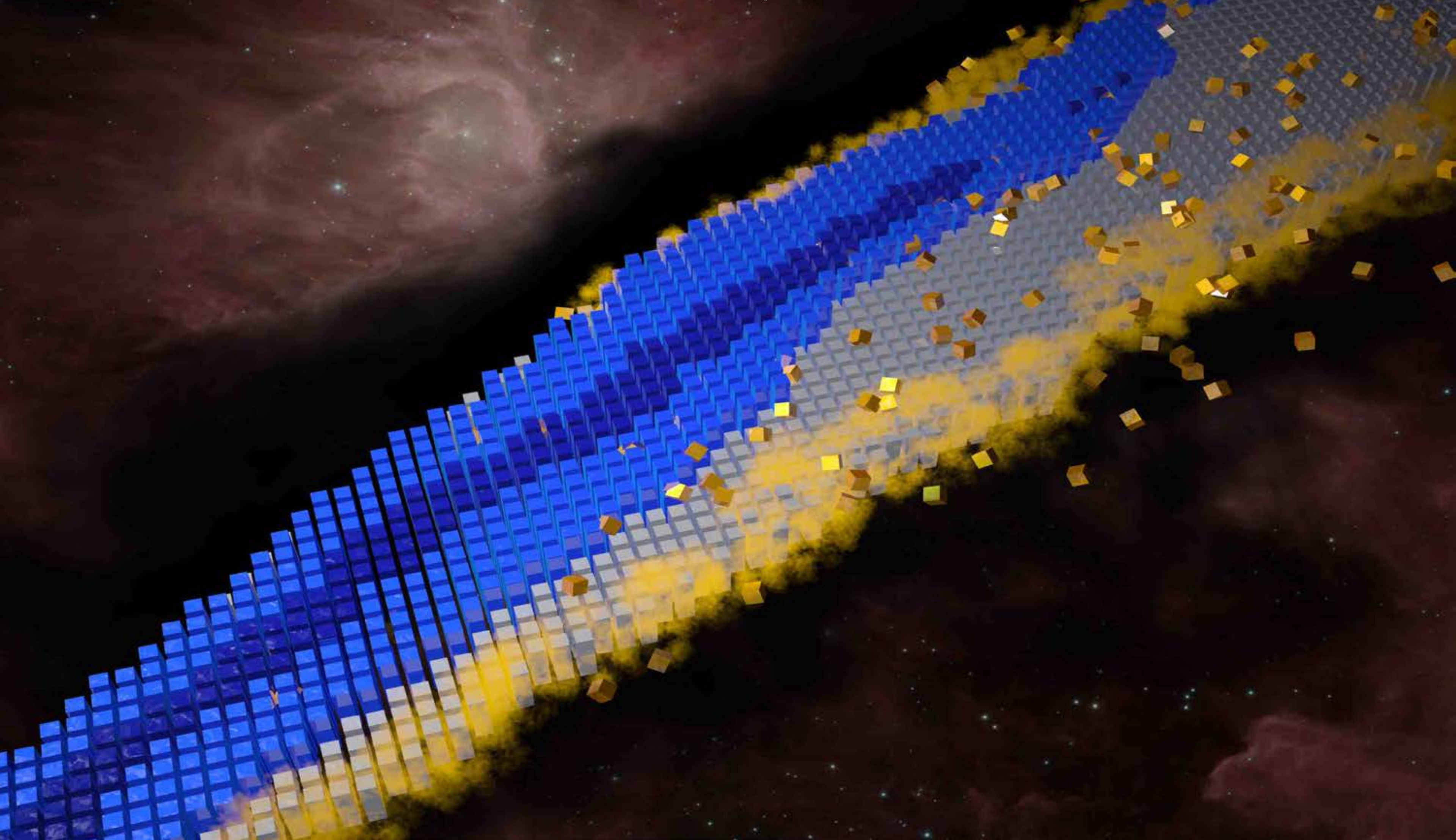
Atomic Nuclei: Many-Body Open Quantum Systems

Witold Nazarewicz (UTK/ORNL)

INT Program INT-13-1a: Computational and Theoretical Advances for Exotic Isotopes in the Medium Mass Region

Seattle, April 5

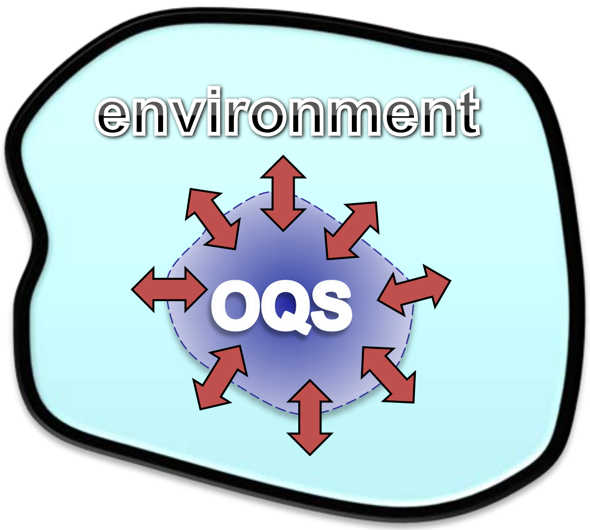


OUTLINE

- General principles
  - Open quantum systems
  - Selected experimental examples
  - Theoretical strategies
- The limits (particle thresholds)
- Continuum CI (real- and complex-energy)
- Continuum CC
- Continuum DFT
- Examples
  - Charge radii of halo nuclei
  - Dipolar molecules
- Origin of clustering
- Conclusions

Wikipedia:

An open quantum system is a quantum system which is found to be in interaction with an external quantum system, the environment.



INTERDISCIPLINARY

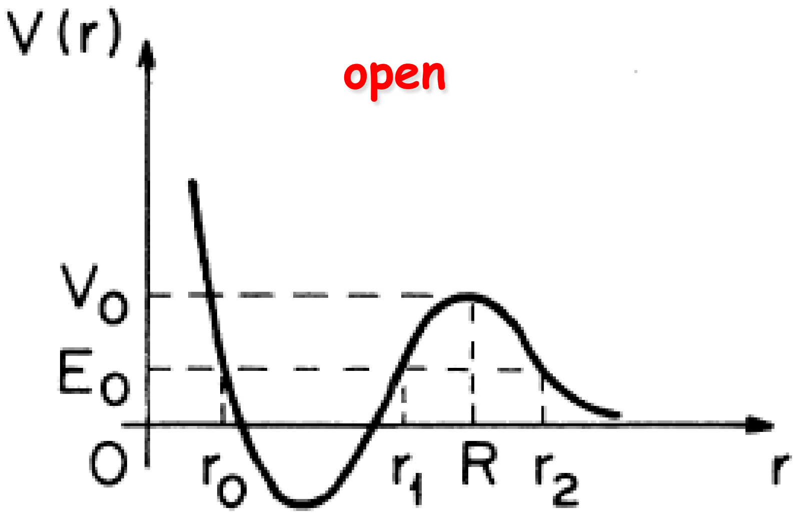
Small quantum systems, whose properties are profoundly affected by environment, i.e., continuum of scattering and decay channels, are intensely studied in various fields of physics: nuclear physics, atomic and molecular physics, nanoscience, quantum optics, etc.

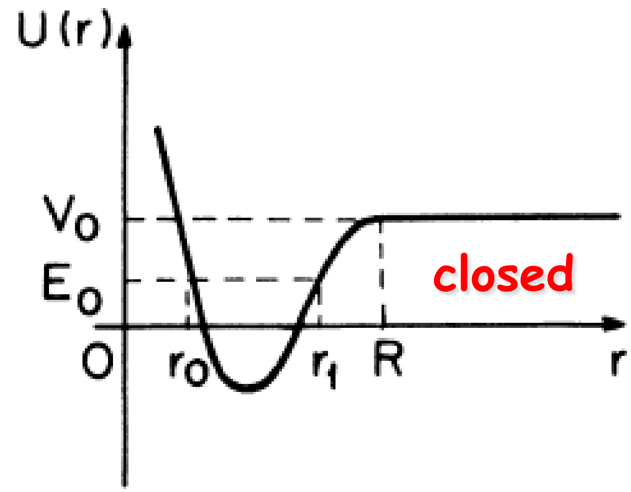
http://www.phy.ornl.gov/theory/MBOQS/Manifesto_09.html

Two potential approach to tunneling

(decay width and shift of an isolated quasistationary state)

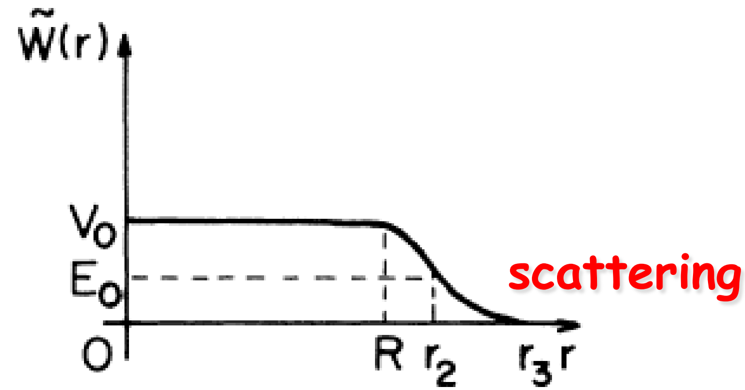
A. Gurvitz, Phys. Rev. A 38, 1747 (1988); A. Gurvitz et al., **Phys. Rev. A69, 042705 (2004)**



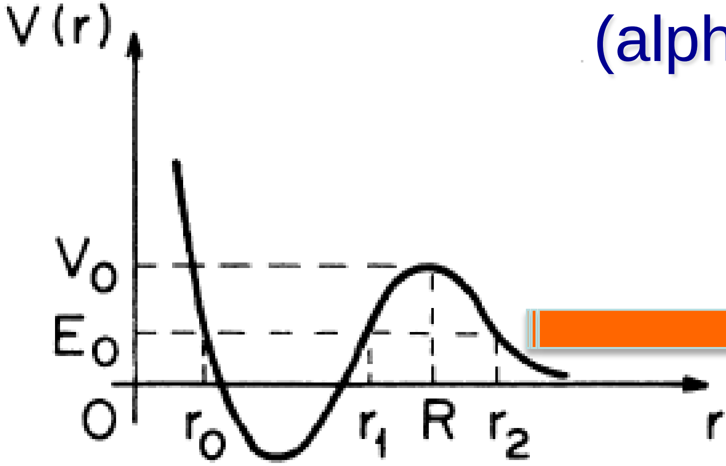


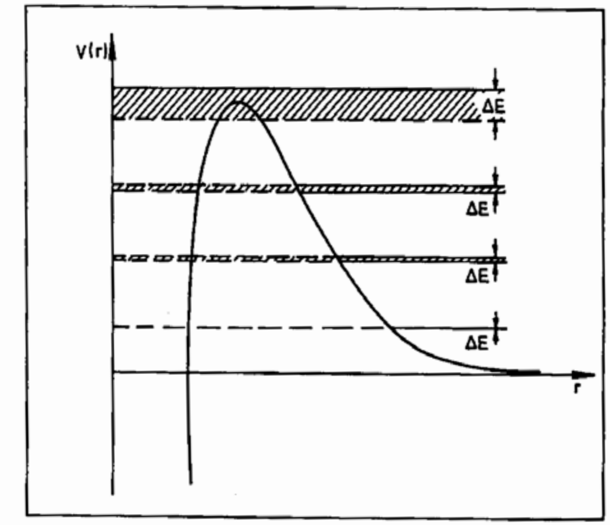
$$V(r) = U(r) + \tilde{W} - V_0$$

decaying state | bound state | scattering state



Quasistationary States
(alpha decay)





$$\frac{dN}{dt} = -wN; \quad N = N_0 e^{-wt}$$

N is a number of radioactive nuclei, i.e., number of particles inside of sphere *r=R:*

$$N \sim \int |\psi^2| d^3 r$$

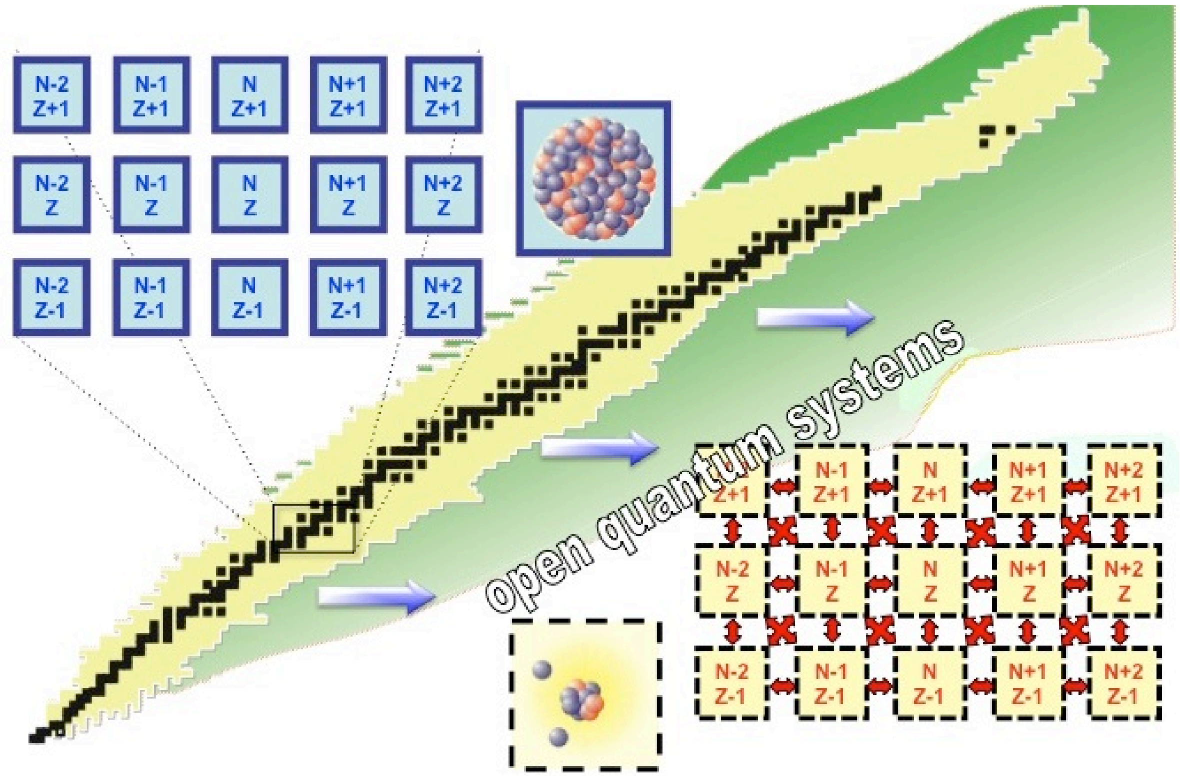
$$\psi = \psi(r) e^{-iE_0 t/\hbar - wt/2} = \psi(r) e^{-iEt/\hbar}$$

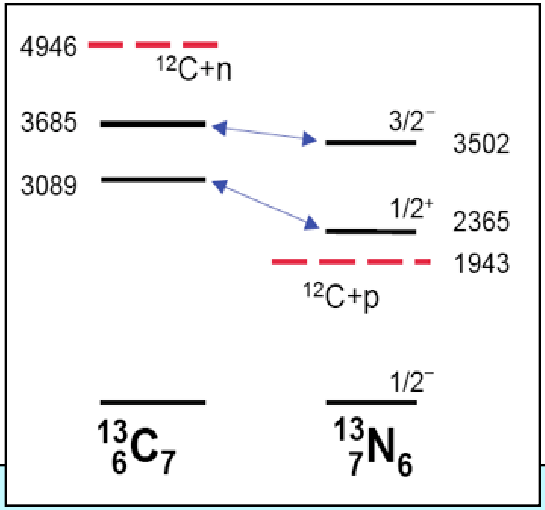
$$E = E_0 - i\frac{\Gamma}{2}; \quad \Gamma = \hbar w$$

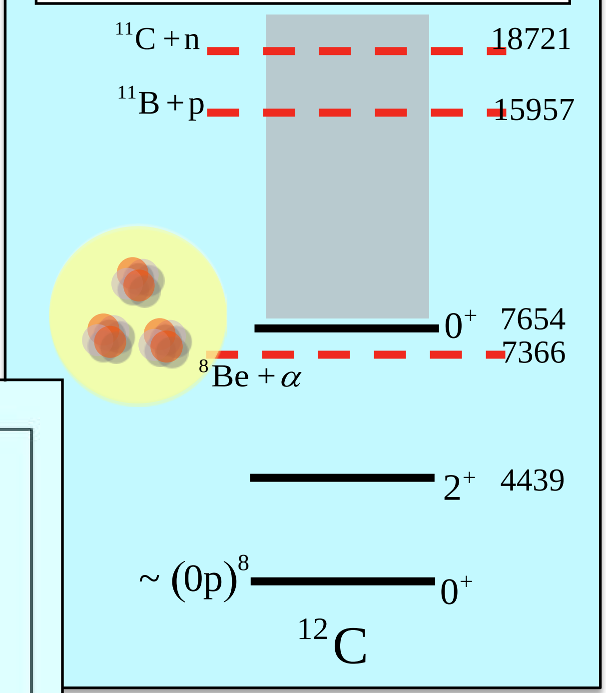
J.J. Thompson, 1884
G. Gamow, 1928

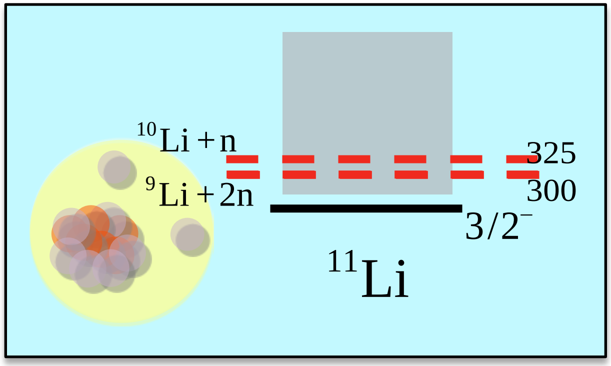
relation between decay width and decay probability (tested!)

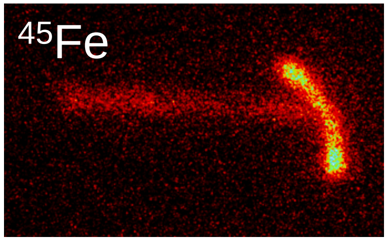
Nucleus as an open quantum system

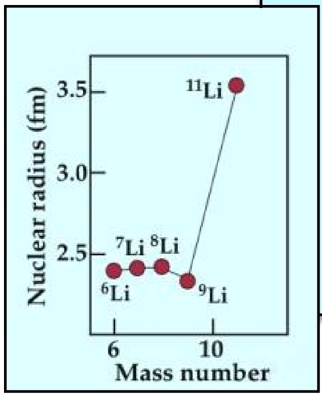




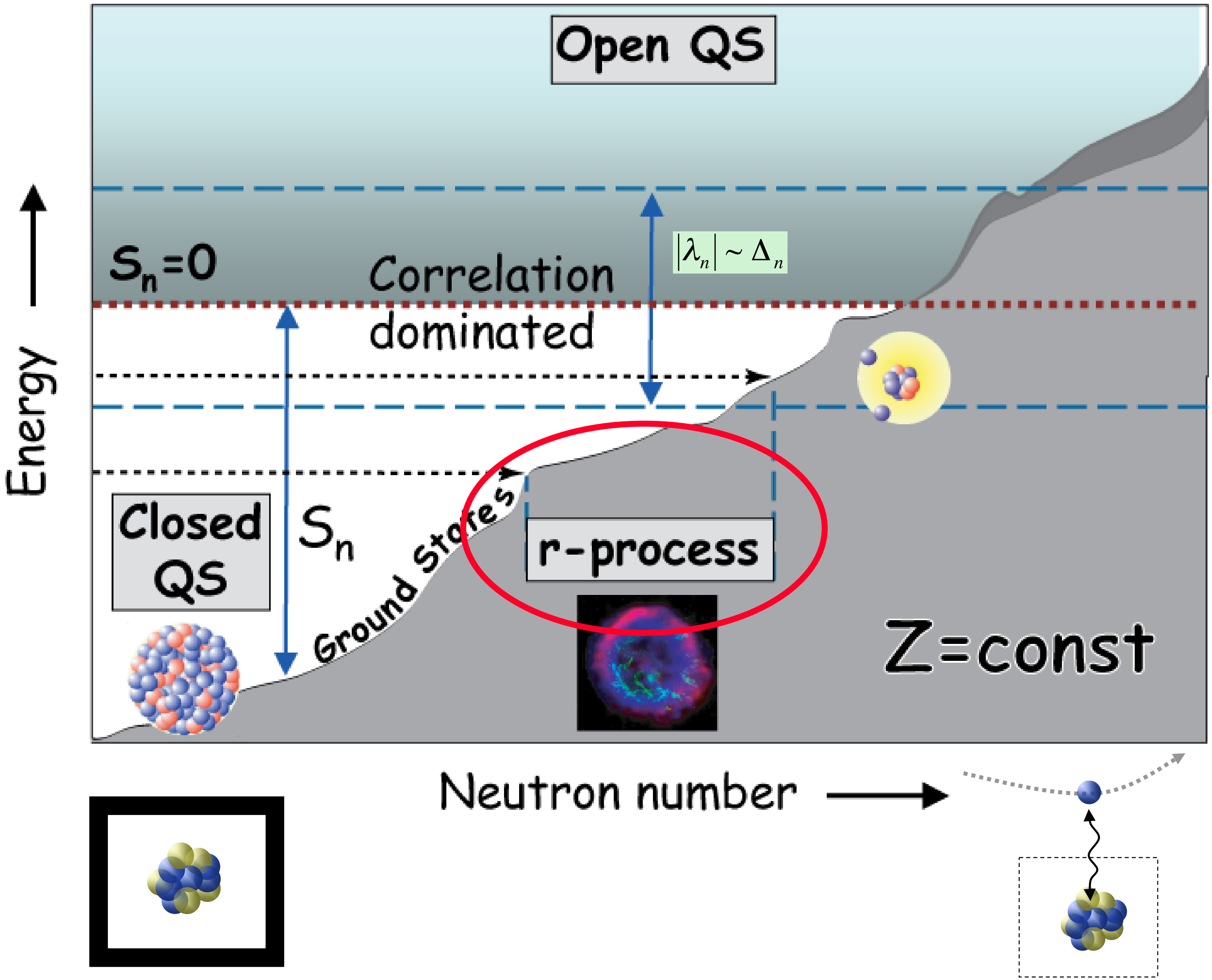




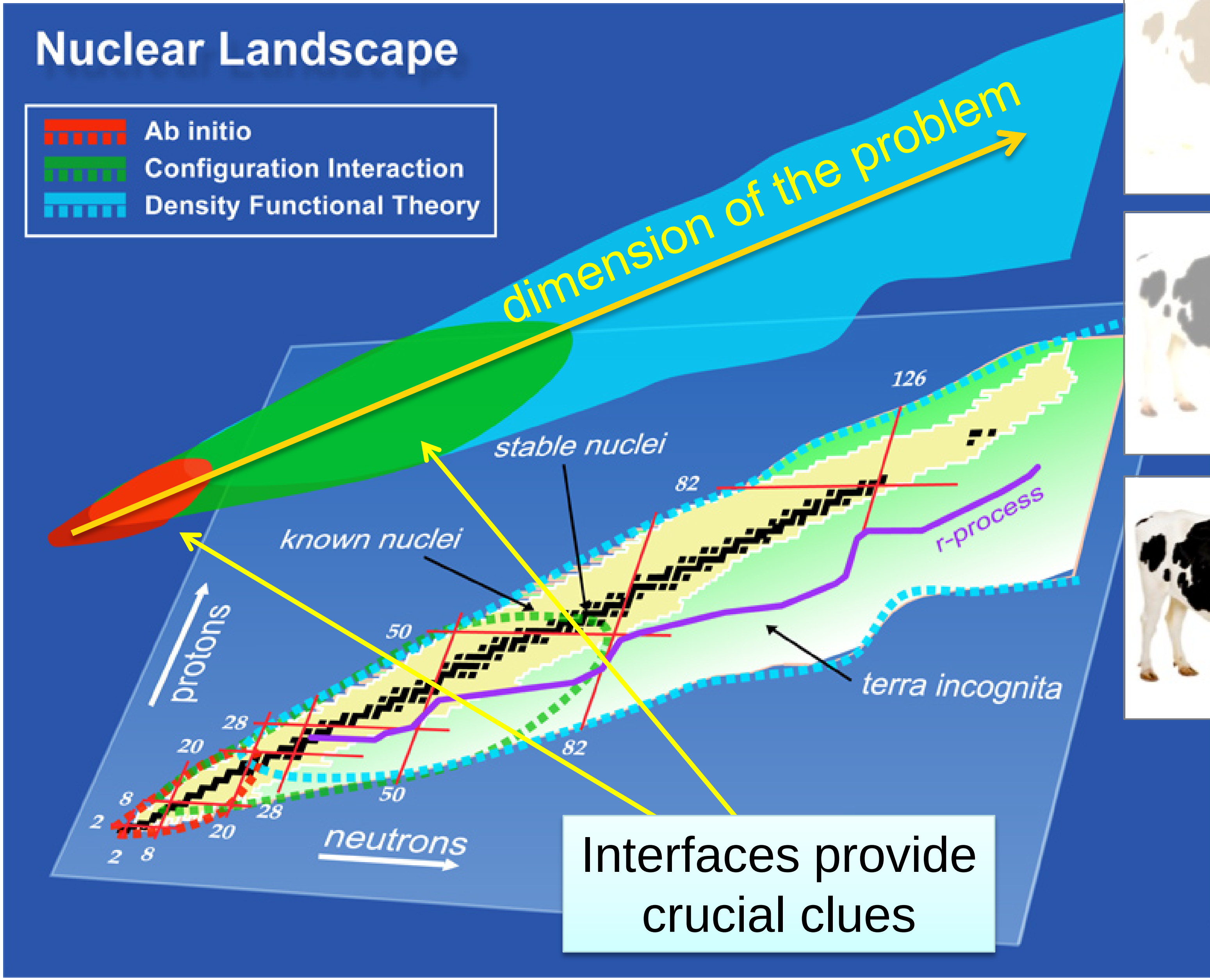




Prog. Part. Nucl. Phys. 59, 432 (2007)

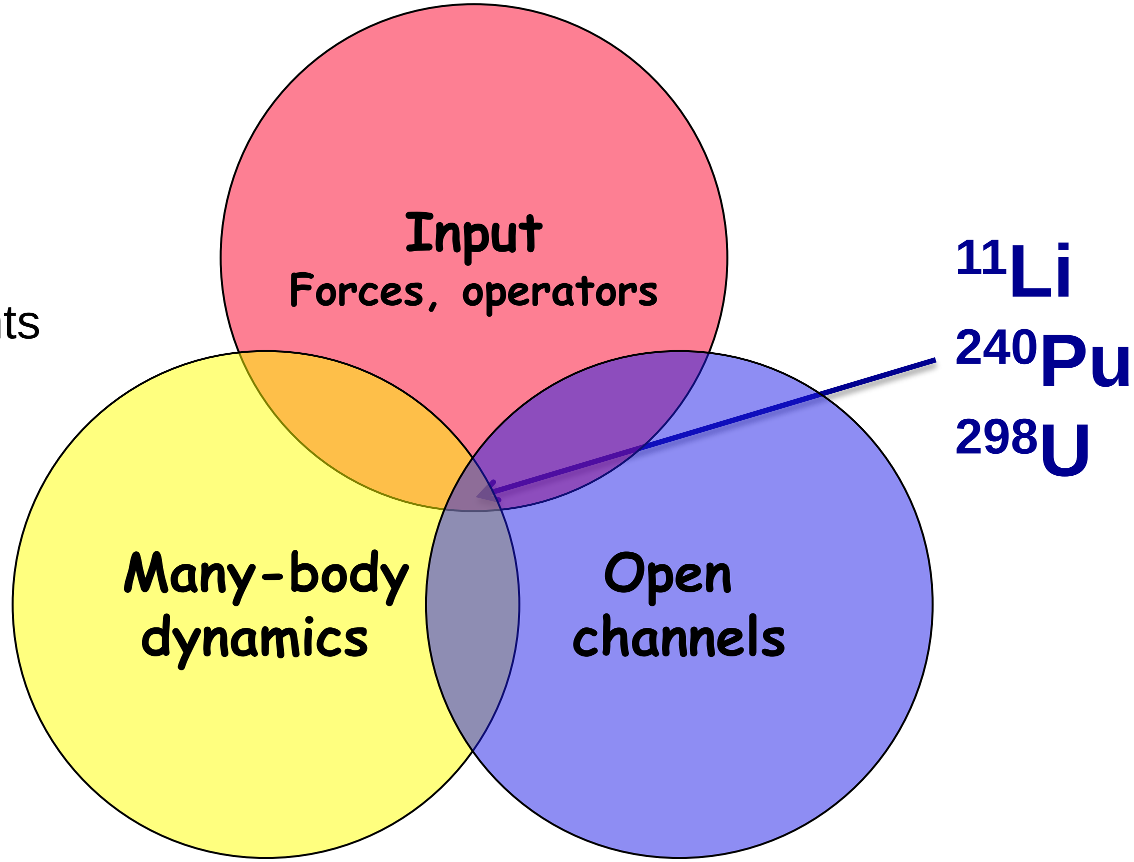


The nuclear landscape as seen by theorists …



Physics of nuclei is demanding

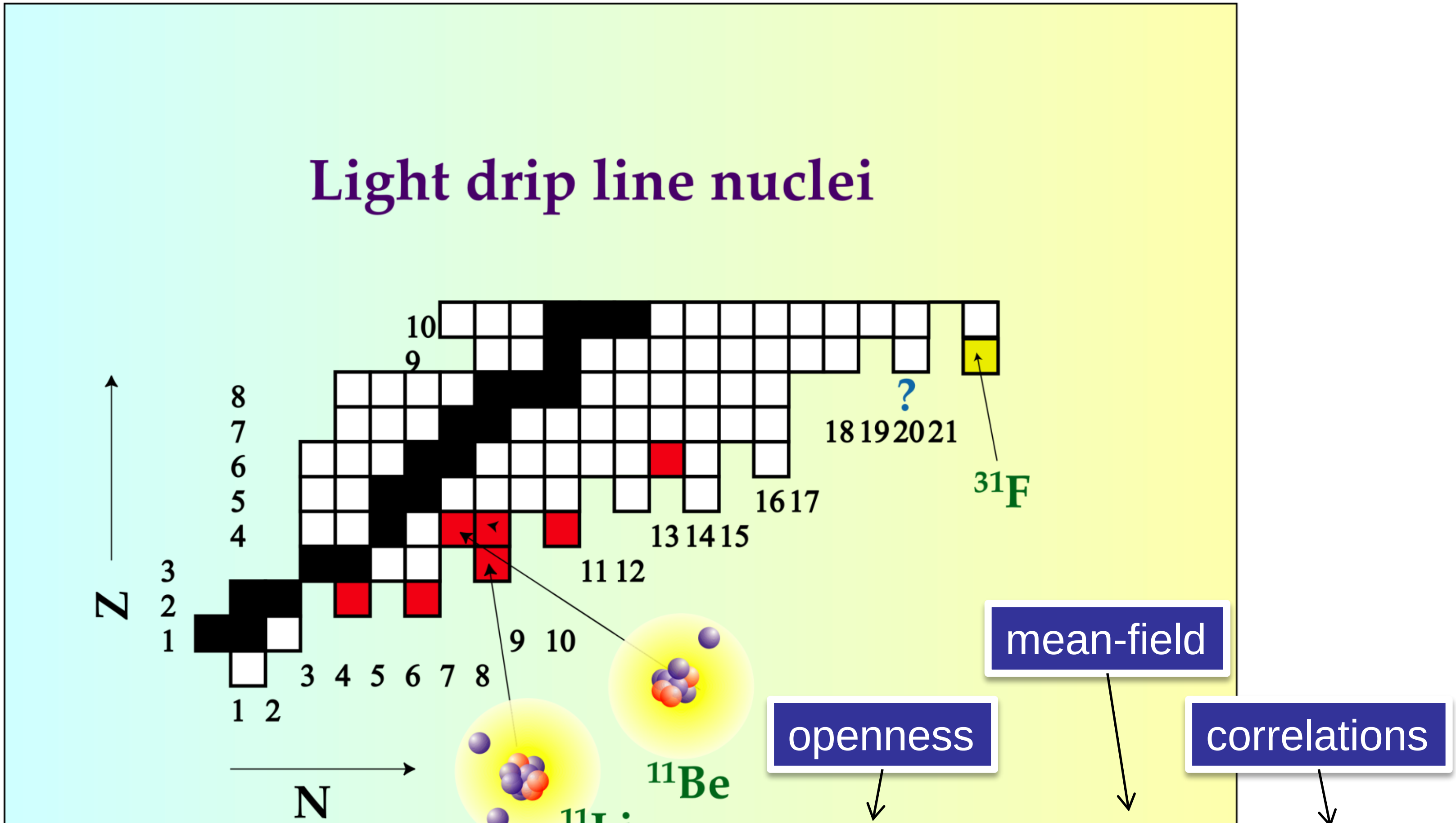
- rooted in QCD
- insights from EFT
- many-body interactions
- in-medium renormalization
- microscopic functionals
- low-energy coupling constants optimized to data
- crucial insights from exotic nuclei



- many-body techniques
  - o direct schemes
  - o symmetry-based truncations
  - o symmetry breaking and restoration
- high-performance computing
- interdisciplinary connections

- nuclear structure impacted by couplings to reaction and decay channels
- clustering, alpha decay, and fission still remain major challenges for theory
- continuum shell model, ab-initio reaction theory and microscopic optical model
- unified picture of structure and reactions

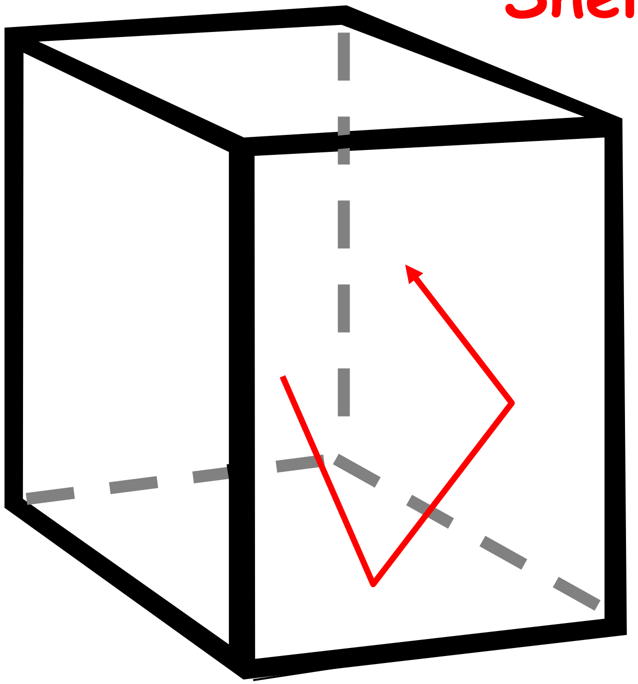
Can we talk about shell structure at extreme N/Z ?



$$S_{1n} \approx -\lambda - \Delta$$

The single-particle field characterized by λ, determined by the p-h component of the effective interaction, and the pairing field Δ determined by the pairing part of the effective interaction are equally important when S_{1n} is small.

Shell effects and classical periodic orbits



$$\hat{H} = \hat{H}_1 + \hat{V}_2 \cdots$$

$$\hat{H}_1 = \sum_{i=1}^{A} \hat{h}_i = \sum_{i=1}^{A} \left(\frac{\hat{p}_i^2}{2m_i} + \hat{V}_i \right)$$

$$\hat{h}\varphi_\alpha = \epsilon_\alpha \varphi_\alpha$$

Shells

One-body field
- Not external (self-bound)
- Hartree-Fock

- Product (independent-particle) state is often an excellent starting point
- Localized densities, currents, fields
- Typical time scale: babyseconds (10^{-22}s)
- Closed orbits and s.p. quantum numbers

But…
- Nuclear box is not rigid: motion is seldom adiabatic
- The walls can be transparent
- In weakly-bound nuclei, residual interaction may dominate the picture: shell-model basis does not govern the physics!
- Shell-model basis not unique (many equivalent Hartree-Fock fields)

Basic Equations

Time Dependent (Many Body) Schödinger Equation

$$i\hbar\frac{\partial\psi}{\partial t}=\hat{H}\psi$$ + boundary conditions

Often impractical to solve but an excellent starting point

^{238}U: $T_{1/2}=10^{16}$ years
^{256}Fm: $T_{1/2}=3$ hours
^{147}Tm: $T_{1/2}=0.57$ s

For very narrow resonances, explicit time propagation impossible!
Nuclear time scale: 10^{-22} s

Time Independent (Many Body) Schödinger Equation

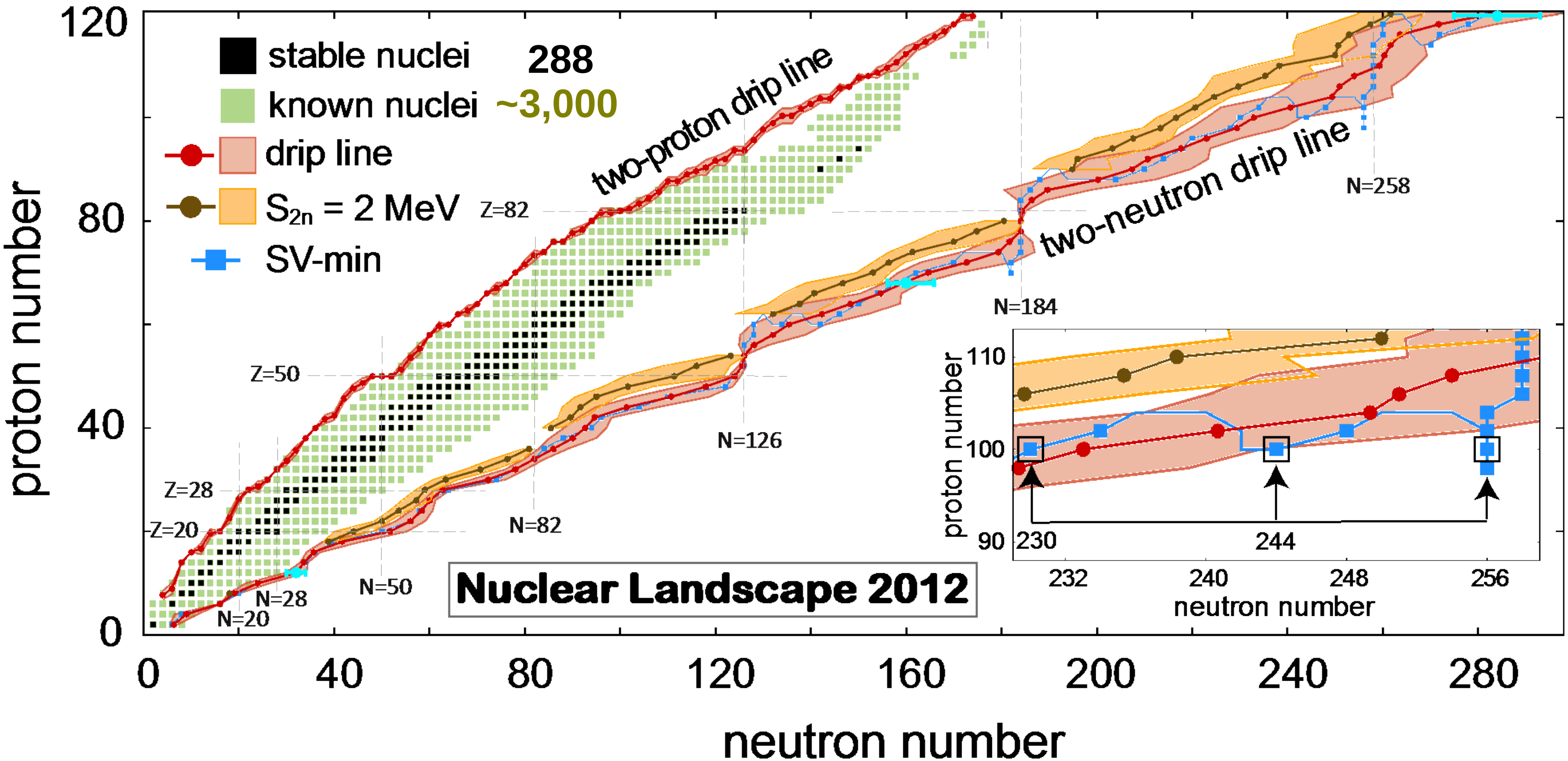
$$\hat{H}\psi=E\psi$$

Box boundary conditions (w.f. vanishes at large distances)
Decaying boundary conditions
Incoming or capturing boundary conditions
Scattering boundary conditions

Absorbing boundary conditions

choice depends on physics case

Quantified Nuclear Landscape



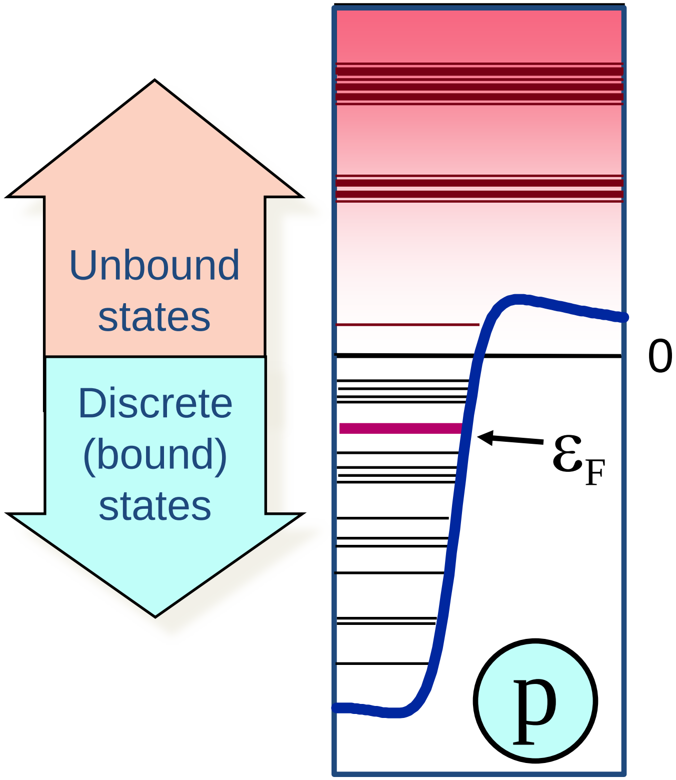
How many protons and neutrons can be bound in a nucleus?

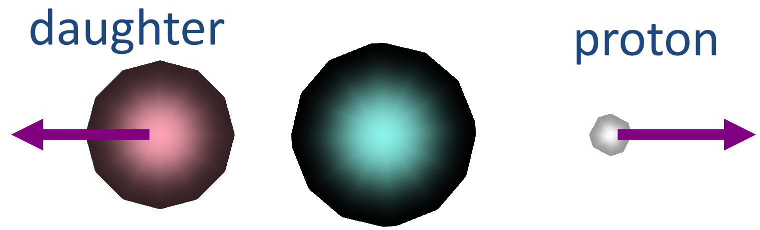
Erler et al.
Nature 486, 509 (2012)

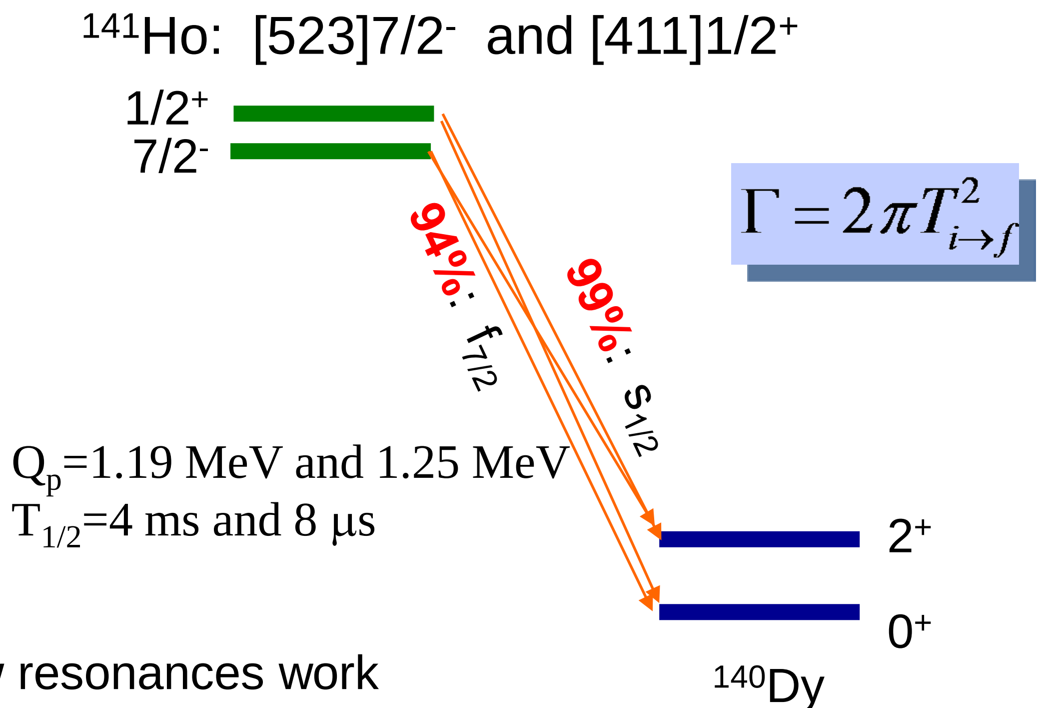
Literature: 5,000-12,000

Skyrme-DFT: 6,900±500$_{syst}$

Description of Proton Emitters



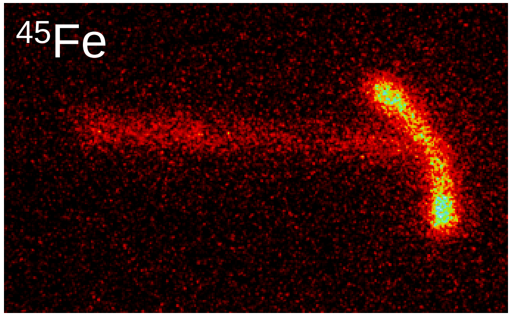




- a simple one-particle case
- many approximations for narrow resonances work
- perfect testing ground for resonant structure approaches
- unique information about nuclear structure beyond the proton drip line

The landscape of two-proton radioactivity

E. Olsen et al, arXiv:1303.1164

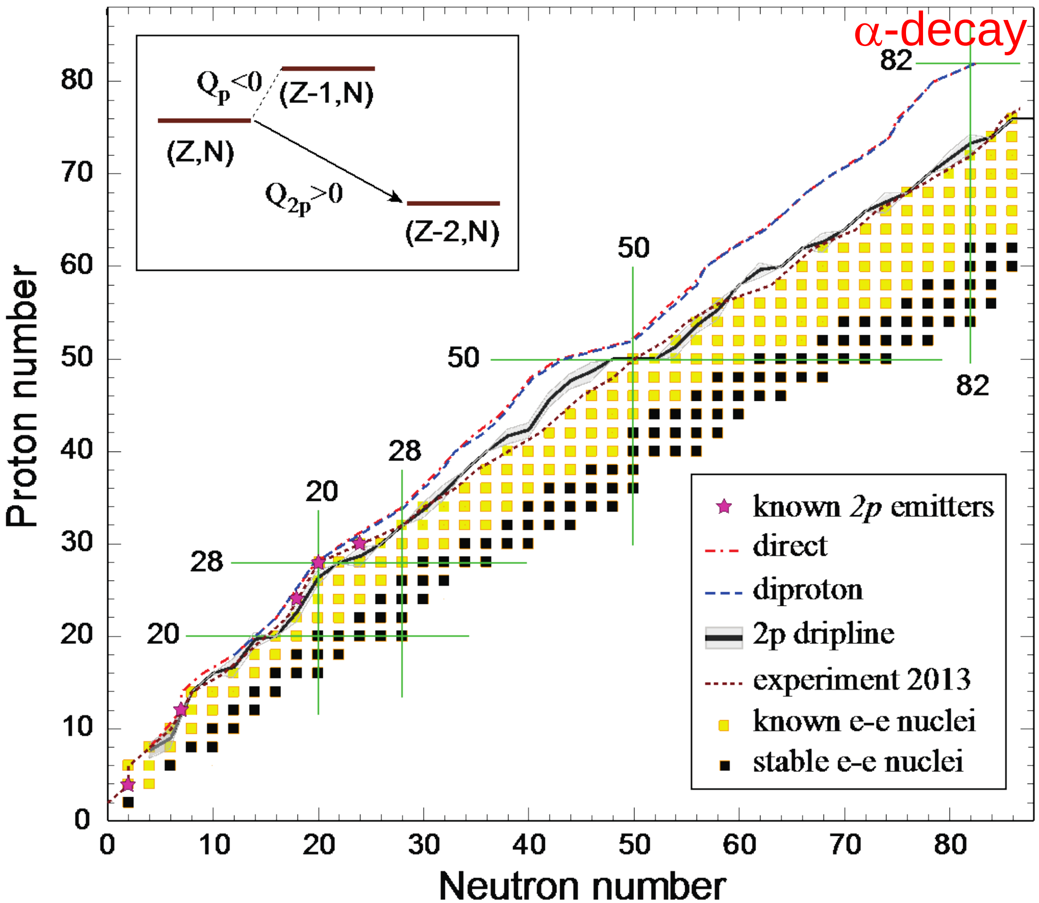


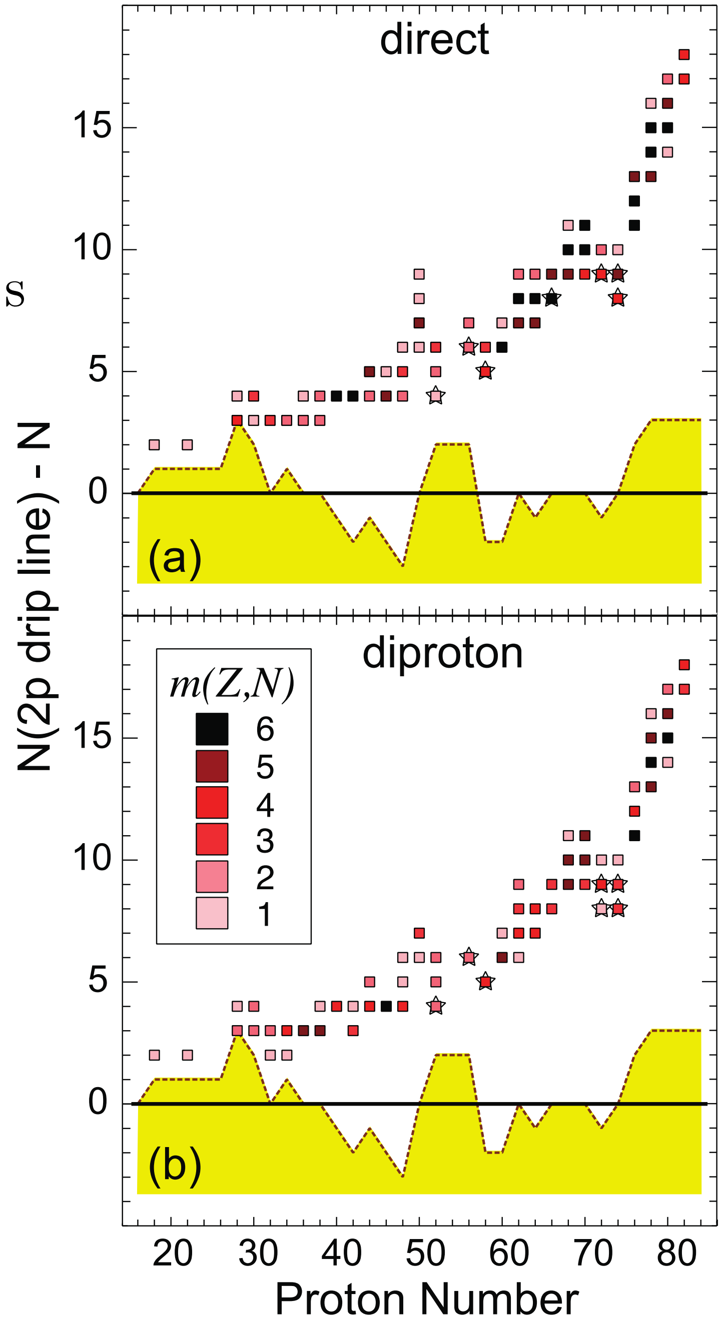
$$Q_{2p} = -S_{2p} > 0$$

$$Q_p = -S_p < 0$$

$$10^{-7}\mathrm{s} < T_{2p} < 10^{-1}\mathrm{s}$$

$$T_{2p} < 10 \cdot T_\alpha$$



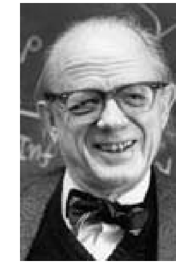


ANNALS OF PHYSICS: 19, 287–313 (1962)

A Unified Theory of Nuclear Reactions. II*

HERMAN FESHBACH

Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

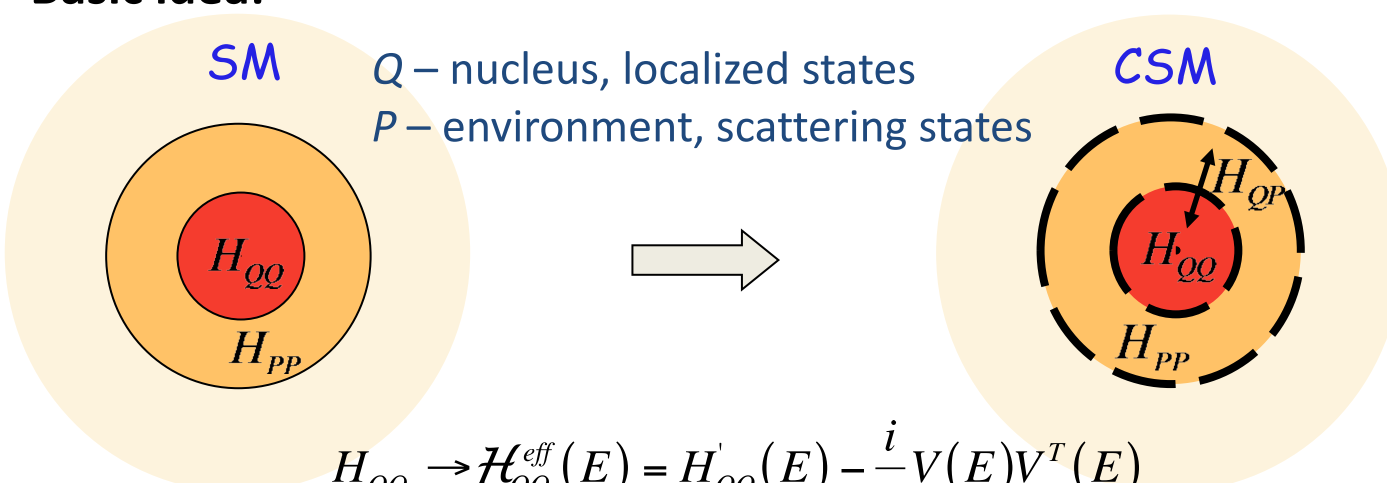


H. Feshbach

The effective Hamiltonian method for nuclear reactions described in an earlier paper with the same title, part I, is generalized so as to include all possible reaction types, as well as the effects arising from the identity of particles.

The principal device employed, as in part I, is the projection operator which selects the open channel components of the wave function...

Basic idea:



$$H_{QQ} \rightarrow \mathcal{H}_{QQ}^{eff}(E) = H'_{QQ}(E) - \frac{i}{2} V(E) V^{T}(E)$$

hermitian

anti-hermitian

closed quantum system

open quantum system

Continuum Shell Model: the unified approach to nuclear structure and reactions

C. Mahaux, H.A. Weidenmüller, *Shell Model Approach to Nuclear Reactions (1969)*
H.W.Bartz et al, Nucl. Phys. A275 (1977) 111
R.J. Philpott, Nucl. Phys. A289 (1977) 109
K. Bennaceur et al, Nucl. Phys. A651 (1999) 289
J. Rotureau et al, Nucl. Phys. A767 (2006) 13
J-B. Faes, M.P., Nucl. Phys. A800 (2008) 21

SMEC
shell model embedded in the continuum

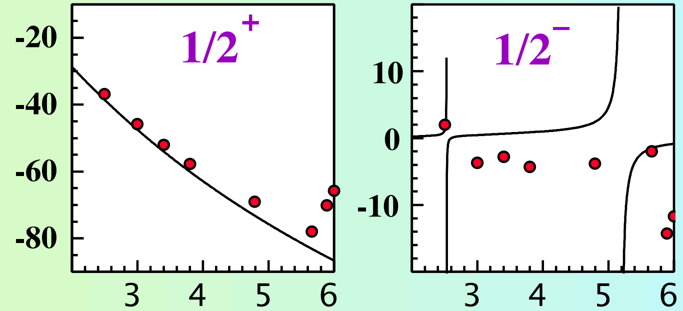
$$\mathcal{H}_{QQ}^{eff}(\mathrm{E}) = \mathrm{H}_{QQ} + \mathrm{H}_{QP}\frac{1}{\mathrm{E}-\mathrm{H}_{PP}}\mathrm{H}_{PQ}$$

$$\mathcal{H}_{\mathrm{QQ}}^{\mathrm{eff}}|\Psi_\alpha\rangle = \mathcal{E}_\alpha(\mathrm{E},\mathrm{V}_0)|\Psi_\alpha\rangle$$

$$\langle\Psi_{\tilde{\alpha}}|\mathcal{H}_{\mathrm{QQ}}^{\mathrm{eff}} = \mathcal{E}_\alpha^{*}(\mathrm{E},\mathrm{V}_0)\langle\Psi_{\tilde{\alpha}}|$$

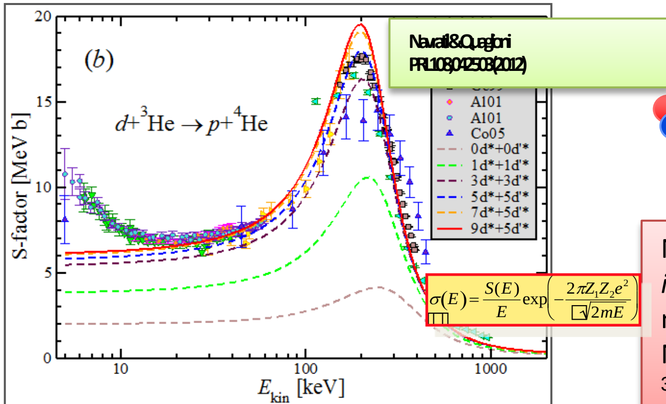
For bound states: $\mathcal{E}_\alpha(E)$ are real and $\mathcal{E}_\alpha(E) = E$

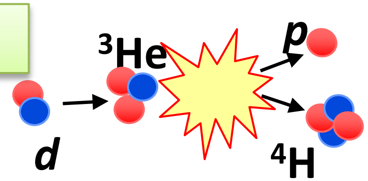
For unbound states: physical resonances are poles of the S-matrix



Lett. B488, 75 (2000)

Nuclear reactions



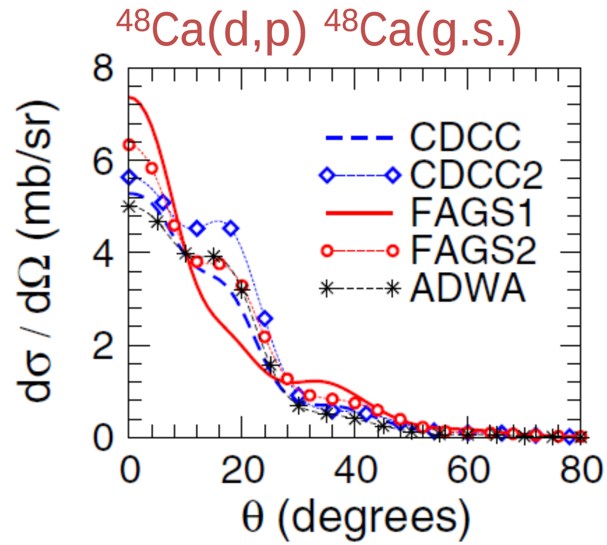


NCSM/RGM is pioneering *ab initio* calculations of light-nuclei fusion reaction with NN interaction. Here, $^3He(d,p)^4He$ S-factor.

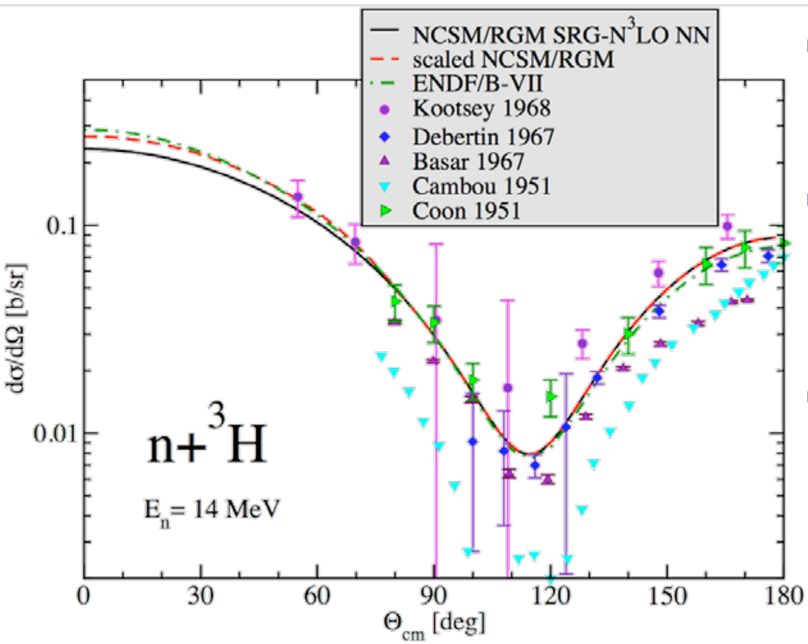
Data deviate from NCSM/RGM results at low energy due to lab. electron-screening

Ab initio theory reduces uncertainty due to conflicting data

TORUS topical collaboration



PRC 84, 034607(2011), PRC 85, 054621 (2012)



- The n-^3H elastic cross section for 14 MeV neutrons, important for NIF, was not known precisely enough.
- Delivered evaluated data with required 5% uncertainty and successfully compared to measurements using an Inertial Confinement Facility
- ``First measurements of the differential cross sections for the elastic n-^2H and n-^3H scattering at 14.1 MeV using an Inertial Confinement Facility", by J.A. Frenje *et al.*, Phys. Rev. Lett. **107,** 122502 (2011)

http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.107.122502

Complex-energy CI
Gamow Shell Model

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

Effects of Configuration Interaction on Intensities and Phase Shifts*

U. Fano
National Bureau of Standards, Washington, D. C.
(Received July 14, 1961)

The actual stationary states may be represented as superpositions of states of different configurations which are "mixed" by the "configuration interaction," i.e., by terms of the Hamiltonian that are disregarded in the independent-particle approximation. The effects of configuration interaction are particularly conspicuous at energy levels above the lowest ionization threshold, where states of different configurations coincide in energy exactly since at least some of them belong to a continuous spectrum.

U. Fano

I.M. Gelfand

T. Berggren

It took over 40 years and required the development of:

- New mathematical concepts: Rigged Hilbert Space (≥1964),…
- Generalized completeness relation including s.p. bound states, resonances, and scattering states (~1968)
- New many-body framework(s): Gamow Shell Model (2002), …

Resonant (Gamow) states

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi$$

outgoing solution

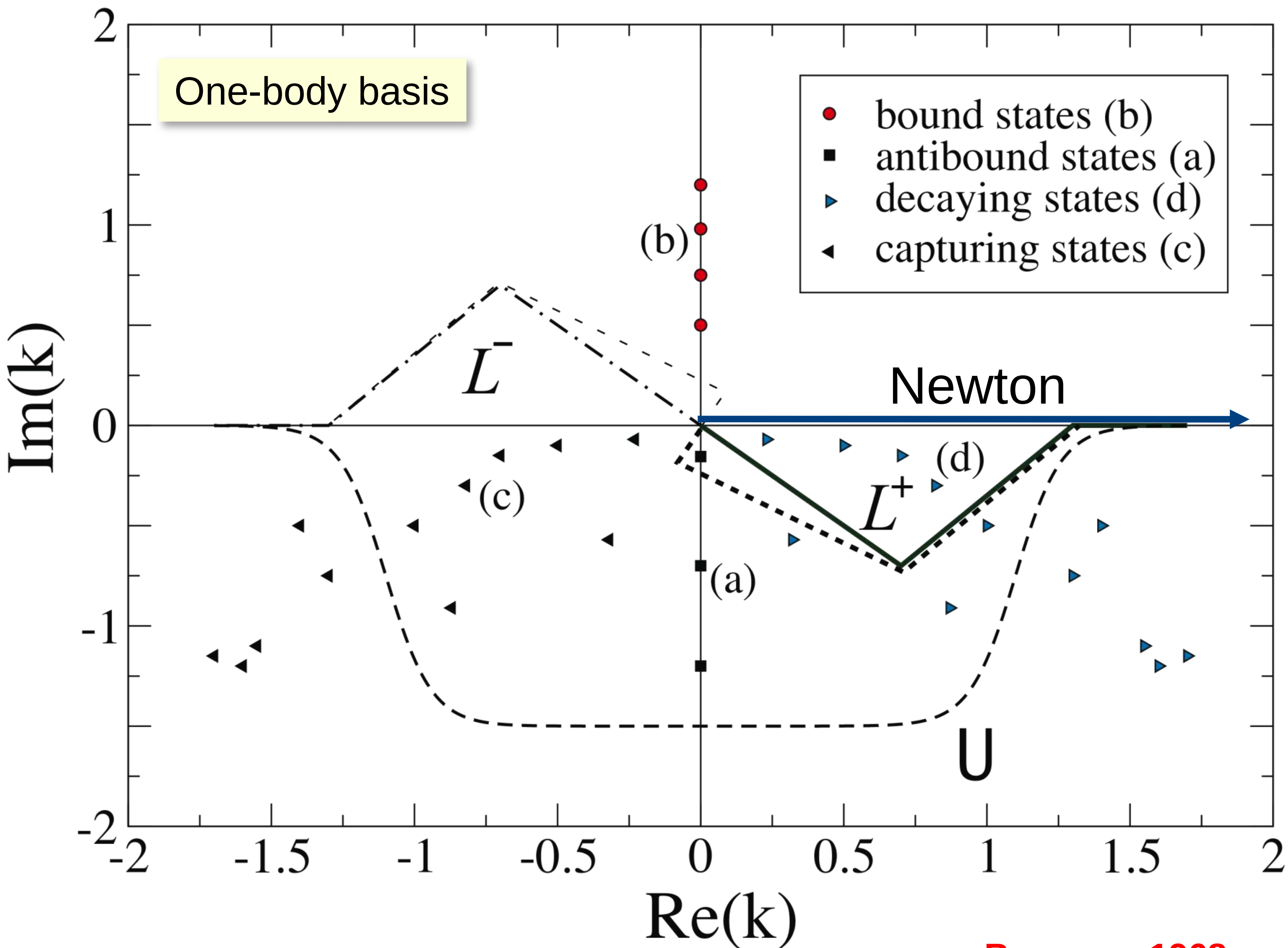
$$\Psi(0,k) = 0, \quad \Psi(\vec{r},k) \xrightarrow[r\to\infty]{} O_l(kr)$$

$$k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$$

complex pole of the S-matrix

- Humblet and Rosenfeld, Nucl. Phys. **26**, 529 (1961)
- Siegert, Phys. Rev. **36**, 750 (1939)
- **Gamow, Z. Phys. 51, 204 (1928)**

Also true in many-channel case!



Berggren 1968

Berggren ensemble for a given *jl* channel:

$$\sum_{n\in(b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|dk = 1.$$

Gamow states and completeness relations

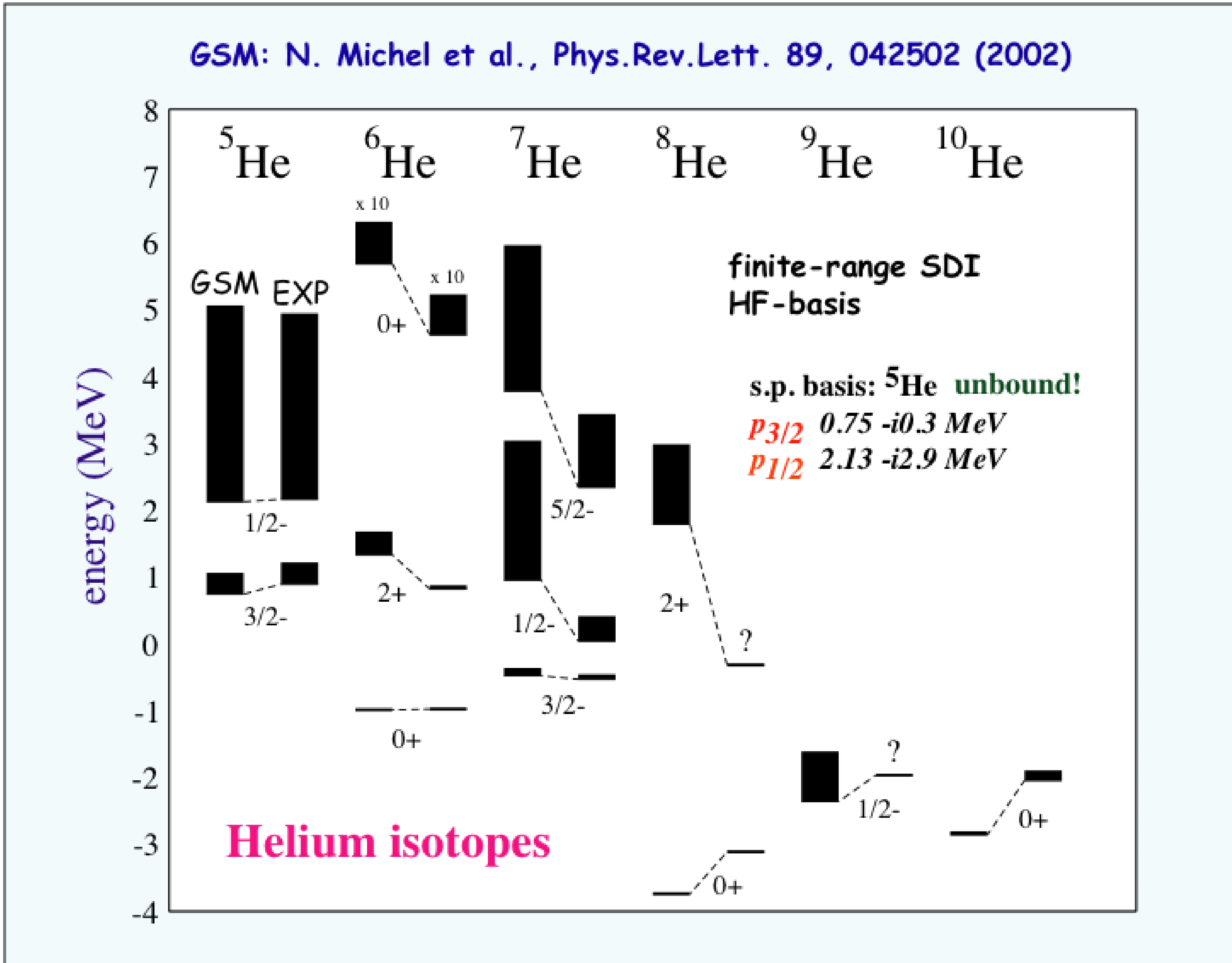
T. Berggren, Nucl. Phys. A109, 265 (1968); A389, 261 (1982)
T. Lind, Phys. Rev. C47, 1903 (1993)

$$\sum_{n\in(b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|dk = 1.$$

Gamow Shell Model:

- Based on the Rigged Hilbert Space formulation of quantum mechanics
- Contour is identified and discretized for each partial wave
- Many-body Slater determinants are built out of resonant and scattering states
- GSM Hamiltonian matrix is computed (using external complex scaling) and Lanczos-diagonalized (the matrix is complex symmetric)
- The many-body Gamow states are identified
- Expectation values of operators are computed

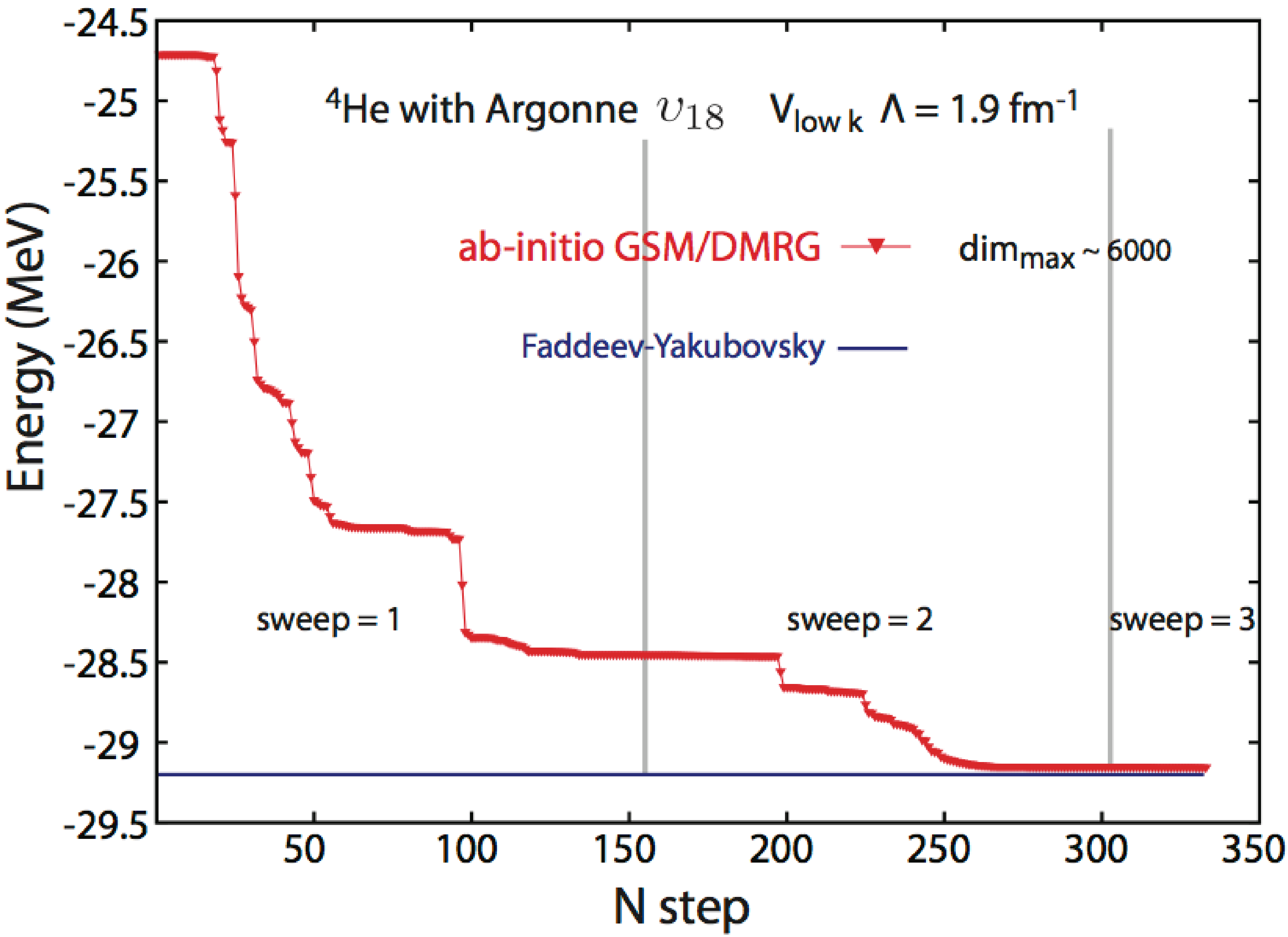
- Discretization/truncation optimized my means of DMRG
- Generalized variational principle



S_{2n} in ^{8}He greater than S_{2n} in ^{6}He

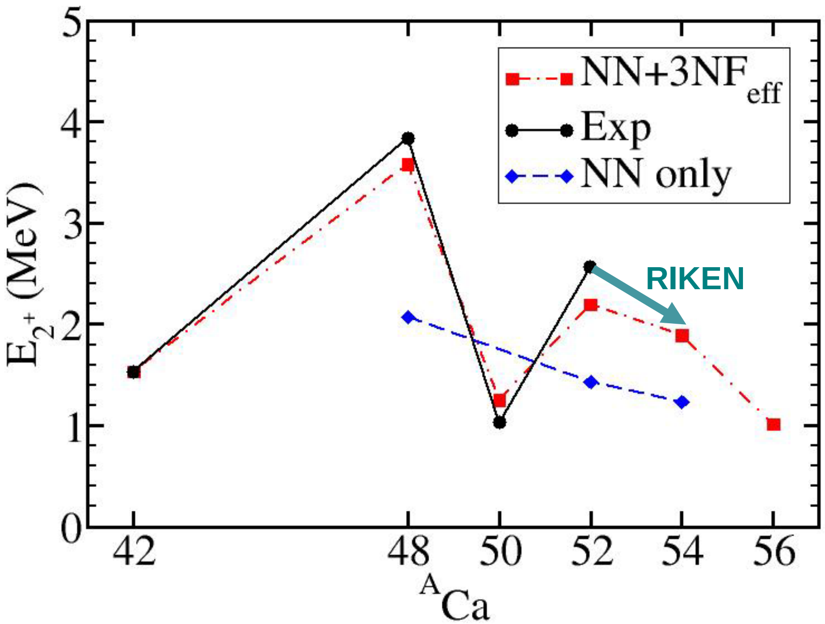
Ab-initio No-Core Gamow Shell Model calculations with realistic interactions

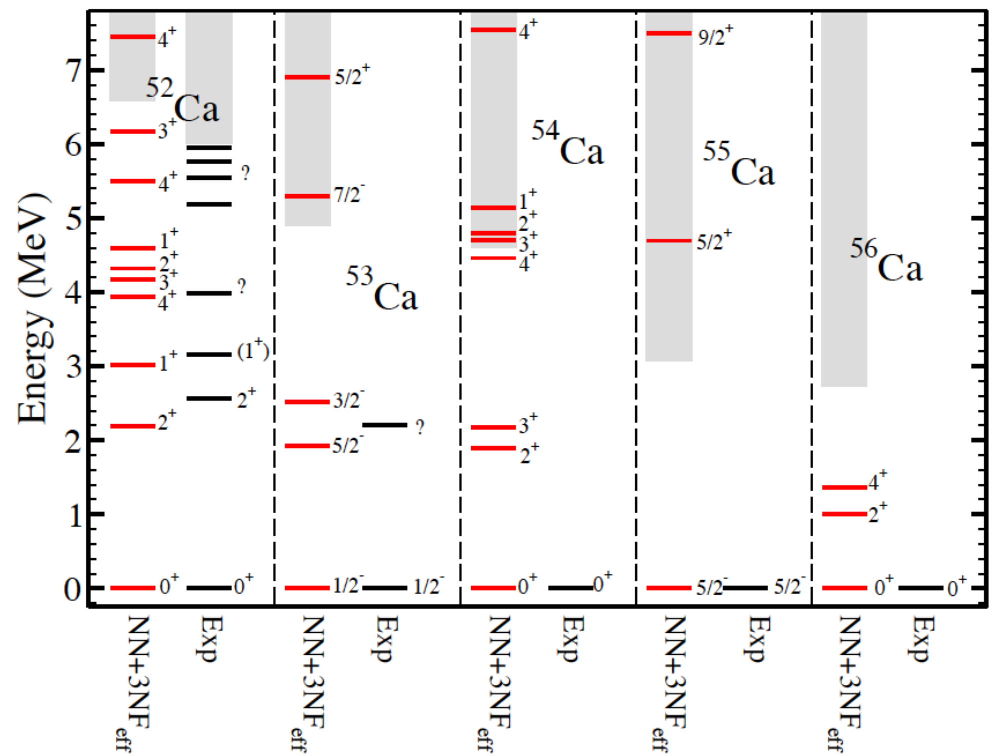
G. Papadimitriou et al., arXiv:1301.7140



Ab-initio description of medium-mass open nuclear systems

G. Hagen et al., Phys. Rev. Lett. 109, 032502 (2012)





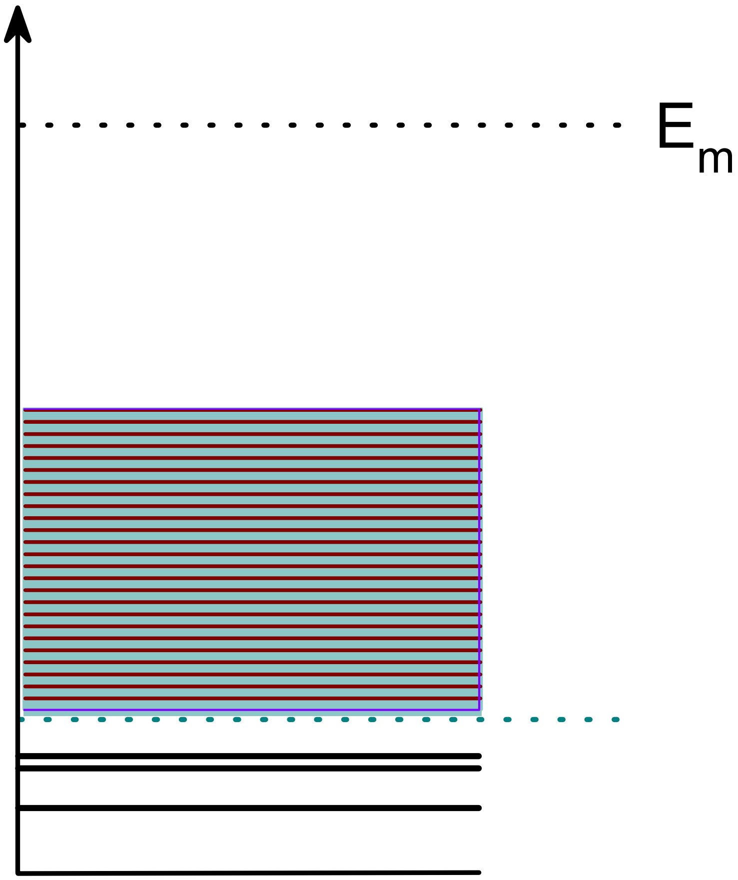
	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

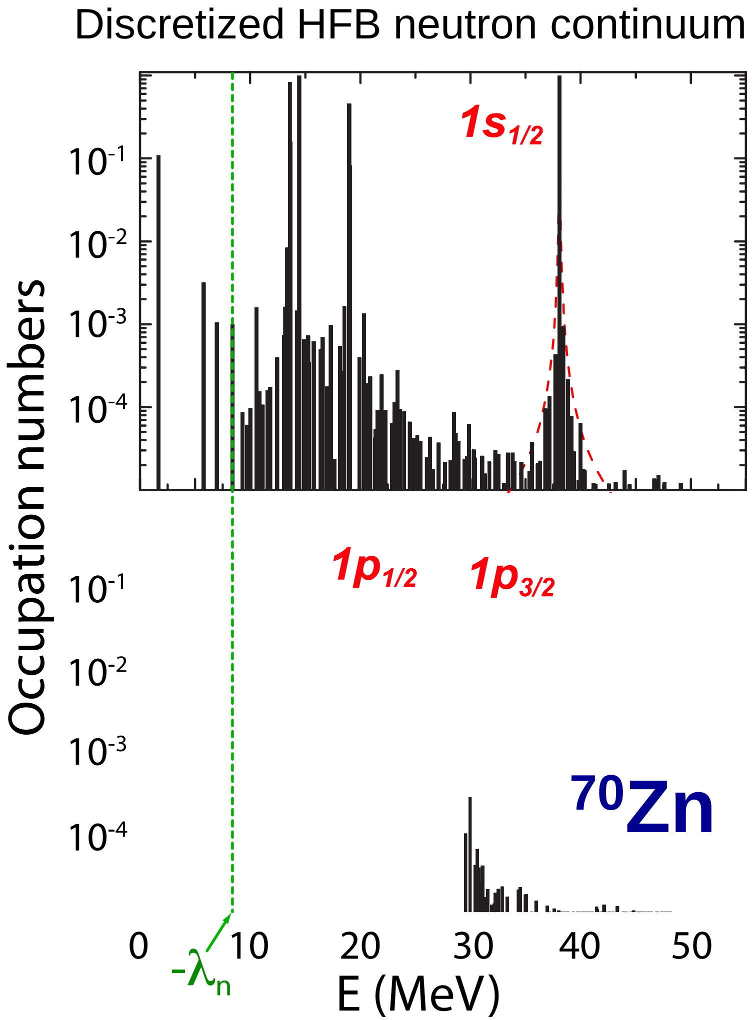
½$^+$ virtual state

- Strong coupling to continuum for neutron rich calcium isotopes
- Level ordering of states in the *gds* shell is contrary to naïve shell model picture

Quasi-particle continuum and resonances in the Hartree-Fock-Bogoliubov theory

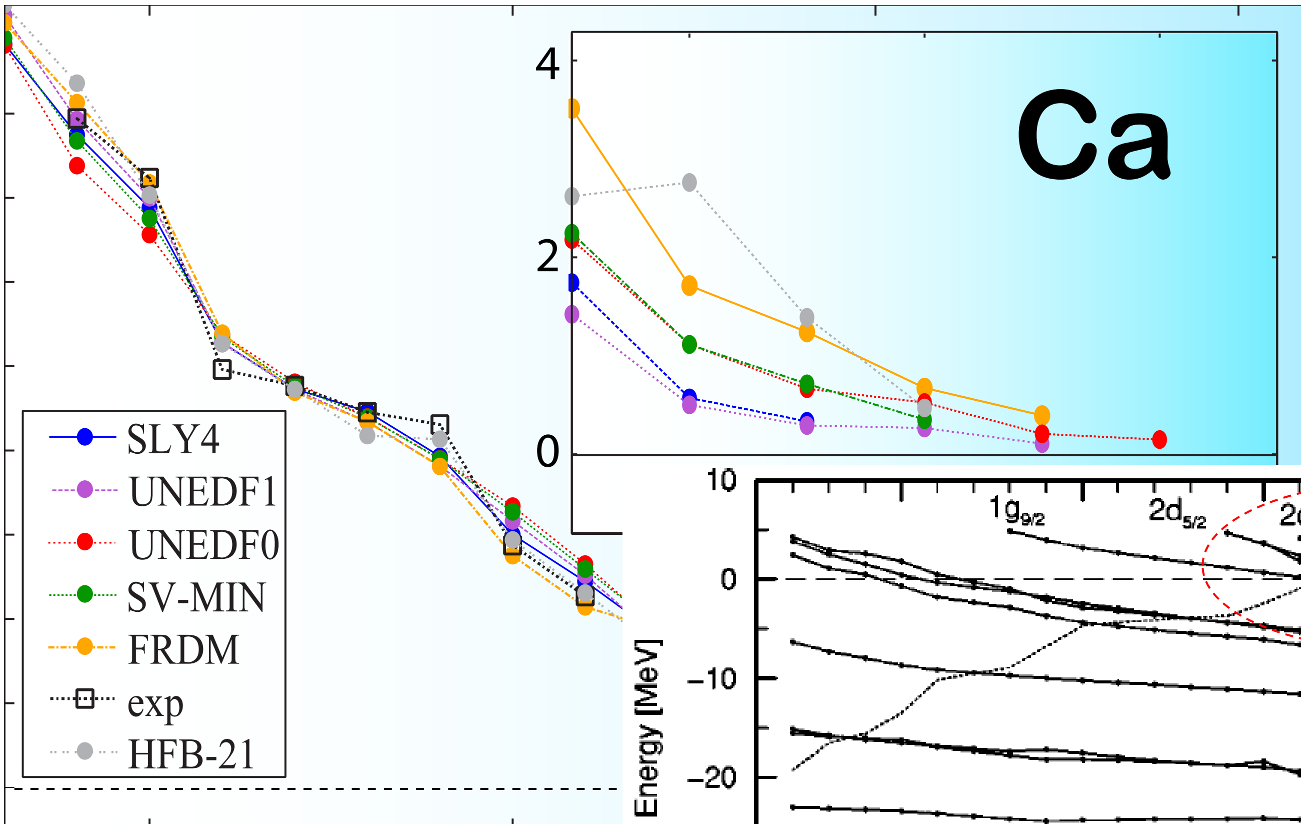
J.C. Pei et al. Phys. Rev. C 84, 024311 (2011)

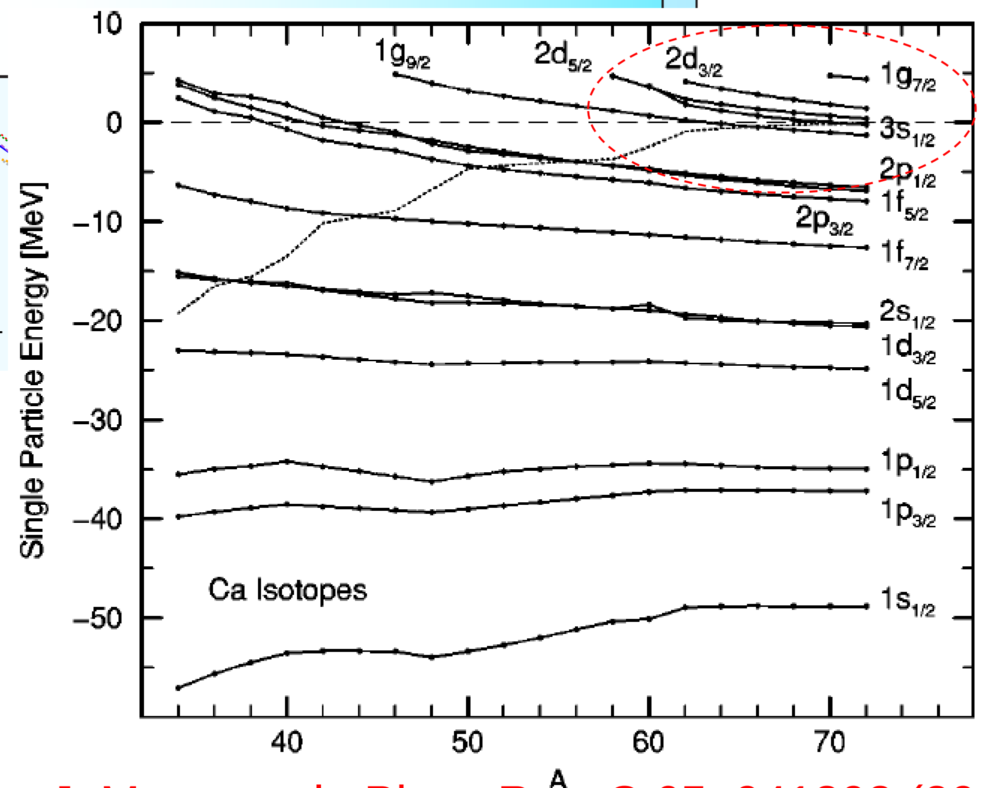




S.T. Belyaev et al., Sov. J. Nucl. Phys. 45, 783 (1987)

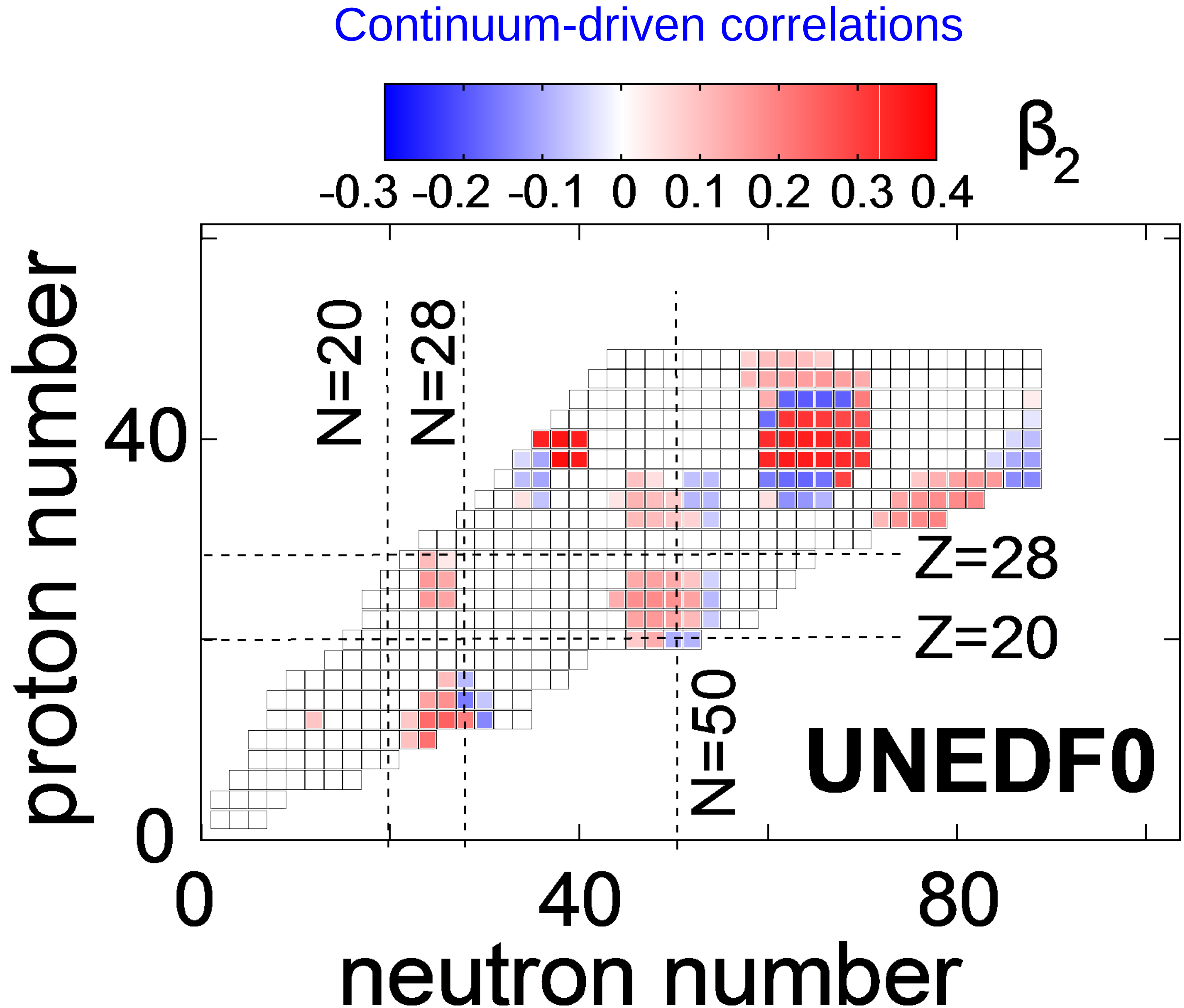
$$\Gamma = 2\pi |\langle u_{0E}|\Delta|v^{\mathrm{HF}}|\rangle|^2$$





See extended discussion in Sec. 6 of arXiv:1206.2600:
Hartree-Fock-Bogoliubov solution of the pairing Hamiltonian in finite nuclei
J. Dobaczewski, WN

J. Meng et al., Phys. Rev. C 65, 041302 (2002)
S. Fayans et al., Phys. Lett. 491 B, 245 (2000)


Continuum-driven correlations
-0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4
β2
proton number
neutron number
0
40
80
N=20
N=28
N=50
Z=28
Z=20
UNEDF0

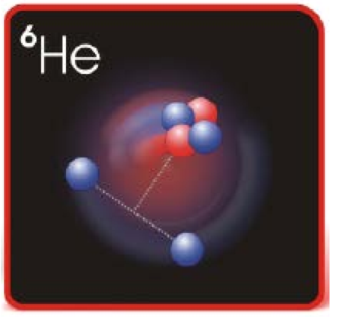
Experimental charge radii of ^{6}He, ^{8}He, ^{11}Li, ^{11}Be

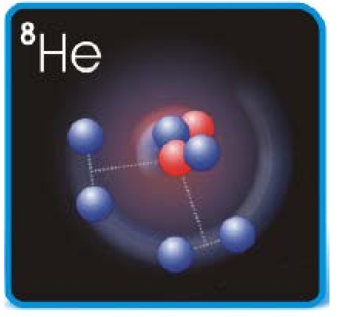
	^{4}He	^{6}He	^{8}He
L.B.Wang *et al*	**1.67fm**	**2.054(18)fm**	
P.Mueller *et al*	**1.67fm**	**2.068(11)fm**	**1.929(26)fm**

	^{9}Li	^{11}Li
R.Sanchez *et al*	**2.217(35)fm**	**2.467(37)fm**

	^{10}Be	^{11}Be
W.Nortershauser *et al*	**2.357(16)fm**	**2.460(16)fm**

L.B.Wang *et al*, PRL **93**, 142501 (2004)
P.Mueller *et al*, PRL **99**, 252501 (2007)
R.Sanchez *et al* PRL **96**, 033002 (2006)
W.Nortershauser *et al* PRL **102**, 062503 (2009)





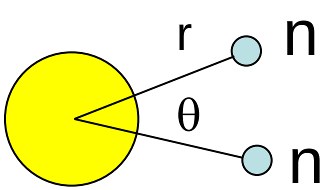
- ✓ High precision measurements based on laser spectroscopy
- ✓ Charge radii were extracted from isotopic shift measurements with the help of atomic theory calculations

GSM calculations for ^{6}He, ^{8}He by G. Papadimitriou et al.

Two-nucleon density distribution:

$$\rho(r_1, r_2, \theta_{12}) = \langle \Psi | \delta(r_1 - r) \delta(r_2 - r') \delta(\theta_{12} - \theta) | \Psi \rangle$$

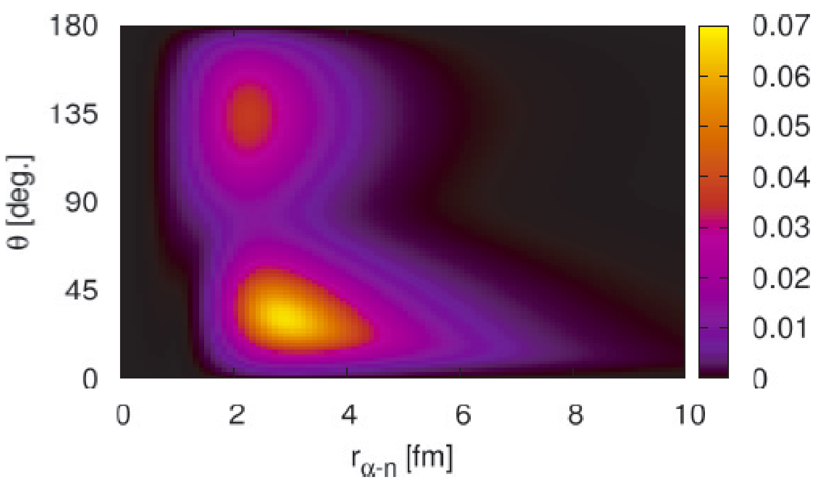
V. Kukulin et al., Nucl.Phys.A 453, 365 (1986).
G. Bertsch and H. Esbensen, Ann.Phys.(N.Y.) 209, 327 (1991).
M. V. Zhukov et al., Phys. Rep. 231, 151 (1993).
E. Nielsen et al., Phys. Rep. 347, 373 (2001).
K. Hagino and H. Sagawa, Phys. Rev. C 72, 044321 (2005).



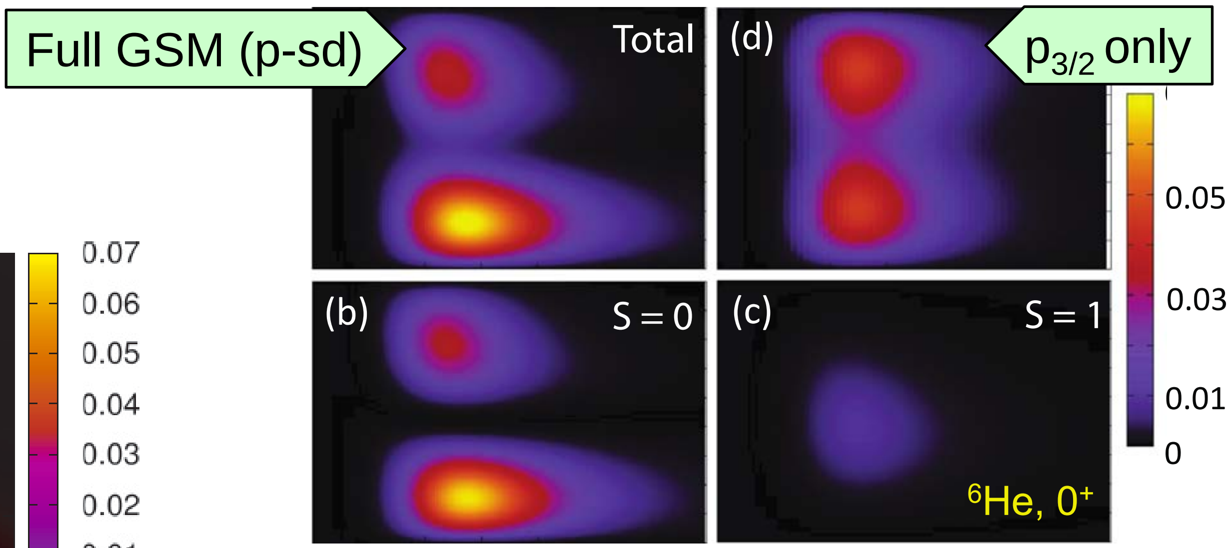
^{6}He ground state

Two-neutron peak enhanced by the continuum coupling

Complex Scaling

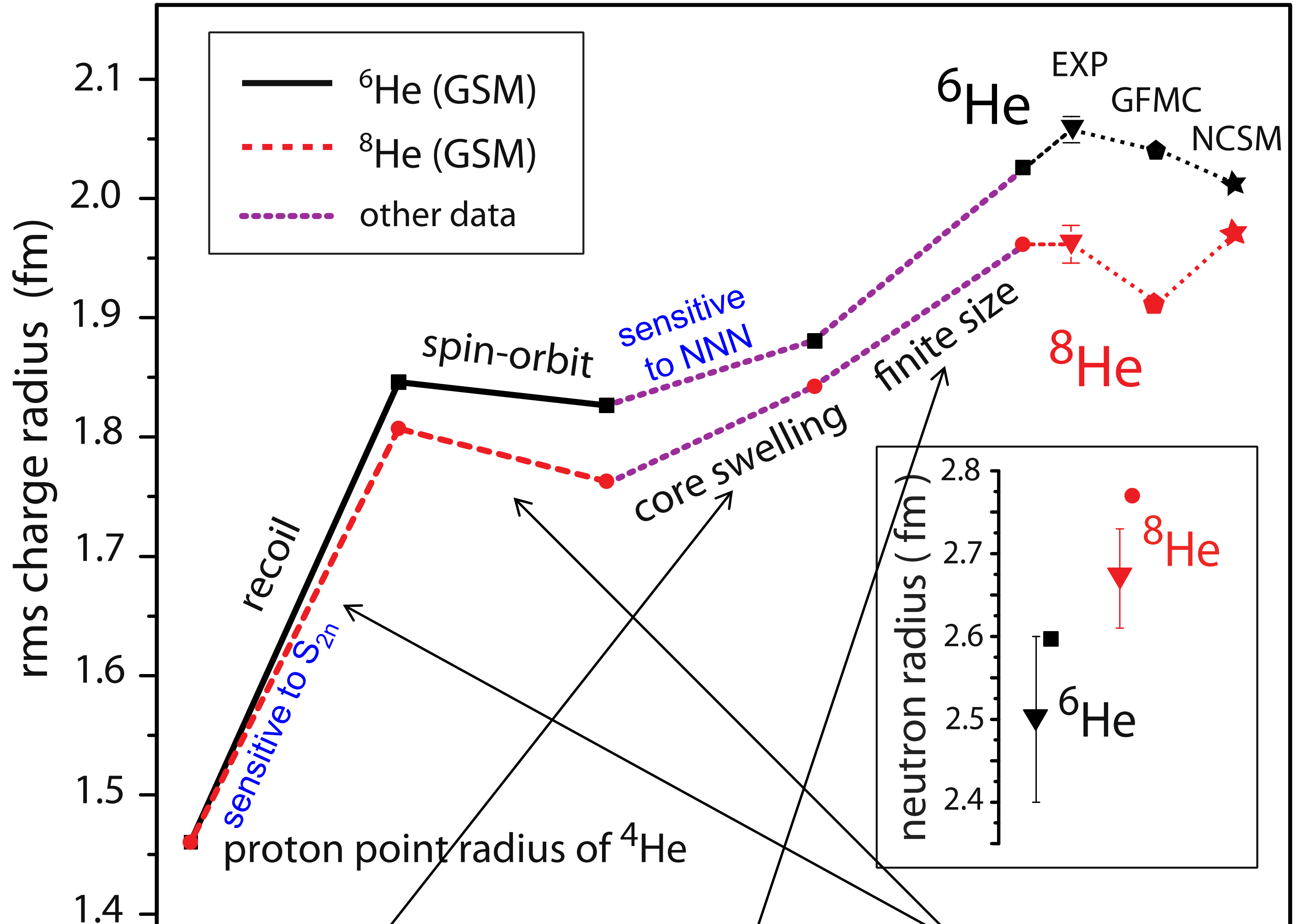


Y. Kikuchi et al., Phys. Rev. C 81, 044308 (2010).



G. Papadimitriou et al., Phys. Rev. C 84, 051304(R) (2011)

G. Papadimitriou et al., Phys. Rev. C 84, 051304(R) (2011)



$$\langle r_{pp}^2(^{A_c+n}X)\rangle = \boxed{\langle r_{pp}^2(^{A_c}X)\rangle} + \boxed{\frac{1}{(A_c+n)^2}\sum_{i=1}^{n}\langle r_i^2\rangle + \frac{2}{(A_c+n)^2}\sum_{i<j}^{n}\langle \mathbf{r}_i\cdot\mathbf{r}_j\rangle}$$

$$\langle r_{ch}^2\rangle = \langle r_{pp}^2\rangle + \boxed{\langle R_p^2\rangle + \frac{N}{Z}\langle R_n^2\rangle + \frac{3}{4M_p^2}} + \boxed{\langle r^2\rangle_{so}}$$

Bound states of dipolar molecules

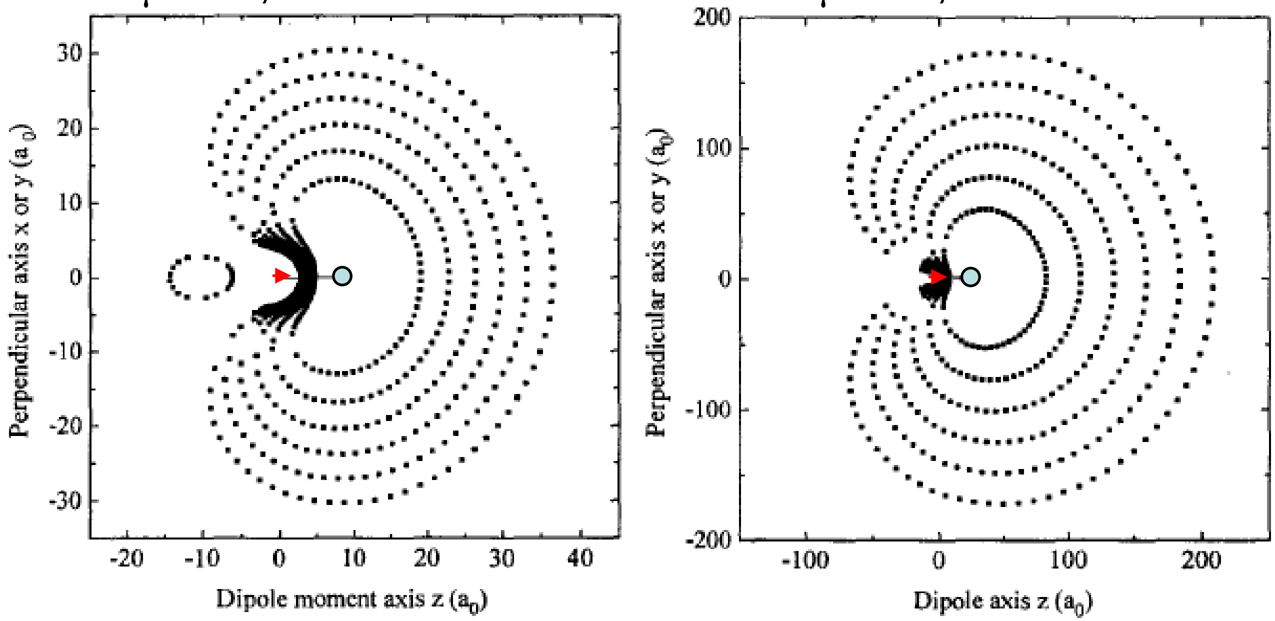
K. Fossez, N. Michel, WN, M. Płoszajczak, arXiv:1303.1928

The mechanism for forming anion states by the long-range dipolar potential has been proposed by Fermi and Teller in 1947, who studied the capture of negatively charged mesons in matter. They found that if a negative meson is captured by a hydrogen nucleus, the binding energy of the electron becomes zero for the electric dipole moment of a meson-proton system μ_{cr} = 1.625D. Lifting the adiabatic approximation by considering the rotational degrees of freedom of the anion turned out to be crucial; it also boosted μ_{cr} to about 2.5 D.

Examples: LiI^-, $LiCl^-$, LiF^-, and LiH^- anions

μ=7D, E=-176 meV

μ=3D, E=-3 meV



Desfrançois et al., IJMP B 10, 1339 (1996)

$$H_{tot} = \frac{\boldsymbol{p}_e^2}{2m_e} + \frac{\boldsymbol{j}^2}{2I} + V$$

W. R. Garrett, J. Chem. Phys. 77, 3666 (1982)

$$V(r,\theta) = V_\mu(r,\theta) + V_\alpha(r,\theta) + V_{Q_{zz}}(r,\theta) + V_{\mathrm{SR}}(r)$$

$$V_\mu(r,\theta) = -\mu e \sum_{\lambda=1,3,\cdots} \left(\frac{r_<}{r_>}\right)^\lambda \frac{1}{s r_>} P_\lambda(\cos\theta)$$

dipole potential of the molecule

$$V_\alpha(r,\theta) = -\frac{e^2}{2r^4}\left[\alpha_0 + \alpha_2 P_2(\cos\theta)\right] f(r)$$

induced dipole potential

$$f(r) = 1 - \exp\{-(r/r_0)^6\}$$

$$V_{Q_{zz}}(r,\theta) = -\frac{e}{r^3} Q_{zz} P_2(\cos\theta) f(r)$$

quadrupole potential of the molecule

$$V_{\mathrm{SR}}(r) = V_0 \exp(-(r/r_c)^6)$$

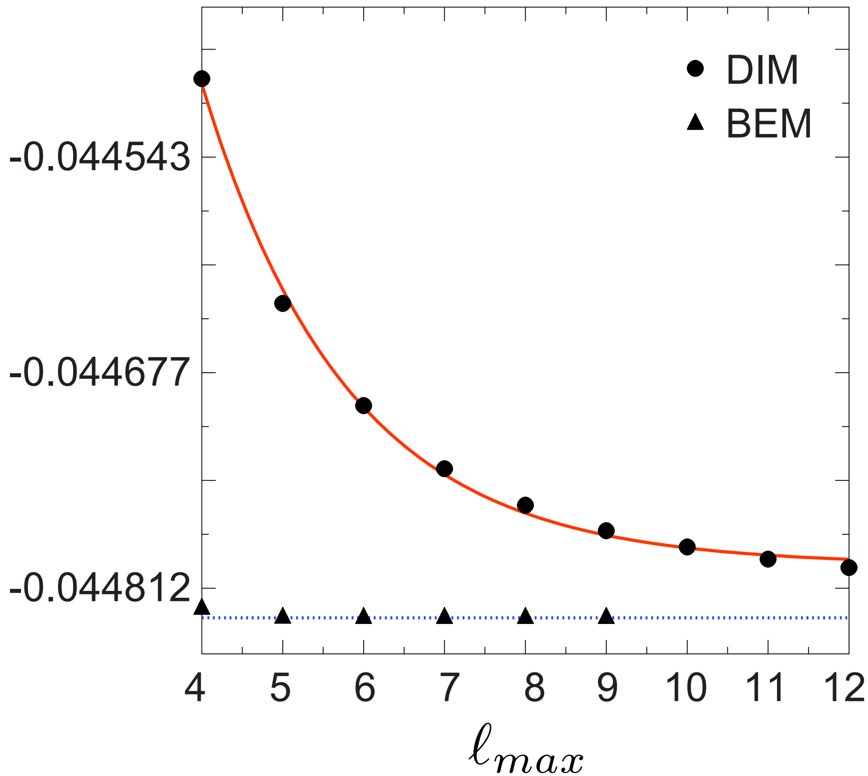
short-range potential

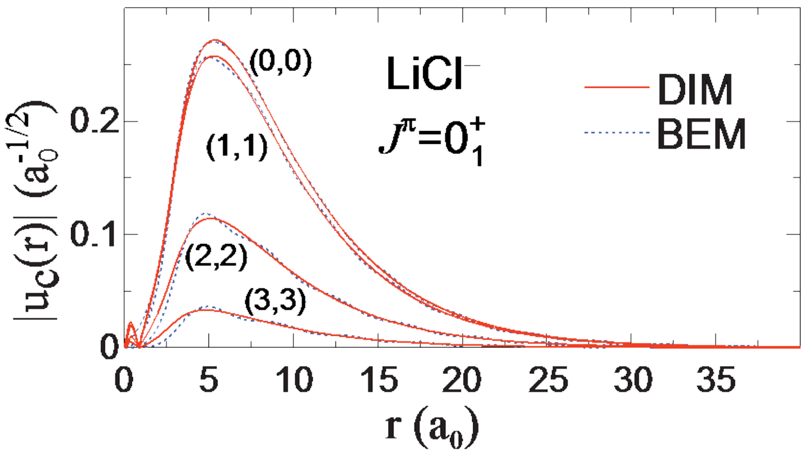
$$\Psi^J = \sum_c u_c^J(r)\Phi^J_{j_c\ell_c} \Longrightarrow$$

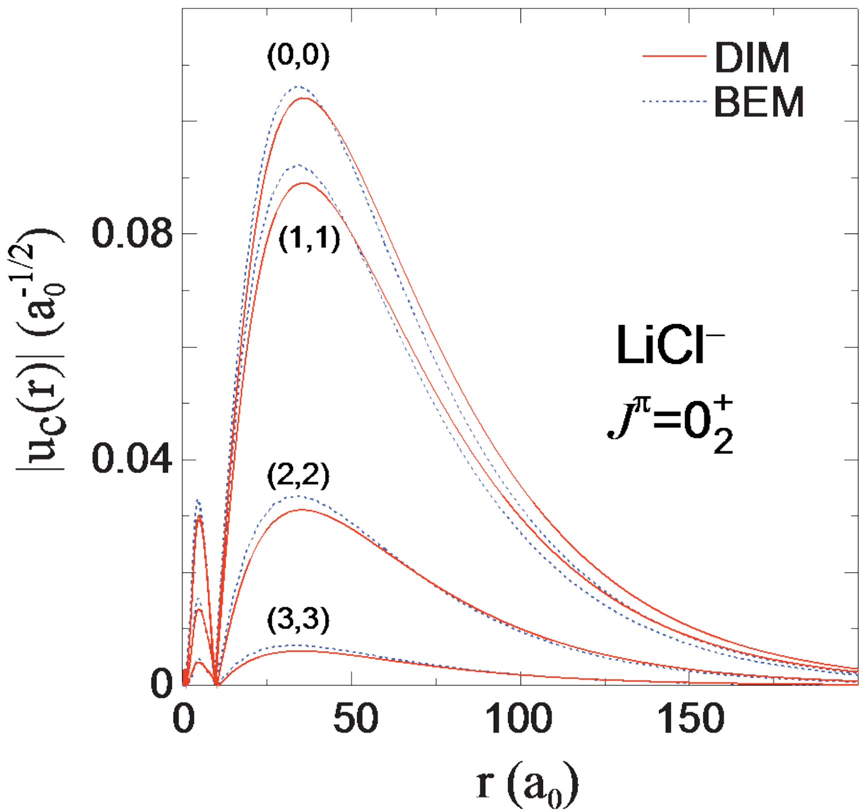
Coupled-channel problem

$$\left[\frac{d^2}{dr^2} - \frac{\ell_c(\ell_c+1)}{r^2} - \frac{j_c(j_c+1)}{I} + E_J\right] u_c^J(r) = \sum_{c'} v^J_{cc'}(r) u^J_{c'}(r)$$

- Direct Integration Method (DIM)
- Berggren Expansion Method (BEM)







The momentum cutoff in BEM softens the interaction and reduces off-diagonal channel couplings!

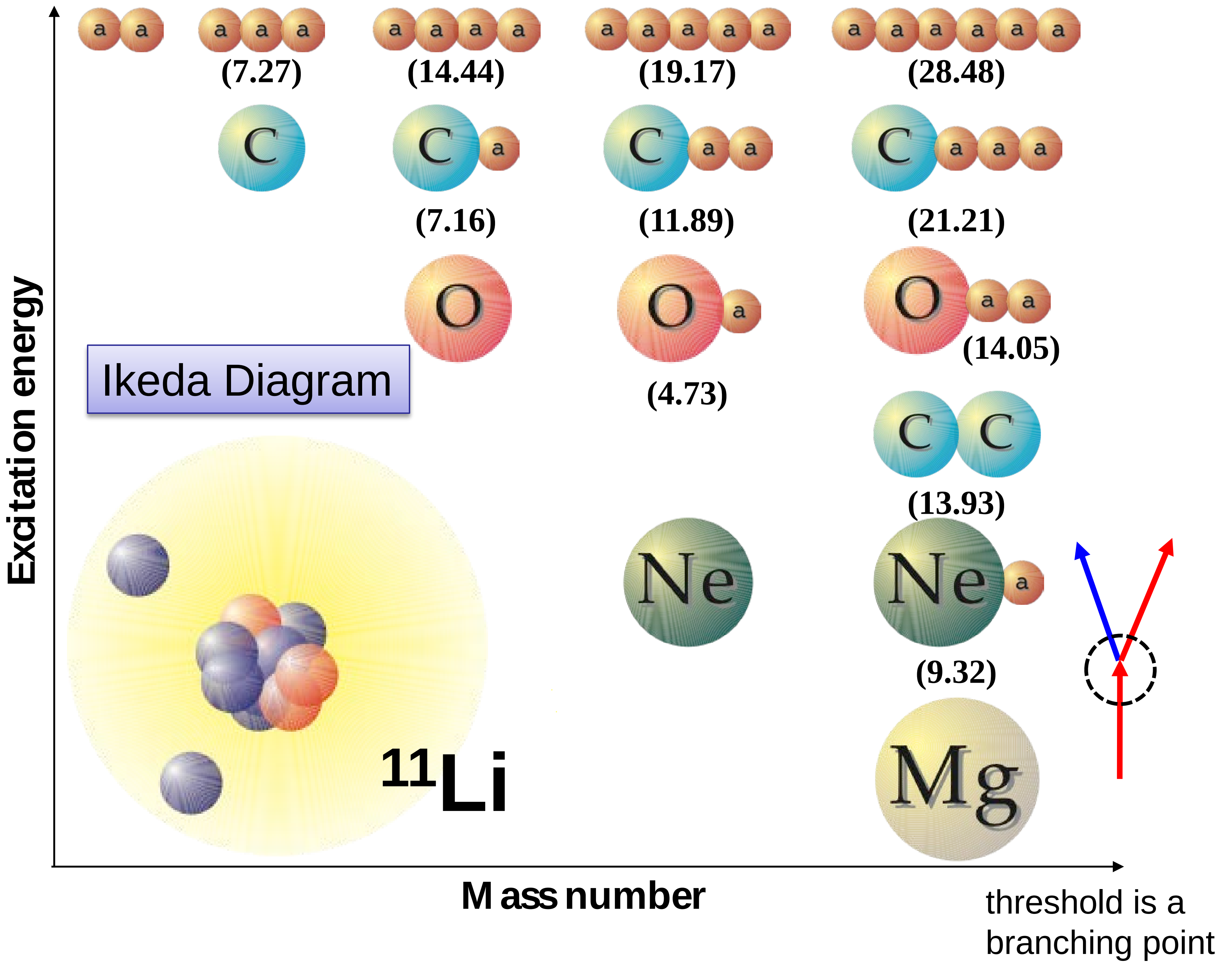
Approaching fixed-dipole limit:

$I(m_e a_0^2)$	$\mu_{cr}^{(0)}$ (ea_0) BEM	$\mu_{cr}^{(0)}$ (ea_0) Garrett	$\mu_{cr}^{(1)}$ (ea_0) BEM	$\mu_{cr}^{(1)}$ (ea_0) Garrett
10^4	0.937	0.843	1.024	1.515
10^6	0.674	0.750	0.633	1.145
10^8	0.639	0.715	0.622	0.974
10^{10}	0.639	—	0.622	—
10^{15}	0.639	—	0.62	—

Energies and radii:

Anion	state	E (Ry) DIM	E (Ry) BEM	$r_{\rm rms}$ (a_0) DIM	$r_{\rm rms}$ (a_0) BEM
LiI$^-$	0_1^+	-5.079(-2)	-5.023(-2)	7.569(0)	7.620(0)
	0_2^+	-9.374(-4)	-1.037(-3)	5.112(1)	4.759(1)
	0_3^+	-1.502(-5)	-1.797(-5)	3.719(2)	3.308(2)
	1_1^-	-5.079(-2)	-4.995(-2)	7.569(0)	7.641(0)
	1_2^-	-9.291(-4)	-1.023(-3)	5.112(1)	4.886(1)
	1_3^-	-1.261(-7)	-1.099(-5)	3.423(3)	3.464(2)
LiCl$^-$	0_1^+	-4.483(-2)	-4.483(-2)	7.885(0)	7.894(0)
	0_2^+	-7.374(-4)	-8.241(-4)	5.632(1)	5.017(1)
	0_3^+	-7.051(-6)	-9.907(-6)	5.124(2)	4.106(2)
	1_1^-	-4.482(-2)	-4.458(-2)	7.885(0)	7.915(0)
	1_2^-	-7.241(-4)	-8.067(-4)	5.633(1)	5.337(1)
	1_3^-	-3.062(-7)	-8.159(-7)	2.066(3)	8.831(2)
LiF$^-$	0_1^+	-2.795(-2)	-2.983(-2)	9.117(0)	8.991(0)
	0_2^+	-3.022(-4)	-3.525(-4)	8.098(1)	7.501(1)
	0_3^+	—	-6.101(-8)	—	3.363(3)
	1_1^-	-2.793(-2)	-2.968(-2)	9.117(0)	9.010(0)
	1_2^-	-2.782(-4)	-3.277(-4)	8.124(1)	7.520(1)
LiH$^-$	0_1^+	-2.149(-2)	-2.370(-2)	1.011(1)	9.698(0)
	0_2^+	-1.491(-4)	-1.922(-4)	1.058(2)	9.297(1)
	1_1^-	-2.142(-2)	-2.353(-2)	1.011(1)	9.717(0)
	1_2^-	-7.942(-5)	-1.231(-4)	1.146(2)	9.591(1)

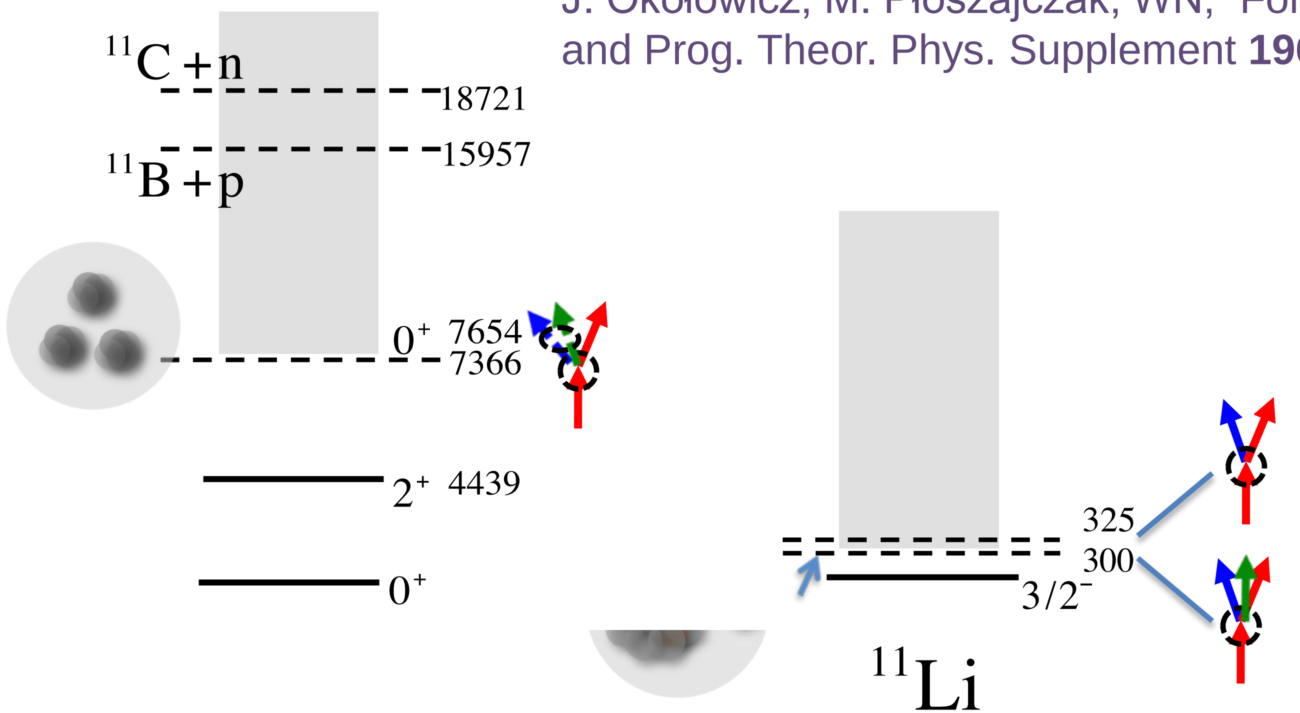
The BEM has been benchmarked by using the traditional DIM. While a fairly good agreement between the two methods has been found for well-bound states, the direct integration technique breaks down for weakly-bound states with energies $|E| < 10^{-4}$ Ry, which is comparable with the rotational energy of the anion. *For those subthreshold configurations, the Berggren expansion is an obvious tool of choice.*



a a
a a a
(7.27)
a a a a
(14.44)
a a a a a
(19.17)
a a a a a a
(28.48)
C
C a
C a a
C a a a
(7.16)
(11.89)
(21.21)
O
O a
O a a
(14.05)
(4.73)
C C
(13.93)
Ne
Ne a
(9.32)
Mg
Ikeda Diagram
^{11}Li
Excitation energy
Mass number
threshold is a
branching point

The origin of nuclear clustering

J. Okołowicz, M. Płoszajczak, WN, Fortschr. Phys. 61, 66 (2013)
and Prog. Theor. Phys. Supplement **196**, 230 (2012)



Correlations in the near-threshold states depend on the topology of the nearest branching point

The clustering is the generic near-threshold phenomenon in open quantum system which does not originate from any particular property of nuclear forces or any dynamical symmetry of the nuclear many-body problem

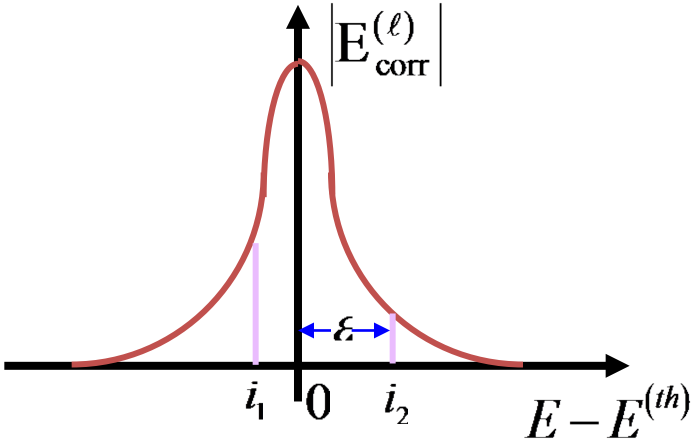
Specific features:

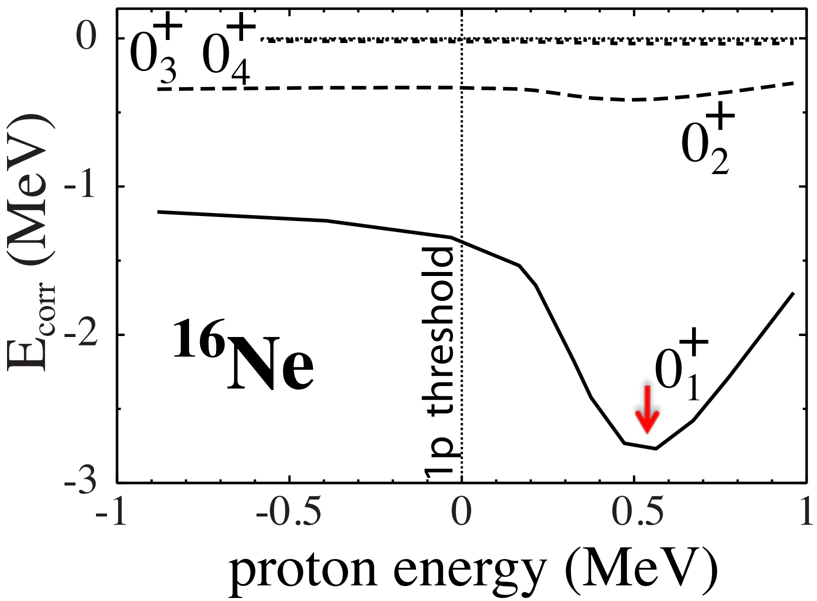
- energetic order of particle emission thresholds
- absence of stable cluster entirely composed of like nucleons

Appearance of the aligned state

Continuum coupling correction to SM eigenstates

$$E^{(\ell)}_{\mathrm{corr};i}(E) \simeq V_0^2 \langle \psi_i | h(E) | \psi_i \rangle$$





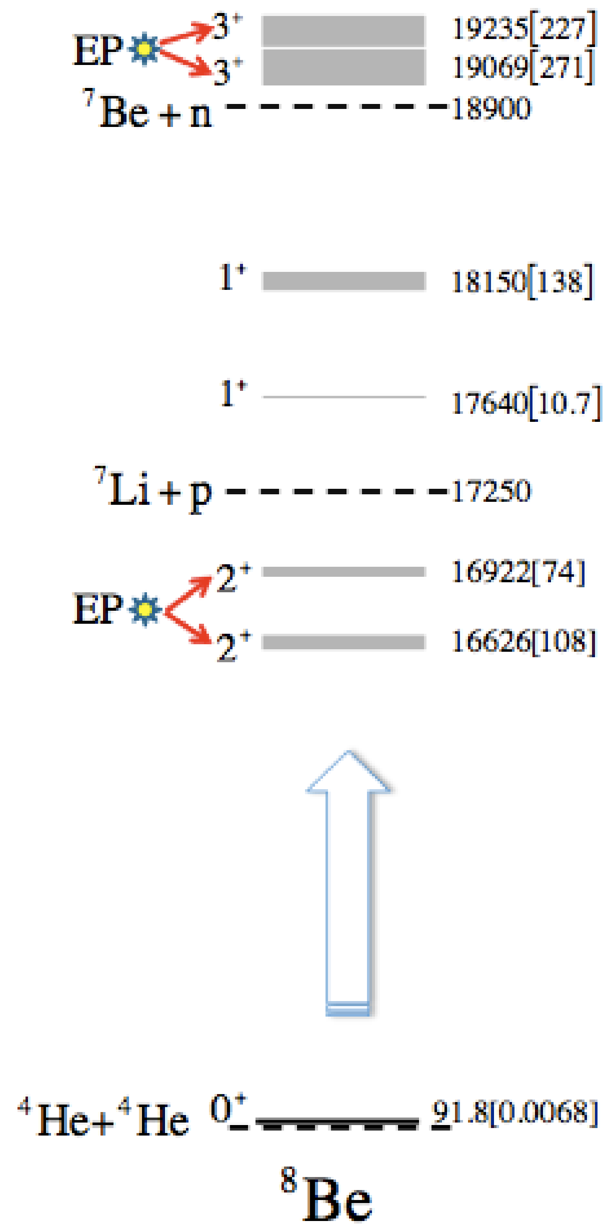
Interaction through the continuum leads to the *collectivization* of SM eigenstates and formation of the *aligned* CSM eigenstate which couples strongly to the decay channel and, hence, carries many of its characteristics.

$$\left[{}^{15}\mathrm{F}\left(1/2^+\right) \oplus \pi s_{1/2} \right]^{0^+}$$

Mixing of SM wave functions via the continuum

- The mixing of eigenfunctions (avoided crossing) is caused by a nearby exceptional point ($\varphi_1 = \varphi_2 (\varphi_1 = \varphi_1^*)$) of the complex-extended Hamiltonian.
- Exceptional point is a generic situation in open quantum systems.
- The configuration mixing of resonances is characterized by lines $\mathcal{E}_{\alpha_1}(E) = \mathcal{E}_{\alpha_2}(E)$ of coalescing eigenvalues (exceptional threads) of the complex-extended CSM Hamiltonian (complex $\mathbf{V}_0$).

Nuclear clustering is a consequence of the collective coupling of SM states via the decay channel which leads to the formation of the OQS state (aligned state). *This state captures most of the continuum coupling and carries many characteristics of the decay channel.* Cluster states may appear in the narrow energy window around the point of maximum continuum coupling. The continuum-coupling correlation energy and collectivity of the aligned state is reduced with increasing Coulomb barrier.



Recent GSM work and in progress

- Isospin mixing: Phys. Rev. C 82, 044315 (2010)

- Properties of Asymptotic Normalization Coefficients: Phys. Rev. C85, 064320 (2012).

- Develop the effective GSM finite-range interaction in the *p-sd* shell model interface

- Develop the GSM-based reaction-theoretical framework

- Apply GSM to resonances in dipole and quadrupole molecules

- Use the GSM formalism in ab-initio approaches

Open problems in the theory of nuclear open quantum systems

N. Michel et al., J. Phys. G **37**, 064042 (2010)

- What is the interplay between mean field and correlations in open quantum systems?
- What are properties of many-body systems around the reaction threshold?
- What is the origin of cluster states, especially those of astrophysical importance?
- What should be the most important steps in developing the theory that will treat nuclear structure and reactions consistently?
  - What is Quantum Mechanics of open quantum systems?
  - How are effective interactions modified in open quantum systems?

in collaboration with <u>M. Płoszajczak</u>, N. Michel, J. Okołowicz, G. Papadimitriou, J. Rotureau, K. Fossez, M. Pfützner, E. Olsen, …

Backup

Rigged Hilbert Space: the natural framework to formulate quantum mechanics

In mathematics, a rigged Hilbert space (Gel'fand triple, nested Hilbert space, equipped Hilbert space) is a construction designed to link the distribution and square-integrable aspects of functional analysis. Such spaces were introduced to study spectral theory in the broad sense. They can bring together the 'bound state' (eigenvector) and 'continuous spectrum', in one place.

Mathematical foundations in the 1960s by Gel'fand et al. who combined Hilbert space with the theory of distributions. Hence, the RHS, rather than the Hilbert space alone, is the natural mathematical setting of Quantum Mechanics

I. M. Gel'fand and N. J. Vilenkin. Generalized Functions, vol. 4: Some Applications of Harmonic Analysis. Rigged Hilbert Spaces. Academic Press, New York, 1964.

The resonance amplitude associated with the Gamow states is proportional to the complex delta function and such amplitude can be approximated in the near resonance region by the Breit-Wigner amplitude (Nucl. Phys. A812, 13 (2008)):

$$\mathcal{A}(E_n \to E) \propto -\frac{1}{2\pi}\frac{1}{E - E_n}$$

For a pedagogical description, see R. de la Madrid, Eur. J. Phys. 26, 287 (2005)

Generalized Variational Principle

(a complex extension of the usual variational principle)

N. Moiseyev, P.R. Certain, and F. Weinhold, Mol. Phys. 36, 1613 (1978).
N. Moiseyev, Phys. Rep. 302, 212 (1998)

$$E[\Phi] = \frac{\langle \Phi^* | \hat{H} | \Phi \rangle}{\langle \Phi^* | \Phi \rangle}$$ is stationary around any eigenstate

$$\hat{H}|\Phi_0\rangle = E[\Phi_0]|\Phi_0\rangle$$

That is, $\delta_\Phi E[\Phi]_{\Phi=\Phi_0} = 0.$

It should be noted that the complex variational principle is a stationary principle rather than an upper of lower bound for either the real or imaginary part of the complex eigenvalue. However, it can be very useful when applied to the squared modulus of the complex eigenvalue. Indeed, $\delta_\Phi |E|^2 = \delta_\Phi (E^* E) = E^* \delta_\Phi E + E \delta_\Phi E^* = 0$

Example: GSM+DMRG calculations for ^{7}Li

J. Rotureau et al., Phys. Rev. C 79, 014304 (2009)

$N_{\text{opt.}}$	$\|E_{\text{max}}\|$	$Re(E_{\text{max}})$	$Im(E_{\text{max}})$
40	22.6489	-22.6475	0.2470
50	22.6605	-22.6591	0.2484
60	22.6631	-22.6617	0.2485
70	22.6634	-22.6620	0.2486
80	22.6634	-22.6620	0.2486

Outgoing flux and width of the Gamow state

Humblet and Rosenfeld: Nucl. Phys. 26, 529 (1961)

$$\left[\hat{T}+\hat{V}\right]\Psi=\left(e-i\frac{\Gamma}{2}\right)\Psi$$

$$\left[\hat{T}+\hat{V}\right]\Psi^*=\left(e+i\frac{\Gamma}{2}\right)\Psi^*$$

$$\hbar\int_S \vec{j}\,d\vec{S}=\Gamma\int_V \rho\, d^3r$$

$$\vec{j}=\frac{\hbar}{2\mu i}\left(\Psi^*\vec{\nabla}\Psi-\Psi\vec{\nabla}\Psi^*\right),\quad \rho=\Psi^*\Psi \qquad \vec{\nabla}\vec{j}-\frac{\Gamma}{\hbar}\rho=0\quad\left(\vec{\nabla}\vec{j}+\frac{\partial\rho}{\partial t}=0\right)$$

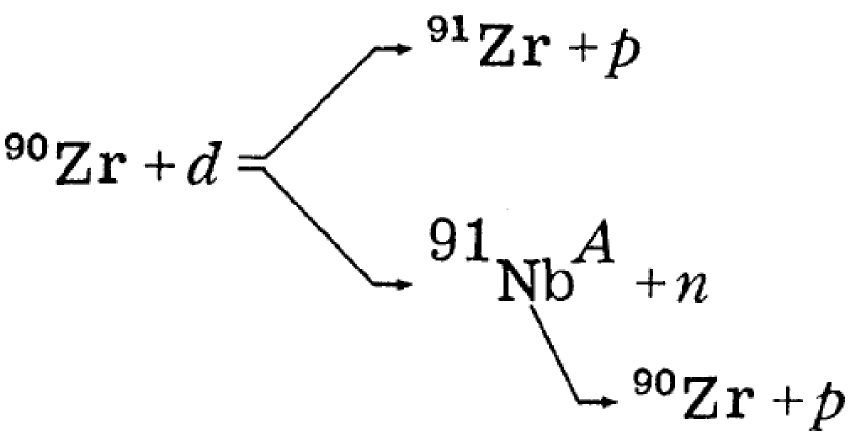
S can be taken as a sphere of radius *R*:

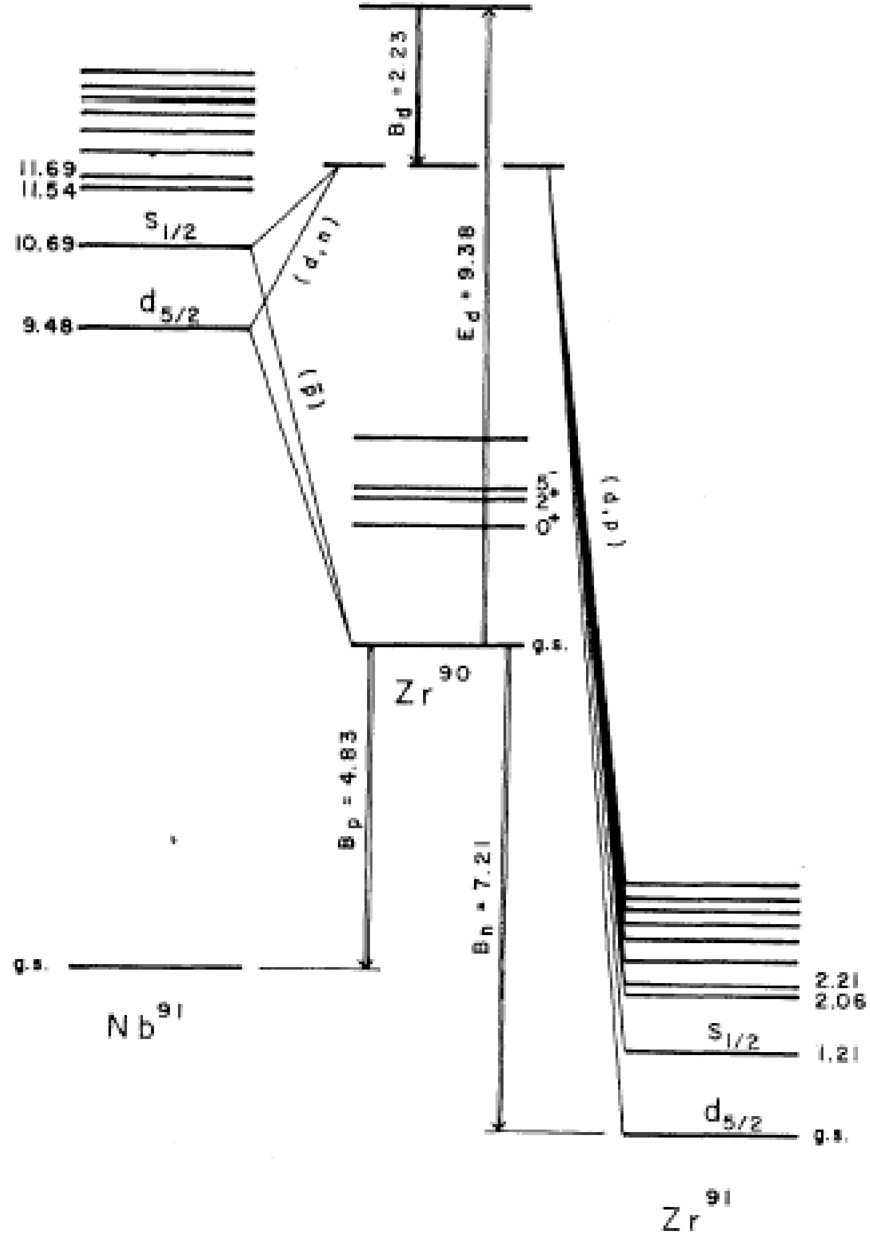
$$\Gamma=\frac{\hbar R^2\int j_r\, d\Omega}{\int_{V_R}\rho\, d^3r}$$

An extremely useful expression!

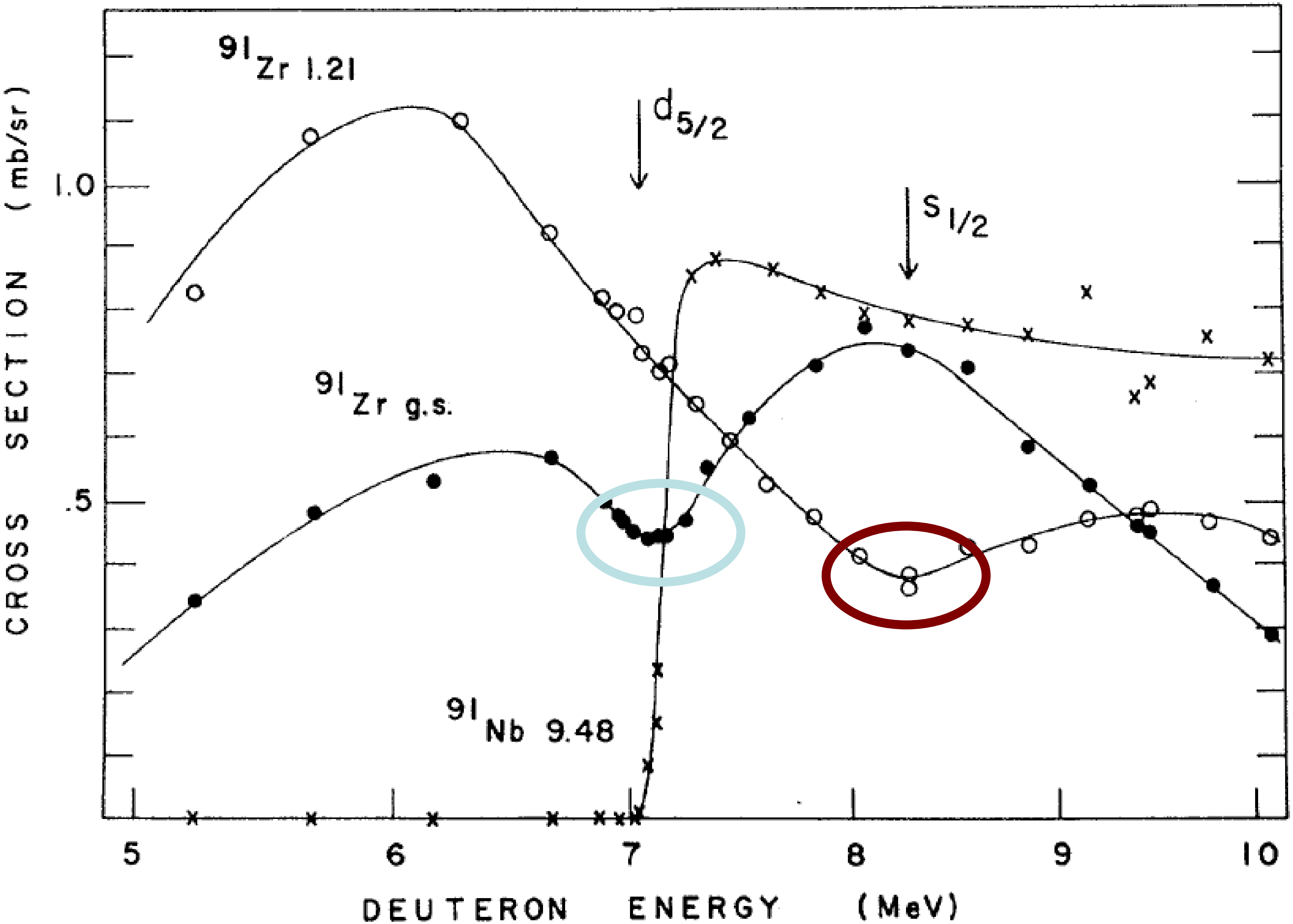
Coupling between analog states in (d,p) and (d,n)

C.F. Moore et al.
Phys. Rev. Lett. 17, 926 (1966)





C.F. Moore et al., Phys. Rev. Lett. 17, 926 (1966)



Threshold anomaly

E.P. Wigner, Phys. Rev. 73, 1002 (1948), the Wigner cusp
G. Breit, Phys. Rev. 107, 923 (1957)
A.I. Baz', JETP 33, 923 (1957)
A.I. Baz', Ya.B. Zel'dovich, and A.M. Perelomov, Scattering Reactions and Decay in Nonrelativistic Quantum Mechanics, Nauka 1966
A.M. Lane, Phys. Lett. 32B, 159 (1970)
S.N. Abramovich, B.Ya. Guzhovskii, and L.M. Lazarev, Part. and Nucl. 23, 305 (1992).

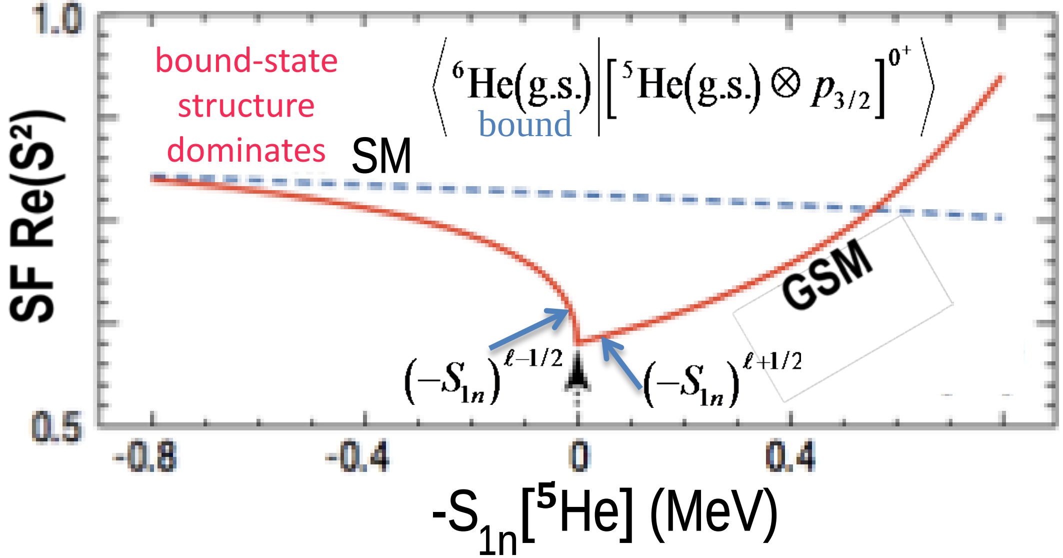
- The threshold is a branching point.
- The threshold effects originate in conservation of the flux.
- If a new channel opens, a redistribution of the flux in other open channels appears, i.e. a modification of their reaction cross-sections.
- The shape of the cusp depends strongly on the orbital angular momentum.

Y(b,a)X	$\sigma_\ell \sim k^{2\ell-1}$
X(a,b)Y	$\sigma_\ell \sim k^{2\ell+1}$

$$a+X \longrightarrow \begin{array}{l} a+X \\ a_1+X_1 \ \text{at } Q_1 \\ a_2+X_2 \ \text{at } Q_2 \\ \\ a_n+X_n \ \text{at } Q_n \end{array}$$

Configuration mixing in weakly bound/unbound many-body states

N. Michel et al. PRC 75, 0311301(R) (2007)



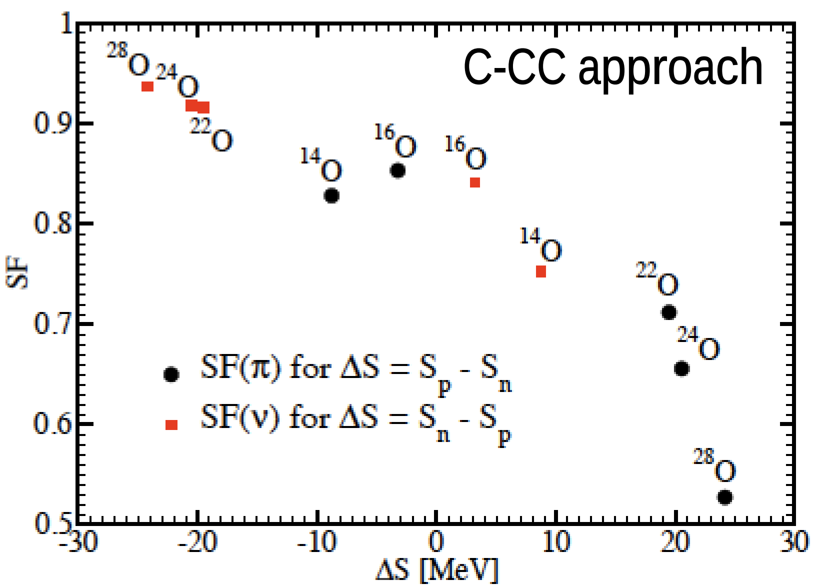
$$S^2 \equiv \int u_{\ell j}^2(r)dr = \sum_{\mathcal{B}} \langle \widetilde{\Psi_A^{J_A}} || a_{\ell j}^{+}(\mathcal{B}) || \Psi_{A-1}^{J_{A-1}} \rangle^2$$

- Anomalies appear at calculated threshold (many-body S-matrix unitary)
- Analogy with the Wigner threshold phenomenon for reaction cross-sections

E.P. Wigner, PR 73, 1002 (1948)

$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \qquad (-S_n)^{\ell-1/2} \text{ for } S_n < 0$$

$$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1} \qquad (-S_n)^{\ell+1/2} \text{ for } S_n > 0$$



Ø. Jensen et al, PRL 107, 032501 (2011)

Respecting unitarity in weakly-bound states leads to significant differences in radial overlap integrals (SFs) as compared to the SM predictions