Momentum-space evolution of 3N interactions and first applications

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Computational and Theoretical Advances for Exotic Isotopes in the Medium Mass Region

in collaboration with S. K. Bogner, A. Ekstroem, R. J. Furnstahl, T. Krueger, J. Lattimer, A. Nogga, C. Pethick, A. Schwenk, I. Tews

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Chiral EFT for nuclear forces, leading order 3N forces

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• goal: generate unitary transformation of "hard" Hamiltonian

 $H_\lambda = U_\lambda H U^\dagger_\lambda \;\;\;$ with the resolution parameter $\;\lambda$

- dH_{λ} • change resolution in small steps: $\frac{a_{\Pi\lambda}}{d\lambda} = [\eta_{\lambda}, H_{\lambda}]$
- transformed wave functions and operators

$$
|\psi_{\lambda}\rangle = U_{\lambda} |\psi\rangle \qquad O_{\lambda} = U_{\lambda} O U_{\lambda}^{\dagger} \quad \Rightarrow \quad \langle \psi | O | \psi \rangle = \langle \psi_{\lambda} | O_{\lambda} | \psi_{\lambda} \rangle
$$

• specifying η_{λ} by generator G_{λ} : $\eta_{\lambda} = [G_{\lambda}, H_{\lambda}]$ $=$ [\cup λ , Π **Two ways to decouple with RG equations**

• common choice for generator

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relative kinetic energy operator $\; G_{\lambda} = T$:

K. Wendt et al, PRC 86, 014003 (2012)

using local projection for representation:

$$
\overline{V}_{\lambda}(r)=\int dr' r'^2 V_{\lambda}(r,r')
$$

- elimination of coupling between low- and high momentum components, many-body problem more perturbative and tractable
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:

RG transformation also changes three-body (and higher-body) interactions.

To what extent are 3N interactions constrained at low resolution?

- only two low-energy constants c_D and c_E
- 3N interactions give only subleading contributions to observables

• So far (in momentum basis):

intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

- $E_{\rm 3H} = -8.482 \,\text{MeV}$ and $r_{\rm 4He} = 1.464 \,\text{fm}$
	- \longrightarrow coupling constants of natural size
	- in neutron matter contributions from c_D , c_E and c_4 terms vanish
	- \bullet long-range 2π contributions assumed to be invariant under RG evolution
	- at low resolution scales nuclear many-body problem more perturbative

Application to infinite nuclear matter: Equation of state

- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

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Hierarchy of many-body contributions

• binding energy results from cancellations of much larger kinetic and potential energy contributions

- chiral hierarchy of many-body terms preserved for considered density range
- cutoff dependence of natural size, consistent with chiral exp. parameter $\sim 1/3$

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Symmetry energy constraints

extend EOS to finite proton fractions *x*

and extract symmetry energy parameters

$$
S_v = \frac{\partial^2 E/N}{\partial^2 x} \bigg|_{\rho = \rho_0, x = 1/2}
$$

$$
L = \frac{3}{8} \left. \frac{\partial^3 E/N}{\partial \rho \partial^2 x} \right|_{\rho = \rho_0, x = 1/2}
$$

KH, Lattimer, Pethick and Schwenk, arXiv:1303.4662

symmetry energy parameters consistent with other constraints

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	- has been achieved in oscillator basis (Jurgenson, Roth)
	- promising results in very light nuclei
	- puzzling effects in heavier nuclei (higher-body forces?)
	- not immediately applicable to infinite systems
	- limitations on $\hbar\Omega$

- 16O as function of *N*max for the three types of Hamiltonians and a rure JI'n consistently p-shell at moderate computational cost. Previously, even the teractions within the SKG calculations in • Ideal case: evolve 3NF consistently with NN interactions within the SRG
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	- \mathbf{B} in the initial chiral 3N interaction, i.e., \mathbf{B} interaction, i.e., by using \mathbf{B} ediately applicable to infinite syst lowered and are in good agreement with experiment for both, ns induced 4N contributions are induced 4N contributions are induced 4N contributions are in starting from the NN interaction only. The NN interaction only. The NN interaction only. The NN+3N-3N-3N-3N-3N-• not immediately applicable to infinite systems
	- limitations on $\hbar\Omega$ α -dependence in the range considered here. We conclude that α

3N interactions at N3LO

Bernard et. al (2007, 2011)

relativistic corrections (2-body-contacts $C_{\mathcal{T}}$, C_{S})

Contributions of 3NFs at N3LO in neutron matter (Hartree-Fock, no RG evolution)

PRL 110, 032504 (2013)

Complete N3LO calculation of neutron matter

- complete neutron matter calculation at N3LO including NN, 3N and 4N forces
- includes uncertainties from bare interactions

Consistent 3NF evolution in momentum basis: Current developments and applications

- application to infinite systems
	- ‣ equation of state (first results for neutron matter)
	- ‣ systematic study of induced many-body contributions, scaling behavior
	- ‣ include initial N3LO 3N interactions
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- study of various generators
	- \blacktriangleright different decoupling patterns (e.g. $V_{\text{low k}}$)
	- ‣ improved efficiency of evolution
	- ‣ suppression of many-body forces?

Lower a cutoff *ⁱ* in *k, k* , Drive the Hamiltonian toward Anderson et al. , PRC 77, 037001 (2008)

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- explicit calculation of unitary 3N transformation
	- ▶ RG evolution of operators
	- \rightarrow study of correlations in nuclear systems \rightarrow factorization

RG evolution of 3N interactions in momentum space

Three-body Faddeev basis:

$$
|pq\alpha\rangle_i \equiv |p_iq_i; [(LS)J(ls_i)j] \mathcal{JJ}_z(Tt_i)TT_z\rangle
$$

 \int *i* $\langle pq\alpha|P|p'q'\alpha'\rangle$ _{*i*} = \int $\langle pq\alpha|p'q'\alpha'\rangle$ $\langle j \rangle$

Faddeev bound-state equation:

 $|\psi_i\rangle = G_0 \left[2t_i P + (1 + t_i G_0) V_{3N}^i (1 + 2P) \right] |\psi_i\rangle$

SRG flow equations of NN and 3N forces in momentum basis

$$
\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \eta_s = [T_{\text{rel}}, H_s]
$$

$$
H = T + V_{12} + V_{13} + V_{23} + V_{123}
$$

- \bullet spectators correspond to delta functions, matrix representation of H_s ill-defined
- solution: explicit separation of NN and 3N flow equations

$$
\frac{dV_{ij}}{ds} = [[T_{ij}, V_{ij}], T_{ij} + V_{ij}],
$$
\n
$$
\frac{dV_{123}}{ds} = [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}] + [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}] + [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}] + [[T_{rel}, V_{123}], H_s]
$$

• only connected terms remain in $\frac{dV_{123}}{d}$, 'dangerous' delta functions cancel *ds*

see Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)

SRG evolution in momentum space

• evolve the antisymmetrized 3N interaction

special thanks to J. Golak, R. Skibinski, K. Topolnicki

$$
\overline{V}_{123} = i \langle pq\alpha | (1 + P_{123} + P_{132}) V_{123}^{(i)} (1 + P_{123} + P_{132}) | p'q'\alpha' \rangle_i
$$

• embed NN interaction in 3N basis:

 $V_{13} = P_{123}V_{12}P_{132}$, $V_{23} = P_{132}V_{12}P_{123}$

with $\frac{3}{pq\alpha}|V_{12}|p'q'\alpha'\rangle_{3} = \langle p\tilde{\alpha}|V_{\rm NN}|p'\tilde{\alpha}'\rangle\,\delta(q-q')/q^{2}$

• use $P_{123}V_{123} = P_{132}V_{123} = V_{123}$

$$
\Rightarrow d\overline{V}_{123}/ds = C_1(s, T, V_{NN}, P)
$$

$$
+ C_2(s, T, V_{NN}, \overline{V}_{123}, P)
$$

$$
+ C_3(s, T, \overline{V}_{123})
$$

It works:

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Transformation to HO basis, Nmax convergence

thanks to A. Ekstroem

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It works:

same decoupling patterns like in NN interactions

Universality in 3N interactions at low resolution

- \bullet remarkably reduced scheme dependence for typical momenta $\sim 1\, {\rm fm}^{-1}$, matrix elements with significant phase space well constrained at low resolution
- new momentum structures induced at low resolution
- study based on $\mathrm{N}^2\mathrm{LO}$ chiral interactions, improved universality at $\mathrm{N}^3\mathrm{LO}$?

First application to neutron matter: Equation of state

- evolve consistently NN + 3NF in the isospin $\mathcal{T} = 3/2$ channel
- calculate EOS by taking all blue-boxed contributions into account
- in this approximation NN and 3NF contributions factorize

• all partial waves included up to $\mathcal{J}=9/2$ in SRG evolution and EOS calculation

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Resolution-scale dependence at saturation density

- solid lines: NN resummed, dashed lines: 2nd order
- variations: NN-only 3.6 MeV, 3N-induced: 390 keV, 3N-full: 650 keV
- indications for 4N forces at small λ ?

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Resolution-scale dependence at saturation density

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- indications for 4N forces at small λ ?

- E_{3N} agrees within 0.4 % with the exact result at saturation density
- E_{3N} converged in partial waves at both scales, $\lambda = \infty$ and $\lambda = 2.0$ fm⁻¹

Matrix elements of evolved 3-neutron interactions (only long-ranged initially!)

$$
\xi^2 = p^2 + \frac{3}{4}q^2 \qquad \tan \theta = \frac{2p}{\sqrt{3}q}
$$

show dominant channel for $J = 1/2$ and positive total parity:

- strong renormalization effects at very small low scales
- moderate effects in range $\lambda = \infty$ to $\lambda = 2.0$ fm⁻¹

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Scaling of three-body contributions

• relative size of 3N contribution grows systematically towards smaller λ

• no obvious trend with density (may be obscured by cancellations among contributions)

Constraints on the nuclear equation of state (EOS) LETTER onstraints on the nuclear equation of state (FOS) $\mathsf{\color{red} \cup}$ onstraints on the nuclear equation of state (EOS) \blacksquare pointing towards Earth, in yellow. At orbital phase \sum

0 10 0

A two-solar-mass neutron star measured using Shapiro delay $C1$ Δ

P. B. Demorest¹, T. Pennucci², S. M. Ransom¹, M. S. E. Roberts³ & J. W. T. Hessels^{4,5}

arrival using standard procedures, with a typical using α We used the measured arrival times to determine key physical para-**Saranacion of theory star** comprehensive timing model that accounts for every rotation of the requires $\mathsf{L}(N)$ un to hi \sim cyull cs \sim count to the rotation and spin-down, astrometric terms (sky position and proper R^{max} mean subjection \mathcal{L} and \mathcal{L} is \mathcal{L} in \mathcal{L} . The \mathcal{L} is the \mathcal{L} Declination (J2000) 222u 309 31.081(7)99 Orbital economic eco In dancitias and 19.17 and 19.17.17 and 19.17 and 19.17.17 and 19.17(2). P equires E \cup S up to nigh densities. Dispersion-derived distance{ 1.2 kpc $\overline{}$ μ and Guber and Guestion signal-to-noise ratio μ P_{P} Inclination and the contract of \overline{a} p to high densit

1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1

1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.155 1.1 parameters, with MCMC extensive in Table 1. Owing to be remarkably edge-on, with an inclination of 89.17u 6 0.02u. r equires FOS un to high dengthed standard x2 find the simulations of the simulations of the simulations of the simulations of the simulations o requires EOS up to high densities. Calculation of neutron star properties

 $F = \max_{\mathbf{C}}$

motion), binary orbital parameters, time-variable interstellar dispersion and general-relativistic effects such as the Shapiro delay (Table 1).

the differences between the observed and the model-predicted pulse

–20

Inclination angle, i (°)

89.1

timescales of the orbital period or less. Additional discussion or less. Additional discussion of the orbital

parameters, with MCMC error estimates, are given in Table 1. Owing to 1. Owing the United States, and the United States, and the United States, and the United Sta

plane, computed from a histogram of MCMC trial values. The ellipses show 1s

From the detected Shapiro delay, we measure a companion mass of (0.500 60.006)M[, which implies that the companion is a helium–

> **Example 21 Strategy:** using an 800-MHz-wide band centred at a radio frequency of 1.5 \pm GHz. The radio frequency of 1.5 GHz. The raw profiles were polarization- \mathcal{C}_t rag-based mass vides noinformation about the neutron starting the neutron starting the neutron starting the neutron starting α In addition, the excellent timing precision achievable from the pulsar Strategy: with signal-to-noise ratio

recorded long-term data set and our new GUPPI data in a single fit. The long-term data determine model parameters (for example spindown and astronomy to a few weeks, whereas the new data best constraints on the package in the bigh d CONSU din the Ingh-de ${\rm S}$ shown in parentheses are separate values for the long-term (first) and new (second) data sets. \Box $\overline{}$ \mathbf{r} ray methods, our result is nearly model independent, as it depends of \mathbf{r} n the high-density bart of the nu In addition, unlike statistical pulsar mass determinations based on Companion mass, M2 (M() **Figure 2 Is a Post of the McMcMCMC extending consuma** two-dimensional posterior probability density function (PDF) in the M2–i measurement of both Shapiro delay parameters within a single orbit. n the high-density part of the companion mass and organisation in the distribution of and describe the dynamics of and 89.18 1.8 1.85 1.9 1.95 2 2.05 2.1 2.15 Probability density Use observations to constrain the high-density part of the nuclear EOS.

system to be remarkably edge-on, with an inclination of 89.17u 6 0.02u.

'clean' binary system—one comprising two stable compact objects—

est precisely measured neutron star mass determined to date. In contrast

 ${\cal C}$ the NE2001 pulsar distance model ${\cal C}$

Credit: NASA/Dana Berry **Credit: NASA/Dana Berry**

nearest to the companion (,240,000 km),

(including Shapiro delay), which have a root mean $\mathcal{S}_\mathcal{S}$

Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter \longrightarrow neutron star matter

parametrize piecewise high-density extensions of EOS:

- \bullet use polytropic ansatz $\ p \sim \rho^\Gamma$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics!

Constraints on the nuclear equation of state

significant reduction of uncertainty band

Constraints on the nuclear equation of state

increased $M_{\rm max}$ systematically reduces width of band

Constraints on neutron star radii

- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint for typical $1.4 M_{\odot}$ neutron star: $9.8 13.4 \, \mathrm{km}$

crophysical EoSs (without temperature and electron fraction dependence) (class included using the Caravitational wave signals from implemented as piecewise polytropes fitting barotropic micro-Gravitational wave signals from neutron star binary mergers

- I gn-density part of nuclear E \cup 5 o $\overline{}$ and $\overline{}$ a · high-density part of nuclear EOS only loosely constrained (e.g. in the sound speed in the speed in
- \mathbf{b} dashed lines unity \mathbf{b} shows the inset shows inset shows that inset shows inset shows in sensitivity in \mathbf{b} mulations of INS binary mergers s excluded by $\frac{1}{3}$. See the symbols for symbols. FIG. 3: Peak frequency of the postmerger GW emission vs. ow strong correlation between be • simulations of NS binary mergers show strong correlation between between $f_{\rm peak}$ of the GW spectrum and the radius of a NS $\mathcal{L}_{\mathcal{A}}$ and the slightly different from the slightly different from those obsimulations of NS binary mergers sh J \rm{peak} or the GVV spectrum and the r T_{c} and to introduce and the set of T_{c} \overline{a} music for the scatter inherent to the presented rela-

3

 \overline{P}

 ϵ because to simulations for the MITGS simulations for the MITGS F both both F and F F and F relationships F • measuring $f_{\rm peak}$ is key step for constraining EOS systematically at large ρ measuring $j_{\rm peak}$ is key step for const $t⁴$ $\min_{\mathbf{g}} \mathbf{g}_k$ EUS systematically at large P rameters to be obtained by a least-square fit. RTOV de-

Summary

- demonstrated the feasibility of SRG evolution of NN+3NF in momentum space
- first results of neutron matter based on consistently evolved NN+3NF interactions
- strong renormalization effects of chiral two-pion exch. interaction in neutron matter
- constraints on equation of state and neutron star properties

Outlook/Work in progress

- extend RG evolution to $\mathcal{T} = 1/2$ channels, application to nuclear matter
- transformation to HO basis, application to finite nuclei (CC, NCSM, SCGF)
- RG evolution of operators: nuclear scaling and correlations in nuclear systems
- include N3LO contributions to 3N interactions