Cosmological Parameters

Kris Sigurdson
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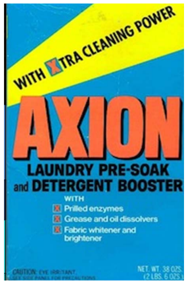
Vistas in Axion Physics
INT, Seattle
April 24, 2012

Cosmological Parameters

from, mostly, the CMB

with implications for Dark Matter and  Physics

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Outline

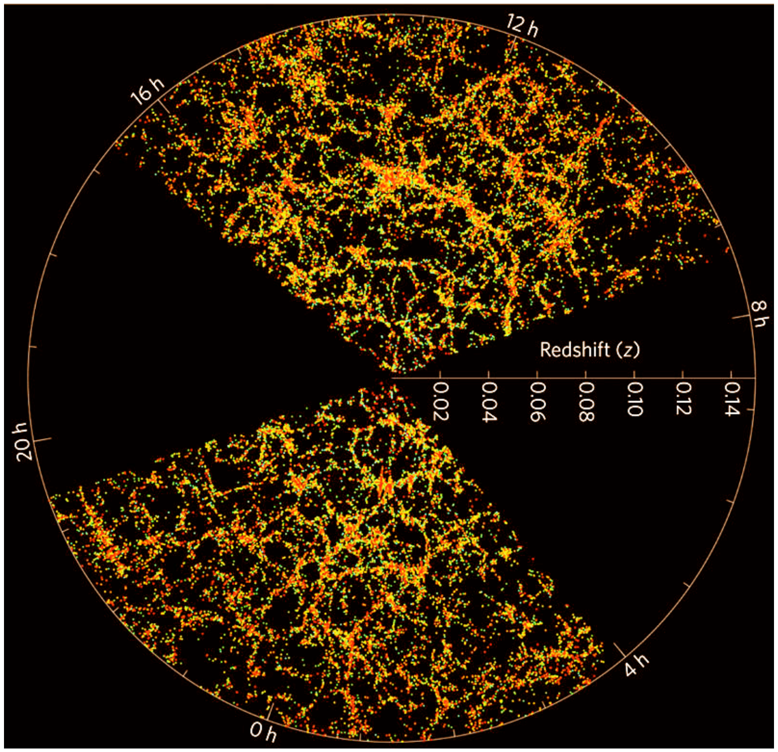
- Cosmological Parameters
- Cosmological Perturbations
- Some CMB Physics
- Dark Radiation?
- Isocurvature Constraints?

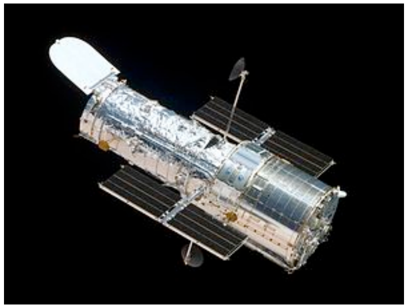
Cosmological Parameters

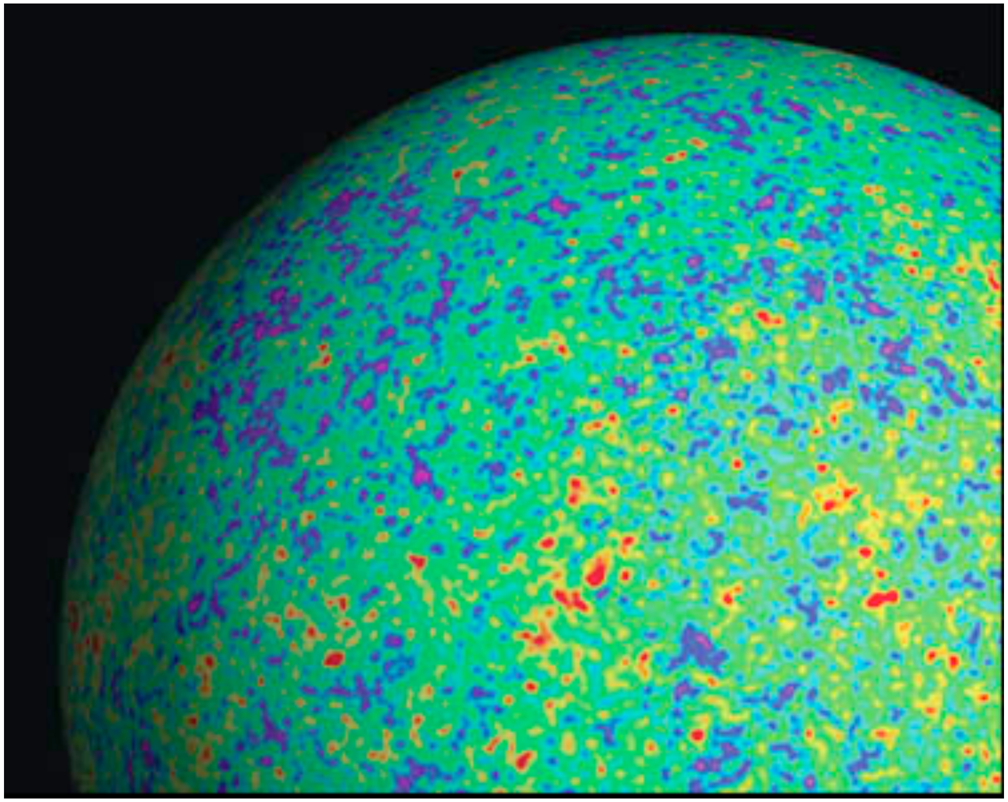
"In the past decade there has been considerable progress towards believable answers, but it may be well to bear in mind that people have been searching for ways to determine the cosmological parameters since the late 1920s."

Jim Peebles, 1993, Principles of Physical Cosmology

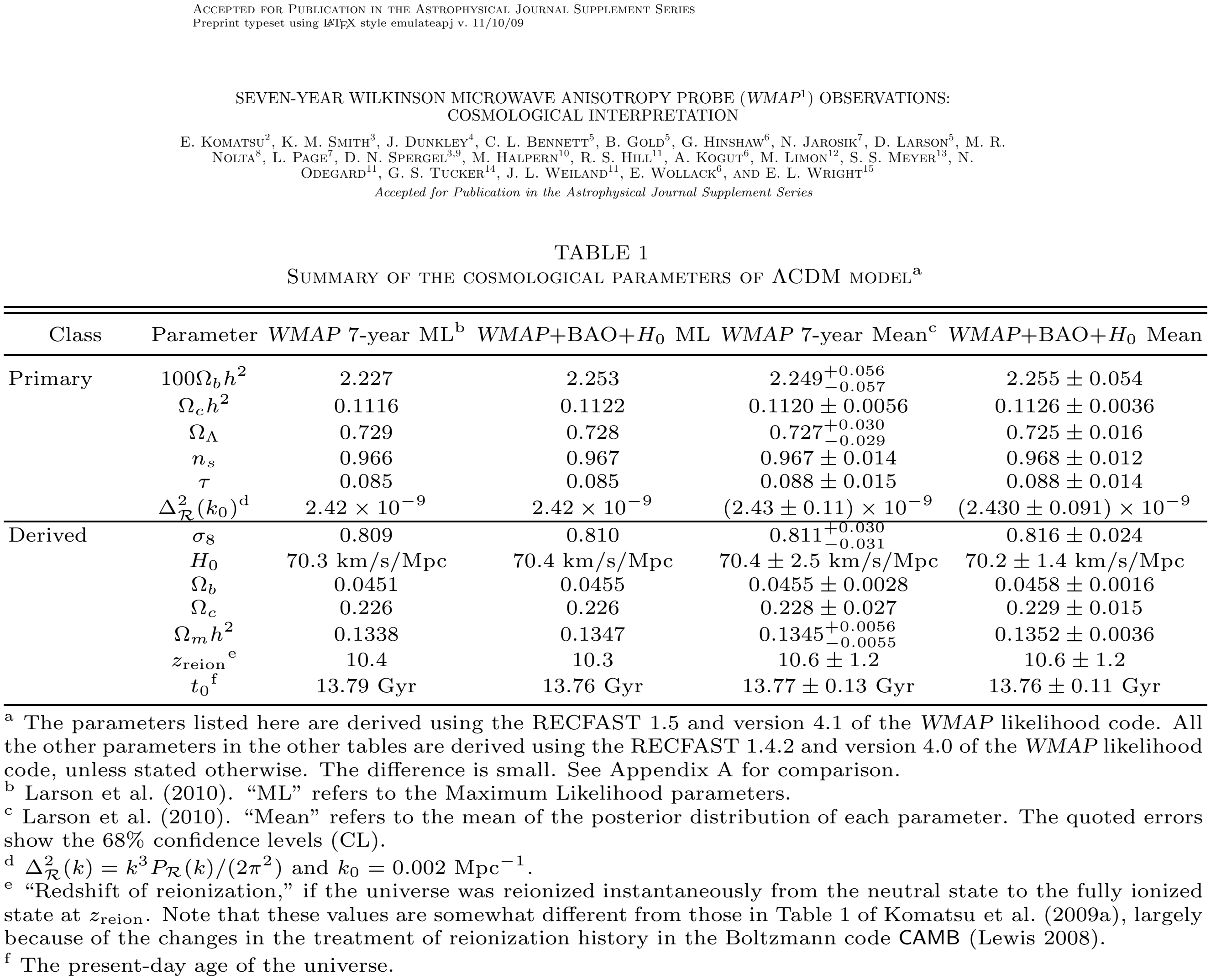
Cosmological Data







Cosmological Parameters



Accepted for Publication in the Astrophysical Journal Supplement Series
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SEVEN-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE (*WMAP*[1]) OBSERVATIONS: COSMOLOGICAL INTERPRETATION

E. Komatsu[2], K. M. Smith[3], J. Dunkley[4], C. L. Bennett[5], B. Gold[5], G. Hinshaw[6], N. Jarosik[7], D. Larson[5], M. R. Nolta[8], L. Page[7], D. N. Spergel[3,9], M. Halpern[10], R. S. Hill[11], A. Kogut[6], M. Limon[12], S. S. Meyer[13], N. Odegard[11], G. S. Tucker[14], J. L. Weiland[11], E. Wollack[6], and E. L. Wright[15]

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TABLE 1
Summary of the cosmological parameters of ΛCDM model[a]

Class	Parameter	*WMAP* 7-year ML[b]	*WMAP*+BAO+H_0 ML	*WMAP* 7-year Mean[c]	*WMAP*+BAO+H_0 Mean
Primary	$100\Omega_b h^2$	2.227	2.253	$2.249^{+0.056}_{-0.057}$	2.255 ± 0.054
	$\Omega_c h^2$	0.1116	0.1122	0.1120 ± 0.0056	0.1126 ± 0.0036
	Ω_Λ	0.729	0.728	$0.727^{+0.030}_{-0.029}$	0.725 ± 0.016
	n_s	0.966	0.967	0.967 ± 0.014	0.968 ± 0.012
	τ	0.085	0.085	0.088 ± 0.015	0.088 ± 0.014
	$\Delta^2_{\mathcal{R}}(k_0)$[d]	2.42×10^{-9}	2.42×10^{-9}	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.430 \pm 0.091) \times 10^{-9}$
Derived	σ_8	0.809	0.810	$0.811^{+0.030}_{-0.031}$	0.816 ± 0.024
	H_0	70.3 km/s/Mpc	70.4 km/s/Mpc	70.4 ± 2.5 km/s/Mpc	70.2 ± 1.4 km/s/Mpc
	Ω_b	0.0451	0.0455	0.0455 ± 0.0028	0.0458 ± 0.0016
	Ω_c	0.226	0.226	0.228 ± 0.027	0.229 ± 0.015
	$\Omega_m h^2$	0.1338	0.1347	$0.1345^{+0.0056}_{-0.0055}$	0.1352 ± 0.0036
	$z_{\rm reion}$[e]	10.4	10.3	10.6 ± 1.2	10.6 ± 1.2
	t_0[f]	13.79 Gyr	13.76 Gyr	13.77 ± 0.13 Gyr	13.76 ± 0.11 Gyr

[a] The parameters listed here are derived using the RECFAST 1.5 and version 4.1 of the *WMAP* likelihood code. All the other parameters in the other tables are derived using the RECFAST 1.4.2 and version 4.0 of the *WMAP* likelihood code, unless stated otherwise. The difference is small. See Appendix A for comparison.

[b] Larson et al. (2010). "ML" refers to the Maximum Likelihood parameters.

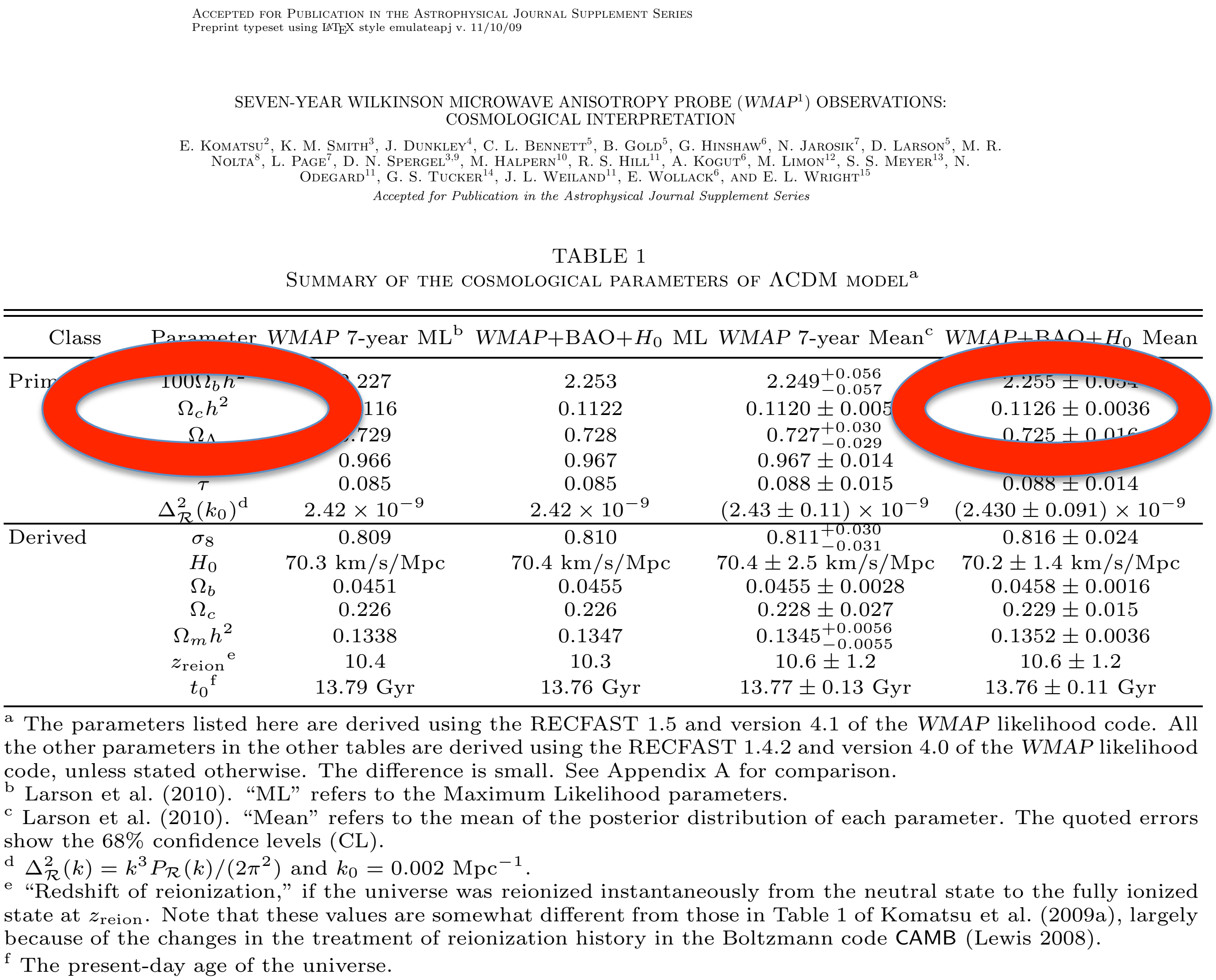
[c] Larson et al. (2010). "Mean" refers to the mean of the posterior distribution of each parameter. The quoted errors show the 68% confidence levels (CL).

[d] $\Delta^2_{\mathcal{R}}(k) = k^3 P_{\mathcal{R}}(k)/(2\pi^2)$ and $k_0 = 0.002$ Mpc^{-1}.

[e] "Redshift of reionization," if the universe was reionized instantaneously from the neutral state to the fully ionized state at $z_{\rm reion}$. Note that these values are somewhat different from those in Table 1 of Komatsu et al. (2009a), largely because of the changes in the treatment of reionization history in the Boltzmann code `CAMB` (Lewis 2008).

[f] The present-day age of the universe.

Cosmological Parameters



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[f] The present-day age of the universe.

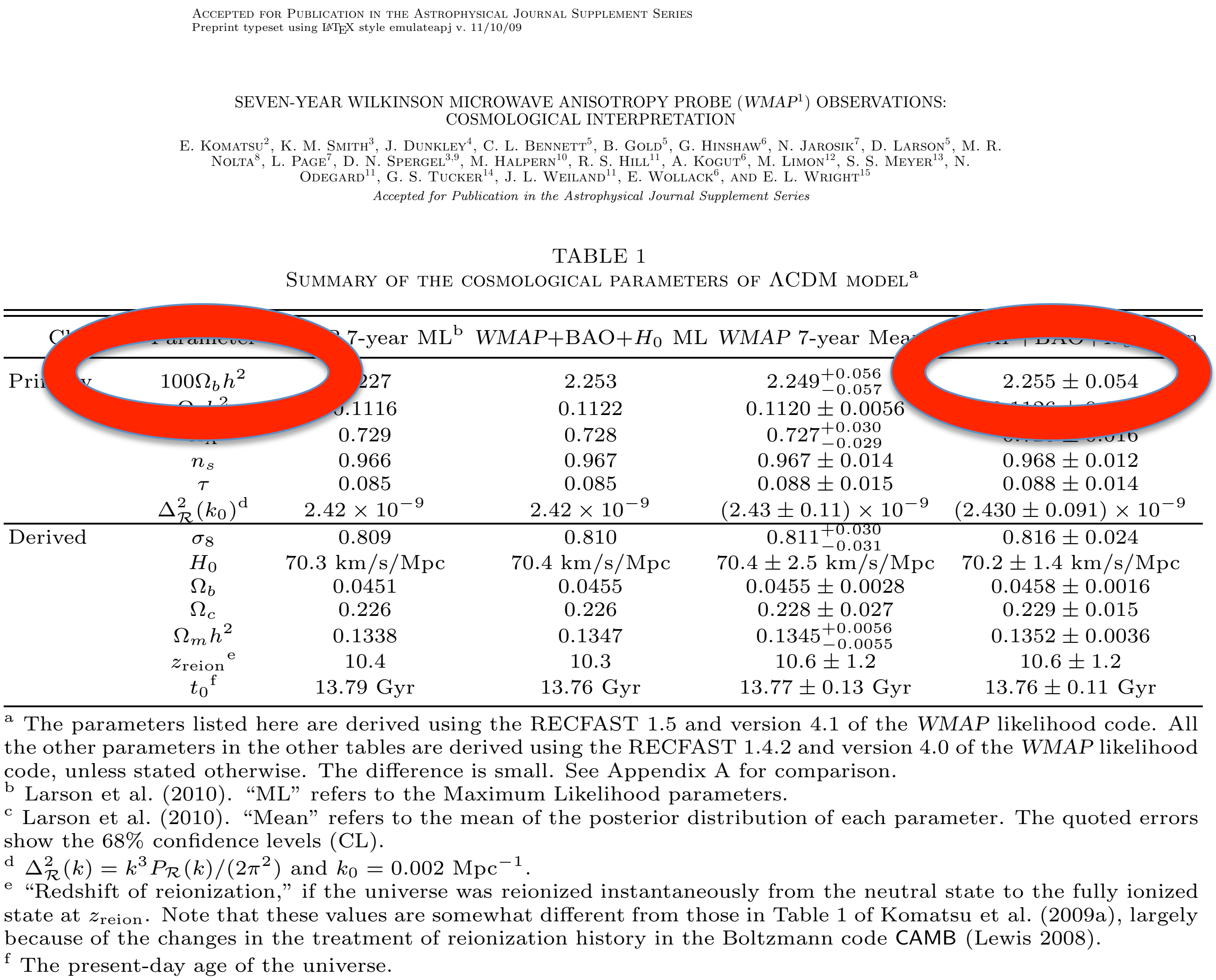
Cosmological Parameters

Something to Remember:

We know the cosmological abundance of dark matter to 3.2% (assuming standard 6-parameter ΛCDM cosmology)

$$\Omega_c h^2 = 0.1126 \pm 0.0036$$

Cosmological Parameters



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Pri[illegible]	$100\Omega_b h^2$	[illegible]227	2.253	$2.249^{+0.056}_{-0.057}$	2.255 ± 0.054
	[illegible]	[illegible].1116	0.1122	0.1120 ± 0.0056	[illegible]
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[f] The present-day age of the universe.

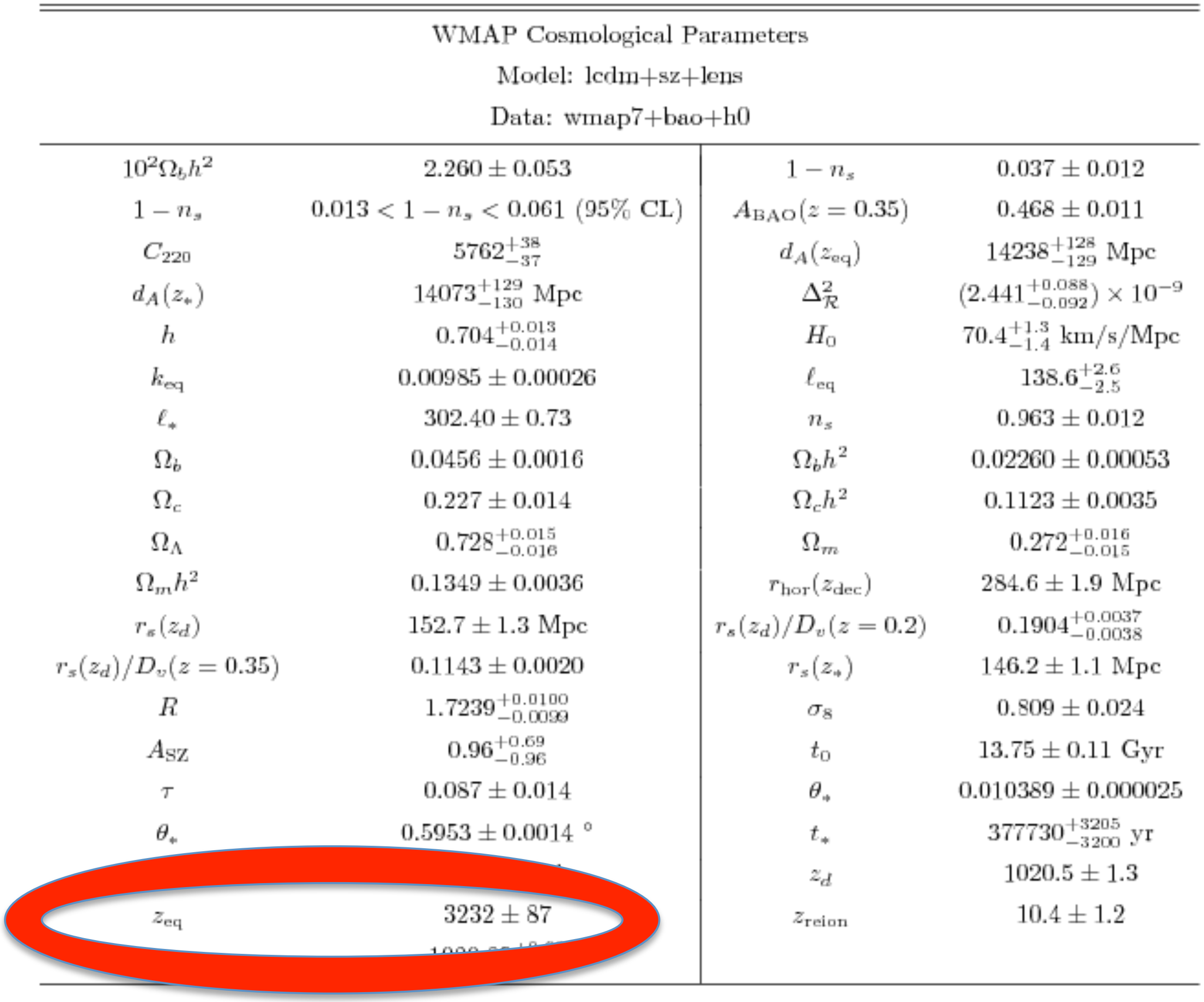
Cosmological Parameters

We know the cosmological abundance of baryons to 2.4% (assuming standard 6-parameter ΛCDM cosmology)

$$100\Omega_b h^2 = 2.255 \pm 0.054$$

Note: CMB is sensitive to photon-to-baryon ratio, but we "know" $\Omega_\gamma h^2$ from first principles

Cosmological Parameters



WMAP Cosmological Parameters

Model: lcdm+sz+lens

Data: wmap7+bao+h0

$10^2\Omega_b h^2$	2.260 ± 0.053	$1 - n_s$	0.037 ± 0.012
$1 - n_s$	$0.013 < 1 - n_s < 0.061$ (95% CL)	$A_{\rm BAO}(z = 0.35)$	0.468 ± 0.011
C_{220}	5762^{+38}_{-37}	$d_A(z_{\rm eq})$	14238^{+128}_{-129} Mpc
$d_A(z_*)$	14073^{+129}_{-130} Mpc	$\Delta^2_{\mathcal{R}}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
h	$0.704^{+0.013}_{-0.014}$	H_0	$70.4^{+1.3}_{-1.4}$ km/s/Mpc
$k_{\rm eq}$	0.00985 ± 0.00026	$\ell_{\rm eq}$	$138.6^{+2.6}_{-2.5}$
ℓ_*	302.40 ± 0.73	n_s	0.963 ± 0.012
Ω_b	0.0456 ± 0.0016	$\Omega_b h^2$	0.02260 ± 0.00053
Ω_c	0.227 ± 0.014	$\Omega_c h^2$	0.1123 ± 0.0035
Ω_Λ	$0.728^{+0.015}_{-0.016}$	Ω_m	$0.272^{+0.016}_{-0.015}$
$\Omega_m h^2$	0.1349 ± 0.0036	$r_{\rm hor}(z_{\rm dec})$	284.6 ± 1.9 Mpc
$r_s(z_d)$	152.7 ± 1.3 Mpc	$r_s(z_d)/D_v(z = 0.2)$	$0.1904^{+0.0037}_{-0.0038}$
$r_s(z_d)/D_v(z = 0.35)$	0.1143 ± 0.0020	$r_s(z_*)$	146.2 ± 1.1 Mpc
R	$1.7239^{+0.0100}_{-0.0099}$	σ_8	0.809 ± 0.024
$A_{\rm SZ}$	$0.96^{+0.69}_{-0.96}$	t_0	13.75 ± 0.11 Gyr
τ	0.087 ± 0.014	θ_*	0.010389 ± 0.000025
θ_*	0.5953 ± 0.0014 °	t_*	377730^{+3205}_{-3200} yr
[illegible]	[illegible]	z_d	1020.5 ± 1.3
$z_{\rm eq}$	3232 ± 87	$z_{\rm reion}$	10.4 ± 1.2
[illegible]	[illegible]		

Courtesy of LAMBDA

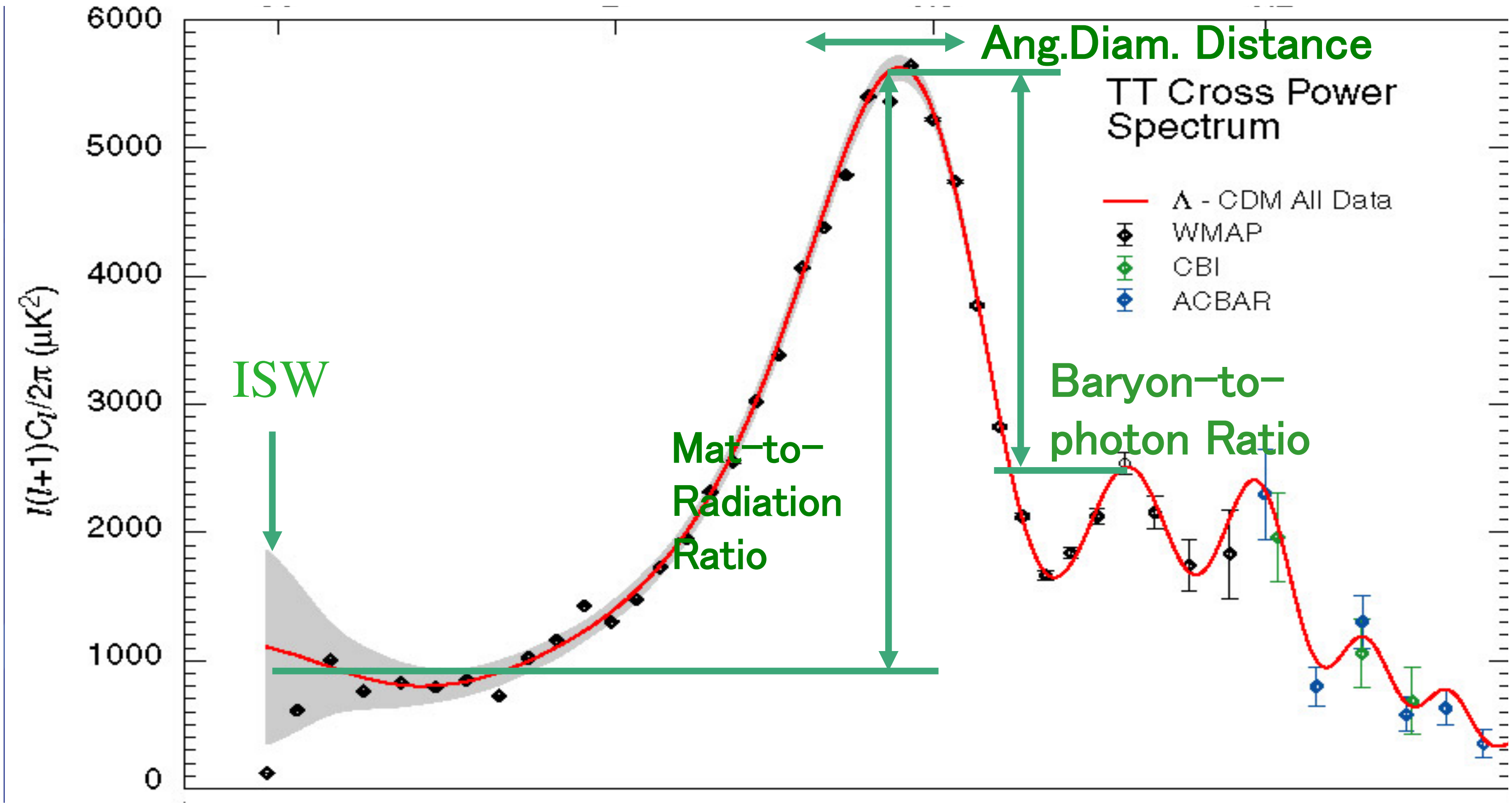
Cosmological Parameters

We know the redshift of matter-radiation equality to 2.7% (assuming standard 6-parameter ΛCDM cosmology)

$$z_{eq} = 3232 \pm 87$$

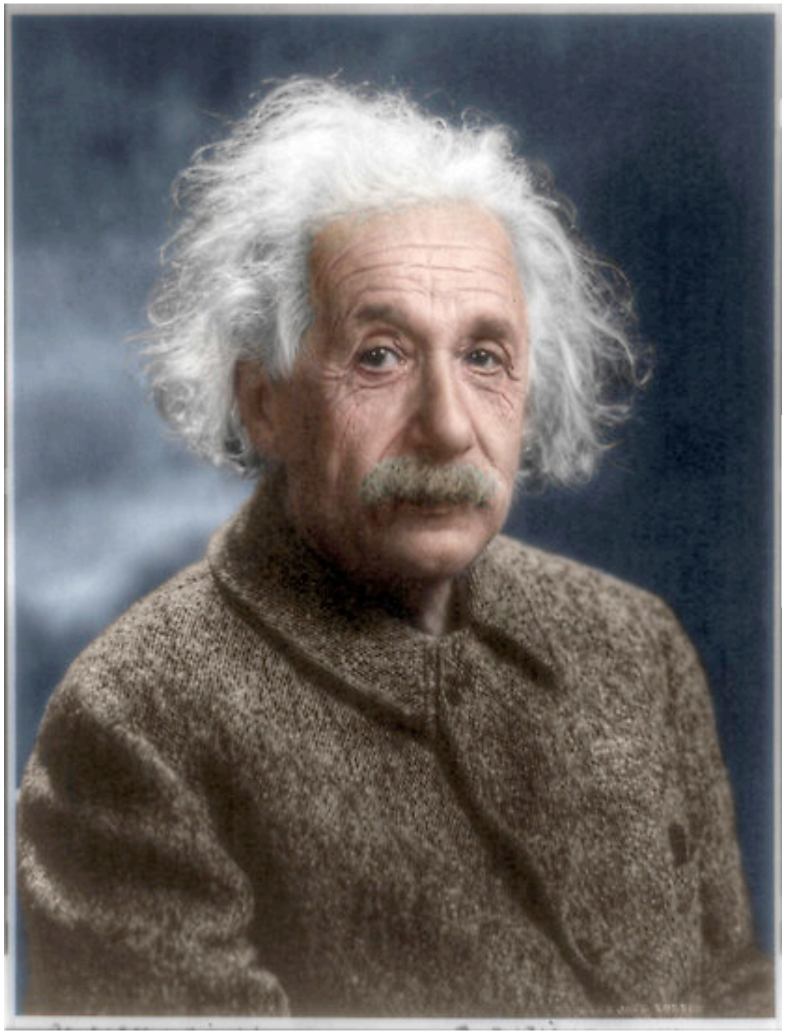
$$1 + z_{eq} = \frac{\Omega_m}{\Omega_r} = \frac{\Omega_m h^2}{4.16 \times 10^{-5} (N_{eff}/3.02)}$$

CMB Physics



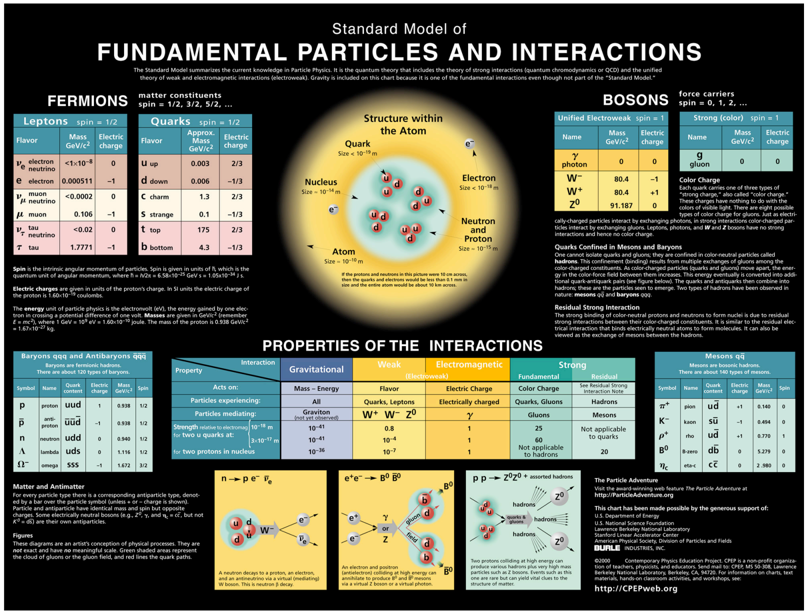
Credit: E. Komatsu

Standard Cosmology



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Standard Cosmology



Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Spin is the intrinsic angular momentum of particles. Spin is given in units of ħ, which is the quantum unit of angular momentum, where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s = 1.05×10^{-34} J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$), where 1 GeV = 10^9 eV = 1.60×10^{-10} joule. The mass of the proton is 0.938 GeV/c^2 = 1.67×10^{-27} kg.

Structure within the Atom

If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	–1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Color Charge
Each quark carries one of three types of "strong charge," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically-charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and ***W*** and ***Z*** bosons have no strong interactions and hence no color charge.

Quarks Confined in Mesons and Baryons
One cannot isolate quarks and gluons; they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: **mesons** $q\bar{q}$ and **baryons** qqq.

Residual Strong Interaction
The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual electrical interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

PROPERTIES OF THE INTERACTIONS

Interaction / Property	Gravitational	Weak (Electroweak)	Electromagnetic (Electroweak)	Strong: Fundamental	Strong: Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at: 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength relative to electromag for two u quarks at: 3×10^{-17} m	10^{-41}	10^{-4}	1	60	
for two protons in nucleus	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$
Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

Mesons $q\bar{q}$
Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

Matter and Antimatter
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$, but not $K^0 = d\bar{s}$) are their own antiparticles.

Figures
These diagrams are an artist's conception of physical processes. They are ***not*** exact and have ***no*** meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

$n \rightarrow p\, e^-\, \bar{\nu}_e$

A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

$e^+e^- \rightarrow B^0\, \bar{B}^0$

An electron and positron (antielectron) colliding at high energy can annihilate to produce B^0 and $\bar{B}^0$ mesons via a virtual Z boson or a virtual photon.

$p\, p \rightarrow Z^0 Z^0$ + assorted hadrons

Two protons colliding at high energy can produce various hadrons plus very high mass particles such as Z bosons. Events such as this one are rare but can yield vital clues to the structure of matter.

The Particle Adventure
Visit the award-winning web feature *The Particle Adventure* at
http://ParticleAdventure.org

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Photons

Neutrinos

Baryons/Atoms

Dark Matter

Dark Energy

Cosmological Perturbations

Metric Perturbations about Friedmann-Robertson-Walker

$$\mathrm{d}s^2 = a^2(\tau)[-(1+2\Psi)\mathrm{d}\tau^2 + \delta_{ij}(1+2\Phi)\mathrm{d}x^i\mathrm{d}x^j]$$

Generally: Euler/Fluid Equations for Photons, Neutrinos, Baryons, and Dark Matter

For a Radiation Fluid:

$$\dot{\delta}_r + \frac{4}{3}\theta_r + 4\dot{\Phi} = 0$$

$$\dot{\theta}_r + \nabla^2\left(\frac{\delta_r}{4} + \Psi\right) = 0$$

$$(\theta_r \equiv \vec{\nabla}\cdot\vec{v}_r)$$

Plus Einstein Equations for Gravitational Potentials

For a Radiation Fluid: $\Phi = -\Psi$

$$\nabla^2\Phi + 3\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}\Psi - \dot{\Phi}\right) = -4\pi G a^2\rho_r\delta_r$$

Cosmological Perturbations: Fourier Space

$$\vec{\nabla} \rightarrow -i\vec{k}$$

Generally: Euler/Fluid Equations for Photons, Neutrinos, Baryons, and Dark Matter

For a Radiation Fluid:

$$\dot{\delta}_r + \frac{4}{3}\theta_r + 4\dot{\Phi} = 0$$

$$\dot{\theta}_r - k^2\left(\frac{\delta_r}{4} + \Psi\right) = 0$$

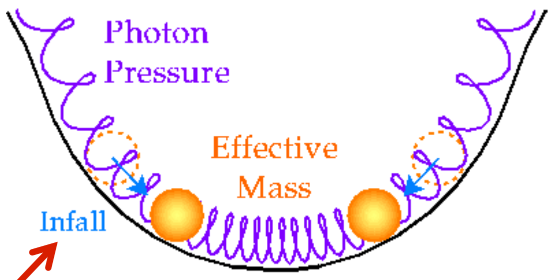
Plus Einstein Equations for Gravitational Potentials

For a Radiation Fluid: $\Phi = -\Psi$

$$-k^2\Phi + 3\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}\Psi - \dot{\Phi}\right) = -4\pi G a^2 \rho_r \delta_r$$

CMB Physics

Maximum fluid
rarification



Credit: Hu et al.

Maximum velocity –
maximum contribution
to the Doppler term

Maximum fluid
compression

$$\frac{d}{d\tau}\left[m_{\rm eff}\frac{d\delta_b}{d\tau}\right]+\frac{k^2}{3}\delta_b = F[\Psi] \qquad m_{\rm eff}=1+3\rho_b/4\rho_\gamma$$

Forced Harmonic Oscillator with a time-dependent “effective-mass”

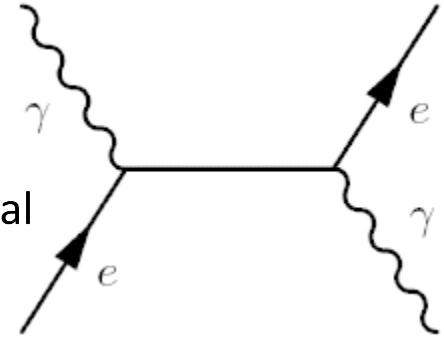
Bottom Line is that Baryon-Photon Fluid Supports Acoustic Waves!

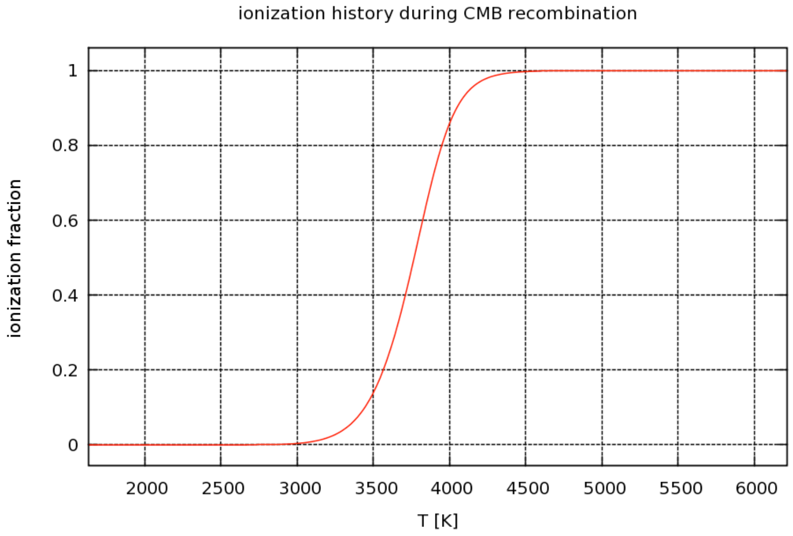
At least until it ceases to exist as a “single” fluid!

CMB Physics

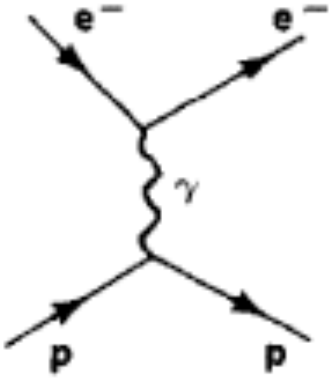
Photons and “Baryons” are Separate But Interact

Compton/Thomson:
Efficient When Ionized
Inefficient When Neutral

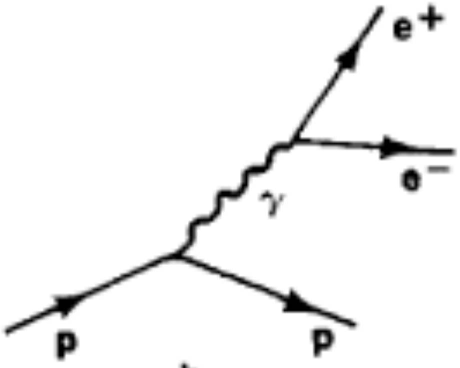




Colulomb:
Always Very Very Efficient!



a



b

CMB Physics: Compton Coupling

$$\begin{aligned}\dot{\delta}_\gamma &= -\frac{4}{3}\theta_\gamma + 4\dot{\phi}\,,\\ \dot{\theta}_\gamma &= k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + k^2\psi + a n_e \sigma_T(\theta_b - \theta_\gamma)\end{aligned}$$

Compton Collision Terms:

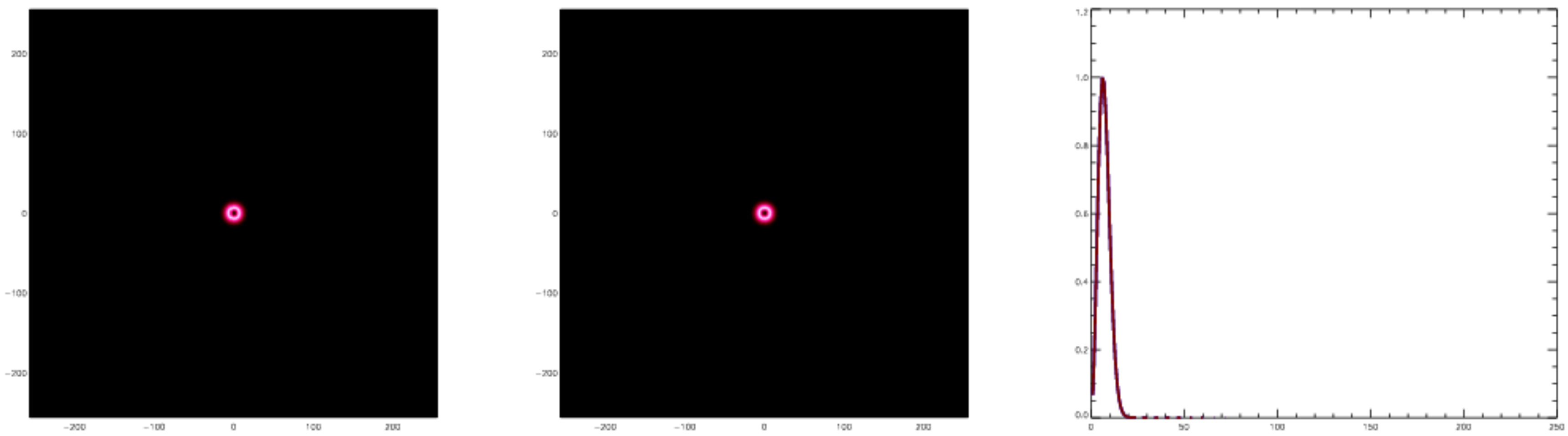
Momentum Transfer Tends to Equalize Bulk Velocities of Photon and Baryon Fluids

$$\begin{aligned}\dot{\delta}_b &= -\theta_b + 3\dot{\phi}\,,\\ \dot{\theta}_b &= -\frac{\dot{a}}{a}\theta_b + c_s^2 k^2 \delta_b + \frac{4\bar{\rho}_\gamma}{3\bar{\rho}_b} a n_e \sigma_T(\theta_\gamma - \theta_b) + k^2\psi\end{aligned}$$

Tight-Coupling into a Single Photon-Baryon Fluid When:

$$a n_e \sigma_T \equiv \tau_c^{-1} \gg \dot{a}/a \sim \tau^{-1}$$

CMB Physics

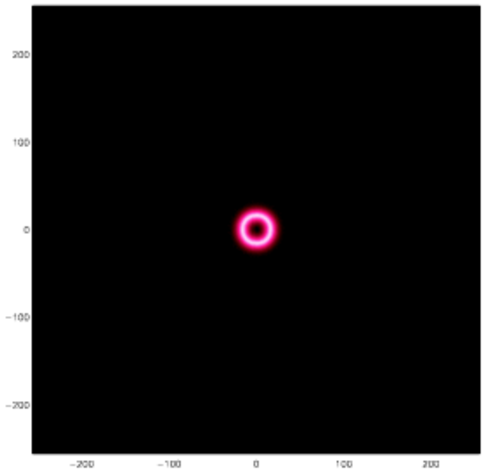

Baryons

Photons

Image Courtesy Martin White

CMB Physics

Baryons



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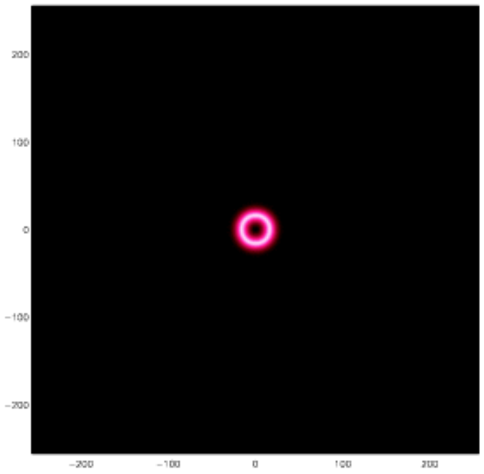


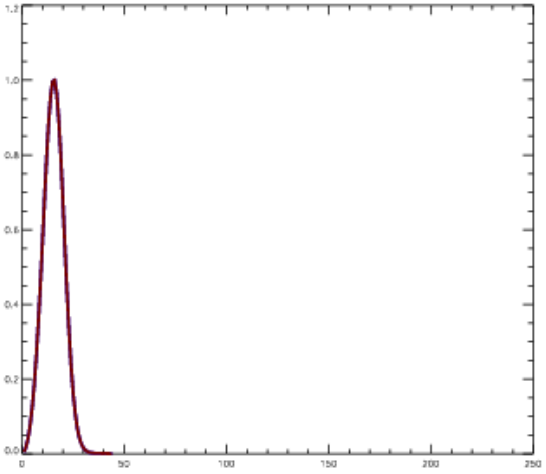
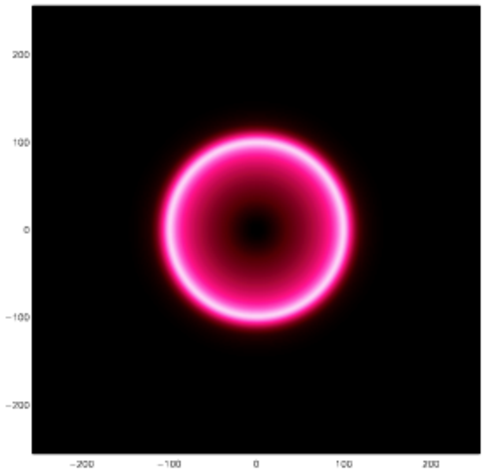


Image Courtesy Martin White

CMB Physics

Baryons



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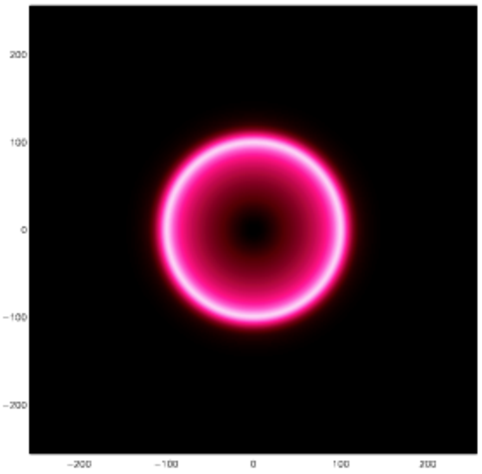


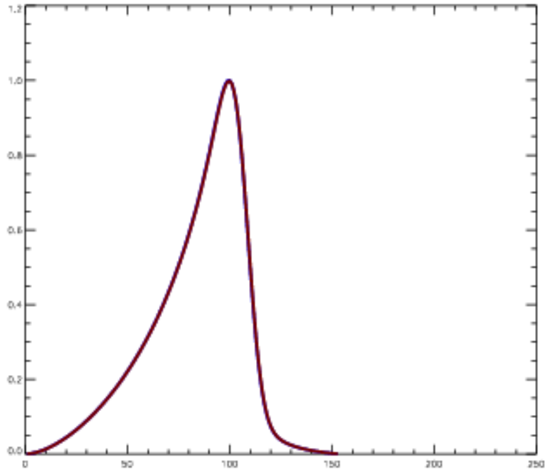
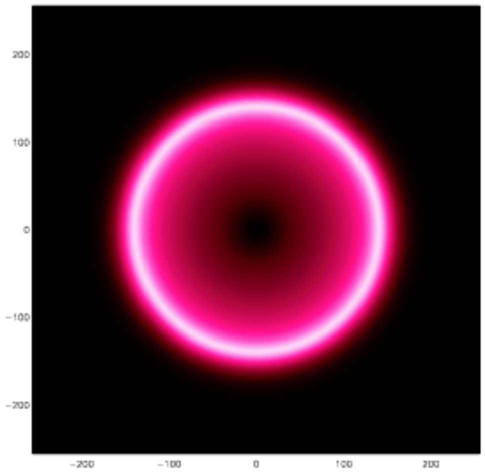


Image Courtesy Martin White

CMB Physics

Baryons



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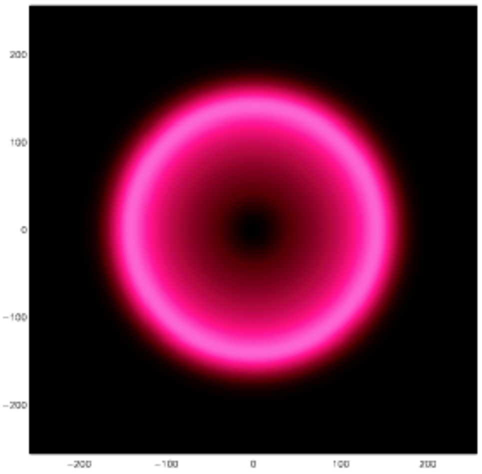


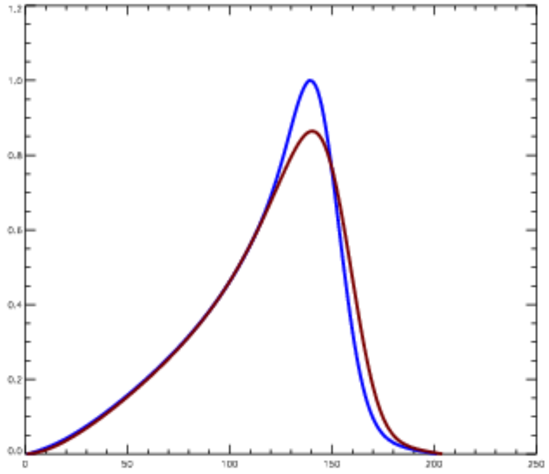
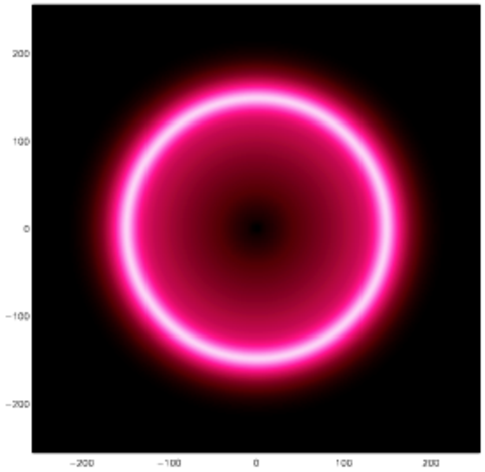


Image Courtesy Martin White

CMB Physics

Baryons



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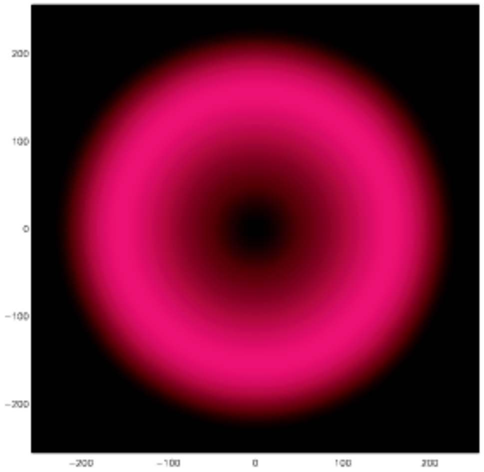


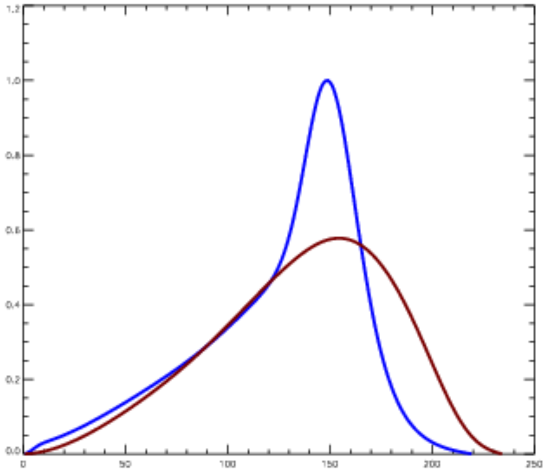
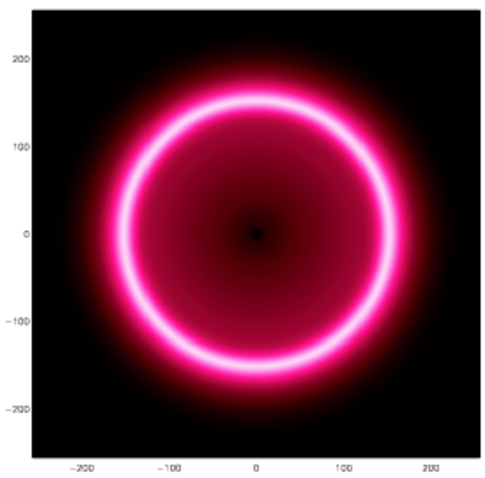


Image Courtesy Martin White

CMB Physics

Baryons



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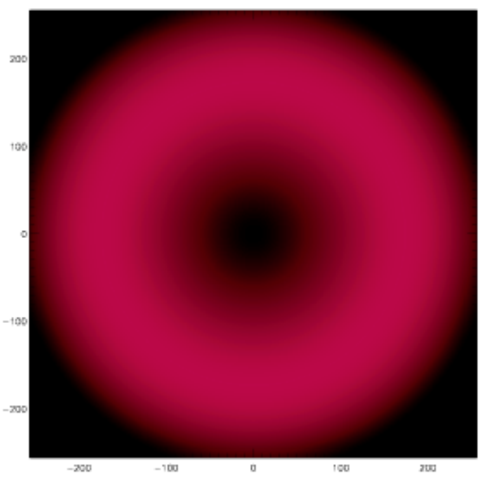


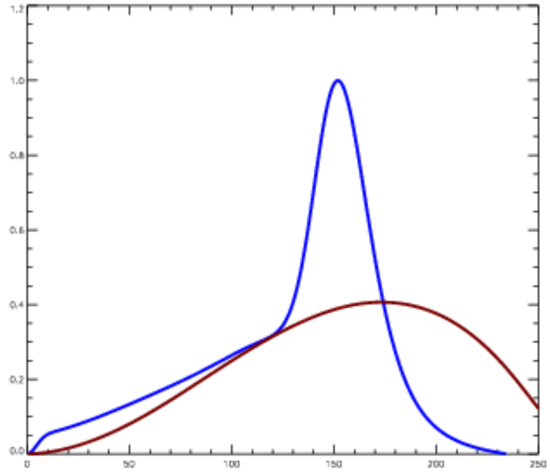
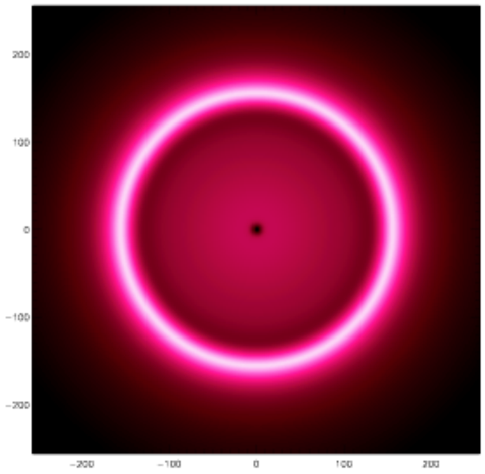


Image Courtesy Martin White

CMB Physics

Baryons



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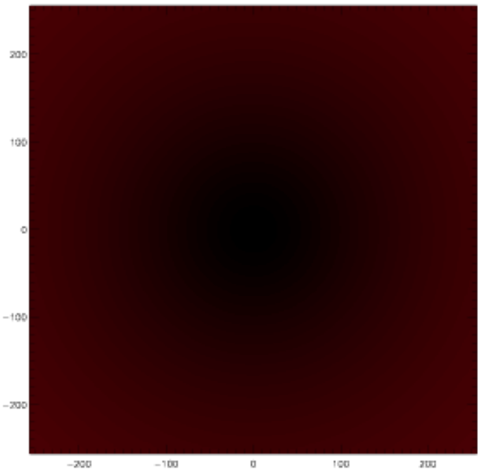


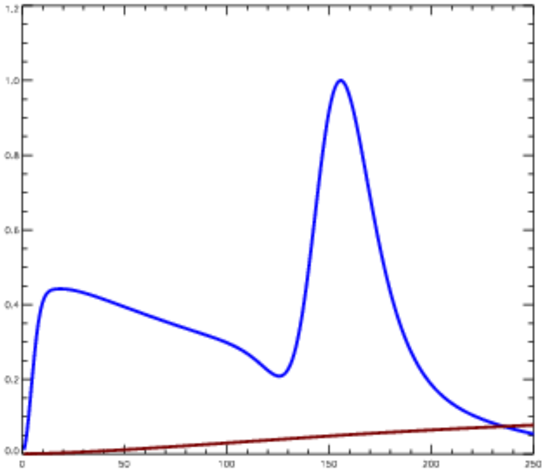
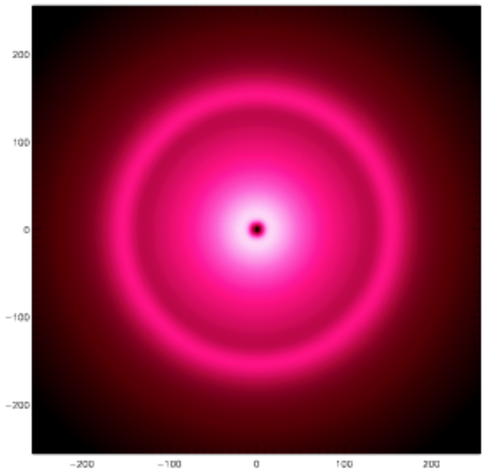


Image Courtesy Martin White

CMB Physics

Baryons



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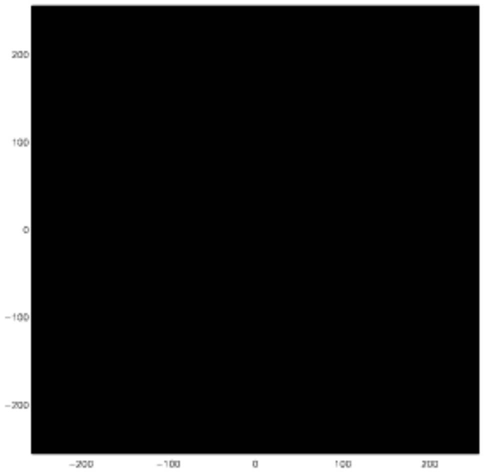


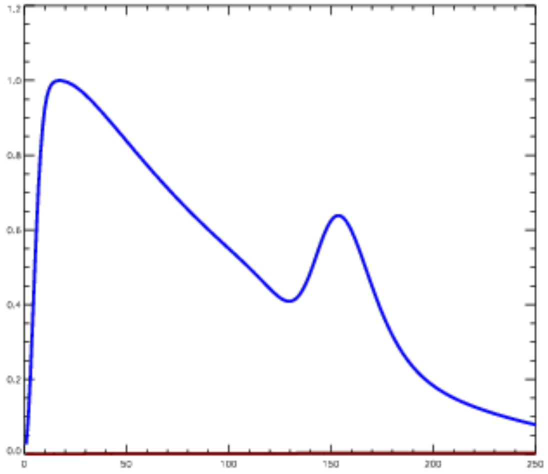


Image Courtesy Martin White

CMB Physics: Photon Memory

Details of Photon-Baryon coupling/decoupling

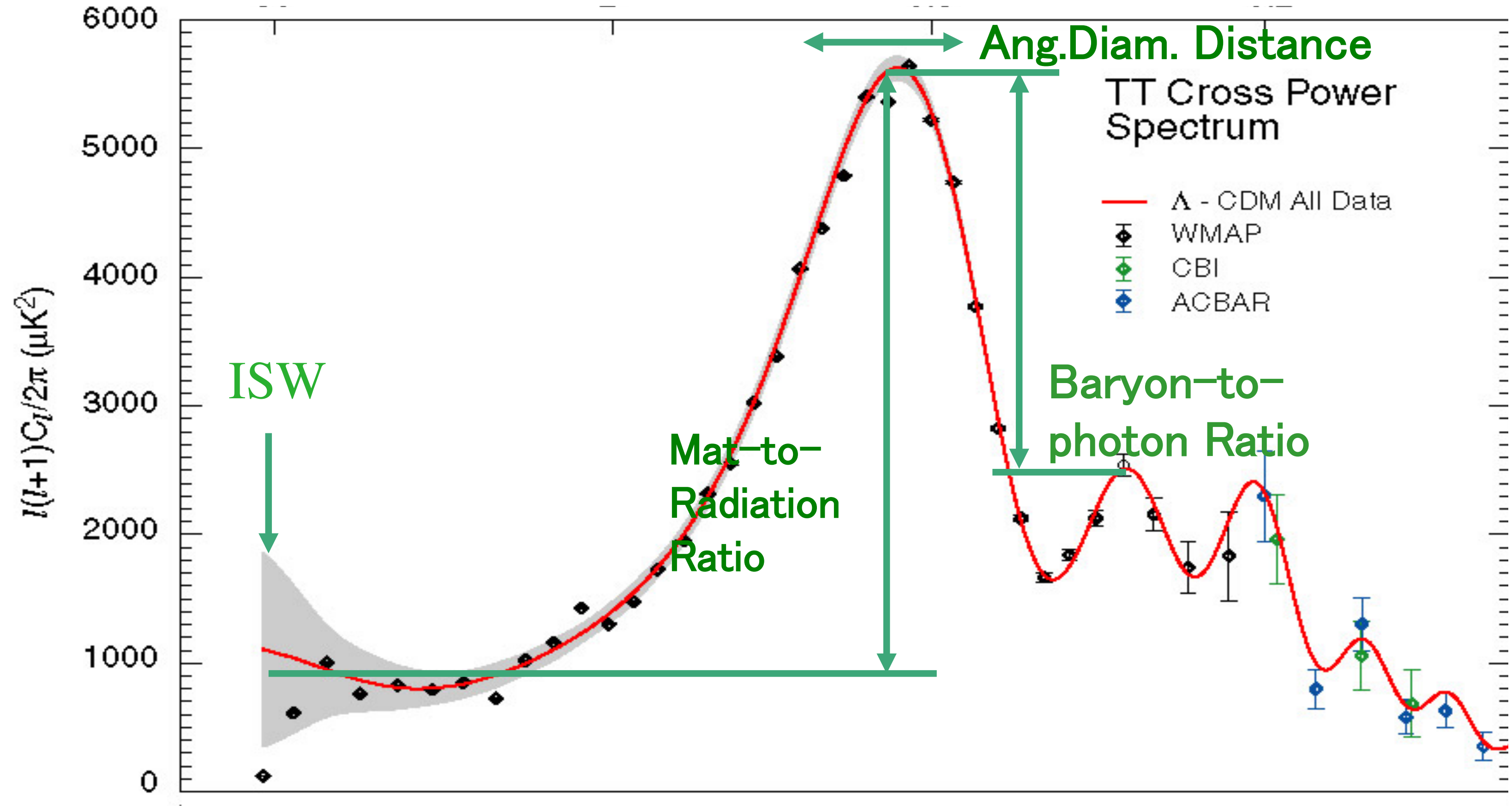
and

Exactly when matter or radiation dominates the background evolution and gravitational potentials

$$\begin{aligned}\dot{\delta}_\gamma &= -\frac{4}{3}\theta_\gamma + 4\dot{\phi}\,, \\ \dot{\theta}_\gamma &= k^2\left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) + k^2\psi + a n_e \sigma_T(\theta_b - \theta_\gamma)\end{aligned}$$

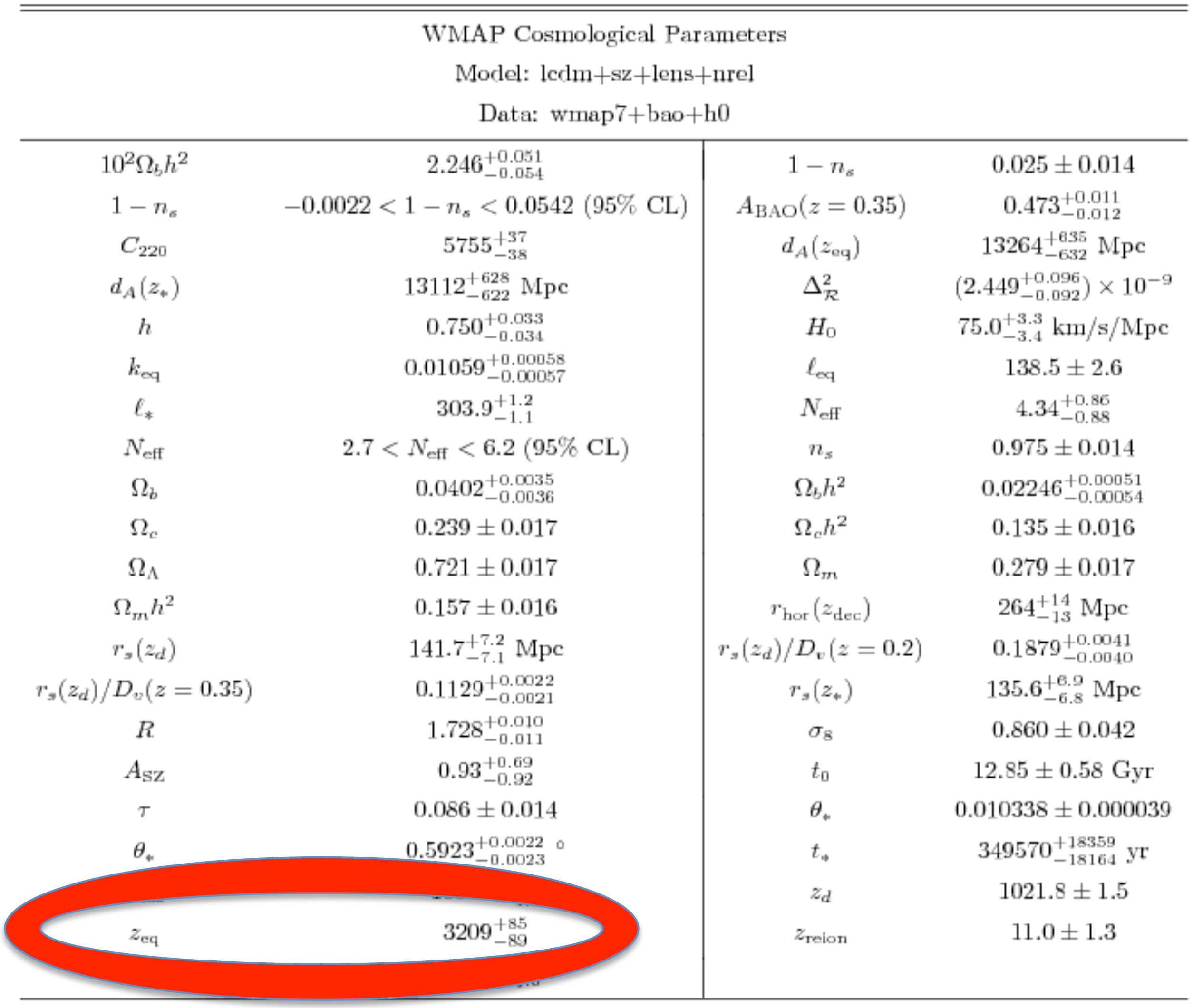
Gets imprinted before photons free-stream to us!

CMB Physics



Credit: E. Komatsu

Cosmological Parameters: Neff ≠ 3.046



WMAP Cosmological Parameters

Model: lcdm+sz+lens+nrel

Data: wmap7+bao+h0

$10^2\Omega_b h^2$	$2.246^{+0.051}_{-0.054}$	$1 - n_s$	0.025 ± 0.014
$1 - n_s$	$-0.0022 < 1 - n_s < 0.0542$ (95% CL)	$A_{\rm BAO}(z = 0.35)$	$0.473^{+0.011}_{-0.012}$
C_{220}	5755^{+37}_{-38}	$d_A(z_{\rm eq})$	13264^{+635}_{-632} Mpc
$d_A(z_*)$	13112^{+628}_{-622} Mpc	$\Delta^2_{\mathcal{R}}$	$(2.449^{+0.096}_{-0.092}) \times 10^{-9}$
h	$0.750^{+0.033}_{-0.034}$	H_0	$75.0^{+3.3}_{-3.4}$ km/s/Mpc
$k_{\rm eq}$	$0.01059^{+0.00058}_{-0.00057}$	$\ell_{\rm eq}$	138.5 ± 2.6
ℓ_*	$303.9^{+1.2}_{-1.1}$	$N_{\rm eff}$	$4.34^{+0.86}_{-0.88}$
$N_{\rm eff}$	$2.7 < N_{\rm eff} < 6.2$ (95% CL)	n_s	0.975 ± 0.014
Ω_b	$0.0402^{+0.0035}_{-0.0036}$	$\Omega_b h^2$	$0.02246^{+0.00051}_{-0.00054}$
Ω_c	0.239 ± 0.017	$\Omega_c h^2$	0.135 ± 0.016
Ω_Λ	0.721 ± 0.017	Ω_m	0.279 ± 0.017
$\Omega_m h^2$	0.157 ± 0.016	$r_{\rm hor}(z_{\rm dec})$	264^{+14}_{-13} Mpc
$r_s(z_d)$	$141.7^{+7.2}_{-7.1}$ Mpc	$r_s(z_d)/D_v(z = 0.2)$	$0.1879^{+0.0041}_{-0.0040}$
$r_s(z_d)/D_v(z = 0.35)$	$0.1129^{+0.0022}_{-0.0021}$	$r_s(z_*)$	$135.6^{+6.9}_{-6.8}$ Mpc
R	$1.728^{+0.010}_{-0.011}$	σ_8	0.860 ± 0.042
$A_{\rm SZ}$	$0.93^{+0.69}_{-0.92}$	t_0	12.85 ± 0.58 Gyr
τ	0.086 ± 0.014	θ_*	0.010338 ± 0.000039
θ_*	$0.5923^{+0.0022}_{-0.0023}$ °	t_*	349570^{+18359}_{-18164} yr
[illegible]	[illegible]	z_d	1021.8 ± 1.5
$z_{\rm eq}$	3209^{+85}_{-89}	$z_{\rm reion}$	11.0 ± 1.3
[illegible]	[illegible]		

Courtesy of LAMBDA

Cosmological Parameters: Neff ≠ 3.046

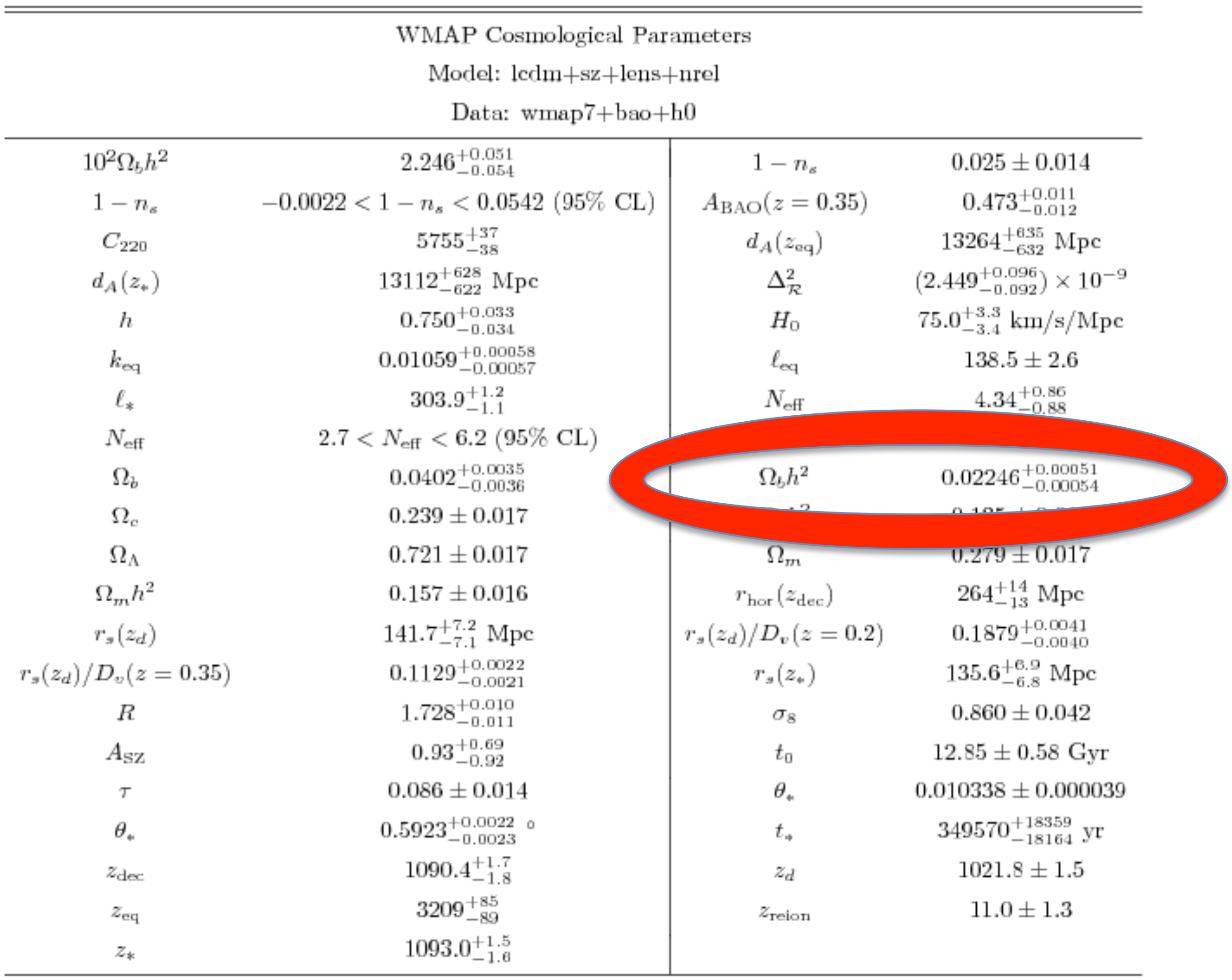
We know the redshift of matter-radiation equality to 2.7% (assuming 7-parameter ΛCDM w/ Neff variation)

$$z_{eq} = 3209 \pm 85$$

$$1 + z_{eq} = \frac{\Omega_m}{\Omega_r} = \frac{\Omega_m h^2}{4.16 \times 10^{-5} (N_{eff}/3.02)}$$

Compare: $z_{eq} = 3232 \pm 87$

Cosmological Parameters: Neff ≠ 3.046



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z_*	$1093.0^{+1.5}_{-1.6}$		

Courtesy of LAMBDA

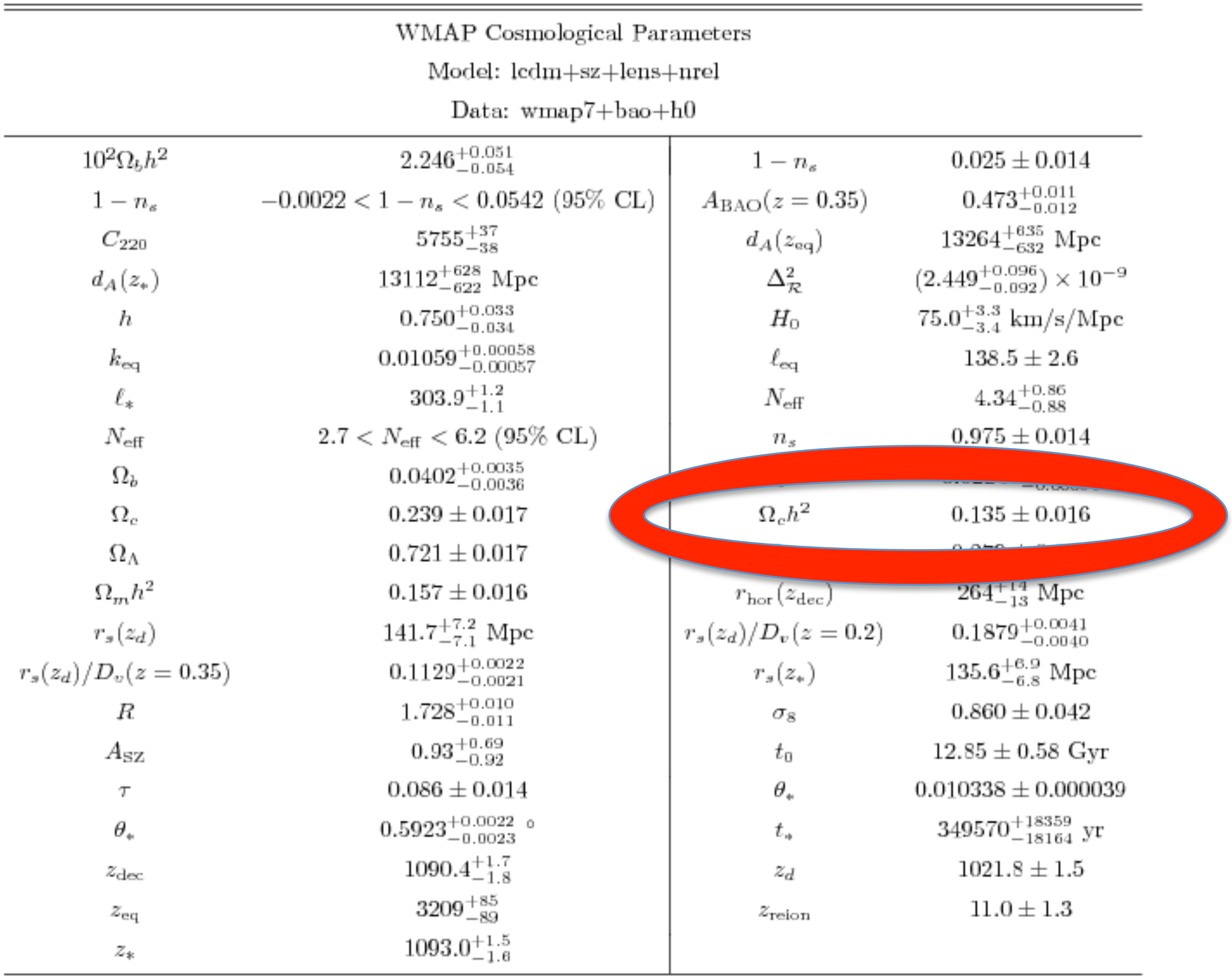
Cosmological Parameters: Neff ≠ 3.046

We know the cosmological abundance of baryons to 2.4%
(assuming 7-parameter ΛCDM w/ Neff variation)

$$100\Omega_b h^2 = 2.246 \pm 0.054$$

Compare: $100\Omega_b h^2 = 2.255 \pm 0.054$

Cosmological Parameters: Neff ≠ 3.046



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z_*	$1093.0^{+1.5}_{-1.6}$		

Courtesy of LAMBDA

Cosmological Parameters: Neff ≠ 3.046

Something to Remember:

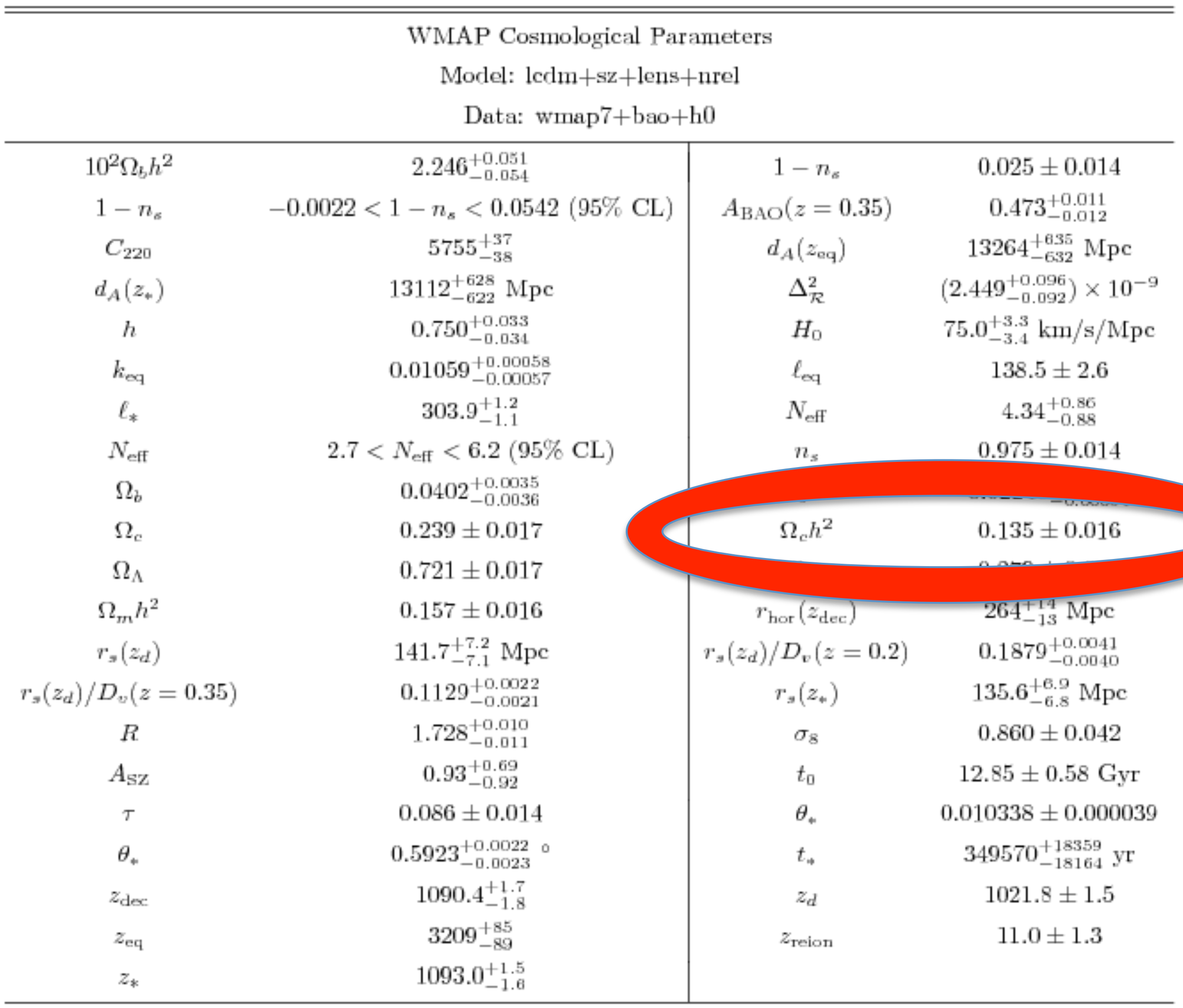
We know the cosmological abundance of dark matter to 12%
(assuming 7-parameter ΛCDM w/ Neff variation)

$$\Omega_c h^2 = 0.135 \pm 0.016$$

Prefers Higher Dark Matter Density?

Compare: $\Omega_c h^2 = 0.1126 \pm 0.0036$

Cosmological Parameters: Neff ≠ 3.046



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Ω_c	0.239 ± 0.017	$\Omega_c h^2$	0.135 ± 0.016
Ω_Λ	0.721 ± 0.017	[illegible]	[illegible]
$\Omega_m h^2$	0.157 ± 0.016	$r_{\rm hor}(z_{\rm dec})$	264^{+14}_{-13} Mpc
$r_s(z_d)$	$141.7^{+7.2}_{-7.1}$ Mpc	$r_s(z_d)/D_v(z=0.2)$	$0.1879^{+0.0041}_{-0.0040}$
$r_s(z_d)/D_v(z=0.35)$	$0.1129^{+0.0022}_{-0.0021}$	$r_s(z_*)$	$135.6^{+6.9}_{-6.8}$ Mpc
R	$1.728^{+0.010}_{-0.011}$	σ_8	0.860 ± 0.042
A_{SZ}	$0.93^{+0.69}_{-0.92}$	t_0	12.85 ± 0.58 Gyr
τ	0.086 ± 0.014	θ_*	0.010338 ± 0.000039
θ_*	$0.5923^{+0.0022}_{-0.0023}$ °	t_*	349570^{+18359}_{-18164} yr
$z_{\rm dec}$	$1090.4^{+1.7}_{-1.8}$	z_d	1021.8 ± 1.5
$z_{\rm eq}$	3209^{+85}_{-89}	$z_{\rm reion}$	11.0 ± 1.3
z_*	$1093.0^{+1.5}_{-1.6}$		

Courtesy of LAMBDA

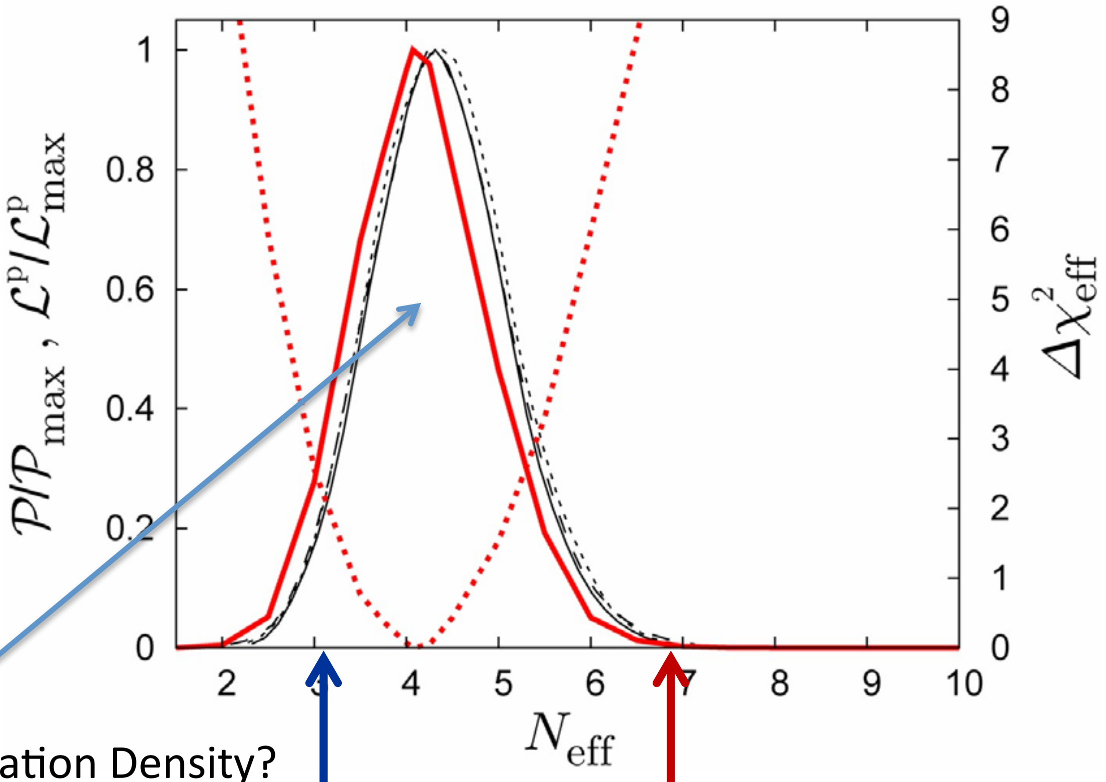
Cosmological Parameters: Neff ≠ 3.046

(assuming 7-parameter ΛCDM w/ Neff variation)

$$2.7 < \mathcal{N}_{eff} < 6.2$$

Cosmological Parameters: Neff ≠ 3.046

WMAP-7+ACT+HST (Hamann, arXiv:1110.427



Prefers Higher Radiation Density?

$N_{\mathrm{eff}} = 3.046$

$N_{\mathrm{eff}} = 6.77$

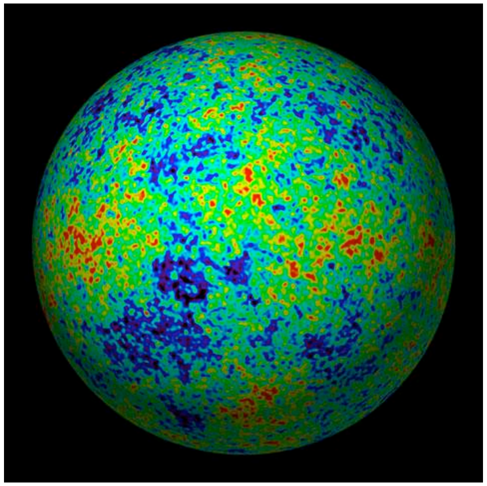
(From Georg Raffelt's Talk Yesterday)

Dark Matter "Knowns"

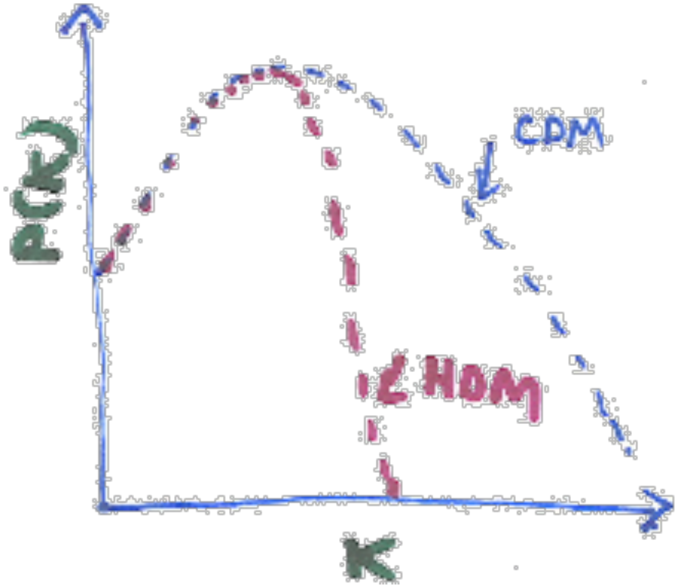
Abundance:

$$\Omega_d h^2 \simeq 0.11$$

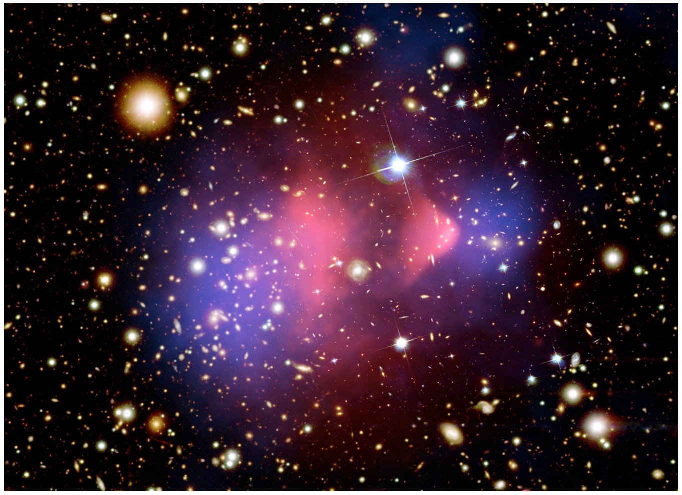
"Small" Interaction Atoms



Not *Too Much* Free Streaming:



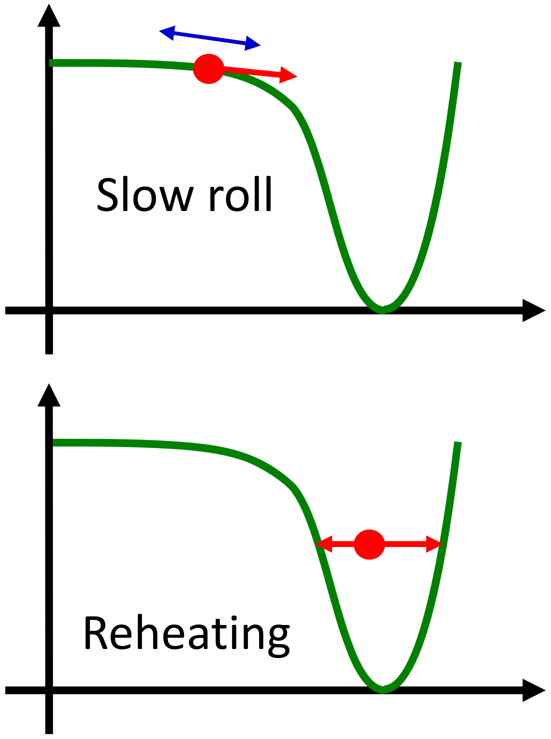
"Small" Self-Interaction:



Creation of Adiabatic vs. Isocurvature Perturbations

Inflaton field

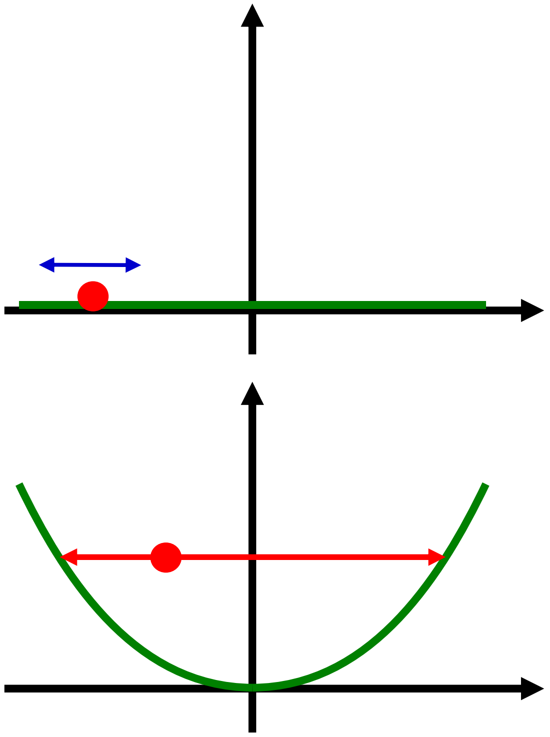
De Sitter expansion imprints
scale invariant fluctuations



Inflaton decay → matter & radiation
Both fluctuate the same:
Adiabatic fluctuations

Axion field

De Sitter expansion imprints
scale invariant fluctuations



Inflaton decay → radiation
Axion field oscillates late → matter
Matter fluctuates relative to radiation:
Entropy fluctuations

DM Isocurvature Fluctuations

$$\mathcal{S}_{c,\gamma} \equiv \frac{\delta\rho_c}{\rho_c} - \frac{3\delta\rho_\gamma}{4\rho_\gamma}$$

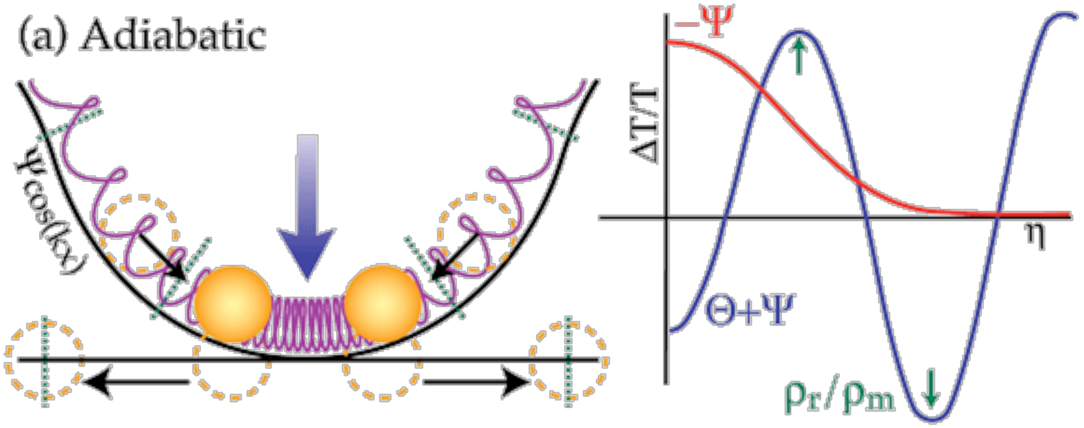
Spatial Variation of the Relative Amount of Dark Matter and Standard Model Plasma

CMB Fluctuations are known to be mostly Adiabatic (not DM Isocurvature)

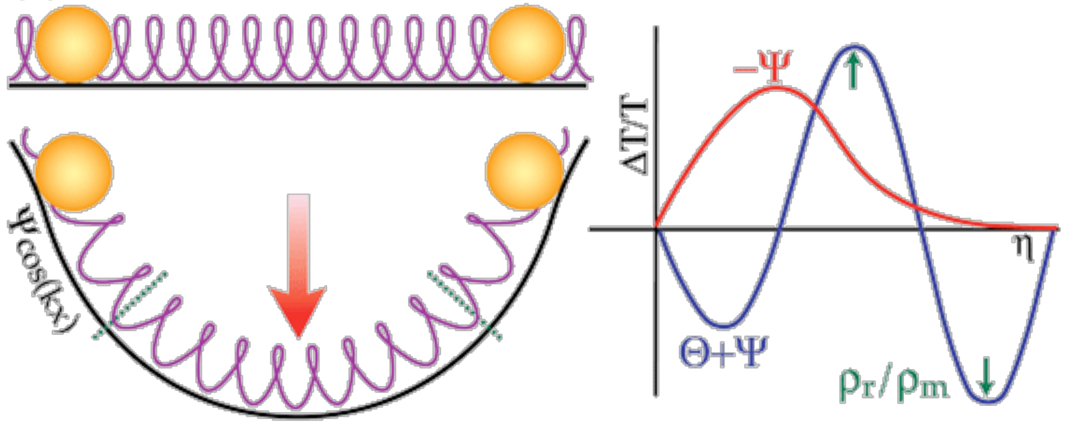
If Inflation before PQ Symmetry Breaking Axion can have Isocurvature

DM Isocurvature Fluctuations

(a) Adiabatic

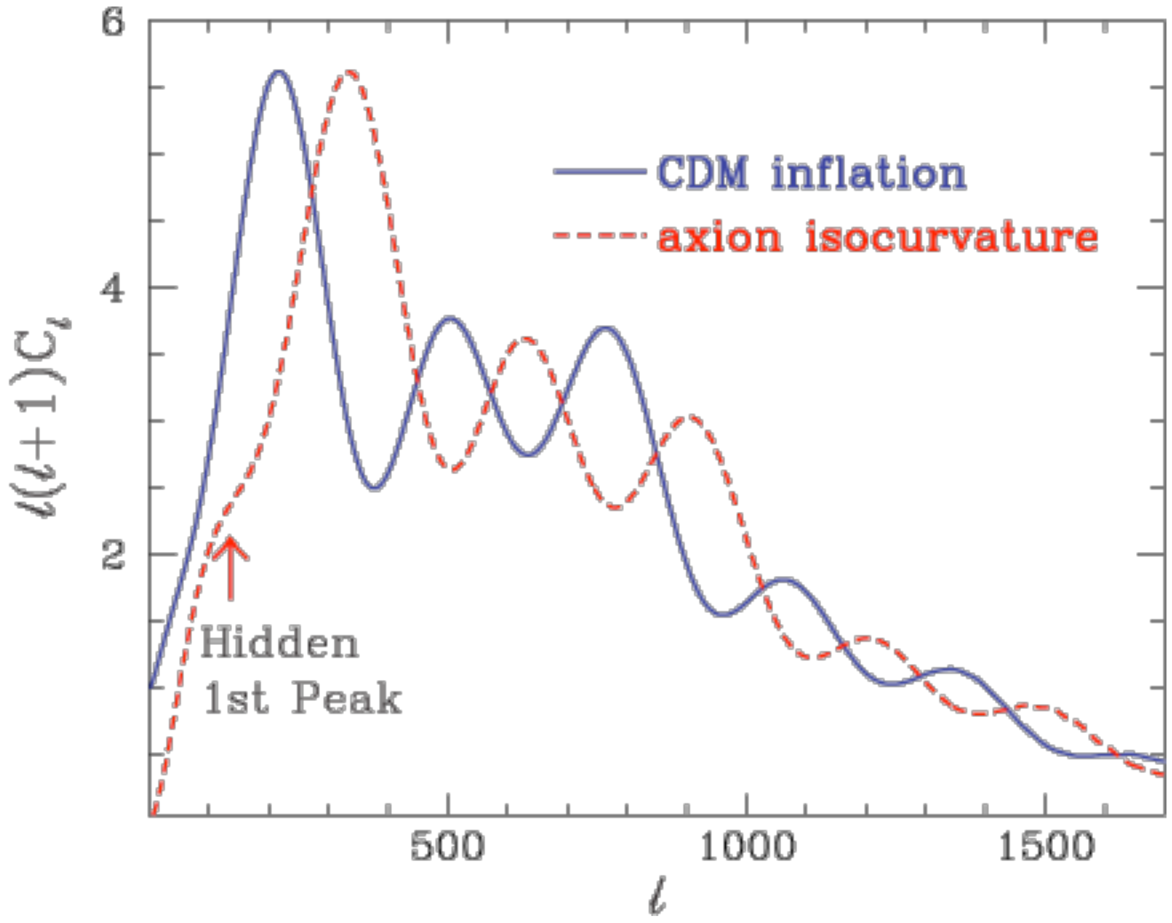


(b) Isocurvature



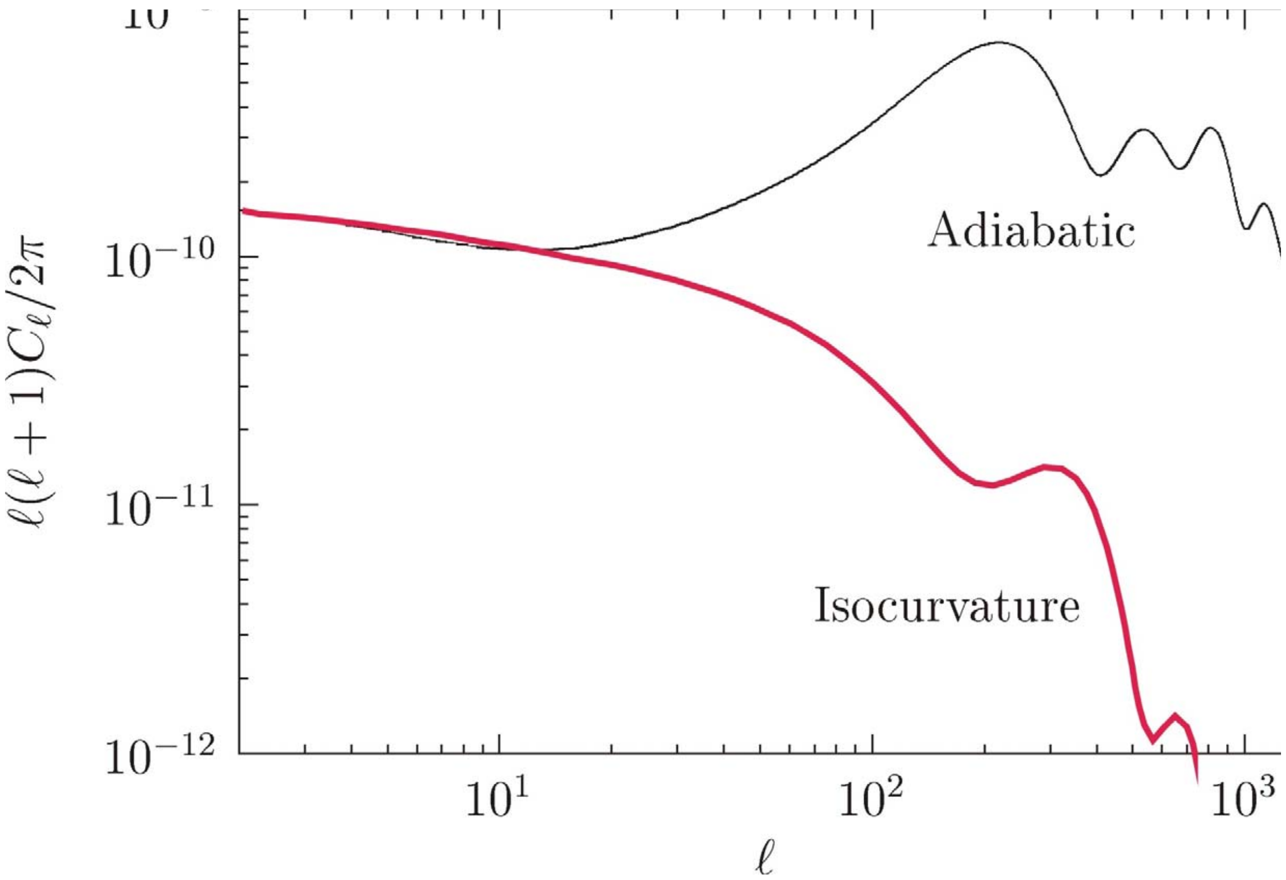
Credit: Wayne Hu

DM Isocurvature Fluctuations



Credit: Wayne Hu; from Hu and White 1996b

DM Isocurvature Fluctuations



DM Isocurvature Fluctuations Alter CMB Angular Power Spectrum

(Georg Raffelt's Talk Yesterday)

From Fox, Pierce, and Thomas hep-th/0409059

DM Isocurvature Fluctuations

Relative Amplitude of Isocurvature Power to Curvature Power

$$\frac{\alpha}{1-\alpha} \equiv \frac{P_{\mathcal{S}}(k_0)}{P_{\mathcal{R}}(k_0)}$$

$$k_0 = 0.002\ \mathrm{Mpc}^{-1}$$

Uncorrelated Isocurvature Fluctuations: $\alpha = \alpha_0$

See, e.g., WMAP7 Year Komatsu et al.

DM Isocurvature Fluctuations

WMAP7 Only

$$\alpha_0 < 0.13 \ (95\% \text{ CL})$$

WMAP7+H0+BAO

$$\alpha_0 < 0.077 \ (95\% \text{ CL})$$

See, e.g., WMAP7 Year Komatsu et al.

Axion / GW Connection

$$\frac{\alpha_0(k)}{1-\alpha_0(k)} = \frac{\Omega_a^2}{\Omega_c^2}\frac{8\epsilon}{\theta_a^2 (f_a/M_{\rm pl})^2}$$

Slow Roll

$$r = 16\epsilon$$

Oscillate Before QCD

$f_a \; < \; \mathcal{O}(10^{-2}) M_{pl}$

$$\Omega_a h^2 = 1.0 \times 10^{-3} \gamma \theta_a^2 \left(\frac{f_a}{10^{10}\ \mathrm{GeV}}\right)^{7/6}$$

$$r = \frac{4.0 \times 10^{-10}}{\theta_a^{2/3}} \left(\frac{\Omega_c h^2}{\gamma}\right)^{4/3} \left(\frac{\Omega_c}{\Omega_a}\right)^{2/3} \frac{\alpha_0}{1-\alpha_0}$$

Oscillate After QCD

$f_a \; > \; \mathcal{O}(10^{-2}) M_{pl}$

$$\Omega_a h^2 = 1.6 \times 10^{5} \gamma \theta_a^2 \left(\frac{f_a}{10^{17}\ \mathrm{GeV}}\right)^{3/2}$$

$$r = \frac{4.0 \times 10^{-10}}{\theta_a^{2/3}} \left(\frac{\Omega_c h^2}{\gamma}\right)^{4/3} \left(\frac{\Omega_c}{\Omega_a}\right)^{2/3} \frac{\alpha_0}{1-\alpha_0}$$

Single Field Slow Roll w/ Axion DM fraction

See, e.g., WMAP7 Year Komatsu et al.

Axion / GW Connection

$$r < \frac{7.6 \times 10^{-15}}{\theta_a^{10/7} \gamma^{12/7}} \quad \text{for } f_a < \mathcal{O}(10^{-2}) M_{pl}$$

$$r < \frac{1.5 \times 10^{-12}}{\theta_a^{2/3} \gamma^{4/3}} \quad \text{for } f_a > \mathcal{O}(10^{-2}) M_{pl}$$

$$\theta_a \gamma^{6/5} < 3.3 \times 10^{-9} \left(\frac{10^{-2}}{r}\right)^{7/10} \text{for } f_a < \mathcal{O}(10^{-2}) M_{pl}$$

$$\theta_a \gamma^{2} < 1.8 \times 10^{-15} \left(\frac{10^{-2}}{r}\right)^{3/2} \text{for } f_a > \mathcal{O}(10^{-2}) M_{pl}$$

See, e.g., WMAP7 Year Komatsu et al.

Axion / GW Connection

$$f_a > 1.8 \times 10^{26}\ \mathrm{GeV}\ \gamma^{6/5}\left(\frac{r}{10^{-2}}\right)^{6/5}$$

$$f < \mathcal{O}(10^{-2})M_{pl}$$

Inconsistent!

$$f_a > 3.2 \times 10^{32}\ \mathrm{GeV}\ \gamma^{2}\left(\frac{r}{10^{-2}}\right)^{2}$$

$$f > \mathcal{O}(10^{-2})M_{pl}$$

for $f_a < M_{pl} = 2.4 \times 10^{18}\ \mathrm{GeV}$

Limit!

$$r < \frac{8.7 \times 10^{-10}}{\gamma}$$

See, e.g., WMAP7 Year Komatsu et al.

See: Hertzberg et al. 2008 Mack 2009,
Mack and Steinhardt 2009

Axion / GW Connection

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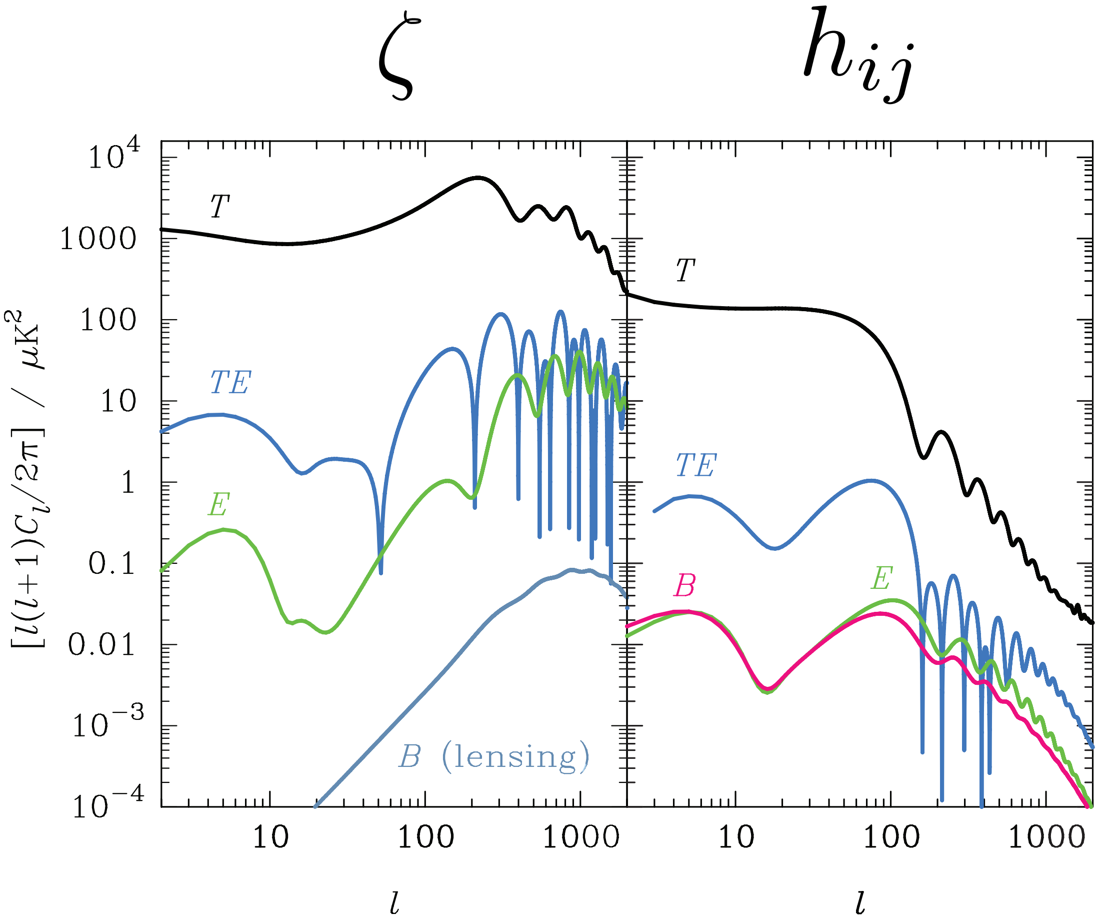
Axion / GW Connection



If detect CMB B-mode Polarization then no misalignment axion scenario.

Credit: Challinor

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SEVEN-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE (*WMAP*[1]) OBSERVATIONS: COSMOLOGICAL INTERPRETATION

E. KOMATSU[2], K. M. SMITH[3], J. DUNKLEY[4], C. L. BENNETT[5], B. GOLD[5], G. HINSHAW[6], N. JAROSIK[7], D. LARSON[5], M. R. NOLTA[8], L. PAGE[7], D. N. SPERGEL[3,9], M. HALPERN[10], R. S. HILL[11], A. KOGUT[6], M. LIMON[12], S. S. MEYER[13], N. ODEGARD[11], G. S. TUCKER[14], J. L. WEILAND[11], E. WOLLACK[6], AND E. L. WRIGHT[15]

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TABLE 1
SUMMARY OF THE COSMOLOGICAL PARAMETERS OF ΛCDM MODEL[a]

Class	Parameter	*WMAP* 7-year ML[b]	*WMAP*+BAO+H_0 ML	*WMAP* 7-year Mean[c]	*WMAP*+BAO+H_0 Mean
Primary	$100\Omega_b h^2$	2.227	2.253	$2.249^{+0.056}_{-0.057}$	2.255 ± 0.054
	$\Omega_c h^2$	0.1116	0.1122	0.1120 ± 0.0056	0.1126 ± 0.0036
	Ω_Λ	0.729	0.728	$0.727^{+0.030}_{-0.029}$	0.725 ± 0.016
	n_s	0.966	0.967	0.967 ± 0.014	0.968 ± 0.012
	τ	0.085	0.085	0.088 ± 0.015	0.088 ± 0.014
	$\Delta^2_{\mathcal{R}}(k_0)^{\rm d}$	2.42×10^{-9}	2.42×10^{-9}	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.430 \pm 0.091) \times 10^{-9}$
Derived	σ_8	0.809	0.810	$0.811^{+0.030}_{-0.031}$	0.816 ± 0.024
	H_0	70.3 km/s/Mpc	70.4 km/s/Mpc	70.4 ± 2.5 km/s/Mpc	70.2 ± 1.4 km/s/Mpc
	Ω_b	0.0451	0.0455	0.0455 ± 0.0028	0.0458 ± 0.0016
	Ω_c	0.226	0.226	0.228 ± 0.027	0.229 ± 0.015
	$\Omega_m h^2$	0.1338	0.1347	$0.1345^{+0.0056}_{-0.0055}$	0.1352 ± 0.0036
	$z_{\rm reion}{}^{\rm e}$	10.4	10.3	10.6 ± 1.2	10.6 ± 1.2
	$t_0{}^{\rm f}$	13.79 Gyr	13.76 Gyr	13.77 ± 0.13 Gyr	13.76 ± 0.11 Gyr

[a] The parameters listed here are derived using the RECFAST 1.5 and version 4.1 of the *WMAP* likelihood code. All the other parameters in the other tables are derived using the RECFAST 1.4.2 and version 4.0 of the *WMAP* likelihood code, unless stated otherwise. The difference is small. See Appendix A for comparison.

[b] Larson et al. (2010). "ML" refers to the Maximum Likelihood parameters.

[c] Larson et al. (2010). "Mean" refers to the mean of the posterior distribution of each parameter. The quoted errors show the 68% confidence levels (CL).

[d] $\Delta^2_{\mathcal{R}}(k) = k^3 P_{\mathcal{R}}(k)/(2\pi^2)$ and $k_0 = 0.002$ Mpc^{-1}.

[e] "Redshift of reionization," if the universe was reionized instantaneously from the neutral state to the fully ionized state at $z_{\rm reion}$. Note that these values are somewhat different from those in Table 1 of Komatsu et al. (2009a), largely because of the changes in the treatment of reionization history in the Boltzmann code `CAMB` (Lewis 2008).

[f] The present-day age of the universe.