Chiral nuclear forces: explicit ∆ scenario

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Computational and Theoretical Advances for Exotic Isotopes in the Medium Mass Region March 25, 2013, INT Program INT-13-1a, Seattle

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Outline

- Nuclear forces in chiral EFT $\overline{}$
- A Role of Δ(1232) resonance
- Long-range part of three-nucleon forces up to $N⁴LO$ \bullet
- \triangle N³LO- \triangle vs. N⁴LO \triangle -less
- Summary & Outlook

Nucleon-Nucleon forces

Phenomenological description by meson-exchange

Boson-Exchange models as basis for NN-force Highly sophisticated phen. NN potentials Excellent description of many experimental data Connection to QCD is unclear

NN force as residual strong interaction between hadrons

QCD Interpretation of NN forces Chiral EFT Interpretation of NN forces

- Model independent treatment
- At low energies NN force dominated by Goldstone Boson dynamics + short range int.
- Systematic perturbative description of few nucleon potentials
- Underlying QCD symmetries implemented by construction

From QCD to nuclear physics

NN interaction is strong: resummations/nonperturbative methods needed

 $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p_i}| \sim M_\pi \ll m_N$) \Longrightarrow the QM A-body problem

$$
\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_{i}^{2}}{2m_{N}} + \mathcal{O}(m_{N}^{-3}) \right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \dots}_{\text{derived within ChPT}} \right] | \Psi \rangle = E | \Psi \rangle \qquad \text{Weinberg '91}
$$

- **unified description of ππ, πN and NN**
- **consistent many-body forces and currents**
- **systematically improvable**
- **bridging different reactions (electroweak, π-prod., ...)**
- **precision physics with/from light nuclei**

Nucleon-nucleon force up to N3LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; …

+ 1/m and isospin-breaking corrections…

How to renormalize the Schrödinger Eq?

$$
\text{Lower-order NN potential:} \quad V_{2N}^{(0)} = -\frac{g_A^2}{4F_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q} \cdot \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + M_\pi^2} + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2
$$

<u>UIT</u> Complication: iterations of the LS equation 4*F*² π !*q* ² + *M*² π

$$
T(\vec{p}',\vec{p}) = V_{2N}^{(0)}(\vec{p}',\vec{p}) + \int \frac{d^3k}{(2\pi)^3} V_{2N}^{(0)}(\vec{p}',\vec{k}) \frac{m_N}{p^2 - k^2 + i\epsilon} T(\vec{k},\vec{p})
$$

!σ¹ *·* !*q* !σ² *·* !*q*

!*q* ² + *M*²

(2 ^π)³*^V* (0)

² [−] ¹⁶⁸ [−] ¹

, ^p!) + ! *^d*³*^k*

! *d*³*k*

(2 π)³

(4)

+ *C^S* + *C^T* !σ¹ *·* !σ2*,* (1)

generate divergences whose subtraction requires infinitely many CTs beyond $V^{(0)}_{\text{2N}}$

*an, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque,
«Alck Payon Valderrama Ruiz Arriola Nogga Timmermanns Fpelbaum Meißner Entem Machleidt Yang Elster Lon M N C* (*discression P* (*direction Committery Titutes)*
 Vart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Bil Nogga, Timmermanns, Epelbaum, Meißner, van Kolck, Pavon Valderrama, Ruiz Arriola, Nogga, Timmermanns, Epelbaum, Meißner, Entem, Machleidt, Yang, Elster, Long, Gegelia, ... *,* (3) *Kaplan, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque,*

² [−] ¹⁶⁸ [−] ¹

τ ¹ *·* τ ²

m[∆] − *m^N* ∼ *M*^π (5) \$2 use a **finite** cutoff (practical solution) *V* (0) *e* cutoπ (practical solution)
———————————————————

 (23.3)

#"

A new, renormalizable approach (yet to be explored...) *, p*!)=*V* (0) *Epelbaum, Gegelia '12*

4*F*² π

V static 3N = \$ *i*=1 *Gi*(!σ1*,* !σ2*,* !σ3*,* τ ¹*,* τ ²*,* τ ³*,* !*r*12*,* !*r*23) *Fi*(*r*12*, r*23*, r*31) + permutations *a*+ and the consequence is an arm act or the non-elativity of the $\frac{1}{2}$ 2N (*p*! ! non-renormalizability of the LO equation is an artifact of the nonrelativistic expansion $\overline{}$ n is an artifact of the no **p**
p
2 + *i* + **tivistic expans**

!*q* ² + *M*²

π

renormalizable LO equation based on manifestly Lorentz-invariant Lagrangian

$$
T(\vec{p}',\vec{p})\!\!=\!\!V^{(0)}_{2N}(\vec{p}',\vec{p})+\frac{m_N^2}{2}\,\int\frac{d^3k}{(2\,\pi)^3}\frac{V^{(0)}_{2N}(\vec{p}',\vec{k})\,T(\vec{k},\vec{p})}{(k^2+m_N^2)\,(E-\sqrt{k^2+m_N^2}+i\,\epsilon)}
$$

 \overline{Q} and the treated nerturbatively in pregress 22 higher-order corrections (e.g. two-pion exchange) to be treated perturbatively *in progress...*∩ → #" \$2

Neutron-proton phase shifts at N3LO

 R *Entem, Machleidt '04; Epelbaum, Glöckle, Meißner '05*

Chiral expansion of NN force *r* ∼ 1 fm *r* 1 **f**m *G*^{*i*} M_N \blacktriangleright **.** *P* ∈*S*³ *Dij* (*P*)*PG, i, j* = 1*,* 2 *^V*˜*^S* ⁺ ^τ ¹ *·* ^τ ²*W*˜ *^S* "σ¹ *·* "σ²

0 $V(\vec{r}) = V_{\alpha} + \tau_1 \cdot \tau_2 W_{\alpha} + V_{\alpha} +$ *r*₃ *³ Epelbaum, Meißner Ann. Rev. Nucl. Part. Sci 62* Bands ($800 \text{ MeV} \leq \tilde{\Lambda}$) visualize estimated -200 -150 -100 -50 0 \tilde{W}_T is governed by 1π-exchange Long-range behavior at $r \geq 2$ fm of short- and long-range contributions scheme-dependence for separation between $\tilde{V}(\vec{r})$ = $\tilde{V}_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_C + \left[\tilde{V}_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_S\right]$ $\tilde{V}(\vec{r}) \;\; = \;\; \tilde{V}_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_C + \left[\tilde{V}_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_S \right] \vec{\sigma}_1 \cdot \vec{\sigma}_2$ $+ \left[\tilde{V}_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_T \right]$ $\frac{1}{\sqrt{1-\bar{\sigma}^2}}\left[V_T+\bm{\tau}_1\cdot\bm{\tau}_2 W_T \right] \left(3\,\vec{\sigma}_1\cdot\hat{r}\,\vec{\sigma}_2\cdot\hat{r}-\vec{\sigma}_1\cdot\vec{\sigma}_2 \right)$ (12) 15.
 $\overline{}$ $\int_0^2 V(\vec{r}) = V_C + \tau_1 \cdot \tau_2 W_C + [V_S + \tau_1 \cdot \tau_2 W_S]$ $V_S + 7$ $(V_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) V_T \left[\right. (\textcolor{black}{\delta \, \sigma_1 \cdot \sigma_2})$ \tilde{V}_C is governed by subleading $2π$ -exchange \tilde{V} *^V*˜*^T* ⁺ ^τ ¹ *·* ^τ ²*W*˜ *^T* (3 "σ¹ *· r*ˆ"σ² *· r*ˆ − "σ¹ *·* "σ2) # *Epelbaum, Meißner Ann. Rev. Nucl. Part. Sci 62 (12) 159* 800 MeV [≤] ^Λ˜

-4

Component Chort-range part of the NN force is scheme-dependent (parametrization)

- left (right) panel shows the results for the results for the results for the EFT with \sim 1232). With \sim 1232 \bullet Long-range part is scheme-independent and is predicted by chiral $\overline{}$ Long-range part is scheme-independent and is predicted by chiral EFT
- model dependence associated with the short-range components as explained in Convergence of chiral expansion is clarified in a theory with Convergence of chiral expansion is clarified in a theory with explicit Δ(1232)

EFT with explicit $\Delta(1232)$

- Standard chiral expansion:
- Small scale expansion:

(Hemmert, Holstein & Kambor '98)

Delta contributions encoded in LECs (Bernard, Kaiser & Meißner 197)

Convergence of EFT potential

The subleading contributions are larger than the leading one!

Few-nucleon forces with the Delta

Isospin-symmetric contributions

NN potential with explicit Δ
Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127

 $V_{\text{eff}} = V_C + W_C \vec{\tau}_1 \cdot \vec{\tau}_2 + [V_S + W_S \vec{\tau}_1 \cdot \vec{\tau}_2] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1 \cdot \vec{\tau}_2] (3 \vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$

• Much better convergence in all potentials

$3F_3$ partial waves up to NNLO with and without Δ

(calculated in the first Born approximation)

Nuclear forces up to N³LO

dimensional analysis counting

Three-nucleon forces

Three-nucleon forces in chiral EFT start to contribute at N^2LO

(Friar & Coon ´86; U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)

 $\left| \begin{array}{l} c_{1,3,4}$ from the fit to πN -scattering data

 D , E from $^3\mathrm{H}, ^4\mathrm{He}, ^{10}\mathrm{B}$ binding energy + *c*1*,*3*,*⁴ ++++++++coherent++++++++E+scaGering+length *nd*

LECs D and E incorporate short-range contr.

Resonance saturation interpretation of LECs

Delta contributions encoded in LECs *(Bernard, Kaiser & Meißner '97)*

Salarization tranctorum at any JUIANZALIUN (RANSIUR $\, {\bm E}_p \, \, = 22.7$ ization transfer: $E^{\rm Lab} = 2$

For references see recent reviews: For references see recent reviews:

Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654 in Cologne in Cologne (292). The Cologne Cologne Cologne Cologne Cologne Cologne Cologne Cologne Cologne Cologne
Final the Cologne Col Entem, Machleidt, Phys. Rept. 503 (11) 1 Epelbaum, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159 Lpeibaum, Meilsher, Anni. Nev. Nucl. Part. Sci. 02 (12) 139
Kalantar et al. Rep. Prog. Phys. 75 (12) 016301 t_{P} (309) t_{P} and the coupled channel calculation in-0.05 A Epelbaum, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159 0.2 Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773

- ϵ consumed interaction ϵ the effect of the Council interaction in ϵ \bullet defierally good description of data. But some discrepancies survive. E.g. break-un observables for SCRF/SST sion. Furthermore, contrary to the Ayconfiguration at low energy \bullet Generally good description of data **proton-deuteron and neutron-deuteron** deuteron deuteron deuteron deuteron deuteron deuteron deuteron deuteron de
1960 e de un deuteron de la production de la catalactica de la production de la catalactica de la production break-up observables for SCRE/SST configuration at low energy $\mathcal{L}_{\mathcal{A}}$ the Coulomb effect was found to be far too small to ex-S (MeV) S (MeV r r ABut some discrepancies survive. E.g. Generally good description of data. configuration at low energy
- \sim 11 \sim S-waves we with any known fine tuning between \sim partial wave been made in the first have been made in the first have been made in the first have been made in the \sim

0

Proton-³He elastic scattering 100

Viviani, Girlanda, Kievsky, Marcucci, Rosati arXiv: 1004.1306

p-³He differential cross section at low energies

$\{f: g \in G\}$ is the group of G and group G and group G and G and G **Fig. 5.** *p* − 3He differential cross sections calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and proton vector analyzing power A_y-puzzle

George 2001 r Eso F and F F and F F to resolve underprediction of A_{y} As in n-d scattering case N²LO 3NF's are not enough

0.5

I-N3LO/N-N2LO

Hope for improvement at higher orders

Three-nucleon forces

Three-nucleon forces at N³LO

Long range contributions

Bernard, Epelbaum, HK, Meißner ´08; Ishikawa, Robilotta ´07

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- $\Delta(1232)$ -contr. are important

Shorter range contributions

Bernard, Epelbaum, HK, Meißner ´11

- \triangle LECs needed for shorter range contr. g_A, F_π, M_π, C_T
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF

Two-pion-exchange 3NF

1-AIAR-AVAALANOA SINI Two-pion-exchange 3NF

N³LO - contr. (leading 1 loop)

╶╶╶╄╌┾╷╎╴┈╬╌┾╷╎╷╎┆┩╷╎╷┡╌┾╷╷╎┆╬╌┾┈╷┆╉╌┾┈┾╷┆╬╌┾╌┾┈╎╁╌┾┈╎╷┆╬╌┾╌┾┈╎┆╄╌┾┈┧╷┆╃╌┾┈┤
┆╣┢╌┾┈╎╎┢╌╆╷╎┟╀╌╄┈╷╎┆╬╌╃╷╷┞╌╀╎╿╷┞╌╄┆╿╷┞╌╄╌╄╌╄┈┦╷┆╫╌╫╷╷╿╌╬╌╄╶╵╊╌╄╌╄┈┦╷┆╂╌╂ (15) (16) (17) (18) (19) (20) (21) !σ¹ *·* !*q*¹ !σ³ *·* !*q*³ *V*2^π τ ¹ *·* τ ³ *A*(*q*2) + τ ¹ × τ ³ *·* τ ² !*q*¹ × !*q*³ *·* !σ² *B*(*q*2) [*q*² ¹ + *M*² π] [*q*² ³ + *M*² π] (22) (23) (24) (25) (26) (27) (28) *, ^B*(3)(*q*2) = *^g*² *^Ac*⁴ renormalization $\begin{bmatrix} 2 & 4 & 4 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix}$ + $\begin{bmatrix} 2 & 4 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ + $\begin{bmatrix} 2 & 4 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ *,* 8*F*⁴ π nontrivial ! ${\cal L}_{\pi N}^{(3)}$ (22) (23) (24) (25) (26) (27) (28) $\mathcal{L}^{(0)}$ π*N* $\sim \pi N$

$$
\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6} \Big[A(q_2) \left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4 \right) + \left(4g_A^2 + 1 \right) M_\pi^3 + 2 \left(g_A^2 + 1 \right) M_\pi q_2^2 \Big],
$$
\n
$$
\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6} \Big[A(q_2) \left(4M_\pi^2 + q_2^2 \right) + (2g_A^2 + 1)M_\pi \Big] \quad \text{Ishikawa, Robilotta '07,}
$$
\nBernard, Epelbaum, HK, Meißner '07

FIG. 2: Two-pion exchange 3N diagrams at N³LO. Solid dots (filled rectangles) denote vertices of dimension ∆ⁱ =0(∆ⁱ = 2). Diagrams which result from the interchange of the nucleon lines and/or application of the time reversal operation are not \bigcap \bigcap scattering amplitude by renormalized ones. This suggests that there are no N³LO corrections to the 3NF from these No unknown parameters at this order and the state of graphs in Fig. 2, diagram (11) does not contribute at the considered order due to the 1/m-suppression caused by the result at the order considered. Here, the time derivative acts either on the pions exchanged between two nucleons No unknown parameters at this order **consider** between two nucleons exchanged between two nucleons leading to a 1/m-suppression or on the pion in the tadpole giving an odd power of the loop momentum l

 \sim Lyci yuning is capicssed in terms of loc $g_{\rm eff}$ exchange 3N diagrams at order α do not generate any nonvanishing 3NF. Given the factor α that nuclear potentials are, in general, not uniquely defined, the above argument based on the (on-shell) scattering α af loop function $A(a) = \frac{1}{a}$ is indeed that the case by explicitly calculated the case by explicitly calculated that the case by explicitly calculated that the case of A σ or loop runction $2I(q) = \frac{aI \cdot b a \cdot I}{2aI}$ Zq at the contribution contribute at the contribute at the ZM_{π} τ and σ the σ the same reason, diagram (25) also leads to a vanishing to a va \bullet Even thing is expressed in terms of loc the corresponding 3NF using the method of unitary transformation along the lines of Ref. [23]. From the remaining leading to a 1/m-suppression or on the pion in the pion in the tadpole giving and power of the loop momentum $2q = 2M_{\pi}$ \mathcal{L} Everything is expressed in terms of loop function $A(q) = \frac{1}{2}$ 2*q* $\arctan \frac{q}{2l}$ $2M_\pi$

and additional unitarity transformations re **graphs include 3** and the 2 do not generate 3 do not general values at α ione required for proper reportion ions required for proper renormalization $\,$ A dditional unitarity transformations re \bullet - and indicate giving an original or on the power of the power of the power of the power of the local Additional unitarity transformations required for proper renormalization

Two-pion-exchange 3NF

N⁴LO - contr. (subleading 1 loop) Epelbaum, Gasparyan, HK, 12

 C_i ´s LECs from ${\cal L}_{\pi N}^{(2)}$, d_i ´s LECs from ${\cal L}_{\pi N}^{(3)}$, e_i ´s LECs from ${\cal L}_{\pi N}^{(4)}$: fitted to πN - scattering data $\; \Big|$

Leading Δ - contributions are taken into account through C_i 's

Vanishing $1/m$ - contributions at this order

Two-pion-exchange 3NF at N4LO The parameter and an remains unfixed. The 3NF, however, does not depend on a substitutely. This leads to an una biguous result for the 3NF at this order. Interestingly, we observe that the 1*/m* contributions to *V* (5) The final provision of \overline{B} and \overline{B} ⁴π² *, K*0(*t*) = [−]tan−¹ √−*^t* 8π √−*^t ,* ! ¹[−] ⁴*M*² The final, renormalized result for the quantities *A* and *B* in Eq. (3.3) has the form: **F**22 **F2** + *g*₂ + *e*₂ +

$$
\mathcal{A}^{(5)}(q_2) = \frac{g_A}{4608\pi^2 F_\pi^6} \Big[M_\pi^2 q_2^2 \big(F_\pi^2 (2304\pi^2 g_A (4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18}c_3 \big) + g_A (144c_1 - 53c_2 - 90c_3) \big) + M_\pi^4 \big(F_\pi^2 (4608\pi^2 \bar{d}_{18} (2c_1 - c_3) + 4608\pi^2 g_A (2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38}) \big) + g_A (72 \left(64\pi^2 \bar{l}_3 + 1\right) c_1 - 24c_2 - 36c_3) \big) + q_2^4 \left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A (5c_2 + 18c_3) \right) \Big] - \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2) \left(M_\pi^2 + 2q_2^2 \right) \big(4M_\pi^2 (6c_1 - c_2 - 3c_3) + q_2^2 (-c_2 - 6c_3) \big) \n\mathcal{B}^{(5)}(q_2) = -\frac{g_A}{2304\pi^2 F_\pi^6} \Big[M_\pi^2 \left(F_\pi^2 (1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A (2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37}) \right) + 108g_A^3 c_4 + 24g_A c_4 \big) + q_2^2 \left(5g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A \right) \Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2) \left(4M_\pi^2 + q_2^2 \right)
$$
\nSome I ECs can be absorbed by shifting c is

\n
$$
L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q} \log \frac
$$

Some LECs can be absorbed by shifting c_i 's

 $c_4 \rightarrow c_4 + 4M_{\pi} (2e_{21} - e_{37})$

 c_4 → $c_4 + 4M_{\pi}^2 (2\bar{e}_{21} - \bar{e}_{37})$

 $c_4 \rightarrow c_4 + 4M^2 (2\bar{e}_{21} - \bar{e}_{37})$

 $c_3 \rightarrow c_3 + 4M_{\pi} (2e_{19} - e_{22} - e_{36})$

 $c_1 \rightarrow c_1 - 2M_\pi^2$

 $c_1 \to c_1 - 2M_{\pi}^{-}$

 $c_1 \rightarrow c_1 - 2N$

Some LECs can be absorbed by shifting
$$
C_i
$$
's\n
$$
L(q) = \frac{\sqrt{q^2 + 4M_\pi^2}}{q} \log \frac{\sqrt{q^2 + 4M_\pi^2}}{2M_\pi} + q
$$

- %*q*² + 4*M*² ^π + *q* 2*M*^π *e* No d_i dependence of TPE-contr. $\overline{e_1}$ $\overline{e_2}$ $\overline{e_3}$ $\overline{e_4}$ $\overline{e_5}$ $\overline{e_6}$ $\overline{e_7}$ $\overline{e_8}$ $\overline{e_9}$ $c_3 \rightarrow c_3 + 4M_{\pi}^2 (2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36})$ b esides d_{18}
- $(2\bar{e}_{21}-\bar{e}_{37})$ PION-NUCLEON fit, as it enters only in linear combinations with \bullet Pio Pion-nucleon scattering does $C_1 - e_{37}$ strongly depend on d_i ´s

 $g_{\pi NN} = \frac{g_{\pi NN}}{E}$ $\left(1 - \frac{\pi}{g_{\pi NN}}\right)$ \leftarrow Violation of Goldberger-Treiman relation $\mathcal{L}^{\prime}\pi$ constants. Here and in what follows, we use the values of values the values of $\mathcal{Y}A$ $g_{\pi NN} =$ *g^A m* F_π $\left(1 - \frac{2M_{\pi}^2 \bar{d}_{18}}{a_{4}}\right)$ *g^A* $\sum_{i=1}^{n}$ Violation of Goldberger-Treiman relation \overline{h} in what follows, we use the values of \overline{h} *g*_a *g*^{*a*} *, Fig. 7 MeV <i>n* and *n* element *n* and *n*

 $\overline{a_3c_1}$

"

 \mathcal{L}

 Δ

 F_{π}^2

 $\frac{F}{\pi}$

 F_{π}^2

 \overline{e}_{38} +

 $\left(\bar{e}_{22} - 4\bar{e}_{38} + \frac{\bar{l}_3c_1}{F^2}\right)$

 $\left(\bar{e}_{22} - 4\bar{e}_{38} + \frac{l_3c_1}{F_{\pi}^2}\right)$

 $\left(\bar{e}_{22} - 4\bar{e}_{38} + \frac{\bar{l}_3c_1}{F^2}\right)$

 $c_1 \rightarrow c_1 - 2M_\pi^2 \left(\bar{e}_{22} - 4\bar{e}_{38} + \frac{l_3c_1}{F^2}\right),$

Two-pion-exchange at N4LO 11

KH -5 -4 -3 -2 -1 0 $\begin{array}{c} \mathsf{A} \end{array}$ $\begin{array}{c} \mathsf{M}_\pi \end{array}$ ب
س N2LO N3LO N4LO 0 50 100 150 200 250 300 $0\frac{C}{D}$ 0.1 0.2 0.3 0.4 0.5 B M_n \sim 0 50 100 150 200 0 5 10 δ [degree] 0 50 100 150 200 -10 -5 0 0 50 100 150 200 -2 0 2 0 50 100 150 200 -2 -1 Ω δ [degree] 0 50 100 150 200 \mathbf{e} -1 0 0 50 100 150 200 $\left(\begin{matrix} 0 \\ 0 \end{matrix} \right)$ 15 30 0 50 100 150 200 $p_{Lab}^{\,}$ [MeV/c] $0\frac{L}{0}$ 0.1 0.2 δ [degree] 0 50 100 150 200 0 0.04 0.08 0 50 100 150 200 $p_{Lab}^{\,}$ [MeV/c] $\left(0 \right)$ 0.1 0.2 0 50 100 150 200 p_{Lab} [MeV/c] $\rm p_{Lab} < 150\,MeV$ -0.2 -0.1 Ω S_{11} S_{31} P_{11} P_{31} P₃₃ P₃₃ P₃₃ P₃₃ D₁₃ $\frac{1}{\sqrt{2}}$ 0.04 D_{33} $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ D_{15} D_{35} Data fitted for *Fettes, Meißner ´00; Epelbaum, Gasparyan, HK,´12*

Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707

 $q_{2}^{\,}$ [MeV]

Similar fit to George-Washington (GW) PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205

 \bullet No dependence on d_i 's $\quad \bullet$ e_i 's are of natural size $\quad \bullet$ Good convergence of TPE 3NF \bullet No dependence on d_i 's perturbation theory inside the Mandelstam triangle [58]. It is also worth mentioning that the values of c3,⁴ are in a

Most general structure of a local 3NF

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

Up to N^4 LO, the computed contributions are local \longrightarrow it is natural to switch to r-space. A meaningful comparison requires a complete set of independent operators range tail of the 3NF. It is clear that all arguments of the previous section can also be applied to operators in A inearingiul companson requires a complet

 \mathbf{V}

 $\tilde{\mathcal{G}}_1 = 1$, $\tilde{\mathcal{G}}_2 \;=\; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\,,$ $\tilde{\mathcal{G}}_3 \;=\; \vec{\sigma}_1 \cdot \vec{\sigma}_3\,,$ $\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \,,$ $\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$, $\tilde{\mathcal{G}}_6 \;=\; \boldsymbol{\tau}_1 \cdot \left(\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3\right) \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 \times \vec{\sigma}_3\right),$ $\tilde{\mathcal{G}}_7 \;=\; \boldsymbol{\tau}_1 \cdot \left(\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3\right) \vec{\sigma}_2 \cdot \left(\hat{r}_{12} \times \hat{r}_{23}\right),$ $\tilde{\mathcal{G}}_8 \,\,=\,\, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_3 \,,$ $\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \,,$ $\hat{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{12} \cdot \vec{\sigma}_3$, $\tilde{G}_{11} = \tau_2 \cdot \tau_3 \hat{r}_{23} \cdot \vec{\sigma}_1 \hat{r}_{23} \cdot \vec{\sigma}_2$, $\tilde{\cal G}_{12}\;=\;{\bm\tau}_2\cdot{\bm\tau}_3\,\hat r_{23}\cdot\vec\sigma_1\,\hat r_{12}\cdot\vec\sigma_2\,,$ $\tilde{\cal G}_{13} \;=\; \bm{\tau}_2 \cdot \bm{\tau}_3 \, \hat r_{12} \cdot \vec{\sigma}_1 \, \hat r_{23} \cdot \vec{\sigma}_2 \,,$ $\ddot{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \,,$ $\tilde{\cal G}_{15} \;=\; \bm{\tau}_1 \cdot \bm{\tau}_3 \, \hat r_{13} \cdot \vec{\sigma}_1 \, \hat r_{13} \cdot \vec{\sigma}_3 \,,$ $\tilde{\cal G}_{16}\;=\;\bm{\tau}_2\cdot\bm{\tau}_3\,\hat r_{12}\cdot\vec{\sigma}_2\,\hat r_{12}\cdot\vec{\sigma}_3\,,$ $\tilde{\cal G}_{17} \;=\; {\bm \tau}_1 \cdot {\bm \tau}_3 \, \hat r_{23} \cdot \vec \sigma_1 \, \hat r_{12} \cdot \vec \sigma_3 \,,$ $\tilde{\cal G}_{18} \;=\; \bm{\tau}_1 \cdot (\bm{\tau}_2 \times \bm{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23}) \,,$ $\tilde {\cal G}_{19} \;=\; \bm{\tau}_1 \cdot (\bm{\tau}_2 \times \bm{\tau}_3) \, \vec{\sigma}_3 \cdot \hat{r}_{23} \, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \,,$ $\tilde{\cal G}_{20} \;=\; {\bm\tau}_1\cdot({\bm\tau}_2\times{\bm\tau}_3)\,\vec{\sigma}_1\cdot\hat{r}_{23}\,\vec{\sigma}_2\cdot\hat{r}_{23}\,\vec{\sigma}_3\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,$ $\tilde{\mathcal{G}}_{21} \;=\; \bm{\tau}_{1} \cdot (\bm{\tau}_{2} \times \bm{\tau}_{3})\, \vec{\sigma}_{1} \cdot \hat{r}_{13}\, \vec{\sigma}_{3} \cdot \hat{r}_{13}\, \vec{\sigma}_{2} \cdot (\hat{r}_{12} \times \hat{r}_{23})\,,$ $\tilde{\mathcal{G}}_{22} \;=\; \bm{\tau}_{1}\cdot(\bm{\tau}_{2}\times\bm{\tau}_{3})\,\vec{\sigma}_{1}\cdot\hat{r}_{23}\,\vec{\sigma}_{3}\cdot\hat{r}_{12}\,\vec{\sigma}_{2}\cdot(\hat{r}_{12}\times\hat{r}_{23})\,,\;\;\left.\rule{0pt}{12pt}\right]$ *, p*!)=*V* (0) 2N (*p*! !


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Building blocks:
```
 τ_1 , τ_2 , τ_3 , $\vec{\sigma}_1$, $\vec{\sigma}_2$, $\vec{\sigma}_3$, \vec{r}_{12} , \vec{r}_{23}

2 Constraints:

4*F*² π τ ¹ *·* τ ² Locality

- \bullet Isospin symmetry
- **2** Parity and t *nd* time-reversal invarial *p*2 + *i i h*2 *ariance* Parity and time-reversal invariance

ivable in ChPT; long*k***₂ +** *k***₂,** *k***₂,** *n***₂,** *k***₂,** *k***_{**} derivable in ChPT; long-range terms **parameter-free predictions**

Two-pion-exchange up to N4LO

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

Excellent convergence of TPE-force at distance $r\geq2\,{\rm fm}$ $\sqrt{1-\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2+\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^2}$ in MeV generated by the two-pion exchange 3NF topology up to the two-pion exchange 3NF topology up to the two-pion exchange 3NF topology up to the two-pion excha \parallel Excellent convergence of TPE-force at distance r $>2\,\mathrm{fm}$ \bigcap

17 Two-pion-one-pion-exchange up to N4LO

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

 \blacksquare In nearly all cases subleading N⁴LO dominate leading N³LO contribu to N⁴LO (in the equilateral triangle configuration). Dashed and solid lines correspond to F(4) ⁱ and ^F(4) ⁱ ⁺ ^F(5) In nearly all cases subleading N^4 LO dominate leading N^3 LO contributions ruc

 $t_{\rm max}$ are spectral of 3 ... $t_{\rm max}$ and $t_{\rm max}$ are $t_{\rm max}$ are $t_{\rm max}$. These are $t_{\rm max}$ are $t_{\rm max}$ Convergence of chiral expansion? Clarification in ChPT with explicit Δ´s

Ring-topology up to N⁴LO

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

Comparison with NN force *r* ∼ 1 fm *r* ∼ 1 fm f*b* NN \overline{a} .
. *P* ∈*S*³ *Dij* (*P*)*PG, i, j* = 1*,* 2

state and the state of the state TT 3
Fnelhaum Meißner Ann Rey Nucl Part Sci 62 (12) 159 *^r*) = *^V*˜*^C* ⁺ ^τ ¹ *·* ^τ ²*W*˜ *^C* ⁺ " *^V*˜*^S* ⁺ ^τ ¹ *·* ^τ ²*W*˜ *^S* # (12) 15.
 $\overline{}$ *^V*˜*^S* ⁺ ^τ ¹ *·* ^τ ²*W*˜ *^S* "σ¹ *·* "σ² *Epelbaum, Meißner Ann. Rev. Nucl. Part. Sci 62 (12) 159*

²

$$
\tilde{V}(\vec{r}) = \tilde{V}_C + \tau_1 \cdot \tau_2 \tilde{W}_C + \left[\tilde{V}_S + \tau_1 \cdot \tau_2 \tilde{W}_S\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2
$$

+
$$
\left[\tilde{V}_T + \tau_1 \cdot \tau_2 \tilde{W}_T\right] \left(3 \vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2\right)
$$

-1 ence for separat rds (800 MeV)
eme-depender Bands ($800 \text{ MeV} \leq \tilde{\Lambda}$) visualize estimated short- and long-range contributions <u>bands</u> (*COO MC* \leq 1*1*, 23 mainted estimated
scheme-dependence for separation between
short-and long range contributions

Long-range behavior at $r \geq 2$ fm of

 \tilde{W}_T is governed by 1π-exchange

 \tilde{V}_C is governed by subleading $2π$ -exchange

-4

Size of various dominant contributions at $r = 2 \, \mathrm{fm}$

NN	$2\pi-3\mathrm{NF}$	$\mid 2\pi - 1\pi - 3NF \mid$ ring-3NF \mid	
	$\sim 34 \text{ MeV}$ $\sim 0.71 \text{ MeV}$ $\sim 50 \text{ keV}$ $\sim 70 \text{ keV}$		

the text (only shown the text of the text (only show the theory theory theory theory theory theory of the text (only shown the text of the text of the text (only shown the text of the text of the text of the text (only sho Long-range 3NFs are considerably weaker than NN forces, but not negligible!

Small scale expansion of 3NF

Pion-nucleon scattering −1*.*41 1*.*40 −3*.*43 1*.*80 0*.*45 −2*.*36 1*.*43 −2*.*18 −1*.*41 1*.*40 −3*.*43 1*.*80 0*.*45 −2*.*36 1*.*43 −2*.*18

Heavy baryon SSE calculation up to ε³: *Fettes & Meißner ´01; Epelbaum, Gasparyan, HK, in preparation*

Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707

N₃LO-Δ

$Δ$ -less $N⁴LO$

Two-pion-exchange 3NF

Similar results for TPE-3NF in N^3LO - Δ and N^4LO Δ -less approaches

We expect small explicit- Δ N⁴LO contributions to two-pion-exchange 3NF

Two-pion-exchange up to N4LO

Epelbaum, Gasparyan, HK, in preparation

Δ-less and Δ-full approaches for TPE-force compared

similar results if contributions are sizeable

slightly different results if contributions are smaller

Two-pion-one-pion-exchange 3NF

Bands indicate physics which is not described by explicit $Δ$ -contributions

Two-pion-one-pion-exchange 3NF \overline{a} **C** on--4 -2

-6 -0.015 $N³$ LO nucleon-contributions are of smaller size

-10 -8 ווטו-פונו. ווו ב-1911/ בס-1955 -2011.
הערכת האוד $\frac{1}{2}$ $\frac{4}{10}$ Dominant effects come from N³LO Δ-/N⁴LO-contr. in Δ-full/Δ-less approach

Ring - 3NFs

Narrow bands

 \Rightarrow Higher order contributions beyond $Δ$ are small

Strong central isoscalar 3NF due to double-Δ excitation

Two different cases:

- 1) Δ-resonance saturation contribution to a given F_i is sizable+
	- \Rightarrow N3LO-Δ and N4LO-Δ-less results are similar
- 2) Δ-resonance saturation contribution to a given F_i is negligible
	- \Rightarrow N3LO-Δ and N4LO-Δ-less results deviate

Explicit-Δ approach is more efficient!

34

Partial wave decomposition Golak et al. Eur. Phys. J. A 43 (2010) 241 Faddeev equation is solved in the partial wave basis $|p,q,\alpha\rangle \equiv |pq(ls)j(\lambda \frac{1}{2})I(jI)JM_J\rangle |(t\frac{1}{2})TM_T\rangle$ Too many terms for doing PWD by hand **New Solut** Automatization $\langle p'q'\alpha'|V|pq\alpha\rangle = \int d\hat{p}' d\hat{q}' d\hat{p} d\hat{q} \sum_{m_l,\dots} (CG \text{ coeffs.}) (Y_{l,m_l}(\hat{p}) Y_{l',m'_l}(\hat{p}') \dots) \langle m'_{s_1} m'_{s_2} m'_{s_3} | V | m_{s_1} m_{s_2} m_{s_3} \rangle$
matrix ~ 10⁵ x 10⁵ can be reduced to 5 dim. integral Ring-diagram-contr. expensive to calculate on the fly We prestore ring-contr. to 3nf's **Numerical interpolation** on a fine momentum grid of ring terms

PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis see talk by Kai Hebeler & Angelo Calci

> Straightforward implementation of high order 3nf's in many-body calc. within No-Core Shell Model

Ay-puzzle in elastic nd scattering

Witala et al. Proceedings of Few Body 20

Right panel: $X = N^3$ LO NN + N²LO 3NF + N³LO 3NF (1π-cont.) + N³LO 3NF (cont.) $\mathbf{v} = \mathbf{X} + \mathbf{N}^3 \mathbf{I} \cap 3\mathbf{N} \mathbf{F}$ (2π -avch) $= X + N³LO 3NF (2π-exch.)$

 \blacksquare = λ + IV LU star (ZII-BXCII.)

 $-$ = X + N³LO 3NF (2π-exch. & 2π-1π-exch.) of 1 α 1π-exchange-contact terms supplemented with long-range terms supplemented with long-range terms: 2 α

 \blacktriangleright = X + N³LO 3NF (2π-exch. & 2π-1π-exch. & ring) and ring (magenta band). The normalised band α full circles) are from β . The normalised band β

Incomplete results: N³LO 3NF (2π-cont. & 1/m-corr.) are missing

- Long-range part of 3NFs is analyzed up to N⁴LO Δ -less/N³LO- Δ
	- Chiral expansion of TPE-3NF seems to be converged
		- TPE-3NF dominates 3NF but does not fill all 22 structures
		- Sizeable contr. are similar for 2π -1 π -3NF in N⁴LO Δ -less and N³LO- Δ approach
		- Dominant effects come from N⁴LO-/N³LO Δ-contr. in Δ-less/Δ-full approach
	- Ring-3NFs fill all 22 structures

b d c d c d c d c d c d c d c d c d c d c d c d c d c

- **a b c d e f** N⁴LO-/N³LO Δ-contr. in Δ-less/Δ-full approach dominate N³LO-nucleon contr.
	- Some missing sizeable Δ -contr. in N⁴LO results like central attractive force \sim O(1/ Δ ²)
	- First (incomplete) results for A_v in nd elastic scattering with N³LO 3NF's

Outlook

- Partial wave decomposition of N^3LO three-nucleon forces
- N⁴LO Δ-less/N³LO-Δ calc. of shorter range part of 3NF
- N⁴LO with explicit-Δ of long range part of 3NF (convergence-test)