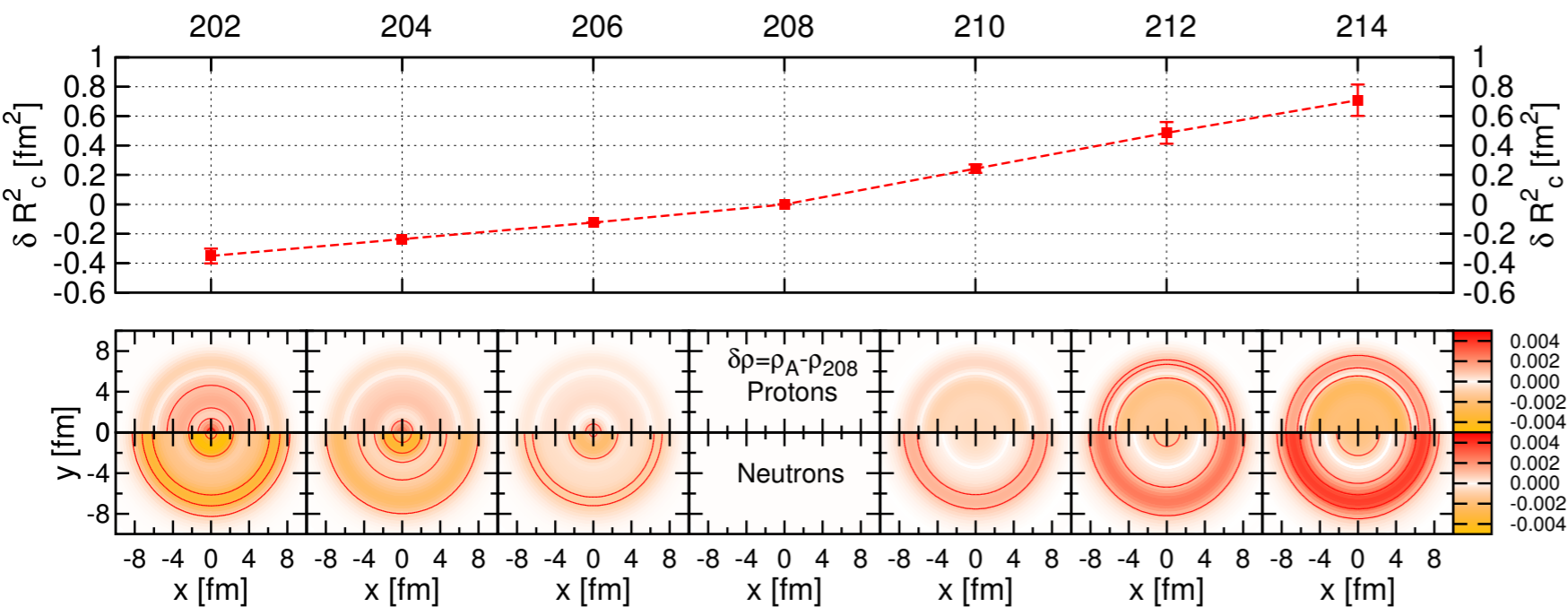


# Why is lead so kinky?

## Charge Radius Isotope Shift Across the N=126 Shell Gap

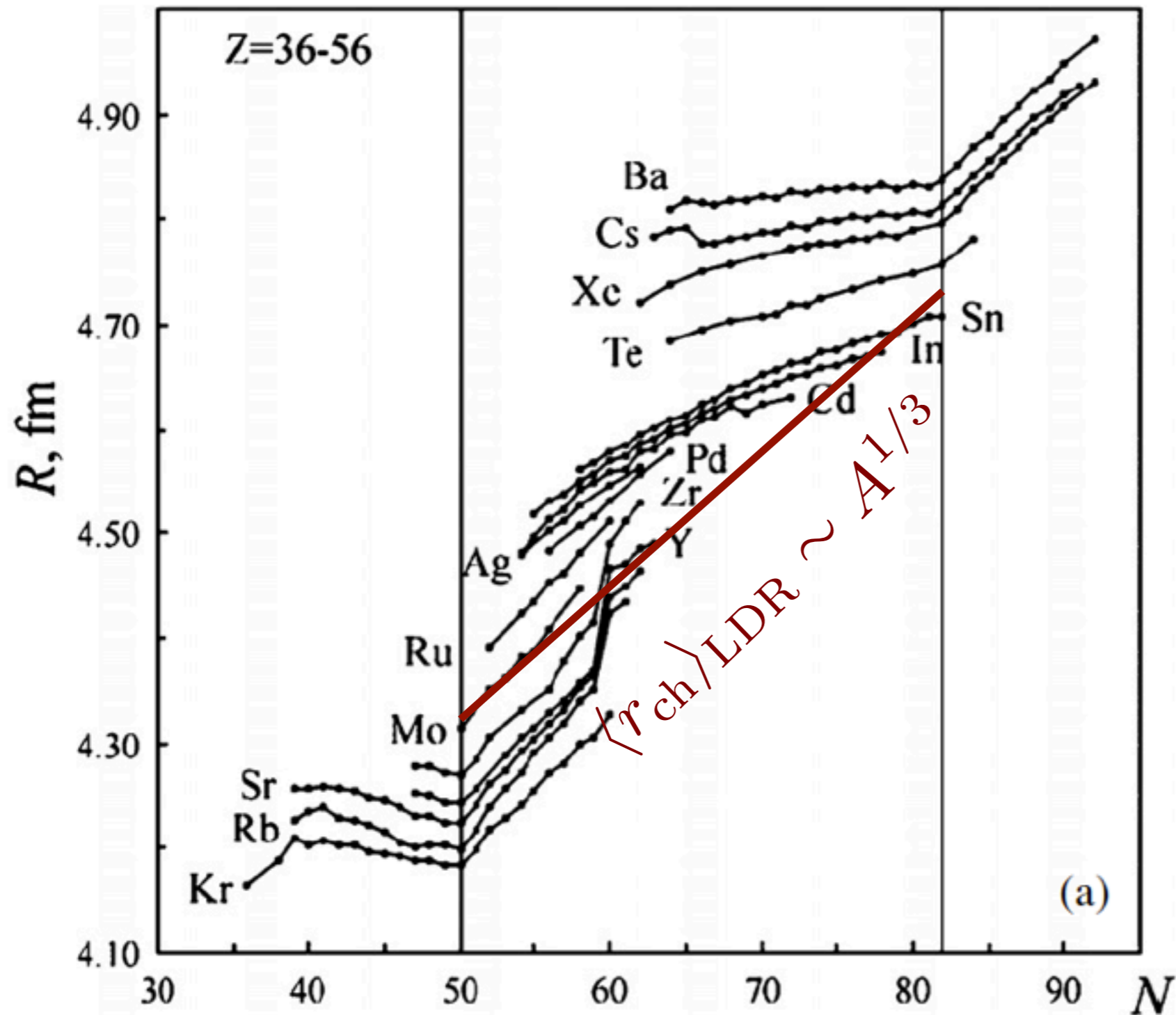


Arnau Rios Huguet  
STFC Advanced Fellow  
Department of Physics  
University of Surrey

P. Goddard, P. Stevenson & A.R., *Phys. Rev. Lett.* **110**, 032503 (2013)

# Nuclear radii

## Medium mass nuclei systematics



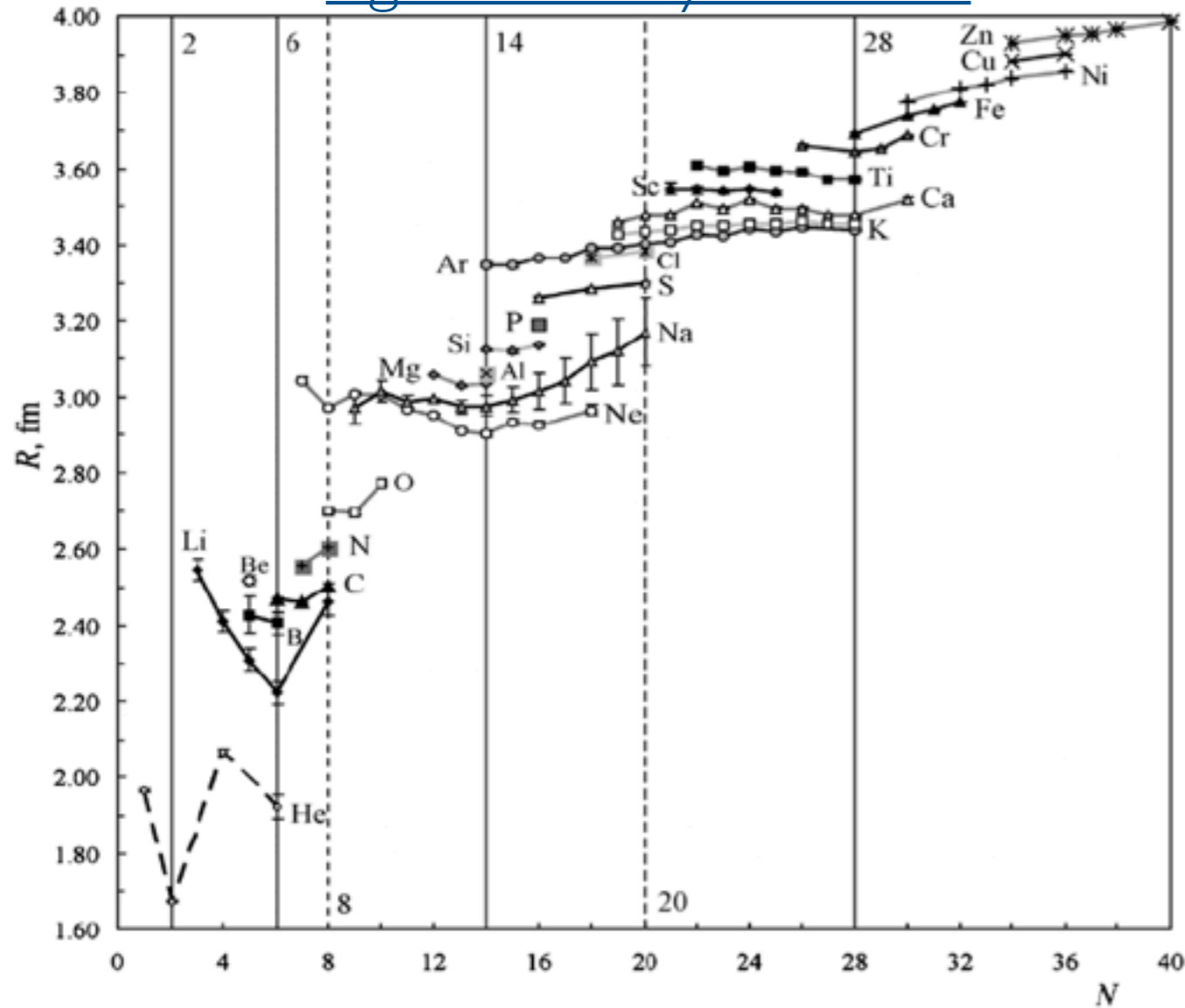
- **Kinks** are ubiquitous
- **Shell** effects influence radii
- **1/3 power** valid in **specific cases**

$$\langle r_{ch} \rangle_{emp} \sim A^{0.003Z}$$

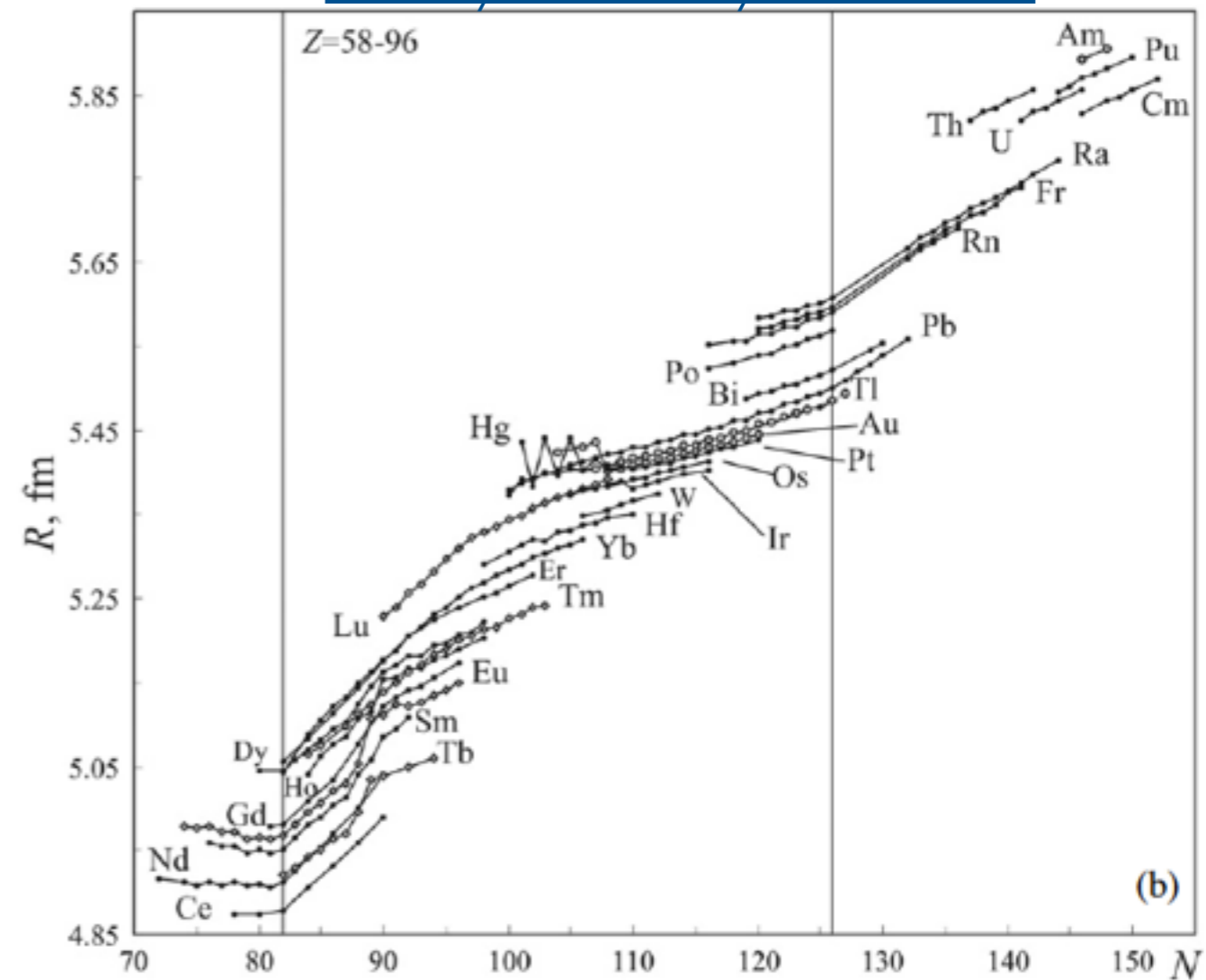
# Nuclear radii

## Experiments

Light nuclei systematics



Heavy nuclei systematics

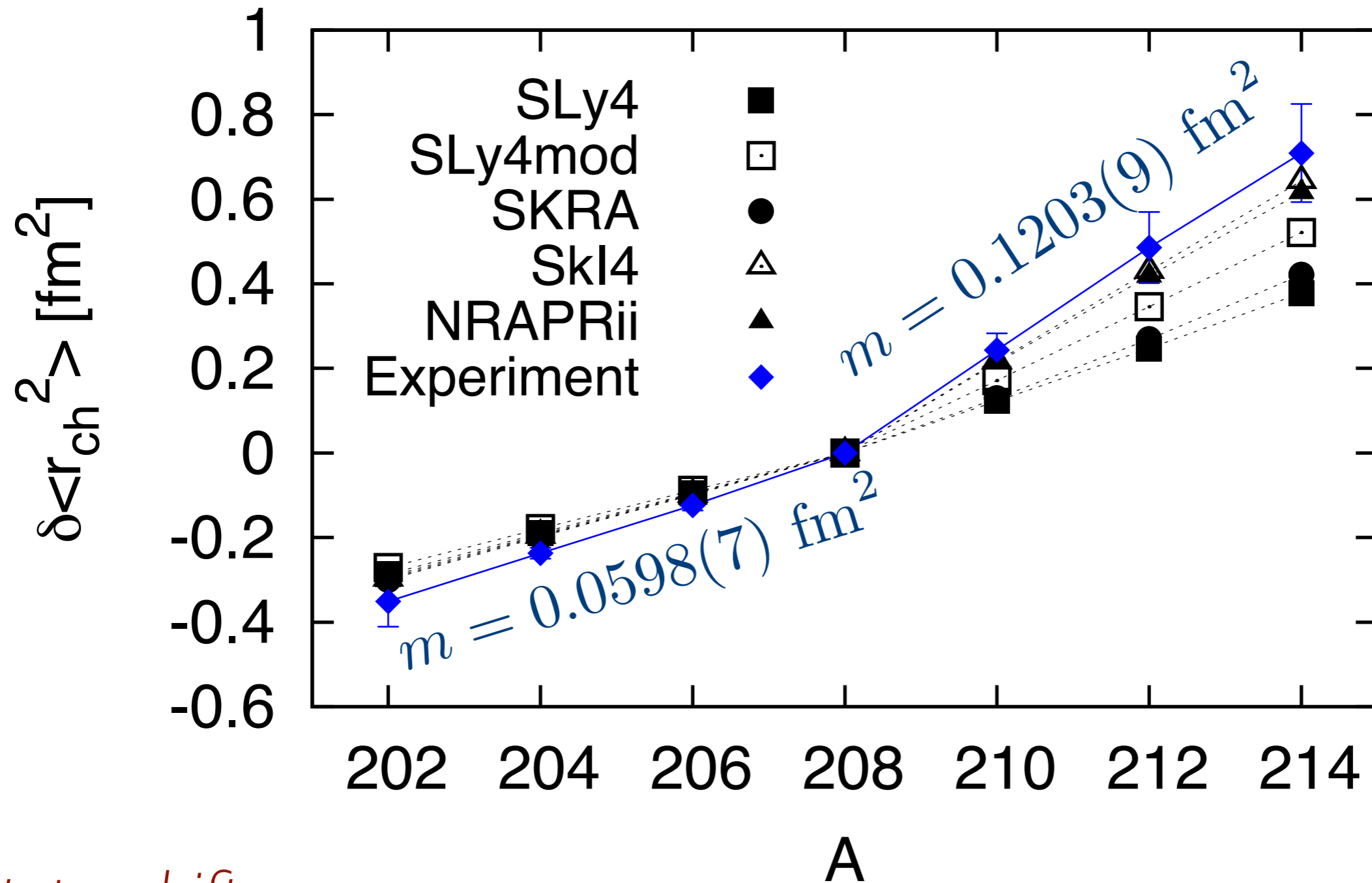


$$\langle r_{\text{ch}} \rangle_{\text{RMS}} \sim A^{1/3}?$$

*Isotope shift in droplet model*

$$\delta \langle r_{\text{ch}}^2 \rangle = \langle r_{\text{ch}}^2 \rangle_A - \langle r_{\text{ch}}^2 \rangle_{A'} \sim 0.575 \frac{\delta A}{A^{1/3}}$$

## Isotope shifts in lead isotopes: theory vs experiment



### Isotope shifts

$$\delta \langle r_{ch}^2 \rangle = \langle r_{ch}^2 \rangle_A - \langle r_{ch}^2 \rangle_{208} = m(A - 208)$$

$$m_{\text{LDR}} = 0.0972 \text{ fm}^2$$

# Nuclear radii: experiments

## 4 methods to extract radii from experiments:

Angeli et al., *J. Phys. G: Nucl. Part. Phys.* **36** 085102 (2009)

### 1. Transition energies in muonic atoms

$$a_{\mu} = \frac{\hbar}{m_{\mu}c\alpha} = \frac{m_e}{m_{\mu}}a_0 \sim 200 \text{ fm}$$

$$E \sim B_{k,\alpha} = \int dr r^k \rho(r) e^{-\alpha r}$$

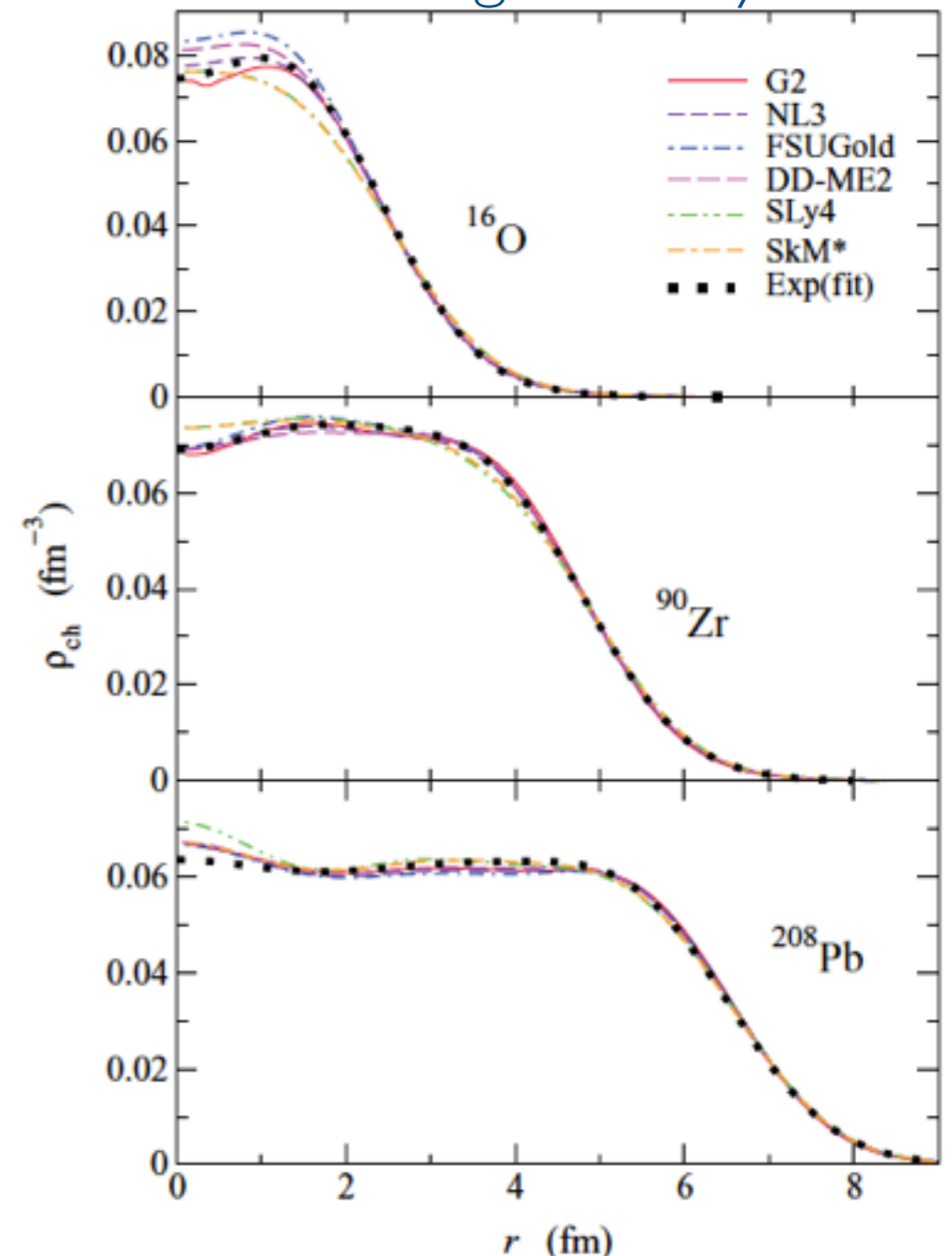
### 2. Elastic electron scattering

$$\langle r_{\text{ch}}^2 \rangle = \frac{\int dr r^4 \rho_{\text{ch}}(r)}{\int dr r^2 \rho_{\text{ch}}(r)}$$

### 3. X-ray isotope shifts

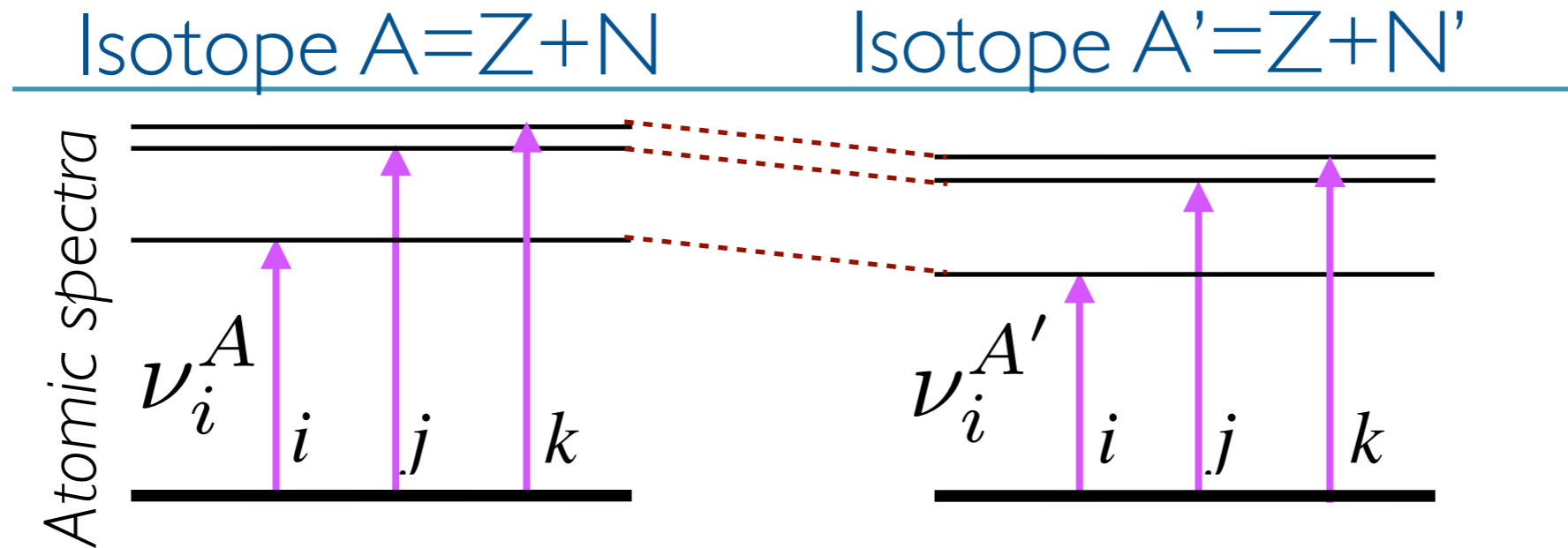
### 4. Optical isotope shifts

### Charge density



# Isotope shifts

Atoms meet nuclei

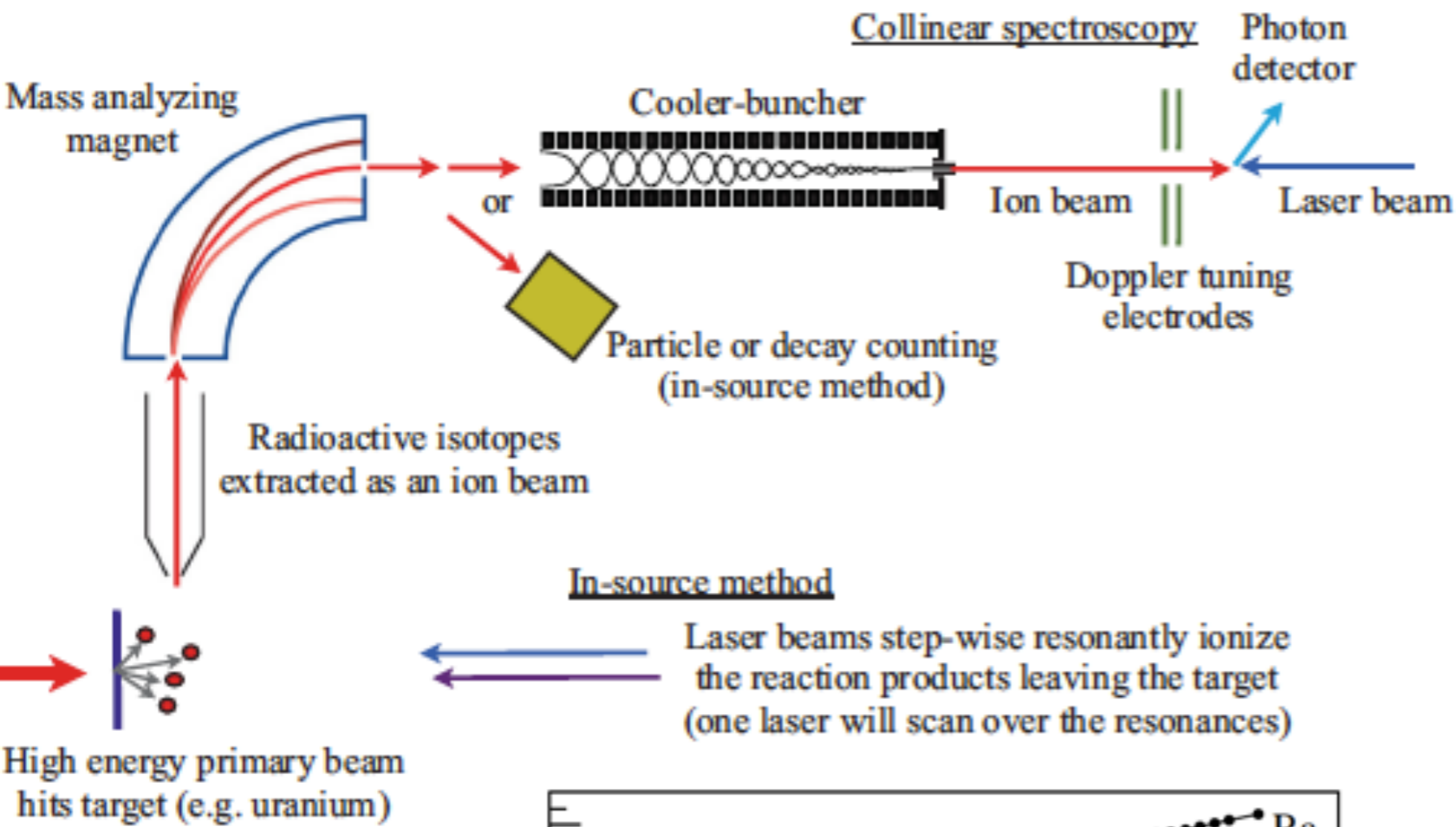


Isotope shifts

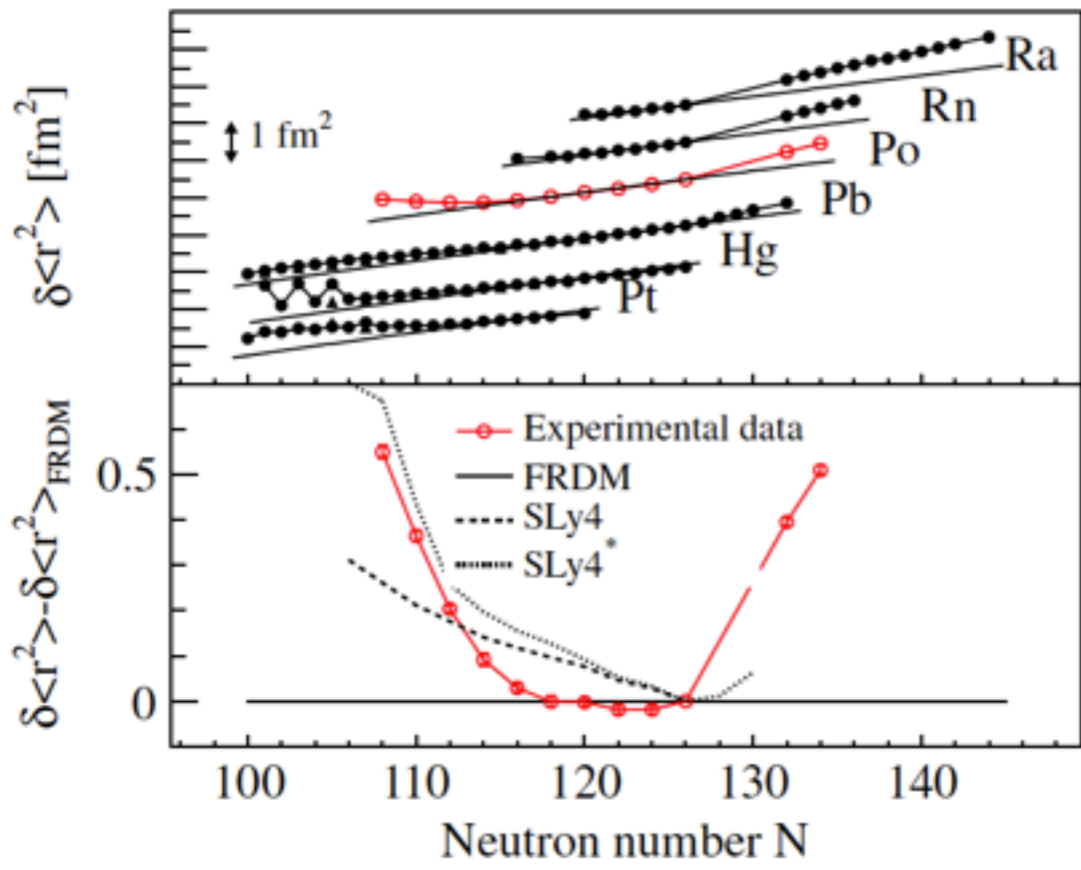
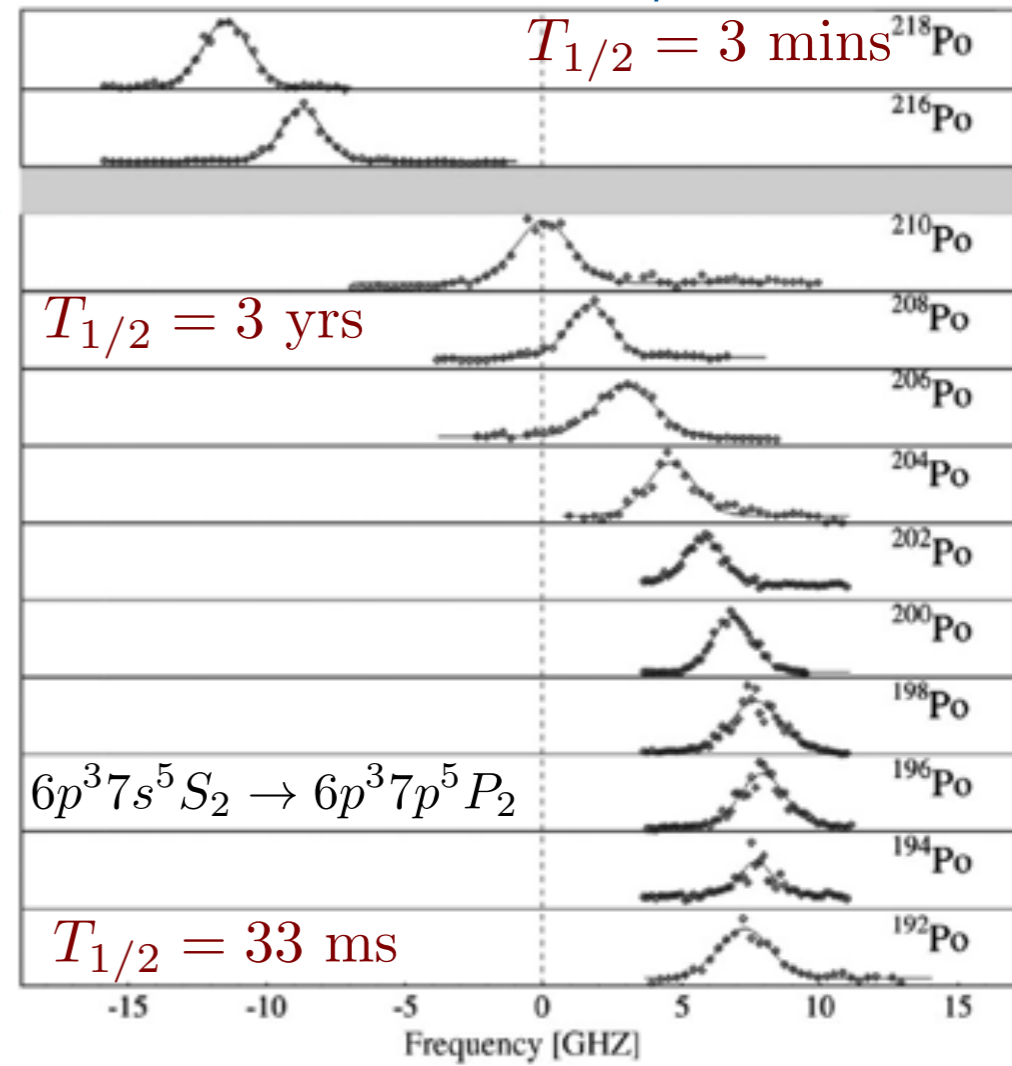
$$\delta\nu_i^{A,A'} = M_i \frac{A' - A}{AA'} + F_i \delta\langle r^2 \rangle^{A,A'}$$

- Mass,  $M_i$ , and field,  $F_i$ , shifts obtained **theoretically** or empirically
- Isotope shift **separation** is possible  $\Rightarrow$  **proliferation** issues

# Laser spectroscopy in unstable beams



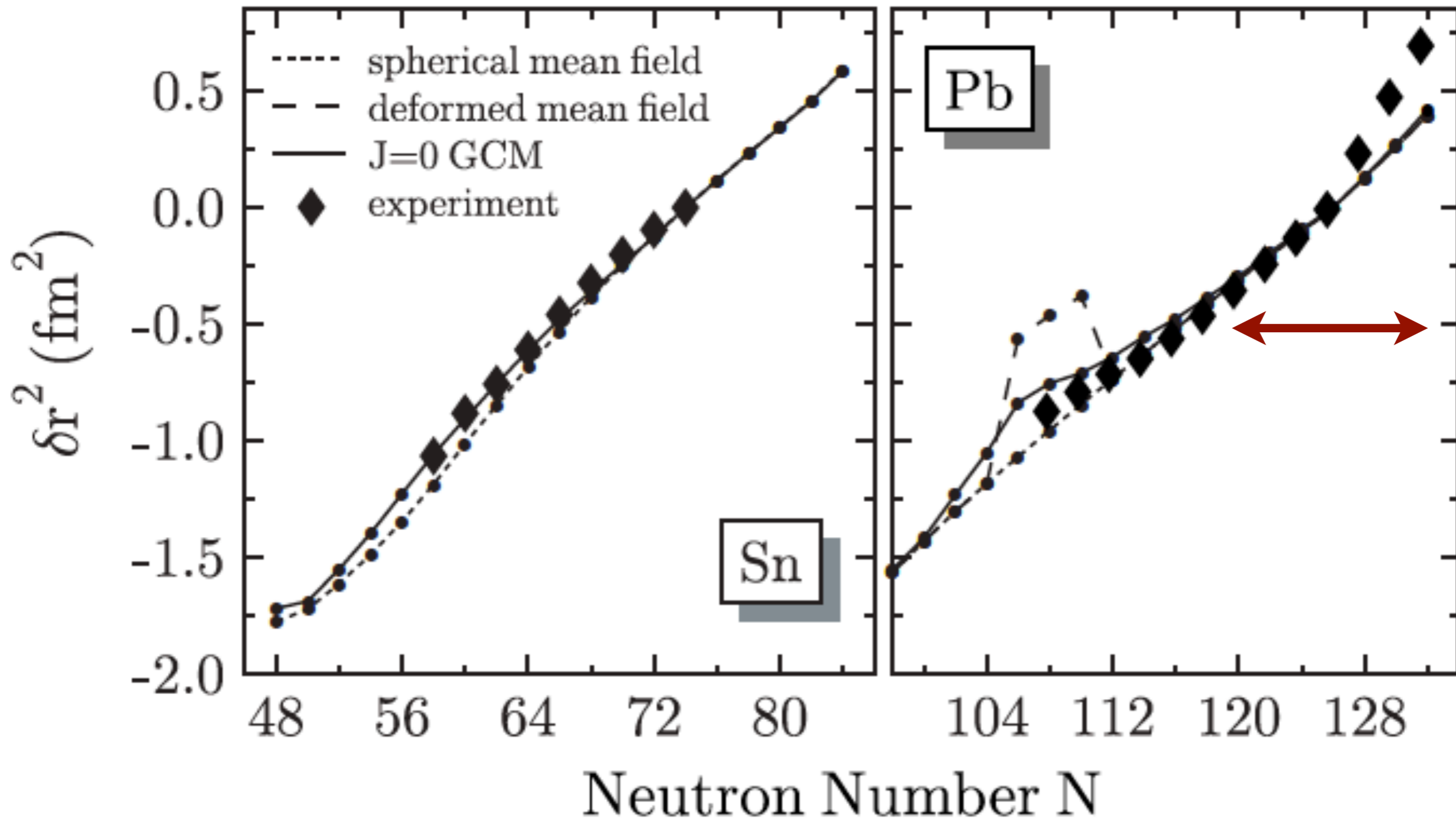
## Polonium isotopic shifts



Cocolios et al., *Phys. Rev. Lett.* **106**, 052503 (2011)

# Quadrupole correlations?

Beyond mean-field calculations of isotope shifts



Observation: **correlations do not affect kink mass region**



# Previous proposal

- Skyrme force yields neutron spin-orbit term:

$$W_{\text{SHF}} = b_4(\nabla\rho + \nabla\rho_n)$$

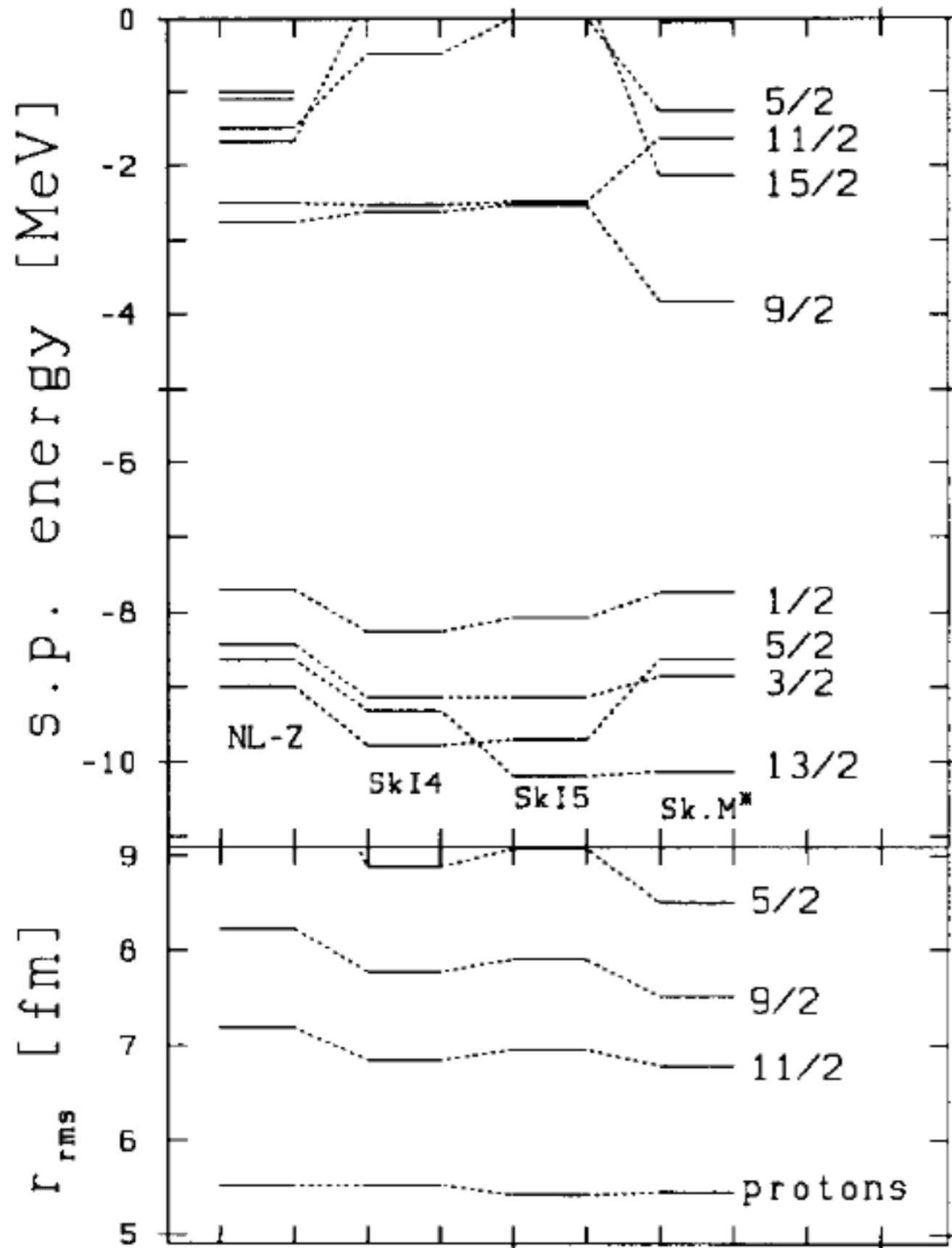
- Relativistic EDF yields:

$$W_{\text{RMF}} = \frac{\hbar^2}{(2m - C\rho)^2} C \nabla\rho$$

- Different isospin dependence?
- Try richer alternative in Skyrme:

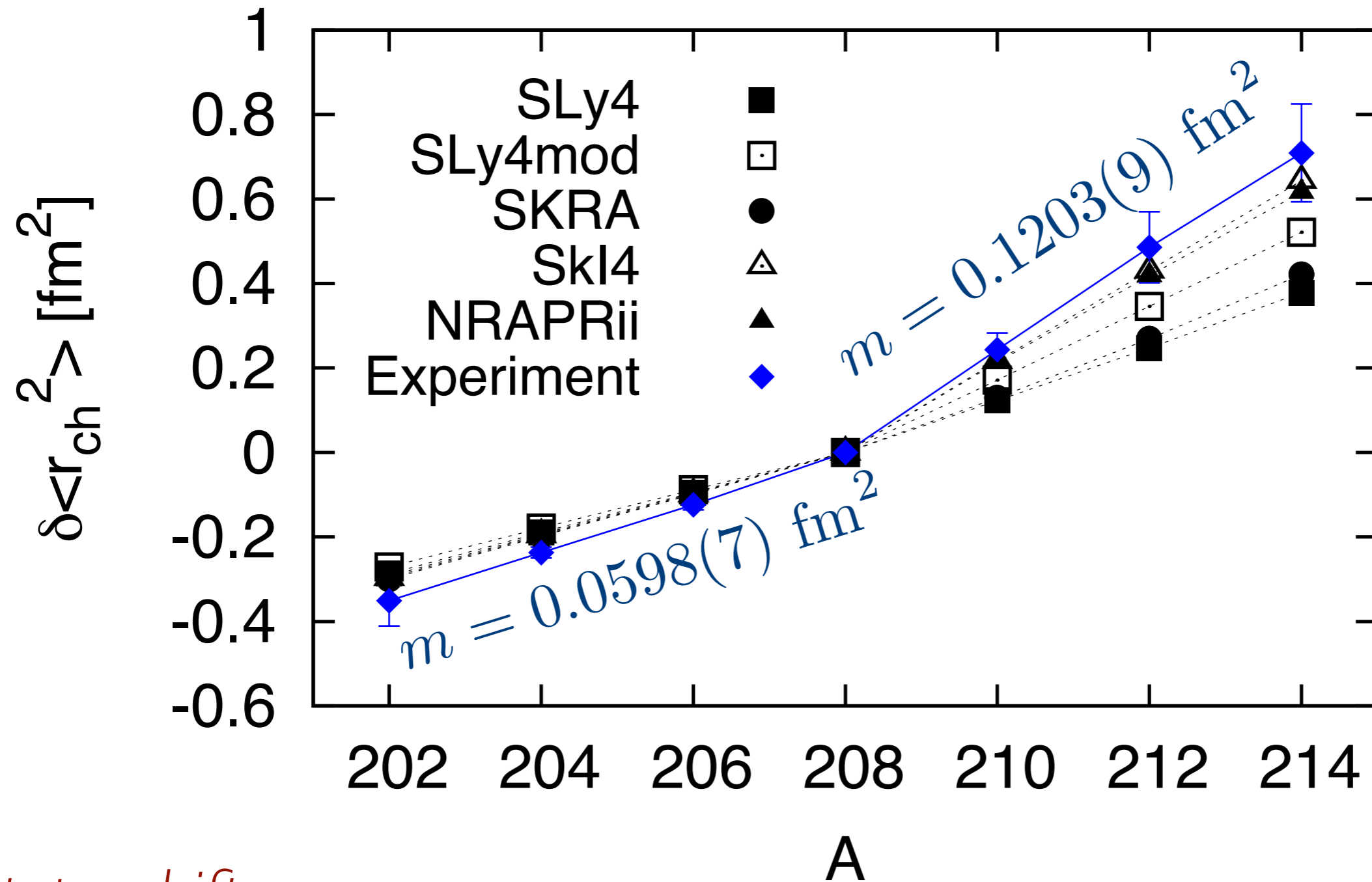
$$W_{\text{SHF}} = b_4 \nabla\rho + b'_4 \nabla\rho_n$$

# Previous proposal II



- Position of  $2g_{9/2}$  relevant
- This state is affected by SO
- When less bound, sp radius is larger
- Pull on protons (via symmetry energy) should be larger
- Charge radius larger when  $2g_{9/2}$  less bound

## Isotope shifts in lead isotopes: theory vs experiment



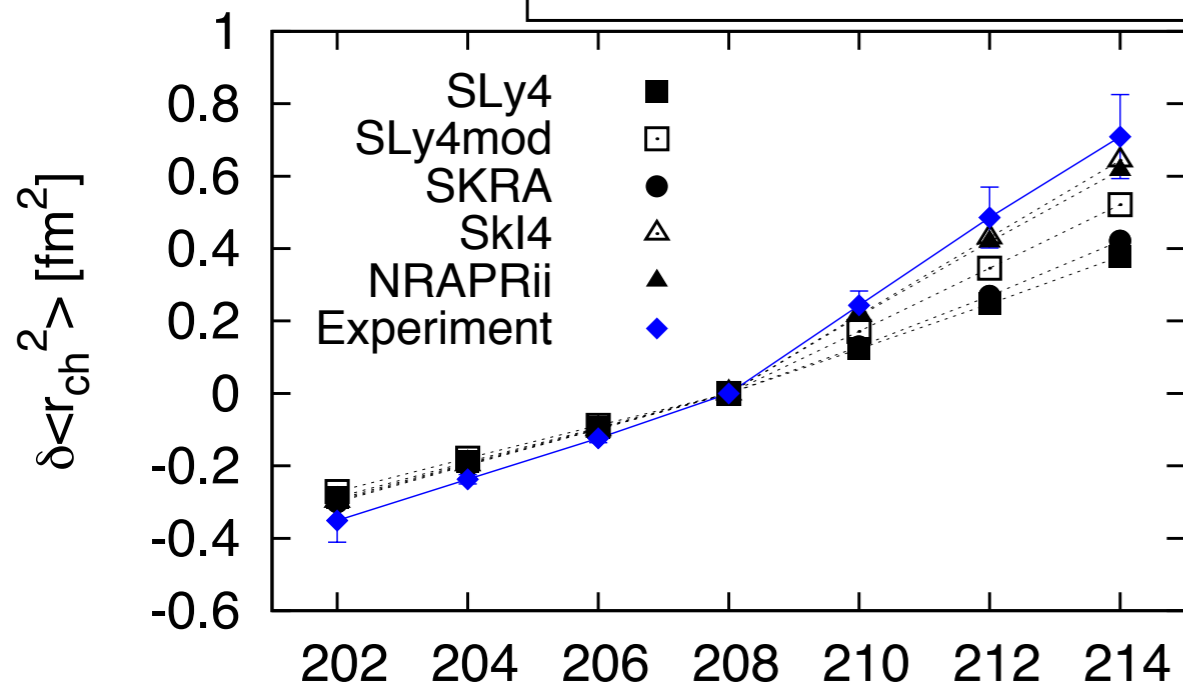
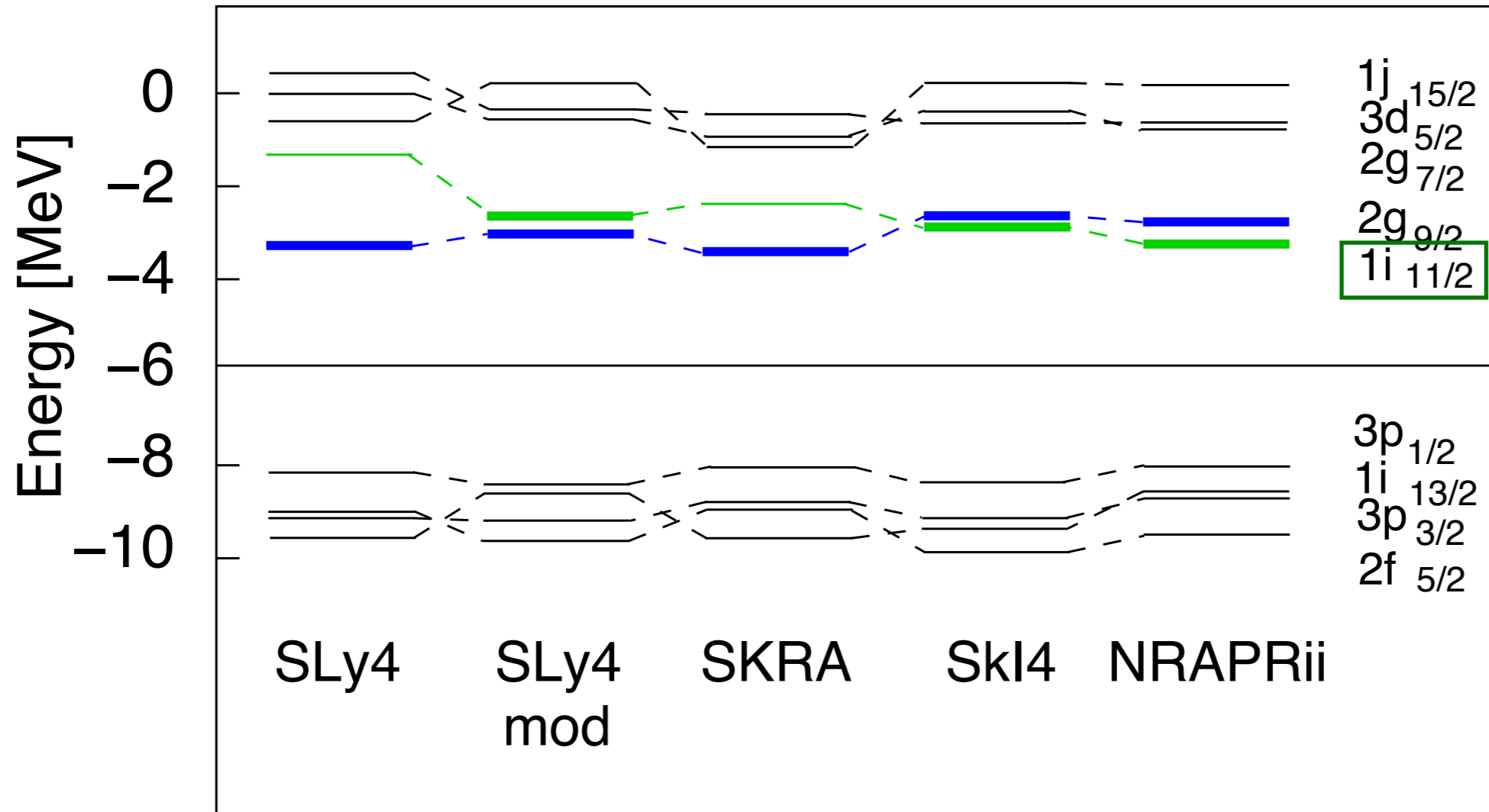
### Isotope shifts

$$\delta \langle r_{ch}^2 \rangle = \langle r_{ch}^2 \rangle_A - \langle r_{ch}^2 \rangle_{208} = m(A - 208)$$

$$m_{\text{LDR}} = 0.0972 \text{ fm}^2$$



## Single-particle spectrum of $^{210}\text{Pb}$ around Fermi surface



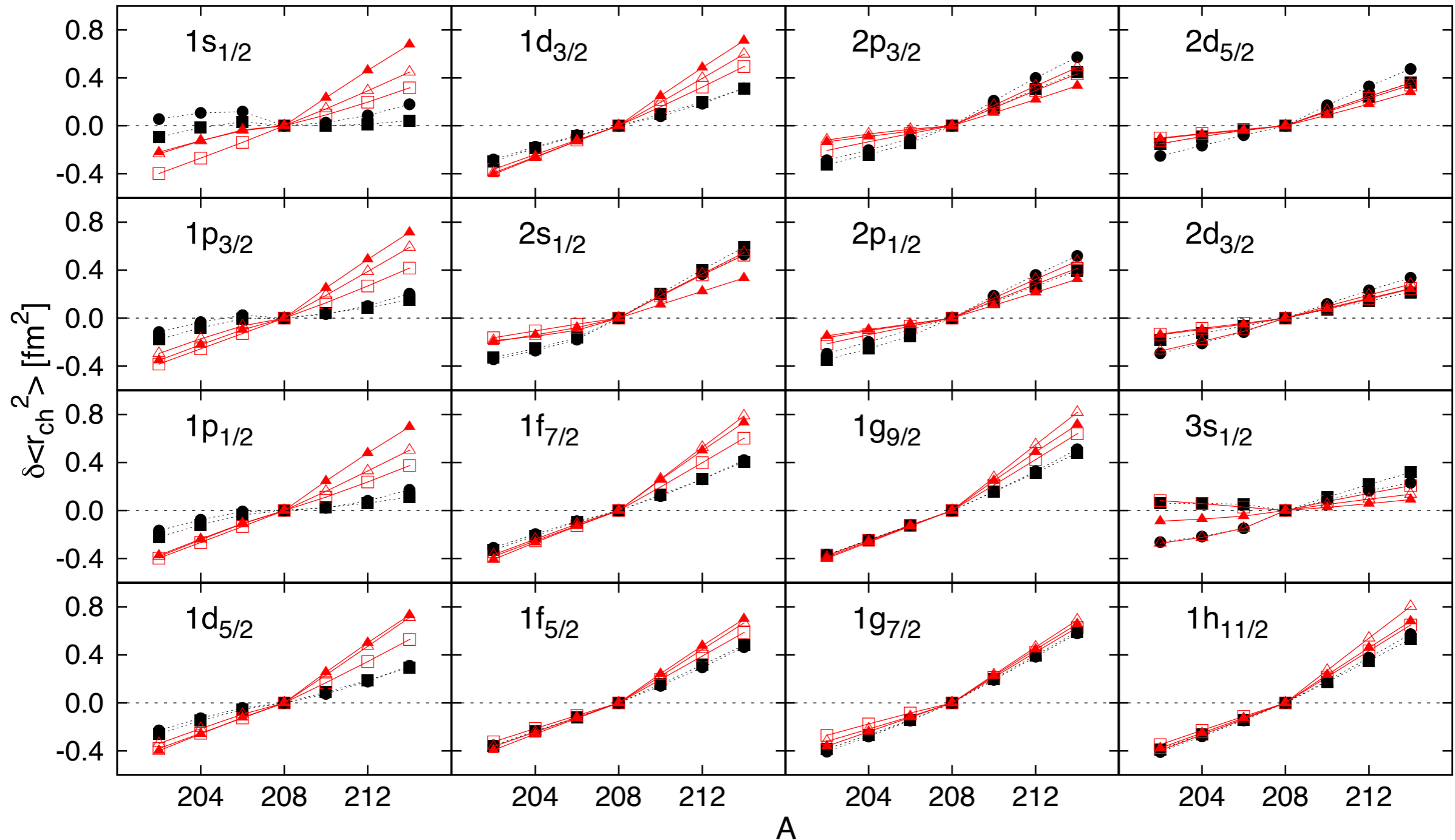
NRAPRii has  $b_4 = b_4'$   
 SLy4mod has  $b_4 \neq b_4'$

$1i_{11/2}$  plays an important role!

# Single-particle isotope shifts

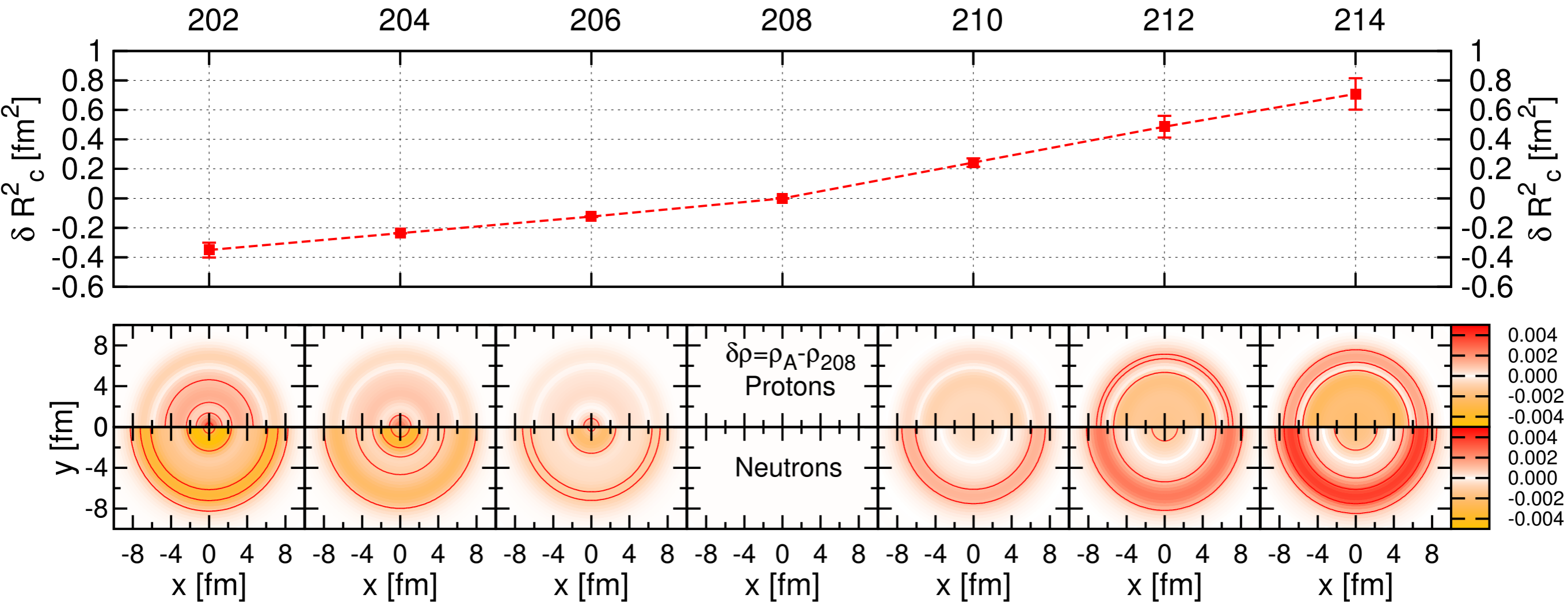
$$\langle r_{\text{ch}}^2 \rangle_A^{nl_j} = \frac{\int dr r^4 |\phi_{nl_j}(r)|^2}{\int dr r^2 |\phi_{nl_j}(r)|^2}$$

- SLy4      ■
- SLy4mod    □
- SKRA      ●
- SkI4      ▲
- NRAPRii    ▲



**Kink in deeply bound states**  $\Leftrightarrow$   **$1i_{11/2}$  is occupied**

# Further proof

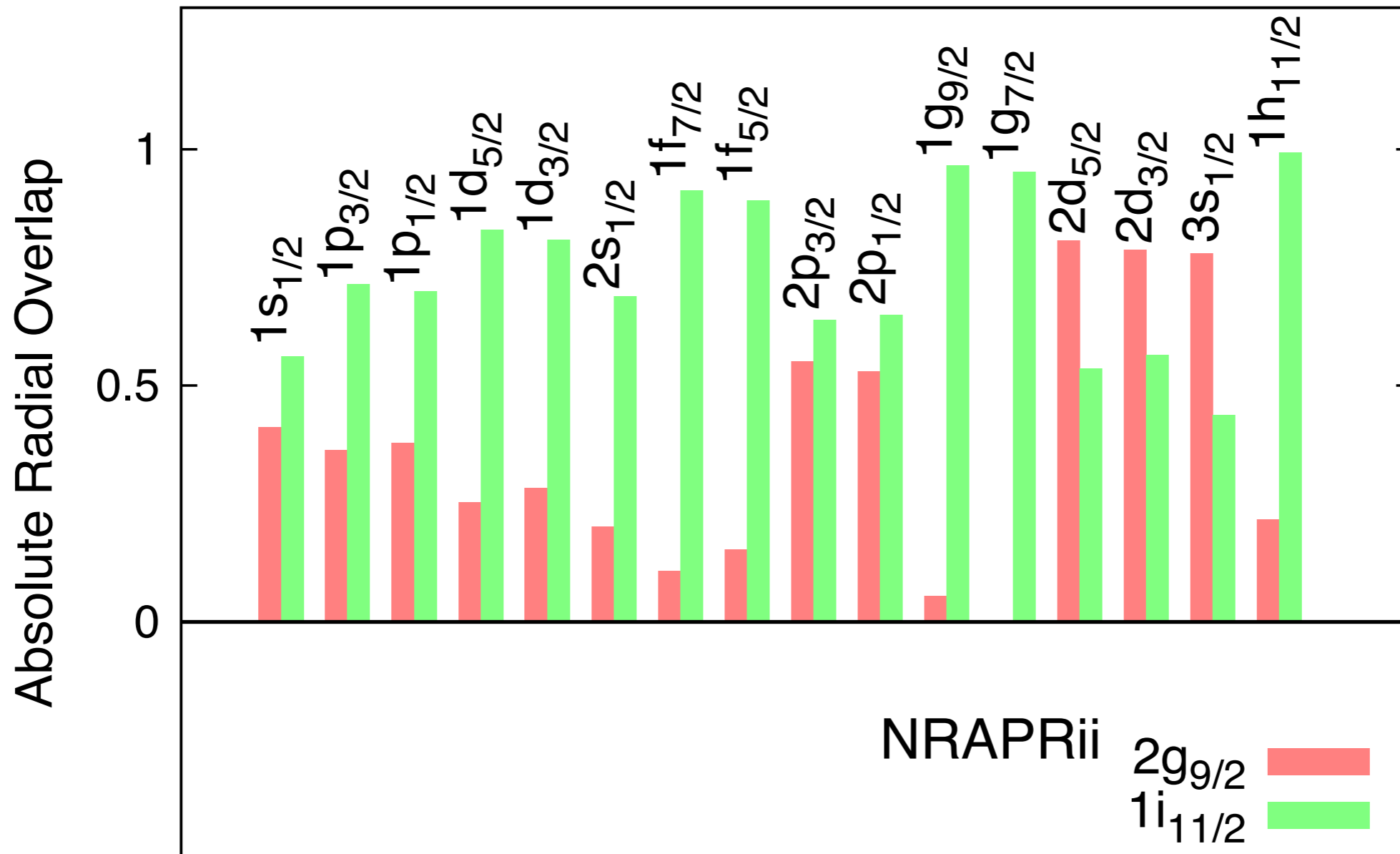


- Neutron density changes mostly at surface
- Proton density change also has interior component
- But  $1i_{11/2}$  is  $\sim 1$  fm more bound than  $2i_{9/2}$

## Radial overlaps

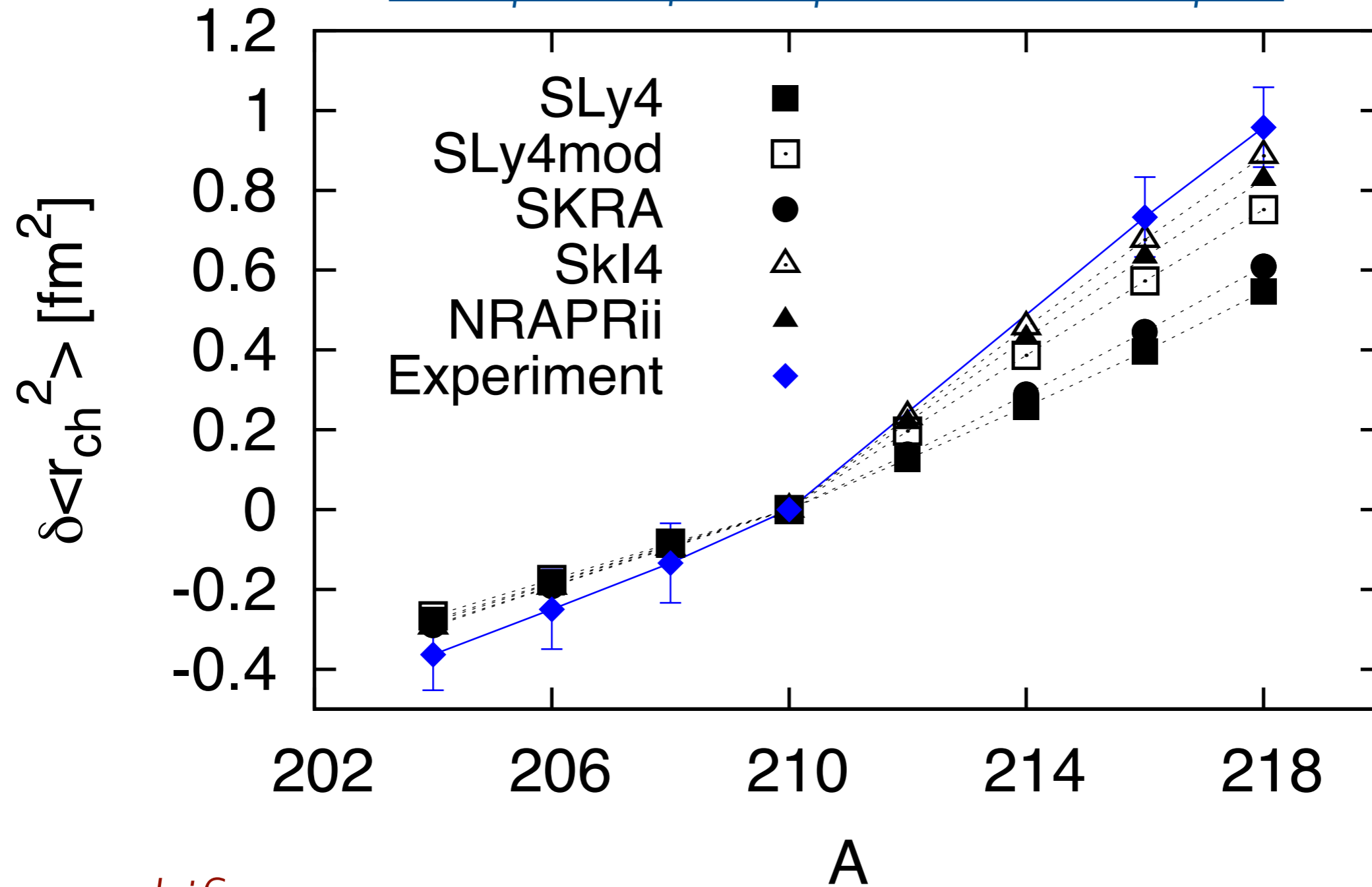
$$\langle \pi, nl_j | \nu, 1i_{11/2} \rangle = \int dr r^2 \phi_{nl_j}^*(r) \phi_{1i_{11/2}}(r)$$

## Proton-neutron overlaps in $^{208}\text{Pb}$



# Same thing in Polonium!

## Isotope shifts in polonium isotopes



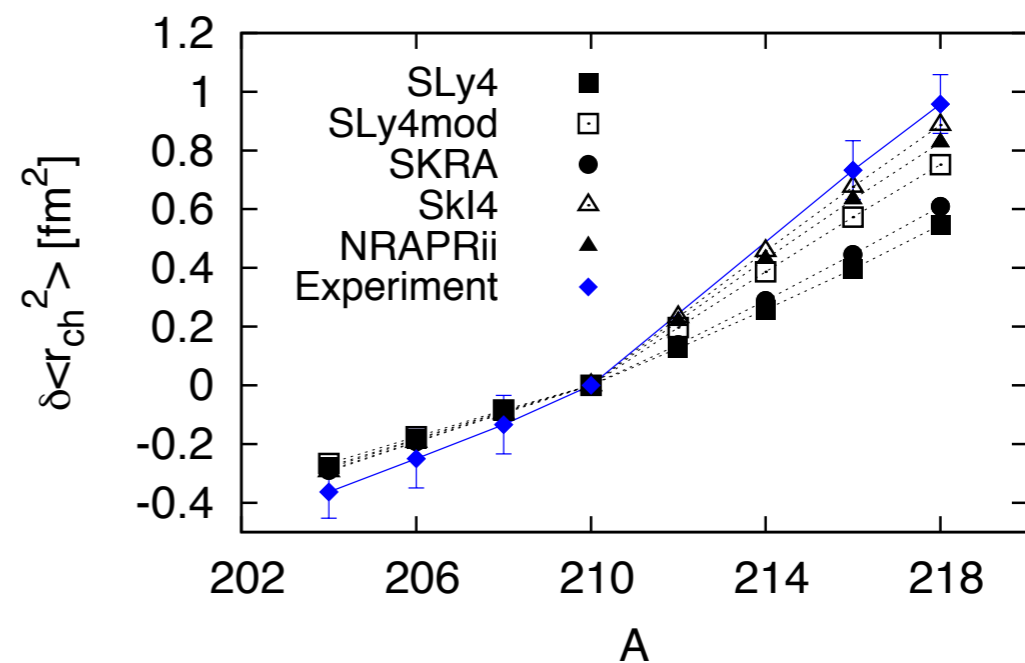
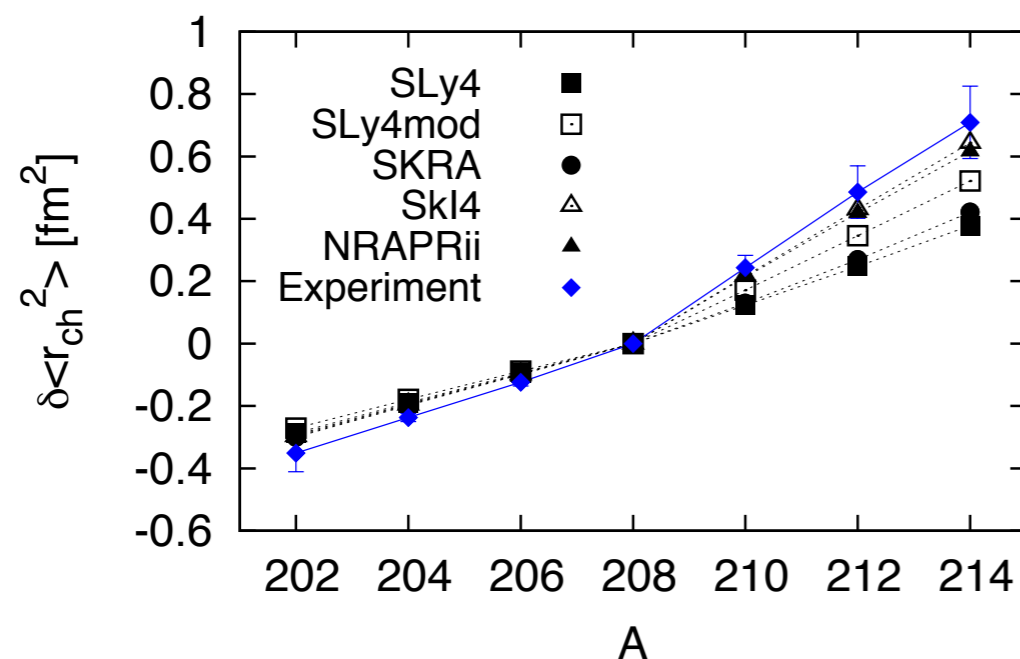
### Isotope shifts

$$\delta \langle r_{ch}^2 \rangle = \langle r_{ch}^2 \rangle_A - \langle r_{ch}^2 \rangle_{210}$$



# Conclusions

- Reproduction of **isotope shift** by and large determined by **occupation** of  $1i_{11/2}$  neutron orbital
- This  $n=1$  orbital has **larger overlap** with deeply bound proton orbitals
- Provides larger **pull** to protons via **symmetry energy**
- Mechanism **general** around  $N=126$



# Future work

- **Why** is  $li_{11/2}$  occupied?
  - Spin-orbit? Tensor? Correlations?
- Experimental spectrum vs postulated  $li_{11/2}$  population?
- Explore other **mass regions** and **kinks**:
  - Tensor in Ca isotopes?
  - Deformation in Hg?
  - Isotone shifts?
- Phil's thesis: **dipole response** with TDHF

# Nuclear & neutron matter

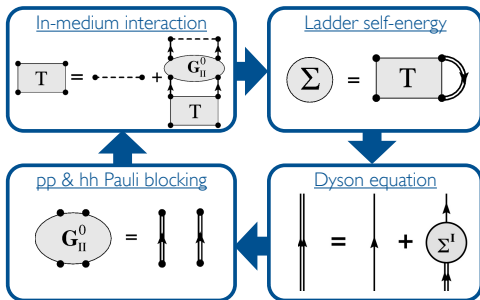
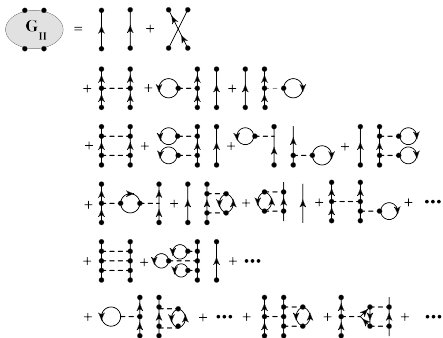
## Beyond a quasi-particle approach

Arnau Rios Huguet  
STFC Advanced Fellow  
Department of Physics  
University of Surrey

INT Program - Medium mass nuclei  
Seattle, 16 April 2013

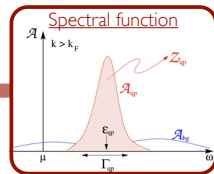
# Self-consistent Green's functions

## Ladder approximation within SCGF



- Ramos, Polls & Dickhoff, NPA **503** 1 (1989)  
 Alm *et al.*, PRC **53** 2181 (1996)  
 Dewulf *et al.*, PRL **90** 152501 (2003)  
 Frick & Muther, PRC **68** 034310 (2003)  
 Rios, PhD Thesis, U. Barcelona (2007)  
 Somà & Božek, PRC **78** 054003 (2008)

One-body properties  
 Momentum distribution  
 Thermodynamics & EoS  
Transport

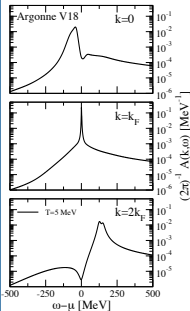


- Self-consistency, pp+hh & full off-shell effects

## Microscopic properties

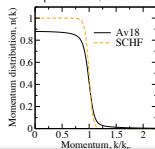
### Spectral function

Spectral function,  $\rho=0.16 \text{ fm}^{-3}$

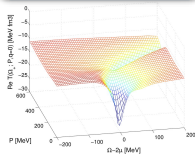


### Momentum distribution

$\rho=0.16 \text{ fm}^{-3}$ ,  $T=4 \text{ MeV}$



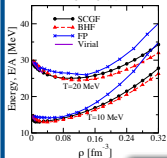
### In-medium interaction



## Bulk properties

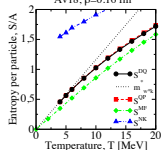
### Total Energy

CDBONN



### Entropy

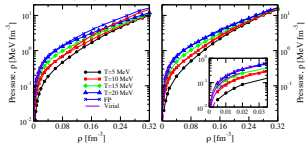
Av18,  $\rho=0.16 \text{ fm}^{-3}$



### Equation of State

CDBONN

Argonne V18

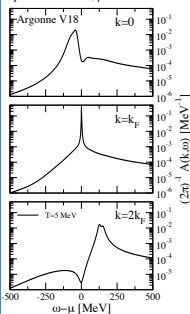


- Self-consistency, pp+hh & full off-shell effects

## Microscopic properties

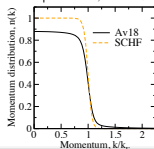
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Spectral function,  $\rho=0.16 \text{ fm}^{-3}$

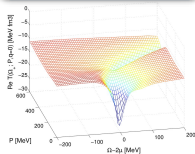


### Momentum distribution

$\rho=0.16 \text{ fm}^{-3}$ ,  $T=4 \text{ MeV}$



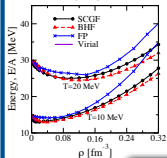
### In-medium interaction



## Bulk properties

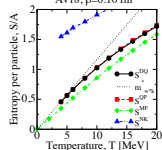
### Total Energy

CDBONN



### Entropy

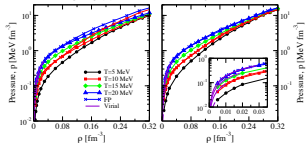
Av18,  $\rho=0.16 \text{ fm}^{-3}$



### Equation of State

CDBONN

Argonne V18

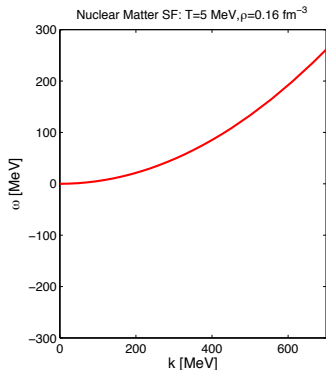


Transport?

- Self-consistency, pp+hh & full off-shell effects

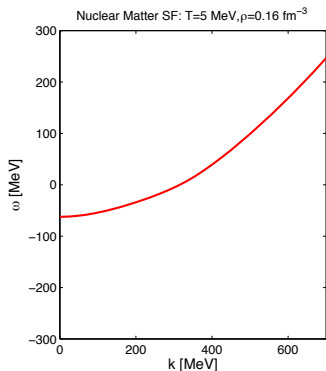
$$\varepsilon_k = \frac{k^2}{2m}$$

$$n_k = \frac{1}{1 + e^{\beta(\varepsilon_k - \mu)}}$$



- Free fermions
- Mean-field approximation: sp spectrum
- Correlations do not allow simple  $\varepsilon_k$  relation!
- A very general approach

$$\varepsilon_k = \frac{k^2}{2m} + U(k)$$
$$n_k = \frac{1}{1 + e^{\beta(\varepsilon_k - \mu)}}$$



- Free fermions
- Mean-field approximation: sp spectrum
- Correlations do not allow simple  $\varepsilon_k$  relation!
- A very general approach



# Correlations & spectral functions

## Spectral function

$$\mathcal{A}^<(k, \omega) = \sum_{n,m} \frac{e^{-\beta(E_n - \mu A)}}{Z} |\langle m | a_{\mathbf{k}} | n \rangle|^2 \delta[\omega - (E_n^A - E_m^A -$$

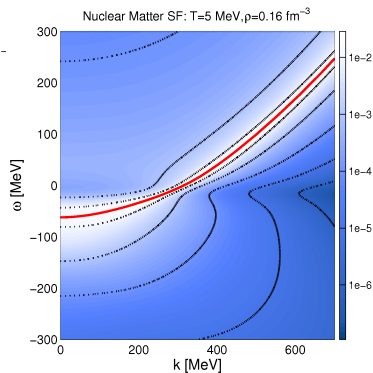
## Momentum distribution

$$n_{\mathbf{k}} = \int \frac{d\omega}{2\pi} f(\omega) \mathcal{A}(k, \omega)$$

## Probability

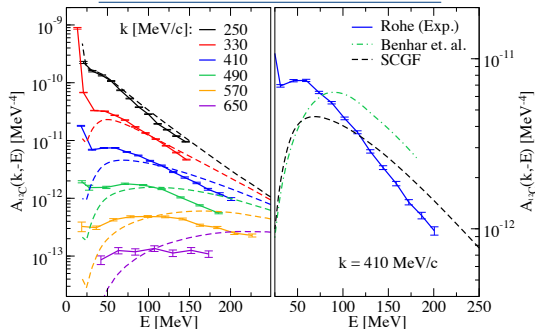
$$\int \frac{d\omega}{2\pi} \mathcal{A}(k, \omega) = 1$$

- Free fermions
- Mean-field approximation: sp spectrum
- Correlations do not allow simple  $\varepsilon_{\mathbf{k}}$  relation!
- A very general approach

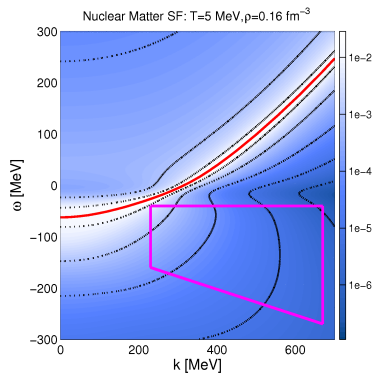


# Correlations & spectral functions

## $^{12}\text{C}$ spectral function from $(e, e'p)$

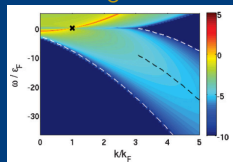


Rohe *et al.*, PRL **93** 182501 (2004)



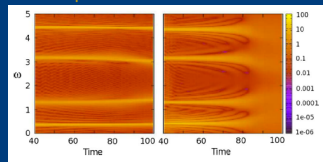
- Free fermions
- Mean-field approximation: sp spectrum
- Correlations do not allow simple  $\varepsilon_k$  relation!
- A very general approach

## Ultracold gases



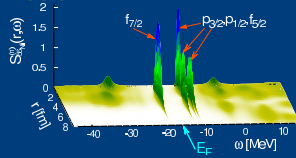
PRA **81**, 021601R (2010)

## Nonequilibrium nanostructures



PRB **80**, 115107 (2009)

## Nuclei



PRC **79**, 064313 (2009)

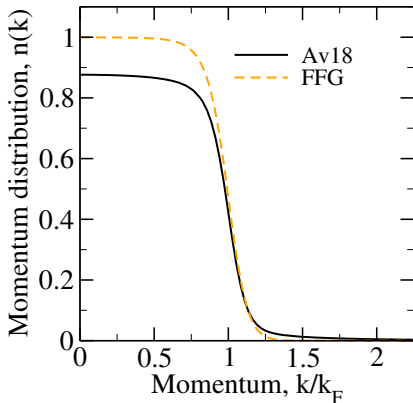
- Free fermions
- Mean-field approximation: sp spectrum
- Correlations do not allow simple  $\epsilon_k$  relation!
- A very general approach

# Momentum distribution

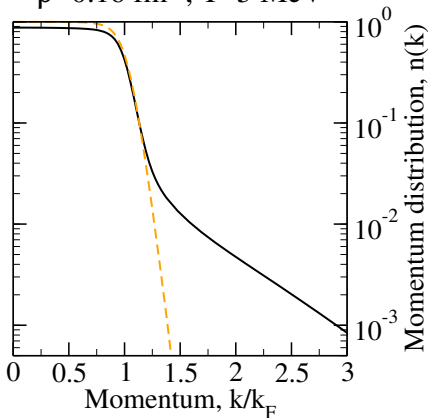
Single-particle occupation

$$n(k) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{A}(k, \omega) f(\omega) \Rightarrow \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$

$\rho = 0.16 \text{ fm}^{-3}$ ,  $T = 5 \text{ MeV}$



$\rho = 0.16 \text{ fm}^{-3}$ ,  $T = 5 \text{ MeV}$



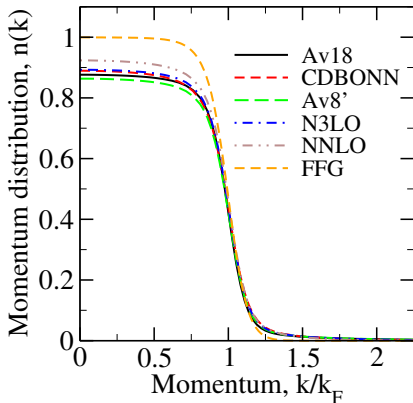
- 11 – 13% depletion at low  $k$ , population at high  $k$
- Dependence on NN interaction under control

# Momentum distribution

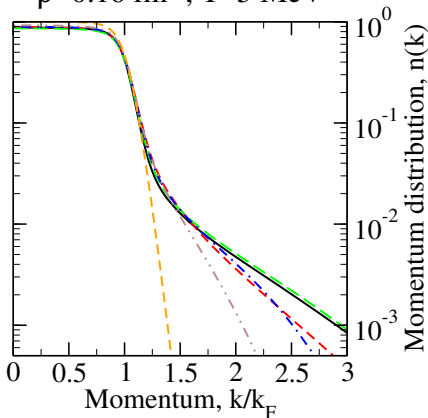
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$\rho = 0.16 \text{ fm}^{-3}$ ,  $T = 5 \text{ MeV}$



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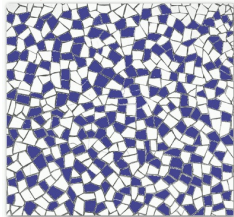
# Isospin asymmetric matter

Tuning correlations

## Nuclear “trencadís”

$\beta=0$

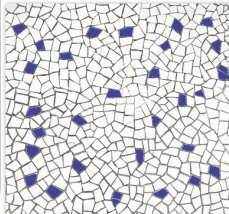
Symmetric matter



SR+Tensor correlations

$\beta \neq 0$

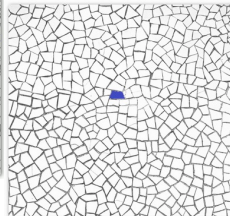
Asymmetric matter



Neutrons **less** correlated  
Protons **more** correlated

$\beta \approx 1$

Polaron



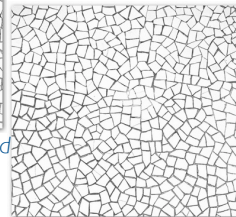
Protons **maximally** correlated  
Hyper-impurities?

Neutron stars



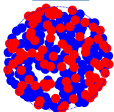
$\beta=1$

Neutron matter



SR correlations

Nuclei

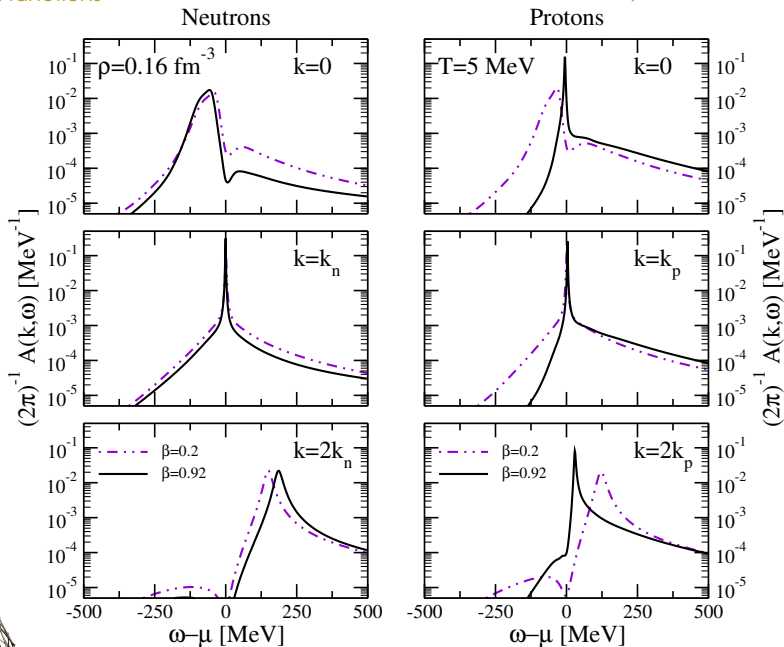


$$\beta = \frac{N - Z}{N + Z}$$

Frick, Rios et al. PRC **71**, 014313 (2005)

Rios et al. PRC **79**, 064308 (2009)

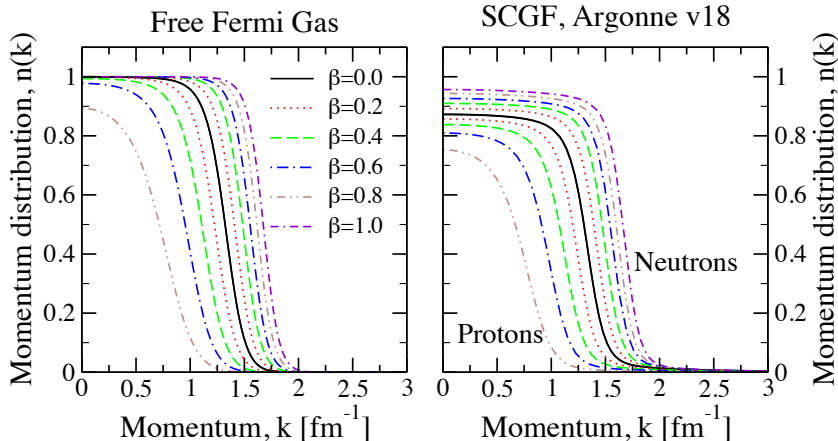
Carbone et al. EPL **97** 22001 (2012)



# Asymmetric nuclear matter

## Momentum distribution

$$\rho = 0.16 \text{ fm}^{-3} \quad T = 5 \text{ MeV}$$



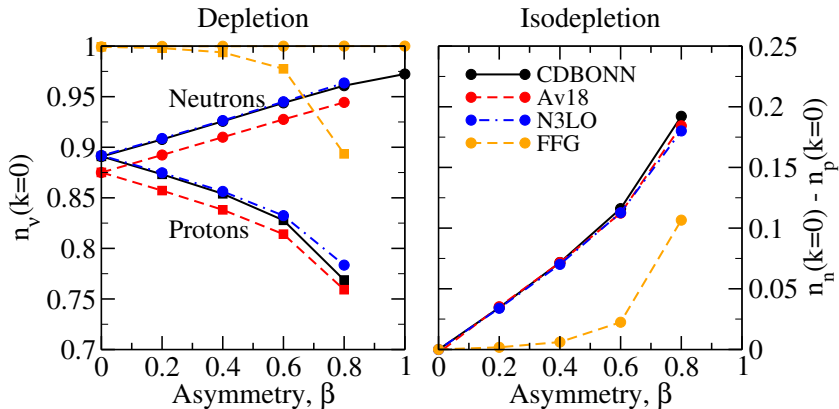
- Correlations affect depletion  $\Rightarrow$  non-perturbative effect
- Neutrons become less correlated
- Protons are more correlated



# Depletion vs. asymmetry

Isodepletion

$\rho=0.16 \text{ fm}^{-3}$   $T=5 \text{ MeV}$



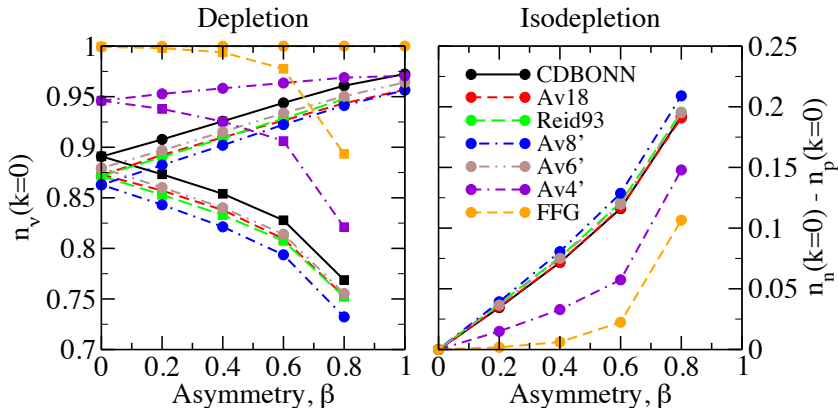
A. Rios *et al.*, PRC 79, 064308 (2009)

- Realistic potentials lie in a narrow iso-depletion band
- Proton depletion has a thermal component

# Depletion vs. asymmetry

Isodepletion

$\rho=0.16 \text{ fm}^{-3}$   $T=5 \text{ MeV}$



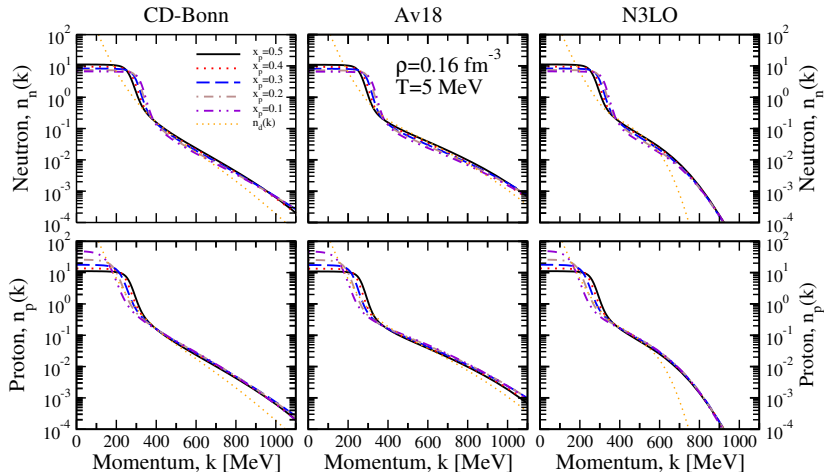
A. Rios *et al.*, PRC 79, 064308 (2009)

- Realistic potentials lie in a narrow iso-depletion band
- Proton depletion has a thermal component

# Asymmetric nuclear matter

High momenta

$$\nu \int \frac{d^3k}{(2\pi)^3} n_\tau(k) = 1$$



- High momentum proportional to species density
- Proportional to deuteron

A. Rios, A. Polls & W. H. Dickhoff, in preparation

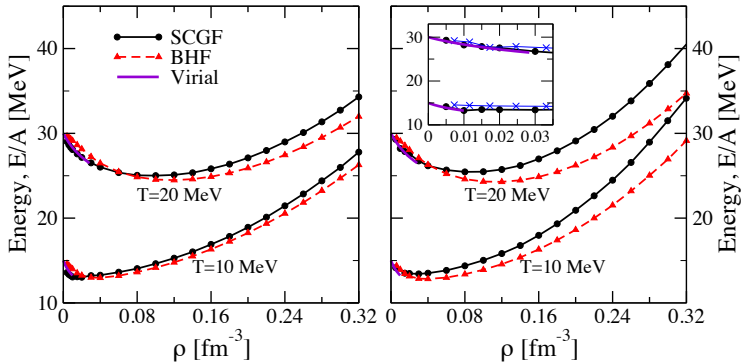
# EoS of neutron matter

Proof of principle

$$E^{GMK} = \sum_k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{2} \left[ \frac{k^2}{2m} + \omega \right] \mathcal{A}(k, \omega) f(\omega)$$

CDBONN

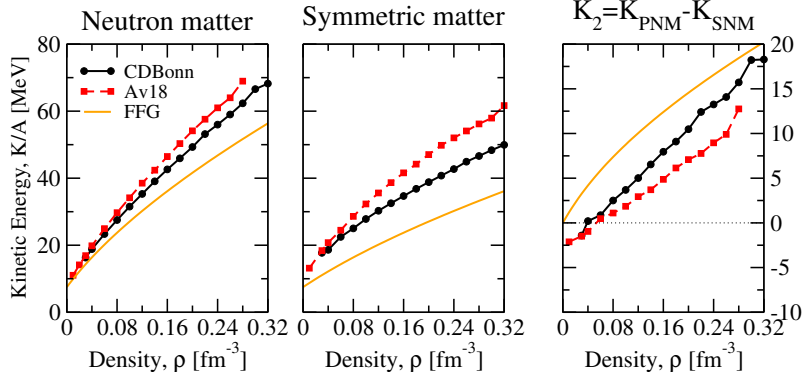
Argonne V18



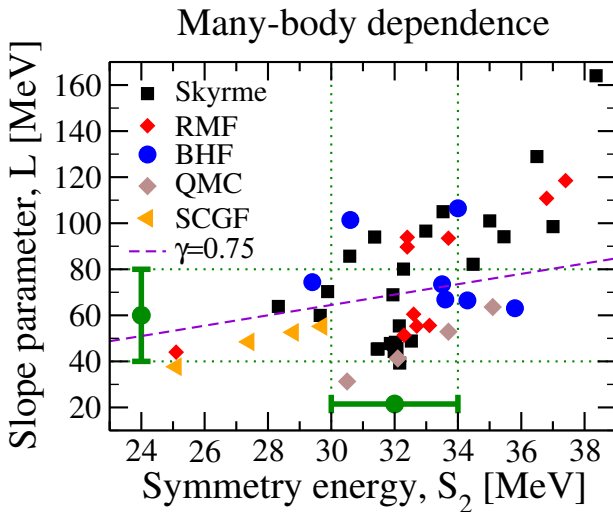
Rios, Polls & Vidaña, PRC **79**, 025802 (2009)

- Potential dependence for  $\rho > \rho_0$
- Agrees with virial expansion at low  $\rho$ 's
- Systematically more repulsive than BHF
- 3BF still needed

# Kinetic symmetry energy



|        | $S_{\text{tot}}$ (MeV) | $S_{\text{kin}}$ (MeV) | $S_{\text{pot}}$ (MeV) | $L$ (MeV) |
|--------|------------------------|------------------------|------------------------|-----------|
| Av18   | 25.1                   | 4.9                    | 20.2                   | 37.7      |
| Nij1   | 27.4                   | 4.6                    | 22.8                   | 48.5      |
| CDBonn | 28.8                   | 7.9                    | 20.9                   | 52.6      |
| N3LO   | 29.7                   | 7.2                    | 22.5                   | 55.2      |
| FFG    | 12.3                   | 12.3                   | 0                      | 24.6      |



## Diagrammatic expansion with 3BF

### Self-energy expansion to 2nd order

$$\Sigma = \text{---}\circ\text{---} + \frac{1}{2} \text{---}\circ\text{---}\circ\text{---}$$

$$+ \frac{1}{2} \text{---}\text{---}\text{---}\text{---} + \frac{1}{2} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---} + \frac{1}{2} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

$$+ \frac{1}{4} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---} + \frac{1}{4} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

$$+ \frac{1}{12} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

- Only **skeleton** 1PI diagrams needed
- **Anti-symmetrized** interactions
- Proper **symmetry** factors included

## Effective interaction expansion

### Rewrite self-energy expansion

$$\Sigma = \text{---}\times\text{---}$$

$$+ \frac{1}{2} \text{---}\text{---}\text{---}\text{---} + \frac{1}{12} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

### Definition of effective 1B and 2B forces

Effective one-body force

$$\text{---}\times\text{---} = \text{---}\circ\text{---} + \frac{1}{4} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

Effective two-body force

$$\text{---}\text{---}\text{---} = \text{---}\text{---} + \text{---}\text{---}\text{---}\text{---}$$

Two-body propagator

$$\text{---}\text{---} = \text{---}\text{---} + \text{---}\text{---} + \text{---}\text{---}\text{---}$$

## Diagrammatic expansion with 3BF

### Self-energy expansion to 2nd order

$$\Sigma = \bullet \text{---} \circ + \frac{1}{2} \bullet \text{---} \circ \text{---} \circ$$

$$+ \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] + \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \circ + \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \circ$$

$$+ \frac{1}{4} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right] + \frac{1}{4} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \circ$$

$$+ \frac{1}{12} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right]$$

- Only skeleton 1PI diagrams needed
- Anti-symmetrized interactions
- Proper symmetry factors included
- Number of diagrams substantially reduced
- Usable in higher order resummations
- Implementation in progress

## Effective interaction expansion

### Rewrite self-energy expansion

$$\Sigma = \bullet \text{---} \times$$

$$+ \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] + \frac{1}{12} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right]$$

### Definition of effective 1B and 2B forces

Effective one-body force

$$\bullet \text{---} \times = \bullet \text{---} \circ + \frac{1}{4} \bullet \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right]$$

Effective two-body force

$$\text{---} \text{---} = \bullet \text{---} \bullet + \bullet \text{---} \bullet \text{---} \circ$$

Two-body propagator

$$\text{---} \text{---} = \text{---} \text{---} + \text{---} \times \text{---} + \text{---} \text{---} \text{---}$$



## Diagrammatic expansion with 3BF

### Self-energy expansion to 2nd order

$$\Sigma = \bullet \text{---} \circ + \frac{1}{2} \bullet \text{---} \circ \text{---} \circ$$

$$+ \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] + \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \circ + \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \circ$$

$$+ \frac{1}{4} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right] + \frac{1}{4} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \circ$$

$$+ \frac{1}{12} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right]$$

- Only **skeleton** 1PI diagrams needed
- **Anti-symmetrized** interactions
- Proper **symmetry** factors included
- **Number** of diagrams **substantially** reduced
- **Usable** in higher order **resummations**
- **Implementation** in **progress**

## Effective interaction expansion

### Rewrite self-energy expansion

$$\Sigma = \bullet \text{---} \times$$

$$+ \frac{1}{2} \text{---} \left[ \text{---} \circ \text{---} \right] + \frac{1}{12} \text{---} \left[ \text{---} \circ \text{---} \right] \text{---} \left[ \text{---} \circ \text{---} \right]$$

### Definition of effective 1B and 2B forces

Effective one-body force

$$\bullet \text{---} \times = \bullet \text{---} \circ + \frac{1}{2} \bullet \text{---} \circ \text{---} \circ$$

Effective two-body force

$$\text{---} \text{---} = \bullet \text{---} \bullet + \bullet \text{---} \bullet \text{---} \circ$$

Two-body propagator

$$\text{---} \text{---} \text{---} = \text{---} \text{---} + \text{---} \text{---} + \text{---} \text{---}$$

## Diagrammatic expansion with 3BF

### Self-energy expansion to 2nd order

$$\Sigma = \text{---}\circ\text{---} + \frac{1}{2} \text{---}\circ\text{---}\circ\text{---}$$

$$+ \frac{1}{2} \text{---}\text{---}\text{---}\text{---} + \frac{1}{2} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---} + \frac{1}{2} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

$$+ \frac{1}{4} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---} + \frac{1}{4} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

$$+ \frac{1}{12} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

- Only **skeleton** 1PI diagrams needed
- **Anti-symmetrized** interactions
- Proper **symmetry** factors included
- **Number** of diagrams **substantially** reduced
- **Usable** in higher order **resummations**
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## Effective interaction expansion

### Rewrite self-energy expansion

$$\Sigma = \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

$$+ \frac{1}{2} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

$$+ \frac{1}{12} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

### Definition of effective 1B and 2B forces

Effective one-body force

$$\text{---}\text{---}\text{---}\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---}\text{---}\text{---}\text{---} + \frac{1}{2} \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

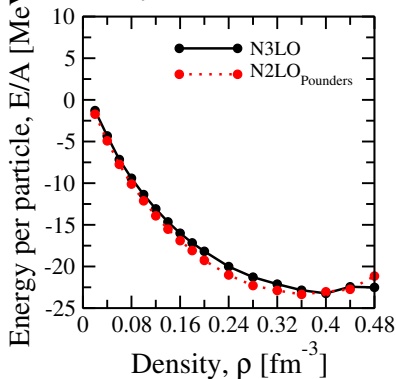
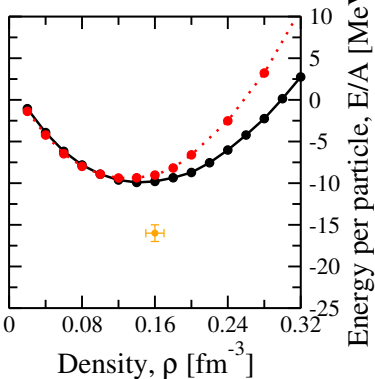
Effective two-body force

$$\text{---}\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}\text{---}\text{---}\text{---}$$

Two-body propagator

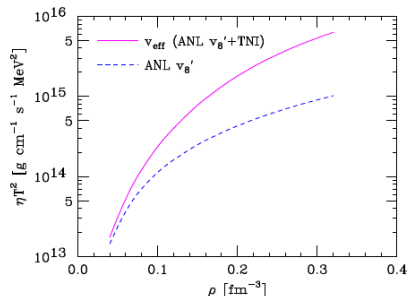
$$\text{---}\text{---}\text{---}\text{---} = \text{---}\text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}\text{---} + \text{---}\text{---}\text{---}\text{---}$$

Symmetric matter

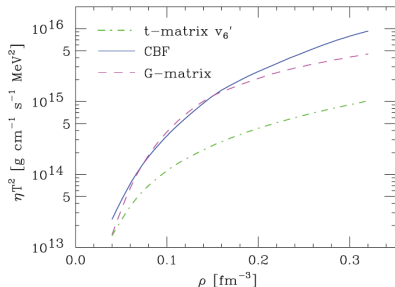
 $\rho=0.16 \text{ fm}^{-3}$ ,  $T=5 \text{ MeV}$ PRELIMINARY, [A. Carbone](#), [A. Rios](#) & [A. Polls](#)

- 3BF can be added  $\Rightarrow$  modified Koltun sum-rule<sup>1</sup>
- NNLO  $\Rightarrow$  2B & 3B same order in  $\chi$  expansion
- Saturation properties of N3LO and NNLO similar
- $S \sim 30 \text{ MeV}$  &  $L \sim 56 \text{ MeV}$

## Shear viscosity of neutron matter in CBF



## Shear viscosity: CBF vs BHF



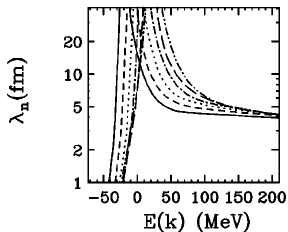
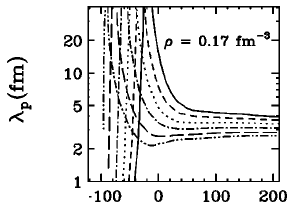
Wambach, Ainsworth & Pines, NPA **555**, 128 (1993)  
Benhar & Valli, PRL **99**, 232501 (2007)

Benhar, Polls, Valli & Vidaña, PRC **81**, 024305 (2010)  
Benhar & Carbone, arxiv:0912.0129

- Don't stick to EoS only, aim at complete NS models!
- Better if experimentally testable
  - 1 Mean-free path  $\Rightarrow$  Optical potentials & scattering
  - 2 Viscosities  $\Rightarrow$  Resonances
  - 3 Neutrino responses  $\Rightarrow \nu$ -A experiments
  - 4 Specific heat  $\Rightarrow$  cooling

# Ab initio quantum transport?

## EBHF calculation of mean-free path



- Often hybrid models are used:
  - Viscosity: CBF / BHF + Landau-Abrikosov-Khalatnikov
  - MFP: EBHF + quasi-particle pole expansion
- Uncontrolled or inconsistent approximations
- Need of fully quantal, many-body calculations
- Beyond Boltzmann
- Green's functions advantages:  
time-dependence is natural & consistency
- Simplest transport coefficient: mean free path
- Key coefficient, underlying in transport
- Closely related to imaginary optical potential

Zuo *et al.*, PRC **60** 024605 (1999)

Negele & Yazaki, PRL **47**, 71 (1981)

Fantoni, Friman & Pandharipande, PLB **104**, 89 (1981)

Mahaux, PRC **28**, 1848 (1983)

## Naive optical potential model

$$\left[ -\frac{\nabla^2}{2m} + \text{Re} \Sigma(\varepsilon_k) + i \text{Im} \Sigma(\varepsilon_k) \right] \psi(r) = \varepsilon_k \psi(r)$$

$$\psi(r) = N e^{-i \left\{ k - \frac{i}{2\lambda_k} \right\} r} \quad \Rightarrow \quad p(r) = |\psi(r)|^2 \sim e^{-\frac{r}{\lambda_k}}$$

$$\lambda_k = -\frac{k}{2m} \frac{1}{\text{Im} \Sigma(E_k)} = \frac{k}{m} \frac{1}{\Gamma_k} = \frac{\partial_k \varepsilon_k}{\Gamma_k}$$

- Mean-free path from quasi-particle properties

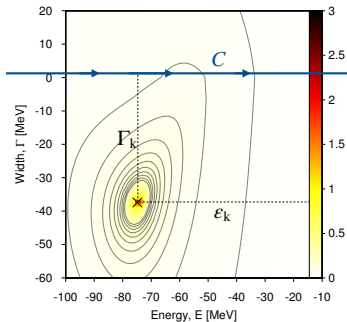
$$\lambda_k = \frac{v_k}{\Gamma_k}$$

- Fundamental asymptotic behavior for propagator in real time

$$\mathcal{G}_R(k, t) \rightarrow -i\eta_k e^{-i\varepsilon_k t} e^{-|\Gamma_k|t}$$



# Quasi-particle "pole"



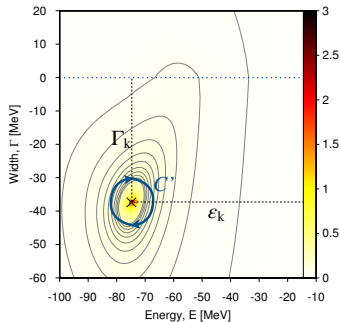
- Fundamental asymptotic behavior for propagator in real time

$$\mathcal{G}_R(k, t) \rightarrow -i\eta_k e^{-i\varepsilon_k t} e^{-|\Gamma_k|t} \Rightarrow \lambda_k = \frac{\partial_k \varepsilon_k}{\Gamma_k}$$

- Time-energy Fourier transform using retarded contour + Cauchy

$$\mathcal{G}_R(k, t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \mathcal{G}_R(k, \omega) \sim \int_{C'} \frac{dz}{2\pi} e^{-izt} \frac{\eta(z)}{z - (\varepsilon_k - i|\Gamma_k|)} = -i\eta_k e^{-i\varepsilon_k t} e^{-|\Gamma_k|t}$$

# Quasi-particle "pole"



- Fundamental asymptotic behavior for propagator in real time

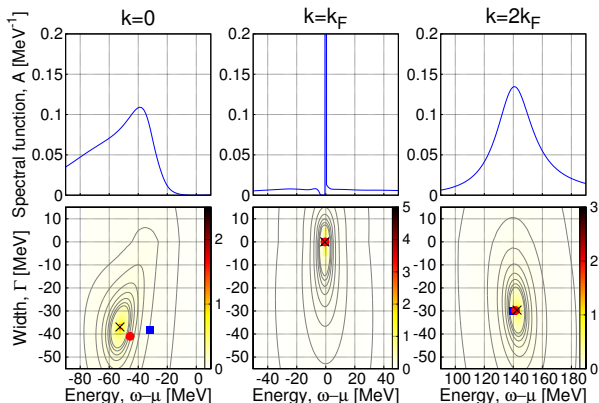
$$\mathcal{G}_R(k, t) \rightarrow -i\eta_k e^{-i\varepsilon_k t} e^{-|\Gamma_k|t} \Rightarrow \lambda_k = \frac{\partial_k \varepsilon_k}{\Gamma_k}$$

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CDBonn,  $T = 0$ ,  $\rho = 0.16 \text{ fm}^{-3}$



- **Circle:** first renormalization (expansion on  $\text{Im } z$  to 1<sup>st</sup> order)

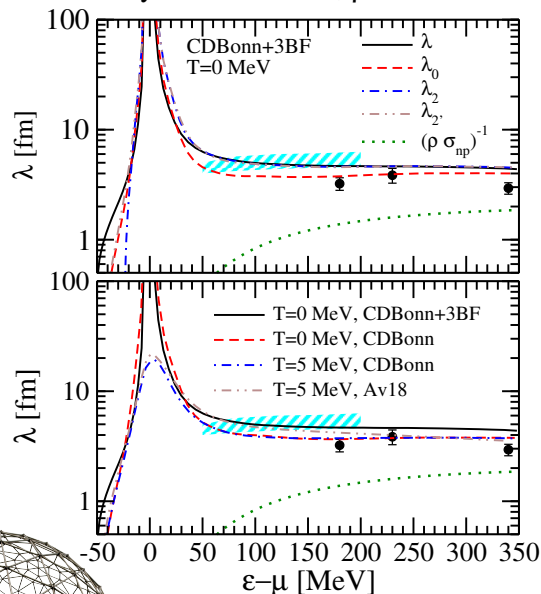
$$\varepsilon_k = \frac{k^2}{2m} + \text{Re}\Sigma_k(\varepsilon_k) \quad \Gamma_k = \text{Im}\Sigma_k(\varepsilon_k)$$

- **Square:** second renormalization (expansion on  $\text{Im } z$  to 2<sup>nd</sup> order)

$$\varepsilon_k = \frac{k^2}{2m} + \text{Re}\Sigma_k(\varepsilon_k) \quad \Gamma_k = \frac{m_k}{m} \text{Im}\Sigma_k(\varepsilon_k)$$

# Model dependence

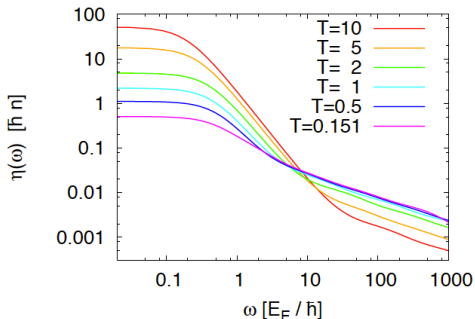
Symmetric matter,  $\rho=0.16 \text{ fm}^{-3}$



$$\lambda_k = \frac{1}{\Gamma_k} \frac{\partial \epsilon_k}{\partial k}$$

- $\lambda \sim 4 - 5 \text{ fm}$  above 50 MeV
- Compatible with  $pA$  experiments
- Small model dependence
  - $\lambda_0 \Rightarrow$  no non-locality
  - $\lambda_2 \Rightarrow$  full non-locality
  - $\lambda'_2 \Rightarrow m_k^*$  non-locality
- Classical approximation is incorrect!
- Little effect of 3BFs

## Viscosity spectral function in unitary gas



Enss, Haussman, Zwirger, *Ann. Phys.* **326**, 770 (2011)

- Viscosity over entropy ratio in ultracold gases

Kadanoff & Martin, *Ann. Phys.* **24**, 419 (1963)  
Taylor & Randeria, *PRA* **81**, 053610 (2010)

- Specific heat in quantum liquids

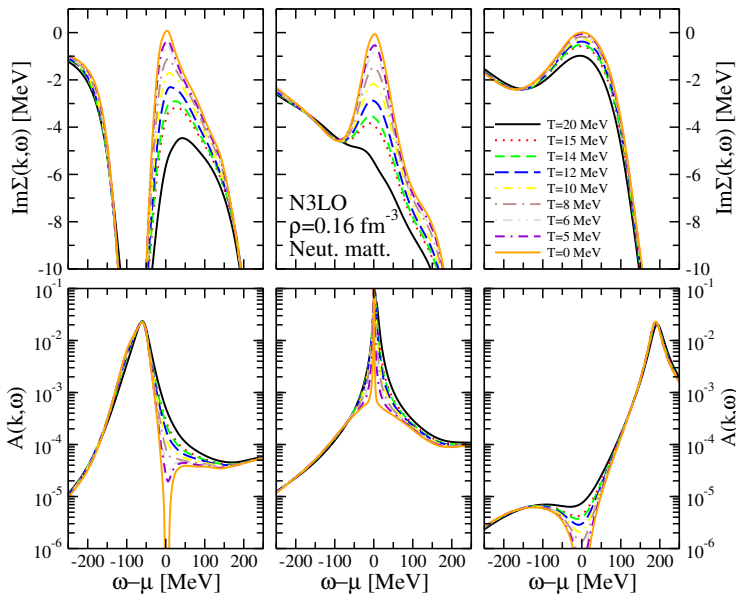
Pethick & Carneiro, *PRA* **7**, 304 (1973)  
Carneiro & Pethick, *PRA* **11**, 1106 (1976)

# Extrapolation to $T=0$

$k=0$

$k=k_F$

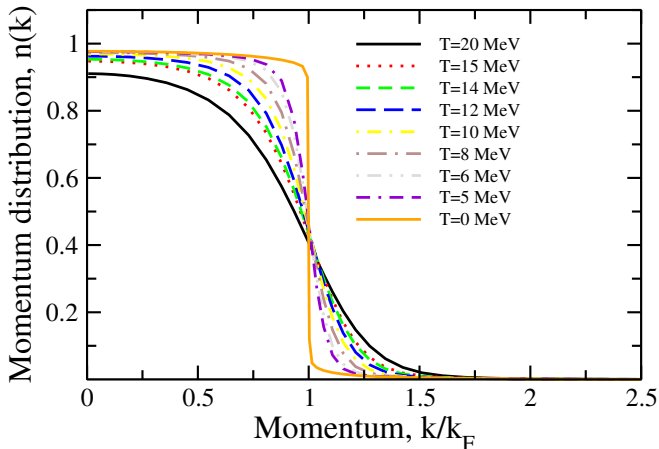
$k=2k_F$



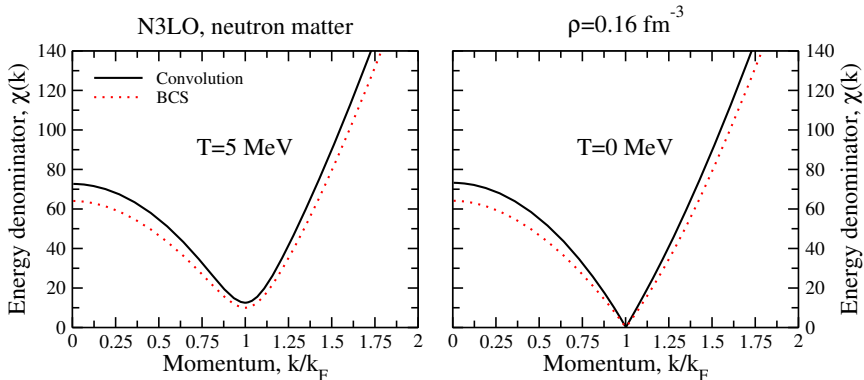
# Extrapolation to T=0

$$n(k) = \int_{-\infty}^{\epsilon_F} \frac{d\omega}{2\pi} \mathcal{A}(k, \omega) \Rightarrow \nu \int \frac{d^3k}{(2\pi)^3} n(k) = \rho$$

N3LO,  $\rho=0.16 \text{ fm}^{-3}$ , neutron matter



## Pairing energy denominator



$$\Delta_{lk}^{JST} = - \sum_{l'} \int_0^\infty dk' k'^2 \langle kl | V^{JST} | k'l' \rangle \frac{\Delta_{l'k'}^{JST}}{2\chi_{k'}}$$

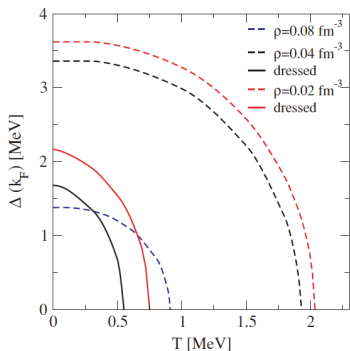
BCS / quasi-particle

$$\frac{1}{2\chi_k} = \frac{1 - 2f(\epsilon_k)}{2\epsilon_k}$$

Beyond quasi-particle

$$\frac{1}{2\chi_k} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A_k(\omega) A_k^s(\omega')$$

## CDBonn neutron matter $^1S_0$ pairing gap



Muther & Dickhoff, PRC **72** 054313 (2005)

$$\Delta_{lk}^{JST} = - \sum_{l'} \int_0^\infty dk' k'^2 \langle kl | V^{JST} | k'l' \rangle \frac{\Delta_{l'k'}^{JST}}{2\chi_{k'}}$$

BCS / quasi-particle

$$\frac{1}{2\chi_k} = \frac{1 - 2f(\epsilon_k)}{2\epsilon_k}$$

Beyond quasi-particle

$$\frac{1}{2\chi_k} = \int_{-\infty}^\infty \frac{d\omega}{2\pi} \int_{-\infty}^\infty \frac{d\omega'}{2\pi} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A_k(\omega) A_k^s(\omega')$$

- Ab initio description of nuclear & neutron matter
- Fully self-consistent & quantum mechanical calculation
- One-body microscopic properties ✓
- Thermodynamic properties ✓
- Mean-free path in correlated matter ✓
- Adding three-body forces consistently
- Other transport properties are coming
- Zero-temperature extrapolation
- Pairing:  $^1S_0, ^3PF_2$
- Two-body properties: relative momentum distribution







V. Somà  
T. U. Darmstadt



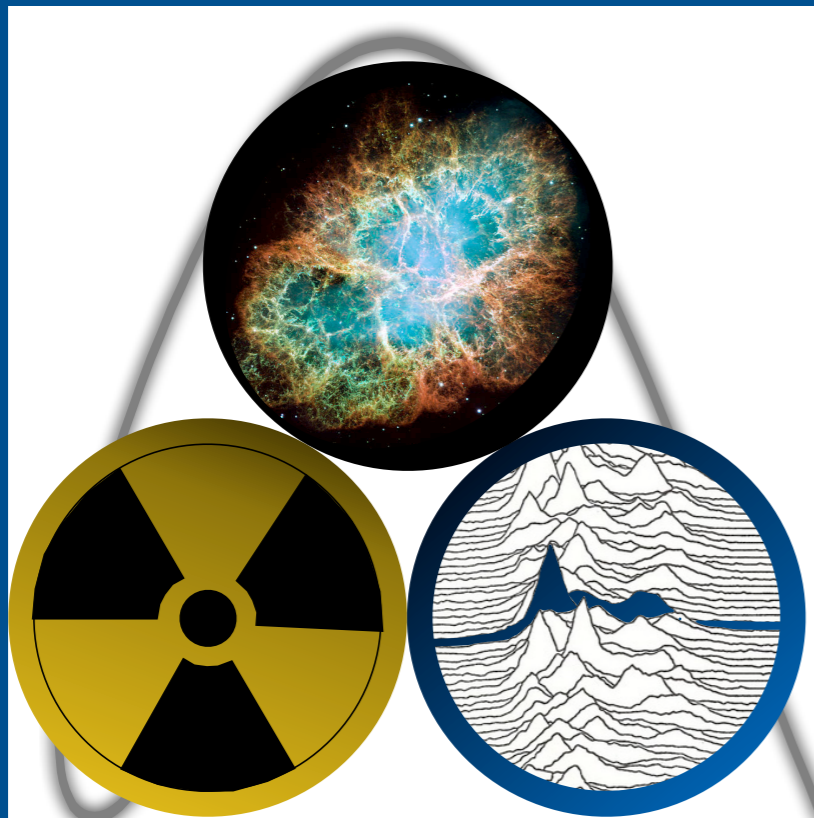
W. H. Dickhoff  
Wash. U. St. Louis



A. Polls, A. Carbone  
University of Barcelona



Thank you!



## *Neutron Stars*

*Nuclear Physics, Gravitational Waves & Astronomy*

*29-30 July 2013*

*Institute of Advanced Studies, University of Surrey*  
<http://www.ias.surrey.ac.uk/workshops/neutstar/>

[a.rios@surrey.ac.uk](mailto:a.rios@surrey.ac.uk)



**Science & Technology**  
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