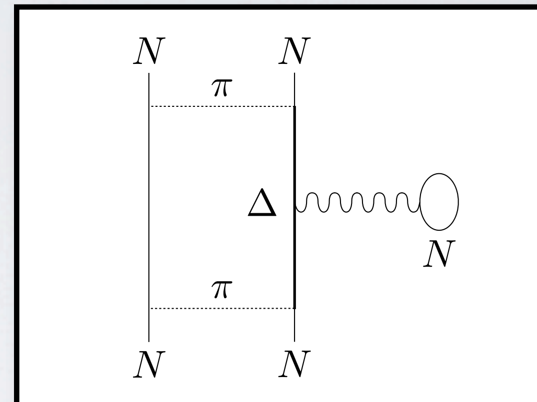
3-body

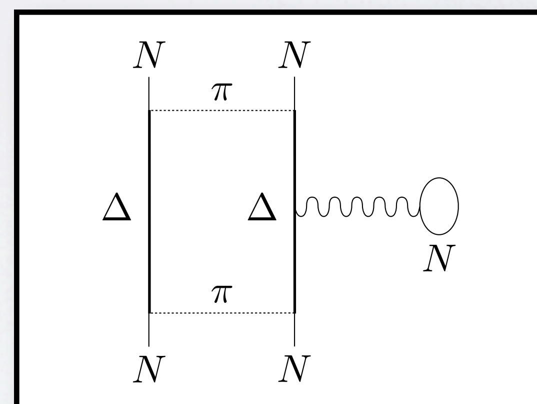
2π



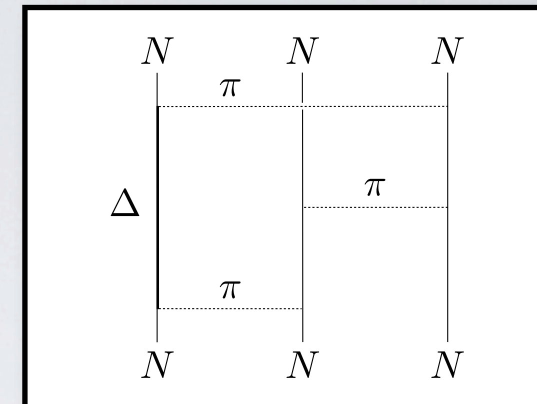


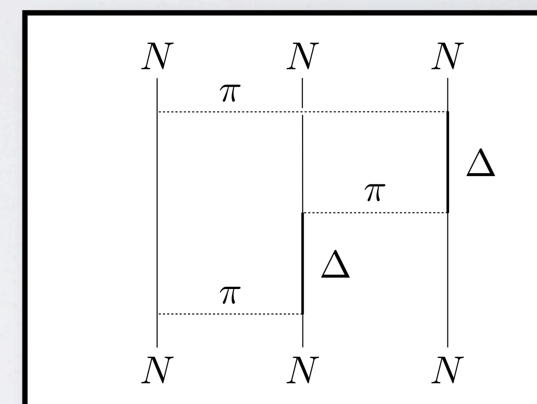
dispersive

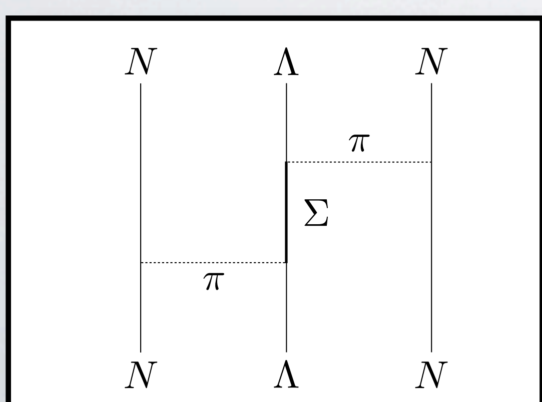


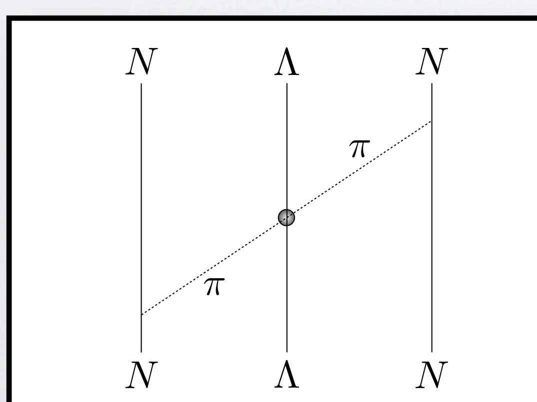


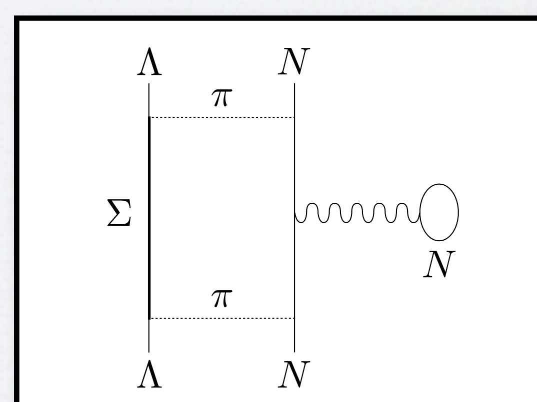
3π











forbidden

The strange AFDMC project: the hyperon-nucleon interaction

✓ 2-body ΛN interaction

- charge symmetric term: $v_{\Lambda i} = v_0(r) + v_0(r)\varepsilon(\mathcal{P}_x - 1) + \frac{1}{4} v_\sigma T_\pi^2(r) \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$

$$\begin{cases} v_0(r) = v_c(r) - v_{2\pi}(r) \\ v_c(r) = W_c \left[1 + \mathrm{e}^{\frac{r-\bar{r}}{a}}\right]^{-1} \\ v_{2\pi}(r) = \bar{v}\, T_\pi^2(r) \end{cases} \qquad \begin{cases} v_\sigma = v_s - v_t \\ \bar{v} = \frac{1}{4}(v_s + 3v_t) \end{cases}$$

parameters fitted on Λp scattering data

- charge symmetry breaking term: $v_{\Lambda i}^{\mathrm{CSB}} = \tau_i^3\, v_0^{\mathrm{CSB}}\, T_\pi(r)$

parameter fitted on $A = 4$ mirror hypernuclei

The strange AFDMC project: the hyperon-nucleon interaction

✓ 3-body ΛNN interaction

$$v_{\Lambda ij} = v_{\Lambda ij}^{2\pi} + v_{\Lambda ij}^{D} = v_{\Lambda ij}^{PW} + v_{\Lambda ij}^{SW} + v_{\Lambda ij}^{D}$$

$$\begin{cases} v_{\Lambda ij}^{PW} & = & -\frac{1}{6} C^P \{X_{i\Lambda}, X_{\Lambda j}\} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\Lambda ij}^{SW} & = & C^S Z_\pi(r_{\Lambda i}) Z_\pi(r_{\Lambda j}) (\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{i\Lambda} \, \boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{j\Lambda}) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \\ v_{\Lambda ij}^{D} & = & W^D T_\pi^2(r_{\Lambda i}) T_\pi^2(r_{\Lambda j}) \left[1 + \frac{1}{6} \boldsymbol{\sigma}_\Lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\right] \end{cases}$$

parameters not yet fixed

↓

fitting of the parameters to reproduce experimental separation energies

The strange AFDMC project: the AFDMC code

✓ Diffusion Monte Carlo

$$-\frac{\partial}{\partial\tau}\psi(\boldsymbol{R},\boldsymbol{S},\tau)=\mathcal{H}\psi(\boldsymbol{R},\boldsymbol{S},\tau) \quad\longrightarrow\quad \psi(\boldsymbol{R},\boldsymbol{S},\tau+d\tau)=\mathrm{e}^{-\mathcal{H}d\tau}\,\psi(\boldsymbol{R},\boldsymbol{S},\tau)$$

$$(\tau=it/\hbar)$$

$$\Big\downarrow\ \tau\to\infty$$

ground state

$$\psi\left(\boldsymbol{R},\boldsymbol{S},\tau+d\tau\right)=\int\langle\boldsymbol{S}\boldsymbol{R}|\,\mathrm{e}^{-(\mathcal{H}-E_0)d\tau}\,|\boldsymbol{R}'\boldsymbol{S}'\rangle\,\langle\boldsymbol{S}'\boldsymbol{R}'|\psi(\tau)\rangle\,d\boldsymbol{R}'d\boldsymbol{S}'$$

walkers

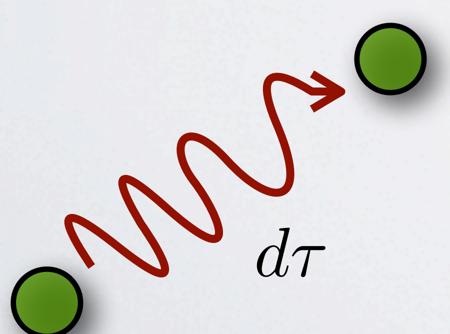
$$D=\hbar^2/2m$$

diffusion

$$(4\pi D d\tau)^{\frac{3A}{2}}\,\mathrm{e}^{-\frac{(\boldsymbol{R}-\boldsymbol{R}')^2}{4Dd\tau}}$$

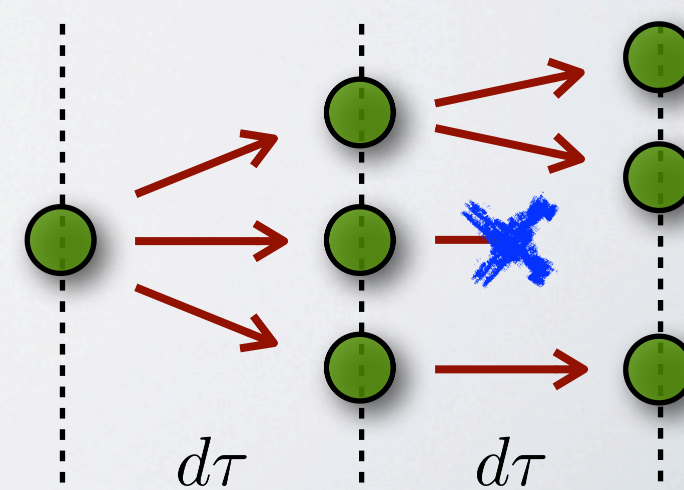
branching

$$\mathrm{e}^{-\left(\frac{\hat{V}(\boldsymbol{R}')+\hat{V}(\boldsymbol{R})}{2}-E_0\right)d\tau}$$



kinetic term

potential term



The strange AFDMC project: the AFDMC code

✓ Auxiliary Field

$$\mathcal{P} \sim \mathrm{e}^{-\frac{1}{2}\lambda d\tau\mathcal{O}^2} \quad \Rightarrow \quad \psi \sim \frac{A!}{Z!(A-Z)!}2^A \text{ terms}$$

GFMC: $A \leq 12$

Idea: Hubbard-Stratonovich transformation

$$\mathrm{e}^{-\frac{1}{2}\lambda d\tau\mathcal{O}^2} = \frac{1}{\sqrt{2\pi}}\int dx\ \mathrm{e}^{-\frac{x^2}{2}+\sqrt{-\lambda d\tau}x\mathcal{O}}$$

auxiliary field

rotation over spin-isospin configurations

potential matrices diagonalization

computational cost $\sim A^3$

The strange AFDMC project: the AFDMC code

- ✓ AFDMC matrices: nuclear systems

$$V_{NN}(\mathrm{AV6}) = \sum_{i<j}\sum_{p=1}^{6} v_p(r)\mathcal{O}_{ij}^{p} \qquad \mathcal{O}_{ij}^{p=1,6} = \{1, \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j, S_{ij}\}\otimes\{1, \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\}$$

$$\begin{aligned} V_{NN}^{sd} &= \frac{1}{2}\sum_{i\neq j}\sum_{\gamma}\tau_i^{\gamma}\left(\mathcal{A}_{ij}^{[\tau]}\right)\tau_j^{\gamma} && A\times A \\ &+ \frac{1}{2}\sum_{i\neq j}\sum_{\alpha\beta}\sigma_i^{\alpha}\left(\mathcal{A}_{i\alpha,j\beta}^{[\sigma]}\right)\sigma_j^{\beta} && 3A\times 3A \\ &+ \frac{1}{2}\sum_{i\neq j}\sum_{\alpha\beta\gamma}\tau_i^{\gamma}\sigma_i^{\alpha}\left(\mathcal{A}_{i\alpha,j\beta}^{[\sigma\tau]}\right)\sigma_j^{\beta}\tau_j^{\gamma} && 3A\times 3A \end{aligned}$$

Ex: $\sum_{j\beta}\mathcal{A}_{i\alpha,j\beta}^{[\sigma]}\,\psi_{n,j\beta}^{[\sigma]} = \lambda_n^{[\sigma]}\,\psi_{n,i\alpha}^{[\sigma]}$ $\Rightarrow$ $\tilde{\mathcal{O}}_n^{[\sigma]} = \sum_{j\beta}\sigma_j^{\beta}\psi_{n,j\beta}^{[\sigma]}$

The strange AFDMC project: the AFDMC code

- ✓ AFDMC matrices: hyper-nuclear systems

$$V_{\Lambda N} = \cancel{\sum_i \left(\mathcal{B}_{(\Lambda)i}^{[\hat{P}_x]} \right)} \hat{P}_{x_{\Lambda i}} + \sum_i \left(\mathcal{B}_i^{[CSB]} \right) \tau_i^{\,z}$$

$$+ \sum_i \sum_\alpha \sigma_\Lambda^\alpha \left(\mathcal{B}_{(\Lambda)i}^{[\sigma]} \right) \sigma_i^\alpha \qquad 1 \times (A-1)$$

$$V_{\Lambda NN}^{D} = \frac{1}{2} \sum_{i \neq j} \sum_\alpha \sigma_\Lambda^\alpha \left(\mathcal{C}_{(\Lambda j)i}^{[\sigma]} \right) \sigma_i^\alpha \qquad 1 \times (A-1)$$

$$V_{\Lambda NN}^{2\pi} = \frac{1}{2} \sum_{i \neq j} \sum_{\alpha\beta\gamma} \tau_i^{\,\gamma} \sigma_i^\alpha \left(\mathcal{C}_{i\alpha, j\beta}^{[\sigma\tau]} \right) \sigma_j^{\,\beta} \tau_j^{\,\gamma} \qquad 3(A-1) \times 3(A-1)$$

good for Hubbard-Stratonovich

The strange AFDMC project: the AFDMC code

- Wave functions

$$\Psi_T(R,S)=\Psi_T^N(R_N,S_N)\otimes\Psi_T^\Lambda(R_\Lambda,S_\Lambda)$$

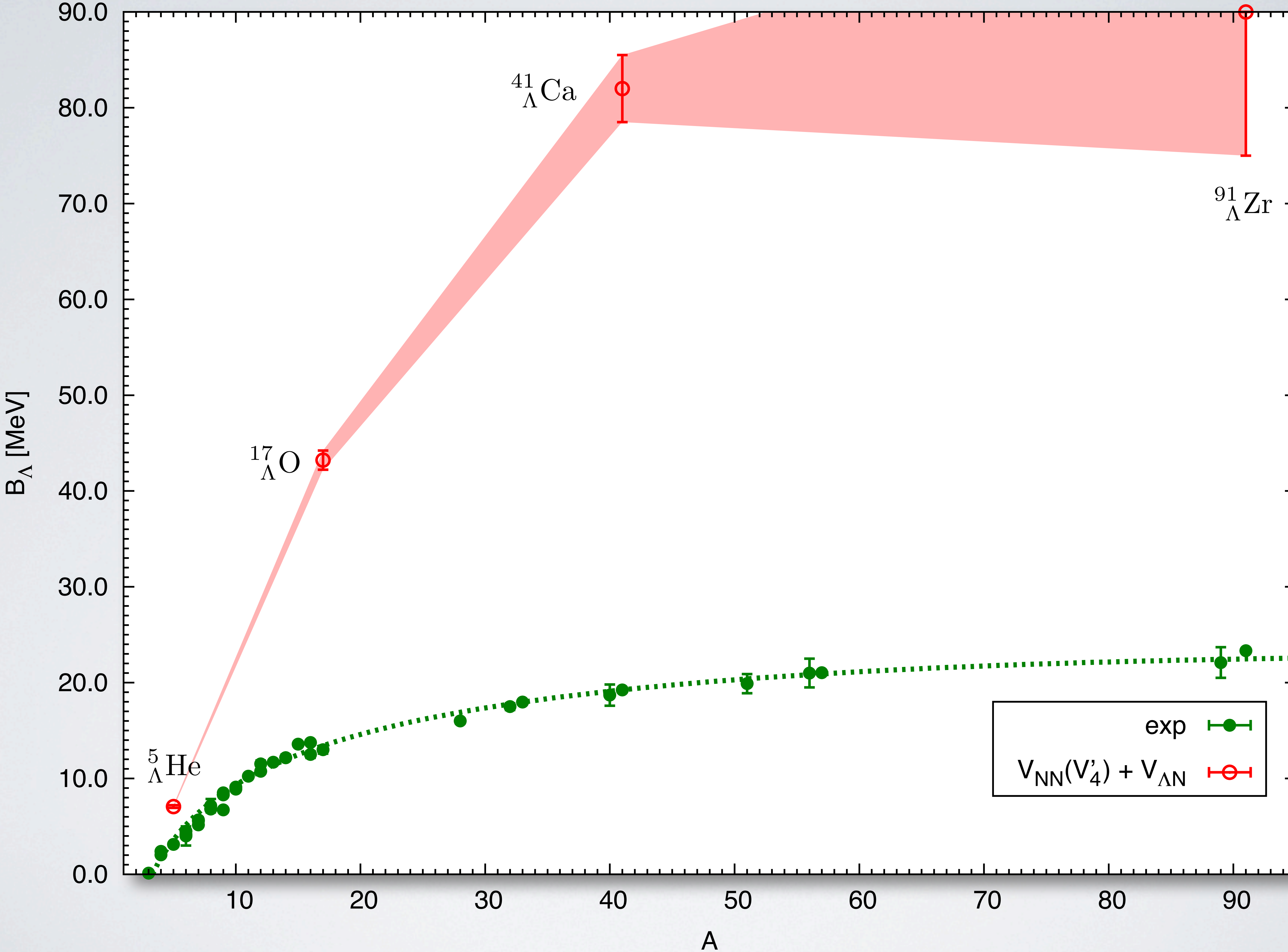
$$\Psi_T^p\left(R_p,S_p\right)=\left[\prod_{i<j}f_{ij}^c\right]_p\mathfrak{A}\left[\prod_i\phi_i\left(\vec{r}_i-\vec{R}_{\mathrm{CM}},\vec{s}_i\right)\right]_p$$

HF s.p. orbitals   plane waves

$$\vec{s}_i^{\,N}=\begin{pmatrix}a_i\\ b_i\\ c_i\\ d_i\end{pmatrix}=a_i|p\uparrow\rangle+b_i|p\downarrow\rangle+c_i|n\uparrow\rangle+d_i|n\downarrow\rangle$$

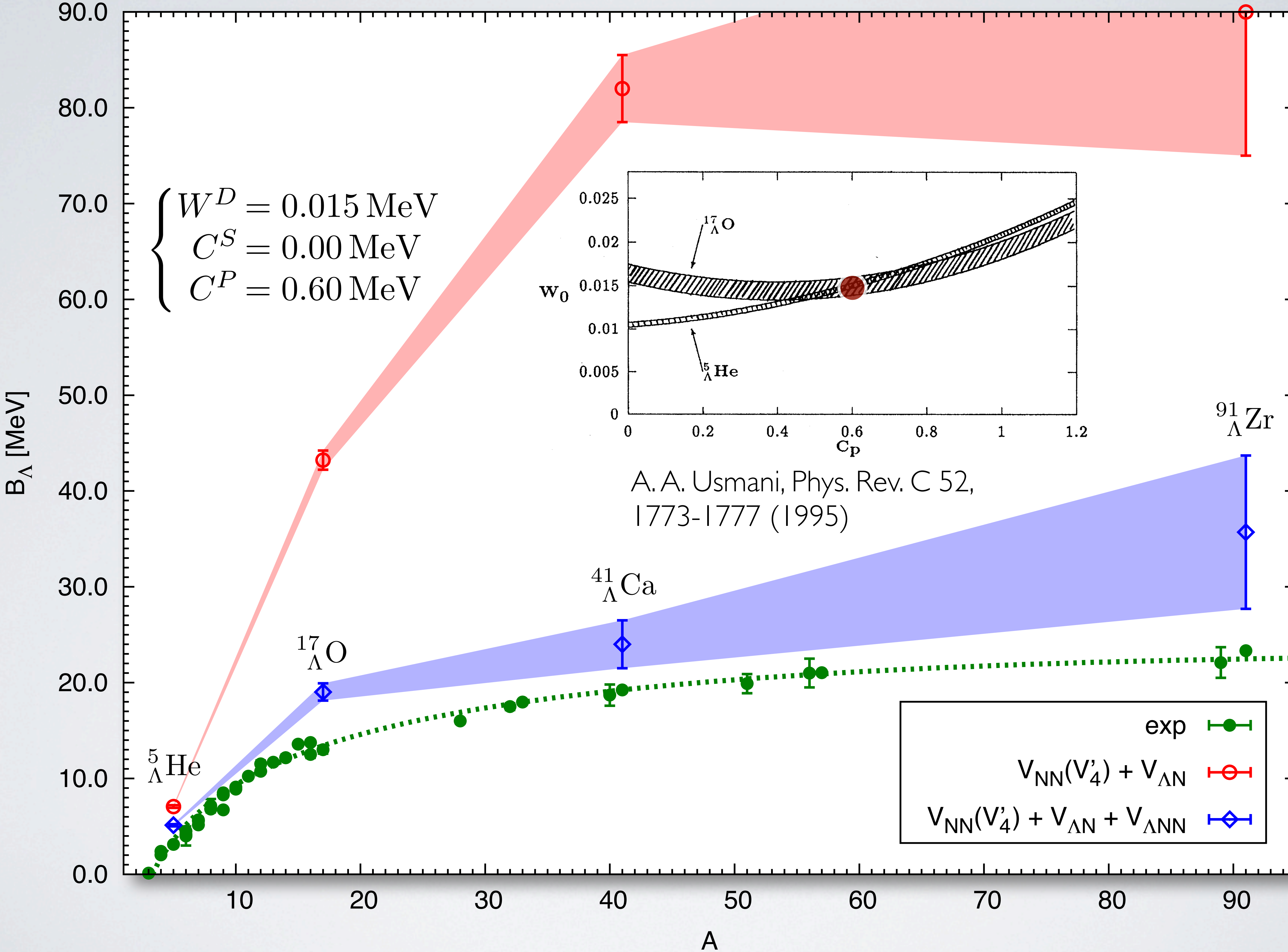
$$\vec{s}_i^{\,\Lambda}=\begin{pmatrix}u_i\\ v_i\end{pmatrix}=u_i|\uparrow\rangle+v_i|\downarrow\rangle$$

Results: Λ-hypernuclei



D. Lonardoni, S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

Results: Λ-hypernuclei



D. Lonardoni, S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

Results: Λ-hypernuclei

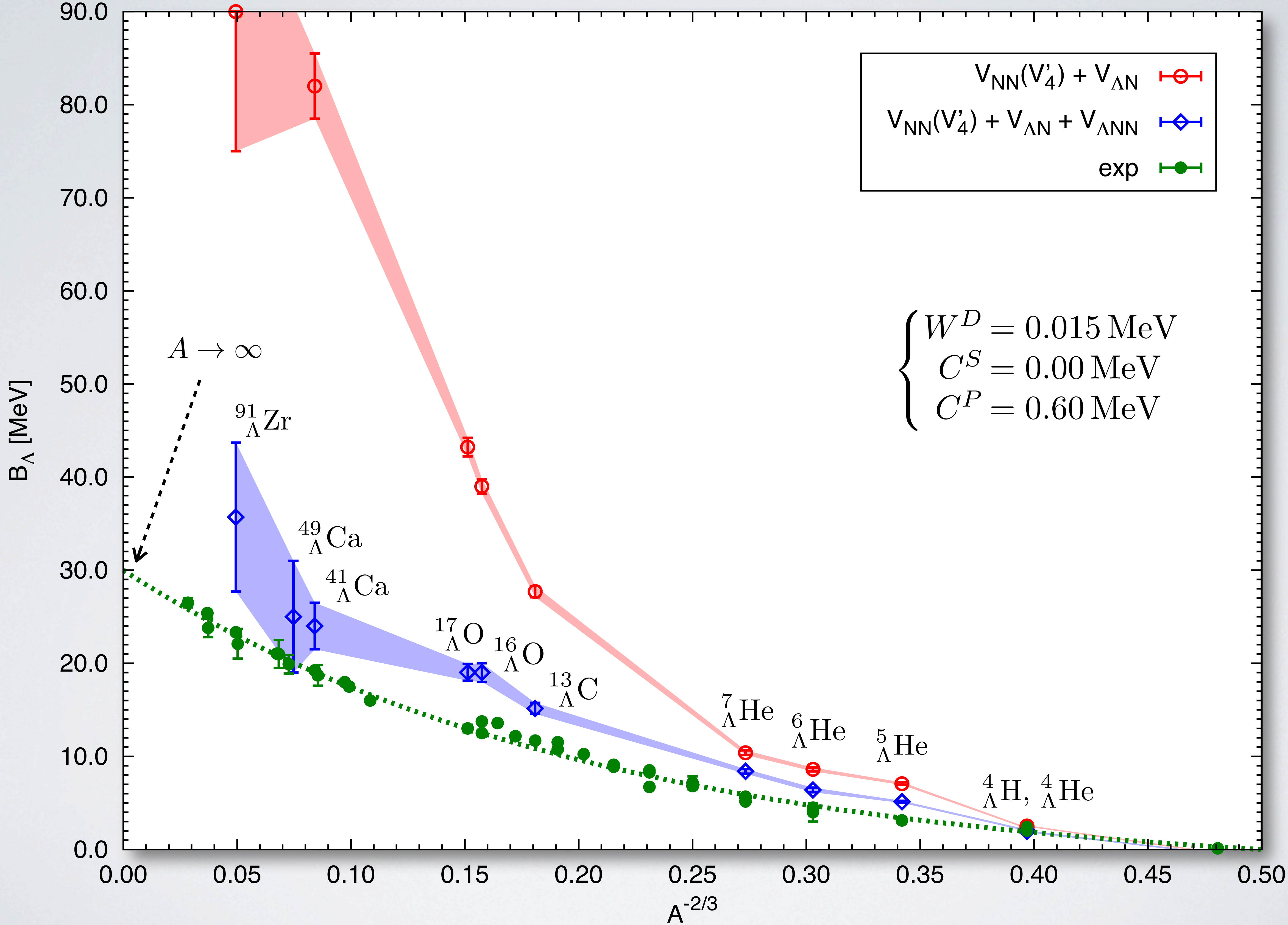
Hyp. : nuclear effects cancel at most ✓

NN potential	${}^{5}_{\Lambda}$He		${}^{17}_{\Lambda}$O	
	$V_{\Lambda N}$	$V_{\Lambda N} + V_{\Lambda NN}$	$V_{\Lambda N}$	$V_{\Lambda N} + V_{\Lambda NN}$
Argonne V4'	7.1(1)	5.1(1)	43(1)	19(1)
Argonne V6'	6.3(1)	5.2(1)	34(1)	21(1)
Minnesota	7.4(1)	5.2(1)	50(1)	17(2)
Expt.	3.12(2)		13.0(4)	

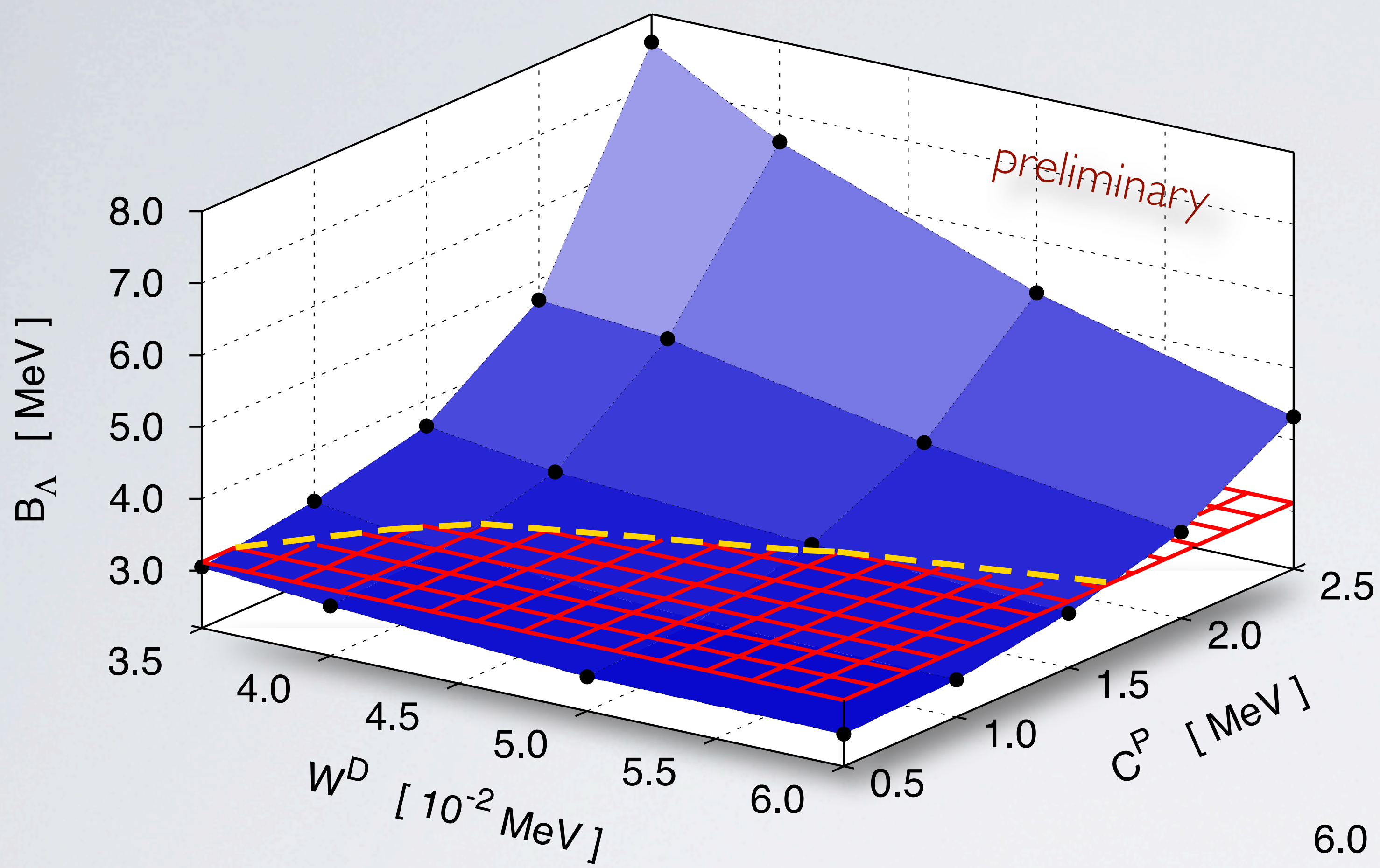
D. Lonardoni, S. Gandolfi, F. Pederiva, Phys. Rev. C 87, 041303(R) (2013)

hyperon separation energy not sensitive
to the details of nuclear interaction

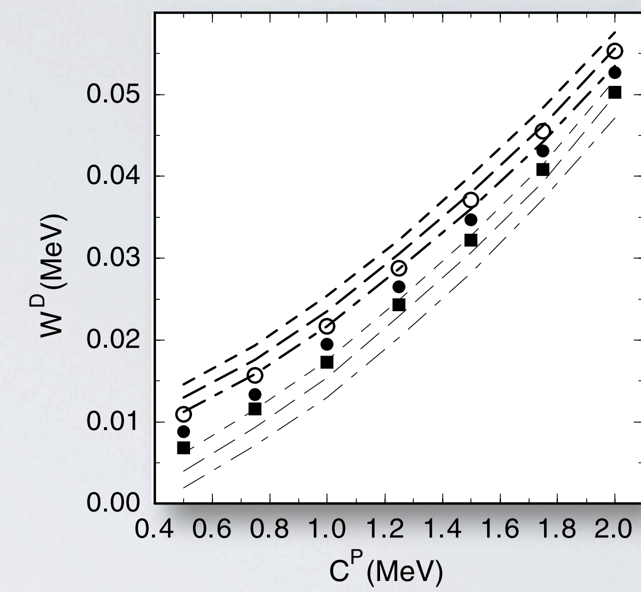
Results: Λ-hypernuclei



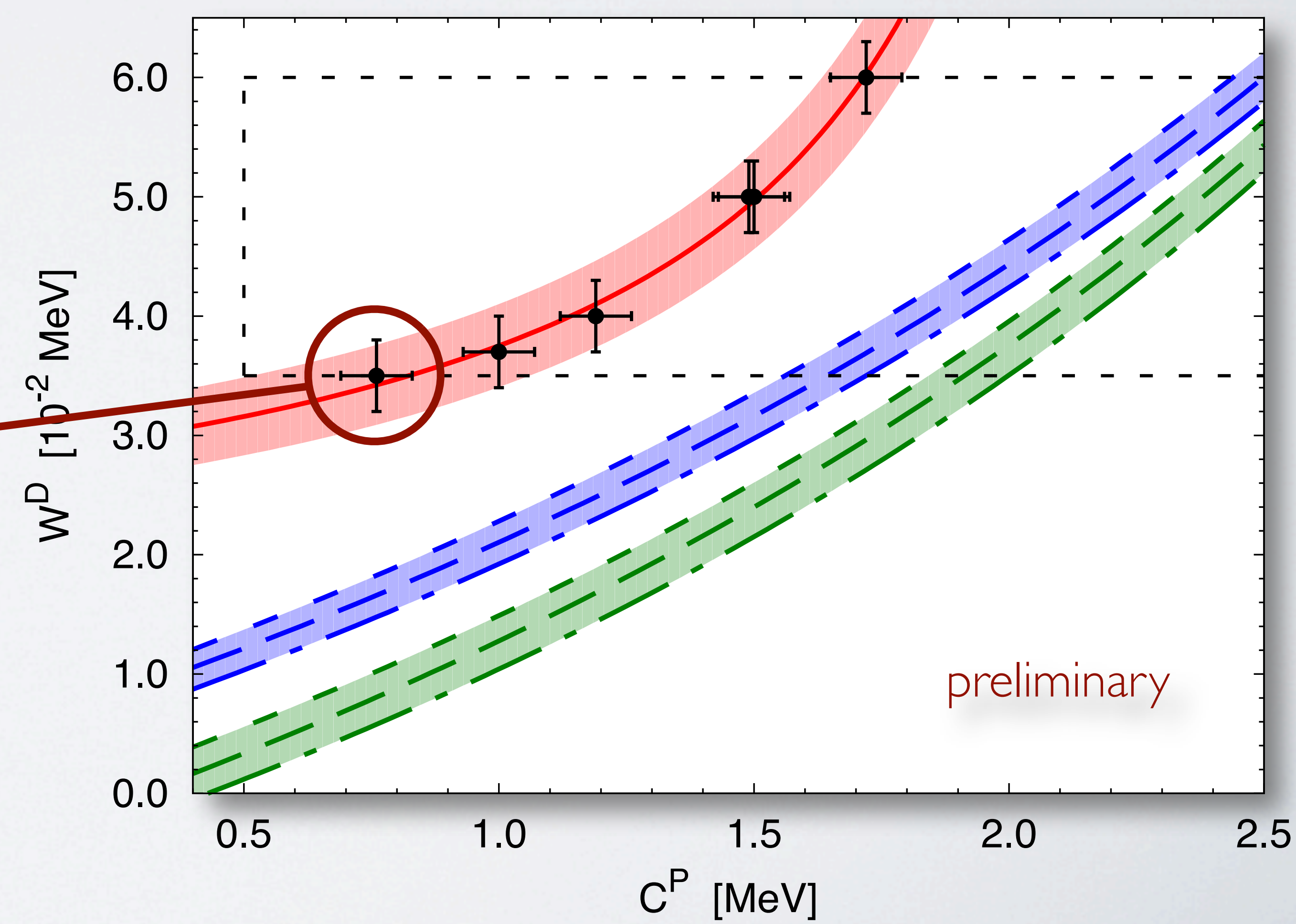
Results: Λ-hypernuclei



$^{5}_{\Lambda}\mathrm{He}$



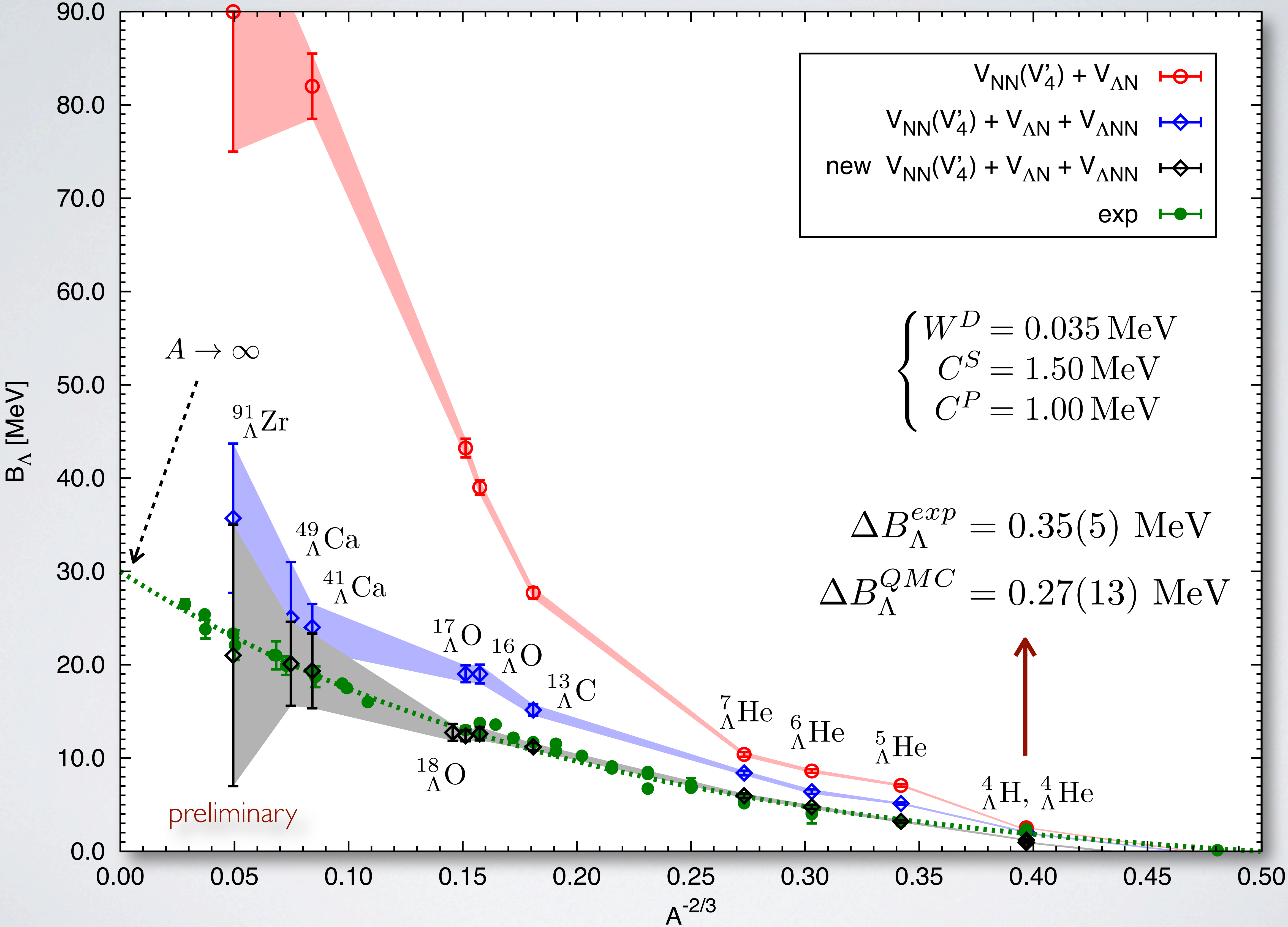
A. A. Usmani, F. C. Khanna,
J. Phys. G: Nucl. Part. Phys. 35, 025105 (2008)



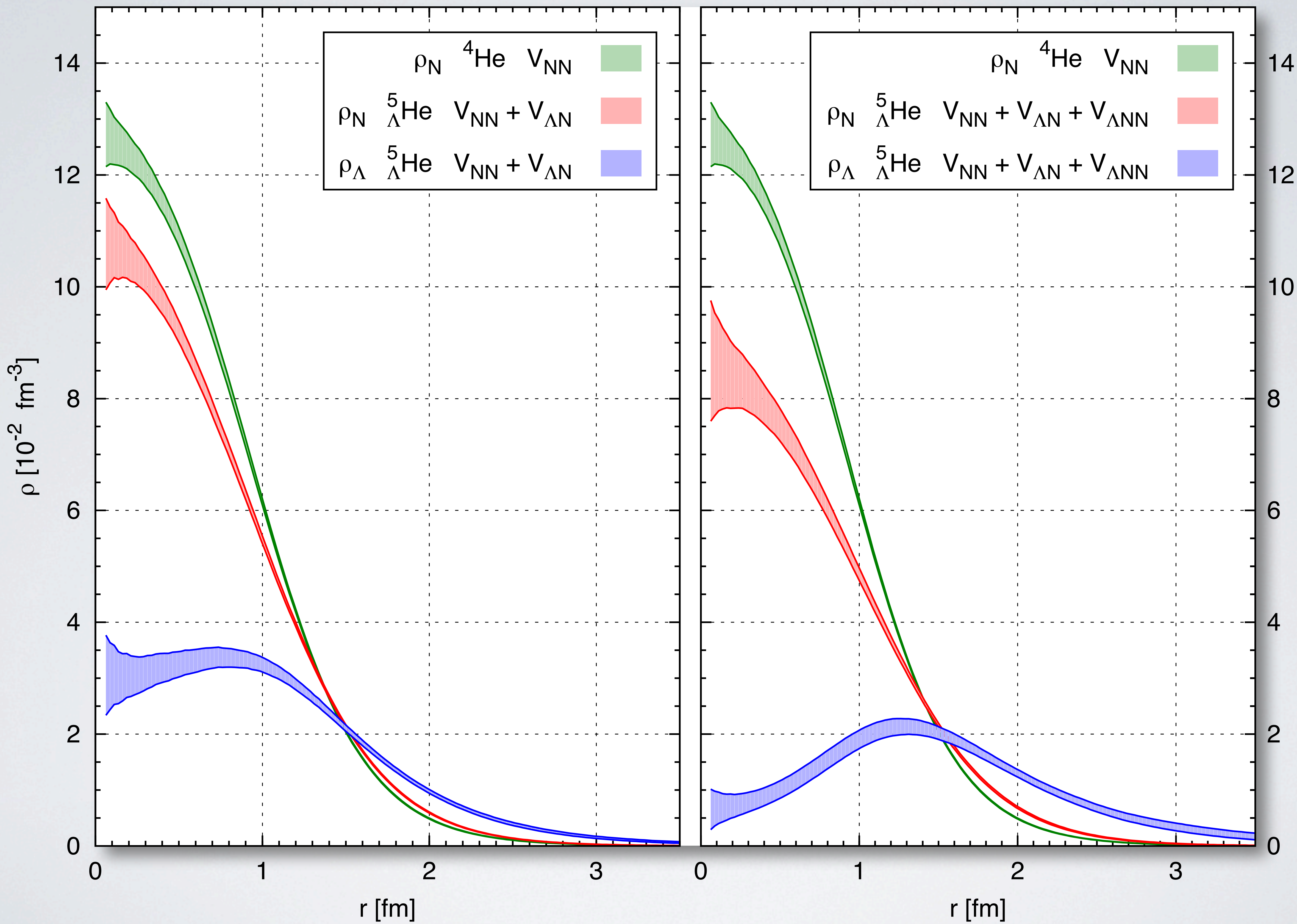
$$\begin{cases} W^D = 0.035\,\mathrm{MeV} \\ C^S = 1.50\,\mathrm{MeV} \\ C^P = 1.00\,\mathrm{MeV} \end{cases}$$

good for $^{5}_{\Lambda}\mathrm{He}$ and $^{17}_{\Lambda}\mathrm{O}$

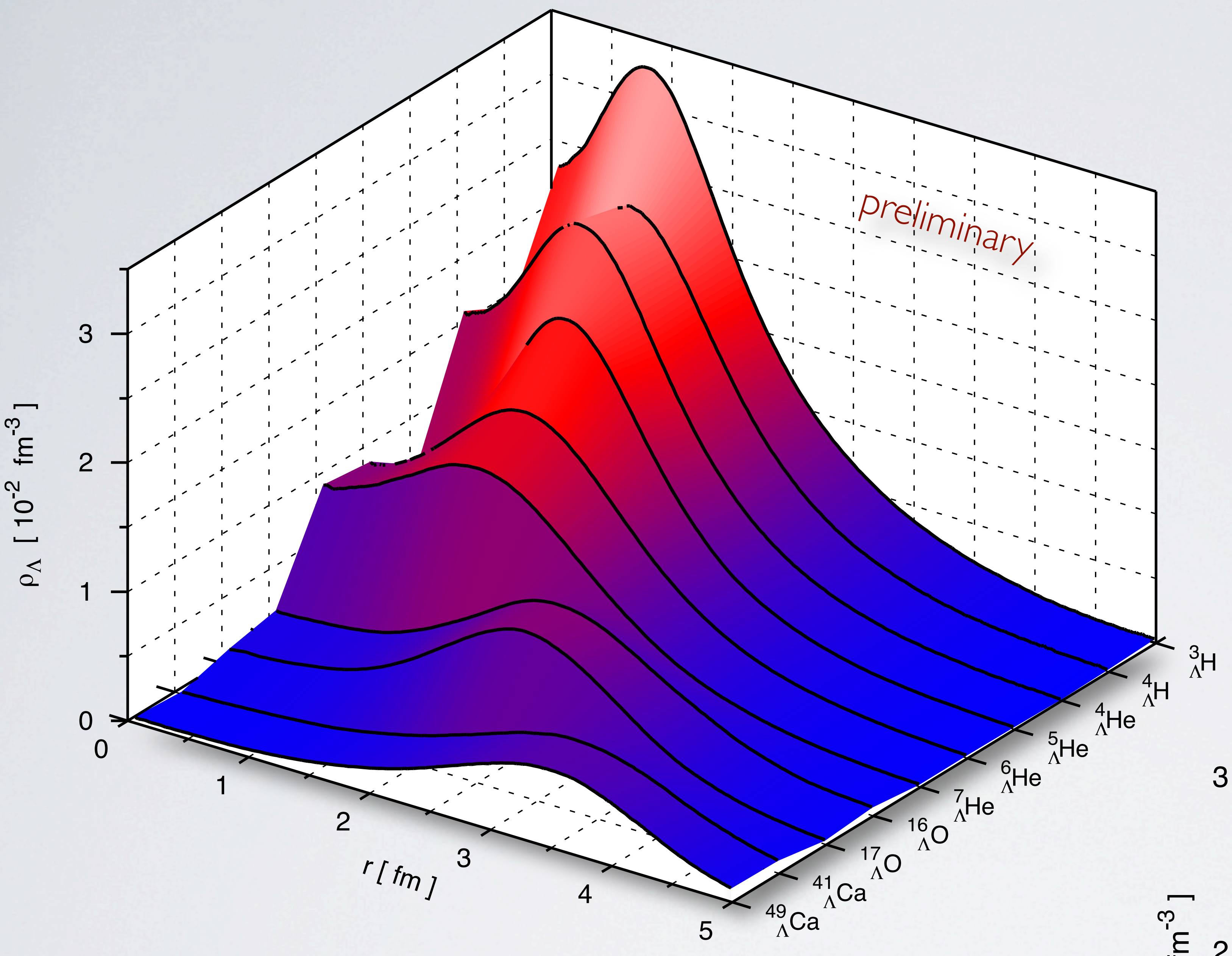
Results: Λ-hypernuclei



Results: Λ-hypernuclei

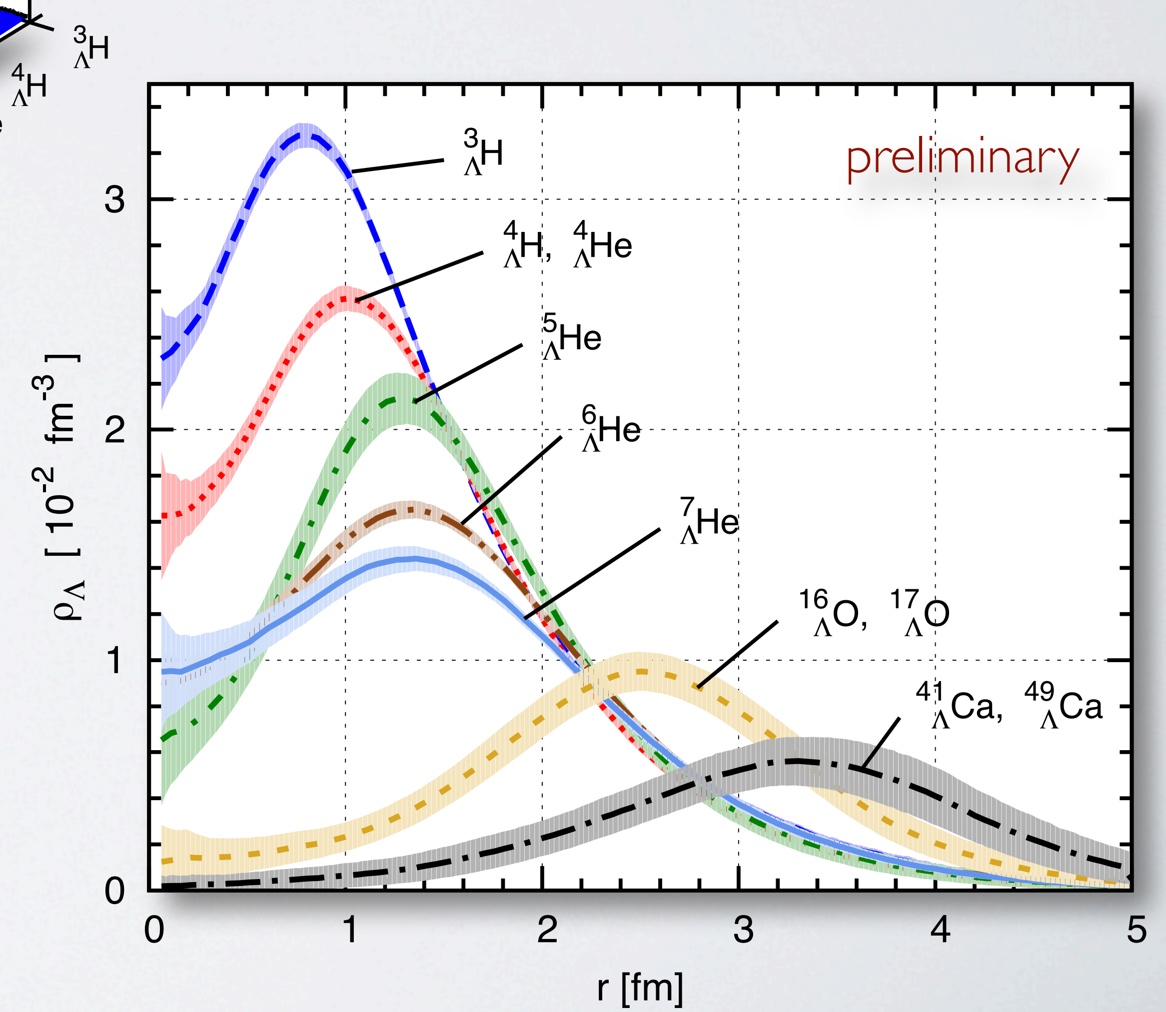


Results: Λ-hypernuclei



Λ single particle densities

significant 3-body effects



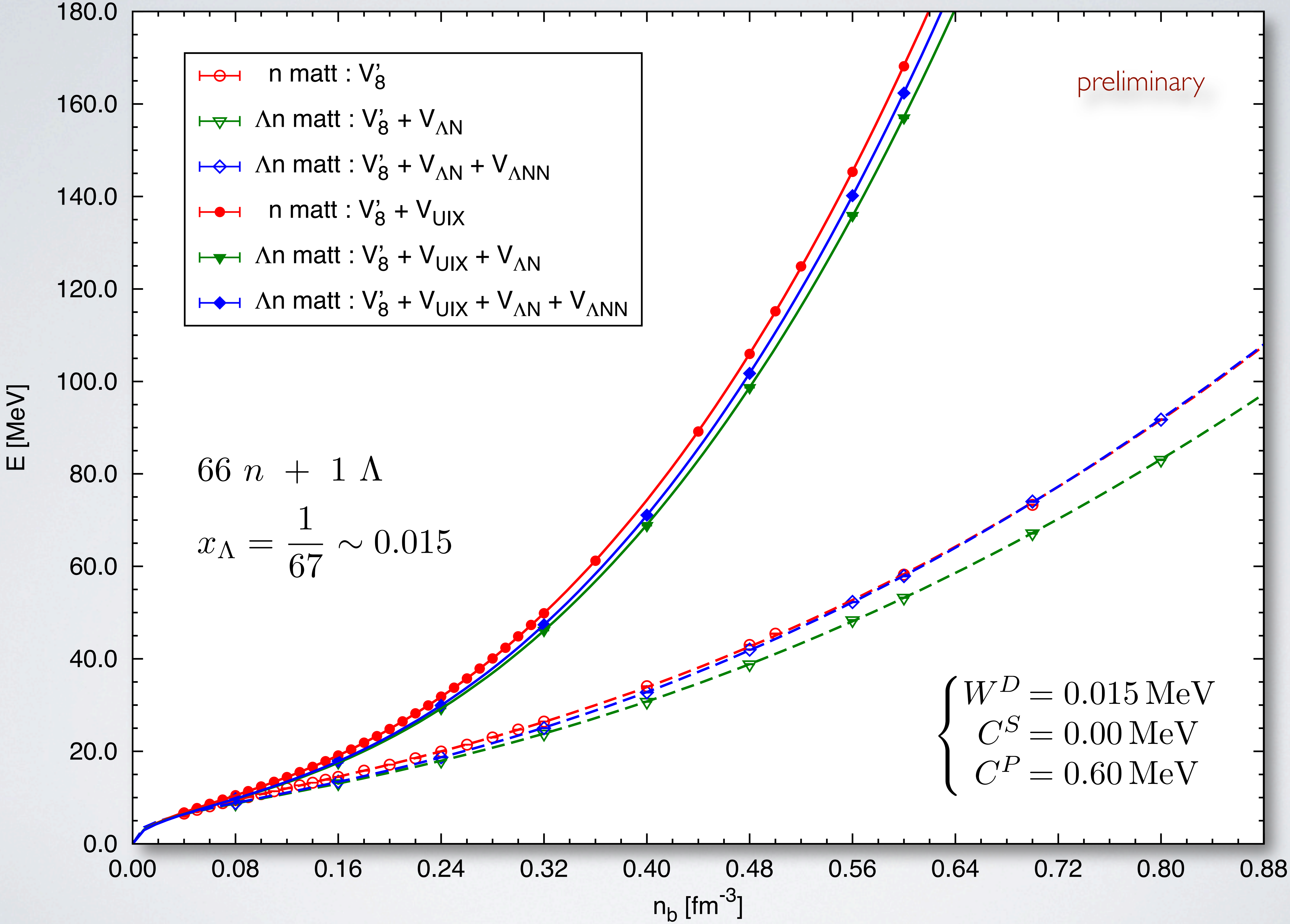
Results: Λ-neutron matter

neutron matter		nuclear matter		Λ-neutron matter
$E(n_b)$	$\xrightarrow{\beta \text{ eq.}}$	$E(n_b, x_p)$		$E(n_b, x_\Lambda)$
		$x_p = \dfrac{n_p}{n_n + n_p}$		$x_\Lambda = \dfrac{n_\Lambda}{n_n + n_\Lambda}$
		$E(n_b, x_p) = E(n_b, \frac{1}{2}) + \Delta E$ $\Delta E = S(n_b)(1 - 2x_p)^2$		?

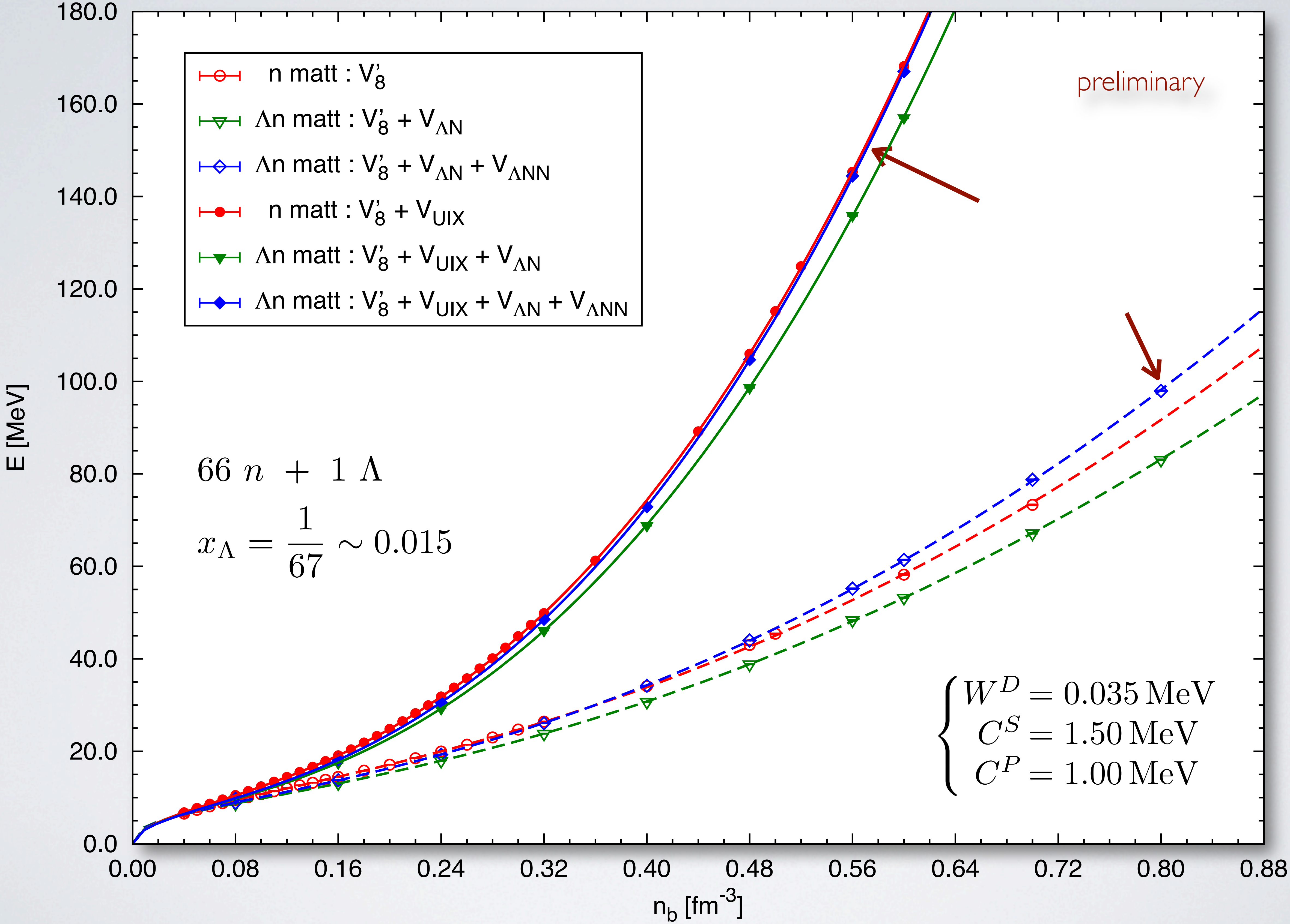
$$\begin{cases} E_n(n_b) \\ E_{n+\Lambda}(n_b) \\ x_\Lambda \ll 1 \end{cases} \Rightarrow E(n_b, x_\Lambda) \longleftrightarrow \begin{cases} \varepsilon(n_b, x_\Lambda) \\ \mu_n(n_b, x_\Lambda) \\ \mu_\Lambda(n_b, x_\Lambda) \end{cases}$$

$\Rightarrow$ solve the TOV $\longrightarrow$ M(R) & Mmax

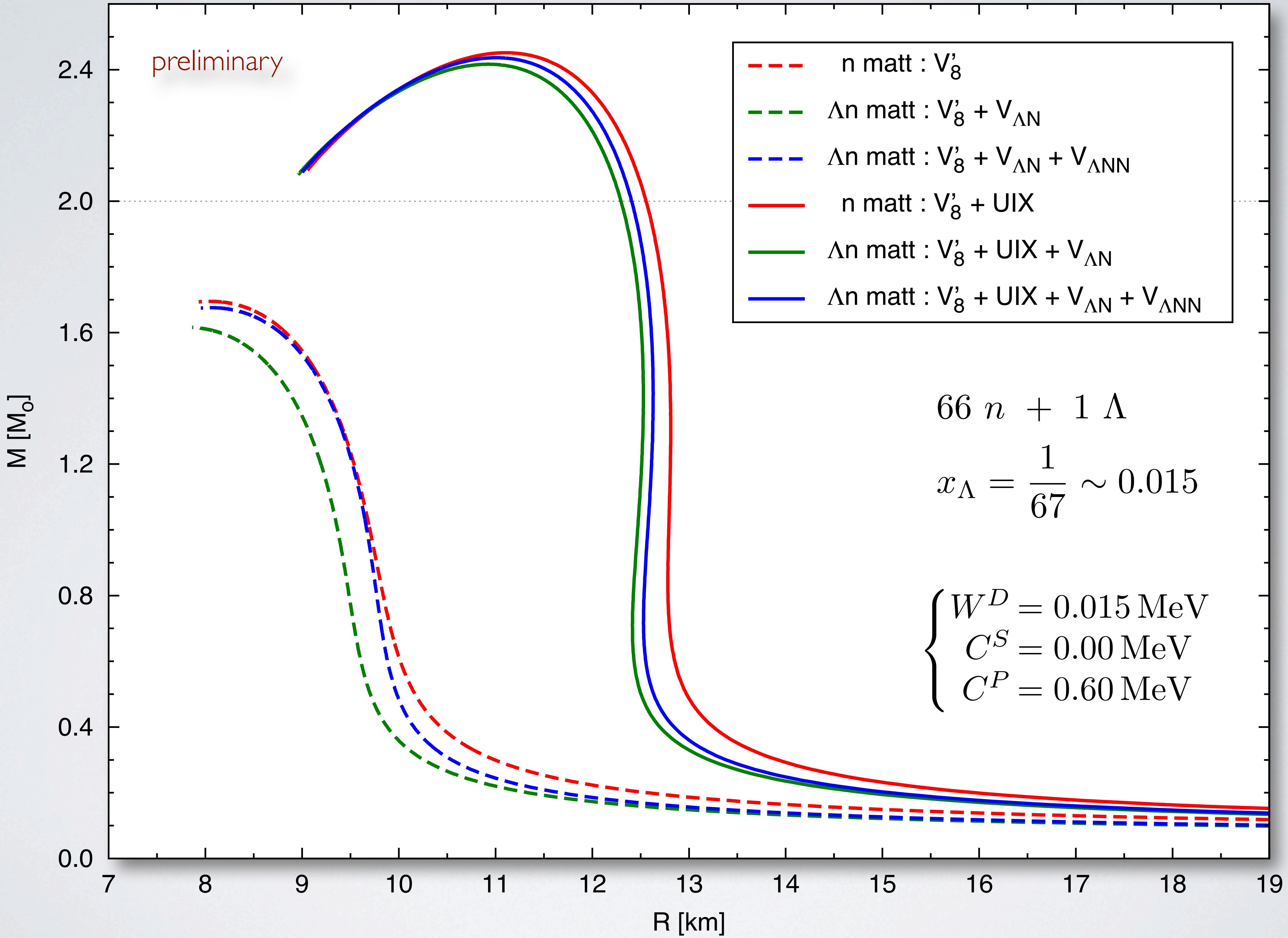
Results: Λ-neutron matter



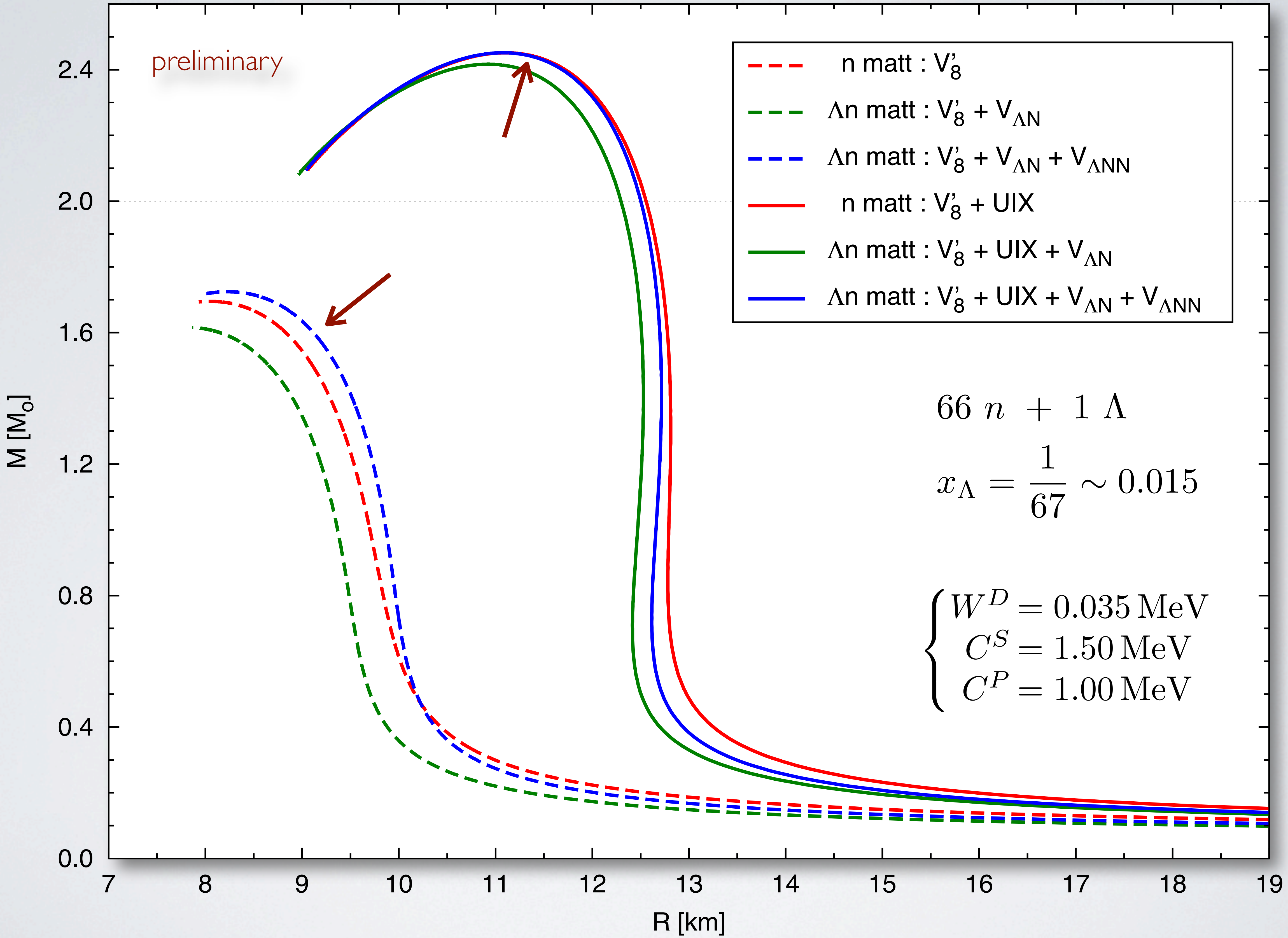
Results: Λ-neutron matter



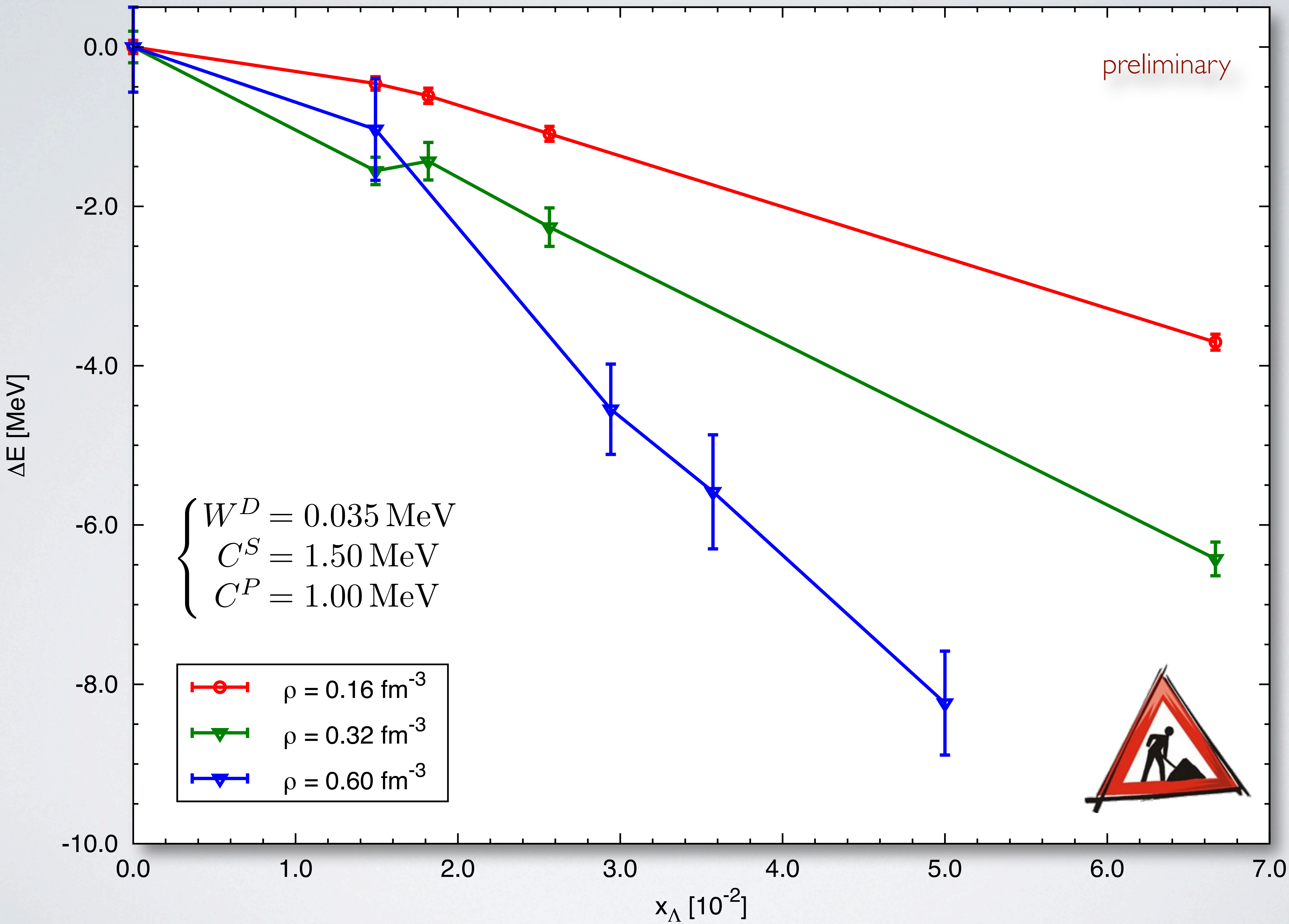
Results: Λ-neutron matter



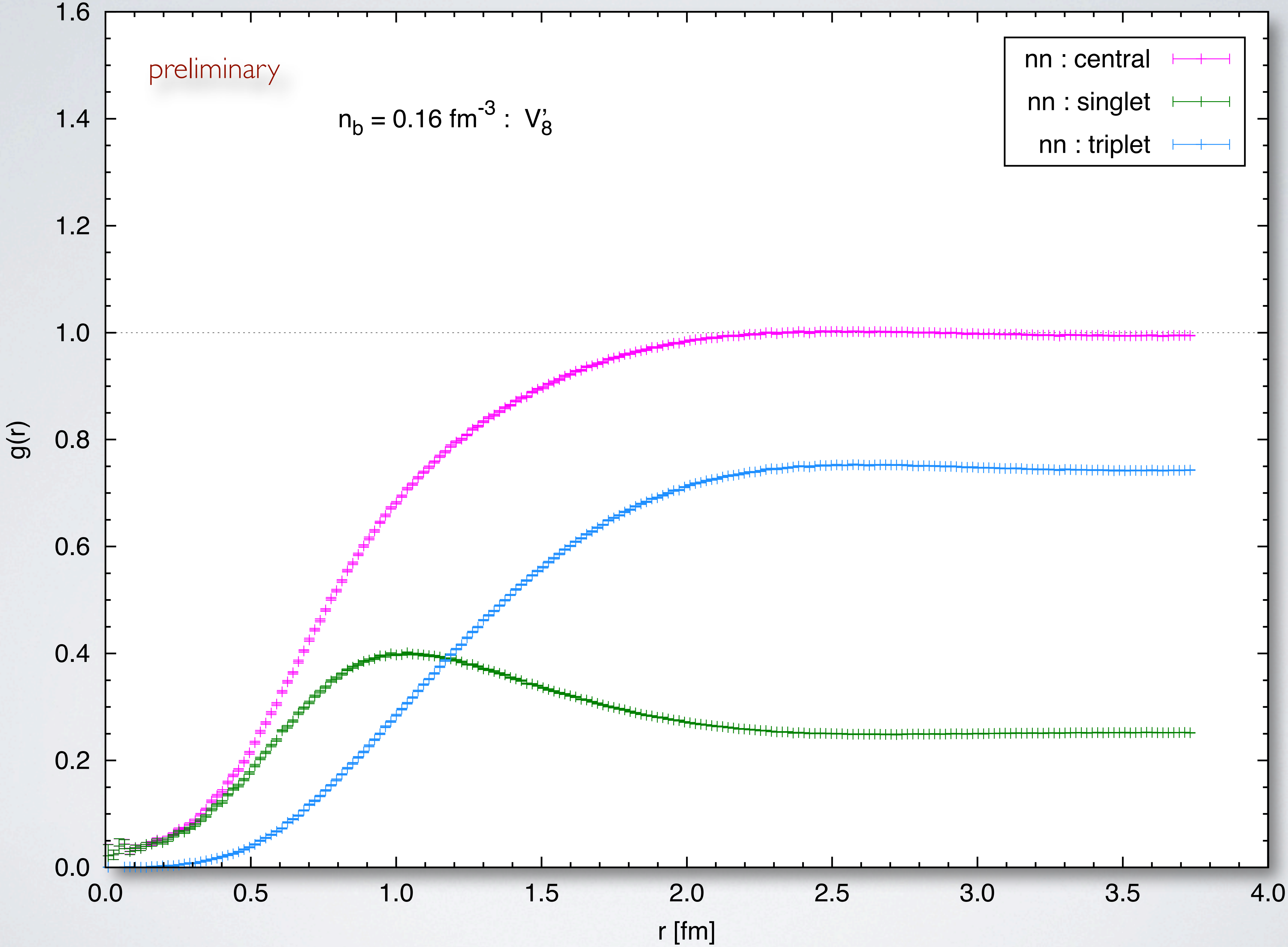
Results: Λ-neutron matter



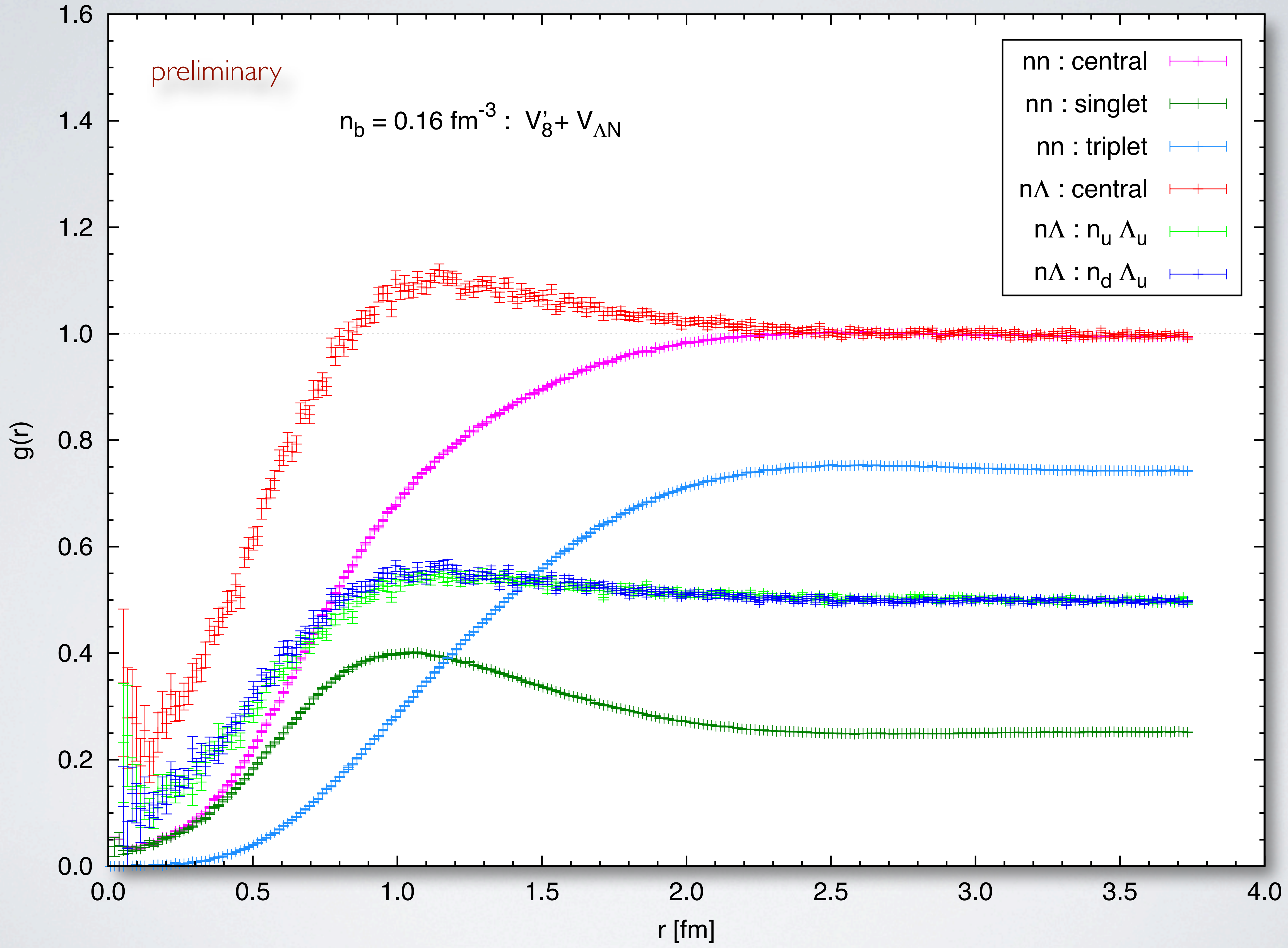
Results: Λ-neutron matter



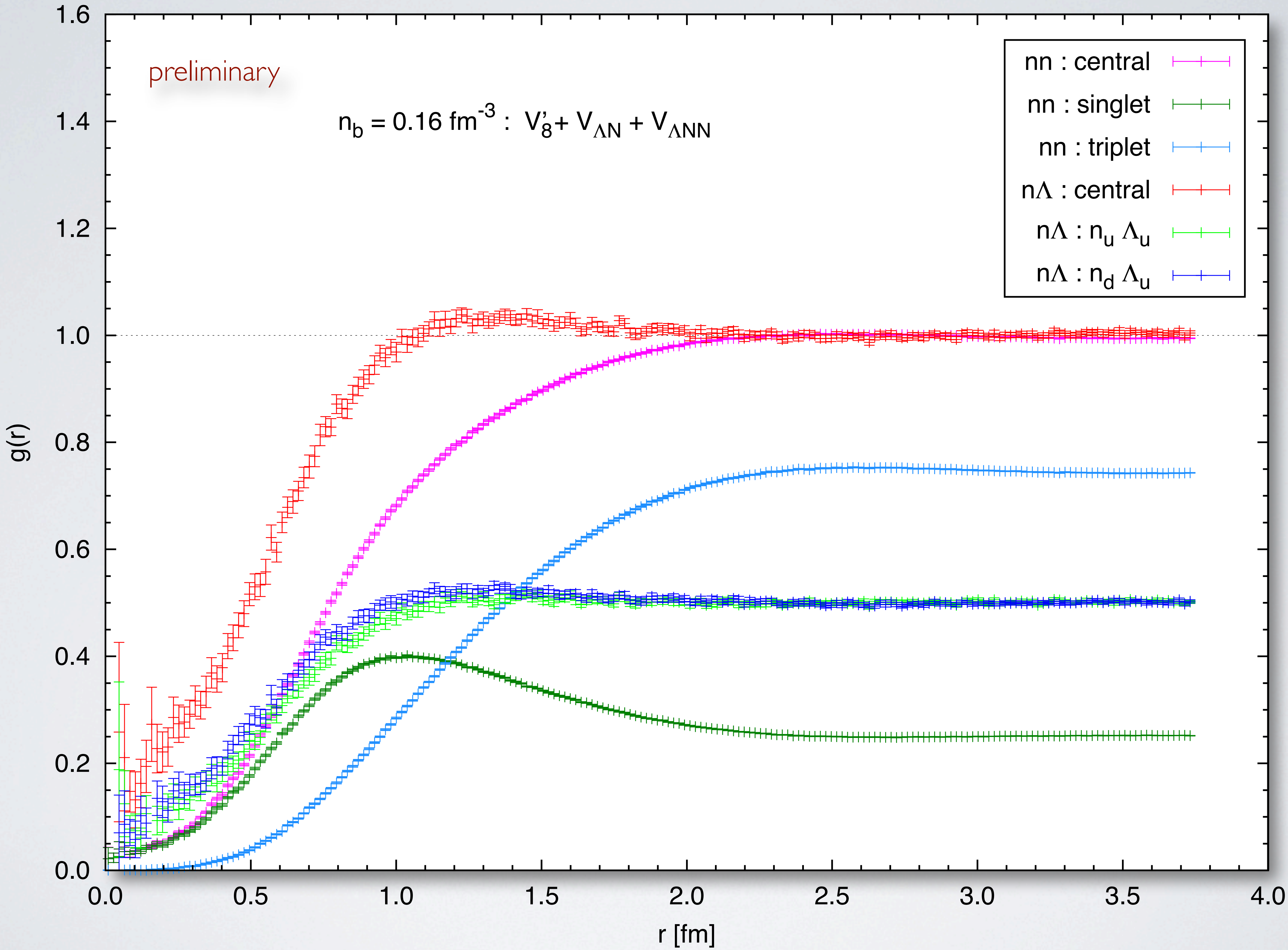
Results: Λ-neutron matter



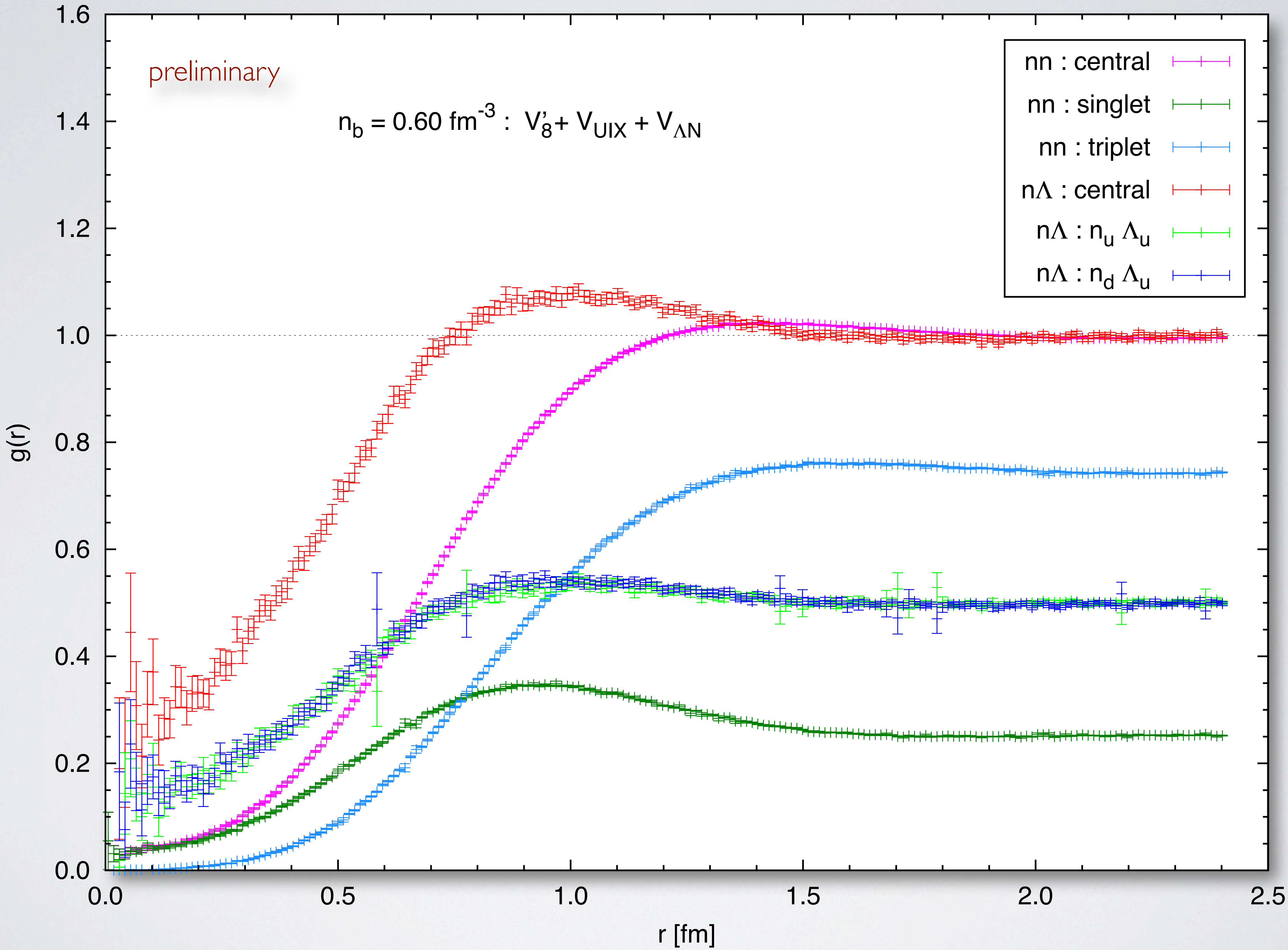
Results: Λ-neutron matter



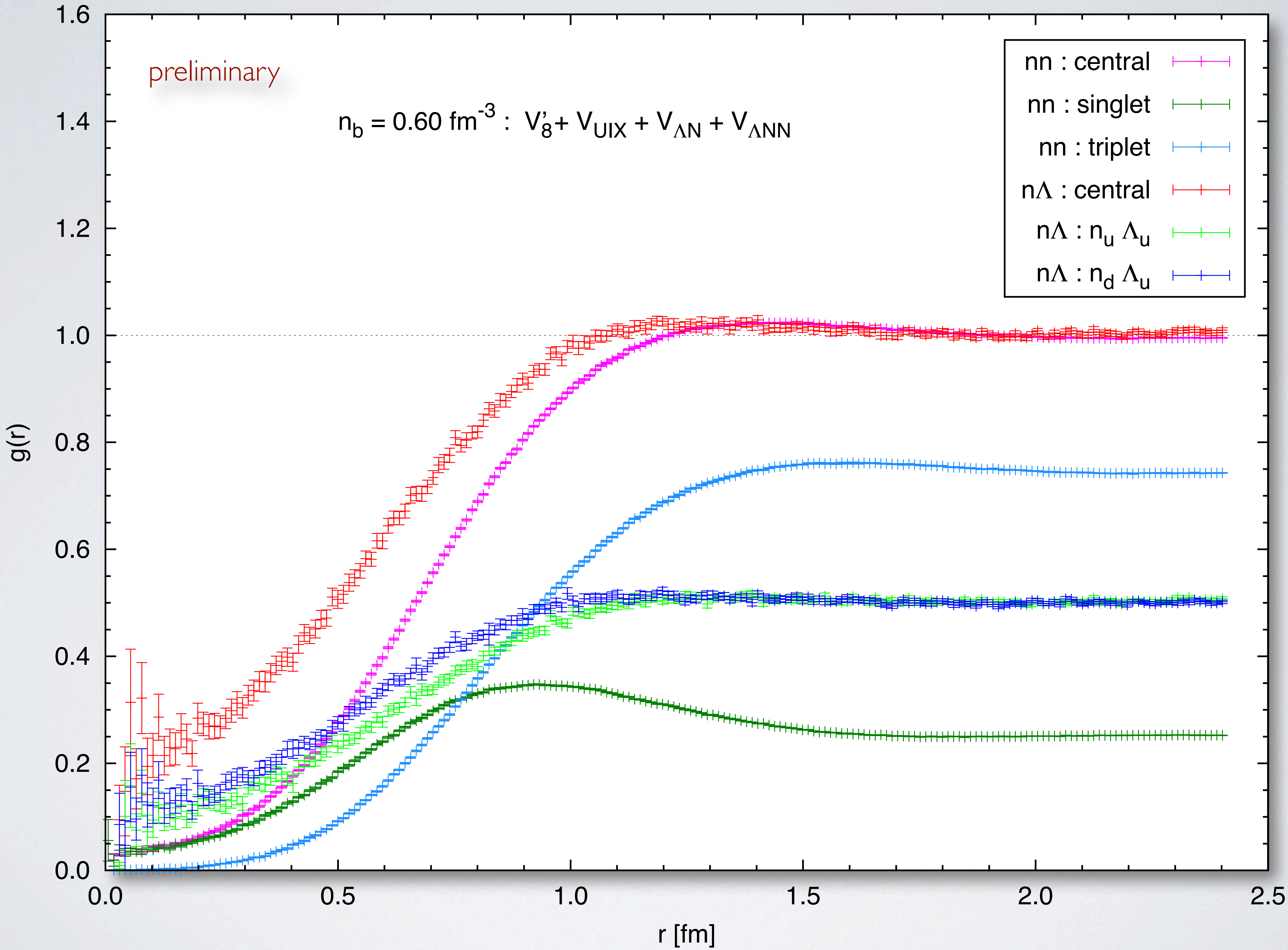
Results: Λ-neutron matter



Results: Λ-neutron matter



Results: Λ-neutron matter



Conclusions

- ✓ Extension of the AFDMC code for strange finite and infinite nuclear systems: develop of an hyperon-nucleon interaction in the Quantum Monte Carlo scheme
  - analysis of the hyperon separation energy in medium-light hypernuclei
  - analysis of the EoS of the hyperon-neutron matter at high density

- ✓ Two-body ΛN interaction not sufficient to describe the hyperon-separation energy of medium-light Λ-hypernuclei: need of a strongly repulsive three-body ΛNN interaction

- ✓ EoS for the Λ-neutron matter not too soft: chance for a NS maximum mass up to $2\ M_{\odot}$ even in presence of hyperons

work in progress

Thank you for
your attention !!

Backup: the hyperon-nucleon interaction

$$T_\pi(x) = \left(1 + \frac{3}{m_\pi x} + \frac{3}{(m_\pi x)^2}\right) Y_\pi(x)\xi(x)$$

$$Y_\pi(x) = \frac{\mathrm{e}^{-m_\pi x}}{m_\pi x}\xi(x) \qquad \xi(x) = 1 - \mathrm{e}^{-cx^2}$$

$$Z_\pi(x) = \frac{x}{3}\left[Y_\pi(x) - T_\pi(x)\right]$$

$$X_{\Lambda i} = (\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i)\, Y_\pi\left(r_{\Lambda i}\right) + S_{\Lambda i}\, Y_\pi\left(r_{\Lambda i}\right)$$

$$S_{\Lambda i} = 3\left(\boldsymbol{\sigma}_\Lambda \cdot \hat{\boldsymbol{r}}_{\Lambda i}\right)\left(\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{\Lambda i}\right) - \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$$

constant	value	unit
m_π	138.03899	MeV
W_c	2137	MeV
$\bar{r}$	0.5	fm
a	0.2	fm
v_s	6.33, 6.28, 6.23	MeV
v_t	6.09, 6.04, 5.99	MeV
$\bar{v}$	6.15(5)	MeV
v_σ	0.24	MeV
c	2.0	fm^{-2}
ε	$0.1 \div 0.38$	–
W^D	$0.002 \div 0.058$	MeV
C^P	$0.5 \div 2.5$	MeV
C^S	~ 1.5	MeV
C^{CSB}	-0.050(5)	MeV

Backup: the hyperon-hyperon interaction

$$v_{\lambda\mu} = \sum_{k=1}^{3} \left(v_0^{(k)} + v_\sigma^{(k)} \, \boldsymbol{\sigma}_\lambda \cdot \boldsymbol{\sigma}_\mu \right) \mathrm{e}^{-\mu^{(k)} r_{\lambda\mu}^2}$$

$\mu^{(k)}$	0.555	1.656	8.163
$v_0^{(k)}$	-10.67	-93.51	4884
$v_\sigma^{(k)}$	0.0966	16.08	915.8

E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, Y. Yamamoto,
Phys. Rev. C 66, 024007 (2002)

$$V_{\Lambda\Lambda} = \frac{1}{2} \sum_{\lambda \neq \mu} \sum_{\alpha} \sigma_\lambda^\alpha \left(\mathcal{D}_{\lambda\mu}^{[\sigma]} \right) \sigma_\mu^\alpha$$