Chiral nuclear forces: explicit Δ scenario

Hermann Krebs
Ruhr-Universität-Bochum

Computational and Theoretical Advances for Exotic Isotopes
in the Medium Mass Region
March 25, 2013, INT Program INT-13-1a, Seattle

With V. Bernard, E. Epelbaum, A. Gasparyan, U.-G. Meißner

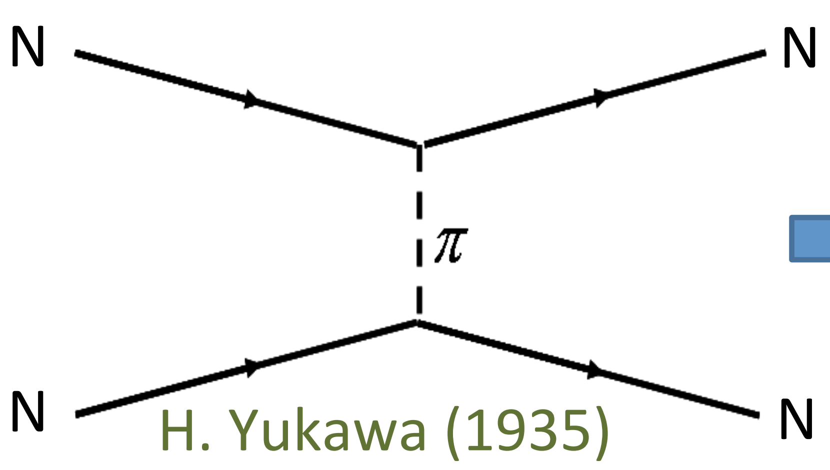
RUB

Outline

- Nuclear forces in chiral EFT
- Role of Δ(1232) resonance
- Long-range part of three-nucleon forces up to N^4LO
- N^3LO-Δ vs. N^4LO Δ-less
- Summary & Outlook

Nucleon-Nucleon forces

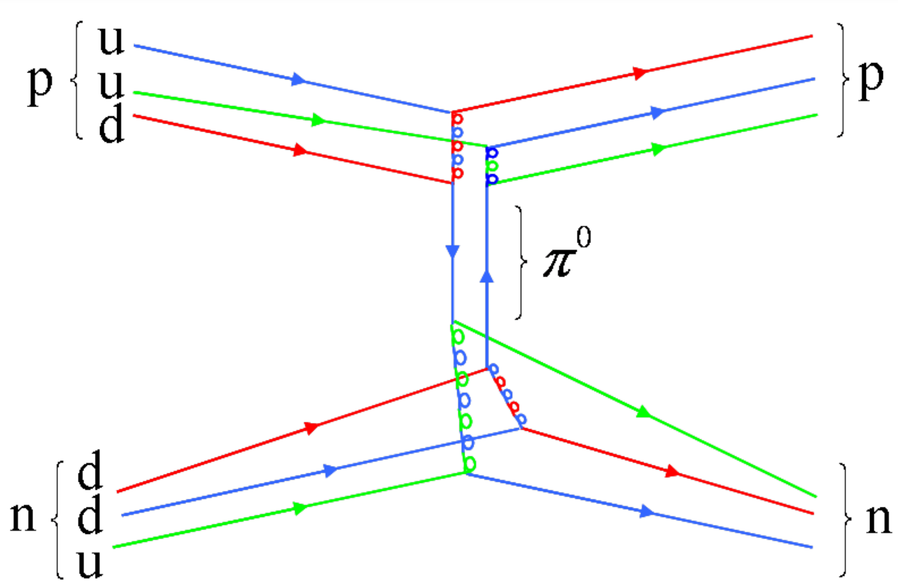
Phenomenological description by meson-exchange



- Boson-Exchange models as basis for NN-force
- Highly sophisticated phen. NN potentials
- Excellent description of many experimental data
- Connection to QCD is unclear

QCD Interpretation of NN forces

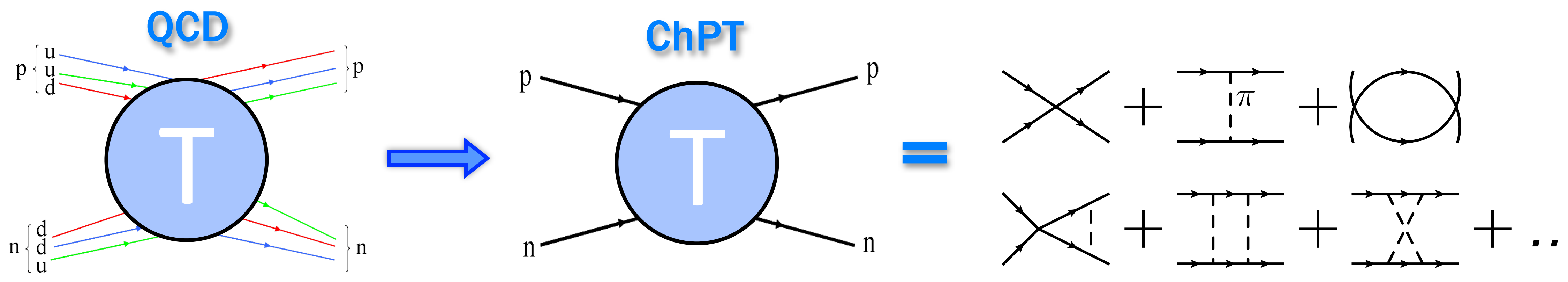
- NN force as residual strong interaction between hadrons



Chiral EFT Interpretation of NN forces

- Model independent treatment
- At low energies NN force dominated by Goldstone Boson dynamics + short range int.
- Systematic perturbative description of few nucleon potentials
- Underlying QCD symmetries implemented by construction

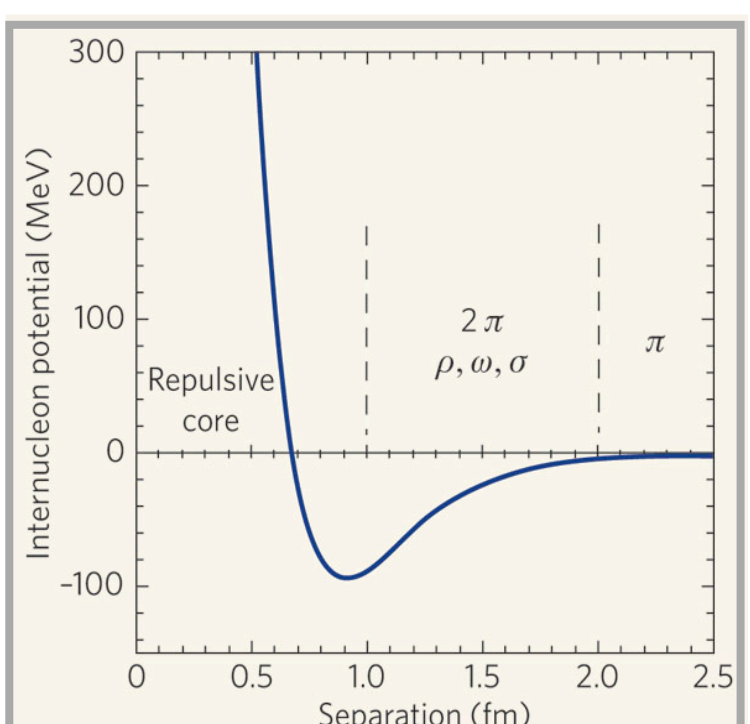
From QCD to nuclear physics

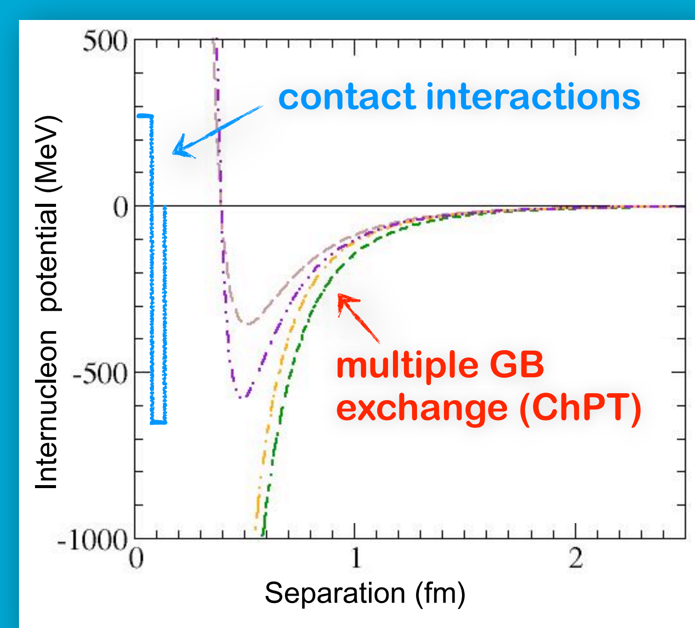


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i\,| \sim M_\pi \ll m_N$) ⟹ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

Weinberg '91





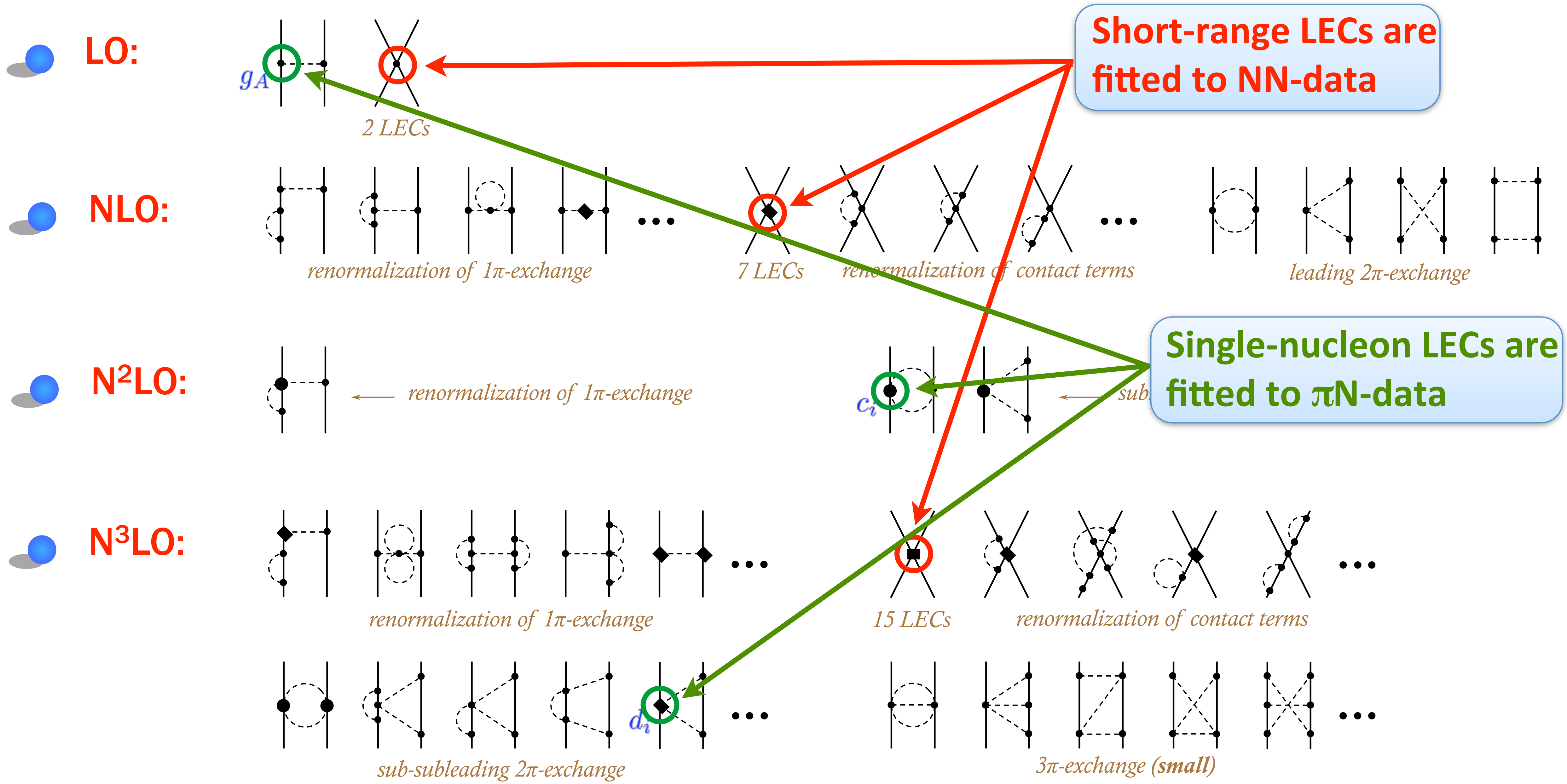
- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Nucleon-nucleon force up to N³LO

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; Epelbaum et al. ’98,‘03; Kaiser ’99-’01; Higa et al. ’03; …

Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$$



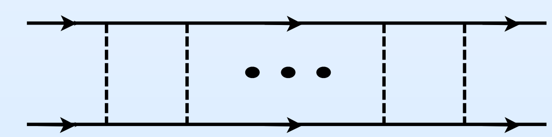
+ 1/m and isospin-breaking corrections…

How to renormalize the Schrödinger Eq?

Lowest-order NN potential: $$V_{2N}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M_\pi^2} + C_S + C_T\vec{\sigma}_1\cdot\vec{\sigma}_2$$

Complication: iterations of the LS equation

$$T(\vec{p}\,',\vec{p}) = V_{2N}^{(0)}(\vec{p}\,',\vec{p}) + \int\frac{d^3k}{(2\,\pi)^3}V_{2N}^{(0)}(\vec{p}\,',\vec{k})\,\frac{m_N}{p^2-k^2+i\,\epsilon}\,T(\vec{k},\vec{p})$$



generate divergences whose subtraction requires infinitely many CTs beyond $V_{2N}^{(0)}$

Kaplan, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque, van Kolck, Pavon Valderrama, Ruiz Arriola, Nogga, Timmermanns, Epelbaum, Meißner, Entem, Machleidt, Yang, Elster, Long, Gegelia, ...

⟶ use a **finite** cutoff (practical solution)

A new, renormalizable approach (yet to be explored...) *Epelbaum, Gegelia '12*

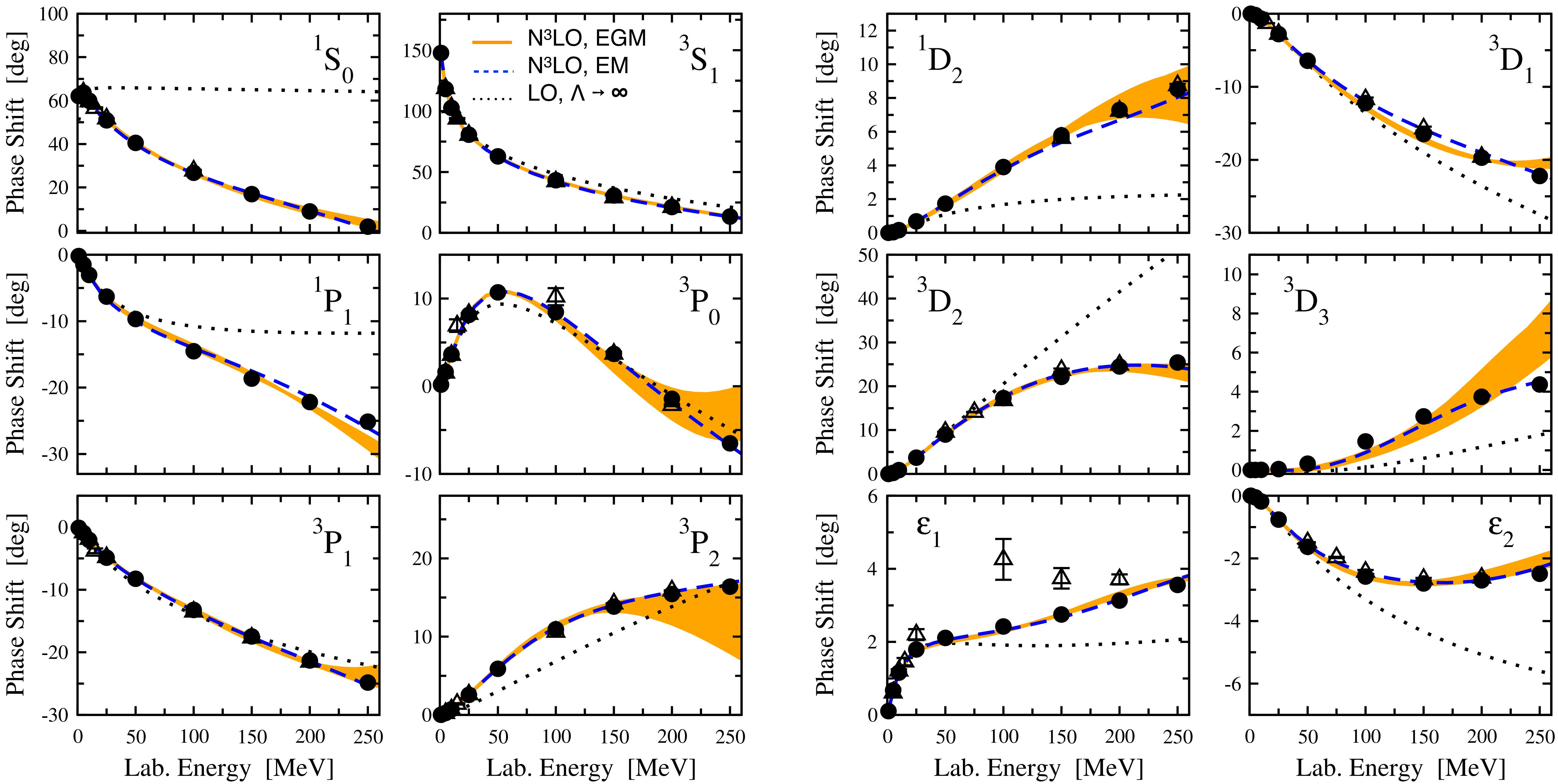
- non-renormalizability of the LO equation is an artifact of the nonrelativistic expansion
- renormalizable LO equation based on manifestly Lorentz-invariant Lagrangian

$$T(\vec{p}\,',\vec{p}) = V_{2N}^{(0)}(\vec{p}\,',\vec{p}) + \frac{m_N^2}{2}\int\frac{d^3k}{(2\,\pi)^3}\frac{V_{2N}^{(0)}(\vec{p}\,',\vec{k})\,T(\vec{k},\vec{p})}{(k^2+m_N^2)\,(E-\sqrt{k^2+m_N^2}+i\,\epsilon)}$$

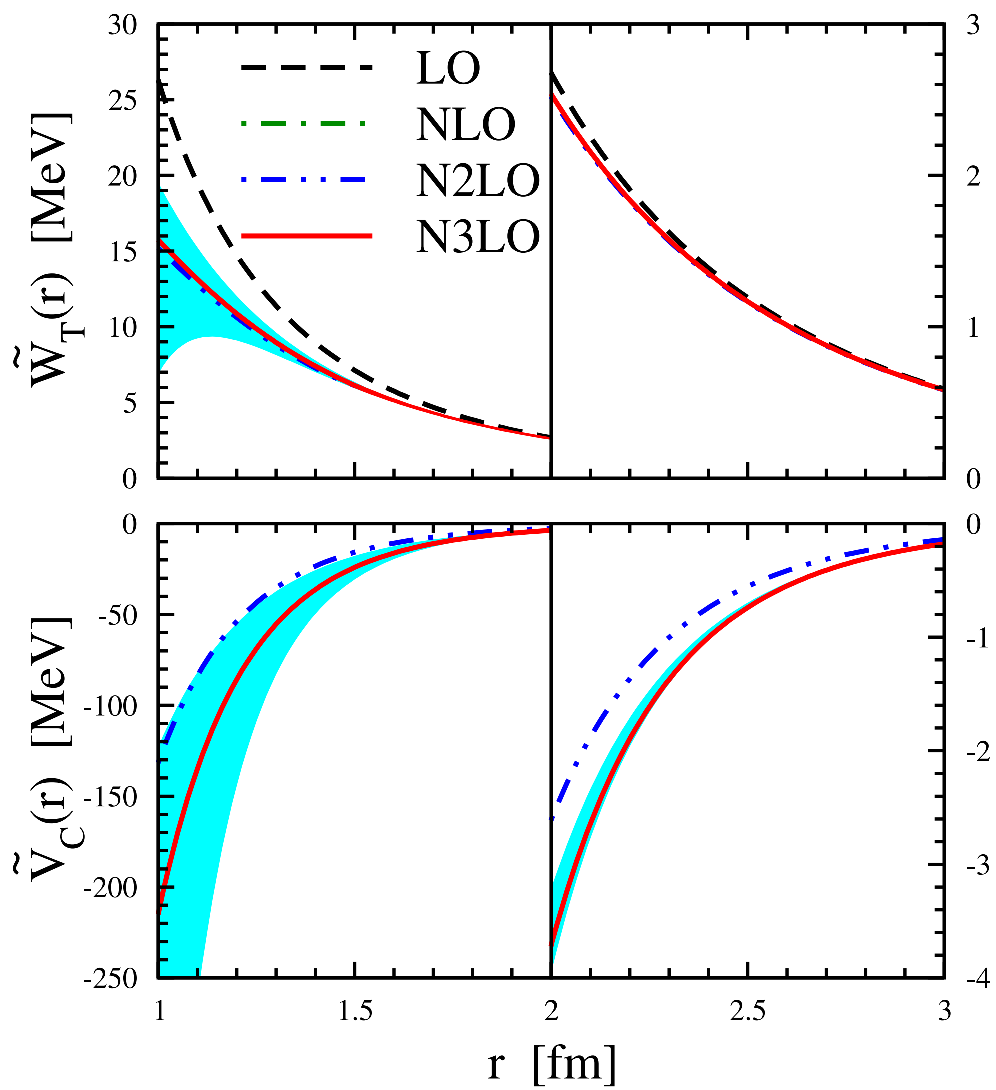
- higher-order corrections (e.g. two-pion exchange) to be treated perturbatively *in progress...*

Neutron-proton phase shifts at N^3LO

Entem, Machleidt ’04; Epelbaum, Glöckle, Meißner ’05



Chiral expansion of NN force



Epelbaum, Meißner Ann. Rev. Nucl. Part. Sci 62 (12) 159

$$\tilde{V}(\vec{r}) = \tilde{V}_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_C + \left[\tilde{V}_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_S\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left[\tilde{V}_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_T\right] (3\, \vec{\sigma}_1 \cdot \hat{r}\, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

Bands ($800\ \mathrm{MeV} \leq \tilde{\Lambda}$) visualize estimated scheme-dependence for separation between short- and long-range contributions

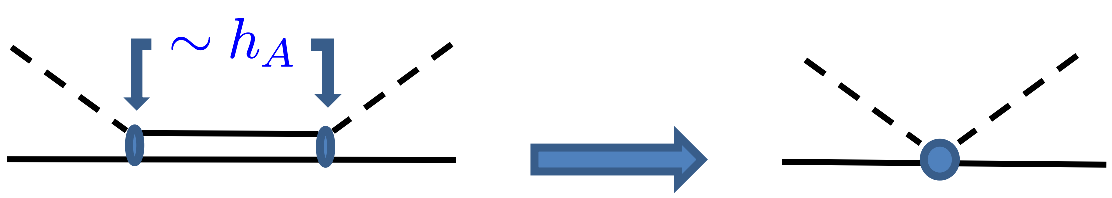
Long-range behavior at $r \geq 2\,\mathrm{fm}$ of

- $\tilde{W}_T$ is governed by 1π-exchange
- $\tilde{V}_C$ is governed by subleading 2π-exchange

- Short-range part of the NN force is scheme-dependent (parametrization)
- Long-range part is scheme-independent and is predicted by chiral EFT
- Convergence of chiral expansion is clarified in a theory with explicit Δ(1232)

EFT with explicit Δ(1232)

- Standard chiral expansion: $Q \sim M_\pi \ll \Delta \equiv m_\Delta - m_N = 293\ \mathrm{MeV}$
- Small scale expansion: $Q \sim M_\pi \sim \Delta \ll \Lambda_\chi$ *(Hemmert, Holstein & Kambor '98)*
- Delta contributions encoded in LECs
  (Bernard, Kaiser & Meißner ´97)

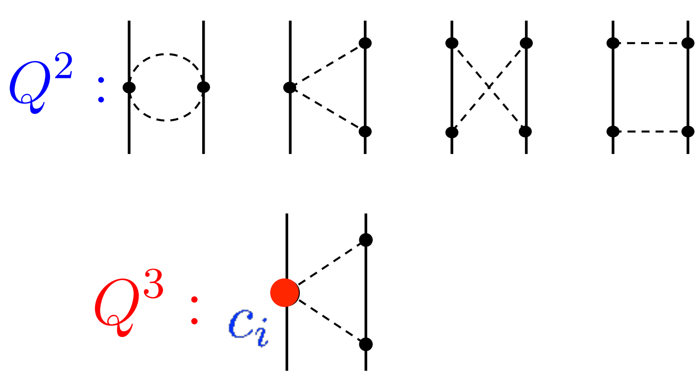


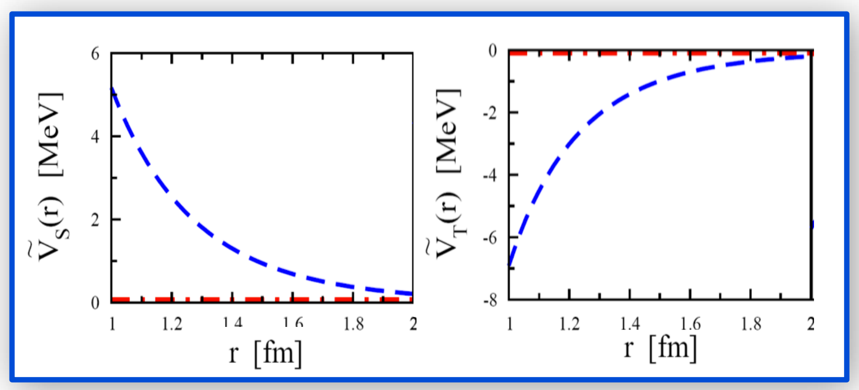
Delta-resonance saturation

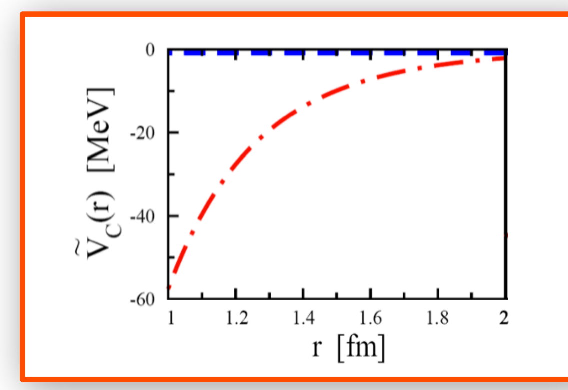
$$c_3 = -2c_4 = c_3(\not\Delta) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

- Convergence of EFT potential







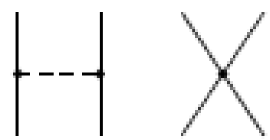
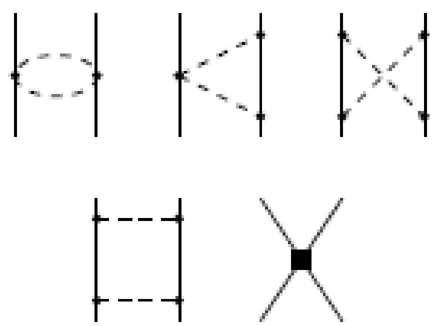
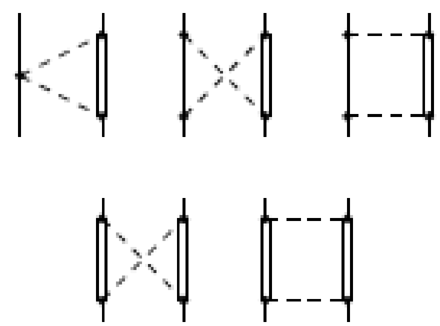
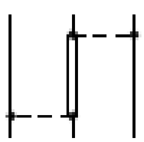
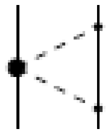
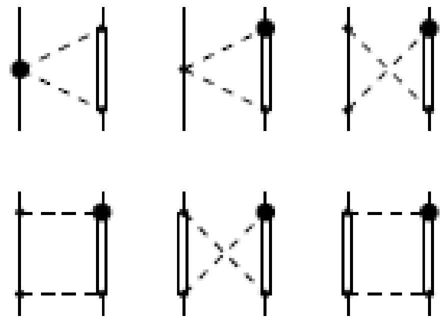
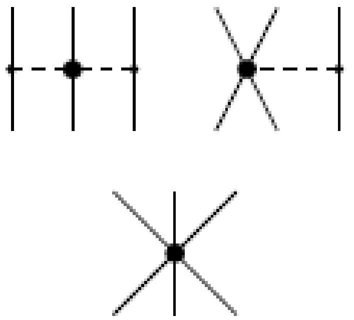
The subleading contributions are larger than the leading one!

Expectation from inclusion of Δ explicitely

- more natural size of LECs
- better convergence
- applicability at higher energies

Few-nucleon forces with the Delta

Isospin-symmetric contributions

	Two-nucleon force		Three-nucleon force	
	Δ–less EFT	Δ–contributions	Δ–less EFT	Δ–contributions
LO		—	—	—
NLO		 Ordonez et al.'96, Kaiser et al. '98	—	
NNLO		 H.K., Epelbaum & Meißner '07		—

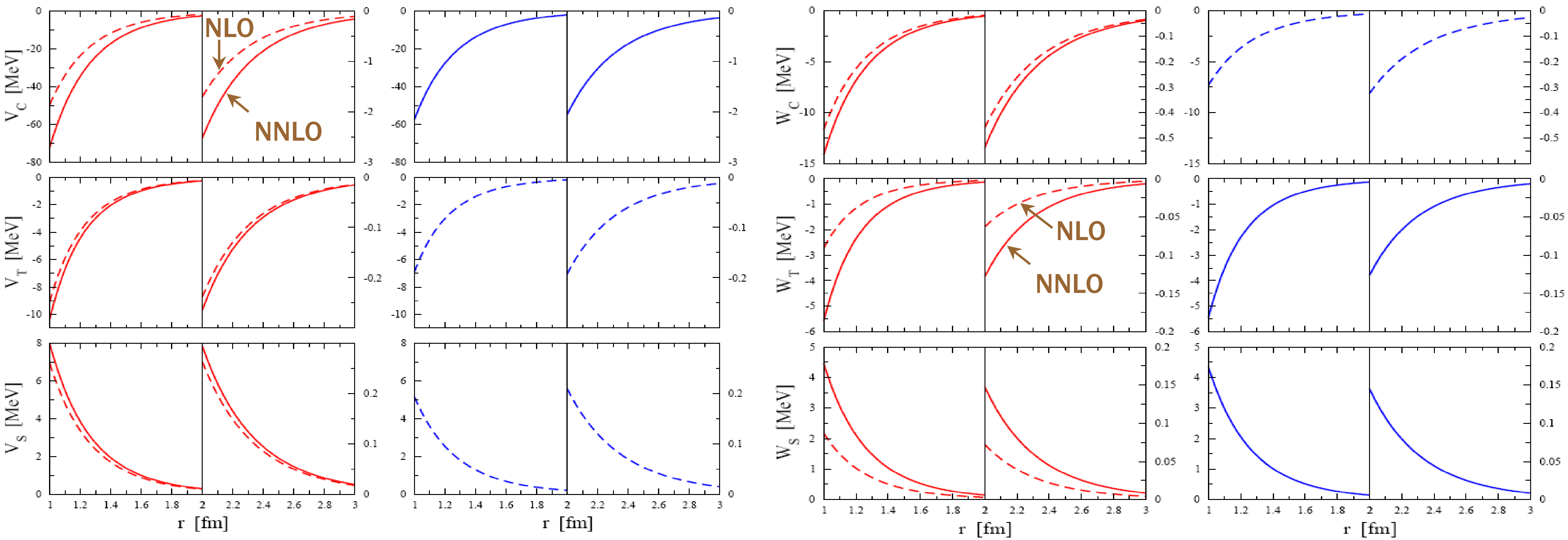
NN potential with explicit Δ

Epelbaum, H.K., Meißner, Eur. Phys. J. A32 (2007) 127

$$V_{\rm eff} = V_C + W_C \vec{\tau}_1 \cdot \vec{\tau}_2 + [V_S + W_S \vec{\tau}_1 \cdot \vec{\tau}_2] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + [V_T + W_T \vec{\tau}_1 \cdot \vec{\tau}_2](3\, \vec{\sigma}_1 \cdot \hat{r} \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

Chiral 2π- exchange potential up to NNLO

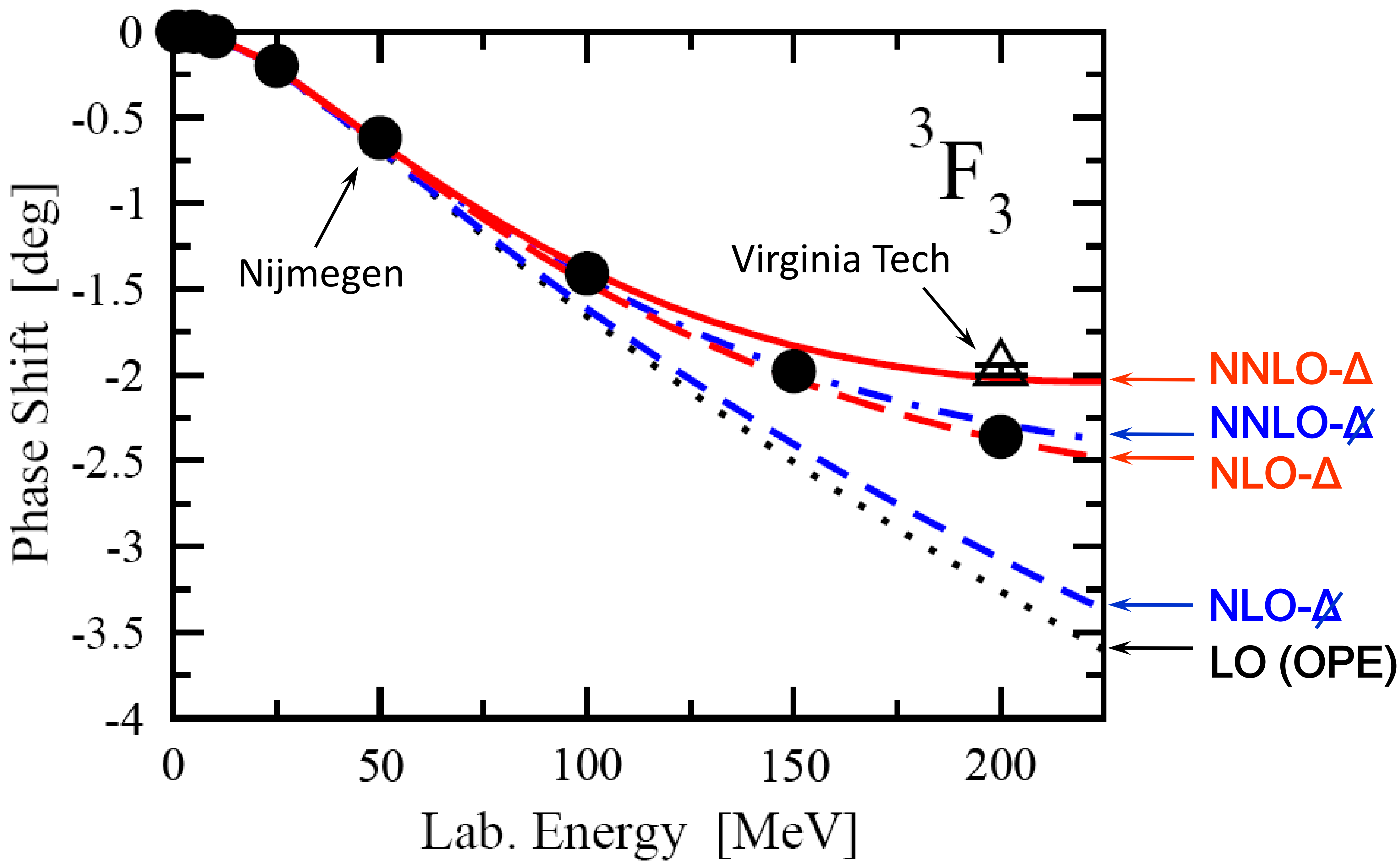
with Δ | without Δ | with Δ | without Δ



Advantages when Δ is included explicitly

- Dominant contributions already at NLO
- Much better convergence in all potentials

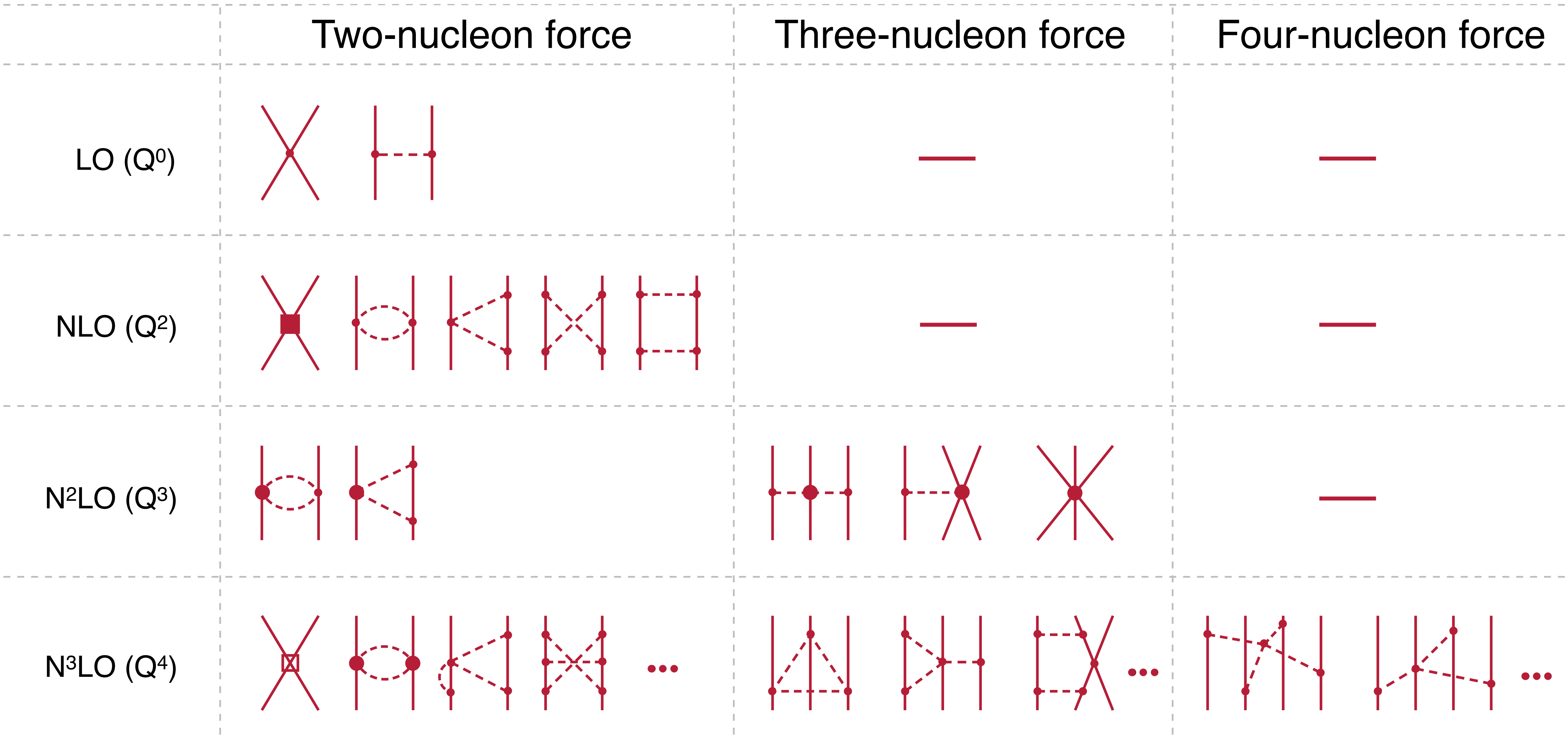
$^3F_3$ partial waves up to NNLO with and without Δ



(calculated in the first Born approximation)

Nuclear forces up to N^3LO

dimensional analysis counting



- converged
- accurate description of NN at least up to E_{lab} ~ 200 MeV

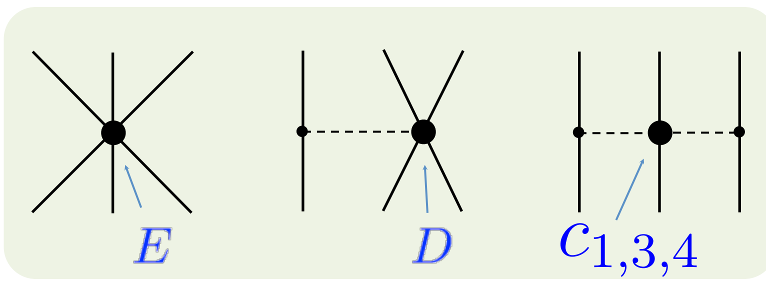
- not yet converged
- higher orders in progress
- impact on few- & many-N systems?

- converged ??

Three-nucleon forces

- Three-nucleon forces in chiral EFT start to contribute at N^2LO

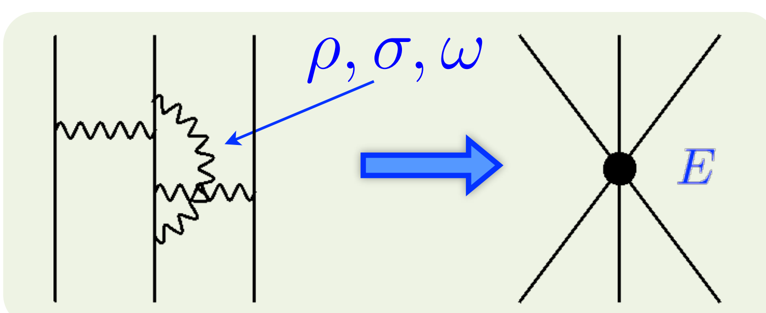
  (Friar & Coon ´86; U. van Kolck ´94; Epelbaum et al. ´02; Nogga et al. ´05; Navratil et al. ´07)



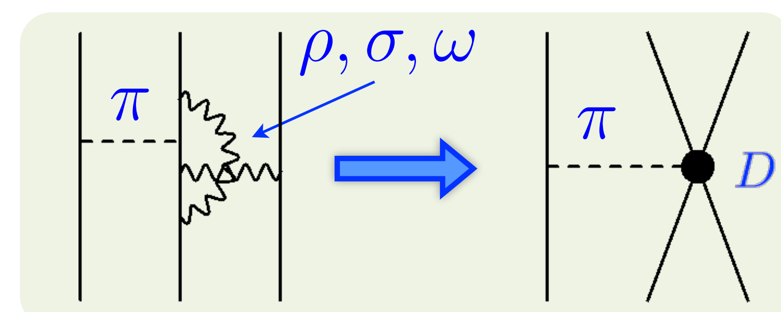
$c_{1,3,4}$ from the fit to πN-scattering data

D, E from $^3\mathrm{H}, ^4\mathrm{He}, ^{10}\mathrm{B}$ binding energy + coherent nd - scattering length

- LECs D and E incorporate short-range contr.



Resonance saturation interpretation of LECs



- Delta contributions encoded in LECs

  (Bernard, Kaiser & Meißner '97)

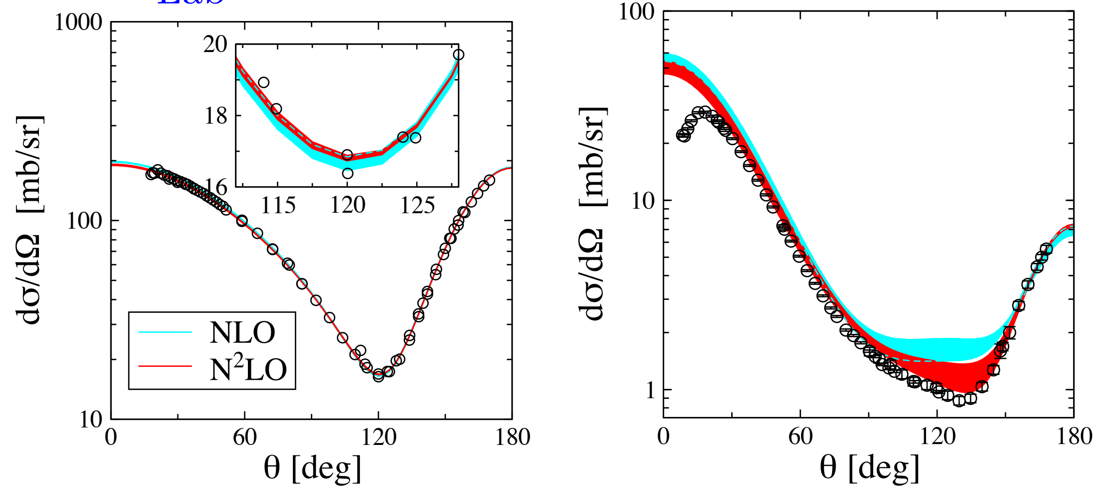
Delta-resonance saturation

$$c_3 = -2c_4 = c_3(\not{\Delta}) - \boxed{\frac{4h_A^2}{9\Delta}}$$

Enlargement due to Delta contribution

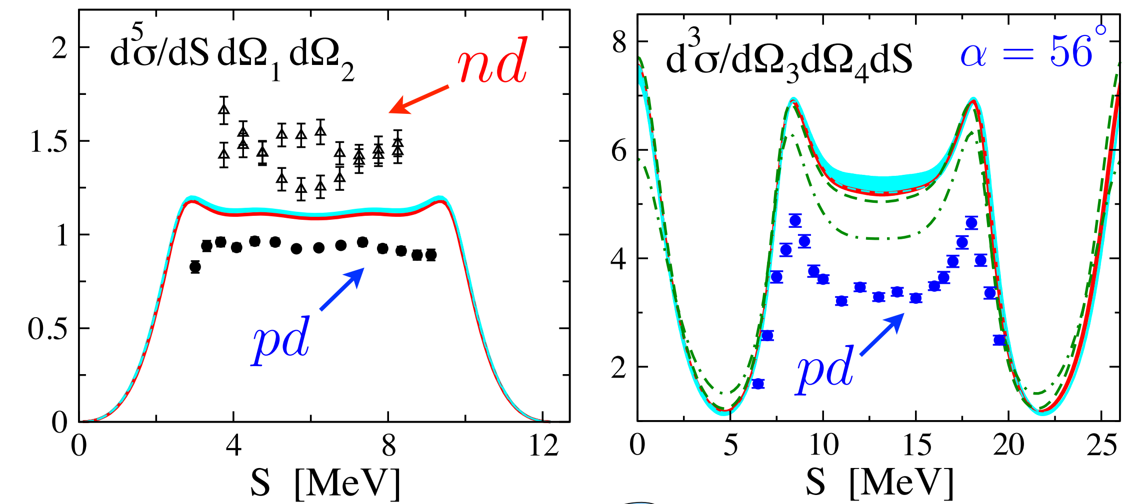
nd elastic scattering

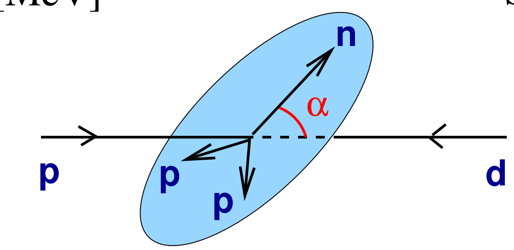
$E_{\mathrm{Lab}} = 10\,\mathrm{MeV}$ $E_{\mathrm{Lab}} = 65\,\mathrm{MeV}$



nd break-up $[\mathrm{mb\,MeV^{-1}sr^{-2}}]$

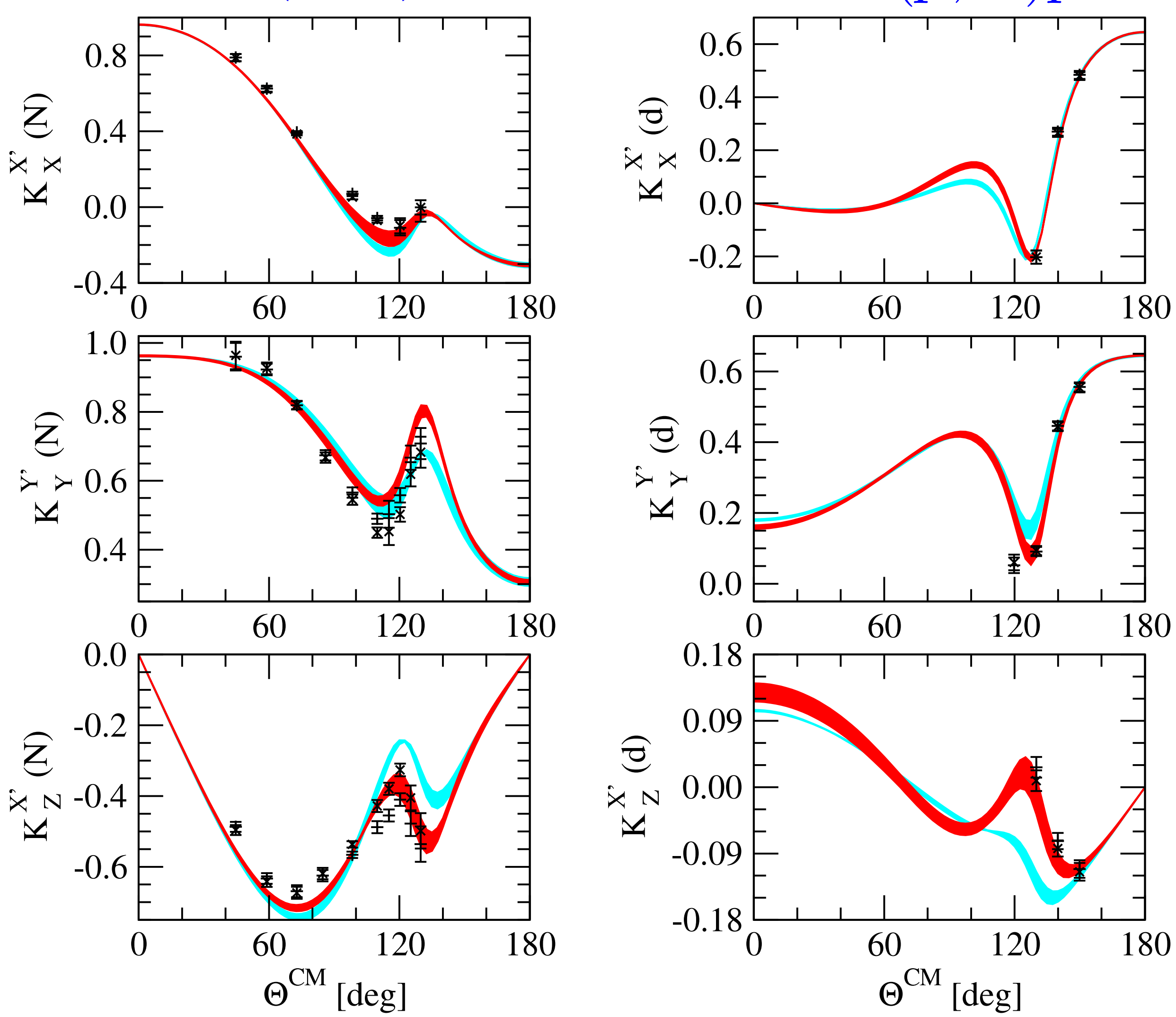
SST at $E_N = 12\,\mathrm{MeV}$ SCRE at $E_N = 19\,\mathrm{MeV}$





polarization transfer: $E_p^{\mathrm{Lab}} = 22.7\,\mathrm{MeV}$

$d(\vec{p},\vec{p})d$ $d(\vec{p},\vec{d})p$



For references see recent reviews:

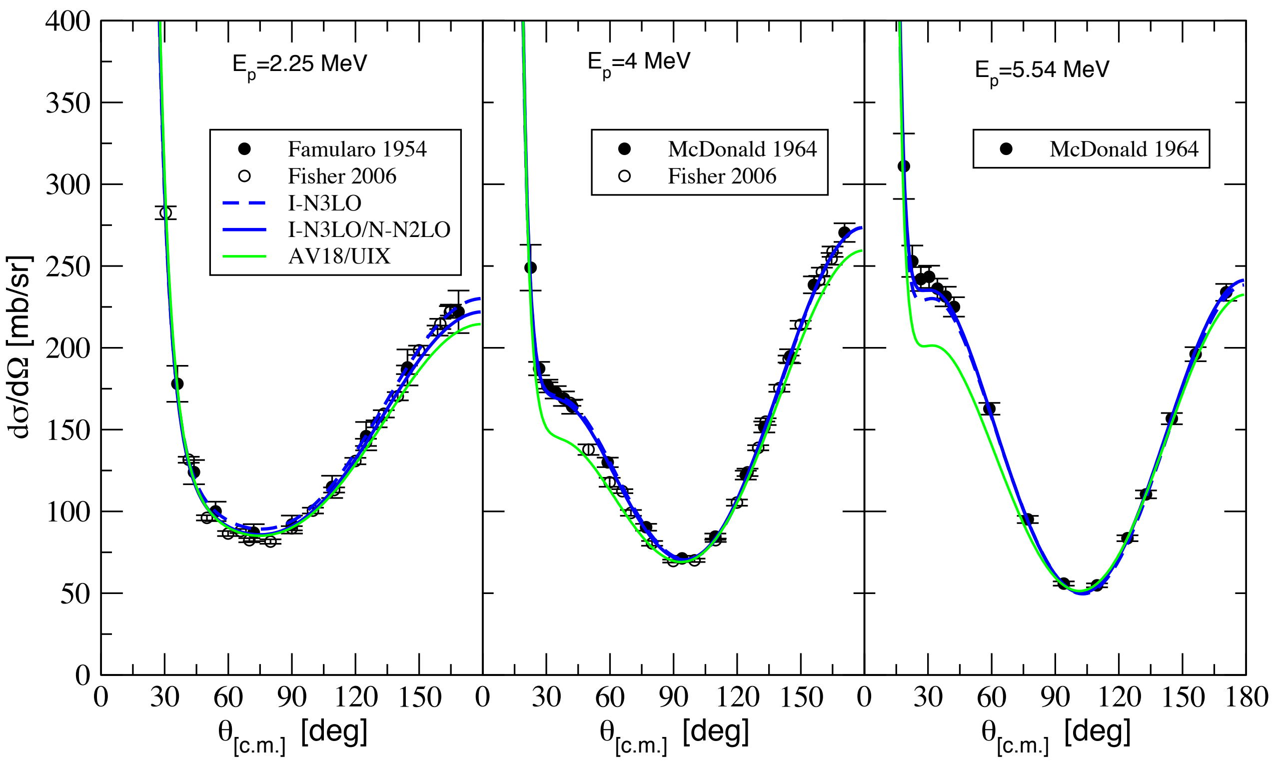
- Epelbaum, Prog. Part Nucl. Phys. 57 (06) 654
- Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81 (09) 1773
- Entem, Machleidt, Phys. Rept. 503 (11) 1
- Epelbaum, Meißner, Ann. Rev. Nucl. Part. Sci. 62 (12) 159
- Kalantar et al. Rep. Prog. Phys. 75 (12) 016301

- Generally good description of data. But some discrepancies survive. E.g. break-up observables for SCRE/SST configuration at low energy
- Hope for improvement at N^3LO

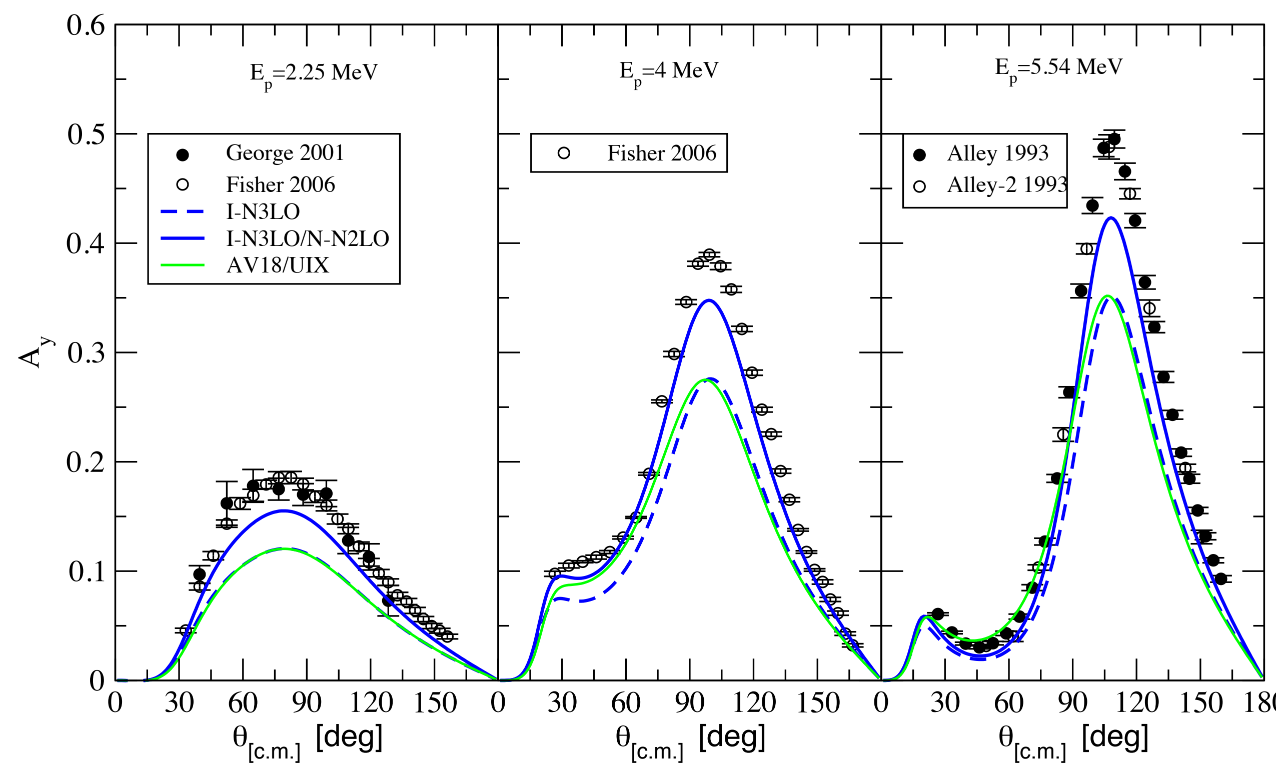
Proton-^{3}He elastic scattering

Viviani, Girlanda, Kievsky, Marcucci, Rosati arXiv: 1004.1306

p-^{3}He differential cross section at low energies



proton vector analyzing power A_y-puzzle



As in n-d scattering case N^{2}LO 3NF's are not enough to resolve underprediction of A_y

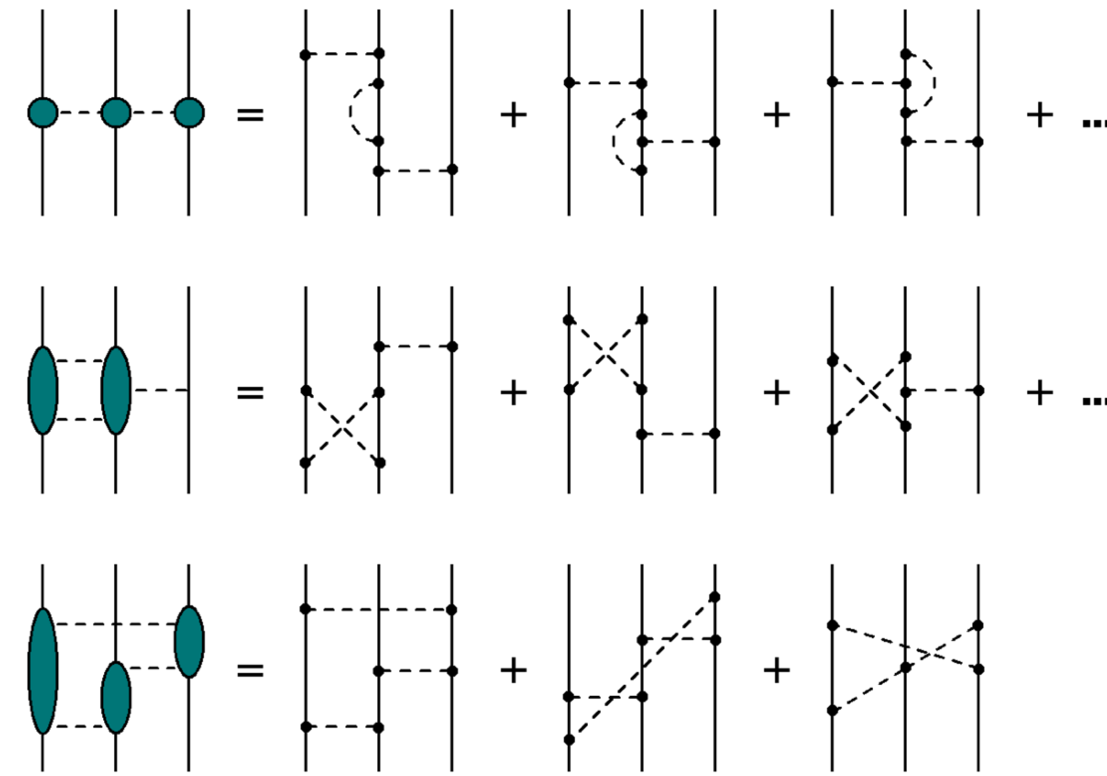
→ Hope for improvement at higher orders

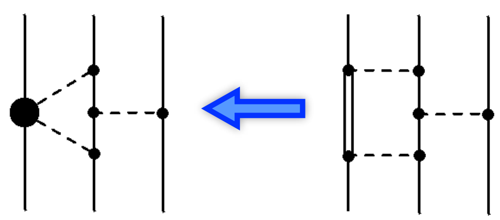
Three-nucleon forces

- Three-nucleon forces at N³LO

Long range contributions *Bernard, Epelbaum, HK, Meißner ´08; Ishikawa, Robilotta ´07*

- No additional free parameters
- Expressed in terms of g_A, F_π, M_π
- Rich isospin-spin-orbit structure
- Δ(1232)-contr. are important

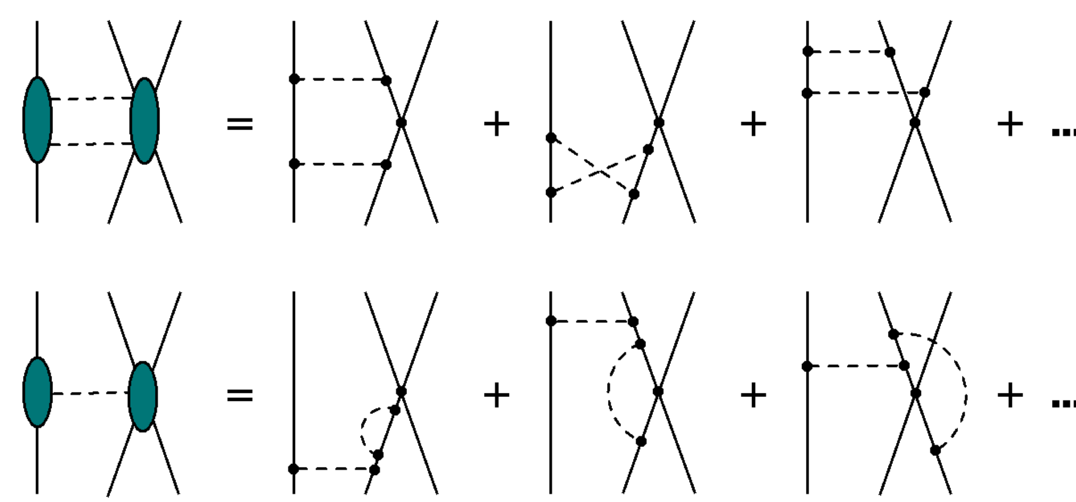


Large $c_i \sim$ 

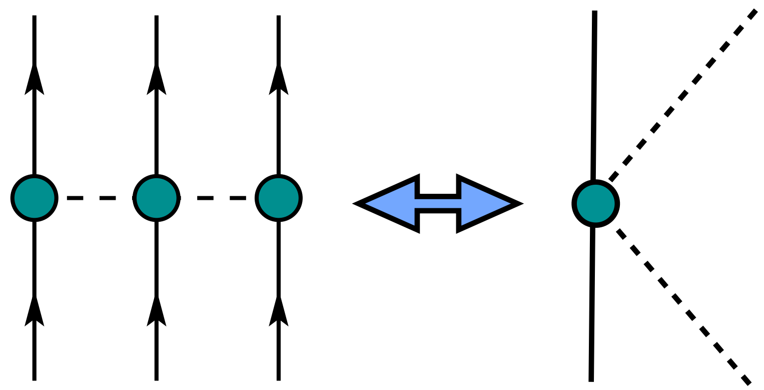
Shorter range contributions *Bernard, Epelbaum, HK, Meißner ´11*

- LECs needed for shorter range contr.

  g_A, F_π, M_π, C_T
- Central NN contact interaction does not contribute
- Unique expressions in the static limit for a renormalizable 3NF



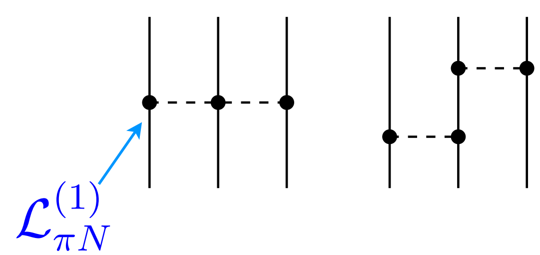
Two-pion-exchange 3NF



- Two-pion-exchange 3NF is connected to pion-nucleon scattering amplitude
  Ishikawa, Robilotta ´07
  - The same linear combinations of LECs
  - The same renormalization

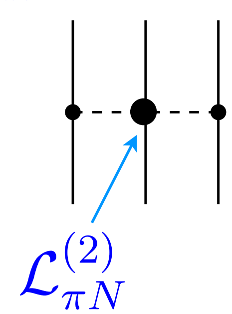
$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

NLO - contr.

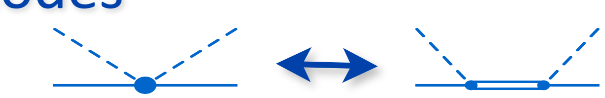


⟵ yield vanishing 3NF contributions

N²LO - contr.



⟵ first nonvanishing 3NF, encodes information about the Δ:

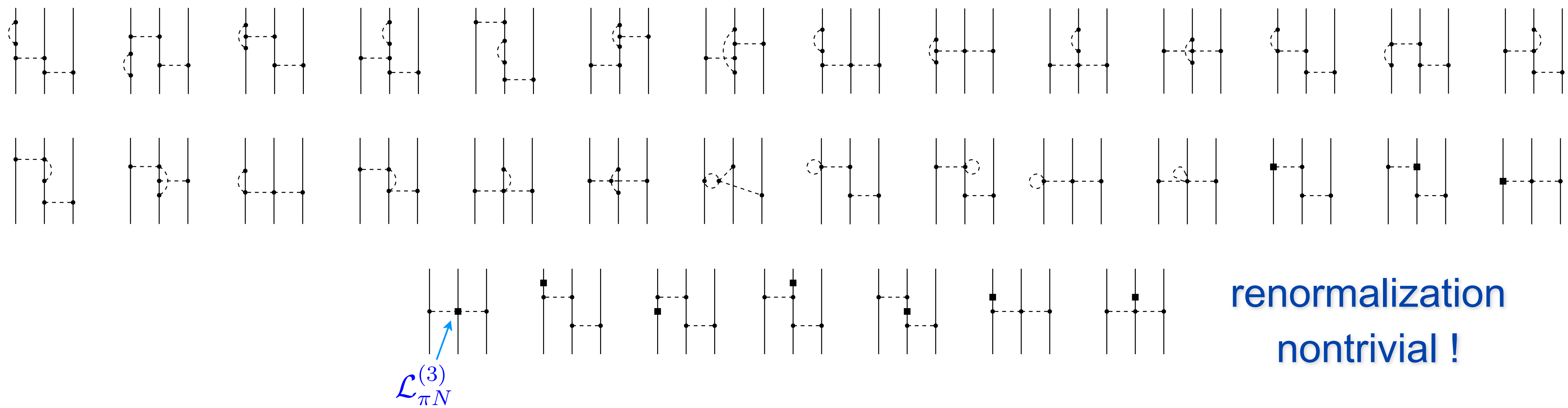


$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big((2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big)\,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

U. van Kolck ´94

Two-pion-exchange 3NF

N[3]LO - contr. (leading 1 loop)



renormalization nontrivial !

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2+1\right)M_\pi^3 + 2\left(g_A^2+1\right)M_\pi q_2^2\right],$$

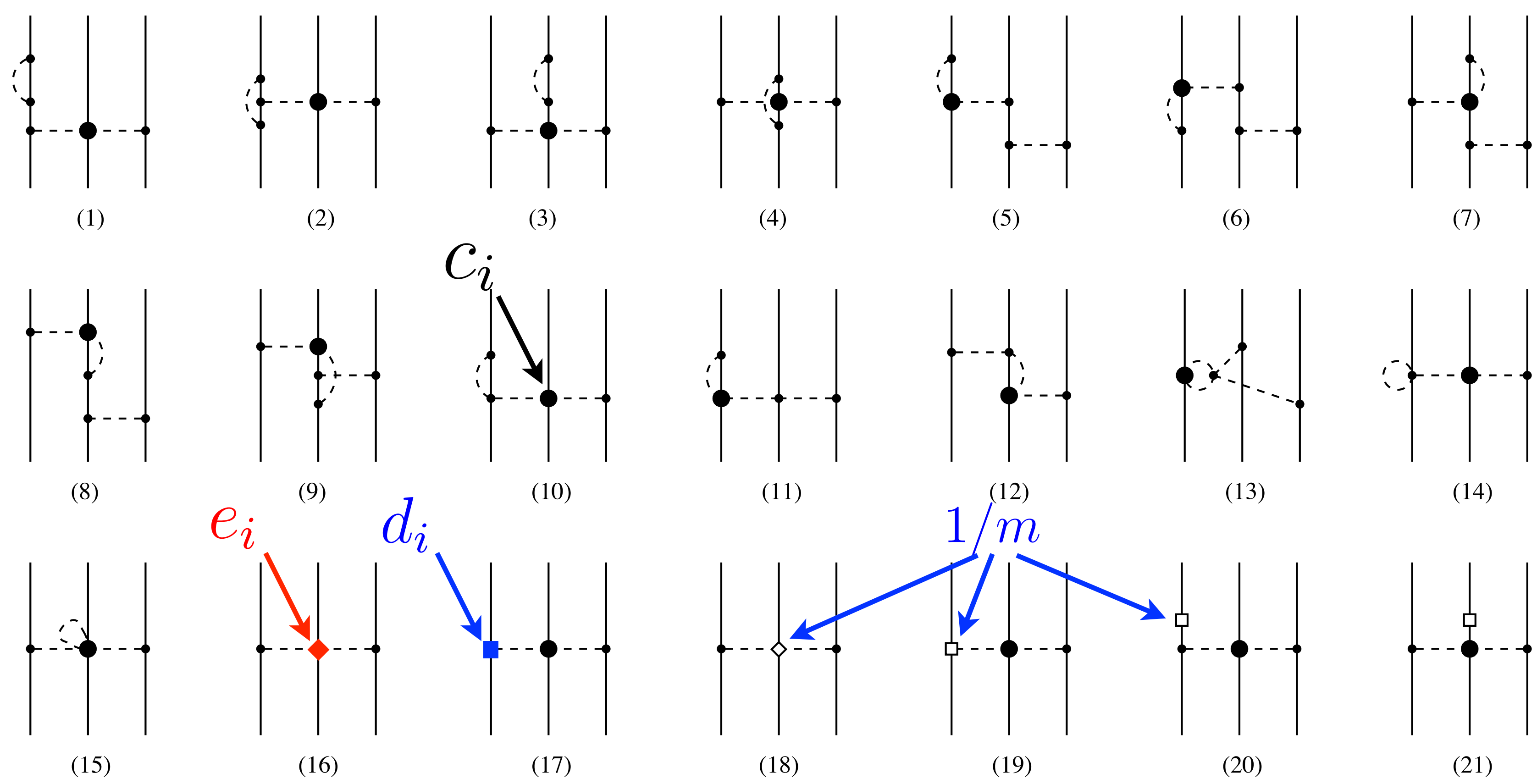
$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\left[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2+1)M_\pi\right]$$

Ishikawa, Robilotta ´07,
Bernard, Epelbaum, HK, Meißner ´07

- No unknown parameters at this order
- Everything is expressed in terms of loop function $A(q) = \frac{1}{2q}\arctan\frac{q}{2M_\pi}$
- Additional unitarity transformations required for proper renormalization

Two-pion-exchange 3NF

N^4LO - contr. (subleading 1 loop) *Epelbaum, Gasparyan, HK,´12*



c_i´s LECs from $\mathcal{L}_{\pi N}^{(2)}$, d_i´s LECs from $\mathcal{L}_{\pi N}^{(3)}$, e_i´s LECs from $\mathcal{L}_{\pi N}^{(4)}$: fitted to πN - scattering data

- Leading Δ - contributions are taken into account through c_i´s
- Vanishing $1/m$ - contributions at this order

Two-pion-exchange 3NF at N^{4}LO

$$\mathcal{A}^{(5)}(q_2) = \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2\big(F_\pi^2\left(2304\pi^2 g_A(4\bar{e}_{14}+2\bar{e}_{19}-\bar{e}_{22}-\bar{e}_{36}) - 2304\pi^2\bar{d}_{18}c_3\right)$$

$$+\ g_A(144c_1 - 53c_2 - 90c_3)\big) + M_\pi^4\left(F_\pi^2\left(4608\pi^2\bar{d}_{18}(2c_1-c_3) + 4608\pi^2 g_A(2\bar{e}_{14}+2\bar{e}_{19}-\bar{e}_{36}-4\bar{e}_{38})\right)\right.$$

$$\left. +\ g_A\left(72\left(64\pi^2\bar{l}_3+1\right)c_1 - 24c_2 - 36c_3\right)\right) + q_2^4\left(2304\pi^2\bar{e}_{14}F_\pi^2 g_A - 2g_A(5c_2+18c_3)\right)\Big]$$

$$-\ \frac{g_A^2}{768\pi^2 F_\pi^6}L(q_2)\left(M_\pi^2+2q_2^2\right)\left(4M_\pi^2(6c_1-c_2-3c_3) + q_2^2(-c_2-6c_3)\right)$$

$$\mathcal{B}^{(5)}(q_2) = -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2\left(F_\pi^2\left(1152\pi^2\bar{d}_{18}c_4 - 1152\pi^2 g_A(2\bar{e}_{17}+2\bar{e}_{21}-\bar{e}_{37})\right) + 108g_A^3c_4 + 24g_Ac_4\right)$$

$$+\ q_2^2\left(5g_Ac_4 - 1152\pi^2\bar{e}_{17}F_\pi^2 g_A\right)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6}L(q_2)\left(4M_\pi^2+q_2^2\right)$$

$$L(q) = \frac{\sqrt{q^2+4M_\pi^2}}{q}\log\frac{\sqrt{q^2+4M_\pi^2}+q}{2M_\pi}$$

Some LECs can be absorbed by shifting c_i´s

$$c_1 \to c_1 - 2M_\pi^2\left(\bar{e}_{22} - 4\bar{e}_{38} + \frac{\bar{l}_3 c_1}{F_\pi^2}\right)$$

$$c_3 \to c_3 + 4M_\pi^2\left(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}\right)$$

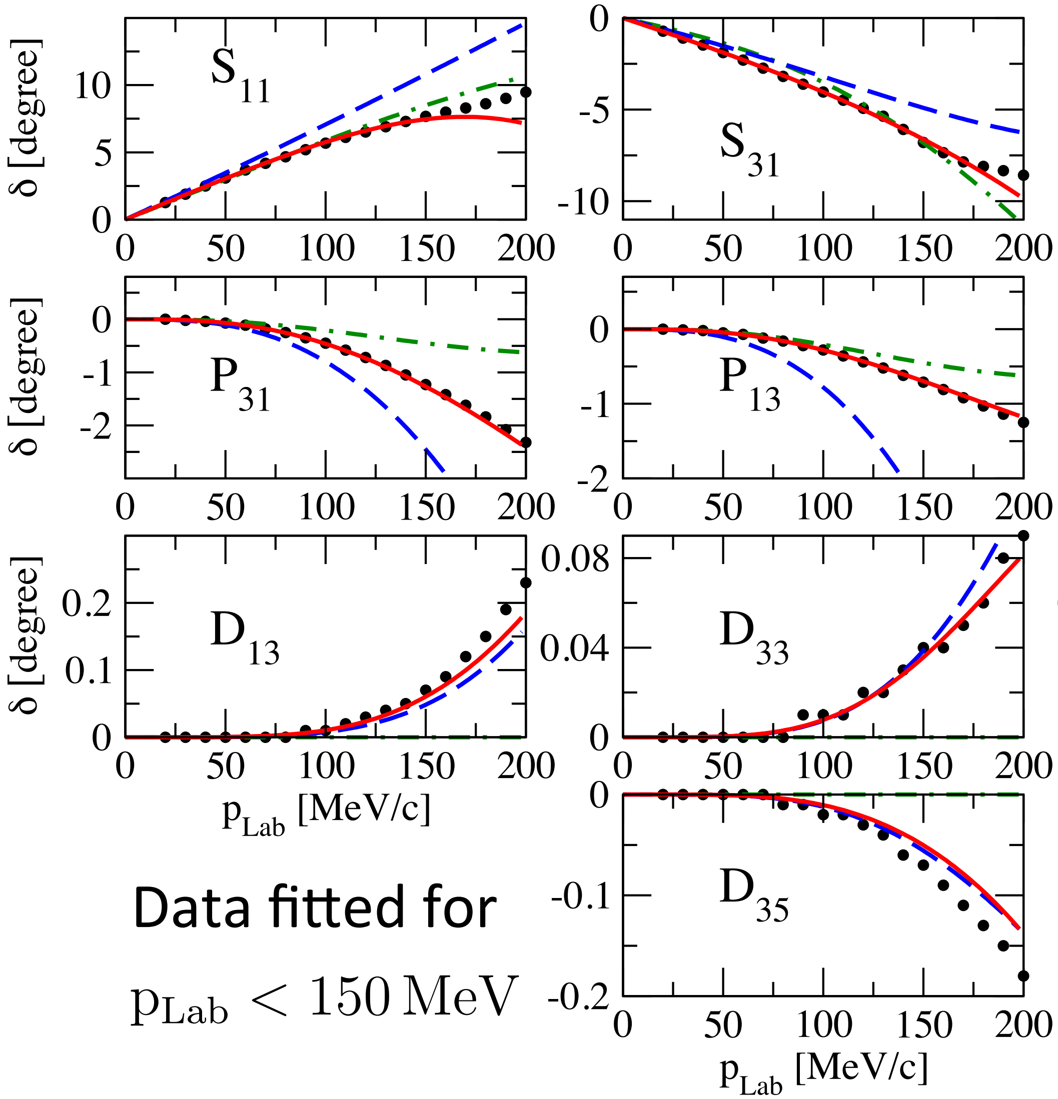
$$c_4 \to c_4 + 4M_\pi^2\left(2\bar{e}_{21} - \bar{e}_{37}\right)$$

- No d_i dependence of TPE-contr. besides d_{18}
- Pion-nucleon scattering does strongly depend on d_i´s

$$g_{\pi NN} = \frac{g_A\, m}{F_\pi}\left(1 - \frac{2M_\pi^2\,\bar{d}_{18}}{g_A}\right)$$ ⇐ Violation of Goldberger-Treiman relation

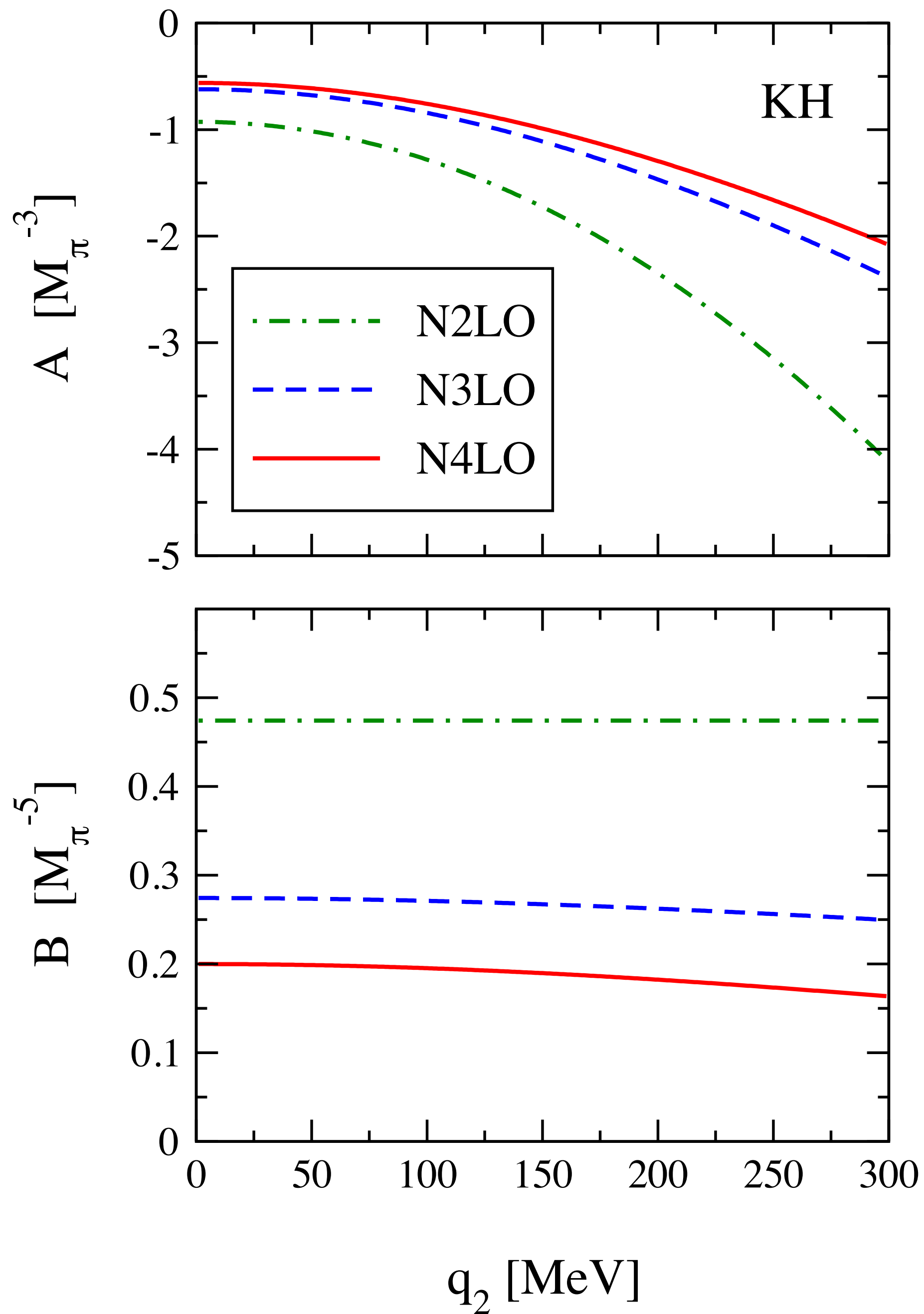
Two-pion-exchange at N^4LO

Fettes, Meißner ´00; Epelbaum, Gasparyan, HK, ´12



Data fitted for

$p_{\mathrm{Lab}} < 150$ MeV



Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707

Similar fit to George-Washington (GW) PWA: Arndt et al. Phys. Rev. C 74 (2006) 045205

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
GW-fit	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
KH-fit	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

- No dependence on d_i´s
- e_i´s are of natural size
- Good convergence of TPE 3NF

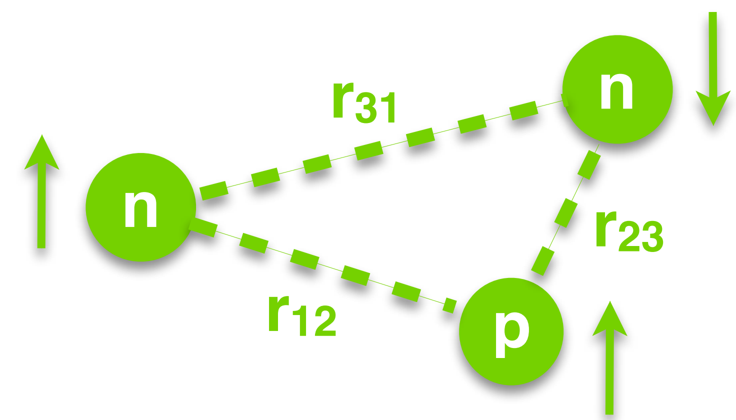
Most general structure of a local 3NF

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

Up to N^4LO, the computed contributions are local $\longrightarrow$ it is natural to switch to r-space.

A meaningful comparison requires a complete set of independent operators

$$
\begin{aligned}
\tilde{\mathcal{G}}_1 &= 1\,, \\
\tilde{\mathcal{G}}_2 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\,, \\
\tilde{\mathcal{G}}_3 &= \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_4 &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_5 &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_6 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)\,, \\
\tilde{\mathcal{G}}_7 &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_8 &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_9 &= \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\,, \\
\tilde{\mathcal{G}}_{10} &= \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{11} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{12} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{13} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{14} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2\,, \\
\tilde{\mathcal{G}}_{15} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{16} &= \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{17} &= \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3\,, \\
\tilde{\mathcal{G}}_{18} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{19} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\,, \\
\tilde{\mathcal{G}}_{20} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_2 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{21} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,, \\
\tilde{\mathcal{G}}_{22} &= \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})\,,
\end{aligned}
$$



Building blocks:

$$\boldsymbol{\tau}_1,\ \boldsymbol{\tau}_2,\ \boldsymbol{\tau}_3,\ \vec{\sigma}_1,\ \vec{\sigma}_2,\ \vec{\sigma}_3,\ \vec{r}_{12},\ \vec{r}_{23}$$

Constraints:

- Locality
- Isospin symmetry
- Parity and time-reversal invariance

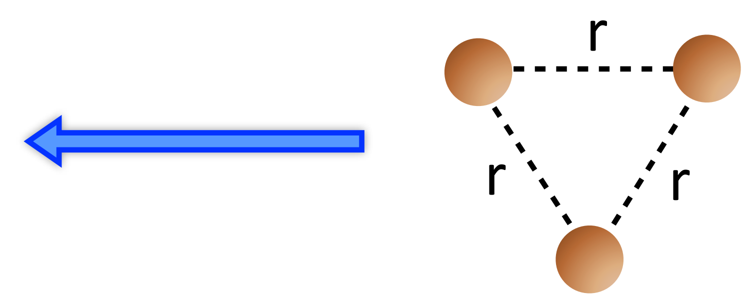
$$\longrightarrow \quad V_{3\mathrm{N}} = \sum_{i=1}^{22} \mathcal{G}_i\, F_i(r_{12}, r_{23}, r_{31}) + \ 5 \text{ perm.}$$

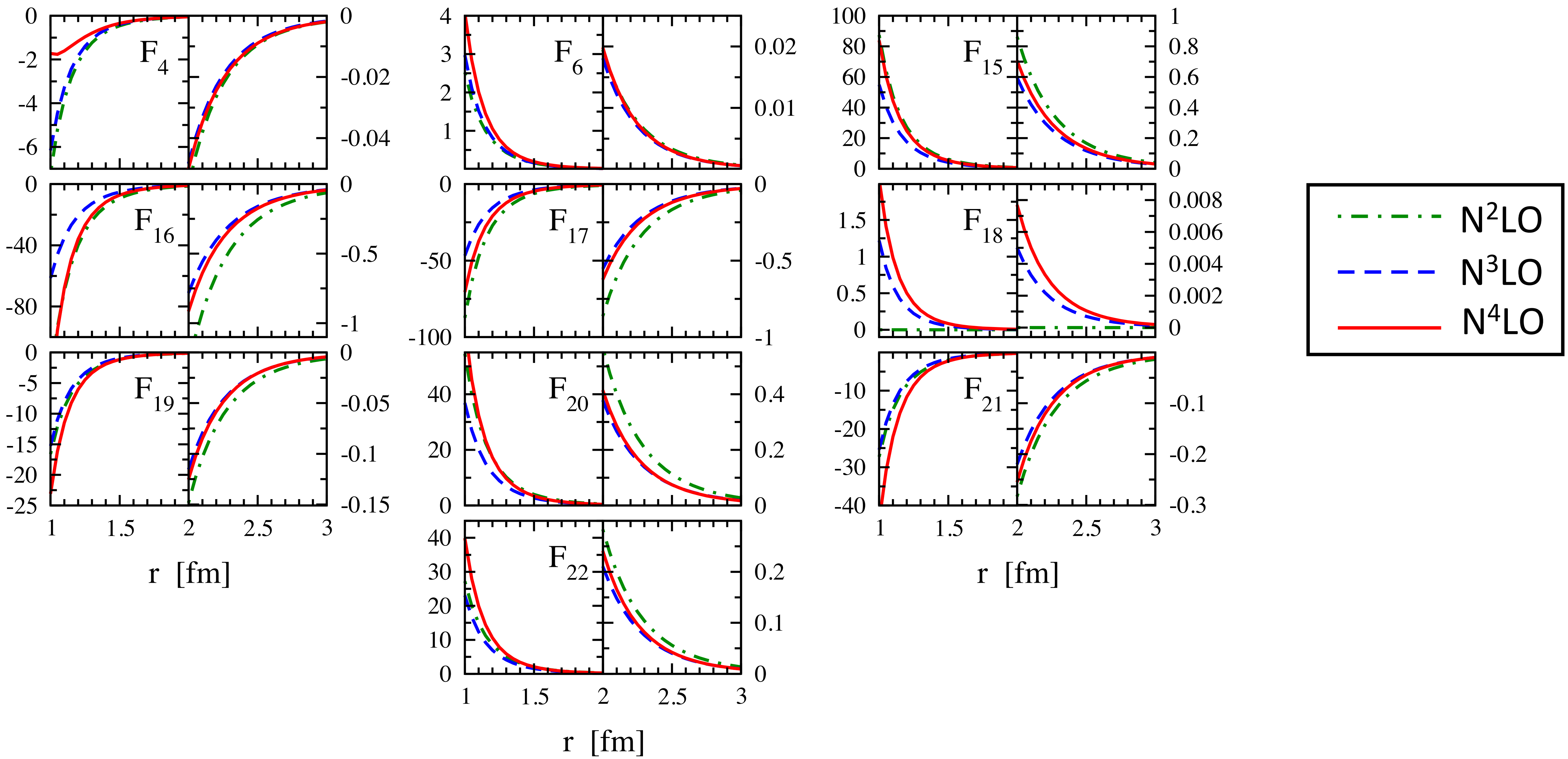
derivable in ChPT; long-range terms parameter-free predictions

Two-pion-exchange up to N^4LO

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration

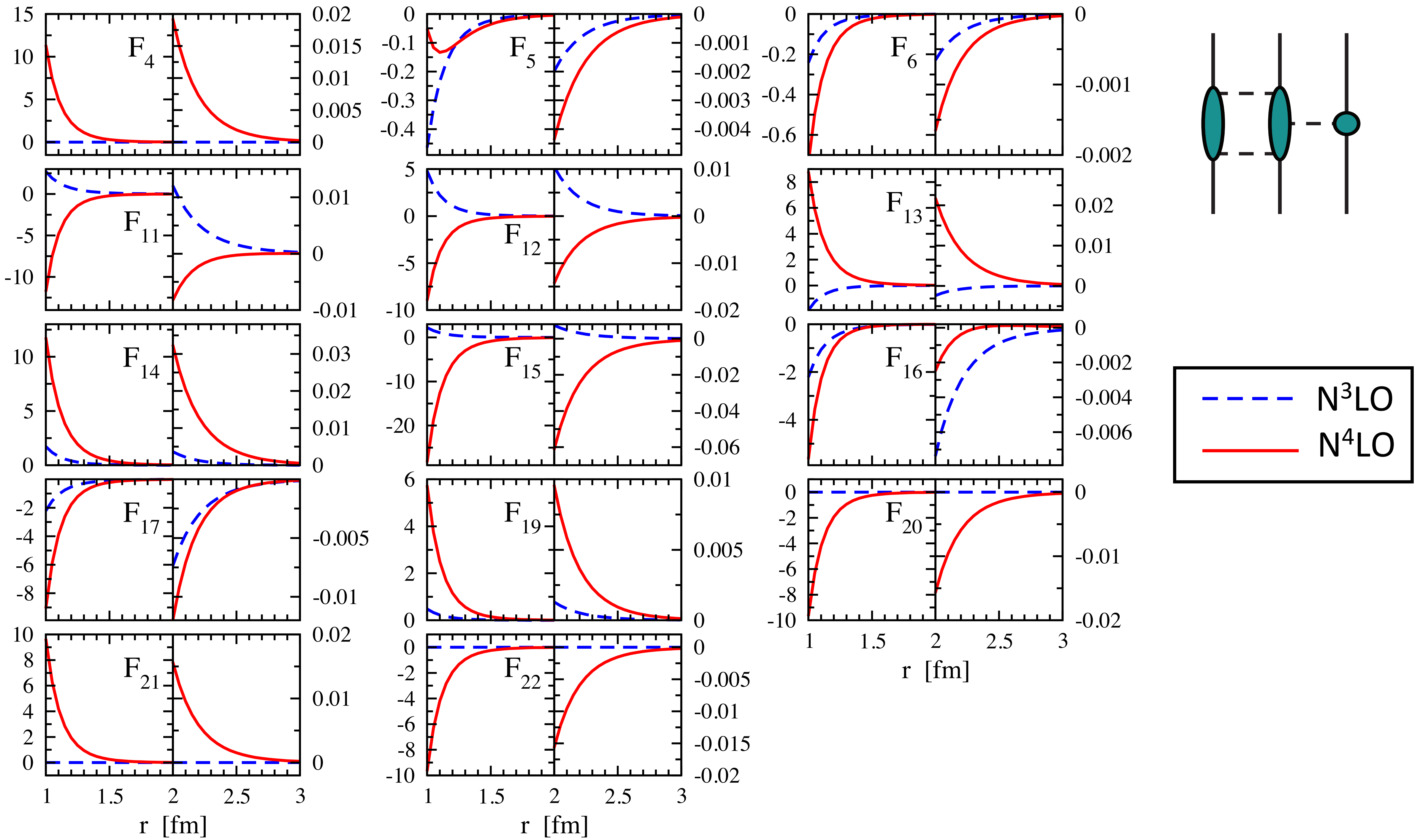




Excellent convergence of TPE-force at distance r $\geq 2\,\mathrm{fm}$

Two-pion-one-pion-exchange up to N^4LO

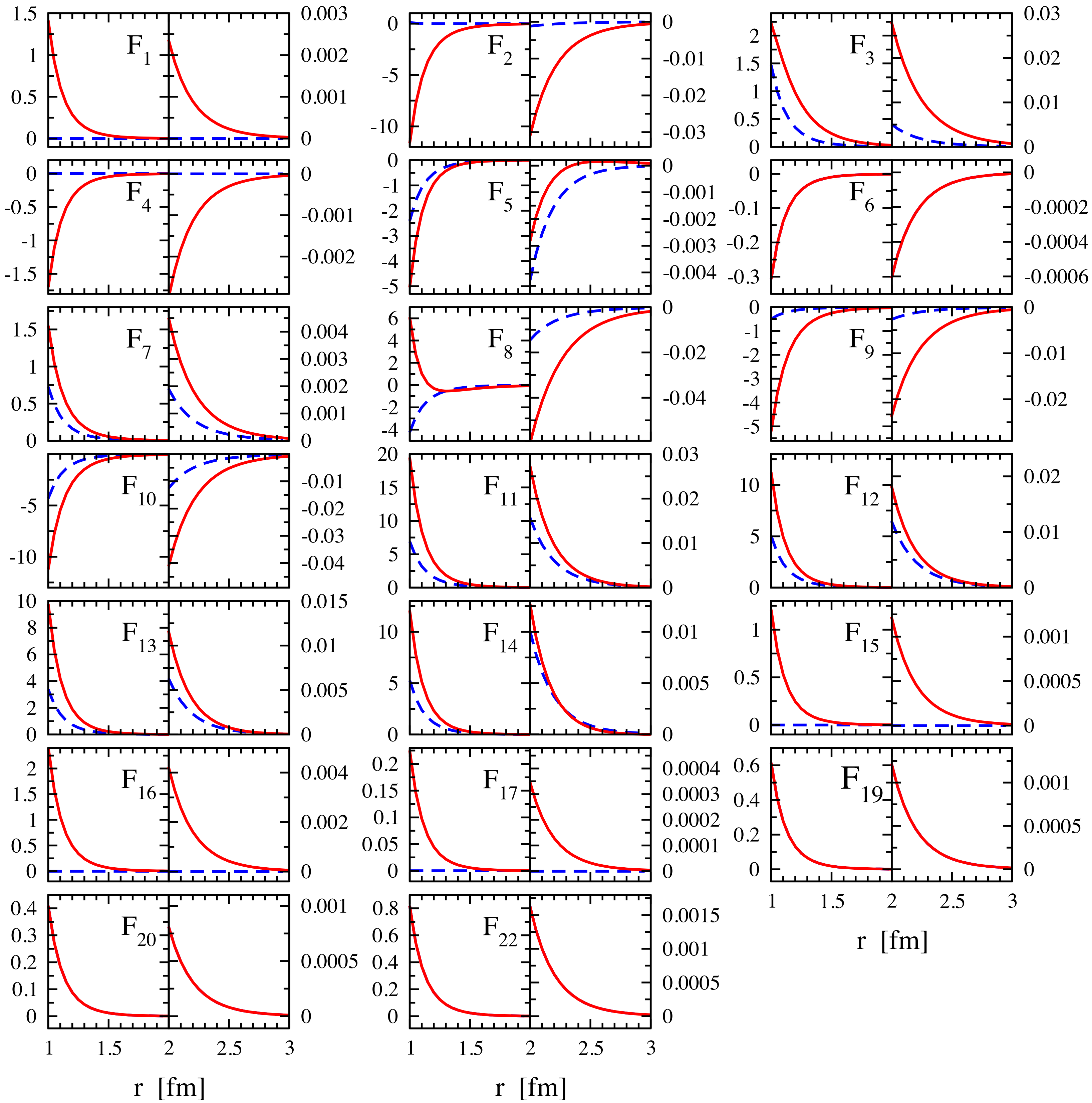
Epelbaum, Gasparyan, HK, arXiv: 1302.2872

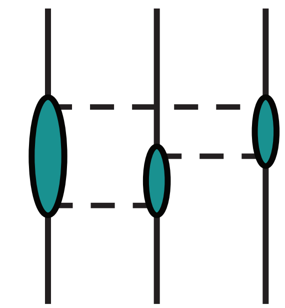


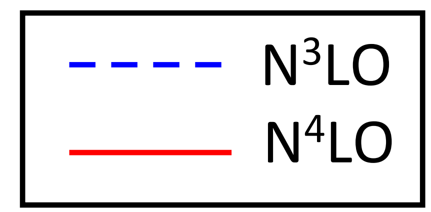
- In nearly all cases subleading N^4LO dominate leading N^3LO contributions
- Convergence of chiral expansion? Clarification in ChPT with explicit Δ´s

Ring-topology up to N^4LO

Epelbaum, Gasparyan, HK, arXiv: 1302.2872

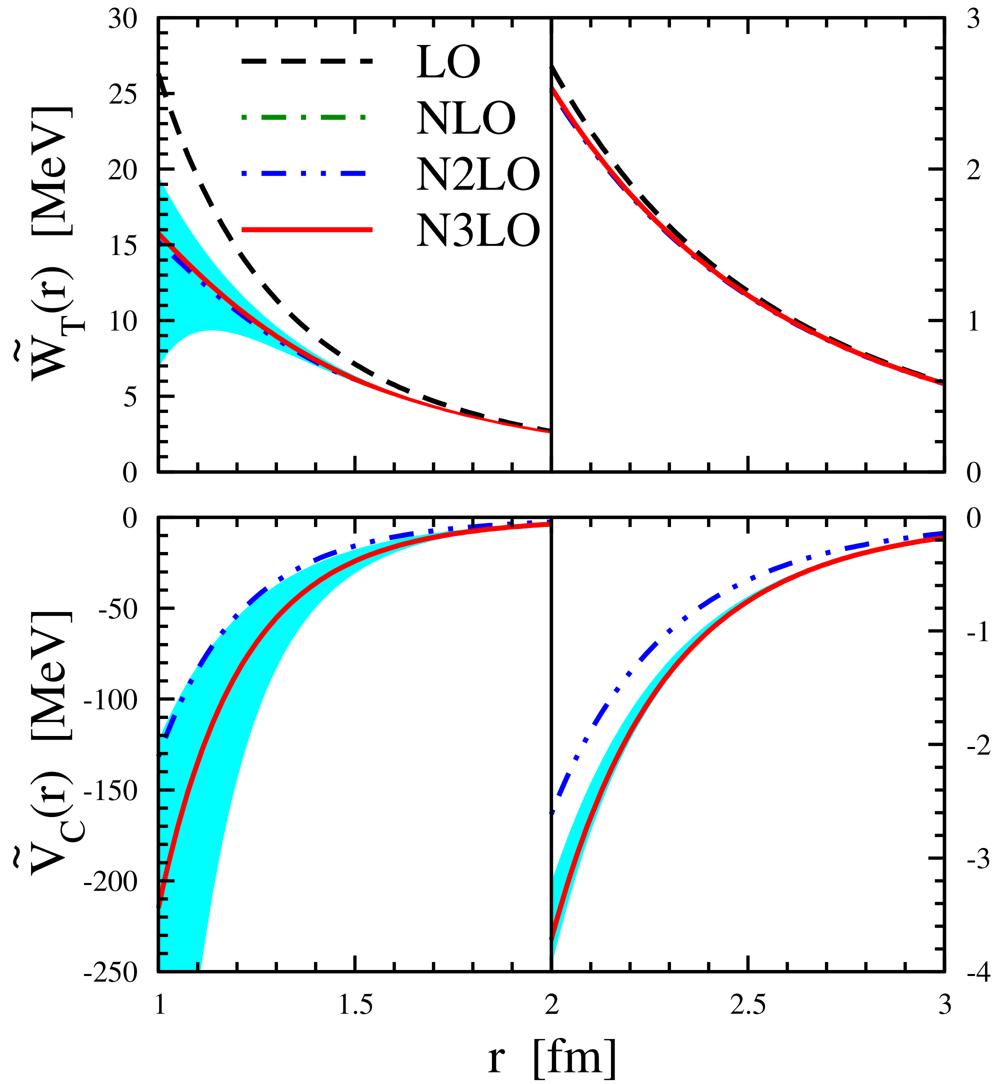






- Rings fill all 22 structures
- Often subleading N^4LO dominate leading N^3LO contributions
- Convergence? ChPT with explicit Δ´s

Comparison with NN force



Epelbaum, Meißner Ann. Rev. Nucl. Part. Sci 62 (12) 159

$$\tilde{V}(\vec{r}) = \tilde{V}_C + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_C + \left[\tilde{V}_S + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_S\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left[\tilde{V}_T + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tilde{W}_T\right] (3\, \vec{\sigma}_1 \cdot \hat{r}\, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

Bands ($800 \text{ MeV} \leq \tilde{\Lambda}$) visualize estimated scheme-dependence for separation between short- and long-range contributions

Long-range behavior at $r \geq 2 \text{ fm}$ of

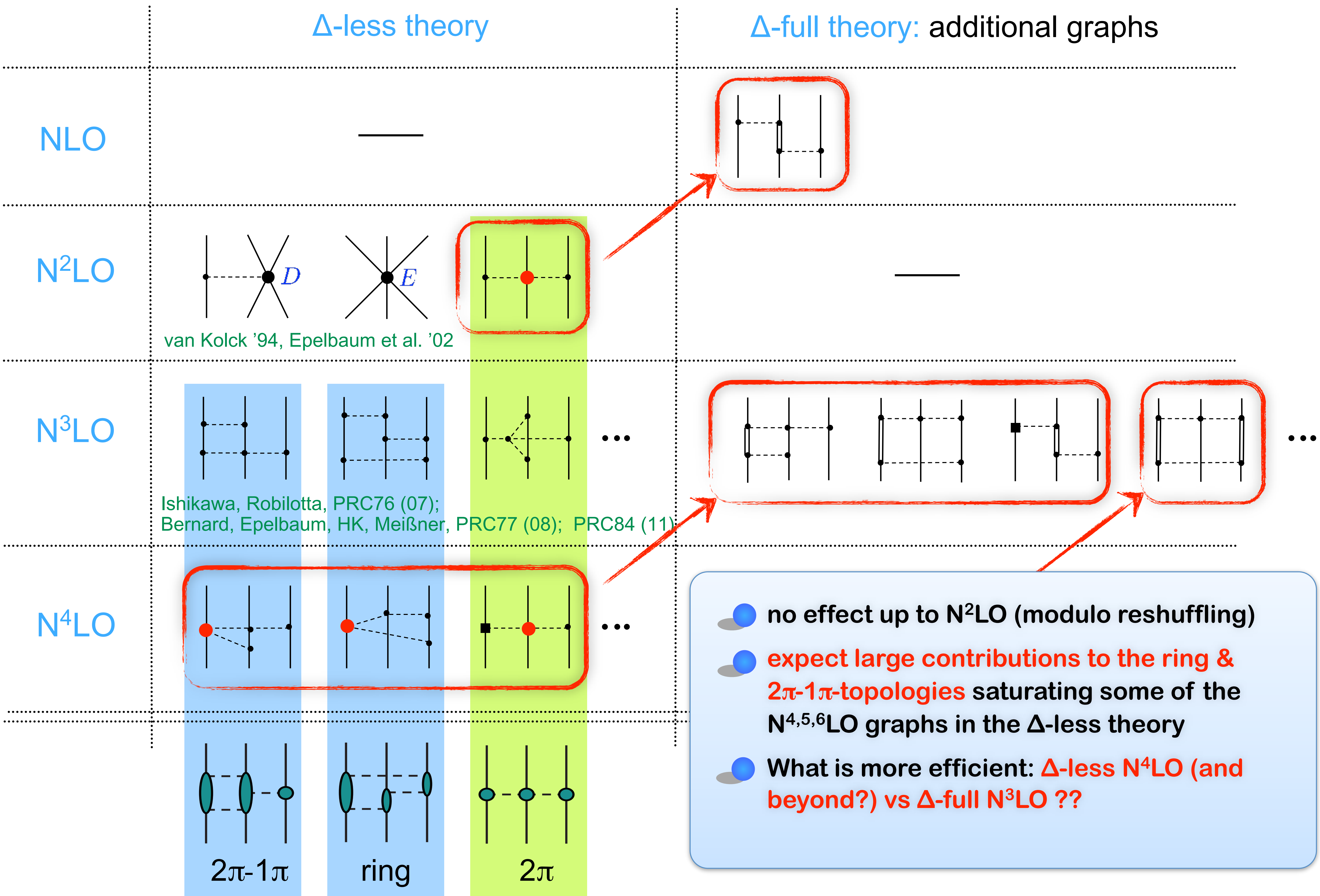
- $\tilde{W}_T$ is governed by 1π-exchange
- $\tilde{V}_C$ is governed by subleading 2π-exchange

Size of various dominant contributions at $r = 2 \text{ fm}$

NN	2π–3NF	$2\pi - 1\pi$–3NF	ring-3NF
$\sim 3 \ldots 4$ MeV	$\sim 0.7 \ldots 1$ MeV	~ 50 keV	~ 70 keV

Long-range 3NFs are considerably weaker than NN forces, but not negligible!

Small scale expansion of 3NF

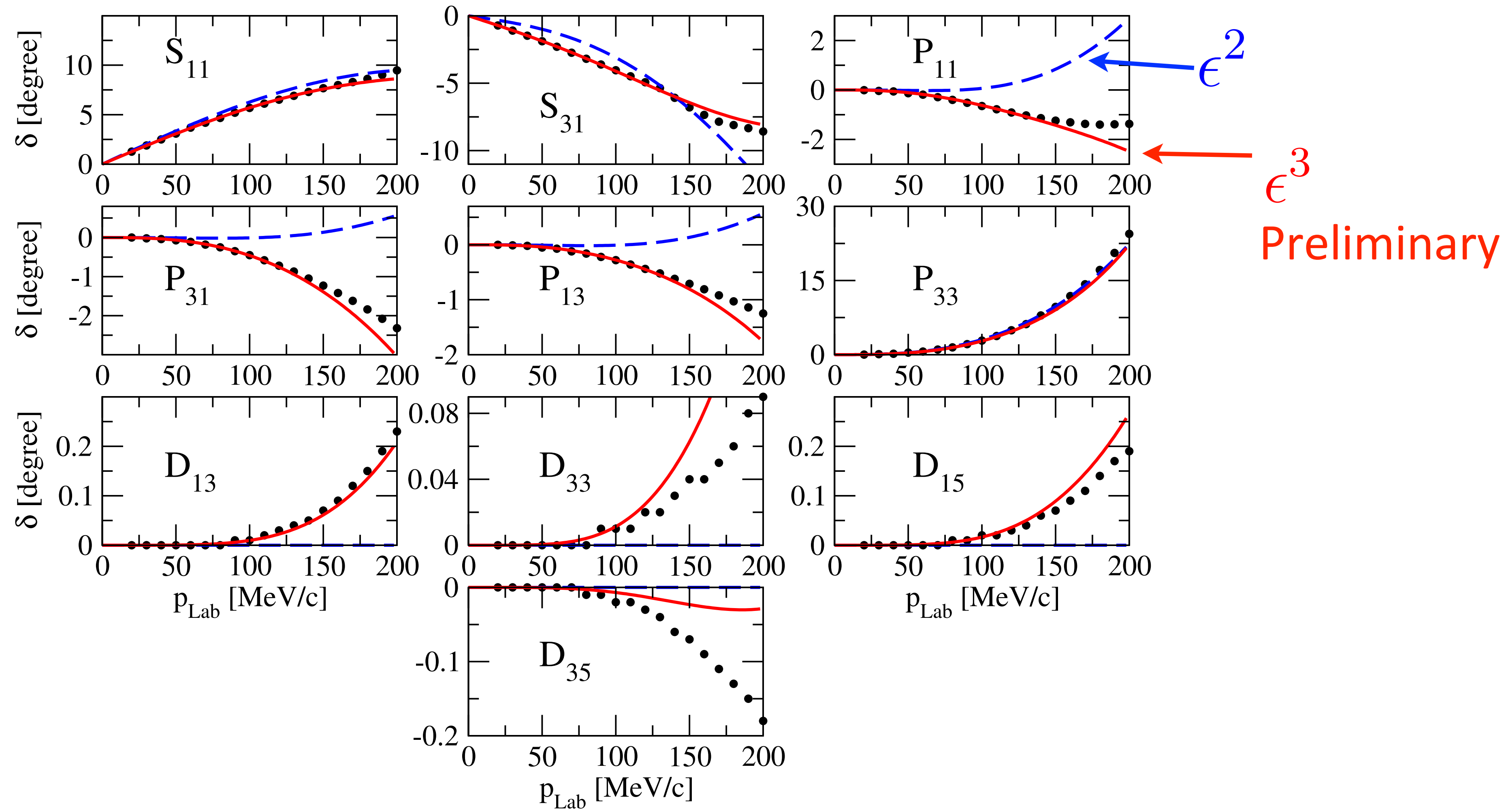


- no effect up to N^{2}LO (modulo reshuffling)
- expect large contributions to the ring & 2π-1π-topologies saturating some of the N^{4,5,6}LO graphs in the Δ-less theory
- What is more efficient: Δ-less N^{4}LO (and beyond?) vs Δ-full N^{3}LO ??

Pion-nucleon scattering

Heavy baryon SSE calculation up to ε^3 : *Fettes & Meißner ´01; Epelbaum, Gasparyan, HK, in preparation*

Karlsruhe-Helsinki (KH) PWA: R. Koch Nucl. Phys. A 448 (1986) 707



N^{3}LO-Δ

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$
GW-fit	−1.70	1.19	−3.52	1.85	0.10	−1.26	0.71	−1.17
KH-fit	−1.41	1.40	−3.43	1.80	0.45	−2.36	1.43	−2.18

Δ-less N^{4}LO

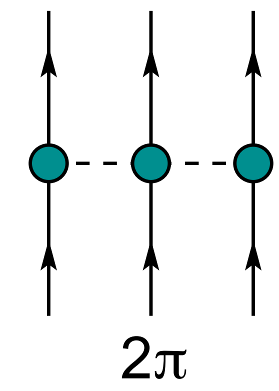
	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
GW-fit	−1.13	3.69	−5.51	3.71	5.57	−5.35	0.02	−10.26	1.75	−5.80	1.76	−0.58	0.96
KH-fit	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−10.41	6.08	−0.37	3.26

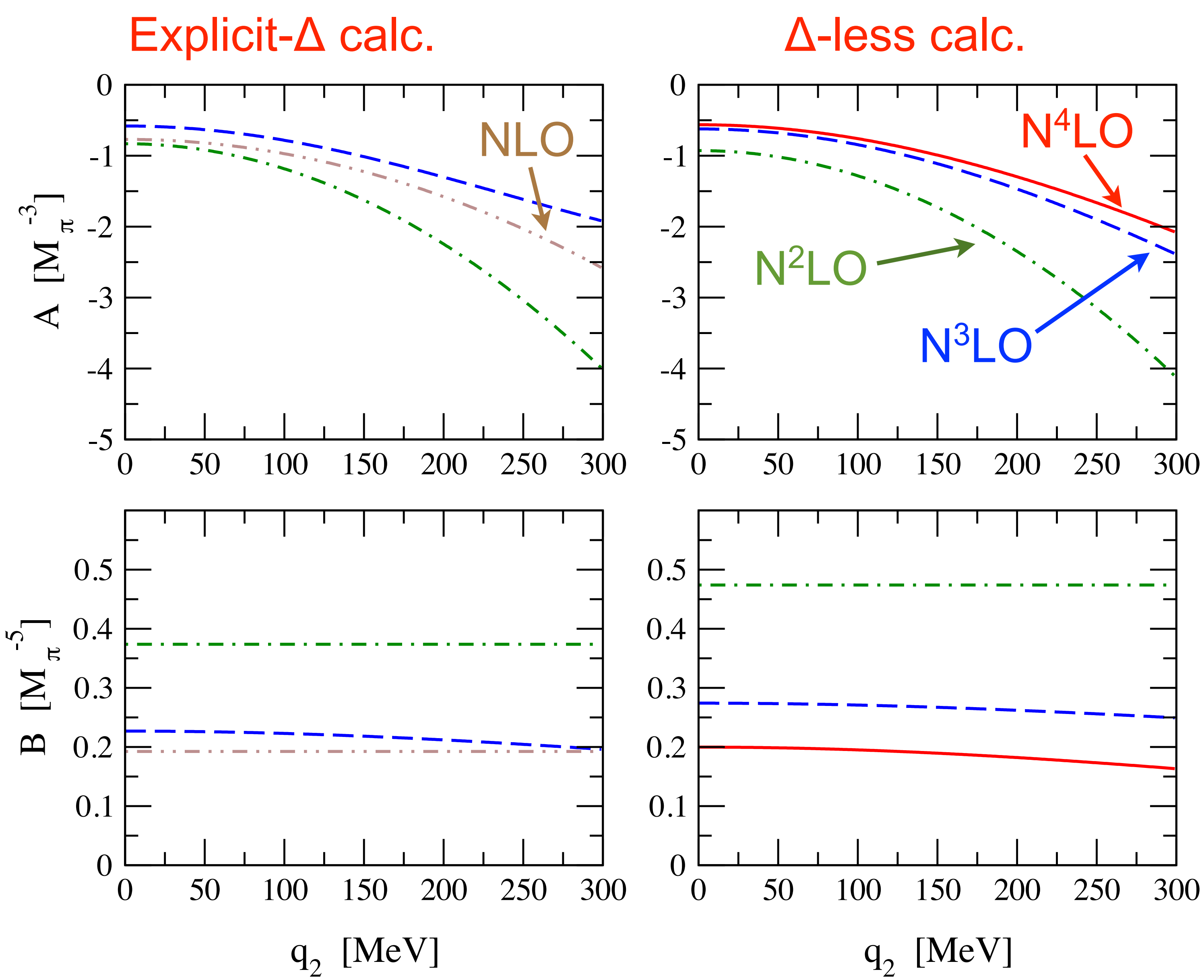
Two-pion-exchange 3NF

Preliminary

Epelbaum, Gasparyan, HK. forthcoming

$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$



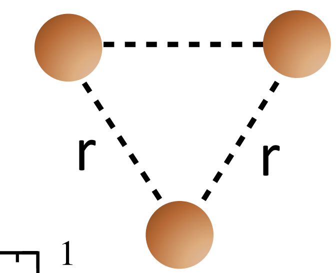


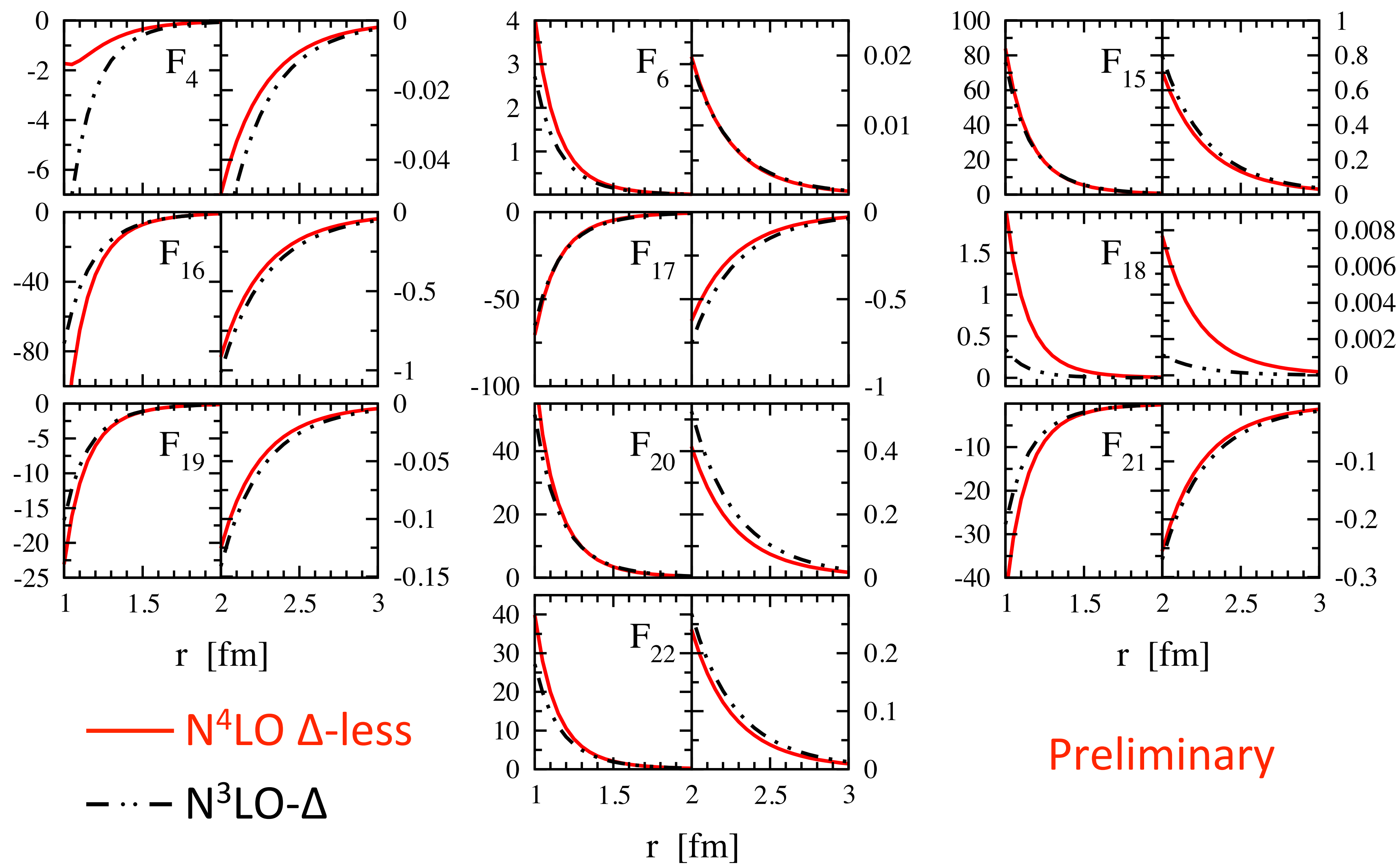
- Similar results for TPE-3NF in N^{3}LO-Δ and N^{4}LO Δ-less approaches
- We expect small explicit-Δ N^{4}LO contributions to two-pion-exchange 3NF

Two-pion-exchange up to N^{4}LO

Epelbaum, Gasparyan, HK, in preparation

Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration



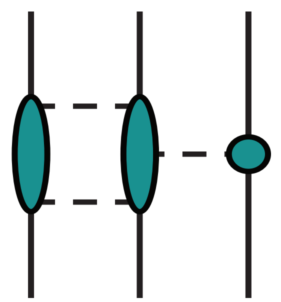


Preliminary

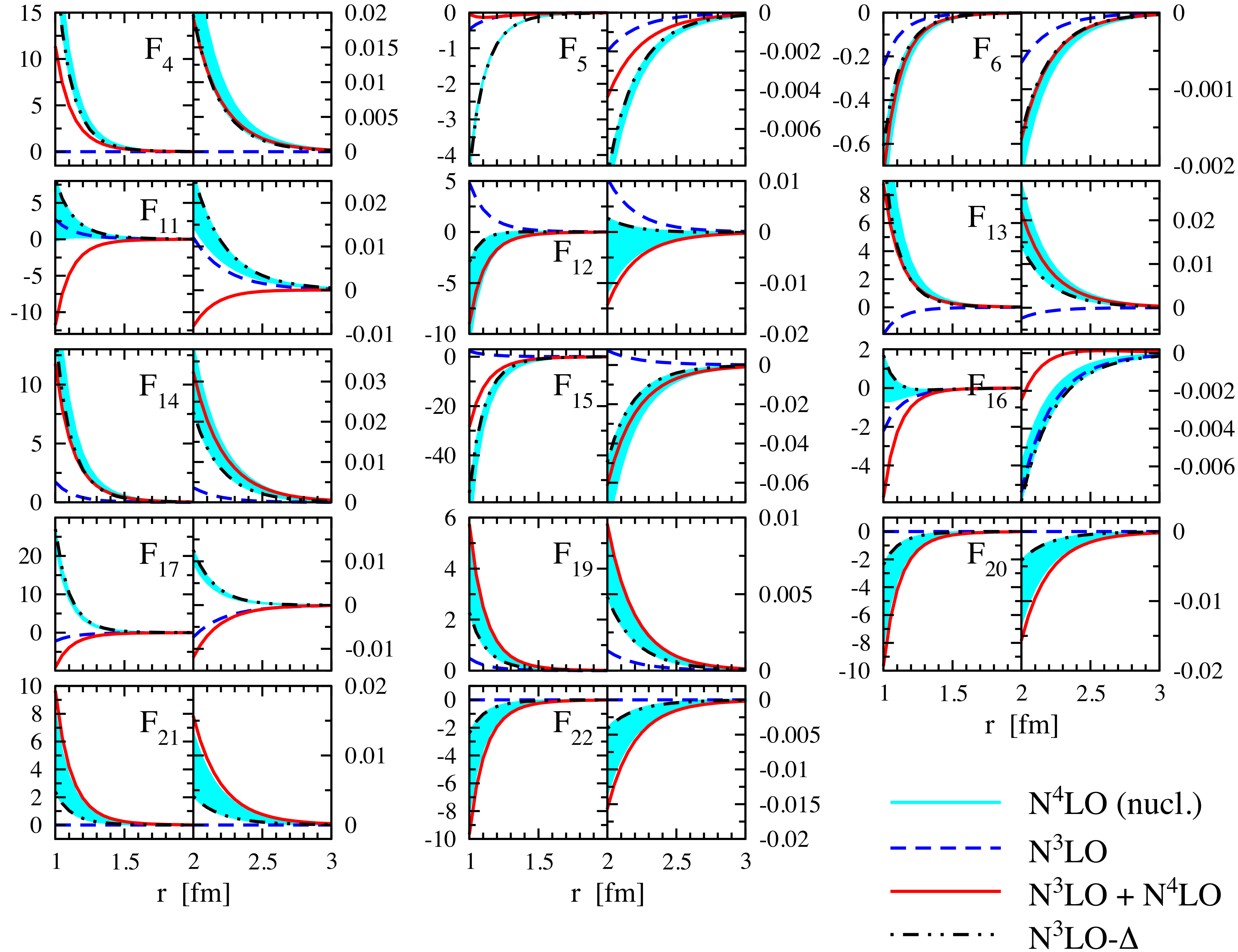
Δ-less and Δ-full approaches for TPE-force compared

- similar results if contributions are sizeable
- slightly different results if contributions are smaller

Two-pion-one-pion-exchange 3NF

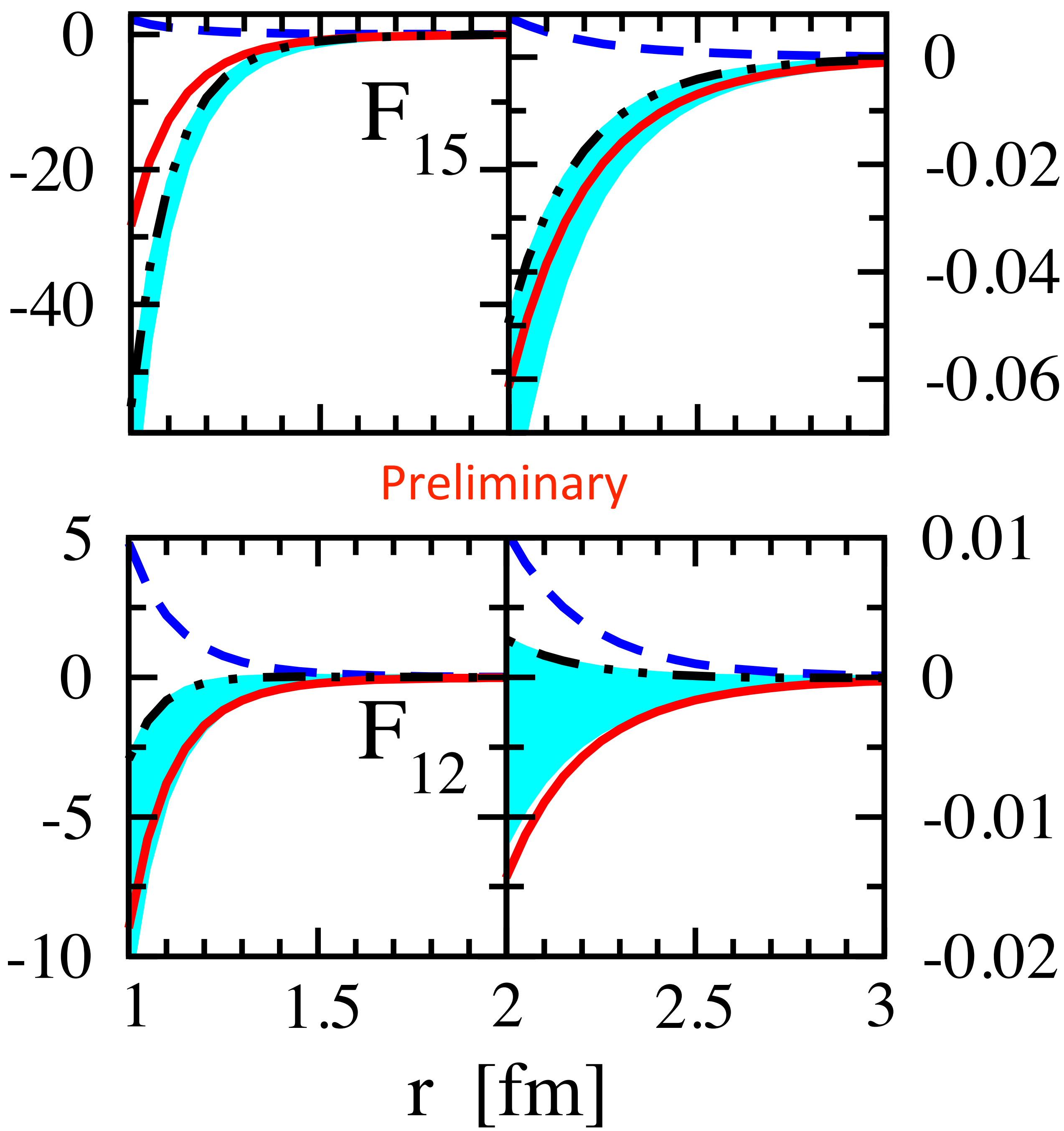


Preliminary



Bands indicate physics which is not described by explicit Δ-contributions

Two-pion-one-pion-exchange 3NF



- Most sizable contribution
- Δ-less/full results are similar
- Band is narrow

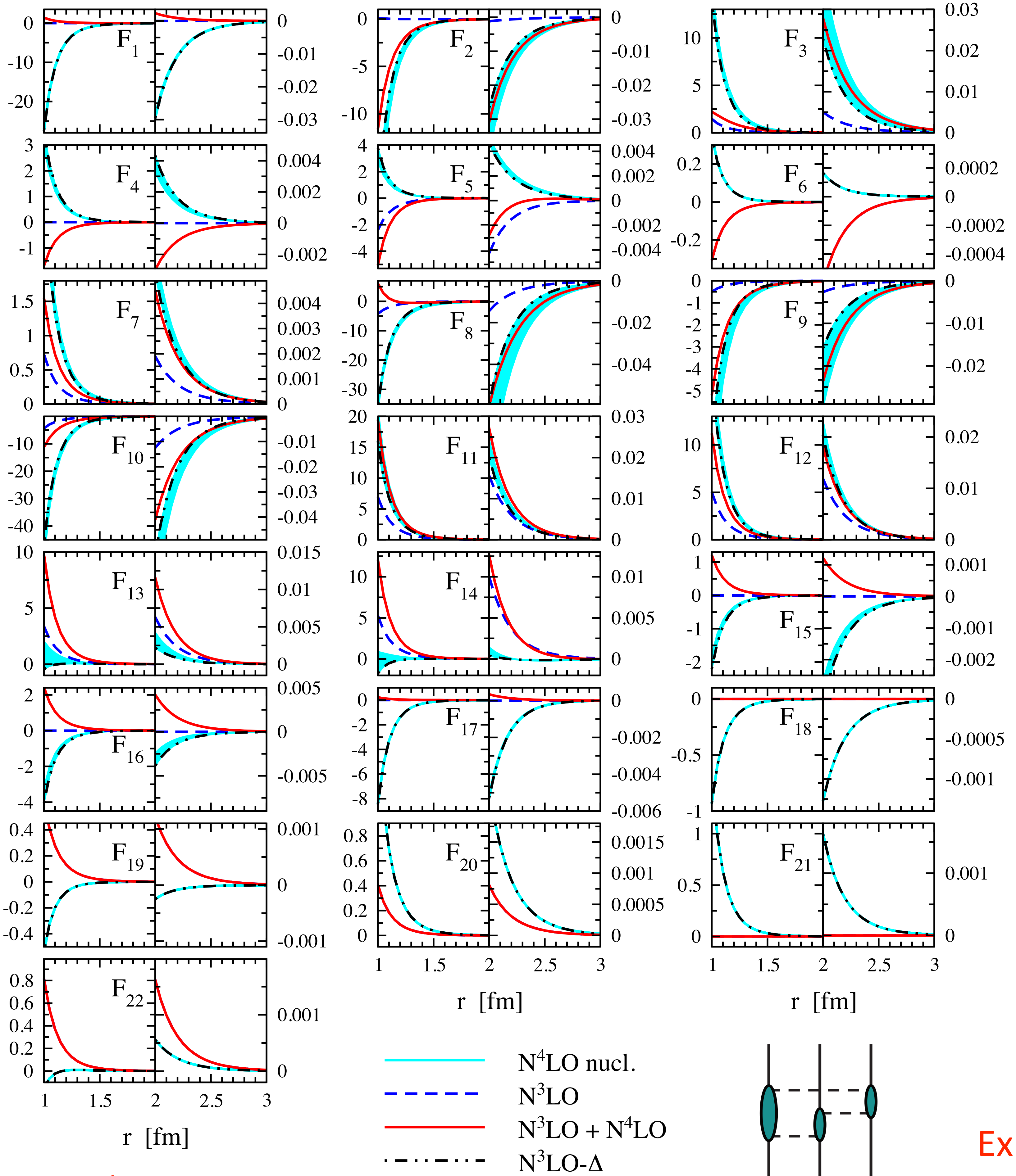
N^4LO Effects beyond Δ-contr. are small

- Small contribution
- Δ-less/full results differ
- Band is broad

N^4LO Effects beyond Δ-contr. are important

- N^3LO nucleon-contributions are of smaller size
- Dominant effects come from N^3LO Δ-/N^4LO-contr. in Δ-full/Δ-less approach

Ring - 3NFs



Preliminary

- Narrow bands
  - ⇨ Higher order contributions beyond Δ are small
- Strong central isoscalar 3NF due to double-Δ excitation

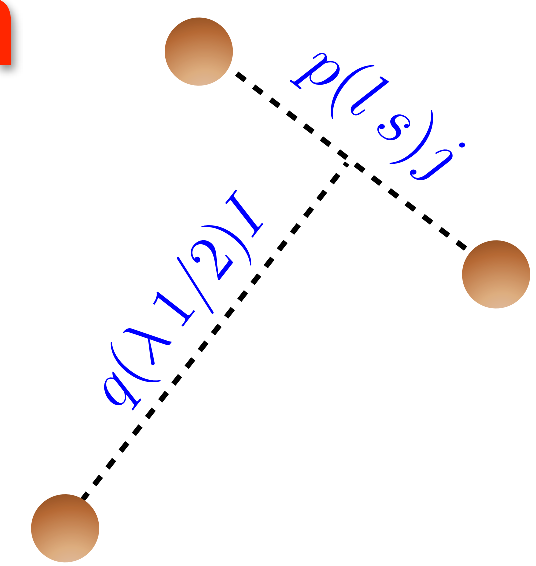
Two different cases:

1) Δ-resonance saturation contribution to a given F_i is sizable
   - ⇨ N3LO-Δ and N4LO-Δ-less results are similar
2) Δ-resonance saturation contribution to a given F_i is negligible
   - ⇨ N3LO-Δ and N4LO-Δ-less results deviate

Explicit-Δ approach is more efficient!

Partial wave decomposition

Golak et al. Eur. Phys. J. A 43 (2010) 241



- Faddeev equation is solved in the partial wave basis

$$|p,q,\alpha\rangle \equiv |pq(ls)j(\lambda\tfrac{1}{2})I(jI)JM_J\rangle\,|(t\tfrac{1}{2})TM_T\rangle$$

- Too many terms for doing PWD by hand ⟹ Automatization

$$\underbrace{\langle p'q'\alpha'|V|pq\alpha\rangle}_{\text{matrix} \sim 10^5 \times 10^5} = \int \underbrace{d\hat{p}'\, d\hat{q}'\, d\hat{p}\, d\hat{q}}_{\text{can be reduced to 5 dim. integral}} \sum_{m_l,\ldots} \Big(\text{CG coeffs.}\Big)\Big(Y_{l,m_l}(\hat{p})\, Y_{l',m_l'}(\hat{p}')\ \ldots\Big)\underbrace{\langle m'_{s_1} m'_{s_2} m'_{s_3}|V|m_{s_1} m_{s_2} m_{s_3}\rangle}_{\text{depends on spin \& isospin}}$$

- Ring-diagram-contr. expensive to calculate on the fly

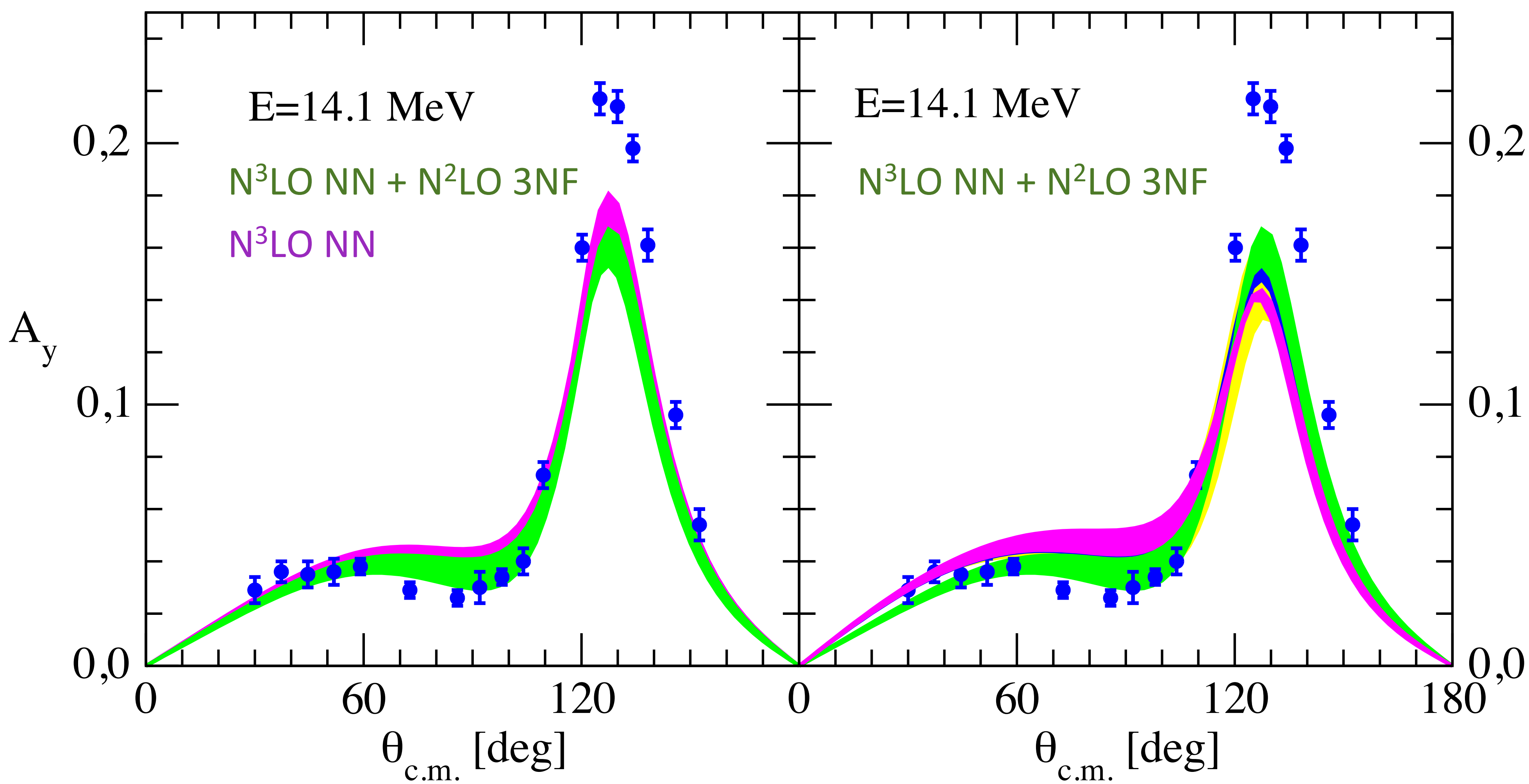
We prestore ring-contr. to 3nf's on a fine momentum grid ⟹ Numerical interpolation of ring terms

- PWD matrix-elements can be used to produce matrix-elements in harmonic oscillator basis
  see talk by Kai Hebeler & Angelo Calci

Straightforward implementation of high order 3nf's in many-body calc.
within No-Core Shell Model

A_y-puzzle in elastic nd scattering

Witala et al. Proceedings of Few Body 20



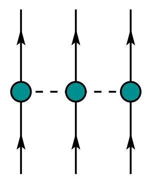
Right panel: X = N³LO NN + N²LO 3NF + N³LO 3NF (1π-cont.) + N³LO 3NF (cont.)

- (yellow) = X + N³LO 3NF (2π-exch.)
- (blue) = X + N³LO 3NF (2π-exch. & 2π-1π-exch.)
- (magenta) = X + N³LO 3NF (2π-exch. & 2π-1π-exch. & ring)

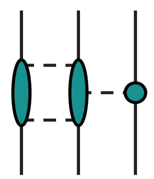
Incomplete results: N³LO 3NF (2π-cont. & 1/m-corr.) are missing

Summary

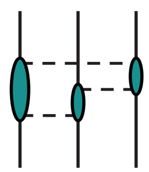
- Long-range part of 3NFs is analyzed up to N^4LO Δ-less/N^3LO-Δ



- Chiral expansion of TPE-3NF seems to be converged
- TPE-3NF dominates 3NF but does not fill all 22 structures



- Sizeable contr. are similar for 2π-1π-3NF in N^4LO Δ-less and N^3LO-Δ approach
- Dominant effects come from N^4LO-/N^3LO Δ-contr. in Δ-less/Δ-full approach



- Ring-3NFs fill all 22 structures
- N^4LO-/N^3LO Δ-contr. in Δ-less/Δ-full approach dominate N^3LO-nucleon contr.
- Some missing sizeable Δ-contr. in N^4LO results like central attractive force ~$O(1/\Delta^2)$

- First (incomplete) results for A_y in nd elastic scattering with N^3LO 3NF's

Outlook

- Partial wave decomposition of N^3LO three-nucleon forces
- N^4LO Δ-less/N^3LO-Δ calc. of shorter range part of 3NF
- N^4LO with explicit-Δ of long range part of 3NF (convergence-test)