
Vacuum Polarization

Vistas in Axion Physics

April 25, 2012

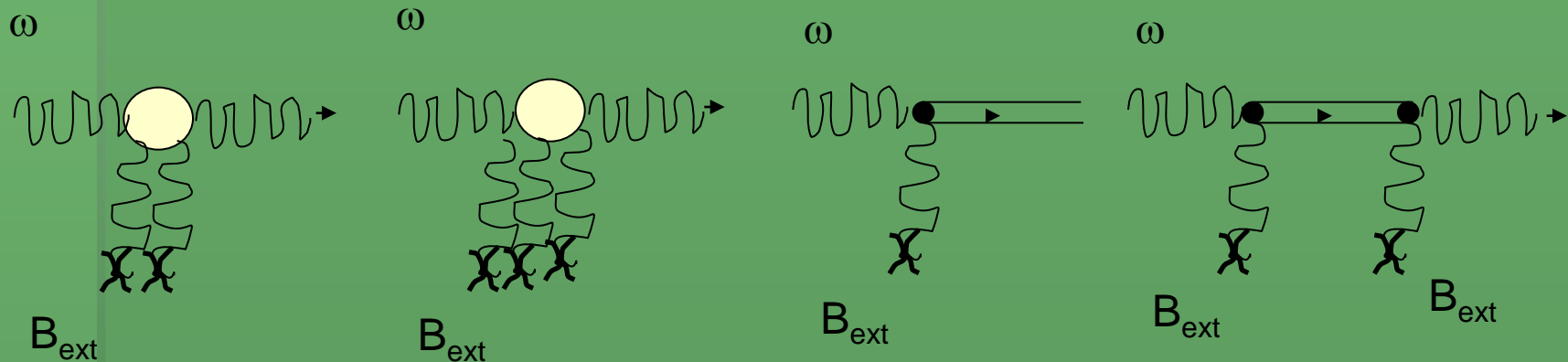
Carol Scarlett

Florida A&M University

Experimental Searches

- Photon-Axion Coupling Diagrams
- BNL's E840
- PVLAS
- BMV
- Beam Splitting
- Bifurcation

Photon Coupling to B_{ext}



- Diagrams: a.) QED vacuum polarization, b.) photon splitting, c.) axion real production and d.) axion virtual production

Index of Refraction

- Diagrams alter the index of refraction for photons polarized along direction of \mathbf{B}_{ext}
- Lagrangian:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_{\mu}a\partial^{\mu}a - m_a^2a^2) + \frac{1}{4}g_a aF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\alpha^2}{90m_e^4} \left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \right], \quad (1)$$

Photon-EM Coupling

- In the presence of an externally applied Magnetic field:

$$L = \frac{1}{8} (E^2 + B^2) + \frac{A}{4\pi} [(E^2 - B^2)^2 + 7 (E \cdot B)^2]$$

- Vacuum becomes birefringent

Photon-EM Coupling

- A photon propagating through an external field will acquire an ellipticity:

$$n_{\parallel} = 1 + 7 A B_{\text{ext}}^2 \sin^2\theta$$

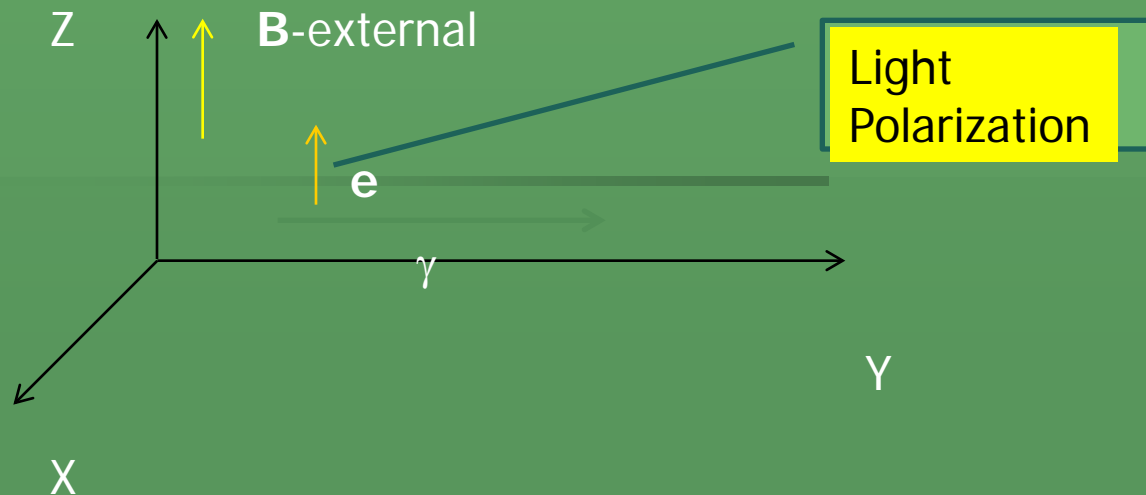
$$n_{\perp} = 1 + 4 A B_{\text{ext}}^2 \sin^2\theta$$

$$\psi = \pi \frac{L}{\lambda} (n_{\parallel} - n_{\perp}) = \frac{\pi L}{\lambda} 3 A B_{\text{ext}}^2$$

Axion Coupling Term

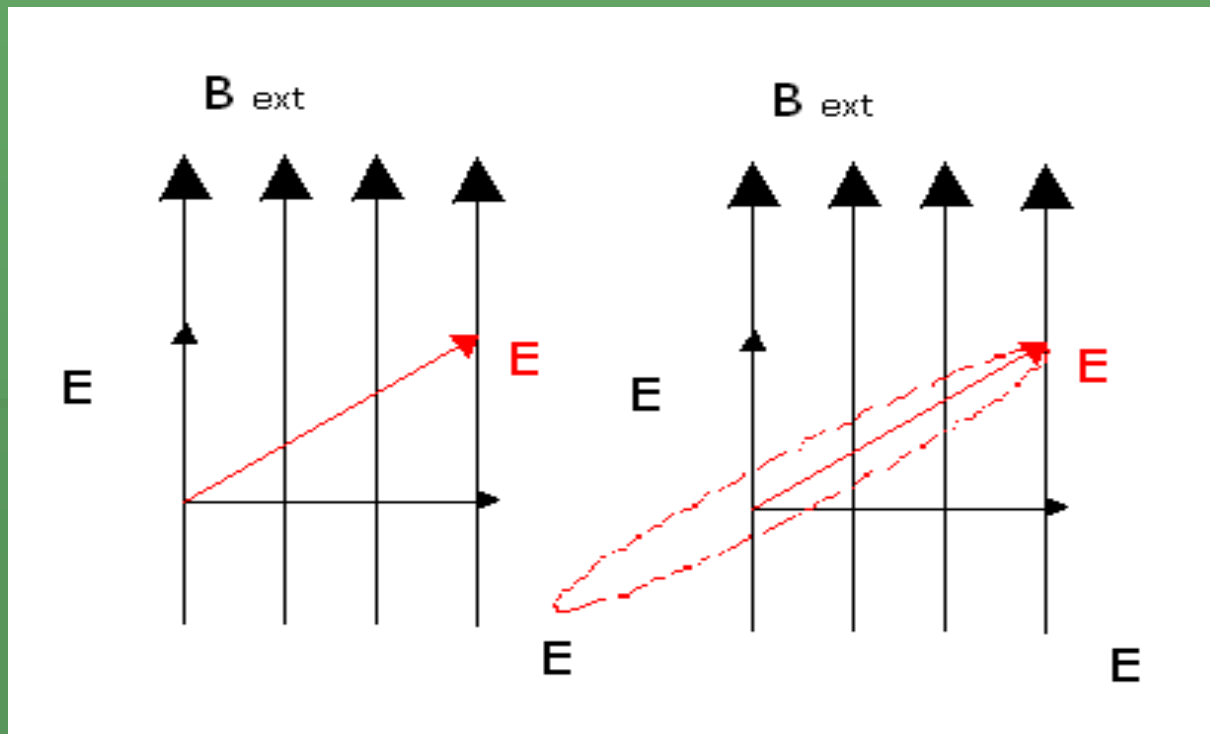
$$L_{int} = \frac{1}{4} g_a a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_a a (\vec{B}^e \vec{E}).$$

Magnetic field geometry and coordinates



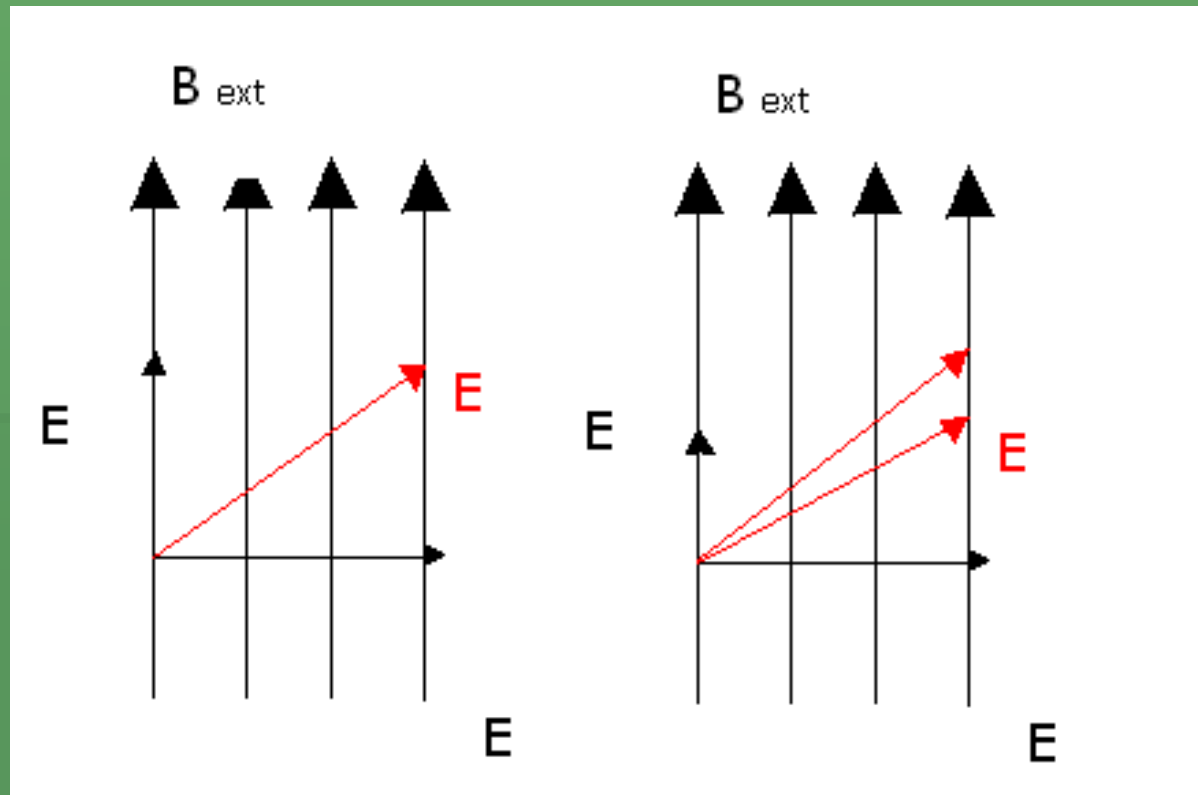
Measurable Effects

- Ellipticity:

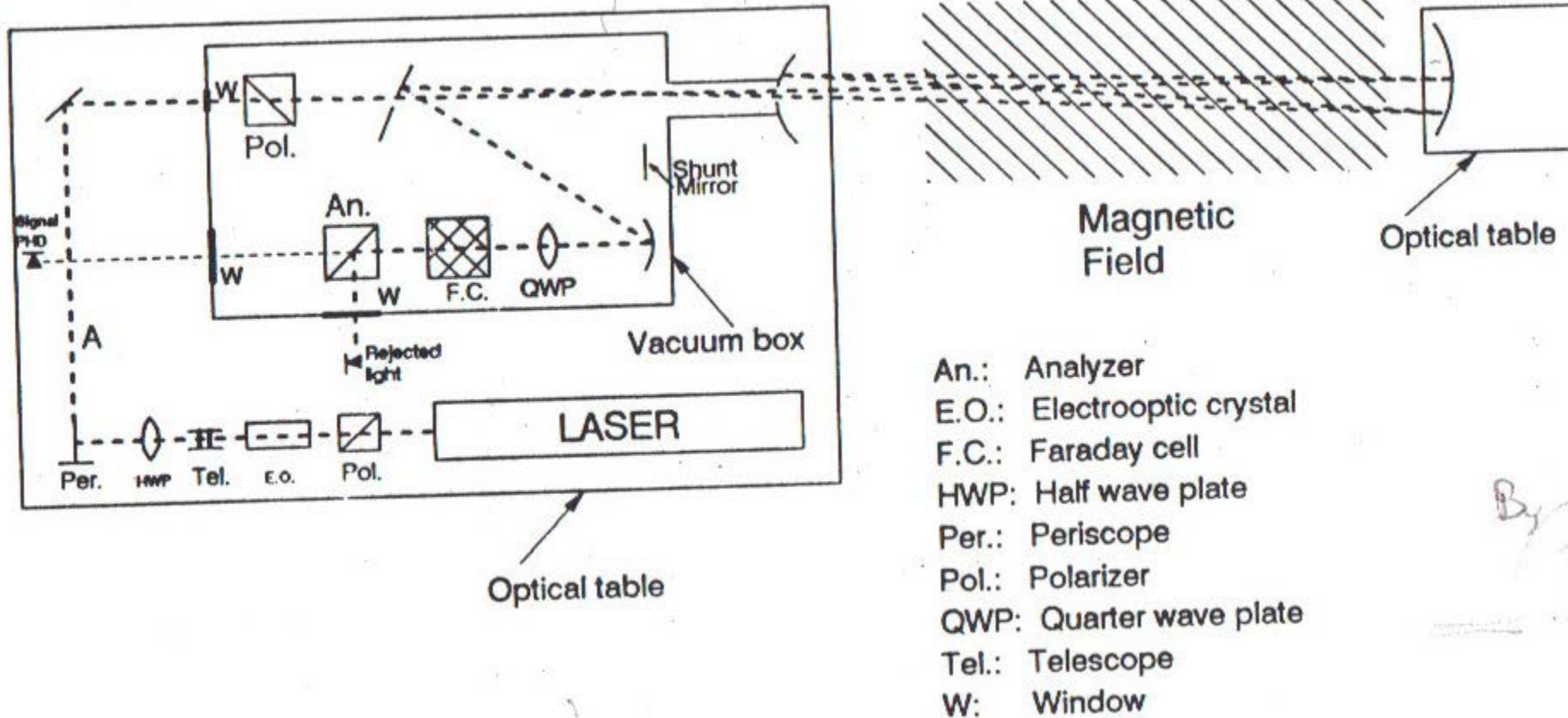


Measurable Effects

- Dichroism:



BFRT [BNL - E840]

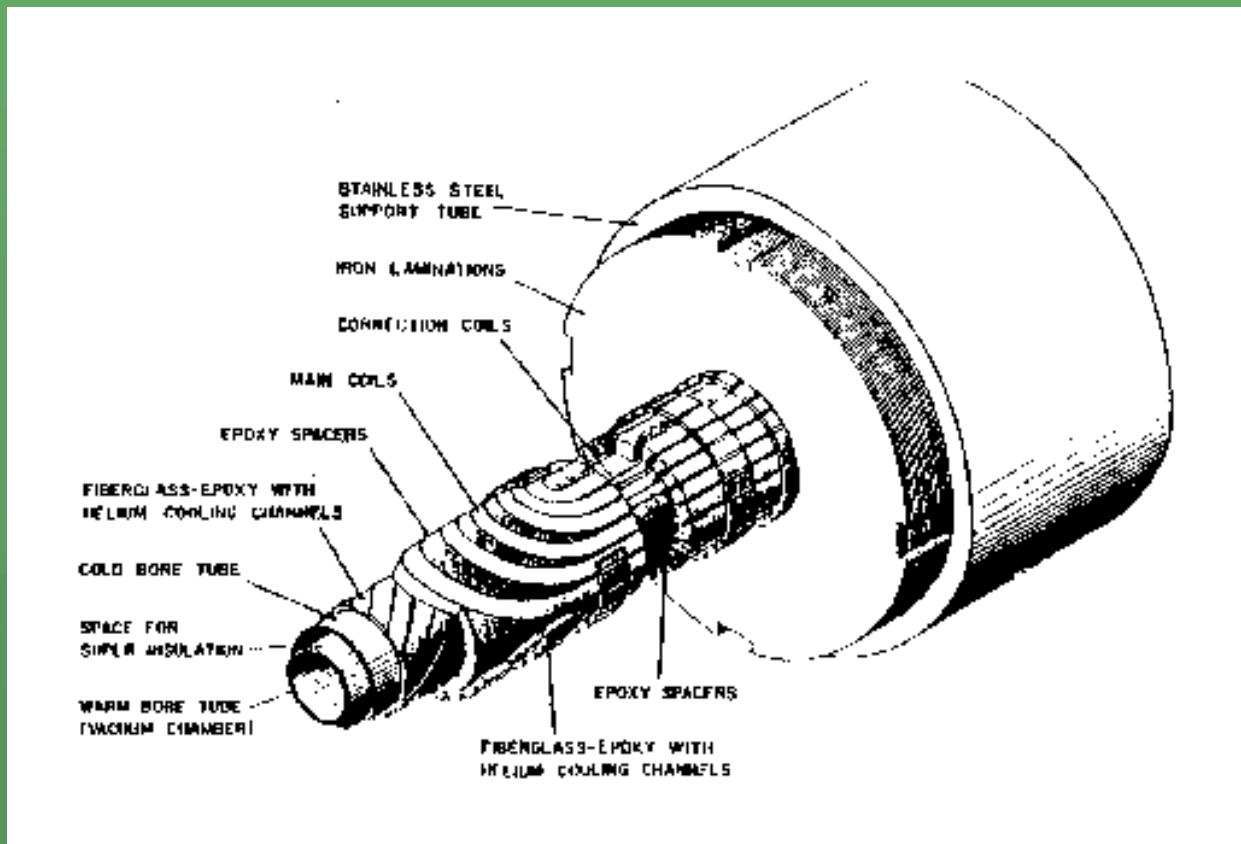


- E840 looked for evidence of axions through measuring both induced ellipticity changes and selected absorption

BNL – E840

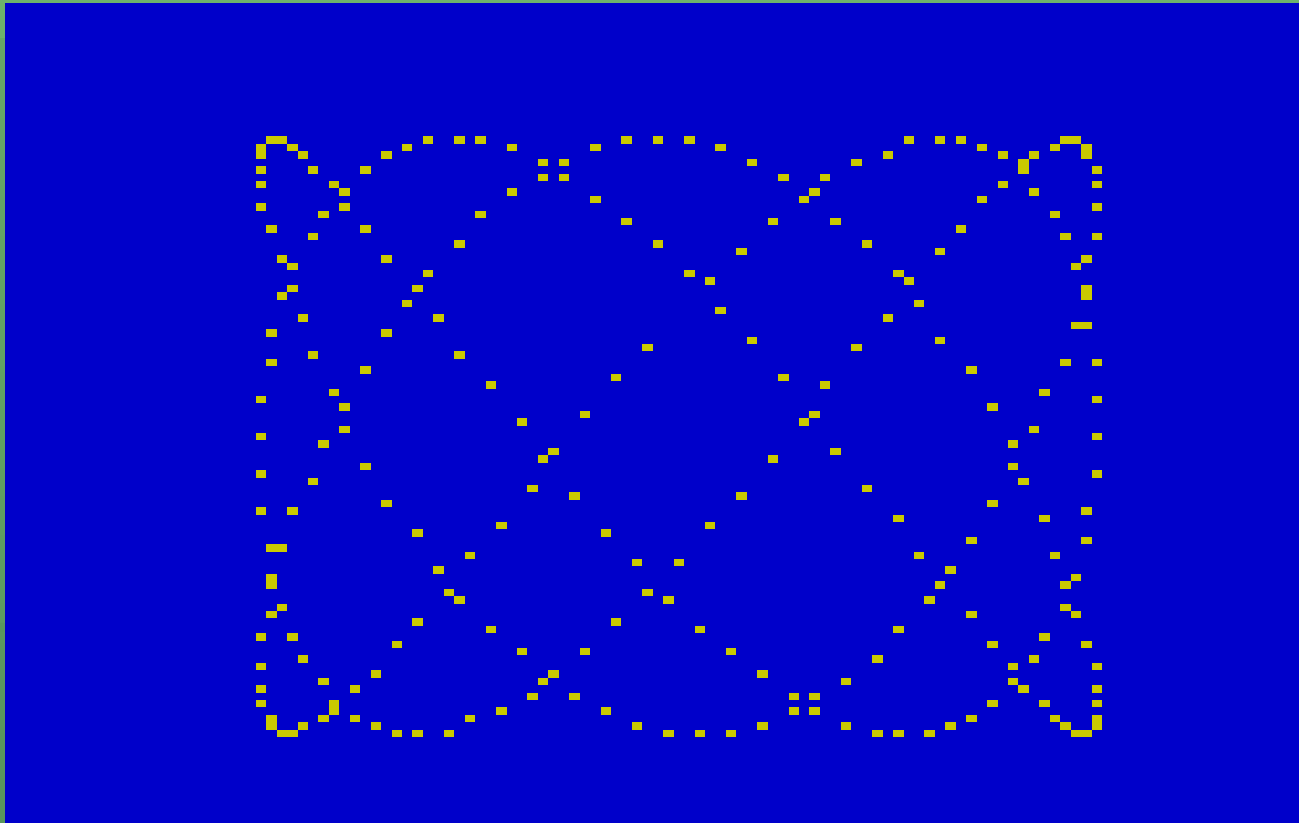
- 2 CBA Superconducting magnets each 4.4m long
- Dipole Field of 2.2 T at full Strength
- Field is Vertical to direction of incoming beam
- Field extends some 10 m
- Modulated at $\sim 16.5\text{mHz}$ with B field given by:
$$B[\text{T}] = T_F \times I[\text{Amps}] \times 10^{-4}$$
where T_F refers to the transfer function
- Homogeneity of $> 10^{-4}$
- Laser 90_5 Argon Ion Laser delivered $\lambda = 514.5\text{nm}$ with 2 watts
- Optics under vacuum produced by turbo pump
- Cavity composed of two mirrors with Diameter = 11.25 cm and Radius of curvature = 19.03 m (this is an optical delay line)
- Output fed to silicon photodiode
- Signal sought at magnet modulation frequency ω_m

Superconduction CBA



- End section for magnet used in E840 Setup
- Coil orientation shows that the field is aligned vertically relative to the direction of the incoming beam projected along the warm bore tube

Optical Delay Line



- Optical cavity consists of two mirrors one of which has an entrance hole
- One mirror deformed so that pattern closes but misses hole, tracing out more ellipses
- Lissajous pattern forms on the end mirrors similar to seen above

Sensitivity & Signals

- Ellipticity after analyzer using Jones Matrices

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \eta & \sin \eta \\ -\sin \eta & \cos \eta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} \cdot \\ \begin{bmatrix} \cos^2 \theta + \sin^2 \theta e^{-i\phi} & \cos \theta \sin \theta (1 - e^{-i\phi}) \\ \cos \theta \sin \theta (1 - e^{-i\phi}) & \cos^2 \theta e^{-i\phi} + \sin^2 \theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_x e^{i\theta_x} \\ A_y e^{i\theta_y} \end{bmatrix}$$

here ϕ is the phase shift from the cavity due to the magnetic field

- This gives a current at the photodiode:

$$I_T = I_0 \left[\alpha^2 + \frac{\eta_0^2}{2} + \frac{\eta_0^2}{2} \cos 2\omega_F t + 2\alpha\eta_0 \cos \omega_F t \right. \\ \left. - \psi_0 \eta_0 \sin 2\theta \{ \cos(\omega_F - \omega_M)t + \cos(\omega_F + \omega_M)t \} \right]$$

here $\psi = \phi/2$ represents the ellipticity induced by the field

Sensitivity & Signals

- A similar analysis reveals the expected photodiode current for the rotation setup:

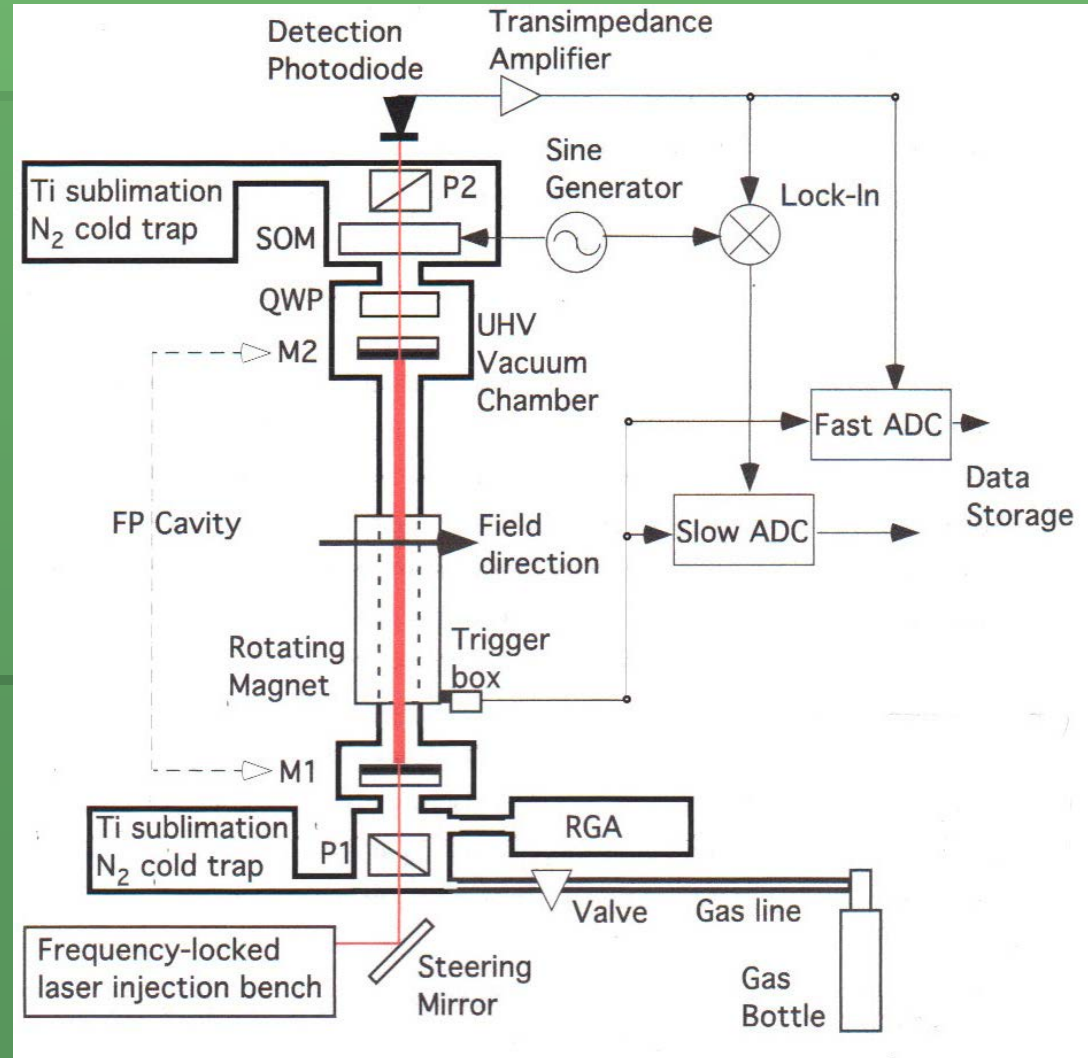
$$I_T = I_0 \left[\alpha^2 + \frac{\eta_0^2}{2} + \frac{\eta_0^2}{2} \cos 2\omega_F t + 2\alpha\eta_0 \cos \omega_F t \right. \\ \left. + \varepsilon_0 \eta_0 \{ \cos(\omega_F - \omega_M)t + \cos(\omega_F + \omega_M)t \} \right]$$

here the rotation parameter is ε_0 . Note: there is actually an attenuation given as ε'

- In the final analysis, the signal to noise took into account shot noise (as a limit) along with electronic sources (e.g. laser amplitude)

PVLAS

- PVLAS looked for ellipticity and rotation as well
- Enhanced cavity compared to E840
- Magnet rotates in lieu of modulation

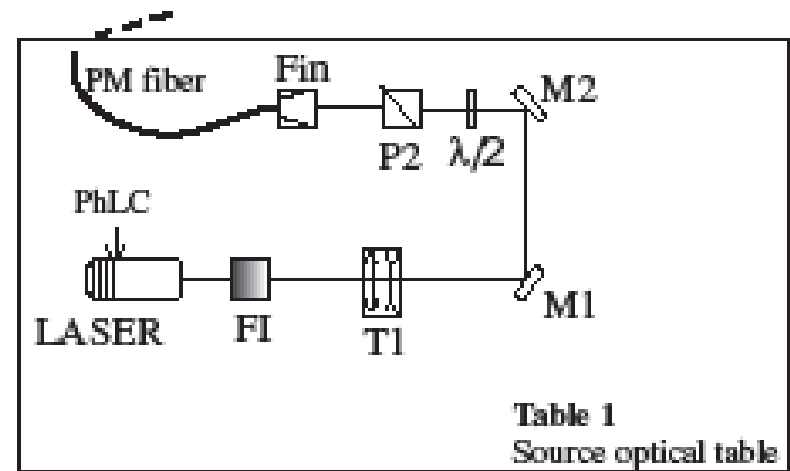
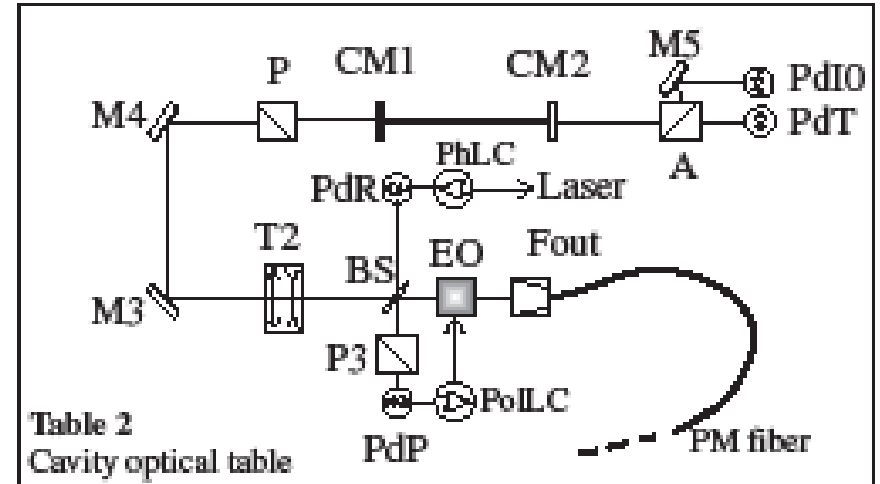


PVLAS

- Dipole Field of 2.5 T and 5 T achieved
- Magnet is positioned vertically and on rotating table
- Magnet length is 1 m long
- Modulated achieved through rotation of magnet $\Omega_{\text{mag}} \approx 0.3$ Hz
- Stray field reduced to $< 50 \cdot 10^{-3}$ gauss as part of 2007 upgrades... following which a previously observed signal disappeared
- Laser Source Nd: Yag emits 800mW at 1064nm
- Optics cavity is a Fabry-Perot optical resonator
- Cavity mirrors are multilayer, highly reflective dielectrics with a Radius of curvature = 11 m and are 6.4 m apart
- Some $4.5 \cdot 10^4$ gauss traversals of the magnet are achieved
- Ellipticity Modulator used (heterodyne technique)
- Signal sought at magnet modulation frequency ω_m

BMV

- Ultra high finesse cavity achieving up to $\cdot 10^6$ traversals
- Used 25.4 mm diameter mirrors
- Capable of using both hetero- and homo- dyne techniques
- Designed to look for regeneration of photons



Vacuum Experiments

Table 1:

	B Field	Length	QED Effect	Sensitivity
E840	5 T	~ 1 km	~ 10^{-12} rad	~ 10^{-8} rad/ $\sqrt{\text{Hz}}$
PVLAS	5.5 T	~ 10 km	~ 10^{-11} rad	~ 10^{-9} rad/ $\sqrt{\text{Hz}}$
BMV	25 T	~ 3×10^3 km	~ 10^{-10} rad	~ 10^{-9} rad/ $\sqrt{\text{Hz}}$

- Can anymore be gained from cavity experiments?

Mixing of Photon-Axion

- George Raffelt and Leo Stodolsky : Mixing of photons with low-mass particles , Phys. Rev. D 37 (1988) 1237

- Eduardo Guendelman :

Photon and axion splitting in an inhomogeneous magnetic field,

Phys. Lett. B 662(2008) 445 ;

Axions and photons in terms of “particles “ and “antiparticles”

$$\psi = \frac{1}{\sqrt{2}}(a + iA_1), \psi^* = \frac{1}{\sqrt{2}}(a - iA_1)$$
$$a = \frac{1}{\sqrt{2}}(\psi + \psi^*), A = \frac{1}{i\sqrt{2}}(\psi - \psi^*)$$

The charge is associated with the following conserved quantity

$$Q = \int d\vec{k} \{ \psi^{*+}(\vec{k})\psi^{-}(\vec{k}) - \psi^{*-}(\vec{k})\psi^{+}(\vec{k}) \}, \quad (9)$$

Beam Splitting

- The unit tangent vector

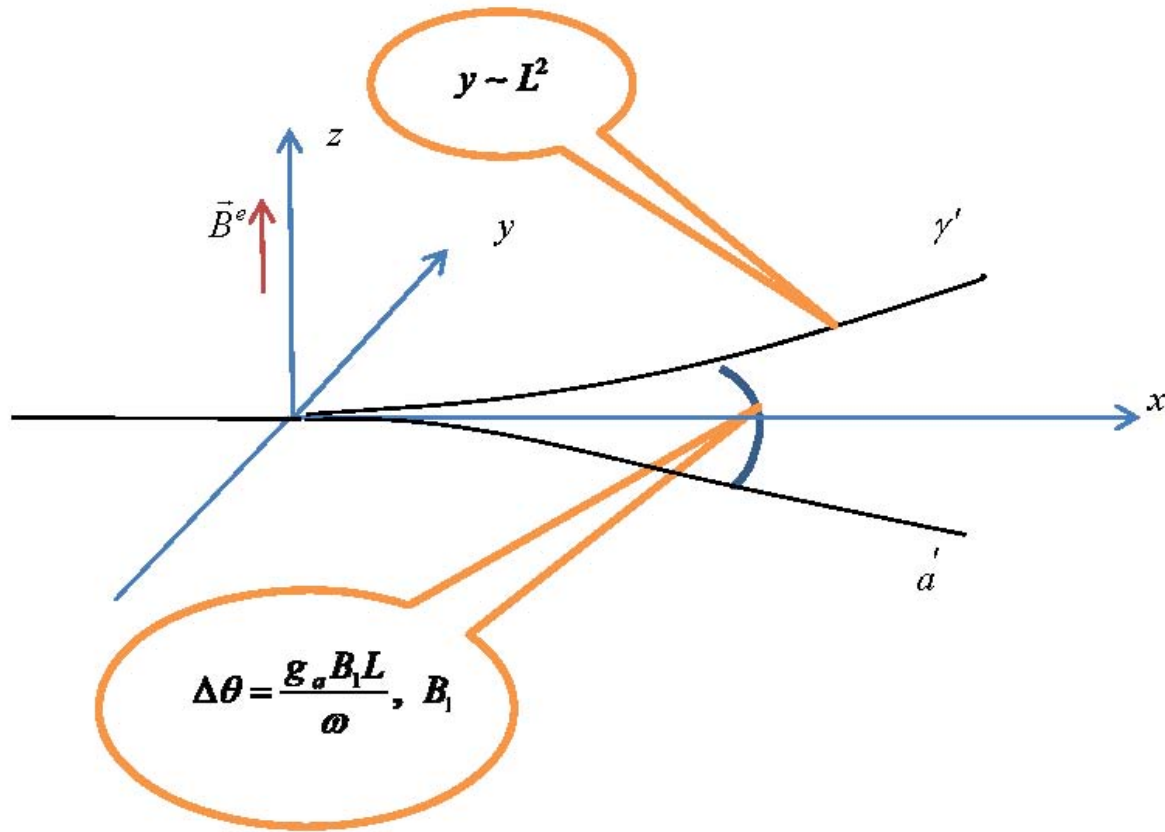
$$l_y \begin{cases} \sqrt{1 - \frac{n_0^2}{n^2}} & n > n_0, \\ -\sqrt{1 - \frac{n^2}{n_0^2}} & n < n_0, \end{cases}$$

- Beam trajectory: _____

$$z(y) = \int_{y_0}^y \frac{n_0 dy}{\sqrt{n^2(y) - n_0^2}} + z_0, \quad n > n_0;$$

$$z(y) = - \int_{y_0}^y \frac{n(y) dy}{\sqrt{n_0^2 - n^2(y)}} + z_0, \quad n < n_0;$$

Beam Splitting



Beam Splitting

Input Parameters

$$B^e \approx 10^4 - 10^5 \text{ G}$$

$$L_0 \approx 1 \text{ m}$$

$$\frac{\partial B^e}{\partial y} \approx 10^6 \text{ G} \quad (\text{The magnetic field gradient})$$

$$\lambda \approx 10^{-5} \text{ m} \quad (\text{A photon's wave length})$$

Beam splitting

- $L = 1 \text{ m} : \Delta\theta \approx 10^{-5} g_a$ (The splitting angle as function of g_a)

$$\Delta\theta \approx 10^{-15} \text{ rad} \quad \text{for } g_a = 10^{-10} \text{ GeV}^{-1}.$$

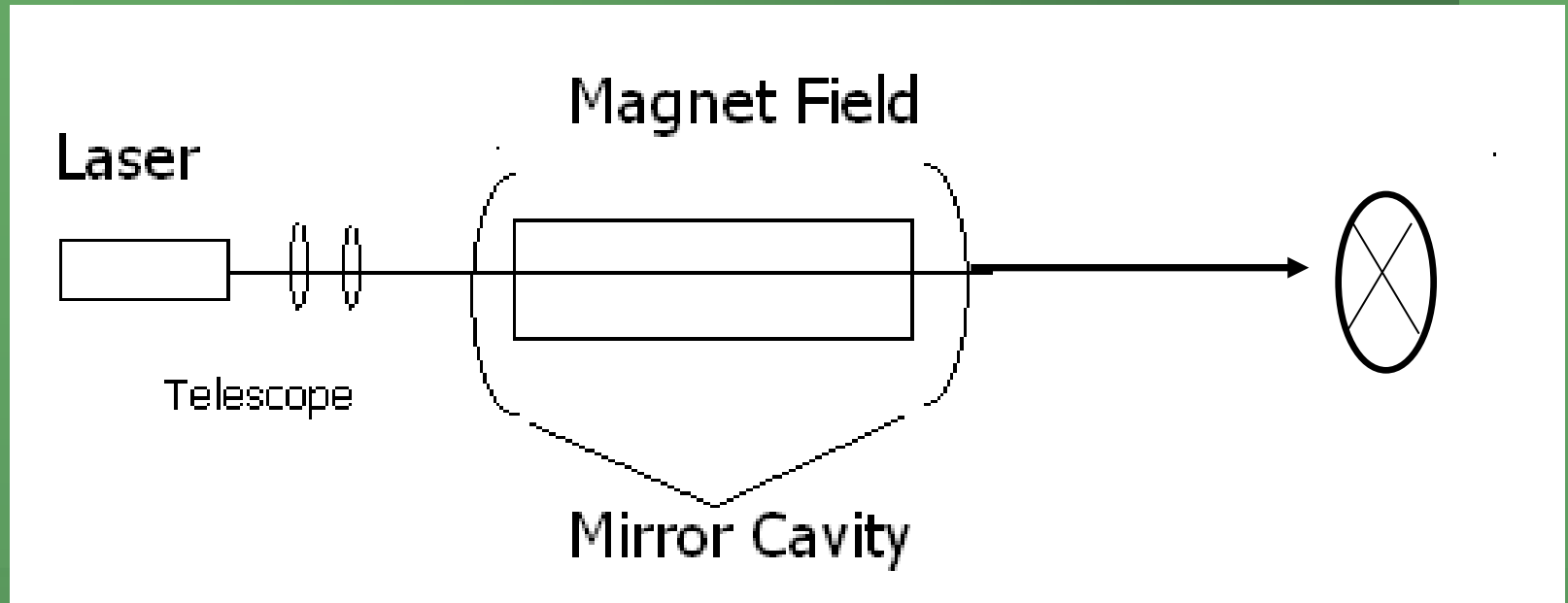
- $L = 10^5 \text{ m}$ (10^5 bounces!)

$$\Delta\theta \approx 10^{-10} \text{ rad}$$

Beam splitting + Bifurcation

$$\Delta\theta \approx 10^{-9} \text{ rad} \quad \text{and The FWHM drop of } \approx 10^{-6} E_0$$

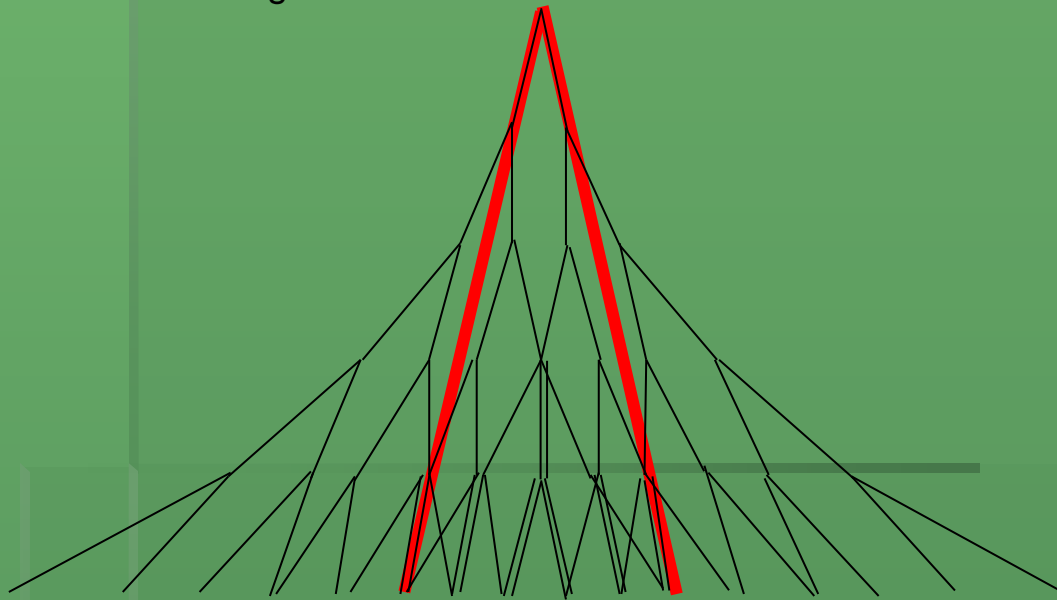
Splitting in a Cavity



- Consider a cavity such as the one used in PVLAS/E840 (optical delay line)

Bifurcation

Bifurcation Diagram



Linear Separation of two beams occurs along red line

- At the mirror, the photon-axion state dissolves
- The reflected light re-enters the cavity and divides again

Simulations

- Treat beam using Jones Matrices

$$\text{Focusing: } \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \quad (3)$$

where f represents the focal length of a lens.

$$\text{Propagation: } \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \quad (2)$$

where d represents the distance traveled, and

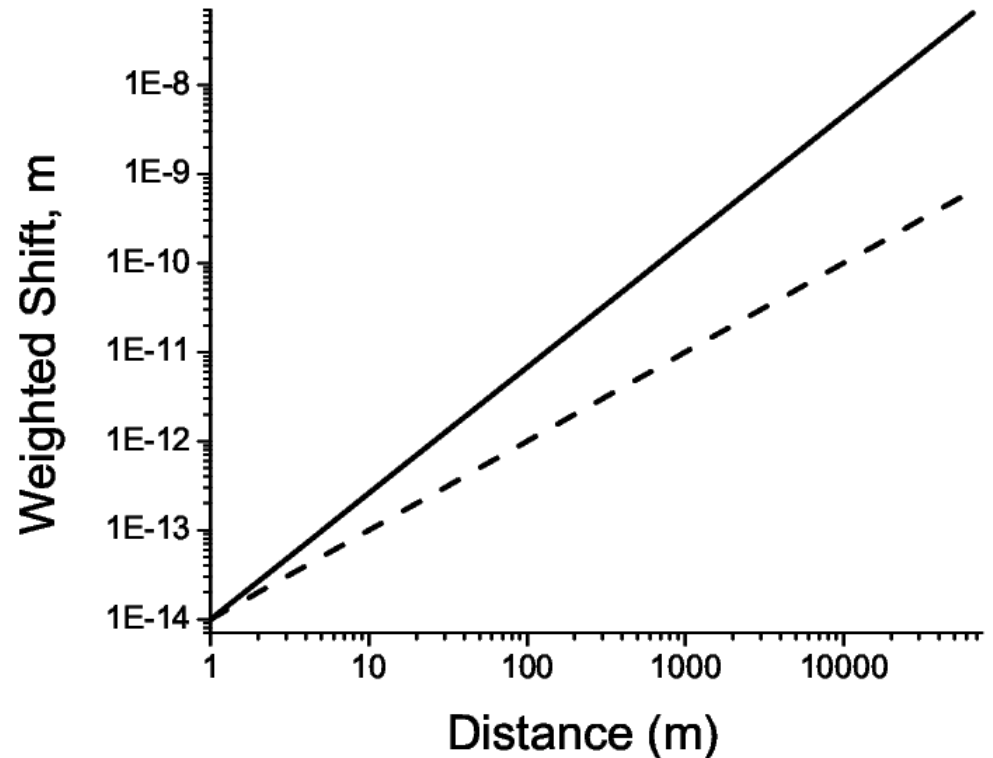
- Introduce splitting matrix

$$\text{Splitting: } \begin{bmatrix} 1 & 0 \\ \pm \theta_{\text{split}}/X_0 & 1 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \pm \theta_{\text{split}}/X_0 & 1 \end{bmatrix} \begin{bmatrix} X_0 \\ \theta_i \end{bmatrix}$$

Bifurcation vs Linear Spreading

- For a splitting of $\sim 10^{-15}$ rad
- The position of the rays are weighted by their relative density and summed
- The solid line represents the bifurcated distribution and the dashed line represents a linear spreading



Future Cavity Searches ?

- Future cavity searches could search for bifurcation in the form of energy loss
 - Development of an interferometer to interfere a cavity beam with reference source
-