

# High finesse optical cavities

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# References

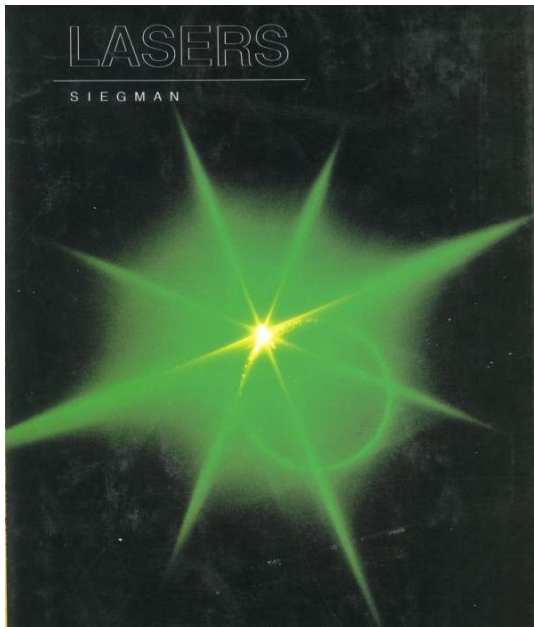
## Laser Beams and Resonators

H. KOGELNIK AND T. LI

**Abstract**—This paper is a review of the theory of laser beams and resonators. It is meant to be tutorial in nature and useful in scope. No attempt is made to be exhaustive in the treatment. Rather, emphasis is placed on formulations and derivations which lead to basic understanding and on results which bear practical significance.

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### LASERS

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*Lasers* by A.E. Siegman is both a textbook and general reference book on lasers, with an emphasis on basic laser principles and laser theory. It brings together into a unified and carefully laid out exposition all the fundamental and important physical principles and properties of laser devices, including both the atomic physics of laser materials and the optical physics and practical performance of laser devices. A unique feature of this book is that it gives a complete, detailed, and accurate treatment of laser physics, building only on classical models, without requiring a quantum mechanical background of the reader.

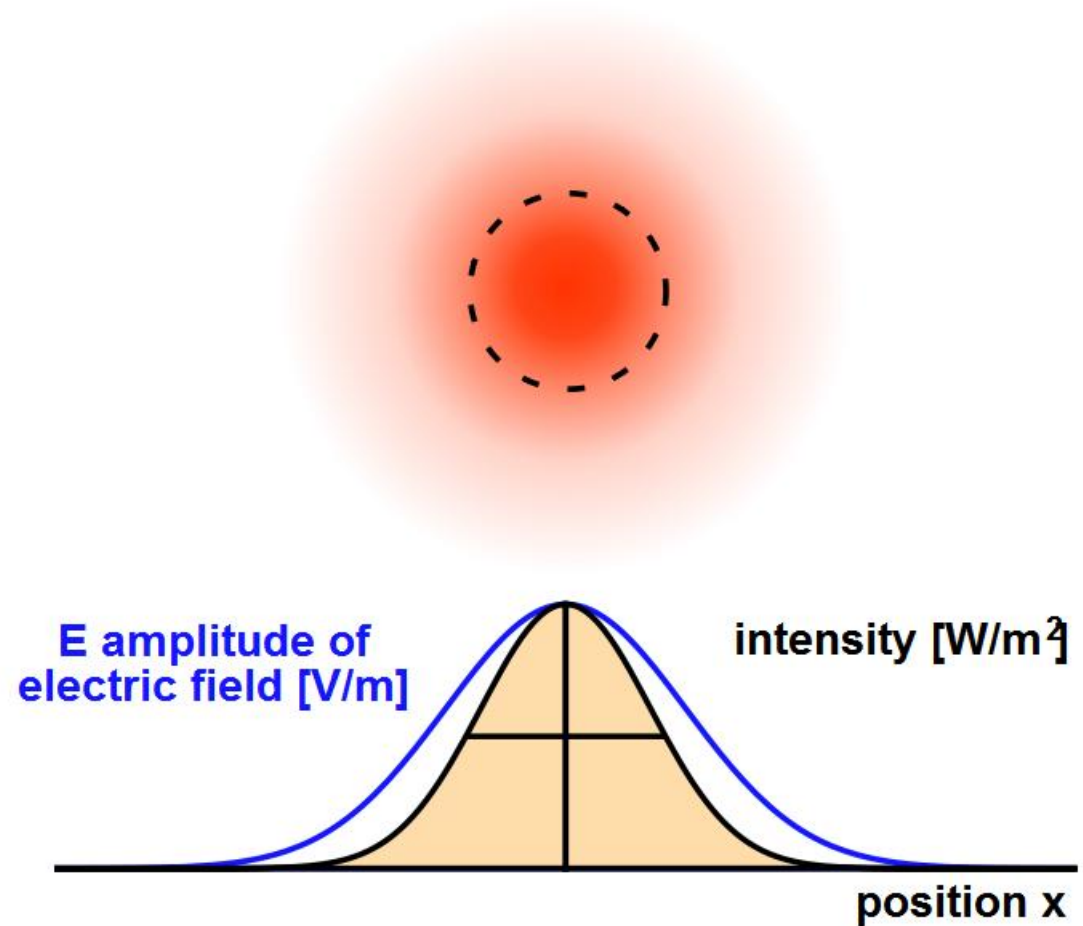


[http://en.wikipedia.org/wiki/Gaussian\\_beam](http://en.wikipedia.org/wiki/Gaussian_beam)

A screenshot of the Wikipedia article for 'Gaussian beam'. The page features the Wikipedia logo at the top left, a search bar, and a navigation menu. The main content area includes a table of contents with sections like '1 Mathematical form', '2 Beam parameters', and '3 Power and intensity'. The article text begins with 'In optics, a Gaussian beam is a beam of electromagnetic radiation whose wavefronts are spherical surfaces centered on a point on the beam's axis...'. The page also includes a 'References' section at the bottom right. A small image of a person in a dark coat is visible in the bottom right corner of the screenshot.

# Cavity parameters

- Gaussian beams
- Strawman parameters
- Items governing finesse
- Items governing length



# Paraxial approximation

A gaussian beam is described in the *paraxial* approximation ( $\sin \theta = \theta$ ) by

$$E(\rho, z) = A \frac{w_0}{w(z)} e^{ikz} e^{-\tan^{-1}(z/z_0)} e^{ik\rho^2/2R(z)} e^{-\rho^2/w^2(z)}$$

where  $w_0$  is the beam waist dimension (a *radius*) and

$$z_0 = \frac{\pi w_0^2}{\lambda}$$

is the Rayleigh range. The beam is  $\sqrt{2}$  bigger at  $z = z_0$  from the waist.

The beam has a “diameter” of  $2w(z)$ , with

$$w^2(z) = w_0^2 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right] = w_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]$$

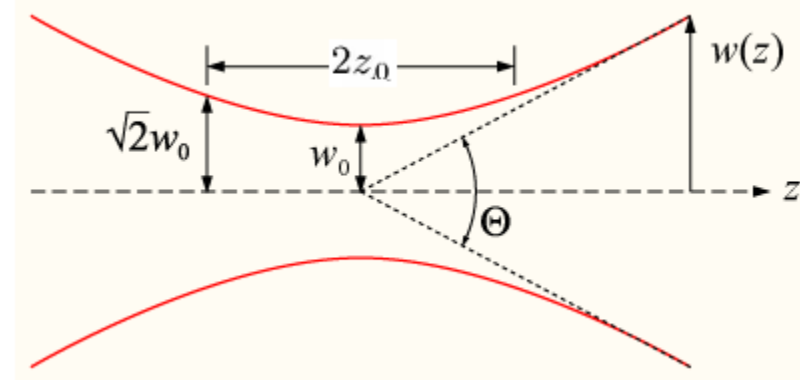
the beam “size,” and a curvature

$$R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right] = z + \frac{z_0^2}{z}.$$

Finally,

$$\theta = \frac{\lambda}{\pi w_0}$$

is the beam divergence angle.



# Intensities

At the waist,  $z = 0$ ,  $w = w_0$ ,  $R = \infty$ , and

$$E = Ae^{-\rho^2/w_0^2}$$

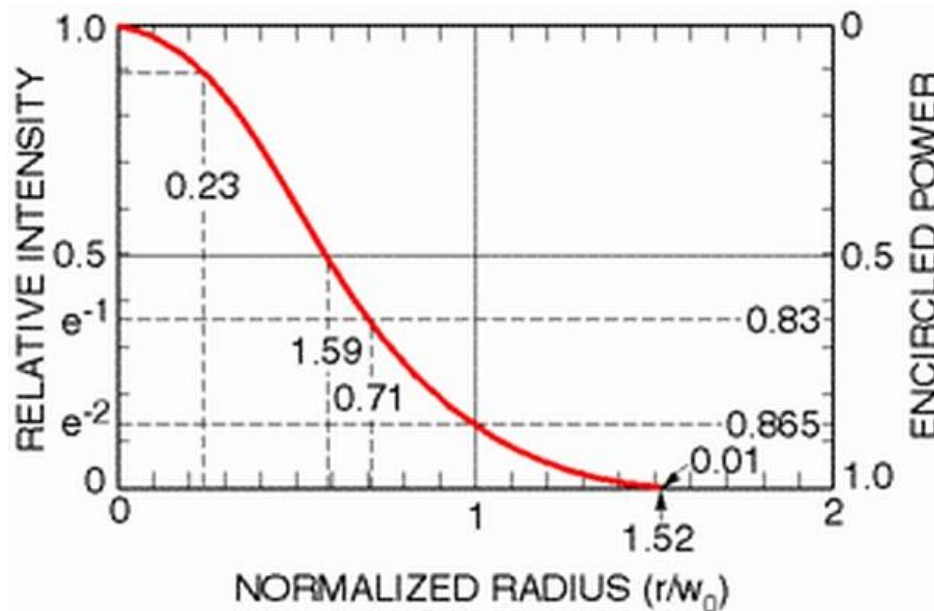
The intensity  $\propto E^2$ , so

$$I = I_0e^{-2\rho^2/w_0^2}$$

and the power enclosed by a circle of diameter  $D$  is

$$P(D) = P_0 \left[ 1 - e^{-D^2/2w_0^2} \right]$$

with  $P_0$  the total power of the beam.



# Cavities

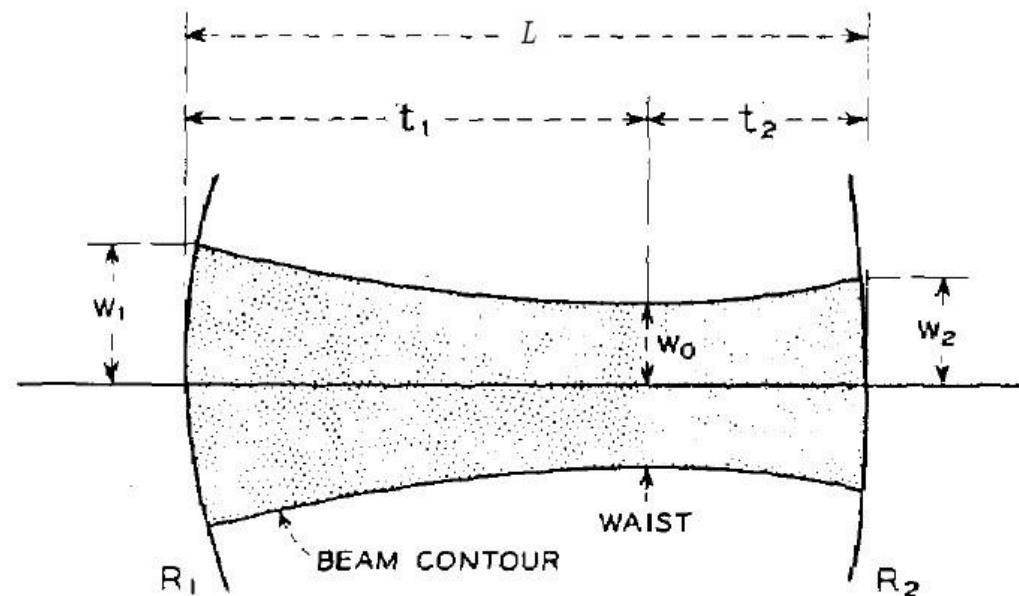
How do we find the waist? Set up a cavity, with curved mirrors of radii  $R_1$  and  $R_2$  and with a distance  $L$  between them. The resonant beam will have radii of curvature of  $R_i$  at each mirror, and a waist between them. For us, with curve/flat,  $R_1 = R$  and  $R_2 = \infty$ . Then,

$$g = 1 - \frac{L}{R}$$

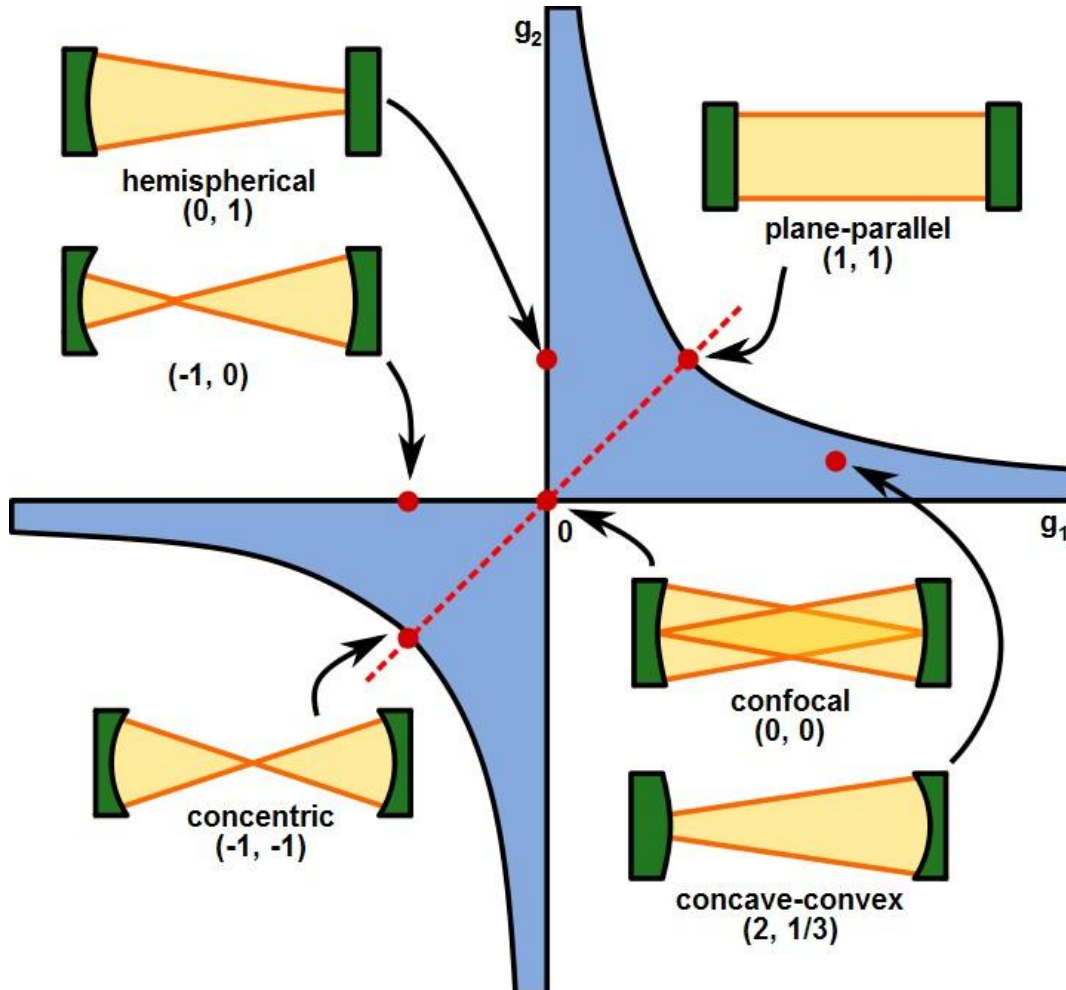
and

$$w_o^2 = \frac{\lambda L}{\pi} \sqrt{\frac{g}{1-g}}$$

$g(= g_1 g_2)$  is called the stability product. We have  $g_2 = 1$ . Want  $0 < |g| < 1$ .

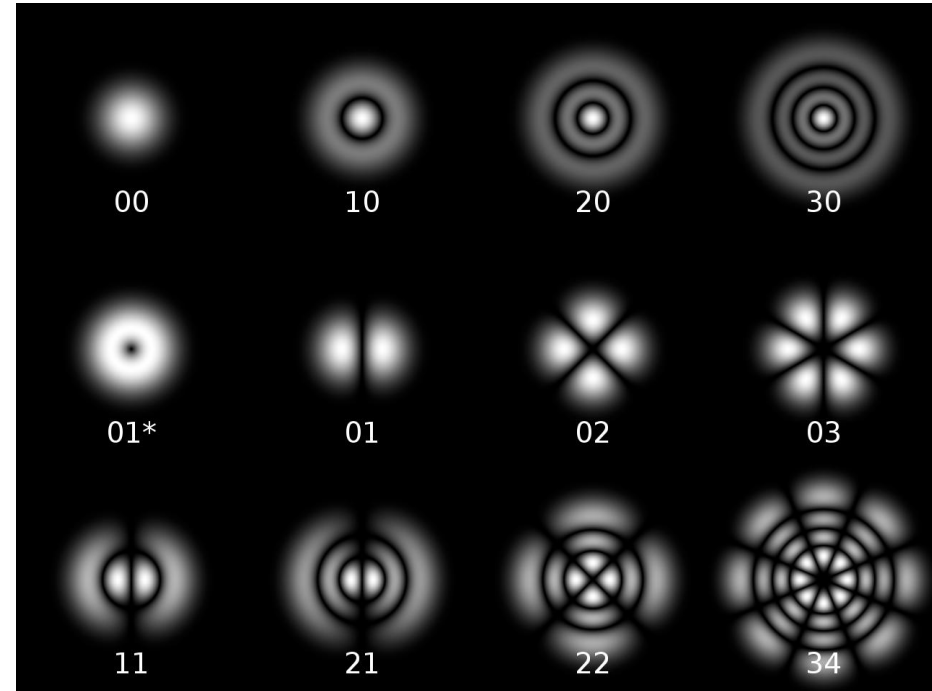
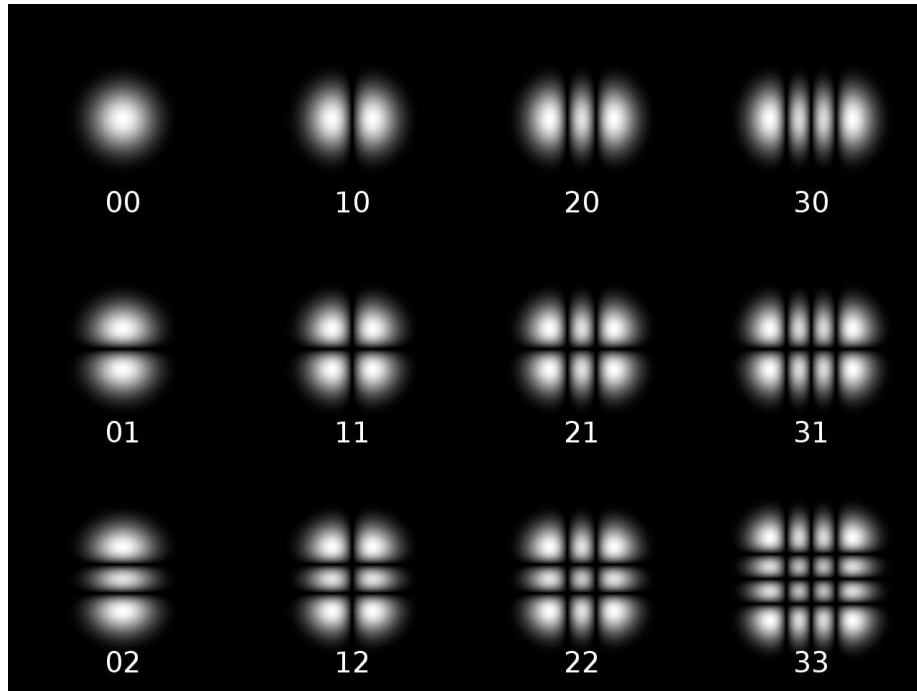


# Stability



# Cavities have an infinite set of modes

- Hermite Gauss (r) or Laguerre Gauss (l)





# Finesse and free spectral range

- Finesse is like  $Q$  of a metal cavity
- Effectively, it is the number of bounces a beam makes before leaking out or being absorbed.

$$\mathcal{F} = \frac{4\pi T_1}{(T_1 + V)^2}$$

- with  $T_1$  the transmission of the input mirror (assumed to be larger than that of the end mirror) and  $V$  the round-trip fractional power loss from power absorption in both mirrors, scattering due to mirror defects, diffraction from the finite mirror size, etc.
- FSR:

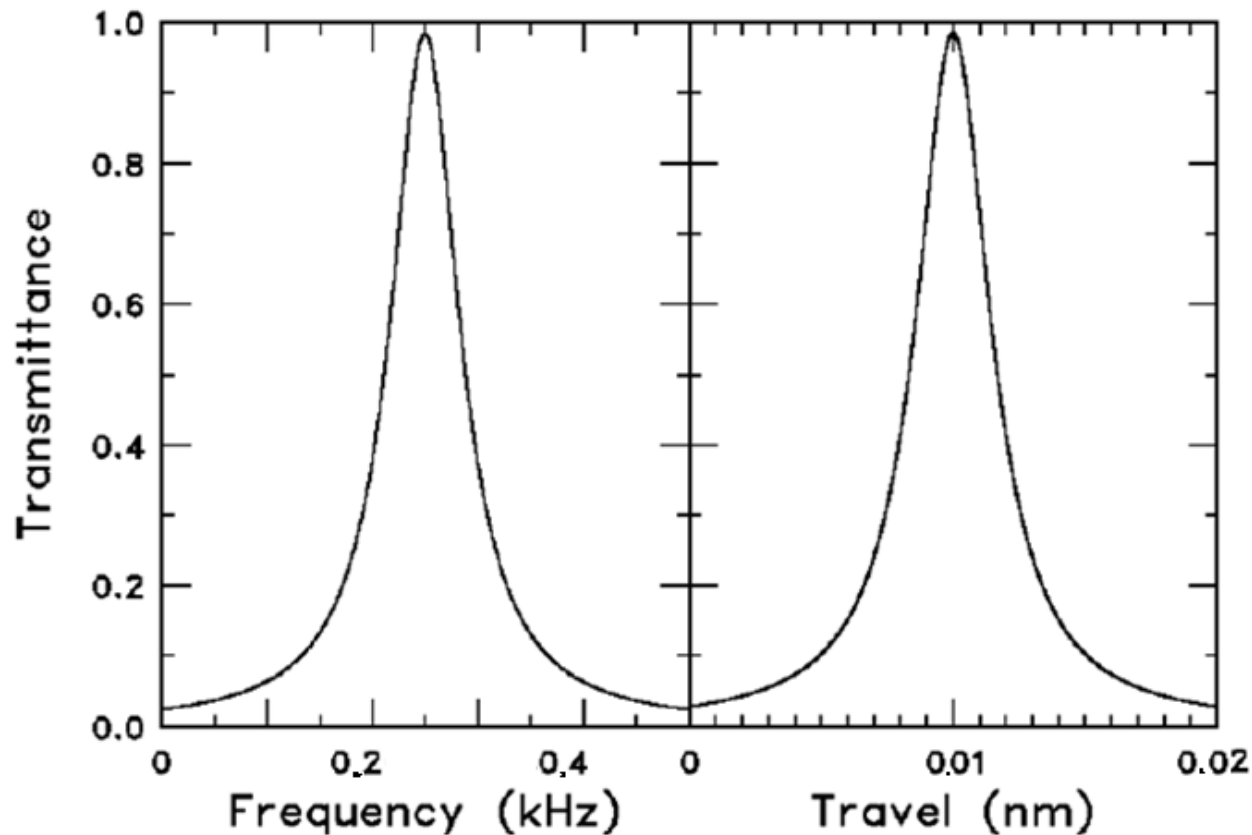
$$FSR = \frac{c}{2L}$$

- Finesse is equal to FSR divided by the FWHM of the resonance.



# Cavity with Finesse = 100,000

- Cavity transmission as function of frequency (left) or motion of one mirror (right)
- Cavity is 12 m in length, with modes at 12 MHz (=FSR)

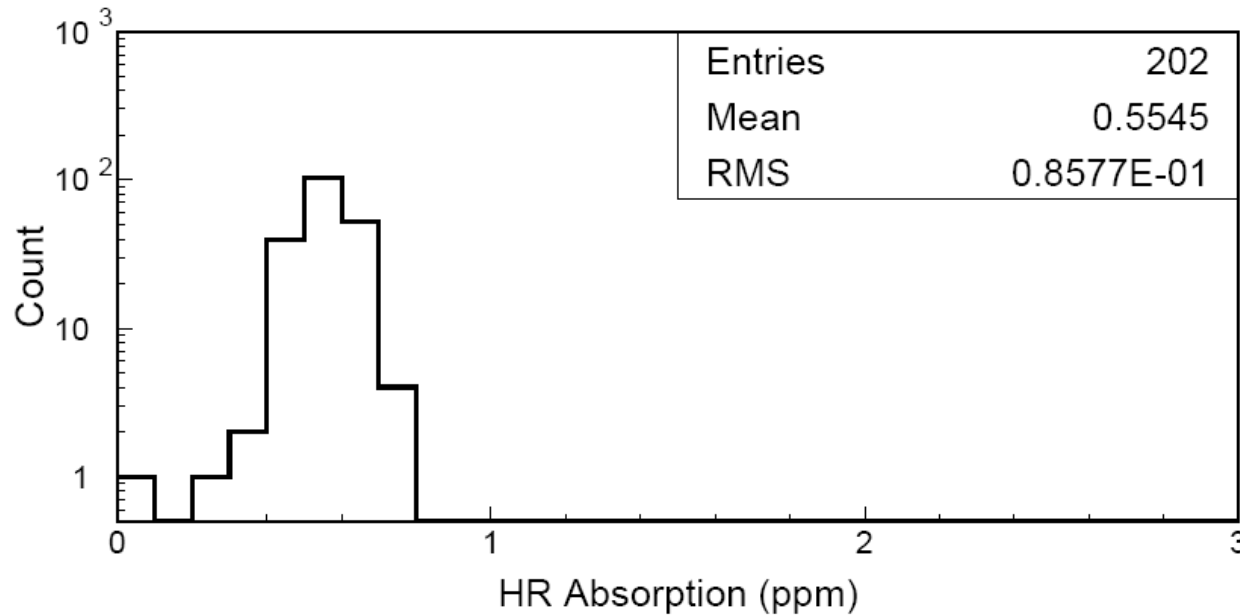
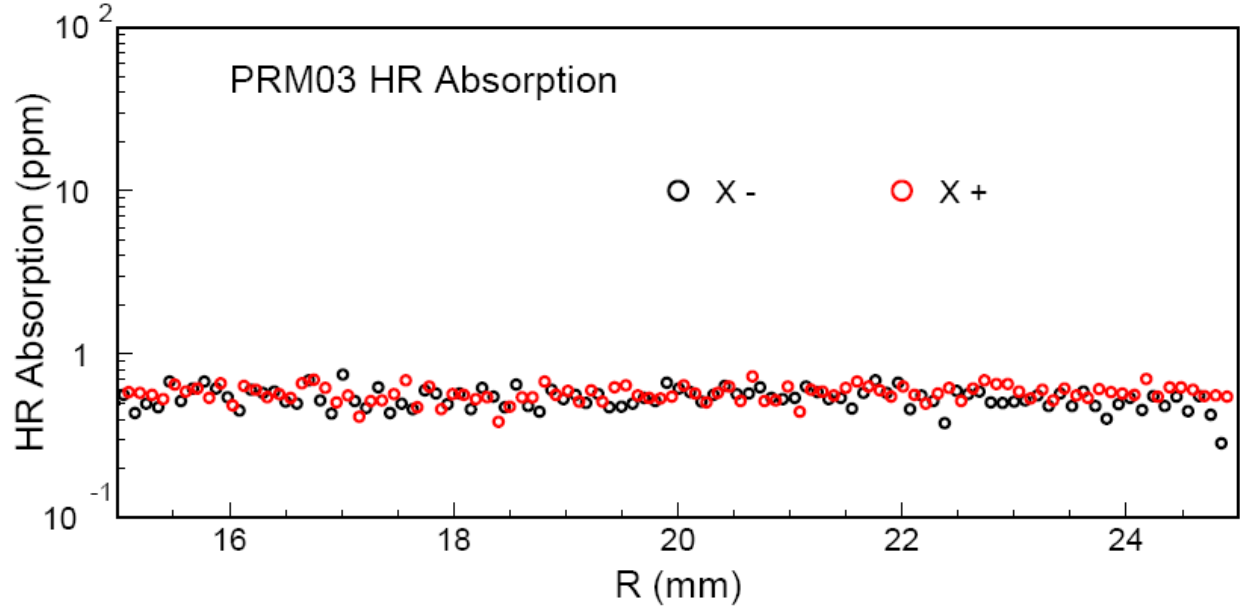


# Factors controlling finesse

- Loss
- Scatter
- Microroughness
- Coating nonuniformity
- Ultimately, transmission of input mirrors

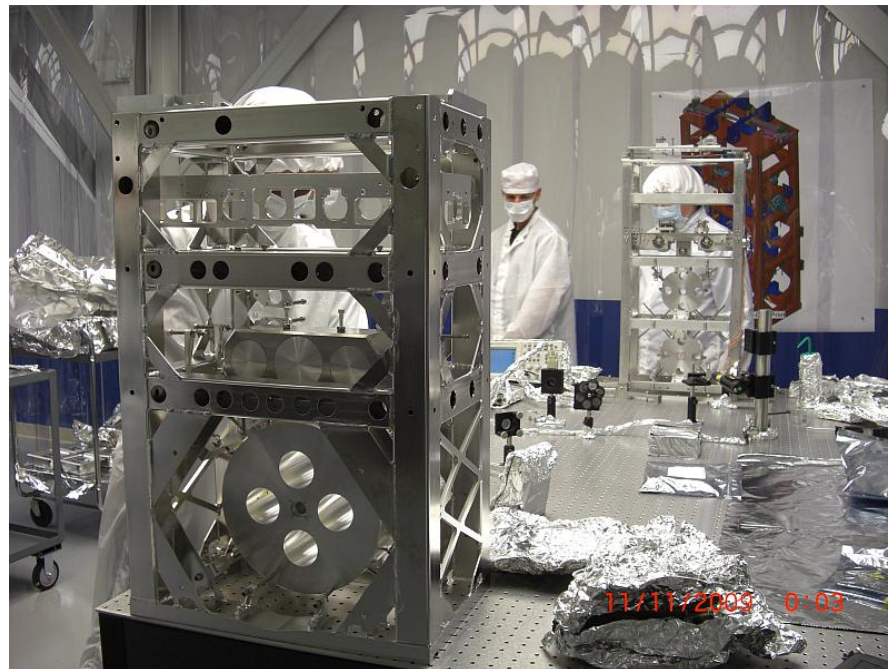


# Loss



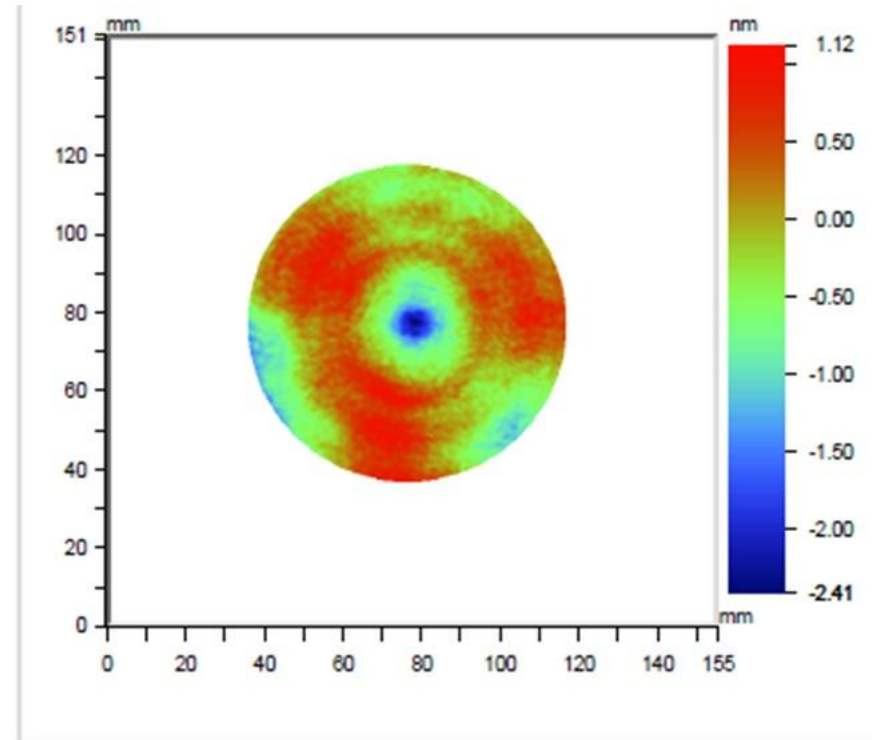
# Scatter

- Determined by dust particulates on surface, as well as by defects from polishing
- Scatter from 100 particles of  $10\ \mu$  diameter already dominates the loss budget
- Cleanliness!



# Microroughness

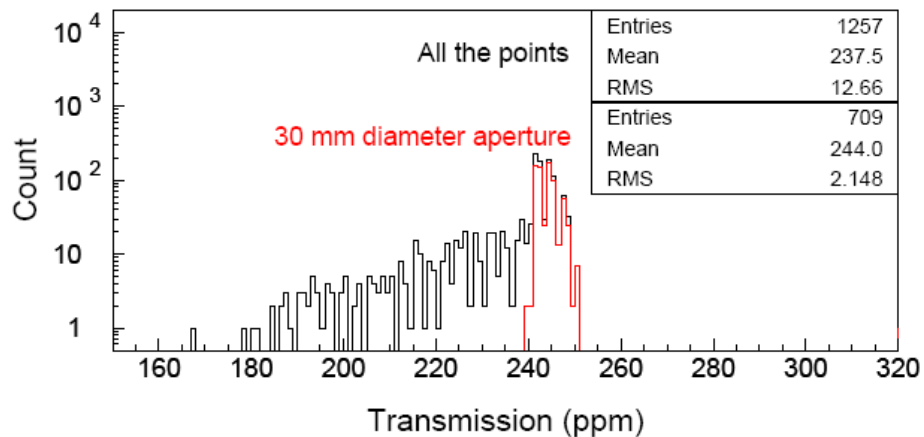
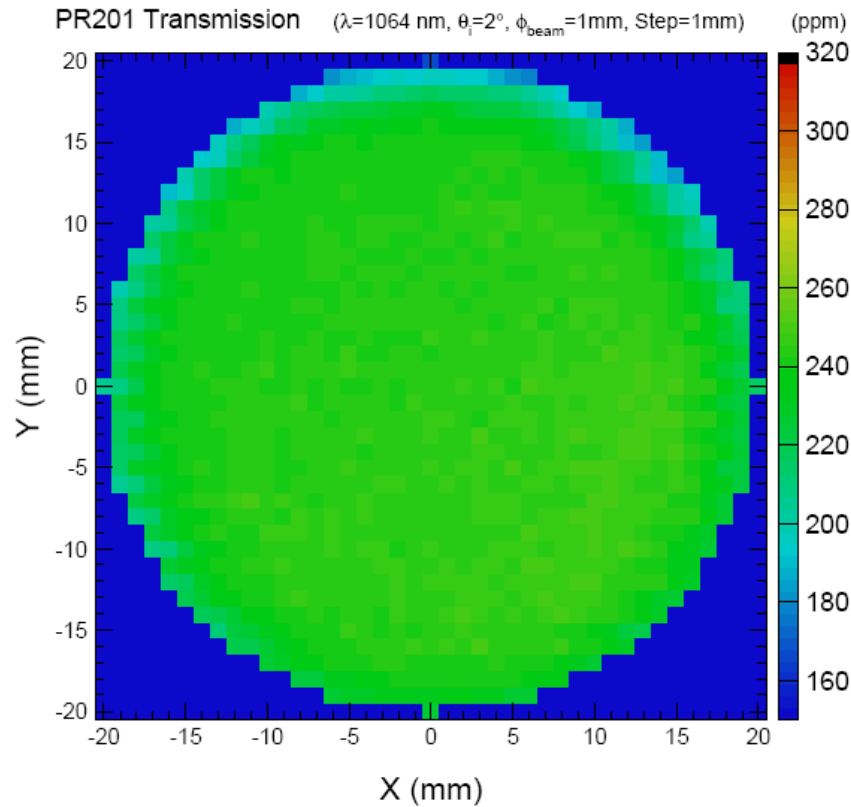
- Low-angle scatter
- Rms ~ 0.5 nm
- “superpolish”



Date: 03/22/2012	X Center: 283.00
Time: 14:48:48	Y Center: 244.00
Wavelength: 1 $\mu$ m	Radius: 150.0 pix
Pupil: 100.0 %	Terms: Tilt Power Astig
<b>PV: 3.5339 nm</b>	Filters: None
<b>RMS: 0.5417 nm</b>	Masks: Analysis Mask
Rad of curv: 84.882 km	Ref Sub:



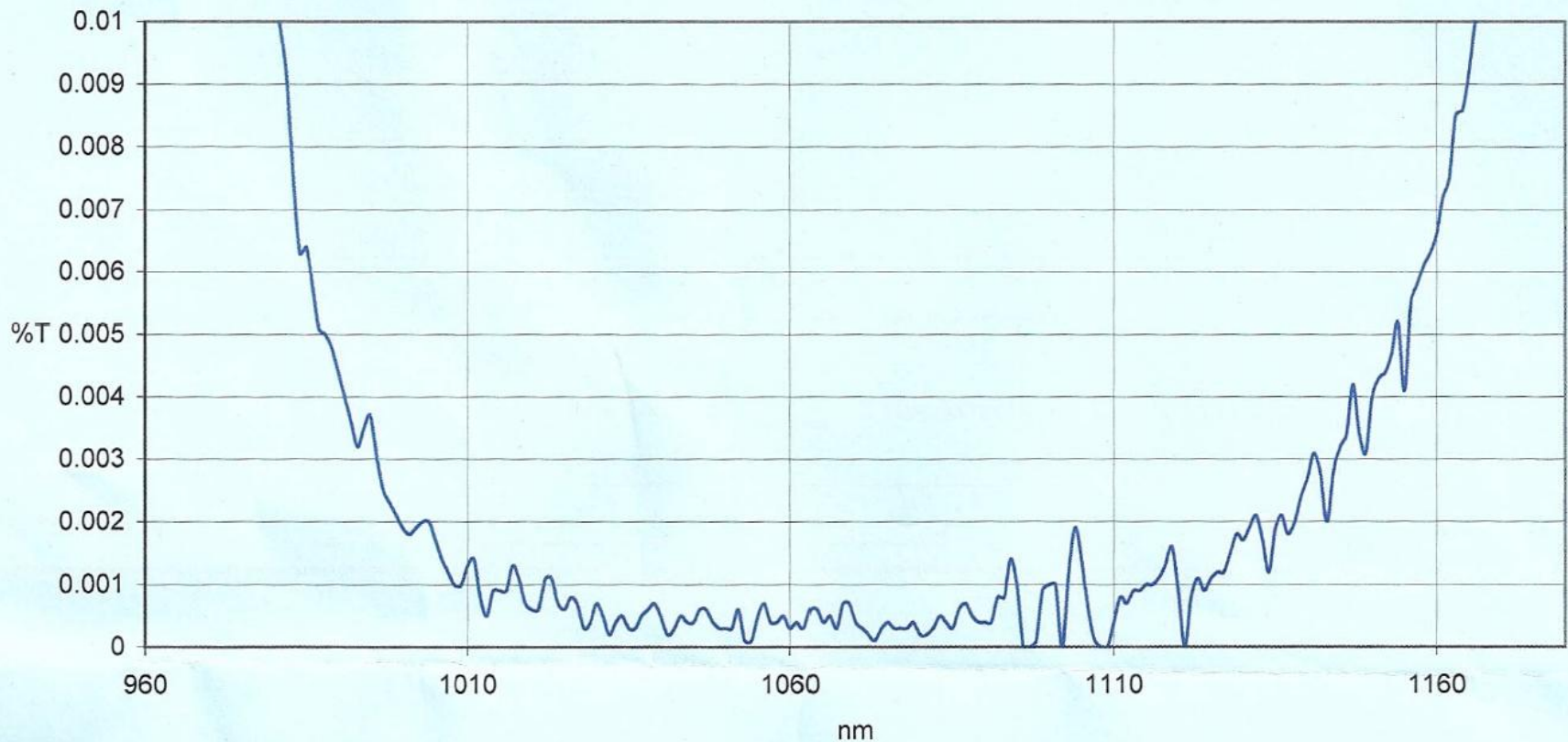
# Coating nonuniformity





# Transmission

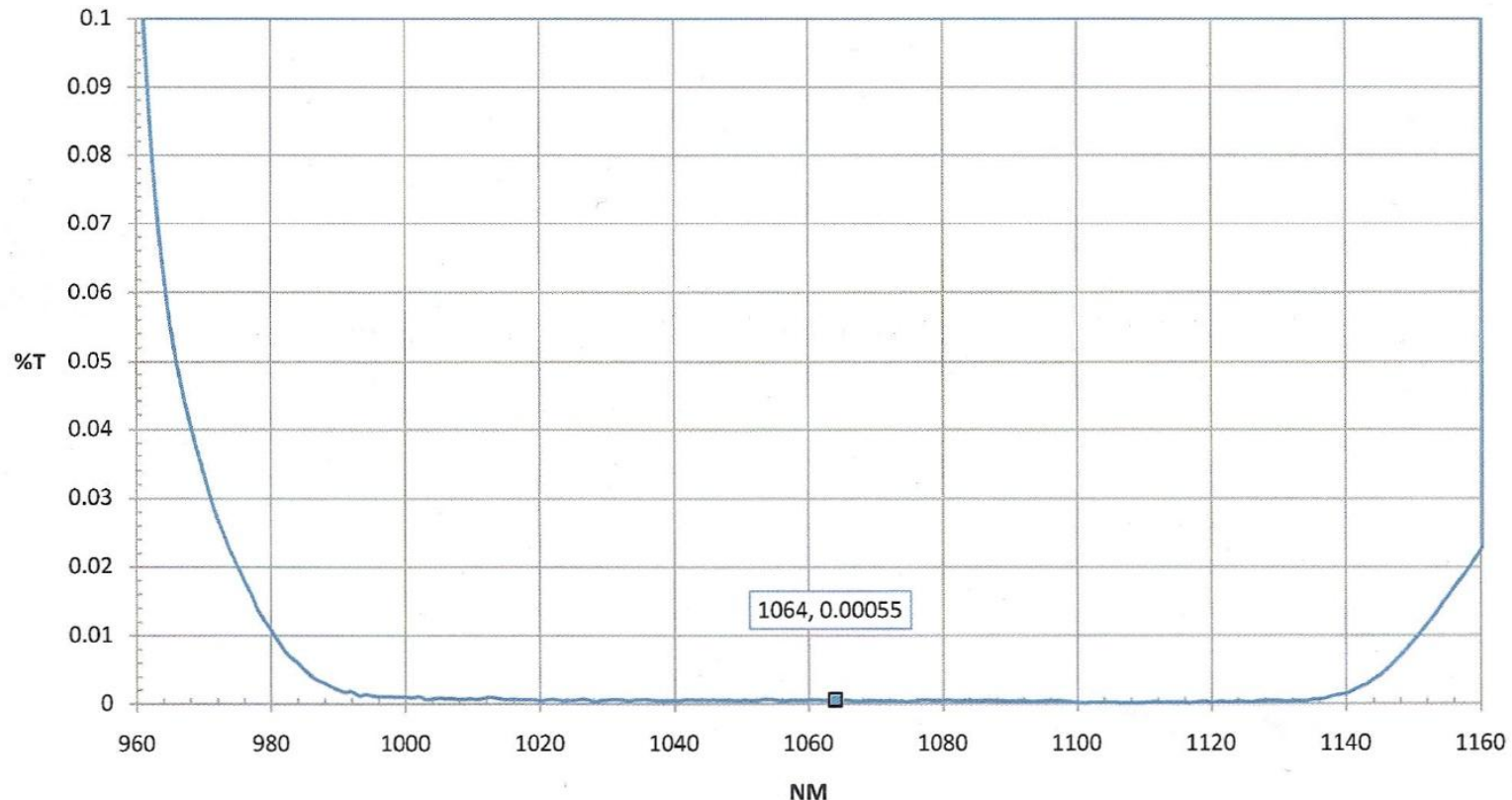
- At 1064 nm,  $T \sim 3$  ppm
- Finesse  $\sim 2$  million (ignoring scattering, which you cannot do)





# Transmission 2

- At 1064 nm,  $T \sim 5.5$  ppm
- Finesse  $\sim 1$  million (ignoring scattering, which you cannot do)



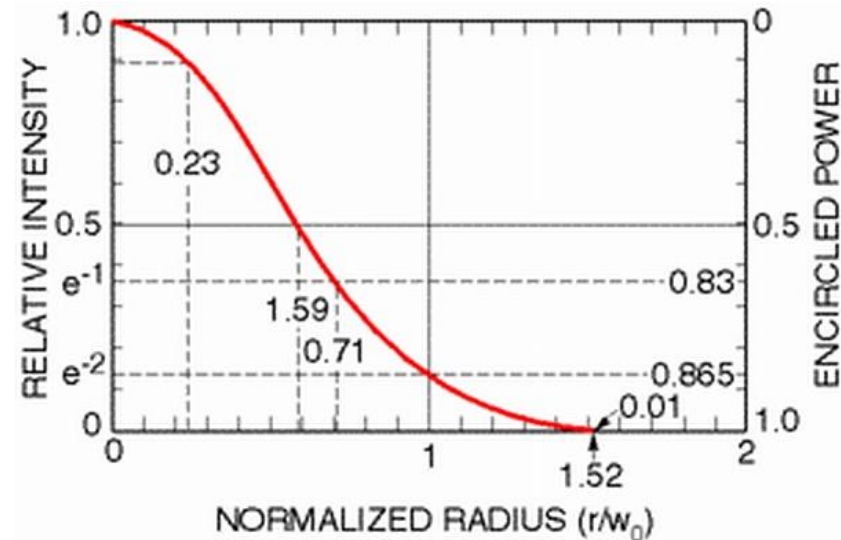
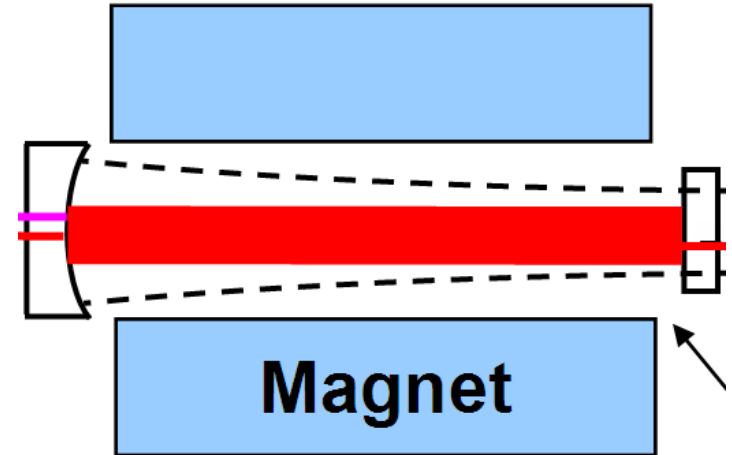
# Strawman cavity design

## Magnets: 6+6 Tevatron dipoles

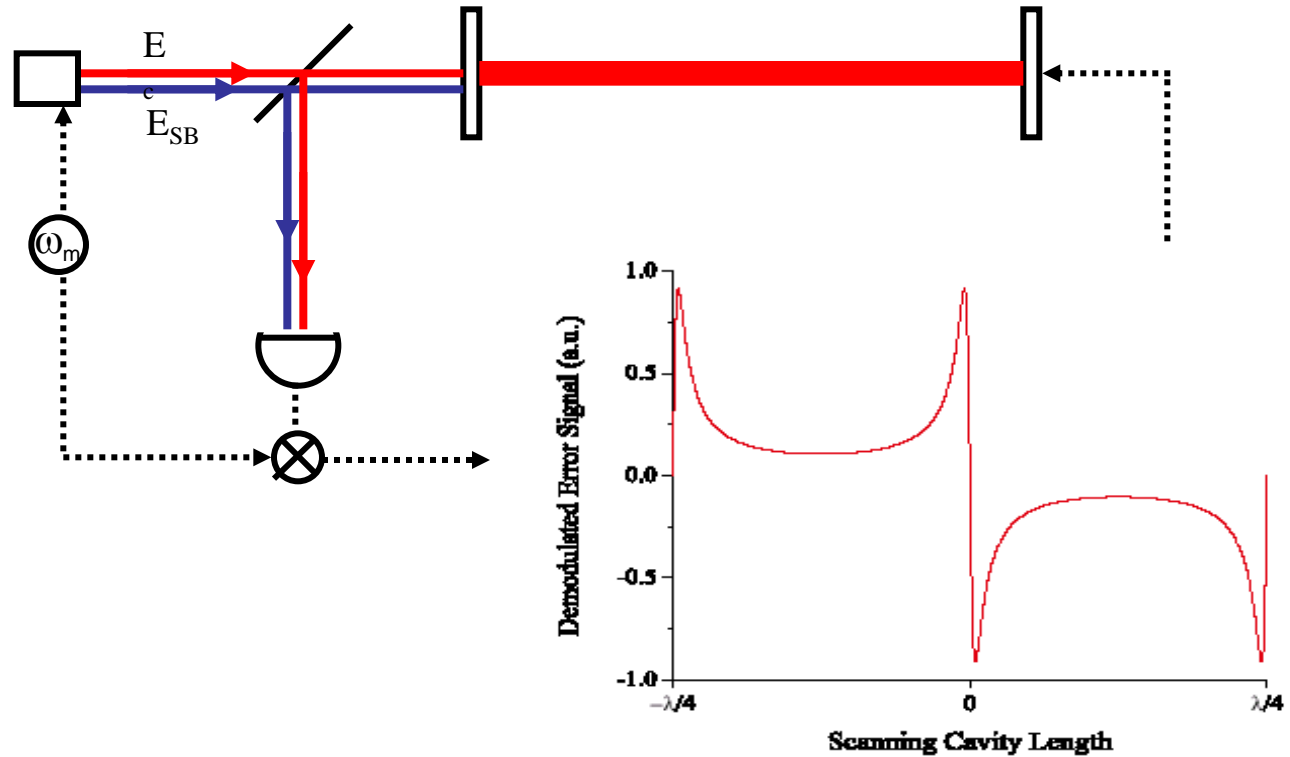
- 5 T field
- 6 m length each
- 48 mm diameter
- $B_0^* L_{mag} = 180 \text{ T}\cdot\text{m}$

## Cavity: curved-flat FP

- 45 m length;  $\text{FSR} = c/2L_{cav} \sim 3.3 \text{ MHz}$
- Mirror radii: 114 m (outer) and -4500 m (inner);  $g = 0.59$
- Gaussian beam radii (field): 5.5 mm (outer); 4.3 mm (inner)
- 1 ppm diameter = 30 mm
- Finesse =  $3.1 \times 10^5$ ;  $T = 10 \text{ ppm}$ ;  
     $A < 1 \text{ ppm/mirror}$
- Stored power  $\sim 1 \text{ MW}$
- Intensity  $2.2 \text{ MW/cm}^2$



# Locking the cavities

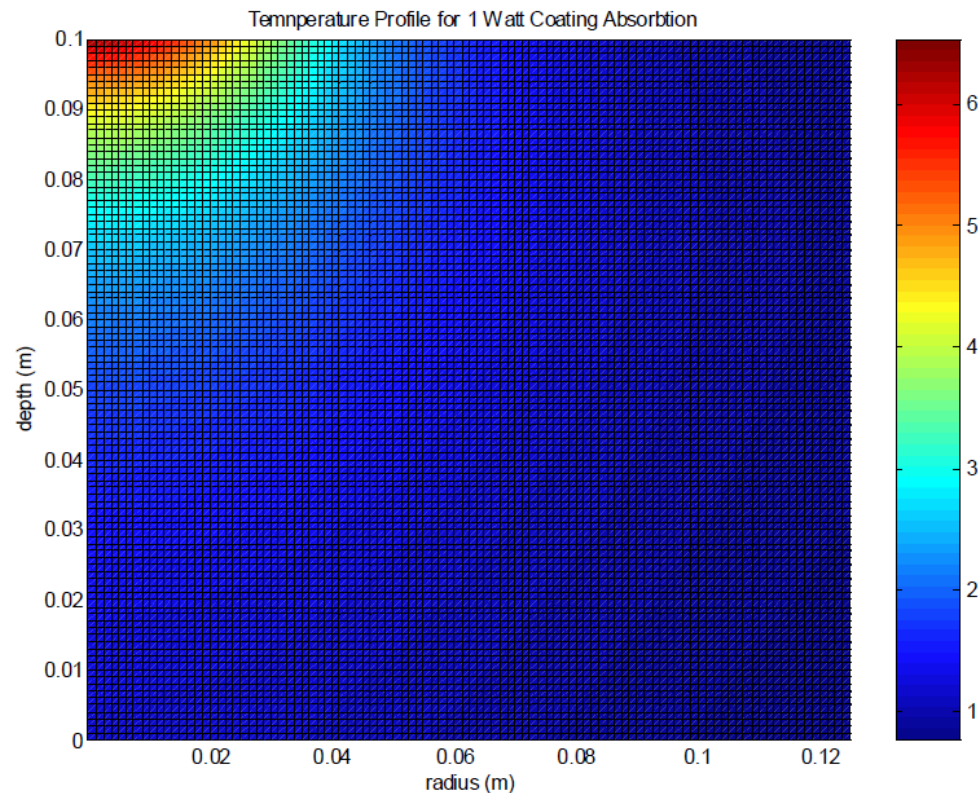


- Pound-Drever-Hall locking
- Resonant regeneration experiment is complex:
  - 2 length degrees of freedom + alignment
  - Absolute position must be held to  $\sim 10^{-13}$  m



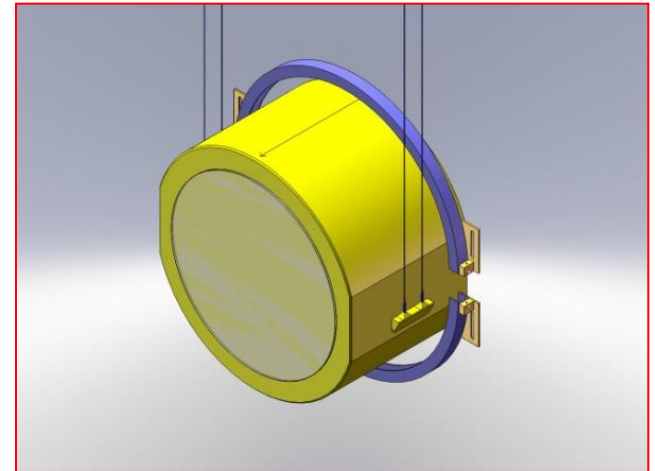
# Heating

- With 1 MW incident, 1 ppm loss, absorbed power would be 1 W
- Heating, deformation of mirror surface, loss of mode



# Thermal Compensation

- Add heaters to perimeter of mirror
- Heat reduces thermal gradients
- Used successfully in eLIGO



# DAMAGE

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- An unknown unknown
- Damage thresholds said to be  $1 \text{ GW/cm}^2$ 
  - vs  $0.002 \text{ GW/cm}^2$  in strawman
- Dust, sleeks, point defects seed damage



# Summary

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- With care, optical cavities with lengths of  $\sim 50$  m and finessees of  $\sim 100,000$  are well within the state of the art.
- Peter's 105-110 m is OK. (1 ppm spot on curved mirror is 46.5 mm diameter. [Spot on flat is 34 mm.],  $g \sim 0.64$ )



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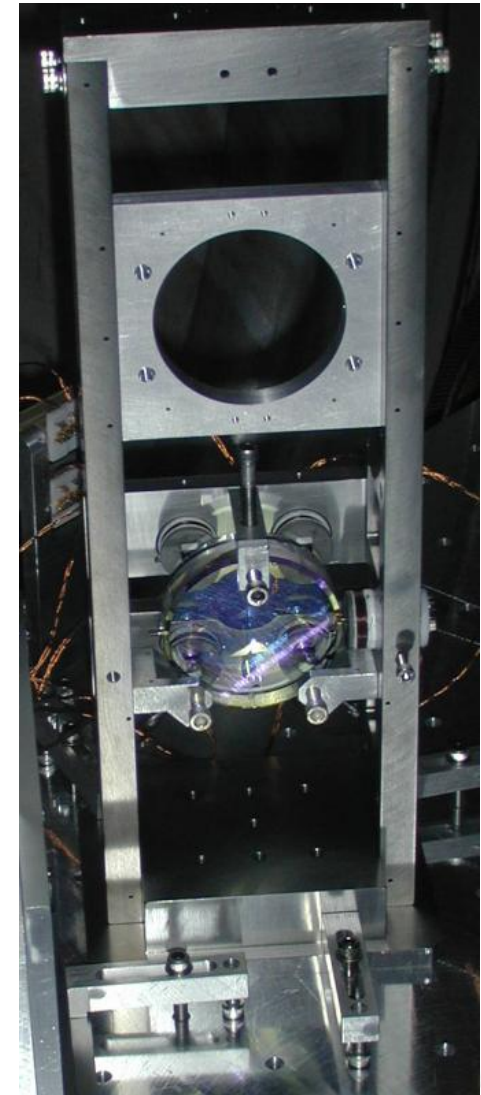
THE END





## Other issues

- Can avoid zeros of sinc function in conversion rate by alternating field directions.
- To go beyond  $L \sim 90$  m would require first removing sagitta and then using larger diameter magnets. Km scales  $\Rightarrow$  200 mm diameters.
- For high power in production cavity, thermal management/thermal lenses become important.
- Avoid stray light.
- Must run in UHV.
- Dust elimination is critical; scatter from 100 particles of  $10 \mu$  diameter already dominates the loss budget.
- Need vibration-free mirror suspensions. Possibly suspended.
- Include quantum efficiency, photodetector dark current.

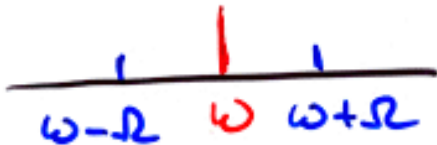


- Phase modulated light

$$E = e^{-i\omega t + i\Gamma \cos \Omega t}$$

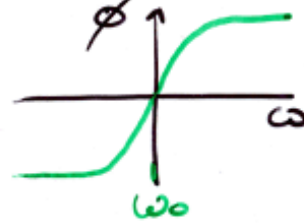
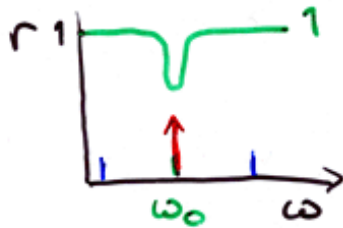
$$= e^{-i\omega t} [1 + i\Gamma \cos \Omega t + \dots]$$

$$= e^{-i\omega t} + \frac{i\Gamma}{2} e^{-i(\omega+\Omega)t} + \frac{i\Gamma}{2} e^{-i(\omega-\Omega)t}$$



# PDH 2

- Reflect from cavity



$$\phi = \begin{cases} \frac{\omega - \omega_0}{\Delta\omega} \equiv \epsilon & \omega \approx \omega_0 \\ \pm\pi & \omega \gg \omega_0 \\ & \omega \ll \omega_0 \end{cases}$$

$$E_r = (r e^{i\epsilon} - i\Gamma \cos \Omega t) e^{-i\omega t}$$

$$I_r = R + \Gamma^2 \cos^2 \Omega t - 2\Gamma r \sin \epsilon \cos \Omega t$$

$r^2$

- Demodulate at  $\cos \Omega t$

$$\langle \cos^2 \rangle = 1/2$$

$$\langle I_m \rangle = -\Gamma r \sin \epsilon$$

