Understanding/calculating the nucleon self-energy at positive and negative energy

INT 4/16/2013

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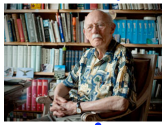
- Why do Green's functions?
- Question: "What does a nucleon do in the nucleus?"
- Ab initio minimum for nuclei

  SRC + LRC = FRPA but without momenta beyond mean field

  Fix: Finite nuclei with SRC --> high (modest) momenta
- Dispersive Optical Model (Framework of Green's functions <--> data)
- Conclusions and Outlook

Why do Green's functions?

- Properly executed --> answers an old question from Sir Denys Wilkinson: "What does a nucleon do in the nucleus?"



- Nucleon self-energy --> elastic nucleon scattering data --> input for the analysis of many nuclear reactions
- Ab initio: miserable status (see later) needs urgent improvement
- Nucleon self-energy --> bound-state overlap functions with their normalization --> also used in the analysis of nuclear reactions
- Nucleon self-energy: density distribution & E/A from V_{NN}
- Ab initio: FRPA good but can be further improved
- Self-energy Σ --> nucleon propagator G ---> $\dfrac{\delta\Sigma}{\delta G}$ ---> excited states
- Self-energy <--> data --> dispersive optical model (DOM)

Understanding/Calculating Self-energy

Propagator / Green's function

- Lehmann representation

$$G_{\ell j}(k,k';E) = \sum_m \frac{\langle \Psi_0^A | a_{k\ell j} | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_{k'\ell j}^\dagger | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} + \sum_n \frac{\langle \Psi_0^A | a_{k'\ell j}^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta}$$

- Any single-particle basis can be used
- Overlap functions --> numerator
- Corresponding eigenvalues --> denominator
- Spectral function

$$S_{\ell j}(k;E) = \frac{1}{\pi} \,\mathrm{Im}\, G_{\ell j}(k,k;E) = \sum_n \left| \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle \right|^2 \delta(E - (E_0^A - E_n^{A-1})) \qquad E \le \varepsilon_F^-$$

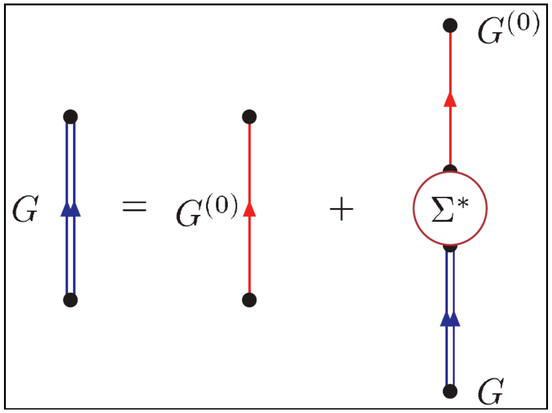
- Spectral strength in the continuum

$$S_{\ell j}(E) = \int_0^\infty dk\; k^2\; S_{\ell j}(k;E)$$

- Discrete transitions $\sqrt{S_{\ell j}^n}\; \phi_{\ell j}^n(k) = \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle$

Understanding/Calculating Self-energy

Propagator from Dyson Equation and "experiment"



Equivalent to …

Schrödinger-like equation with: $E_n^- = E_0^A - E_n^{A-1}$

Self-energy: non-local, energy-dependent potential

With energy dependence: spectroscopic factors < 1

$\Rightarrow$ as observed in (e,e'p)

$$\frac{k^2}{2m}\phi_{\ell j}^n(k) + \int dq\ q^2\ \Sigma_{\ell j}^*(k,q;E_n^-)\ \phi_{\ell j}^n(q) = E_n^-\ \phi_{\ell j}^n(k)$$

Spectroscopic factor $\quad S_{\ell j}^n = \int dk\ k^2\ \left|\langle \Psi_n^{A-1} |\, a_{k\ell j}\, | \Psi_0^A \rangle\right|^2 < 1$

Dyson equation also yields $\quad \chi_c^{A+1}(\boldsymbol{r}\sigma;E) = \langle \Psi_E^{c,A+1} |\, a_{\boldsymbol{r}\sigma}^\dagger\, | \Psi_0^A \rangle$ for positive energies

Elastic scattering wave function for protons or neutrons

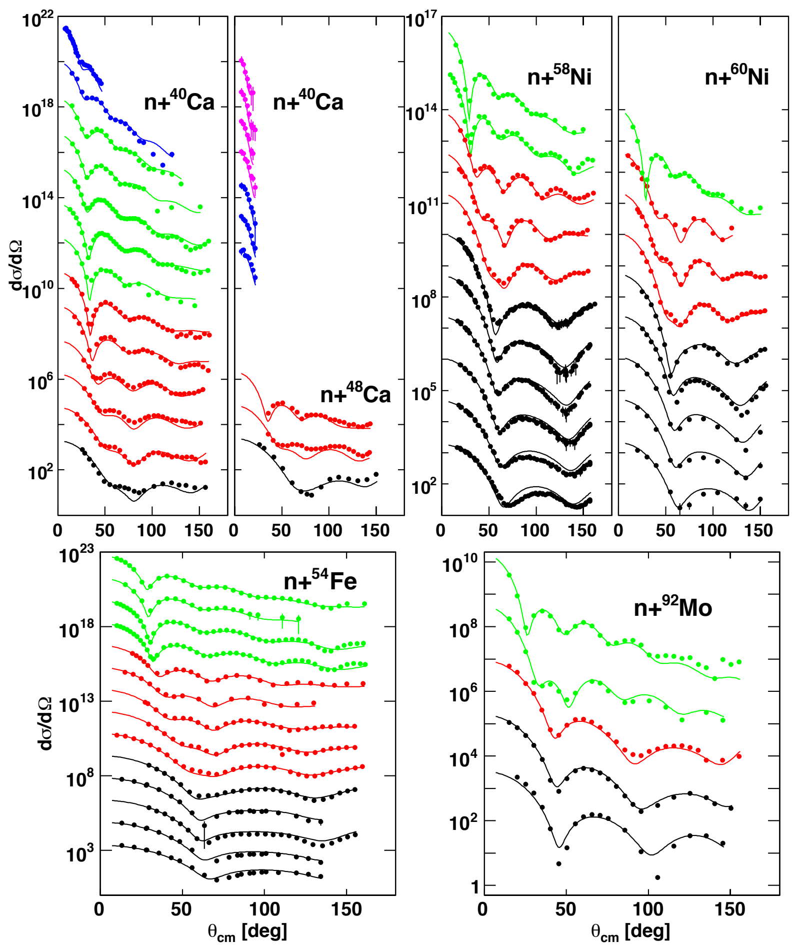
Dyson equation provides:

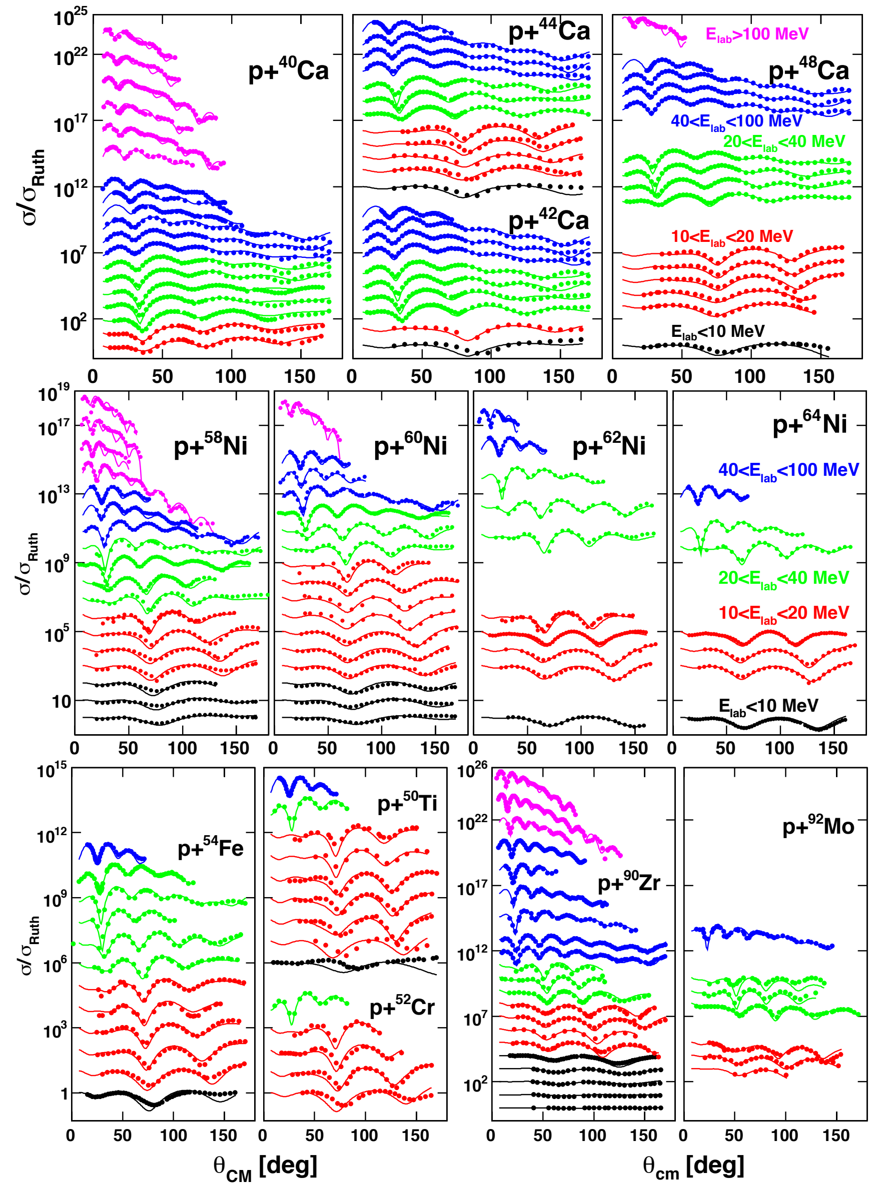
Link between scattering and structure data from **dispersion relations**

Understanding/Calculating Self-energy

Elastic scattering data for protons and neutrons

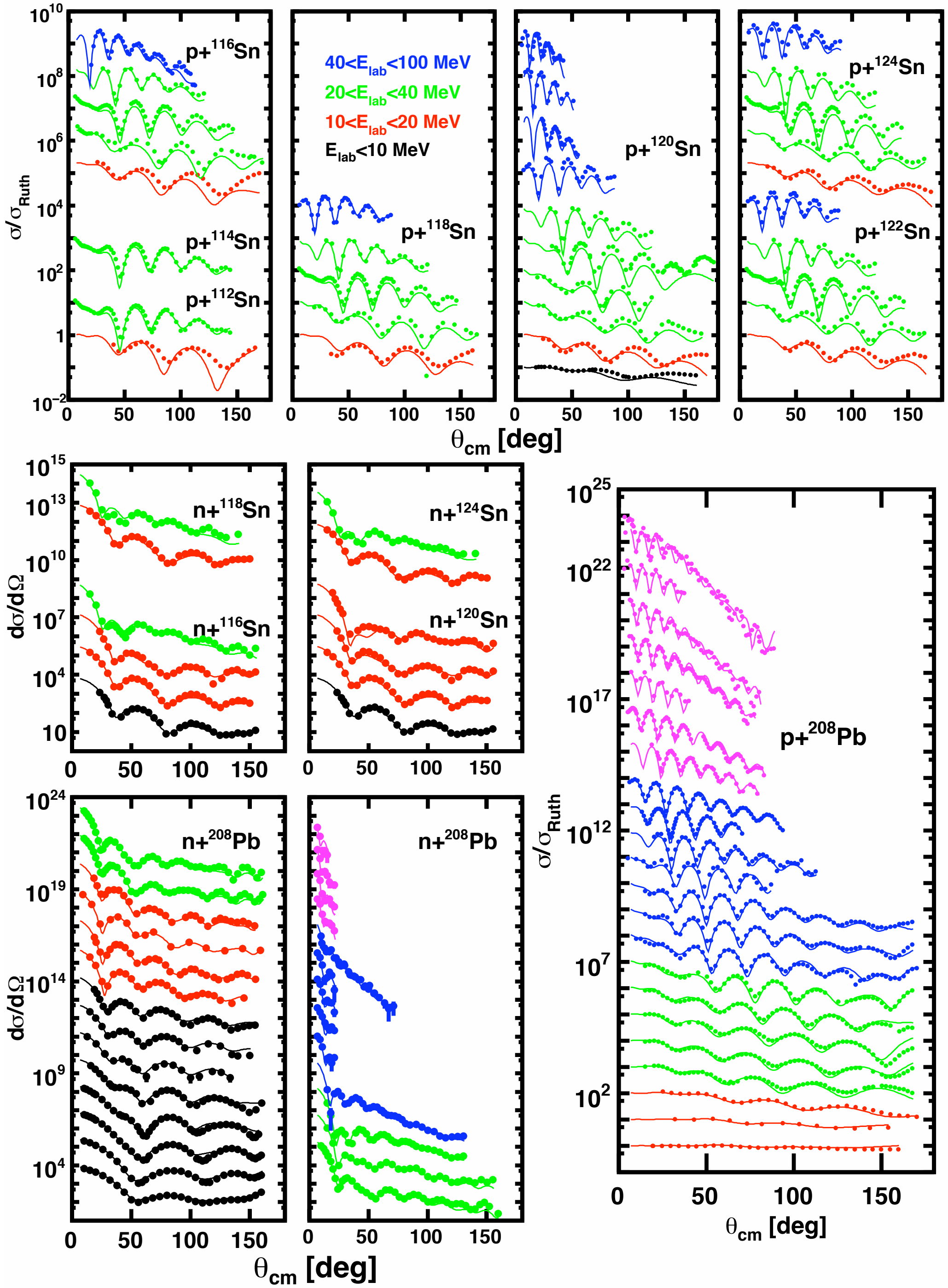
- Abundant for stable targets





Phys. Rev. C83, 064605 (2011)

Understanding/Calculating Self-energy

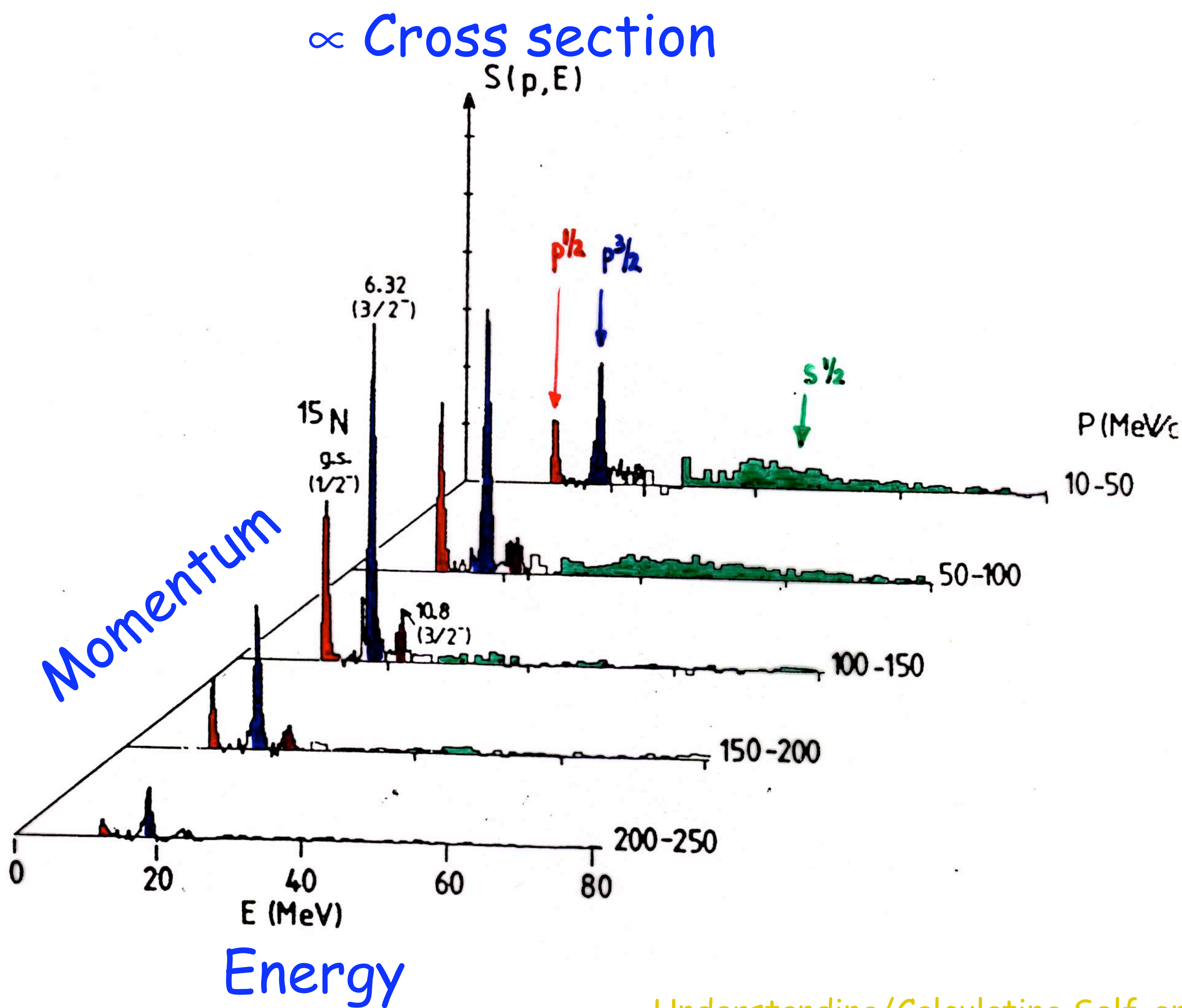


Recent DOM analysis --> towards global

J. Mueller et al.
PRC83,064605 (2011), 1-32

Mougey et al., Nucl. Phys. A335, 35 (1980)

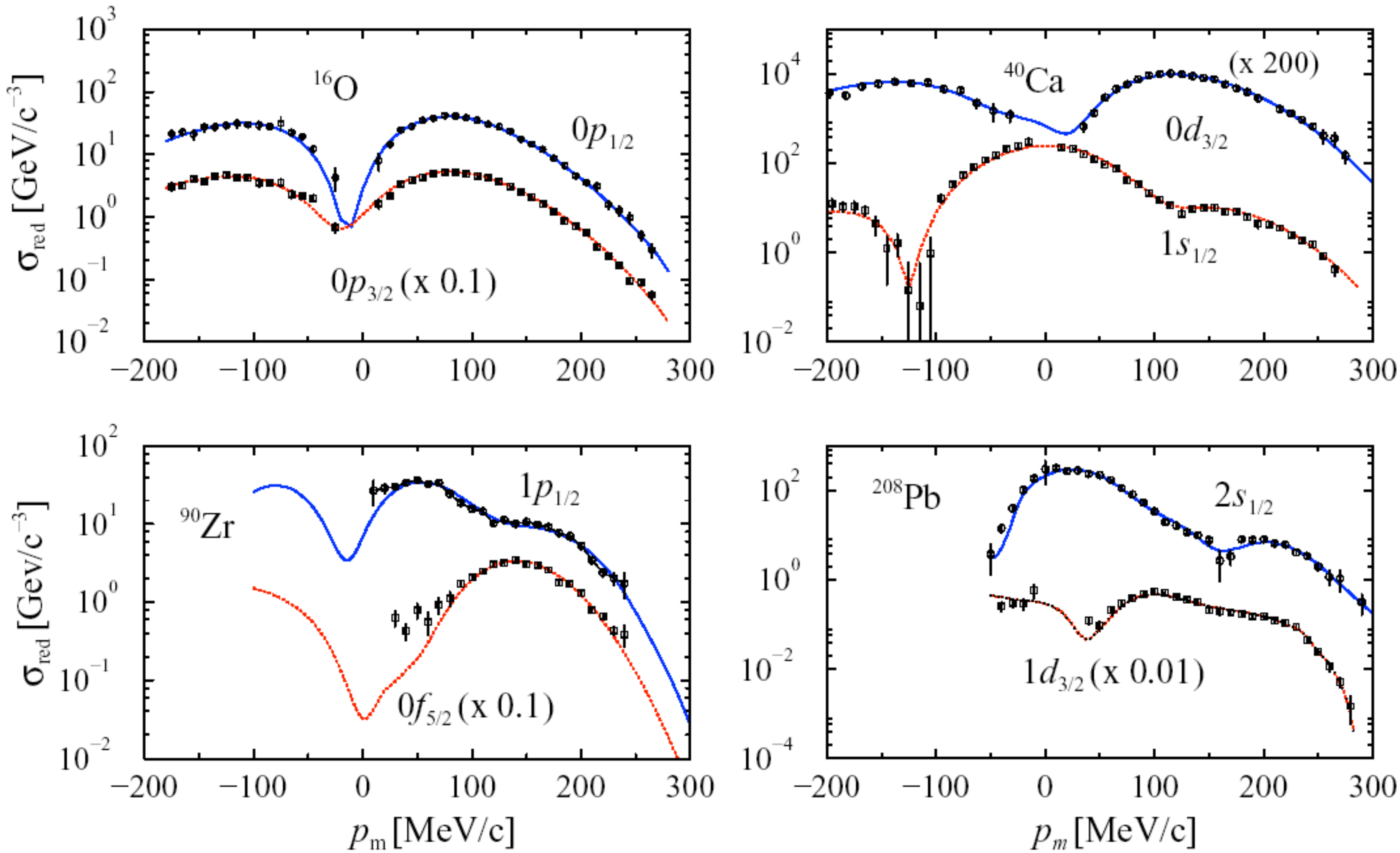
$^{16}O(e,e'p)$



Understanding/Calculating Self-energy

Nuclei (e,e'p) reaction

NIKHEF data, L. Lapikás, Nucl. Phys. *A553*, 297c (1993)



Wave functions as expected, except ….Understanding/Calculating Self-energy

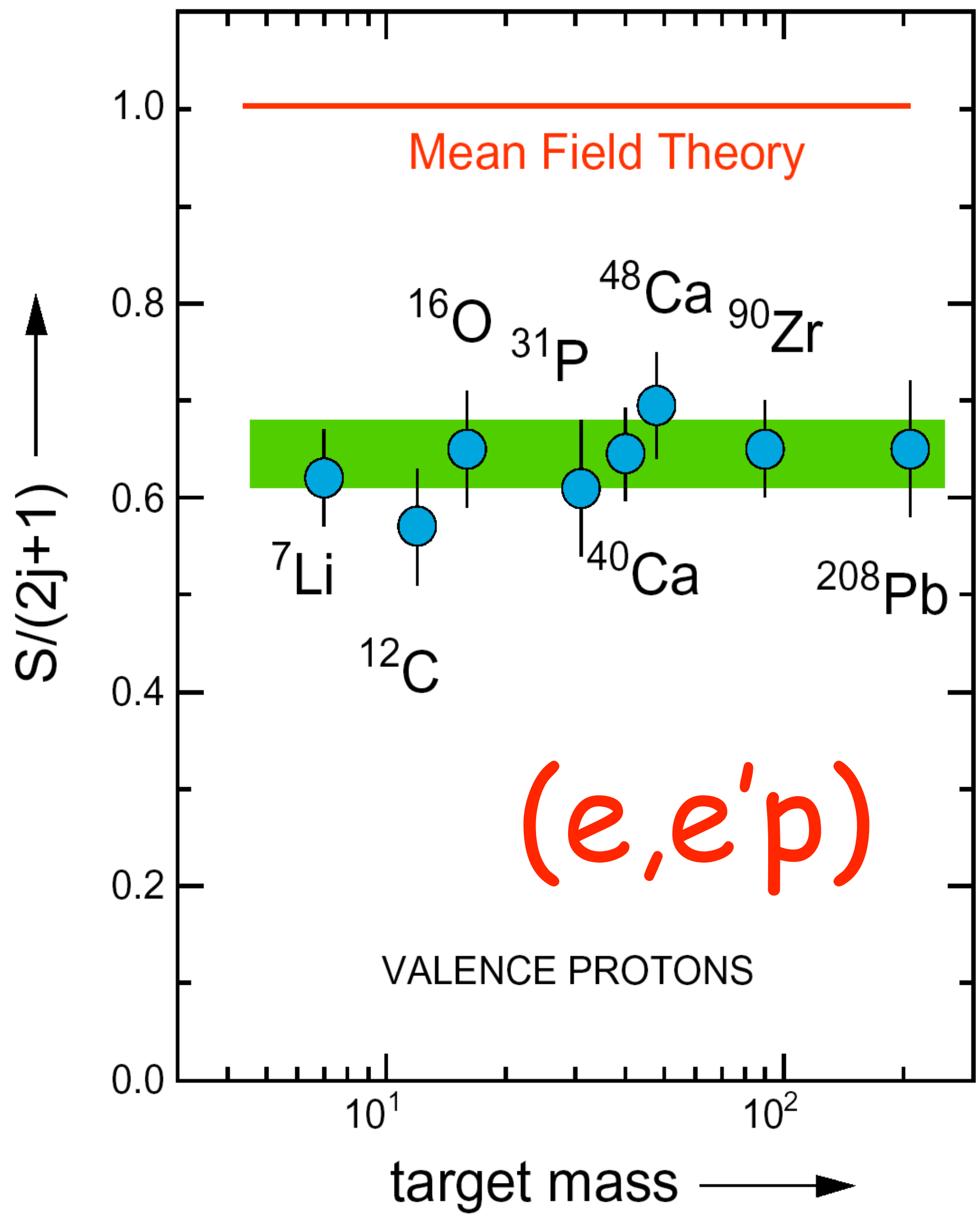
Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

S ≈ 0.65 for valence protons
Reduction ⇒ both SRC and LRC

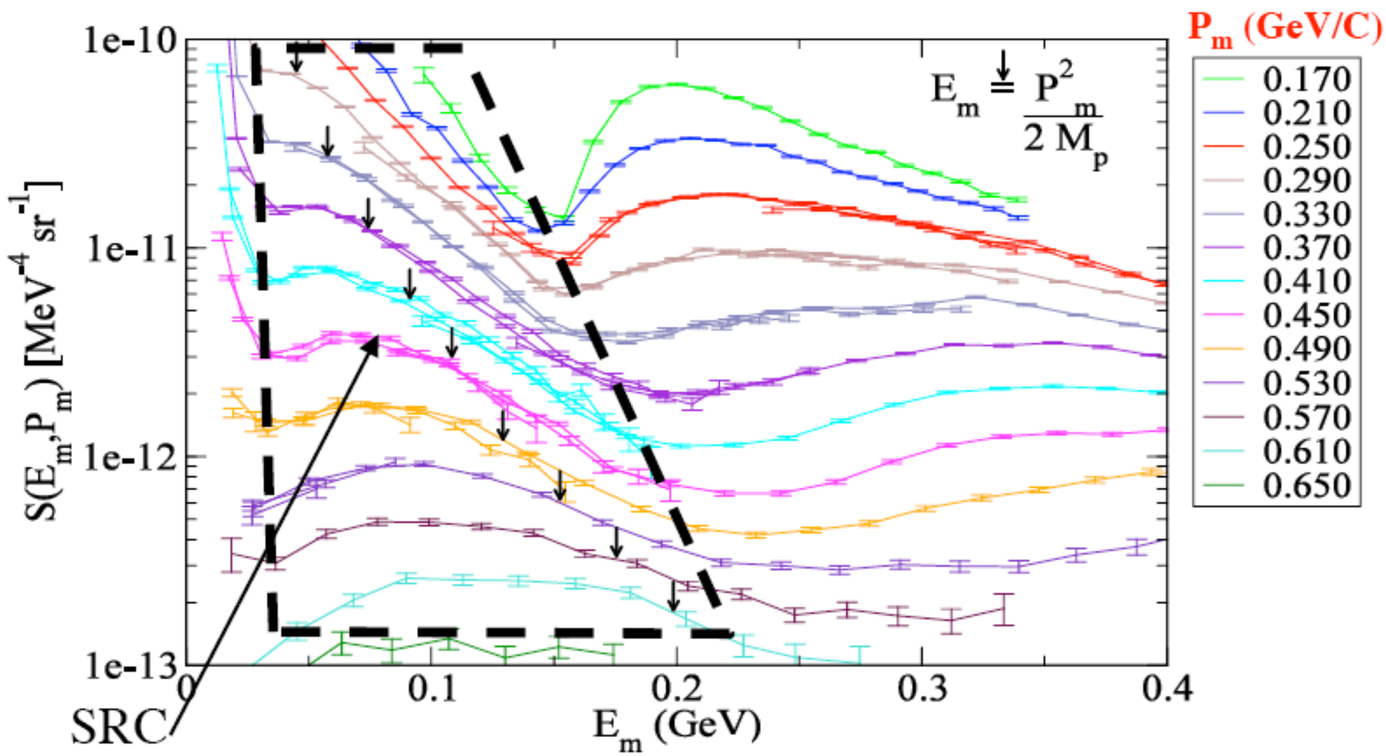
Weak probe but propagation in the nucleus of removed proton using standard optical potentials to generate distorted wave --> associated uncertainty ~ 5-10%

Why: details of the interior scattering wave function uncertain since non-locality is not constrained (so far)



High-momentum protons have been seen in nuclei!

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



^{12}C

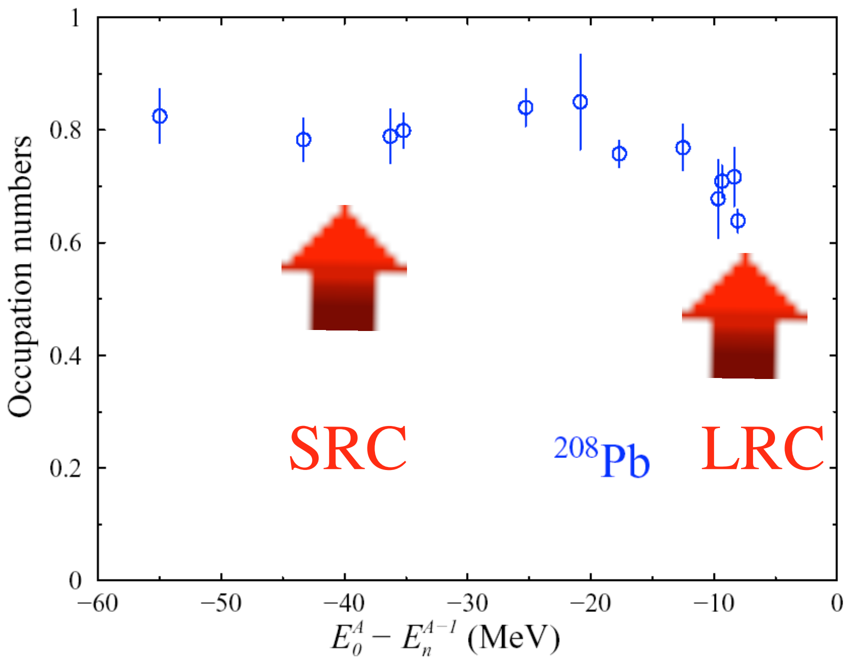
- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

$\Rightarrow$ ~0.6 protons for ^{12}C $\Rightarrow$ **~10%**

Understanding/Calculating Self-energy

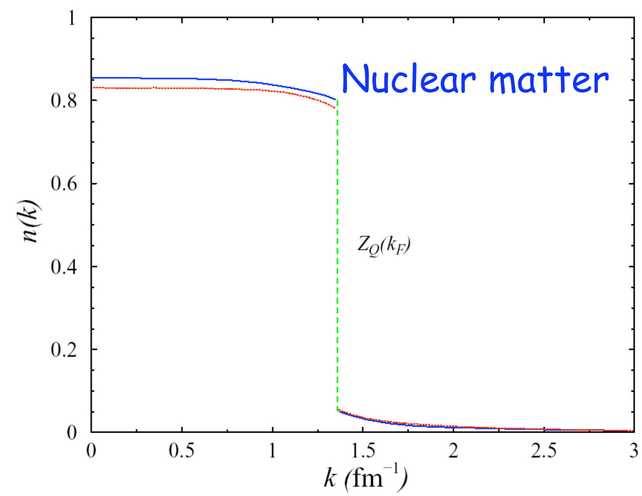
M. van Batenburg & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl
NIKHEF 2001 data (one of the last experiments)

Occupation of deeply-bound proton levels from **EXPERIMENT**



Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the **whole** mean-field domain!!



Understanding/Calculating Self-energy

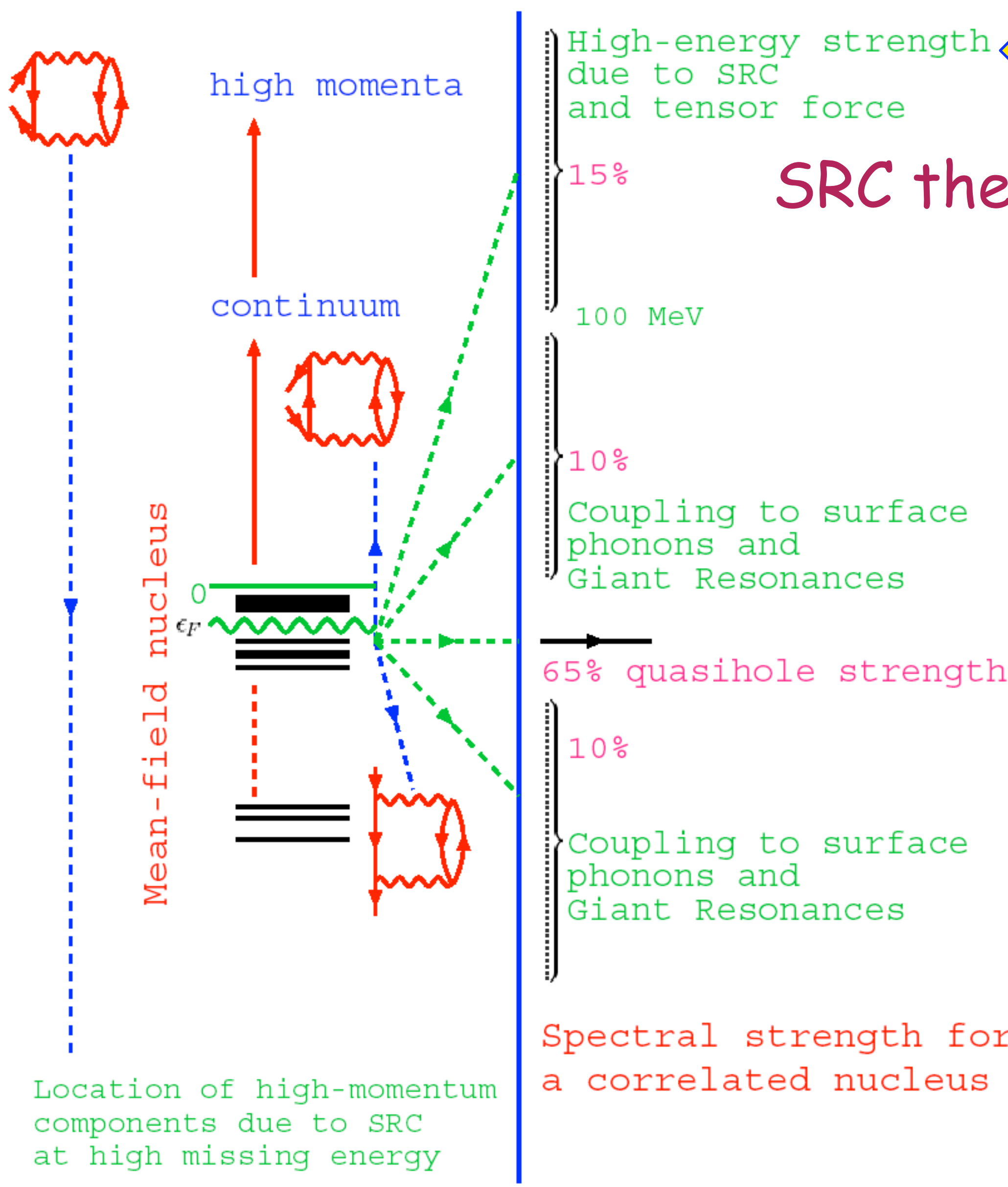
Confirms predictions for depletion

$n(0) \Rightarrow$ 0.85 Reid
0.87 Argonne V18
0.89 CDBonn/N3LO

Reviewed in Prog. Part. Nucl. Phys. 52 (2004) 377-496

Location of single-particle strength in closed-shell (stable) nuclei

For example: protons in ^{208}Pb



SRC theory

Elastic nucleon scattering

NIKHEF (e,e'p) data

L. Lapikás

Nucl. Phys. A553,297c (1993)

SRC

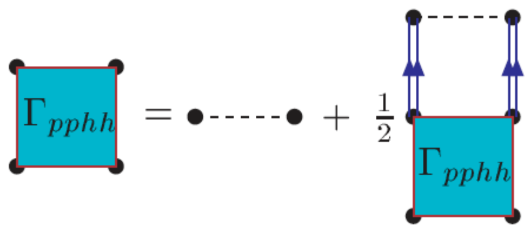
JLab E97-006

Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.

Understanding/Calculating Self-energy

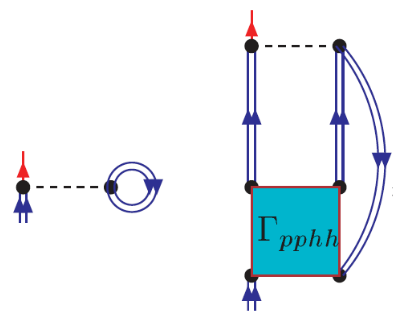
Full off-shell propagation in infinite matter

SCGF:
self-consistent
Green's functions
for SRC and tensor
effects

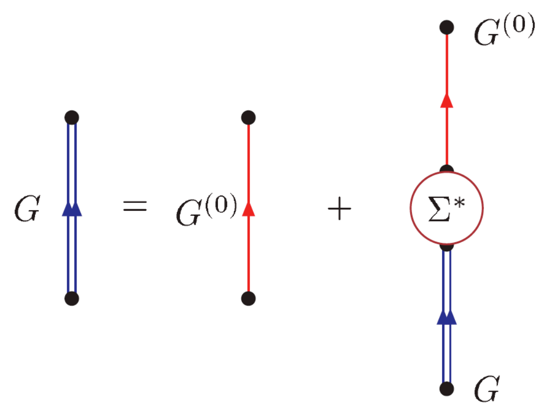


Interaction in the medium properly treating short-range and tensor correlations

self-consistency
=> thermodynamically consistent



Self-energy = complex potential in nuclear matter



Dyson equation ⇒ Schrödinger equation for dressed nucleons

Arnau Rios
Arturo Polls
W.D.
finite T avoids pairing (with in progress)

Understanding/Calculating Self-energy

Fetter & Walecka

The Bethe-Goldstone theory described above still differs in principle from the Brueckner theory because the Brueckner theory relies on a self-consistent single-particle potential. In terms of Green's functions, this result can be achieved by replacing $G^0(p)$ with a $G(p)$ that includes self-energy effects associated with Γ. Furthermore, Γ must itself be determined with G and not G^0. The equations for this self-consistent theory are shown schematically in Fig. 42.4.

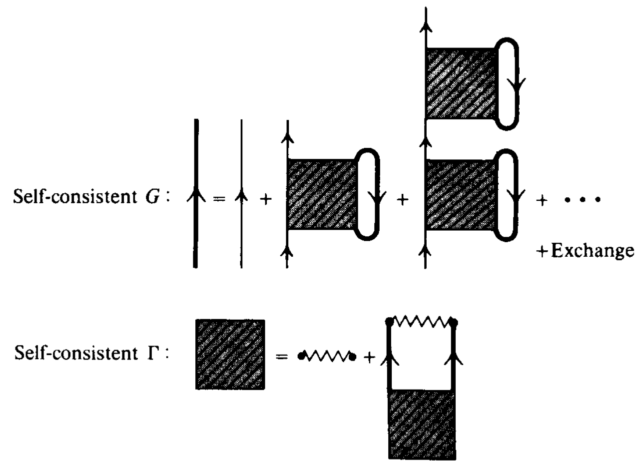


Fig. 42.4 Self-consistent theory for G and Γ.

As they stand, these equations are quite intractable because the frequency dependence of $\Sigma^\star(\mathbf{p},p_0)$ complicates the integral equation for Γ immensely. (This difficulty is sometimes known as *propagation off the energy shell.*) The simple Brueckner-Goldstone theory can be obtained from these equations in a series of approximations. First, the self-consistency is treated only on the average, and we use a frequency-independent self-energy $\Sigma^\star_{sc}(\mathbf{p}) \equiv \Sigma^\star(\mathbf{p},\epsilon_\mathbf{p}/\hbar)$, obtained by setting $p_0 = \epsilon_\mathbf{p}/\hbar$, where $\epsilon_\mathbf{p}$ satisfies the self-consistent equation

$$\epsilon_\mathbf{p} = \epsilon^0_\mathbf{p} + \hbar\Sigma^\star(\mathbf{p},\epsilon_\mathbf{p}/\hbar) \equiv \epsilon^0_\mathbf{p} + \hbar\Sigma^\star_{sc}(\mathbf{p}) \tag{42.13}$$

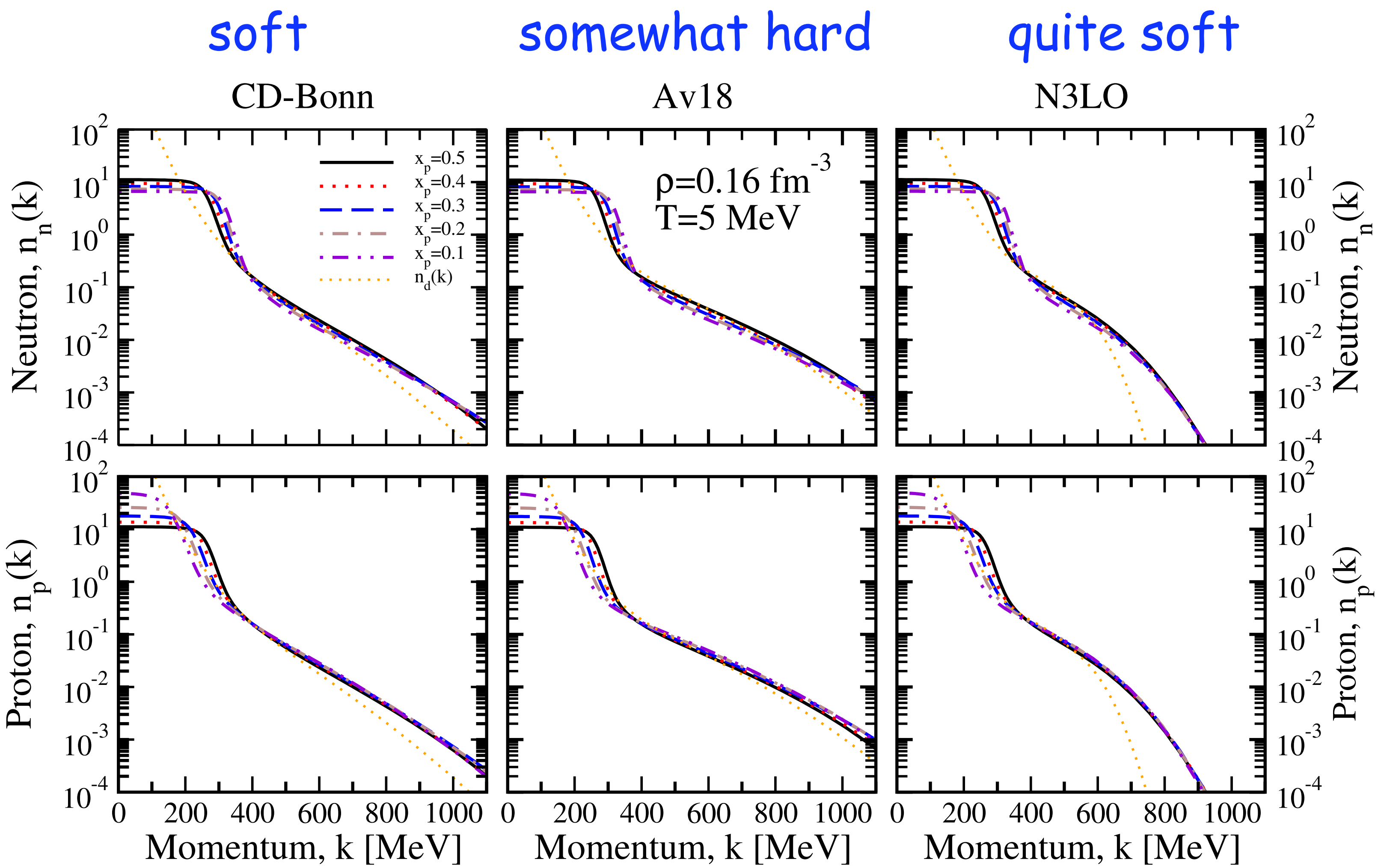
In this way, the Green's function is given approximately as

$$G_{sc}(\mathbf{p},p_0) = \frac{\theta(|\mathbf{p}| - k_F)}{p_0 - \epsilon_\mathbf{p}/\hbar + i\eta} + \frac{\theta(k_F - |\mathbf{p}|)}{p_0 - \epsilon_\mathbf{p}/\hbar - i\eta} \tag{42.14}$$

Second, this Green's function is used to evaluate both the proper self-energy [Eq. (42.4)] and the scattering amplitude [Eqs. (42.5) and (42.6)]. We again obtain χ_m by omitting the hole-hole scattering, which is presumed small in the low-density limit. The only effect on the self-consistent wave function is to change the denominator in Eq. (42.6) from $mP_0/\hbar - \frac{1}{2}(\frac{1}{2}\mathbf{P} + \mathbf{q})^2 - \frac{1}{2}(\frac{1}{2}\mathbf{P} - \mathbf{q})^2 + i\eta$

Understanding/Calculating Self-energy

Three different interactions (Rios, Polls recently)



- Normalization of n(k) --> 1 (not density)

SRC/high momenta ab initio for heavier nuclei

- In the beginning stages of proper sophistication…

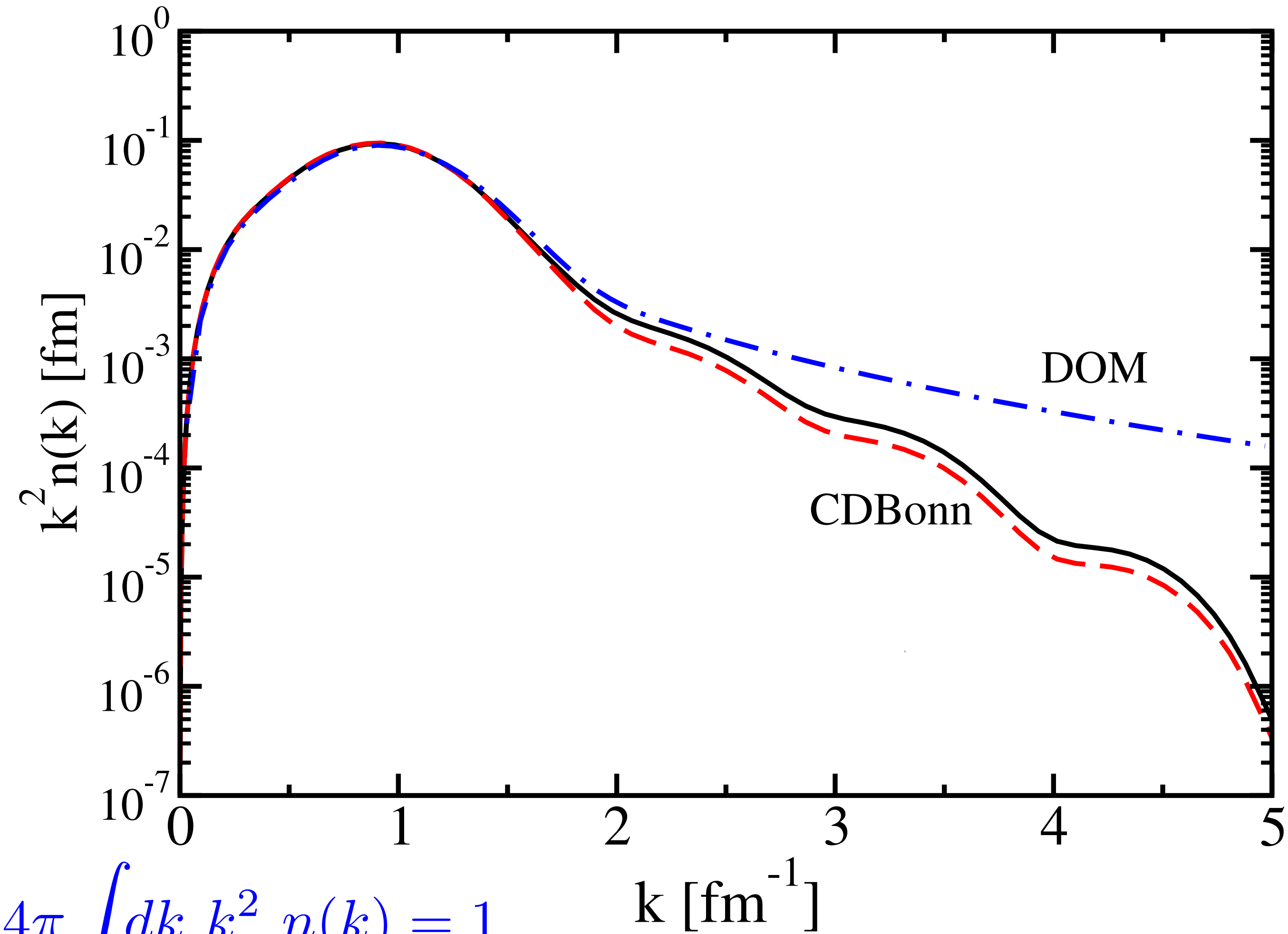
- But in progress!

- One-body scattering --> momentum vector - spin basis
- Two-body scattering --> momentum vector - spins basis
- Initial step: use nuclear matter detour --> PRC51, 3040 (1995)

- How "bad" is it?

Ab initio with CDBonn for ^{40}Ca

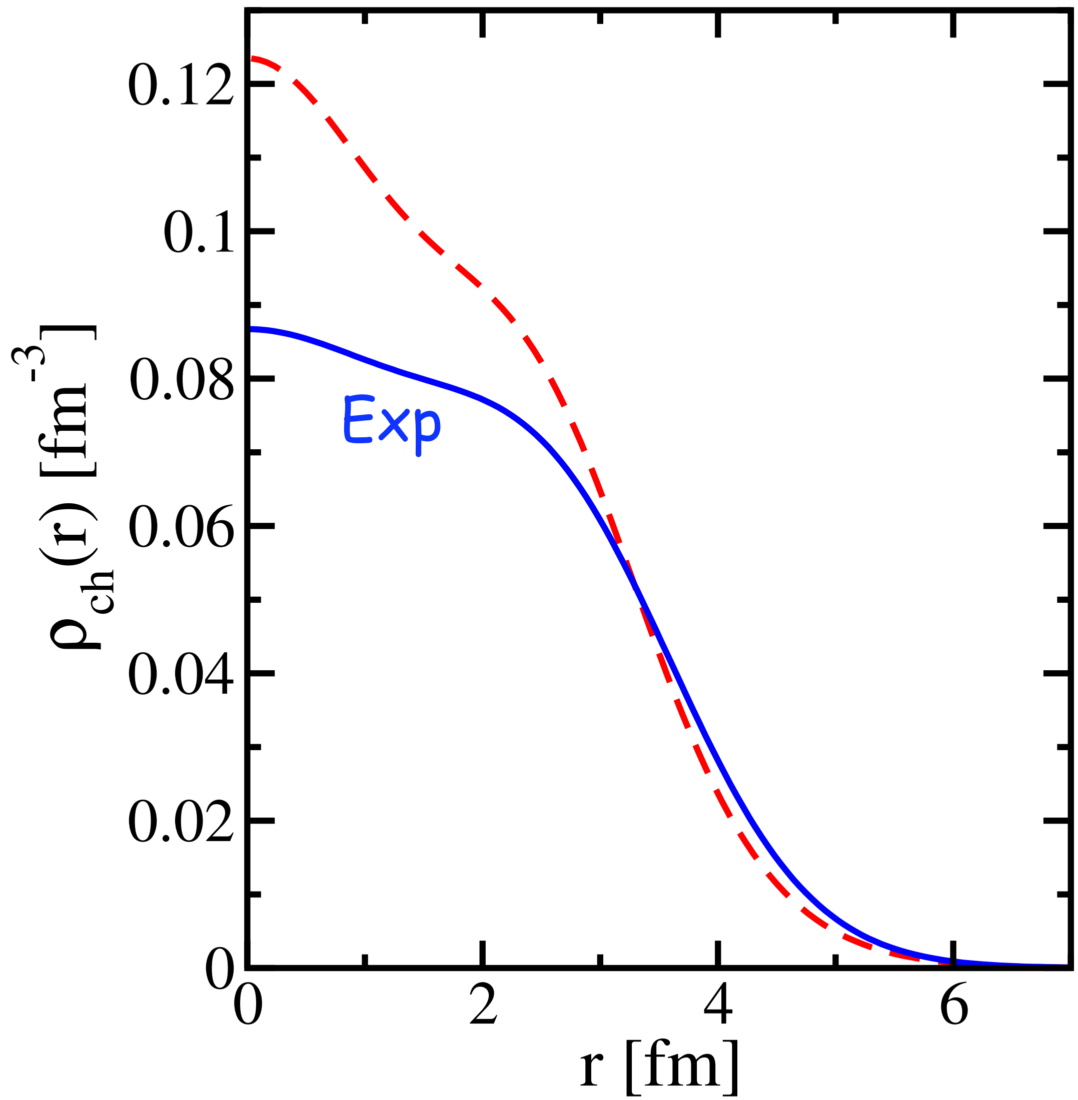
- Dussan et al. PRC84, 044319 (2011); spectral functions available



$$4\pi \int dk\ k^2\ n(k) = 1$$

CDBonn

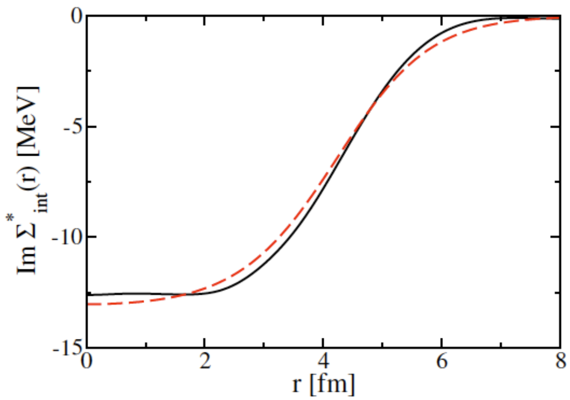
- Density ....



Non-locality of imaginary part

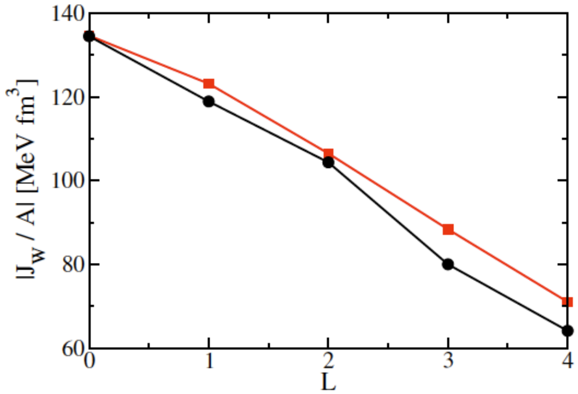
- Fit non-local imaginary part for ℓ=0

$$W_{NL}(\boldsymbol{r},\boldsymbol{r}') = W_0 \sqrt{f(r)}\sqrt{f(r')} H\left(\frac{\boldsymbol{r}-\boldsymbol{r}'}{\beta}\right)$$



- Integrate over one radial variable

- Predict volume integrals for higher ℓ

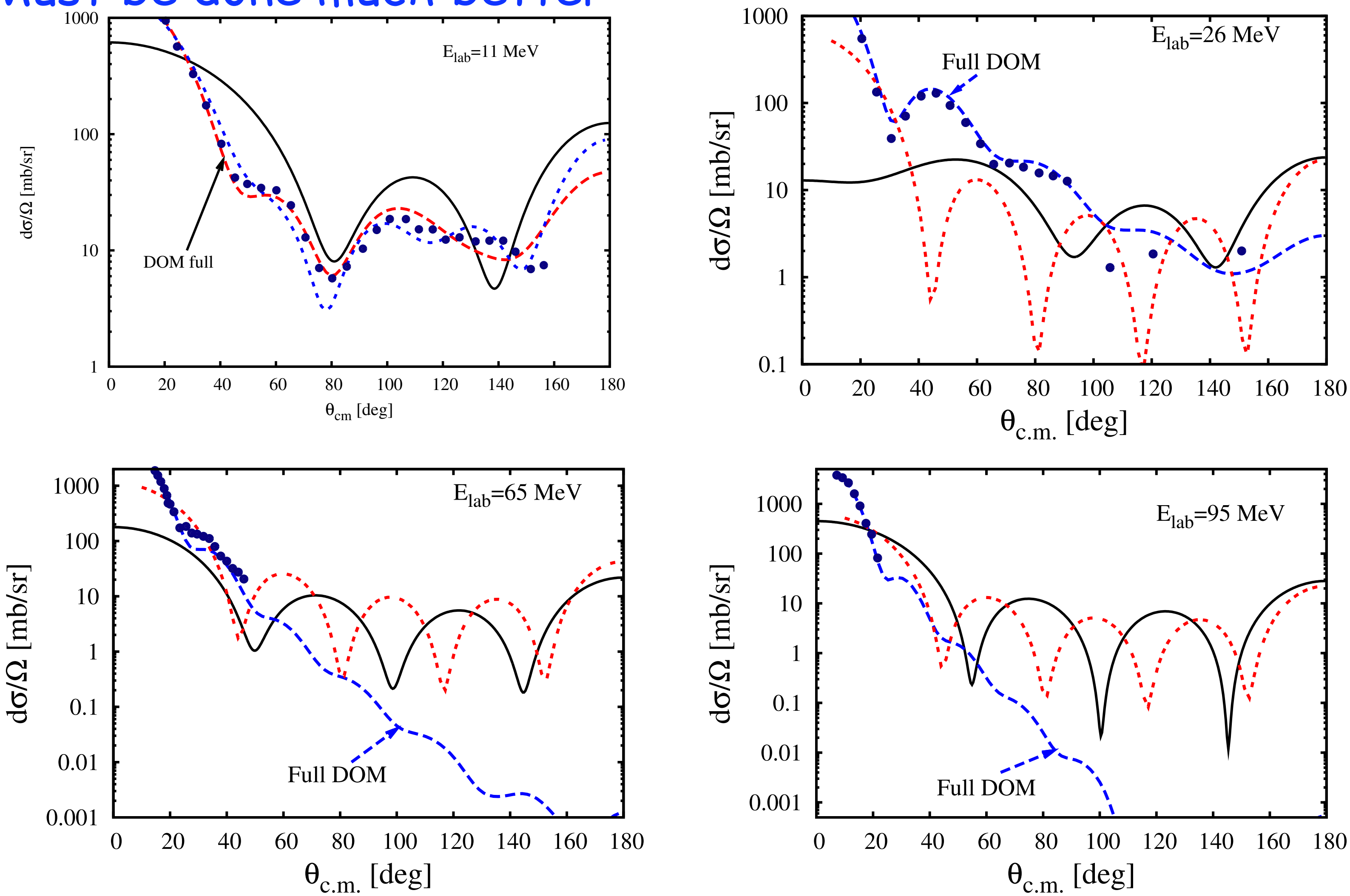


Parameters

Energy MeV	W_0	r_0	a_0	β	$\lvert J_W/A\rvert$	$\lvert J_W/A\rvert$ CDBonn
-76	36.30	0.90	0.90	1.33	193	193
49	6.51	1.25	0.91	1.43	73	73
65	13.21	1.27	0.70	1.29	135	135
81	23.90	1.22	0.67	1.21	215	215

Ab initio description of elastic scattering

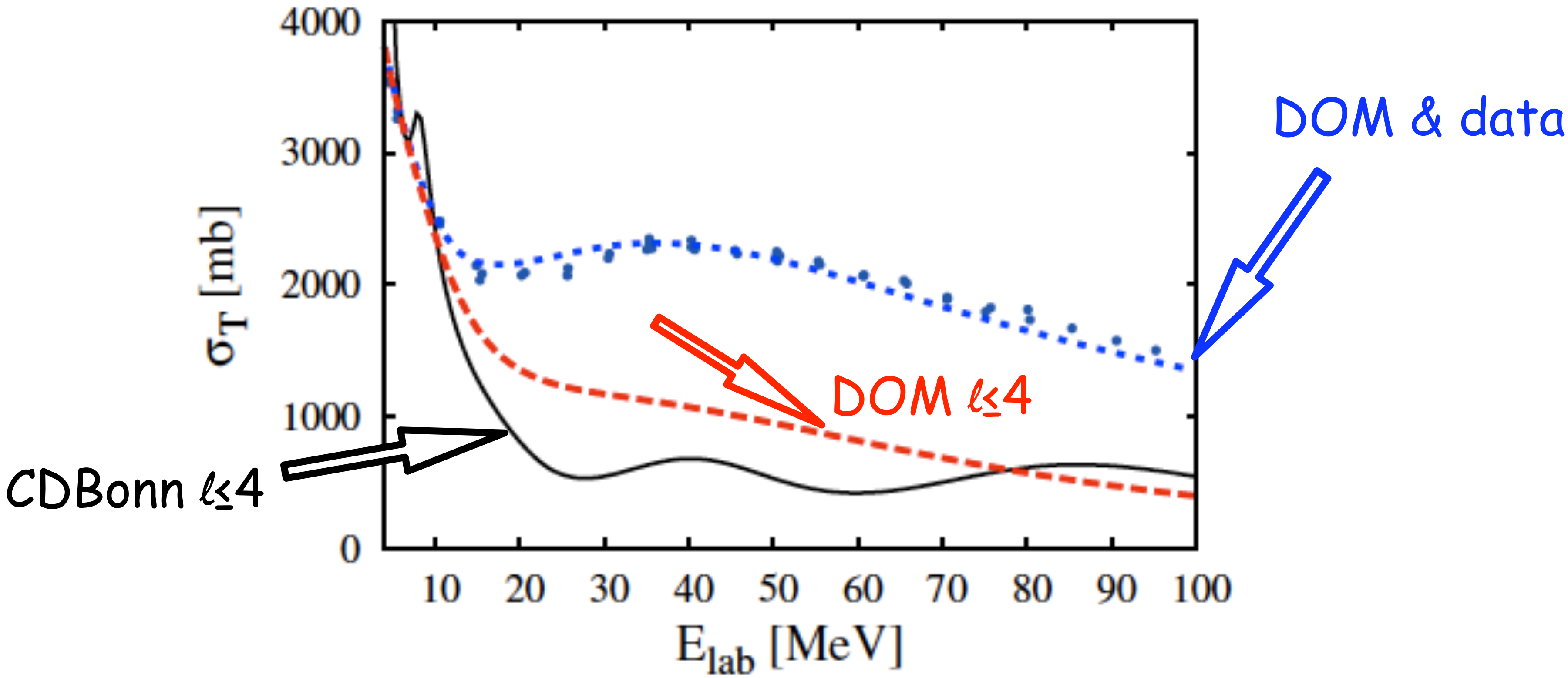
- Must be done much better



Understanding/Calculating Self-energy

Ab initio calculation of elastic scattering n+^{40}Ca

- ONLY treatment of short-range and tensor correlations



Drip-line nuclear physics

- Many reactions necessarily involve strongly interacting particles
  - (p,2p) perhaps (p,pn)
  - (d,p) or (p,d)
  - HI knock-out reactions
- Interactions of “projectiles” with “target” are not experimentally constrained at this time --> no unambiguous information
- Empirical Green’s function project: Dispersive Optical Model (DOM)
  - intends to provide a frame work for such constraints
  - simultaneous treatment of negative (structure) and positive energies (reactions) for nucleons PLUS a reaction description
  - linking information below and above the Fermi energy such as elastic scattering cross sections, level structure, charge densities, knock-out cross sections etc. ---> constrained description of p or n distorted waves

Understanding/Calculating Self-energy

Optical potential <--> nucleon self-energy

- e.g. Bell and Squires --> elastic T-matrix = reducible self-energy
- Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\text{Re }\Sigma(E) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im }\Sigma(E')}{E-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im }\Sigma(E')}{E-E'}$$

Calculated at the Fermi energy $\varepsilon_F = \frac{1}{2}\left\{(E_0^{A+1}-E_0^A)+(E_0^A-E_0^{A-1})\right\}$

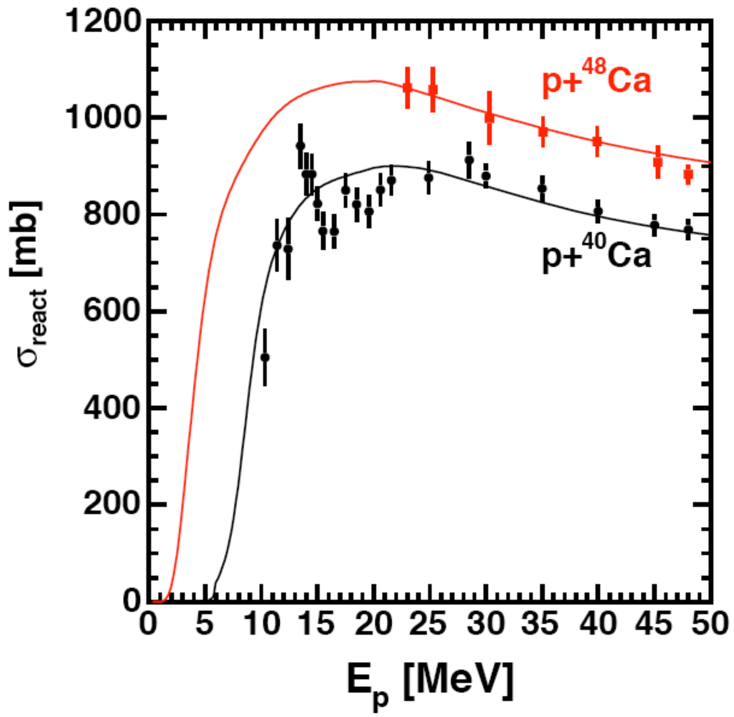
$$\text{Re }\Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi}\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im }\Sigma(E')}{\varepsilon_F-E'} + \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im }\Sigma(E')}{\varepsilon_F-E'}$$

Subtract

$$\text{Re }\Sigma(E) = \text{Re }\Sigma^{\widetilde{HF}}(\varepsilon_F)$$

$$- \quad \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{E_T^+}^{\infty} dE' \frac{\text{Im }\Sigma(E')}{(E-E')(\varepsilon_F-E')} + \frac{1}{\pi}(\varepsilon_F - E)\mathcal{P}\int_{-\infty}^{E_T^-} dE' \frac{\text{Im }\Sigma(E')}{(E-E')(\varepsilon_F-E')}$$

Does the nucleon self-energy also have an imaginary part above the Fermi energy?



Loss of flux in the elastic channel

Answer: YES!

Potentials assumed to have standard forms: including surface and volume absorption; parameters determined by fit to data. Potentials assumed local or "made" local. Assumptions are made about surface absorption above and below the Fermi energy.

DOM = Dispersive Optical Model

C. Mahaux and R. Sartor, Adv. Nucl. Phys. **20**, 1 (1991)

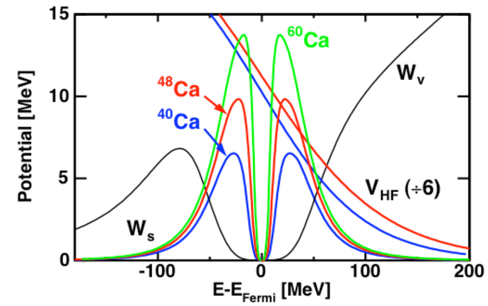
Goal: extract "propagator"/"self-energy" from data

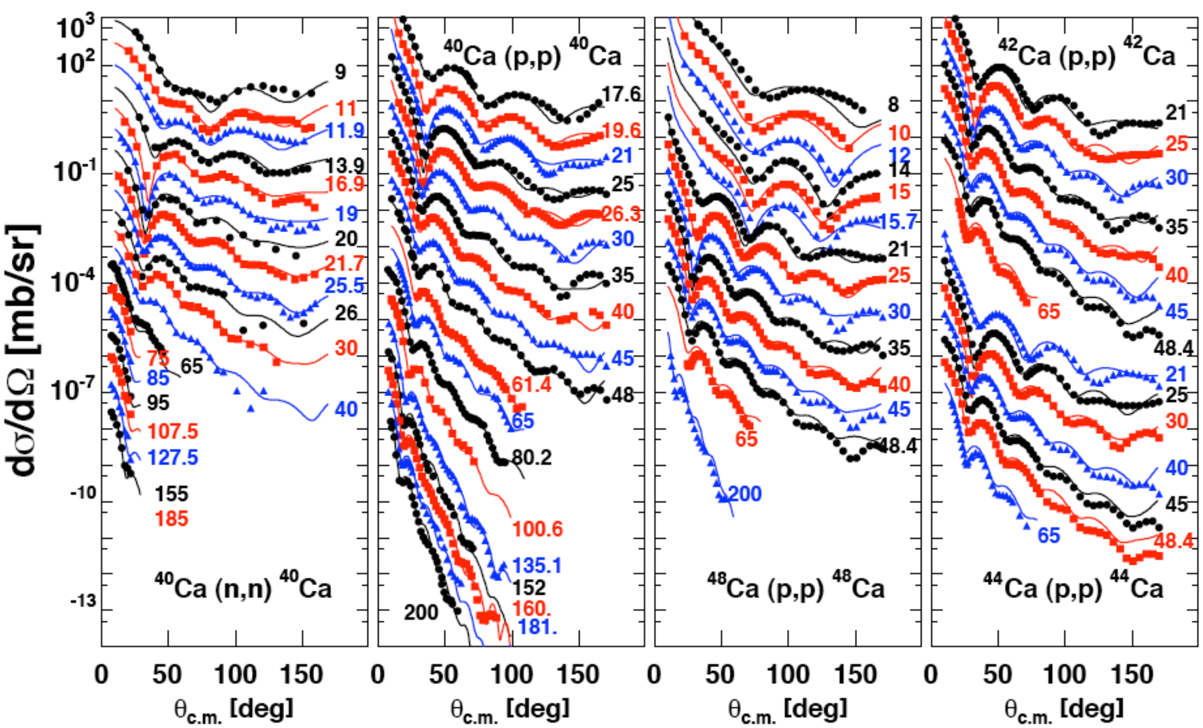
Vehicle for data-driven extrapolations / predictions to the drip lines

Combined analysis of protons/neutrons in ^{40}Ca and ^{48}Ca

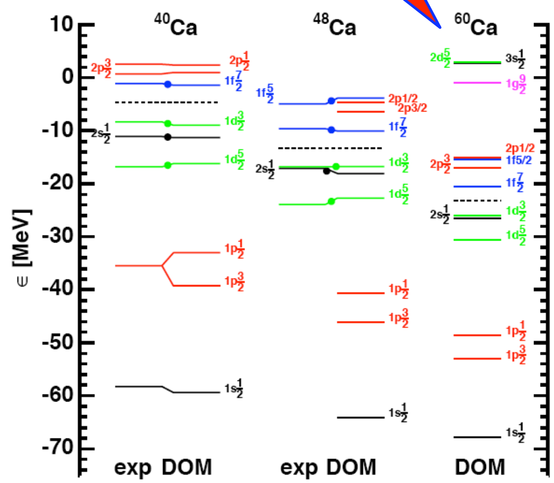
Charity, Sobotka, & WD, PRL **97**, 162503 (2006)

Charity, Mueller, Sobotka, & WD, PRC**76**, 044314 (2007)





Predict



Understanding/Calculating Self-energy

DOM improvements

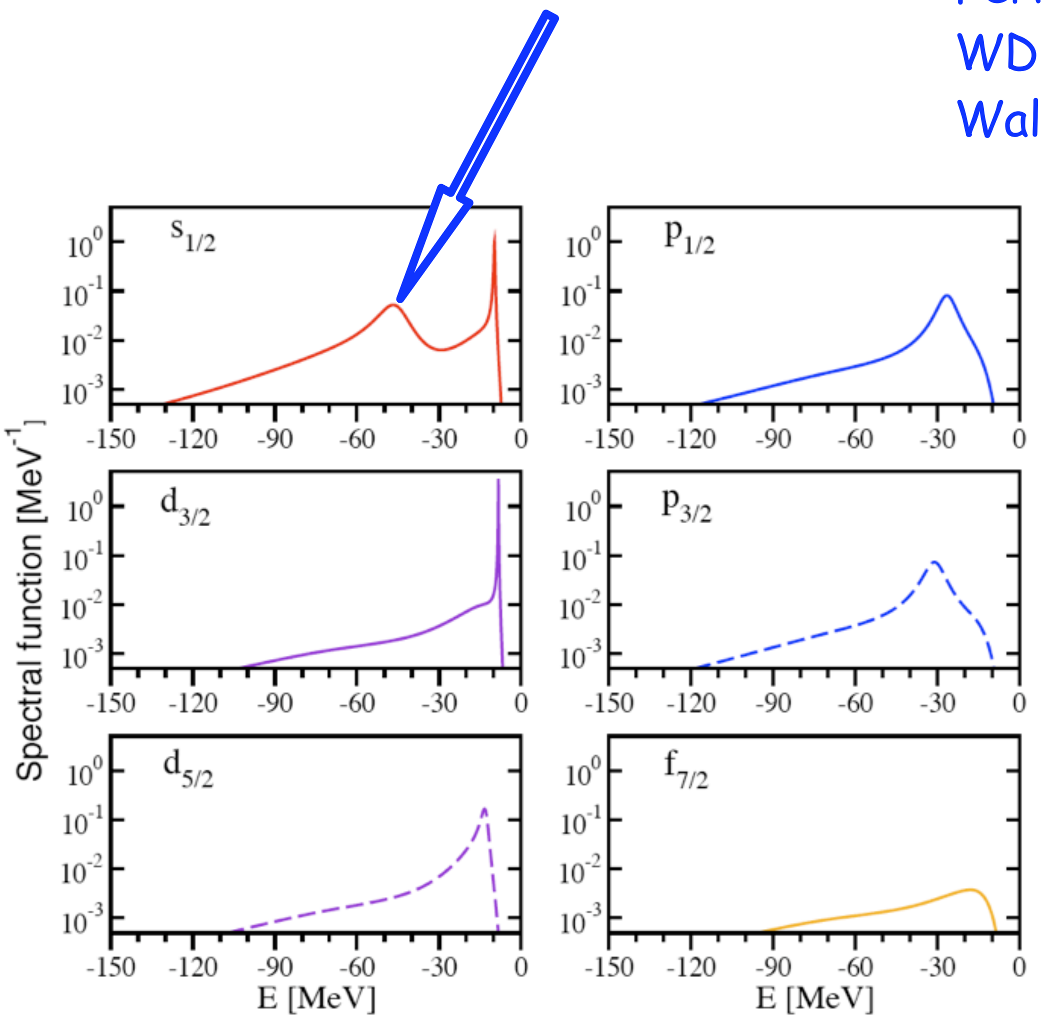
- Replace local energy-dependent HF potential by non-local (energy-independent potential) in order to calculate more properties below the Fermi energy like the charge density and spectral functions --> PRC82, 054306 (2010)

DOModel --> DOMethod-->DSelf-energyMethod

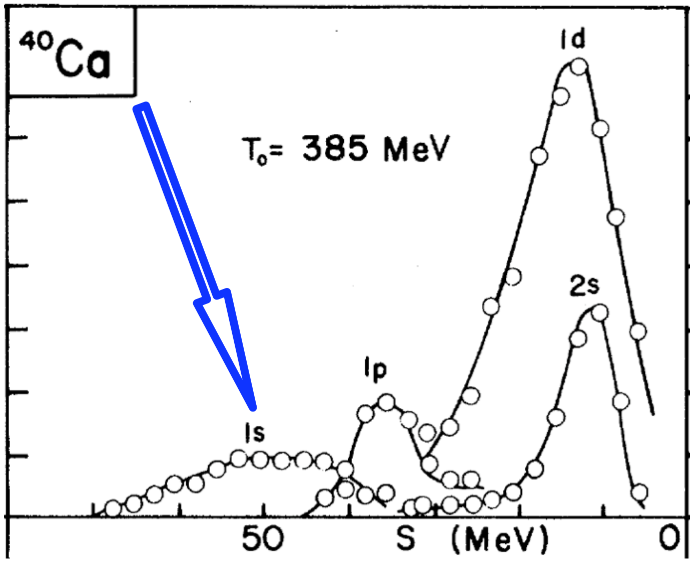
Below εF

^{40}Ca spectral function

Recent theoretical development: nonlocal "HF" self-energy --> below the Fermi energy
WD, Van Neck, Charity, Sobotka, Waldecker, PRC82, 054306 (2010)



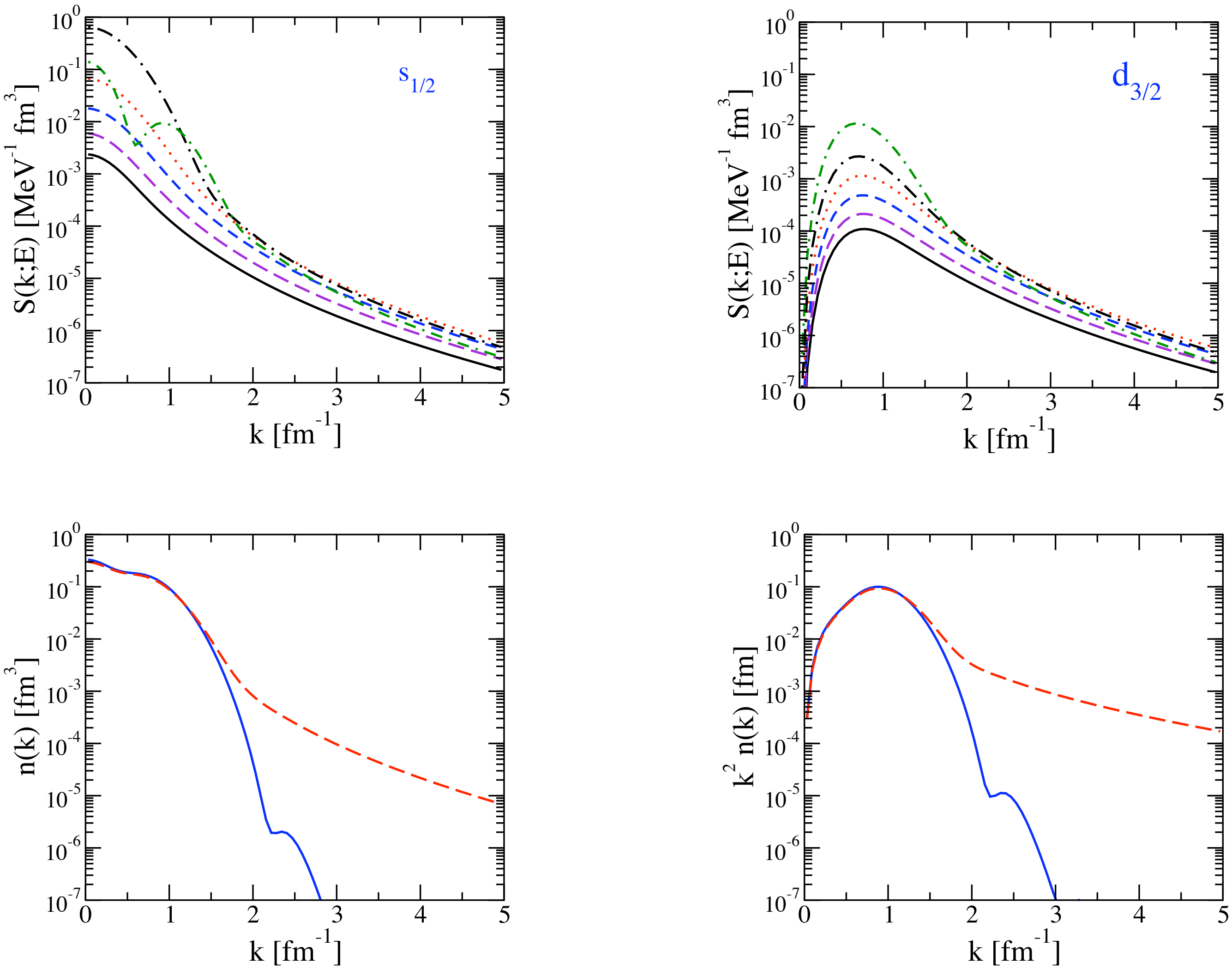
Old (p,2p) data from Liverpool



Spectral functions and momentum distributions

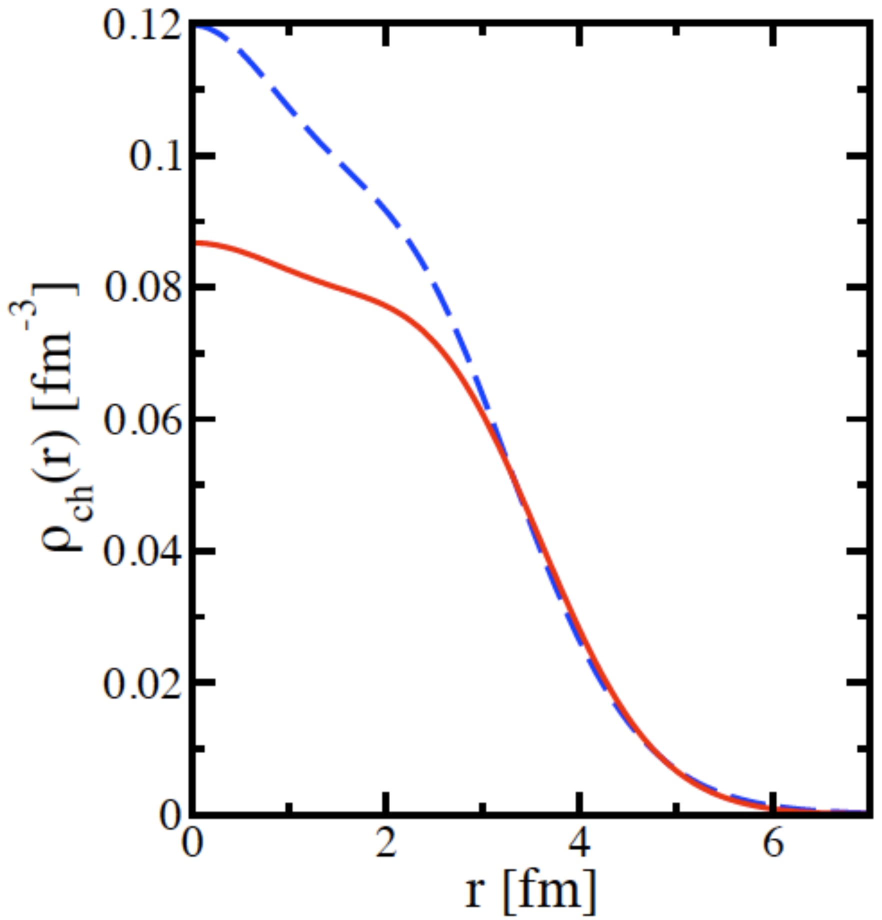
- ^{40}Ca

PRC 82, 054306 (2010)



Understanding/Calculating Self-energy

Charge density



Not a good reproduction of charge density even though mean square radius was fitted.

Related to local representation of the imaginary part of the self-energy --> independent of angular momentum --> must be abandoned to represent particle number correctly as well.

DOM extensions linked to ab initio FRPA

- Employ microscopic FRPA calculations of the nucleon self-energy to gain insight into future improvements of the DOM -->

S. J. Waldecker, C. Barbieri and W. H. Dickhoff

Phys. Rev. C84, 034616 (2011), 1-11

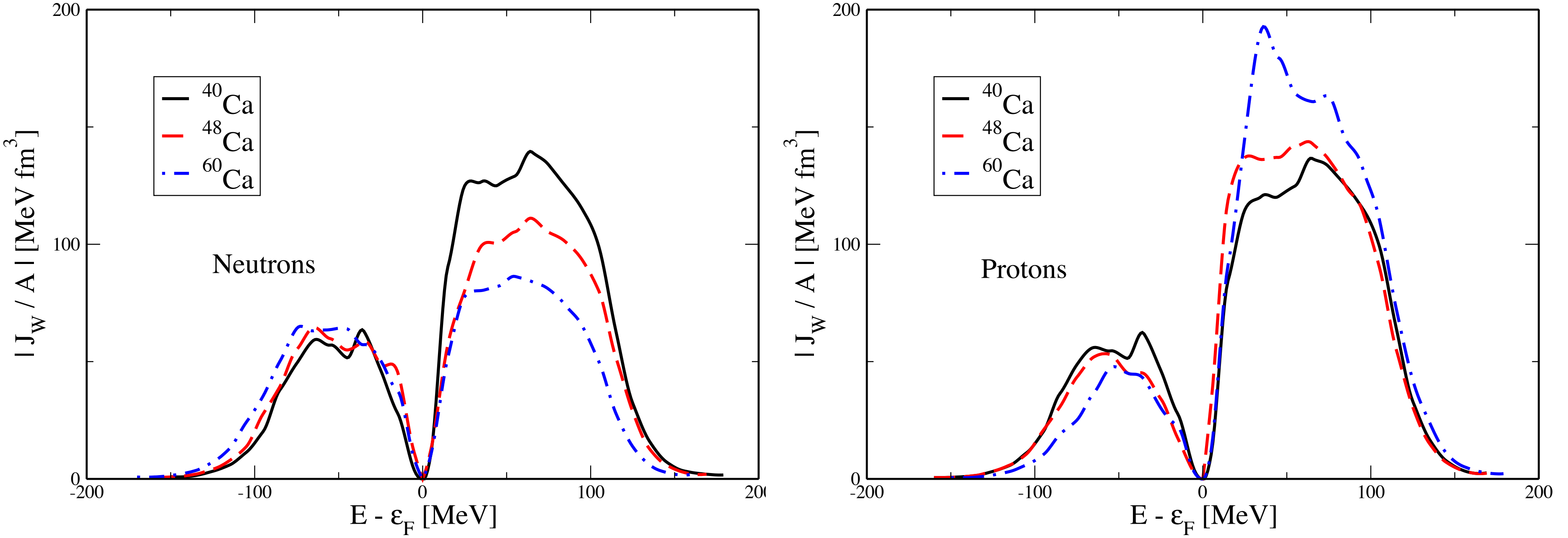
- FRPA = Faddeev RPA --> Barbieri for a recent application see e.g. PRL103,202502(2009)
- Most important conclusions
  - Ab initio self-energy has imaginary part with a substantial non-locality
  - Tensor force already operative for low-energy imaginary part
  - Absorption above and below Fermi energy not symmetric

Volume integrals from microscopic FRPA

relevant up to ~ 75 MeV

Volume integral for local imaginary potentials

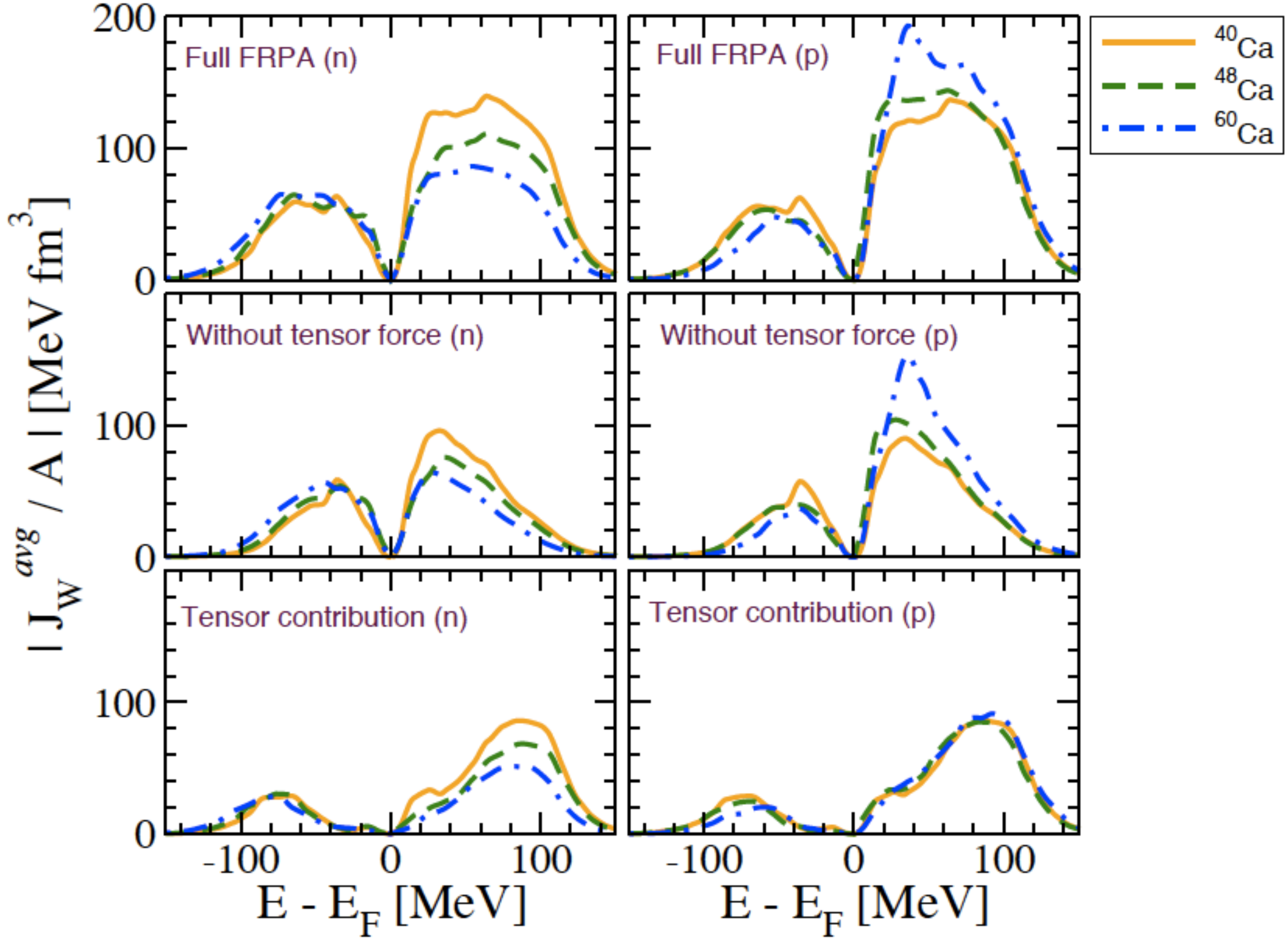
$$J_W(E) = 4\pi \int dr\ r^2\ W(r, E)$$



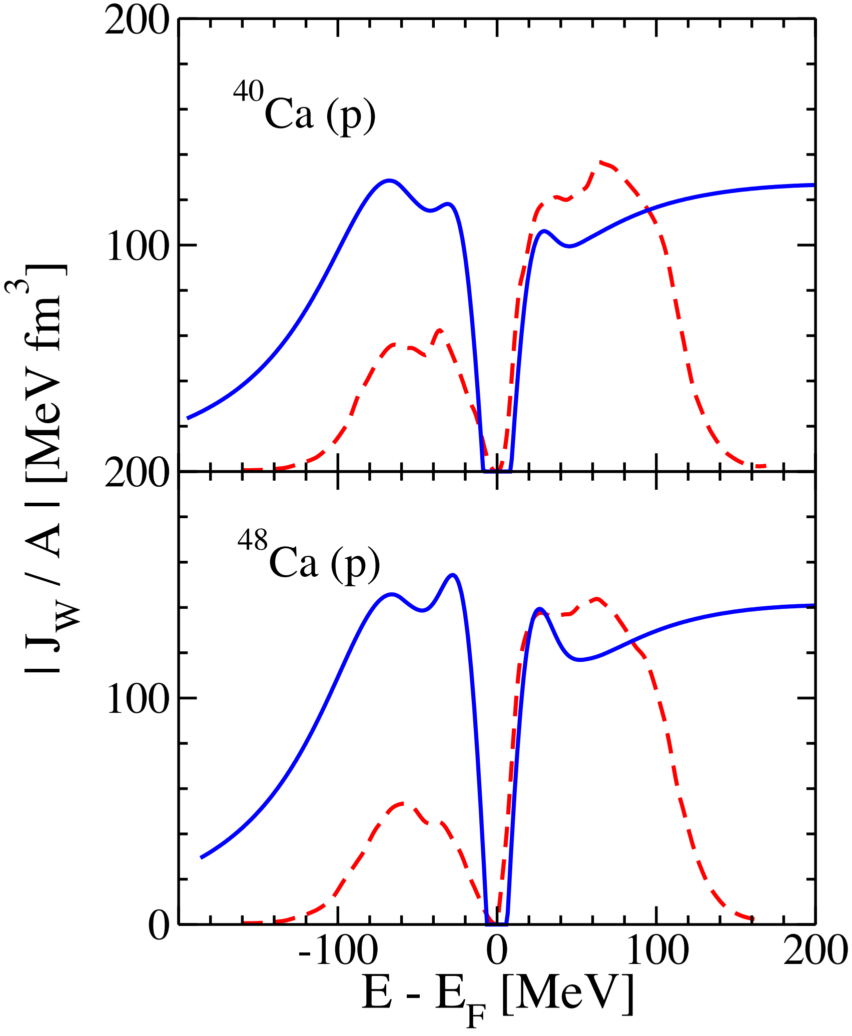
Microscopic potentials: nonlocal --> depend strongly on ℓ

Here averaged

Tensor force



Comparison with DOM for ^{40,48}Ca



New DOM implementation in progress

- Particle number --> **nonlocal** imaginary part
- Microscopic FRPA & SRC --> different nonlocal properties above and below the Fermi energy
- Include charge density in fit
- Describe high-momentum nucleons <--> (e,e'p) data from JLab

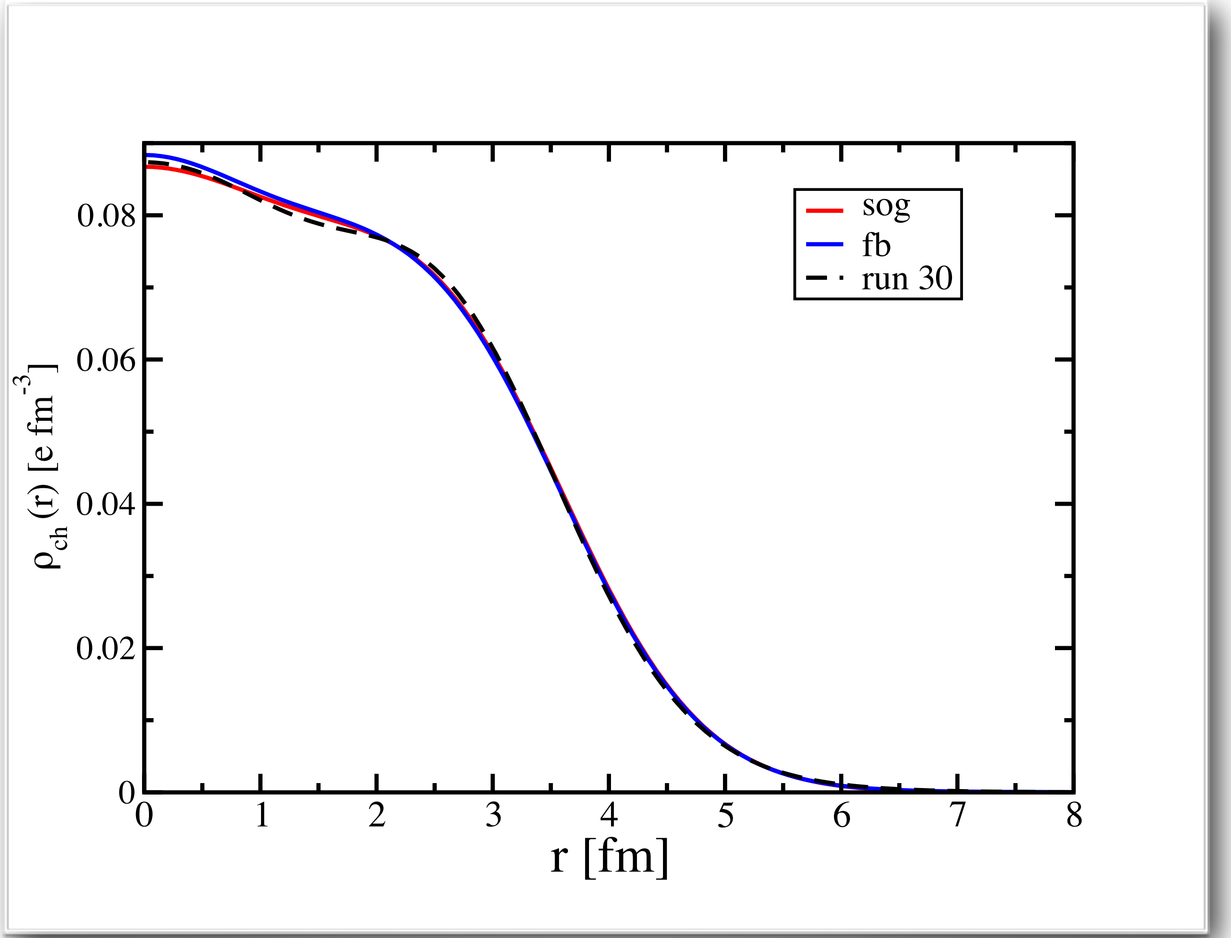
Implications

- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of the interpretation of (e,e'p) possible
- Independent "experimental" statement on size of three-body contribution to the energy of the ground state--> two-body only:

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ E S_{\ell j}(k;E)$$

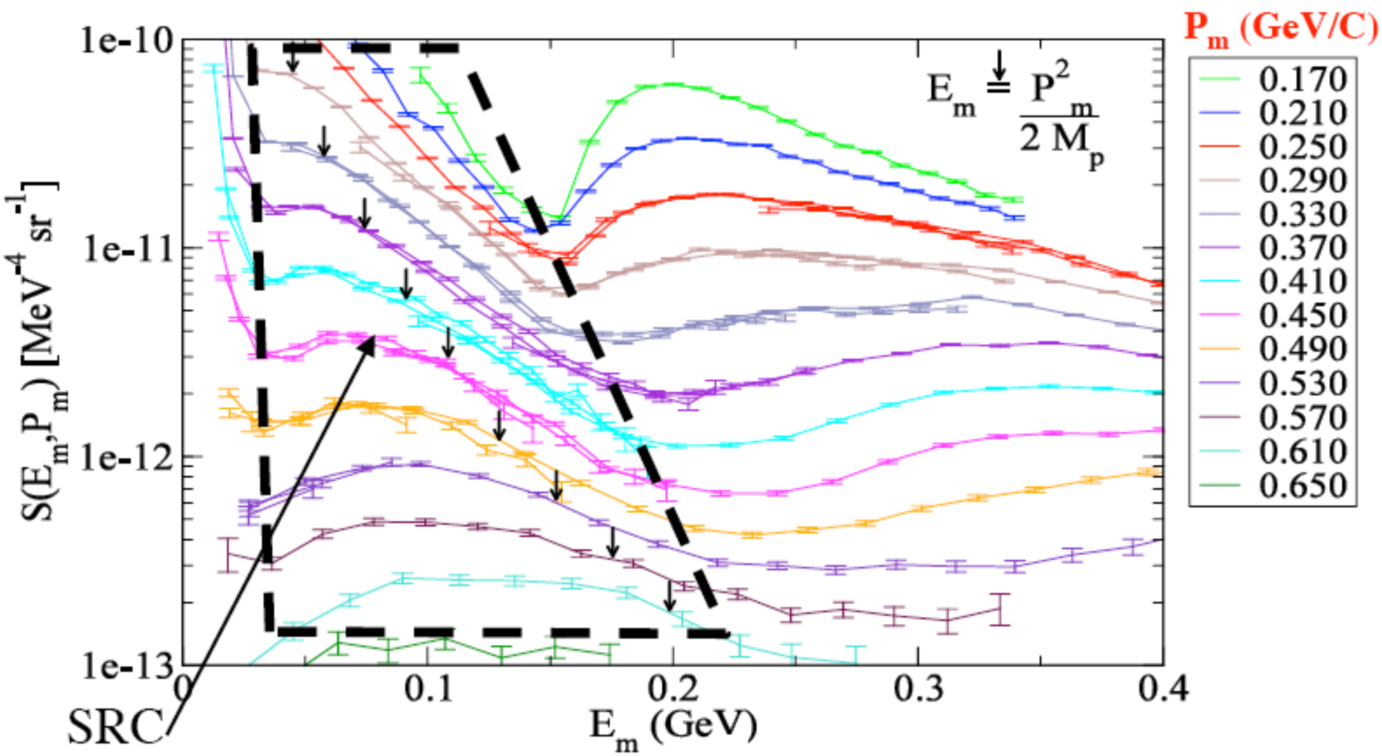
Critical experimental data

Charge density ^{40}Ca



High-momentum protons have been seen in nuclei!

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



^{12}C

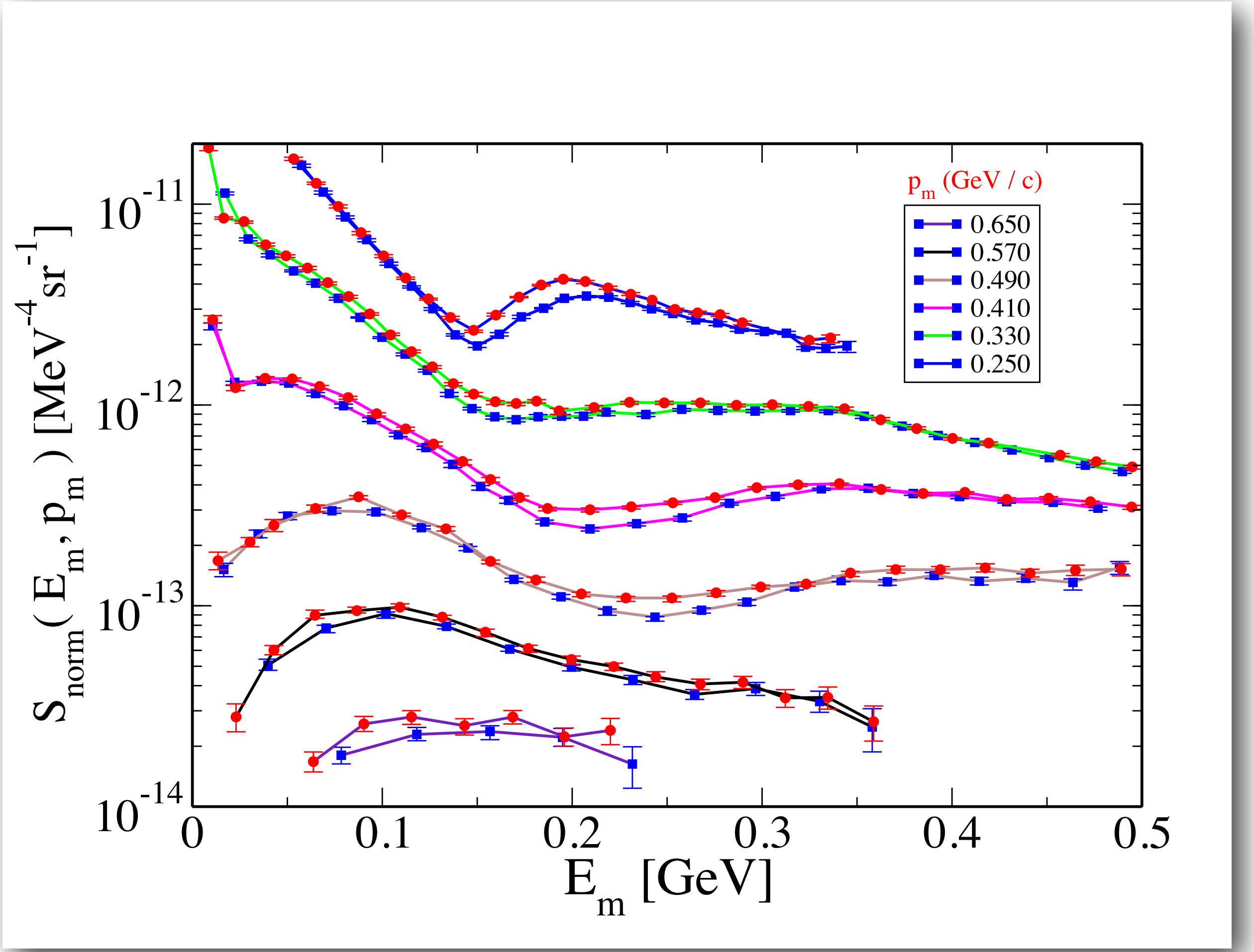
- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

$\Rightarrow$ ~0.6 protons for ^{12}C $\Rightarrow$ **~10%**

Understanding/Calculating Self-energy

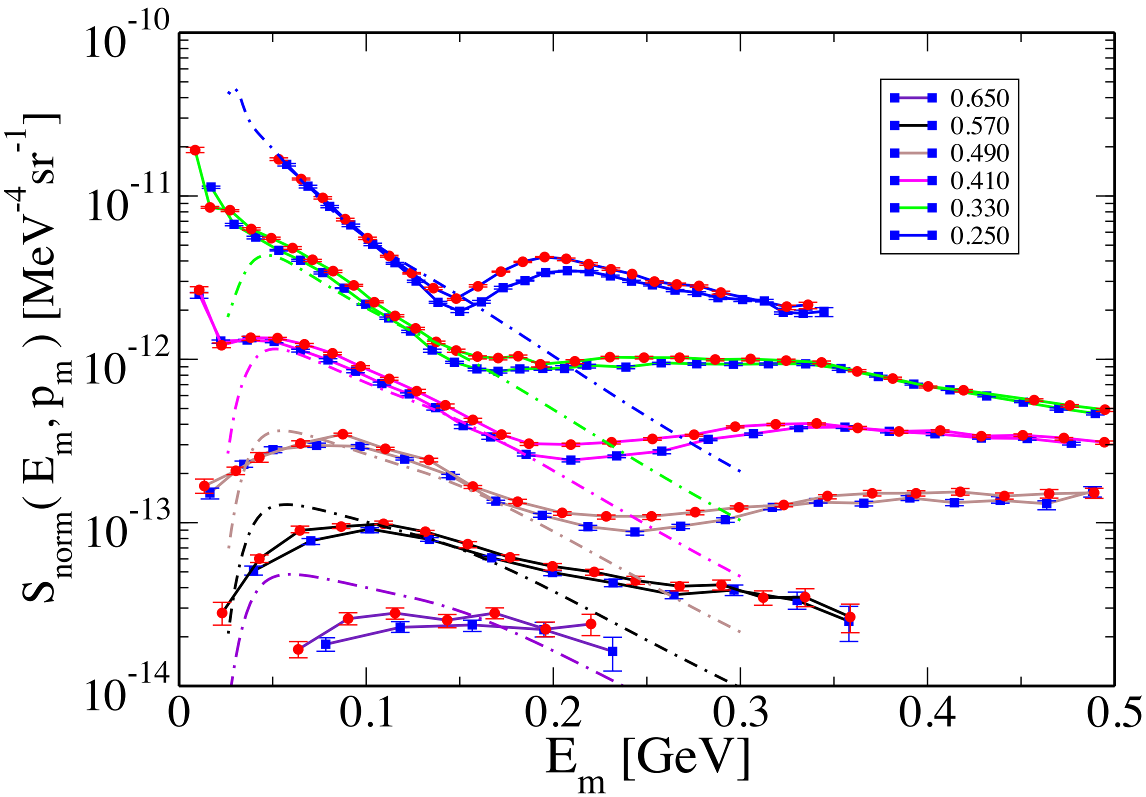
High-momentum components

Rohe, Sick et al. Al and Fe (e,e'p) data per proton



Preliminary results

- Mahzoon, Waldecker, Charity, Dussan, WD (2013)



Note: their location in energy yields an important contribution to the energy of the ground state

$$E/A = \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A}\sum_{\ell j}(2j+1)\int_0^\infty dk k^2 \int_{-\infty}^{\varepsilon_F} dE\ ES_{\ell j}(k;E)$$

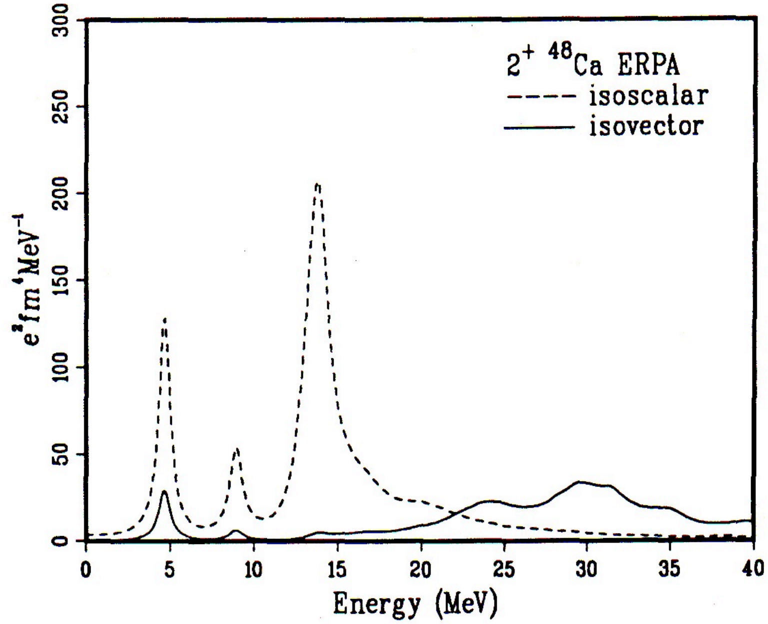
Understanding/Calculating Self-energy

Conclusions and Outlook

- Sir Denys has been answered
- Given a realistic NN interaction, its implications for the role of short-range and tensor correlations can be calculated reliably for infinite matter of any nucleon asymmetry, density, and temperatures above the critical temperature for pairing
- For finite nuclei this is not yet the case but some insight has been gained
  - Is a difficult challenge but in progress right now...
- Long-range correlations --> FRPA identifies relevant properties of the self-energy near the Fermi energy
- Alternative approach for finite nuclei: correlate a lot of data --> DOM --> drip line
  - Will be a tool for FRIB physics as well

Giant Quadrupole

Example of --> $\frac{\delta\Sigma}{\delta G}$

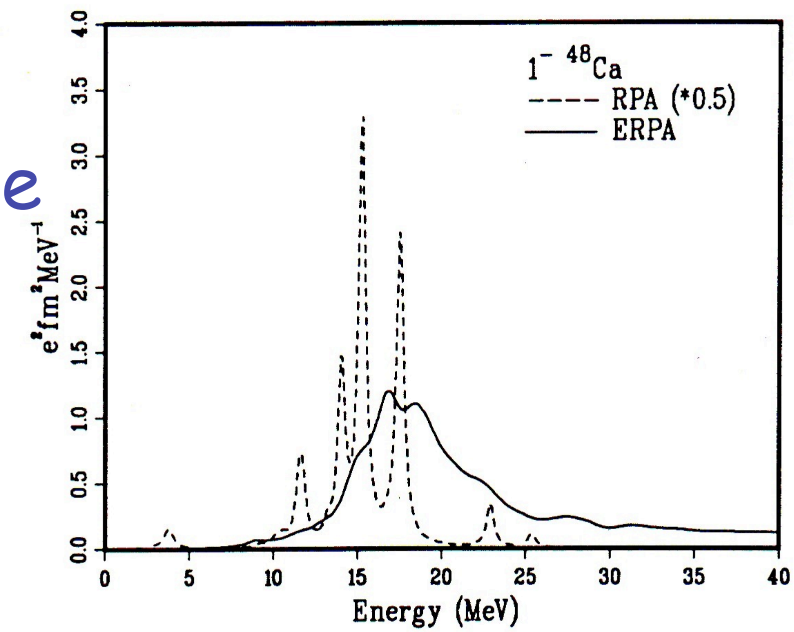


Giant Resonances only correct when sp fragmentation is included!

In turn:

Excited states determine sp fragmentation

Giant Dipole



M. G. E. Brand, K. Allaart, and W. D.
Phys. Lett. **214B** , 483 (1988);
Nucl. Phys. **A509** , 1 (1990).