

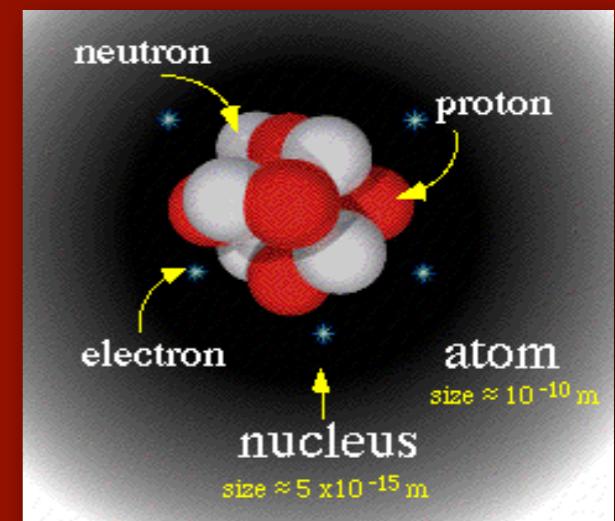
# First Principle Calculation of Nuclear Response Functions

**Sonia Bacca**

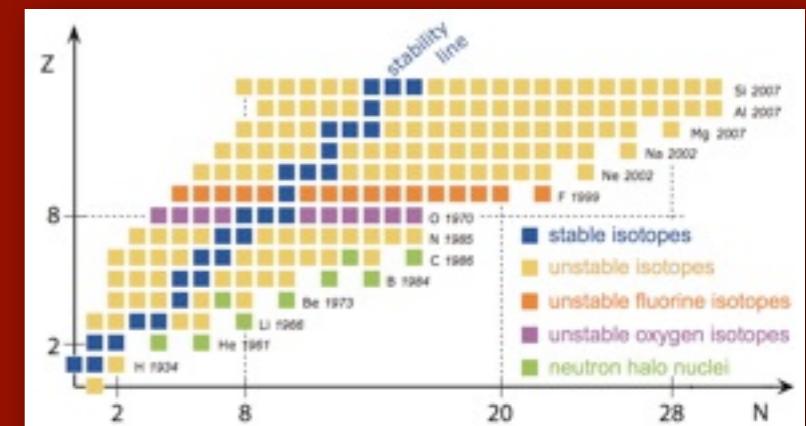
**TRIUMF Theory Group**

INT program on “Computational and Theoretical advances for Exotic isotopes in the Medium Mass Region”

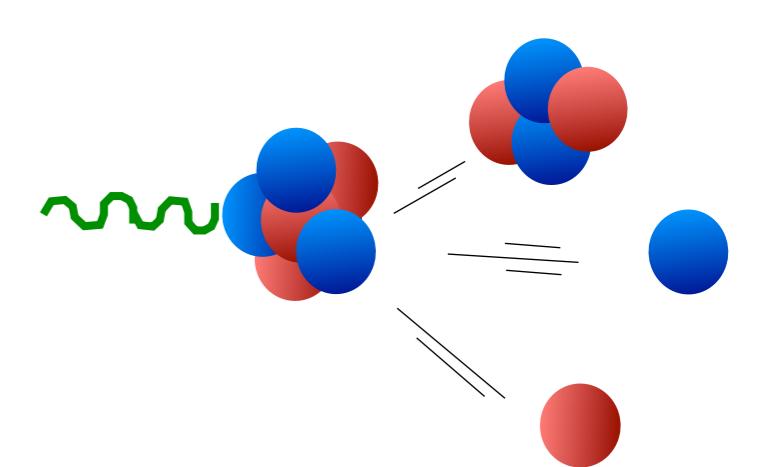
April 19, 2013



The Atomic Nucleus



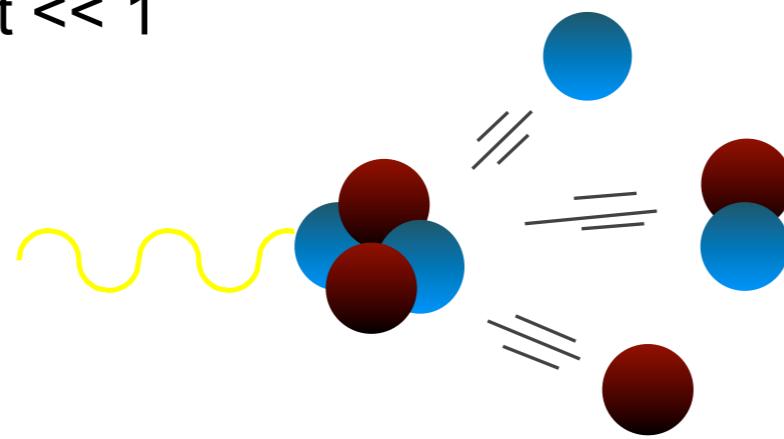
Nuclear Chart



Nuclear Reactions

# Electro-weak reactions

- The coupling constant  $\ll 1$



*“With the electro-weak probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself”*  
 [De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto \left| \langle \psi_f | \hat{J}^\mu | \psi_0 \rangle \right|^2$$

$$\sigma \propto R(q, \omega) \quad \text{with} \quad R(q, \omega) = \sum_f \left| \langle \psi_f | \hat{J}^\mu(q) | \psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$

Nuclear Response Function

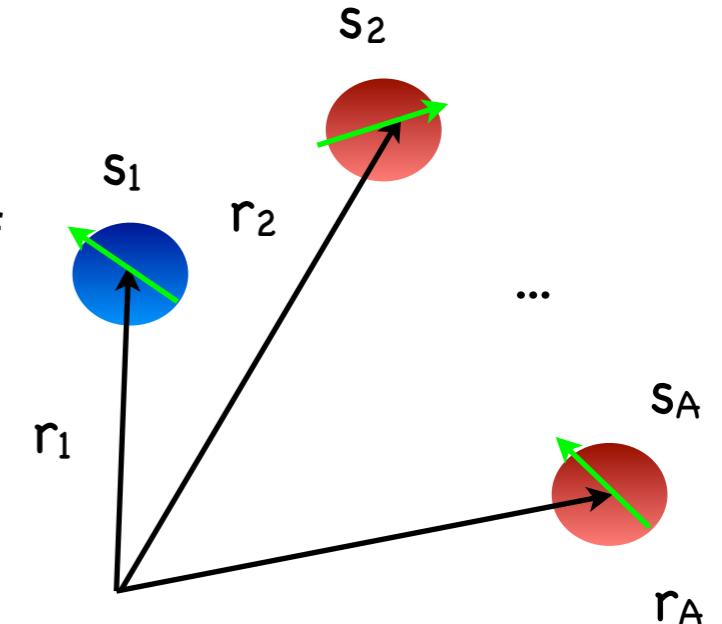
# From “First Principles”

- Start from neutrons and protons as building blocks  
(centre of mass coordinates, spins, isospins)
- Solve the (non-relativistic) quantum mechanical problem of A-interacting nucleons

$$H|\psi\rangle = E|\psi\rangle$$

$$H = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$

- Find numerical solutions with no approximations or controllable approximations

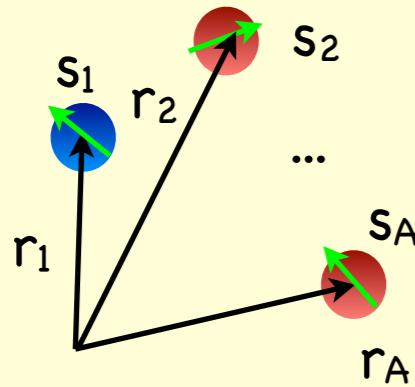


- Calculate low-energy observables from the  $\infty$ -body wave function and compare with experiment to **test nuclear forces** and **provide predictions** when experiments are hard or even not possible or help interpret new experiments
- Develop a **strong predictive theory** in the framework of light nuclei and then extend it towards heavier and neutron-rich systems



# Ingredients

## Nuclear Forces

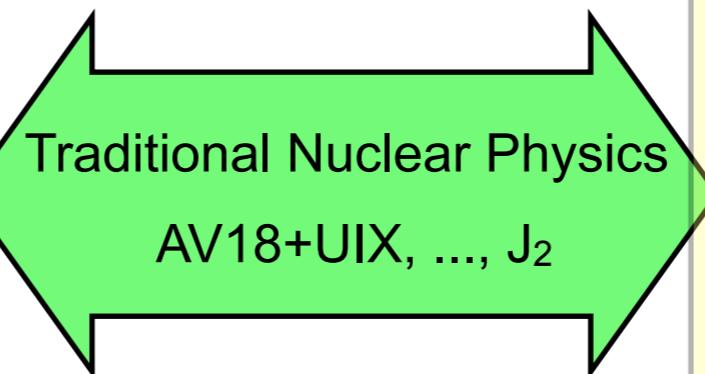


$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN} + V_{3N} + \dots$$

High precision two-nucleon potentials:  
well constraint on NN phase shifts

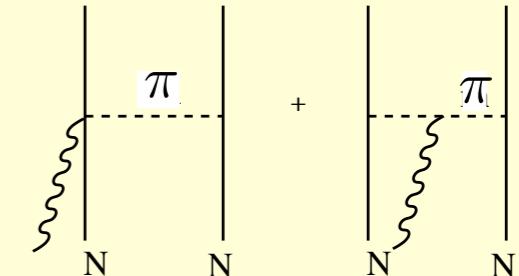
Three nucleon forces:  
less known, constraint on  $A > 2$  observables



Effective Field Theory  
N<sup>2</sup>LO, N<sup>3</sup>LO ...

## Nuclear Currents

$$J^\mu = J_N^\mu + J_{NN}^\mu + \dots$$



exchange currents  
subnuclear d.o.f.

$J^\mu$  consistent with  $V$   
 $\nabla \cdot J = -i[V, \rho]$

## Final State Interaction

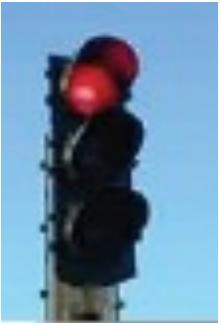
$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Final state in the continuum at different energies and for different  $A$

# Final State Interaction

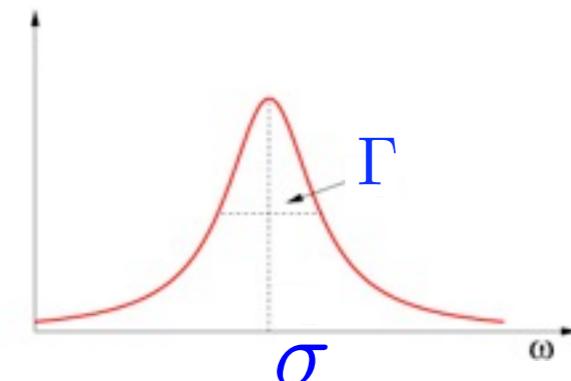
Exact evaluation of the final state in the continuum is limited in energy and A

Solution: **The Lorentz Integral Transform Method** Efros *et al.*, Nucl.Part.Phys. **34** (2007) R459



Response in the continuum

$$R(\omega) = \sum_f \left| \langle \psi_f | \hat{O} | \psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega)$$



$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \langle \tilde{\psi} | \tilde{\psi} \rangle$$

$$(H - E_0 - \sigma + i\Gamma) | \tilde{\psi} \rangle = \hat{O} | \psi_0 \rangle$$

- Due to imaginary part  $\Gamma$  the solution  $|\tilde{\psi}\rangle$  is unique
- Since the r.h.s. is finite, then  $|\tilde{\psi}\rangle$  has bound state asymptotic behavior

$$L(\sigma, \Gamma) \xrightarrow{\text{inversion}} R(\omega) \text{ with the exact final state interaction}$$

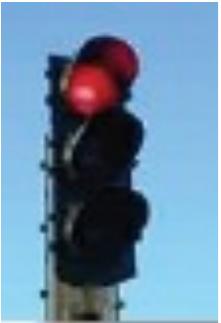


You can use any good bound state method! e.g. Hyperspherical Harmonics, No Core Shell Model, Coupled Cluster Theory

# Final State Interaction

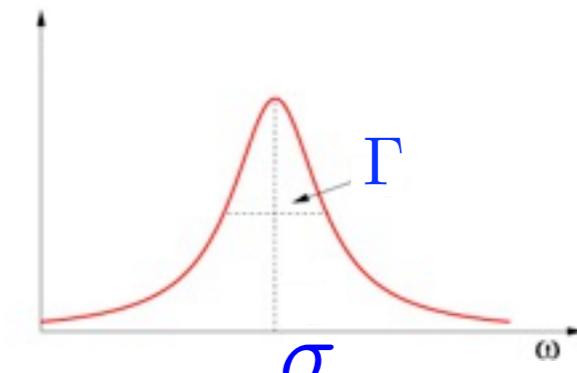
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$(\omega - \sigma - i\Gamma)(\omega - \sigma + i\Gamma)$

$$= \sum_f \left\langle \psi_0 | \hat{O} \frac{1}{E_f - E_0 - \sigma - i\Gamma} | \psi_f \right\rangle \left\langle \psi_f | \frac{1}{E_f - E_0 - \sigma + i\Gamma} \hat{O} | \psi_0 \right\rangle$$

$$= \sum_f \left\langle \psi_0 | \hat{O} \frac{1}{H - E_0 - \sigma - i\Gamma} | \psi_f \right\rangle \left\langle \psi_f | \frac{1}{H - E_0 - \sigma + i\Gamma} \hat{O} | \psi_0 \right\rangle$$

$$= \left\langle \psi_0 | \hat{O} \frac{1}{H - E_0 - \sigma - i\Gamma} \frac{1}{H - E_0 - \sigma + i\Gamma} \hat{O} | \psi_0 \right\rangle$$

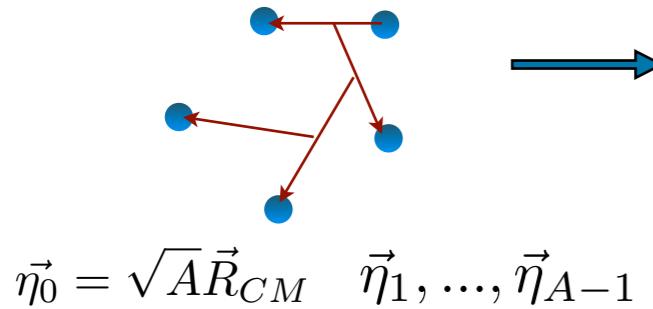
$\tilde{\psi}$

# Hyperspherical Harmonics

A basis set, mostly used for A=3, 4. Challenge to go up to A=7,8

- Few-body method - uses relative coordinates

$$|\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle = |\varphi(\vec{R}_{CM})\Psi(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})\rangle$$



Recursive definition of hyper-spherical coordinates

$$\rho, \Omega \quad \rho^2 = \sum_{i=1}^A r_i^2 = \sum_{i=1}^{A-1} \eta_i^2$$

$$H_0(\rho, \Omega) = T_\rho - \frac{K^2(\Omega)}{\rho^2}$$

$$\Psi = \sum_{[K], \nu}^{K_{max}, \nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n(\frac{\rho}{b}) [\mathcal{Y}_{[K]}^\mu(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$



Asymptotic	$e^{-a\rho}$	$\rho \rightarrow \infty$
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Model space truncation     $K \leq K_{max}$     Matrix Diagonalization

$$\langle \psi | H_{(2)} | \psi \rangle = \frac{A(A-1)}{2} \langle \psi | H_{(A,A-1)} | \psi \rangle$$

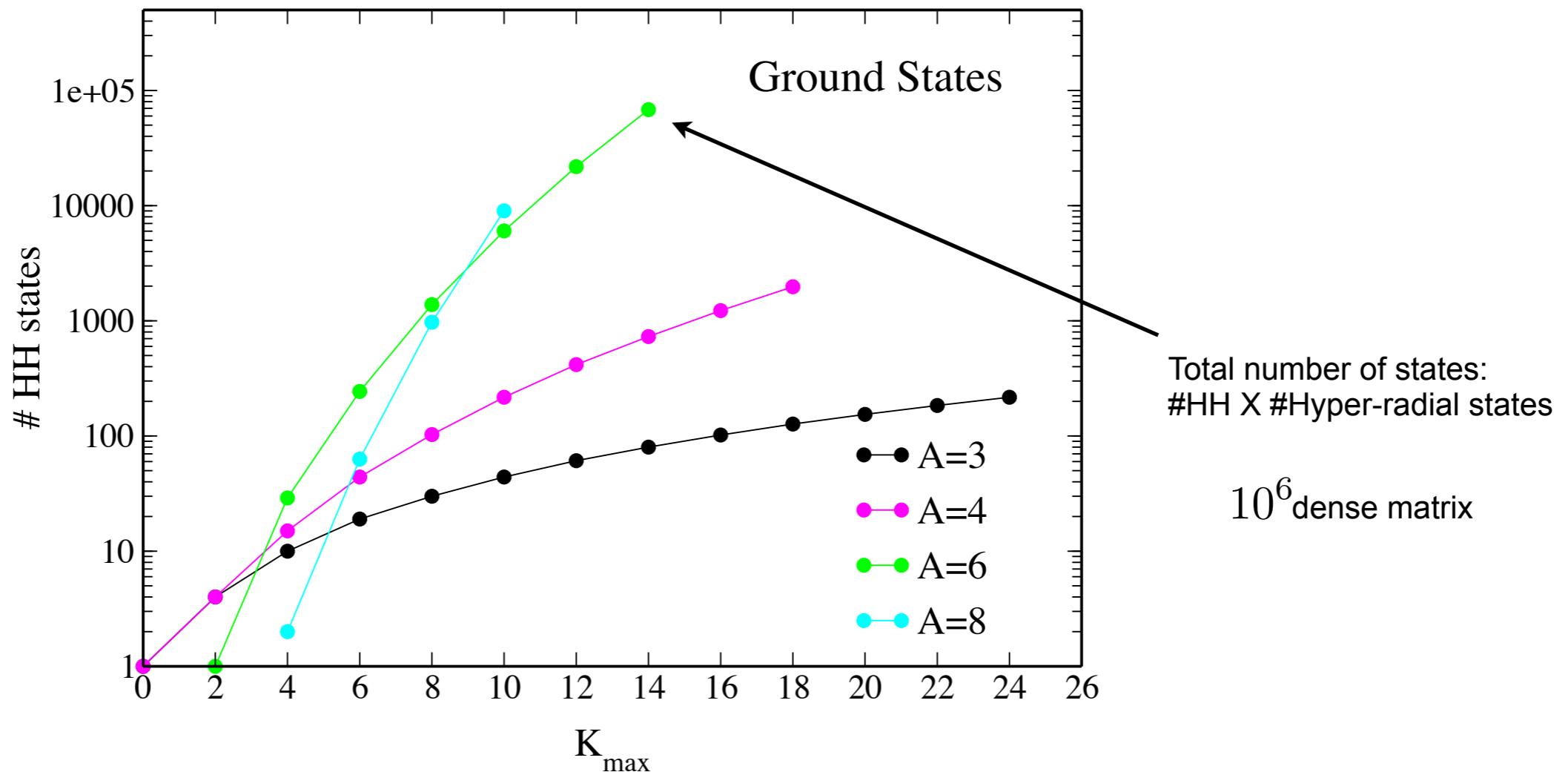
Can use local and non-local interactions

Most applications in few-body; challenge in A>4

Barnea and Novoselsky, Ann. Phys. 256 (1997) 192

# Hyperspherical Harmonics Expansions

$$\Psi = \sum_{[K],\nu}^{K_{max},\nu_{max}} c_\nu^{[K]} e^{-\rho/2} \rho^{n/2} L_\nu^n(\rho) [\mathcal{Y}_{[K]}^\mu(\Omega) \chi_{ST}^{\bar{\mu}}]_{JT}^a$$



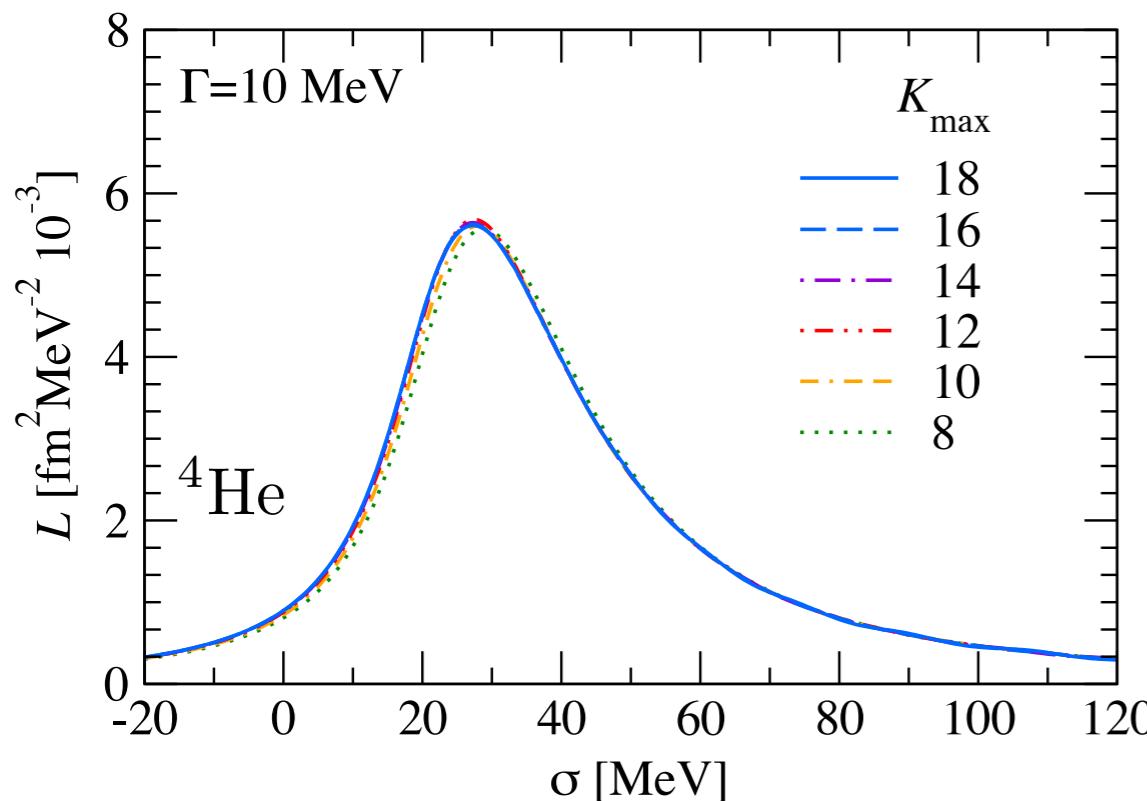
$$(H - E_0 - \sigma + i\Gamma) | \tilde{\psi} \rangle = \hat{O} | \psi_0 \rangle$$

For the reactions expanding  $\hat{O} | \psi_0 \rangle$   
increase of dimension by at least one order of magnitude of angular momentum is changed

# The LIT with Hyperspherical Harmonics

Numerical example: Dipole Response Function of  ${}^4\text{He}$

$$\hat{O} = \hat{D}_z = \sum_i^Z (z_i - Z_{\text{cm}}) \quad \text{with NN(N}^3\text{LO)}$$



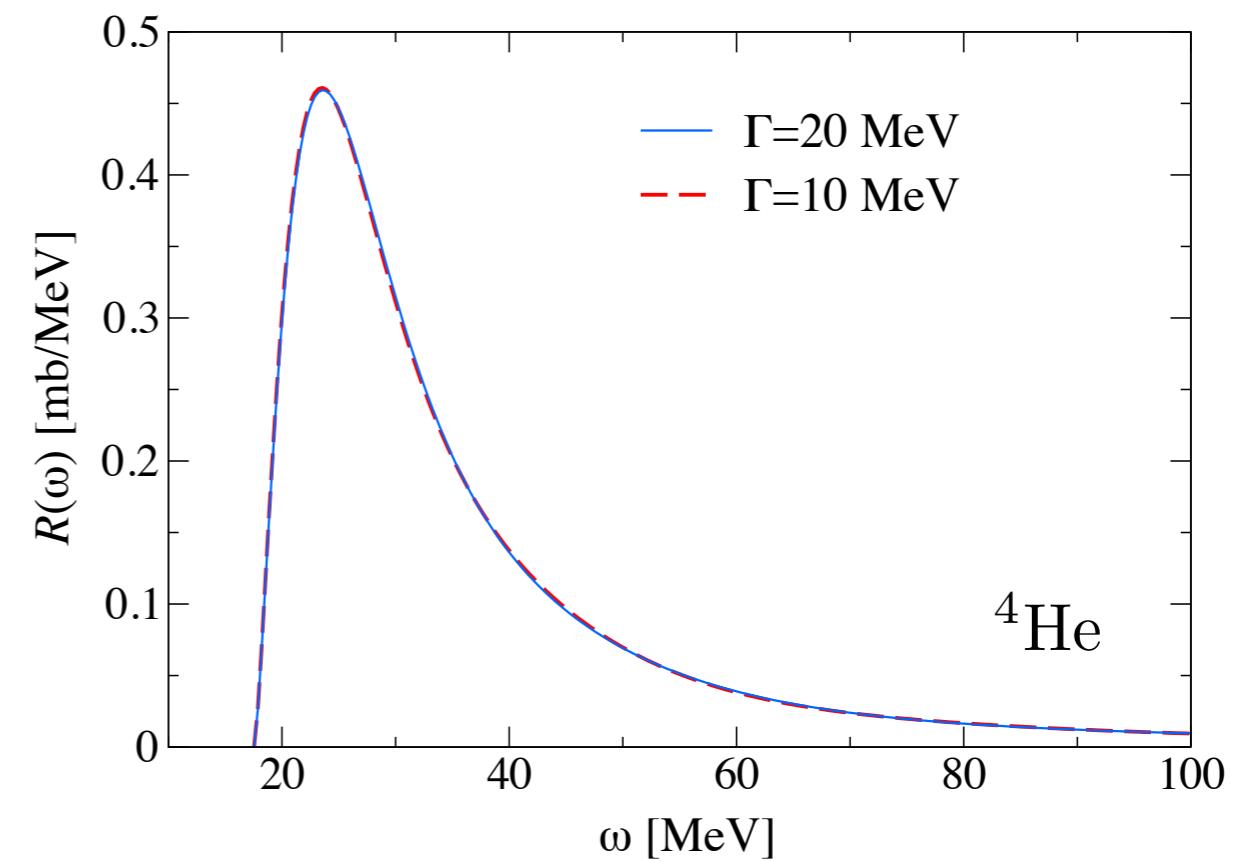
Inversion of the LIT

Ansatz

$$R(\omega) = \sum_i^{I_{\text{max}}} c_i \chi_i(\omega, \alpha)$$

$$L(\sigma, \Gamma) = \sum_i^{I_{\text{max}}} c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

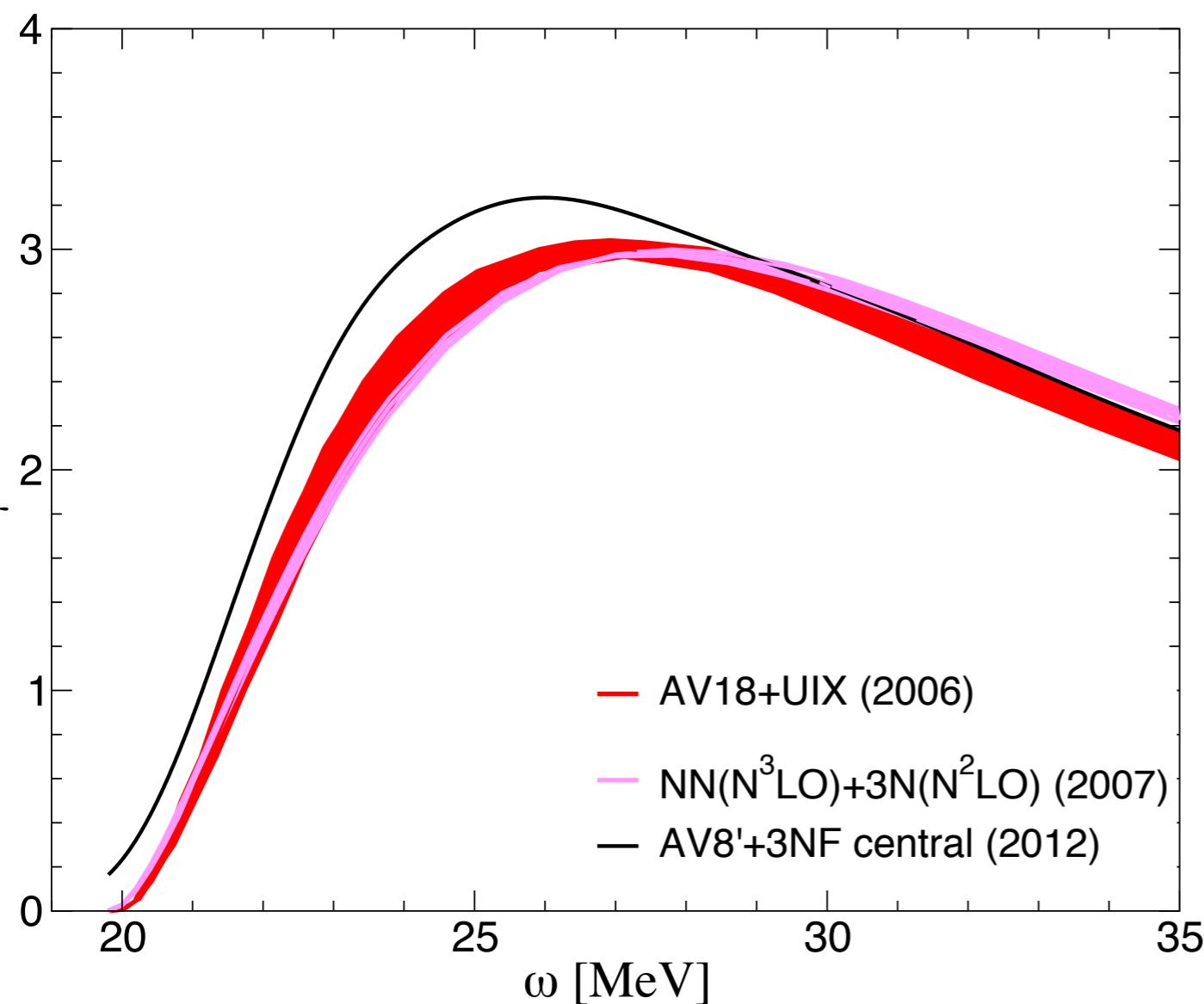
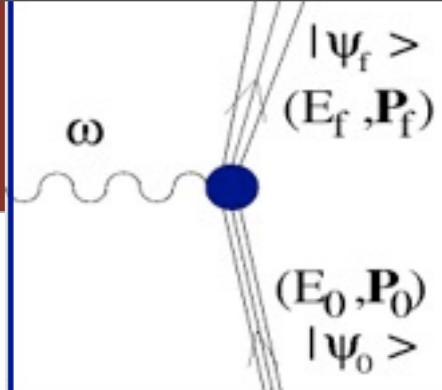
Least square fit of the coefficients  $c_i$  to reconstruct the response function



## Some examples for A=4

- Photo-absorption
- Electron Scattering

# Theory



$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

Traditional Hamiltonian

D.Gazit, S.B. et al. PRL 96 112301 (2006)

$E_0 = -28.40$  MeV

Hamiltonian from EFT

S.Quaglioni and P.Navratil PLB 652 (2007)

NN(N<sup>3</sup>LO) Entem-Machleidt PRC68, 041001(R) (2003)

3N(N<sup>3</sup>LO) local version from Navratil with  
 $C_D=1$   $C_E=-0.029$

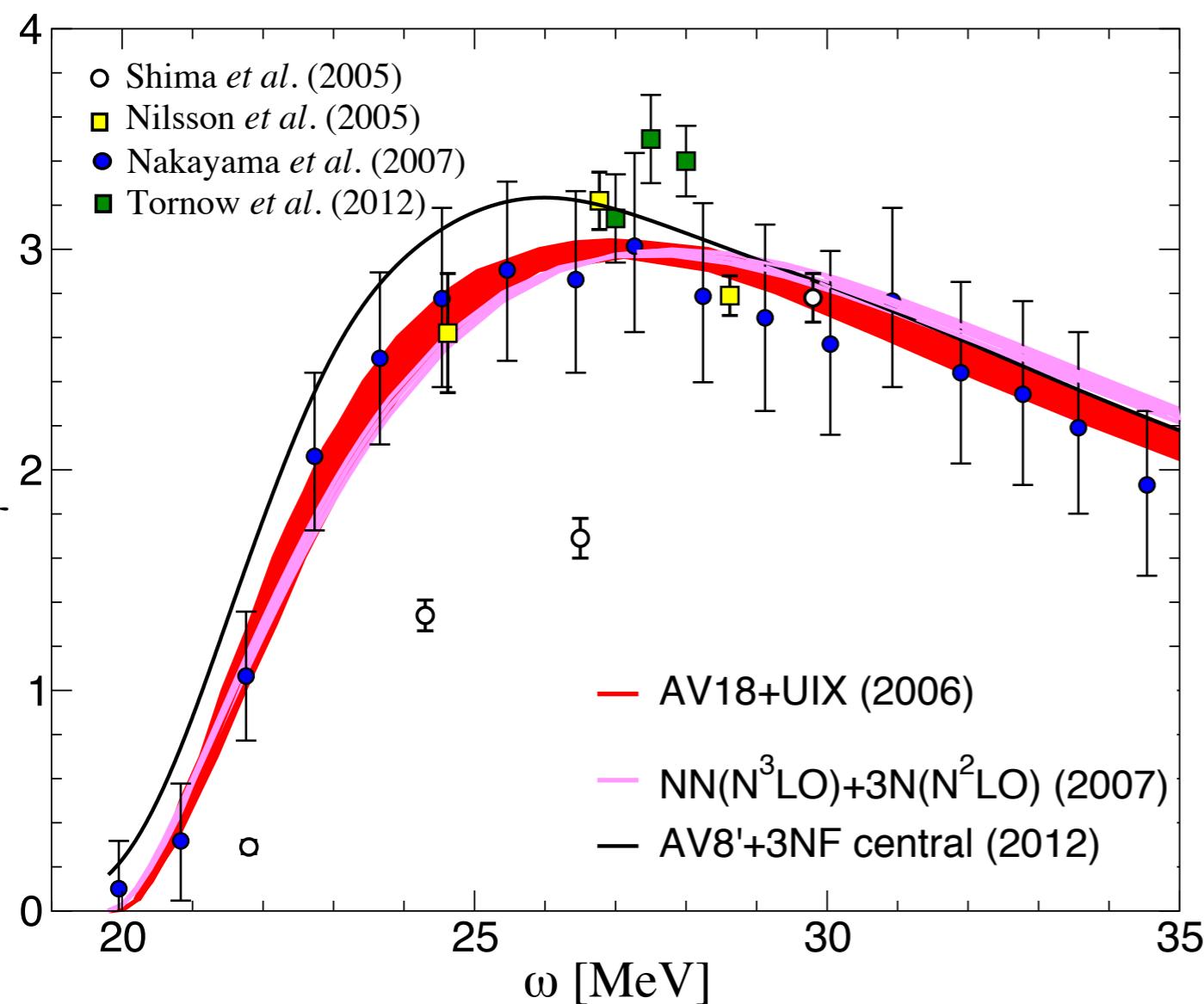
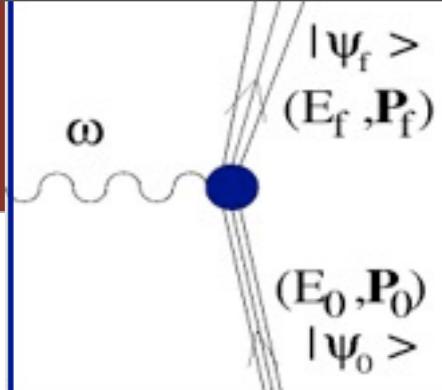
$E_0 = -28.36$  MeV

Realistic NN + phenomenological central 3NF  
W.Horiuchi et al. PRC 85 054002 (2012)

$E_0 = -28.44$  MeV

→ Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak

# Theory/Experiment



$$\sigma_\gamma = \frac{4\pi^2\alpha}{3}\omega R^{E1}(\omega)$$

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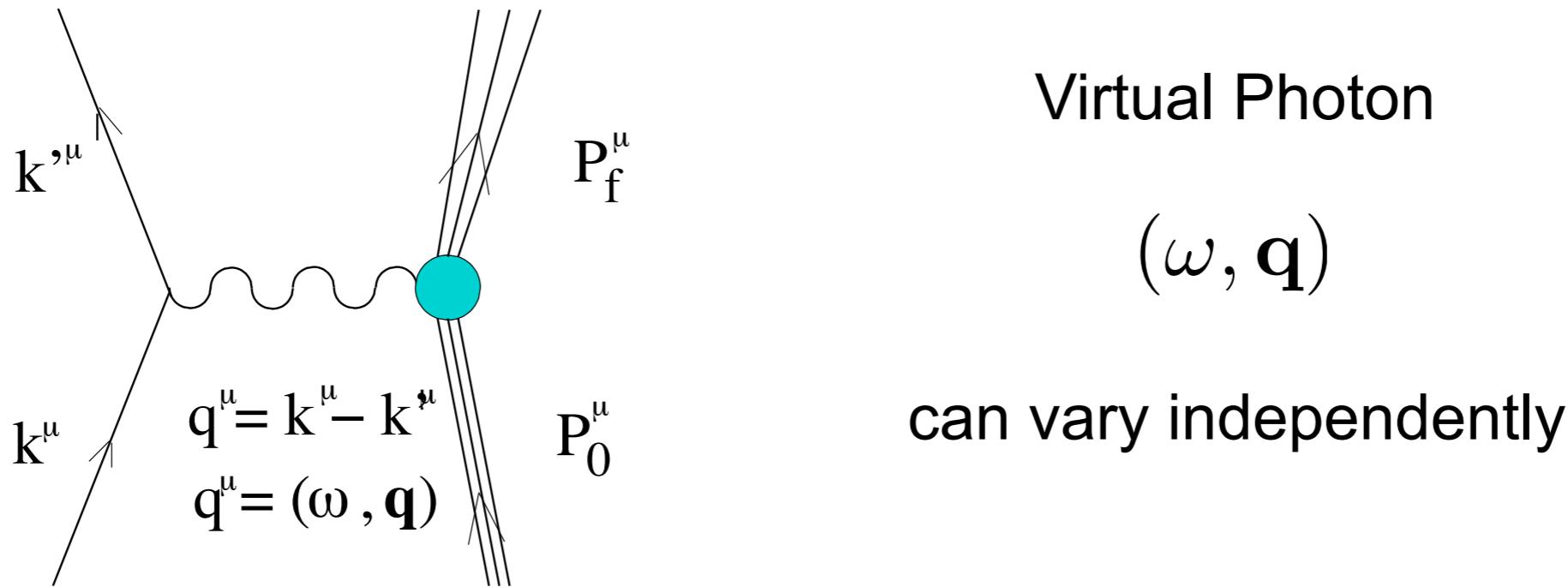
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Realistic NN + phenomenological central 3NF  
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$E_0 = -28.44$  MeV

- ➡ Moderate sensitivity to the Hamiltonian used; theory variation about 10% in peak
- ➡ More recent experimental activity seems to confirm higher data with peak around 27 MeV
- ➡ Theoretical precision is better than experimental error

# Inelastic e-Scattering ${}^4\text{He}(e,e')X$

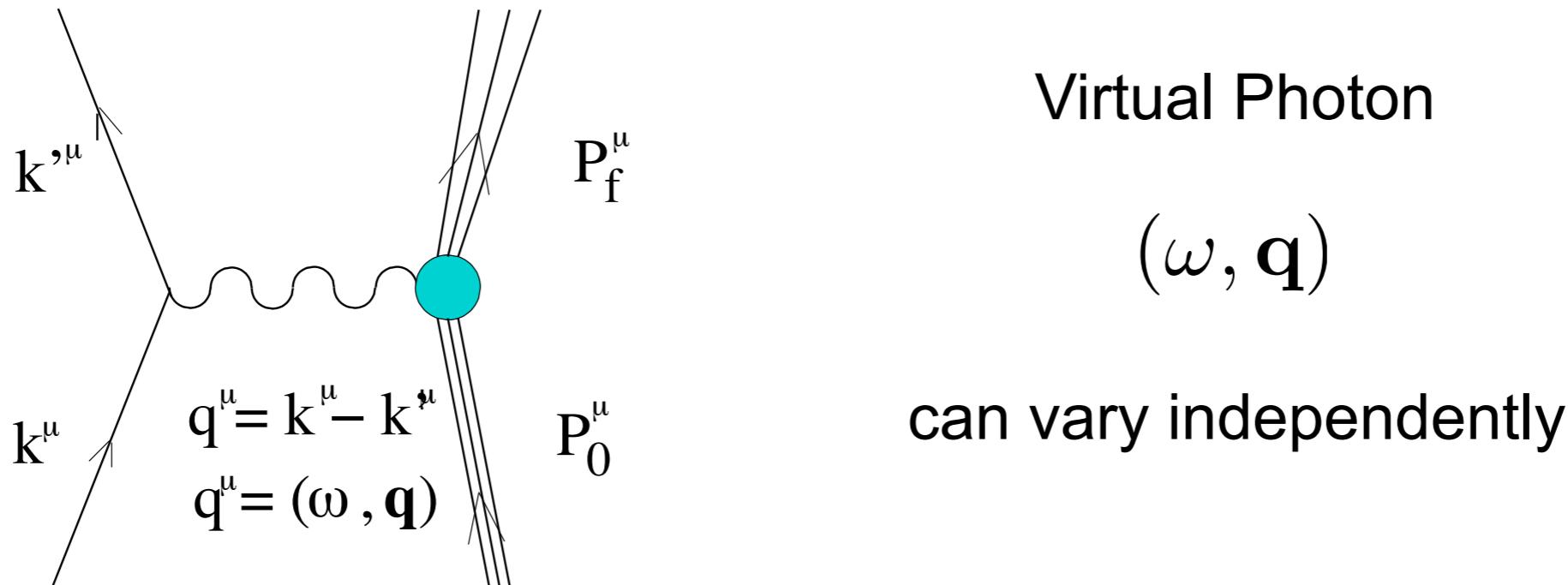


$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[ \frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left( \frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

with  $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$  and  $\theta$  scattering angle

and  $\sigma_M$  Mott cross section

# Inelastic e-Scattering ${}^4\text{He}(e,e')X$



Inclusive cross section  $A(e,e')X$

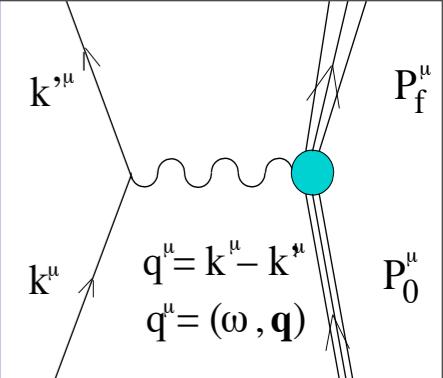
$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[ \frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left( \frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

$$R_L(\omega, \mathbf{q}) = \sum_f |\langle \Psi_f | \rho(\mathbf{q}) | \Psi_0 \rangle|^2 \delta \left( E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$

$$R_T(\omega, \mathbf{q}) = \sum_f |\langle \Psi_f | J_T(\mathbf{q}) | \Psi_0 \rangle|^2 \delta \left( E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M} \right)$$

two-body currents  
are not important

# Inelastic e-Scattering ${}^4\text{He}(e,e')X$

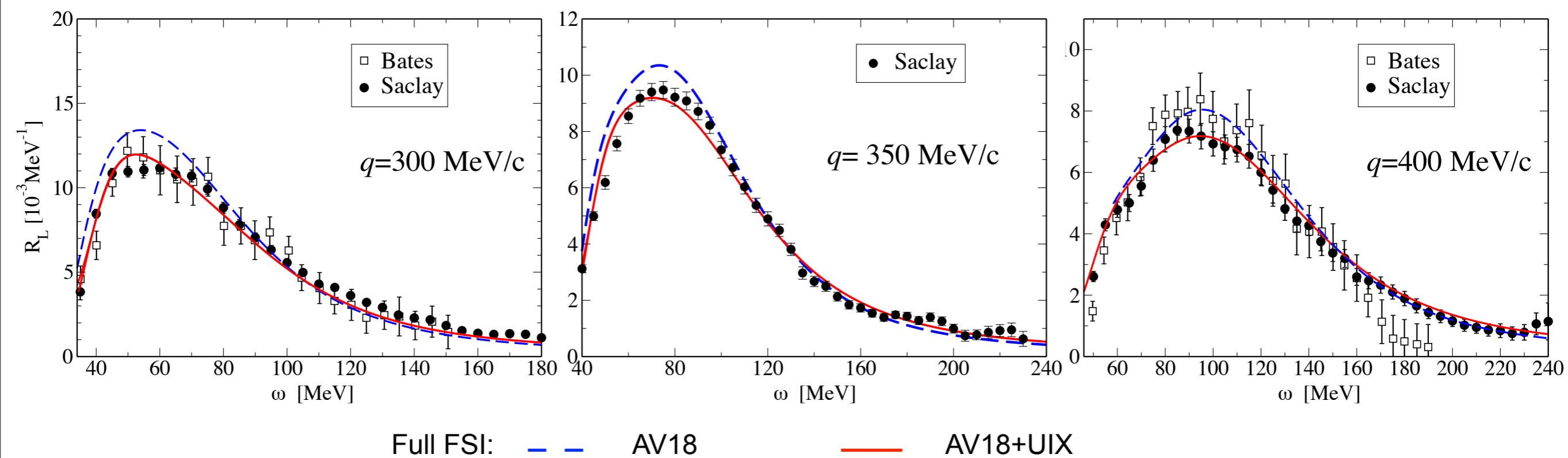


Calculation of  $R_L(\omega, \mathbf{q})$  with the LIT

Medium-q kinematics

Searching for 3NF effects

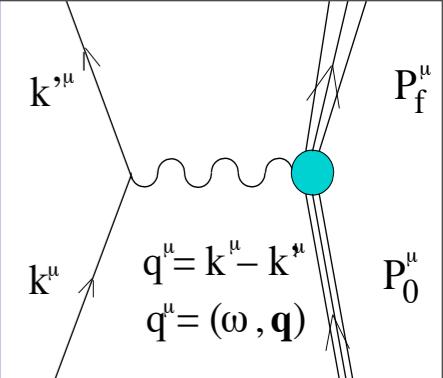
S.B. et al., PRL 102, 162501 (2009)



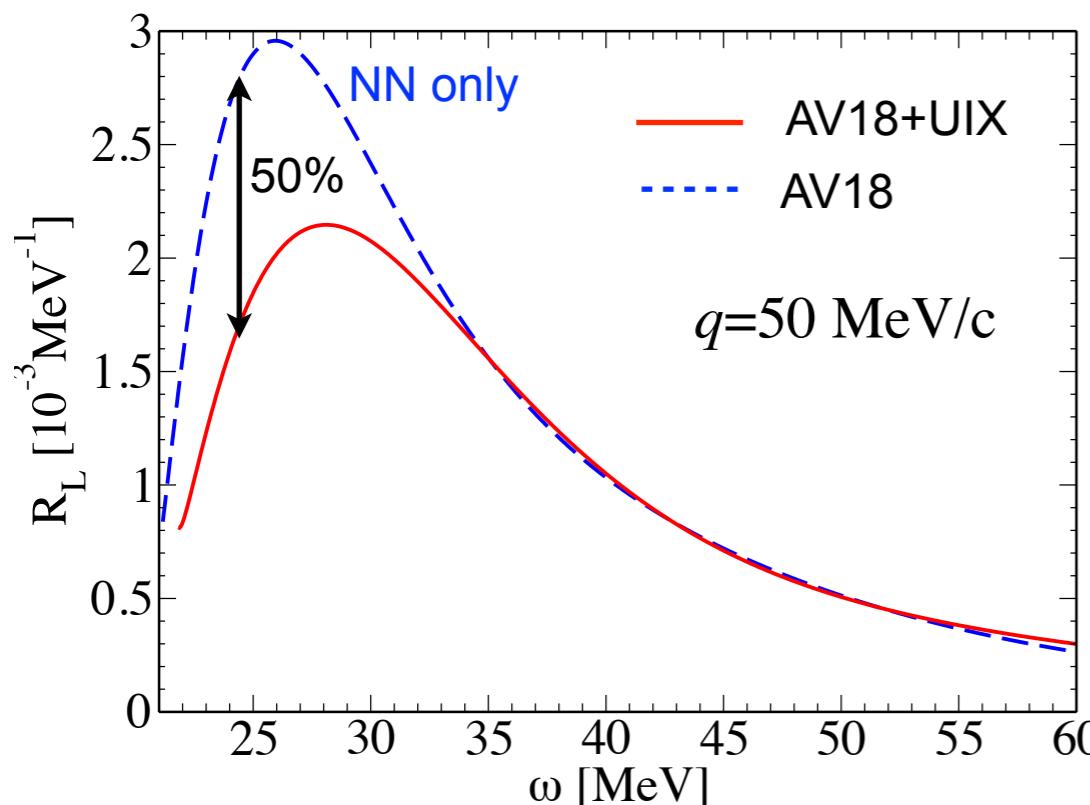
→ 3NF reduce the peak of 10%

→ Comparison with experiment improves with 3NF

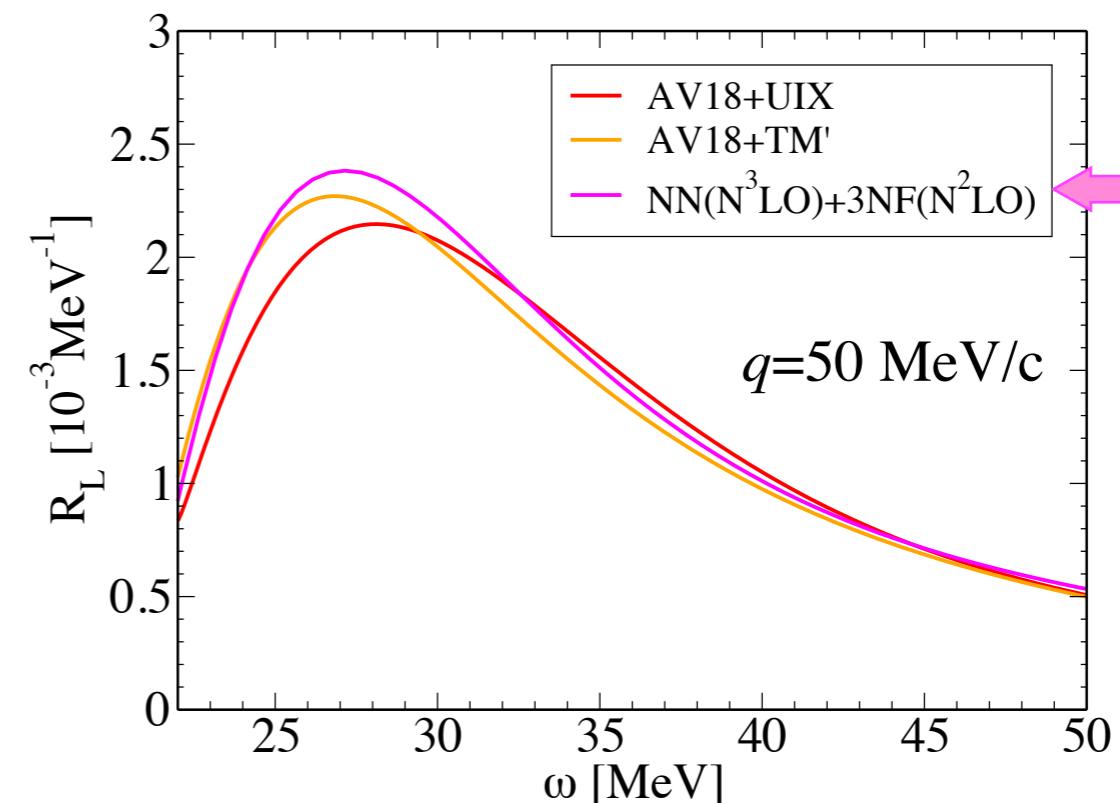
# Inelastic e-Scattering ${}^4\text{He}(e,e')X$



## Low-q kinematics

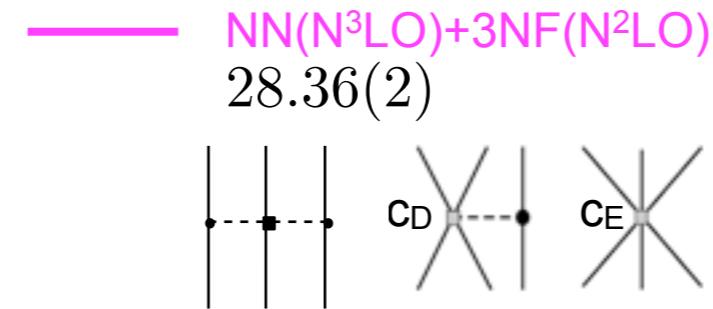
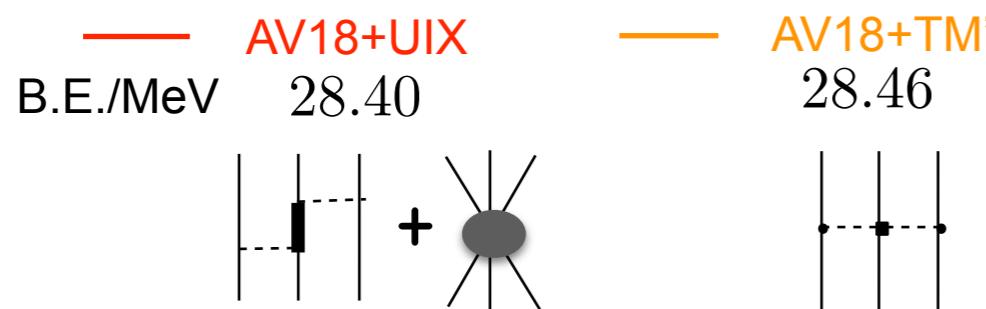


## different 3N Hamiltonians



Strong 3NF effect at low q

It is not a simple binding effect!

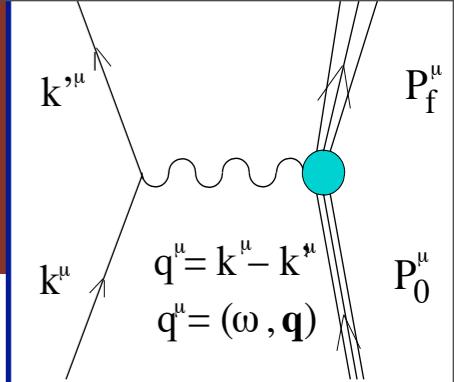


Stimulating new experiments: MAMI taken data  $q \geq 150 \text{ MeV}/c$ ; S-DALINAC will maybe take data at lower  $q$

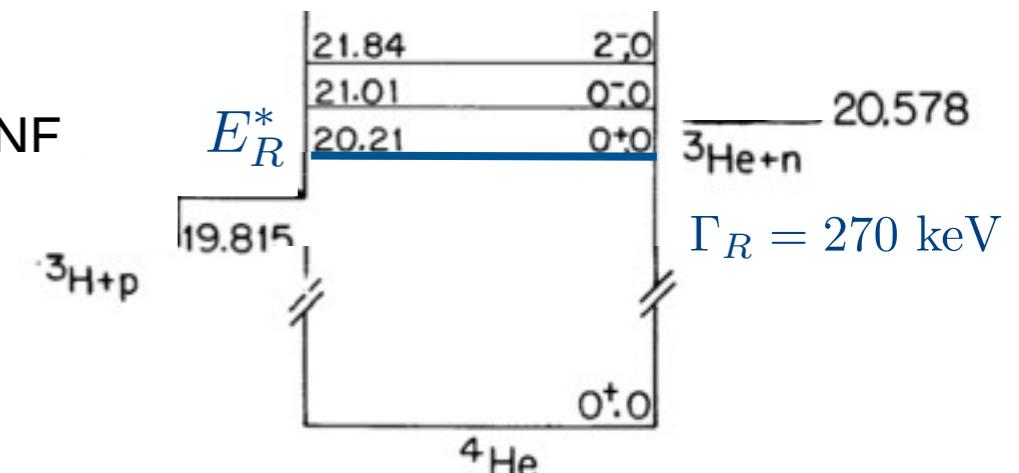
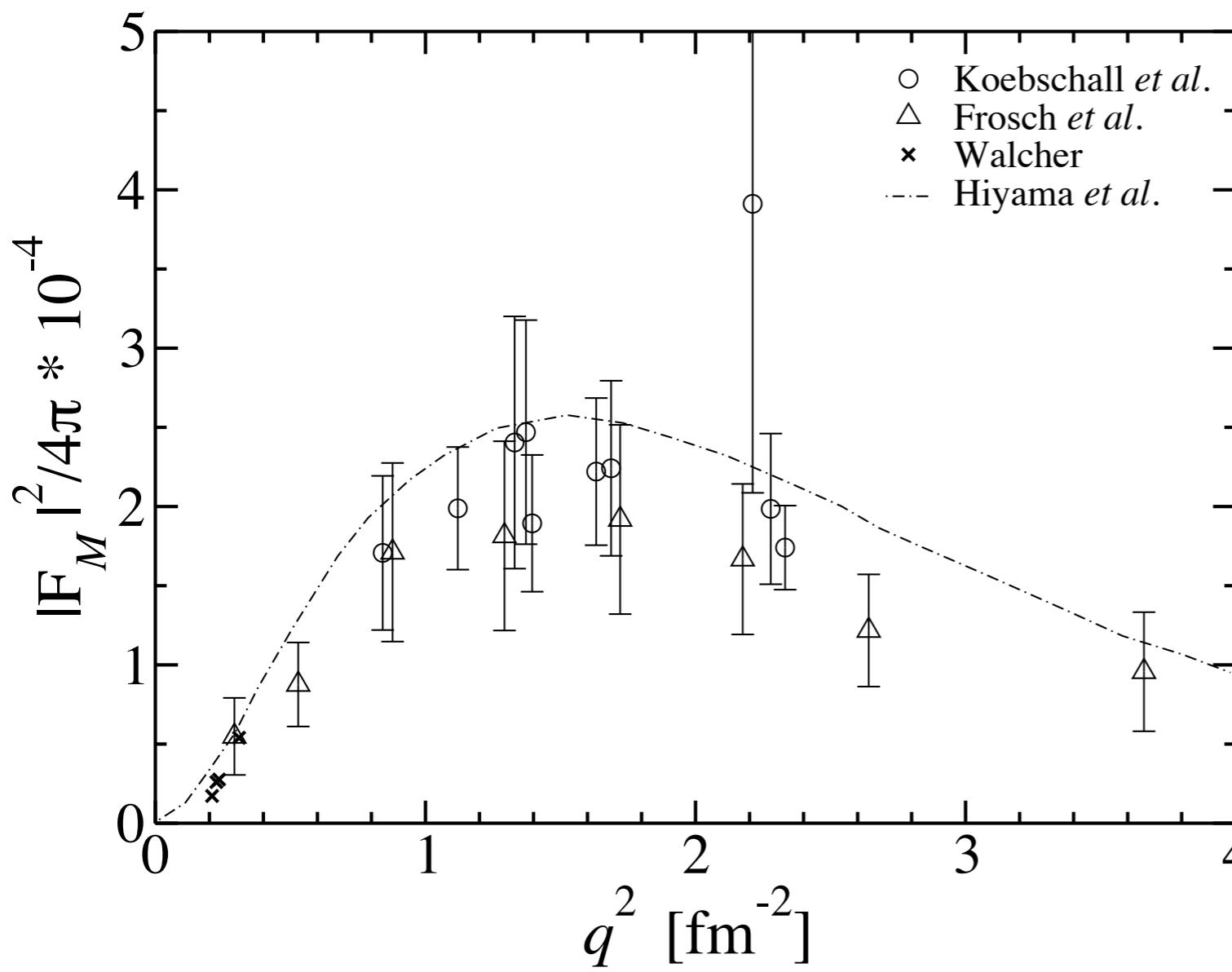
# Monopole Resonance ${}^4\text{He}(\text{e},\text{e}')0^+$

Resonant Transition Form Factor  
 $0_1^+ \rightarrow 0_2^+$

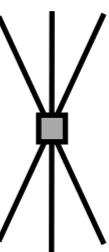
$$|F_M(q)|^2 = \frac{1}{Z^2} \int d\omega R_M^{\text{res}}(q, \omega)$$



First ab-initio calculation: Hiyama *et al.*, PRC **70** 031001 (2004)  
obtained good description of data with phenomenological central 3NF



AV8' + central 3NF  
 $E_0 = -28.44 \text{ MeV}$   
 $E_0^{\text{exp}} = -28.30 \text{ MeV}$



# Monopole Resonance ${}^4\text{He}(\text{e},\text{e}')0^+$

In proximity of the resonance  
both in theory and experiment

$$R_{\mathcal{M}}(q, \omega) = R_{\mathcal{M}}^{\text{res}}(q, \omega) + R_{\mathcal{M}}^{\text{bg}}(q, \omega) \quad (\star)$$

We use a square integrable basis (HH) to  
calculate the LIT (not the response)  
rigorous because of finite  $\Gamma$

$$\mathcal{L}_{\mathcal{M}}(q, \sigma, \Gamma) = \frac{\Gamma}{\pi} \sum_{\nu=1}^N \frac{|\langle \Psi_{\nu} | \mathcal{M}(q) | \Psi_0 \rangle|^2}{(\sigma - e_{\nu} + E_0)^2 + \Gamma^2}$$

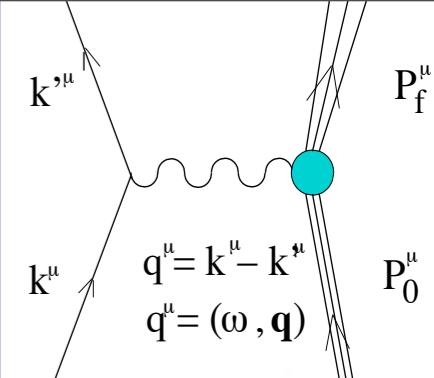
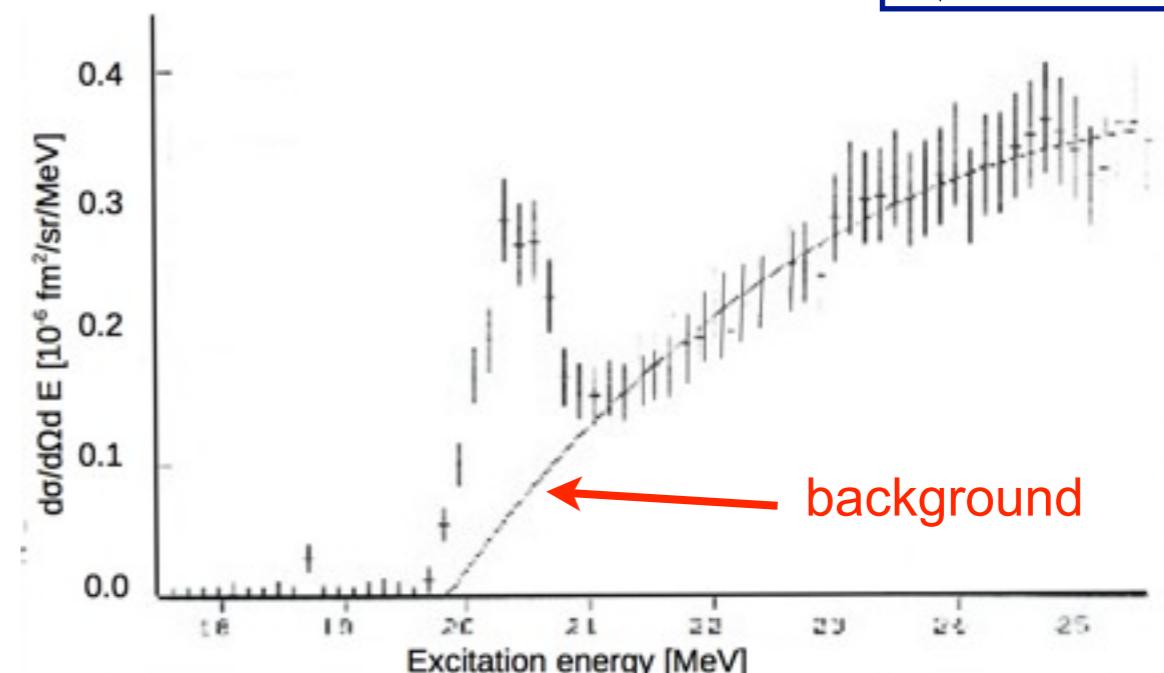
where  $\Psi_{\nu}, e_{\nu}$  are eigenstate and eigenvalues of  $H$  on our basis

We see ONE very pronounced strength  $|\langle \Psi_{\nu_R} | \mathcal{M}(q) | \Psi_0 \rangle|^2$  located at the energy

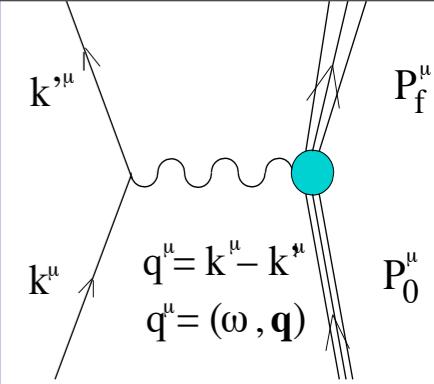
$$e_{\nu} - E_0 = E_R^*$$

Exploit the power of the LIT method (calculate the far continuum) to subtract the background

Nucl. Phys. A405, 648 (1983)



# Monopole Resonance ${}^4\text{He}(\text{e},\text{e}')0^+$



In proximity of the resonance  
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$$R_M(q, \omega) = R_M^{\text{res}}(q, \omega) + R_M^{\text{bg}}(q, \omega) \quad (\star)$$

Inversion of the LIT

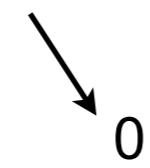
ansatz

$$\mathcal{R}_M(q, \omega) = \sum_i c_i \chi_i(\omega, \alpha)$$



$$\mathcal{L}_M(\sigma, \Gamma) = \sum_i c_i \mathcal{L}[\chi_i(\omega, \alpha)]$$

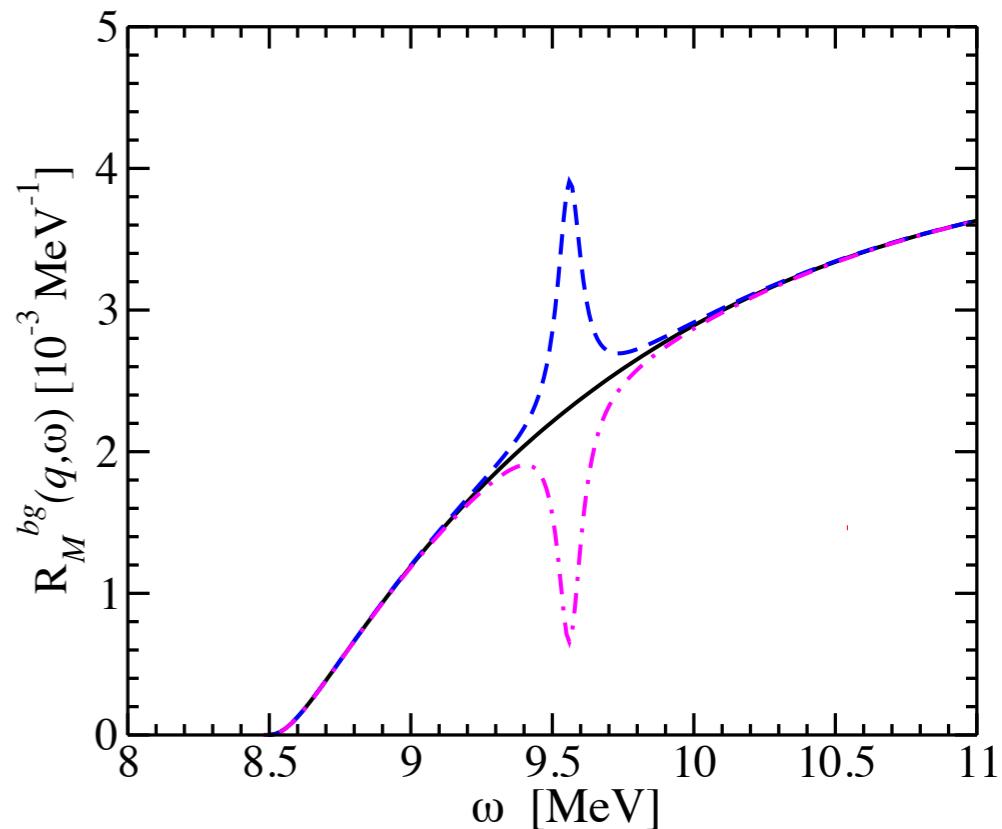
least square fit of  $c_i$



$$f_R(q) \frac{\Gamma}{\pi} \frac{1}{(\sigma - E_R + E_0)^2 + \Gamma^2}$$

LIT of a delta by  
numerically  
choosing  
 $\gamma \ll \Gamma$

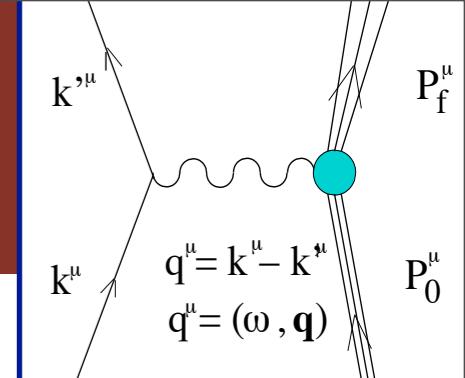
Fit  $f_R(q)$  to obtain a smooth **background**  $\rightarrow f_R(q)$  is related to the resonant form factor



# Monopole Resonance ${}^4\text{He}(\text{e},\text{e}')0^+$

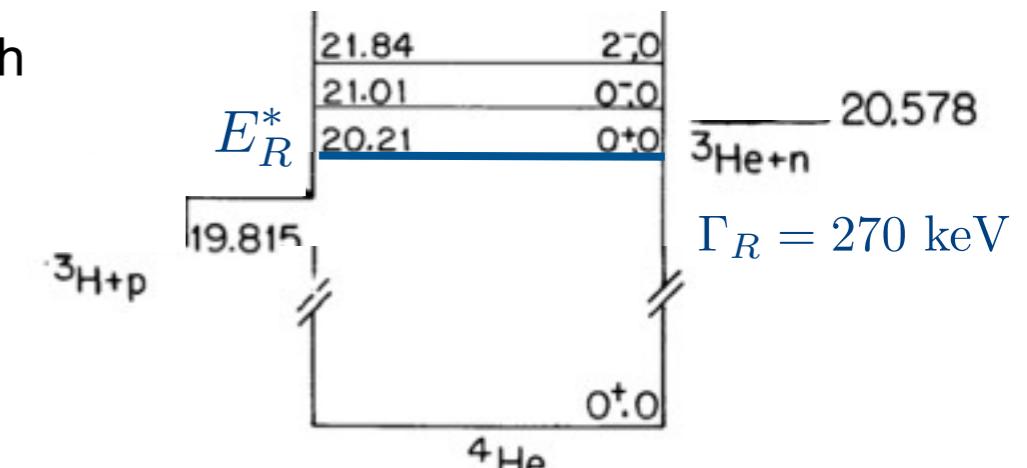
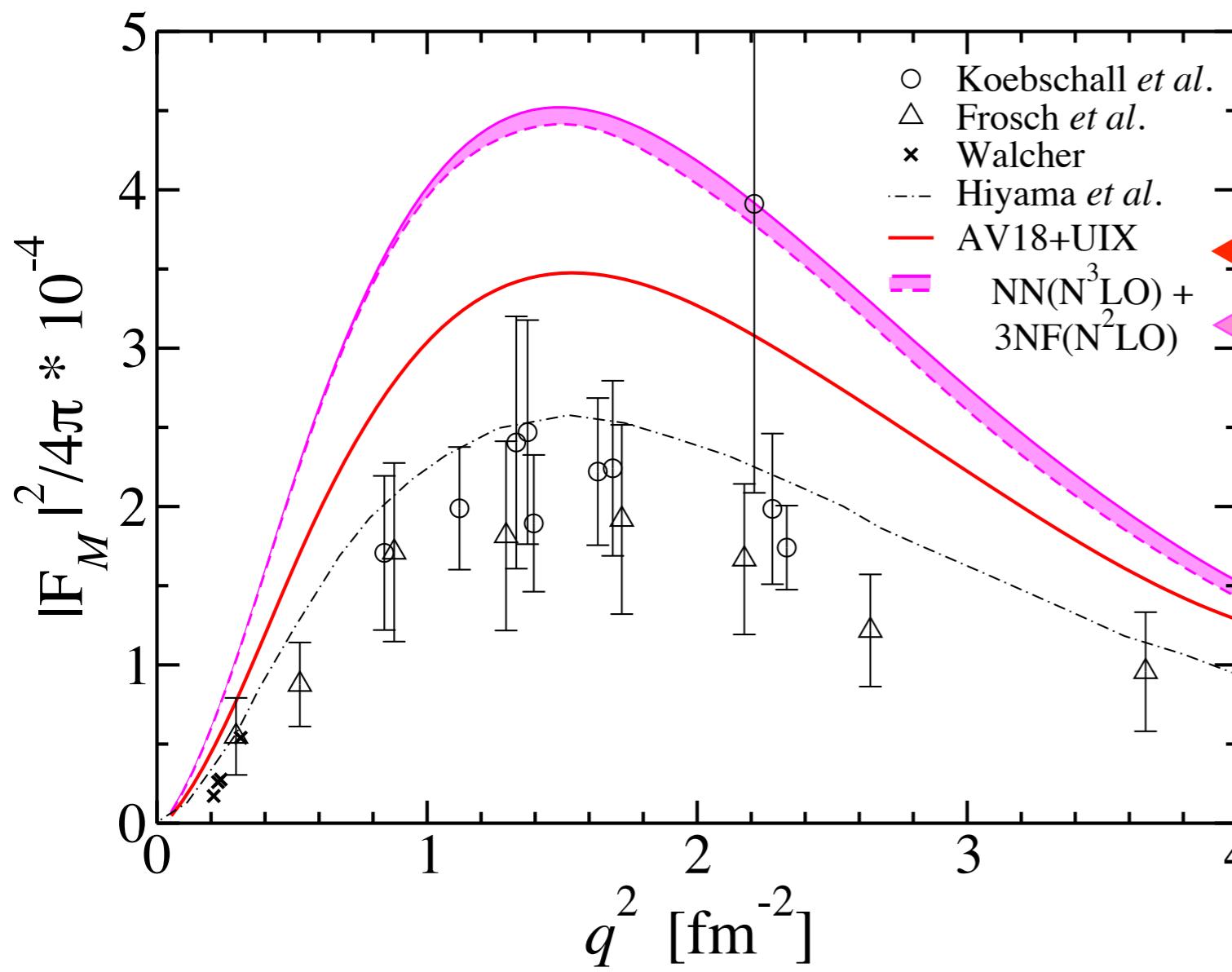
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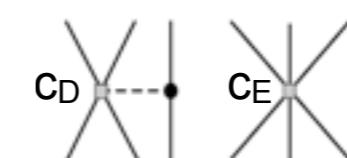


First ab-initio calculation with realistic three-nucleon forces and with the Lorentz Integral Transform method

S.B. et al., PRL 110, 042503 (2013)



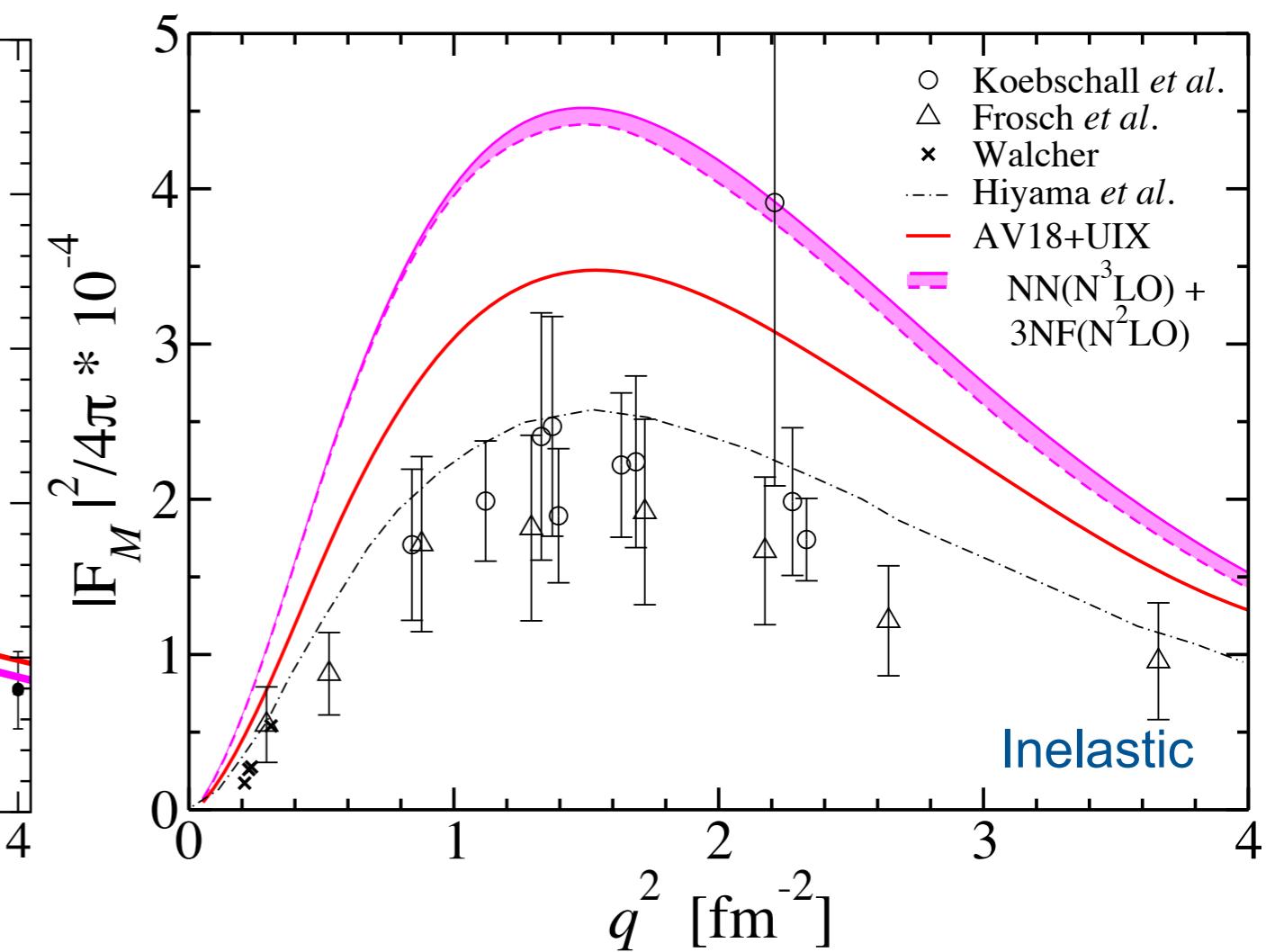
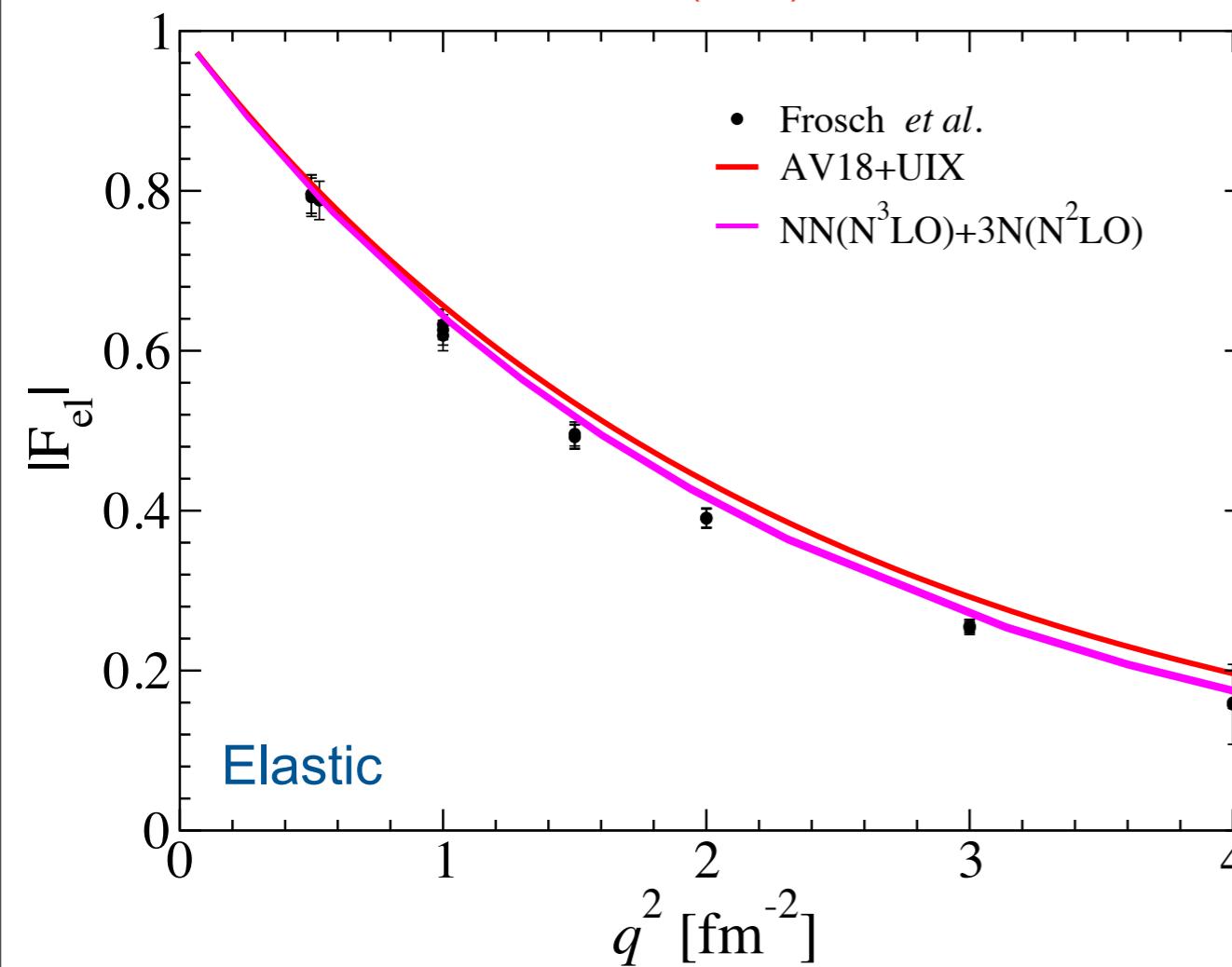
conventional forces  
 $\chi\text{EFT}$  forces



Two-sets of value of the three-nucleon forces low energy constants  $C_D$ ,  $C_E$   
difference in the short-range part

# Sensitivity to Nuclear Hamiltonians

S.B. et al., PRL 110, 042503 (2013)



→ The inelastic monopole resonance acts as a prism to nuclear Hamiltonians.

AV8' + central 3NF

AV18+UIX

NN( $N^3\text{LO}$ )+3NF( $N^2\text{LO}$ )

$E_0 = -28.44 \text{ MeV}$

$E_0 = -28.40 \text{ MeV}$

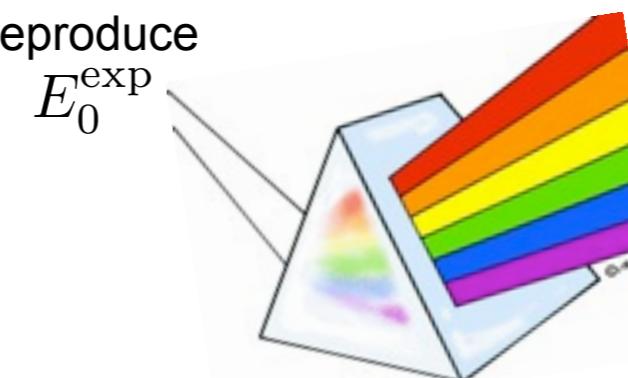
$E_0 = -28.36 \text{ MeV}$

$E_0^{\text{exp}} = -28.30 \text{ MeV}$

H reproduce

$E_0^{\text{exp}}$

spectrum  
in  $|F_M|^2$



# Analysis of this result

Realistic three-nucleon forces do not reproduce the data for  $|F_M|^2$   
 Particularly large difference are found with chiral EFT potentials.  
**This is unexpected!** What can be the source of this behaviour?

- **Numerics?** Our calculations are well converged (few % level) in the HH basis

$K_{\max}$	12	14	16	18
$10^4  F_M ^2$	4.59	4.75	4.85	4.87

- **Many-body charge operators?**

Conventional Nuclear Physics

Impulse approximation valid for elastic form factor below  $2 \text{ fm}^{-1}$

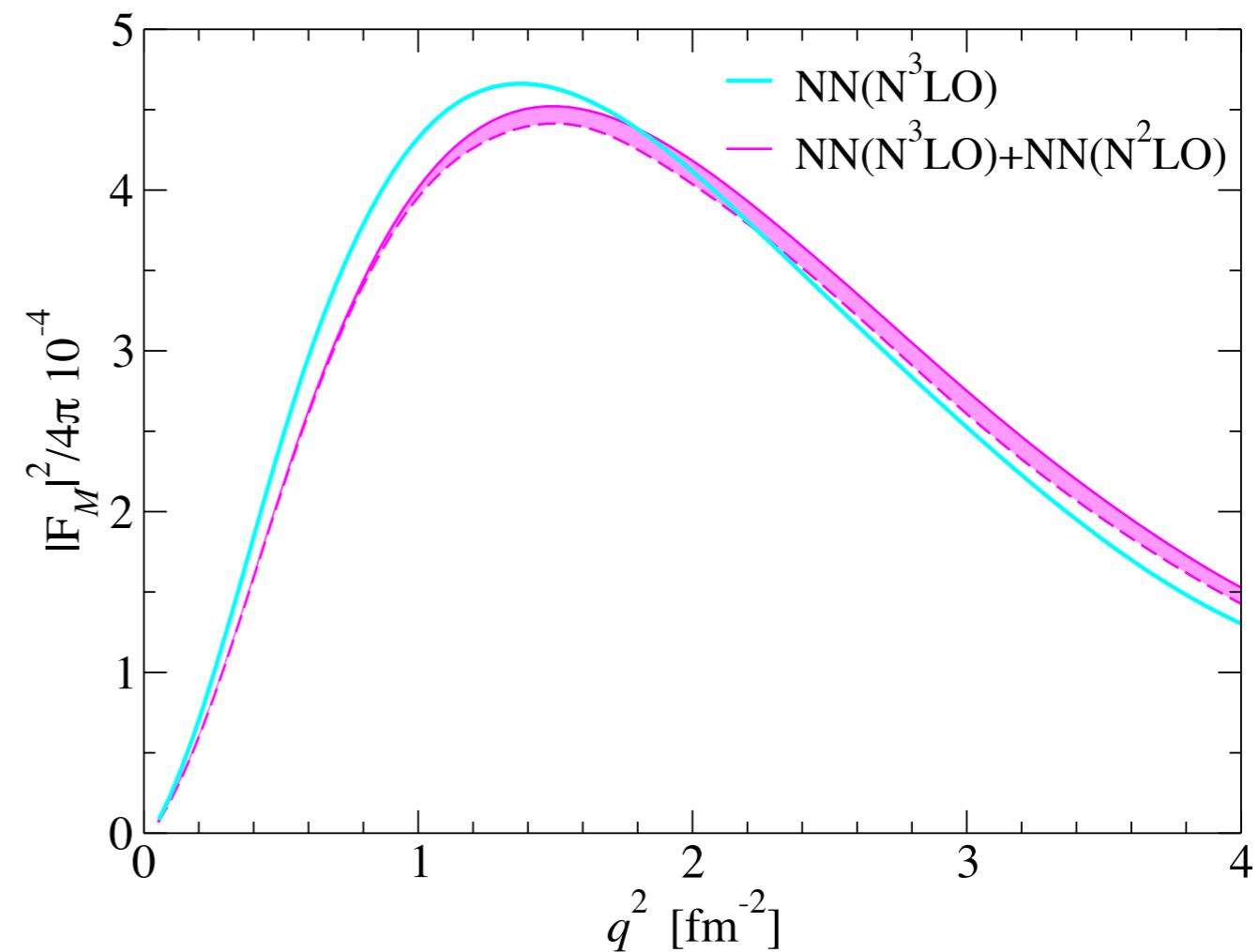
Viviani *et al.*, PRL **99** (2007) 112002

EFT approach

work done by Park *et al.*, Epelbaum, Koelling *et al.*, Pastore *et al.*, many-body operators appear at high order in EFT

- **Higher order 3NF ( $N^3LO$ )?**

Unlikely...



# Analysis of this result

- Location of the resonance?

AV8' + central 3NF

$$E_R^* = 20.25 \text{ MeV}$$

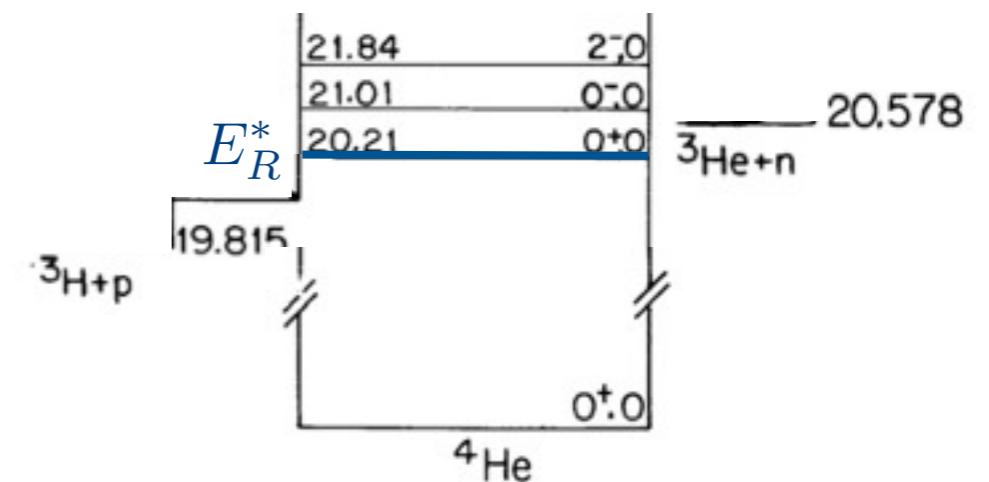
AV18+UIX

$$E_R^* = 21.00(20) \text{ MeV}$$

NN(N<sup>3</sup>LO)+3NF(N<sup>2</sup>LO)

$$E_R^* = 21.01(30) \text{ MeV}$$

$$E_R^* = 20.21 \text{ MeV}$$



The “realistic Hamiltonians” fail to reproduce the correct position of the  $0^+_2$  resonance

More theoretical work needed to understand this.

- Can this be measured again?

# Extension to medium-mass nuclei

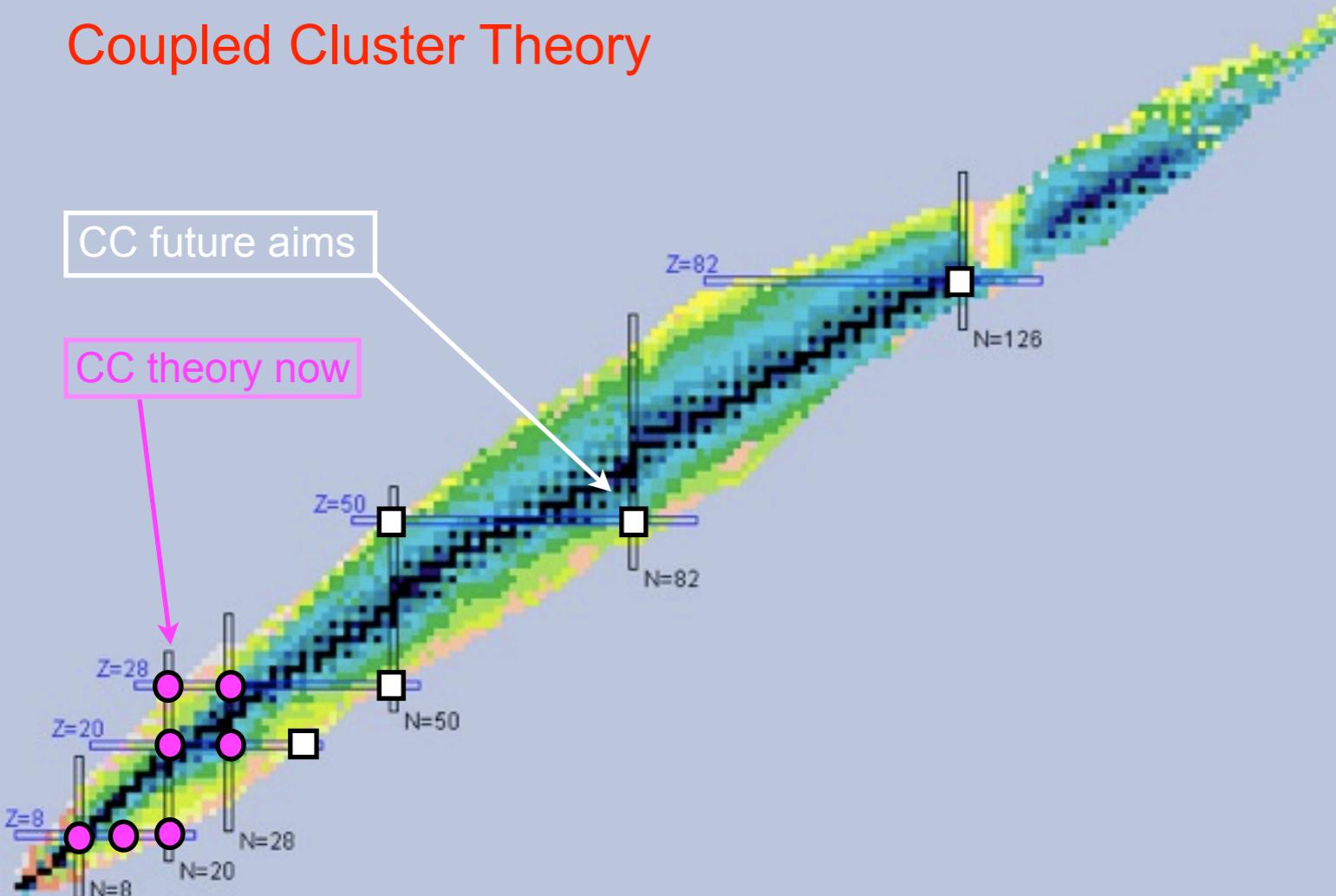
One example for A=16

- Photo-absorption

# Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei

## Coupled Cluster Theory



For the ground state energy

$$E_0 = \langle \phi_0 | e^{-T} H e^T | \phi_0 \rangle \quad \bar{H} = e^{-T} H e^T \quad \text{similarity transformed Hamiltonian}$$

$$0 = \langle \phi_i^a | e^{-T} H e^T | \phi_0 \rangle$$

$$0 = \langle \phi_{ij}^{ab} | e^{-T} H e^T | \phi_0 \rangle$$

Leads to CCSD equations for the t-amplitudes

- CC is optimal for closed shell nuclei ( $\pm 1, \pm 2$ )

Uses particle coordinates

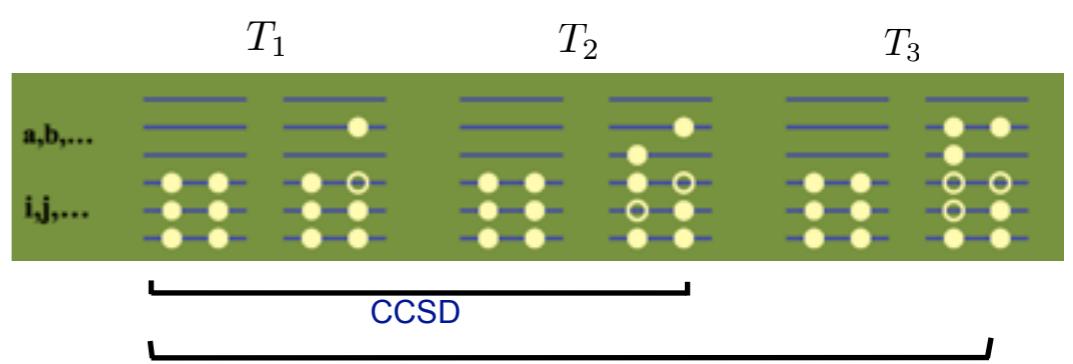
$$|\psi_0(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle$$

↳ reference SD with any sp states

$$T = \sum T_{(A)} \quad \text{cluster expansion}$$

$$T_1 = \sum_{ia} t_i^a a_a^\dagger a_i$$

$$T_2 = \frac{1}{4} \sum_{ij,ab} t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i \quad \dots$$



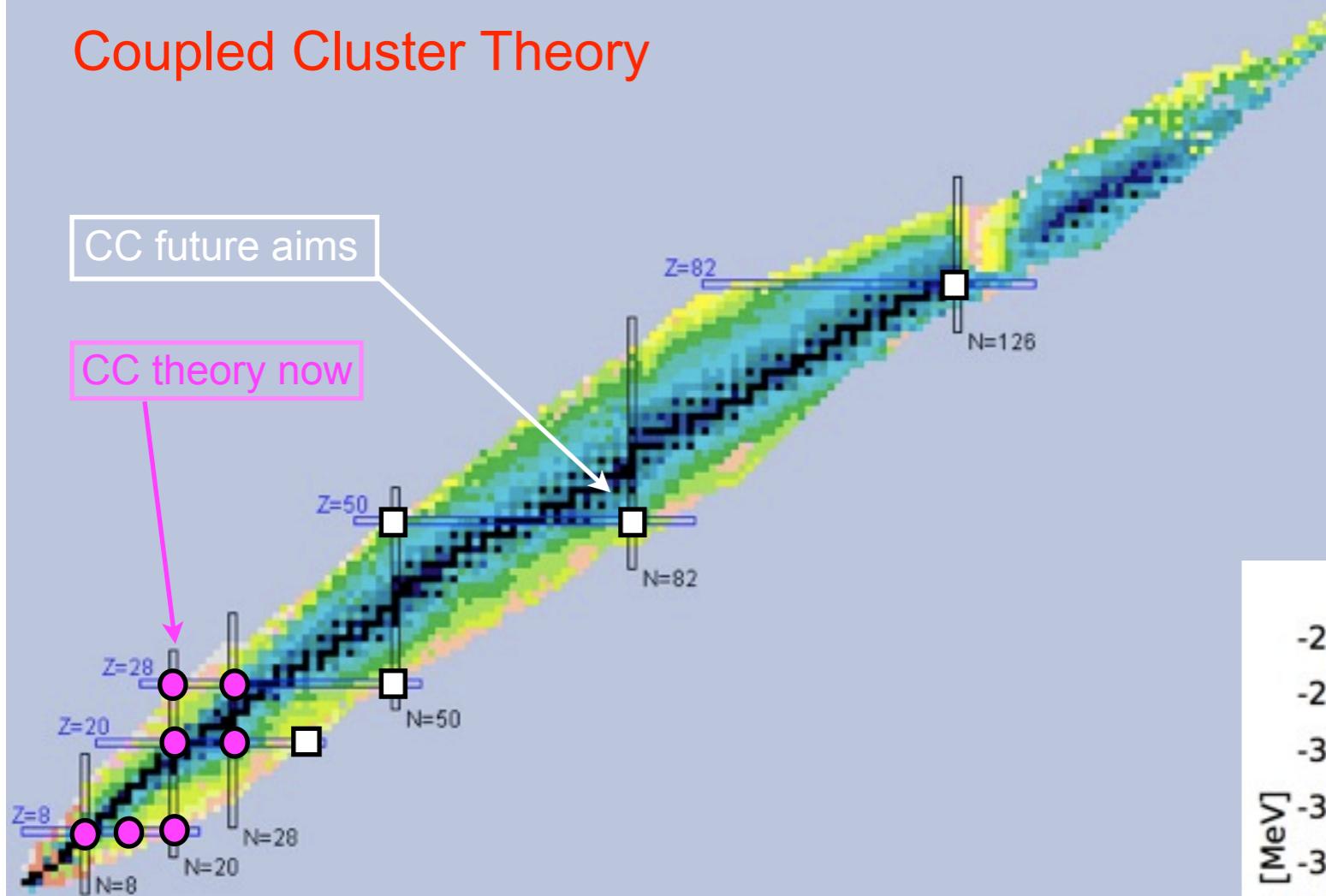
Model space truncation  $N \leq N_{max}$

Computational load  $n_o^2 n_u^4$

# Extension to medium-mass nuclei

Develop new many-body methods that can extend the frontiers to heavier and neutron nuclei

## Coupled Cluster Theory



CC is a very mature theory for g.s., see e.g.

Hagen *et al.* PRL 101, 092502 (2008), PRC 82, 03433 (2010)  
PRL 108, 242501 (2012), PRL 109, 032502 (2012)

What about electro-weak reactions?

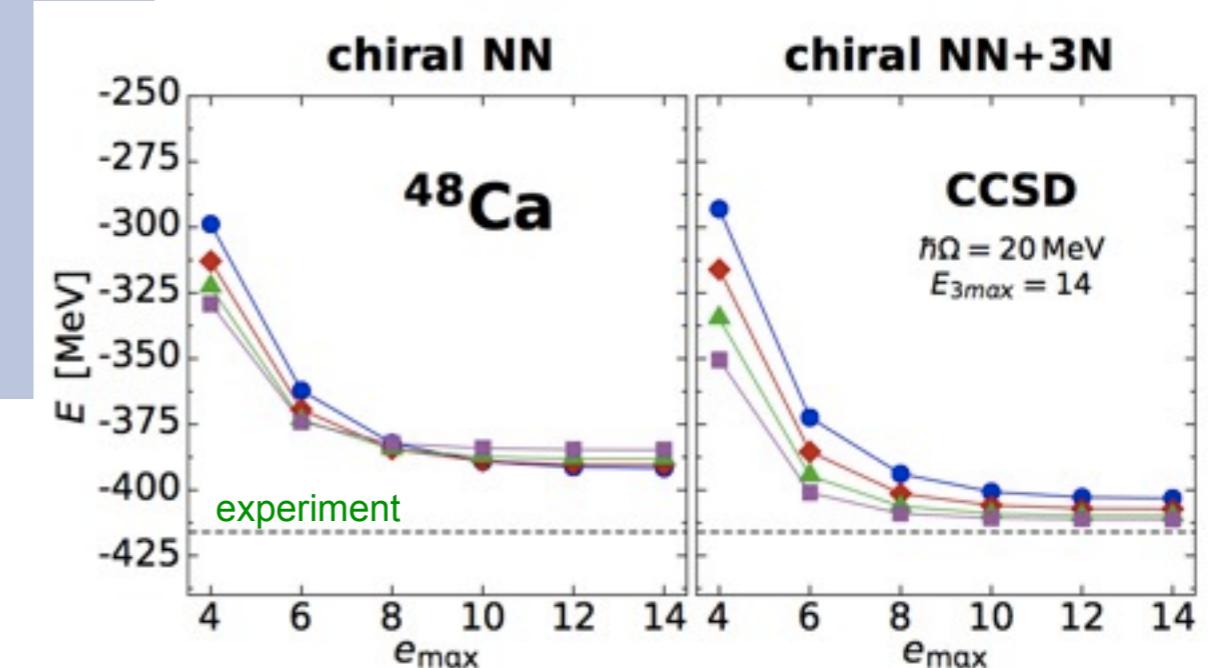
LIT+CC can possibly extend calculations of inelastic reactions into medium-mass nuclei!

- CC is optimal for closed shell nuclei ( $\pm 1, \pm 2$ )

Uses particle coordinates

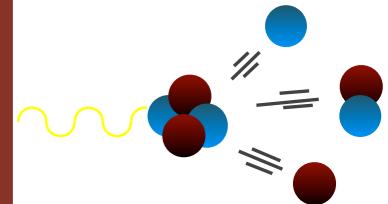
$$|\psi_0(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle = e^T |\phi_0(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A)\rangle$$

↳ reference SD with  
any sp states



R. Roth *et al.*, Phys. Rev. Lett. 109, 052501 (2012)

# LIT with Coupled Cluster Theory



$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2}$$

S.B., N.Barnea, G.Hagen, G.Orlandini, T.Papenbrock, [arXiv:1303.7446](https://arxiv.org/abs/1303.7446)

$$L(\sigma, \Gamma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle \rightarrow \langle \tilde{\Psi}_L | \tilde{\Psi}_R \rangle = \langle \Phi_0 | \hat{L}(z) | \hat{R}(z^*) \Phi_0 \rangle \quad \text{with} \quad z = E_0 + \sigma + i\Gamma$$

$$\begin{aligned} & \downarrow \quad \downarrow \\ \hat{R}_0 + \sum_{ia} \hat{R}_i^a \hat{c}_a^\dagger \hat{c}_i &+ \frac{1}{4} \sum_{ijab} \hat{R}_{ij}^{ab} \hat{c}_a^\dagger \hat{c}_b^\dagger \hat{c}_j \hat{c}_i + \dots \\ \hat{L}_0 + \sum_{ia} \hat{L}_i^a \hat{c}_i^\dagger \hat{c}_a &+ \frac{1}{4} \sum_{ijab} \hat{L}_{ij}^{ab} \hat{c}_i^\dagger \hat{c}_j^\dagger \hat{c}_b \hat{c}_a + \dots \end{aligned}$$

The Schrödinger-like eq. becomes

$$(\bar{H} - z^*) \overbrace{\hat{R}(z^*)}^{| \tilde{\Psi}_R \rangle} | \Phi_0 \rangle = \bar{\Theta} | \Phi_0 \rangle \quad \text{with} \quad \bar{\Theta} = e^{-T} \Theta e^T \quad \text{similarity transformed operator}$$

$$\hat{R}(z^*) \bar{H} | \Phi_0 \rangle = E_0 \hat{R}(z^*) | \Phi_0 \rangle$$

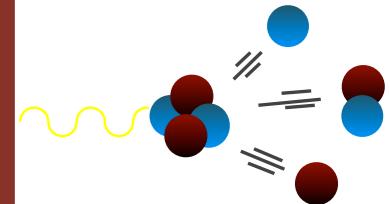
$$[\bar{H}, \hat{R}(z^*)] | \Phi_0 \rangle = (z^* - E_0) \hat{R}(z^*) | \Phi_0 \rangle + \bar{\Theta} | \Phi_0 \rangle$$

Right EoM to find the amplitudes of  $\hat{R}$

$$\langle \Phi_0 | [\hat{L}(z), \bar{H}] = \langle \Phi_0 | \hat{L}(z)(z - E_0) + \langle \Phi_0 | \bar{\Theta}^\dagger$$

Right EoM to find the amplitudes of  $\hat{L}$

# LIT with Coupled Cluster Theory

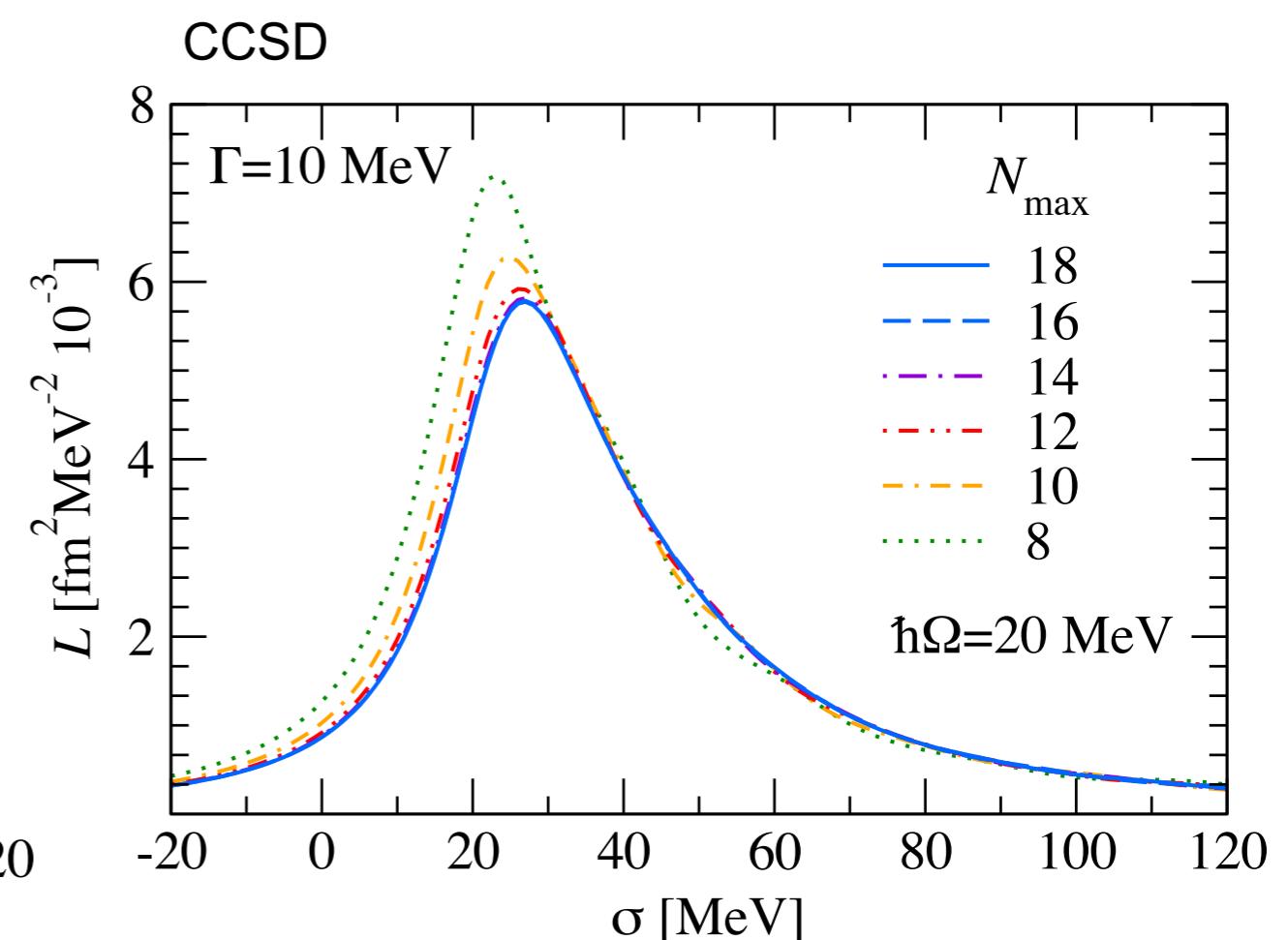
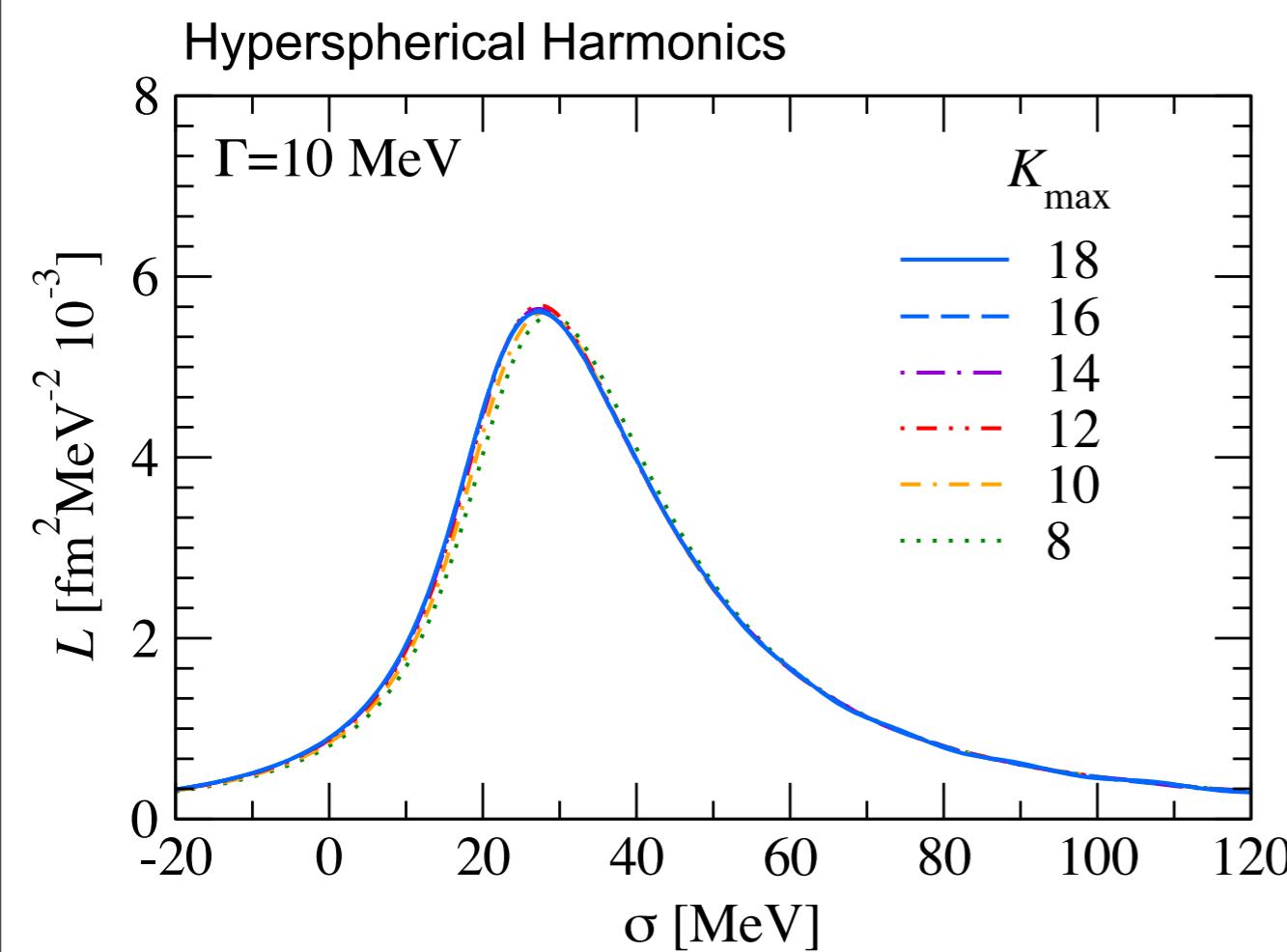


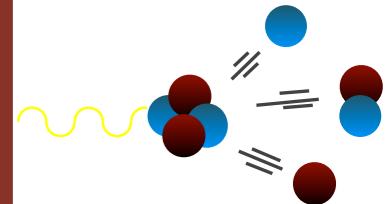
Dipole Response Function

S.B., N.Barnea, G.Hagen, G.Orlandini, T.Papenbrock, [arXiv:1303.7446](https://arxiv.org/abs/1303.7446)

Validation on  ${}^4\text{He}$  with NN forces derived from  $\chi$ EFT (N<sup>3</sup>LO)

→ Convergence in the model space expansion

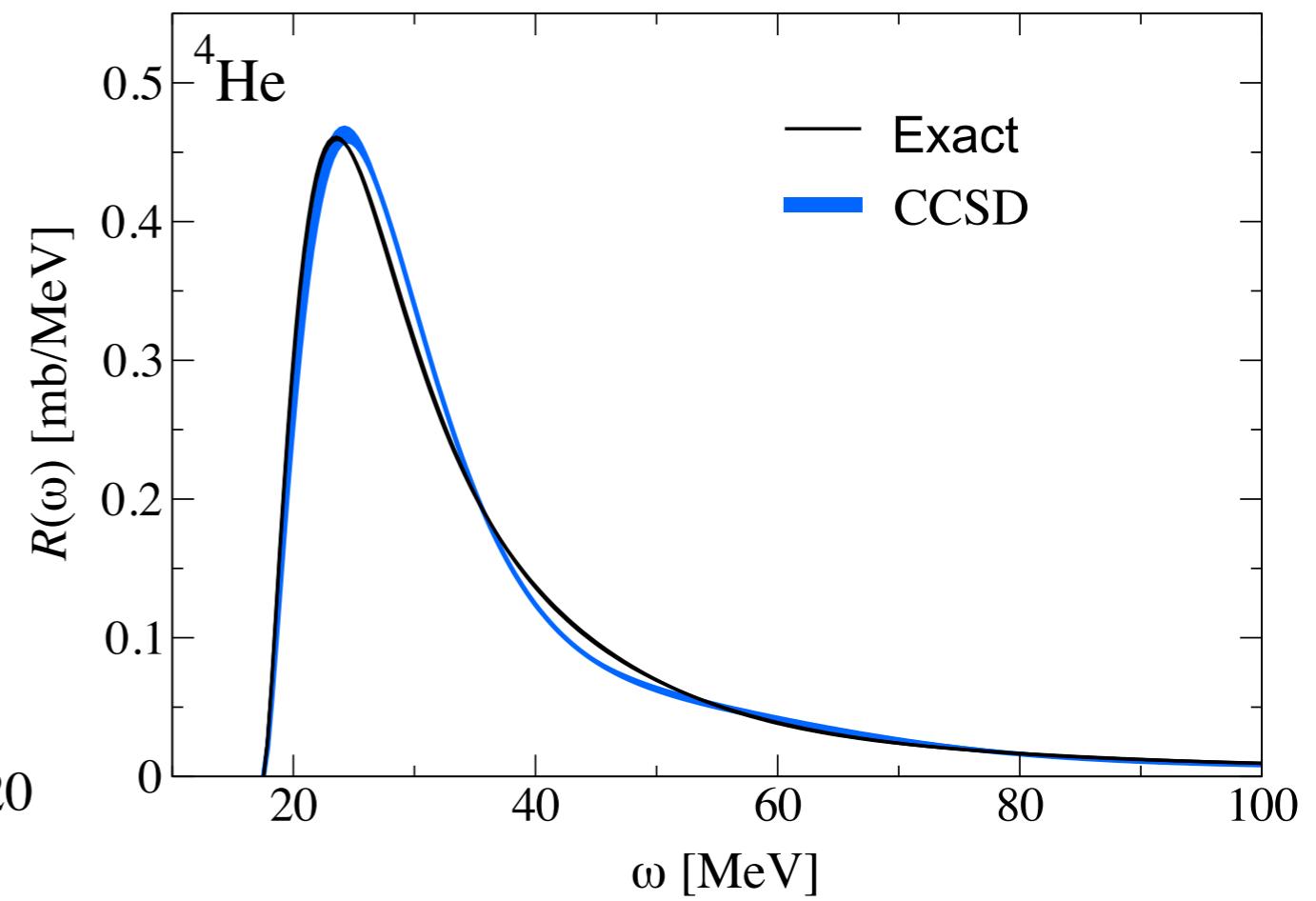
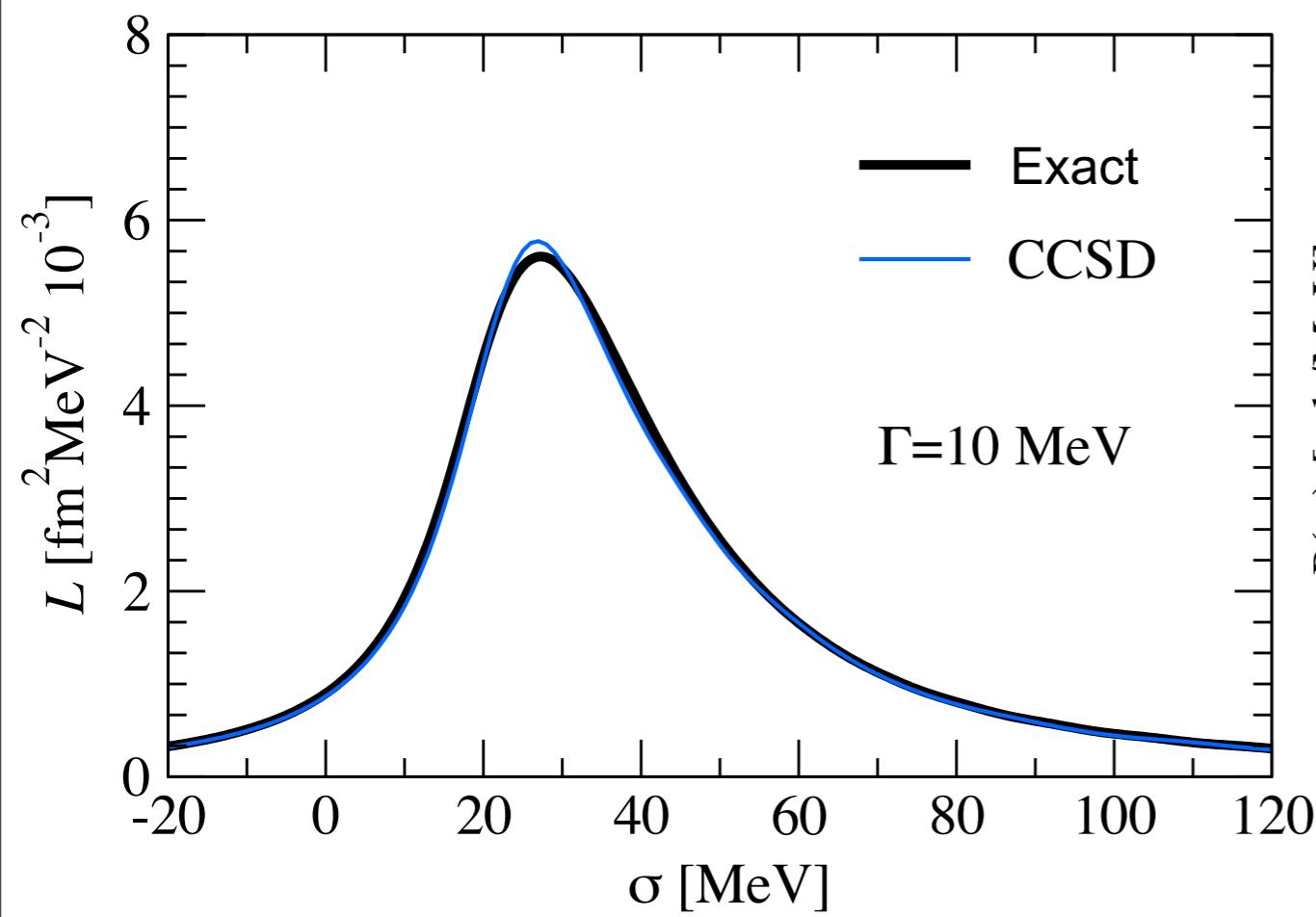




## Dipole Response Function

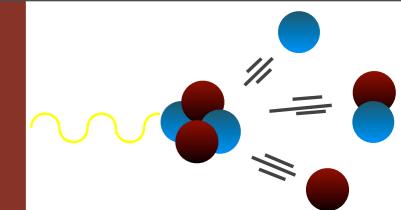
S.B., N.Barnea, G.Hagen, G.Orlandini, T.Papenbrock, [arXiv:1303.7446](https://arxiv.org/abs/1303.7446)Validation on  ${}^4\text{He}$  with NN forces derived from  $\chi$ EFT ( $N^3\text{LO}$ )

→ Comparison of CCSD with exact hyperspherical harmonics



The comparison with exact theory is very good!

# LIT with Coupled Cluster Theory

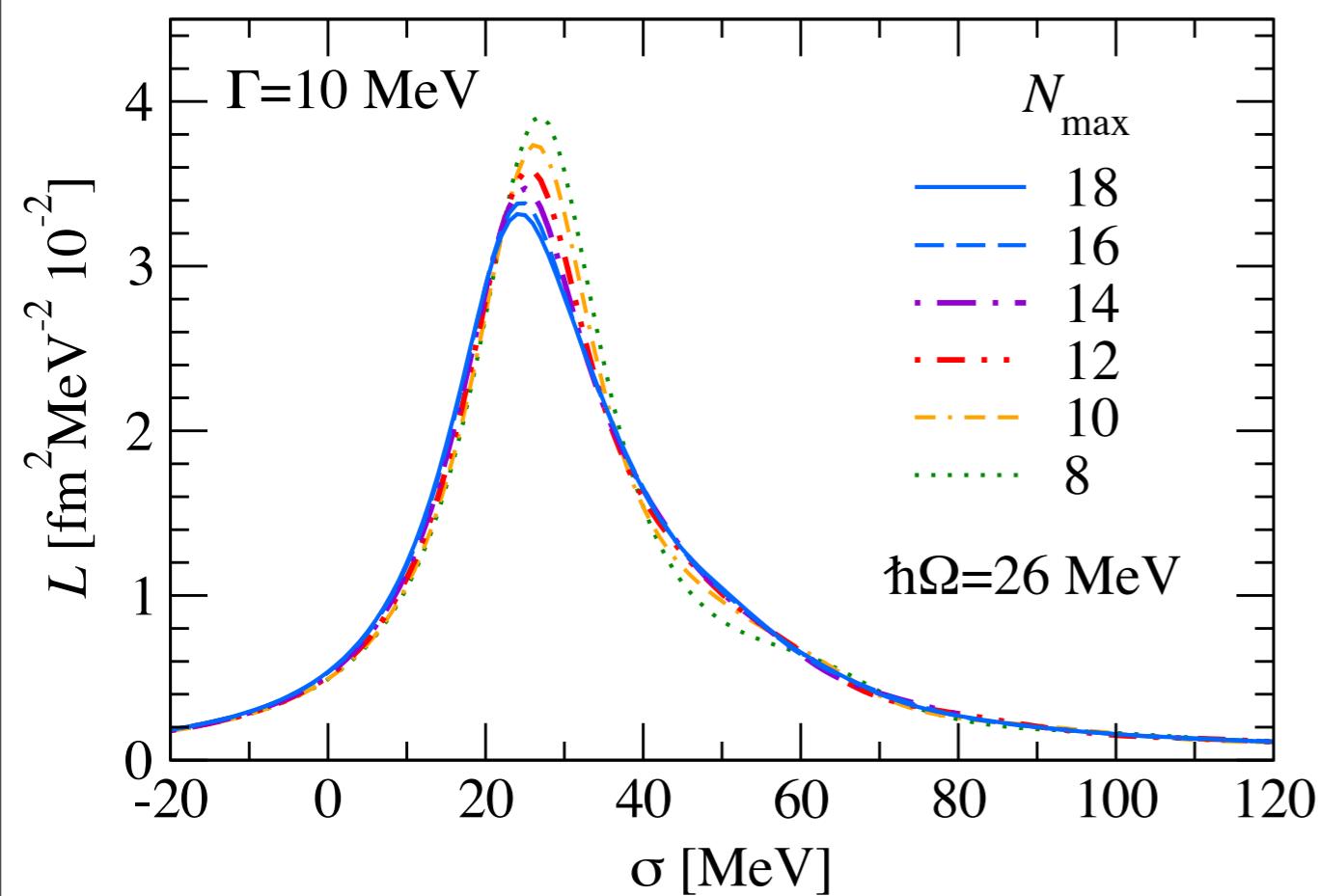


Dipole Response Function

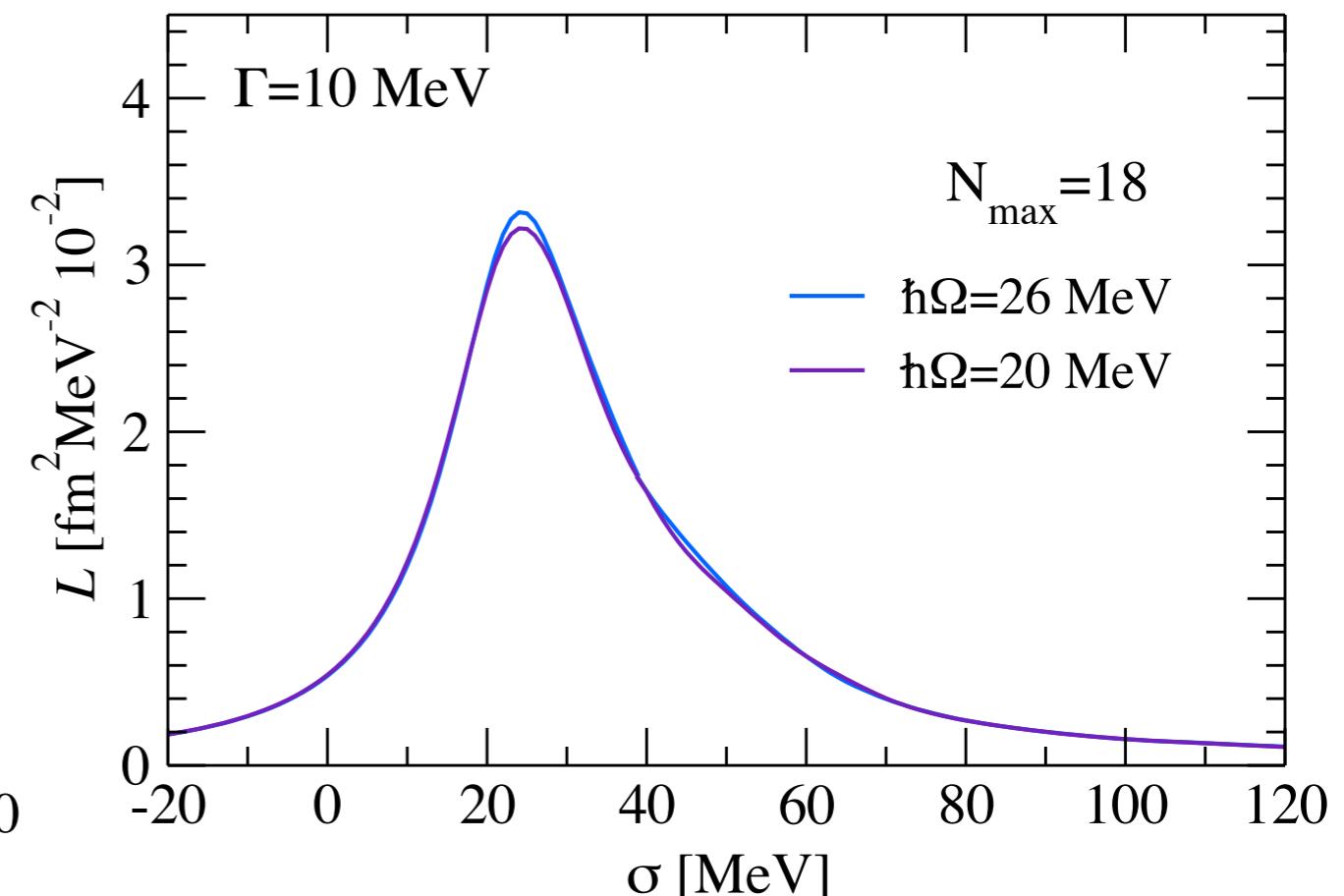
S.B., N.Barnea, G.Hagen, G.Orlandini, T.Papenbrock, [arXiv:1303.7446](https://arxiv.org/abs/1303.7446)

Extension to  $^{16}\text{O}$  with NN forces derived from  $\chi\text{EFT}$  ( $N^3\text{LO}$ )

Convergence in the model space expansion

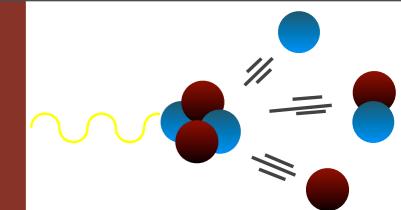


Good convergence!



Small HO dependence: use it as error bar

# LIT with Coupled Cluster Theory

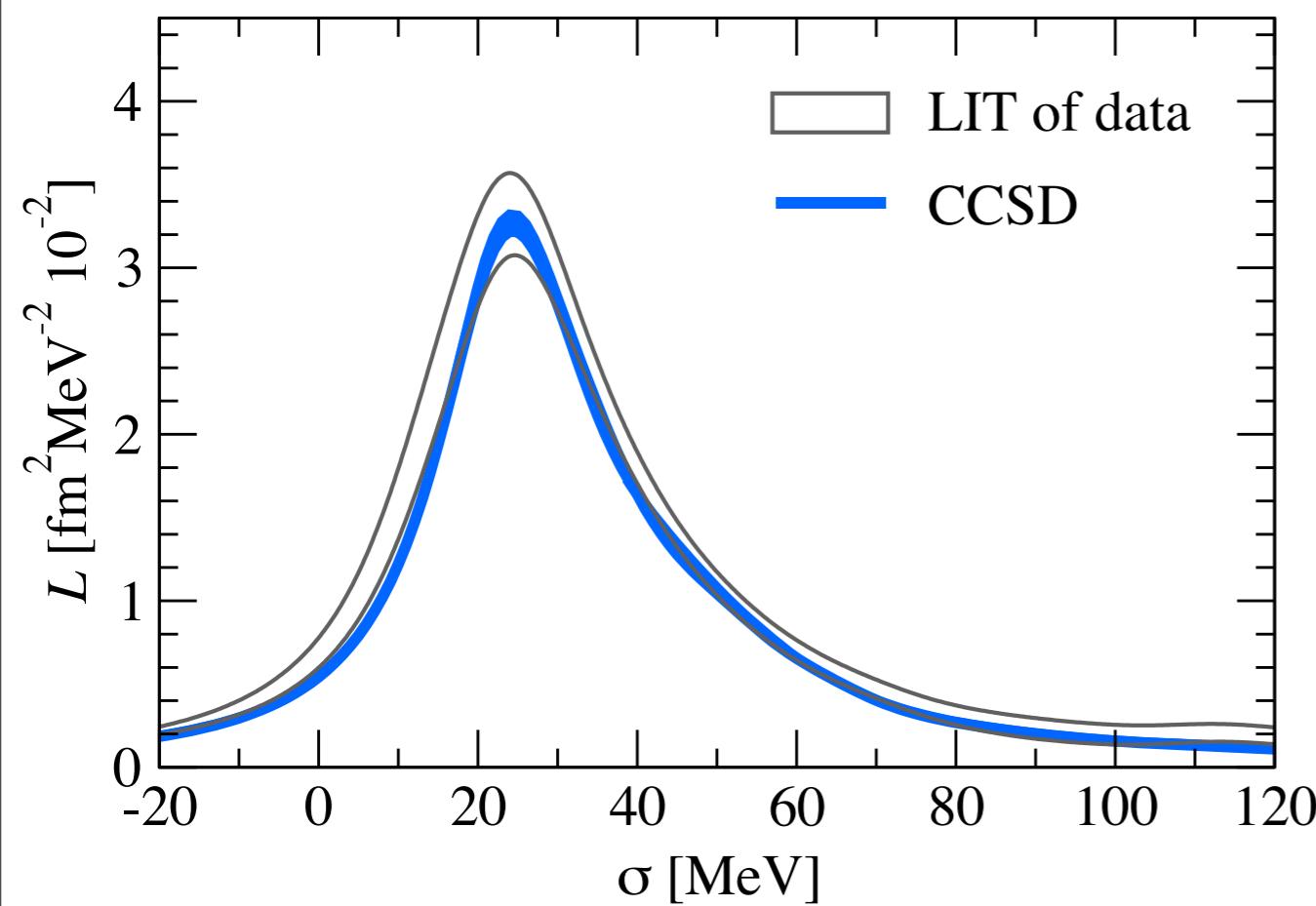


$$L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2}$$

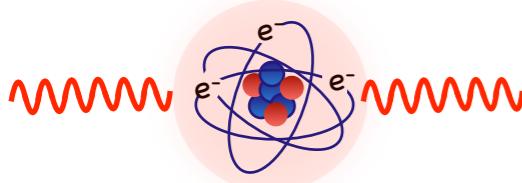
S.B., N.Barnea, G.Hagen, G.Orlandini, T.Papenbrock, [arXiv:1303.7446](https://arxiv.org/abs/1303.7446)

Extension to  $^{16}\text{O}$  with NN forces derived from  $\chi$ EFT ( $N^3\text{LO}$ )

→ Comparison with experiment

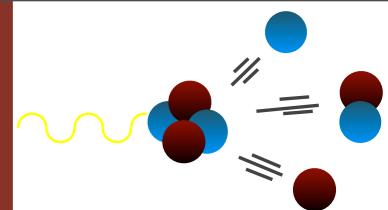


Data: target absorption experiment with  $\gamma$



Sonia Bacca

# LIT with Coupled Cluster Theory

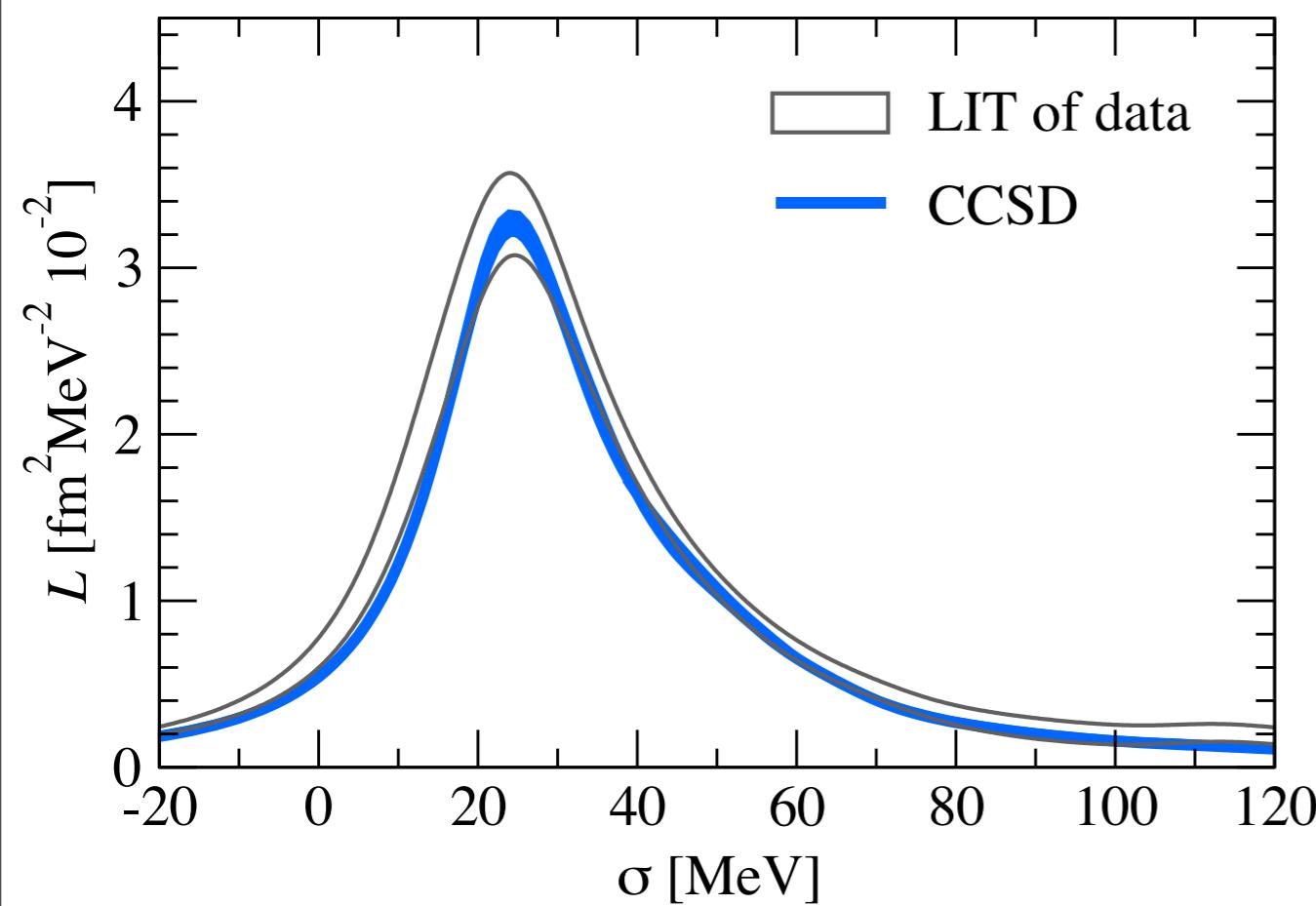


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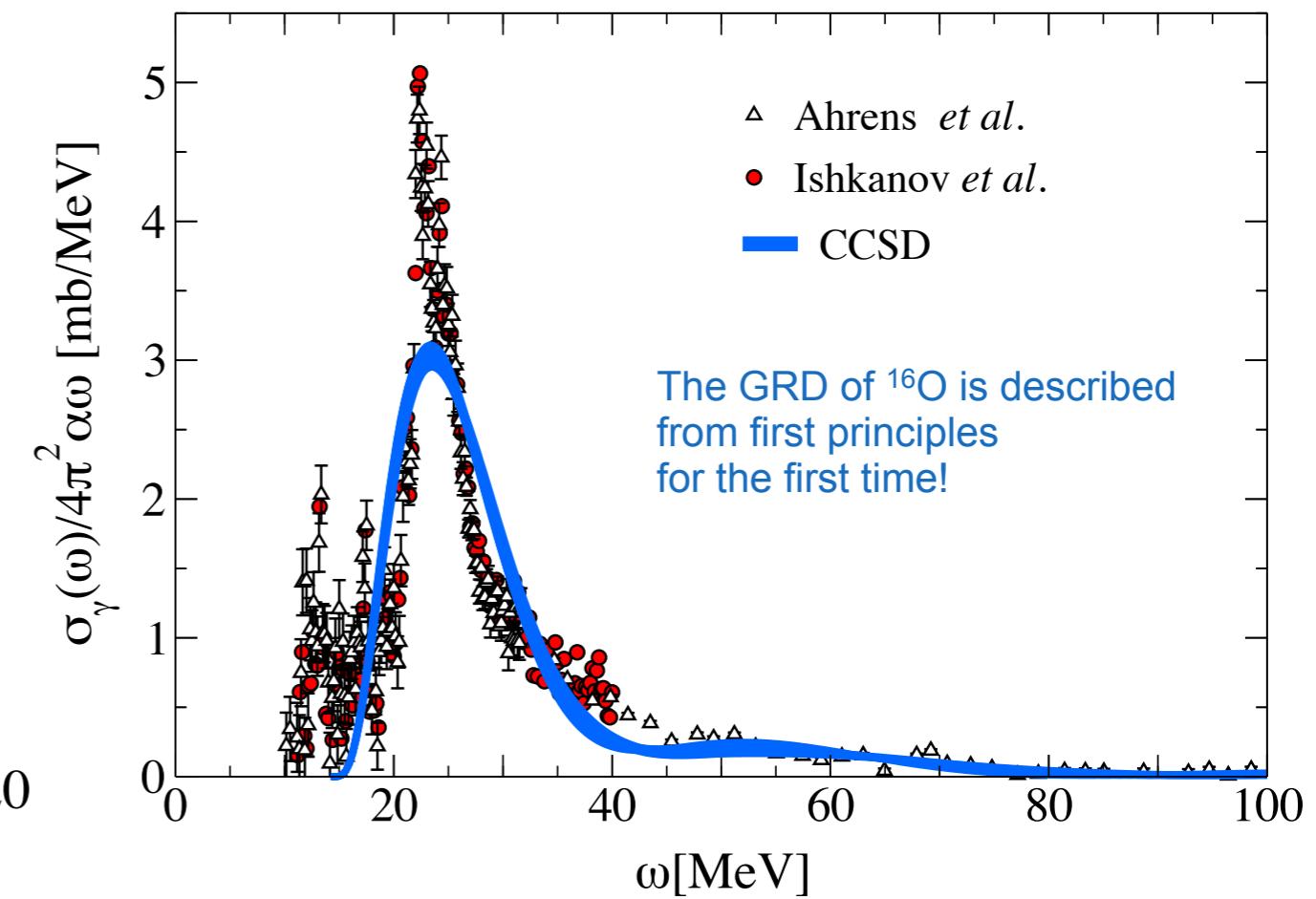
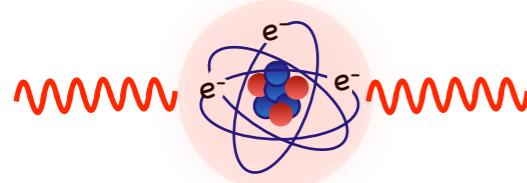
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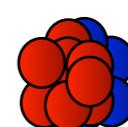
Comparison with experiment



Data: target absorption experiment with  $\gamma$



Giant Dipole Resonance



Sonia Bacca

protons  $\longleftrightarrow$  neutrons

# Conclusions and Outlook

- Nuclear response functions are very rich observables to test our understanding on nuclear forces
- Extending these calculations to medium mass nuclei is possible and very exciting

## Perspectives

- Dipole response function in  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$ , electric dipole polarizability  
Dipole response function of neutron-rich Oxygen isotope
- Quadrupole or monopole excitation of medium mass nuclei  
→ need extension of LIT/CCSD to two-body operator
- Magnetic transitions for medium mass nuclei with two-body currents  
→ need extension of LIT/CCSD to two-body operator
- Add triples and three-nucleon forces

# Thanks to my collaborators



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Nir Barnea, Doron Gazit



Gaute Hagen, Thomas Papenbrock



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# Thank you!