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## THE ISOOR BEDING ONLIGAL EFT

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# Why?

Chiral "EFT" potentials based on Weinberg's power counting widely used in nuclear physics because of their supposed link to QCD

Problem: Weinberg's power counting inconsistent with renormalization Solution: Certain counterterms appear at lower order than expected; subleading terms should be treated in perturbation theory

Kaplan, Savage + Wise '96, …, Nogga, Timmermans + Nogga '05, …

## VS.

The problem doesn't exist: Renormalization not important Epelbaum + Meissner '06, …, Epelbaum + Gegelia '09, …

Anyway, there is a solution for the problem that doesn't exist: Relativity essential in a non-relativistic problem Epelbaum + Gegelia '12

(Oh, yeah, this solution doesn't completely solve the problem that doesn't exist ---counterterms still need to be promoted--- but that is a detail which barely needs acknowledgement…)

the talk yesterday

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# **Outline**

**Effective field theory & model spaces** 

- O Pre-story: ChiPT
- $\Box$  The story
- Conclusion & Outlook

Weinberg, Wilson, ...



$$
T = T^{(\infty)}(Q) \sim N(M) \sum_{v=v_{\min}}^{\infty} \sum_{i} \tilde{c}_{v,i}(\Lambda) \left[ \frac{Q}{M} \right]^{v} F_{v,i} \left( \frac{Q}{m}, \frac{\Lambda}{m} \right)
$$
  
normalization  

$$
\frac{\partial T}{\partial \Lambda} = 0
$$
from loops  

$$
v = v(d, n, ...)
$$
power counting"  
log. # loops L

For 
$$
Q \sim m
$$
, truncate ...   
\n
$$
T = T^{(\overline{v})} \left[ 1 + \mathcal{O}\left(\frac{Q}{M}, \frac{Q}{\Lambda}\right) \right] \implies \frac{\Lambda}{T^{(\overline{v})}} \frac{\partial T^{(\overline{v})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q}{\Lambda}\right) \ll 1
$$
\ncontrolled  
\n
$$
\frac{\text{controlled} \times M}{\text{realistic estimate of errors comes from variation } \Lambda \in [M, \infty)} \left\{ \begin{array}{l} \text{want } \Lambda \geq M \\ \text{realistic estimate of errors comes from variation } \Lambda \in [M, \infty) \end{array} \right\}
$$

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#### Cutoffs define "model spaces"



 $\Lambda \geq M$  $\tilde{\lambda} \stackrel{\sim}{\ll} Q$ To minimize "model space" error (to "converge"), want

### Popular examples



o  $\Box$ **0000000000** n  $0.1$ ö  $3<sub>H</sub>$ **COOCCOOO**  $\circ$  $\circ$  $\circ$ Idaho $\mathrm{N}^3\mathrm{LO}$  $|\Delta E/E|$  $0.1 \begin{array}{ccccc}\n\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\Delta\end{array}$  $0.01 \hbar\omega$  $\lambda$  $\rm ^3H$  $(MeV/c)$  $(MeV)$ Idaho ${\rm N^3LO}$ 162  $\blacksquare$  28  $•35$ 181  $\frac{|\Delta E/E|}{0.01}$  $\blacktriangle$  45 206  $0.001 \Lambda$  (MeV/c) 247  $\bullet$  65  $\infty$  $\Box$  80 274 400  $\frac{1}{2}$  100 306 □  $\Delta$ 150 375 500  $\sqrt{200}$ 433 600 0.0001  $\frac{1500}{\Lambda (\text{MeV/c})}$ 2000 2500 700 500 1000 3000  $0.001$ 800 scaling 1000 1200  $\begin{tabular}{l} \hline \hline \hline \end{tabular} \begin{tabular}{l} \hline \end{tabular} \begin{tabular}{l} \hline \end{tabular} \begin{tabular}{l} \hline \end{tabular} \end{tabular} \begin{tabular}{l} \hline \end{tabular} \begin{tabular}{l} \hline \end{tabular} \end{tabular} \begin{tabular}{l} \hline \end{tabular} \begin{tabular}{l} \hline \end{tabular} \end{tabular} \begin{tabular}{l} \hline \end{tabular} \end{tabular} \begin{tabular}{l} \hline \end{tabular} \begin{tabular}{l} \hline \end{tabular} \end{$  $0.0001 \overline{20}$ गाममा ┯┯┯┱ 50 60 70 80  $0.1 10$ 30 40 90 **AAAAA**AAAAAAAAA  $\lambda_{sc}$  (MeV/c)  ${}^{3}H$  $\lambda^2$  $=\frac{\lambda^2}{\Lambda}\sim\frac{1}{L}$ Idaho  $N^3LO$  $|\Delta E/E|$  $\Lambda$  (MeV/c)  $0.01$ 400  $\Box$ 000000 500  $\circ$ 600 for much more see 700 Furnstahl, Hagen + Papenbrock '12 800 0.001 1000 More et al. '13 1200  $0.0001 -$ 200 250 300 8 50 100 350 150  $\lambda$  (MeV/c) www.getcliparts.com

#### Extrapolations in a HO basis

## Chiral EFT

### $Q \sim m_{\pi} \ll M_{\text{oCD}} \sim 1$  GeV

 $N = \left(\begin{array}{c} p \end{array}\right)$ 

*n*

 $=\begin{pmatrix} P \\ n \end{pmatrix} \qquad \Delta = \begin{pmatrix} \Delta \\ \Delta^0 \end{pmatrix}$ 

 $\left( p\right)$ 

+ d.o.f.s: pions, nucleons, deltas  $(m_\Delta - m_N \sim 2 m_\pi)$ 

2

• symmetries: Lorentz, P, T, chiral  $=\begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}=\begin{pmatrix} (\pi^+ + \pi^-)/\sqrt{2} \\ -i(\pi^+ - \pi^-)/\sqrt{2} \\ \pi^0 \end{pmatrix}$ 

 $(\pi^+ + \pi^-)$ 

 $+$  +  $-$ 

 $\pi$  +  $\pi$ 

 $\pi$ ,  $=$   $-$ **l**  $\pi$   $\pi$ 

 $\pi$ <sub>3</sub> )  $\pi$ 

 $(\pi^+ - \pi^-)$ 

 $+$   $-$ 

0

 $i(\pi^+ - \pi^-)/\sqrt{2}$ 

$$
f_{\pi} \simeq 92 \,\text{MeV} = \mathcal{O}\left(M_{\text{QCD}}/4\pi\right)
$$

1

π

**π**

2

3

spontaneously broken: non-linear realization

Weinberg '68 Callan, Coleman, Wess + Zumino '69

9

#### chiral invariants

(chiral) covariant

$$
\mathbf{D}_{\mu} = \left(\frac{\partial_{\mu}\pi}{2f_{\pi}}\right) \left(1 - \frac{\pi^2}{4f_{\pi}^2} + \ldots\right)
$$

2 2 *f*  $\partial_{\mu} \equiv \partial_{\mu} - 2i \mathbf{T} \cdot \frac{\partial}{\partial c} \times D_{\mu}$ π  $\begin{pmatrix} \pi & \pi \end{pmatrix}$  $\equiv \partial_{\mu} - 2i \mathbf{T} \cdot \left( \frac{\kappa}{2 f_{\pi}} \times \mathbf{D}_{\mu} \right)$ derivatives  $D_{\mu} \equiv \partial_{\mu} - 2i \mathbf{T} \cdot \left( \frac{\pi}{2 \epsilon} \times D \right)$ baryon, isospin **T**

+ chiral breaking as in quark mass terms

++

 $\left(\right. \Delta^{++} \left. \right)$ 

 $\Delta = \left | \begin{array}{c} \Delta^+ \ \Delta^0 \end{array} \right |$ 

+

−

 $\begin{pmatrix} 1 \ 0 \end{pmatrix}$ 

non-derivative interactions proportional to masses

$$
m_{\pi}^2 = \mathcal{O}\left(\left(m_u + m_d\right)M_{\mathcal{QCD}}\right)
$$

### Pre-story: ChiPT

Weinberg '79 Gasser + Leutwyler '84 Manohar + Georgi '84

…

Example: pion sector (similar in one-nucleon sector)

$$
\mathcal{L}_{f=0} = 2f_{\pi}^{2} \mathbf{D}_{\mu} \cdot \mathbf{D}^{\mu} - \frac{1}{2} m_{\pi}^{2} \pi^{2} \left( 1 - \frac{\pi^{2}}{4f_{\pi}^{2}} + ... \right) \mathbf{R}
$$
  
+  $c_{1} f_{\pi}^{2} \left( \mathbf{D}_{\mu} \cdot \mathbf{D}^{\mu} \right)^{2} + c_{2} f_{\pi}^{2} \mathbf{D}_{\mu} \cdot \mathbf{D}_{\nu} \mathbf{D}^{\mu} \cdot \mathbf{D}^{\nu} + c_{3} m_{\pi}^{2} \mathbf{D}_{\mu} \cdot \mathbf{D}^{\mu} \pi^{2} (1 + ...) + c_{4} \frac{m_{\pi}^{4}}{f_{\pi}^{2}} \pi^{4} (1 + ...)$   
+...



$$
\sum_{\substack{n=1\\n \text{odd } n}}^{\infty} + ... = \frac{1}{f_{\pi}^{4}} \int_{0}^{4} \frac{d^{4}l}{(2\pi)^{4}} \frac{(l, k, m_{\pi})^{2}}{l^{2} - m_{\pi}^{2} - i\varepsilon} \frac{(l, k, m_{\pi})^{2}}{(l + k)^{2} - m_{\pi}^{2} - i\varepsilon}
$$
\n
$$
\sum_{\substack{n=1\\n \text{odd } n}}^{\infty} \frac{1}{f_{\pi}^{2}(4\pi f_{\pi})^{2}} \left\{ \frac{\mathbf{\Phi}}{\mathbf{A}}^{4} + \Lambda^{2}(\#\kappa^{2} + \#\m_{\pi}^{2}) + (\#\kappa^{4} + \#\m_{\pi}^{2}\kappa^{2} + \#\m_{\pi}^{4}) \left[ \ln\left(\frac{\Lambda}{m_{\pi}}\right) + \#\ln\left(\frac{k}{m_{\pi}}\right) \right] + \mathcal{O}\left(\frac{Q^{6}}{\Lambda^{2}}\right) \right\}
$$
\n
$$
\text{forbidden by} \qquad \sum_{\substack{n=1\\n \text{odd } n}}^{\infty} \frac{1}{\int_{0}^{2} (4\pi f_{\pi})^{2}} \left( \frac{\mathbf{\Phi}}{\mathbf{A}} \right)^{2} \left( \frac{\mathbf
$$

Generalizing,

$$
\mathcal{L}_{EFT} = \sum_{\{n,p,f\}} c_{\{n,p,f\}} \left( \frac{\mathbf{D}, \mathbf{D}, m_{\Delta} - m_{N}}{M_{QCD}} \right)^{n} \left( \frac{m_{\pi}^{2}}{M_{QCD}^{2}} \frac{\pi^{2}}{f_{\pi}^{2}} \right)^{n/2} \left( \frac{\psi^{+}\psi}{f_{\pi}^{2}M_{QCD}^{2}} \right)^{1/2} f_{\pi}^{2}M_{QCD}^{2}
$$
\n
$$
\text{calculated from QCD: lattice, ...} = \mathcal{O}\left(\varepsilon, \frac{\alpha}{4\pi}\right) \qquad \text{isospin conserving} \qquad \text{(NDA)}
$$
\n
$$
= \sum_{\Delta=0}^{\infty} \mathcal{L}^{(\Delta)} \qquad \Delta \equiv n + p + \frac{f}{2} - 2 \equiv d + \frac{f}{2} - 2 \geq 0 \qquad \text{``chiral symmetry} \qquad \text{chiral symmetry}
$$
\n
$$
T = T^{(\infty)}(Q) \sim N(M_{QCD}) \sum_{\nu = \nu_{\min}}^{\infty} \sum_{i} \tilde{c}_{\nu,i}(\Lambda) \left[ \frac{Q}{M_{QCD}} \right]^{\nu} F_{\nu,i} \left( \frac{Q}{m_{\pi}}, \frac{\Lambda}{m_{\pi}} \right)
$$
\n
$$
\nu = 2 - A + 2L + \sum_{i} V_{i} \Delta_{i} \geq \nu_{\min} = 2 - A
$$
\n
$$
\# \text{ nucleons} = 0, 1 \quad \text{# loops} \quad \# \text{ vertices of type } i
$$

## The story\*

The era of the scriptures The era of the scriptures

Weinberg '90, '91, '92 Ordonez + v.K. '92 v.K. '94 Ordonez, Ray + v.K. '94, '96 Brockmann, Kaiser + Weise '96 Gerstendoerfer, Kaiser + Weise '97 Friar '99

Kaiser '99 …

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 $\underline{1}$ 

…

0 *l*

$$
\left\{\frac{V}{V}\right\} = i \int \frac{d^4 l}{(2\pi)^4} V \frac{1}{l^0 + k^2 / m_N - l^2 / m_N - i\epsilon} \frac{1}{-l^0 + k^2 / m_N - l^2 / m_N - i\epsilon} V
$$
  

$$
\left\{\frac{1}{K} \sum_{m_N} \frac{d^3 l}{(2\pi)^3} V \frac{m_N}{l^2 - k^2} V \right\} \dots
$$

infrared enhancement: no ChiPT expansion for  $T$  for  $A \geq 2$ 

potential = sum of subdiagrams without IR enhancement: amenable to ChiPT expansion, cutoff absorbed in counterterms of NDA size

Weinberg's recipe ("W PC"): truncate potential, solve dynamical equation exactly [and, as always, check assumptions…]

\* Not a history, not even Whiggish

$$
V(\Lambda) \sim N(M) \sum_{v=v_{\min}}^{\infty} \sum_{i} \hat{c}_{v,i}(\Lambda) \left[ \frac{Q}{M} \right]^{\nu} f_{v,i} \left( \frac{Q}{m}, \frac{\Lambda}{m} \right)
$$
  

$$
\nu = 2 - A + 2L + \sum_{i} V_{i} \Delta_{i} \ge \nu_{\min} = 2 - A
$$

not an observable: in general depends on cutoff, form of dynamical equation, choice of nucleon fields, etc.



$$
V(\Lambda) \sim N(M) \sum_{v=v_{\min}}^{\infty} \sum_{i} \hat{c}_{v,i}(\Lambda) \left[ \frac{Q}{M} \right]^{\nu} f_{v,i} \left( \frac{Q}{m}, \frac{\Lambda}{m} \right)
$$
  

$$
\nu = 2 - A + 2L + \sum_{i} V_{i} \Delta_{i} \ge \nu_{\min} = 2 - A
$$

not an observable: in general depends on cutoff, form of dynamical equation, choice of nucleon fields, etc.

- $\triangleright$  Potential to O(Q^3) with and to O(Q^4) without delta isobar derived
- Fit of NN phase shifts to  $O(Q^3)$  with delta encouraging; similar accuracy (or lack thereof) for three cutoffs from 500 to 1000 MeV
- TPE potential to O(Q^3) without delta improves Nijmegen PWA
- $\triangleright$  Pions perturbative in F waves and higher

Weinberg '92 Rho '93 Park, Min + Rho '94 … Beane, Lee + v.K. '95

…

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Also, many processes with external probes:

- o pion elastic scattering
- o electroweak currents
- o pion photoproduction
- o pion production
- o Compton scattering

o …

### The Reformation The Reformation

 $\binom{I}{I}$  =  $\binom{I}{I_Y}$ 

 $=$   $($   $)$   $+$ 

 $\widehat{\chi}\bigg\rangle = / \sqrt{1 + \widehat{\chi}}$ 

 $\left(T_{Y}\right)$ 

Amplitude in 1S0 solved in semi-analytic form for W LO:

 $T_Y$ 

χ

Kaplan, Savage + Weise '96 Cohen + Phillips '97 Kaplan '97

… v.K. '97 Kaplan, Savage + Weise '98 Gegelia '98 Bedaque, Hammer + v.K. '98, ...

…



$$
\frac{4\pi}{m_N}I(k) = \#\Lambda + \#\frac{m_N}{4\pi f_\pi} \frac{m_\pi^2}{f_\pi} \ln\left(\frac{\Lambda}{m_\pi}\right) + \mathcal{O}\left(\frac{k^2}{\Lambda}\right)
$$

$$
c(m_{\pi}^{2}) = C_0 + D_2 m_{\pi}^{2} + \dots
$$
  
W PC: LO NLO

NDA fails for chiral symmetry-breaking operators: W PC not entirely correct

Detailed study of

renormalization, validity of NDA, perturbativity of subLOs, power counting, etc. in simpler pionless  $\mathsf{EFT}$  for  $Q < m_{\pi}$ 

Some lessons:

- 1) fine-tuning necessary for large scattering lengths can be incorporated into PC for amplitude
- 2) non-perturbative renormalization intrinsically different from renormalization of corresponding perturbative series
- 3) one gains no understanding of the renormalization of the A-body system by just monkeying around with higher-order terms in the A-1-body system
- 4) NDA has very limited usefulness; e.g., three-body force of very high order by NDA, but renormalization requires it at LO
- 5) subleading interactions must be treated in perturbation theory
- 6) fully consistent theory works well for very low-energy processes involving (at least) light nuclei and cold atoms, incorporating universal properties such as the Efimov effect, Phillips and Tjon lines, Wigner SU(4), ...; yet, mostly ignored by nuclear physics community

Moral: faced with W PC vs RG, choose RG

Proposal for perturbation approach to pion exchange in chiral EFT ("KSW PC")

#### Some Results

- 1) manifestly consistent PC
- 2) rescues NDA for chiral symmetry-breaking operators
- 3) converges only for Q < 100-150 MeV; at that point pion tensor force no longer perturbative







But, since Weinberg's PC inconsistent, then what?

Epelbaum, Gloeckle + Meissner '98 … Entem + Machleidt '03

#### The Counter-Reformation

I he Counter-Retormation

Ekstroem et al., last week

…

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Elevate cutoff to physical quantity constrained to  $M_{_{NN}} < \Lambda < M_{_{OCD}}$ 

Faced with W PC vs RG, choose W's PC

Countless improvements under W PC:

- 1) elimination of redundant operators
- 2) correction of some mistakes
- 3) smart choice of regulator (cutoff not on transferred momentum, to decouple effects of short-range interactions on various partial waves)
- 4) careful treatment of relativistic corrections

… N) fits to NN data at O(Q^4) without delta of similar quality as purely phenomenological pots

(But also some steps back, e.g., no deltas until recently, different regulators for different loops)

… Goes Viral Chiral "EFT" becomes input of choice for a new generation of ab initio methods for light and medium-mass nuclei **GOES VIPGI** 

### The Reckoning? The Reckoning?

Beane, Bedaque, Savage + v.K. '02 Nogga, Timmermans + v.K. '06 Pavon Valderrama + Ruiz Arriola '06 Birse '06

…

…

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Conjecture:  $M_{NN} > m_{\pi}$ 

Long + v.K. '08 Yang, Elster + Phillips '09 Pavon Valderrama '10, '11 Long + Yang '11, '12

so that one can think of T as an expansion around the chiral limit, only necessary resummation being that of the tensor force:

- $\circ$  singlet channels ~ KSW (solves the W problem with chiral symmetry breaking)
- $\circ$  triplet channels  $\sim$  W (solves the KSW problem of convergence)

However, W's PC fails also in triplets!

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That means some counterterms deemed to be subLO because of NDA are actually LO!

Nogga, Timmermans + v.K. '05

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Add needed 200 counterterms 150 at this order, 100 e.g., 50  $\begin{bmatrix} 50 \\ -50 \\ -50 \end{bmatrix}$  $V_{l=1, i=0} = \frac{c_1}{(2, 3)} pp$  $_{=1, j=0} = \frac{c_1}{(2\pi)^3} pp'$ 1  $\textbf{-}100$  $\overline{\phantom{i}^{l=1,~j=0}}$   $\overline{\phantom{i}^{l=1,~j=0}}$   $\overline{\phantom{i}^{l=1,~j=0}}$  $-150$  $-200$ 10 12 14 16 18 20  $\overline{2}$ 6 8 4  $\Lambda$  [fm<sup>-1</sup>] 200 cf. 150 3 100  $\left[\text{fm}^2\right]$ 50  $V_{l=0, j=1} = \frac{c}{\sqrt{2}}$  $t_{0,j=1} = \frac{c_t}{(2\pi)^{2}}$ *t*  $\boldsymbol{0}$  $\overline{^{0,j=1}}^{\,-\,}\left( 2\pi\right) ^{3}$  $\frac{1}{2}$  -50  $-100$  $-150$  $-200$ 12 14 16 18 20 2 6 8 4 10  $\Lambda \left[ \text{fm}^{-1} \right]$ 



0.012

0.011

1000

 $\boldsymbol{P}_0$ č  $\boldsymbol{P}_2$  $0.2$ ۰. 0.15  $0.1$ 2000 3000 4000 5000 1000 2000 3000 4000 5000  $\Lambda$  [MeV]  $\Lambda$  [MeV]

Yang, Elster + Phillips '09

28 That means some counterterms deemed to be subNNLO because of NDA are actually NNLO or lower!



Root of the problem:

pion exchanges (long-ranged, contribute to waves higher than S) are singular (sensitive to short-range physics, require counterterms)

This has **P.** Othing to do with relativity...

(For the opposite opinion, see Epelbaum + Gegelia '12)

New, emerging PC:

LO:

OPE plus needed counterterms

(one per wave where OPE is non-perturbative, singular, attractive)

subLOs:

 NPE given by ChPT plus counterterms given by NDA with respect to the lowest order they appear at, treated in perturbation theory

(contrast with Epelbaum + Gegelia '09, who suggest: if you cannot take a large cutoff when treating certain subLOs non-perturbatively, don't take a large cutoff. )



(Details still being worked out, e.g. at ESNT Saclay workshop two weeks ago)













 $\boxed{T^{(0)}}$ 

…

 $E^{(1)} = \langle \psi^{(0)} | V^{(1)} | \psi^{(0)} \rangle$ 

smaller



new PC



#### Fits to data Pavon Valderrama '10, '11 Long + Yang '11, 12

bands (not error estimates): coordinate-space cutoff variation 0.6 – 0.9 fm

cyan: NNLO in Weinberg's scheme

Pavon Valderrama '10



### Conclusion & Outlook

- $\triangleright$  much has been learned about  $EFT$  in a non-perturbative context
- $\triangleright$  non-analytic parts of long-range pots derived
- $\triangleright$  a chiral EFT NN amplitude consistent with RG being constructed
- $\triangleright$  compared to the NN amplitude obtained with W PC: it contains more counterterms (thus parameters) at a given order but subLOs require perturbation theory (sorry, but that is what physics asks of you)
- > details still being worked out, but first results suggest possibility of better fits to data than W PC; perhaps a "realistic" amplitude emerges at NNNLO?
- $\triangleright$  few-body forces and currents remain to be studied; effects could be substantial since they are tied to NN amplitude