# Properties of homogeneous and inhomogeneous neutron matter

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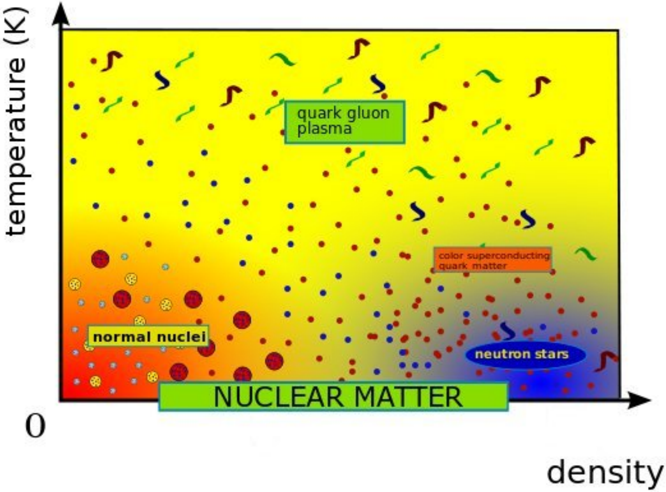
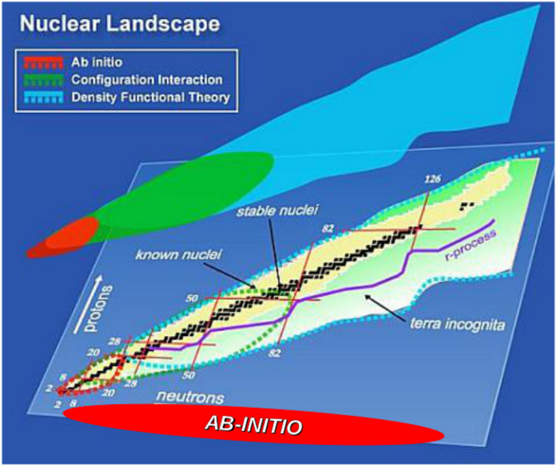
Computational and Theoretical Advances for Exotic Isotopes
in the Medium Mass Region
INT, University of Washington, Seattle, March 25 - April 19, 2013

# Homogeneous neutron matter



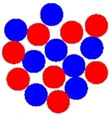
# Inhomogeneous neutron matter



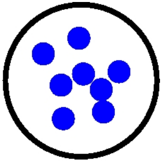

W. Nazarewicz – UNEDF

# Neutron drops

Why study neutron drops?
Are they nothing more than a pure simple toy model?



NP self–bound



N confined

Neutron drops are interesting because:

- Provide a strong benchmark for microscopic calculations
- Model neutron-rich nuclei
- Calibrate Skyrme models for neutron-rich systems (useful to check $\nabla\rho$ terms in different geometries)

# Outline

- The model and the method
- **Inhomogeneous neutron matter: Skyrme vs ab-initio.**
  - Energy
  - Density and radii
- **Homogeneous neutron matter**
  - Role of three-neutron force
  - Symmetry energy
  - Neutron star structure
- Conclusions

# Nuclear Hamiltonian

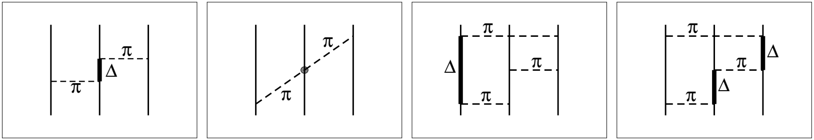
Model: non-relativistic nucleons interacting with an effective nucleon-nucleon force (NN) and three-nucleon interaction (TNI).

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

$v_{ij}$ NN (Argonne AV8') fitted on scattering data. Sum of operators:

$$v_{ij} = \sum O_{ij}^{p=1,8} v^p(r_{ij}), \quad O_{ij}^p = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}, \vec{L}_{ij} \cdot \vec{S}_{ij}) \times (1, \vec{\tau}_i \cdot \vec{\tau}_j)$$

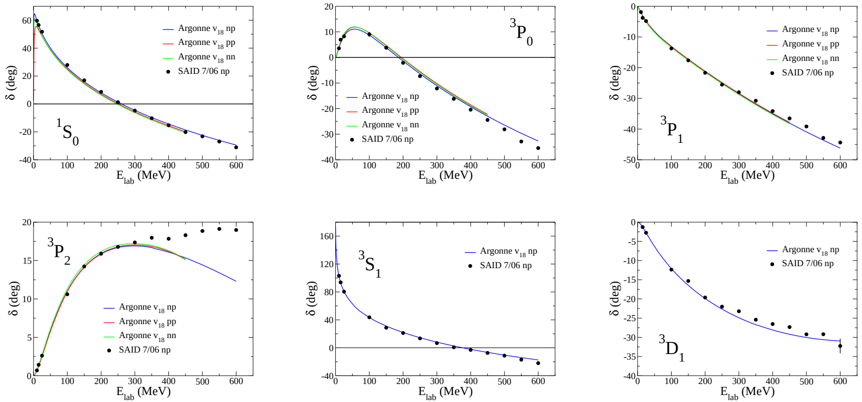
Urbana–Illinois $V_{ijk}$ models processes like




$+$ short-range correlations (spin/isospin independent).

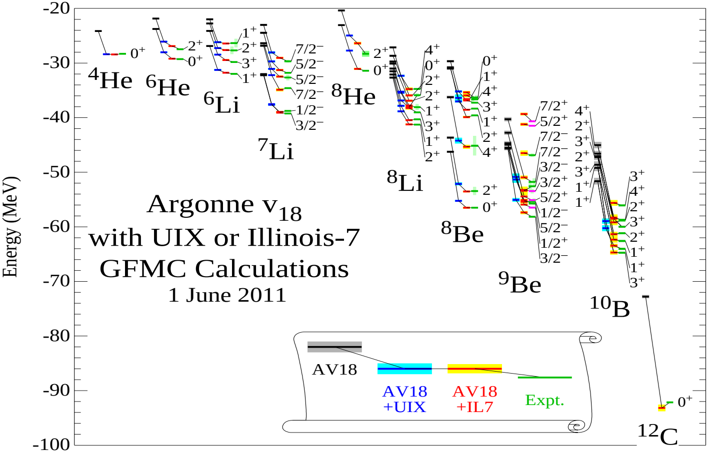
# Nuclear Hamiltonian

## Argonne NN interaction




Wiringa, Stoks, Schiavilla (1995)

# Light nuclei spectrum computed with GFMC




Carlson, Pieper, Wiringa, many papers

# Quantum Monte Carlo

Evolution of Schrodinger equation in imaginary time **t**:

$$\psi(R,t) = e^{-(H-E_T)t}\psi(R,0)$$

In the limit of $t \to \infty$ it approaches to the lowest energy eigenstate (not orthogonal to $\psi(R,0)$).

Propagation performed by

$$\psi(R,t) = \langle R|\psi(t)\rangle = \int dR' G(R,R',t)\psi(R',0)$$

$G(R,R',t)$ is an approximate propagator (small-time limit). We iterate the above integral equation many times in the small time-step limit.
$\to$ parallel codes and supercomputers.

For a given microscopic Hamiltonian, this method solves the ground–state within a systematic uncertainty of **1–2%** in a **non-perturbative way.**

# Quantum Monte Carlo

Recall: propagation in imaginary-time

$$e^{-(T+V)\Delta\tau}\psi \approx e^{-T\Delta\tau}e^{-V\Delta\tau}\psi$$

Kinetic energy is sampled as a diffusion of particles:

$$e^{-\nabla^2\Delta\tau}\psi(R) = e^{-(R-R')^2/2\Delta\tau}\psi(R) = \psi(R')$$

The (scalar, local) potential gives the weight of the configuration:

$$e^{-V(R)\Delta\tau}\psi(R) = w\psi(R)$$

Algorithm for each time-step:

- do the diffusion: $R' = R + \xi$
- compute the weight $w$
- compute observables using the configuration $R'$ weighted using $w$ over a trial wave function $\psi_T$.

For spin-dependent potentials things are much worse!

# GFMC and AFDMC

Because the Hamiltonian is state dependent, all spin/isospin states of nucleons must be included in the wave-function.

Example: spin for 3 neutrons (radial parts also needed in real life):

**GFMC wave-function:**

$$\psi = \begin{pmatrix} a_{\uparrow\uparrow\uparrow} \\ a_{\uparrow\uparrow\downarrow} \\ a_{\uparrow\downarrow\uparrow} \\ a_{\uparrow\downarrow\downarrow} \\ a_{\downarrow\uparrow\uparrow} \\ a_{\downarrow\uparrow\downarrow} \\ a_{\downarrow\downarrow\uparrow} \\ a_{\downarrow\downarrow\downarrow} \end{pmatrix}$$

A correlation like

$$1 + f(r)\sigma_1 \cdot \sigma_2$$

can be used, and the variational wave function can be very good. Any operator accurately computed.

**AFDMC wave-function:**

$$\psi = \mathcal{A}\left[\xi_{s_1}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}\xi_{s_2}\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}\xi_{s_3}\begin{pmatrix} a_3 \\ b_3 \end{pmatrix}\right]$$

We must change the propagator by using the Hubbard-Stratonovich transformation:

$$e^{\frac{1}{2}\Delta t O^2} = \frac{1}{\sqrt{2\pi}}\int dx e^{-\frac{x^2}{2}+x\sqrt{\Delta t}O}$$

Auxiliary fields $x$ must also be sampled. The wave-function is pretty bad, but we can deal to large systems (up to $A \approx 100$). Operators (except the energy) are very hard to be computed, but in some case there is some trick!

# Neutron drops

Now let's study **inhomogeneous neutron matter.**

We confine neutrons by adding an external potential:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \sum_i V_{ext}(r_i)$$

$V_{ext}$ is a Wood-Saxon or Harmonic well:

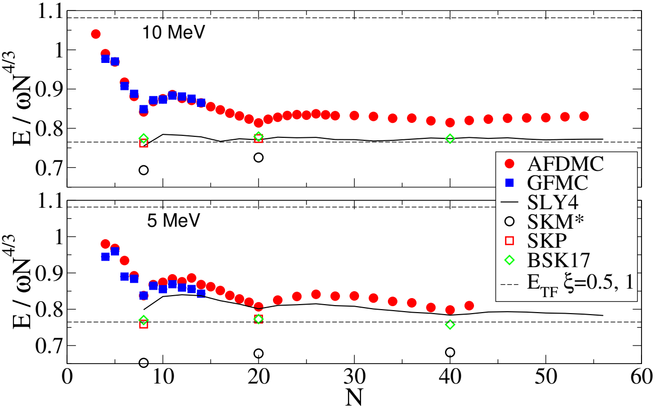
$$V_{WS} = -\frac{V_0}{1 + exp[(r - R)/a]}$$

$$V_{HO} = \frac{1}{2} m\omega^2 r^2$$

$\Longrightarrow$ different geometries and densities.

# Neutron drops, harmonic oscillator well

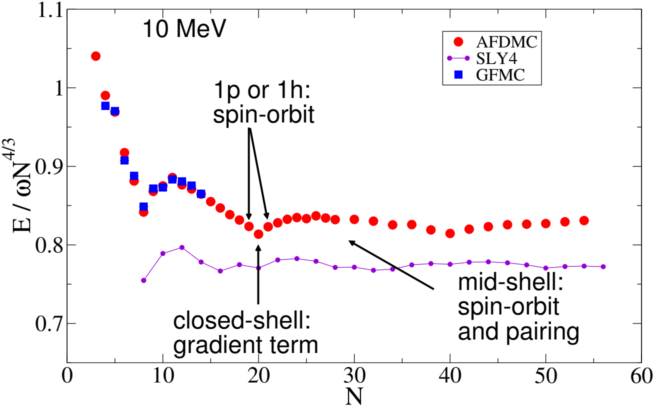
External well: harmonic oscillator with $\hbar\omega$=5, 10 MeV.




Skyrme systematically overbind neutron drops.

# Neutron drops, harmonic oscillator well

Fixing Skyrme force:

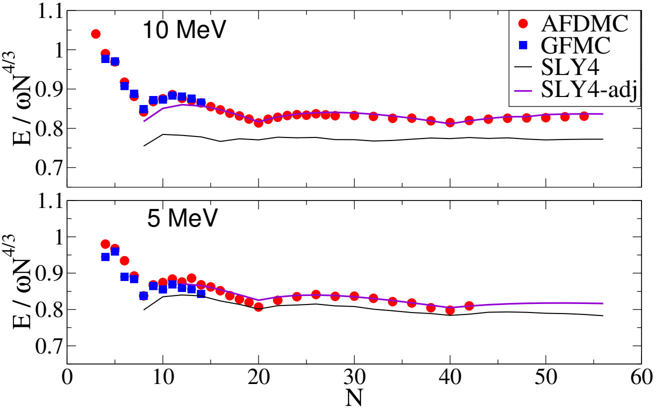



The correction is very similar in all the Skyrme forces we considered.

# Neutron drops, adjusted Skyrme force

Note: bulk term of Skyrme fit neutron matter.

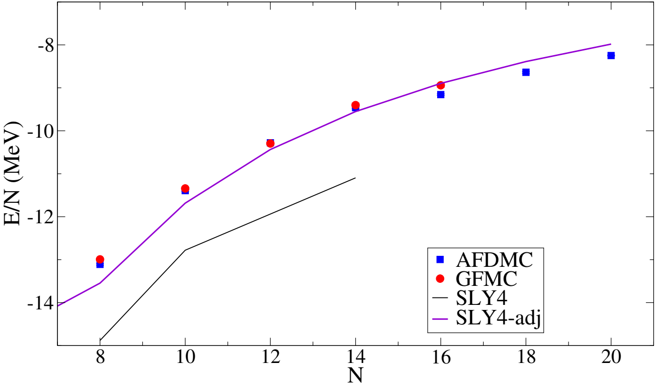
We add the **missing repulsion** by adjusting the gradient term $G_d[\nabla\rho_n]^2$, the pairing and spin-orbit terms.




Gandolfi, Carlson, Pieper (2011).

# Neutron drops, adjusted Skyrme force

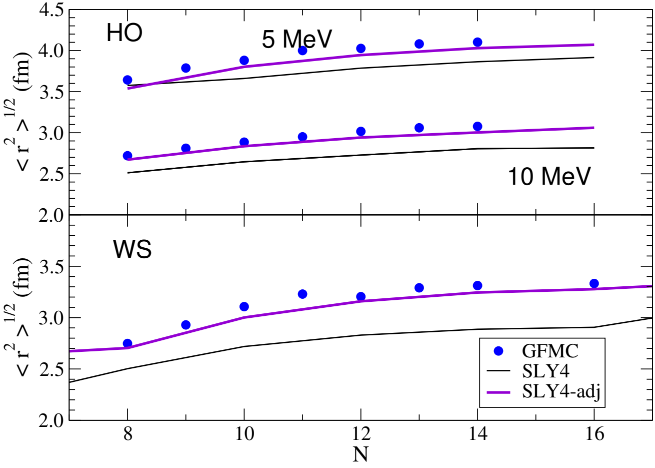
Neutrons in the Wood-Saxon well are also better reproduced by the adjusted SLY4.




Gandolfi, Carlson, Pieper (2011).

# Neutron drops: radii

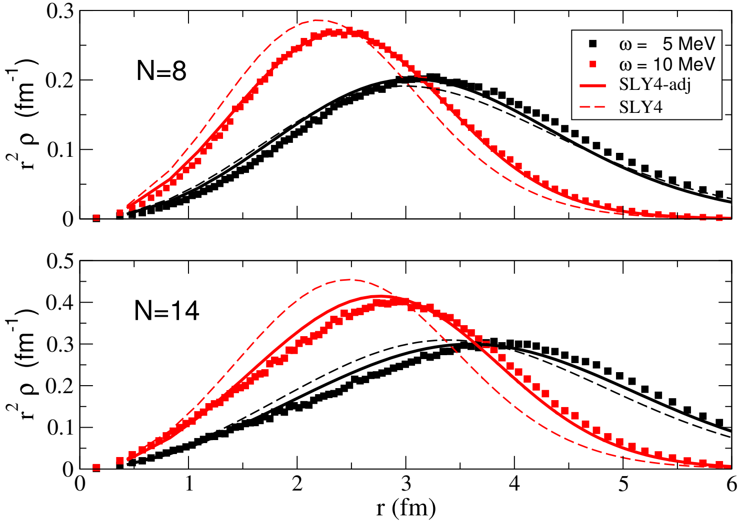
Correction to radii using the adjusted-SLY4.




Gandolfi, Carlson, Pieper (2011).

# Neutron drops: radial density

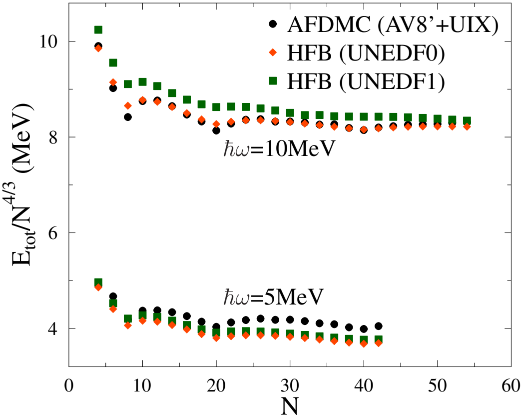
Neutron radial density:




Gandolfi, Carlson, Pieper (2011).

# Neutron drops

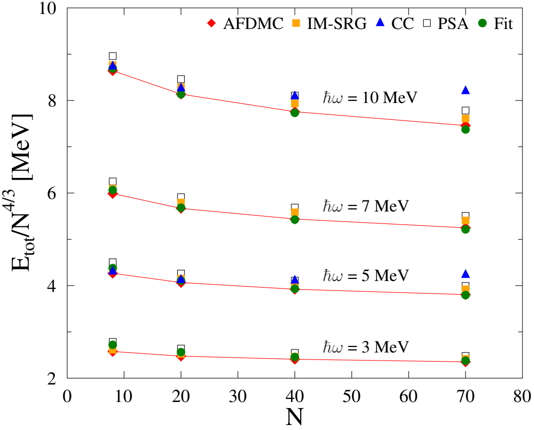
Ab-initio calculations meant as experimental data:




M. Kortelainen, et al. (2012).

# Neutron drops: comparison

Comparison using the softer Minnesota interaction:




Kortelainen, Holt, Hergert, Bogner, Furnstahl, Gandolfi, in preparation.

## Gradient term

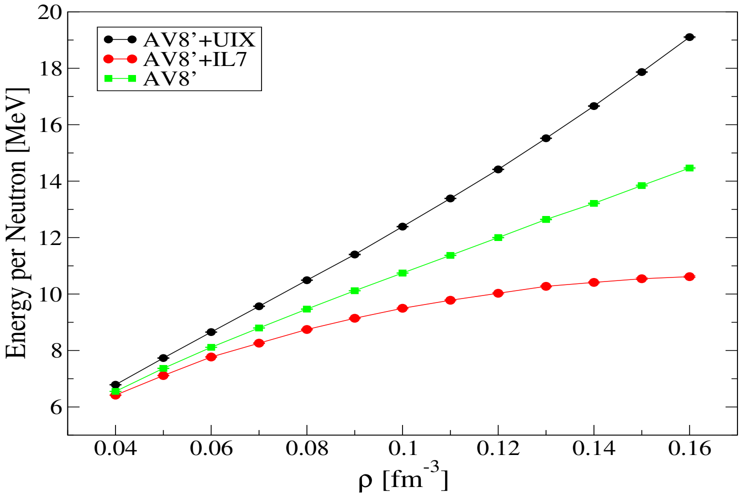
Where is the gradient term important?

Just few examples:

- Medium large neutron-rich nuclei
- Phases in the crust of neutron stars
- Isospin-asymmetry energy of nuclear matter

Note: in the pasta phase the volume vs surface energy process is critical. Role of the gradient?

# Neutron matter and the puzzle of the three-body force




Note: AV8'+UIX and (almost) AV8' are stiff enough to support observed neutron stars. $\rightarrow$ How to reconcile with nuclei???

# Neutron matter

Assumptions:

- The two-nucleon interaction reproduces well (elastic) *pp*, *np* and *nn* scattering data up to high energies ($E_{lab} \sim 600$MeV).
- The three-neutron force ($T = 3/2$) very weak in light nuclei, while $T = 1/2$ is the dominant part (but zero in neutron matter). **Difficult to study in light nuclei.**

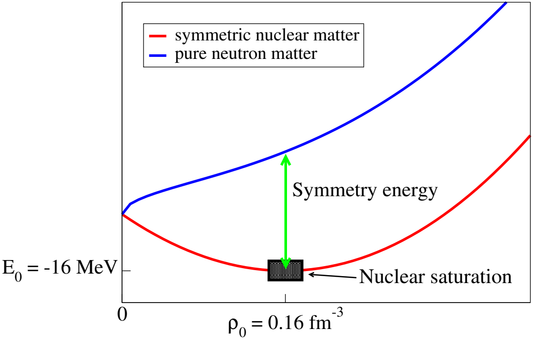
# Symmetry energy

Nuclear matter EOS:

$$E(\rho, x) = E_{SNM}(\rho) + E_{sym}^{(2)}(\rho)(1-2x)^2 + \cdots$$

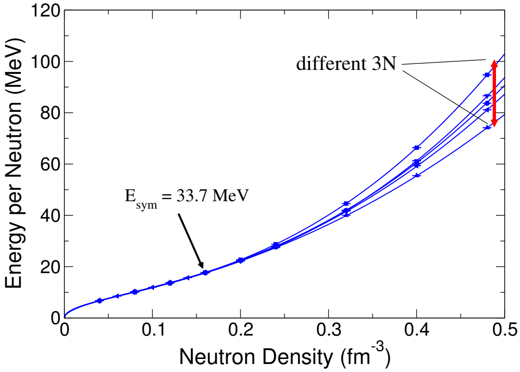
where

$$\rho = \rho_n + \rho_p\,, \quad x = \frac{\rho_p}{\rho}$$



# Neutron matter

We consider different forms of three-neutron interaction by only requiring a particular value of $E_{sym}$ at saturation.

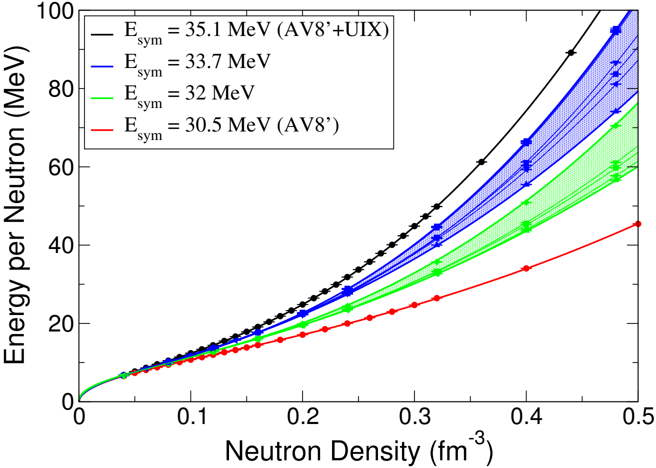



different 3N:

- $V_{2\pi} + \alpha V_R$
- $V_{2\pi} + \alpha V_R^{\mu}$ (several $\mu$)
- $V_{2\pi} + \alpha \tilde{V}_R$
- $V_{3\pi} + \alpha V_R$

# Neutron matter and symmetry energy

We then try to change the neutron matter energy at saturation:

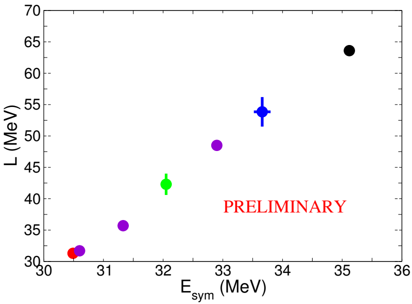


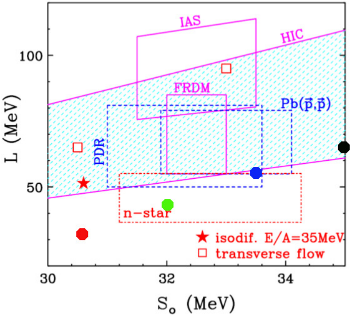

Gandolfi, Carlson, Reddy, (2012).

# Neutron matter and symmetry energy

From the EOS, we can fit the symmetry energy around $\rho_0$ using

$$E_{sym}(\rho) = E_{sym} + \frac{L}{3}\frac{\rho - 0.16}{0.16} + \cdots$$



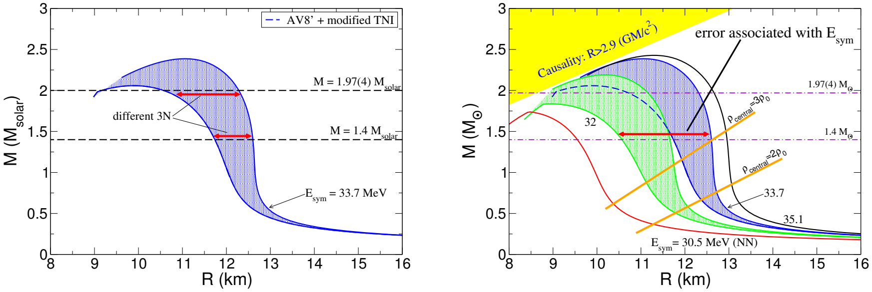




Tsang *et al.*, (2012)

Very weak dependence to the model of 3N force for a given $E_{sym}$.
Role of NN will be investigated next.

# Neutron star structure

EOS used to solve the TOV equations.

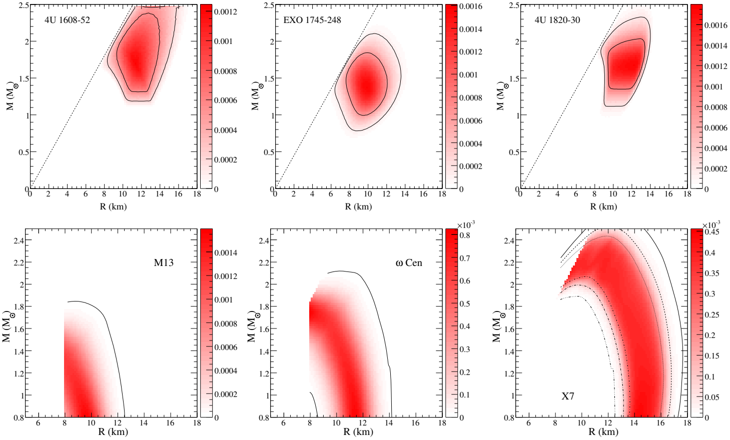



Accurate measurement of $E_{sym}$ would put a constraint to the radius of neutron stars, **OR** observation of M and R would constrain $E_{sym}$!

$M = 1.97\ M_{solar}$ observed – Demorest et al., Nature (2010).

# Neutron stars

Observations of the mass-radius relation are available:




Steiner, Lattimer, Brown, ApJ (2010)

We can use neutron star observations to 'measure' the EOS and constrain $E_{sym}$ and $L$.

# Neutron star matter

We model neutron star matter as

$$E_{NSM} = a \left( \frac{\rho}{\rho_0} \right)^{\alpha} + b \left( \frac{\rho}{\rho_0} \right)^{\beta}, \qquad \rho < \rho_t$$

(form suggested by QMC simulations),

and a high density model for $\rho > \rho_t$

i) two polytropes

ii) polytrope+quark matter model, Alford et al., ApJ (2005).

By changing $\rho_t$ and the high density model we can understand systematic errors in $E_{NSM}$ parametrization.

We also add a correction to account for the proton fraction present in neutron stars.

# Observations

What can we learn by fitting our model to observations?

- Symmetry energy and its slope:

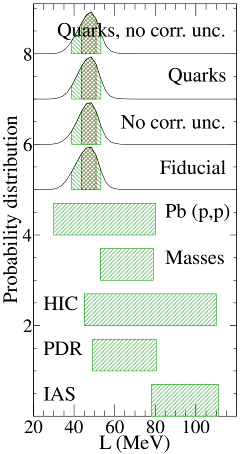
$$E_{sym} = a + b + 16\,, \quad L = 3(a\alpha + b\beta)$$

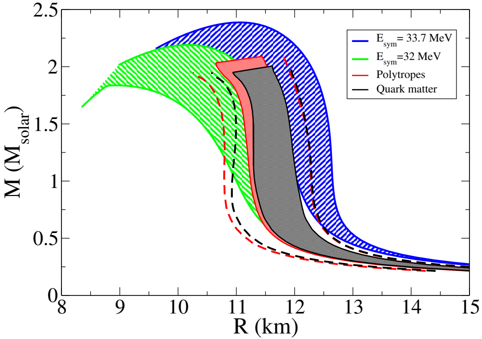
- Strength of 3N:

| 3N force | $E_{\mathrm{sym}}$ (MeV) | L (MeV) | $a$ (MeV) | $\alpha$ | $b$ (MeV) | $\beta$ |
|---|---|---|---|---|---|---|
| none | 30.5 | 31.3 | 12.7 | 0.49 | 1.78 | 2.26 |
| $V_{2\pi} + V^R_{\mu=300}$ | 32.0 | 40.6 | 12.8 | 0.488 | 3.19 | 2.20 |
| $V_{2\pi} + V^R_{\mu=600}$ | 32.0 | 41.3 | 12.8 | 0.488 | 3.19 | 2.20 |
| $V_{2\pi} + V_R$ | 32.1 | 41.3 | 12.7 | 0.476 | 3.34 | 2.22 |
| $V_{3\pi} + V_R$ | 32.0 | 44.0 | 13.0 | 0.49 | 3.21 | 2.47 |
| $V_{2\pi} + V_R$ | 33.7 | 52.9 | 13.3 | 0.512 | 4.38 | 2.39 |
| $V_{3\pi} + V_R$ | 33.8 | 56.2 | 13.0 | 0.50 | 4.71 | 2.49 |
| UIX | 35.1 | 63.6 | 13.4 | 0.514 | 5.62 | 2.436 |

Note: $a$ and $\alpha$ don't depend too much to the model of 3N!

# Neutron star observations

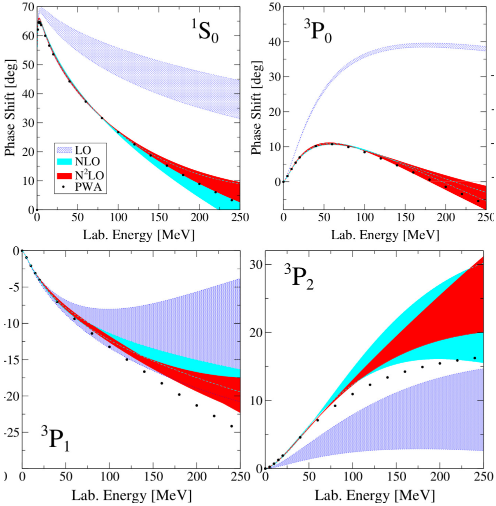



$32 < E_{sym} < 34\, MeV,\ 43 < L < 52\, MeV$

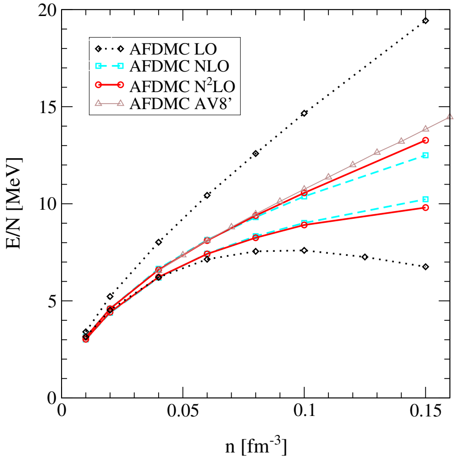
Steiner, Gandolfi (2012).

# Local N$^2$LO potential




Bands: short-range cutoff $R_0$ varied from 400 to 600 MeV

# Neutron matter with QMC/EFT




Gezerlis, Tews, Epelbaum, SG, Hebeler, Nogga, Schwenk (2013)

Inclusion of TNI in progress.

# Conclusions

- Isovector parts pf Skyrme forces can be better constrained by ab-initio calculations.
- Effect of three–neutron forces to high-density neutron matter is (reasonably) under control.
- $E_{sym}$ strongly constrain $L$. Weak dependence to the model of 3N.
- Uncertainty of the radius of neutron stars mainly due $E_{sym}$ rather than 3N.
- Neutron star observations becoming competitive with terrestrial experiments.
- QMC calculations using chiral forces now possible.

Thanks for the attention