Momentum-space evolution of 3N interactions and first applications

Kai Hebeler (OSU)

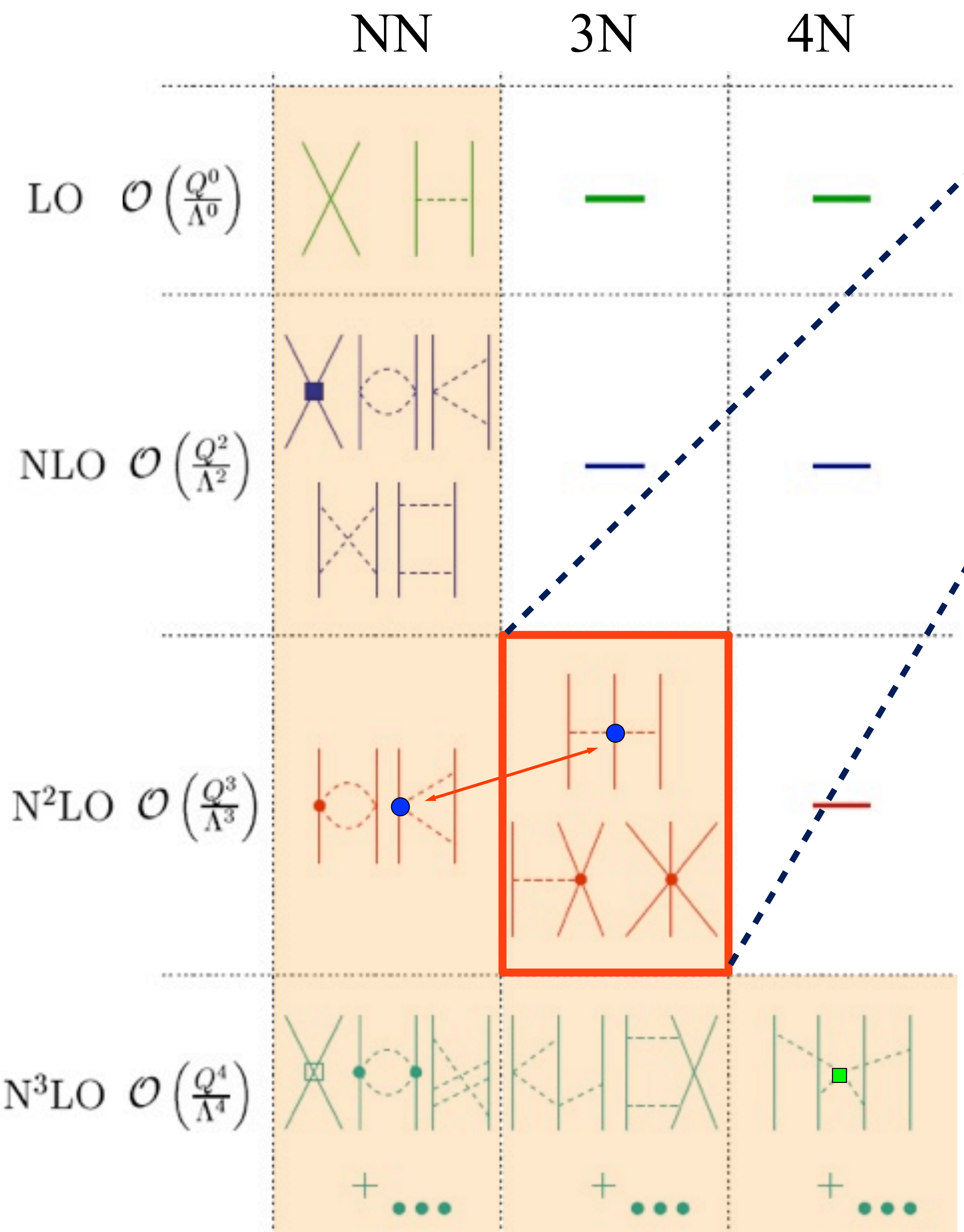
Computational and Theoretical Advances for Exotic Isotopes in the Medium Mass Region

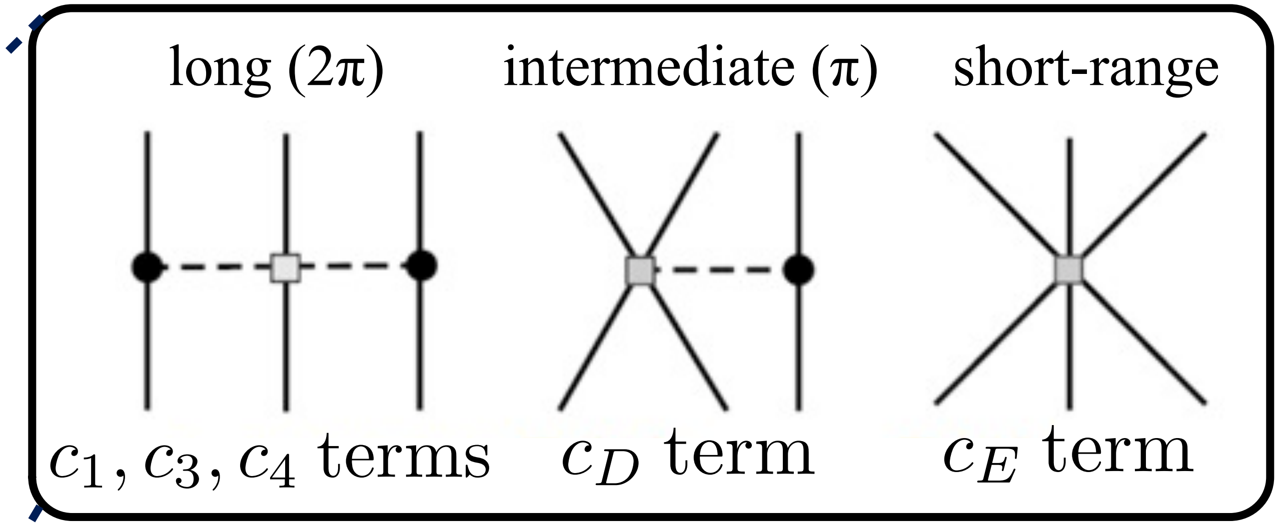
in collaboration with
S. K. Bogner, A. Ekstroem, R. J. Furnstahl, T. Krueger, J. Lattimer, A. Nogga, C. Pethick, A. Schwenk, I. Tews

Seattle, March 28, 2013

THE OHIO STATE UNIVERSITY

Chiral EFT for nuclear forces, leading order 3N forces





large uncertainties in coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\ ,\ c_3 = -4.7^{+1.5}_{-1.0}\ ,\ c_4 = 3.5^{+0.5}_{-0.2}$$

lead to theoretical uncertainties in many-body observables

use chiral interactions as input for RG evolution

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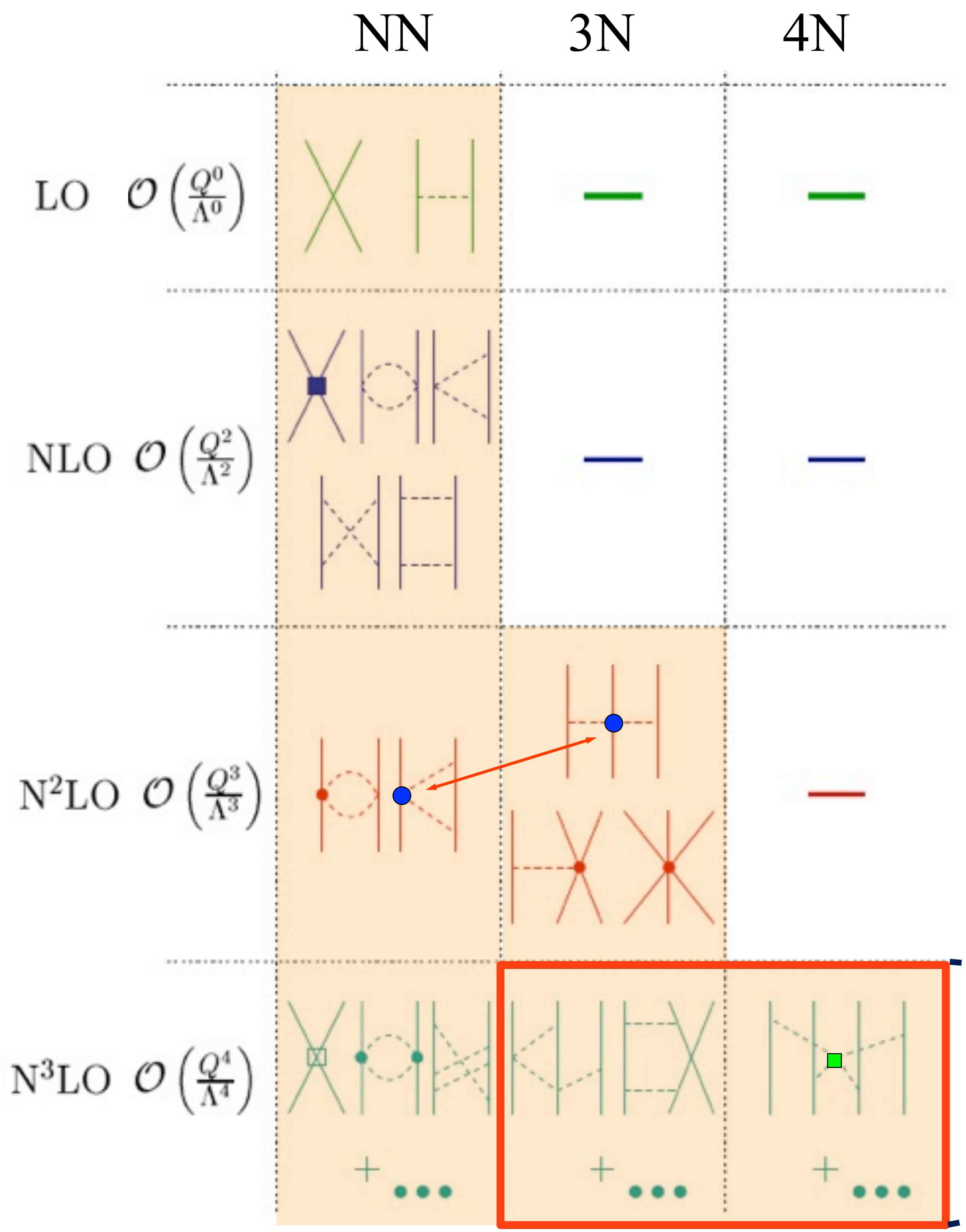

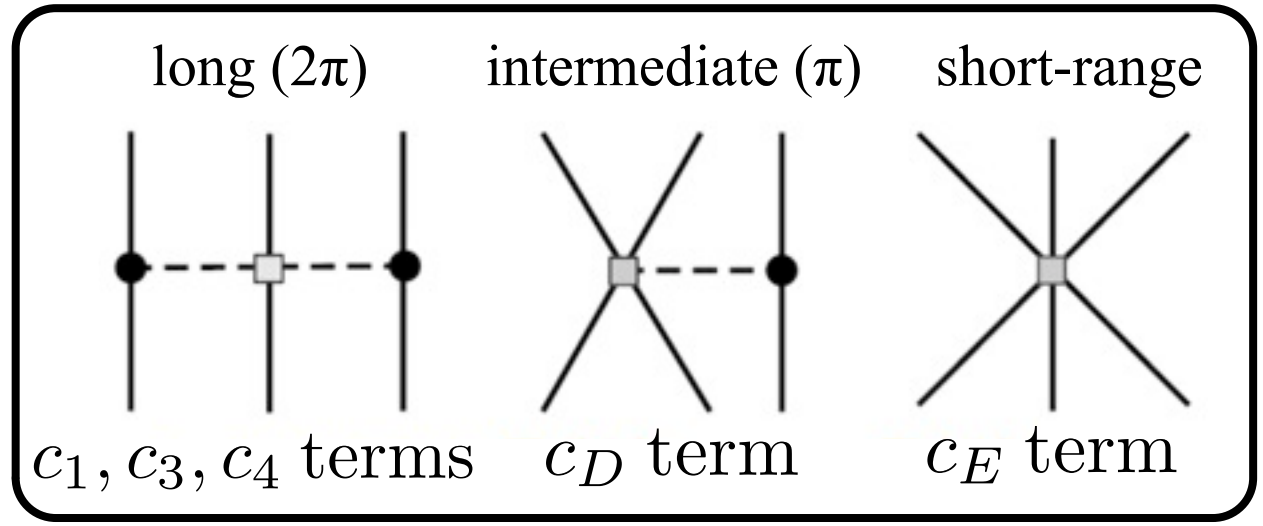

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first incorporation in calculations of neutron matter

Tews, Krueger, KH, Schwenk
PRL 110, 032504 (2013)

Changing the resolution: The (Similarity) Renormalization Group

- goal: generate unitary transformation of "hard" Hamiltonian

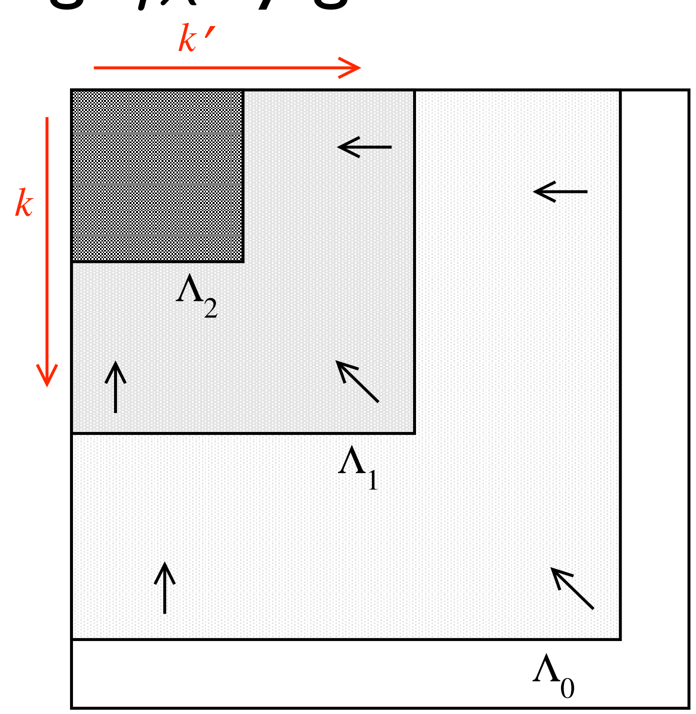
$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

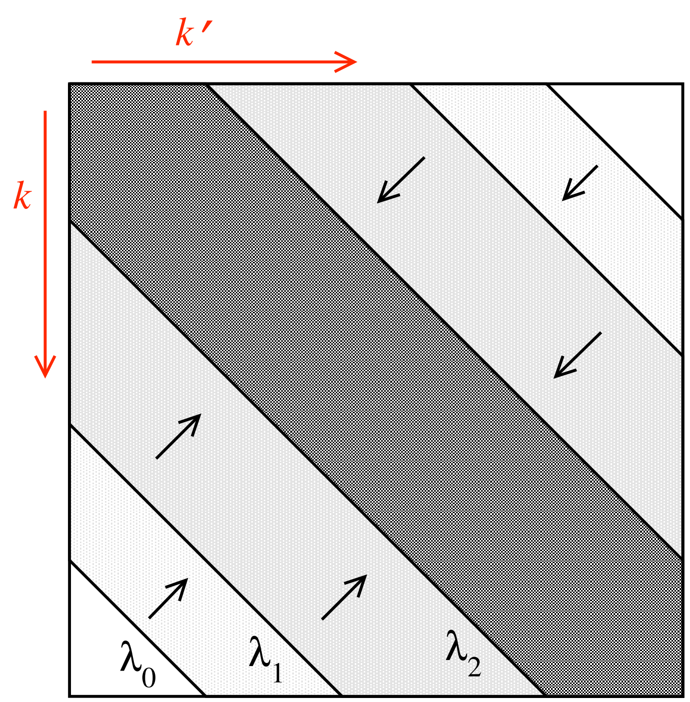
- change resolution in small steps: $$\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$$

- transformed wave functions and operators

$$|\psi_\lambda\rangle = U_\lambda |\psi\rangle \qquad O_\lambda = U_\lambda O U_\lambda^\dagger \quad \Rightarrow \quad \langle\psi| O |\psi\rangle = \langle\psi_\lambda| O_\lambda |\psi_\lambda\rangle$$

- specifying η_λ by generator G_λ: $\eta_\lambda = [G_\lambda, H_\lambda]$

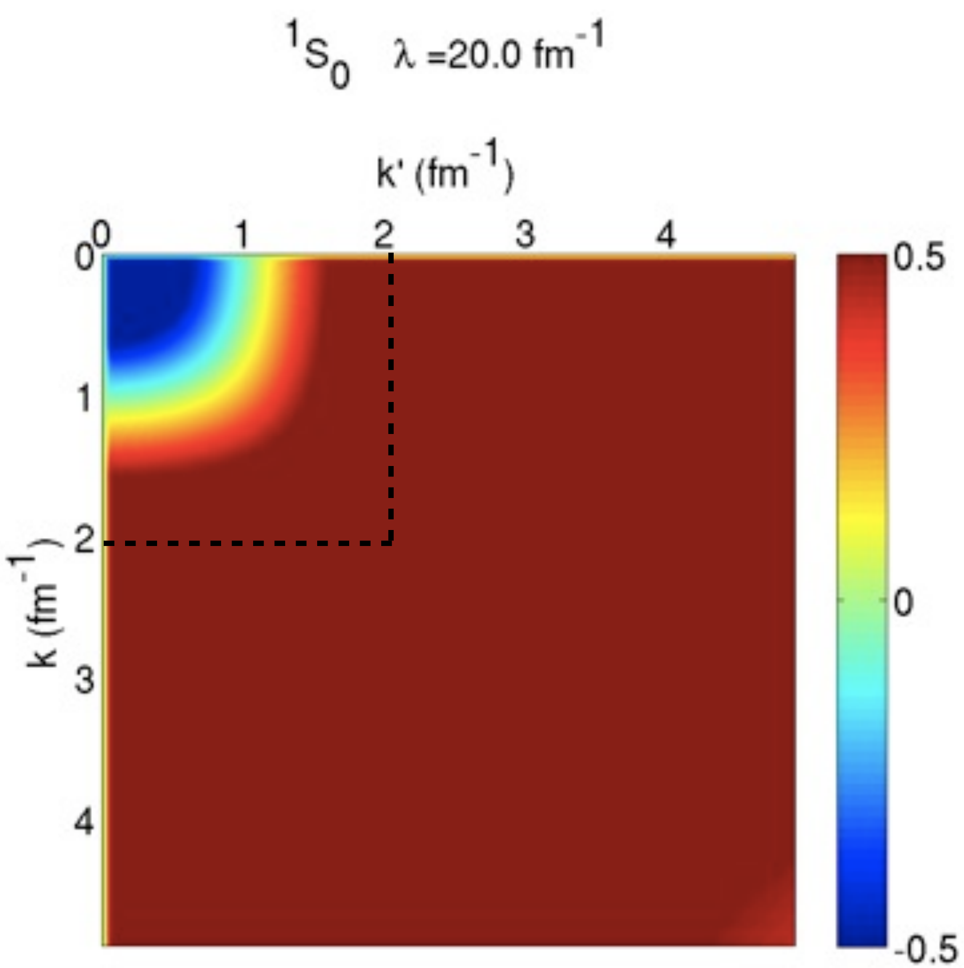


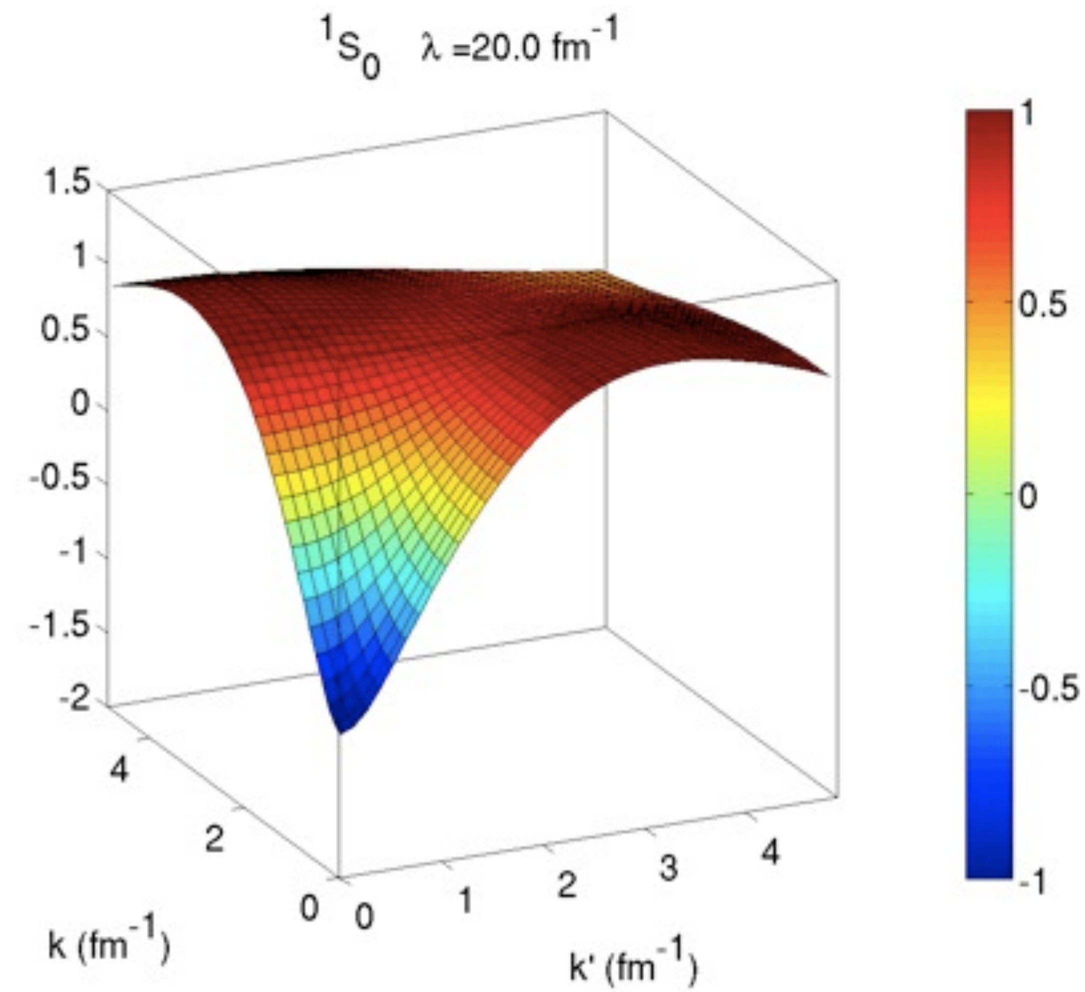


Changing the resolution: The (Similarity) Renormalization Group

- common choice for generator

  relative kinetic energy operator $G_\lambda = T$:





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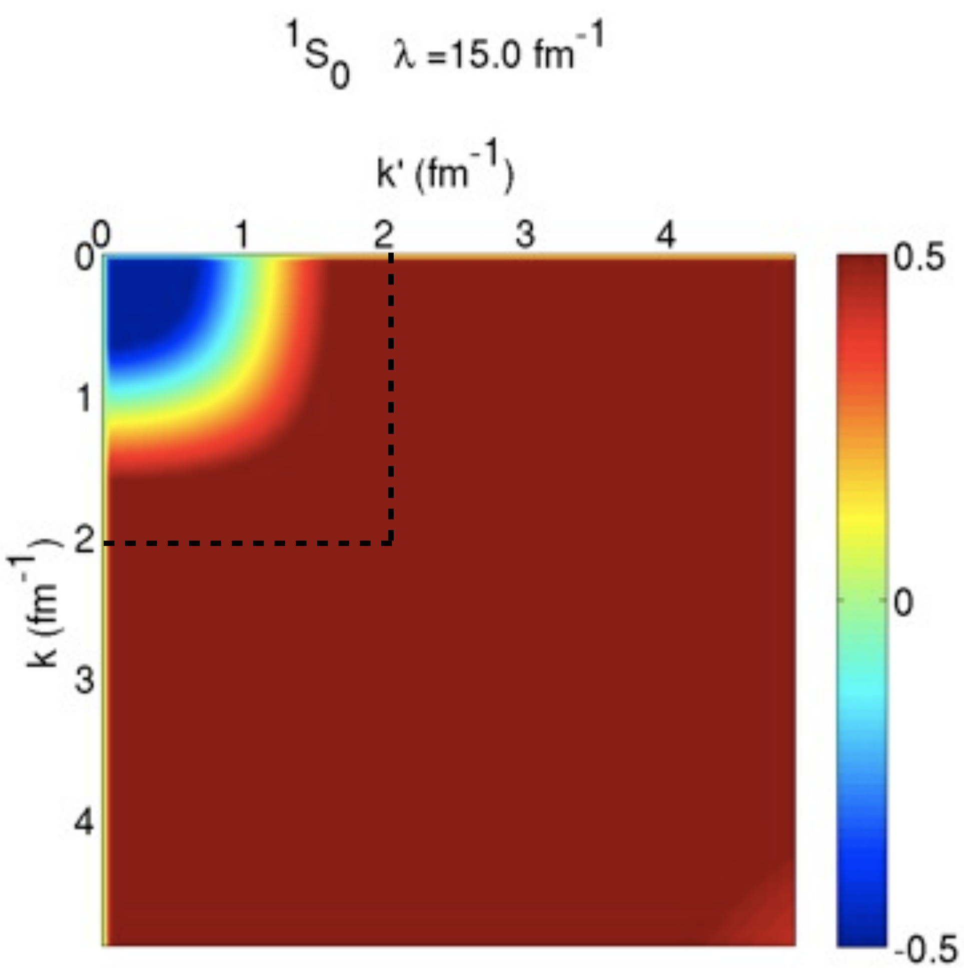

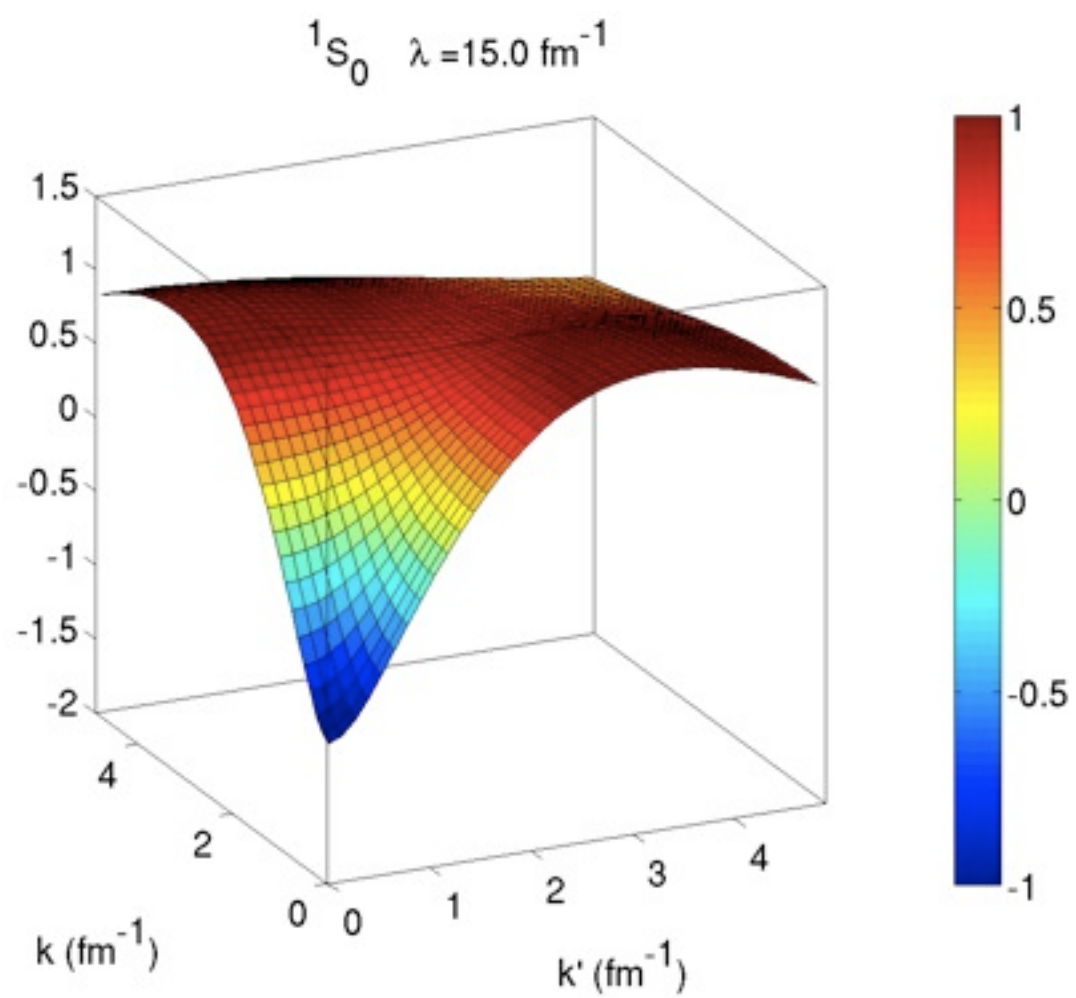

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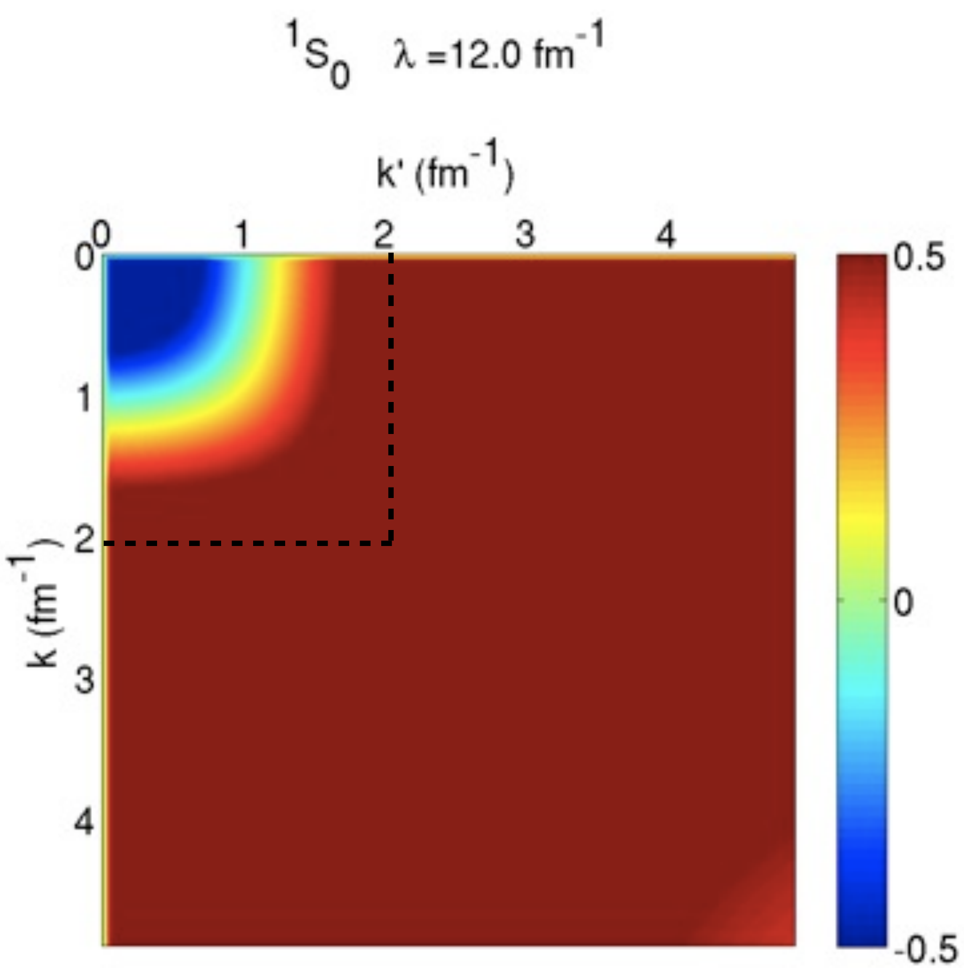

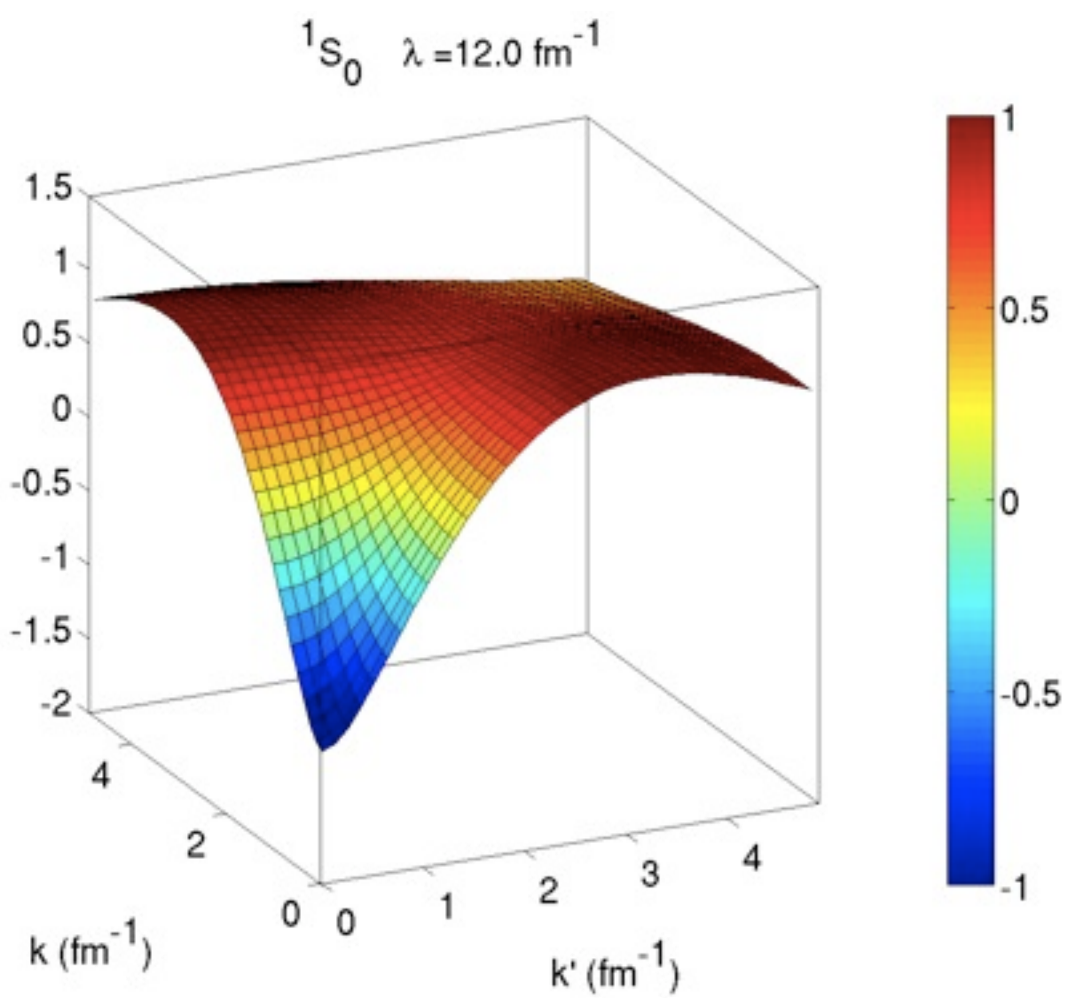

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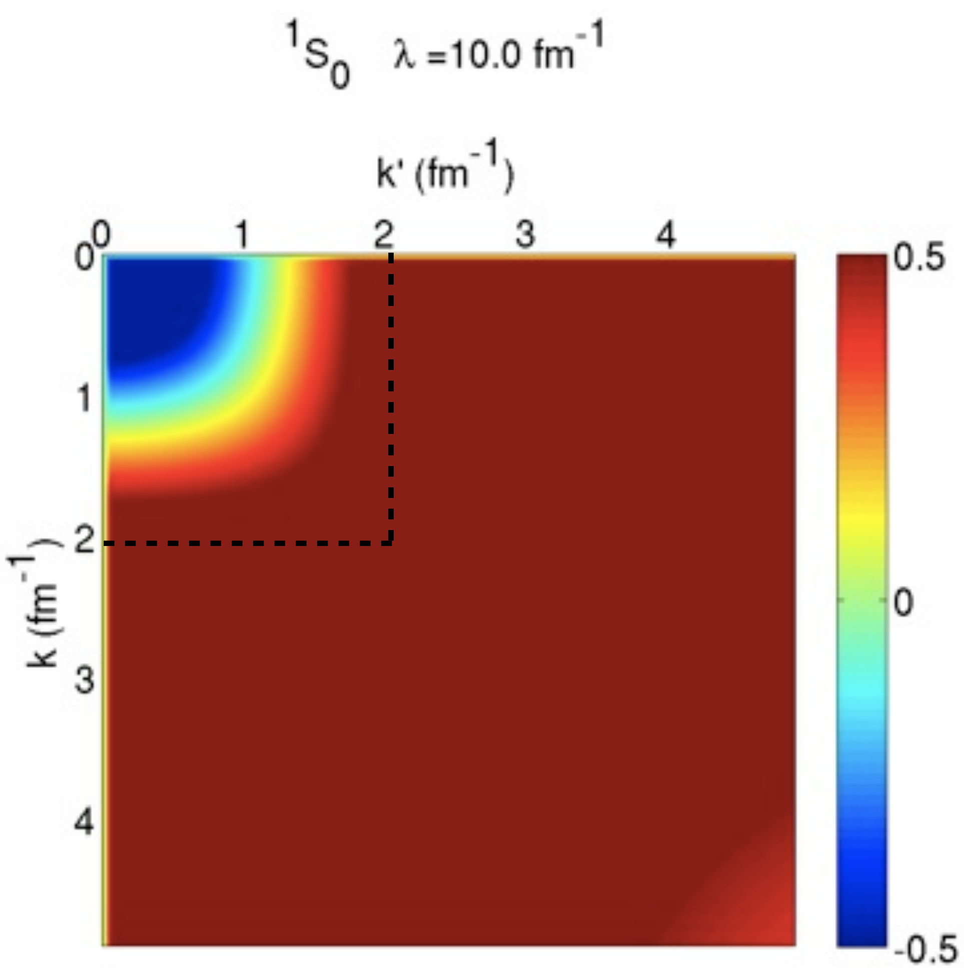

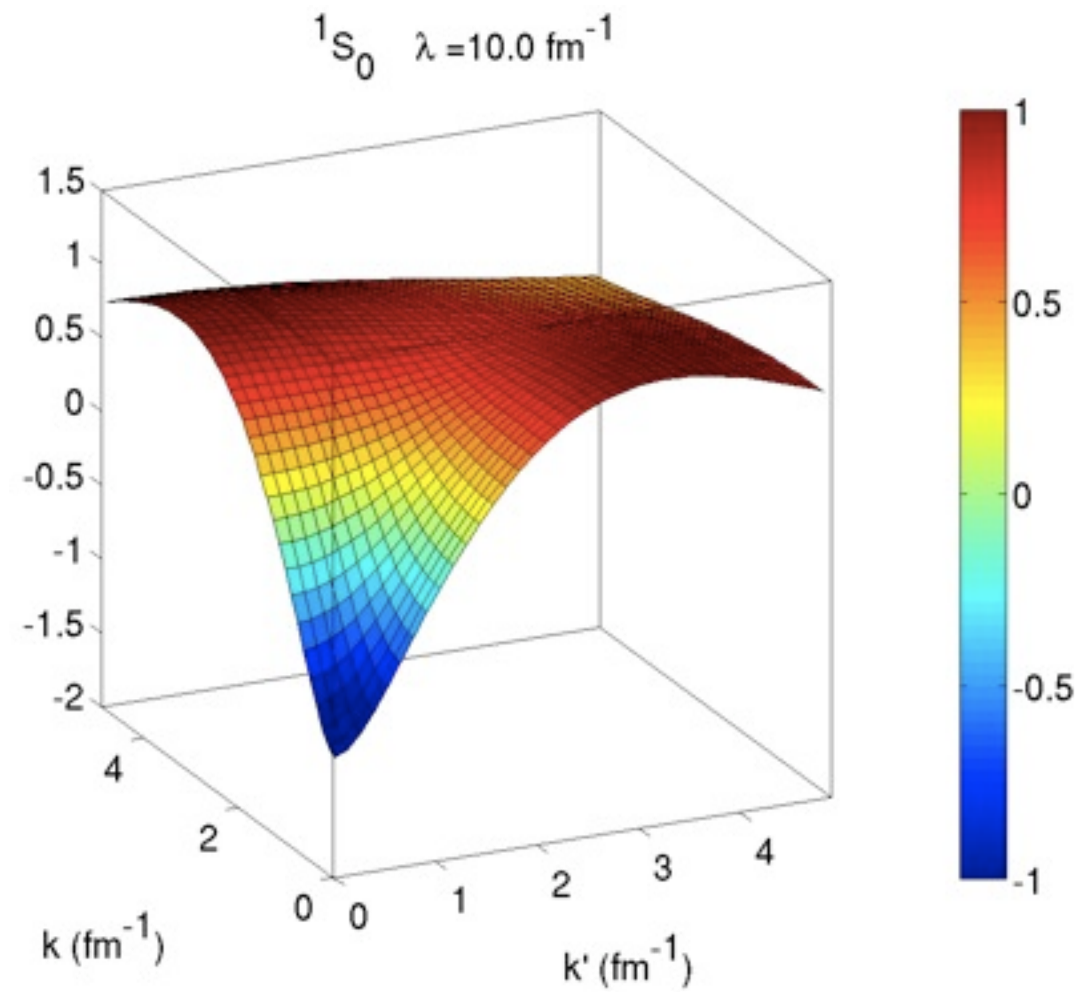

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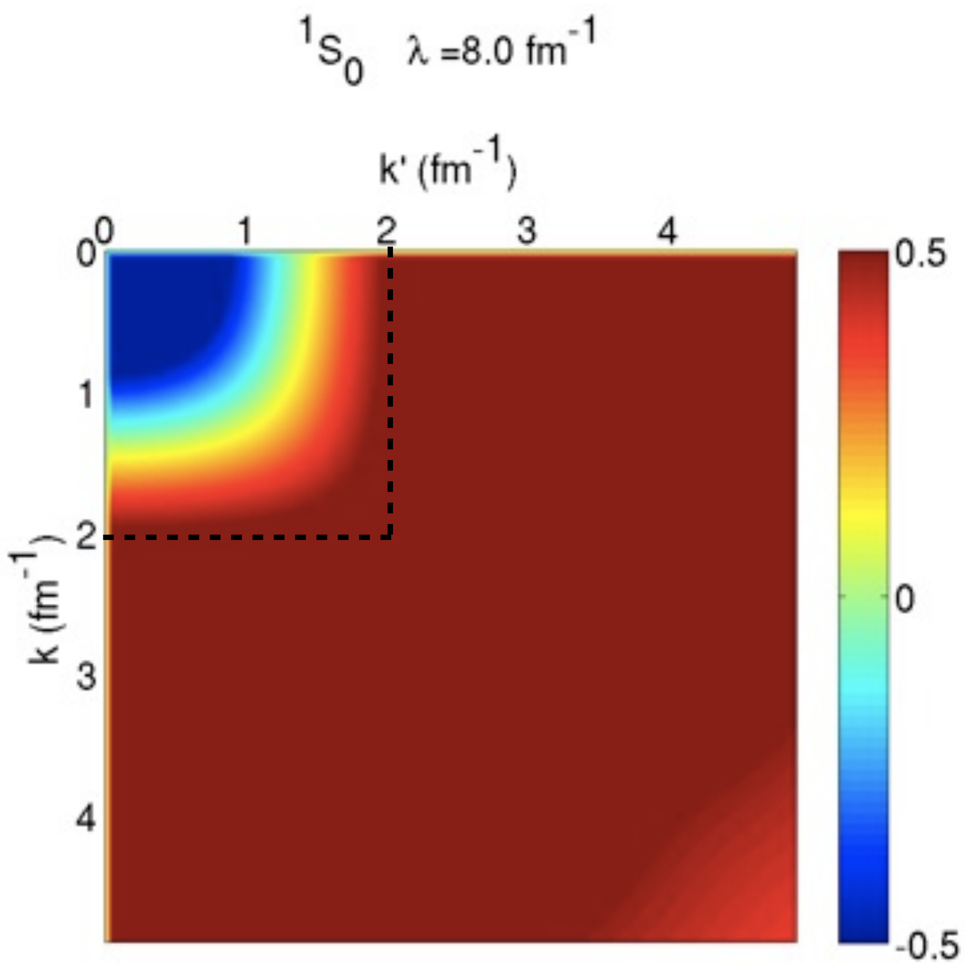

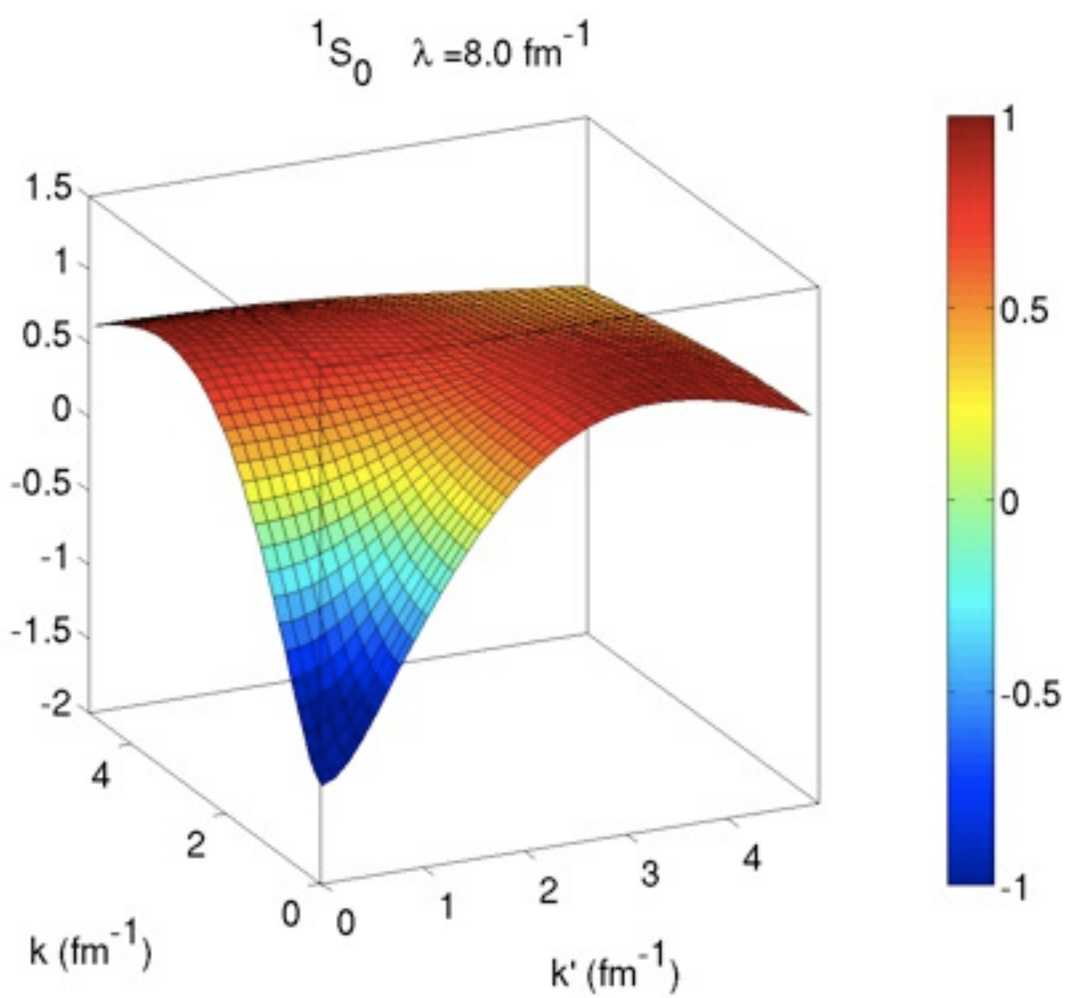

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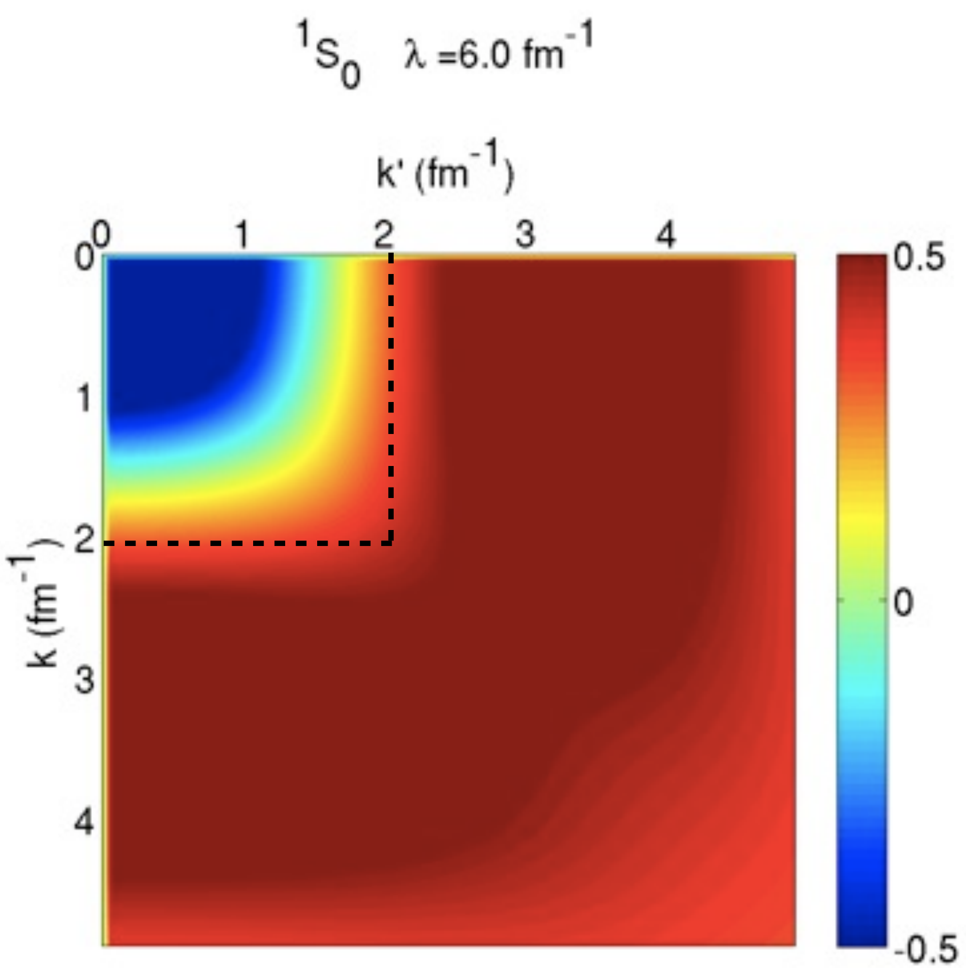

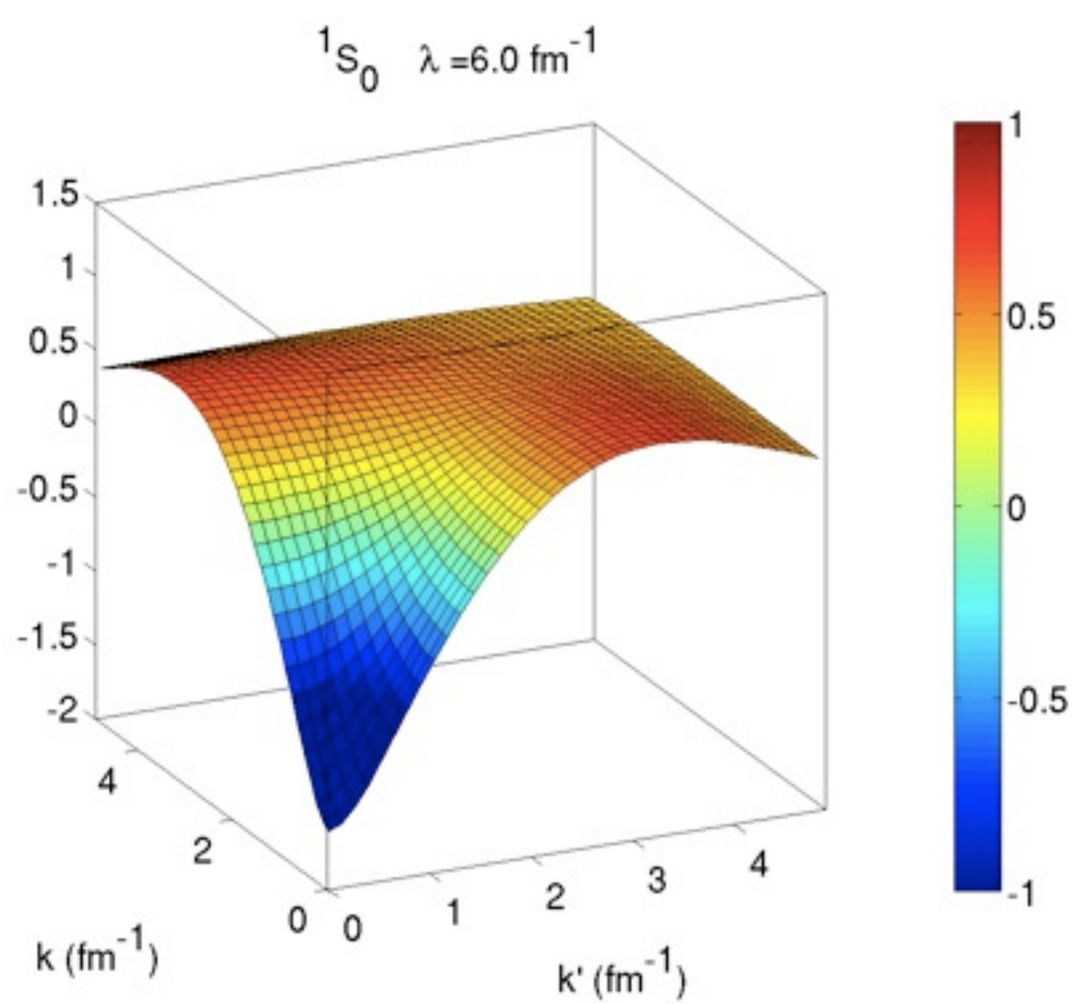

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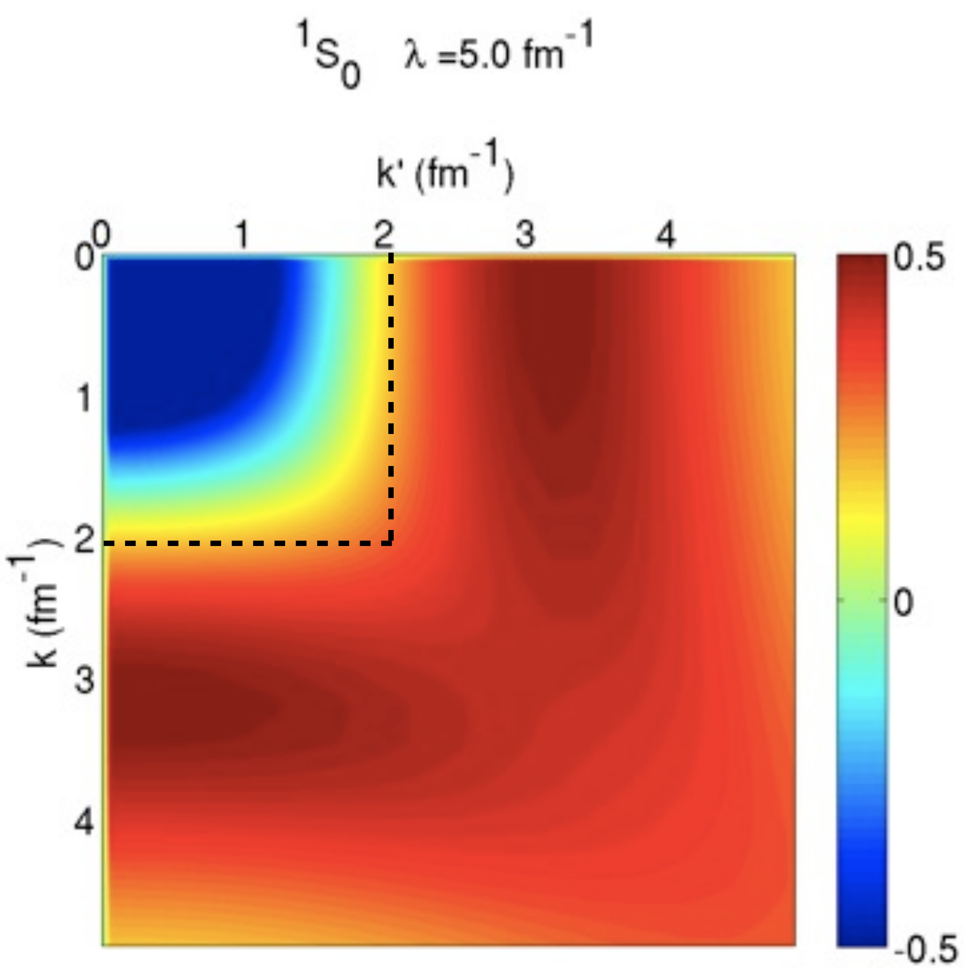

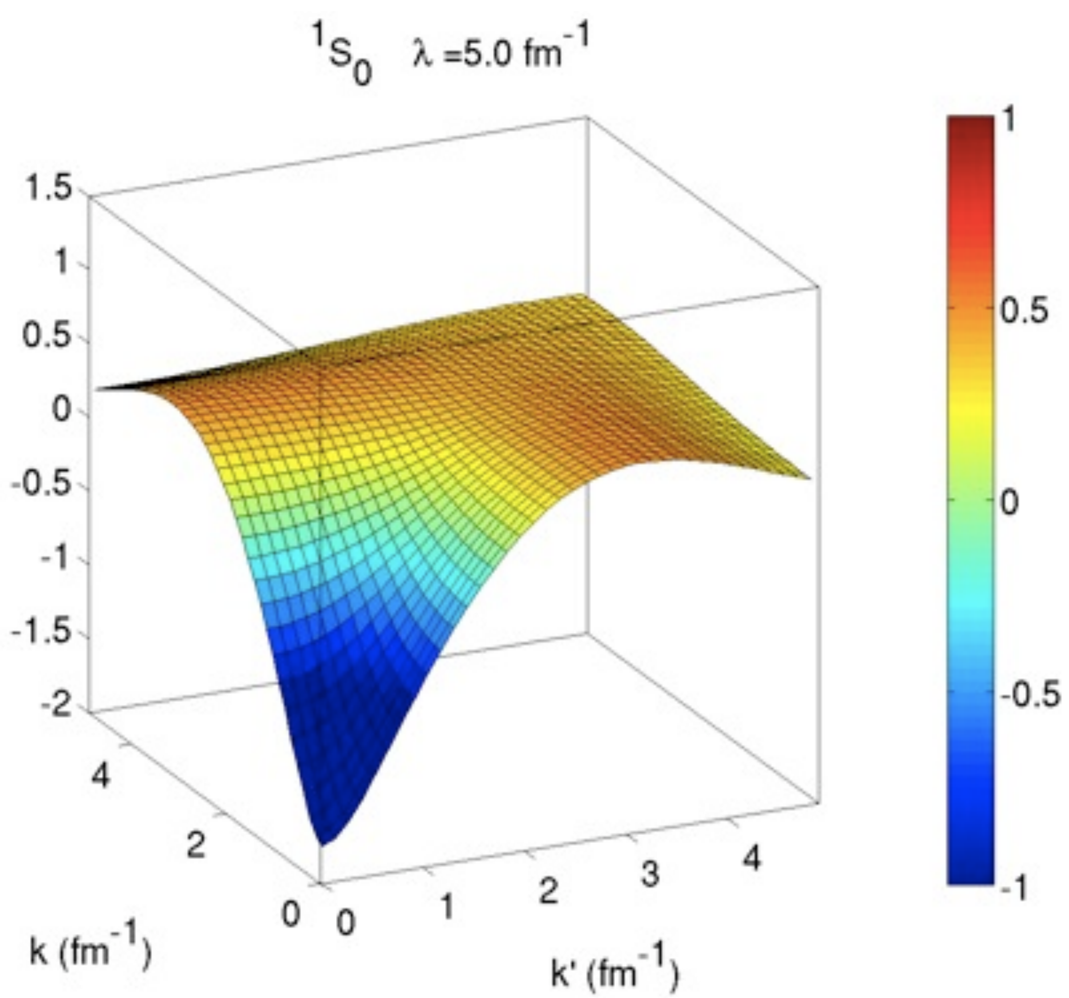

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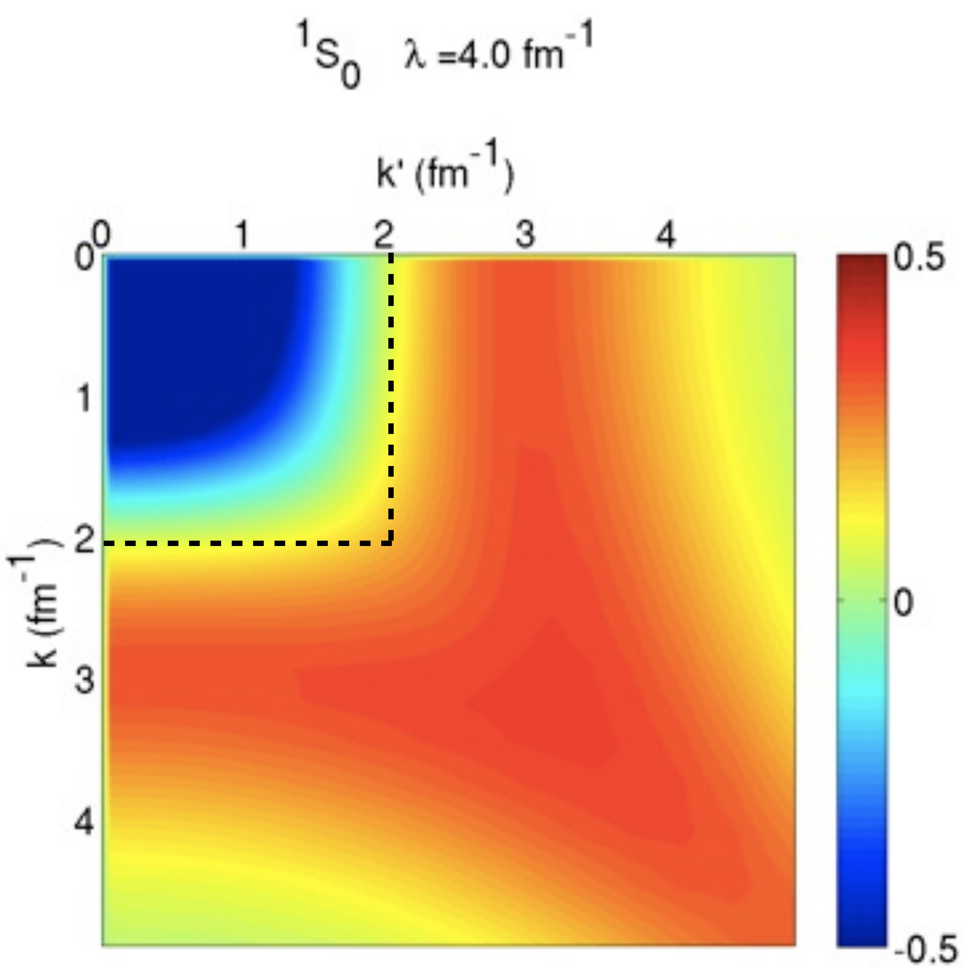

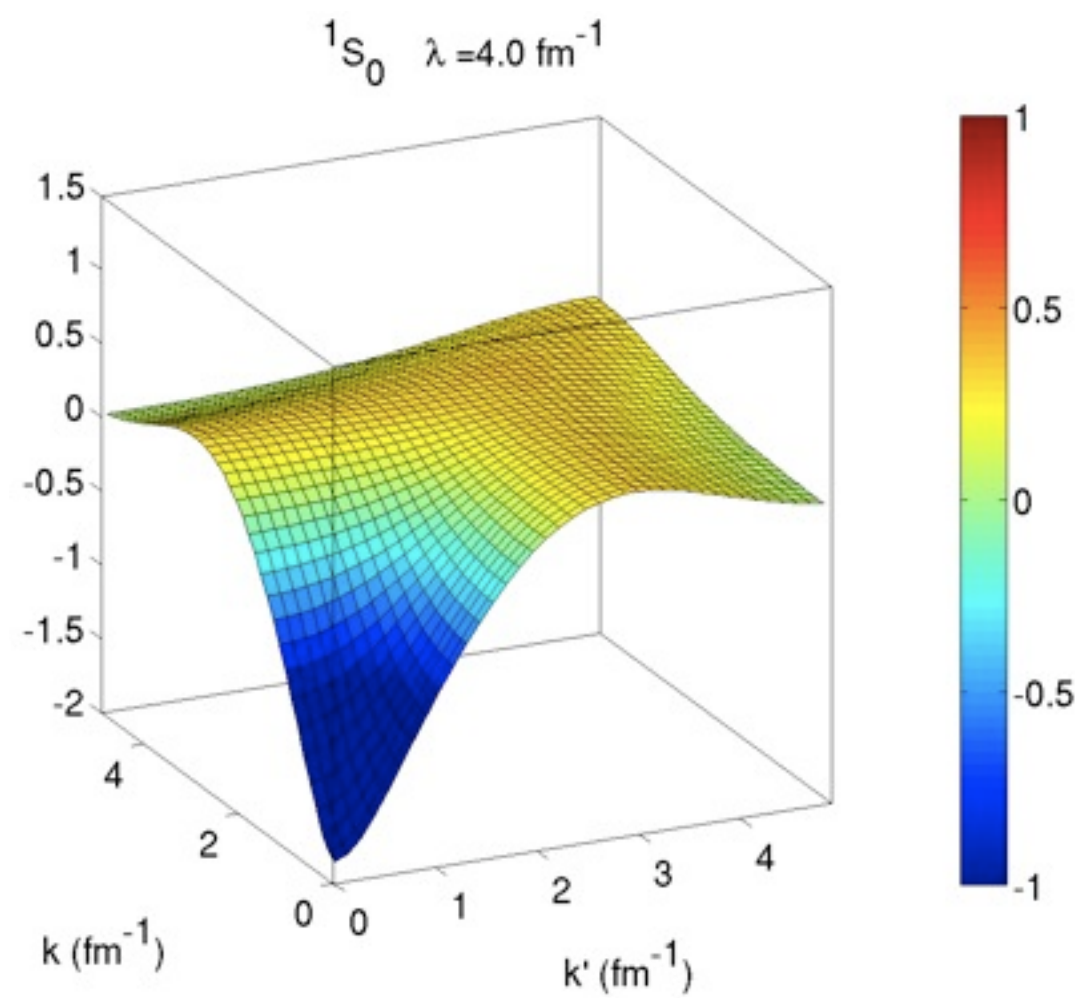

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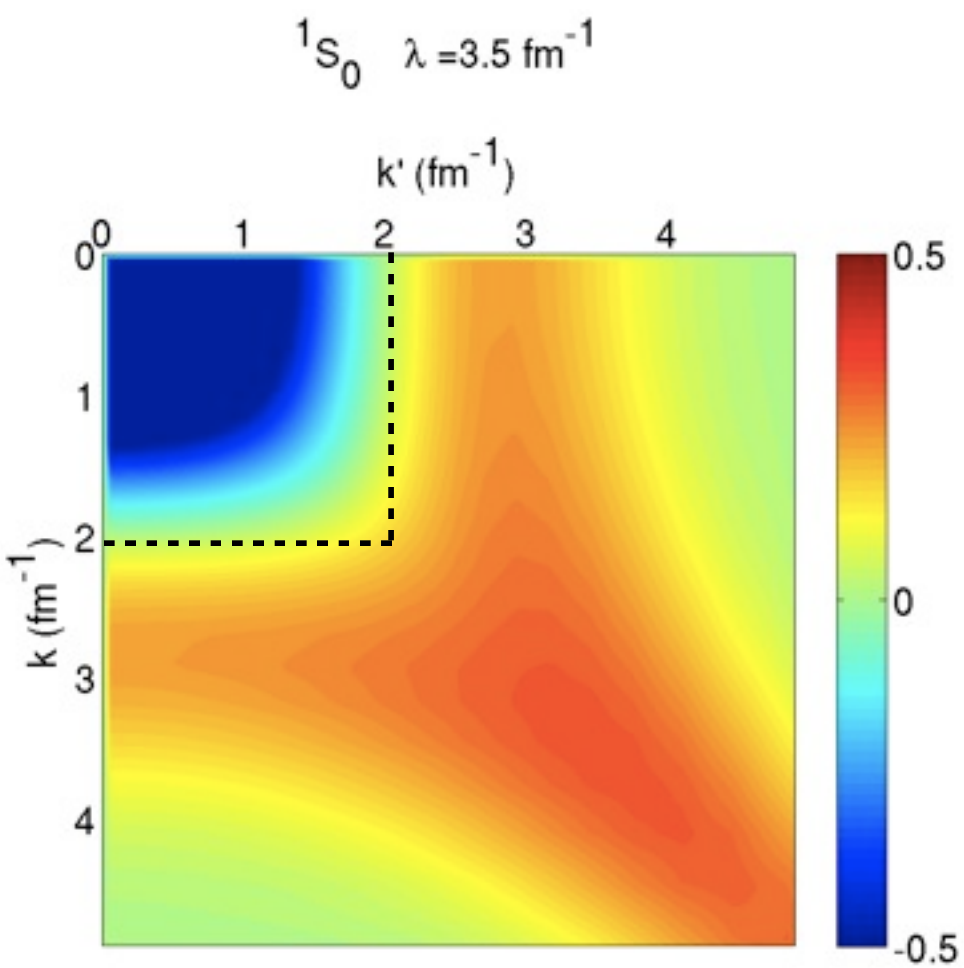

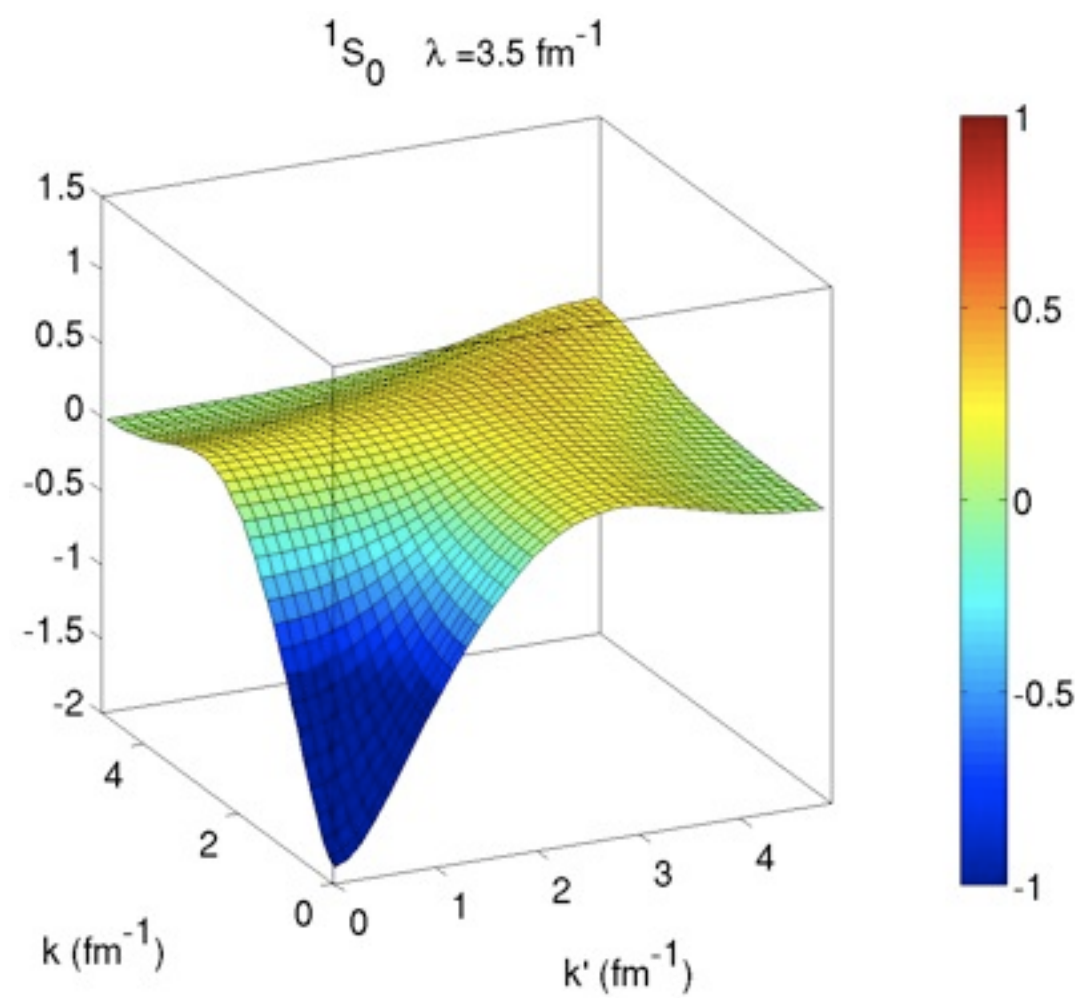

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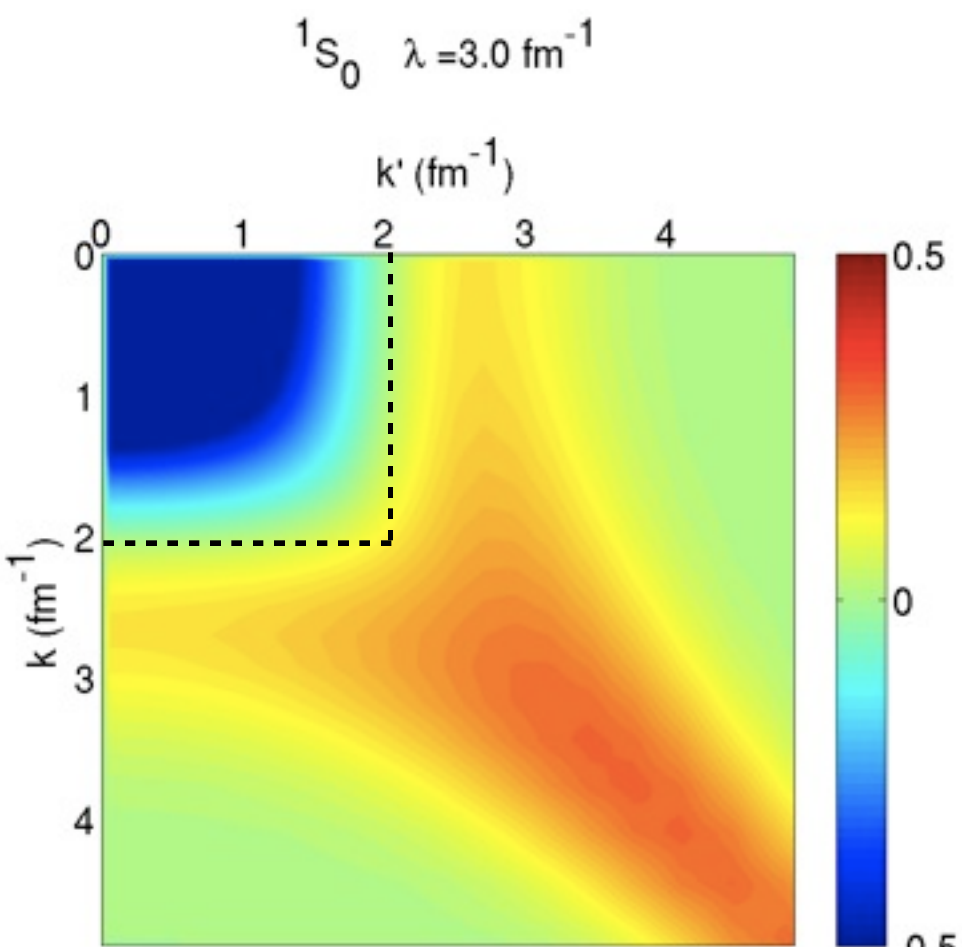

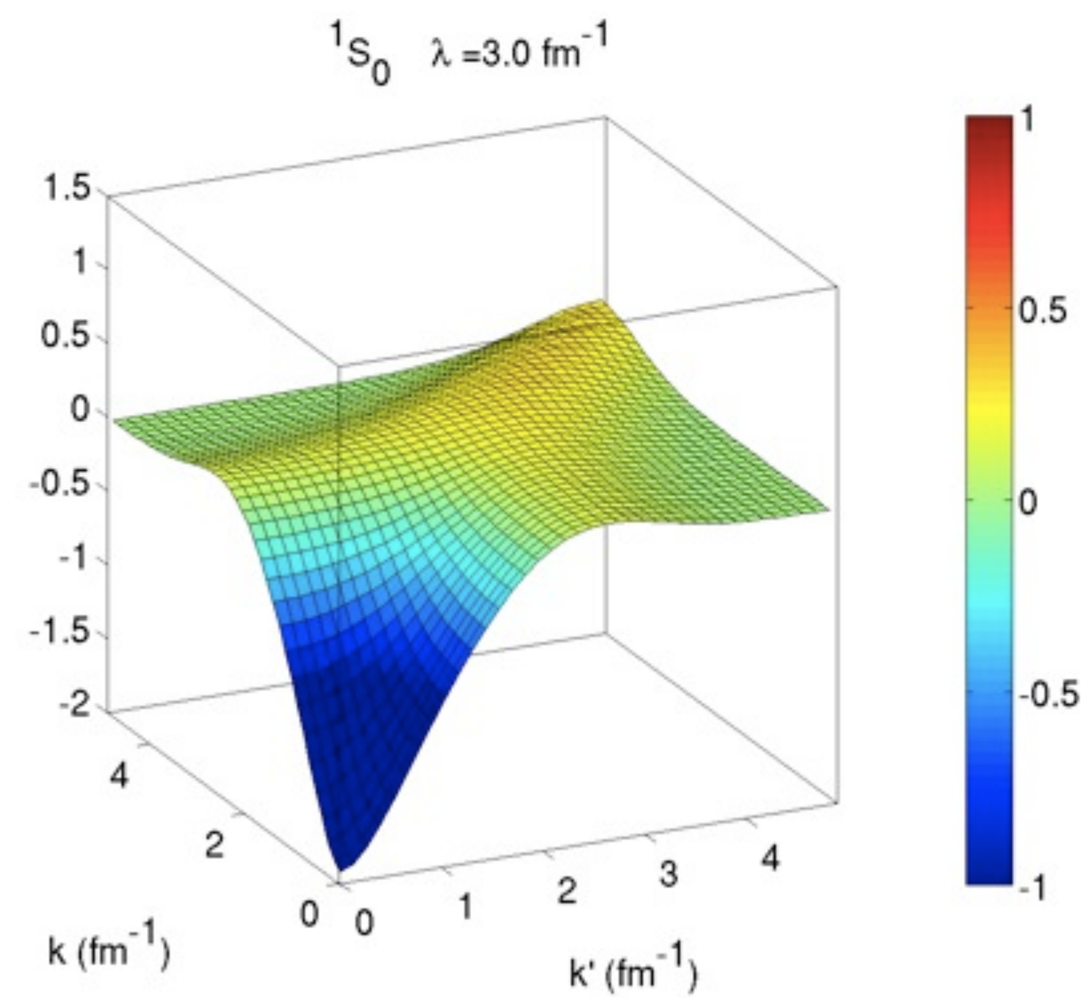

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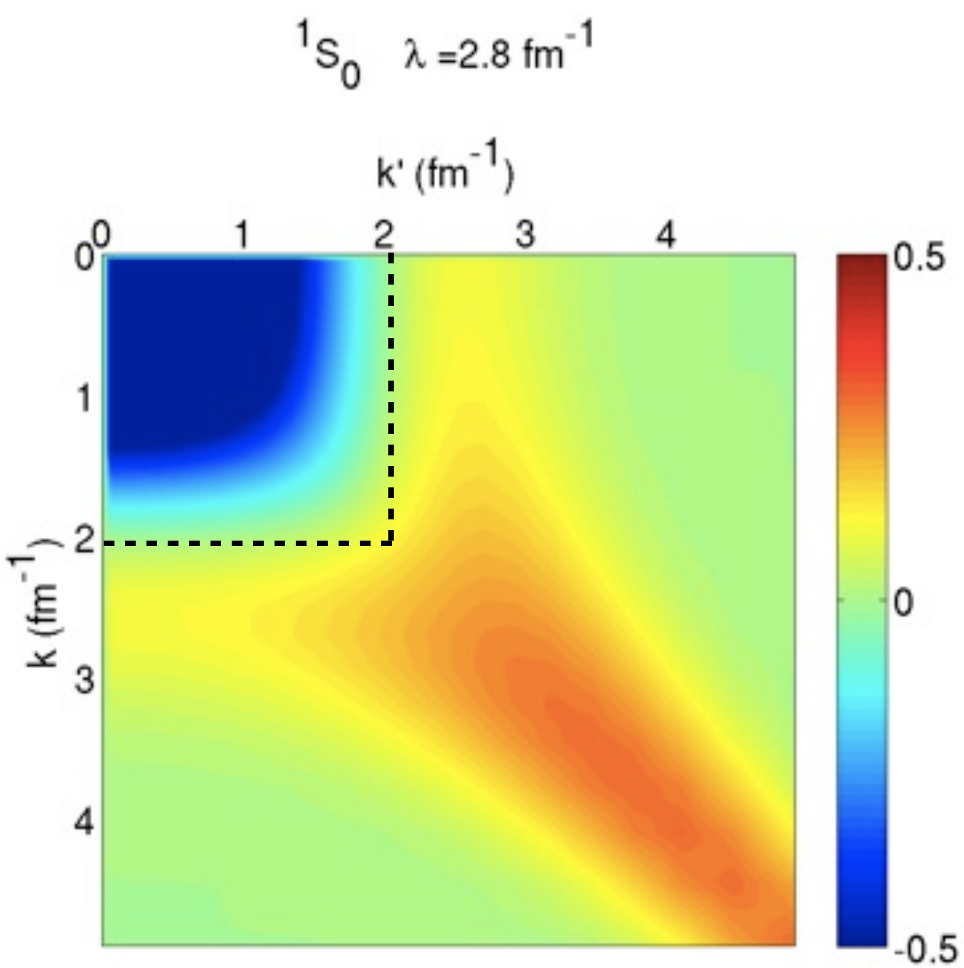

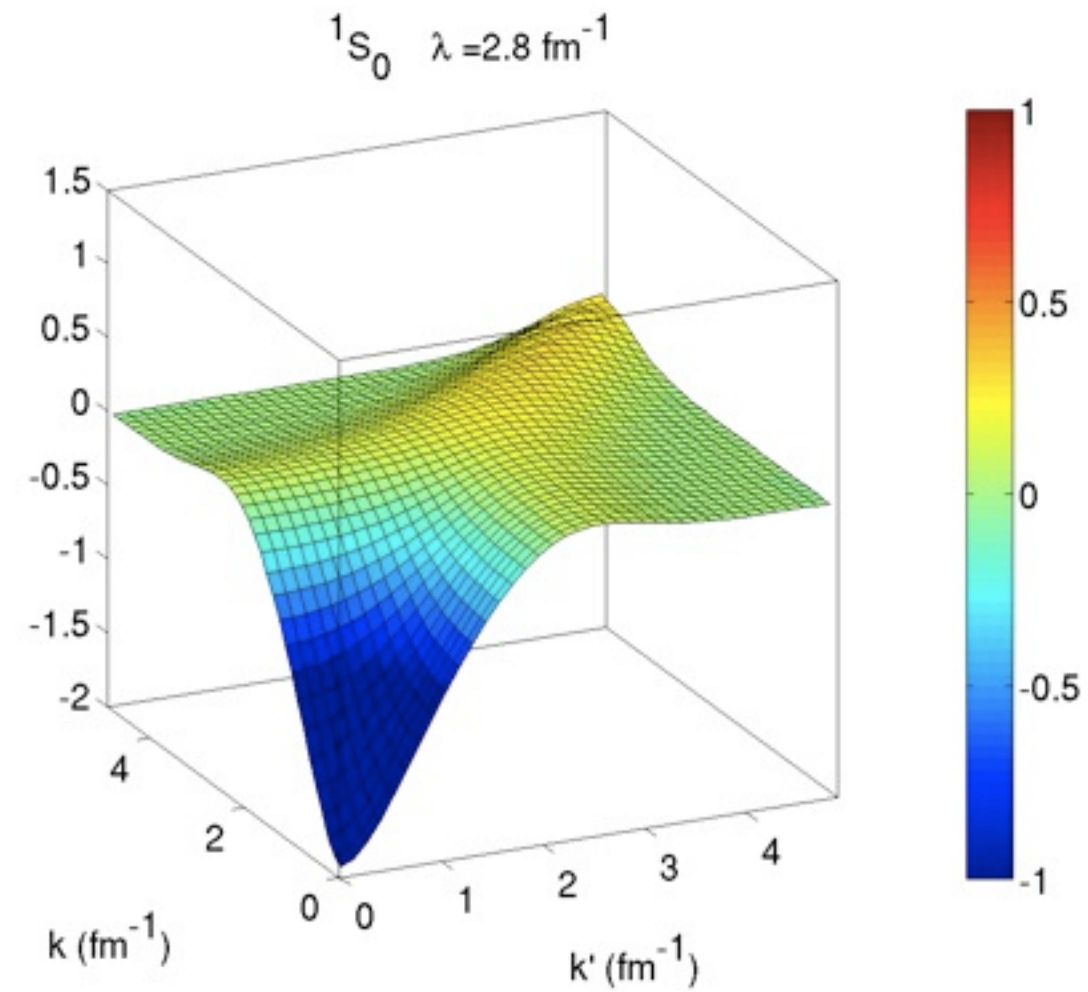

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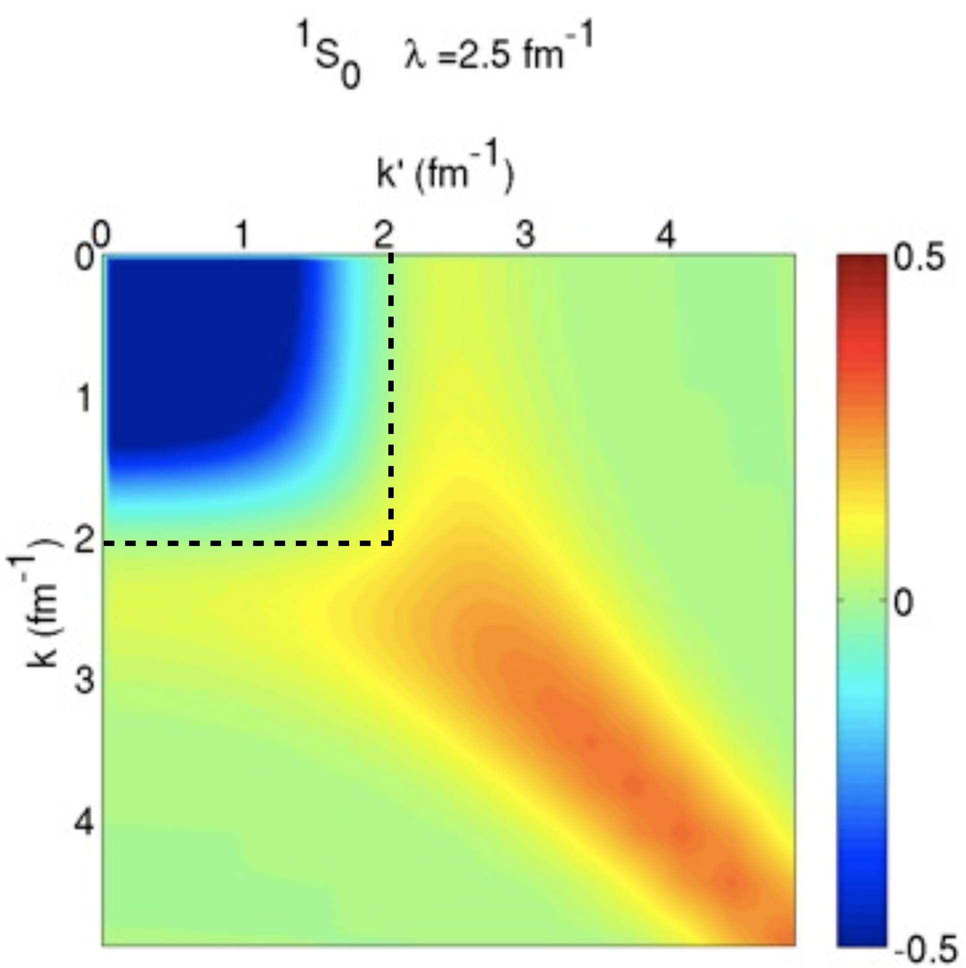

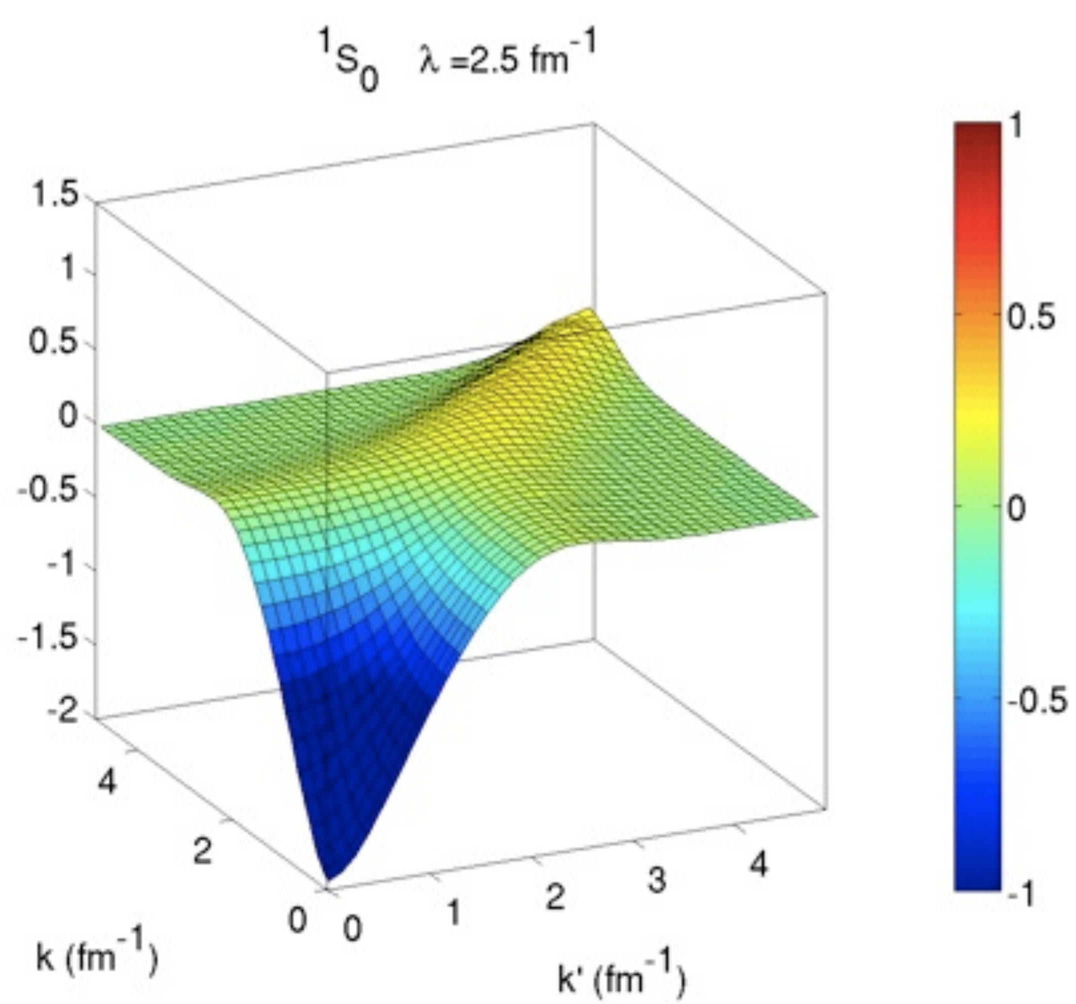

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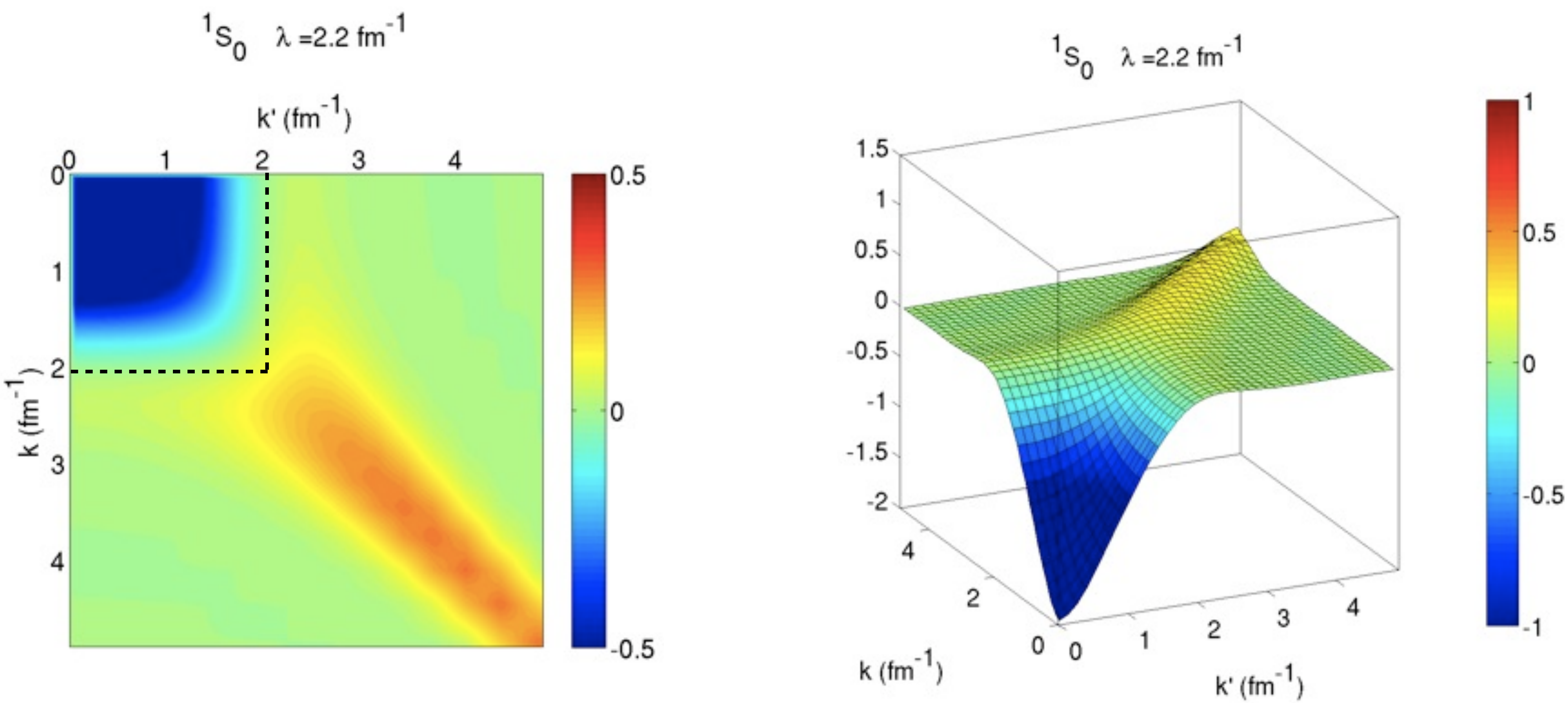

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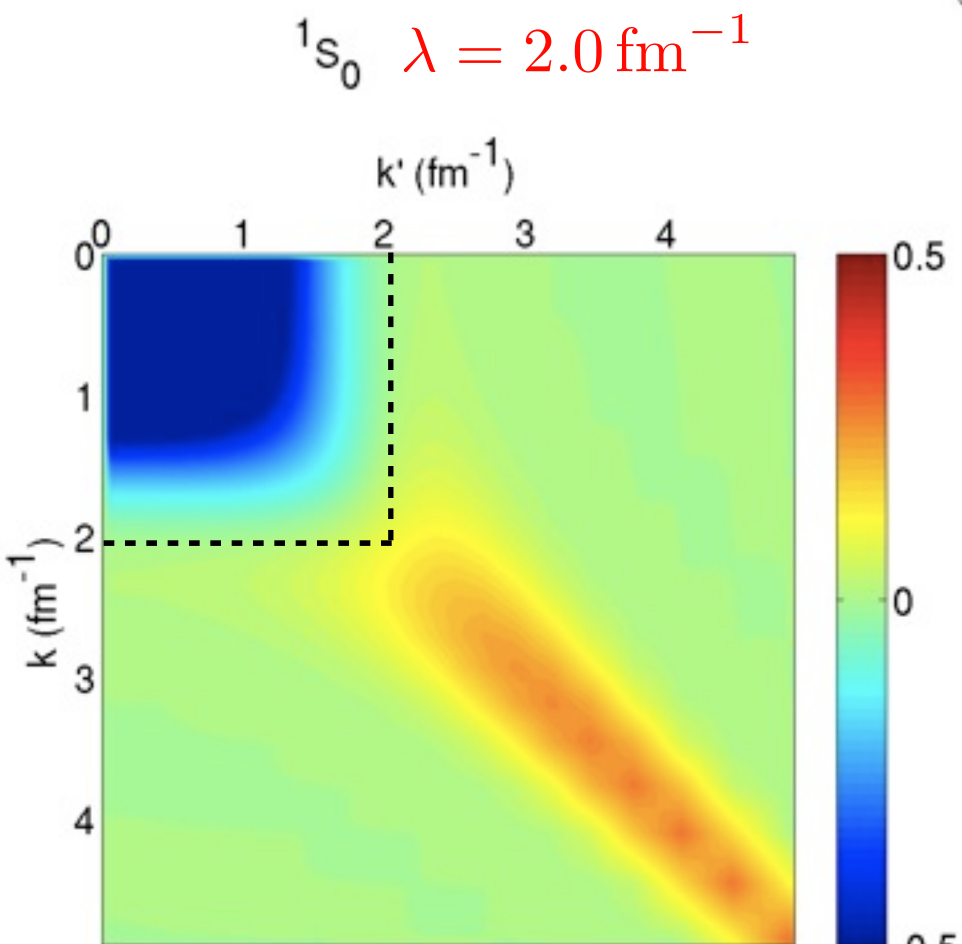

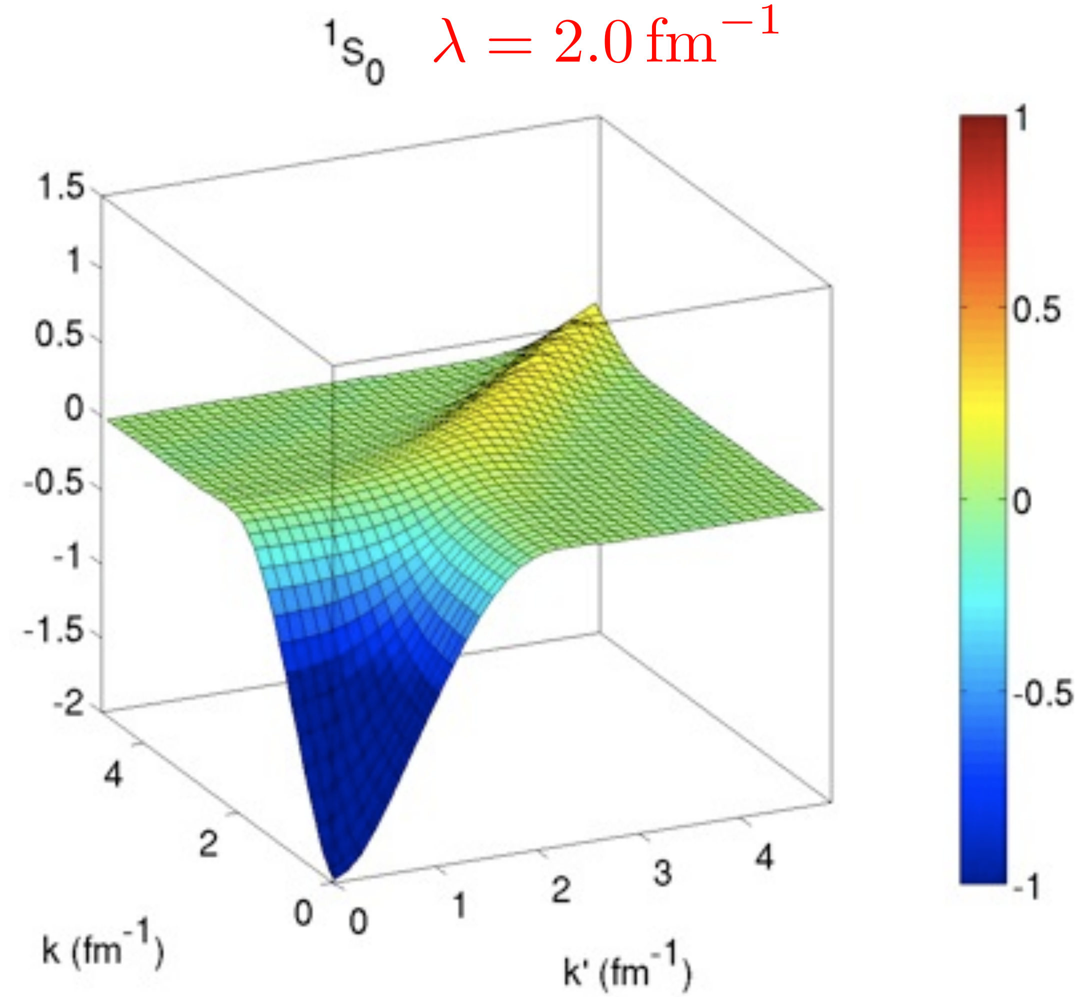

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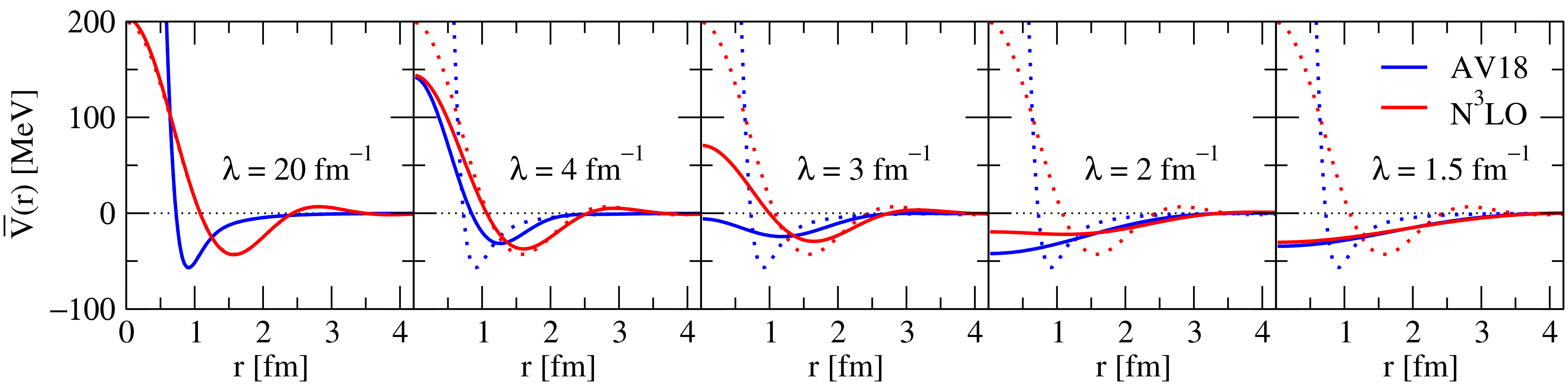

K. Wendt et al, PRC 86, 014003 (2012)

using local projection for representation:

$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r, r')$$

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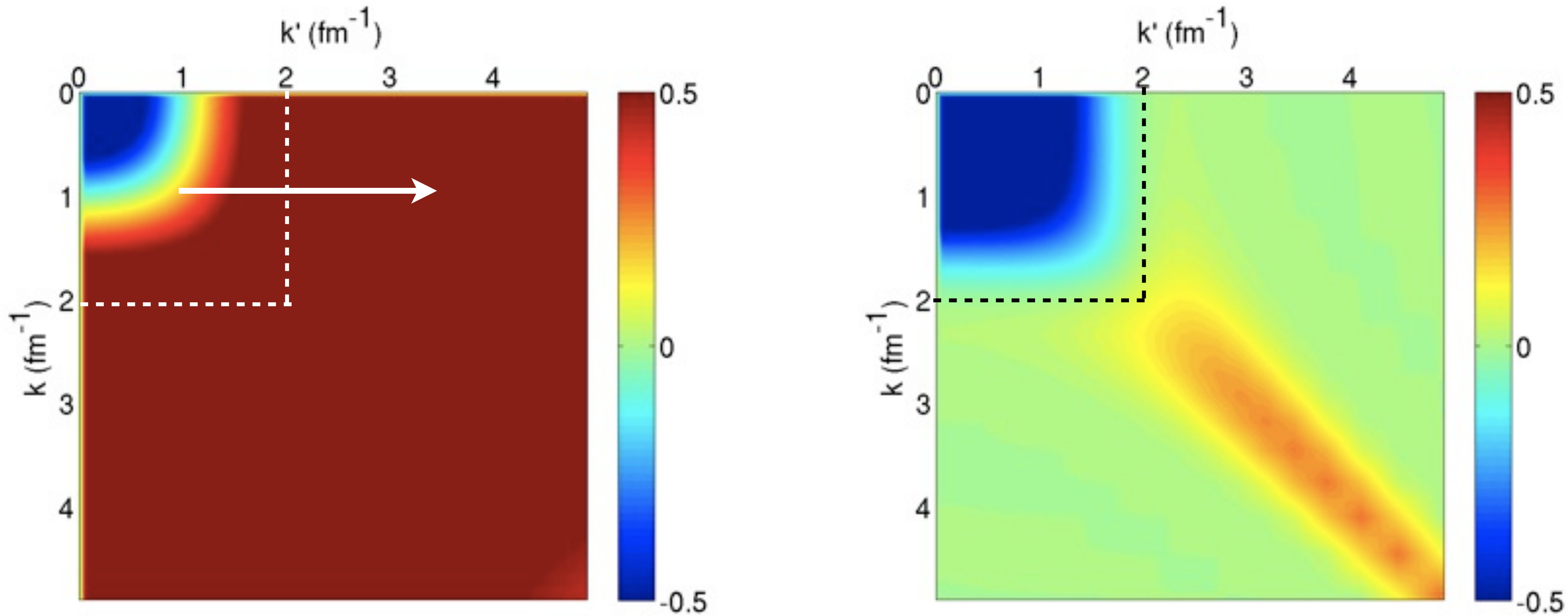


- elimination of coupling between low- and high momentum components, many-body problem more perturbative and tractable
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

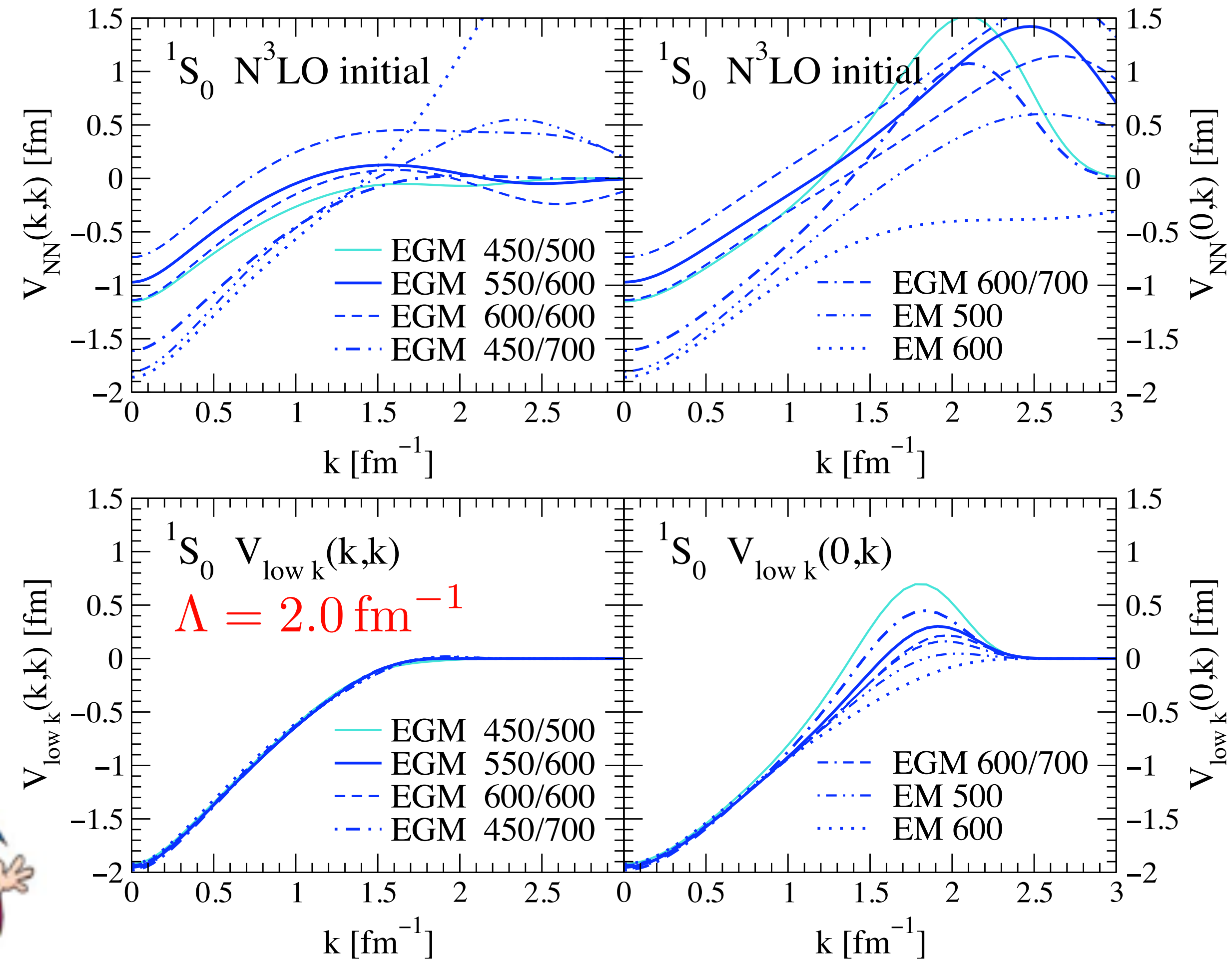
Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

Universality of nuclear interactions at low resolution

phase-shift equivalence

common long-range physics

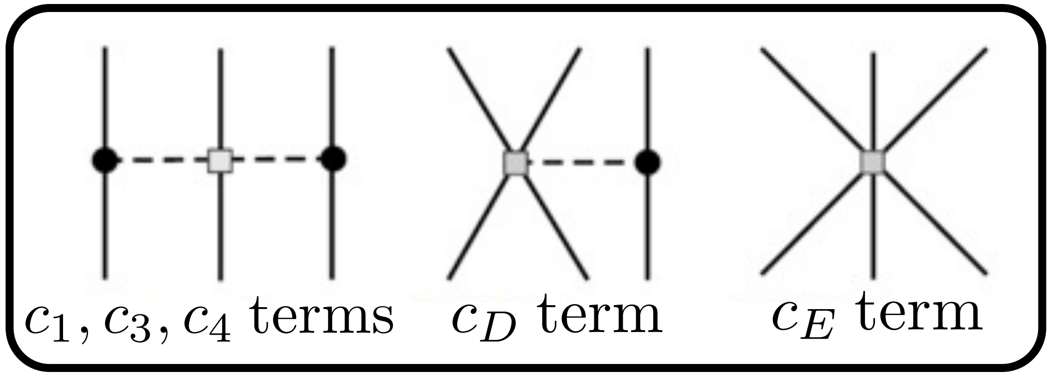
(approximate) universality of low-resolution NN interactions



To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

RG evolution of 3N interactions



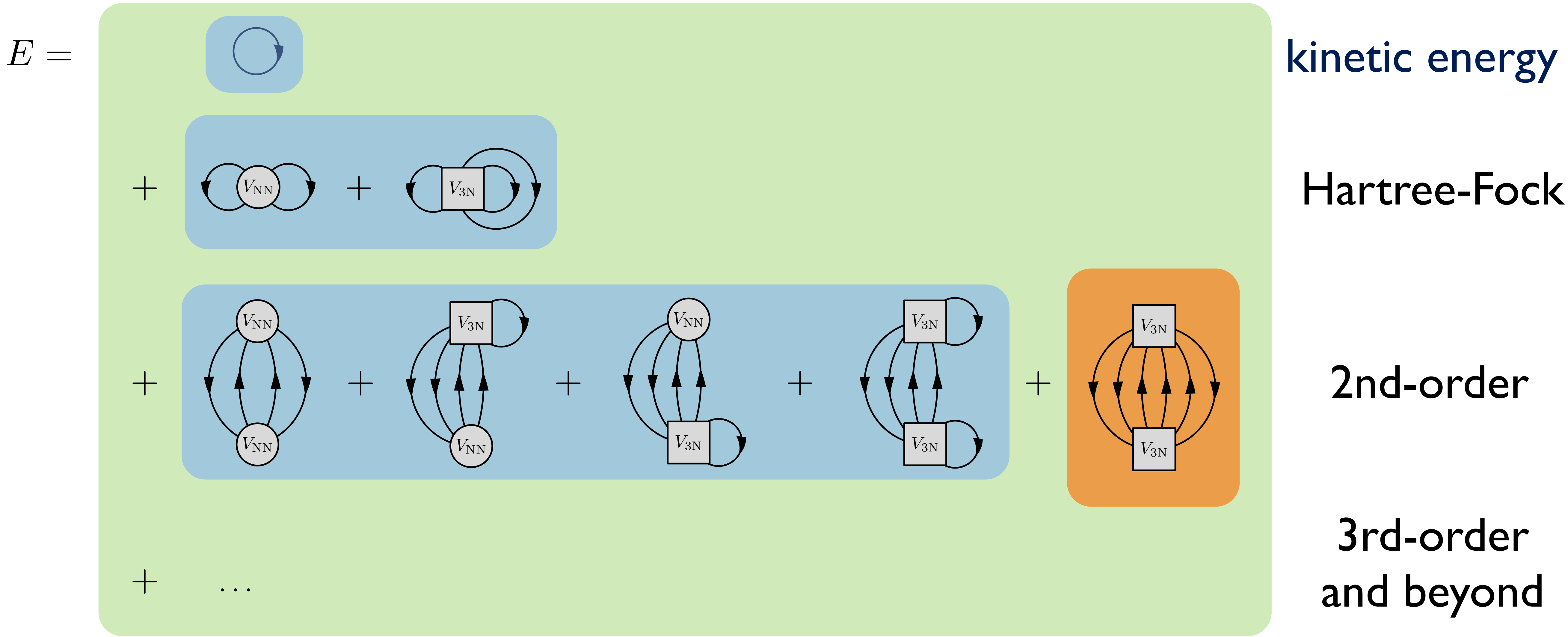
• So far (in momentum basis):
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV}$ and $r_{^4\mathrm{He}} = 1.464\,\mathrm{fm}$

$\longrightarrow$ coupling constants of natural size

- in neutron matter contributions from c_D, c_E and c_4 terms vanish
- long-range 2π contributions assumed to be invariant under RG evolution
- at low resolution scales nuclear many-body problem more perturbative

Application to infinite nuclear matter: Equation of state



- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

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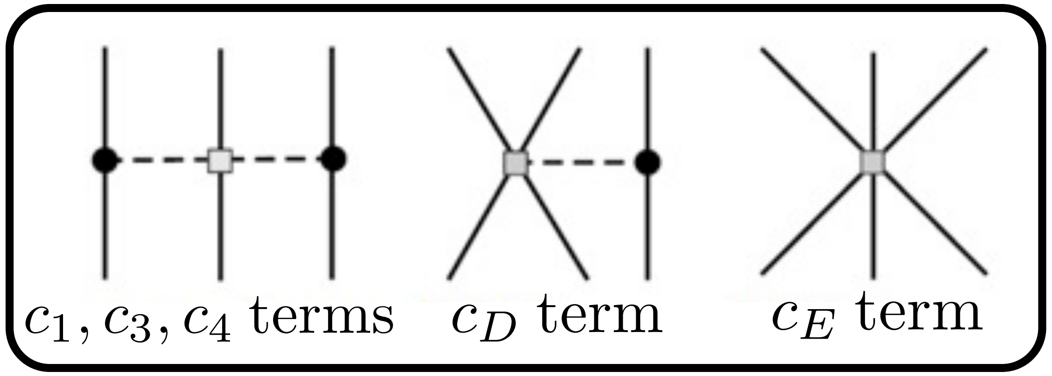

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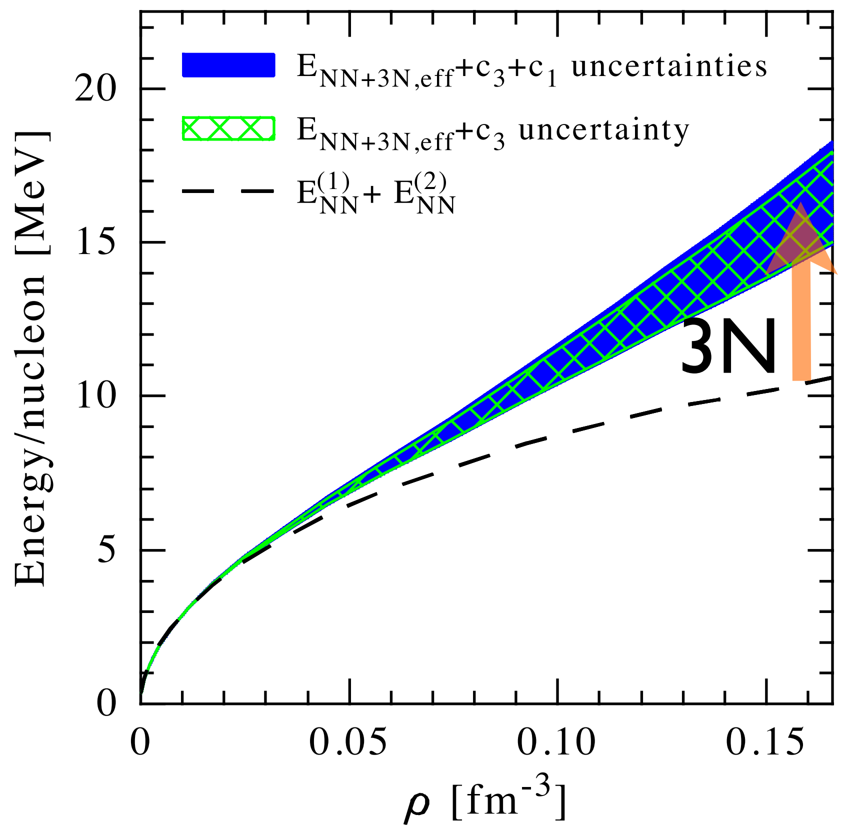
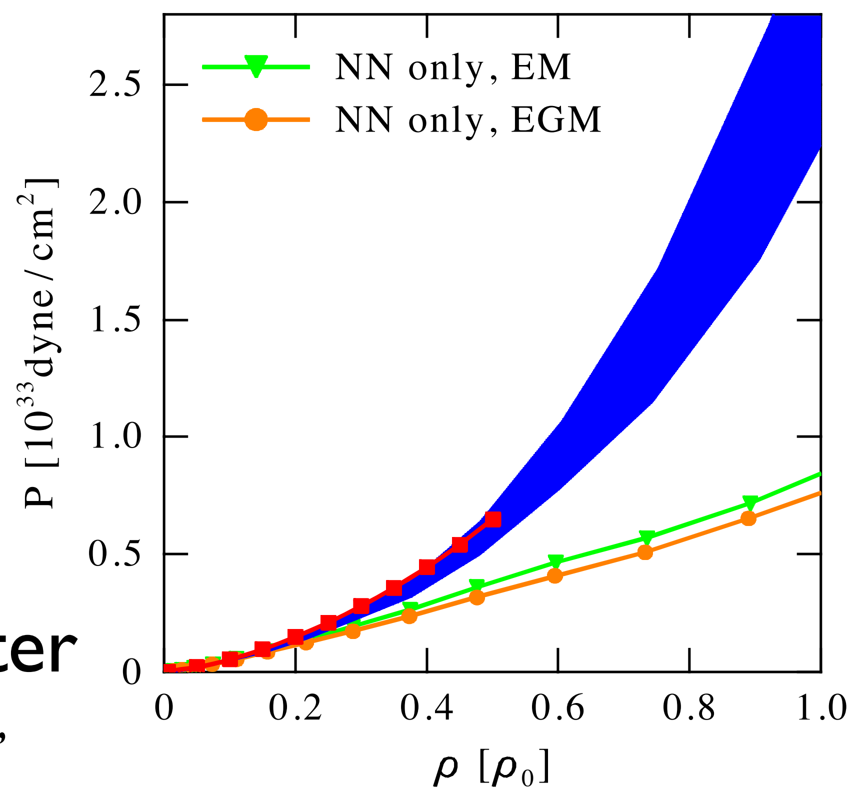


pure neutron matter

KH and Schwenk PRC 82, 014314 (2010)

neutron star matter

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)



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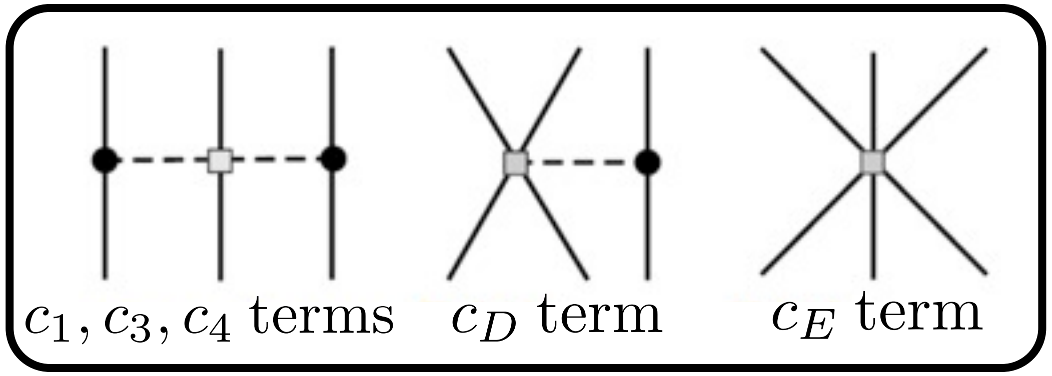

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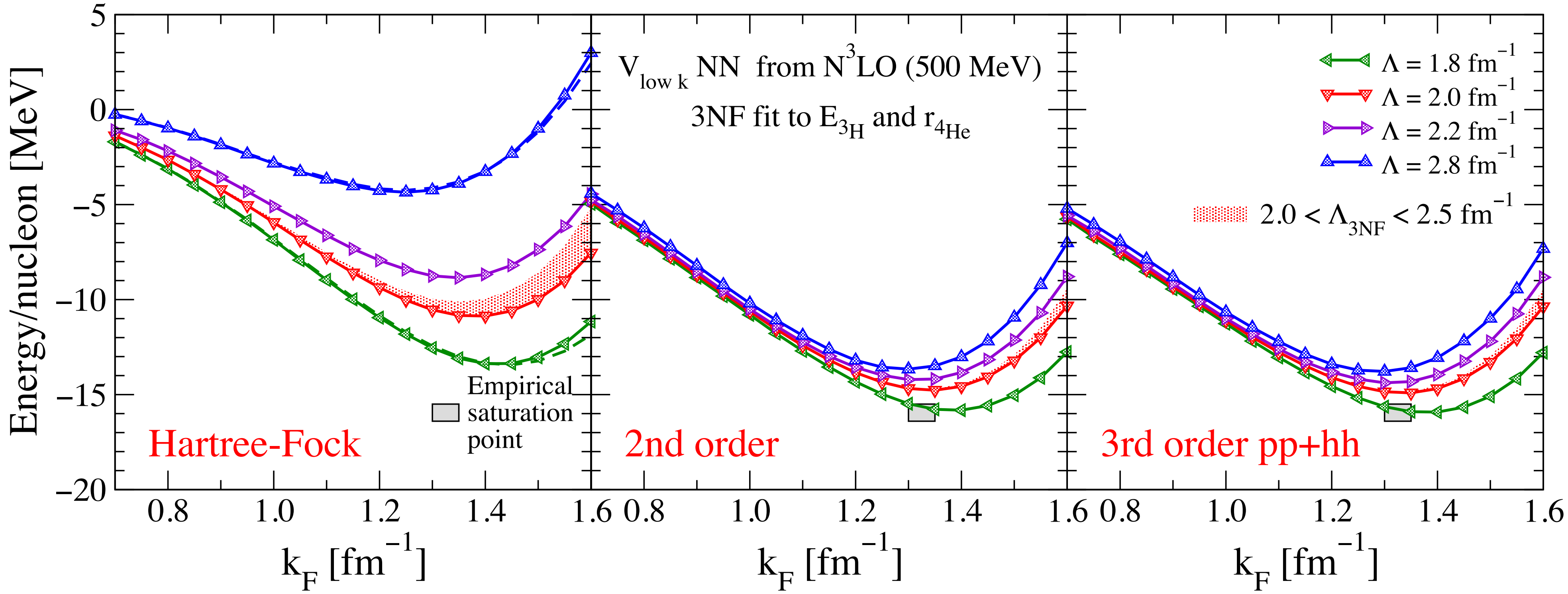
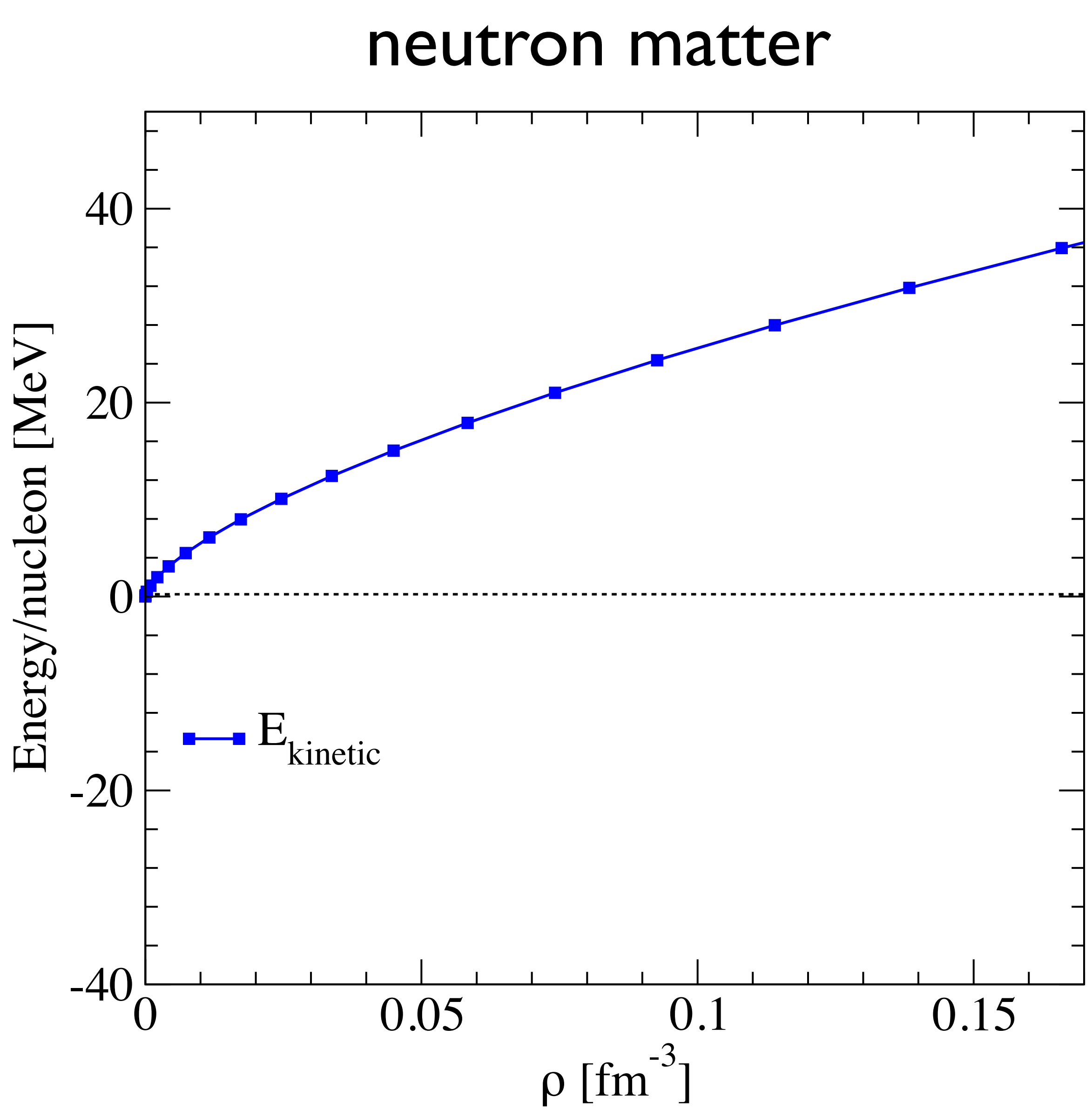
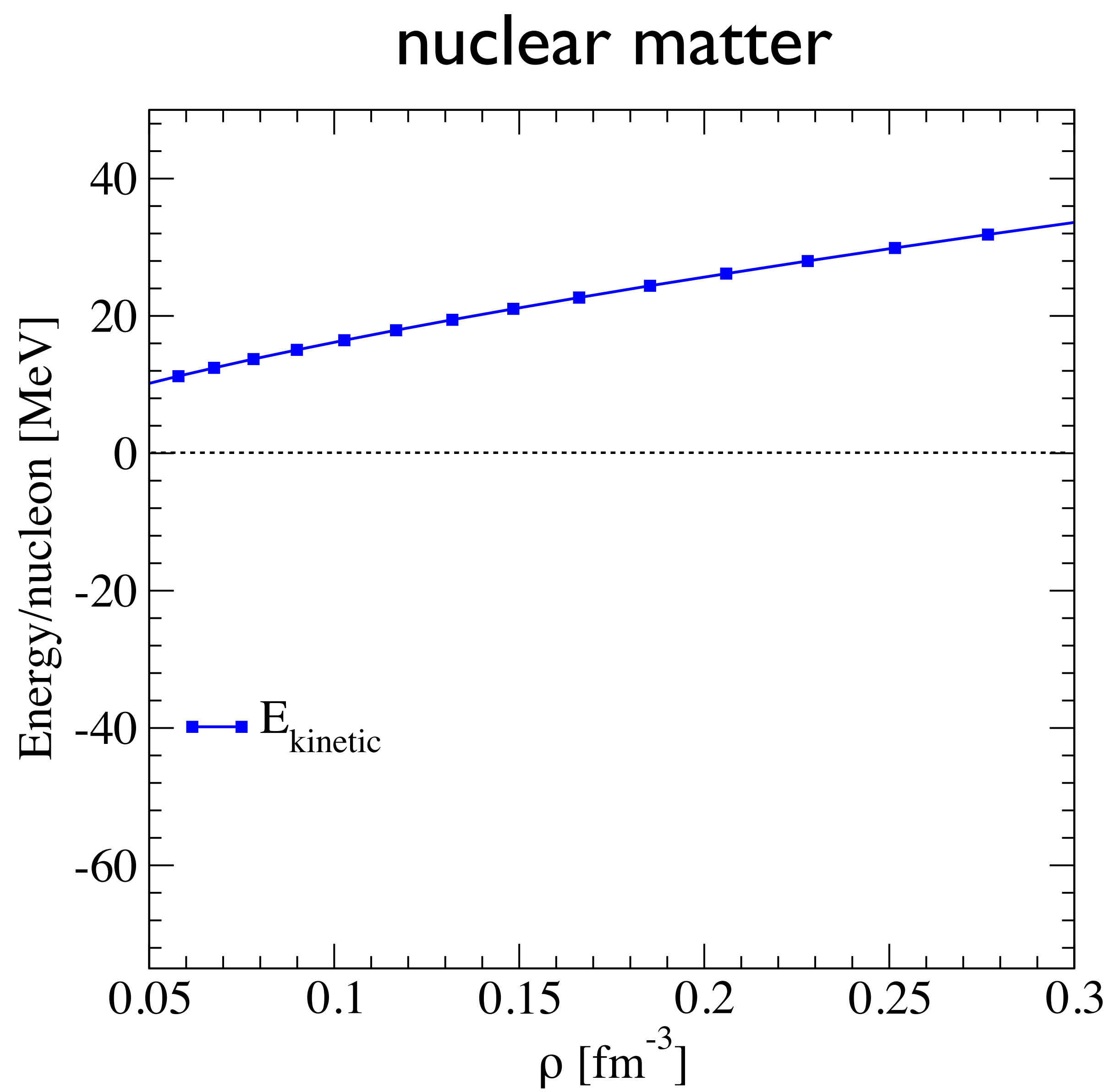


symmetric nuclear matter

KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

Hierarchy of many-body contributions





- binding energy results from cancellations of much larger kinetic and potential energy contributions
- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

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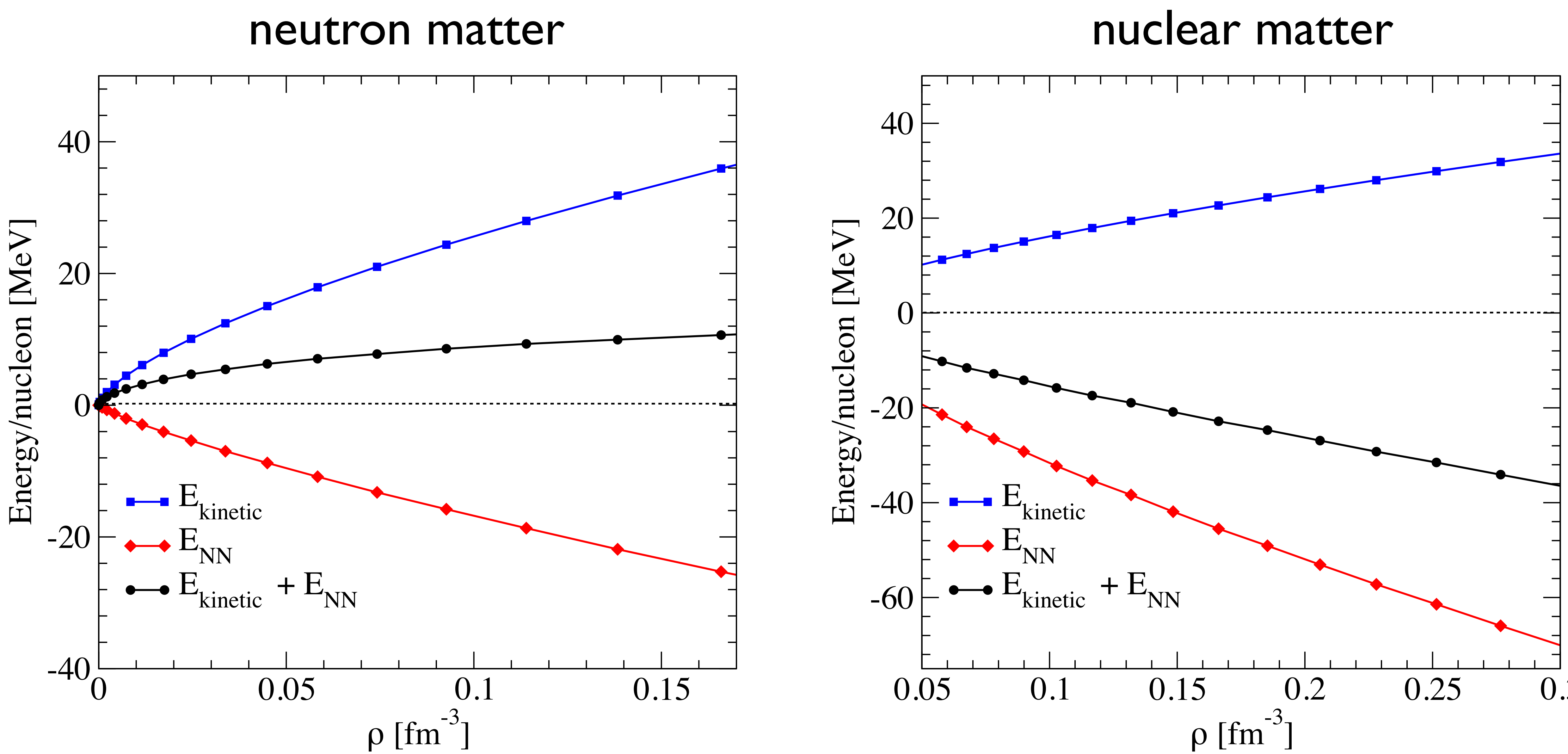

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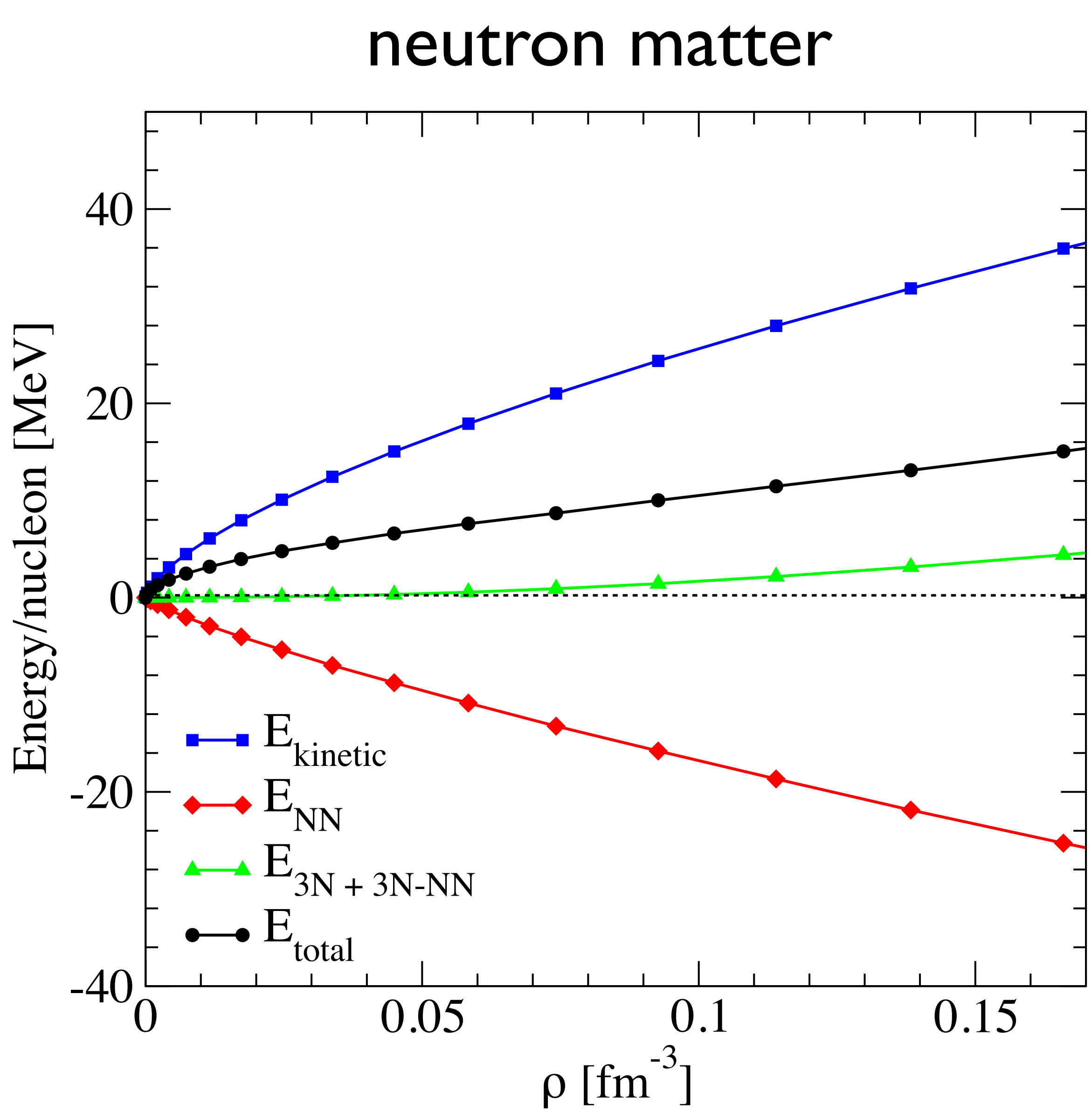

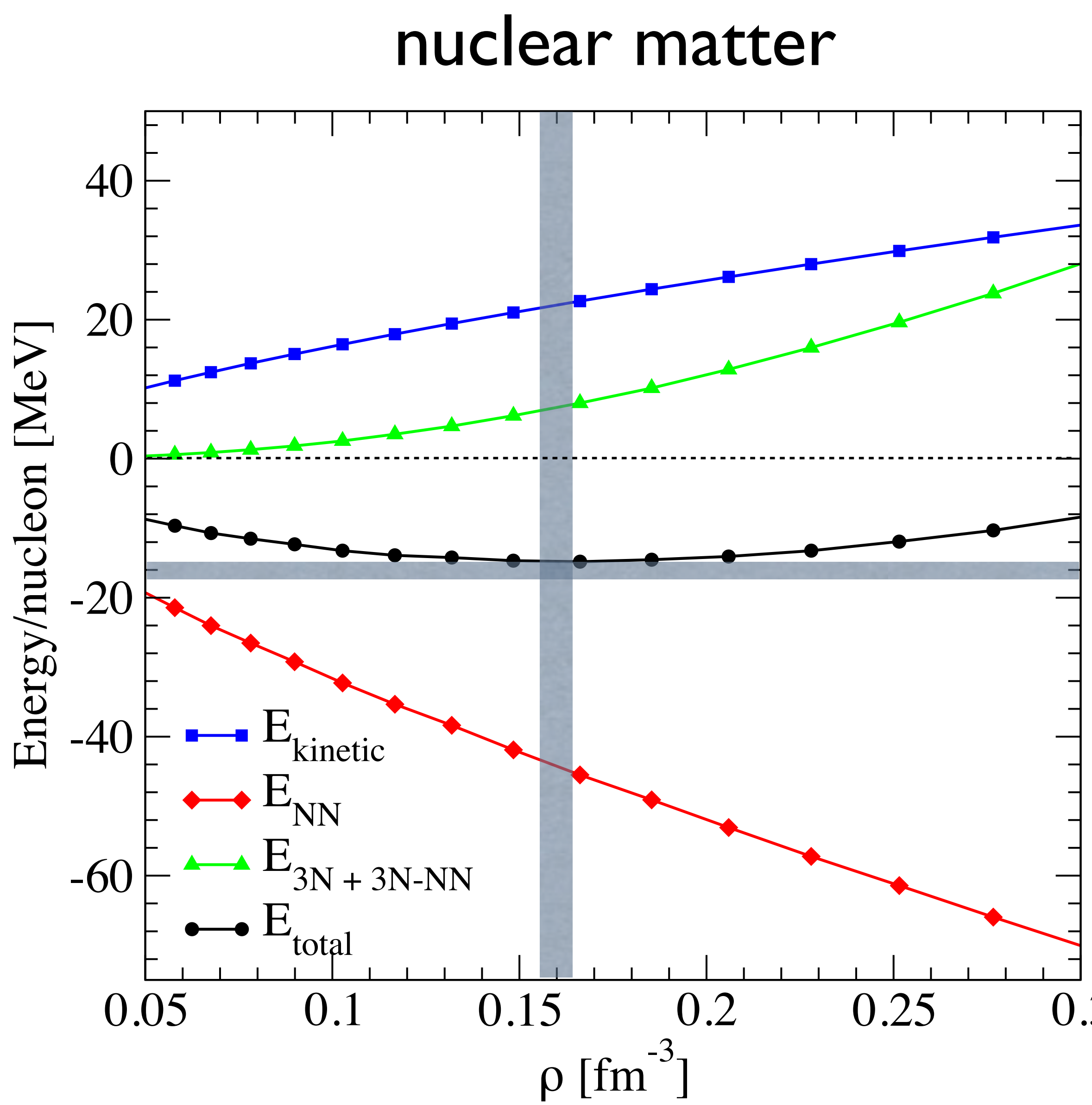

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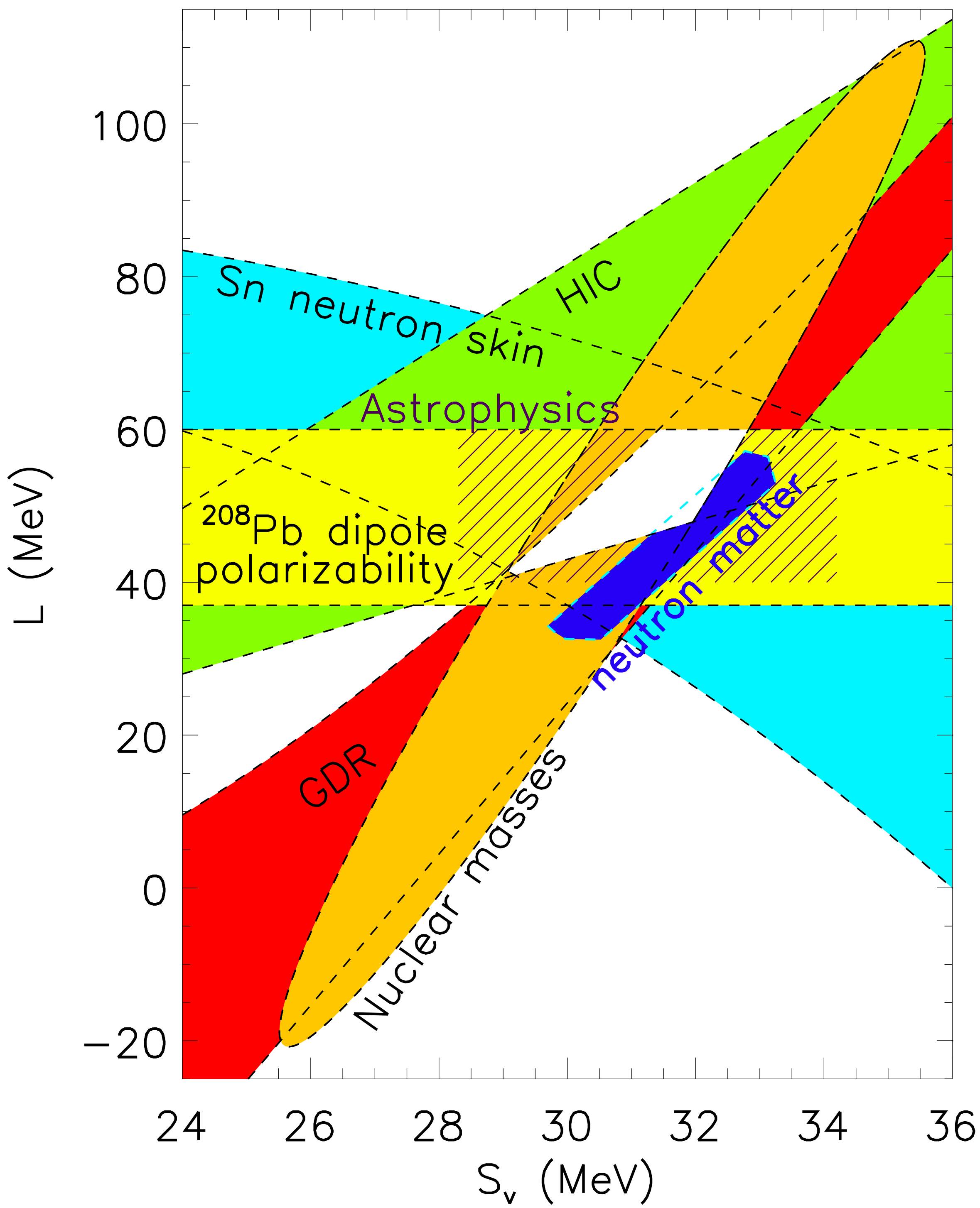
Symmetry energy constraints



extend EOS to finite proton fractions x

and extract symmetry energy parameters

$$S_v = \left. \frac{\partial^2 E/N}{\partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

$$L = \frac{3}{8} \left. \frac{\partial^3 E/N}{\partial \rho \partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

KH, Lattimer, Pethick and Schwenk, arXiv:1303.4662

symmetry energy parameters consistent with other constraints

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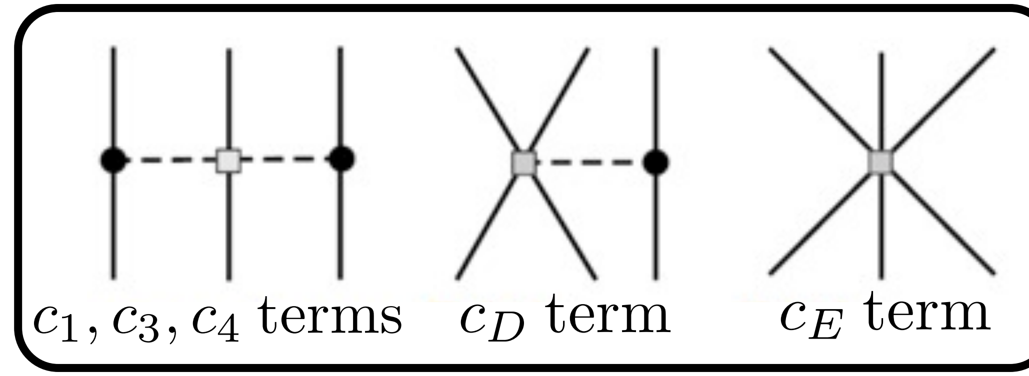

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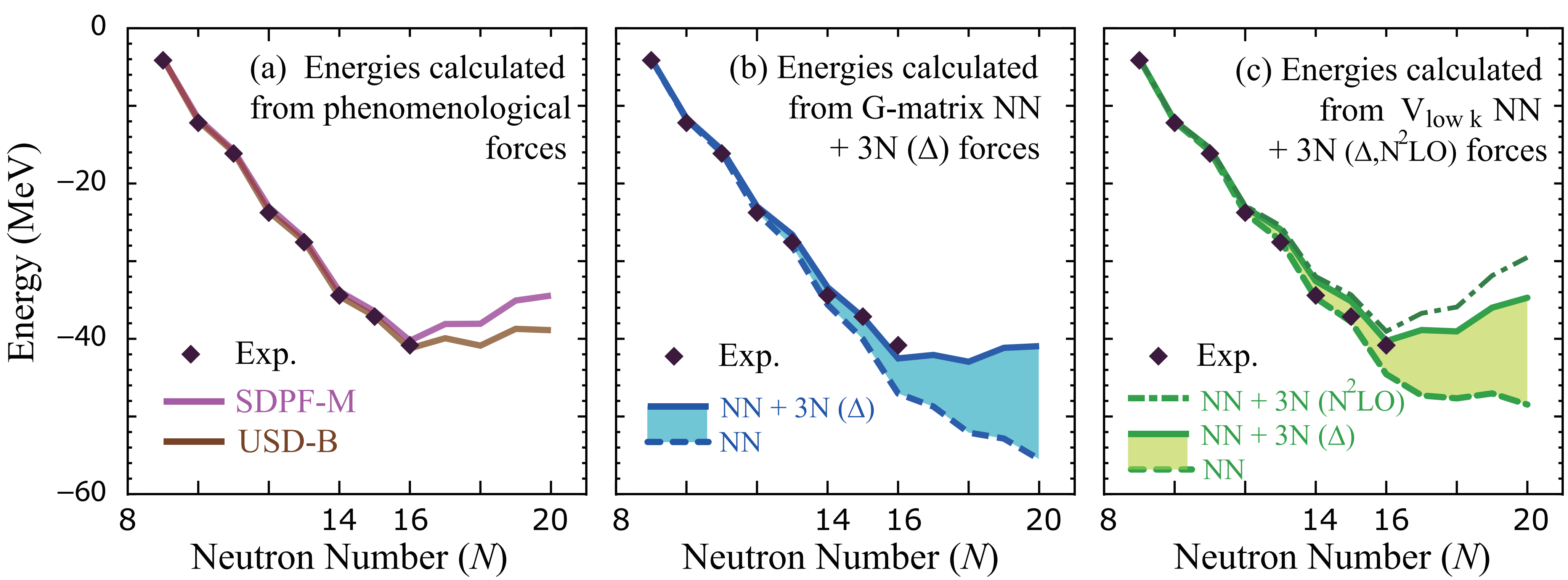

nuclear shell model

Otsuka et al., PRL 105, 032501 (2010)

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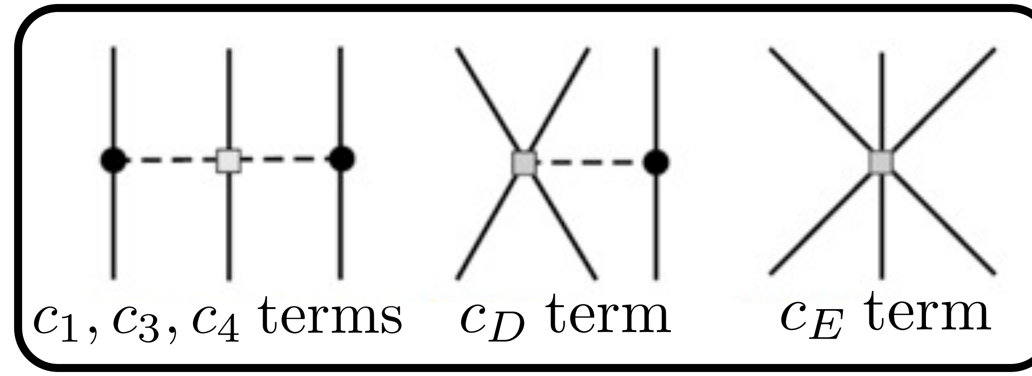

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- Ideal case: evolve 3NF consistently with NN interactions within the SRG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - puzzling effects in heavier nuclei (higher-body forces?)
  - not immediately applicable to infinite systems
  - limitations on $\hbar\Omega$

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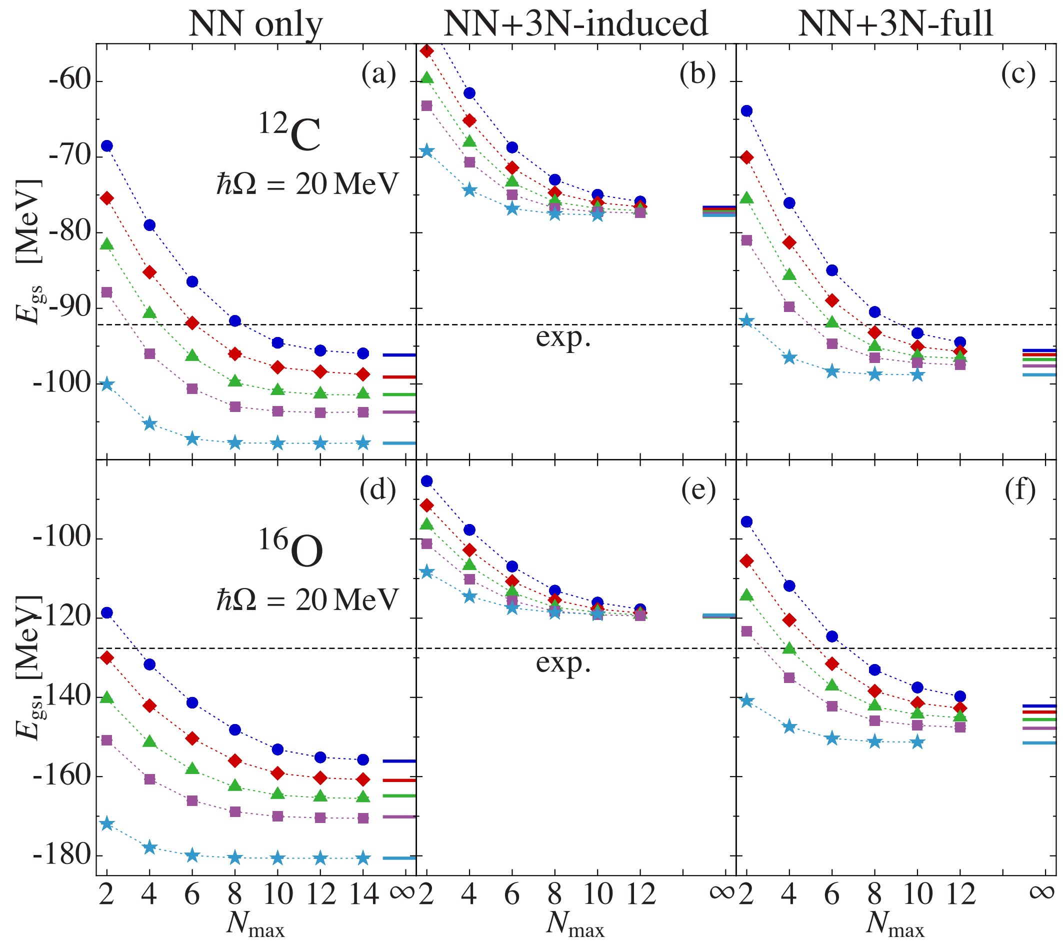
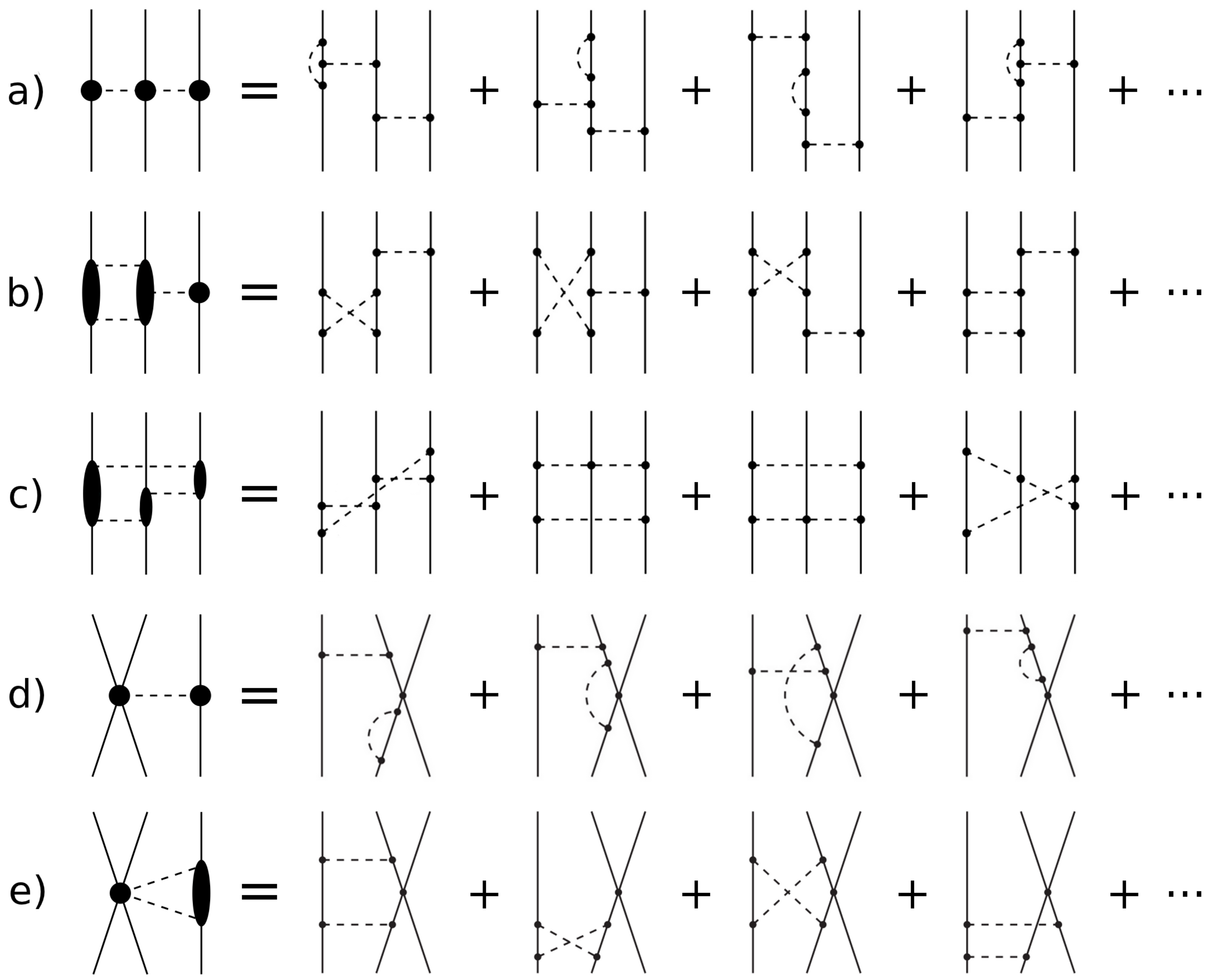


Roth et al. PRL 107, 072501 (2011)

- Ideal case: evolve 3NF consistently with NN interactions within the SRG
  - has been achieved in oscillator basis (Jurgenson, Roth)
  - promising results in very light nuclei
  - puzzling effects in heavier nuclei (higher-body forces?)
  - not immediately applicable to infinite systems
  - limitations on $\hbar\Omega$

3N interactions at N3LO



2π exchange

2π-1π exchange

pion ring (involved)

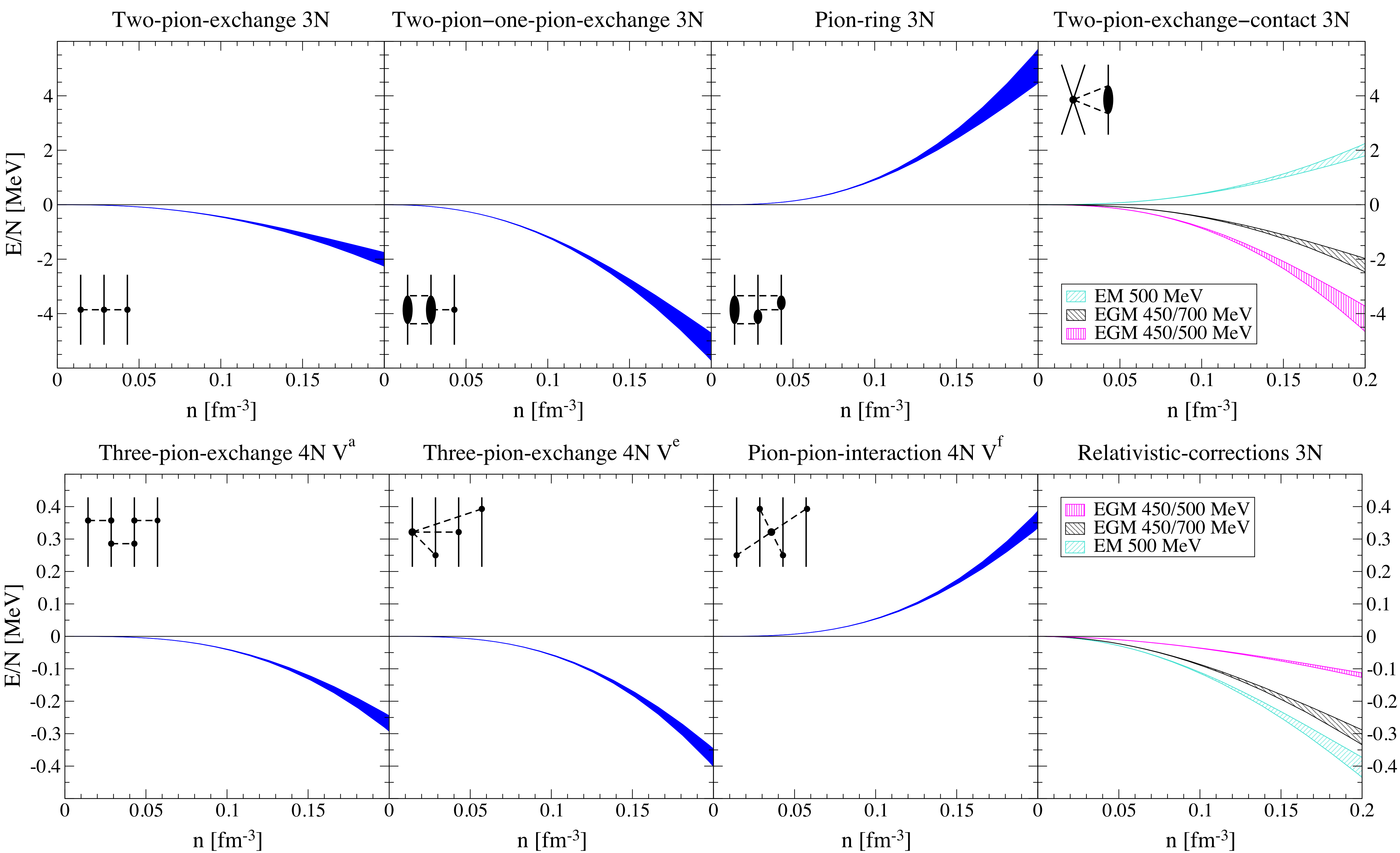
1π-exchange contact
(vanishes)

2π-exchange contact
(2-body-contact C_T)

Bernard et. al (2007, 2011)

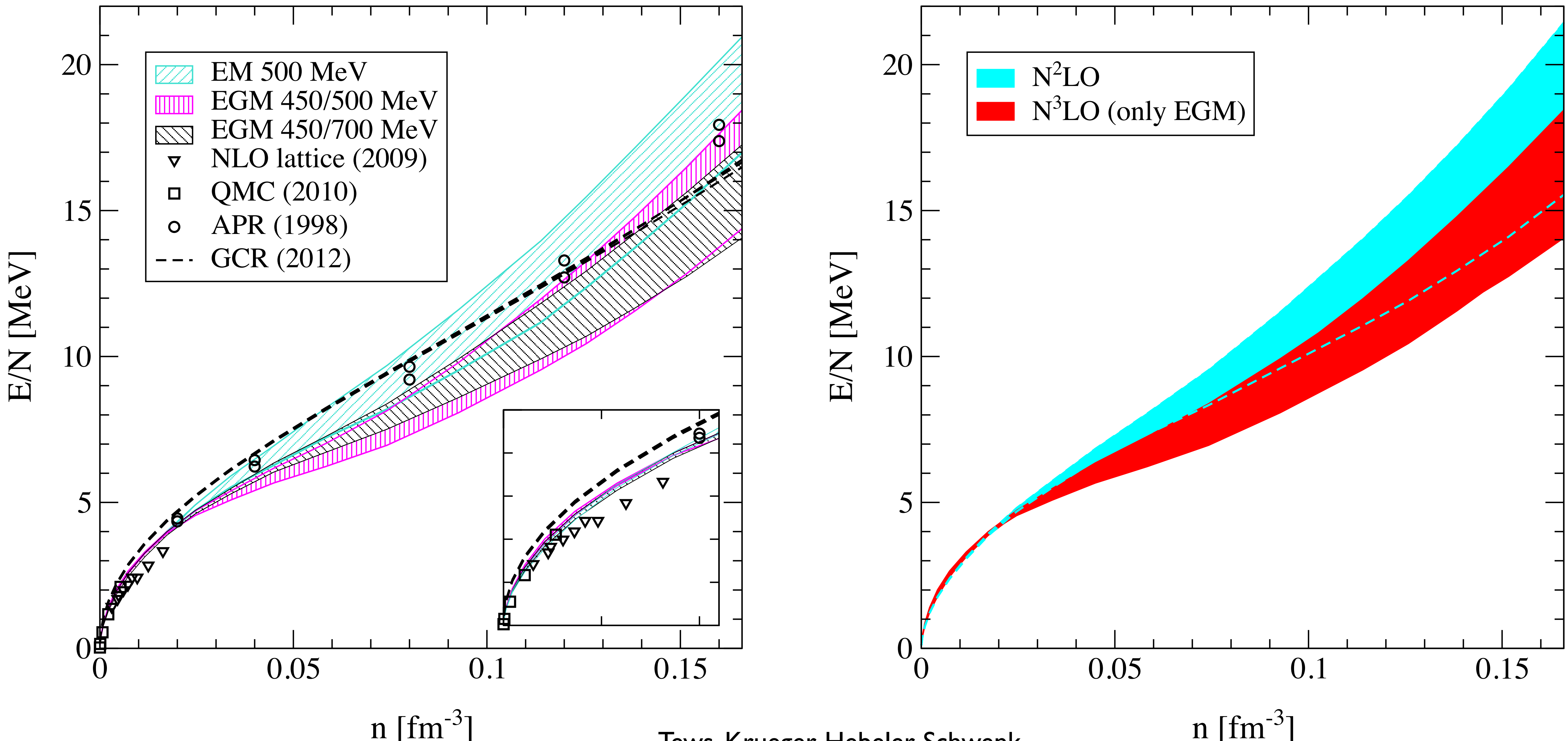
relativistic corrections
(2-body-contacts C_T, C_S)

Contributions of 3NFs at N3LO in neutron matter (Hartree-Fock, no RG evolution)



Tews, Krueger, Hebeler, Schwenk
PRL 110, 032504 (2013)

Complete N3LO calculation of neutron matter



Tews, Krueger, Hebeler, Schwenk
PRL 110, 032504 (2013)

- complete neutron matter calculation at N3LO including NN, 3N and 4N forces
- includes uncertainties from bare interactions

Consistent 3NF evolution in momentum basis: Current developments and applications

- application to infinite systems
  - equation of state (first results for neutron matter)
  - systematic study of induced many-body contributions, scaling behavior
  - include initial N3LO 3N interactions

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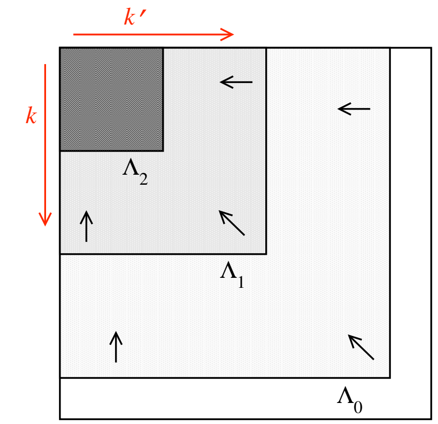

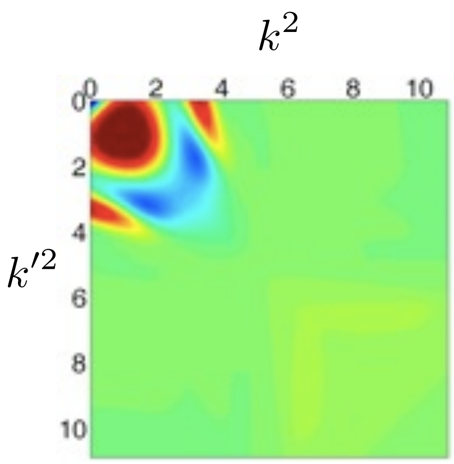

Anderson et al., PRC 77, 037001 (2008)

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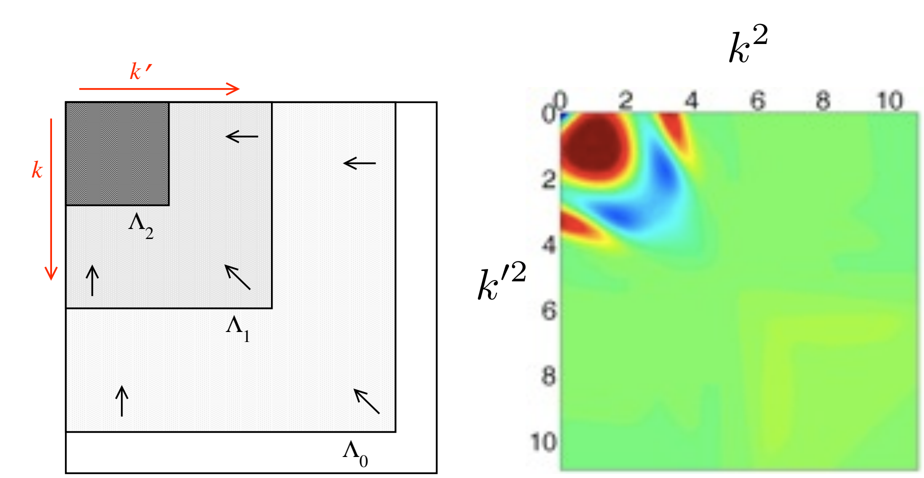


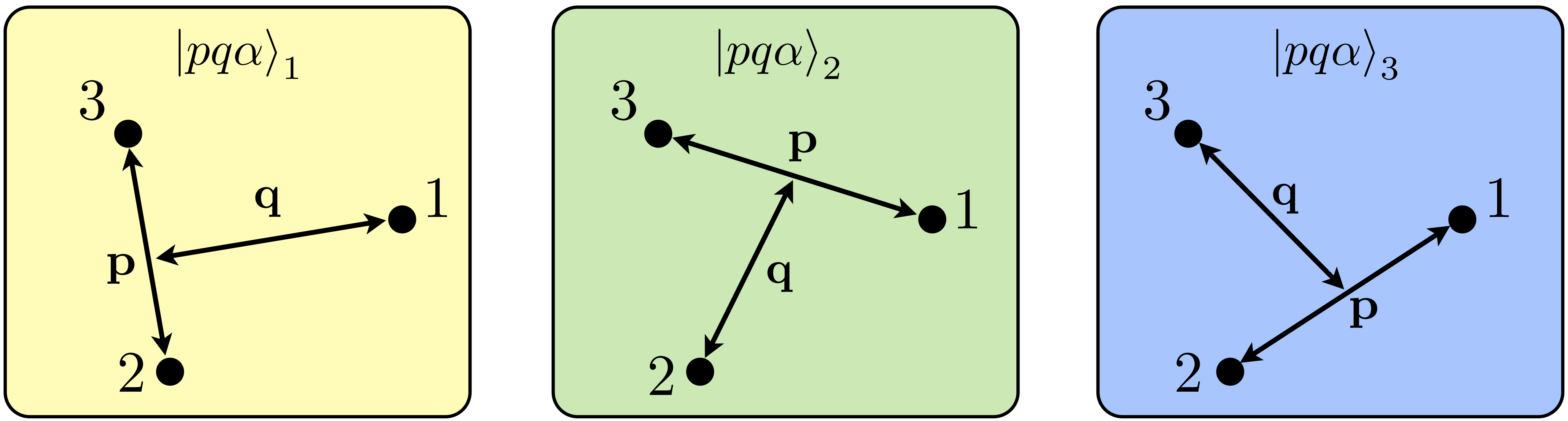
Anderson et al. , PRC 77, 037001 (2008)

- explicit calculation of unitary 3N transformation
  - RG evolution of operators
  - study of correlations in nuclear systems ⟶ factorization

RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$$



$${}_i\langle pq\alpha|P|p'q'\alpha'\rangle_i = {}_i\langle pq\alpha|p'q'\alpha'\rangle_j$$

Faddeev bound-state equation:

$$|\psi_i\rangle = G_0\left[2t_iP + (1+t_iG_0)\,V_{3N}^i(1+2P)\right]|\psi_i\rangle$$

SRG flow equations of NN and 3N forces in momentum basis

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\rm rel}, H_s]$$

$$H = T + V_{12} + V_{13} + V_{23} + V_{123}$$

- spectators correspond to delta functions, matrix representation of H_s ill-defined
- solution: explicit separation of NN and 3N flow equations

$$\begin{aligned}
\frac{dV_{ij}}{ds} &= \left[\left[T_{ij}, V_{ij}\right], T_{ij} + V_{ij}\right], \\
\frac{dV_{123}}{ds} &= \left[\left[T_{12}, V_{12}\right], V_{13} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{13}, V_{13}\right], V_{12} + V_{23} + V_{123}\right] \\
&+ \left[\left[T_{23}, V_{23}\right], V_{12} + V_{13} + V_{123}\right] \\
&+ \left[\left[T_{\rm rel}, V_{123}\right], H_s\right]
\end{aligned}$$

- only connected terms remain in $\frac{dV_{123}}{ds}$, 'dangerous' delta functions cancel

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

SRG evolution in momentum space

- evolve the antisymmetrized 3N interaction

special thanks to J. Golak, R. Skibinski, K. Topolnicki

$$\overline{V}_{123} = {}_i\langle pq\alpha| \, (1 + P_{123} + P_{132}) V_{123}^{(i)} (1 + P_{123} + P_{132}) \, |p'q'\alpha'\rangle_i$$

- embed NN interaction in 3N basis:

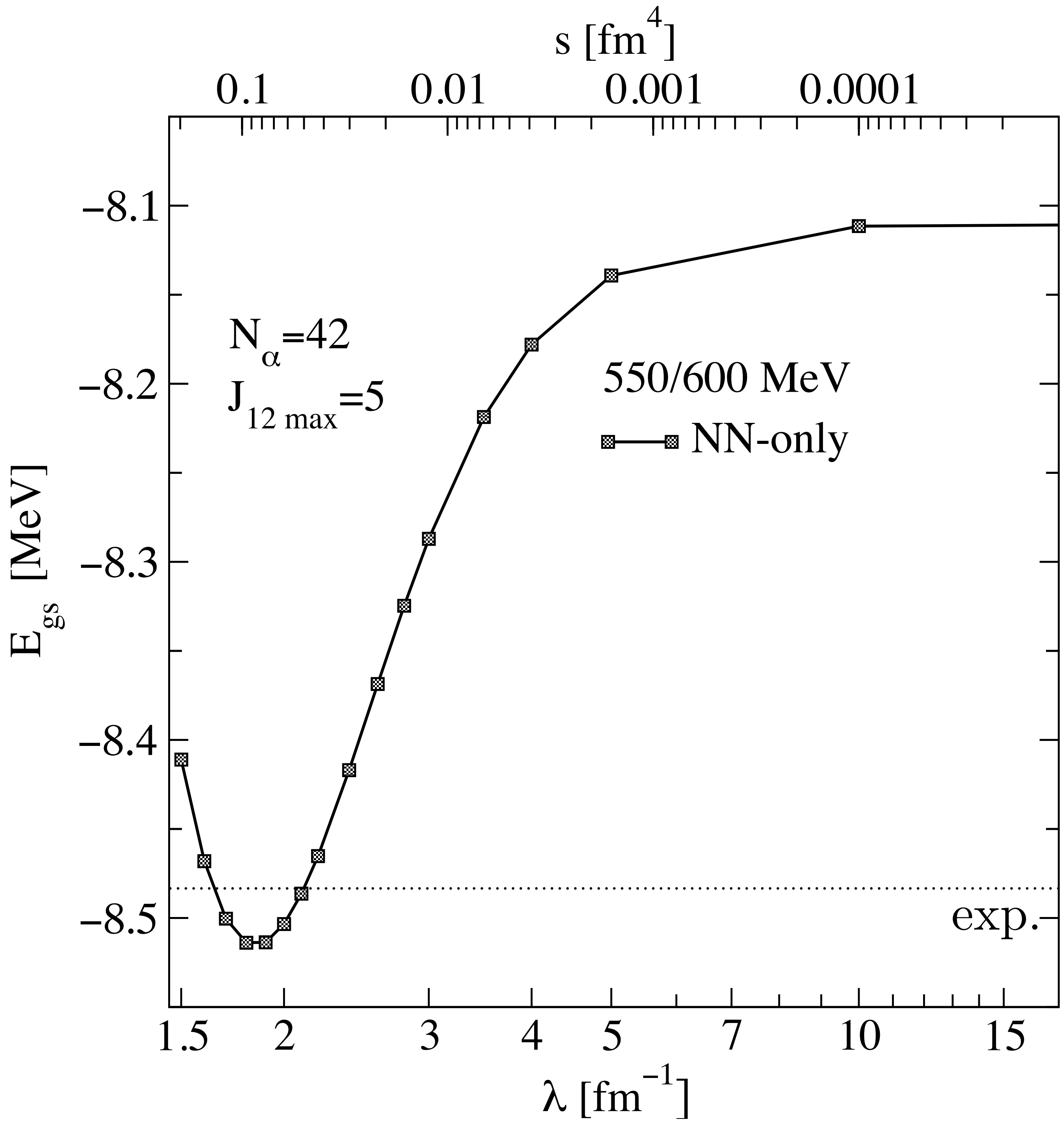
$$V_{13} = P_{123} V_{12} P_{132}, \quad V_{23} = P_{132} V_{12} P_{123}$$

with $${}_3\langle pq\alpha|V_{12}|p'q'\alpha'\rangle_3 = \langle p\tilde{\alpha}|V_{\mathrm{NN}}|p'\tilde{\alpha}'\rangle \, \delta(q - q')/q^2$$

- use $P_{123}\overline{V}_{123} = P_{132}\overline{V}_{123} = \overline{V}_{123}$

$$\Rightarrow \quad d\overline{V}_{123}/ds = C_1(s, T, V_{\mathrm{NN}}, P) + C_2(s, T, V_{\mathrm{NN}}, \overline{V}_{123}, P) + C_3(s, T, \overline{V}_{123})$$

SRG evolution of 3N interactions in momentum space: Results for the Triton



Hebeler PRC(R) 85, 021002 (2012)

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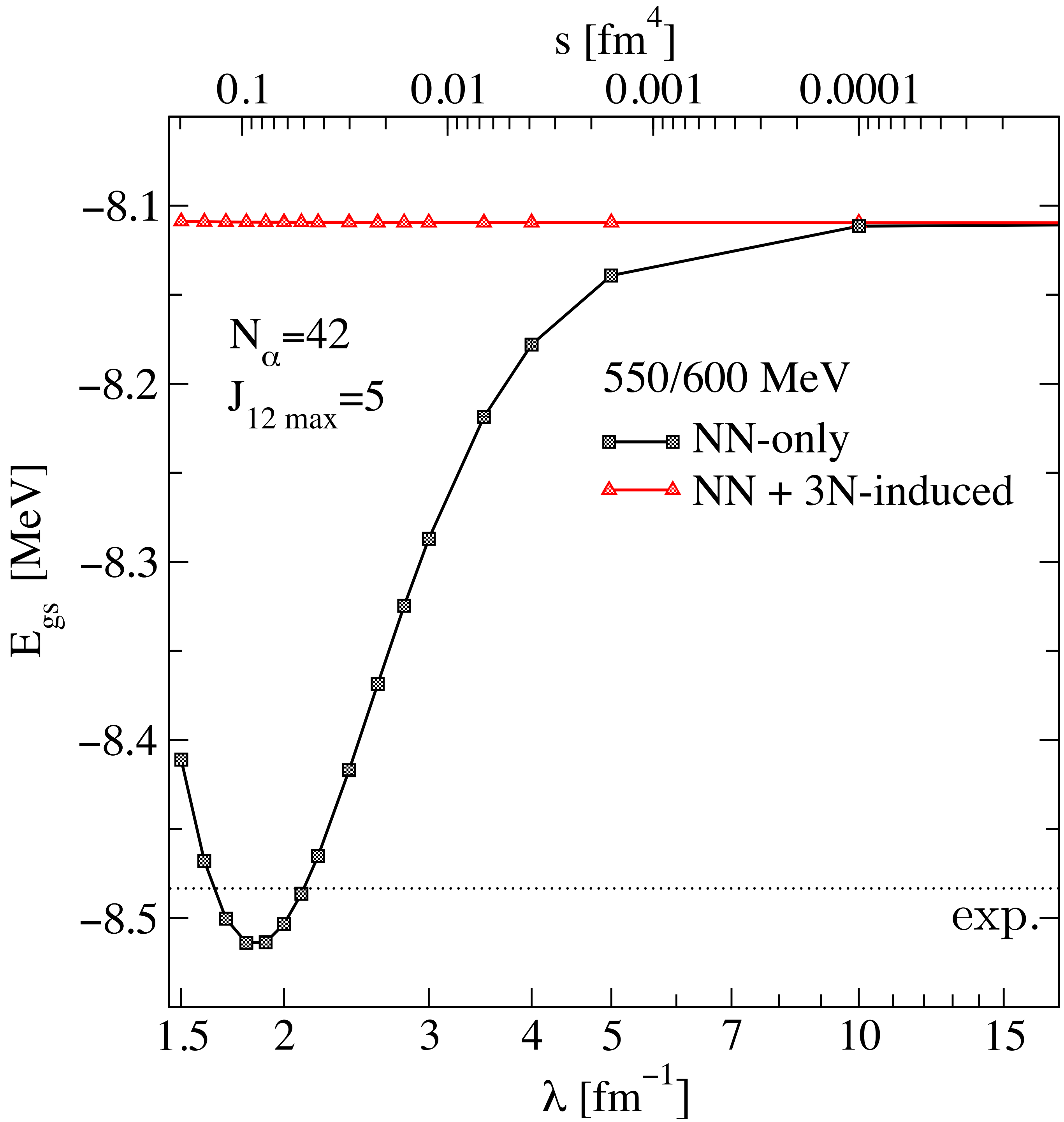

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It works:

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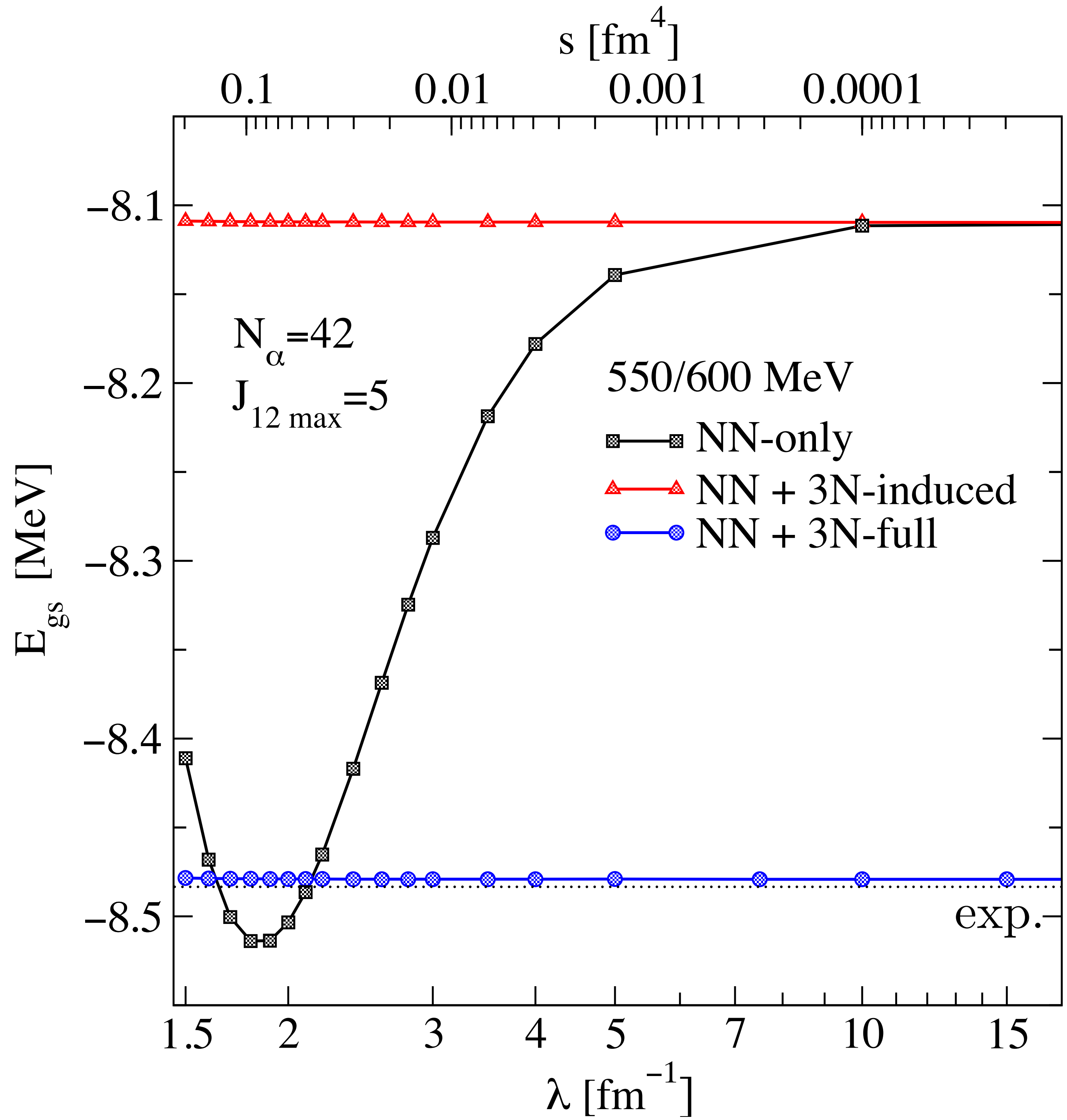

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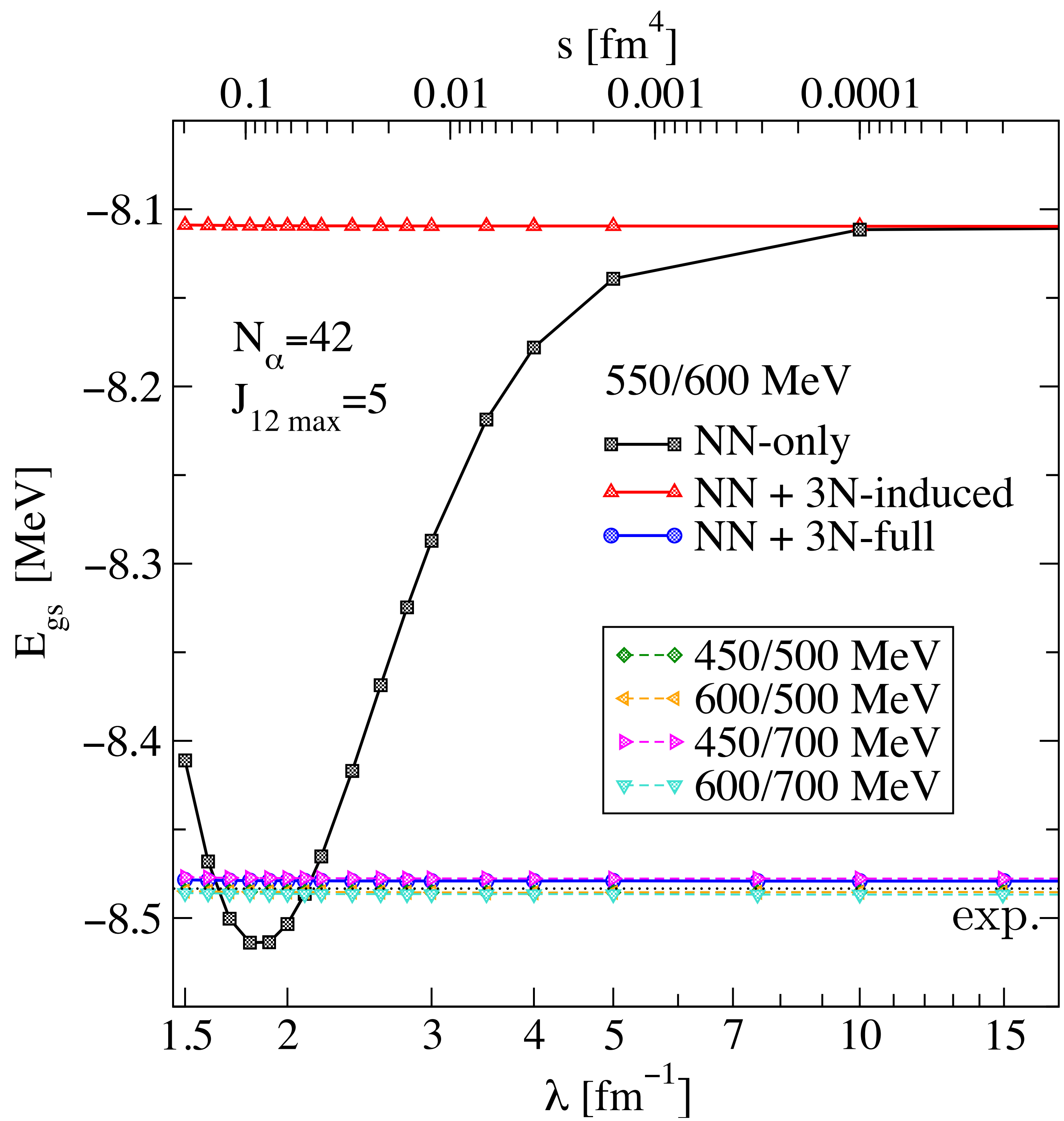

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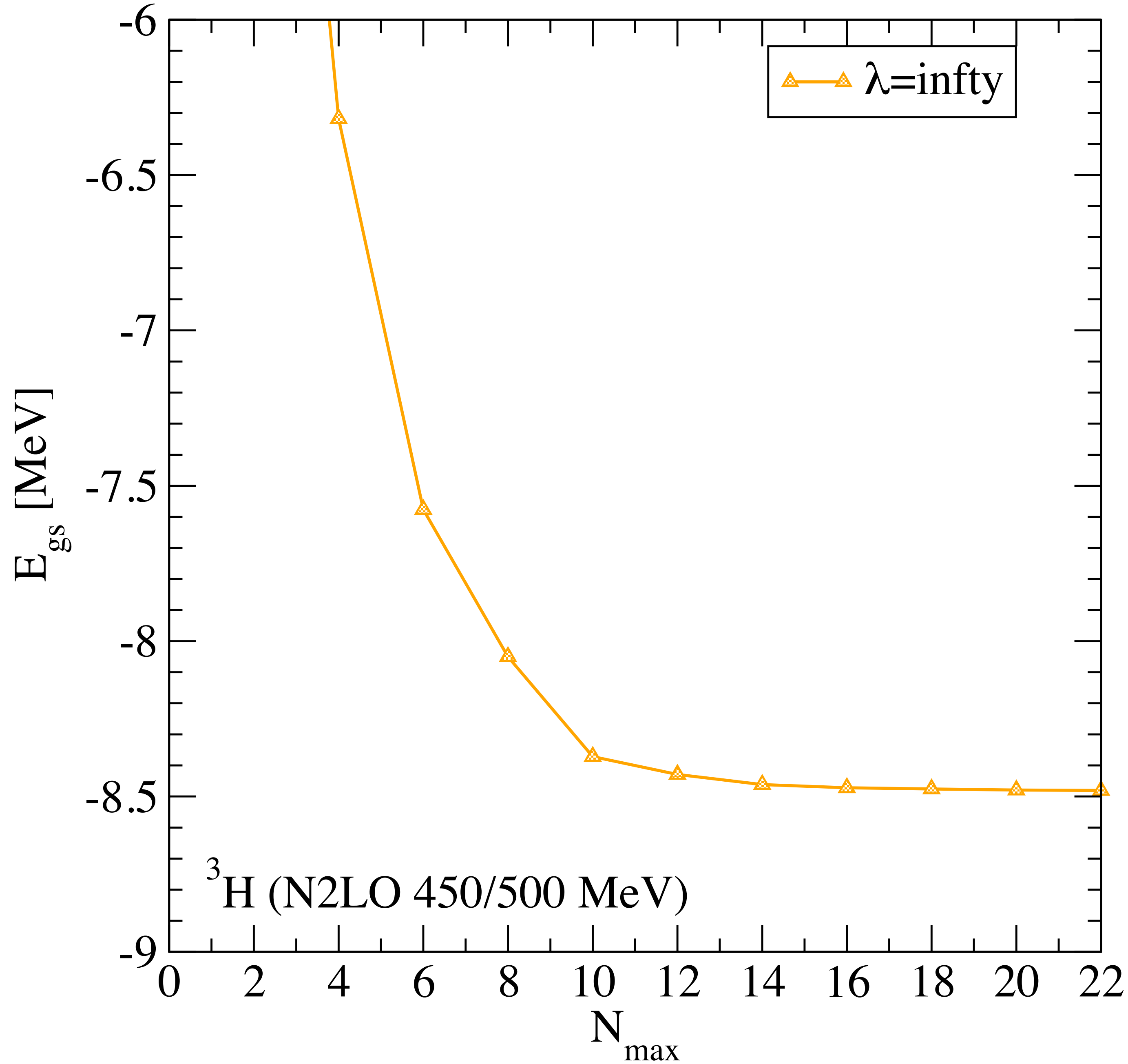

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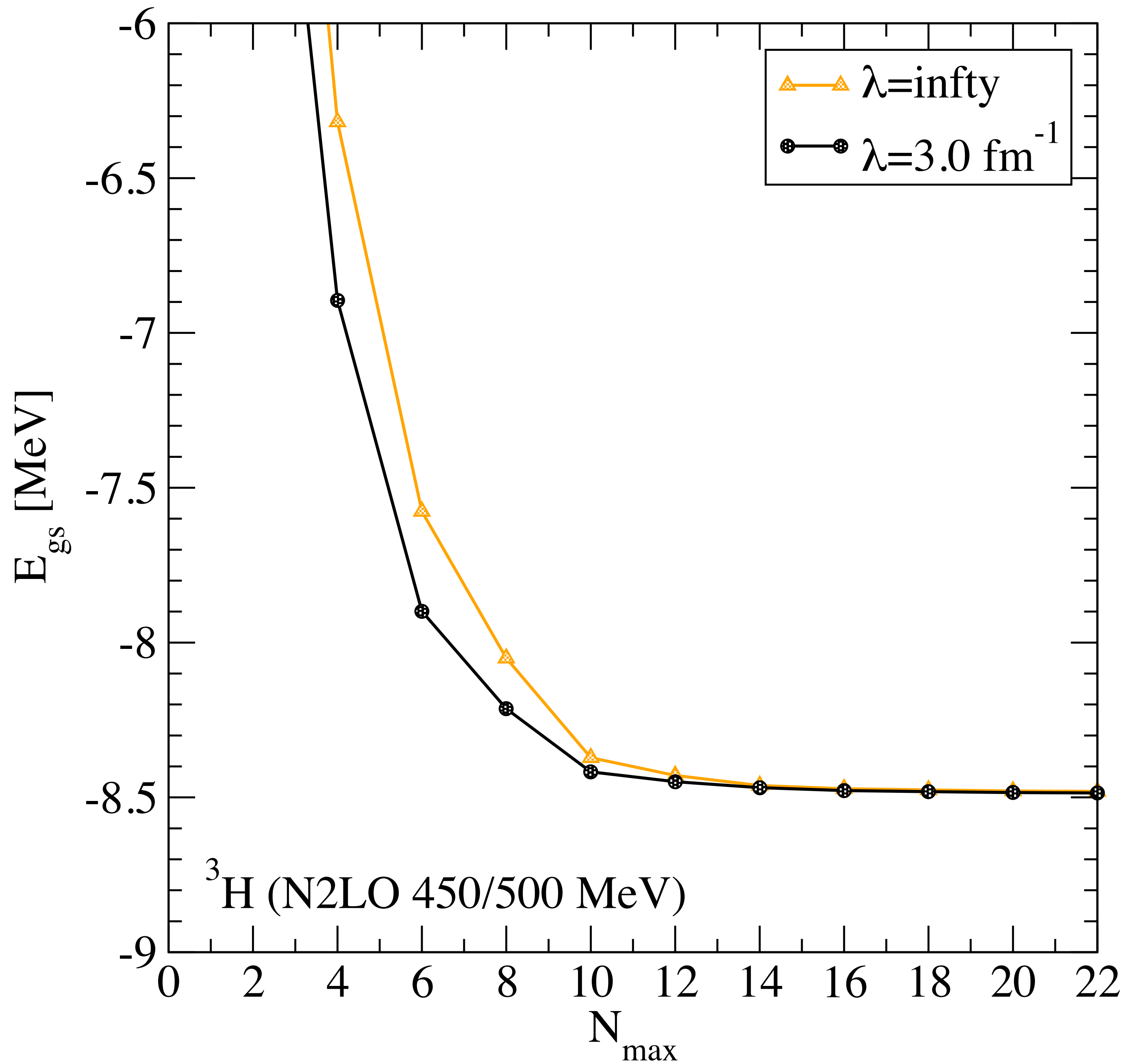

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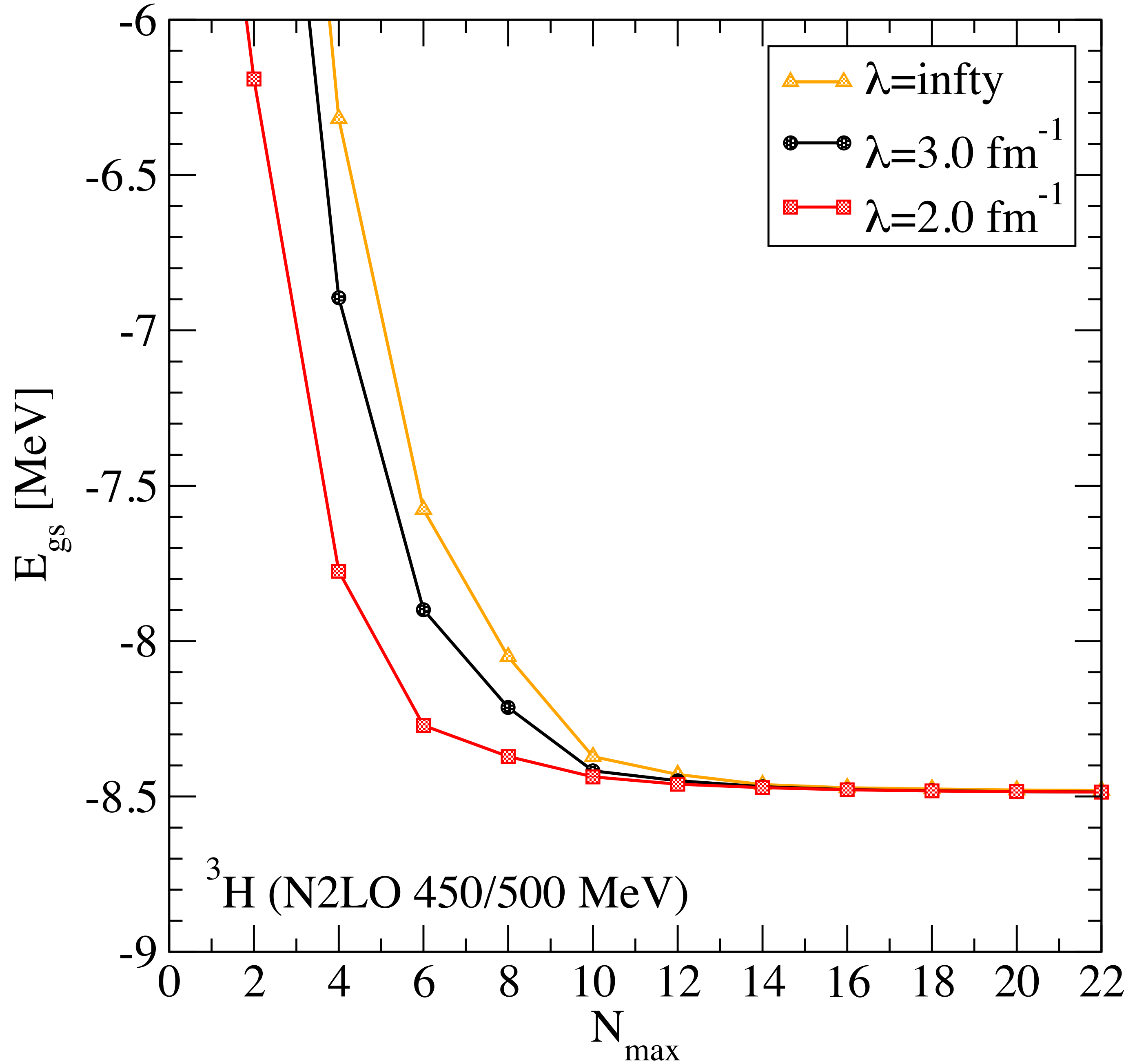

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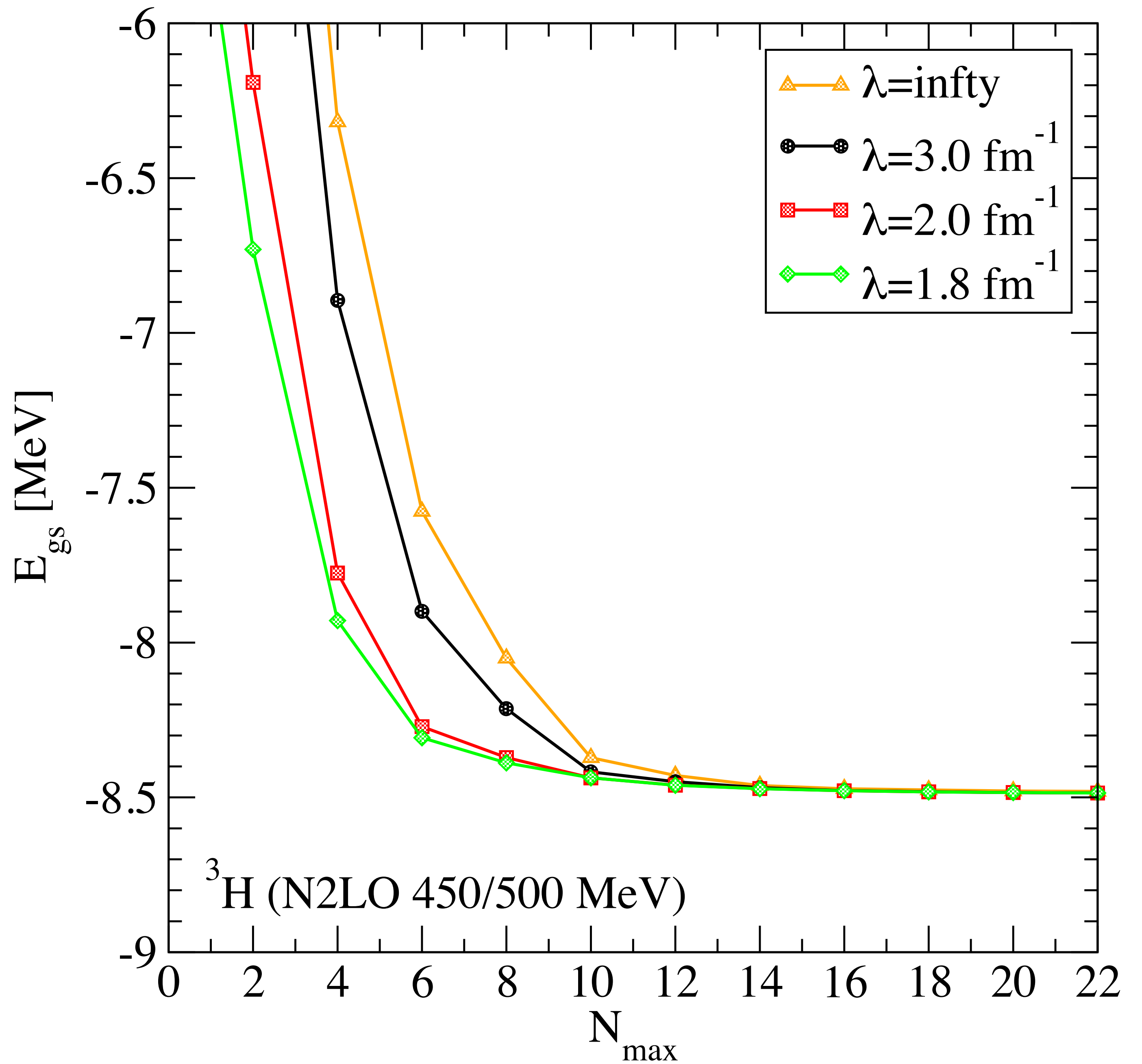

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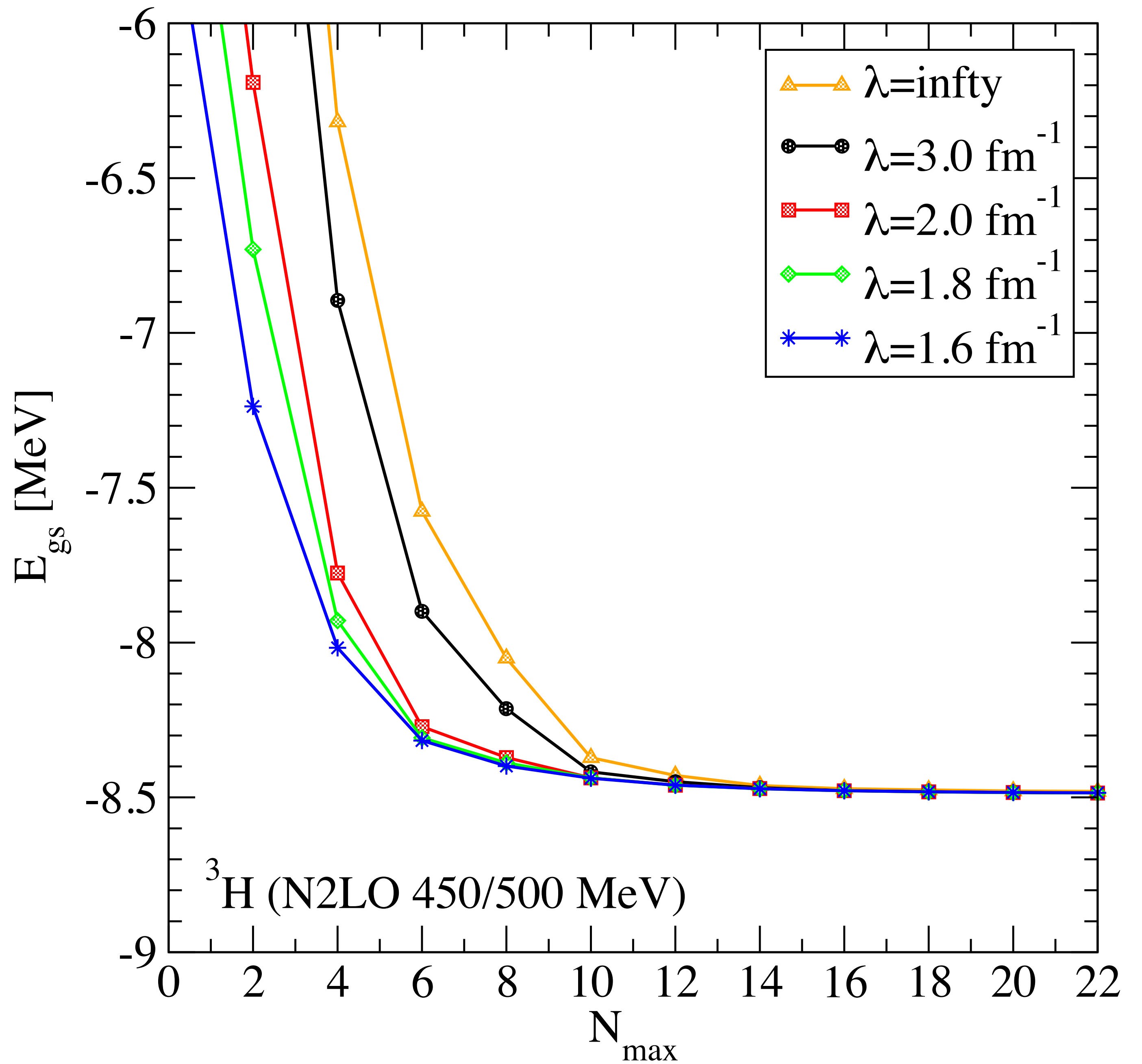

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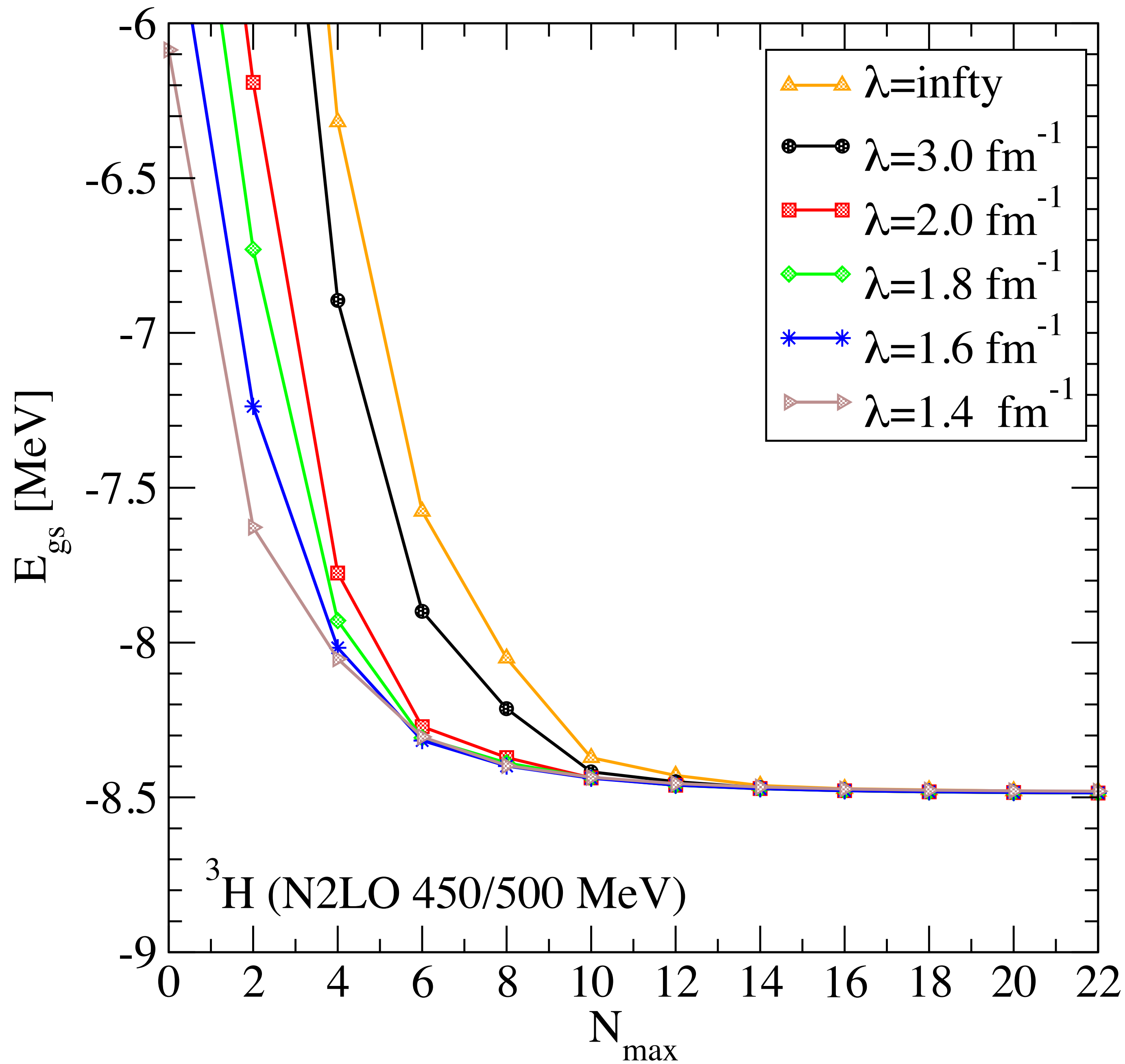

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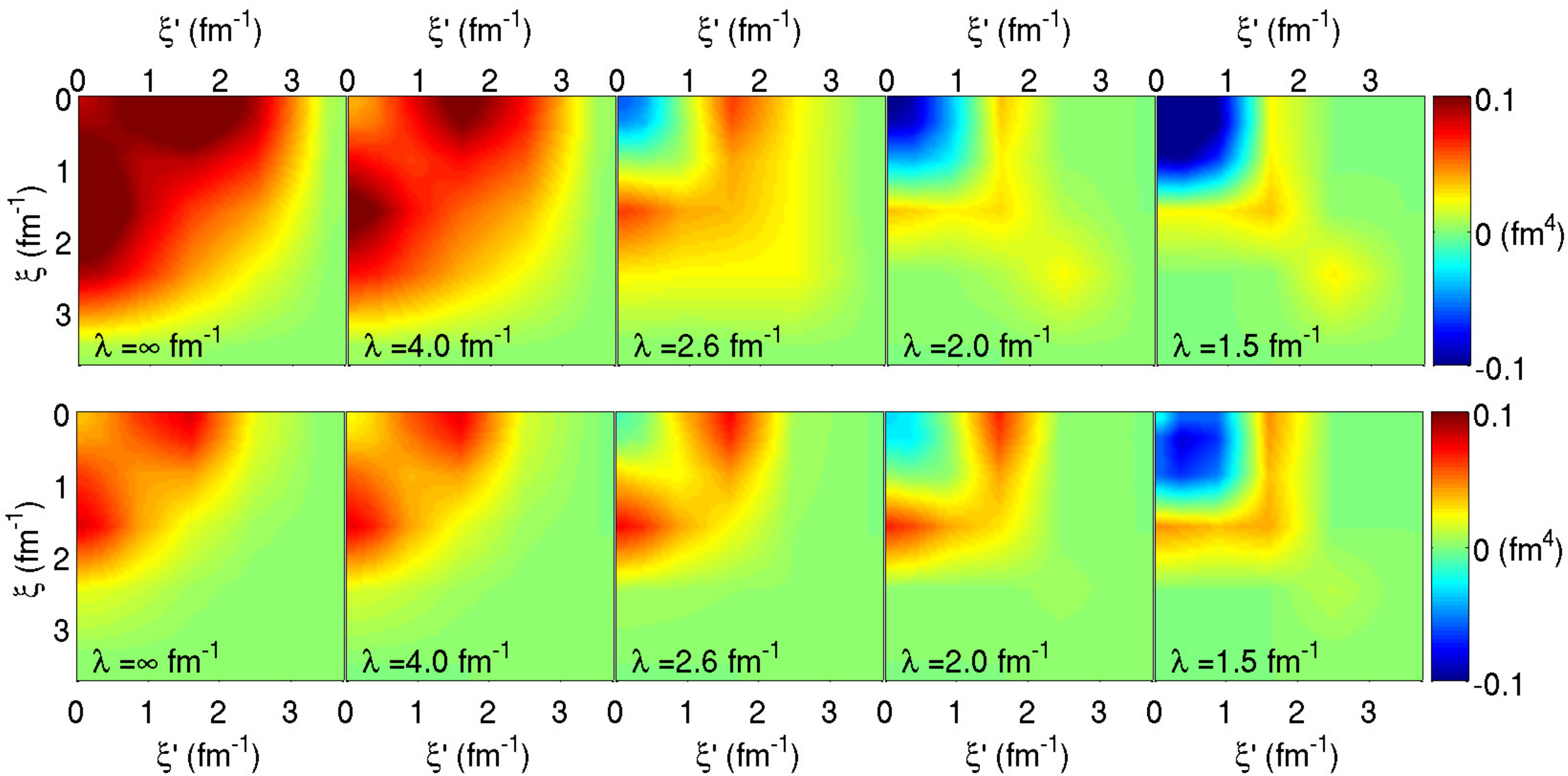
Decoupling of matrix elements

$$\theta = \frac{\pi}{12} \qquad \mathcal{T} = \mathcal{J} = \frac{1}{2}$$

$\Lambda/\tilde{\Lambda}$

$550/600\,\mathrm{MeV}$

$450/500\,\mathrm{MeV}$



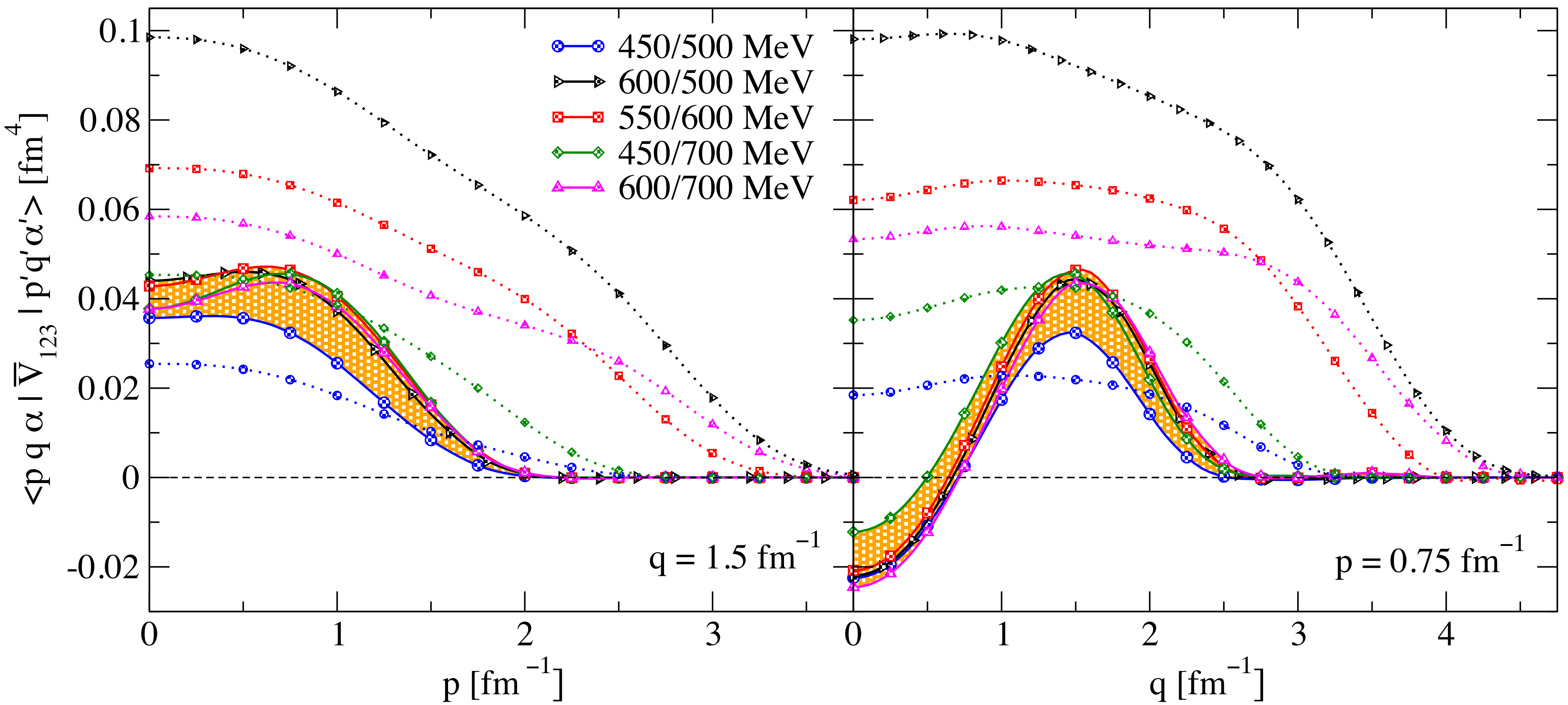
KH, PRC(R) 85, 021002 (2012)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$

hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

same decoupling patterns like in NN interactions

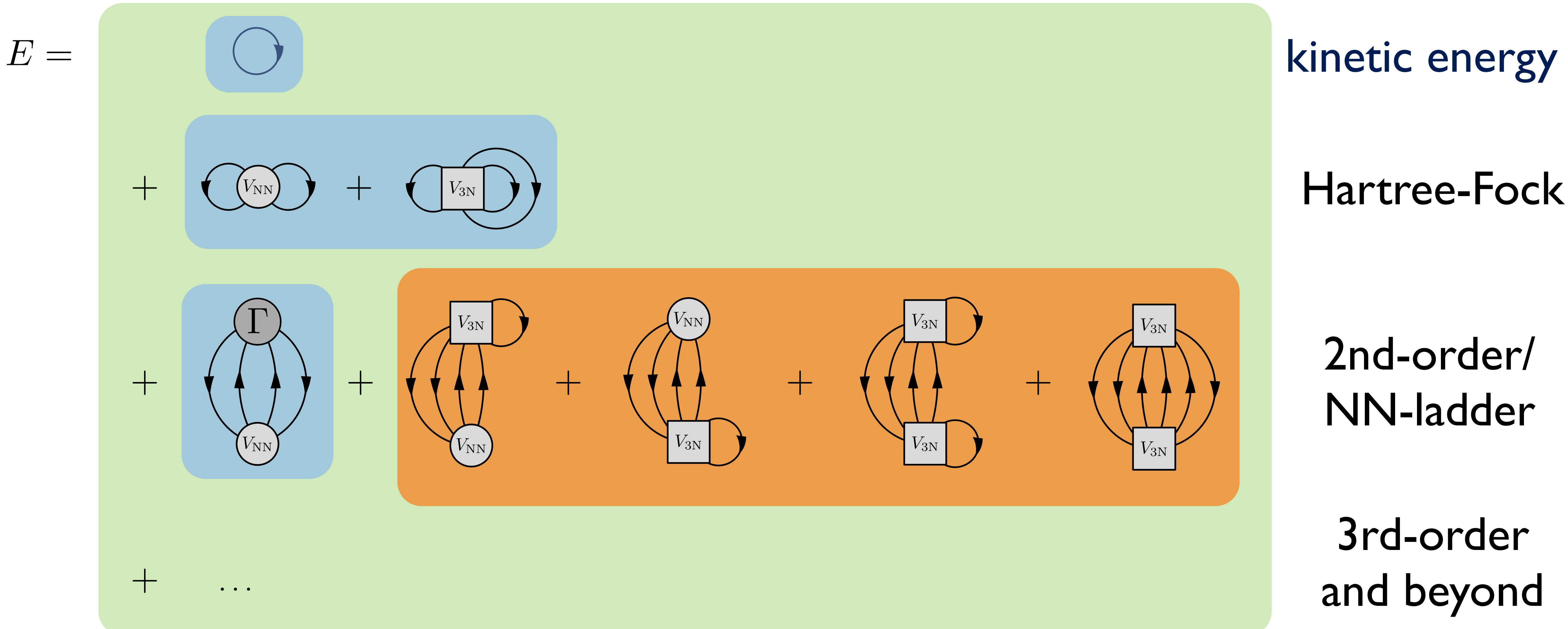
Universality in 3N interactions at low resolution



KH, PRC(R) 85, 021002 (2012)

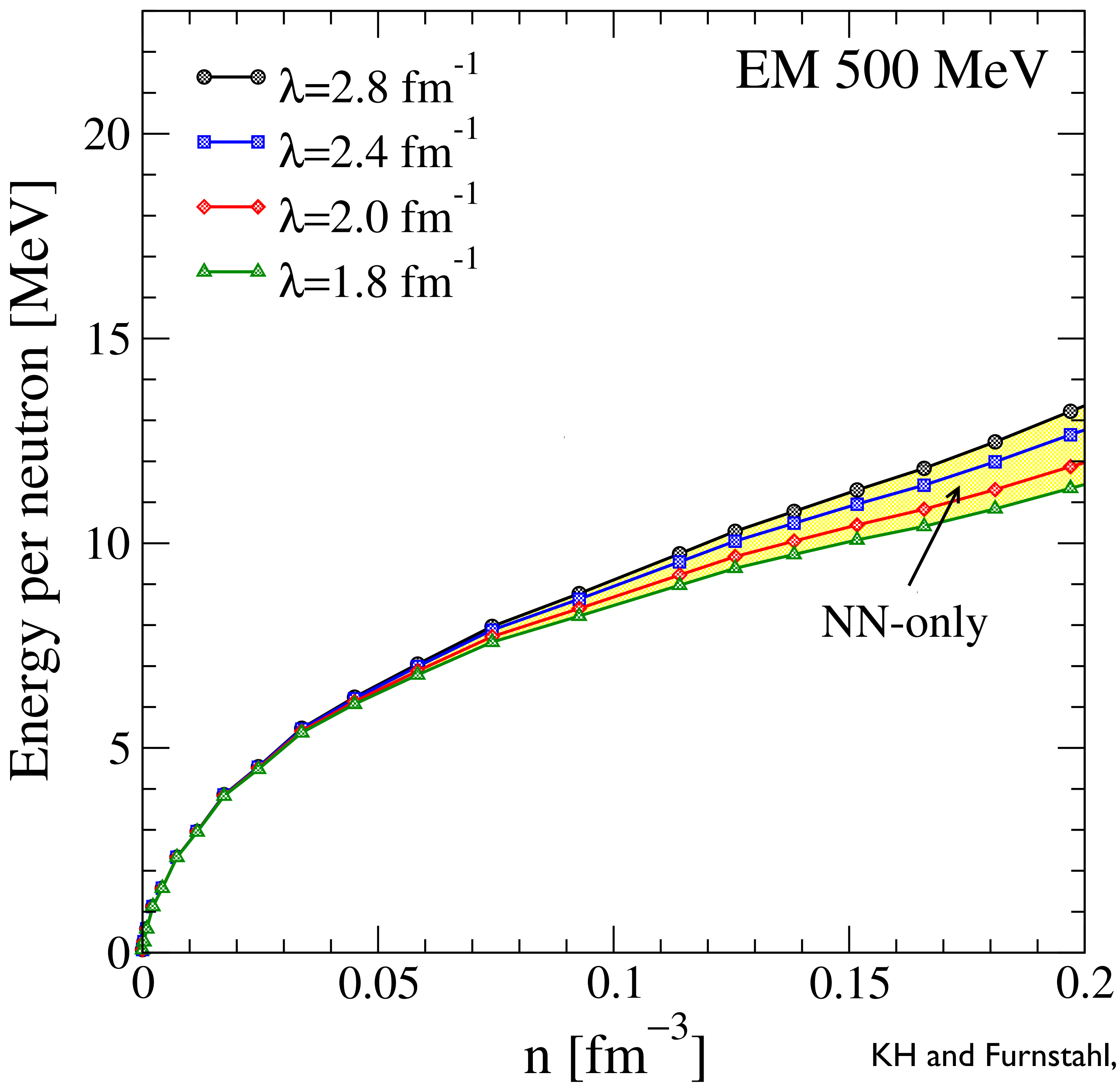
- remarkably reduced scheme dependence for typical momenta $\sim 1\,\mathrm{fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution
- new momentum structures induced at low resolution
- study based on $\mathrm{N}^2\mathrm{LO}$ chiral interactions, improved universality at $\mathrm{N}^3\mathrm{LO}$?

First application to neutron matter: Equation of state



- evolve consistently NN + 3NF in the isospin $\mathcal{T} = 3/2$ channel
- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

First results for neutron matter



KH and Furnstahl, PRC 87, 031302(R) (2013)

- all partial waves included up to $\mathcal{J} = 9/2$ in SRG evolution and EOS calculation
- consistent 3NF with $c_1 = -0.81\,\mathrm{GeV}^{-1}$ and $c_3 = -3.2\,\mathrm{GeV}^{-1}$

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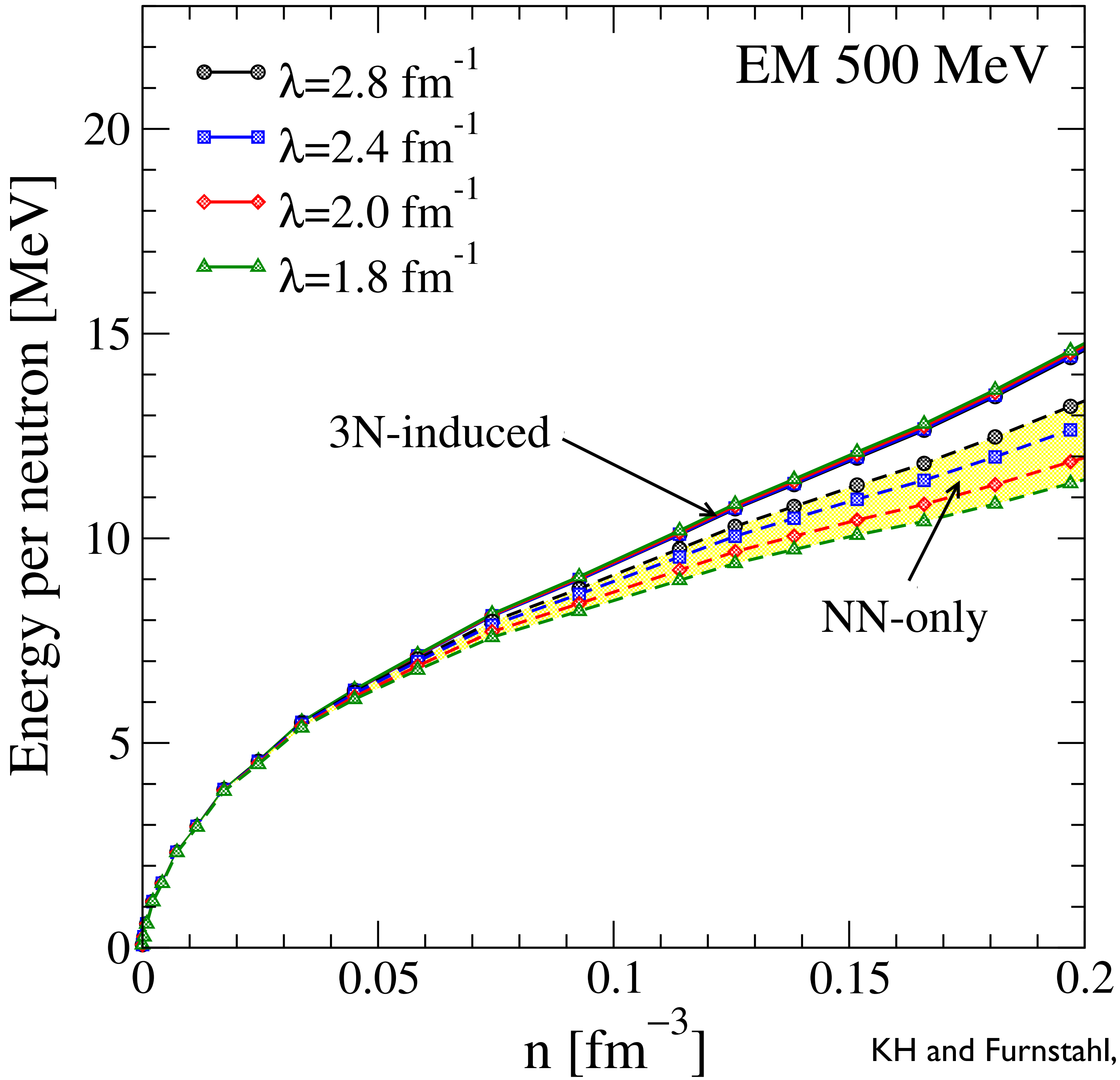

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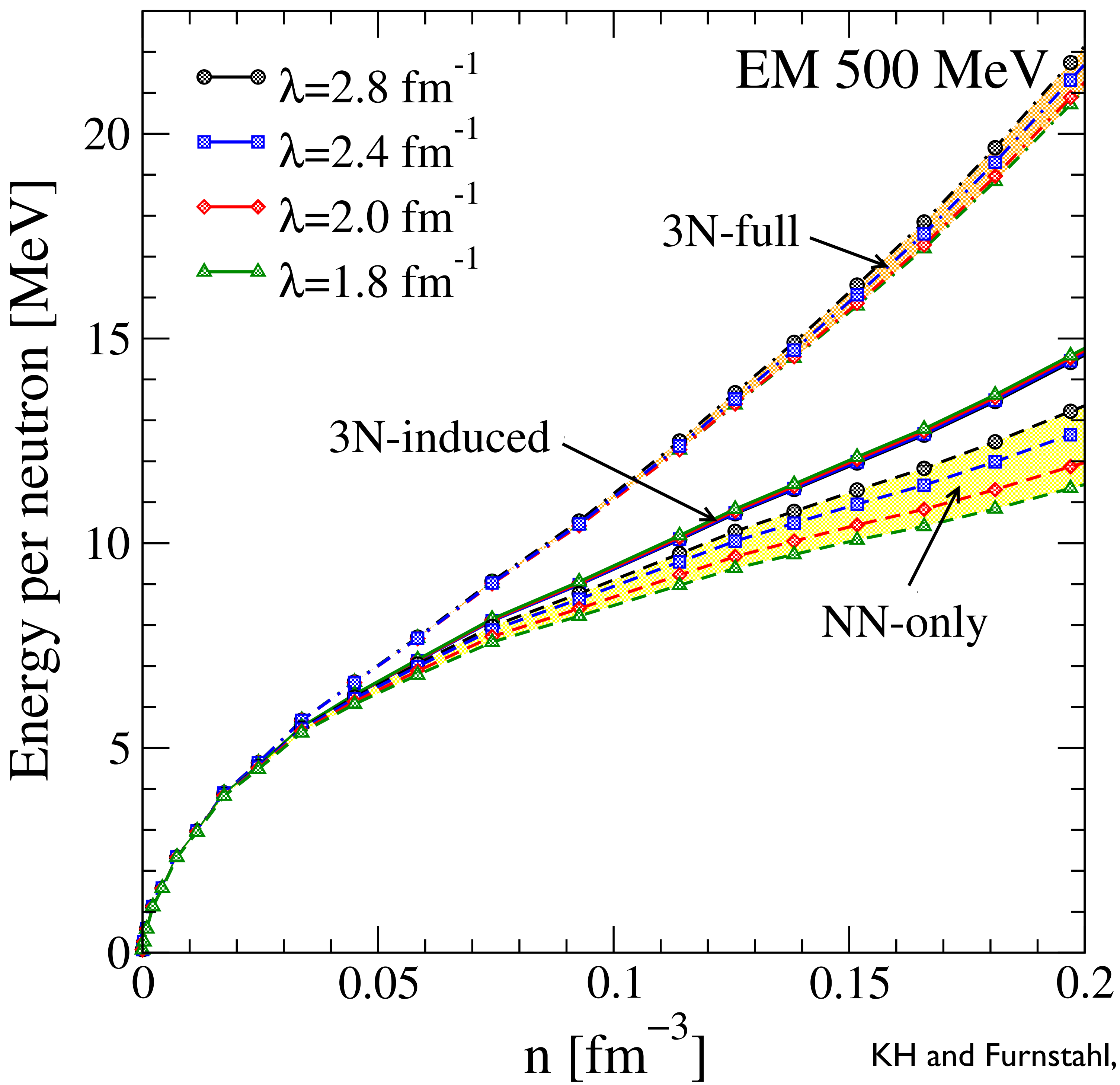

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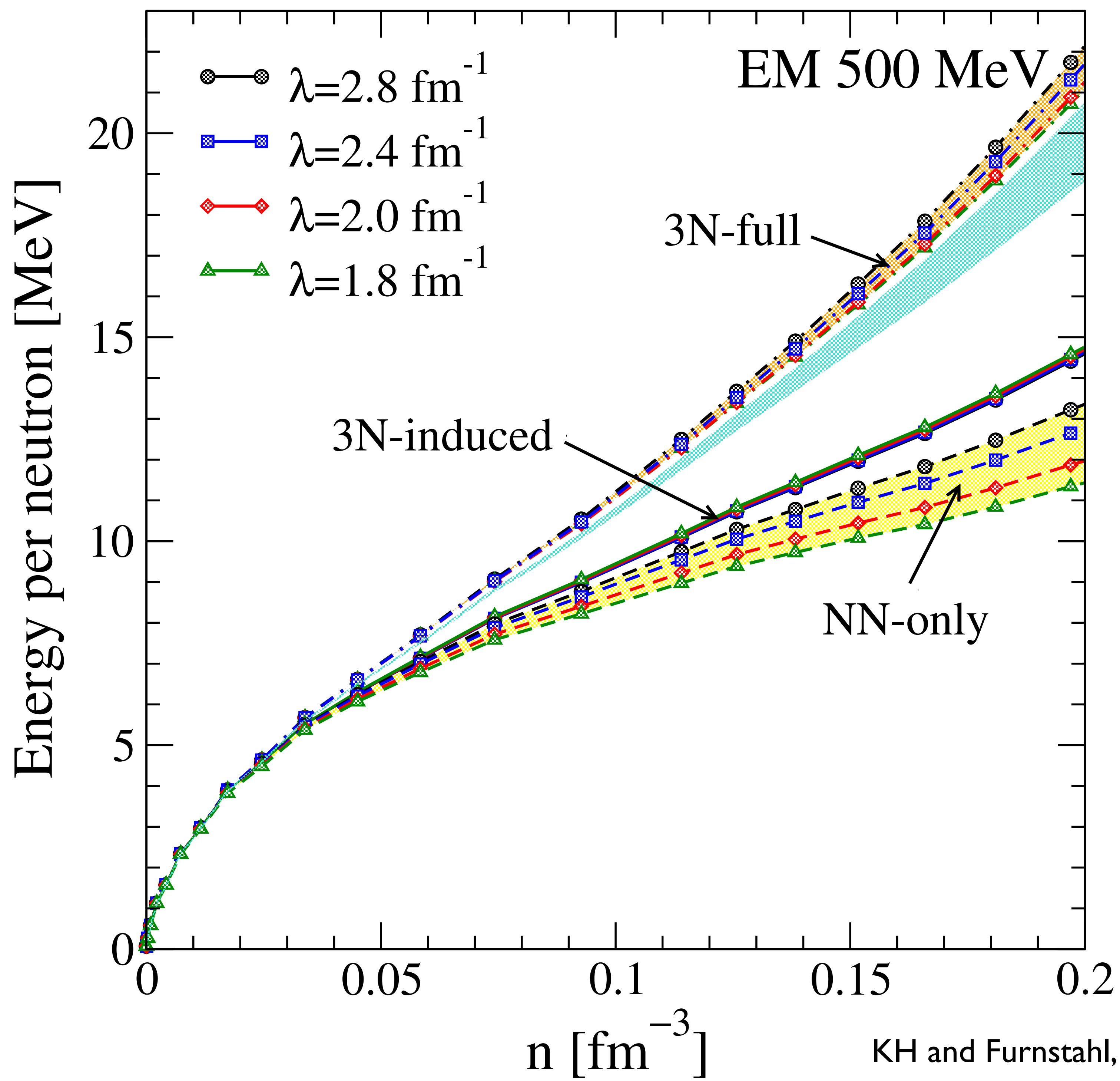

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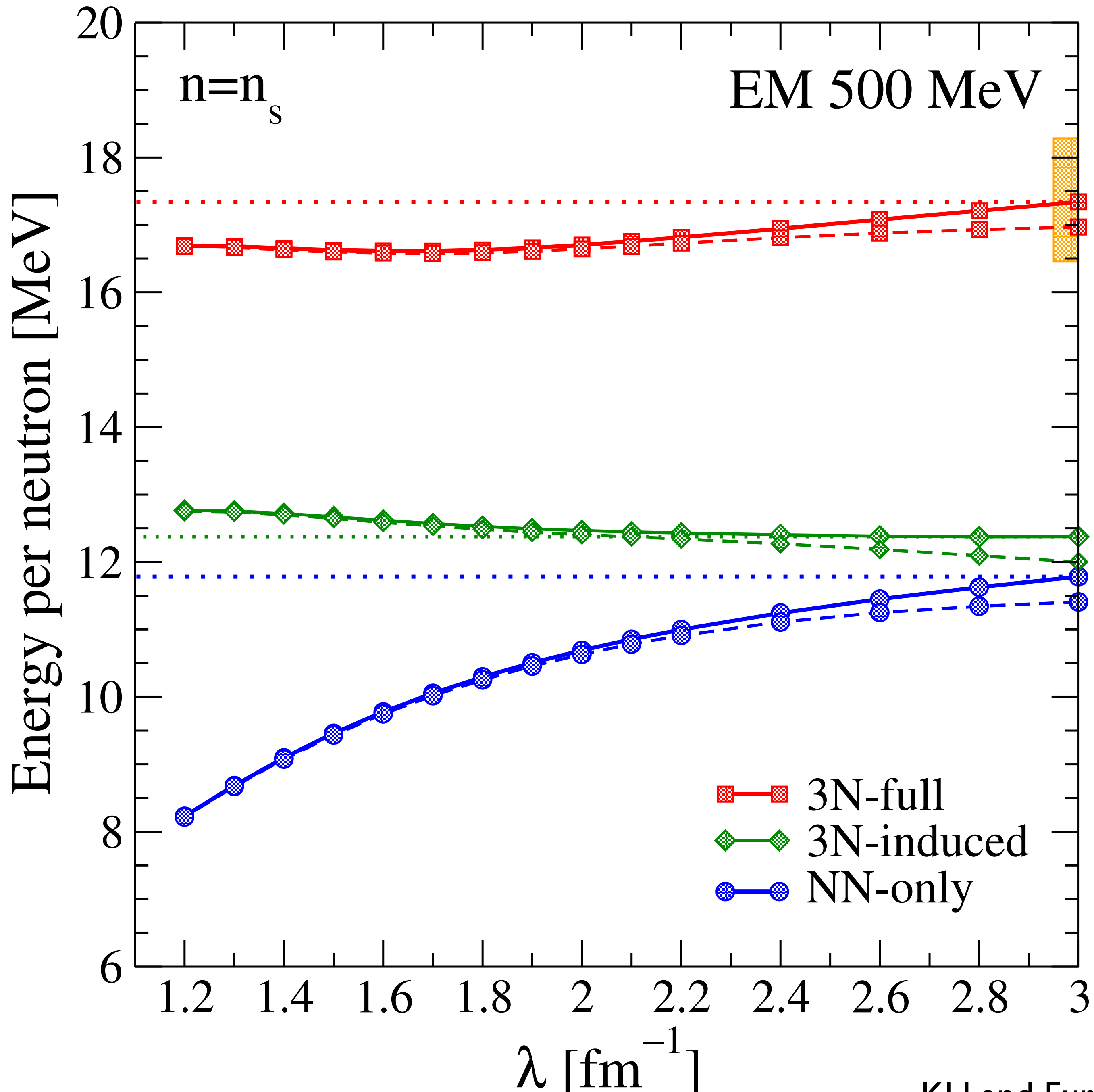
Resolution-scale dependence at saturation density

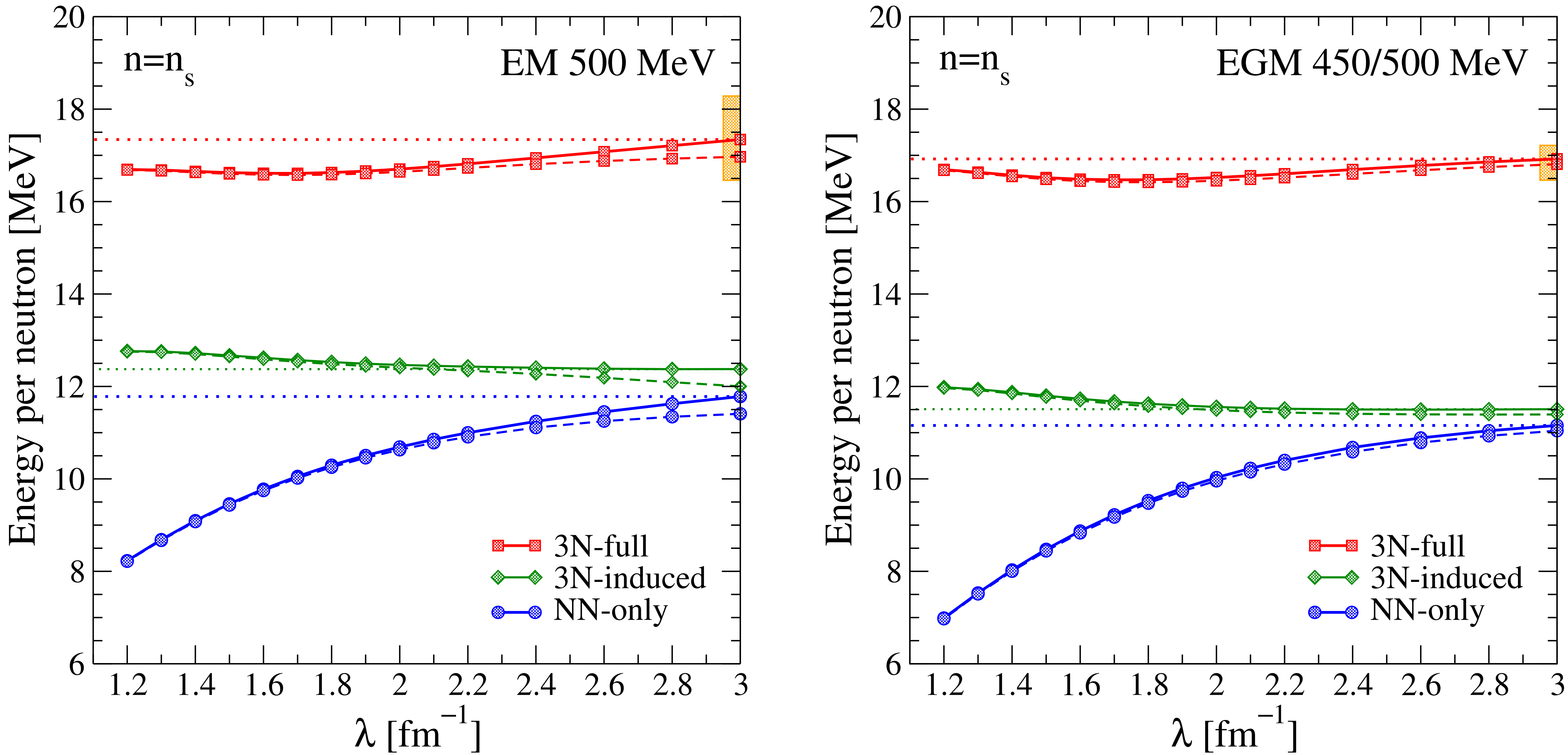


2nd order NN-3N contributions at $\lambda = \infty$

KH and Furnstahl, PRC 87, 031302(R) (2013)

- solid lines: NN resummed, dashed lines: 2nd order
- variations: NN-only 3.6 MeV, 3N-induced: 390 keV, 3N-full: 650 keV
- indications for 4N forces at small λ?

Resolution-scale dependence at saturation density



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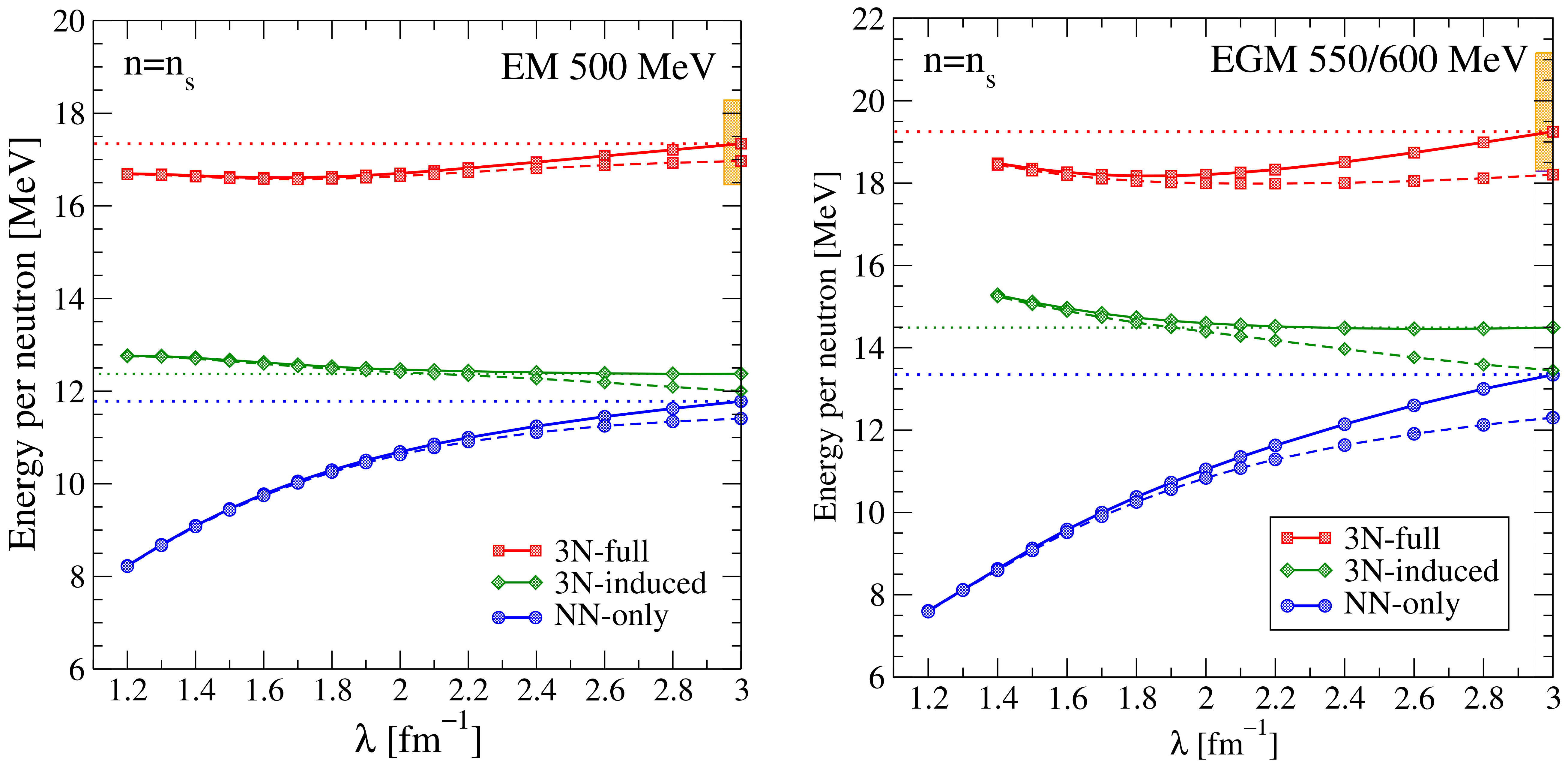

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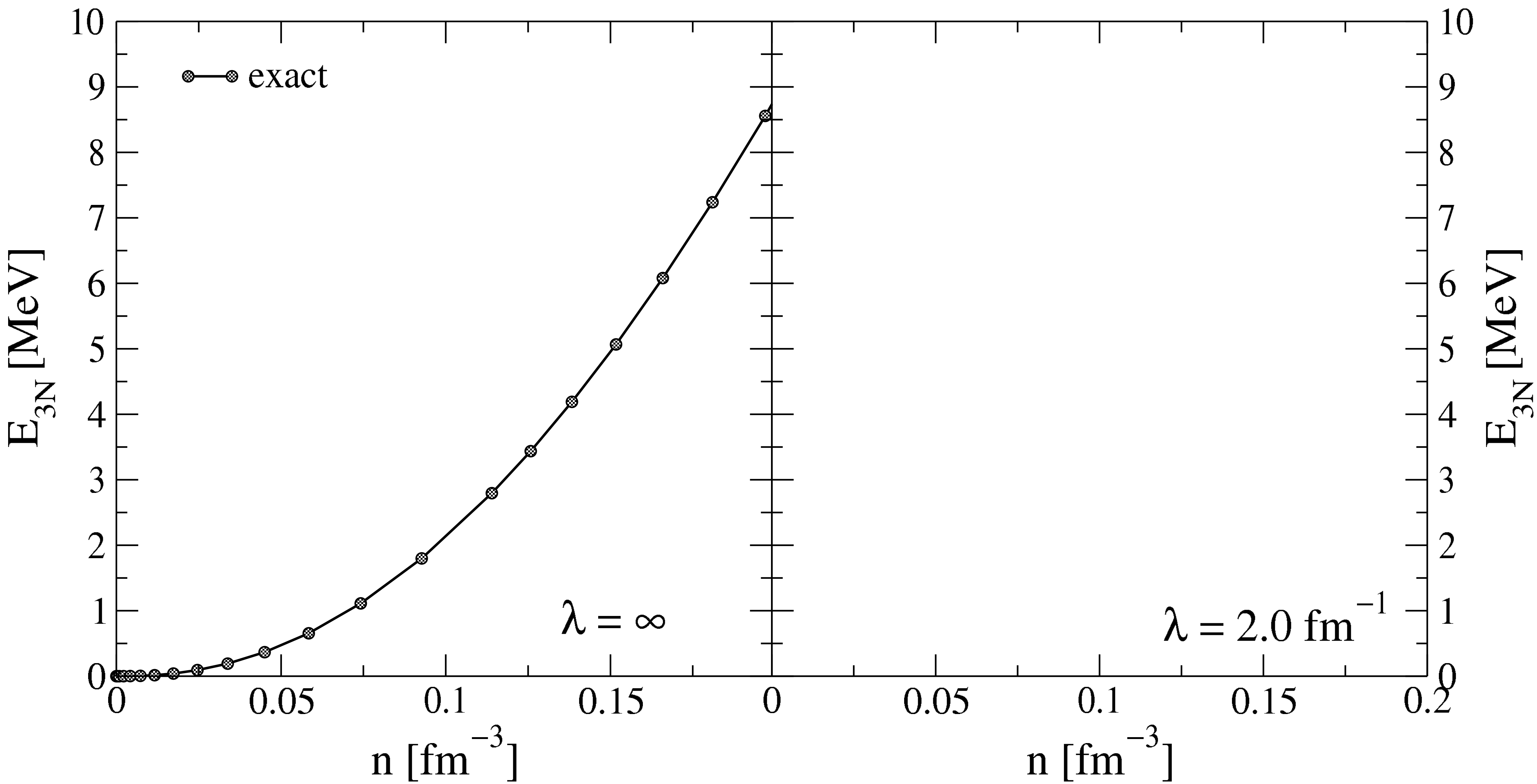

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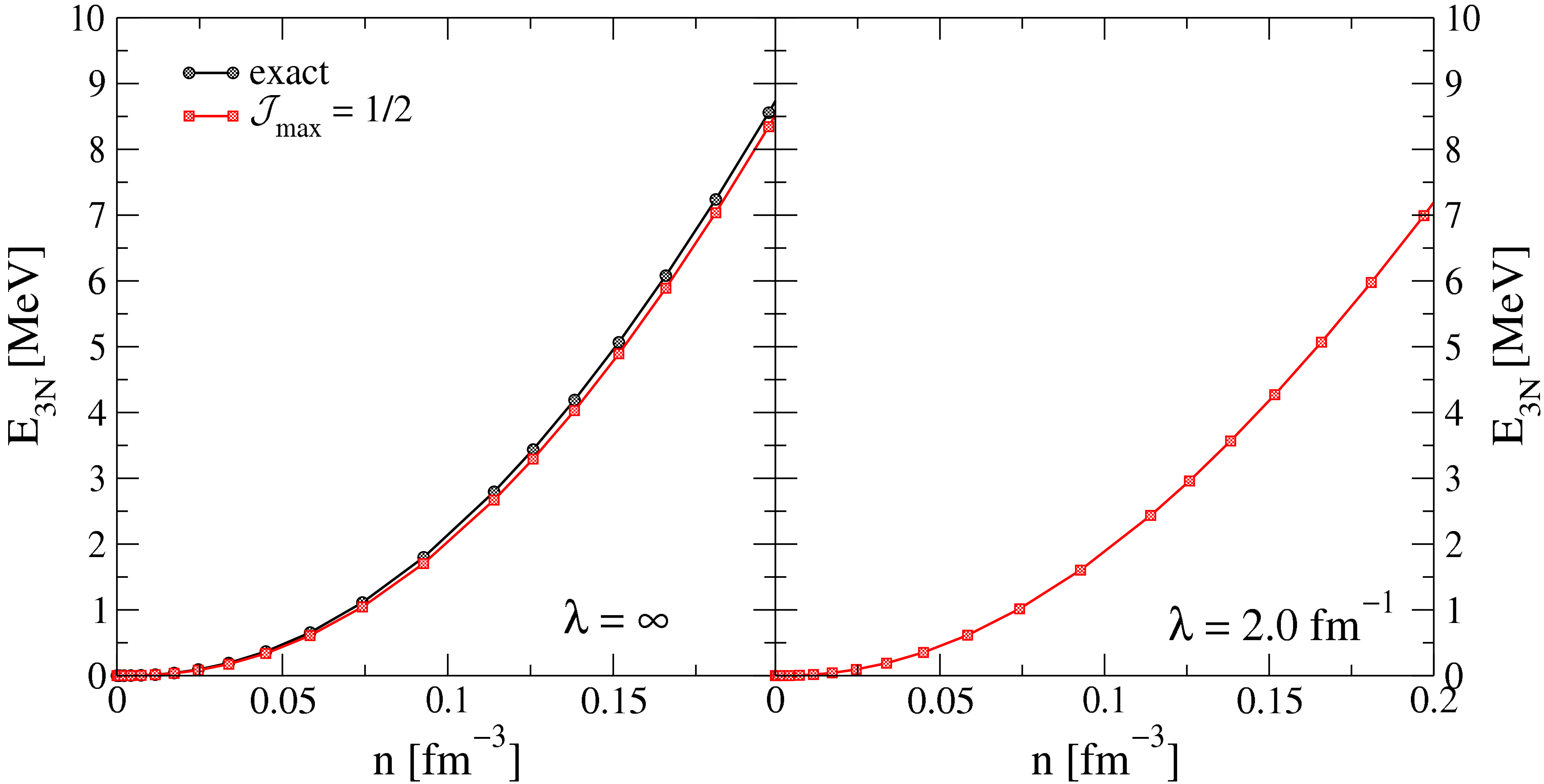

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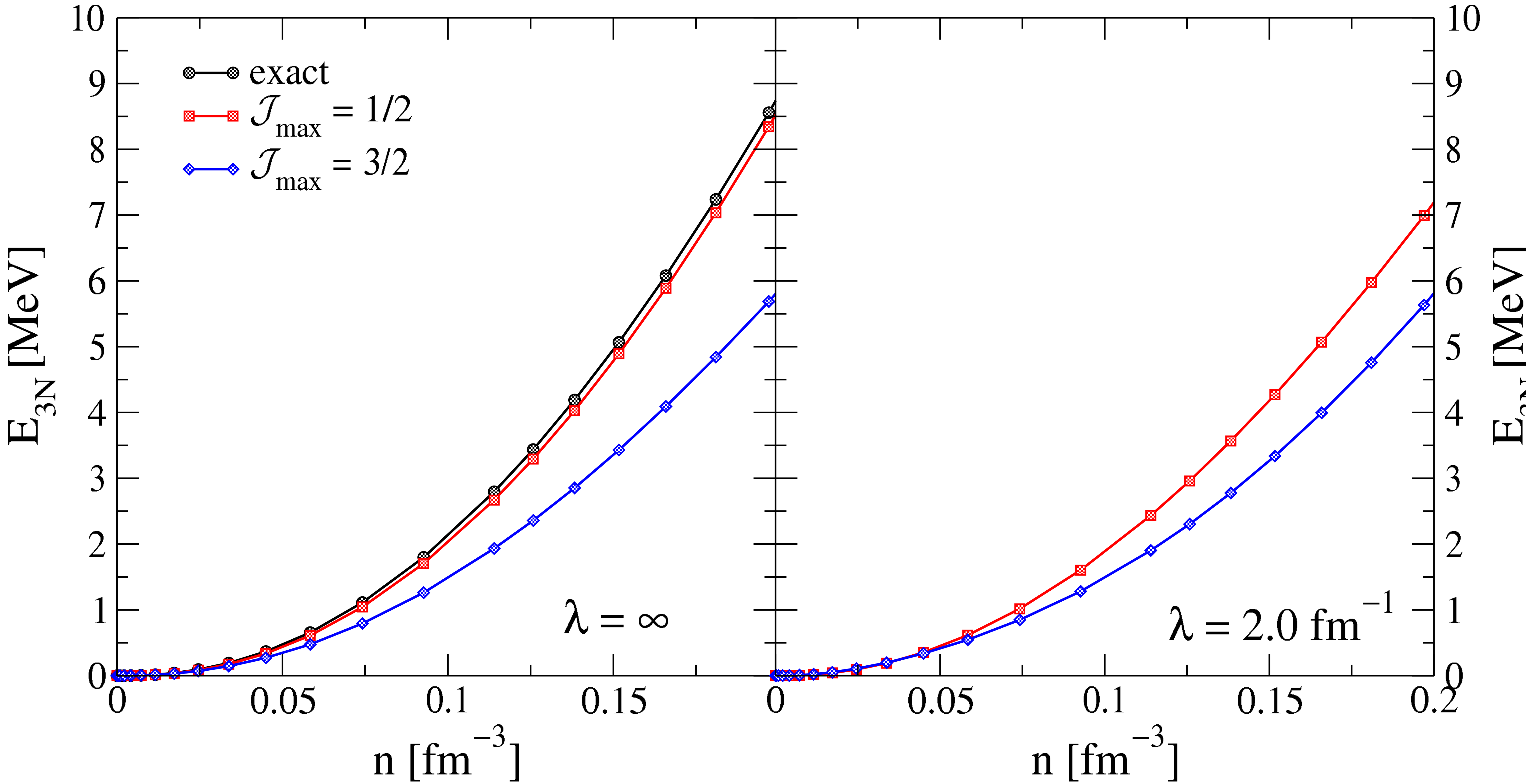

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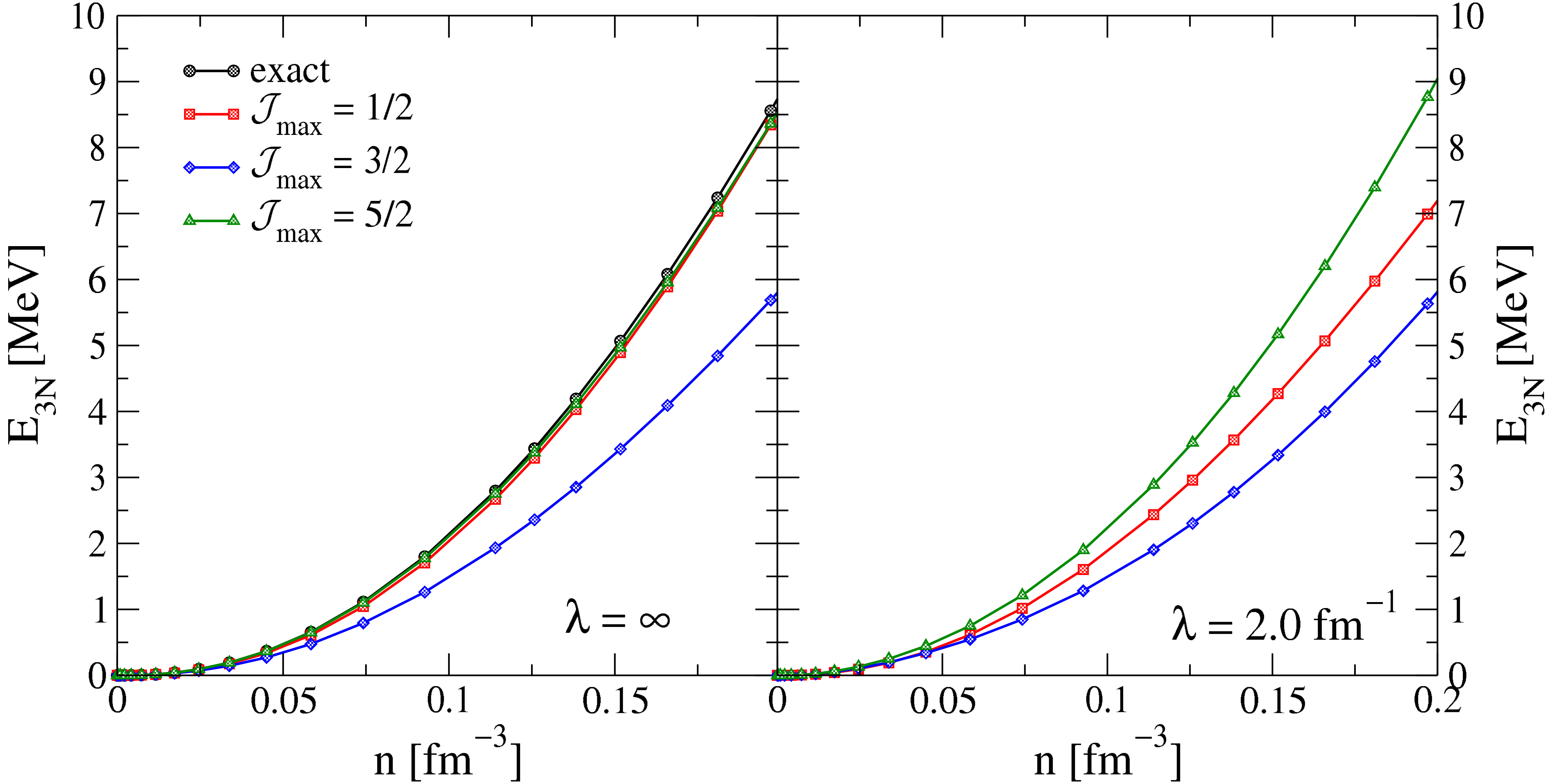

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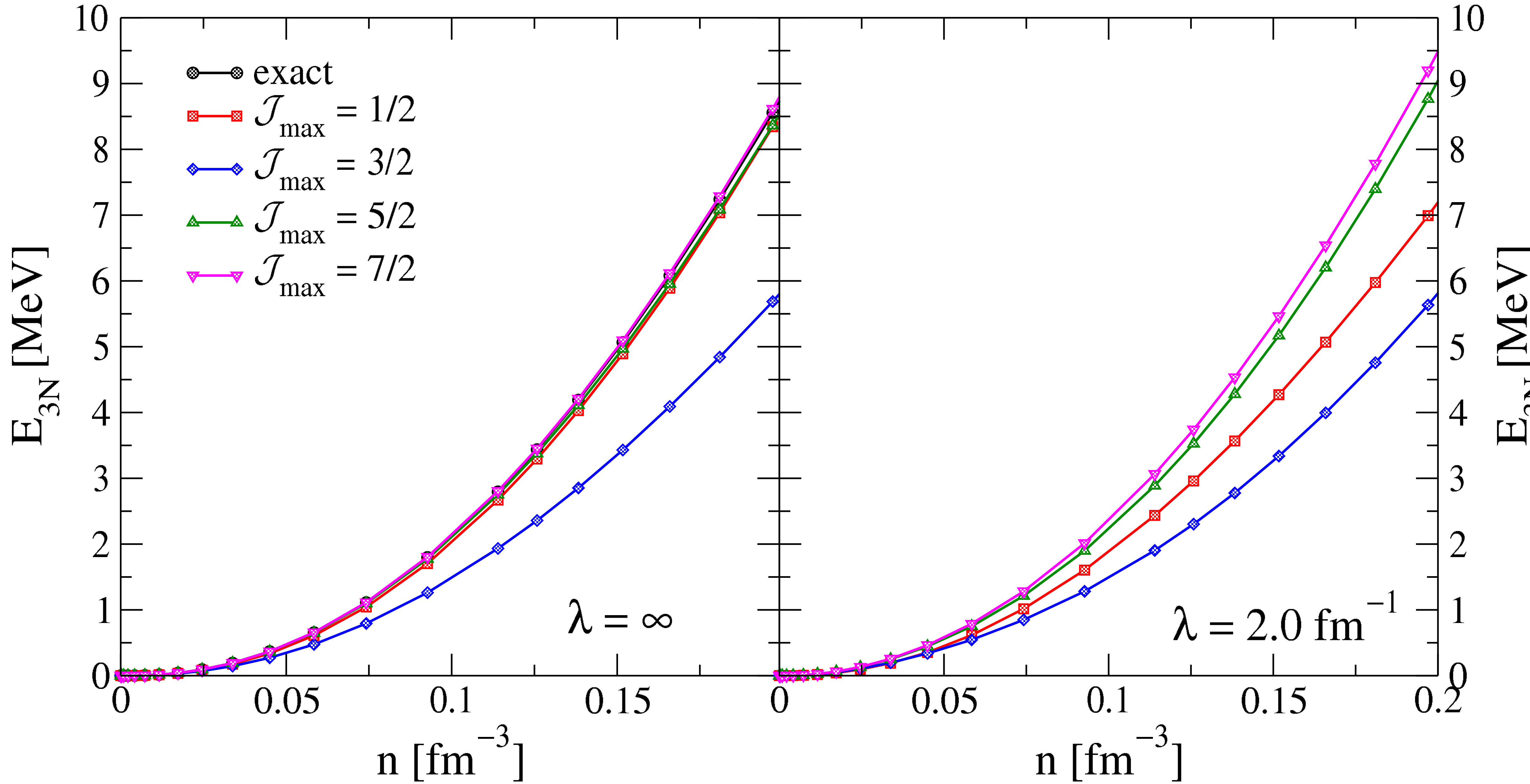

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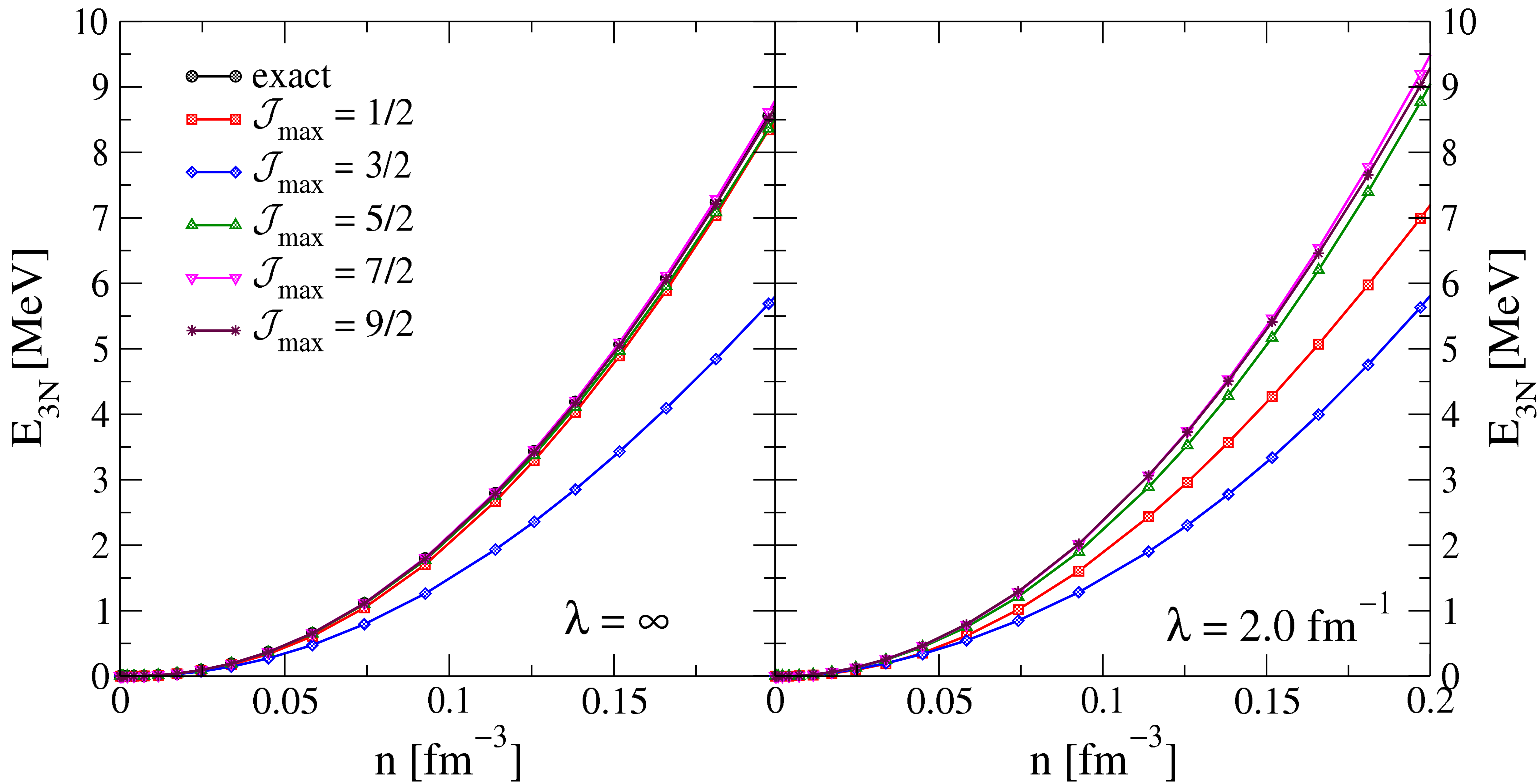


KH and Furnstahl, PRC 87, 031302(R) (2013)

- E_{3N} agrees within 0.4 % with the exact result at saturation density
- E_{3N} converged in partial waves at both scales, $\lambda = \infty$ and $\lambda = 2.0\ \mathrm{fm}^{-1}$

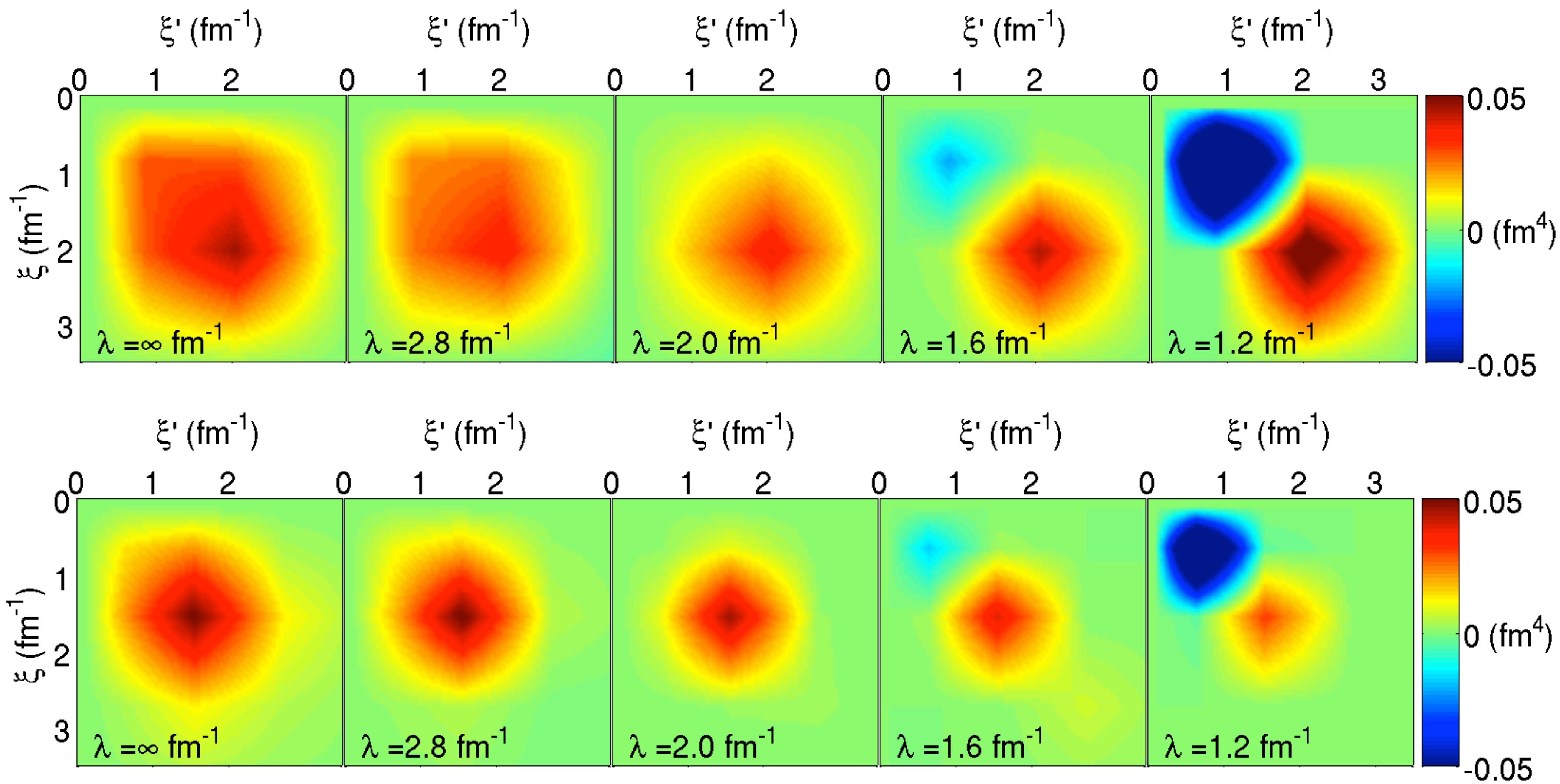
Matrix elements of evolved 3-neutron interactions (only long-ranged initially!)

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

$\theta = \dfrac{\pi}{20}$

$\theta = \dfrac{\pi}{15}$



- strong renormalization effects at very small low scales
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0\,\mathrm{fm}^{-1}$

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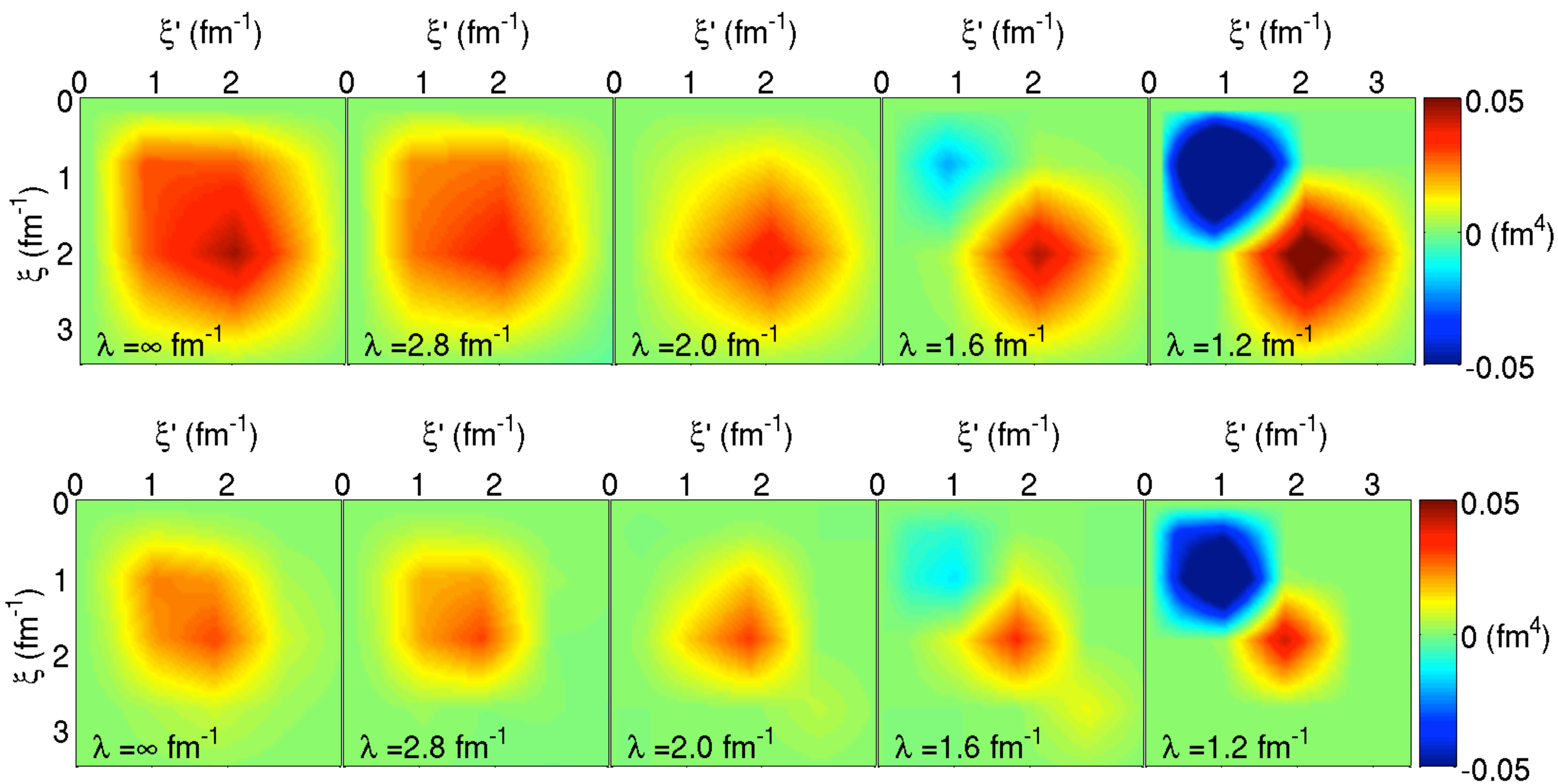

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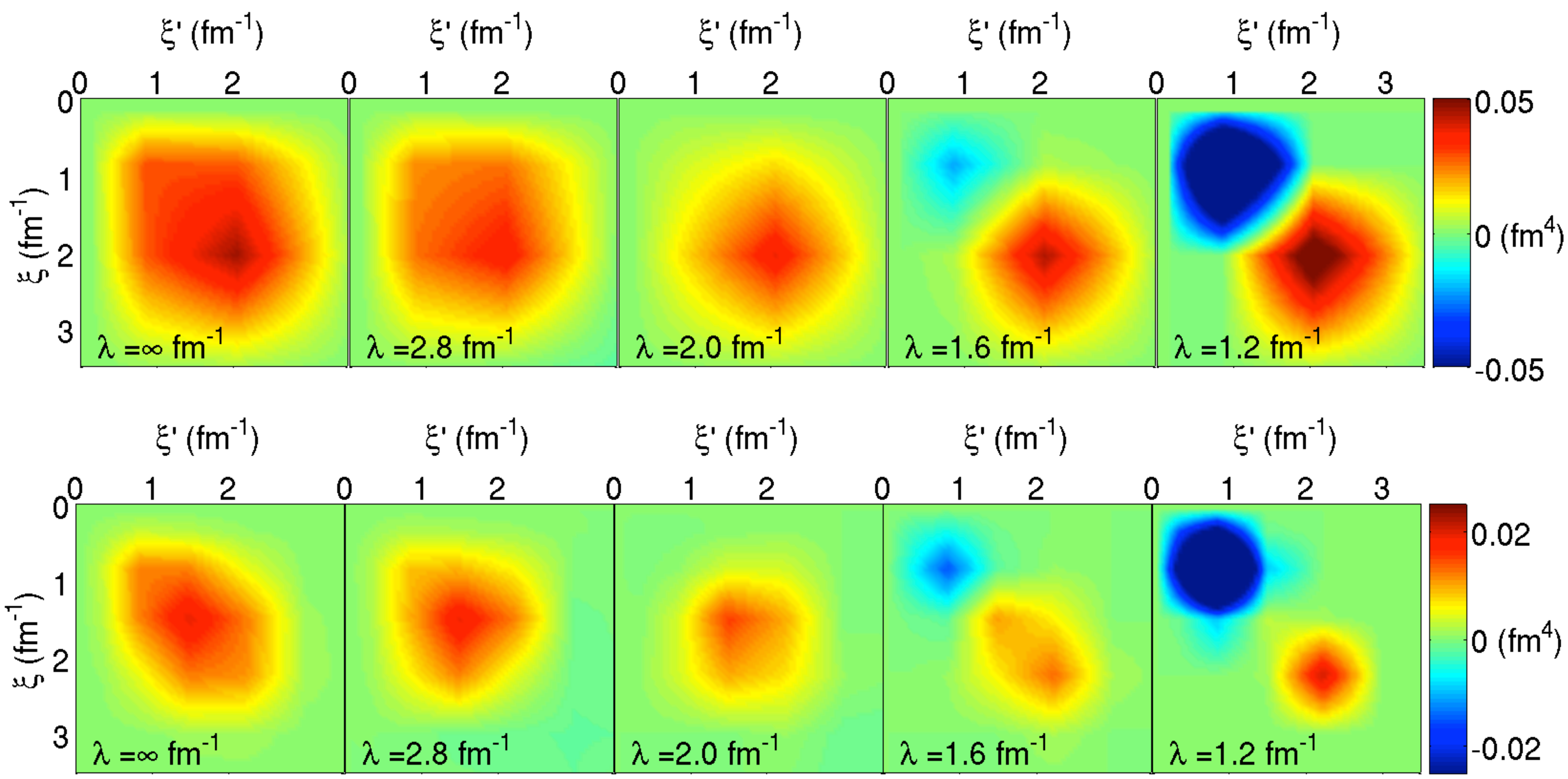
$\theta = \frac{\pi}{8}$



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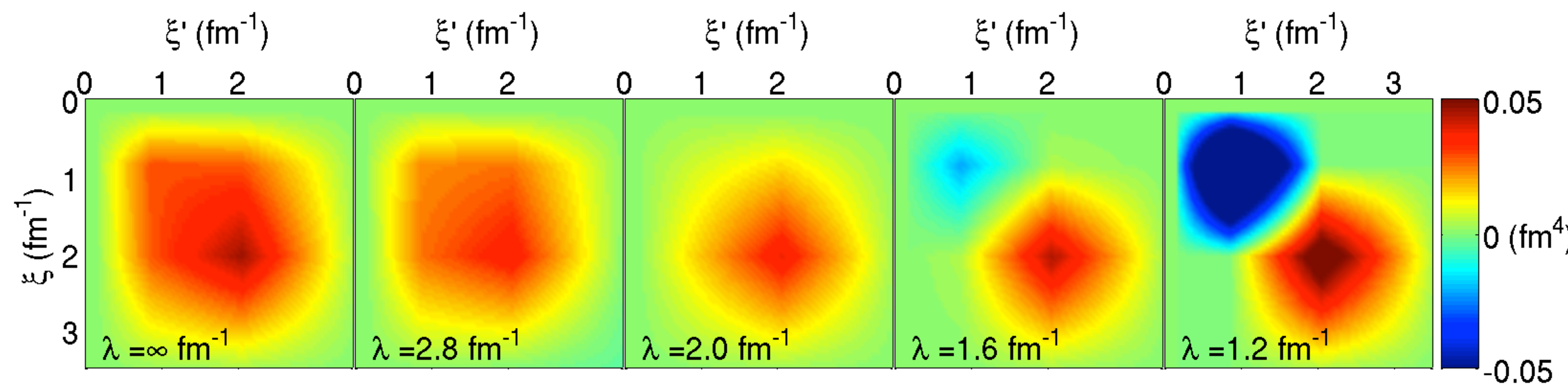
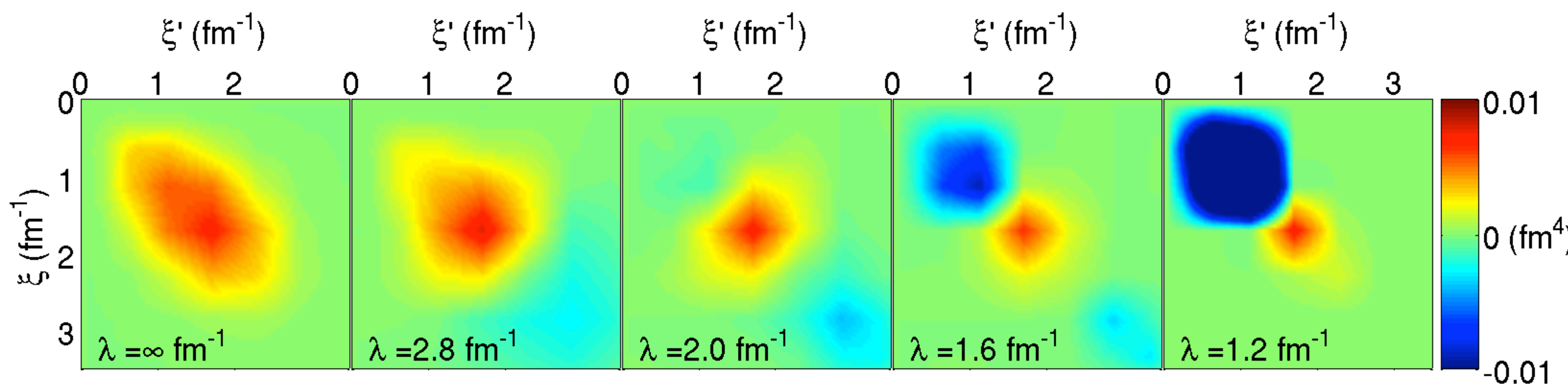


$\theta = \frac{\pi}{6}$



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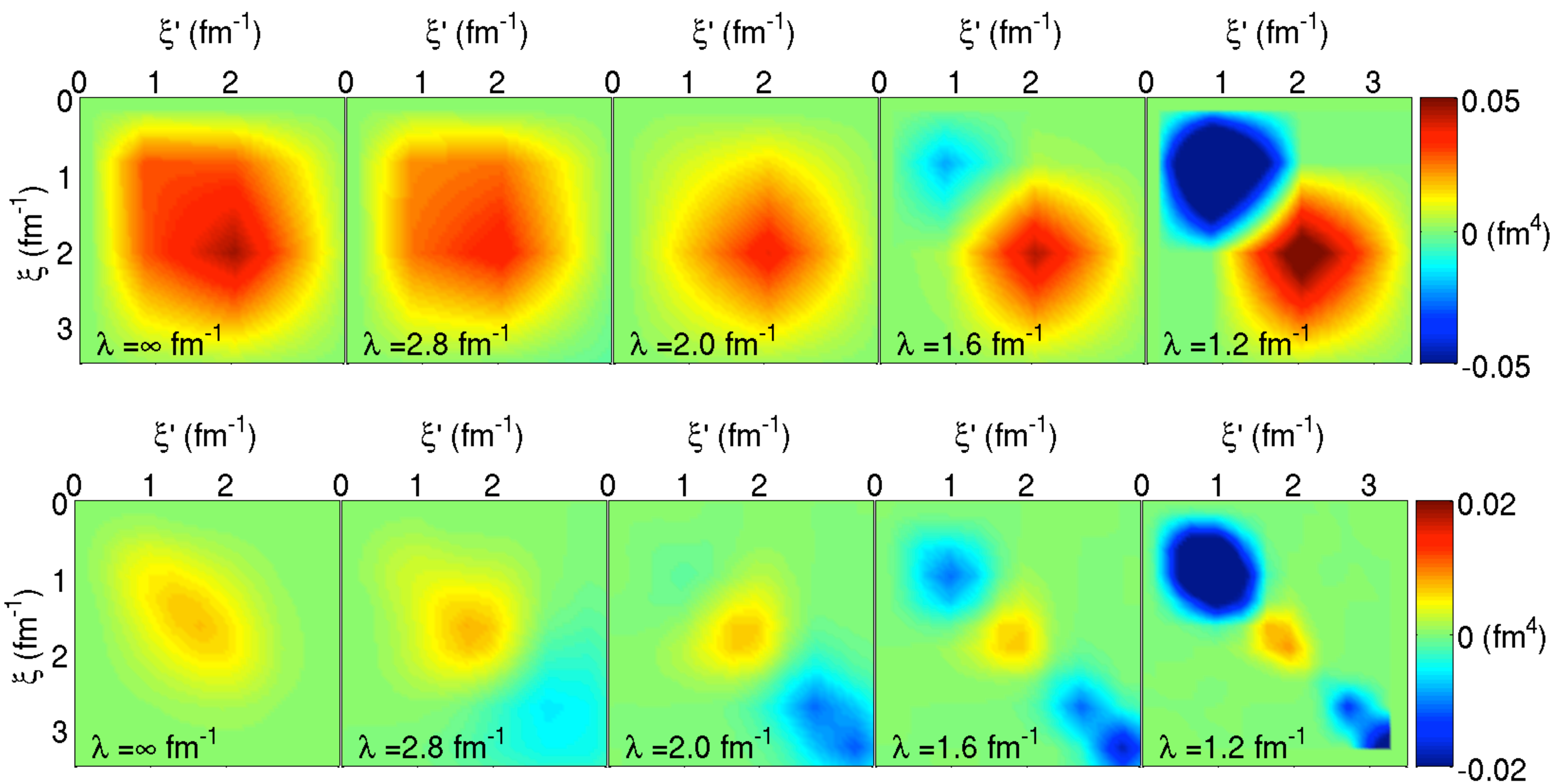
Matrix elements of evolved 3-neutron interactions (only long-ranged initially!)

$$\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan\theta = \frac{2\,p}{\sqrt{3}\,q}$$

show dominant channel for $\mathcal{J} = 1/2$ and positive total parity:

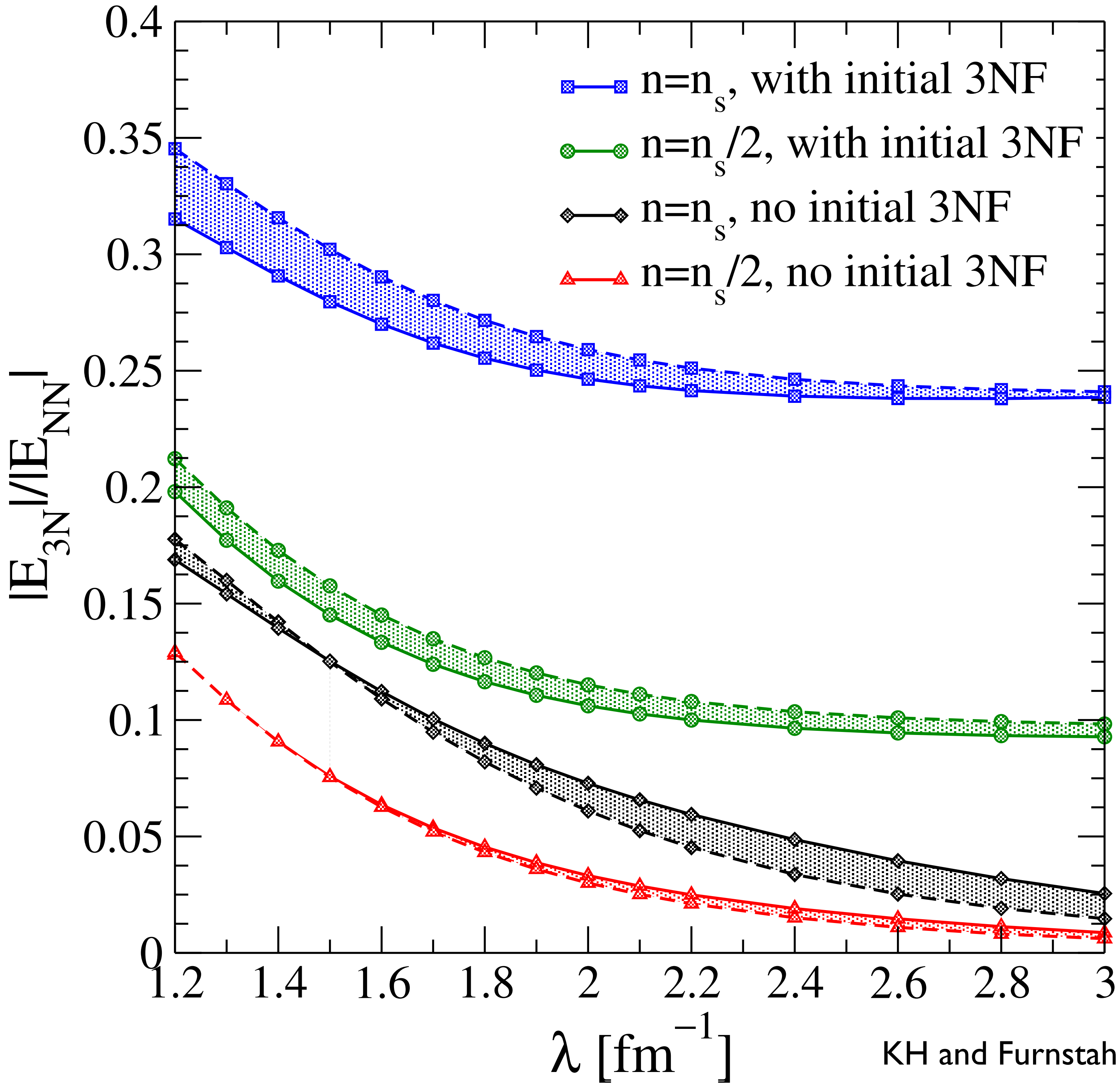
$\theta = \frac{\pi}{20}$

$\theta = \frac{\pi}{3}$



- strong renormalization effects at very small low scales
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0\,\mathrm{fm}^{-1}$

Scaling of three-body contributions



KH and Furnstahl, PRC 87, 031302(R) (2013)

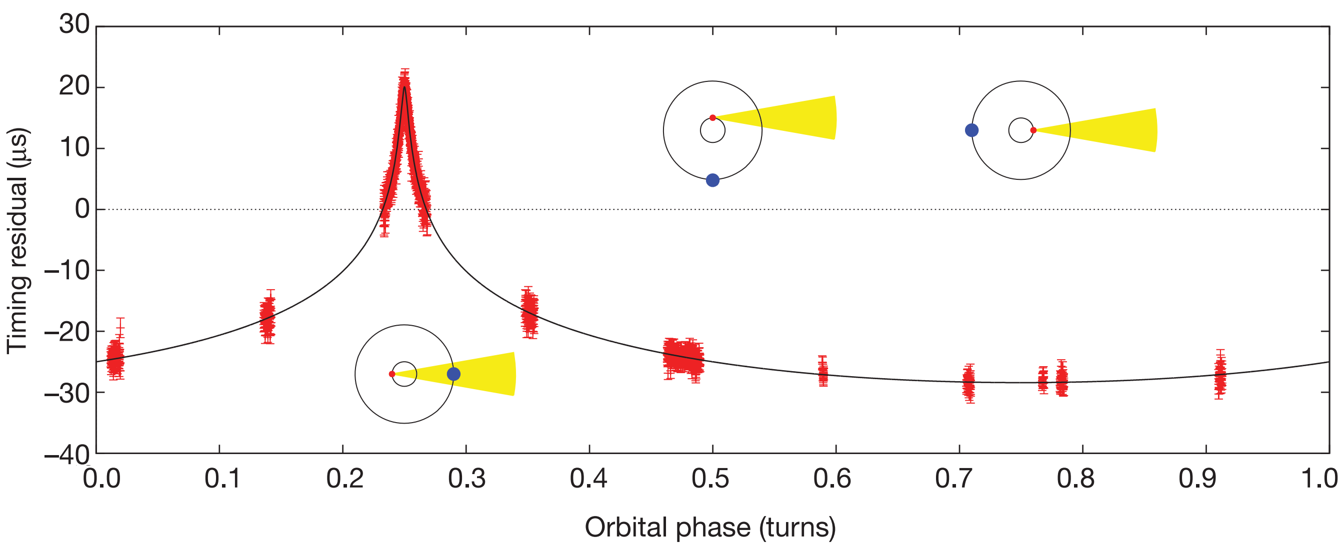
- relative size of 3N contribution grows systematically towards smaller λ
- no obvious trend with density (may be obscured by cancellations among contributions)

Constraints on the nuclear equation of state (EOS)

nature

A two-solar-mass neutron star measured using Shapiro delay

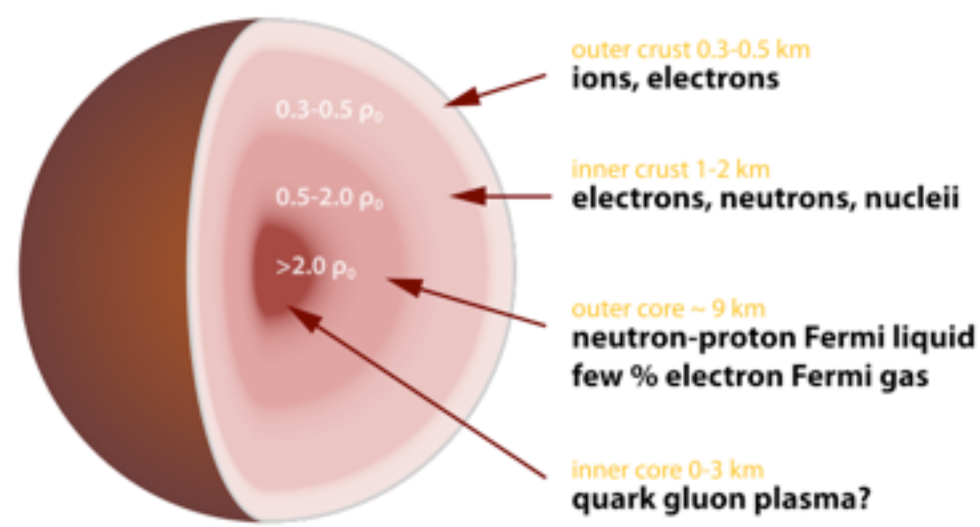
P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



Demorest et al., Nature 467, 1081 (2010)

$$M_{\rm max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$

Credit: NASA/Dana Berry



Calculation of neutron star properties requires EOS up to high densities.

Strategy:

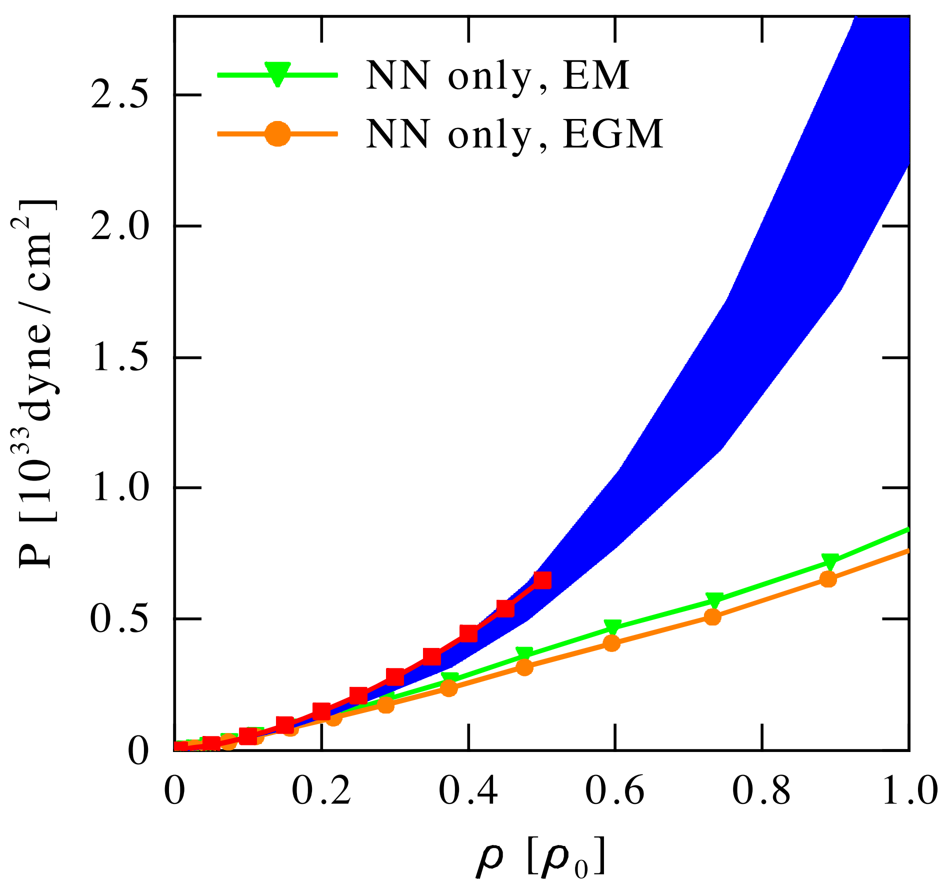
Use observations to constrain the high-density part of the nuclear EOS.

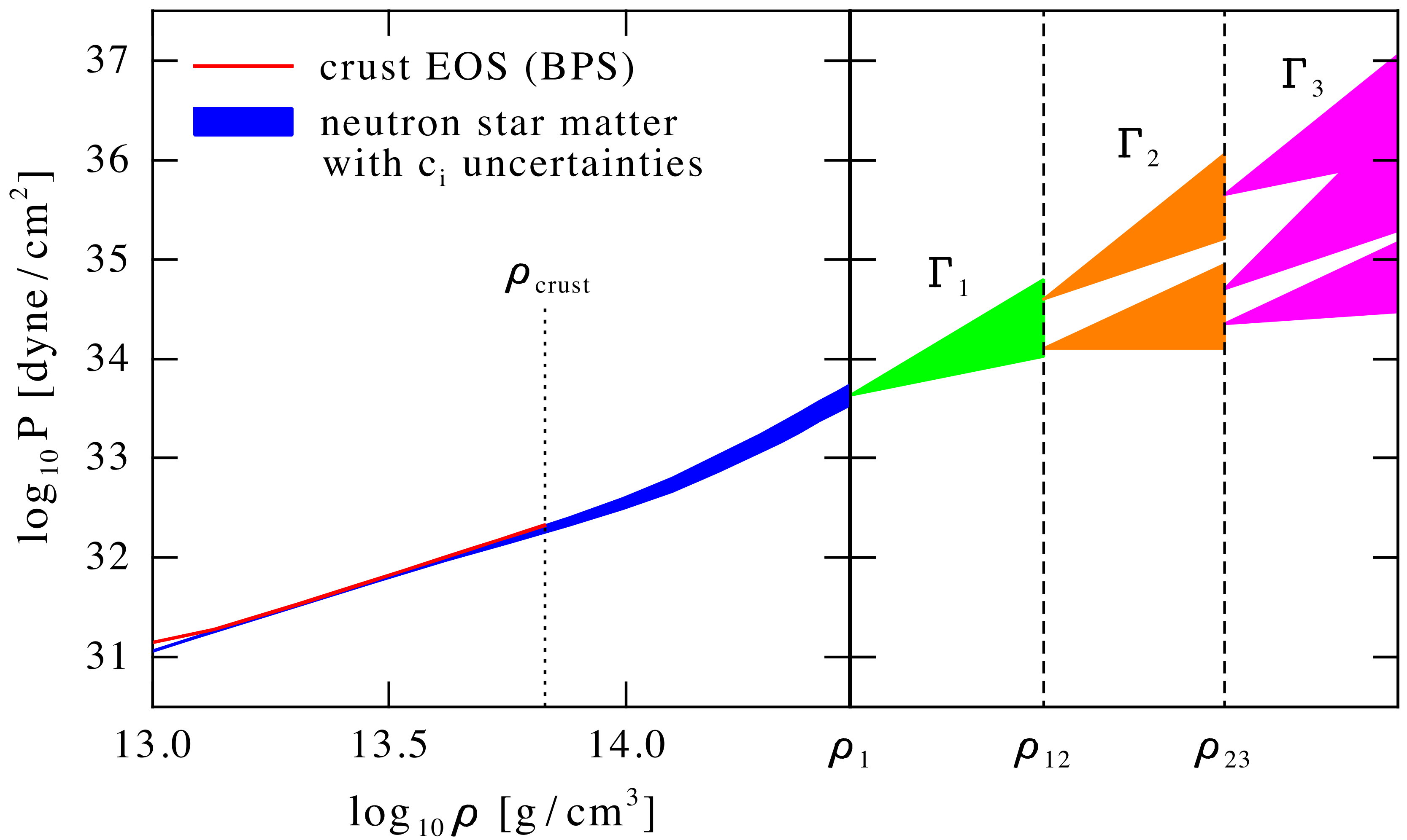
Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!





KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

use the constraints:

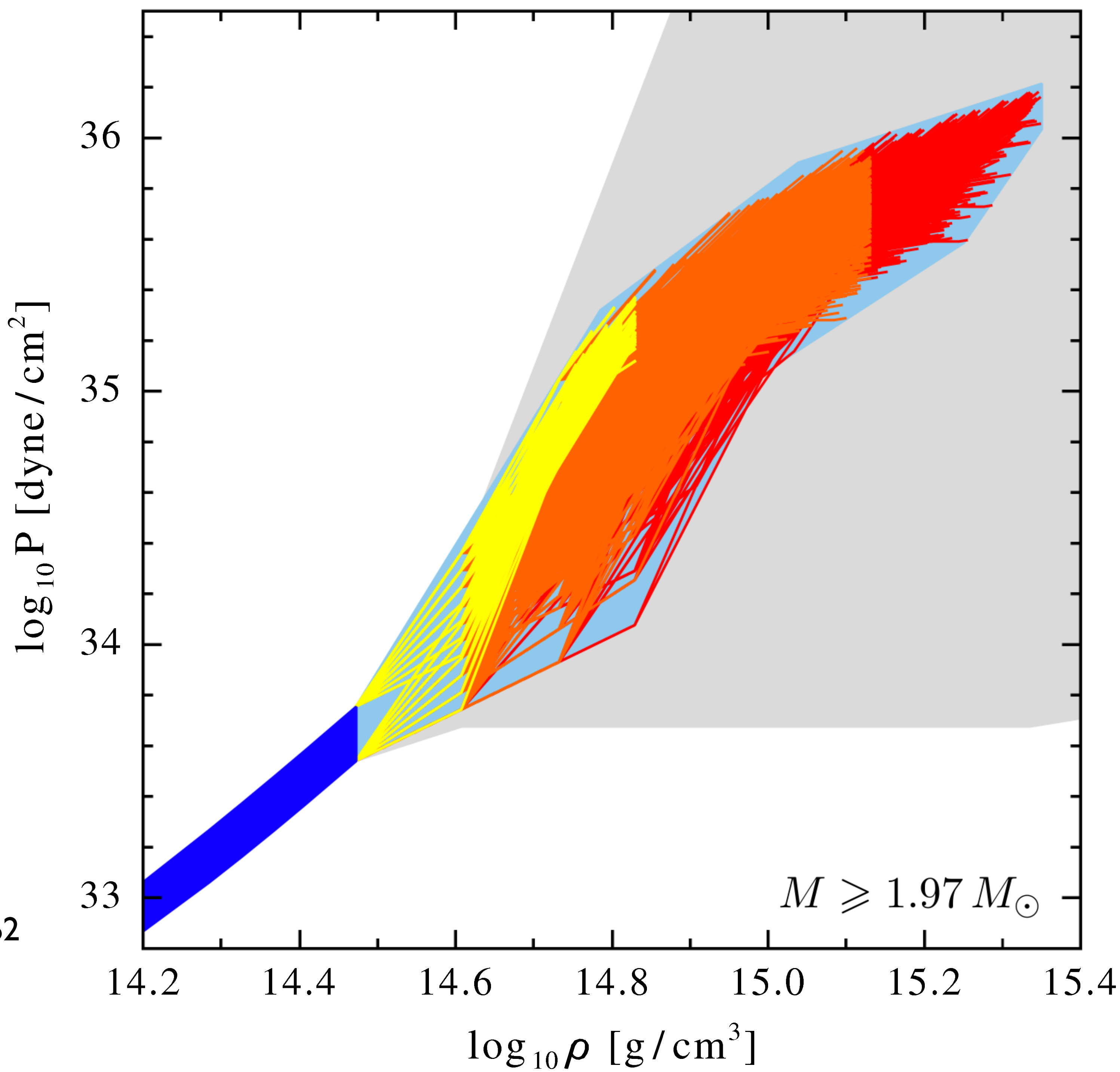
recent NS observation

$$M_{\mathrm{max}} > 1.97\, M_\odot$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662



significant reduction of uncertainty band

Constraints on the nuclear equation of state

use the constraints:

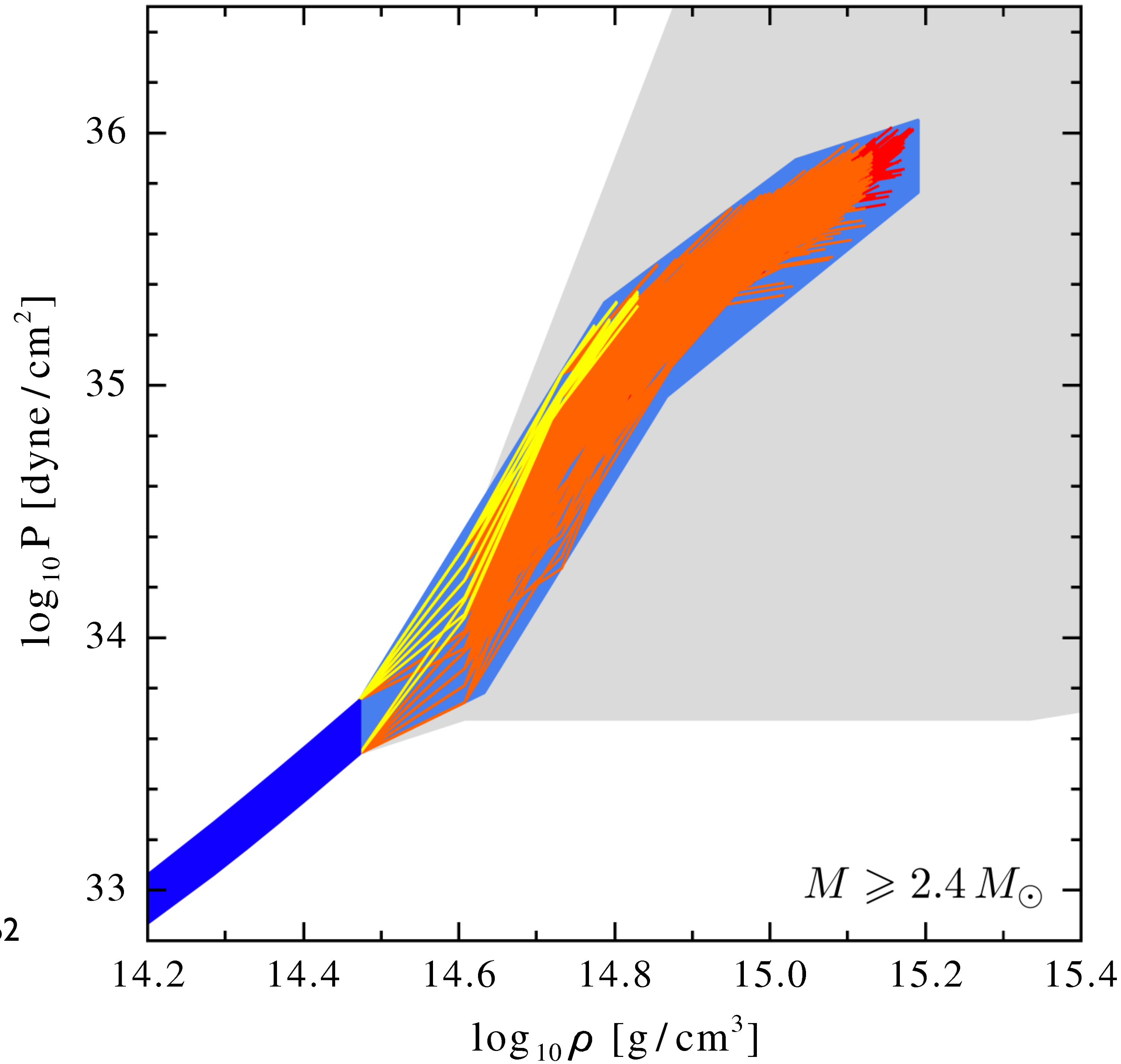
NS mass

$$M_{\rm max} > 2.4\, M_\odot$$

causality

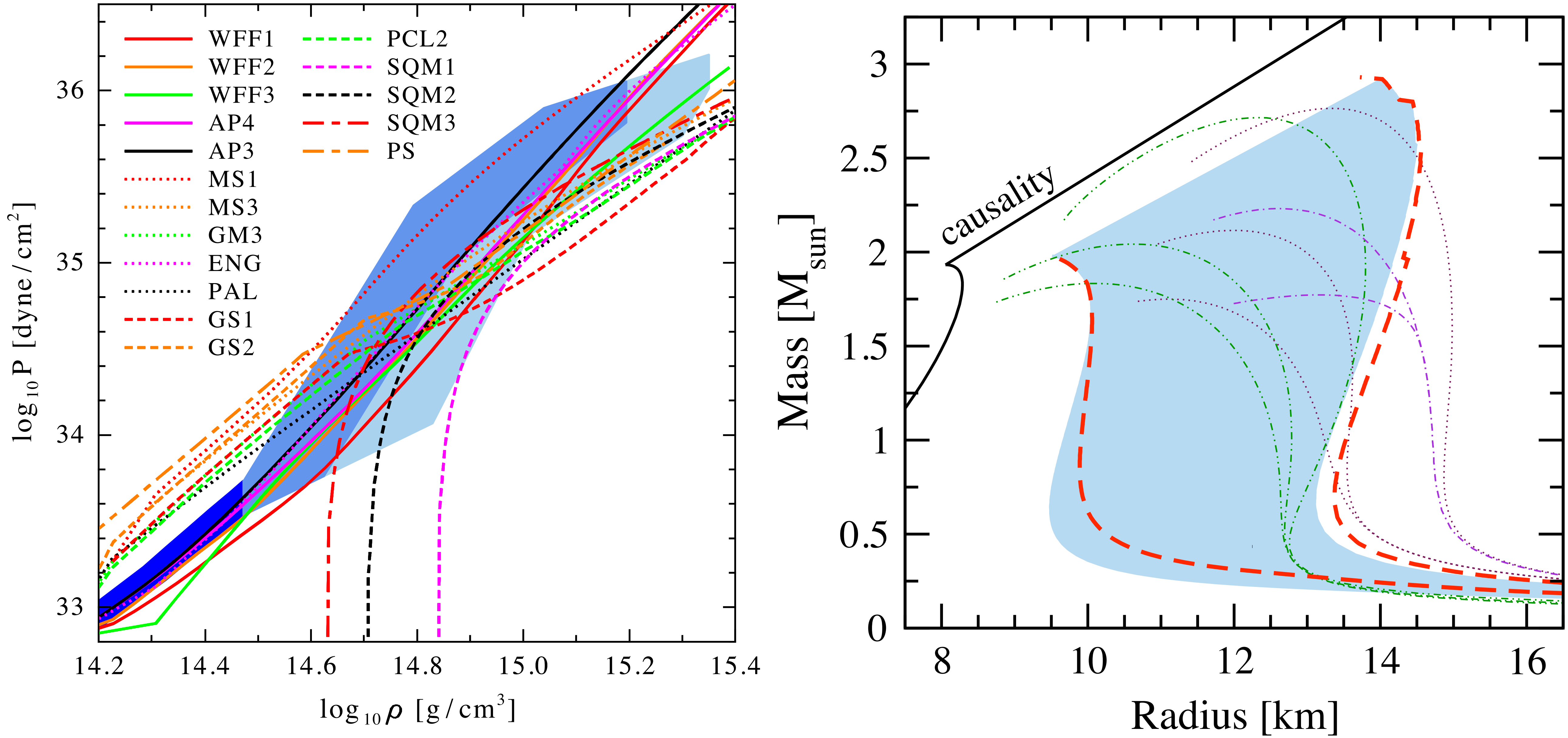
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662



increased $M_{\rm max}$ systematically reduces width of band

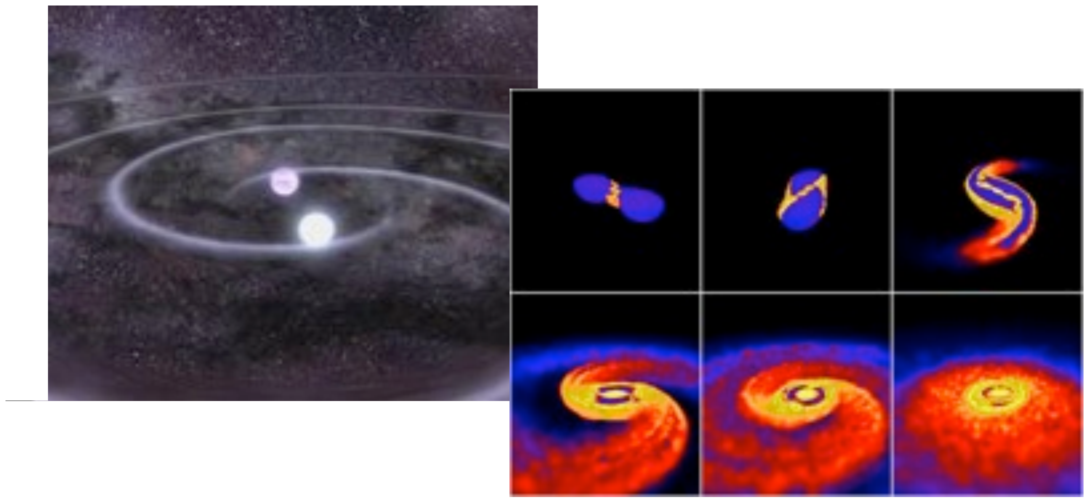
Constraints on neutron star radii

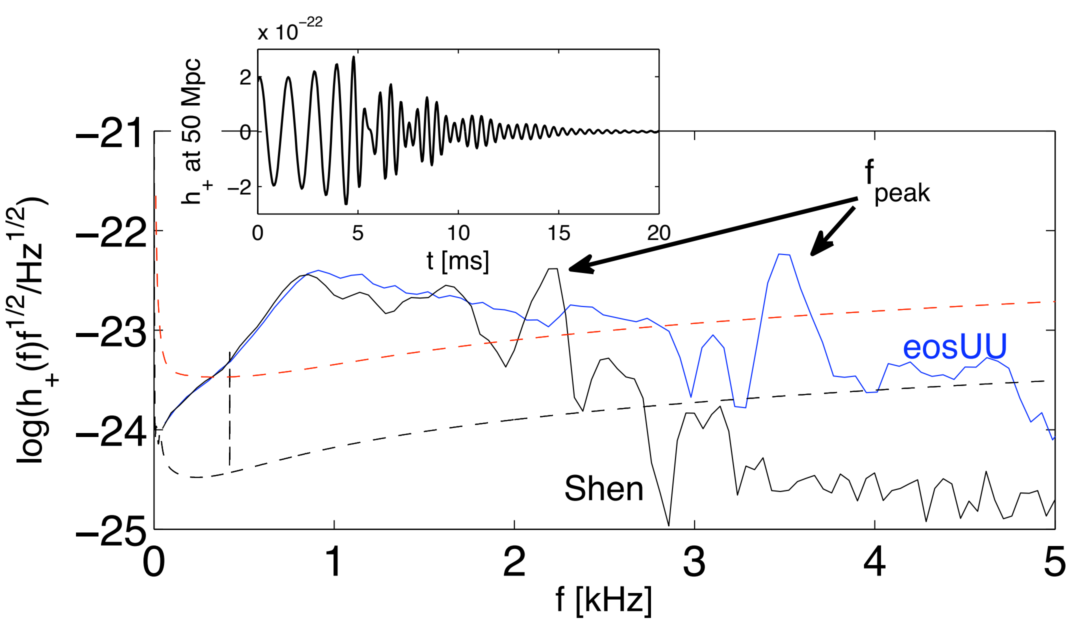


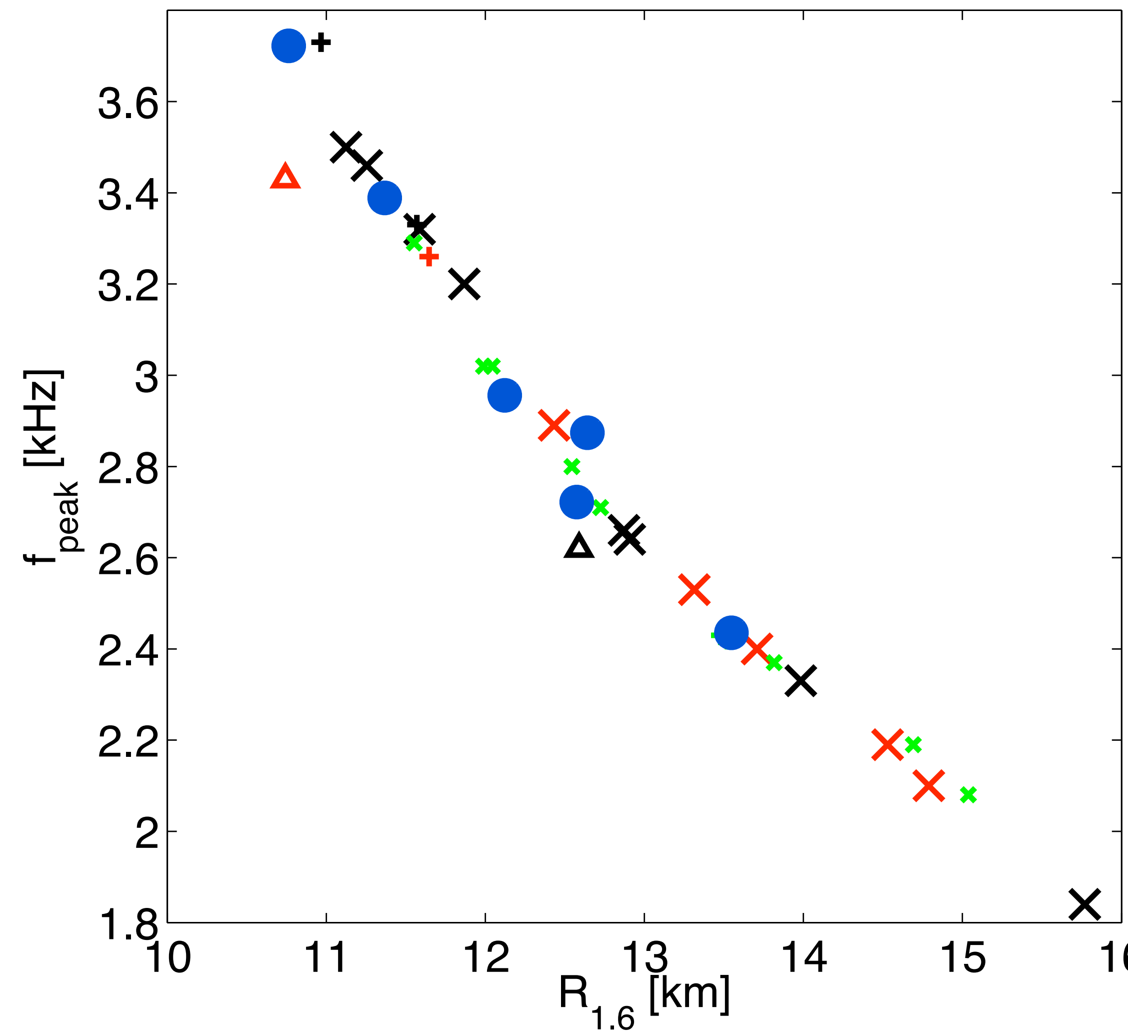
KH, Lattimer, Pethick, Schwenk, arXiv:1303.4662
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint for typical $1.4\,M_{\odot}$ neutron star: $9.8 - 13.4\,\mathrm{km}$

Gravitational wave signals from neutron star binary mergers







Bauswein and Janka PRL 108, 011101 (2012),
Bauswein, Janka, KH, Schwenk arXiv: PRD 86, 063001 (2012)

- high-density part of nuclear EOS only loosely constrained
- simulations of NS binary mergers show strong correlation between between $f_{\rm peak}$ of the GW spectrum and the radius of a NS
- measuring $f_{\rm peak}$ is key step for constraining EOS systematically at large ρ

Summary

- demonstrated the feasibility of SRG evolution of NN+3NF in momentum space
- first results of neutron matter based on consistently evolved NN+3NF interactions
- strong renormalization effects of chiral two-pion exch. interaction in neutron matter
- constraints on equation of state and neutron star properties

Outlook/Work in progress

- extend RG evolution to $\mathcal{T} = 1/2$ channels, application to nuclear matter
- transformation to HO basis, application to finite nuclei (CC, NCSM, SCGF)
- RG evolution of operators: nuclear scaling and correlations in nuclear systems
- include N3LO contributions to 3N interactions