Fermion Bag Approach to Fermion Sign Problems

Shailesh Chandrasekharan
(Duke University)

Collaborator (Anyi Li, INT)

Duke UNIVERSITY

Outline

- Review fermions in Quantum Field Theory
- A new look at Grassmann Integration and Fermion Bags
- Lessons from a Plaquette Model
- New Class of Solvable Models
- Some Recent Results
- Conclusions

Fermions in Quantum Field Theory

Partition function

$$Z = \int [d\phi] \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S_b[\phi]\ -\ S_f(\overline{\psi},\psi,\phi)}$$

Here ψ and $\overline{\psi}$ are two independent

Grassmann valued fields

on a lattice while ϕ is a bosonic field.

What are Grassmann variables?

Grassmann Calculus

Two Grassmann variable anticommute

$$\psi_1 \, \psi_2 = - \, \psi_2 \, \psi_1$$

This implies $\psi^2 \; = \; 0$

Integration rules are very simple

$$\int d\psi \;=\; 0, \qquad \int d\psi \; \psi \;=\; 1$$

This implies

$$\int \prod_i \left(d\overline{\psi}_i \; d\psi_i \right) \; \mathrm{e}^{-\overline{\psi}_i M_{ij} \psi_j} \;=\; \mathrm{Det}(M)$$

Traditional Approach

Scalettar, Scalapino, Sugar, Toussaint (1986,1987)

Rewrite partition functions as

$$Z = \int [d\sigma] \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S_b[\sigma]\ -\ \sum_{i,j} \overline{\psi}_i\ M_{ij}[\sigma]\ \psi_j}$$

Then integrate over the Grassmann variables

$$Z\ =\ \int [d\sigma]\ \mathrm{e}^{-S_b[\sigma]}\ \mathrm{Det}(M[\sigma])$$

If $\mathrm{Det}(M[\sigma])$ is positive then sign problem is solved!

Unfortunately, determinants are NOT always postitive!

Solvable problems (mostly!) are of the type

$$M[\sigma] = \begin{pmatrix} 0 & D[\sigma] \\ -D^\dagger[\sigma] & 0 \end{pmatrix}$$

"solvable form"

(in an appropriate basis!)

$$\mathrm{Det}(M[\sigma]) = |\mathrm{Det}(D[\sigma])|^2$$

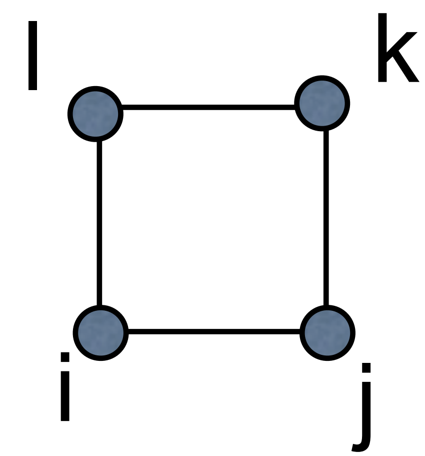
which proves that $\mathrm{Det}(M[\sigma])$ **is positive**

Many interesting problems are not of this type!

Example : A Plaquette model

$$S = \sum_{xy} \overline{\psi}_x \, M_{xy} \, \psi_y - g \sum_{\langle ijkl \rangle} (-\overline{\psi}_i \psi_i)\,(-\overline{\psi}_j \psi_j)\,(-\overline{\psi}_k \psi_k)\,(-\overline{\psi}_l \psi_l)$$

has a “solvable” form



$$Z = \int [d\overline{\psi}\psi]\; \mathrm{e}^{-\overline{\psi} M \psi} \prod_{\langle ijkl \rangle} \mathrm{e}^{g(-\overline{\psi}_i \psi_i)\,(-\overline{\psi}_j \psi_j)\,(-\overline{\psi}_k \psi_k)\,(-\overline{\psi}_l \psi_l)}$$

$$\mathrm{e}^{g(-\overline{\psi}_i \psi_i)\,(-\overline{\psi}_j \psi_j)\,(-\overline{\psi}_k \psi_k)\,(-\overline{\psi}_l \psi_l)} = \sum_{z_{ijkl} \in Z_4} \mathrm{e}^{g^{1/4}\; z_{ijkl}\; \sum_{a=i,j,k,l}(-\overline{\psi}_a \psi_a)}$$

$$Z = \sum_{[z]} \int [d\overline{\psi} d\psi]\; \mathrm{e}^{-\overline{\psi}_i (M_{ij} \, + \, g^{1/4} \; \overline{z} \; \delta_{ij})\psi_j}$$

leads to sign problems

Lesson

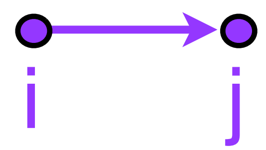
Introducing Auxiliary fields without thought can lead to sign problems!

Are there other ways to approach the Grassmann Integration beyond the "traditional approach"?

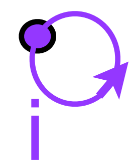
A new look at Grassmann Integration

Grassmann numbers help generate fermion "worldlines"

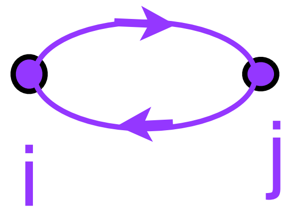
$\overline{\psi}_i \, \psi_j$ =



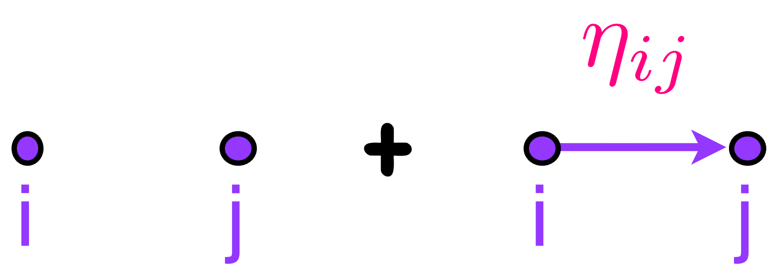
$\overline{\psi}_i \, \psi_i$ =



$\overline{\psi}_i \psi_j \overline{\psi}_j \psi_i$ =

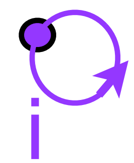


$e^{\eta_{ij} \overline{\psi}_i \psi_j} = 1 + \eta_{ij} \overline{\psi}_i \psi_j$ =

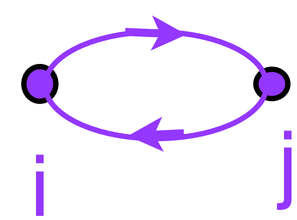


Grassmann Integration is trivial

$$\int d\psi \;=\; 0, \qquad \int d\psi\;\psi \;=\; 1$$

$$\int [d\overline{\psi}_i \; d\psi_i]$$ 

= -1

$$\int [d\overline{\psi_j} \; d\psi_j] \; [d\overline{\psi}_i \; d\psi_i]$$ 

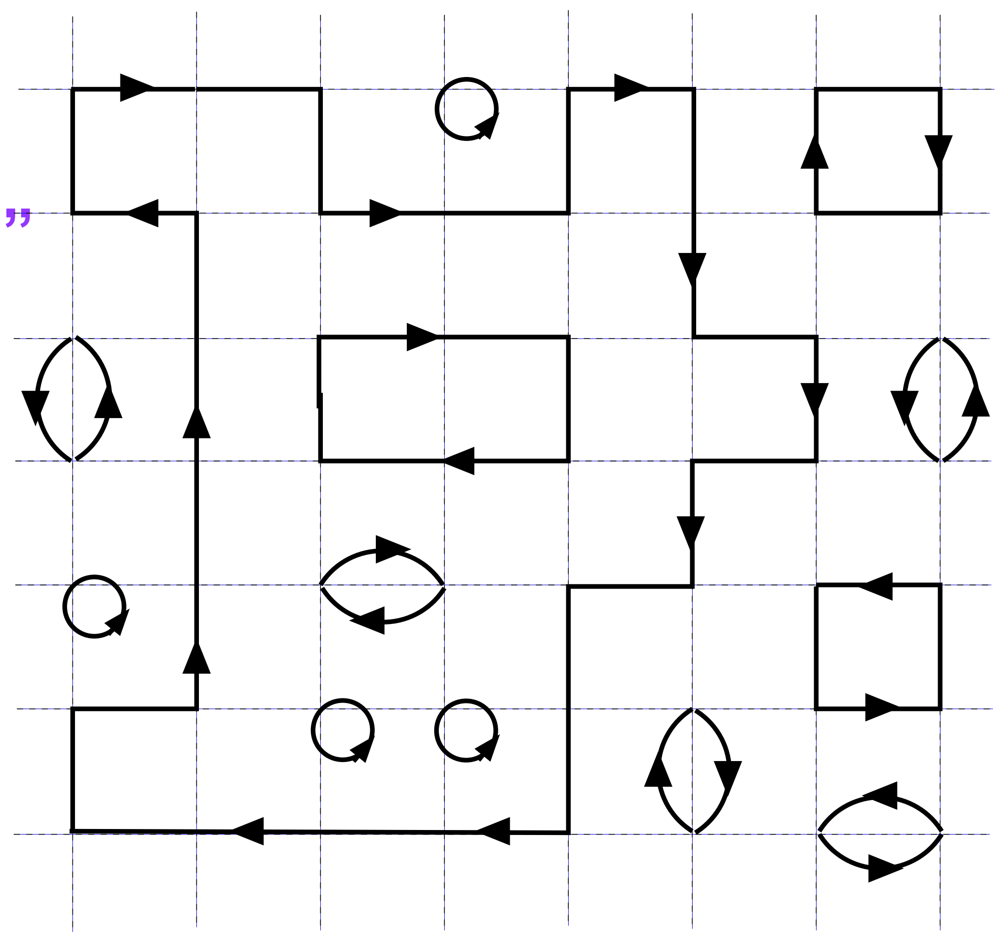
= -1

Every site must have one incoming and one outgoing line

Every closed loop gives a -1

$$\int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}_i M_{ij} \psi_j} = \sum_C \mathrm{W}(C)\ \mathrm{Sign}(C) \ = \ \mathrm{Det}(M)$$

Sum over fermion “worldline” Configuration



Weight of the configuration comes from local weights of bonds

Sign of a configuration comes from local sign factors and number of loops

example of C

Fermion Bag Idea

SC, 2010

Group fermion “worldlines”
inside regions called fermion bags
and sum over them

Choose fermion bags carefully
such that the sum (“path integral”) is postitive

(Extension of the meron cluster idea)

SC, Wiese, 2000

Consider

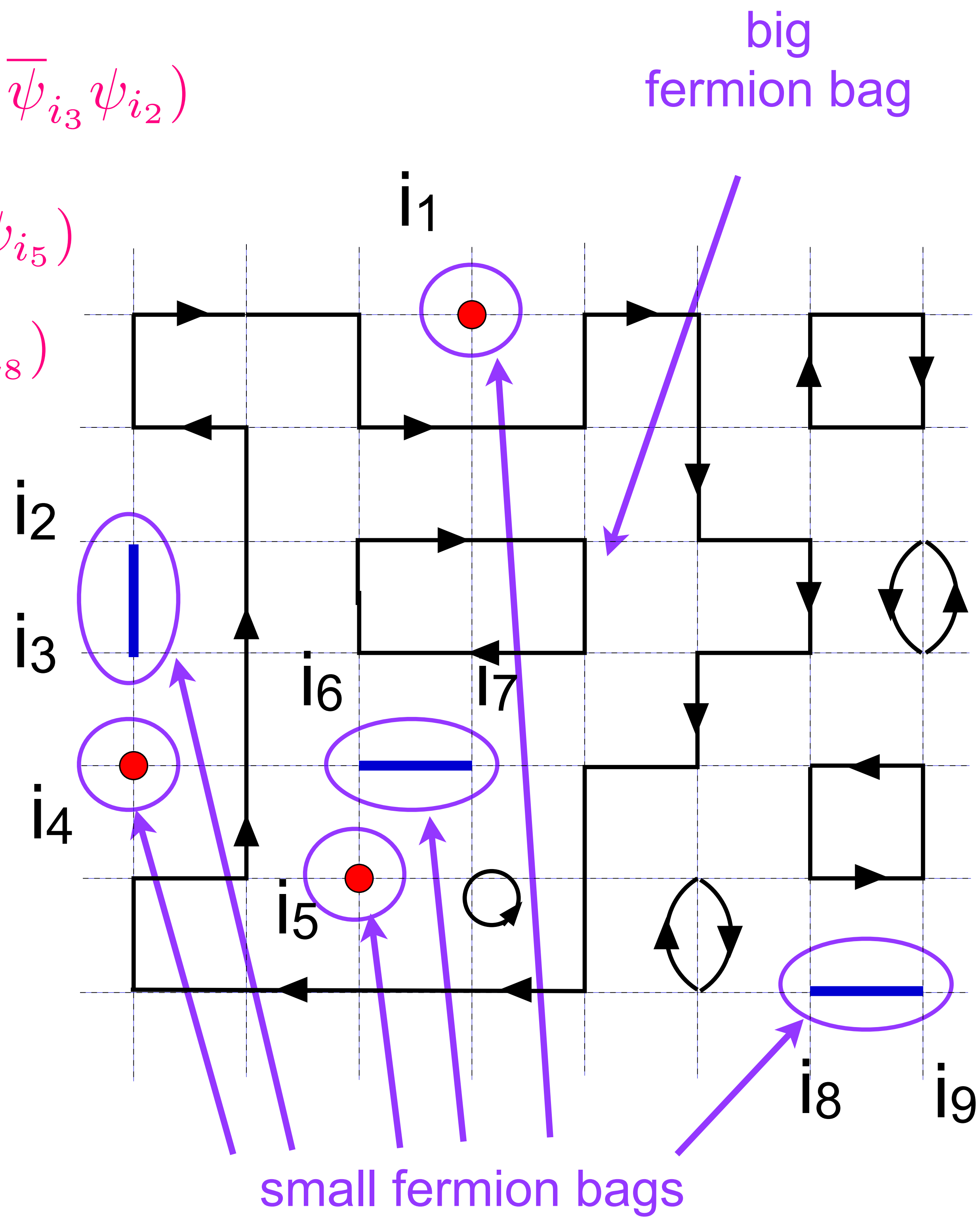
$$\int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}_i M_{ij} \psi_j}\ (-\overline{\psi}_{i_1}\psi_{i_1})(-\overline{\psi}_{i_2}\psi_{i_3}\overline{\psi}_{i_3}\psi_{i_2})$$

$$(-\overline{\psi}_{i_2}\psi_{i_3}\overline{\psi}_{i_3}\psi_{i_2})\ (-\overline{\psi}_{i_4}\psi_{i_4})(-\overline{\psi}_{i_5}\psi_{i_5})$$

$$(-\overline{\psi}_{i_6}\psi_{i_7}\overline{\psi}_{i_7}\psi_{i_6})\quad (-\overline{\psi}_{i_8}\psi_{i_9}\overline{\psi}_{i_9}\psi_{i_8})$$

$$= \quad \mathrm{Det}(W)$$

W is the matrix obtained by dropping some rows and the same columns from M



$$
M = \left(\begin{array}{ccccc|ccccc}
0 & 0 & 0 & \ldots & 0 & M_{11} & M_{12} & M_{13} & \ldots & M_{1N} \\
0 & 0 & 0 & \ldots & 0 & M_{21} & M_{22} & M_{23} & \ldots & M_{2N} \\
0 & 0 & 0 & \ldots & 0 & M_{31} & M_{32} & M_{33} & \ldots & M_{3N} \\
. & . & . & \ldots & . & . & . & . & \ldots & . \\
. & . & . & \ldots & . & . & . & . & \ldots & . \\
. & . & . & \ldots & . & . & . & . & \ldots & . \\
0 & 0 & 0 & \ldots & 0 & M_{N1} & M_{N2} & M_{N3} & \ldots & M_{NN} \\
\hline
-M^*_{11} & -M^*_{21} & -M^*_{31} & \ldots & -M^*_{N1} & 0 & 0 & 0\ldots & 0 & \\
-M^*_{12} & -M^*_{22} & -M^*_{32} & \ldots & -M^*_{N2} & 0 & 0 & 0\ldots & 0 & \\
-M^*_{13} & -M^*_{23} & -M^*_{33} & \ldots & -M^*_{N3} & 0 & 0 & 0\ldots & 0 & \\
. & . & . & \ldots & . & . & . & . & \ldots & . \\
. & . & . & \ldots & . & . & . & . & \ldots & . \\
. & . & . & \ldots & . & . & . & . & \ldots & . \\
-M^*_{1N} & -M^*_{2N} & -M^*_{3N} & \ldots & -M^*_{NN} & 0 & 0 & 0\ldots & 0 &
\end{array}\right)
$$

$$
W = \left(\begin{array}{ccccc|ccccc}
0 & 0 & 0 & \ldots & 0 & M_{11} & M_{12} & M_{13} & \ldots & M_{1N} \\
0 & 0 & 0 & \ldots & 0 & M_{21} & M_{22} & M_{23} & \ldots & M_{2N} \\
0 & 0 & 0 & \ldots & 0 & M_{31} & M_{32} & M_{33} & \ldots & M_{3N} \\
. & & & \ldots & . & . & . & . & \ldots & . \\
. & & & \ldots & . & . & . & . & \ldots & . \\
. & & & \ldots & . & . & . & . & \ldots & . \\
0 & 0 & 0 & \ldots & 0 & M_{N1} & M_{N2} & M_{N3} & \ldots & M_{NN} \\
\hline
-M^*_{11} & -M^*_{21} & -M^*_{31} & \ldots & -M^*_{N1} & 0 & 0 & 0\ldots & 0 & \\
-M^*_{12} & -M^*_{22} & -M^*_{32} & \ldots & -M^*_{N2} & 0 & 0 & 0\ldots & 0 & \\
-M^*_{13} & -M^*_{23} & -M^*_{33} & \ldots & -M^*_{N3} & 0 & 0 & 0\ldots & 0 & \\
. & & & \ldots & . & . & . & . & \ldots & . \\
. & & & \ldots & . & . & . & . & \ldots & . \\
. & & & \ldots & . & . & . & . & \ldots & . \\
-M^*_{1N} & -M^*_{2N} & -M^*_{3N} & \ldots & -M^*_{NN} & 0 & 0 & 0\ldots & 0 &
\end{array}\right)
$$

Thus if

$$M = \begin{pmatrix} 0 & D \\ -D^\dagger & 0 \end{pmatrix}$$

then

$$W = \begin{pmatrix} 0 & \tilde{D} \\ -\tilde{D}^\dagger & 0 \end{pmatrix}$$

$$\Rightarrow \quad \mathrm{Det}(W) \geq 0$$

This means that theories of the type

$$S = \sum_{ij} \overline{\psi}_i M_{ij} \psi_j \; - \; \sum_i U_i(-\overline{\psi}_i \psi_i)$$

$$- \sum_{i_1 i_2} U_{i_1 i_2} \; (-\overline{\psi}_{i_1} \psi_{i_1})(-\overline{\psi}_{i_2} \psi_{i_2}) + ...$$

$$- \sum_{i_1 .. i_k} U_{i_1 ... i_k} \; (-\overline{\psi}_{i_1} \psi_{i_1}) ... (-\overline{\psi}_{i_k} \psi_{i_k}) + ...$$

have no sign problems as long as the couplings U are all positive and M is in the "solvable form"

Can introduce high order fermion couplings easily!

Fermion Bag approach to the plaquette model

$$Z = \int [d\overline{\psi}\psi]\ \mathrm{e}^{-\overline{\psi}M\psi} \prod_{\langle ijkl\rangle} \mathrm{e}^{g(-\overline{\psi}_i\psi_i)\ (-\overline{\psi}_j\psi_j)\ (-\overline{\psi}_k\psi_k)\ (-\overline{\psi}_l\psi_l)}$$

$$= \int [d\overline{\psi}d\psi]\ \mathrm{e}^{-\overline{\psi}_i M_{ij}\psi_j} \prod_{\langle ijkl\rangle} [1 + g(-\overline{\psi}_i\psi_i)(-\overline{\psi}_j\psi_j)(-\overline{\psi}_k\psi_k)(-\overline{\psi}_l\psi_l)]$$

$$= \int [d\overline{\psi}d\psi]\ \mathrm{e}^{-\overline{\psi}_i M_{ij}\psi_j} \prod_{\langle ijkl\rangle} \sum_{n_{ijkl}=0,1} \left\{g(-\overline{\psi}_i\psi_i)(-\overline{\psi}_j\psi_j)(-\overline{\psi}_k\psi_k)(-\overline{\psi}_l\psi_l)\right\}^{n_{ijkl}}$$

$$= \sum_{[n_{ijkl}]} g^{N_p} \int [d\overline{\psi}d\psi]\ \mathrm{e}^{-\overline{\psi}_i M_{ij}\psi_j} \left\{(-\overline{\psi}_i\psi_i)(-\overline{\psi}_j\psi_j)(-\overline{\psi}_k\psi_k)(-\overline{\psi}_l\psi_l)\right\}^{n_{ijkl}}$$

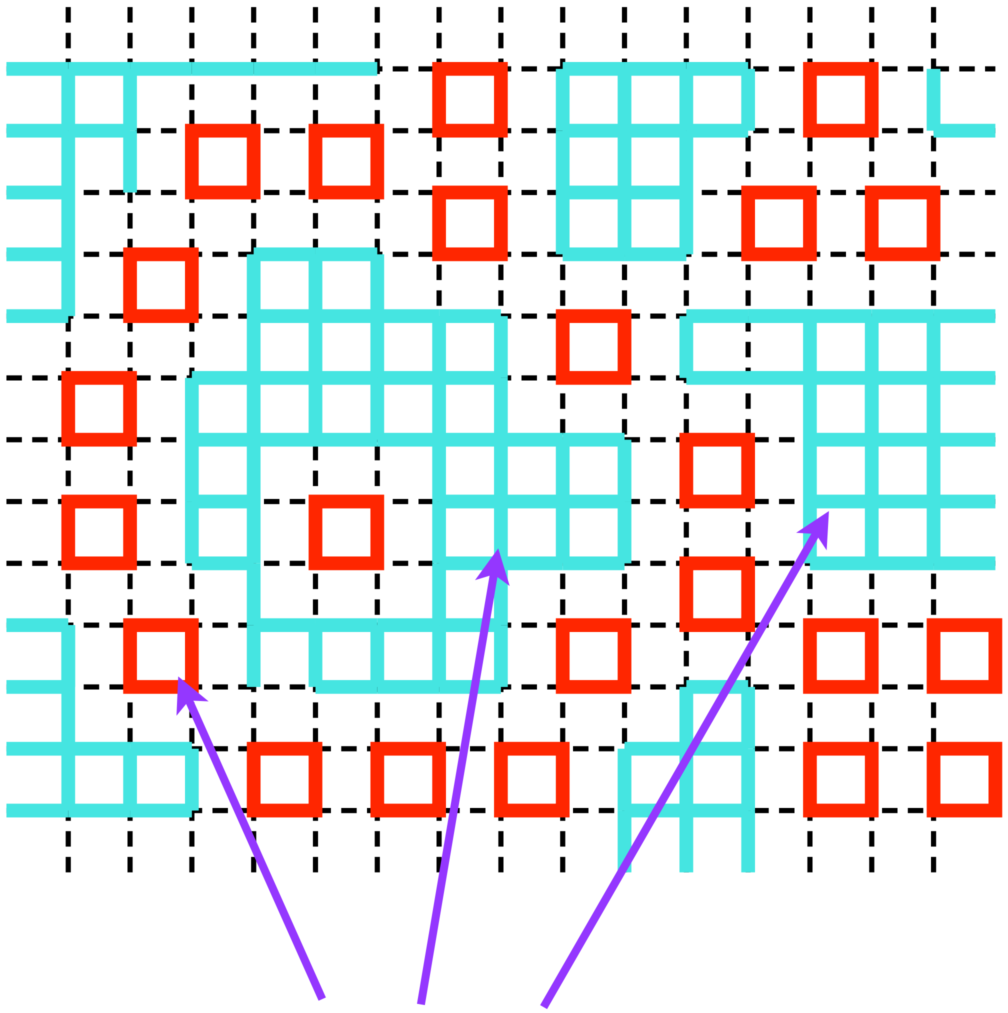
$$= \sum_{[n_{ijkl}]} g^{N_p}\ \mathrm{Det}(W_{[n]})$$

Fermion Bag Partition Function

$$Z = \sum_{[n_{ijkl}]} g^{N_p} \prod_{\text{i} \in \text{blue fbags}} \text{Det}(W_{[n,i]})$$

visualizable configurations

contains fermion physics



fermion bags

Compare with Traditional Partition Function

$$Z = \sum_{[z]} \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}_i (M_{ij} + g^{1/4}\ \overline{z}\ \delta_{ij})\psi_j}$$

$$Z = \sum_{[z]} \text{Det}((M + g^{1/4} z)$$

suffers from sign problem
and
difficult to visualize classically

A new class of “solvable” problems

Consider actions of the form

$$S = \sum_{xy} \overline{\psi}_x \, M_{xy}[\sigma] \, \psi_x \; + \; g \sum_x \phi_x \overline{\psi}_x \psi_x \; + \; S_b(\sigma, \phi)$$

solvable

complex scalar field

space dependent mass term

where the action $S_b[\sigma, \phi]$ is chosen such that

the sign problem in the k-pt correlation function

$$G(z_1, .., z_k, \sigma) \; = \; \int [d\phi] \, \mathrm{e}^{-S_b(\sigma,\phi)} \; \phi_{z_1} \; \phi_{z_2} \; ... \; \phi_{z_k}$$

is solvable.

Solvable bosonic theories are those in which we can write

$$G(z_1, .., z_k, \sigma) \; = \sum_b \int [d\rho] \;\; \Omega(\sigma, b, \rho, n),$$

$$\Omega(\sigma, b, \rho, n) \geq 0$$

where the [n] is a monomer field labeling the location of z_1, z_2,...,z_k

and (b, ρ) are "other" bosonic fields

introduced to solve the sign problem.

These class of models are not solvable with the traditional approach

$$S = \overline{\psi}(M[\sigma] + g\Phi)\psi + S_b(\sigma, \phi)$$

$$M[\sigma] + g\Phi \;=\; \begin{pmatrix} g\ \phi_1 & D[\sigma] \\ -D^\dagger[\sigma] & g\ \phi_2^* \end{pmatrix}$$

$$Z = \int [d\sigma\ d\phi]\mathrm{e}^{-S_b[\sigma,\phi]}\ \ \mathrm{Det}(M[\sigma] + g\Phi)$$

↑ suffers from sign problem

The Fermion bag approach solves the sign problem!

Fermion Bag approach

Rewrite the partition function as

$$Z = \int [d\sigma\ d\phi]\ \mathrm{e}^{-S_b(\sigma,\phi)} \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}\ M[\sigma]\ \psi} \prod_x \left(\mathrm{e}^{-g\ \phi_x\ \overline{\psi}_x \psi_x}\right)$$

Due to the Grassmann nature

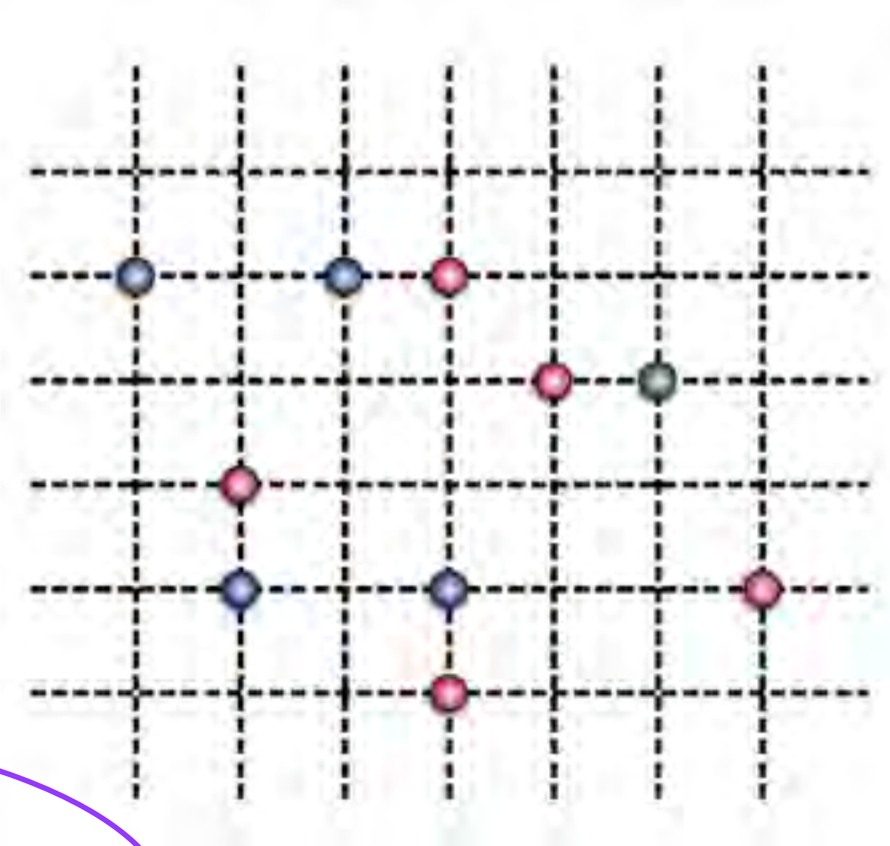
$$\mathrm{e}^{-g\ \phi_x\ \overline{\psi}_x \psi_x} = 1 + g\ \phi_x(-\overline{\psi}_x \psi_x) = \sum_{n_x=0,1} \left(g\ \phi_x\ (-\overline{\psi}_x \psi_x)\right)^{n_x}$$

We can then rewrite

$$Z = \sum_{[n]} \int [d\sigma] \int [d\phi]\ \mathrm{e}^{-S_b(\sigma,\phi)} \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}\ M\ \psi} \prod_x \left(g\ \phi_x\ (-\overline{\psi}_x \psi_x)\right)^{n_x}$$

example of configuration [n] with k = 10

For a given configuration [n]
let z_1 z_2 ... z_k be the k sites
where $n_x = 1$
at all other sites $n_x = 0$



$$Z = \sum_{[n]} g^k \int [d\sigma] \int [d\phi]\; e^{-S_b(\sigma,\phi)}\; \phi_{z_1}\; \phi_{z_2}\; \ldots\; \phi_{z_k}$$

$G(z_1, .., z_k, \sigma)$

$$\int [d\overline{\psi} d\psi]\; e^{-\overline{\psi}\; M[\sigma]\; \psi}\; (-\overline{\psi}_{z_1}\psi_{z_1})\; (-\overline{\psi}_{z_2}\psi_{z_2})\; \ldots\; (-\overline{\psi}_{z_k}\psi_{z_k})$$

Fermion Correlation Function?

Fermion k-point correlation function

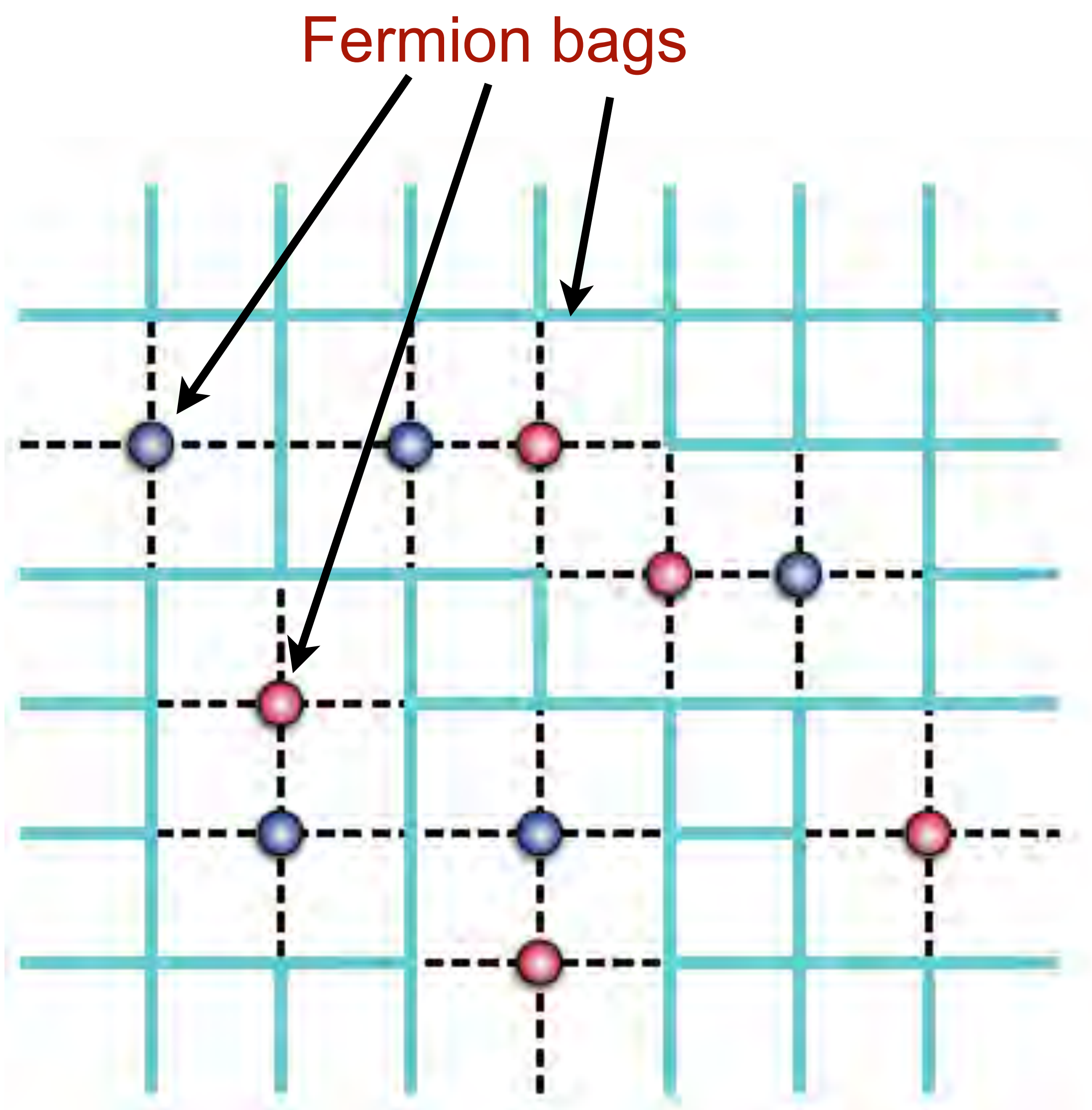
$$\int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}\ M[\sigma]\ \psi}\ \overline{\psi}_{z_1}\psi_{z_1}\ \ldots\ \overline{\psi}_{z_k}\psi_{z_k}$$

$$=\ \mathrm{Det}(W[n,\sigma]) \geq 0$$

W is a (V-k) x (V-k) matrix
obtained by dropping sites z_1 ... z_k in M

$$M[\sigma] = \begin{pmatrix} 0 & D[\sigma] \\ -D^\dagger[\sigma] & 0 \end{pmatrix}$$

$$W[n,\sigma] = \begin{pmatrix} 0 & w[n,\sigma] \\ -w^\dagger[n,\sigma] & 0 \end{pmatrix}$$



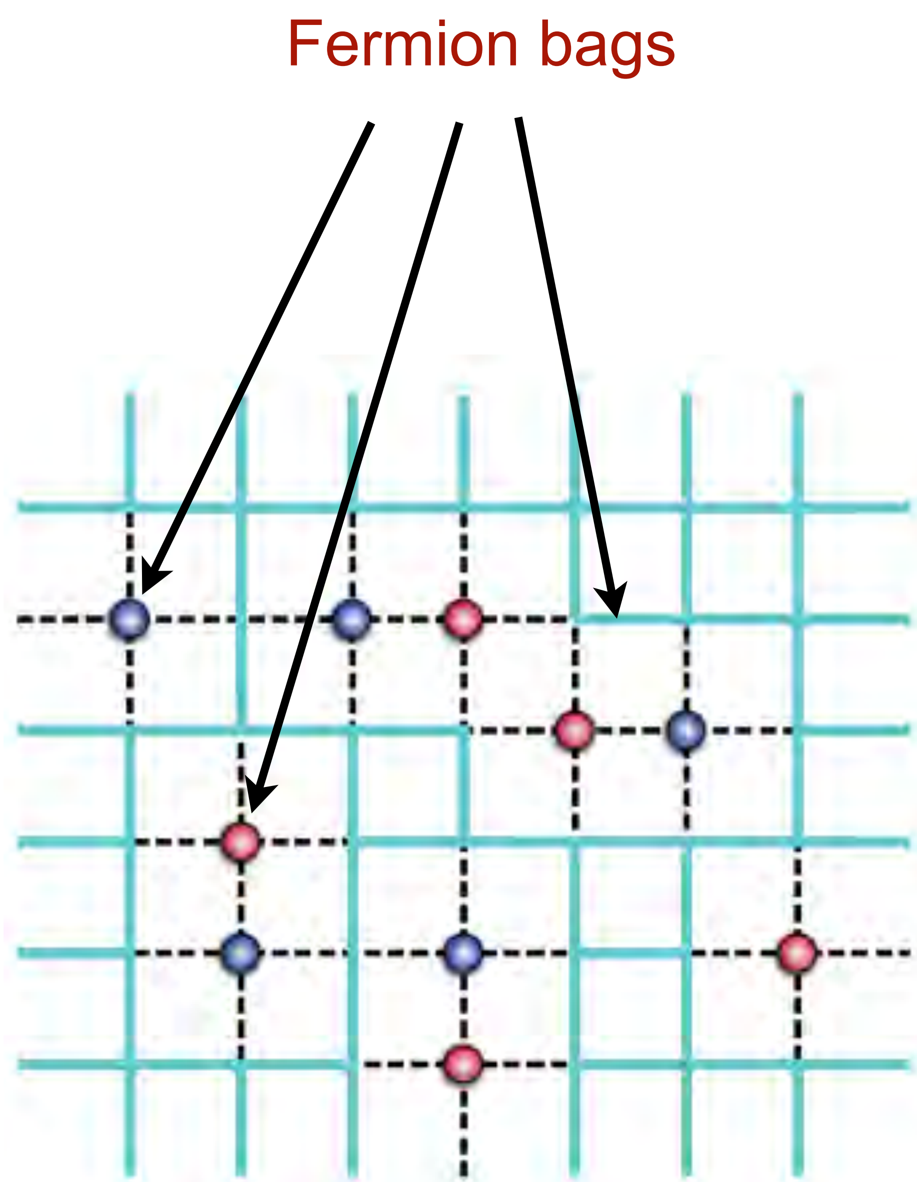
fermion bag configuration

Thus, the partition function is given by

$$Z = \sum_{n,b} \int [d\sigma\ d\rho]\ \ g^k\ \Omega(\sigma, b, \rho, n)\ \mathrm{Det}(W[n,\sigma])$$

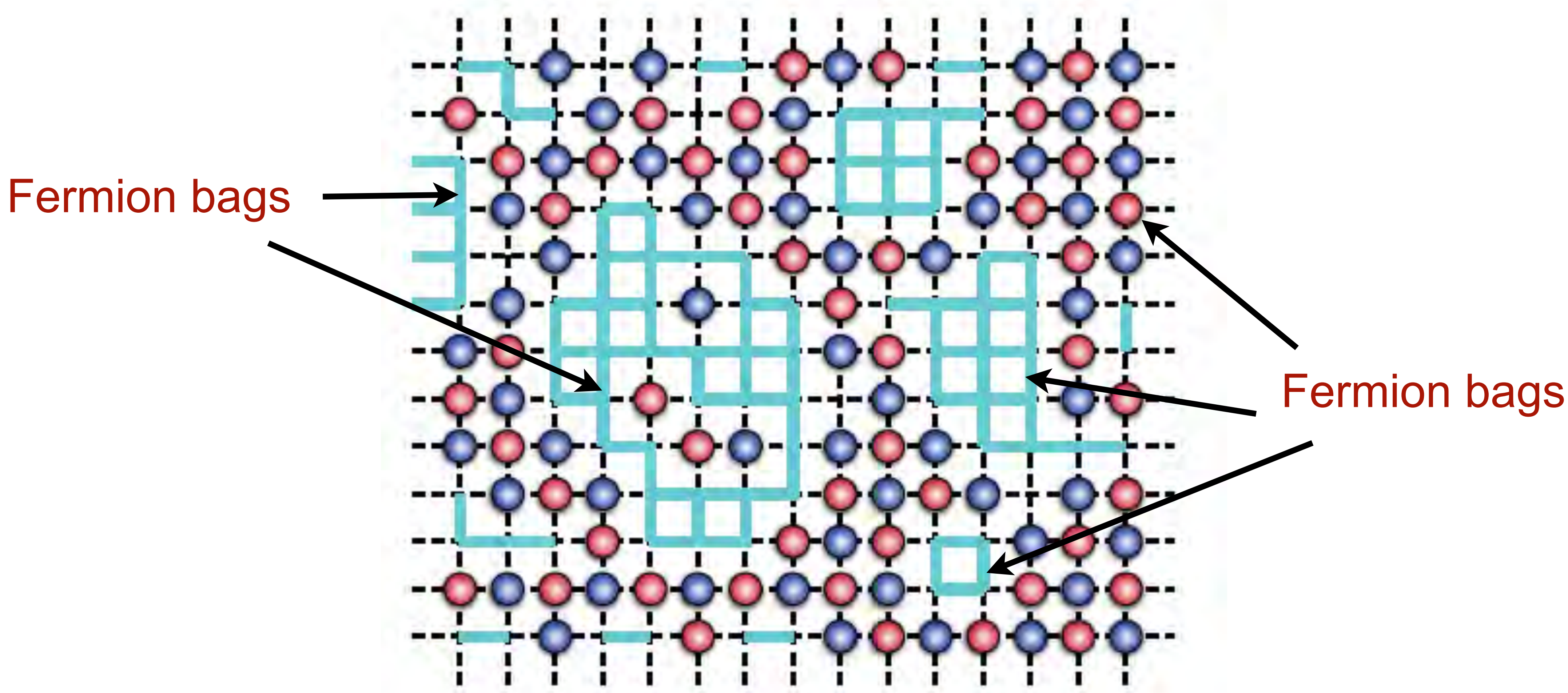
No sign problem!

Mapping into classical statistical mechanics



fermion bag configuration

At large coupling --> many small fermion bags



small fermion bags --> computation is efficient!

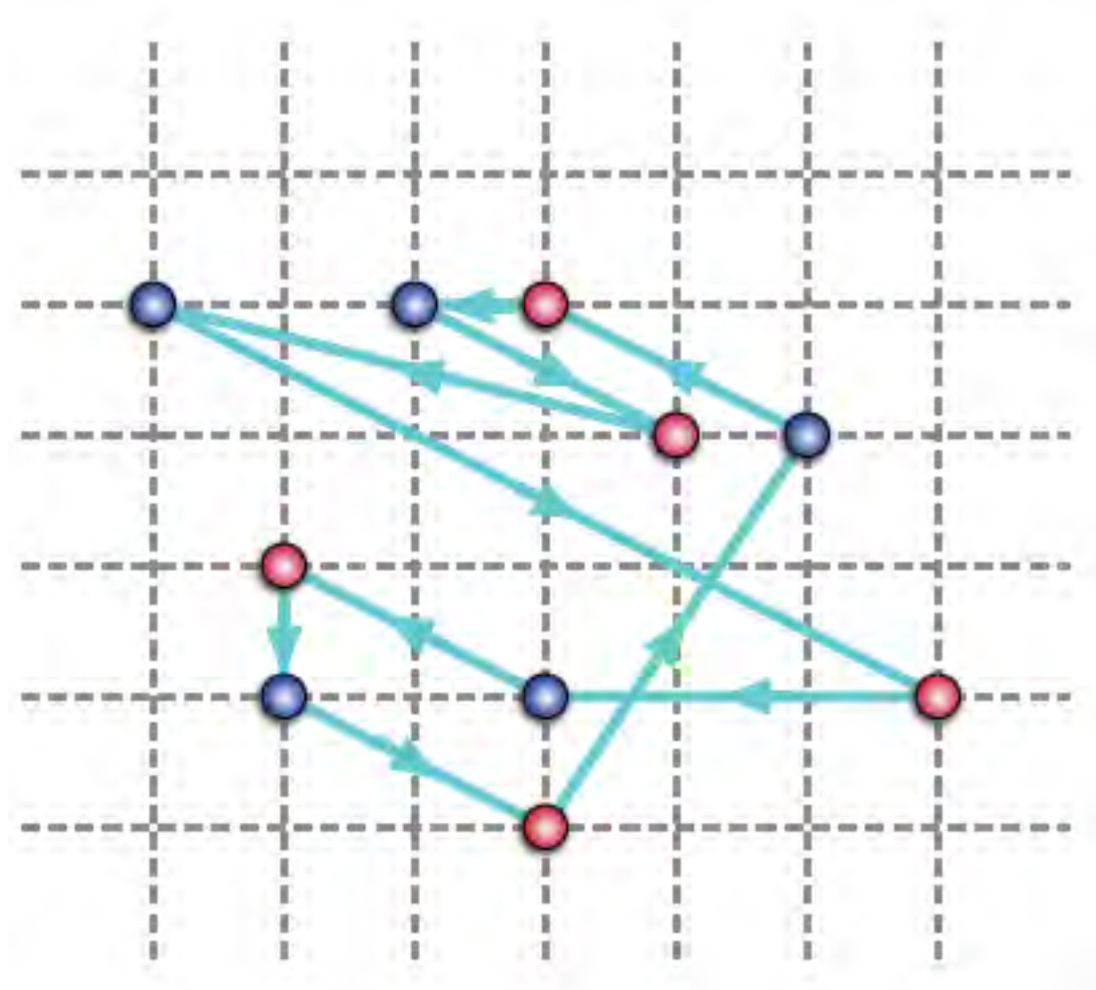
Duality

k-point correlation function

$$\int [d\overline{\psi} d\psi]\ \mathrm{e}^{-\overline{\psi}\ M[\sigma]\ \psi}\ \overline{\psi}_{z_1}\psi_{z_1}\ \ldots\ \overline{\psi}_{z_k}\psi_{z_k}$$

$$= \ \mathrm{Det}\Big(M[\sigma]\Big)\ \mathrm{Det}\Big(G_{[n]}(\sigma)\Big)$$

where $G_{[n]}$ is a (k x k) matrix of propagators



Duality Relation

$\mathrm{Det}\ W^0 \ = \ \mathrm{Det}\ D^0\ \mathrm{Det}\ G_{[n]}$

strong coupling fermion bag

weak coupling fermion Bag

Dual Fermion Bag

Rubtsov, Savkin, Lichtenstein, Prokofev, Svistunov, Troyer, ...

diagrammatic determinantal Monte Carlo

Lesson

A sign problem can be entangled in both fermionic and bosonic variables.

A full solution may require one to solve the sign problems in both the variables!

"Solvable" problems with spin-half

$$S = \sum_{xy} \overline{\psi}_x \; M_{xy} \; \psi_x \; + \; g \; \sum_x (\phi_x \psi_{\uparrow,x} \psi_{\downarrow,x} + \; \phi_x^* \overline{\psi}_{\downarrow,x} \overline{\psi}_{\uparrow,x}) \; + \; S_b$$

If $\sigma_2 M \sigma_2 = M^*$ then $\mathrm{Det}(M) \geq 0$

It is then possible to argue that $\sigma_2 W \sigma_2 = W^*$

so $\mathrm{Det}(W) \geq 0$

Sign problem is solved!

Many interesting lattice field theory models solvable

SU(2) Yukawa models with Wilson Fermions

Gauged NJL models

Models inspired by Graphene

New models with pairing interactions

Some models with repulsive interactions also solvable!

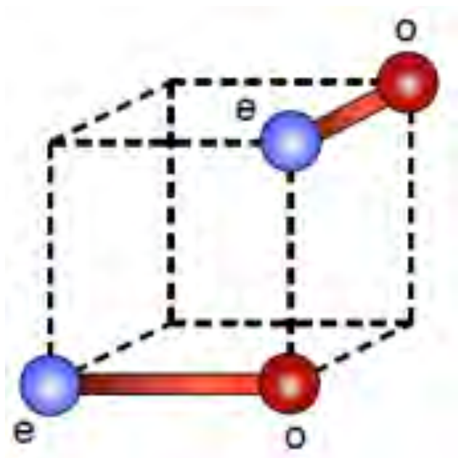
Results: "Graphene" Hubbard Models

S.C. A.Li, PRL (2012), arXiv:1304.7761

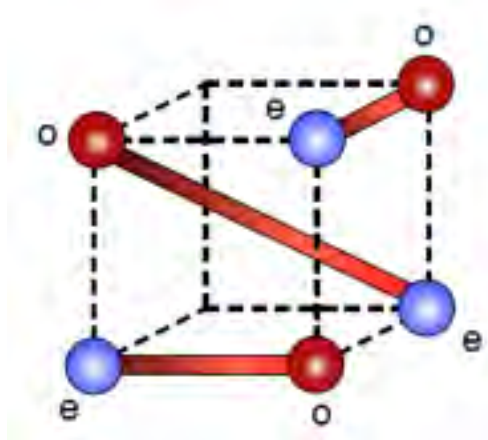
SU(2) x U(1) symmetric models

$$S(\theta,\overline{\psi},\psi) = \sum_{xy} \overline{\psi}_x M_{xy} \psi_y \; - \sum_{\langle xy\rangle} U_{\langle xy\rangle} \overline{\psi}_x \psi_x \; \overline{\psi}_y \psi_y$$

solvable



Thirring

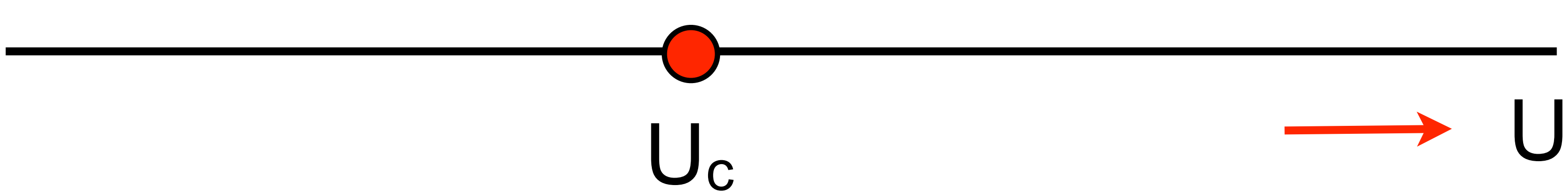


Gross-Neveu

suffers from
sign problems

massless fermions/
U(1) symmetric

massive fermions/
U(1) broken



Observables

chiral susceptibility

$$\chi = \left\langle \frac{\mathbf{1}}{\mathbf{2L^3}} \sum_{\mathbf{x},\mathbf{y}} \overline{\psi}_{\mathbf{x}} \psi_{\mathbf{x}} \, \overline{\psi}_{\mathbf{y}} \psi_{\mathbf{y}} \right\rangle$$

chiral winding susceptibility

$$\mathbf{q}_{\chi}^{\mathbf{2}} = \left\langle \frac{\mathbf{1}}{\mathbf{3}} \sum_{\alpha} (\mathbf{q}_{\chi}^{\mathbf{2}})_{\alpha} \right\rangle$$

fermion correlation ratio

$$\mathbf{C_F}(\mathbf{t}) = \left\langle \frac{\mathbf{1}}{\mathbf{3}} \sum_{\alpha} \overline{\psi}_{\mathbf{0},\mathbf{0},\mathbf{0}} \;\; \psi_{\mathbf{0},\mathbf{0},\mathbf{t}\hat{\alpha}} \right\rangle$$

$$\mathbf{R_F} \; = \; \mathbf{C_F}(\mathbf{L}/\mathbf{2} - \mathbf{1})/\mathbf{C}(\mathbf{1})$$

Critical Finite Size Scaling

$$\chi^{-1}\mathbf{L}^{\mathbf{2}-\eta} = \mathbf{f_0} + \mathbf{f_1}(\mathbf{U} - \mathbf{U_c})\mathbf{L}^{\mathbf{1}/\nu} + \mathbf{f_2}(\mathbf{U} - \mathbf{U_c})^{\mathbf{2}}\mathbf{L}^{\mathbf{2}/\nu} + \ldots$$

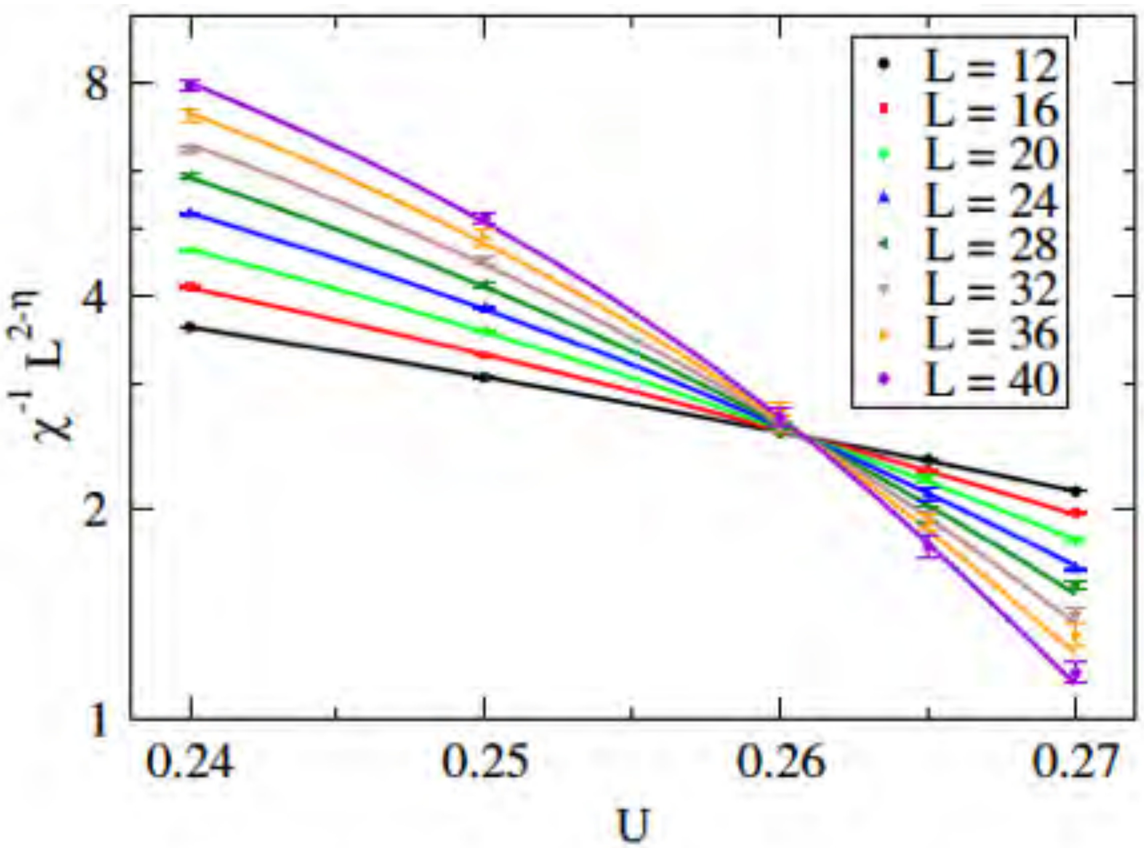
$$\langle \mathbf{q}_\chi^{\mathbf{2}} \rangle = \kappa_{\mathbf{0}} + \kappa_{\mathbf{1}}(\mathbf{U} - \mathbf{U_c})\mathbf{L}^{\mathbf{1}/\nu} + \kappa_{\mathbf{2}}(\mathbf{U} - \mathbf{U_c})^{\mathbf{2}}\mathbf{L}^{\mathbf{2}/\nu} + \ldots$$

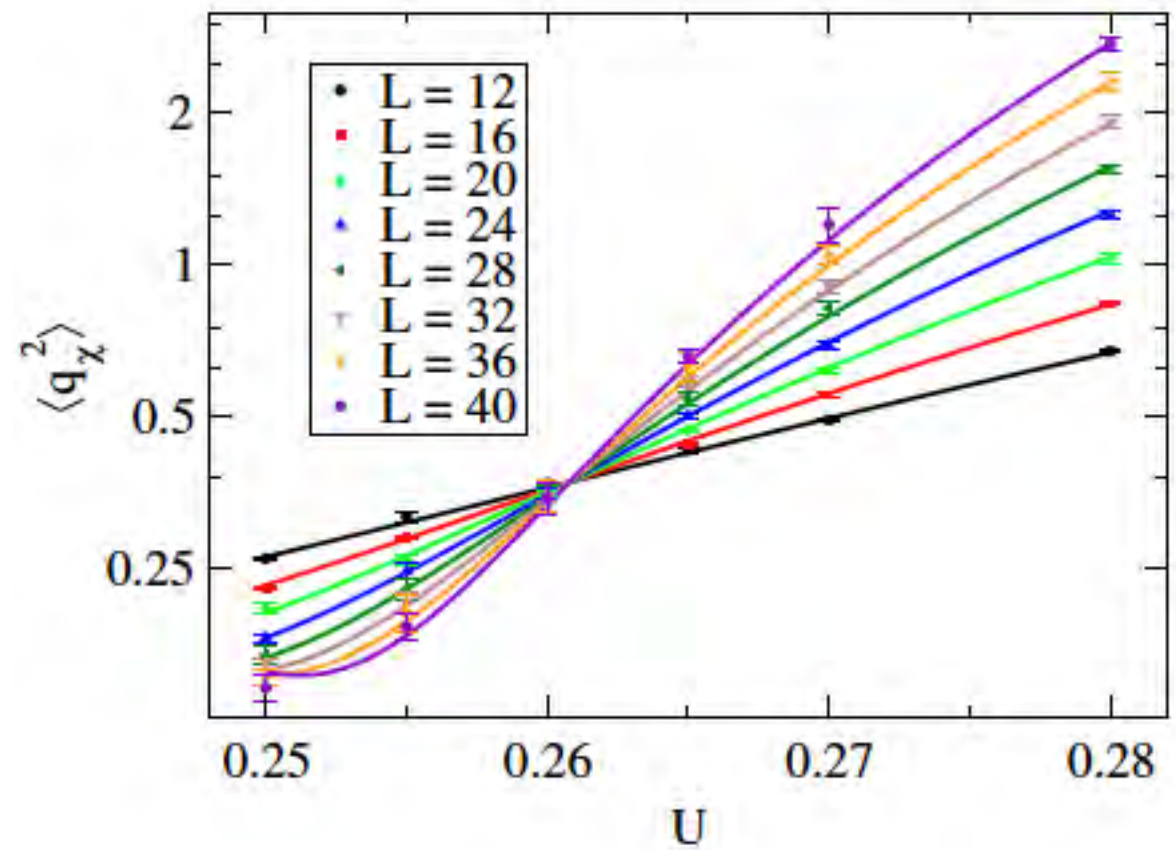
$$\mathbf{R_f}\ \mathbf{L}^{\mathbf{2}+\eta_\psi} = \mathbf{p_0} + \mathbf{p_1}(\mathbf{U} - \mathbf{U_c})\mathbf{L}^{\mathbf{1}/\nu} + \mathbf{p_2}(\mathbf{U} - \mathbf{U_c})^{\mathbf{2}}\mathbf{L}^{\mathbf{2}/\nu} + \ldots$$

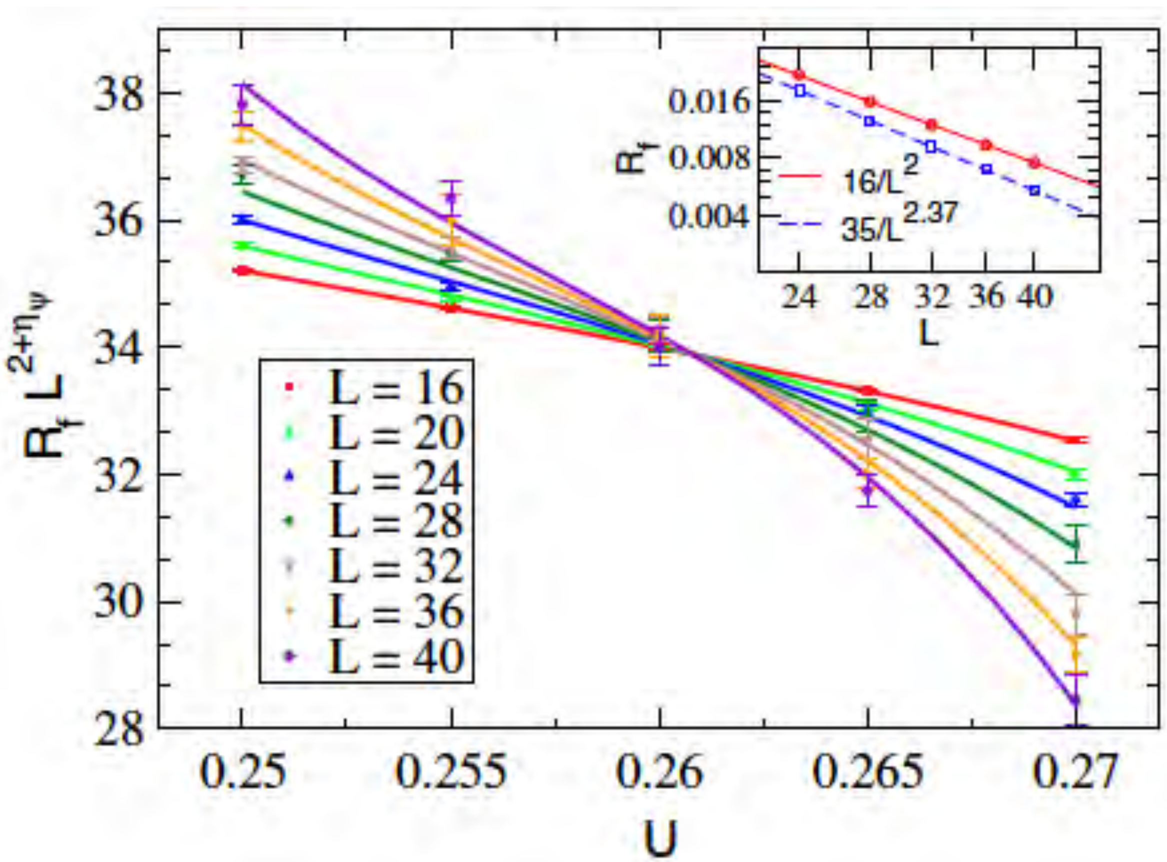
If we plot w.r.t U

all quantities must be independent of L at $U = U_c$

Thirring model results







Combined fit results

$U_c = 0.2608(2)$

$\nu = 0.85(1)$

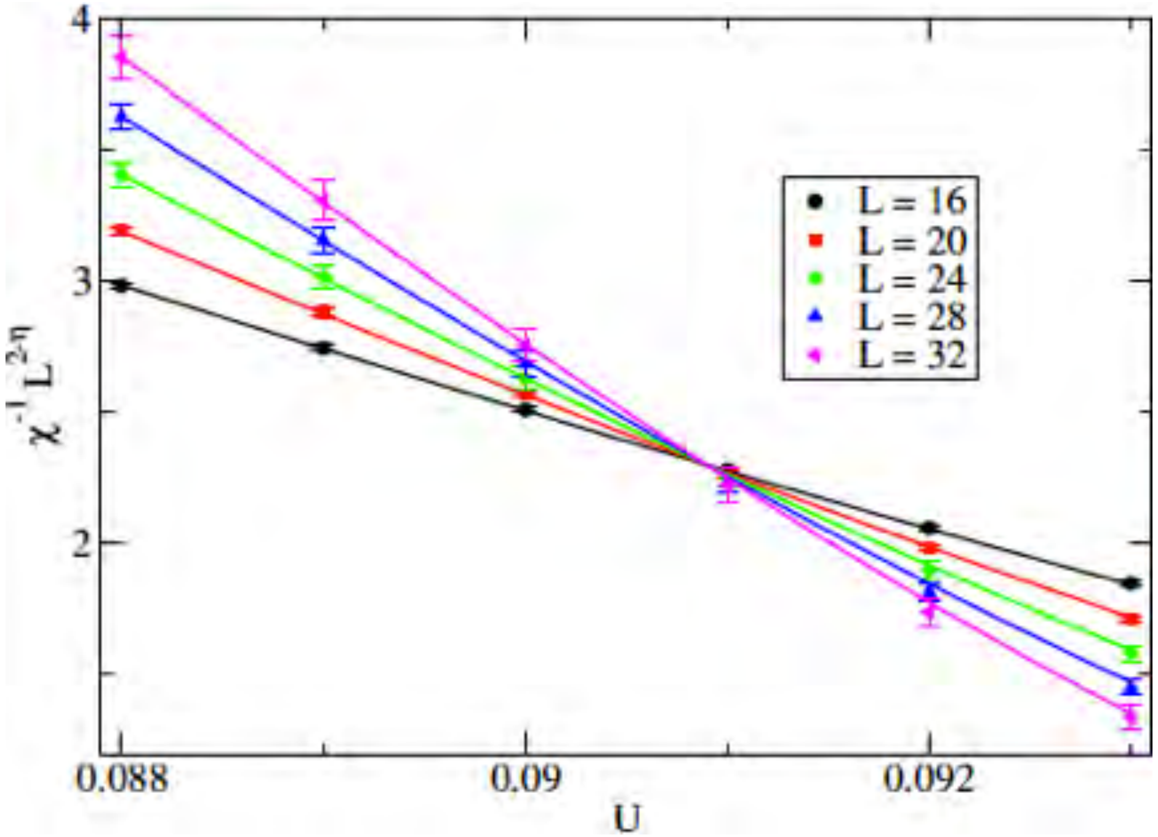
$\eta = 0.65(1)$

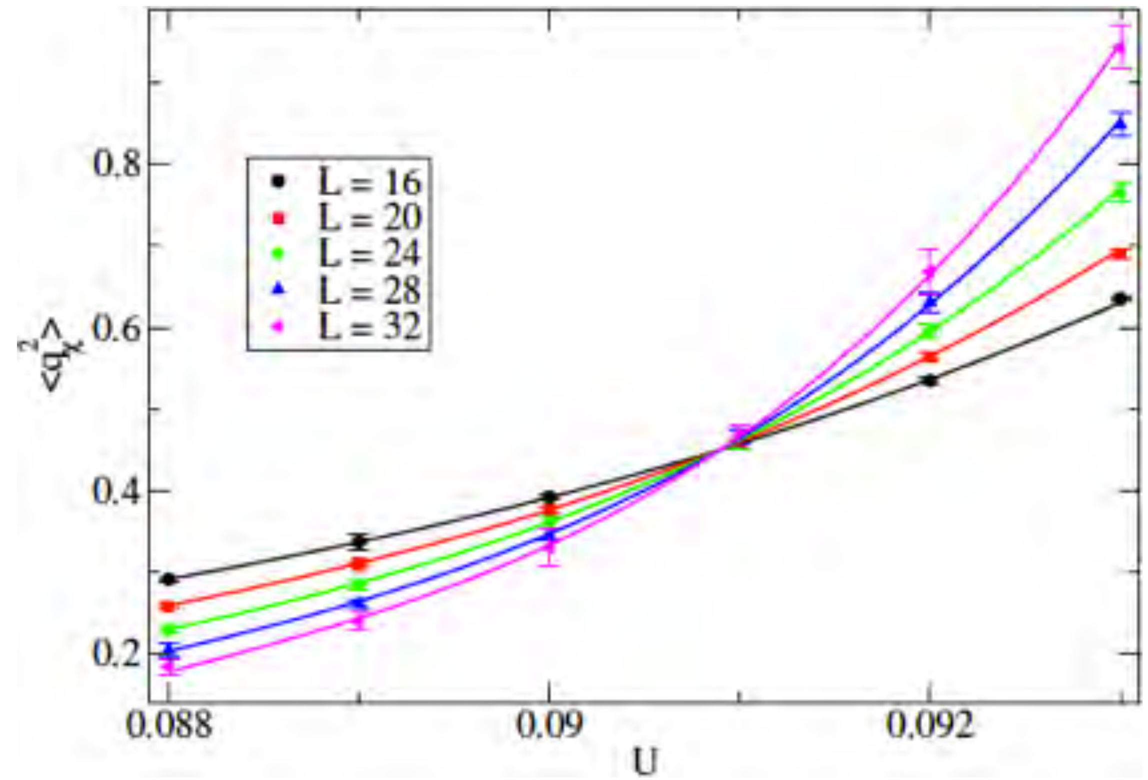
$\eta_\psi = 0.37(1)$

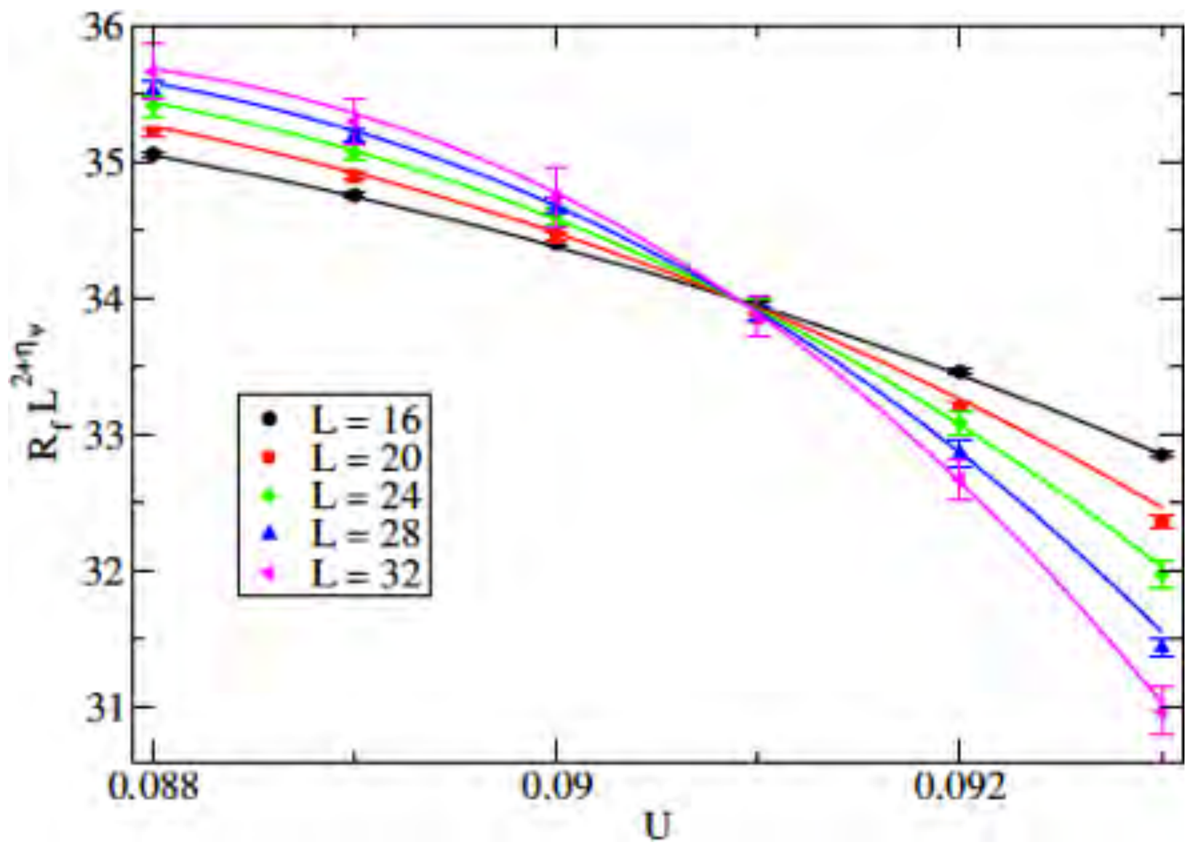
Comparison with previous work

Work	Range of L	Range of m	U_c	ν	η	η_ψ
Mean Field Theory Lee & Shrock PRL (1987)	N/A	0	0.25	1.0	1.0	0.0
Hybrid Monte Carlo Debbio & Hands, PLB (1997)	8-12	0.4-0.02	0.250(10)	0.80(15)	0.7(15)	??
Hybrid Monte Carlo Barbour et. al., PRD (1998)	16-24	0.06-0.01	0.250(06)	0.80(20)	0.4(2)	??
Fermion Bag S.C & A. Li (our work)	12-40	0	0.2608(2)	0.85(1)	0.65(1)	0.37(1)

Gross-Neveu model results







Combined fit results

$U_c = 0.1560(4)$

$\nu = 0.82(2)$

$\eta = 0.62(2)$

$\eta_\psi = 0.37(1)$

Lattice Thirring Model
(no sign problem in the traditional approach)

same universality class

Lattice GN Model
(complex determinant in traditional approach)

surprising?

ALL previous (traditional) MC results for GN models
are well described by Large N_f

(Christoffi & Strouthos, JHEP(2007))

Large N_f : $\nu = 1$, $\eta = 1$, $\eta_\psi = 0$

Usually involve extra “doubling” to solve sign problems

Belief : Sign problems is NOT a problem in GN models

Our results show clear
deviations with the large N_f results!

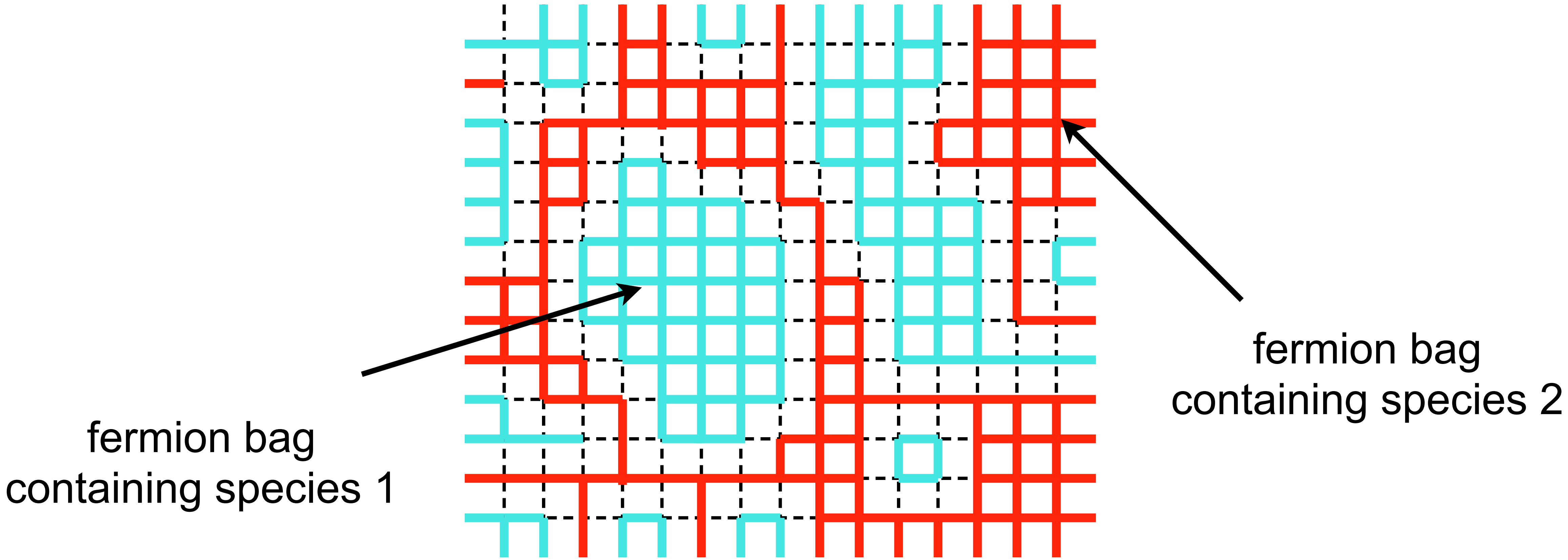
$\nu \approx 0.85$, $\eta \approx 0.65$, $\eta_\psi \approx 0.37$

Our critical exponents in a related model
with $SU(2) \times Z_2$ symmetry
disagrees with earlier work that ignored sign problem!

Karkkainen, Lacaze, Lacock and Petersson, NPB (1994)

Conclusion : Sign Problem is important!

Two spin-half species model with infinite repulsion



Fermion Bag Configuration

$$Z = \sum_{B} |\mathrm{Det}(W_A)|^2 \; |\mathrm{Det}(W_B)|^2$$

Conclusions

Fermion bag approach
is an alternative method to solve
fermion sign problems

Many new sign problems can be solved with it!

Solutions require thought and
understanding of the underlying physics

Solution to sign problems
is an interesting field of research
at the cross roads of
mathematical and computational physics